# Demand Forecasting and Location Optimization of Recharging Stations for Electric Vehicles in Carsharing Industries

Elnaz Moein Namin

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This is to certify that the thesis prepared

By: Elnaz Moein Namin

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Signed by the final examining committee:

Dr. Jeremy Clark ________ Chair

Dr. Anjali Awasthi ________ Supervisor

Dr. Jamal Bentahar ________ Internal Examiner

Dr. Brigitte Jaumard ________ External Examiner

Approved by ________________________________
Chair of Department or Graduate Program Director

________________________________
Dean of Faculty

Date ________ 2015 ________

## Abstract

**Demand Forecasting and Location Optimization of Recharging Stations for Electric Vehicles in Carsharing Industries**

Elnaz Moein Namin

Carsharing is an alternative to private car usage. Using electric-vehicles as a substitute to fuel vehicles is a wiser option which leads to lower fuel emissions, more energy savings and decreased oil dependency. However, there are some barriers in using electric vehicles at large scale in carsharing companies. Battery power limitation and lack of sufficient infrastructures are some of them. Accurate demand forecasting is a must for this purpose.

In the first part of this thesis, we investigate the demand forecasting problem for carsharing industries and apply four techniques namely simple linear regression, seasonally adjusted forecast, Winter's Model and artificial neural networks to decide the right number of vehicles to be made available at each station to meet the customer requests. The results on randomly generated test datasets show that artificial neural networks perform better over the other three.

In the second part, we investigate the location planning problem of recharging stations for electric vehicles. The base model used for this study is the mathematical optimization model proposed by Wang & Lin (2013). Firstly, we improve their MIP model and solve it using AIMMS (Advanced Interactive Multidimensional Modeling System). Secondly, we propose Genetic Algorithm for the same problem and implement it in Matlab. The obtained results are compared with previous work done by Wang and Lin (2013). The comparisons show better performance of the proposed methods.

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I also would like to thank CommunAuto for providing me initial data set for the forecasting study.

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# Chapter 1:

# Introduction

This chapter gives an introduction about carsharing industry, electric vehicles usage, and the dilemmas and challenges related to fleet management in this area. Next, the thesis contribution is discussed and finally in the last part structure of the thesis is explained.

## 2. Background

Nowadays, many governments are involved in reducing air pollution arising from vehicle movements and saving energy at the same time, especially in big cities where it can be a big dilemma (Wang and Linn, 2013). Using public transport is a good alternative to personal cars, however it is not applicable all the times.

Using shared cars is another alternative to this problem, which lets users have their personal privacy. It also decreases the number of cars on the streets. It is also a good alternative for doing urban trips.

There is even a better solution to this problem, i.e. using electric vehicles in the carsharing companies as an alternative to fossil fuel based vehicles. In this way, they can reduce the fuel consumption and decrease the air pollution significantly.

There are some challenges regarding electric vehicles usage which currently limits their application in carsharing companies. Facility planning is one of the main issues. It is the process of identifying the location, number and type of facilities to meet the customer needs.

Considering the charging limit of battery of an electric vehicle and its energy consumption during trip, drivers can think of various charging strategies such as charging the vehicle in the middle of the trip in order to avoid running out of electricity charge (Wang and Lin, 2013). Facing these challenges needs more in-depth studies in order to investigate the best solutions and possible approaches to deal with this issue (Kuby and Lim, 2005).

In this thesis, we go through two main problems in carsharing industry:

- Customer Demand Forecasting. This involves predicting customer demands at various locations (carsharing stations) based on historical data.
- Location Planning of Electric Vehicles Charging Stations. This involves determining best locations for placing charging stations for electric vehicles in order to ensure that vehicles are charged enough for rendering a service when needed.

# 2. Problem Definition

## 2.1. Demand Forecasting

Car sharing organizations are often faced with the dynamic fleet management problem. This involves accurate demand forecasting, optimized location finding for installing the parking lots and in case we are talking about electric vehicles, installing different kinds and different numbers of charging stations' infrastructures, relocation strategies and so on.

Demand management is one of the key challenges in carsharing industry. It includes accurate demand forecasting to better serve the customers through efficient fleet management. In other words, it helps in identifying the right number of vehicle requests to be met at each station to satisfy customer demands. Demand forecasting can be done using quantitative methods and qualitative methods. In this thesis, we focus on some quantitative methods to predict the demand.

We use causal methods (regression forecast and regression forecast with seasonal adjustment) and time series (neural network and Winter's model) of forecasting for this purpose.

There are various challenges in forecasting studies including seasonality, trends and cycles in quantitative approaches and the nature of human behaviour in qualitative approaches. In this thesis we considered some of these quantitative challenges like trends and seasonality as part of our forecasting study.

## 2.2. Recharging Stations Planning and Optimization

Once accurate demands are known, the next step for carsharing companies is to decide how to plan recharging stations, distribute the available infrastructures and services.

Battery power limitations in electric vehicles forces the drivers to keep their cars charged at all times, so finding a charging station for these drivers can be a very important issue in order to complete their trip and also for companies to respond to their customers' needs. Considering budgetary constraints, finding the right number of required charging stations and also installing them in the right places so that they can be accessible for all customers are some of the challenges that carsharing companies need to address. The paths and trips with more demands could be the base for the company to make the distribution of stations, but the question is how to use this information for this purpose.

The challenges in location planning of recharging stations for EVs is not limited to finding the right locations and number of the stations to be installed, but also other issues such as choosing the right kind of recharging technology for each of the stations. Since there are different kinds of technologies for recharging the EVs' battery, it is important to find a trade-off between the different kinds of infrastructures' cost and their recharging rates. For example, while a fast recharging station's rate is 2 km/min, its cost is 20,000 $ per each station. On the other hand, cost of a slow recharging station is only 4000$, but its rate is 0.133 km/min. So, it is important to find the best combination of different available charging types in each of the stations based on customer requirements.

# 3. Thesis contribution

This thesis consists of two main parts. In the first part, we apply forecasting techniques, generate results, and perform analysis and comparisons on randomly generated data sets for the years between 1994 and 2011. This data set was generated based on real data obtained from Communauto for the first 9 months of the years 2011 and 2012. The objective of forecasting is to find the method with least forecasting error or the one which gives the most accurate forecasted data set. The best forecasting technique shall be used to get more realistic assessment, regarding demand in each month of the year, specifically for each station and in total.

In the second part, we address the location planning problem of recharging stations for electric vehicles. The problem objective is to maximize the flow coverage for the electric vehicles through optimizing the number of recharging stations and their specific type at each station. Flow coverage is referred to the ratio of car flows that are covered with regard to a special budget level. It also tries to minimize the locating cost related to these infrastructures. Our work is based on the model developed by Wang and Lin, 2013. We implement the improved version of the problem through branch and cut algorithm in AIMMS (Advanced Interactive Multidimensional Modeling System) and Genetic Algorithm in Matlab. The results of our model show superior performance over the original model.

## 4. Organization of the thesis

This thesis is organized as follows:

Chapter 2 presents literature review on electric vehicles carsharing, issues in managing electric vehicles fleet, demand forecasting and location planning in this industry.

Chapter 3 contains proposed solution approach for demand forecasting and location planning problem for the recharging stations.

Chapter 4 represents the application results for the demand forecasting and the recharging stations location planning solution approaches and compares the achieved results.

Chapter 5 presents the conclusions and future works.

# Chapter 2:

# Literature Review

This chapter discusses about car sharing, how it works, its applications, history, and so on. It also gives an overview of electric vehicles and their application in carsharing industry as well as costs and benefits regarding these vehicles.

Then, we present the approaches available in literature to deal with the demand forecasting and location planning problem of recharging stations for carsharing vehicles. Finally in the last part, performed studies and researches in each of the two areas are reviewed and research gaps are presented.

## 1. What Is Carsharing?

A carsharing company usually consists of small to medium sized fleets of vehicles available for members at specific stations spread over the city (Shaheen *et al.*, 1999). They can be rented for different periods of time from some hours up to some days.

Since it is considered as an alternative to private car usage, its environmental and social objectives are considered more important than business and financial purposes (Car2go, 2012).

Promoting community transit and environmental concerns are some of the main reasons behind governmental support for these sorts of businesses. It also leads to decreasing personal car ownership, reducing the distance travelled by the cars, decrease in fossil fuels consumption and greenhouse gases' emission (Martin and Shaheen, 2011).

## 1.1. History

Car sharing history comes back to as early as 1948 by a company named Sefage (Selbst Fahrer gemeinschaft) in Zurich Switzerland. After that, a series of other CSOs in the 1970/80 era around Europe were established like: PROCOTIP in Montpelier and Witkar in Amsterdam (Harms and Truffer, 1998).

Late 1980's was the start of more successful experiences in carsharing business all over Europe especially in Switzerland, Germany, Austria, the Netherlands, Denmark, Sweden, Norway, Italy and Great Britain (Shaheen *et al.*,1999).

It was demonstrated across North America in the 1980's by two carsharing companies, first one called Mobility which was operated as a Purdue University research program from 1983 to 1986 in West Lafayette Indiana and second one called Short Term Auto Rental (STAR) which was operated as a private enterprise from 1983 to 1985. However, both of them stopped working after a while. Mobility Enterprise was deployed as a research experiment and STAR failed in the middle way of the three year planned program (Shaheen, 2005).

Auto-Com, the first and oldest CSO of North America was located in Quebec City. It was started as a nonprofit cooperative and then after was changed to for-profit business in 1997. Communauto Inc. was established by Auto-Com group in Montreal in September 1995 as a for-profit business. It is now one of the main carsharing companies in Quebec Province in Canada (Shaheen *et al.*, 1999).

## 1.2. Car Sharing vs. Traditional Car Rental

Carsharing is an alternative to private car usage. Its difference with respect to car rental is that people can rent the cars only for special periods of time (some hours), as well as longer times (some days), and pay for that time.

The second difference which makes it more practical for the users who don't have a car is that they can access the cars at any time during the 24 hours of a day. Since the parking stations are spread around the town, everybody has a chance to reach a car at the nearest station (What is Carsharing, 2010).

## 1.3. How Car Sharing Works

First, one needs to figure out which carsharing companies are offering service and where they are located. The list of available carsharing operators is accessible in Wikipedia's website which is categorized by country.

If someone meets the requirements (like the minimum age and possessing a valid driving license), then it will be as simple as filling out the online registration form and picking a rate plan according to their needs and usage.

The next step will be to find a car in a parking which is nearest to them. The reservation for cars is usually done in advance through the websites of the companies. Users can reserve a car for the period and the date that they want.

Membership cards are usually used to unlock the car, thanks to the magic of RFID technology. Usually fuel and insurance are covered in what is paid for the service (Car2go, 2012).

## 1.4. Is carsharing more sustainable than car ownership?

In most of the times, when carsharing makes sense, it is a greener solution than car ownership. Based on the location, each shared car can replace 6 to 20 personal cars. Furthermore, most of the shared vehicles are recent models with the most up to date technologies, emission control and fuel-efficient options like plug-in hybrids and electric vehicles (Bradley and Frank, 2009).

It also encourages the users to use more public transport, bike or walk instead of using personal cars, which leads to more saving in costs rather than using personal cars.

## 1.5. What is an electric vehicle?

- **The motor**

The car is generally have one or more electric motors with a total power ranging from 15 to 100 kW depending on the size, usage and desired performance. E.g. 48 kW (65 hp.) for a small 4-seater saloon.

- **Batteries and range**

The battery bank supplies energy provided:

• Recharging from an external source via cable

• Vehicle deceleration, where the engine works as a generator.

The battery capacity is within a range of 5 to 40 kWh, with a total voltage of between 300 and 500V. The vehicle's range depends directly on the battery's capacity and also on the type of journey (flat, varied, urban, etc.), the driving mode and the accessories used (headlights, heating, air-conditioning, windscreen wipers, other accessories).

The manufacturers announce an average driving range of 150km (Communauto, 2014).

## 1.6. How are electric vehicles charged?

**Mode 1: connecting to a domestic socket**

The electric vehicle is connected to the building's electrical distribution system via connector bases. They are plugged into domestic single phase or three phase AC sockets with earth and power supply conductors. A charging control function is either built into the plug or into a unit fitted to the cable. Charging is restricted to 10A.

**Mode 2: connecting to a specific socket**

The electric vehicle is connected to the building's electrical distribution network via connector bases. They are plugged into specific sockets on a dedicated AC circuit. A charging control function is built into the plug base.

Mode 2 guarantees for users the highest level of safety and the best performance. Danger may arise from:

• A faulty system (damaged cable, faulty or aging installation, etc.)

• Mis-handling by users (a child putting its fingers into the socket, etc.)

• Incorrect usage (the user plugs the connector into the wrong socket, etc.)

In mode 2, the personal protection functions (e.g. differential circuit breaker) are in the fixed part, whereas in mode 1 they are built into the cable. This means that in mode 1, if the cable is damaged, there is no guarantee that these functions will not be affected. In all cases, property protection (e.g. lightning conductor) is not built into the cable (Communauto, 2014).

**Mode 3: DC connection**

The electric vehicle is connected to an external charger fitted with a specific cable and delivering direct current. The charger incorporates the control function and electrical protection.

In Communauto, when the vehicle is connected to a charging dock in a Communauto station and the battery has been completely depleted, the charging time comes to about 7-8 hours for the Nissan LEAF and 3-4 hours for the Ford Focus.

If the vehicle is plugged into a conventional electrical outlet with the kit found in the trunk of the vehicle, charging will take 21 hours for the Nissan LEAF and 18 hours for the Ford Focus (Communauto, 2014).

## 1.7. Electric Vehicles & Fleet Management

The increasing use of electric vehicle fleets is a response to the political commitment to reducing the carbon usage of the transport sector (Touati-Moungla and Jost, 2010).

Electric vehicle can significantly reduce emissions that contribute to smog, particularly in urban areas, and to climate change by displacing the demand for gasoline. The policies regarding environmental and sustainable development issues of governments and private corporations, therefore, drive an interest in electric vehicle technology for fleets (Touati-Moungla and Jost, 2010).

