Short Production Run Control Charts to Monitor Process Variances

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Abstract

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Control chart is one of the most commonly used statistical tools for quality control and improvement. If the process mean and standard deviation are not given or unknown, most Shewhart control charts require sufficient sample data before the control chart can be established. However, in certain industries or processes, it may not be practical to collect adequate amount of data at the beginning of the manufacturing process to build the trial control chart in Phase I. For quality improvement in such or similar processes, some authors developed self-starting control charts for short-run production, e.g. t chart, Q chart, EWMA t chart/Q chart, CUSUM t chart/Q chart. This thesis studies the performance of some short run control charts for monitoring process variances. Numerical simulations are using in this study. The results of the numerical experiments are extensively tested for different combinations of process lengths and starting points of process shifts.

Key words: statistical quality control, short production run, control charts, standard deviation, numerical simulation.

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1 Introduction

1.1 Motivation

1.1.1 Quality and Quality Control Tools

As it is widely known, quality is one of the most important factors for consumers in acquiring different products and services. Consequently, it is significant for companies to understand and improve quality to realise business success, growth, and enhanced competitiveness. In large scale production, a widely used definition of quality is “inversely proportional to variability” (Montgomery, 2013). Accordingly, variability is an important characteristic for quality of the products. Since “variability can only be described in statistical terms” (Montgomery, 2013), statistical methods play a central role in quality improvement process.

As stated in Montgomery (2013), main statistical tools for quality improvement are statistical process control (SPC), design of experiments, and acceptance sampling. Statistical process control (SPC) is a powerful collection of problem-solving tools useful in achieving process stability and improving capability through the reduction of variability. A designed experiment is an approach to systematically varying the controllable input factors in the process and determining their effects on the output. Designed experiments are a major off-line quality-control tool. They are mainly used for product development and in early stages of manufacturing. Acceptance sampling is to inspect a sample of units selected at random from a large batch or lot and decide on the disposition of the lot.

1.1.2 Statistical Process Control (SPC)

"Among the three major statistical tools for quality improvement, SPC is one of the greatest technological developments of the twentieth century because it is based on sound underlying principles, is easy to use, has significant impact and can be applied to any process." (Montgomery, 2013). Its seven major tools are:

- Histogram or stem-and-leaf plot
- Check sheet
- Pareto chart
- Cause-and-effect diagram
- Defect concentration diagram
- Scatter diagram
- Control chart

Among them, Shewhart control chart is probably the most technically sophisticated. It was developed in 1920s by Walter A. Shewhart of the Bell Telephone Laboratories. Different from design of experiment, control chart is used for on-line process monitoring. Some Shewhart control charts are designed for variables, e.g. $\bar{x}$-R charts and $\bar{x}$-s charts. Some others are for attributes, e.g. p chart for fraction nonconforming, c chart and u chart for number of nonconformities.

Normally, there are two general situations when applying Shewhart control charts. In the first case, previous experience and knowledge can provide accurate information on the process mean and standard deviation (SD) in advance, and a Shewhart control chart can be built as soon as the manufacture starts. In the second case, there is no information on the process mean or SD from previous knowledge or experiences. Sample data are required at the beginning of the process to build the control chart, so called Phase I. After Phase I is built, the established chart can be applied to detect the process changes in the process. This is Phase II.

1.1.3 Short-Run Production Control Charts

Most of the Shewhart control chart applications are non-self-starting. As mentioned before, since the process mean and SD are unknown in advance, they require sufficient sample data at the beginning of the process to build the control chart (Phase I). However, in certain industries or processes, it may not be practical to collect adequate amount of data at the beginning of the manufacturing process to build the trial control chart in Phase I. For example, in aerospace industry, production rate of large components can be very low. It takes very long time to collect enough sample data for constructing a control chart. On the other hand, it is often desirable that the quality control process start as early as possible, because the cost of each product is too high to be nonconforming. In such situations, classic Shewhart control charts are less effective. They require Phase I to build trial control charts and may need more sample data to adjust the control limits until they are accurate enough to monitor the production process.

For quality improvement in such or similar processes, some authors developed self-starting control charts for short-run production. For example, Quesensberry (1991) developed Q charts. Q statistic is a standardized individual measurement. It can be plotted to the standard normal control chart with centerline at zero and the control limits at ± 3 without requiring Phase I to build the control limits. Zhang et al. (2009) proposed t control charts to solve the problem of inaccurately estimating the SD of the process in short-run production. EWMA schemes and CUSUM schemes for Q chart and t chart are also developed for short-run production processes.

Most of the existing research work tests the statistical properties of short-run control charts by detecting a shift on process mean. There are not many studies on the detecting ability for SD shift. In practice, the variance of the quality measurement is as important as the process mean. In short-run production, it is necessary to monitor the process variation as well. So in this research the detecting ability of different short-run control charts for SD shift will be studied.

1.2 Objectives

The main purpose of this thesis research is to compare the detecting ability of different short-run control charts for SD shift. More specifically, we have the following objectives:

- To review the papers related to short-run control charts and to summarize the conclusions and observations.
- To summarize the model for each available short-run control chart.
- To identify promising parameter values for certain charts through numerical experiments.
- To compare performances of several short run control charts using common parameter values.
- To test the detecting ability on SD shift of several popular short-run control charts under different conditions and observe their performances.

1.3 Methodology

The main method used in this paper is numerical simulation. We use numerical simulation in searching for optimal values of parameter λ for EWMA t chart, and for the optimal combination of several widely used supplemental rules as they are applied to Q chart. We also compare the performances and effectiveness of those short-run control charts studied in this thesis. We built a simple Microsoft Excel program for conducting required simulation runs.

1.4 Organization

Chapter 2 reviews and summarizes the existing research on both non self-starting control charts and self-starting control charts for short-run production. In Chapter 3, models for different short-run control charts are presented and some of the parameter values are defined based on previous research. In the first two parts of Chapter 4, we present numerical simulations for selecting proper values of parameter λ of EWMA t chart and optimal supplemental rules for Q chart. In the final part of Chapter 4, we run simulations for comparing all the short-run control charts studied in this thesis. Finally, we present conclusions and future research in this area in Chapter 5.

2 Literature Review

2.1 Non Self-Starting Charts

Non self-starting control charts are those requiring Phase I to estimate the unknown process parameters and accordingly to construct the control limits. In other words, non self-starting control charts cannot be constructed without Phase I when the process parameters are unknown. In Phase I, data are collected and analyzed to determine the center line and the trial control limits. If the control chart constructed with Phase I data is in statistical control, Phase II can start. Otherwise, one needs to find out the assignable causes and eliminate all corresponding data to make the control chart in control and then starts Phase II. It has to be mentioned that, in Phase I sufficient amount of data should be collected to construct a reliable control chart. In practice, 20 to 25 subgroups of data with reasonable group sizes are typically required in Phase I (Saleh et al. 2015). In Phase II, after the influence of assignable causes is eliminated, the control chart will be well used to monitor the process. $\bar{x}$-R charts and $\bar{x}$-s charts are the most commonly used non self-starting control charts for variables. $\bar{x}$ chart is usually used to monitor the process mean. While R chart and s chart are to monitor the process variability. R chart is the control chart based on range of the subgroups, while s chart uses the sample SD. $\bar{x}$ chart is always combined with R chart or s chart to monitor the process mean and variance at the same time. All of the above three charts require the sample size greater than 1, normally 4 or 5. Under the conditions that the sample size is equal to 1, there are also individual X chart and Moving Range (MR) chart for monitoring individual observations. Some of the more recent developments on these Shewhart control charts are discussed below.

Ma et al. (2010) conducted research to improve the detecting capability of s chart when the shift size was small. They established two supplemental rules and illustrated that with the implementation of those rules, s chart has better average run length (ARL) performance when detecting small shift and its detecting capability maintains satisfied when the shift size is large.

Yang et al. (2012) studied individual X chart and compared it with 3-Cumulative Sum Control (3-CUSUM) chart proposed in Reynolds and Stoumbos (2004). Yang et al. (2012) found that the individual X chart outperforms $\bar{x}$ charts with sample sizes larger than one on detecting capability when both mean and variance shift. It also has better performance than 3-CUSUM chart in almost all cases except that the shift size is quite small. Chen and Yeh (2010) conducted the economic statistical design for $\bar{x}$ chart with genetic algorithm.

In addition to the above mentioned 3 commonly used non self-starting control charts for variables, non self-starting control charts for attributes are also available such as p chart for fraction nonconforming, c chart and u chart for nonconformities.

Recently, Noskievicova et al. (2014) used MATLAB to program cumulative count of conforming (CCC) chart and cumulative quantity of conforming (CQC) chart for attributes and provided software support for these two attribute control charts. CCC chart is utilised to monitor the cumulative count of conforming while CQC chart is the alternative of c chart and u chart. The two charts are able to detect smaller defect rate in a manufacturing process compared to that by traditional attribute control charts. The software design provides help on implementing CCC and CQC charts in practice.

All of the above discussed traditional control charts are non self-starting control charts. They require Phase I to estimate the parameters of the processes to establish relatively reliable control limits. They are widely used in industries of mass production which are capable of providing adequate samples in a certain period of time for control chart implementation.

2.2 Self-Starting Charts

As discussed, commonly used control charts require that the process mean and SD are known or well estimated before they can be constructed. However, this may be difficult to satisfy in certain situations. For instance, in short-run production processes without knowing the process mean and SD, one may not have enough data to estimate the process mean and SD. Even for long-run processes, at the beginning of the production, there may not be sufficient data to estimate the process parameters. A control chart can only be built after a certain period of time. However, we always hope to start the control chart as early as possible. For this purpose, some self-starting control charts have been developed. Typical such charts are Q charts and t charts.

2.2.1 Q Charts

Shewhart Q Charts

Quesensberry (1991) first proposed Q charts in 1991 for quality control of short-run processes. He presented the Q statistic which is the standardized individual measurement considering four different cases

- both process mean and SD are known;
- process mean is known and SD is unknown;
- process mean is unknown and SD is known; and
- both process mean and SD are unknown.

The proposed Q statistic is a standard normal variable transformed from t-statistics. The Q statistic can be plotted to the standard normal control chart with center line at zero and control limits at ±3. It is also possible for a Q chart to plot different parts in one chart because of its standardized control limits, which may to some extent simplify the work related to process control charts for front-line workers.

Castillo and Montgomery (1994) believed that implementing Q charts for various applications should be further studied. They pointed out that the average run length (ARL) performance of Q charts is not satisfactory in some cases. One of the main concerns is, if the Q chart cannot detect the shift of the process mean immediately after it occurs, the plotted data will quickly become steady at a new level and the chart tends to "miss" the shift. They used numerical simulation to investigate the statistical properties of Q charts for a normally-distributed variable for all the four cases when both or either of the mean and SD were not known. They proposed to use EWMA charts and adaptive Kalman filtering method for the processes when the mean is known and process SD is unknown. Their tests showed that these tools have better ARL performances than Q charts. For the case that both the mean and SD are unknown, they proposed to use adaptive Kalman filtering method together with a tracking signal to improve the ARL performance. However Quesenberrry (1996) pointed out that results of some cases are incorrect in Castillo and Montgomery (1994).

The same problem was also studied in Zantek (2005). He compared the signal probability of each observation following a shift of the mean and observed that the signal probability decreased when the number of observations following process mean shift increased. This indicates that the signal for a shift of the mean may not persist in Q charts. In addition, based on run length (RL) distribution study, he also demonstrated that if the out-of-control signal is missed, the RL would increase and the shift may be masked.

He et al. (2008) conducted further investigations on ARL of Q charts. They considered that the control chart is biased if the out-of-control ARL (ARL_1) is larger than the in-control ARL (ARL_0). They used simulation and showed that for Q charts ARL_1 is larger than ARL_0 when the shift of mean appears at the beginning of the process when its mean and SD are unknown. They also pointed out that when the shift of the mean happens in the later part of the process, Q charts have similar performance comparing to classic Shewhart charts without the bias. Regarding the bias problem, they explored two alternative Q charts to decrease the bias. The main reason for the existing bias is that the estimation of process variance will be biased after a shift happens in the process mean. An alternative Q chart, named Q_I chart, was proposed subsequently. It revises the method of estimating sample variance. The new estimator would be less affected by the shift of the mean happened at the beginning of the process. But it may still be affected by the shift occurred at the later time. They further improved the Q_I chart to Q_{II} chart by performing a test for determining the shift likely to occur in which subgroup. Such subgroup would be discarded when estimating the process variance.

Recently, several researchers proposed different methods to address some of the disadvantages of Q charts. They include using Q charts in combination with other charts. Different versions of revised Q charts were also proposed.

Roes et al. (1999) investigated Q chart performance with a set of Western Electric type of rules. They also used tightened control limits on Q chart and compared with EWMA Q chart. They concluded that comparing with applying individual rules, applying combinations of the rules on Q chart has higher signal probability at the first observation after shift. An alternative control chart - Q(R) chart was also proposed. It uses the average moving range as the estimator of process SD.

The authors also developed an economic model for short-run processes. Champ and Woodball (1987) conducted similar investigation with Shewahrt $\bar{x}$ chart.

Wen and Zhao (2012) used Q charts in conjunction with variable sampling interval (VSI) to improve ARL performance of Q charts with parameters unknown. Although this can improve the Q chart performance to some extent, the early stage of the control chart may not be stable. The authors recommended smaller sampling intervals at the early part of the process.

Zhu and Zhou (2010) proposed weighted Q control charts based on difference-declining weight parameters. They used simulation and showed that ARL performance of weighted Q charts was better than that of the classic Shewhart Q charts when both process mean and SD are unknown.

Lampreia and Requeijo (2012) proposed a Modified Q control chart. It was designed for vibration monitoring of repairable systems. The Modified Q chart can be applied to monitor vibration processes online.

Chang and Tong (2013) used Q chart in software development processes where sufficient data were not available for statistical quality control practice. The authors showed that Q chart is more effective than other conventional control charts because of its self-starting characteristic and its standardized control limits.

Kawamura et al. (2013) proposed a method of applying Q chart to autocorrelated data. They combined Q statistic with residuals from a time series model. To illustrate the use of the method, they applied it in a horizontal low-pressure chemical vapor deposition (LPCVD) process. The authors showed the effectiveness of the method for quality control of the considered semiconductor manufacturing processes through a practical study. Snoussi et al. (2005) showed that using Q statistic in conjunction with residuals control charts is another possible tool for monitoring autocorrelated data.

CUSUM Q Charts and EWMA Q Charts

Quesenberry (1995) also proposed for short production run EWMA Q charts and CUSUM Q charts. He conducted a series of simulation tests to identify their capabilities of detecting the shifts of process mean and SD for Shewhart Q charts, EWMA Q charts and CUSUM Q charts. The results showed that EWMA and CUSUM Q charts are more sensitive to one-step permanent shifts on process mean or SD than classic Shewhart Q chart.

Zantek (2006) improved the design of CUSUM Q chart to enhance its capability of detecting a larger range of shifts of process mean. Different ranges for more promising parameter values to construct CUSUM Q charts were identified based on simulation experiments.

One problem of using CUSUM Q charts is that the constant value k is determined by the shift size of process mean. When the shift size is unknown, it may be difficult to apply CUSUM Q chart directly. Li and Wang (2010) developed an adaptive CUSUM Q chart (ACQ). It does not need to have a given shift size in advance. They estimated the mean shift using an EWMA scheme with a reflecting boundary as a one-step-ahead forecast. It was shown through simulation that the ACQ charts perform better than CUSUM Q charts especially when the shift size is small and happens in later part of the monitored process.

Li et al. (2010) presented another adaptive CUSUM Q charts. They adopted variable sampling intervals (VSIACQ) in them. The authors studied the distribution of CUSUM of Q statistics by simulation tests to solve the “mask” problem of Q charts. They believed that the VSIACQ charts are able to detect a range of shifts rather than only a fixed size of shifts compared with conventional CUSUM charts. The VSIACQ charts were developed assuming normal distribution.

Capizzi and Masarotto (2012) explored the adaptive cumulative score (ACUSCORE) control charts. They estimated the process mean using both adaptive EWMA and adaptive CUSUM control charts to address the dynamic pattern of mean change. Compared with traditional CUSUM Q and EWMA Q charts, ACUSCORE charts have stronger detecting ability when there is a small shift on mean. When the shift on mean is large, the performance of ACUSCORE charts are similarly to those of CUSUM Q and EWMA Q charts. The author believed that ACUSCORE charts outperform other control charts on detecting ability of mean shift.

2.2.2 t Charts

Zhang et al. (2009) first proposed t control charts to solve the problem of inaccurately estimating process SD. t charts plot t statistics following Student's t-distribution. They illustrated that t charts are more robust against changes in the process SD than $\bar{x}$ chart. Yet when the SD has no change, $\bar{x}$ chart has better ARL performances than t chart. They also compared ARL performances of EWMA t charts and EWMA $\bar{x}$ charts to show that EWMA t charts are more robust than the EWMA $\bar{x}$ charts against changes in process SD.

Celano et al. (2011) showed the possibility of implementing t control chart for short-run production processes when the setup is perfect (the estimated mean is equal to the target) or imperfect (otherwise). The authors believed that one can use t charts to monitor the short-run production because it does not require Phase I data and it is easy to implement. They compared the statistical properties of several t type charts (Shewhart t charts and EWMA t charts) and $\bar{x}$ type charts (Shewhart $\bar{x}$ charts and EWMA $\bar{x}$ charts) under the conditions of perfect setup and imperfect setup.

The simulation tests illustrated that EWMA t charts are more effective on mean shift detection compared with Shewhart t chart.

In short run productions, the process mean and SD are often unknown in advance due to the lack of Phase I data. Furthermore, when the mean of the process shifts, the shift size is most likely unavailable in practice as well. Celano et al. (2013) investigated statistical performances of Shewhart t, EWMA t and CUSUM t charts for short production runs when the shift size is unknown. They proposed an approach of modeling the unknown shift size using statistical distribution. They showed that CUSUM t and EWMA t charts perform better than Shewhart t chart when the shift size is in a certain range.

Castagliola et al. (2013) proposed a variable sample size (VSS) t chart and investigated the performance of the variable sample size strategy. They compared it with fixed-parameter (FP) t chart considering both fixed and unknown shift sizes. When the shift size is fixed and occurs at the start of the run, the tests showed that the VSS t chart is more effective than FP t chart. When the shift size is unknown, it was illustrated that the VSS t chart is more sensitive on the shift than FP t chart.

Sitt et al. (2014) proposed another revised t chart, the run sum t chart. Run sum t chart is a zone chart. It divides the interval between upper control limit (UCL) and lower control limit (LCL) into several different zones. The authors demonstrated that as an addition to the EWMA t charts and t charts, run sum t charts perform better than EWMA t charts for medium to large shifts. When compared with run sum X charts and EWMA X charts, run sum t charts perform better for large shifts while run sum X charts and EWMA X charts are more effective for small shifts. They also conducted further research on economic optimal design for t type charts and $\bar{x}$ type charts. It is found that $\bar{x}$ type charts have the lower minimum cost than t type charts. Considering that $\bar{x}$ charts can only be used when the mean and SD are accurately estimated, the authors suggested that t type charts be selected if there are estimation errors.

Celano et al. (2012) studied economic design of CUSUM t charts for short production runs and compared CUSUM t charts and CUSUM $\bar{x}$ charts for different scenarios. The numerical analysis show that the economic loss of CUSUM t charts due to imperfect implementation of the chart is insignificant when the process parameters are not accurately estimated.

Besides Q charts and t charts, other self-starting control charts with different features are presented as well. Some more recent development can be found in, for example, Zhang et al. (2012), Li et al. (2014), Li et al. (2010), Liu et al. (2015). In addition, Garjani and Noorossana (2010) proposed a control scheme for monitor start-up processes and short runs.

2.3 Summary

The literature discussed above is summarized in Table 2.1 with information on the authors, the year of publishing, main work, method used, and the result of the research.