Electric vehicles use expensive and new technology. Since they are partly- or fully-powered by electricity, and because their mechanical design is different, the higher upfront capital costs can be offset through reduced fuel and maintenance expenses over their service life in a fleet (Wen-liang *et al.*, 2009).

Because vehicles in public or commercial fleet service often are used for more hours and mileage than personal vehicles, and fleet managers are eager to use electric vehicles within their operations, it is expected that the return on the investment in electric vehicles will be more economically attractive for fleet application (Wen-liang *et al.*, 2009).

## 1.8. Benefits & Costs of Electric Vehicle in Carsharing

Car share schemes increase the flexibility of public transport (PT) offer in a city and provide quality of life benefits further enhanced by use of electric powered cars.

Less pollution i.e. emissions and noise, due to less traffic and use of electric vehicles, reduced environmental impacts resulting from reduced mileage, clean, quiet vehicles, reduced congestion and delays, and more sustainable transport leads to improved quality of life in the city are the main advantages of using electric vehicles in carsharing organizations (Ion *et al.*, 2009).

On the other hand, costs are associated with the provision, and subsequent maintenance of the electric vehicles. A back-office system for handling charges and membership, and with the provision in the city for infrastructures of parking spaces equipped with facilities for re-charging the electric vehicle's batteries are some of them (He *et al.*, 2013).

## 1.9. Challenges in electric vehicles carsharing

When it comes to the use of electric vehicles in carsharing, various challenges are faced by the industry. Customers' attitudes and priorities intuitively have effects on the demand. On the other hand, media and education affect the way people think of using such a system. Encouraging people to rent the electric vehicles instead of the fuel ones is one of the challenges that companies are constantly facing. Environmental awareness and promotional offers may be some of the ways to face these sorts of human related issues.

From the industry aspect, there is still a lot to deal with electric vehicles. Battery power shortage, long time of battery recharging, and dealing with the possible technical issues after depreciation are some of them.

While there are some fast charging technologies for EVs, they are still very expensive for many companies to install and offer these options to their customers. Cutting battery costs while improving its efficiency and power, reducing the vehicle weight, and reducing the cost of electric drive systems are some of the technical issues that industries are facing with.

Battery limitations make it crucial for the companies to provide recharging stations as much as possible accessible for the customers. The right number of recharging docs at each station and choosing the right type of technology according to the customers' needs, installation costs and budgetary constraints are some the crucial issues that business companies in the carsharing industry have to overcome. Therefore, solving these issues is a great help for extending the use of EVs in the future and considering it as a future business model for the carsharing industries.

## 1.10. About Communauto

Communauto Inc. is a carsharing company privately owned and founded in Quebec City in 1994. It was then merged with its competitor, Auto-com in 2000. After that, Communauto expanded its activity in four areas of Quebec province namely Québec, Montréal, Gatineau and Sherbrooke. Its members reached to the number of 36000 in total in May 2012 with 883 vehicles in use across 331 stations spread through the city (Communauto, 2014).

The network map of the stations across Montreal is shown below:

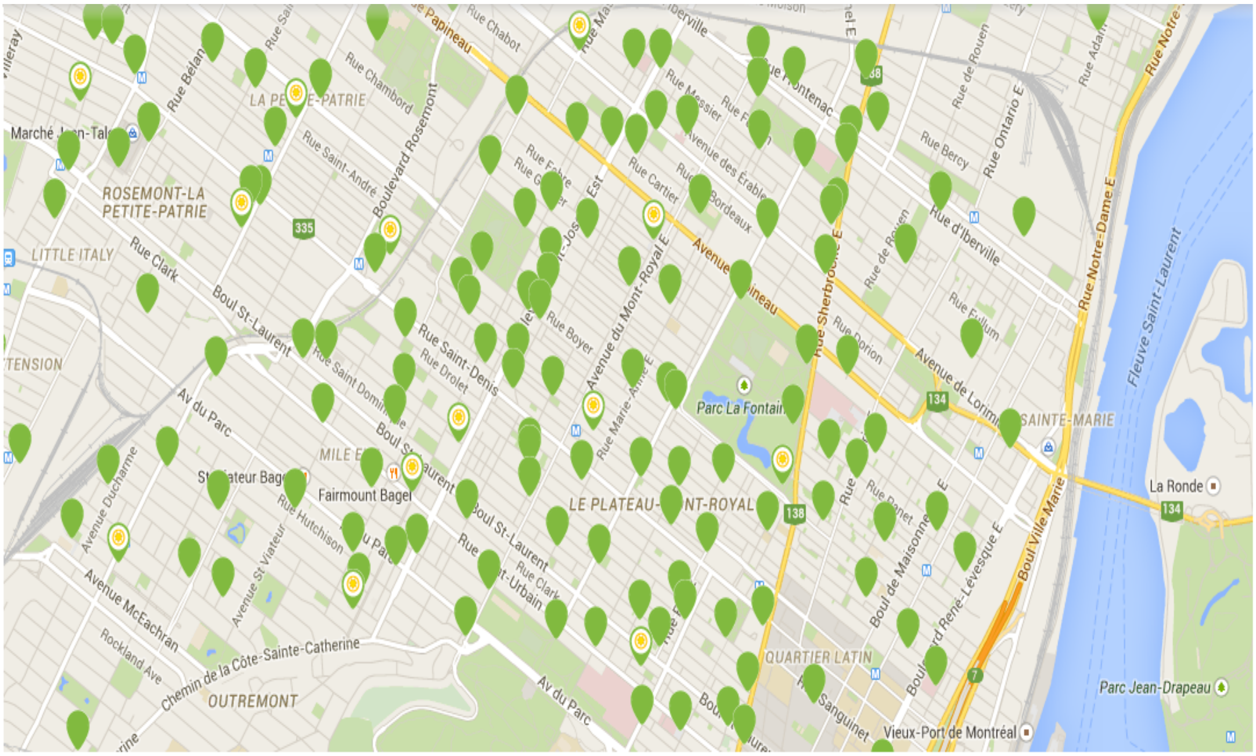


Figure 1: Network map of Communauto's stations across Montreal (Communauto, 2014)

## 1.11. How Communauto works?

Communauto offers a simple process to its members for using their services. It is possible to reserve a car for half-hour, an hour, a day or longer. The vehicles are available without delay, 24/7. There are some steps for this purpose as follows:

1. Register online or in person
2. Reserve a car online (free of charge) or through a phone
3. Pick up the car at the selected station, its self-service
4. Use the car and at the end of your reservation return it to the specified station
5. Pay for the distance you travelled, a detailed invoice is sent to the customers at the end of each month

Communauto pays all the related costs for the normal operations of the required services like purchasing, financing, insurance, registrations, repairs and maintenance, vehicle gas also administrative costs.

The fleets in Communauto are mostly composed of Toyotas Yaris, Nissan Versa, Toyotas Prius C and Note. It also includes 100% electric vehicles Nissan LEAF and Ford Focus**.** All of the vehicles have non-smoking conditions.

All EVs have 5 seats, a gear level and a max speed of 144 km/h. When EVs are fully charged, they can travel over 100 km. When you start the vehicle, the driving range is displayed on the dashboard.

The charging time for a completely depleted battery of an EV in a Communauto station is about 7-8 hours for the Nissan LEAF and 3-4 hours for the Ford Focus.

There is also a Quick Start Guide, inside the glove compartment of all the cars which can simply guide the drivers who want to use the car for the first time.

According to a survey by Communauto (Communauto, 2014), the most important challenges for this company in using electric vehicles are driving range, availability of the charged cars at the time of order and associated costs. These costs are related to electricity, maintenance and purchase costs of electric vehicles.

# 2. Customer Demand Forecasting

In this section, after a literature review on demand forecasting in carsharing, the solution approaches in different categories are explained and their application for customer demand forecasting are illustrated.

## 2.1. Research Reviews

The concept of carsharing as an alternative to private car ownership and use has come into picture in the last decades. Among different parts of the world, Europe, North America, Japan and Singapore were the pioneers in this business. According to Shaheen *et al.* (2006), there were 40 programs deployed in North America, 18 in Japan and 4 in Singapore (Barth *et al.*, 2006). The key concept behind carsharing is to share a small number of vehicles among a large number of people. The vehicles are reserved and used individually as required.

While users are only responsible for the time used and traveled distance and in some cases fueling, the company pays for the vehicle expenses, all repairs, gas and insurance. In cities with large population, carsharing plays a determinant role in lowering pollution, and improving transportation and mobility system (Britton *et al.*, 1999).

Carsharing helps to reduce car ownership. Each vehicle used in carsharing replaces 14.9 private cars which makes carsharing a viable solution to parking management strategy (Millard-Ball *et al.*, 2006). It is also more economical to use shared vehicles rather than personal cars. Based on a simulation research, carsharing has the potential to be profitably economical. Rapid growth of carsharing companies like Zipcar and Flexcar in the U.S. and Canada since 1999 is a proof of this finding (Bart and Todd 1999). Also, there is a direct relationship between the population size and sustainability of the positive effects of carsharing (Nobis, 2006).

According to a survey by Sioui *et al.* (2013), vehicle usage for a person who joins a carsharing organization is significantly less (30% less) than a person who uses his/her personal vehicle. This result comes from a comparison study between two surveys which were done simultaneously in

Montreal area. The first one was a web-based survey targeting customers of carsharings and the second one was a survey targeting household travels in the large-scale.

Communauto is one of the pioneers in carsharing business, especially electric vehciles usage, in Montreal. Lorimier and El-Geneidy (2011), conducted a study for investigating the factors affecting the vehicle usage and availability in Communauto Inc. They developed two regression model namely monthly usage model and availability model and concluded that the size of a carsharing station has a significant impact on both vehicle availability and usage. The larger stations can offer more options to choose between vehicles. Moreover, there is a clear seasonal impact which affects the demand during the year. According to their research, while there is more customer demand for Communauto during the summers, in winters they face lack of vehicles at specific stations due to more one-way trips. The most important factor affecting vehicle usage and availability is customer demands. According to a literature review by Jorge and Correia (2013), demand forecasting in carsharing is a challenging area since the number of available vehicles is vital for responding the customers.

Several researches have tried to identify and estimate the factors affecting the customer demand. Varagouli *et al.* (2005) developed a model to identify and estimate the main variables affecting the travel demand. Their model was based on multiple linear regression analysis. The achieved results from their demand models were within the acceptable range and conceptually reasonable.

Also, Koegst *et al.* (2008) implemented a factor analysis in order to identify relevant parameters in water demand forecasting. They used a multi regression analysis for this purpose and compared the impacts of demographic changes, socio-economic variability and population age structures on water demand forecasting.

Artificial neural network is also widely used in customer demand forecasting studies. Benkachcha *et al.* (2013) defined two model as a base for neural network in order to forecast the future sales of a product. The first model was based on causal methods namely regression and multilayer perceptron-based model and the second model was based on time series and multilayer perceptron. According to their results the performance of the both models were almost the same, however the cost of the prediction method parameter in time series model was lower than the Causal method, so it was preferable according to their results.

Also, Law and Au (1999) represented a new feed-forward neural network model with five layers for demand forecasting of Japanese tourism arrivals to Hong Kong. The performance then is compared with multiple regression, naive, moving average, and exponent smoothing and according to their corresponding (MAPE) mean absolute percentage error the proposed NN model outperforms the other methods.

Exponential smoothing approaches are other approaches which are popularly used for the forecasting studies. Their potential to consider both seasonality and trends in the data are of interest for many researchers. Taylor (2003) recommended a new method for the Holt-Winters exponential smoothing formulation in order to take in to account the both two seasonality cycles, one within the day and the other one within a week.

Abd Jalil *et al.* (2013), compared five exponential smoothing methods namely Standard Holt-winters Exponential Smoothing for Intraday Seasonality and Intraweek Seasonality, HWT Exponential Smoothing**,** Modified Holt-winters Exponential Smoothing by Kotsialos *et al.* (2005) and McKenzie and Gardner (2010) and a modified version proposed by the authors together based on their corresponding MAPE (mean absolute percentage error). They utilized these methods for forecasting lead times for periods from half-hour-ahead to a year-ahead. According to the results HWT Exponential Smoothing method was the best among the four others.

Moving averages including auto-regressive moving averages are also widely utilized for demand forecasting studies. Babu and Reddy (2014) proposed a new hybrid ARIMA–ANN model which is a combination of a linear ARIMA (autoregressive integrated moving average) and nonlinear artificial neural network (ANN) model. It is used in their research to predict the time series data. They compared the results based on their MAE and MSE with ARIMA and ANN models and some existing hybrid models to investigate the accuracy of their proposed model. The hybrid ARIMA–ANN model achieved the best result according to their study.

Mitrea *et al.* (2009) performed a comparison study between different traditional forecasting approaches and neural network. They used Moving Average (MA), Autoregressive Integrated Moving Average (ARIMA), Neural Networks (NN) model as Feed-forward NN and Nonlinear Autoregressive network with eXogenous inputs (NARX) for this study. Their corresponding MSE (Mean Square Errors) results show that forecasting with NN performs better than traditional methods.

Performing such comparison studies as mentioned above are a very important step in oder to find more efficient ways for forecasting customer demands. Without an accurate demand forecasting, optimizing the number of infrastructures, facilities and associated cost is useless since it is the base for facility planning and fleet management studies.

## 2.2. Approaches

Accurate demand forecasting is a vital challenge for carsharing companies, especially when it comes to the electric vehicles. According to the recharging time of the electric vehicles which in most of the cases are more than fuel ones and also limited available facilities for this purpose, determining the right number of customer demand become more vital for carsharing companies (Jorge and Correia, 2013).

There are two main types of demand forecasting practices called qualitative methods and quantitative methods.

Qualitative Methods are based on experience, judgment and expert knowledge, while quantitative Methods are based on data, statistics and numerical methods.

In this thesis, we go through some quantitative methods to predict the demand. Figure 1 shows two of the popular categories of approaches in quantitative forecasting field. We explain each of them in detail and choose some of them according to our needs for this purpose.

We use some approaches from Causal methods (regression forecast and regression forecast with seasonal adjustment) and some from time series categories (neural network and Winter's model).

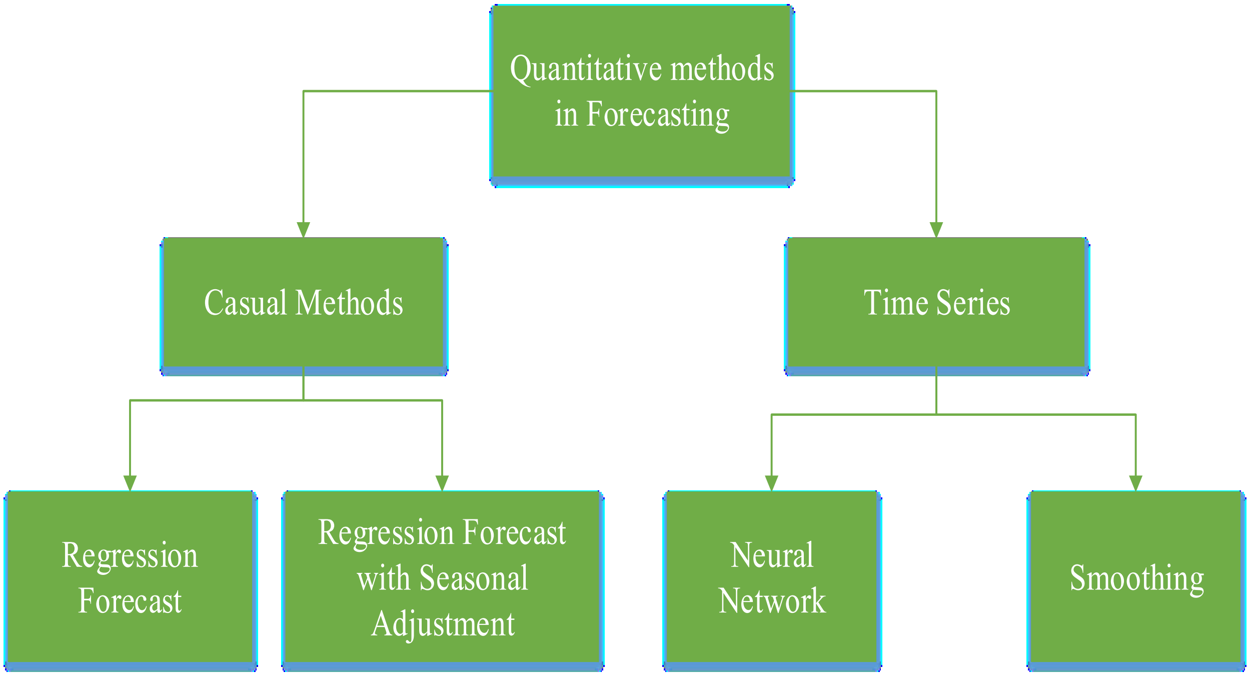


Figure 2: Classification of solution approaches

### 2.2.1. Causal Methods

Causal methods are referred to estimating techniques that assume the forecasted variable (dependent variable) are related to other (independent) variables with a cause-and-effect relationship.

It is used for demand forecasting when it is believed that something has caused the demand to act in a certain way. When demand varies drastically, planned or unplanned events happen in the system. An example of a planned event is a sale or promotion and an example of an unplanned event is a snow storm, severe weather, a strike, or shortage of materials or equipment. Regression forecasting is one of the popular types of Causal forecasting. It can also come with seasonal adjustments which take into account the seasonal impacts as well. Benkachcha *et al.* (2013), Lorimier and El-Geneidy (2011), Varagouli *et al.* (2005), Koegst *et al.* (2008) and Law and Au (1999) are some of the authors who have used the Causal methods in their studies.

#### 2.2.1.1. Regression Forecast

Regression analysis is known as a statistical technique for investigating the relationship between a set of variables and modeling this relationship. If we consider $y$ as a dependent variables and $x$ as an independent variable, then a linear equation relates these two variables to each other.

#### 2.2.1.2. Seasonally Adjusted Forecast

Seasonal adjustment is a statistical method which takes into account the seasonal impact in a set of data to have a more realistic estimate and forecast for the future. Many economic phenomena have seasonal cycles, such as agricultural production and consumer consumption like carsharing business, e.g. greater demands for renting cars in months leading up to Christmas and in spring when the weather is warm enough for short trips.

### 2.2.2. Time series

Time series is referred to a sequence of data points that are typically obtained through successive measurements over a time period. Time series forecasting is performed through defining a model to predict the values for the coming periods based on observed values in the past. Exponential smoothing and neural network are two of the main tools in time series for predicting the future demands. Benkachcha *et al.* (2013), Babu and Reddy (2014) have used time series in their studies.

### 2.2.2.1. Smoothing

In statistics, smoothing is the process of creating an approximating function to capture important patterns that effect the future trend of the data. Two popular types of smoothing algorithms are moving average and exponential smoothing. While moving average can help in diagnosing trends in statistical surveys with repetitive trends, exponential smoothing is a good tool for predicting future values of a time series.

Exponential Smoothing and moving average are two of the main approaches in smoothing category. You can find some explanations regarding both approaches in the following sections.

### Exponential Smoothing

Exponential smoothing is applied to time series data, either to forecast or to produce smoothed data. There are three main types of simple exponential smoothing, double exponential smoothing and triple exponential smoothing. While simple exponential smoothing cannot consider any trend or seasonality, double exponential smoothing takes into account the possible trends of the data set, and triple exponential smoothing considers both trend and seasonality. Taylor (2003) and Abd Jalil *et al.* (2013) implemented exponential smoothing in their forecasting studies.

### Moving Averages

Moving averages can give an overall idea of the current trend in the data. It is calculated by getting the average of any subset of the actual values for the previous periods. However it cannot take in to account the seasonality of the data in time series due to its nature which relies solely on the previous data sets.

It can be calculated for any period of time and is widely used for forecasting projects. Simple moving average and weighted moving average are two popular types of moving average methods. Babu and Reddy (2014), Mitrea *et al.* (2009) and Law and Au (1999) have used this method in their studies.

#### 2.2.2.2. Artificial Neural Network

Neural network structure is a combination of input nodes (independent variables), a number of hidden nodes and an output node. The number of hidden layers is usually determined through try and errors. It has been used successfully to forecast many non-linear time series. It is a general purpose model that has a universal functional approximate (Hornik *et al.* 1989).

Scientific studies have used this method to forecast various non-linear time series (Hill *et al.* 1996, Tang *et al.* 1991 and Zhang 2003). Many recent applications of this model can also be founded in Ghiassi *et al.* (2005) and Hippert *et al.* (2005).

Neural networks are organized though some layers including input layers, output layers and hidden layers. A typical structure of the layer is displayed in Figure 3.

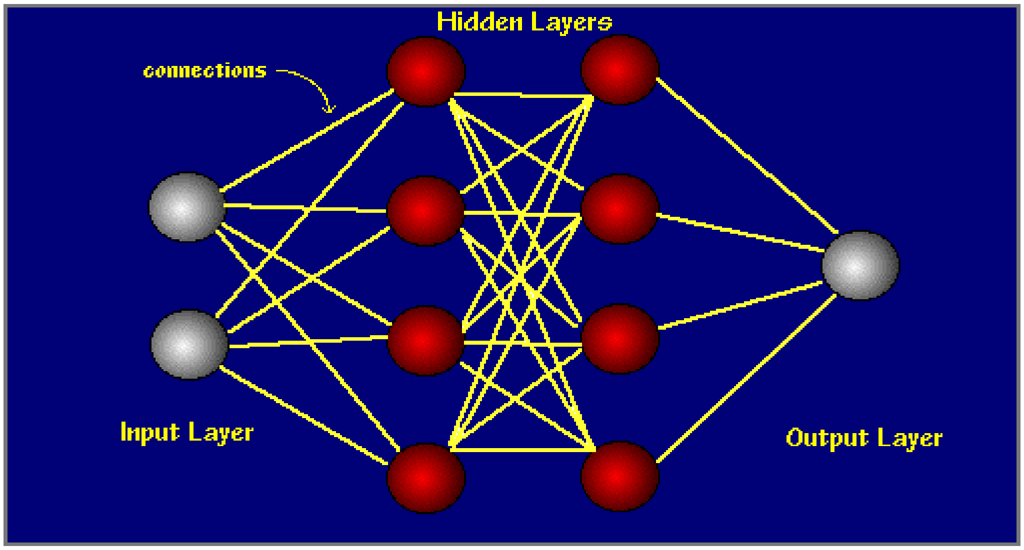


Figure 3: 2 Layer NN model (A basic introduction to Neural Network, 2004)

Benkachcha *et al.* (2013), Babu and Reddy (2014), Mitrea *et al.* (2009) and Law and Au (1999) used this approach in their forecasting studies.

# 3. Recharging Stations Planning and Optimization

In this section, after a literature review on location planning optimization in carsharing, the solution approaches are explained in detail under different categories.

## 3.1. Research Reviews

It is more challenging in electric vehicles than fuel vehicles to manage the facilities, their associated costs and infrastructures in the carsharing industries. A limited number of researches have been done in the field of electric vehicles location allocation and refueling strategies till now. Optimization plays a main role when it comes to fleet management in electric vehicles. Electric vehicles produce very low emissions, but they are still offered in a limited range. They can be used for short term trips and they can be charged at their holding locations. Basically electric vehicles in carsharing systems operating in standard roadways must meet Federal Motor Safety Standards (FMVSS), (Barth & Todd, 1999).

While air pollution, gas emissions and fossil fuels increasing consumption are pushing governments to use more sustainable means of transport, limitations and dilemmas makes it more challenging to use these source of energies for vehicles.

Establishing an accessible recharging system for electric vehicles is a very important factor in encouraging people to use more EVs when choosing which kind of vehicle to rent. As a result, due to critical challenges regarding the installation costs and procurement of infrastructures, developing a comprehensive model for recharging station planning is a concern for both government and researchers (Wang & Lin, 2013).

Alternative-fuel stations availability increases the customer acceptance of the electric vehicles (Kim & Kuby, 2012). In this study, a MIP model was developed for optimizing the location planning of hydrogen refueling stations. Their model maximized the number of flows which can be refueled in the network. It is assumed in their model that the customers do not deviate from their pre-planned path in order to refuel their vehicle. They assumed that there is an upper tolerance limit for drivers to deviate from their original path.

There are several algorithms in order to solve optimization and mathematical problems. While, some of them like branch and bound and branch and cut algorithms give exact solutions to the problems, some others like heuristic algorithms give local optimum solutions that in many cases are close to the exact optimum solutions. While some studies have used exact algorithms, some other have preferred to use heuristic algorithms and in some case multiplication of different heuristics in order to solve optimization problems. For large scale cases, while the exact approaches take long time to solve the problem, heuristics take shorter time to solve the problem. Also they are good substitute when there is no exact solution for a kind of optimization problems.

Careful planning is a must to locate the stations in the nearest distance from customers' needs (Wang, 2011).

Fontana (2007) solved a routing problem for electric vehicles through robust optimization, while taking in to account uncertainty, congestion and energy constraints. Wang (2007) considered the charging time as a decision variable in order to calculate the minimum required charge to complete the journey. It can be defined based on the required charge that each car needs to complete the journey. On the other hand, Wang (2008) and Wang & Lin (2009) used the available charge and used added charge at a station as a substitute to recharging time variables. None of them considered the percentage of flow coverage on the base of different available budgets. All of these researches have used branch and bound algorithm to solve the location planning problems.

Wang and Lin (2013) assumed all three types of recharging stations, including slow-recharging stations, fast-recharging stations and battery-exchange stations. Furthermore they formulated a separate optimization problem to maximize the flow coverage based on the available budget. They also used branch and bound algorithm.

Exact algorithms are also used in Labbe *et al.* (2004). They implemented a branch and cut algorithm for solving a plant cycle location problem. They developed a MILP problem with the objective of minimizing the total cost. According to their result, the branch and cut algorithm in their problem could solve up to 120 customers and 16 potential plants in a real-world data set.

Also, Catanzaro *et al.* (2011) solved a Partitioning-Hub-Location-Routing Problem through developing an Integer Programming Model (IP Model) and solved the problem with branch and cut algorithm. According to their computational experiments, the proposed approach was able to solve up to 20 vertices of instances in PHLRP problem.

Heuristic algorithms are also widely used in recent location planning studies. Kuby and Lim (2004) assumed multiple combinations of refueling facilities. Kuby and Lim (2010) solved a proposed MILP problem using three different heuristics algorithms. According to their results, all of these methods perform fairly well, but they give only local optimum solutions. They compared two cases of basic scenario in which each vehicle passes from each station at most once and extended scenario in which vehicles can visit each node more than once as well.

Duron *et al.* (2000) used heuristic algorithms to find the stations with more probability of running out of cars based on the current status of the system.  The first algorithm was to use a truck with a fixed circuit to cover the lack of vehicles in the system. The objective of the system was to minimize the number of not responded demands.  The second algorithm which named reactive algorithm does not consider the circuit fixed, but is designed to visit the next station by the truck based on the state of the whole system. According to the results, the second algorithm works much better than the first algorithm in terms of minimum not satisfied demands.

Genetic algorithm is one of the popular heuristic methods used in this research area. Tohyama *et al.* (2011), developed a model for incapacitated facility location planning and solved it through genetic algorithm. They have solved an NP_hard problem through a genetic algorithm model based on a rough estimation of a of optimal facility locations. The estimation is performed based on the ratio of the facility placement cost and the cost to users of the facility. The mutation function then searches the solution spaces with more probable good solution. In this way the whole space is being searched more efficiently. A comparative study with other proposed methods in the past are done in order to verify their methods performance.