Table 2.1 Summary of short production run charts

Author	Year	Main Work	Method	Result
Non Self-Starting Control Charts				
Ma et al.	2010	Established two supplemental rules for s chart.	Analytical	Improved the detecting capability of s chart when the shift size is small.
Yang et al.	2012	Studied X chart and compared it with 3-CUSUM chart.	Numerical simulation	Showed the advantage of simple X chart with sample size n=1.
Noskievicova et al.	2014	Developed the MATLAB application for CCC and CQC charts for attributes.	Analytical and coding	Provided the aids on implementation of the CCC and CQC charts in practice.
Self-Starting Control Charts				
Q Charts				
Quesensberry	1991	Proposed Q chart.	Analytical	Solved the difficulties of estimating the process mean and SD in short-run productions.
Castillo and Montgomery	1994	Explained the problems of Q charts and proposed weighted moving average method and an adaptive Kalman filtering method.	Numerical simulation	Pointed out the problem of Q charts and proposed the alternative methods.
Zantek	2005	Compared the signal probabilities of Q chart on each observation following a shift on mean.	Analytical and numerical simulation	Showed the problem of Q charts.
He et al.	2008	Investigated the bias of Shewhart Q charts. Explored two alternative Q charts to decrease the bias.	Analytical and numerical simulation	Showed the problem of Q charts.
Roes et al.	1999	Investigated the performance of supplemental run rules. Presented Q(R) chart and developed an economic model.	Numerical simulation	Illustrated the effectiveness of the supplemental rules of Q chart.
Wen and Zhao	2012	Designed Q chart in conjunction with Various Sampling Interval (VSI).	Analytical and numerical simulation	Showed that the method could improve the Q chart performance to some extent.
Kawamura et al.	2013	Combined Q statistic with residuals of a time series model and applied Q charts to autocorrelated data.	-	Showed how to apply Q charts when the data are autocorrelated rather than independent.

Table 2.2 Summary of short production run charts (continued)

Author	Year	Main Work	Method	Result
Zhu and Zhou	2010	Presented Weighted Q control chart.	Analytical	Showed the advantage of weighted Q chart in ARL performance.
Lampreia and Requeijo	2012	Designed MQ chart for vibration monitoring of repairable system.	-	Illustrated that the MQ chart can monitor the state of equipment online.
Chang and Tong	2013	Applied Q chart in software industry.	-	Concluded Q chart is more effective in software industry than other conventional control charts.
CUSUM Q Charts and EWMA Q Charts				
Quesenberry	1995	Designed EWMA Q charts and CUSUM Q charts.	Numerical simulation	Showed that EWMA Q and CUSUM Q charts are more sensitive to detect a shift on mean or SD than Shewhart Q charts.
Zantek	2006	Improved the design of CUSUM Q chart with considering the changing of distribution of Q statistics after a shift.	Numerical simulation	Enhanced the capability of CUSUM Q charts for detecting a broad range of shifts.
Li and Wang	2010	Developed an ACQ charts which do not need to be designed with a given shift size.	Numerical Simulation	Showed that the ACQ charts perform better than CUSUM Q charts.
Li et al.	2010	Presented another adaptive CUSUM of Q chart – VSIACQ charts.	Numerical simulation	Showed that the VSIACQ charts were able to detect a range of shifts rather than a fixed shift size.
Capizzi and Masarotto	2012	Explored the ACUSCORE control charts.	Numerical simulation	Showed that ACUSCORE charts have stronger detection power than CUSUM Q and EWMA Q chart.
t Charts				
Zhang et al.	2009	Proposed t control chart.	Numerical simulation	Showed that t charts were more robust against changes in the process SD than the $\bar{x}$ chart.
Celano et al.	2011	Implemented t control chart into short-run production process under perfect setup and imperfect setup.	Numerical Simulation	Illustrated that EWMA t charts are the most effective chart on mean shift detection compared with Shewhart t chart.

Table 2.3 Summary of short production run charts (continued)

Author	Year	Main Work	Method	Result
Celano et al.	2013	Investigated the statistical performance of the Shewhart t, EWMA t and CUSUM t charts when the shift size was unknown.	Analytical	Demonstrated that CUSUM t and EWMA t charts perform better than Shewhart t chart when the shift size is in a certain range.
Castagliola et al.	2013	Proposed the VSS t chart.	Analytical and numerical simulation	Showed that the VSS t chart is more effective than FP t chart.
Sitt et al.	2014	Explored the run sum t chart. Conducted research on economic optimal design for t type charts and $\bar{x}$ type charts.	Analytical and numerical simulation	Showed advantage of run sum t chart.
Celano et al.	2012	Conducted economic design of the CUSUM t chart.	Numerical analysis.	Illustrated that the economic loss of CUSUM t chart corresponding to the imperfect implementation.

As can be seen in the reviewed literature in this area, different versions of control charts based on t chart and Q chart were proposed and tested for their detecting capability on mean shift. It is apparent that most of the studies on control charts for short-run production focus on monitoring the process mean shifts. A less number of research papers studied the detecting ability for SD shifts of short-run control charts. In practice, the variance of a certain measurement in a manufacturing process is as important as the process mean, and there is a need to apply the short-run control charts to monitor the process variation as well. In this thesis, we study the detecting ability of Q type charts and t type charts for detecting process SD shifts. We also compare various performances of t type charts and Q type charts. Finally we identify several effective control charts for detecting SD shift in short production runs.

3 Modeling Short Production Processes

3.1 Introduction

From the literature review, most of the existing research aims at studying detecting ability of control charts on mean shift, but few of the published papers study the detecting ability on SD shift. However, in many manufacturing processes, the variance of the measurements is as important as the mean and the variance control is an inevitable aspect in statistical quality control as well. When the fluctuation of a certain character increases significantly, the potential problem must be identified and further studied. Consequently, considering its practical value in manufacturing, numerical simulation used in this thesis focuses on the performance of different short-run control charts in detecting SD shift.

This chapter presents several statistical models for the control charts to be compared in the simulation study. They include Q chart, Q chart with supplemental rules, EWMA Q chart, CUSUM Q chart, t chart, EWMA t chart, CUSUM t chart and individual X chart. Before the detailed numerical experiments are presented, we briefly explain the general settings of the parameters used in this simulation study.

3.2 Q and Q type Charts Setting

3.2.1 Q Chart

Quesensberry (1991) first proposed Q chart in an attempt to overcome the difficulties in estimating the process mean and SD in short production runs. As explained in Quesensberry (1991) as well as reported in Castillo and Montgomery (1994), Q chart can be well used for certain types of short-run productions. The basics of setting up a Q chart are described below.

Considering a normally and independently distributed process with mean μ and SD σ, collect a sample of $\{X_1, X_2, \dots, X_r\}, r = 1,2, \dots, n$. The Q statistics for monitoring the process mean are calculated under the following four cases:

<u>Case I (KK):</u> Both μ and σ are known, $\mu = \mu_0, \sigma = \sigma_0$, the Q statistic is calculated as follows:

$$Q_r(X_r) = \frac{X_r - \mu_0}{\sigma_0}, (r = 1,2, \dots) \tag{3.1}$$

<u>Case II (UK):</u> μ is unknown and σ is known, $\sigma = \sigma_0$, the Q statistic is calculated with the estimator $\overline{X}_r$:

$$Q_r(X_r) = \left(\frac{r-1}{r}\right)^{\frac{1}{2}} \left(\frac{X_r - \bar{X}_{r-1}}{\sigma_0}\right), (r = 2,3, \dots) \tag{3.2}$$

where:

$$\bar{X}_r = \frac{1}{r} \sum_{j=1}^{r} X_j \tag{3.3}$$

<u>Case III (KU):</u> μ is known and σ is unknown, $\mu = \mu_0$, Q statistic is calculated with the estimator $S_{0,r}$

$$Q_r(X_r) = \Phi^{-1}\left\{G_{r-1}\left(\frac{X_r - \mu_0}{S_{0,r-1}}\right)\right\}, (r = 2,3, \dots) \tag{3.4}$$

where: Φ^{-1} is the inverse of the standard normal distribution, G_{r-1} is the t distribution with r-1 degrees of freedom and

$$S_{0,r}^2 = \frac{1}{r}\sum_{j=1}^{r}(X_j - \mu_0)^2 \quad (3.5)$$

<u>Case IV (UU):</u> Both μ and σ are unknown, the Q statistic is calculated as follows:

$$Q_r(X_r) = \Phi^{-1}\left\{G_{r-2}\left[\left(\frac{r-1}{r}\right)^{1/2}\left(\frac{X_r - \bar{X}_{r-1}}{S_{r-1}}\right)\right]\right\}, (r = 3,4,\ldots) \quad (3.6)$$

where: Φ^{-1} is the inverse of the standard normal distribution, G_{r-2} is the t distribution with r-2 degrees of freedom and

$$\bar{X}_r = \frac{1}{r}\sum_{j=1}^{r} X_j \quad (3.7)$$

$$S_r^2 = \frac{1}{r-1}\sum_{j=1}^{r}\left(X_j - \bar{X}_r\right)^2 \quad (3.8)$$

The Q statistics are independently and identically distributed N(0, 1) random variables. They can be plotted on a Shewhart chart with:

$$UCL_Q = +3 \quad (3.9a)$$

$$Center\ Line = 0 \quad (3.9b)$$

$$LCL_Q = -3 \quad (3.9c)$$

In short production runs, it mainly is Case IV (both μ and σ are unknown), since most of the time the process mean and SD are not known in advance. In this research, the simulation experiments focus on the study of Q statistic of Case IV.

3.2.2 Supplemental Rules of Q Chart

Using supplemental run rules may, to some extent, improve the detecting capability of Q chart. Roes et al. (1999) tested the detecting capability of supplemental run rules (originally recommended by Western Electric Company) for mean shift of Q chart. In this thesis, we test the effectiveness of the following rules on detecting capability of SD shift for Q chart to be discussed in Chapter 4.The supplemental rules are listed below.

(A) 1-of-1 test - signals if the last point is beyond the control limits (± 3);

(B) 2-of-3 test - signals if two out of the last three points are beyond the same warning limit (± 2);

(C) 4-of-5 test - signals if four of the last five points are beyond the same auxiliary limit (± 1);

(D) 8-of-8 test - signals if eight consecutive points fall on the same side of the central line (0).

3.2.3 CUSUM Q Chart and EWMA Q Chart

Quesenberry (1995) pointed out that the Q statistics can be used as the input data of CUSUM (Cumulative Sum) and EWMA (Exponentially Weighted Moving Average) charts as well.

Originally, CUSUM chart was designed to detect the small shift of mean which may be difficult to be captured by standard Shewhart control chart. Let μ_0 be the target for the process mean and x_j be the jth sample. The CUSUM control chart is formed by plotting the quantity of

$$C_i = \sum_{j=1}^{i} (x_j - u_0) \tag{3.10}$$

The statistics of C_i are further computed as follows:

$$C_i^+ = max[0, x_i - (\mu_0 + K) + C_{i-1}^+] \tag{3.11a}$$

$$C_i^- = max[0, (\mu_0 - K) - x_i + C_{i-1}^-] \tag{3.11b}$$

with $C_0^+ = C_0^- = 0$. K is the reference value and H is the decision interval. The control limits are $\pm$H . K and H are constant parameters. They can be optimized according to the desired average run length.

For CUSUM Q chart, Quesenberry (1991) defined the CUSUM statistics as S_i^+ and S_i^- :

$$S_i^+ = max[0, S_{i-1}^+ + Q_i - k_s] \tag{3.12a}$$

$$S_i^- = min[0, S_{i-1}^- + Q_i + k_s] \tag{3.12b}$$

with $S_0^+ = S_0^- = 0$

Based on the results in Quesenberry (1991), we set the reference value k_s and decision interval h_s at $k_s = 0.75$ and $h_s = 3.34$ in our simulation study. These values provide CUSUM Q chart an in control average run length (ARL) of 370.5 for detecting a mean shift of 1.5σ in a normal process. In our simulation experiments, we also use this average run length to search parameter values for CUSUM and EWMA control charts.

Similar to CUSUM control charts, EWMA control charts were first developed for detecting small shifts of mean.

EWMA statistics are constructed as:

$$z_i = \lambda x_i + (1 - \lambda) z_{i-1} \tag{3.13}$$

where $0 < \lambda \leq 1, z_0 = \mu_0$, the known process mean or the estimated process mean.

EWMA chart control limits and centre line are defined by:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \quad (3.14a)$$

$$Center\ Line = \mu_0 \quad (3.14b)$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \quad (3.14c)$$

L and λ are constant parameters which can be designed according to the desired average run length.

For EWMA Q chart, the EWMA statistics given by Quesenberry (1991) are:

$$z_i = \lambda Q_i + (1-\lambda)z_{i-1} \quad (3.15)$$

with $z_0 = 0$. The control limits are $\pm K\sqrt{\lambda/(2-\lambda)}$.

Following Quesenberry (1991), parameters λ and K can be designed according to the same ARL used for CUSUM Q chart (ARL=370.5). The values obtained are $\lambda = 0.25$ and $K = 2.90$, which give an ARL of 372.6 (the difference with 370.5 is very small and can be ignored) to detect a mean shift of 1.5σ in a normal process.

It may be noticed that the parameter settings for CUSUM Q chart and EWMA Q chart here are the optimized settings for detecting mean shift rather than SD shift, while the simulation in this thesis is aiming at detecting SD shift. Through the research work in this thesis, we would like to compare different short-run control charts rather than design optimal control charts. We used consistent parameter settings in comparing detecting capabilities of different charts developed for short production runs.

3.3 t and t Type Charts Setting

3.3.1 t Chart

t chart was first proposed in Zhang et al. (2009) to more accurately estimate the of SD of a process or to estimate SD when the process is not stable. The details of setting up a t chart can be described below.

Assume that we take several subgroups $X_{i,1}, X_{i,2}, \ldots, X_{i,n}$ of size n at time $i = 1,2, \ldots$. Typically n is small, say n=5. We assume that the subgroups are independent with each other and $X_{i,j} \sim \mathrm{N}(\mu_0, \sigma_0)$ $i = 1,2, \ldots, 1 \leq j \leq n$,where μ_0 and σ_0 are the nominal process mean and SD, respectively.

The subgroup mean $\bar{X}_i$ and the subgroup SD S_i at time i can be calculated as following:

$$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{i,j} \tag{3.16}$$

$$S_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(X_{i,j} - \bar{X}_i)^2} \tag{3.17}$$

Then the t statistics T_i can be defined by the following function:

$$T_i = \frac{\bar{X}_i - \mu_0}{S_i/\sqrt{n}}, i = 1,2, \ldots \tag{3.18}$$

T_i follows Student's t-distribution with n-1 degrees of freedom. So the control limits are:

$$UCL_t = F_t^{-1}(1 - \frac{\alpha}{2} | n - 1) \tag{3.19a}$$

$$LCL_t = -UCL_t \tag{3.19b}$$

Where $F_t^{-1}(\cdot\ |n-1)$ is the inverse distribution function of the Student's t-distribution with n-1 degrees of freedom, and α is the false alarm rate (the probability of Type I error). Different from $\overline{x}$ chart, implementing a t chart may not be necessary to know or to estimate SD of the process. So using t chart for short-run process is possible without knowing or estimating process SD and mean.

For short-run production applications, Quesenberry (1991) utilised t statistics as part of the Q statistics under the condition that both μ_0 and σ_0 are unknown. According to Quesenberry (1991), we may use the current data to estimate the unknown u_0 and update it when having a new data each time. The control limits provided by Zhang et al. (2009) should be implemented when sample size n>1. However in most short-run processes, to have sample size n>1 may not be practical. So for short-run production, the number of the samples i will replace the sample size n in the formula. It can also be understood that there is only one sample and the sample size i keeps updating as new data are collected. t statistics and the control limits of t chart can be revised as following. Assume i stands for the ith sample with sample size n=1, then:

$$T_i = \frac{a_i(X_i - \bar{X}_{i-1})}{S_{i-1}} \quad i = 2,3, \ldots, \tag{3.20}$$

where $a_i = \sqrt{(i-1)/i}$

$$UCL_t = F_t^{-1}\left(1 - \frac{\alpha}{2}\middle|i-1\right) \tag{3.21a}$$

$$LCL_t = -UCL_t \tag{3.21b}$$

where the false alarm rate α will be set at 0.0027 , the same as for a Shewhart $\bar{x}$ chart.

3.3.2 CUSUM t Chart and EWMA t Chart

Celano et al. (2012) utilised t statistic as the input data of CUSUM scheme and proposed CUSUM t chart. The plotted statistic C_i is given by:

$$C_0 = 0 \tag{3.22a}$$

$$C_i = max\{0, C_{i-1} + T_i - E(T) - k\}, i = 1,2, \dots \tag{3.22b}$$

where $E(T)$ is the mean of the Student t distribution function with n-1 degrees of freedom.

The control limits are $\pm H\ where\ H = h\sigma_0$; h is a parameter to calculate the decision interval; k is a parameter and typically $\mathrm{k} = \frac{\delta}{2}\sigma$, δ is the shift size of the mean, σ is the SD used to generate the normally distributed random numbers. For short-run production, σ_0 will be replaced by S_i which is the estimated SD from the first i points.

Zhang et al. (2009) first plotted t statistics in the EWMA charting scheme and presented EWMA t chart. The plotted statistic Y_i is given by:

$$Y_0 = 0 \tag{3.23a}$$

$$Y_i = \lambda T_i + (1 - \lambda) Y_{i-1} \quad i = 1,2, \dots \tag{3.23b}$$

where $T_i = \frac{a_i(X_i - \bar{X}_{i-1})}{S_{i-1}}$ $i = 2,3, \dots.$ $\lambda \in (0,1]$ is a parameter, usually takes very small values.

The lower and upper control limits of EWMA t chart satisfy $LCL_t = -UCL_t$ and the center line is 0. The control limits are $\pm K\sqrt{\lambda/(2-\lambda)}$, where $K = L\sigma_0$.For short-run production, σ_0 will be replaced by S_i which is the estimated SD from the first i points. L is the constant parameter.

Unlike CUSUM Q chart and EWMA Q chart, research publications focusing on design of CUSUM t chart, EWMA t chart and their parameter values are very limited. We follow the common Shewhart control chart structure and use h=3 for CUSUM t chart and L=3 for EWMA t chart to construct the control limits in our simulation study. We conduct numerical experiments to be presented in Chapter 4 to select proper values of parameter λ for EWMA t chart.

3.4 Individual X Charts and Setting

In short run productions, the sample size is typically equal to one. So the individual X charts designed for individual measurement is possible to apply as well. The theoretical support for implementing individual X chart in short-run production is not strong and study on this possibility is less. From practical point of view, if the performance of individual X charts is similar or slightly weaker than other short-run production control charts, it may still be a preferred choice for many processes, since implementation of Individual X charts is much simpler. Thus, we included individual X chart in our study to compare it with other charts.

Individual X chart is the plot of the individual observations. Assume that there are m samples, $x_1, x_2, \dots x_m$, The control limits and the center line are:

$$UCL = \bar{x} + 3\frac{\overline{MR}}{d_2} \tag{3.24a}$$

$$Center\ Line = \bar{x} \tag{3.24b}$$

$$LCL = \bar{x} - 3\frac{\overline{MR}}{d_2} \tag{3.24c}$$

where $MR_i = |x_i - x_{i-1}|$, $\overline{MR} = \frac{\sum_{i=1}^{m} MR_i}{m}$, $d_2 = 1.128$

The limitations of Individual X chart can be seen from the formulas. Control limits and the center line can only be constructed after all the samples are collected, unlike other self-starting control charts. In our short-run production simulation, we used the first i data to estimate the $\overline{MR}$ to build the control limits from the very beginning of the process. The $\overline{MR}$ was updated with the number of samples being collected each time.

3.5 Setting Simulation Experiments

All simulation experiments were run using Microsoft Excel programme. We assumed that a shift of process SD happened at a certain point of the process. Such shift is then implanted in all the tested control charts to evaluate their performances of detecting ability accordingly. The general simulation and parameter settings are explained below.

Distribution function

We used normal distribution functions built in Excel to simulate short run processes studied in this research. Considering that normal distribution is the most common distribution in manufacturing process.

Number of samples

30 random variable values following normal distribution N(1, 1) were generated in each simulation run by Excel based on the following considerations. In many short-run manufacturing processes, it may be difficult or impossible to obtain sufficient number of samples typically required for statistical quality control purpose, because it may take too long time to collect enough data. On the other hand, in order to observe the existence of the signals and to compare the detecting ability of different control charts, the number of samples could not be too small, or there would rarely

have any signal in any control charts. However, to study some specific aspects of the processes in more detail, the number of samples was increased or decreased around 30 in some simulation runs. Details will be given in the next chapter.