According to Lim and Kuby (2010), since in real cases there is a need for combinations of stations, it may be impractical to generate and solve the facility location problem with MILP methods. Therefore, they developed some heuristic algorithms for locating the refueling stations in carsharing industry. They compared the performance of greedy-adding, greedy-adding with substitution and genetic algorithm and implemented them for a case study in Florida.

Along with theses algorithms, there are some other approaches which are popularly used in location planning as well. As an example, Di Febbraro *et al.* (2012) proposed the complex dynamics of the system, using a discrete event system simulation. They formulated a Discrete Event Model for this purpose which was based on a user-based methodology in order to achieve an optimal relocation policy. This methodology also reduces the number of required staff to relocate vehicles and identifies the minimum number of vehicles needed to respond system demands.

Also, Ion et al (2009) used a hybrid approach to select the station sites for electric vehicles of a carsharing service. The if-then rules were used to aliment a fuzzy logic through modeling the customers' preferences. Preferences of habitants of each urban area were considered in this study.

While there have been some studies which have proposed some models to achieve a solution for location allocation problem of recharging stations for electric vehicles, very few have tried to do a comparison study to figure out which approach gives better solution than the others. Proposing a new optimization problem for these sort of problems may not be a new achievement in this area, since there are lots of optimization problems that are proposed for this purpose and they are in some cases more or less similar to each other and even adaptable to other real world cases as well. In this thesis, after a forecasting study, we go through some comparative studies in order to find the best approach to solve this problem. Considering the model proposed in Wang and Lin (2013) as a base, we implement our proposed model after some editions on the base model through an exact algorithm and a heuristic algorithm and compare the results.

## 3.2. Approaches

Fleet management has been a concern for carsharing companies for many decades. There are many ways used to manage and optimize the location planning of facilities in carsharing industry. Some of the main ones are optimization approaches, empirical studies and simulation. In Figure 4, the different categories of solution approaches to solve fleet management problems, are shown through a chart.

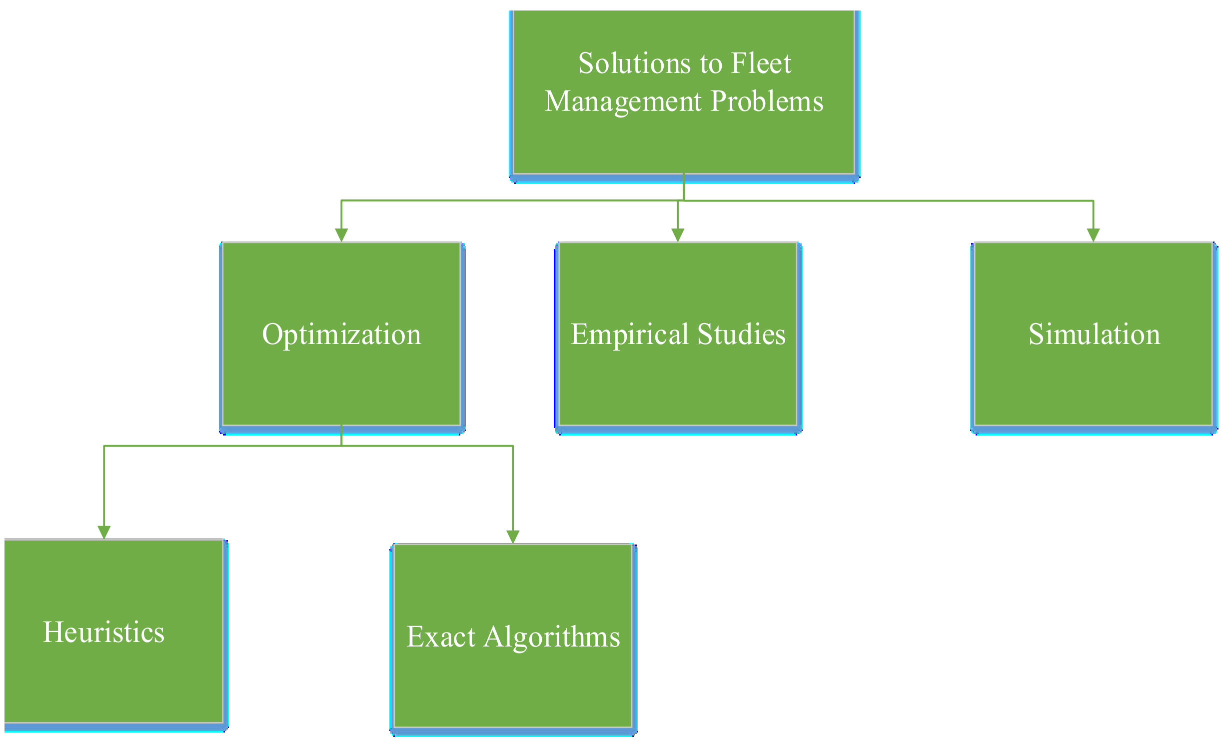


Figure 4: Fleet Management solution approaches

### 3.2.1. Simulation

Simulation is a tool used for optimization. Its history backs to as old as 1979 (Bonsall, 1979). It is a method to mimic the real system behavior. Most of the time it is done using a software which is designed for this purpose. It helps to examine the real systems in a safe, speedy way without interrupting the actual process. There are three dimensions for classifying the simulation studies including Static vs Dynamic, Continuous vs Discrete and Deterministic vs Stochastic.

While in static model, time is not considered as a variable, in dynamic model it has an important role as a variable. In a discrete simulation model, occurrence of an event is a discrete happening, while in a continuous system, it is a continuous process over time. Finally in deterministic type of model, there is no random factor or input, while in the stochastic models, there are always random inputs or parameters which bring probability and uncertainty to the model (Kelton, 2010). Di Febbraro *et al.* (2012), Barth, M., & Todd, M. (1999), Bonsall, P.W. (1979), El Fassi *et al.* (2012 are some of the recent studies who have used simulation for carsharing organizations.

### 3.2.2. Empirical Studies

Empirical studies refer to a method of gaining knowledge through direct and indirect observations or experiments. It can be analyzed quantitatively and qualitatively. A researcher answers the empirical questions through quantifying the evidence or making sense of it in a qualitative form. Combination of quantitative and qualitative studies is a better way to answer the questions which cannot be studied in laboratories, particularly in the education and in social sciences.

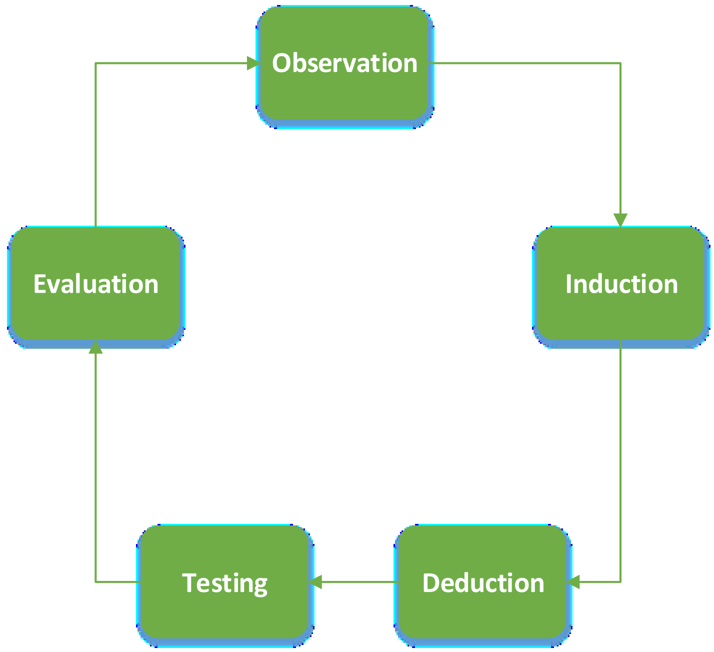


Figure 5: Groot's empirical cycle

According to Figure 5, Groot's empirical cycle (Empirical Research, 2014) is as follows:

1. Observation: Collecting, organizing and forming hypothesis based on the facts.
2. Induction: hypothesis formulation.
3. Deduction: Reducing hypothesis consequences in the form of testable predictions.
4. Testing: Using the new empirical material for testing the hypothesis.
5. Evaluation: Testing outcome evaluation

Arostegui *et al.* (2006), Yeh, S. (2007), Yu *et al.* (2013) are some of the recent works who have used empirical analysis for solving facility location problems and carsharing businesses.

### 3.2.3. Optimization

Optimization or mathematical programming is referred to the process of finding the best feasible values for some objective functions (maximizing or minimizing), while satisfying some given domains or constraints.

Some literatures divide optimization in to two categories of algorithms including exact algorithms and heuristic algorithms.

### 3.2.4. Heuristics

Heuristics approaches are algorithms that find local optimum solutions among all possible ones, but they do not have any guarantee to find the best possible solution. Therefore they may be considered as approximately accurate algorithms. They are typically used for solving real-life issues because of their acceptable speed and ability to solve large instances. Lim & Kuby (2010), Tohyama *et al.* (2011) and Arostegui *et al.* (2006), Yang *et al.* (2007) and Zhang *et al.* (2006) have used some sorts of heuristic approaches for facility location problems in their works.

### Genetic Algorithm

Genetic Algorithm is one of the popular heuristic algorithms which fins local optimum solutions near the global optimums. It is an evolutionary algorithm that is considered as a subset of artificial intelligence. It is a search heuristic (sometimes called metaheuristic), that mimics the natural selection process of genes. The techniques used in this method are inspired by natural evolution, like inheritance, mutation, selection, and crossover.

It is one of the heuristic algorithms that can give a locally optimized solution to optimization problems. Genetic algorithm continuously modifies a population of individual solutions. In each iteration, the genetic selects individuals from the current population based on a cross over definition to be parents and perform the crossover and mutation to produce off-springs for the next generation.

After successive generations, the generated population evolves to an optimal solution. It can be applied to the mixed integer programming problems, while some components are restricted to be integer-valued (Wikipedia: Evolutionary algorithm, 2014). Lim & Kuby (2010), Tohyama *et al.* (2011) and Arostegui *et al.* (2006) have used genetic algorithm approach for their location planning optimization problems.

### 3.2.5. Exact methods

Exact methods are known as approaches which find the exact optimum solutions for optimization problems. They guarantee to find the best possible response for the problem in case that there is enough time and space.

A simple enumeration like what is done in many other approaches is out of question, so exact methods must utilize more clever approaches (Wikipedia: Cutting Plane Methods, 2014).

Hewitt *et al.* (2010), Li *et al.* (2011) and Nagy, and Salhi (2007) have used some sorts of exact algorithms in their studies.

### Branch and Cut Algorithm

Branch and cut algorithm is an exact algorithm that gives the global optimum solution for a problem. It is widely used for solving integer programming problems. It consists of a combination of a cutting plane method and branch and bound algorithm. The procedure involves running a branch and bound algorithm and applying cutting planes to tighten the LP problem relaxations of the main MILP problem. It is a very successful method in solving MIP problems and guarantees the optimality in the solutions.

It is not easily possible to solve a general integer programming problem efficiently through only using a cutting plane method. Therefore it is necessary to branch the problem which results in branch and cut algorithm. Also a pure branch and bound algorithm can be sped up using a cutting plane algorithm. So, a combination of them would give a boost to the efficiency and speed of the solution approach. Labbe *et al.* (2004), Catanzaro *et al.* (2011) and Rodriguez-Martin *et al.* (2014) have implemented branch and cut algorithm for solving their facility location problems.

In the following paragraphs, the branch and bound algorithm and cutting plane method are described in details:

## Branch and Bound Algorithm

Branch and bound algorithm starts with modeling the solution space as a tree and traverses through the tree to explore the most promising subtrees. This is called branching. Bounding is done through computing the upper and lower bounds for checking the optimum value of the objective function in each of the subtree in each iteration. It is continued till the point that either there are no more subtrees to break the problem or if it is continued, the later responses will be inferior.

This method was first suggested by A. H. Land and A. G. Doig in 1960. The name first was introduced in a work by Little *et al.* on traveling salesman problem in 1963. Goel *et al.* (2006), Verter (2011) are some of the recent works who have used branch and bound algorithm for solving location allocation problems.

## Cutting plane Algorithm

Cutting Planes Algorithm is one of the exact methods that are used for integer programming problems. They have been proven to be a very useful approach from the computational aspect, especially when it is combined with branch and bound algorithm.

This method works through solving some sequences of linear programming relaxations of the main integer programming problem. The relaxations in some steps, improve in order to give better approximation to the integer programming model, at least in the neighborhood of the optimum solution.

It has been used to solve various integer programming problems including traveling salesman and maximum cut problems.

It is very popular to use cutting planes algorithm in order to solve mixed integer programming problems (MILP), as well as general, not necessarily differentiable convex optimization models. Applying cutting planes method to MILP was first introduced by Ralph E. Gomory and Václav Chvatal in 1990 (Wikipedia: Cutting Plane Methods, 2014). Laporte *et al.* (1986) and Gollowitzer *et al.* (2013) have used this algorithm in their location planning researches.

In this thesis we are going to improve a MILP problem proposed by Wang & Lin (2013), and then we implement it through two different approaches to compare the results both with each other and with the Wang & Lin (2013).

Two approaches selected for this purpose are as follows:

1. Using CPLEX through AIMMS software which gives a global optimum solution through using a combination of cutting planes algorithm and heuristics.
2. Genetic Algorithm which heuristically searches for a local optimum solution near to the global optimum

# Chapter 3:

# Solution Approach

In this chapter, we present comparative evaluation of four approaches for customer demand forecasting and apply it on the data from Communauto Inc. In the second part, location planning of recharging stations is performed, descriptions regarding the initial assumptions of the problem are given, the improved version of it is presented and taken other solution approaches to solve the problem is provided.

## 1. Customer Demand Forecasting

In this section we present the four methods considered for demand forecasting in our study.

### 1.1. Solution Approaches

In this section some methods which are popularly used in demand forecasting are explained in detail and their application is illustrated.

#### 1.1.1. Simple Linear Regression

Regression analysis is known as a statistical technique for investigating the relationship between a set of variables and modeling this relationship. If we consider $y$ as a dependent variables and $x$ as an independent variable, then the linear equation relating these two variables to each other is as follows:

$$y = a + bX$$

where $a$ is the intercept and $b$ is the slope. In this equation $Y$ is a predicted variable like demand in supply chain and logistic issues, and $X$ is a predictor variable like time sequences in time series which regression is used to predict their behavior.

Estimation of $a$ and $b$:

The least squares method is used to estimate slope and intercept.

$$b = \frac{\sum_{i=1}^{n} x_i\, y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}} \quad \text{and} \quad a = \bar{y} - b\bar{X}$$

In this thesis MS Excel is used for these calculations.

### 1.1.2. Seasonally Adjusted Forecast

Seasonal adjustment is a statistical method which takes in to account the seasonal impact in a set of data to have a more realistic estimate and forecast for the future. Many economic phenomena have seasonal cycles, such as agricultural production and consumer consumption like carsharing business, e.g. greater demands for renting cars in months leading up to Christmas and in spring when the weather is warm enough for short trips. MS Excel is used for the related calculations as well.

Seasonal adjustment in regression forecasting follows the same steps as regression forecasts, but there is a need to calculate the seasonal indices for each of the seasonal cycles. Steps include:

1. (Actual / Forecast) for each of the seasonal cycles for each repetition of the seasonal cycle
2. Average of the values calculated from step 1 for each of the seasons
3. Multiplying the calculated seasonal index in to the forecasted value from simple regression forecast to get the seasonality adjusted forecast

This method can be usually used in forecasting when there is a trend of seasonality in the data that needs to be considered in regression forecasting.

### 1.1.3. Exponential Smoothing

Exponential smoothing is applied to time series data, either to forecast or to produce smoothed data. There are three main types of simple exponential smoothing, double exponential smoothing and triple exponential smoothing which are explained as follows.

### Simple exponential smoothing

Exponential smoothing was proposed by Robert Goodell Brown in 1956. The simplest form of simple exponential smoothing is given by the formula:

$$F_{t+1} = F_t + \alpha(D_t - F_t)$$

where:

$F_t$ = Forecast for the current period *t*

$D_t$= Actual demand for the current period *t*

$\alpha$ = smoothing factor, and $0 < \alpha < 1$.

### Double Exponential Smoothing (Holt's Model (with trend))

Simple exponential smoothing does not work well when there is a trend in the data. Double exponential smoothing is used in such situation. The basic idea behind this method is to include a term in order to take in to account the possibility of existence of any kind of trends in the data.

One method, sometimes referred to as "Holt's double exponential smoothing model is presented as follows:

$$T_{t+1} = \beta * (F_{t+1} - F_t) + (1 - \beta) * T_t$$

while *F* comes from simple exponential smoothing, $\beta$ is the trend factor in this model.

### Triple Exponential Smoothing (Winter's Model with Trend and Seasonality)

When we are dealing with a set of data which includes seasonal changes as well as trends, Holt's model cannot respond to our forecasting needs, so one should use triple exponential smoothing which takes into account seasonal changes as well as trends.

Seasonality is in fact the tendency of time-series data to show a behavior that repeats every L periods.

This method calculates a trend line for the data, also includes seasonal indices. These indices weight the values in a trend line according to the place the time point falls in the cycle of length.

$$L_t = \alpha\left(\frac{D_t}{S_{t-s}}\right) + (1-\alpha)(L_{t-1} + T_{t-1}) \qquad S_t = \gamma\left(\frac{D_t}{L_t}\right) + (1-\gamma)S_{t-s}$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \qquad F_{t+m} = (L + mT_t) * S_{t+m-s}$$

## 1.1.4. Moving Average

Moving average can give an overall idea of the current trend in the data. It is calculated by getting the average of any subset of the actual values for the previous periods. It can be calculated for any period of time and is widely used for forecasting projects. Simple moving average and weighted moving average are two popular types of moving average methods. The weights in weighted moving average method can be defined in different ways according to the specialists' point of view.

Simple moving average: $F_{t+1} = \frac{D_t + D_{t-1} + \cdots + D_{t-n+1}}{n}$

Weighted Moving Average: $F_{t+1} = \frac{W_t * D_t + W_{t-1} * D_{t-1} + \cdots + W_{t-n+1} * D_{t-n+1}}{W_t + W_{t-1} + \cdots + W_{t-n+1}}$

### Why Winter's Model?

According to the applications of the winters model and nature of carsharing demand which is a data that is affected by seasonality and any possible trends, we chose to use this method of forecasting among other two forecasting methods in time series exponential smoothing models and moving average model. Since, these factors are considered in Winter's model more professionally, it is the approach which can cover any possible trend and/or seasonality. MS Excel is used for the related calculations.

### 1.1.5. Artificial Neural Network

Neural network model has been used successfully to forecast many non-linear time series. It is on the other hand a general purpose model that is as a universal functional approximator (Hornik *et al.* 1989).

An artificial neural network is a structure that is designed to solve certain types of problems by attempting to emulate the way the human brain would solve the problem. The general form of a neural network is a black box type of model that is often used to model high-dimensional nonlinear data. Typically, most neural networks are used to solve prediction problems for some systems.

Scientific studies have used this method to forecast various non-linear time series (Hill *et al.* 1996, Tang *et al.* 1991 and Zhang 2003). Many recent applications of this model can also be founded in Ghiassi *et al.* (2005) and Hippert *et al.* (2005).

Neural networks are organized though some layers including input layers, output layers and hidden layers. They are made up of nodes which are interconnected through an 'activation function'. Input layers present a pattern to the whole network, including the hidden layers, where the main processing is done though a system of connections whish are weighted in a basic initialization.

ANNs usually contain a form of learning rule, which is used to modify the weights of the links according to the patterns that the input layers present to the network.

They learn through following their biological counterparts; which refers to a learning rule or a learning algorithm.

For neural network, we have used the Matlab toolbox which uses the Levenberg-Marquardt algorithm as a learning rule. We defined different portions of training data set and testing & validation data set which gives different MSEs indicating different errors. We have used 90/10, 80/20, 70/30, 75/25, 60/40 and 50/50 randomly divided portions for training, testing & validation data sets respectively.

Based on the actual data set of the years 2011 and 2012 for the first 9 months of Communauto Company, random data generation was performed for the years between 1994 and 2011 for the first 9 months of each year. For this purpose we used the randbetween function in MS Excel using the 2011 and 2012 data as the lower bounds and the upper bounds.

The objective of this forecasting practice is to find the method with the least forecasting error that gives the most accurate forecasted data set for the next years.

For this purpose, we have used four different methods of forecasting including linear regression, seasonally adjusted forecasting method, Winter’s model and neural network. In each method after implementing, the forecasting error is measured through some performance evaluation methods including MSE, MAD and RMSE.

## 1.2. Performance Evaluation

We have used two indicators of evaluation for each forecast method including MSE, and RMSE. MSE and RMSE are widely used approaches for forecasting approaches performance evaluation.

- **Mean Square Error**

Mean square error is one of the common methods of error measurements used to estimate the accuracy of the forecasting methods, especially in business analysis and supply chain management.

(Et) = Actual demand (Dt) – Forecast (Ft) MSE= $\frac{\sum (E_t)^2}{n}$

- **Mean Absolute Deviation**

The mean absolute deviation for a set of data is equal to the average of absolute values of distances between each data value and the mean.

MAD= $\frac{\sum |E_t|}{n}$

- **Root Mean Square Error**

RMSE is a method used to measure the differences between values predicted through a model or an estimator and actual values which are observed. It is the square root of the MSE.

RMSE= $\sqrt{\frac{\sum (E_t)^2}{n}}$

A comparative evaluation of the three forecasting methods on Communauto data using these three indicators is provided in the next chapter.

# 2. Recharging Stations Planning Optimization

Optimization (also referred to mathematical programming or mathematical optimization) is the process of selecting the best set of elements out of some available sets of alternatives with regard to some criteria.

More generally, in applied mathematics concepts, optimization consists of finding the best available values for some objective functions (maximizing or minimizing), while satisfying some given domains or constraints.

In this thesis, we are going to improve a MIP problem proposed by Wang & Lin (2013), and then implement it through two different approaches, compare the results with each other, and with the Wang & Lin (2013).

While there have been a number of papers which have proposed a mathematical problem for optimizing the carsharing services for electric vehicles, there are less who have compared different approaches to achieve a more reliable answer. In this thesis we try to cover this gap through a comparison study to find the best approach.

Two approaches for this purpose have been selected:

1. MIP model development using AIMMS software which uses CPLEX to give an optimum value for the problem
2. Genetic Algorithm which heuristically searches for a local optimum solution near to the global optimum

## 2.1. Locating multiple types of recharging stations for battery-powered electric vehicle transport (Wang & Lin, 2013)

In the beginning, we start with explaining the mathematical problem proposed by Wang & Lin (2013) in order to improve it and use the modified model as a base for the comparison study. The reason to choose the proposed optimization model in this research was that the author found this models comprehensive enough in order to be applied to other case studies as well. In their model, different combination of recharging technologies based on their recharging rate and locating cost, along with different combination of paths and nodes can be considered. So, there is enough flexibility in chosen models in order to apply them for other cases as well.

Wang and Lin (2013) proposed a mixed integer programming for maximizing the available carsharing services for the customers for a carsharing company. They proposed their model specifically for the electric vehicles which have the challenge of having enough charge for desired distances according to the limitations of installing electricity recharging stations in the right places in order to be accessible for the users.

They solved the problem through a LINGO's BRANCH and BOUND method (Thornburg and Hummel, 2003). The results have been brought in the application results section in order to make the comparison study.

There are two models proposed in this study. The first one is to solve a MIP problem which minimizes the installation cost of the recharging stations according to different locating capacities. This model consists of four sets of variables including 1 binary, 1 integer and 2 continuous.

The second model is also a MIP problem that maximizes the number of covered flows in the model. The output of the first model is the optimum number/local optimum number of locating capacity.

It brings the minimum cost that is used as an input in this model in order to find the percentage of covered flows based on the different budgetary constraints.

The second model consists of five sets of variables including 2 binaries, 1 integer and 2 continuous. In this problem, we have *k* kinds of charging stations with different charging rates and installation costs.

These models were implemented on a project for the recharging station planning in an Island called Penghu, located in Taiwan.

The key elements used in these models are as follows:

**Sets:**

*p*: Paths

*k*: Recharging station types

*i,j*: Nodes or locations

**Variables:**

*F(p)*: binary, 1 if flows in path *p* is covered, 0 otherwise

*Y(p,i,k)*: binary, 1 if a vehicle is recharged using a type *k* station at node *i* on path *p*, 0 otherwise

*X(i,k)*: Integer variable, the number of type *k* stations located at node *i*

*B(i,p)*: Non-integer variable, available range (km) in the battery at node *i* on path *p*

*R(i,p)*: Non-integer Variable, increased range (km) which is gained by being charged at node *i* on path *p*

**Parameters:**

*W(p)*: Flows per path

*t(i,p)*: time of being charged at node *i* on path *p*

$\beta(k)$: Recharging rate for each kind of recharging stations (km/min)

$\gamma$: Battery's maximal charge (km)

$L$: Available Budget

$U$(i,k): the maximum number of type $k$ stations that can be located at node $i$

$C(i,k)$: locating cost of installation of a type $k$ recharging station at node $i$

$d(j,p,i)$: distance between nodes $i$ and $j$ on path $p$

*S(k)*: Vehicle sharing of a type $k$ recharging station

## ➢ 1st Model: Minimizing the installation cost of the recharging stations

(1) $\quad Minimize \sum_{i\in N}\sum_{k\in K} C_i^k X_i^k$

*Subject to*

(2) $\quad B_j^p = (B_i^p + R_i^p) - d_{ij}^p \qquad \forall i,j\in N,\ \forall p\in P$

(3) $\quad R_i^p \le \gamma - B_i^p \quad \forall i\in N, \forall p\in P$

(4) $\quad R_i^p \le \sum_{k\in K}(Y_{ik}^p t_i^p \beta^k) \quad \forall i\in N, \forall p\in P$

(5) $\quad \sum_{k\in K} Y_{ik}^p \le 1 \quad \forall i\in N, \forall p\in P$

(6) $\quad S^k X_i^k \le \sum_{p\in P}(Y_{ik}^p w^p) \quad \forall k\in K, \forall i\in N$

(7) $\quad X_i^k \le u_i^k \quad \forall k\in K, \forall i\in N$

(8) $\quad \sum_{k\in K}\sum_{i\in N} C_i^k X_i^k \le \ell$

(9) $\quad Y_{ik}^p \in \{0,1\} \quad \forall k\in K, \forall i\in N, \forall p\in P$

(10) $\quad X_i^k, B_i^p, R_i^p \ge 0 \quad \forall k\in K, \forall i\in N, \forall p\in P$

- **$2^{nd}$ Model: Maximizing the flow of electric vehicles passing each of the paths per hour:**

(1) $Maximize \sum_{i \in N} F^p w_i^p$

*Subject to*

(2) $B_j^p = (B_i^p + R_i^p) - F^p d_{ij}^p \quad \forall i, j \in N, \forall p \in P$

(3) $R_i^p \le \gamma - B_i^p \quad \forall i \in N, \forall p \in P$

(4) $R_i^p \le \sum_{k \in K} (Y_{ik}^p t_i^p \beta^k) \quad \forall i \in N, \forall p \in P$

(5) $R_i^p \le F^p \gamma \quad \forall i \in N, \forall p \in P$

(6) $\sum_{k \in K} Y_{ik}^p \le 1 \quad \forall i \in N, \forall p \in P$

(7) $S^k X_i^k \le \sum_{p \in P} (Y_{ik}^p w^p) \quad \forall k \in K, \forall i \in N$

(8) $X_i^k \le u_i^k \quad \forall k \in K, \forall i \in N$

(9) $\sum_{k \in K} \sum_{i \in N} C_i^k X_i^k \le \ell$

(10) $F^p, Y_{ik}^p \in \{0,1\} \quad \forall k \in K, \forall i \in N, \forall p \in P$

(11) $X_i^k, B_i^p, R_i^p \ge 0 \quad \forall k \in K, \forall i \in N, \forall p \in P$

### 2.1.1. Improved Model

In the improved version, we have added one more constraint (constrain 7 in the minimization and constraint 9 in maximization) which limits the number of recharging stations built at each node to the number of consumed charging station which are really needed for each station. In this way, the approach which is trying to find a solution does not substitute any value for the X (number of charging stations) other than the required number of them which were consumed by the customers. While in first model, X is a variable which can take any number between 0 and U (the locating capacity at each node) only if it satisfies the other constraints even at the max level, in the improved version it is limited to the required number of charging stations according to when there is a need for that (when Y=1). Especially in heuristic algorithms, the process of finding the local optimum solution can face more challenges because there is only an upper bound limit for the variable X. Improved version can avoid many occasions when randomly the value of X increases in the feasible criteria and makes the locating cost increase unnecessarily.