Starting time of shifts

For mean shift, a single permanent shift would be implanted at the 7th sample, as to the mean shift is typically the first one to monitor by most control charts. Meanwhile, enough space should be left between the shift sample and the last sample so that a signal will possibly show. For SD shift, a single permanent shift would be implanted at the 10th sample, as we assume that a change of the process SD, if it occurs, should do so, after a shift of the process mean occurs.

Shift size

Define $\sigma' = \delta_\sigma \sigma_0$, $\mu' = \mu_0 + \delta_\mu \sigma_0$, δ_σ and δ_μ is the shift size of SD and mean. For mean shift, we set $\delta_\mu = 1.5$. For SD shift, we set $\delta_\sigma = 1.3, 1.7\ and\ 2.0$.

Replications per experiment

For each experiment, 30 replications were run. If 2 or more points are out of control before the shift actually happened, the group of data would be abandoned and re-generated. We assume that such a data stream may not be representative to those from the actual processes.

3.6 Summary

This chapter mainly introduces the control charts to be studied in the simulation experiments in this research and the parameter settings for their implementations. Results from simulation experiments on Q type charts and t type charts, the main control charts for short-run production, will be presented and analyzed in Chapter 4. In addition, simulation results on individual X charts will be analyzed and compared as well.

4 Simulation Experiments

4.1 Introduction

In the first part of Chapter 4, we identify the more promising values of the parameter for setting up EWMA t chart through numerical experiments with a common criterion that the in control ARL is equal to 370 for the shift of the mean being 1.5 σ.

In the second part of this chapter, we evaluate the supplemental rules for Q chart also through numerical simulation as discussed in Section 3.1.2. We notice that several other researchers had conducted similar investigations. Their work, however, was mainly aiming at studying the detecting ability on the shift of the mean rather than process SD. Through our simulation study, we identify the best combination of the supplemental rules for detecting SD shift and apply it to Q chart. We then conduct simulation experiments to evaluate the performance of Q chart using the best supplemental rules.

The simulation experiments for all short-run control charts conducted in this research are presented in the third part of this chapter. We study the detecting ability for SD shift of the following ten charts through simulation:

- Q Chart
- Q Chart with optimal combination of supplemental rules
- EWMA Q Chart
- CUSUM Q Chart
- t Chart
- EWMA t Chart

- CUSUM t Chart
- Individual X Chart

We present and analyse the simulation results in Section 4.4.

4.2 EWMA t Chart Parameter Setting

As stated in Montgomery (2013), $0.05 \leq \lambda \leq 0.25$ generally works well for EWMA scheme in many applications. We conduct several numerical experiments for short run EWMA t chart with different λ values in this range in order to identify better λ values.

In each of the simulation runs, 30 random variable values following normal distribution N(1, 1) are generated by the normal distribution generation function built in Microsoft Excel. As discussed in Chapter 3, the simulation study is set with the considerations that ARL=370 and mean shift δ=1.5σ. A shift of the mean δ=1.5σ, is implanted at the 7th sample as we assume that in short-run productions the mean shifts tend to happen in early stage of the process. So from the 7th point, the distribution will shift to N(2.5, 1). Accordingly, the EWMA control charts may correctively signal such shift at any time from the 7th point onwards while we would like to see if the out of control signal will appear before the 30th point. We start our simulation with λ=0.05. We increase the value of λ by 0.01 each time until λ=0.25. Therefore, there are 21 experiments in total with each experiment has 30 replications. The best λ value corresponding to the highest success rate are identified for setting up EWMA t chart.

The experiments results are presented in Table 4.1.

Table 4.1 λ performances for short-run EWMA t chart

	λ=0.05	**λ=0.06**	**λ=0.07**	**λ=0.08**	**λ=0.09**	**λ=0.10**	**λ=0.11**
N.O.S.*	5.0	5.0	11.0	10.0	13.0	7.0	6.0
A.A.P.*	17.4	16.2	14.3	15.8	16.9	18.6	15.7
N.O.R.*	30.0	30.0	30.0	30.0	30.0	30.0	30.0
S.R.*	0.2	0.2	0.4	0.3	0.4	0.2	0.2
	λ=0.12	**λ=0.13**	**λ=0.14**	**λ=0.15**	**λ=0.16**	**λ=0.17**	**λ=0.18**
N.O.S.	8.0	6.0	5.0	5.0	6.0	5.0	2.0
A.A.P.	15.5	14.5	14.8	15.6	13.5	13.4	19.5
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.3	0.2	0.2	0.2	0.2	0.2	0.1
	λ=0.19	**λ=0.20**	**λ=0.21**	**λ=0.22**	**λ=0.23**	**λ=0.24**	**λ=0.25**
N.O.S.	6.0	4.0	9.0	4.0	3.0	8.0	5.0
A.A.P.	16.3	12.0	11.9	16.5	11.0	12.0	15.4
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.2	0.1	0.3	0.1	0.1	0.3	0.2

*N.O.S.: Number of successes among 30 replications
A.A.P.: Average alarm point for 30 replications
N.O.R.: Number of replications
S.R.: Success rate

The data shown in Table 4.1 are also plotted in Figure A.4.1, presented in the Appendix of this thesis. Similarly, we also plotted the results shown in other tables in this chapter and presented the corresponding figures in the Appendix.

As can be seen in Table 4.1, when λ=0.07 ~ 0.09, we have higher number of successes (11.0, 10.0 and 13.0) than the cases when λ takes other values. In addition, when λ=0.07 ~ 0.09, EWMA t chart has better detecting performance. Accordingly, we select λ=0.09 (which has the highest number of successes in this case) for EWMA t chart in the following simulation experiments.

4.3 Identify Supplemental Rules of Q Charts

Roes et al. (1999) tested the following supplemental rules for Q chart on its detecting ability on mean shift. In this section, we present simulation experiments for its detecting ability on SD shift.

The following supplemental rules for Q chart are considered in this study:

(A) 1-of-1 test - signals if the last point is beyond the control limits (±3);

(B) 2-of-3 test - signals if two out of the last three points are beyond the same warning limit (±2);

(C) 4-of-5 test - signals if four of the last five points are beyond the same auxiliary limit (±1);

(D) 8-of-8 test - signals if eight consecutive points fall on the same side of the central line (0).

If the chart shows an out-of-control signal before the shift point, this will be treated as a false alarm. The first effective signal would be counted after the shift point.

We can expect that the combination of four rules tends to have the strongest detecting ability among all the other possible combinations. However, the main objective of the experiments of this section is to find other simpler combinations having similar or equal effectiveness. If so, the four rules could be replaced by less rules. The application would be much simpler.

4.3.1 Original Experiments

30 random variable values following N(1, 1) are generated for each replication by MS-Excel. To distinguish the starting time of mean shift and SD shift, we implant the latter from the 10th point rather than the 7th point, considering that a SD shift may happen after the mean shift. In practice, it may take longer time for the process SD to change than the process mean. For example, the worn out of a lathe tool and increased vibration may cause the SD of the turned diameter to change. Normally, this type of system changes should not occur at the early stage of the process. To observe the effectiveness of different individual rules on the sizes of SD shifts, we used 3 different shift sizes by letting $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ in our simulation experiments. Each test has 30

replications. The first effective signal is counted if it is plotted outside the control limit, warning limit or auxiliary limit after the shift is implanted at the10th plot. For example, for applying supplemental Rule B, the signal will be considered if the 2 out of limits points appear after the 10th point. If one happens at the 9th point and another one happens at the 11th point, this will not be counted as an out of control signal.

Results of these simulation experiments for $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ are presented in Tables 4.2, 4.3 and 4.4, respectively. Each of the table presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the individual or combinations of different supplemental rules. For example, in Tables 4.1 to 4.5, "AB" means that the shift is detected by supplemental Rule A or Rule B and "ABCD" means that the shift is detected by any of the supplemental Rules A, B, C, or D. Table 4.5 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.2 to 4.4.

Table 4.2 Supplemental rules performances: n=30, shift size 1.3σ

	A	**B**	**C**	**D**		
N.O.S.	8.0	9.0	17.0	1.0		
A.A.P.	17.8	15.1	14.7	15.0		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.3	0.3	0.6	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	14.0	22.0	10.0	21.0	9.0	21.0
A.A.P.	14.3	15.2	17.5	13.6	17.8	16.0
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.5	0.7	0.3	0.7	0.3	0.7
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	22.0	13.0	19.0	20.0	22.0	
A.A.P.	15.9	18.8	15.8	14.4	16.7	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.7	0.4	0.6	0.7	0.7	

Table 4.3 Supplemental rules performances: n=30, shift size 1.7σ

	A	B	C	D		
N.O.S.	18.0	6.0	21.0	5.0		
A.A.P.	15.2	15.2	13.4	15.6		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.6	0.2	0.7	0.2		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	18.0	24.0	13.0	24.0	13.0	21.0
A.A.P.	13.7	12.8	16.8	14.5	15.7	15.3
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.6	0.8	0.4	0.8	0.4	0.7
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	29.0	24.0	24.0	26.0	27.0	
A.A.P.	14.7	15.2	12.8	14.3	14.6	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	1.0	0.8	0.8	0.9	0.9	

Table 4.4 Supplemental rules performances: n=30, shift size 2σ

	A	B	C	D		
N.O.S.	13.0	9.0	23.0	3.0		
A.A.P.	13.9	14.6	14.5	15.0		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.4	0.3	0.8	0.1		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	22.0	29.0	16.0	22.0	17.0	21.0
A.A.P.	14.9	13.8	15.1	14.9	14.3	14.3
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.7	1	0.5	0.7	0.6	0.7
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	24.0	23.0	24.0	26.0	26.0	
A.A.P.	13.2	14.8	13.2	13.5	12.5	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.8	0.8	0.8	0.9	0.9	

Observing from the experiment results of the 3 shift sizes of the SD, we can see that the combination of Rules A and C has the best performance on N.O.S. among all combinations of any two rules. In addition, when the shift sizes are 1.3σ and 2σ as shown in Table 4.2 and 4.4, respectively, AC has the best N.O.S. performance among all the tested individual rules and all combinations. For all the SD shift sizes, there is no significant difference on A.A.P. between different individual rules or rule combinations. Still, when SD shift is at 1.7σ as shown in Table 4.3, AC is one of the combinations which have the best performance on A.A.P. When SD shift is at 1.3σ or 2σ as shown in Table 4.2 and Table 4.4, respectively, the A.A.P. performance of combination AC is still comparable with the best ones.

As can be seen from Tables 4.2, 4.3 and 4.4, for most supplemental rule combinations, the N.O.S. performances become better when the shift size of SD increases. For example, AB has the N.O.S. of 14.0 when the shift size is 1.3σ, 18.0 when the shift size is 1.7σ, 22.0 when the shift size is 2σ. For individual rules, such trend is not obvious. In terms of A.A.P., most of the individual rules and combined rules detect the shift faster when the shift size of SD is larger. For example, the A.A.P. by ABCD is 16.7 when the shift size is 1.3σ, 14.6 when the shift size is 1.7σ, and 12.5 when the shift size is 2σ.

Table 4.5 Average values of N.O.S. and A.A.P. when n=30

	A	**B**	**C**	**D**		
Average N.O.S.	13.0	8.0	20.3	3.0		
Average A.A.P.	15.6	14.9	14.2	15.2		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
Average N.O.S.	18.0	25.0	13.0	22.3	13.0	21.0
Average A.A.P.	14.3	14.0	16.4	14.3	15.9	15.2
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
Average N.O.S.	25.0	20.0	22.3	24.0	25.0	
Average A.A.P.	14.6	16.3	13.9	14.0	14.6	

It is apparent from Table 4.5 that for the 3 SD shift sizes, the combinations of Rules ABCD, Rules ABC and Rules AC have the best average N.O.S. performance among other rule combinations or individual rules. For practicality considerations, AC is the best combination of rules to use since it is simpler and more effective. Rule C is the most effective individual rule and performs better than Rules A, B and D. At the same time, any other rule or rules combining with Rule C have good performance as shown in Table 4.5.

In terms of average A.A.P, we can see from Table 4.5 that there is no significant difference between different combinations and individual rules. Although combinations of Rules ABCD, Rules ABC and Rules AC no longer have the best performance, they still have good performance comparing to all other individual rules and rule combinations.

4.3.2 Experiments for Different Lengths of Tested Processes

The length of the tested processes may affect the detecting capability of the rules investigated in this research. For example, using Rule C and Rule D requires longer run lengths to identify the signal. It means that the number of samples should be larger when these rules are used. With this consideration, we conducted two groups of experiments to observe the influence of process length on the detecting ability of the considered rules with all other conditions kept the same. Similar to those discussed earlier, we used 3 different shift sizes by letting $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ in our simulation experiments. Normal distribution function in Microsoft Excel is used to generate 30 random variable values following N(1,1) in each simulation run. A shift of the SD is implanted at the 10th sample. Each test has 30 simulation runs or replications. The first effective signal is counted if it is plotted outside the control limit, warning limit or auxiliary limit after the shift is implanted at the10th plot.

For the first group of experiments, we reduced the length of the tested process from 30 points to 20 points without changing the shift starting point, the 10th plot.

Results of these experiments for $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ are presented in Tables 4.6, 4.7 and 4.8, respectively. Each of the table presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the individual or combinations of different supplemental rules. Table 4.9 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.6 to 4.8.

Table 4.6 Supplemental rules performances: n=20, shift size 1.3σ

	A	**B**	**C**	**D**		
N.O.S.	2.0	5.0	11.0	0.0		
A.A.P.	11.0	14.0	11.6	-		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.1	0.2	0.4	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	9.0	14.0	7.0	11.0	8.0	13.0
A.A.P.	12.7	12.4	15.9	12.5	13.3	11.7
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.3	0.5	0.2	0.4	0.3	0.4
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	14.0	9.0	10.0	14.0	22.0	
A.A.P.	12.9	13.8	11.7	10.7	12.5	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.5	0.3	0.3	0.5	0.7	

Table 4.7 Supplemental rules performances: n=20, shift size 1.7σ

	A	**B**	**C**	**D**		
N.O.S.	6.0	13.0	12.0	0.0		
A.A.P.	14.8	11.5	12.3	-		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.2	0.4	0.4	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	12.0	24.0	7.0	12.0	9.0	13.0
A.A.P.	13.7	12.6	12.7	11.3	11.7	11.2
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.4	0.8	0.2	0.4	0.3	0.4
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	19.0	14.0	18.0	19.0	21.0	
A.A.P.	12.4	12.7	11.9	11.9	12.8	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.6	0.5	0.6	0.6	0.7	

Table 4.8 Supplemental rules performances: n=20, shift size 2σ

	A	**B**	**C**	**D**		
N.O.S.	15.0	13.0	18.0	0.0		
A.A.P.	12.9	12.2	10.9	-		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.5	0.4	0.6	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	20.0	22.0	9.0	20.0	19.0	18.0
A.A.P.	12.5	11.8	14.1	11.3	12.8	11.6
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.7	0.7	0.3	0.7	0.6	0.6
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	22.0	16.0	19.0	21.0	24.0	
A.A.P.	11.5	11.7	13.4	10.8	11.3	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.7	0.5	0.6	0.7	0.8	

Combination of Rules AC still has the best performance on N.O.S. among all the 2 rule combinations. When the shift size is 1.7σ as shown in Table 4.7, AC has the best N.O.S. performance among all the tested individual rules and combinations. When the shift sizes are 1.3σ and 2σ as shown in Table 4.6 and Table 4.8 respectively, Rules ABCD performs best. It is worth mentioning that Rules AC and other combinations show limited effectiveness when shift size is 1.3σ, while Rules ABCD still has the N.O.S at 22.0, the largest among all individual rules or rule combinations. Rules AC follow it with 14.0. For A.A.P., when shift size is 1.3σ (Table 4.6), Rule A, Rule C, Rules CD and Rules ACD have better A.A.P. performance. When shift size is 1.7σ, Rule B, Rules BC, Rules BD, Rules CD, Rules ACD and Rules BCD have better A.A.P performance. When shift size is 2σ, Rules AC, Rules BC, Rules CD, Rules ABC, Rules ABD, Rules BCD and Rules ABCD have better A.A.P. performance. We can see from above observations, Rules AC always has better A.A.P. performance for all the SD shift sizes with its value at around 12.0.

As can be seen in Table 4.6, 4.7 and 4.8, the N.O.S. performances in this set of experiments are similar to those in the original experiments. For almost all the individual rules and combinations, the N.O.S. becomes better when the SD shift size increases. It also shows that Rule D is less effective for all the 3 SD shift sizes. This is understandable as using Rule D requires 8 points to identify the signal. After reducing the length of the tested process to 20 points, only 10 points after the shift could be observed. In terms of A.A.P., there is no apparent increasing or decreasing trend when the shift size of SD becomes larger.

Table 4.9 Average values of N.O.S. and A.A.P. when n=20

	A	**B**	**C**	**D**		
Average N.O.S.	7.7	10.3	13.7	0.0		
Average A.A.P.	12.9	12.6	11.6	-		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
Average N.O.S.	13.7	20.0	7.7	14.3	12.0	14.7
Average A.A.P.	12.9	12.3	14.2	11.7	12.6	11.5
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
Average N.O.S.	18.3	13.0	15.7	18.0	22.3	
Average A.A.P.	12.3	12.7	12.3	11.2	12.2	

Observing the averaged values of N.O.S. and A.A.P shown in Table 4.9, for the tested 3 shift sizes of SD, the combination of Rules ABCD is the most effective one in terms of N.O.S. Rules AC, similar to the results from our original experiments, has a competitive performance on N.O.S. as well. Rule C still has the best performance among individual rules. The averaged A.A.P. value does not show significant differences corresponding to different individual rules and rule combinations, except the weakness of Rule D, as discussed earlier.

For the second group of experiments, we increased the length of the tested process from 30 points to 40 points without changing the shift starting point at the 10th plot.

Results of these experiments for $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ are presented in Tables 4.10, 4.11 and 4.12, respectively. Each of the tables presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the individual or combinations of different supplemental rules. Table 4.13 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.10 to 4.12.

Table 4.10 Supplemental rules performances: n=40, shift size 1.3σ

	A	**B**	**C**	**D**		
N.O.S.	5.0	7.0	17.0	3.0		
A.A.P.	16.6	20.1	17.5	18.7		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.2	0.2	0.6	0.1		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	20.0	27.0	14.0	25.0	21.0	21.0
A.A.P.	20.1	15.5	20.3	18.5	17.5	18.5
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.7	0.9	0.5	0.8	0.7	0.7
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	28.0	25.0	26.0	26.0	29.0	
A.A.P.	14.3	15.6	15.5	15.3	13.4	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.9	0.8	0.9	0.9	1.0	

Table 4.11 Supplemental rules performances: n=40, shift size 1.7σ

	A	**B**	**C**	**D**		
N.O.S.	13.0	17.0	25.0	2.0		
A.A.P.	18.8	16.2	19.2	20.5		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.4	0.6	0.8	0.1		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	22.0	26.0	13.0	25.0	14.0	24.0
A.A.P.	16.5	14.7	18.9	15.0	17.0	17.3
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.7	0.9	0.4	0.8	0.5	0.8
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	28.0	23.0	26.0	26.0	29.0	
A.A.P.	18.1	18.3	19.9	17.5	14.4	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.9	0.8	0.9	0.9	1.0	

Table 4.12 Supplemental rules performances: n=40, shift size 2σ

	A	**B**	**C**	**D**		
N.O.S.	11.0	17.0	23.0	3.0		
A.A.P.	15.3	18.4	16.2	21.7		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.4	0.6	0.8	0.1		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	25.0	29.0	20.0	27.0	15.0	24.0
A.A.P.	12.9	14.1	16.3	13.7	17.3	17.5
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.8	1.0	0.7	0.9	0.5	0.8
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	28.0	25.0	26.0	26.0	29.0	
A.A.P.	14.3	15.6	15.5	15.3	13.4	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.9	0.8	0.9	0.9	1.0	

It is clear from Tables 4.10, 4.11 and 4.12 that combination of Rules AC still has the best performance on N.O.S. among all the 2 rule combinations. When the shift size is 2σ as shown in Table 4.12, AC and ABCD have the best N.O.S. performance compared to all the tested individual rules and rule combinations. When the shift sizes are 1.3σ and 1.7σ as shown in Tables 4.10 and Table 4.11 respectively, Rules ABCD performs best. After increasing the length of the tested process, the performance of Rule D on N.O.S. for the 3 SD shift sizes does not improve significantly. It is less effective than other individual rules and combined rules. In contrast, Rule C keeps a higher level of N.O.S. performance among other individual rules. In terms of A.A.P., when shift size is 1.3σ as shown in Table 4.10, Rule ABCD has the best A.A.P. performance at 13.4. When shift size is 1.7σ, Rules AC and Rules ABCD have better A.A.P performance at 14.7 and 14.4, respectively. When shift size is 2σ, Rules AB has the best A.A.P. performance at 12.9. Compared with experiment results from the tested shorter processes, we can see that Rule AC no longer has the best performance in terms of A.A.P., but it is steady at the level of 14.0 to 15.0 for the 3 SD shift sizes.