## Revised 1st Model:

(1) $\quad Minimize \sum_{i \in N} \sum_{k \in K} C_i^k X_i^k$

*Subject to*

(2) $\quad B_j^p = (B_i^p + R_i^p) - d_{ij}^p \qquad \forall i, j \in N, \ \forall p \in P$

(3) $\quad R_i^p \leq \gamma - B_i^p \quad \forall i \in N, \forall p \in P$

(4) $\quad R_i^p \leq \sum_{k \in K} (Y_{ik}^p t_i^p \beta^k) \quad \forall i \in N, \forall p \in P$

(5) $\quad \sum_{k \in K} Y_{ik}^p \leq 1 \quad \forall i \in N, \forall p \in P$

(6) $\quad S^k X_i^k \leq \sum_{p \in P} (Y_{ik}^p w^p) \quad \forall k \in K, \forall i \in N$

(7) $\quad X_i^k \leq u_i^k \quad \forall k \in K, \forall i \in N$

(8) $\quad \sum_{k \in K} \sum_{i \in N} C_i^k X_i^k \leq \ell$

(9) $\quad Y_{ik}^p \in \{0,1\} \ \ \forall k \in K, \forall i \in N, \forall p \in P$

(10) $\quad X_i^k, B_i^p, R_i^p \geq 0 \ \ \forall k \in K, \forall i \in N, \forall p \in P$

$+$

$$\boldsymbol{X(i,k) = \sum_{p} Y(p,i,k)}$$

**Revised 2nd Model:**

(1) $\quad Maximize \sum_{i \in N} F^{p} w_{i}^{p}$

*Subject to*

(2) $\quad B_{j}^{p} = (B_{i}^{p} + R_{i}^{p}) - F^{p} d_{ij}^{p} \qquad \forall i, j \in N, \ \forall p \in P$

(3) $\quad R_{i}^{p} \leq \gamma - B_{i}^{p} \quad \forall i \in N, \forall p \in P$

(4) $\quad R_{i}^{p} \leq \sum_{k \in K} (Y_{ik}^{p} t_{i}^{p} \beta^{k}) \quad \forall i \in N, \forall p \in P$

(5) $\quad R_{i}^{p} \leq F^{p} \gamma \quad \forall i \in N, \forall p \in P$

(6) $\quad \sum_{k \in K} Y_{ik}^{p} \leq 1 \quad \forall i \in N, \forall p \in P$

(7) $\quad S^{k} X_{i}^{k} \leq \sum_{p \in P} (Y_{ik}^{p} w^{p}) \quad \forall k \in K, \forall i \in N$

(8) $\quad X_{i}^{k} \leq u_{i}^{k} \quad \forall k \in K, \forall i \in N$

(9) $\quad \sum_{k \in K} \sum_{i \in N} C_{i}^{k} X_{i}^{k} \leq \ell$

(10) $\quad F^{p}, Y_{ik}^{p} \in \{0,1\} \ \ \forall k \in K, \forall i \in N, \forall p \in P$

(11) $\quad X_{i}^{k}, B_{i}^{p}, R_{i}^{p} \geq 0 \ \ \forall k \in K, \forall i \in N, \forall p \in P$

$$\boldsymbol{X(i,k) = \sum_{p} Y(p,i,k)}$$

## 2.2. Solution Approaches

In this section some methods which are popularly used in location planning optimization are explained in detail and their application is illustrated.

## 2.2.1. Branch & Cut Algorithm

Branch and cut algorithm is an exact method that guarantees the global optimum solution for a problem. It has been very successful in solving integer programming problems. This method consists of a combination of a cutting plane method and branch and bound algorithm.

It works through solving a sequence of LP relaxations of the main MIP problem. Cutting plane method improves the relaxation of the problem to more close or approximate integer values for the main integer programming problem. Branch and bound algorithm proceeds by divide-and-conquer procedure to achieve the optimum solution.

It solves the linear program without the integer constraints through regular simplex algorithm. After achieving an optimal solution, if there is a non-integer value for a variable which was supposed to be integer, a cutting plane algorithm is used to find more linear constraints that satisfies the feasible integer points. At this step, the branch and bound part starts. It splits the problem in to two (or more) versions. The new LP programs are again solved by simplex method and the process continues.

Perhaps the most popular branch and cut algorithm is the one that has been used to solve traveling salesman problems.

### 2.2.1.1. Software's Used

### AIMMS

**Advanced Interactive Multidimensional Modeling System** or **AIMMS** is referred to a software system which is used for modeling and solving large-scale scheduling and optimization problems. AIMMS Supported solvers include CPLEX, Gurobi, MOSEK, CBC, Conopt, MINOS, IPOPT, SNOPT, KNITRO and CP Optimizer.

AIMMS has one of the five most important algebraic modeling languages (besides AMPL, GAMS, LINDO/LINGO, and MPL) and its creator (Johannes J. Bisschop) has been awarded by INFORMS impact prize for his contribution in this language.

AIMMS is a flexible platform which is suited exceptionally for supply chain, production and scheduling and many other optimization challenges.

AIMMS has links to many powerful solvers that allow users to solve problems of all major mathematical programming types such as linear programming, mixed-integer programming and nonlinear programming (AIMMS, 2014).

## CPLEX:

AIMMS goes through CPLEX for solving MILP problems. For the problem we are seeking to solve here, CPLEX is a suitable option. It is a high level performance solver for Linear Programming problems (LP), Mixed Integer Programming problems (MIP) and Quadratic Programming problems.

CPLEX branch-and-bound algorithm for solving mixed-integer programming problems advantages from modern methods like cutting planes and heuristics to find integer solutions. Sometimes it is also called branch-and-cut algorithm. This method can be combined with the state-of-the-art presolver which enables it to solve large and tough mixed-integer-programming problems.

Cutting planes algorithm is an exact method used for finding exact optimum solutions for optimization problems. Using it together with other heuristics algorithms makes it more reliable and applicable for finding optimal solutions to MILP problems (AIMMS: CPLEX, 2014).

### 2.2.2. Genetic Algorithm

Genetic algorithm is a heuristic approach which is used for solving the optimization problems so popularly and through approximate methods. It searches rapidly among all possible solutions and finds the best one among the achieved responses. There are some main steps to create the next generation from the current population and repeat it to reach an optimal solution (Genetic Algorithm, 2014):

1. Choosing the right fitness function that can be used as a base for scoring the chromosomes and choosing good ones in order to find a more complete solution

1. Generating an initial population of chromosomes to start with and use them as the first generation
2. Selection of which chromosomes will be chosen to be parent of the next generation off-springs.
3. Crossover to produce next generation
4. Random mutation in produced off-springs in new generation

In our case, we define the fitness function the same as the objective function. Since we are seeking to maximize (minimize) the objective function, in the selection part, we give the priority to bigger (smaller) values of the fitness function.

For our optimization problem, we have 5 sets of variables including 2 binaries, 1 integer and 2 continuous. Based on the objective function (fitness function), the only variable in the objective function that we have to optimize is a binary variable called F, containing 12 binary values indicating whether or not each one of our popular paths is covered.

In Generation phase, each of the five variables are generated through a random function in MATLAB R2012b. A population size of 200 is used. Then each produced string of chromosomes is evaluated based on the fitness function to verify its score in order to achieve the best value to cover whole paths as much as possible.

In selection phase, we keep the fitter half (0.5) of the current population based on the ranking method for a maximization problem, make it double to reach to the same number of chromosomes and then choose randomly among them. We use the first half to select first parent and the second half to select second parent, so in this way all the best fitted values chosen, have equal chance.

To perform mating and producing the off-springs, we randomly choose a crossover point and generate the off-springs using that point.

Finally random mutation is done with the rate of 0.2. The maximum iteration is 100.

In the next chapter, the applications results for the exact algorithm and heuristic algorithm are explained in details and the achieved results are compared.

# Chapter 4:

# Numerical Application

In this chapter, application of the proposed methods for demand forecasting and optimization of recharging stations planning is provided and comparison of the results is performed.

## 1. Input Data

The customer demand for a specific station is randomly generated based on actual data from Communauto for the first nine months of the years between 1994 and 2011. The results can be seen in Table 1. Similar demand calculations can be done for other stations as well.

| | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Jan | 406 | 400 | 411 | 409 | 384 | 429 | 414 | 410 | 403 | 411 | 398 | 404 | 392 | 397 | 384 | 428 | 431 | 380 |
| Feb | 369 | 381 | 376 | 375 | 373 | 370 | 384 | 379 | 384 | 383 | 374 | 374 | 384 | 375 | 379 | 370 | 377 | 372 |
| Mar | 427 | 458 | 436 | 472 | 447 | 462 | 419 | 431 | 427 | 472 | 472 | 428 | 450 | 435 | 430 | 473 | 481 | 444 |
| Apr | 467 | 478 | 487 | 479 | 462 | 493 | 433 | 441 | 504 | 522 | 446 | 500 | 482 | 516 | 497 | 494 | 465 | 451 |
| May | 565 | 419 | 485 | 561 | 447 | 470 | 448 | 525 | 456 | 438 | 532 | 469 | 538 | 548 | 479 | 551 | 540 | 504 |
| Jun | 441 | 458 | 457 | 460 | 500 | 428 | 435 | 417 | 444 | 470 | 450 | 469 | 463 | 428 | 431 | 420 | 432 | 479 |
| Jul | 420 | 416 | 419 | 407 | 415 | 441 | 424 | 403 | 434 | 444 | 419 | 413 | 407 | 407 | 429 | 421 | 412 | 414 |
| Aug | 456 | 417 | 438 | 470 | 451 | 519 | 445 | 438 | 430 | 444 | 409 | 501 | 484 | 518 | 483 | 520 | 484 | 433 |
| Sep | 418 | 426 | 438 | 472 | 453 | 432 | 463 | 473 | 474 | 459 | 462 | 463 | 420 | 450 | 424 | 444 | 466 | 413 |

Table 1: Demand between 1994 and 2011

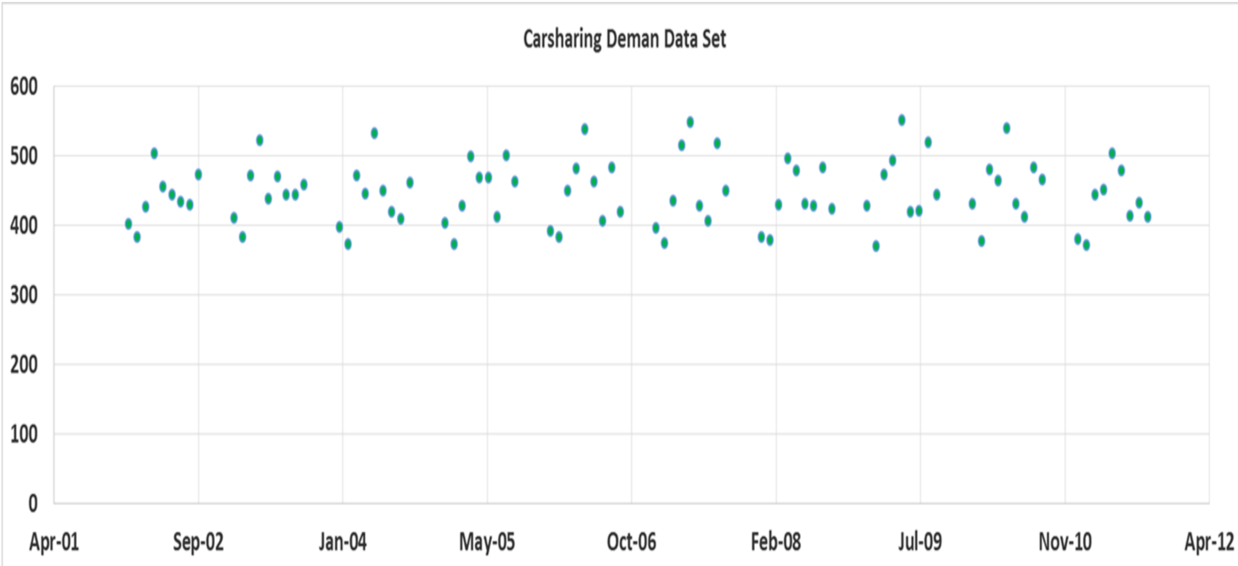

Figure 6: Demand data set trend between 1994 and 2011

## 2. Results for Customer Demand Forecasting

In this section, the results obtained from each of the applied approaches are illustrated in the tables. Then the performance evaluators including MSE, RMSE and MAD regarding each of these methods are illustrated in the related tables.