As can be seen in Tables 4.10, 4.11 and 4.12, for individual rules, except for Rule D showing less effectiveness on N.O.S. for the 3 SD shift sizes, the N.O.S. of Rule A, B and C tend to increase with the SD shift size. For the 2 rule combinations, the N.O.S. of Rules BD shows a trend of decrease when the SD shift size is larger. N.O.S. of other 2 rule combinations increase with the SD shift size. For 3 and 4 rule combinations, the N.O.S. performances are relatively steady for the 3 SD shift sizes. In terms of A.A.P., there is no apparent increasing or decreasing trend for individual rules or for combinations of 3 and 4 rules when the shift size of SD becomes larger. For most of the 2 rule combinations, A.A.P. decreases when the SD shift size increases.

Table 4.13 Average values of N.O.S. and A.A.P. when n=40

	A	**B**	**C**	**D**		
Average N.O.S.	9.7	13.7	21.7	2.7		
Average A.A.P.	16.9	18.3	17.7	20.3		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
Average N.O.S.	22.3	27.3	15.7	25.7	16.7	23.0
Average A.A.P.	16.5	14.7	18.5	15.7	17.3	17.7
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
Average N.O.S.	28.0	24.3	26.0	26.0	29.0	
Average A.A.P.	15.5	16.5	17.0	16.0	13.7	

Observing the averaged values of N.O.S. and A.A.P shown in Table 4.13, for the tested 3 shift sizes of SD, the combination of Rules ABCD is the most effective ones in terms of N.O.S. Rules AC, similar to the results from our original experiments, has a competitive performance on N.O.S. as well. Rule C still has the best performance among individual rules. Rules ABCD has the best A.A.P. performance at 13.7, followed by Rules AC, Rules ABC and Rules BC at 14.7, 15.5 and 15.7, respectively. To summarize, when increase the length of the tested process, Rules AC is the combination with simpler application and competitive performance on both N.O.S. and A.A.P.

Comparing the results with that of the original experiment, for most of the rule combinations, the average N.O.S. of different shift sizes increases with the increase of the process length. For example, as can be seen from Table 4.9, when the number of samples is 20, the average N.O.S. of Rules AB is 13.7. From Tables 4.5 and 4.13, when the number of samples are 30 and 40, the average N.O.S. of Rules AB are 18 and 22.3, respectively. However, individual rules A and B do not show such trend. When the length of tested process increases from 20 to 30 and 40, the average N.O.S. of rule A are 7.7, 13.0 and 9.7, respectively and the average N.O.S. of rule B are 10.3, 8.0 and 13.7, respectively.

For this set of experiments, we cannot compare the average A.A.P. for different rules or rule combinations since the run lengths after the shift are different so that such comparison will not be meaningful.

We can summarize that different lengths of tested process affect the detecting performance for most of the tested rules and rule combinations. The combination of Rules AC in most cases always has the highest N.O.S. among all the individual rules and 2 rule combinations. In addition, the combination of Rules AC has a comparable performance with 3 rule combinations and even 4 rule combination while 2 rule combination will be much easier to implement in practice.

4.3.3 Experiments for Different Shift Starting Points

We conducted other two groups of experiments to observe if changing the starting point of the shift will lead to different performances of the rules or rule combinations. Similar to those discussed earlier, we used 3 different shift sizes by letting $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ in our simulation experiments. Normal distribution function in MS-Excel is used to generate 30 random variable values following N(1,1) in each simulation run. A shift of the SD is implanted at the 10th sample. Each test has 30 simulation runs or replications. The first effective signal is counted if it is plotted outside the control limit, warning limit or auxiliary limit after the shift is implanted at the10th plot.

For the first group, we implant SD shift at the 10th point, same as we did in the original set of experiments. In this case, the number of samples is kept at 30. For the second group, we implanted the shift at the 20th point, the number of samples is also kept at 30. Considering that the shifted process of the second group lasts only for 10 points (from the 20th point to 30th point), for the first group, we only observed 10 points after the shift (from the 10th point to the 20th point), not all the 20 points after the shift.

Results of the first group of experiments for $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ are presented in Tables 4.14, 4.15 and 4.16, respectively. Each of the table presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the individual rules or combinations of different supplemental rules. Table 4.17 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.14, 4.15 and 4.16.

As can be seen from the simulation results, combination of Rules AC still has the best performance on N.O.S. among all the 2 rule combinations. When the shift size is 1.7σ and 2σ as shown in Tables 4.15 and 4.16, respectively, Rules AC has the best N.O.S. performance among all the tested individual rules and rule combinations. When shift size is 1.3σ as shown in Table 4.14, Rules ABCD performs best. For A.A.P., when shift size is 1.3σ (Table 4.14), Rule D and Rules AD have the best A.A.P. performance. We notice that the N.O.S. of Rule D and Rules AD is 1.0, it means that Rule D and Rules AD are not effective even they have best A.A.P. performance. Except for

Rule D and Rules AD, Rule C and Rule ABC perform well on A.A.P. When shift size is 1.7σ, Rule C and Rules CD have the best A.A.P. performance. When shift size is 2σ, A.A.P. performance of all individual rules and rule combinations are similar.

Table 4.14 Supplemental rules performances: SD shifts from the 10th point, shift size 1.3σ

	A	**B**	**C**	**D**		
N.O.S.	4.0	5.0	12.0	1.0		
A.A.P.	16.3	14.4	11.7	10.0		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.1	0.2	0.4	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	8.0	16.0	1.0	13.0	3.0	9.0
A.A.P.	13.8	12.4	10.0	12.2	11.7	12.1
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.3	0.5	0.0	0.4	0.1	0.3
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	11.0	7.0	15.0	16.0	18.0	
A.A.P.	11.2	13.7	12.2	11.6	12.4	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.4	0.2	0.5	0.5	0.6	

Table 4.15 Supplemental rules performances: SD shifts from the 10th point, shift size 1.7σ

	A	**B**	**C**	**D**		
N.O.S.	6.0	11.0	11.0	0.0		
A.A.P.	12.8	13.6	10.9	-		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.2	0.4	0.4	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	13.0	20.0	8.0	19.0	10.0	10.0
A.A.P.	13.2	11.9	12.5	12.0	11.6	11.0
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.4	0.7	0.3	0.6	0.3	0.3
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	20.0	18.0	21.0	21.0	20.0	
A.A.P.	12.7	12.8	12.5	12.0	12.1	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.7	0.6	0.7	0.7	0.7	

Table 4.16 Supplemental rules performances: SD shifts from the 10th point, shift size 2σ

	A	B	C	D		
N.O.S.	12.0	12.0	19.0	0.0		
A.A.P.	13.2	12.0	11.1	-		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.4	0.4	0.6	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	18.0	26.0	11.0	20.0	12.0	24.0
A.A.P.	12.8	11.3	12.1	12.8	11.4	11.3
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.6	0.9	0.4	0.7	0.4	0.8
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	24.0	16.0	19.0	21.0	22.0	
A.A.P.	12.3	11.2	11.6	11.5	11.5	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.8	0.5	0.6	0.7	0.7	

As can be seen in Tables 4.6, 4.7 and 4.8, the N.O.S. performances in this set of experiments are similar to those in the original experiments. For almost all the individual rules and rule combinations, the N.O.S. becomes better when the SD shift size increases. It also shows that Rule D is less effective for all the 3 SD shift sizes. As mentioned earlier, using Rule D requires 8 points to identify the signal. In our first group of experiments, although the total number of samples is still 30, we only observe 10 points after shift. In terms of A.A.P., there is no apparent increasing or decreasing trend when the shift size of SD becomes larger. For the 3 SD shift sizes, the A.A.P. performances of all the individual rules and combined rules are similar.

Table 4.17 Average values of N.O.S. and A.A.P. when SD shifts from the 10th point

	A	B	C	D		
Average N.O.S.	7.3	9.3	14.0	0.3		
Average A.A.P.	14.1	13.3	11.2	-		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
Average N.O.S.	13.0	20.7	6.7	17.3	8.3	14.3
Average A.A.P.	13.2	11.9	11.5	12.3	11.6	11.5
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
Average N.O.S.	18.3	13.7	18.3	19.3	20.0	
Average A.A.P.	12.1	12.6	12.1	11.7	12.0	

Observing the averaged values of N.O.S. and A.A.P shown in Table 4.17, for the tested 3 shift sizes of SD, Rules AC and Rules ABCD are the most effective ones in terms of N.O.S. Rules AC performs slightly better than Rules ABCD (20.7 compare to 20.0). Rule C still has the best performance among all individual rules. Rules C also has the best A.A.P. performance at 11.2.

The effectiveness of Rules AC is clearly showed from the experiment results again.

For the second group of experiments, we implanted the shift at the 20th point, the number of samples is kept at 30.

Results of the second group of experiments for $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ are presented in Tables 4.18, 4.19 and 4.20, respectively. Each of the tables presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the individual or combinations of different supplemental rules. Table 4.21 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.18, 4.19 and 4.20.

Table 4.18 Supplemental rules performances: SD shifts from the 20th point, shift size 1.3σ

	A	**B**	**C**	**D**		
N.O.S.	5.0	10.0	8.0	1.0		
A.A.P.	25.0	23.5	22.4	23.0		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.2	0.3	0.3	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	5.0	15.0	3.0	13.0	7.0	10.0
A.A.P.	25.0	22.1	25.3	22.1	23.6	21.9
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.2	0.5	0.1	0.4	0.2	0.3
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	15.0	10.0	13.0	11.0	19.0	
A.A.P.	22.1	22.7	22.6	21.3	22.5	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.5	0.3	0.4	0.4	0.6	

Table 4.19 Supplemental rules performances: SD shifts from the 20th point, shift size 1.7σ

	A	B	C	D		
N.O.S.	8.0	15.0	20.0	0.0		
A.A.P.	23.3	22.2	21.8	-		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.3	0.5	0.7	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	19.0	24.0	9.0	16.0	12.0	19.0
A.A.P.	23.7	21.3	22.9	21.7	22.6	21.9
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.6	0.8	0.3	0.5	0.4	0.6
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	25.0	17.0	18.0	23.0	23.0	
A.A.P.	21.2	23.0	21.5	21.6	21.9	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.8	0.6	0.6	0.8	0.8	

Table 4.20 Supplemental rules performances: SD shifts from the 20th point, shift size 2σ

	A	B	C	D		
N.O.S.	17.0	13.0	19.0	1.0		
A.A.P.	22.8	21.4	21.6	21.0		
N.O.R.	30.0	30.0	30.0	30.0		
S.R.	0.6	0.4	0.6	0.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
N.O.S.	21.0	26.0	16.0	24.0	11.0	17.0
A.A.P.	22.4	21.7	23.3	21.1	22.2	20.9
N.O.R.	30.0	30.0	30.0	30.0	30.0	30.0
S.R.	0.7	0.9	0.5	0.8	0.4	0.6
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
N.O.S.	27.0	21.0	25.0	23.0	27.0	
A.A.P.	21.6	23.1	22.2	21.3	22.0	
N.O.R.	30.0	30.0	30.0	30.0	30.0	
S.R.	0.9	0.7	0.8	0.8	0.9	

Combination of Rules AC has the best performance on N.O.S. among all the 2 rule combinations. When the shift size is 1.7σ as shown in Table 4.19, Rules AC has the best N.O.S. performance among all the tested individual rules and combinations. When the shift sizes are 1.3σ and 2σ as shown in Tables 4.18 and 4.20, respectively, Rules ABCD performs best. For A.A.P., when shift size is 1.3σ (Table 4.18), Rule AC, Rules BC, Rules ABC and Rules BCD have the best A.A.P.

performance. When shift sizes are 1.7σ and 2σ, the A.A.P. performances of different individual rules and combined rules are similar, around 21.0 to 23.0.

As can be seen in Tables 4.6, 4.7 and 4.8, the N.O.S. performances are similar to those in the original experiments. For almost all the individual rules and rule combinations, the N.O.S. becomes better when the SD shift size increases. It also shows that Rule D is less effective for all the 3 SD shift sizes. In terms of A.A.P., there is no apparent increasing or decreasing trend when the shift size of SD becomes larger. The A.A.P. performances of all the individual rules and 2 rule combinations tend to decrease with the shift size of SD. While the A.A.P. for 3 and 4 rule combinations are relatively steady and do not change dramatically with the shift sizes of SD.

Table 4.21 Average values of N.O.S. and A.A.P. when SD shifts from the 20th point

	A	**B**	**C**	**D**		
Average N.O.S.	10.0	12.7	15.7	0.7		
Average A.A.P.	23.7	22.4	21.9	22.0		
	AB	**AC**	**AD**	**BC**	**BD**	**CD**
Average N.O.S.	15.0	21.7	9.3	17.7	10.0	15.3
Average A.A.P.	23.7	21.7	23.8	21.6	22.8	21.6
	ABC	**ABD**	**ACD**	**BCD**	**ABCD**	
Average N.O.S.	22.3	16.0	18.7	19.0	23.0	
Average A.A.P.	21.6	22.9	22.1	21.4	22.1	

Observing the averaged values of N.O.S. and A.A.P shown in Table 4.21, for the tested 3 shift sizes of SD, Rules ABCD is the most effective one in terms of N.O.S at 23.0. Rules AC has a slightly lower N.O.S. at 21.7. Rule C still has the best performance among individual rules. The average A.A.P performances are similar between different individual rules and combined rules. Among them, Rule C, Rules AC, Rules BC, Rules CD, Rules ABC, Rules ACD, Rules BCD and Rules ABCD have slightly better A.A.P. performances.

Comparing the results of the two groups of experiments, we can see from Tables 4.17 and 4.21 that the average N.O.S. performances of different individual rules and combined rules improve when the SD shift starts from a later point. For example, for Rules AB, the total number of successes is 13.0 when the shift starts from the 10th point as shown in Table 4.17 and 15.0 when the shift starts from the 20th point as shown from Table 4.21. A possible reason for this phenomenon is, when SD shift starts from a later point, more data are collected before the shift happens and the control chart is steadier when the shift actually occurs. This may improve the effectiveness of detection.

In terms of average A.A.P. performance, the average alarm point reflects the out-of-control average run length (ARL_1) to some extent. Since we observe the same length of tested process here, it is reasonable to compare the average alarm point of different rules under different shift sizes. For example, from Table 4.17, rule A has an average alarm point of 14.1 when the shift starts from the 10th point. This means that the corresponding ARL_1 is 4.1. From Table 4.21, rule A has an average alarm point of 23.7 when the shift starts from the 20th point. This means that the ARL_1 is 3.7. It appears that the ARL_1 of different rules are similar when the shift starts at different points.

As well, combination of Rules AC still has the best performance among all the individual rules and 2 rule combinations. Meanwhile, AC also has a comparable performance with 3 and 4 rule combinations.

The experiments in Sections 4.3.2 and 4.3.3 illustrate that the combination of rules AC has the best performance on detecting a shift on SD for all 3 shift sizes, for different lengths of the tested processes and for different starting points of the shift. In the following simulation experiments in

Section 4.4, we will test different short run control charts using some of the results on Q chart shown in this section.

4.4 Simulation Experiments for Short-Run Control Charts

In this section, we conducted simulation experiments for Q and other short-run control charts discussed in this thesis. The objective is to identify more effective control charts for detecting SD shift in short-run production.

The tested control charts using simulation are:

- t Chart
- CUSUM t Chart
- EWMA t Chart
- Q Chart
- CUSUM Q Chart
- EWMA Q Chart
- Q Chart with supplemental rules A&C
- Individual X Chart

As discussed in Chapter 3 the parameter values used in the simulation for some of the tested charts are listed below:

Table 4.22 The parameter values identified for different charts

t Chart		CUSUM t Chart		EWMA t Chart		CUSUM Q Chart		EWMA Q Chart		X Chart	
α	0.0027	h	3	λ	0.09	h_S	3.34	λ	0.25	d_2	1.128
		k	0.75	L	3	k_S	0.75	K	2.9		

Similar to that discussed in Section 4.3, we investigate the detecting abilities of the tested control charts to see if they vary with the increase or decrease of the lengths of the tested process; or change with the starting point of the SD shift.

4.4.1 Original Experiments

To observe the effectiveness of different short-run control charts on the sizes of SD shifts, we used 3 different shift sizes by letting $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ in our simulation experiments. 30 random variable values following N(1, 1) are generated for each replication by MS-Excel. Each test has 30 replications. The first effective signal is counted if it is plotted outside the control limit after the shift is implanted at the10th plot.

Results of these simulation experiments for $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ are presented in Tables 4.23, 4.24 and 4.25, respectively. Each of the tables presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the control charts tested in our thesis. Table 4.5 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.23, 4.24 and 4.25.

Table 4.23 Short-run control charts performances: n=30, shift size 1.3σ

	t	**EWMA t**	**CUSUM t**	**Q**
N.O.S.	2.0	-	2.0	6.0
A.A.P.	19.0	-	26.0	17.7
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.1	0.0	0.1	0.2
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	2.0	6.0	22.0	10.0
A.A.P.	17.0	14.2	16.6	16.6
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.1	0.2	0.7	0.3

Table 4.24 Short-run control charts performances: n=30, shift size 1.7σ

	t	**EWMA t**	**CUSUM t**	**Q**
N.O.S.	14.0	-	4.0	11.0
A.A.P.	18.3	-	13.8	16.9
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.5	0.0	0.1	0.4
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	8.0	14.0	22.0	17.0
A.A.P.	19.6	16.4	13.3	17.0
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.3	0.5	0.7	0.6

Table 4.25 Short-run control charts performances: n=30, shift size 2σ

	t	**EWMA t**	**CUSUM t**	**Q**
N.O.S.	15.0	1.0	6.0	10.0
A.A.P.	16.7	11.0	16.8	20.6
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.5	0.0	0.2	0.3
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	10.0	11.0	27.0	17.0
A.A.P.	17.1	17.8	14.2	13.4
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.3	0.4	0.9	0.6

It is apparent that Q chart with Rules AC always has the best N.O.S. performance for 3 shift sizes of SD as shown in Tables 4.23, 4.24 and 4.25. Individual X chart also has better performance than the rest of the control charts and the difference between Individual X chart and Q chart with Rules AC is significant in most of the times. When the SD shift sizes are 1.3σ and 2σ, the differences of N.O.S between Individual X chart and Q chart with Rules AC are 12.0 and 10.0, respectively. When the SD shift size is 1.7σ, the difference is smaller at 5.0. In terms of A.A.P., when the SD shift size is 1.3σ as shown in Table 4.23, CUSUM Q chart has the best A.A.P performance at 14.2. When the SD shift size is 1.7σ, Q chart with Rules AC and CUSUM t have the best A.A.P. performances at 13.3 and 13.8, respectively. When the SD shift size is 2σ, EWMA t chart and individual X chart have the best A.A.P. performance at 11.0 and 13.4, respectively. When the SD is at different shift sizes, there is no obvious trend in terms of A.A.P. performance for different control charts.

Table 4.26 Average values of N.O.S. and A.A.P. when n=30

	t	**EWMA t**	**CUSUM t**	**Q**
Average N.O.S.	10.3	1.0	4.0	9.0
Average A.A.P.	18.0	11.0	18.9	18.4
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
Average N.O.S.	6.7	10.3	23.7	14.7
Average A.A.P.	17.9	16.1	14.7	15.7

From Tables 4.23 to 4.25, for most of the charts (t Chart, CUSUM t Chart, Q Chart, EWMA Q Chart, CUSUM Q Chart, Q Chart with supplemental rules A&C, individual X Chart), the number of successes increases with the shift size of SD. It means that larger shift size corresponds to better detecting ability. However, EWMA t Chart has a weak detecting ability for the 3 SD shift sizes. When it comes to A.A.P. performance, there is no apparent increasing or decreasing trend for each control chart when the shift size of SD becomes larger.

From Table 4.26, Q chart with Rules AC and individual X chart have the best average N.O.S. performance. t chart, Q chart and CUSUM Q chart have similar performances at 10.3, 9.0 and 10.3, respectively. It can be noticed that the average N.O.S. for EWMA t Chart is 1.0. It is much smaller than those for other charts. Besides, EWMA Q chart did not perform well with the average N.O.S. being 6.7. While, on the other hand, both CUSUM t chart and CUSUM Q chart have better performances than EWMA t and EWMA Q charts, respectively. This may show to some extent that when detecting the SD shift in short-run production, CUSUM scheme charts are more effective than corresponding EWMA scheme charts. From the perspective of average alarm point, although EWMA t chart detects the shift fastest from as shown in Table 4.26, the corresponding average

N.O.S. is almost zero. It seems that EWMA t chart is less effective in SD shift detection. The results show that Q chart with Rules AC and individual X chart have the best performance on average A.A.P. with the average A.A.P. being 14.7 and 15.7, respectively.

Overall, Q chart with Rules AC and Individual X chart have better performance on both number of successes and average alarm point.

4.4.2 Experiments for Different Lengths of Tested Processes

Similar to that discussed in Section 4.3.2, we conducted simulation experiments with different process lengths to see if they will affect the detecting ability of some of the short run control charts. When the tested process has more plotted data, the control charts can have more time to detect the shift. However, when applying Q chart to capture the shift of the mean, for short-run production, if it cannot detect the shift immediately after the shift takes place, it has higher probability to miss the shift in the rest of the shifted process. As mentioned in literature review, some authors studied this issue and analysed the reason theoretically, while few papers illustrates if such phenomenon exists on other control charts as well.

In this section, to observe the effectiveness of different short-run control charts on the sizes of SD shifts, we also used 3 different shift sizes by letting $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ in our simulation experiments. We used 30 random variable values following N(1,1) generated by Excel. Each test has 30 replications. The first effective signal is counted if it is plotted outside the control limit after the shift is implanted at the10th plot.

For the first group of experiments, we reduced the length of the tested process from 30 points to 20 points without changing the shift starting point at the 10th plot.

Results of these experiments for $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ are presented in Tables 4.27, 4.28 and 4.29, respectively. Each of the tables presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the individual or combinations of different supplemental rules. Table 4.30 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.27, 4.28 and 4.29.

Table 4.27 Short-run control charts performances: n=20, shift size 1.3σ

	t	**EWMA t**	**CUSUM t**	**Q**
N.O.S.	2.0	-	4.0	5.0
A.A.P.	19.0	-	15.5	12.4
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.1	-	0.1	0.2
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	4.0	4.0	13.0	10.0
A.A.P.	14.5	18.3	11.9	13.5
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.1	0.1	0.4	0.3

Table 4.28 Short-run control charts performances: n=20, shift size 1.7σ

	t	**EWMA t**	**CUSUM t**	**Q**
N.O.S.	9.0	1.0	4.0	9.0
A.A.P.	13.4	16.0	14.0	13.2
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.3	-	0.1	0.3
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	5.0	10.0	13.0	10.0
A.A.P.	13.2	13.6	11.4	12.9
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.2	0.3	0.4	0.3

Table 4.29 Short-run control charts performances: n=20, shift size 2σ

	t	**EWMA t**	**CUSUM t**	**Q**
N.O.S.	11.0	-	4.0	9.0
A.A.P.	12.5	-	14.8	14.1
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.4	0.0	0.1	0.3
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	8.0	8.0	23.0	18.0
A.A.P.	12.4	12.4	12.1	13.3
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.3	0.3	0.8	0.6

Q chart with Rules AC has the best N.O.S. performance for the 3 SD shift sizes shown in Tables 4.27, 4.28 and 4.29, similar to those shown in the original experiments. Individual X chart also has a better N.O.S. performance than other charts. Compare with the original experiments, when the length of the tested process decreases, the difference of N.O.S. performance between individual X chart and Q chart with Rules AC reduces as well. When the SD shift sizes are 1.3σ and 1.7σ shown in Table 4.27 and 4.28, respectively, the differences of N.O.S. between Individual X chart and Q chart with Rules AC are both 3.0. When the SD shift size is 2σ, this difference is 5.0 as shown in Table 4.29. EWMA t chart still shows lack of effectiveness for all the 3 SD shift sizes. For A.A.P. performance, Q chart with Rules AC shows more advances in the tested shorter process of 20 plots. When the SD shift sizes are 1.3σ and 1.7σ, Q chart with Rules AC has the best A.A.P. performance for both conditions. When the SD shift size is 2σ, t chart, EWMA Q chart, CUSUM Q chart and Q chart with Rules AC have the best A.A.P. performance at around 12. Individual X chart, as well, performs well at 13.3 when SD shift is 2σ.

As can be seen form Tables 4.27, 4.28 and 4.29, the N.O.S. performances of almost all the control charts tested here are improved when the SD shift size becomes larger. The N.O.S. of CUSUM t chart keeps steady when the SD shift size increases. In terms of A.A.P., t chart, EWMA Q chart and CUSUM Q chart show a decreasing trend when the SD shift size increases. The A.A.P. of Q chart tends to increase when the SD shift size increases. The A.A.P. of CUSUM t chart, Q chart with Rules AC and individual X chart almost keep constant for the 3 SD shift sizes.

Table 4.30 Average values of N.O.S. and A.A.P. when n=20

	t	**EWMA t**	**CUSUM t**	**Q**
Average N.O.S.	7.3	1.0	4.0	7.7
Average A.A.P.	15.0	16.0	14.8	13.2
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
Average N.O.S.	5.7	7.3	16.3	12.7
Average A.A.P.	13.4	14.7	11.8	13.2

It is clear from Table 4.30 that Q chart with Rules AC and individual X chart have the best performances on both average N.O.S. and average A.A.P. CUSUM scheme charts have better performances than the corresponding EWMA scheme charts.

For the second group of experiments, we increased the length of the tested process from 30 points to 40 points without changing the shift starting point at the 10th plot.

Results of these experiments for $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ are presented in Tables 4.31, 4.32 and 4.33, respectively. Each of the tables presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the individual or combinations of different supplemental rules. Table 4.34 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.31, 4.32 and 4.33.

Table 4.31 Short-run control charts performances: n=40, shift size 1.3σ

	t	EWMA t	CUSUM t	Q
N.O.S.	6.0	2.0	6.0	7.0
A.A.P.	25.8	16.0	17.2	22.6
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.2	0.1	0.2	0.2
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	3.0	5.0	17.0	16.0
A.A.P.	22.7	24.0	16.3	20.1
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.1	0.2	0.6	0.5

Table 4.32 Short-run control charts performances: n=40, shift size 1.7σ

	t	EWMA t	CUSUM t	Q
N.O.S.	13.0	1.0	6.0	8.0
A.A.P.	18.3	10.0	13.2	18.0
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.4	0.0	0.2	0.3
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	12.0	11.0	27.0	15.0
A.A.P.	18.8	15.6	16.3	16.1
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.4	0.4	0.9	0.5

Table 4.33 Short-run control charts performances: n=40, shift size 2σ

	t	EWMA t	CUSUM t	Q
N.O.S.	17.0	1.0	6.0	13.0
A.A.P.	19.4	11.0	15.8	14.2
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.6	0.0	0.2	0.4
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	10.0	18.0	25.0	21.0
A.A.P.	18.8	18.5	12.4	14.8
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.3	0.6	0.8	0.7

As shown in the results of this set of simulation experiments, Q chart with Rules AC and individual X chart still have the best performances on N.O.S. for the 3 SD shift sizes. When SD shift is 1.7σ, the difference between Q chart with Rules AC and Individual X chart is 12. For other 2 shift sizes, the differences between the N.O.S. for Q chart with Rules AC and Individual X chart are smaller. The N.O.S. performance of EWMA t chart improves slightly with longer process, but it is still lack of effectiveness compared with other charts. For A.A.P. performance, except for EWMA t chart, when the SD shift sizes are 1.3σ and 2σ, Q chart with Rules AC performs best in both cases. When the SD shift size is 1.7σ, CUSUM t chart has the best A.A.P. performance.

As can be seen from Tables 4.31, 4.32 and 4.33, the N.O.S. of most of the charts increase with the SD shift size, except for CUSUM t chart. The N.O.S. of CUSUM t chart keeps unchanged when the SD shift size increases. We had the same observation from the results of the first group of experiments. In terms of A.A.P. performance, all the tested control charts have trends to decrease with the SD shift size increasing. This means for longer processes, the control chart can detect the shift earlier if the size of the SD shift is larger.

Table 4.34 Average values of N.O.S. and A.A.P. when n=40

	t	**EWMA t**	**CUSUM t**	**Q**
Average N.O.S.	12.0	1.3	6.0	9.3
Average A.A.P.	21.2	13.0	15.4	18.3
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
Average N.O.S.	8.3	11.3	23.0	17.3
Average A.A.P.	20.1	19.4	15.0	17.0

As shown in Table 4.34, both Q chart with Rules AC and X chart have better average N.O.S. performance than the rest of the control charts. EWMA t chart is lack of effectiveness compared with other charts. CUSUM scheme charts have better performances than their EWMA counterparts. Expect for EWMA t chart, Q chart with Rules AC has the best average A.A.P. performance as well. The A.A.P. performance of Individual X chart is also comparable to that of Q chart with Rules AC.

Comparing the results in Tables 4.30 and 4.34 with those in Table 4.26 (the result of the original experiments), we can see that most of the control charts have a better performance on average N.O.S. when the length of the tested process is longer. Among them, the performance of Q chart with Rules AC has more visible improvement with the average N.O.S. increased from 16.3 to 23.0. Consequently, compared with other control charts, we can see that the performance of Q chart with Rules AC is more easily to be affected by the length of the tested process. Yet, we also notice that from Table 4.26, when the length of the tested process is 30, the average N.O.S. for Q chart with Rules AC has already increased to 23.7. It is almost the same with the result when the process length is 40 points which is 23.0 as shown in Table 4.34. For A.A.P. performance, as mentioned before in Section 4.3.2, we cannot compare the average A.A.P. meaningfully since the lengths of the tested processes are different.

Overall, Q chart with rules AC and Individual X chart are still the most effective control charts among all the tested charts under the given conditions discussed in this section.

4.4.3 Experiments for Different Shift Starting Points

We conducted other two groups of experiments to observe if changing the starting point of the shift would lead to different results. Similar to the experiments discussed in Section 4.3.3, we used 3 different shift sizes by letting $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ in our simulation experiments. We used normal distribution function in MS-Excel to generate 30 random variable values following N(1, 1). A shift of the SD is implanted at the 10th sample. Each test has 30 replications. The first effective signal is counted if it is plotted outside the control limit after the shift is implanted.

For the first group, we implanted SD shift at the 10th point, just as we did in the original setting. In this case, the number of samples is kept at 30. For the second group, we implant the shift at the 20th point, the number of samples is also kept at 30. The processes of the second group after the shift have only 10 points To make these two groups of experiments comparable, we only observed the 10 points following the shift (from the 10th point to the 20th point) in the first group of experiments.

Results of the first group of experiments for $\sigma_1{}' = 1.3\sigma_0$, $\sigma_2{}' = 1.7\sigma_0$ and $\sigma_3{}' = 2\sigma_0$ are presented in Tables 4.35, 4.36 and 4.37, respectively. Each of the tables presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the different control charts tested in this thesis. Table 4.38 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.35, 4.36 and 4.37.

The N.O.S. performances of Q chart with Rules AC and individual X chart are the best for the 3 SD shift sizes among all the charts as shown in Tables 4.35, 4.36 and 4.37. When the SD shift sizes are 1.3σ and 1.7σ as shown in Tables 4.35 and 4.36, respectively, the N.O.S. performances of individual X chart are as good as Q chart with Rules AC. EWMA t chart is still less effective for most of the experiments. For A.A.P. performance, t chart has the best performance when the SD shift sizes are 1.7σ and 2σ as shown in Tables 4.36 and 4.37, respectively. When SD shifts to 1.3σ, Q chart with Rules AC has the best A.A.P. performance. Q chart with Rules AC always has best A.A.P. performance for the 3 SD shift sizes. On the other hand, individual X chart tends to perform better when the SD shift size is larger. For example, when the shift size of SD is 2σ as shown in Table 4.37, individual X chart has the A.A.P. of 12.8. It is very close to that of Q chart with Rules AC which is12.3. But when the shift sizes are 1.3σ and 1.7σ as shown in Tables 4.35 and 4.36 respectively, the performances of individual X chart are below average.

Table 4.35 Short-run control charts performances: SD shifts from the 10th point, shift size 1.3σ

	t	EWMA t	CUSUM t	Q
N.O.S.	4.0	-	-	2.0
A.A.P.	17.0	-	-	14.0
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.1	0.0	0.0	0.1
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	3.0	4.0	11.0	11.0
A.A.P.	15.0	14.3	12.6	15.2
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.1	0.1	0.4	0.4

Table 4.36 Short-run control charts performances: SD shifts from the 10th point, shift size 1.7σ

	t	EWMA t	CUSUM t	Q
N.O.S.	9.0	-	4.0	10.0
A.A.P.	12.3	-	13.3	14.7
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.3	0.0	0.1	0.3
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	4.0	2.0	14.0	14.0
A.A.P.	13.0	13.5	13.4	14.4
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.1	0.1	0.5	0.5

Table 4.37 Short-run control charts performances: SD shifts from the 10th point, shift size 2σ

	t	EWMA t	CUSUM t	Q
N.O.S.	11.0	1.0	4.0	9.0
A.A.P.	11.7	12.0	13.3	12.7
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.4	0.0	0.1	0.3
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	12.0	11.0	21.0	18.0
A.A.P.	13.3	14.0	12.3	12.8
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.4	0.4	0.7	0.6

As can be seen from Tables 4.35, 4.36 and 4.37, all the control charts tested here have a trend of having better N.O.S. performances when the SD shift size increases. In terms of A.A.P., t chart, Q chart, EWMA Q chart and Individual X chart have the decreasing A.A.P. when the shift size of SD increases. For CUSUM t chart, CUSUM Q chart and Q chart with Rules AC, their A.A.P. performances tend to be at a steady level when the shift size of SD changes.

Table 4.38 Average values of N.O.S. and A.A.P. when SD shifts from the 10th point

	t	**EWMA t**	**CUSUM t**	**Q**
Average N.O.S.	8.0	1.0	4.0	7.0
Average A.A.P.	13.7	12.0	13.3	13.8
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
Average N.O.S.	6.3	5.7	15.3	14.3
Average A.A.P.	13.8	13.9	12.8	14.1

From Table 4.38, it is observed that Q chart with Rules AC has the best average N.O.S. performance, followed by individual X chart. CUSUM t chart has better performance than EWMA t chart, while CUSUM Q chart does not show such advantage over EWMA Q chart. They have similar performance on average N.O.S. When it comes to average A.A.P., EWMA t chart performs best followed by Q chart with Rules AC. Individual X chart performs worst in this case. But we can notice that there are no significant differences between the average values of A.A.P. for different control charts.

For the second group of experiments, we implanted the SD shift at the 20th point. The number of total plots is kept at 30.

Results of the second group of experiments for $\sigma_1' = 1.3\sigma_0$, $\sigma_2' = 1.7\sigma_0$ and $\sigma_3' = 2\sigma_0$ are presented in Tables 4.39, 4.40 and 4.41, respectively. Each of the tables presents the number of successes (N.O.S.), average alarm point (A.A.P.), number of replications (N.O.R.) and success rate (S.R.) corresponding to the different control charts tested in this section. Table 4.42 presents the averaged values of N.O.S. and A.A.P. based on the corresponding ones listed in Tables 4.39, 4.40 and 4.41.

Table 4.39 Short-run control charts performances: SD shifts from the 20th point, shift size 1.3σ

	t	EWMA t	CUSUM t	Q
N.O.S.	6.0	1.0	-	5.0
A.A.P.	24.0	22.0	-	26.8
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.2	0.0	0.0	0.2
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	7.0	7.0	9.0	10.0
A.A.P.	23.7	24.4	22.8	23.8
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.2	0.2	0.3	0.3

Table 4.40 Short-run control charts performances: SD shifts from the 20th point, shift size 1.7σ

	t	EWMA t	CUSUM t	Q
N.O.S.	9.0	-	5.0	11.0
A.A.P.	24.0	-	23.4	23.9
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.3	0.0	0.2	0.4
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	6.0	9.0	22.0	11.0
A.A.P.	24.8	26.1	22.4	23.3
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.2	0.3	0.7	0.4

Table 4.41 Short-run control charts performances: SD shifts from the 20th point, shift size 2σ

	t	EWMA t	CUSUM t	Q
N.O.S.	18.0	1.0	4.0	21.0
A.A.P.	23.1	20.0	23.3	22.6
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.6	0.0	0.1	0.7
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
N.O.S.	8.0	15.0	22.0	16.0
A.A.P.	24.5	24.7	22.2	22.8
N.O.R.	30.0	30.0	30.0	30.0
S.R.	0.3	0.5	0.7	0.5

The results in Tables 4.39, 4.40 and 4.41 show that Q chart with Rules AC and individual X chart still have the best performance on N.O.S. for the 3 SD shift sizes. When the SD shift size is 1.3σ, individual X chart performs slightly better than Q chart with Rules AC. When the shift size of SD is larger at 1.7σ and 2σ, the N.O.S. of Q chart with Rules AC performs much better than individual X chart again and the difference between Q chart with Rules AC and Individual X chart become larger as well. EWMA t chart is less effective for the 3 SD shift sizes. In terms of A.A.P., EWMA t chart performs best followed by Q chart with Rules AC. Comparing with those of the first group of experiments, the A.A.P. performance of individual X chart improves significantly. It closely follow that of Q chart with Rules AC.

Comparing the results in Tables 4.39, 4.40 and 4.41, we can see that the N.O.S. performances of most charts improve with the increases of SD shift size. For A.A.P., the performances of t chart, CUSUM t chart, EWMA Q chart, Q chart with Rules AC and individual X chart tend to keep at a steady level. The A.A.P. of Q chart decreases when the SD shift size increases.

Table 4.42 Average values of N.O.S. and A.A.P. when SD shifts from the 20th point

	t	EWMA t	CUSUM t	Q
Average N.O.S.	11.0	1.0	4.5	12.3
Average A.A.P.	23.7	21.0	23.3	24.4
	EWMA Q	**CUSUM Q**	**Q & Rules AC**	**X**
Average N.O.S.	7.0	10.3	17.7	12.3
Average A.A.P.	24.3	25.1	22.5	23.3

As can be seen from Table 4.42, Q chart with Rules AC has the best average value of N.O.S. at 17.7 followed by individual X chart and Q chart at 12.3. CUSUM scheme charts perform better than the corresponding EWMA scheme charts on N.O.S. For average A.A.P., Q chart with Rules AC and individual X chart still have the best performances.

Comparing the average N.O.S. shown in Tables 4.38 and 4.42, we can find that most of the control charts have similar number of successes when the shift starts at different points. For example, for

CUSUM t chart, when the shift starts at the 10th point, it detects the shift 4 times out of the 30 replications. When the shift starts at the 20th point, it detects the shift 4.5 times, in average, out of the 30 replications. However, t chart, Q chart and CUSUM Q chart show a different trend. When the shift happens at a later point (the 20th point), the average N.O.S. becomes much larger for both charts with Q chart increasing from 7.0 to 12.3 and CUSUM Q chart increasing from 5.7 to 10.3. Such phenomenon might indicate that t chart, Q chart and CUSUM Q chart need longer time to become steady. Although the SD shift starts from a later point, the control charts are still built from the very beginning of the process. If a SD shift happens at a later point, this may lead to longer warmup time periods for these charts. For t chart, Q chart and CUSUM Q chart, the longer warmup period probably leads to better detecting ability.