### 2.1. Regression Forecast

Table 2 represents the regression line including the coefficients *a* and *b* which are the slope and intercept of the related regression line.

| ***a*** | ***b*** |
|---|---|
| **416.87** | **0.32** |
| $y = 416.87 + 0.32X$ | |

Table 2: Slope, intercept & the regression line

Table 3 represents the results of linear regression forecast approach for each of the nine months for the following 18 years after 2011.

| Regression Foreast | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Jan | 417 | 420 | 423 | 426 | 429 | 432 | 434 | 437 | 440 | 443 | 446 | 449 | 452 | 454 | 457 | 460 | 463 | 466 |
| Feb | 418 | 420 | 423 | 426 | 429 | 432 | 435 | 438 | 440 | 443 | 446 | 449 | 452 | 455 | 458 | 461 | 463 | 466 |
| Mar | 418 | 421 | 424 | 426 | 429 | 432 | 435 | 438 | 441 | 444 | 447 | 449 | 452 | 455 | 458 | 461 | 464 | 467 |
| Apr | 418 | 421 | 424 | 427 | 430 | 432 | 435 | 438 | 441 | 444 | 447 | 450 | 453 | 455 | 458 | 461 | 464 | 467 |
| May | 418 | 421 | 424 | 427 | 430 | 433 | 436 | 439 | 441 | 444 | 447 | 450 | 453 | 456 | 459 | 462 | 464 | 467 |
| Jun | 419 | 422 | 425 | 427 | 430 | 433 | 436 | 439 | 442 | 445 | 447 | 450 | 453 | 456 | 459 | 462 | 465 | 468 |
| Jul | 419 | 422 | 425 | 428 | 431 | 433 | 436 | 439 | 442 | 445 | 448 | 451 | 454 | 456 | 459 | 462 | 465 | 468 |
| Aug | 419 | 422 | 425 | 428 | 431 | 434 | 437 | 440 | 442 | 445 | 448 | 451 | 454 | 457 | 460 | 462 | 465 | 468 |
| Sep | 420 | 423 | 425 | 428 | 431 | 434 | 437 | 440 | 443 | 446 | 448 | 451 | 454 | 457 | 460 | 463 | 466 | 469 |

Table 3: Regression forecast results

## Evaluation

The MSE, RMSE and MAD values can be seen in Table 4 below. As you can see from the table, the respected values are relatively large. There is a final value for each of MSE, RMSE and MAD indicating the performance of the approach for the whole date set regardless of the months.

| Month | MSE | RMSE | M.A.D |
|---|---|---|---|
| Jan | 209.02 | 14.46 | 11.25 |
| Feb | 24.13 | 4.91 | 4.23 |
| Mar | 365.20 | 19.11 | 17.39 |
| Apr | 609.92 | 24.70 | 20.71 |
| May | 1963.73 | 44.31 | 39.34 |
| Jun | 457.26 | 21.38 | 17.77 |
| Jul | 125.87 | 11.22 | 8.82 |
| Aug | 1021.64 | 31.96 | 26.02 |
| Sep | 406.81 | 20.17 | 18.11 |
| Final MSE, RMSE & MAD | 1,982.09 | 44.52 | 35.14 |

Table 4: Performance evaluation of the regression forecast method

## 2.2. Linear Regression with Seasonality Adjustment (I)

The results regarding the approach of linear regression with the seasonality adjustment are shown in Table 5 below. There are also the calculated seasonal indices which are calculated based on the steps explained in the previous chapter.

| Month | Seasonal Index | Seasonally Adjusted Forecast (I) | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Jan | 0.92 | 383 | 386 | 388 | 391 | 394 | 396 | 399 | 402 | 404 | 407 | 410 | 412 | 415 | 417 | 420 | 423 | 425 | 428 |
| Feb | 0.85 | 356 | 359 | 361 | 364 | 366 | 368 | 371 | 373 | 376 | 378 | 381 | 383 | 386 | 388 | 391 | 393 | 395 | 398 |
| Mar | 1.01 | 424 | 427 | 429 | 432 | 435 | 438 | 441 | 444 | 447 | 450 | 453 | 456 | 459 | 461 | 464 | 467 | 470 | 473 |
| Apr | 1.08 | 453 | 456 | 459 | 462 | 465 | 468 | 471 | 474 | 478 | 481 | 484 | 487 | 490 | 493 | 496 | 499 | 502 | 506 |
| May | 1.13 | 471 | 475 | 478 | 481 | 484 | 487 | 491 | 494 | 497 | 500 | 504 | 507 | 510 | 513 | 517 | 520 | 523 | 526 |
| Jun | 1.01 | 425 | 428 | 431 | 434 | 437 | 439 | 442 | 445 | 448 | 451 | 454 | 457 | 460 | 463 | 466 | 469 | 471 | 474 |
| Jul | 0.95 | 397 | 399 | 402 | 405 | 407 | 410 | 413 | 416 | 418 | 421 | 424 | 426 | 429 | 432 | 435 | 437 | 440 | 443 |
| Aug | 1.04 | 438 | 441 | 444 | 447 | 450 | 453 | 456 | 459 | 462 | 465 | 468 | 471 | 474 | 477 | 480 | 483 | 486 | 489 |
| Sep | 1.01 | 423 | 426 | 429 | 432 | 435 | 438 | 440 | 443 | 446 | 449 | 452 | 455 | 458 | 461 | 464 | 467 | 469 | 472 |

Table 5: Seasonality Adjusted forecast (I) results

### Evaluation

The final MSE, RMSE and MAD can be seen in Table 6 as follows. As it can be seen from the table, they all are smaller than the previous regular regression forecast approach which shows better performance of linear regression with seasonality adjustment.

| Seasonally Adjusted Forecast (I) | | | |
|---|---|---|---|
| Month | MSE | RMSE | MAD |
| Jan | 209.02 | 14.46 | 11.25 |
| Feb | 24.13 | 4.91 | 4.23 |
| Mar | 365.20 | 19.11 | 17.39 |
| Apr | 609.92 | 24.70 | 20.71 |
| May | 1963.73 | 44.31 | 39.34 |
| Jun | 457.26 | 21.38 | 17.77 |
| Jul | 125.87 | 11.22 | 8.82 |
| Aug | 1021.64 | 31.96 | 26.02 |
| Sep | 406.81 | 20.17 | 18.11 |
| Final MSE , RMSE & MAD | 746.42 | 27.32 | 21.89 |

Table 6: Performance evaluation of the seasonality adjusted forecast (I)

## 2.3. Linear Regression with Seasonality Adjustment (II)

We propose a second method for calculating seasonal indices. In this method, we calculate the regression line for each month seperately and generate the regression line for each of the months. This method is one of the contributions of this thesis and does not follow any other approach. The calculated slopes and intercepts for each months individually are illustrated in the Table 7. The final forecasted values are illustrated in the Table 8.

| Month | b | a |
|---|---|---|
| Jan | -0.26 | 407.52 |
| Feb | 0.01 | 376.54 |
| Mar | 0.70 | 441.39 |
| Apr | 0.62 | 472.85 |
| May | 2.30 | 476.78 |
| Jun | -0.67 | 455.39 |
| Jul | -0.21 | 421.14 |
| Aug | 2.46 | 439.92 |
| Sep | -0.10 | 448.14 |

Table 7: Slopes & intercepts of the seasonality adjusted forecast (II)

| Month | Seasonally Adjusted Forecast(II) | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Jan | 407 | 407 | 407 | 406 | 406 | 406 | 406 | 405 | 405 | 405 | 405 | 404 | 404 | 404 | 404 | 403 | 403 | 403 |
| Feb | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 | 377 |
| Mar | 442 | 443 | 443 | 444 | 445 | 446 | 446 | 447 | 448 | 448 | 449 | 450 | 450 | 451 | 452 | 453 | 453 | 454 |
| Apr | 473 | 474 | 475 | 475 | 476 | 477 | 477 | 478 | 478 | 479 | 480 | 480 | 481 | 482 | 482 | 483 | 483 | 484 |
| May | 479 | 481 | 484 | 486 | 488 | 491 | 493 | 495 | 497 | 500 | 502 | 504 | 507 | 509 | 511 | 514 | 516 | 518 |
| Jun | 455 | 454 | 453 | 453 | 452 | 451 | 451 | 450 | 449 | 449 | 448 | 447 | 447 | 446 | 445 | 445 | 444 | 443 |
| Jul | 421 | 421 | 421 | 420 | 420 | 420 | 420 | 419 | 419 | 419 | 419 | 419 | 418 | 418 | 418 | 418 | 418 | 417 |
| Aug | 442 | 445 | 447 | 450 | 452 | 455 | 457 | 460 | 462 | 465 | 467 | 469 | 472 | 474 | 477 | 479 | 482 | 484 |
| Sep | 448 | 448 | 448 | 448 | 448 | 448 | 447 | 447 | 447 | 447 | 447 | 447 | 447 | 447 | 447 | 447 | 446 | 446 |

Table 8: Seasonality adjusted forecast (II) results

## Evaluation

According to Table 9, the final MSE, RMSE and MAD in second approach are significantly less than the original seasonality adjustment method. It shows better performance of the proposed method for seasonally adjustment in regression forecast approach.

| Seasonally Adjusted Forecast(II) | | | |
|---|---|---|---|
| Month | MSE | RMSE | MAD |
| Jan | 209.02 | 14.46 | 11.25 |
| Feb | 24.13 | 4.91 | 4.23 |
| Mar | 365.20 | 19.11 | 17.39 |
| Apr | 609.92 | 24.70 | 20.71 |
| May | 1963.73 | 44.31 | 39.34 |
| Jun | 457.26 | 21.38 | 17.77 |
| Jul | 125.87 | 11.22 | 8.82 |
| Aug | 1021.64 | 31.96 | 26.02 |
| Sep | 406.81 | 20.17 | 18.11 |
| Final MSE, RMSE & MAD | 575.95 | 24.00 | 18.18 |

Table 9: performance evaluation of the seasonality adjusted forecast (II)

## 2.4. Winter's Model

## Initial Results

Based on an initial assumption for alpha, beta and gamma as follows, Table 10 shows the result of the Winter's model. The forecasted values for the following 17 years are described in the table. The first year data have been used an initial data for the Winter's model.

$\boldsymbol{\alpha = 0.3 \quad \beta = 0.3 \quad \gamma = 0.3}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | 406 | 387 | 411 | 434 | 404 | 418 | 395 | 400 | 406 | 411 | 398 | 430 | 399 | 421 | 390 | 419 | 413 |
| Feb | 367 | 363 | 376 | 381 | 388 | 382 | 369 | 367 | 375 | 373 | 370 | 389 | 373 | 387 | 388 | 396 | 367 |
| Mar | 430 | 429 | 436 | 444 | 456 | 461 | 442 | 435 | 434 | 430 | 438 | 456 | 436 | 450 | 449 | 466 | 445 |
| Apr | 483 | 468 | 488 | 469 | 487 | 471 | 468 | 456 | 485 | 476 | 449 | 485 | 462 | 483 | 512 | 516 | 470 |
| May | 587 | 542 | 536 | 515 | 534 | 477 | 487 | 517 | 522 | 457 | 480 | 484 | 492 | 497 | 515 | 515 | 491 |
| Jun | 410 | 449 | 483 | 418 | 473 | 426 | 475 | 452 | 460 | 466 | 442 | 475 | 471 | 428 | 471 | 445 | 412 |
| Jul | 400 | 425 | 447 | 407 | 405 | 391 | 429 | 428 | 445 | 442 | 431 | 441 | 423 | 396 | 431 | 423 | 419 |
| Aug | 436 | 452 | 461 | 445 | 450 | 437 | 454 | 470 | 477 | 459 | 444 | 463 | 462 | 463 | 484 | 482 | 489 |
| Sep | 390 | 421 | 444 | 431 | 462 | 405 | 424 | 447 | 472 | 450 | 492 | 477 | 473 | 444 | 461 | 434 | 431 |

Table 10: Winter's model, initial results

## After Optimization

After achieving the initial results, through MS Excel solver, we optimized the amounts for alpha, beta and gamma based on the value of MSE. The optimized resulted forecasting values are shown in Table 11 below. Optimized values for alpha, beta and gamma are also described as follows:

$\boldsymbol{\alpha = 0.00 \quad \beta = 0.51 \quad \gamma = 0.26}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | 406 | 404 | 406 | 407 | 401 | 408 | 410 | 410 | 408 | 409 | 406 | 405 | 402 | 401 | 396 | 405 | 411 |
| Feb | 369 | 372 | 373 | 374 | 373 | 373 | 376 | 376 | 378 | 380 | 378 | 377 | 379 | 378 | 378 | 376 | 376 |
| Mar | 427 | 435 | 435 | 445 | 445 | 450 | 442 | 439 | 436 | 445 | 452 | 446 | 447 | 444 | 440 | 449 | 457 |
| Apr | 467 | 470 | 474 | 476 | 472 | 478 | 466 | 459 | 471 | 484 | 474 | 481 | 481 | 490 | 492 | 493 | 485 |
| May | 565 | 527 | 516 | 528 | 507 | 497 | 484 | 495 | 485 | 472 | 488 | 483 | 497 | 511 | 502 | 515 | 522 |
| Jun | 441 | 445 | 448 | 451 | 464 | 455 | 450 | 441 | 442 | 449 | 449 | 455 | 457 | 449 | 444 | 438 | 436 |
| Jul | 420 | 419 | 419 | 416 | 416 | 422 | 423 | 418 | 422 | 428 | 425 | 422 | 418 | 415 | 419 | 419 | 417 |
| Aug | 456 | 446 | 444 | 451 | 451 | 469 | 462 | 456 | 449 | 448 | 438 | 454 | 462 | 477 | 478 | 489 | 488 |
| Sep | 418 | 420 | 425 | 437 | 441 | 439 | 445 | 452 | 458 | 458 | 459 | 460 | 450 | 450 | 443 | 443 | 449 |

Table 11: Winter's model, optimized results

## Evaluation:

The MSE, RMSE and MAD in the optimized model is significantly less in the optimized model, however it is relatively large in compare with seasonality adjustment methods. It shows better performance of the model after the optimization.

| Initial Results | | | |
|---|---|---|---|
| Alpha | 0.3 | MSE | 1,167.84 |
| Beta | 0.3 | RMSE | 34.17 |
| Gamma | 0.3 | MAD | 25.66 |

Table 12: Winter's model, Performance evaluation for initial model

| After Optimization | | | |
|---|---|---|---|
| Alpha | 0.00 | MSE | 835.71 |
| Beta | 30.5 | RMSE | 28.91 |
| Gamma | 0.26 | MAD | 21.12 |

Table 13: Winter's model, Performance evaluation for optimized model

As a final point, you can see from the tables that MADs in all cases are less than RMSEs which shows that the absolute deviation from the mean among the forecasted data set is less than root of squared deviation from the mean for the same data set.