The average alarm point can, to some extent, reflect the out of control average run length (ARL_1). If the average alarm point is 3 points after the shift point, the ARL_1 will be 3 in this particular simulation run. Comparing the results of the two group of experiments, it can be seen that the ARL_1 has been kept the same for all tested charts.

Overall, Q chart with Rules AC and individual X chart are the most effective control charts as shown in the results of this set of experiments.

We notice that although individual X charts show better performances than many other charts, the false alarm rate of individual X charts is higher than other charts. In the simulation experiments, we found that individual X charts have more chances to signal before the SD shift happens. The reason may be that at the beginning of the process, there is no sufficient data to better estimate the process mean in establishing more accurate control limits. Individual X charts are quite unstable in monitoring the early stage of the tested short-run processes.

4.5 Summary

In Chapter 4, first we identify the best range of the parameter λ for EWMA t chart and we select the one with the highest success rate in the simulation experiments in Section 4.2. In Section 4.3, we find best combination of supplemental rules (Rules AC) for Q chart through numerical experiments. We further test the result with different process lengths and by changing the points of the implanted process shifts. In Section 4.4, we apply the optimal parameter λ found in Section 4.2 to EWMA t chart and use the best rule combinations found in the experiments in Sections 4.3 and run the simulation experiments for other short-run control charts considered in this thesis. The experiment results show that Q chart with Rules AC and individual X chart are the most effective short-run control charts for SD shift compared with other tested control charts.

5 Conclusions and Future Research

5.1 Summary

This thesis mainly discusses the detecting ability of short-run control charts for SD shifts. In Chapter 2, we reviewed research literature related control charts for short production run. In Chapter 3, different models for short-run control charts were presented and some of the more promising parameter values were identified based on related work done by other researchers. Numerical simulations were conducted with results and their analysis presented in Chapter 4. The simulation runs were to select better parameter values for EWMA t chart; more effective supplemental rules for Q chart and to test other short-run control charts studied in this thesis for different settings and conditions.

5.2 Contributions of the Thesis

We tested the detecting ability of short-run control charts for SD shift through numerical simulation in this thesis.

Q chart is different from other charts, some papers proposed supplemental rules for Q charts to improve its detecting ability in short production run. In our thesis, we identify the best supplemental rule combination for detecting the SD shift using simulation. We applied it to Q chart to compare the detecting ability with that of other short-run control charts.

From the simulation results, we have the following general observations:

- In most cases, Q chart with Rules AC has the best performance on N.O.S. and good performance on A.A.P.

- Individual X chart has the second best performance on N.O.S., while in terms of A.A.P., Its A.A.P. performance varies for different conditions. Individual X chart is much simpler to be implemented in practice than Q chart with Rules AC. One must pay attention to the high rate of false alarms at the beginning of the process.
- For most tested control charts for short-run production, their N.O.S. performances will increase with the increase of SD shift.
- In most cases, we found that CUSUM scheme charts (CUSUM t charts and CUSUM Q charts) perform better than corresponding EWMA scheme charts (EWMA t charts and EWMA Q charts) on N.O.S.
- Most of the tested control charts have better N.O.S. performances when the length of the tested processes increase, especially for Q chart with Rules AC. The N.O.S. of Q chart with Rules AC increases largely when the length of the tested process is longer.
- Most of the tested control charts have similar N.O.S. performances when we change the starting point of the shift. The N.O.S. values of t chart, Q chart and CUSUM Q chart clearly increase when the SD shift occurs at a later point. This may indicate that t chart, Q chart and CUSUM Q chart may not be steady if the shift occurs at an early stage (e.g. the 10th point) and they need longer warmup time period. The A.A.P. of all the tested control chart keep unchanged when the starting point of SD shift changes. This means that the out of control ARL (or ARL_1) of the control charts will not change with the starting point of the shift.

5.3 Future Research

This thesis studies the detecting ability of several short-run control charts for SD shift through numerical simulation. Future research can further explore this topic from theoretical perspective. The numerical results from simulation experiments can be better explained with theoretical analysis.

The parameter values we used in setting up EWMA and CUSUM scheme charts are based on the results reported in the literature on short-run control charts to detect process mean shift. Research on optimal design of EWMA and CUSUM scheme charts for detecting SD shift is very limited. Studies on short-run control chart design for detecting SD shift using EWMA and CUSUM scheme charts are needed.

Future research in this area may also be to study the performances of EWMA X chart and CUSUM X chart for short-run production processes. They are simpler than EWMA t/Q and CUSUM t/Q charts, and are easier to implement in practical applications.

Future research also could include case studies with real data of certain short-run productions to test the possibility and effectiveness of the short-run control charts mentioned in this thesis.

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Appendix

Figures Plotting Simulation Results in Corresponding Tables in Chapter 4

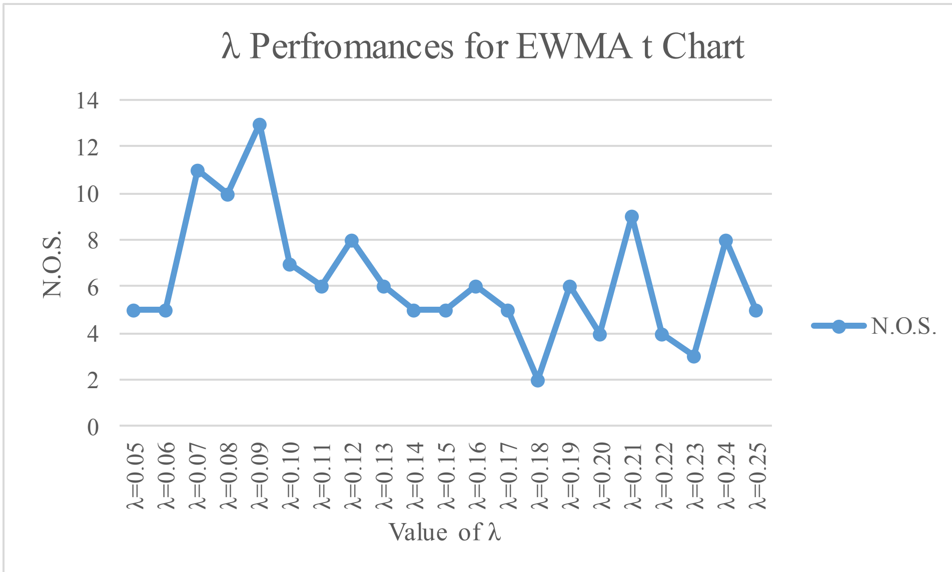

Figure A.4.1: Results in Table 4.1

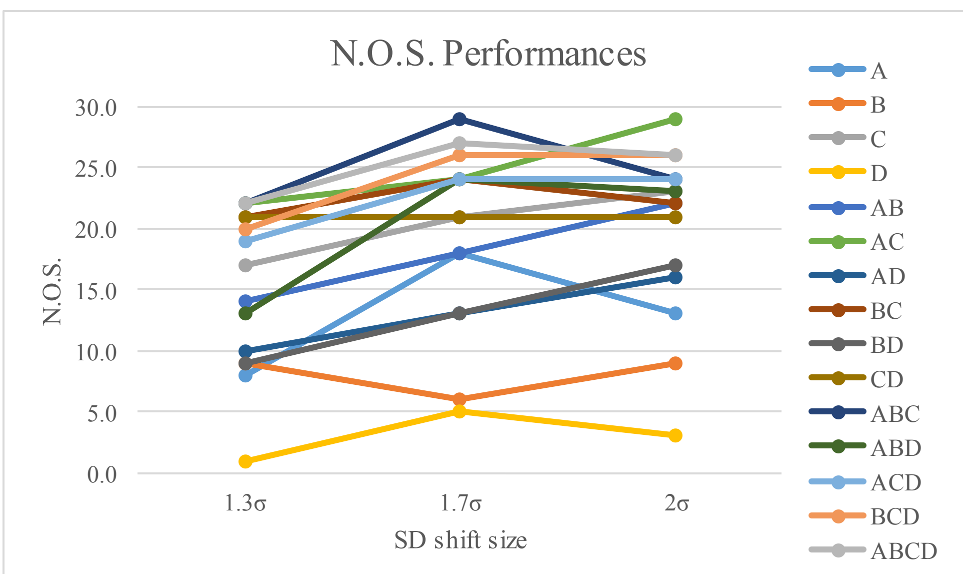

Figure A.4.2-4.4 (a): Results in Table 4.2-4.4

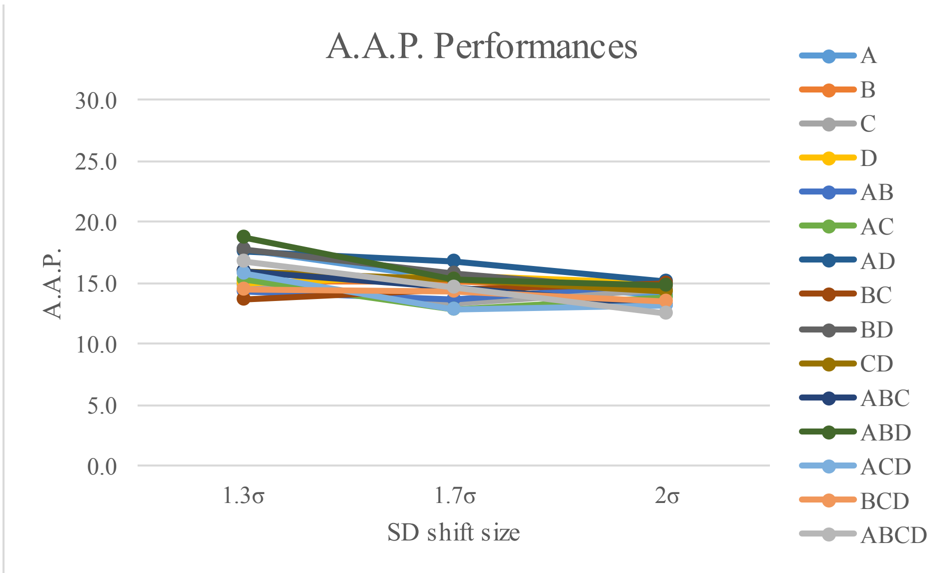

Figure A.4.2-4.4 (b): Results in Table 4.2-4.4

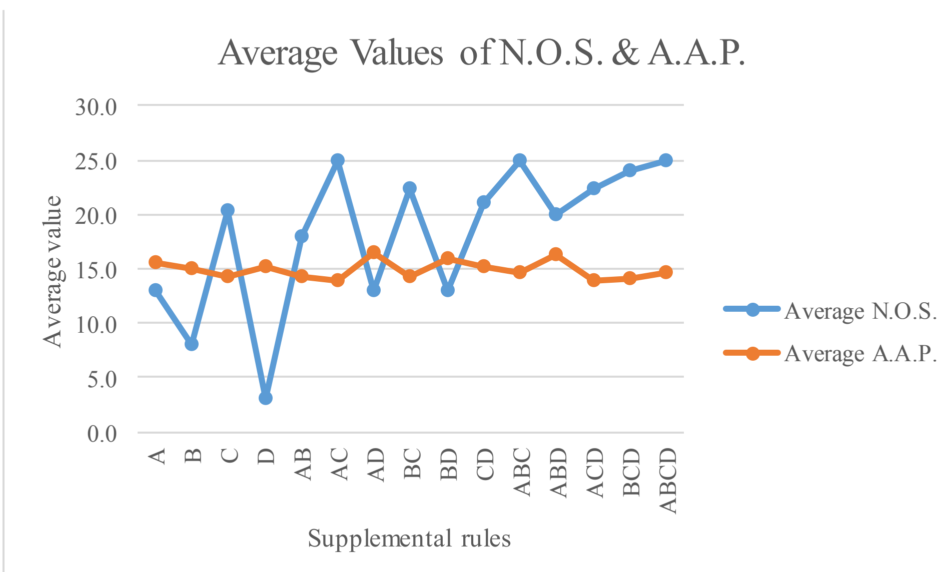

Figure A.4.5: Results in Table 4.5

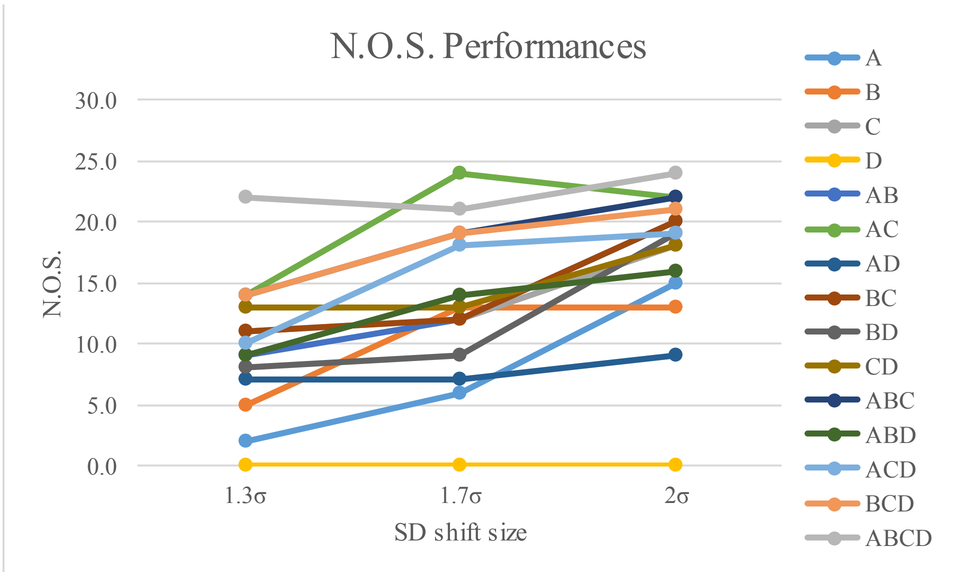

Figure A.4.6-4.8 (a): Results in Table 4.6-4.8

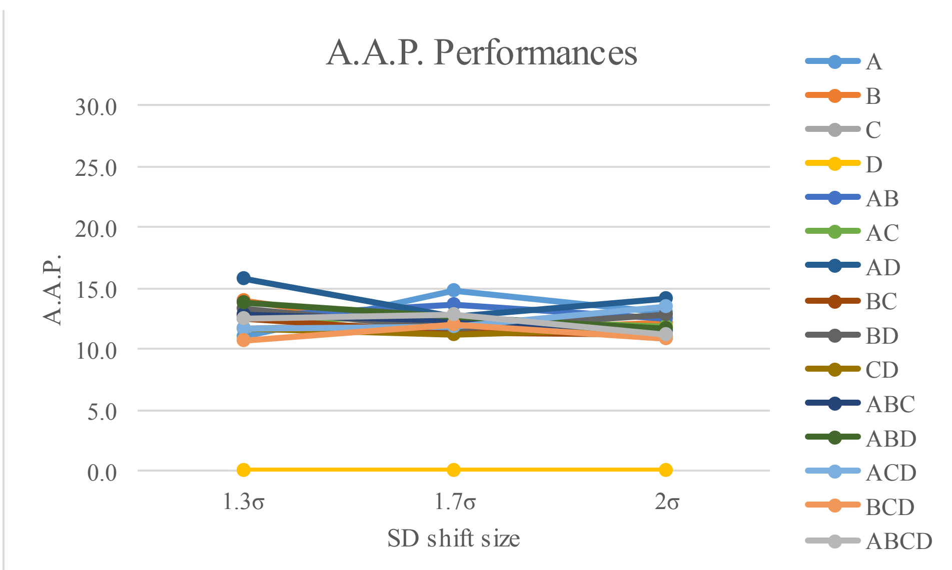

Figure A.4.6-4.8 (b): Results in Table 4.6-4.8

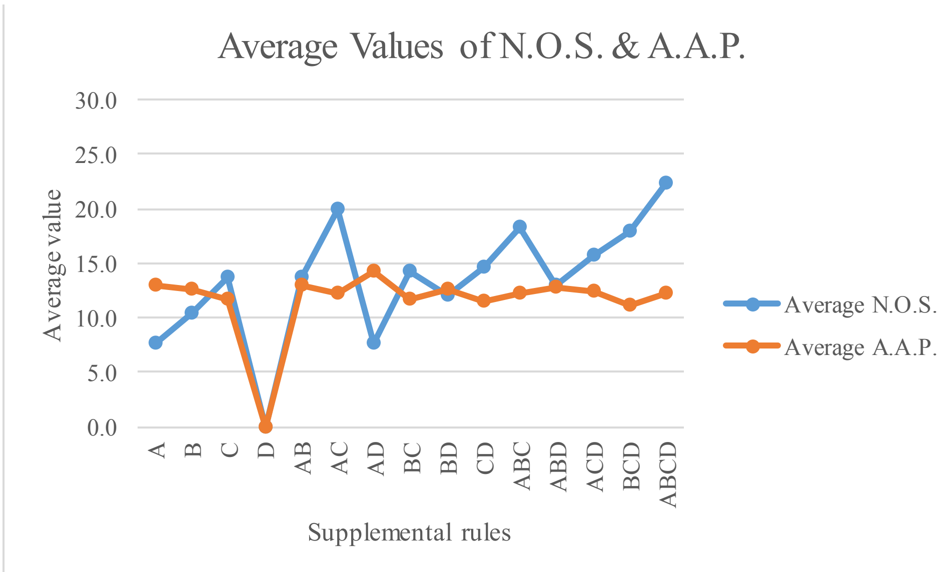

Figure A.4.9: Results in Table 4.9

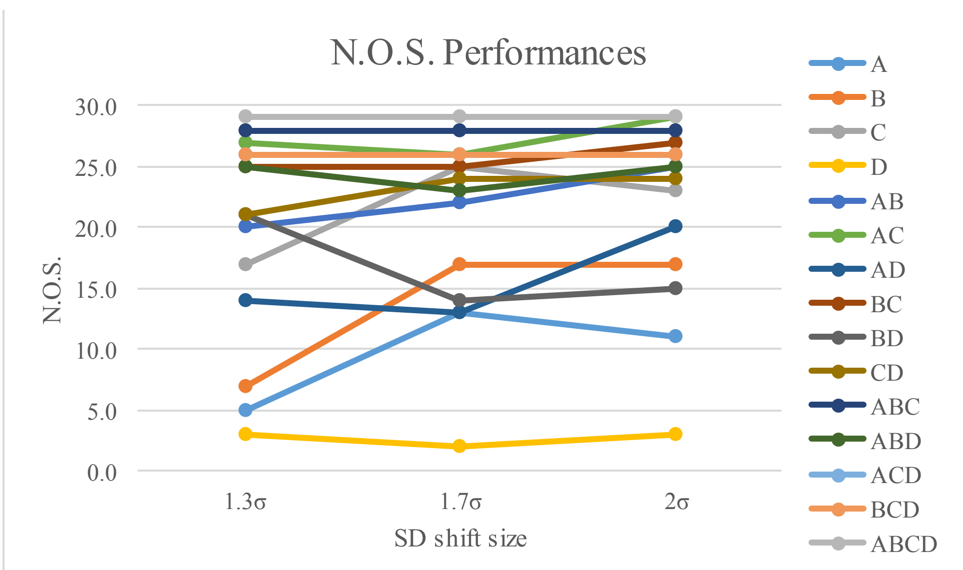

Figure A.4.10-4.12 (a): Results in Table 4.10-4.12

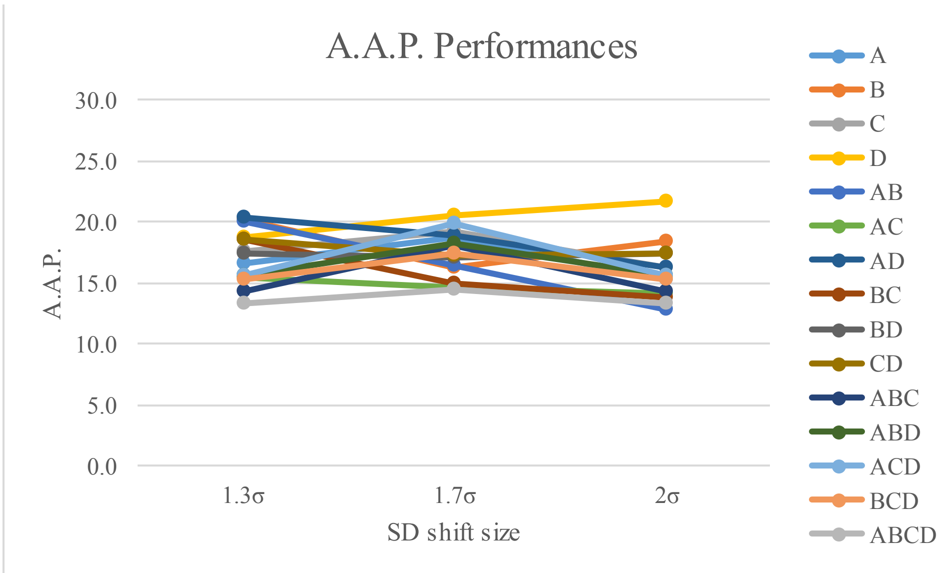

Figure A.4.10-4.12 (b): Results in Table 4.10-4.12

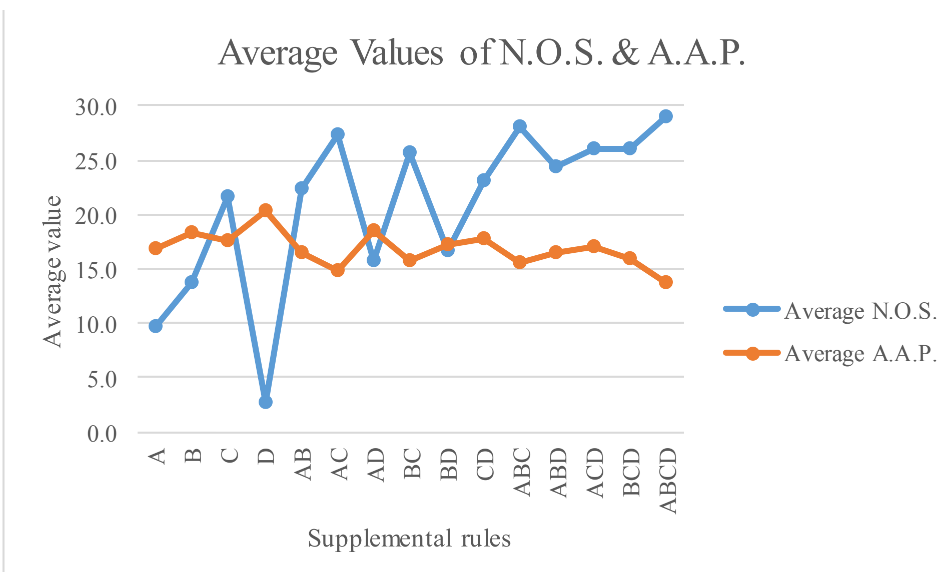

Figure A.4.13: Results in Table 4.13

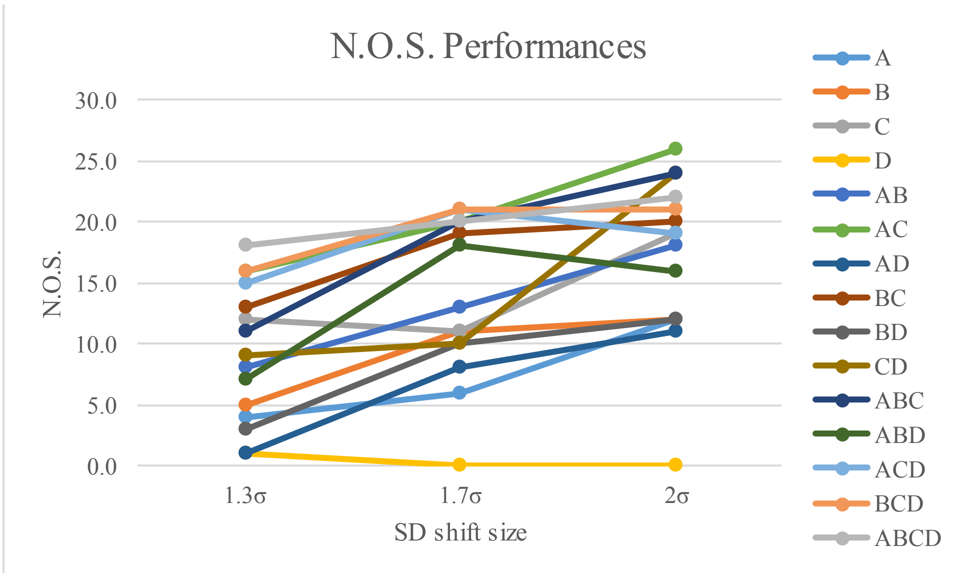

Figure A.4.14-4.16 (a): Results in Table 4.14-4.16

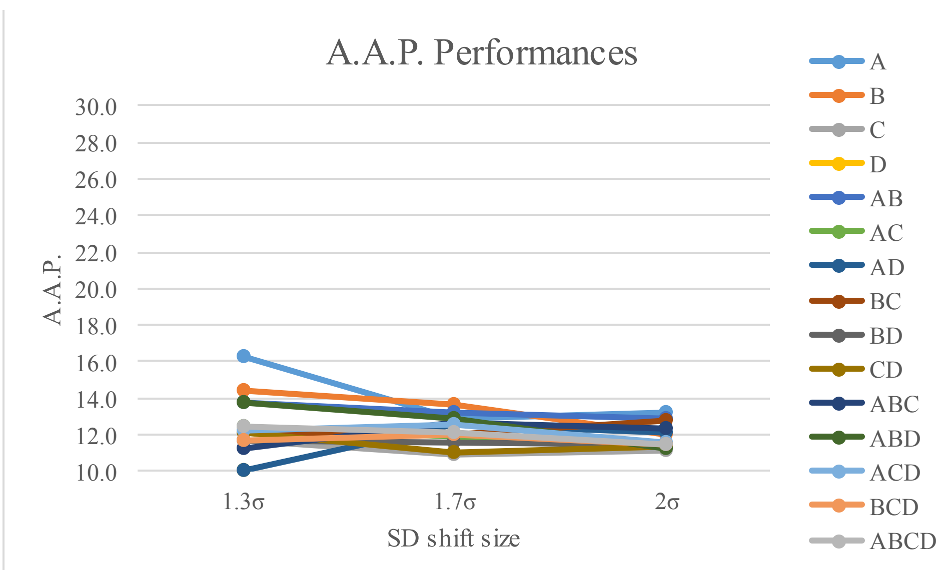

Figure A.4.14-4.16 (b): Results in Table 4.14-4.16

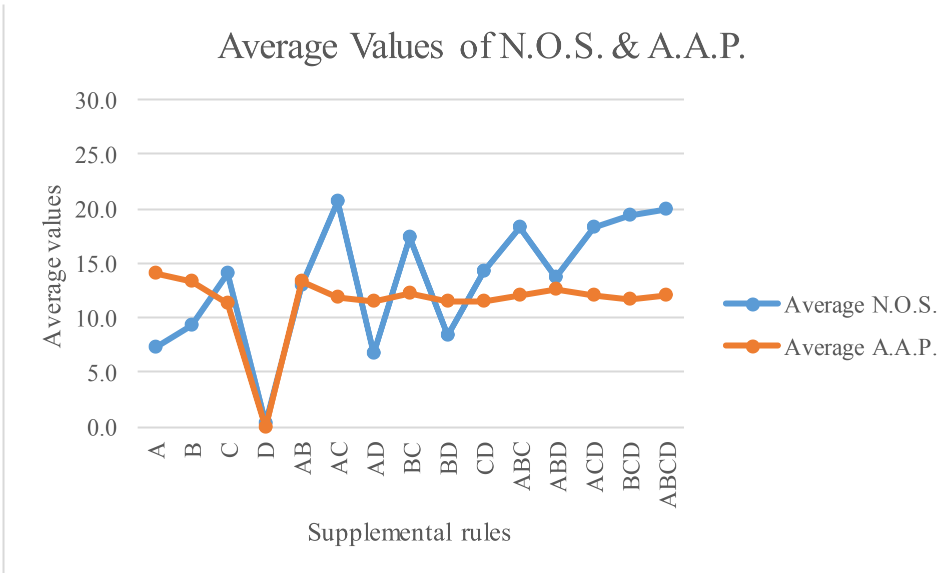

Figure A.4.17: Results in Table 4.17

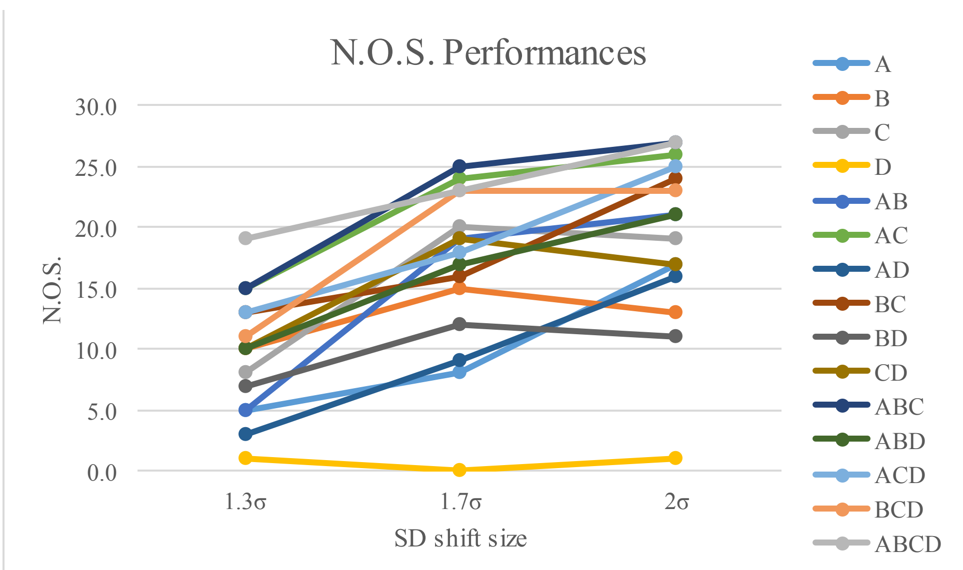

Figure A.4.18-4.20 (a): Results in Table 4.18-4.20

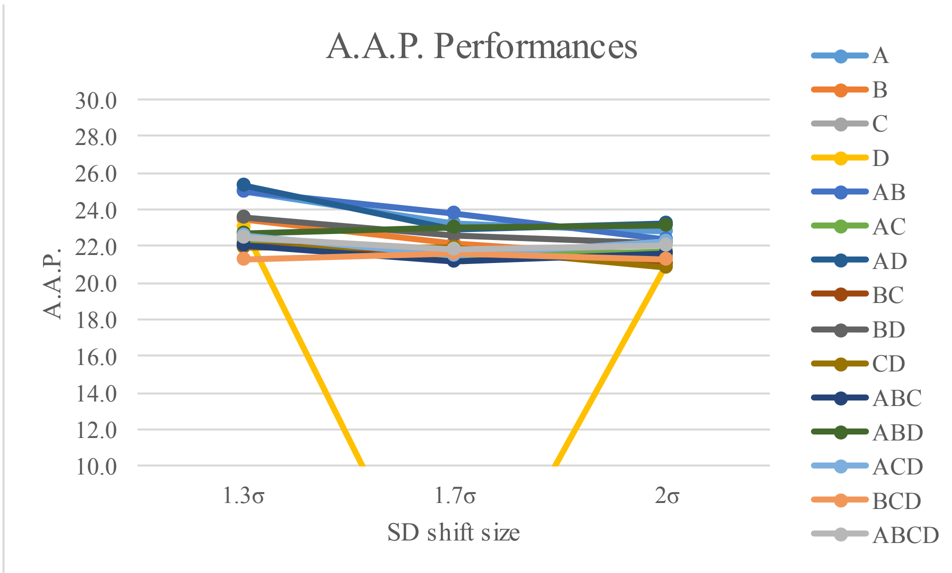

Figure A.4.18-4.20 (b): Results in Table 4.18-4.20

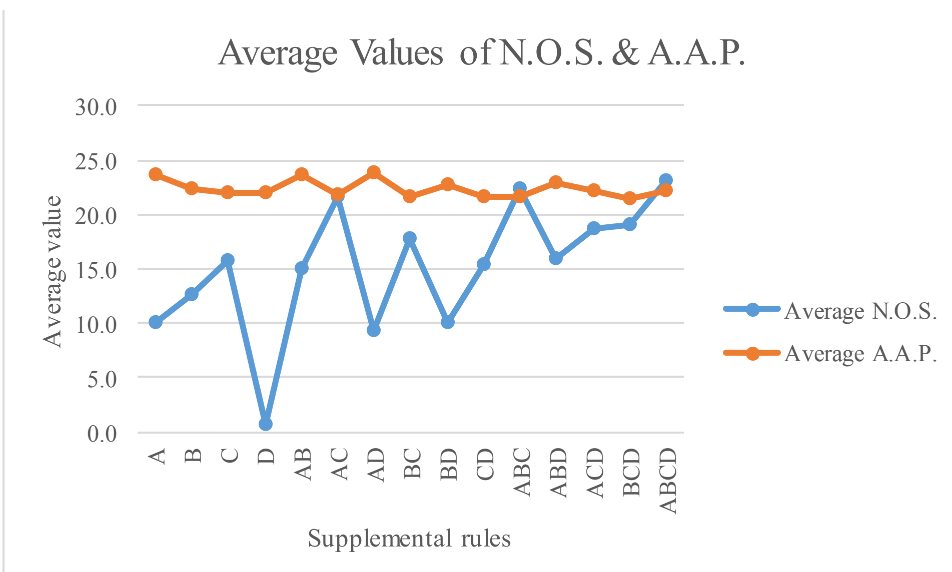

Figure A.4.21: Results in Table 4.21

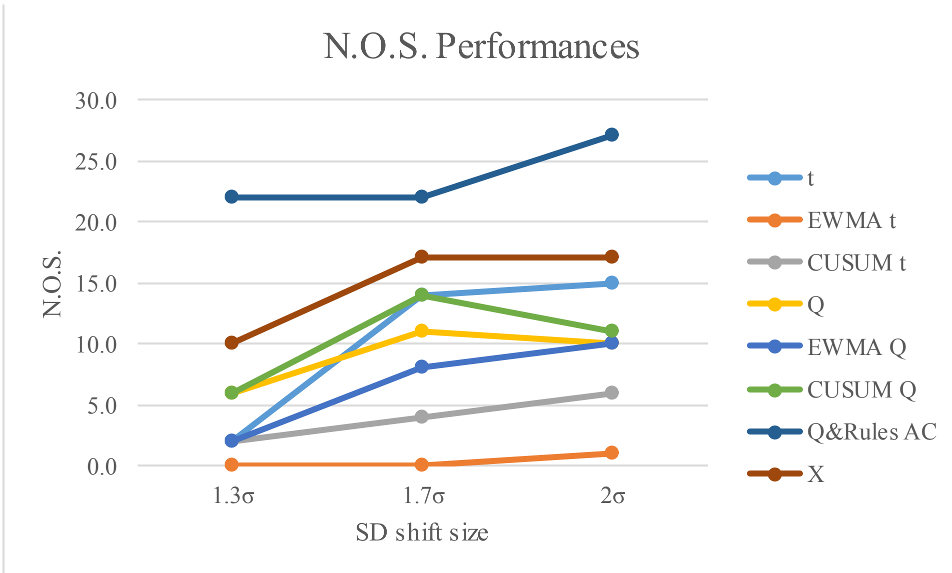

Figure A.4.23-4.25 (a): Results in Table 4.23-4.25

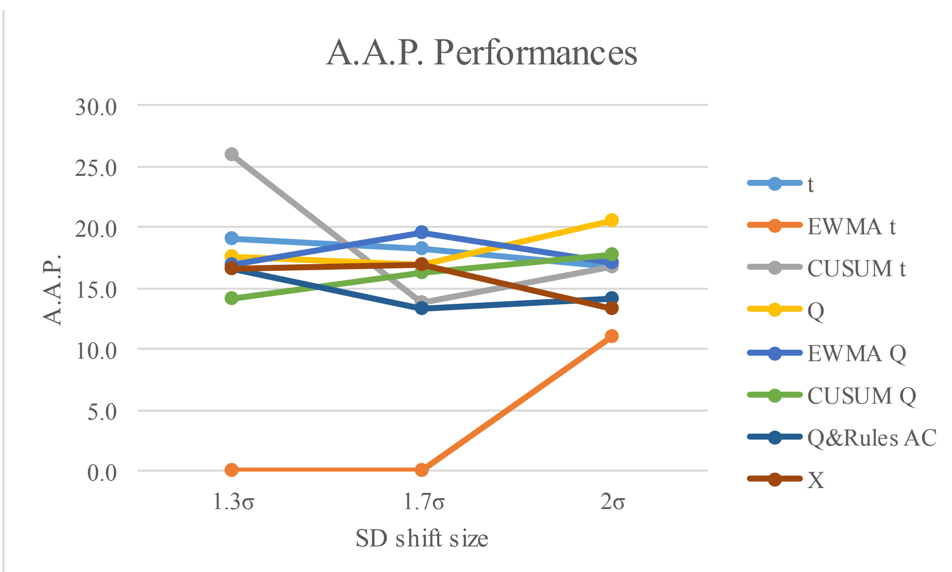

Figure A.4.23-4.25 (b): Results in Table 4.23-4.25

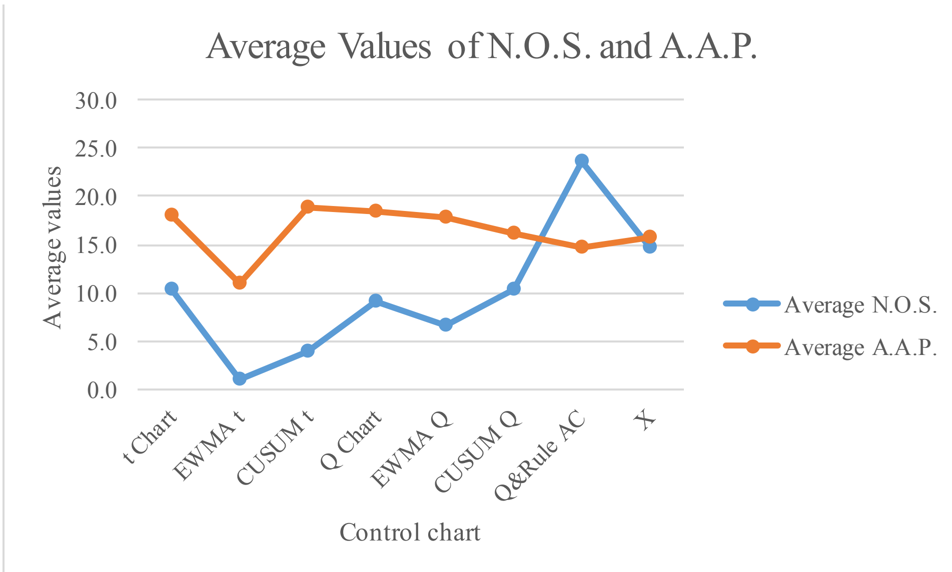

Figure A.4.26: Results in Table 4.26

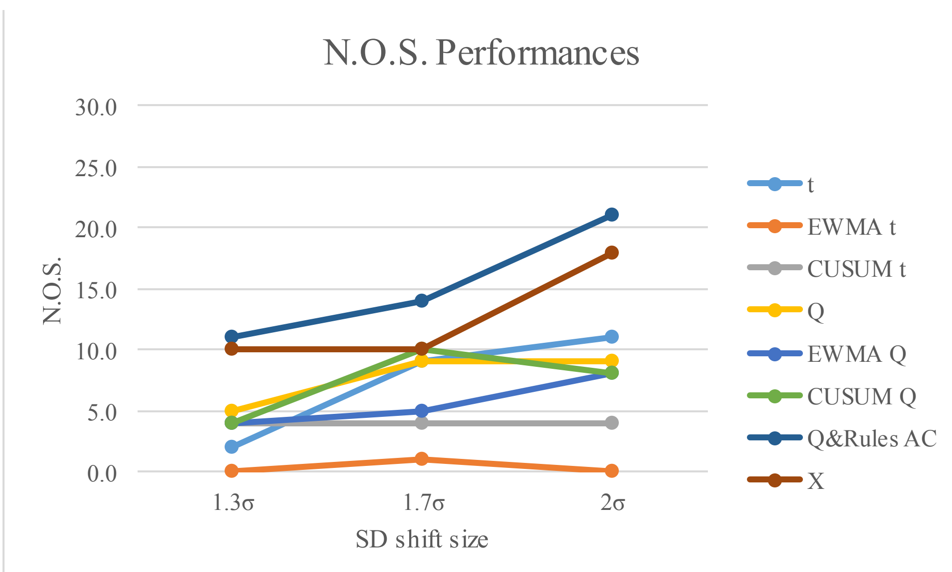

Figure A.4.27-4.29 (a): Results in Table 4.27-4.29

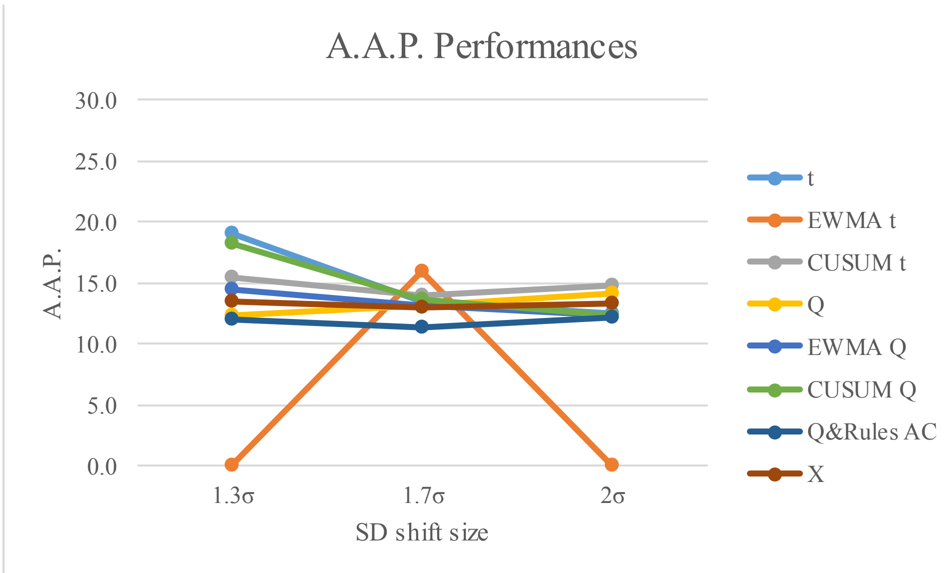

Figure A.4.27-4.29 (b): Results in Table 4.27-4.29

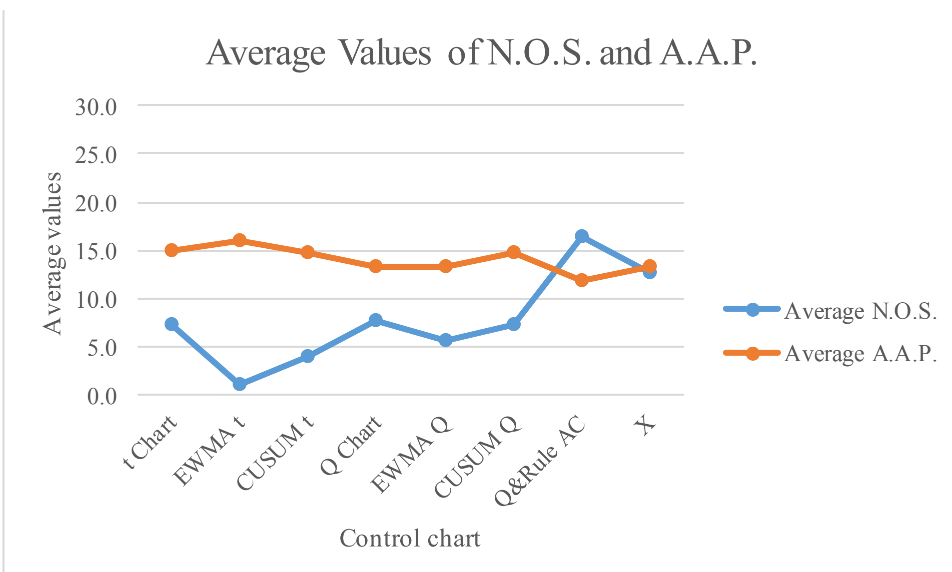

Figure A.4.30: Results in Table 4.30

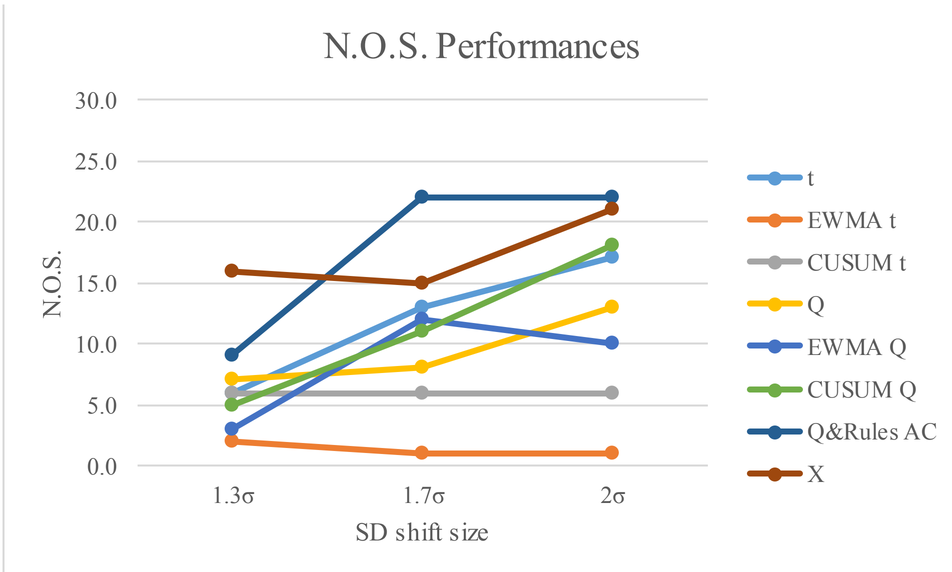

Figure A.4.31-4.33 (a): Results in Table 4.31-4.33

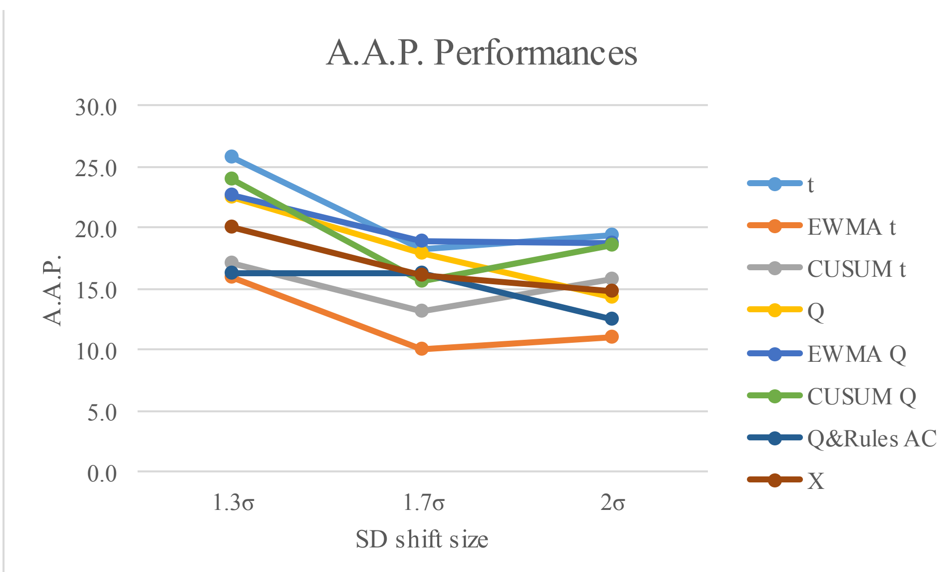

Figure A.4.31-4.33 (b): Results in Table 4.31-4.33

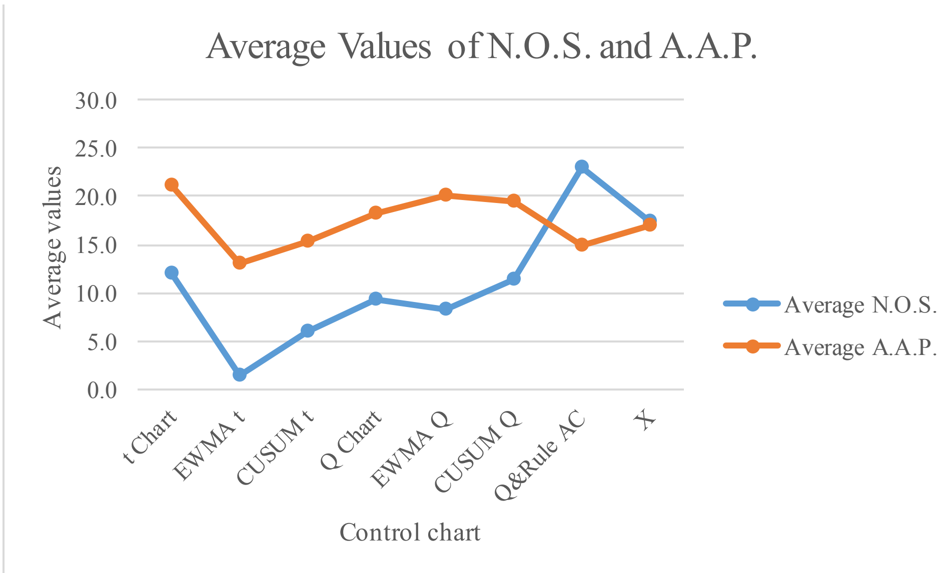

Figure A.4.34: Results in Table 4.34

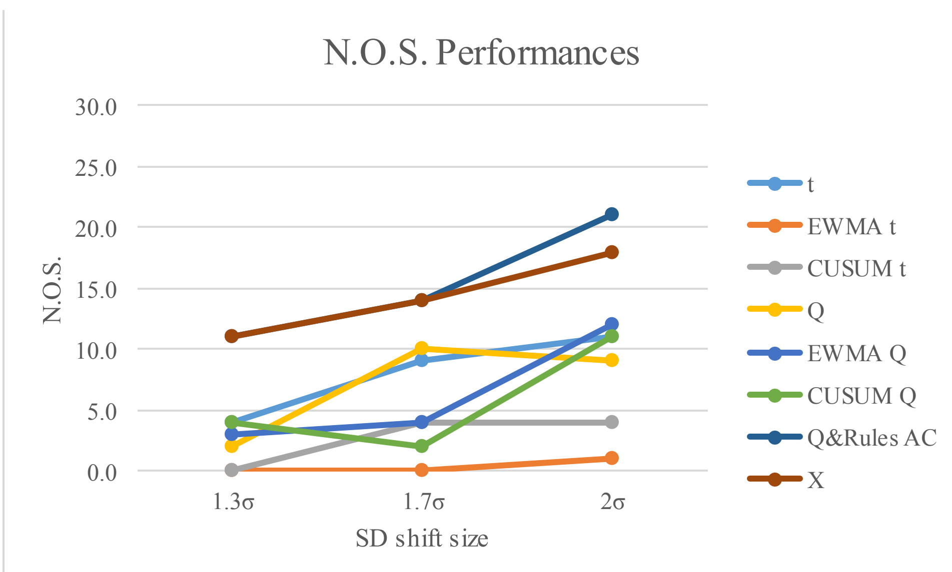

Figure A.4.35-4.37 (a): Results in Table 4.35-4.37

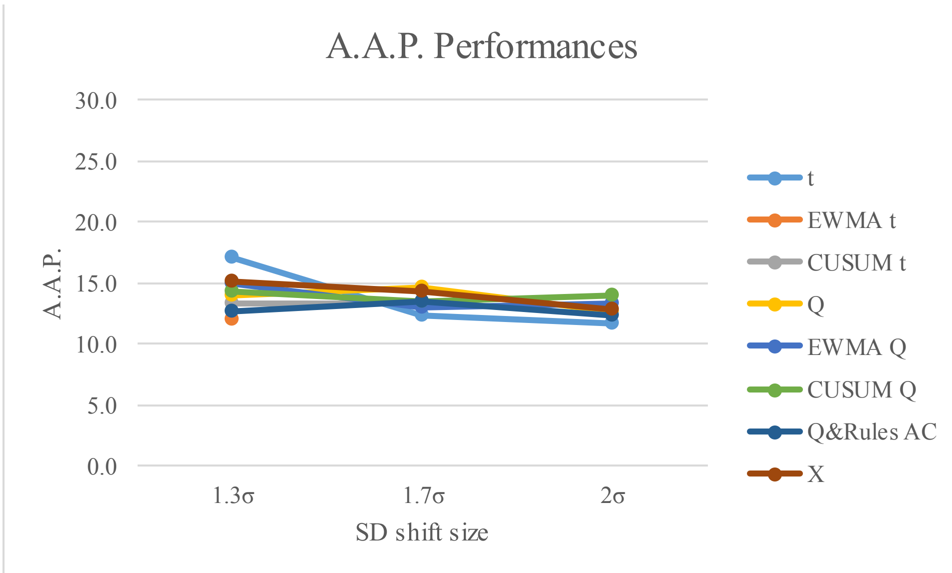

Figure A.4.35-4.37 (b): Results in Table 4.35-4.37

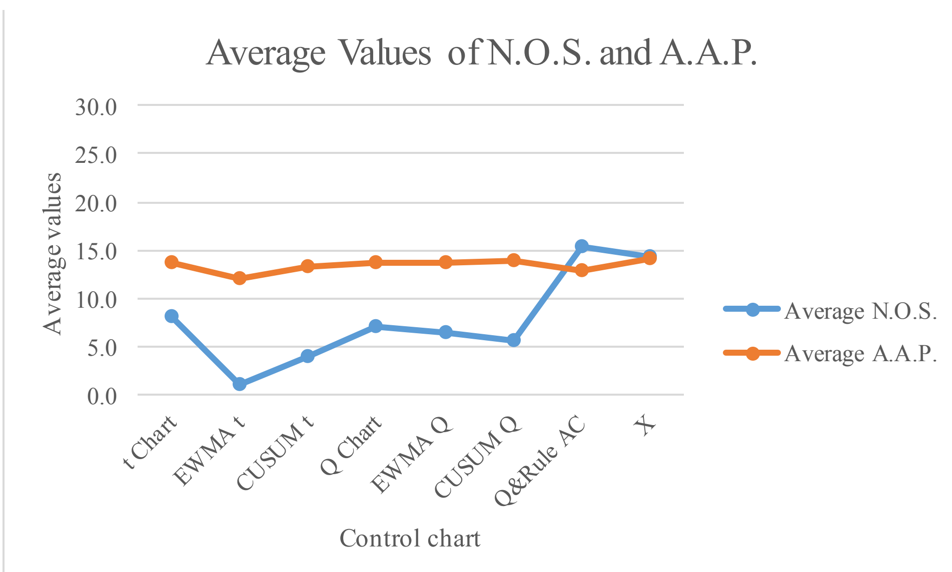

Figure A.4.38: Results in Table 4.38

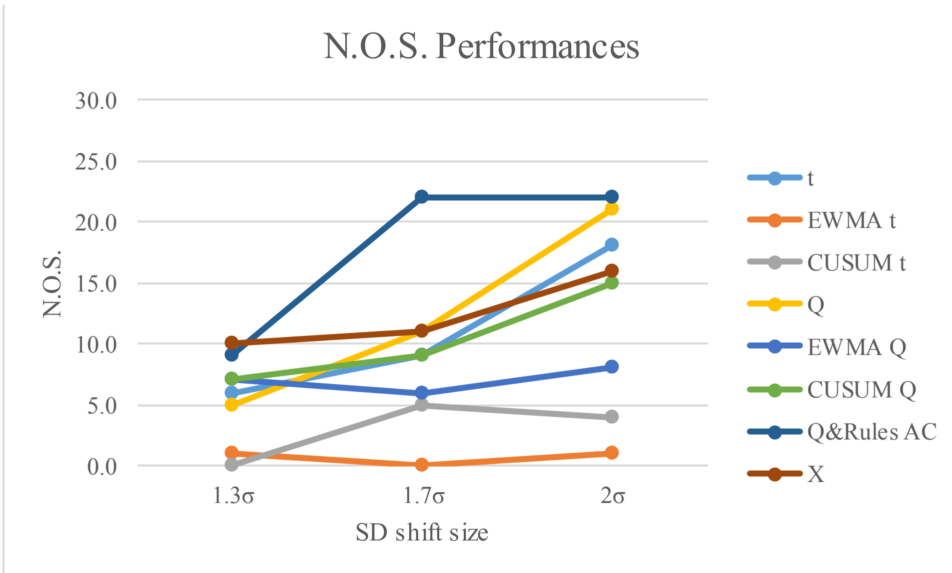

Figure A.4.39-4.41 (a): Results in Table 4.39-4.41

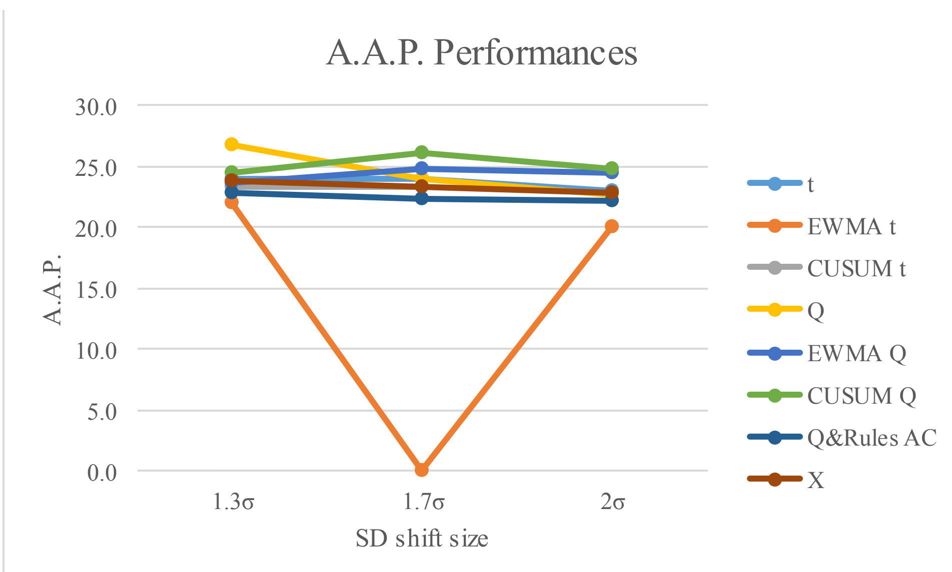

Figure A.4.39-4.41 (b): Results in Table 4.39-4.41

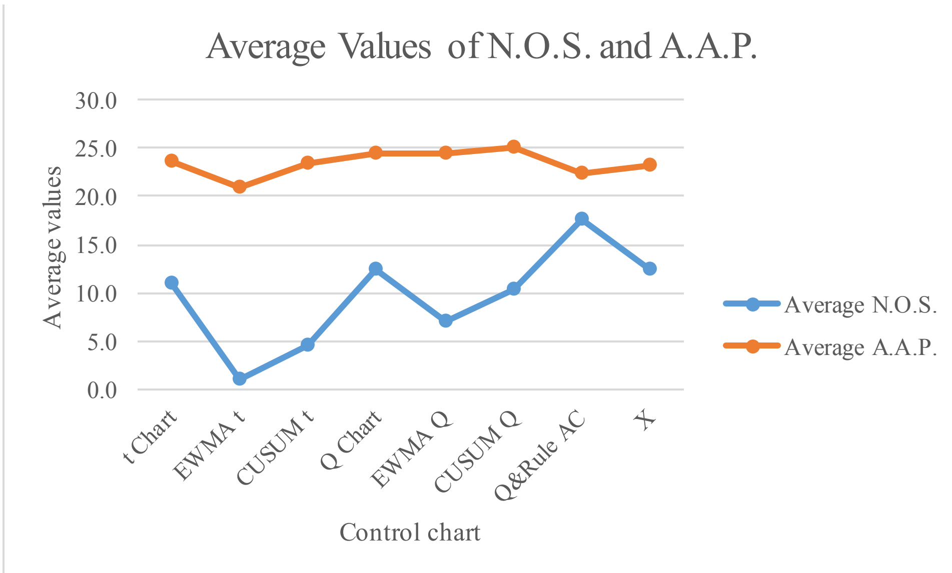

Figure A.4.42: Results in Table 4.42