## 2.5. Artificial Neural Network Implementation

Table 14-25 show that the neural network method has been implemented for different training / (validation & testing) ratios. Using Matlab R2012b Software, different ratios starting from 90% of target time steps allocating for the training and 10% for the validation & testing, and ending to 50:50 ratio, results are illustrated in the following tables:

**(90:10)**

Tables 14 -15 present the neural network forecasted results for 90:10 ratio. In Table 14, the results for the *y* value which is our forecasted demand are illustrated. The final MSE (Table 15) is calculated using the partial MSE for the ratio of 90% training, 5% validation and 5 % testing. The resulted MSE is fairly small which shows the power of neural network in this ratio.

| 90:10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | 395 | 403 | 403 | 396 | 402 | 402 | 405 | 395 | 395 | 403 | 399 | 404 | 403 | 400 | 402 | 405 | 401 | 402 |
| Feb | 379 | 377 | 376 | 378 | 378 | 378 | 376 | 379 | 379 | 376 | 379 | 377 | 378 | 379 | 378 | 376 | 376 | 378 |
| Mar | 446 | 445 | 460 | 452 | 440 | 440 | 454 | 445 | 446 | 461 | 439 | 449 | 443 | 438 | 440 | 454 | 458 | 440 |
| Apr | 492 | 480 | 461 | 482 | 488 | 488 | 464 | 493 | 493 | 459 | 494 | 473 | 483 | 494 | 488 | 464 | 465 | 488 |
| May | 437 | 507 | 523 | 456 | 493 | 493 | 538 | 435 | 435 | 527 | 465 | 521 | 502 | 474 | 493 | 538 | 508 | 493 |
| Jun | 471 | 434 | 461 | 474 | 431 | 431 | 441 | 470 | 470 | 460 | 441 | 437 | 433 | 434 | 431 | 441 | 464 | 431 |
| Jul | 428 | 413 | 412 | 425 | 415 | 415 | 408 | 428 | 428 | 411 | 421 | 411 | 414 | 419 | 415 | 408 | 415 | 415 |
| Aug | 438 | 474 | 439 | 432 | 480 | 480 | 463 | 439 | 439 | 440 | 471 | 469 | 476 | 478 | 480 | 462 | 438 | 480 |
| Sep | 445 | 441 | 460 | 452 | 436 | 436 | 452 | 445 | 445 | 460 | 436 | 446 | 440 | 434 | 436 | 452 | 458 | 436 |

Table 14: Neural Network 90:10 results

| | Target Value | MSE | Final MSE |
|---|---|---|---|
| Training | 153 | 418.14 | 442.32 |
| Validation | 9 | 446.63 | |
| Testing | 9 | 849.13 | |

Table 15: Performance Evaluation of Neural Network 90:10

**(80:20)**

Tables 16 -17 present the neural network forecasted results for 80:20 (Training: Testing) ratio. It can be seen that the final MSE for the ratio of 80:20 (80% for training, 10 % validation and 10% testing) is slightly bigger than the MSE for 90:10 ratio. However it is still not so large.

| 80:20 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | 398 | 407 | 410 | 399 | 401 | 401 | 409 | 406 | 406 | 410 | 401 | 401 | 406 | 405 | 406 | 401 | 401 | 406 |
| Feb | 379 | 378 | 377 | 378 | 378 | 378 | 377 | 378 | 378 | 377 | 378 | 378 | 378 | 378 | 378 | 378 | 378 | 378 |
| Mar | 447 | 450 | 449 | 447 | 446 | 446 | 448 | 450 | 450 | 449 | 446 | 446 | 450 | 449 | 450 | 446 | 446 | 450 |
| Apr | 494 | 491 | 460 | 482 | 459 | 459 | 460 | 496 | 496 | 460 | 459 | 459 | 496 | 496 | 496 | 459 | 459 | 495 |
| May | 436 | 485 | 538 | 456 | 495 | 495 | 534 | 478 | 477 | 539 | 495 | 495 | 477 | 470 | 477 | 495 | 495 | 479 |
| Jun | 471 | 443 | 432 | 467 | 459 | 459 | 435 | 445 | 445 | 432 | 459 | 459 | 445 | 450 | 445 | 459 | 459 | 445 |
| Jul | 423 | 426 | 417 | 419 | 412 | 412 | 416 | 428 | 428 | 417 | 412 | 412 | 428 | 427 | 428 | 412 | 412 | 428 |
| Aug | 464 | 452 | 447 | 462 | 459 | 459 | 448 | 452 | 452 | 447 | 459 | 459 | 452 | 454 | 452 | 459 | 459 | 452 |
| Sep | 431 | 452 | 466 | 436 | 447 | 447 | 464 | 450 | 450 | 466 | 447 | 447 | 449 | 446 | 449 | 447 | 447 | 450 |

Table 16: Neural Network 80:20 results

| | Target Value | MSE | Final MSE |
|---|---|---|---|
| Training | 137 | 422.62 | 552.64 |
| Validation | 17 | 1082.13 | |
| Testing | 17 | 1071.00 | |

Table 17: Performance Evaluation of Neural Network 80:20

Table 16 shows the results for 80:20 ratios. As can be seen in table 17, the final MSE for the ratio of 80:20 (80% for training, 10 % validation and 10% testing) is slightly bigger than the MSE for 90:10 ratio. However it is still not so large.

**(75:25)**

Tables 18 -19 present the neural network forecasted results for 75:25 ratio. For the ratio of 75:25 (75% training data set, 15% validation data set and 10 % testing data set) the resulted MSE is slightly more than 80:20 and significantly more than 90:10 ratio.

| 75:25 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | 394 | 399 | 410 | 385 | 385 | 412 | 385 | 385 | 413 | 413 | 410 | 385 | 413 | 412 | 413 | 413 | 387 | 385 |
| Feb | 378 | 378 | 377 | 378 | 378 | 377 | 378 | 378 | 377 | 377 | 377 | 378 | 377 | 377 | 377 | 377 | 378 | 378 |
| Mar | 443 | 447 | 458 | 432 | 432 | 460 | 432 | 433 | 461 | 461 | 458 | 432 | 461 | 459 | 461 | 461 | 434 | 432 |
| Apr | 476 | 475 | 470 | 478 | 478 | 469 | 478 | 478 | 469 | 469 | 471 | 478 | 469 | 469 | 469 | 469 | 478 | 478 |
| May | 477 | 477 | 494 | 473 | 473 | 496 | 473 | 471 | 496 | 496 | 491 | 473 | 496 | 495 | 496 | 496 | 474 | 473 |
| Jun | 438 | 441 | 446 | 434 | 434 | 447 | 434 | 434 | 448 | 448 | 446 | 434 | 448 | 447 | 448 | 448 | 435 | 434 |
| Jul | 419 | 420 | 418 | 419 | 419 | 418 | 419 | 419 | 418 | 418 | 418 | 419 | 418 | 418 | 418 | 418 | 419 | 419 |
| Aug | 456 | 456 | 465 | 454 | 454 | 466 | 454 | 454 | 466 | 466 | 464 | 454 | 466 | 466 | 466 | 466 | 455 | 454 |
| Sep | 431 | 432 | 439 | 427 | 427 | 440 | 427 | 427 | 441 | 441 | 439 | 427 | 441 | 440 | 441 | 441 | 428 | 427 |

Table 18: Neural Network 75:25 results

| | Target Value | MSE | Final MSE |
|---|---|---|---|
| Training | 128 | 670.29 | 679.94 |
| Validation | 26 | 673.36 | |
| Testing | 17 | 762.62 | |

Table 19: Performance Evaluation of Neural Network 75:25

**(70:30)**

Tables 20 -21 present the neural network forecasted results for 70:30 ratio. It can be seen that the final MSE in this ratio (70% training, 15% validation and 15% testing) is almost the same as 75:25 ratio.

| 70:30 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | [redacted] | | | | | | | | | | | | | | | | | |
| Feb | | | | | | | | | | | | | | | | | | |
| Mar | | | | | | | | | | | | | | | | | | |
| Apr | | | | | | | | | | | | | | | | | | |
| May | | | | | | | | | | | | | | | | | | |
| Jun | | | | | | | | | | | | | | | | | | |
| Jul | | | | | | | | | | | | | | | | | | |
| Aug | | | | | | | | | | | | | | | | | | |
| Sep | | | | | | | | | | | | | | | | | | |

Table 20: Neural Network 70:30 results

| | Target Value | MSE | Final MSE |
|---|---|---|---|
| Training | 119 | 657.57 | 677.97 |
| Validation | 26 | 631.30 | |
| Testing | 26 | 817.99 | |

Table 21: Performance Evaluation of Neural Network 70:30

**(60:40)**

Tables 22 -23 present the neural network forecasted results for 60:40 ratio. In 60:40 ratio (60% fir training, 20% validation and 20% testing) the MSE is slightly bigger than the 70:30 and 75:25 ratios. Here the trend acts vice versa.

| 60:40 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | 402 | 402 | 408 | 391 | 402 | 397 | 397 | 408 | 397 | 397 | 402 | 397 | 391 | 397 | 397 | 397 | 391 | 402 |
| Feb | 377 | 377 | 378 | 376 | 377 | 377 | 377 | 378 | 377 | 377 | 377 | 377 | 376 | 377 | 377 | 377 | 376 | 377 |
| Mar | 444 | 444 | 460 | 433 | 444 | 449 | 449 | 460 | 449 | 449 | 444 | 449 | 433 | 449 | 449 | 449 | 433 | 444 |
| Apr | 478 | 478 | 465 | 479 | 478 | 466 | 466 | 465 | 466 | 466 | 478 | 466 | 479 | 466 | 466 | 466 | 479 | 478 |
| May | 470 | 470 | 506 | 493 | 470 | 530 | 530 | 506 | 530 | 530 | 470 | 530 | 493 | 530 | 530 | 530 | 493 | 470 |
| Jun | 440 | 440 | 437 | 443 | 440 | 441 | 441 | 437 | 441 | 441 | 440 | 441 | 443 | 441 | 441 | 441 | 443 | 440 |
| Jul | 416 | 416 | 415 | 415 | 416 | 414 | 414 | 415 | 414 | 414 | 416 | 414 | 415 | 414 | 414 | 414 | 415 | 416 |
| Aug | 466 | 466 | 442 | 473 | 466 | 449 | 449 | 442 | 449 | 449 | 466 | 449 | 473 | 449 | 449 | 449 | 473 | 466 |
| Sep | 441 | 441 | 453 | 447 | 441 | 459 | 459 | 453 | 459 | 459 | 441 | 459 | 447 | 459 | 459 | 459 | 447 | 441 |

Table 22: Neural Network 60:40 results

| | Target Value | MSE | Final MSE |
|---|---|---|---|
| Training | 103 | 633.50 | 669.23 |
| Validation | 34 | 569.51 | |
| Testing | 34 | 877.20 | |

Table 23: Performance Evaluation of Neural Network 60:40

**(50:50)**

Tables 24 -25 present the neural network forecasted results for 50:50 ratio. For 50:50 ratio (50% of training, 25% validation and 25% testing), the value of MSE significantly increases as shown in the table above.

| 50:50 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | 398 | 399 | 398 | 398 | 407 | 399 | 407 | 407 | 407 | 407 | 407 | 403 | 399 | 398 | 399 | 399 | 398 | 399 |
| Feb | 375 | 375 | 375 | 375 | 377 | 375 | 376 | 376 | 376 | 377 | 377 | 376 | 376 | 375 | 375 | 376 | 375 | 376 |
| Mar | 452 | 451 | 455 | 452 | 448 | 448 | 448 | 448 | 448 | 447 | 441 | 451 | 443 | 452 | 445 | 444 | 453 | 443 |
| Apr | 480 | 481 | 477 | 480 | 491 | 484 | 491 | 491 | 491 | 492 | 499 | 485 | 490 | 480 | 488 | 489 | 479 | 490 |
| May | 459 | 463 | 442 | 458 | 509 | 481 | 507 | 507 | 507 | 513 | 551 | 479 | 513 | 457 | 502 | 507 | 453 | 512 |
| Jun | 469 | 468 | 480 | 470 | 480 | 457 | 481 | 481 | 481 | 477 | 456 | 480 | 440 | 470 | 445 | 443 | 472 | 440 |
| Jul | 414 | 414 | 415 | 414 | 414 | 413 | 414 | 414 | 414 | 413 | 411 | 414 | 410 | 414 | 411 | 411 | 415 | 411 |
| Aug | 428 | 432 | 405 | 426 | 478 | 456 | 475 | 475 | 475 | 483 | 532 | 445 | 497 | 425 | 483 | 490 | 420 | 496 |
| Sep | 438 | 439 | 436 | 437 | 464 | 441 | 463 | 463 | 463 | 464 | 469 | 452 | 445 | 437 | 444 | 445 | 437 | 445 |

Table 24: Neural Network 50:50 results

| | Target Value | MSE | Final MSE |
|---|---|---|---|
| **Training** | 85 | 639.92 | 841.81 |
| **Validation** | 43 | 1203.97 | |
| **Testing** | 43 | 878.73 | |

Table 25: Performance Evaluation of Neural Network 50:50

## 3. Demand Forecasting Results Comparison

- ### Trend Evaluation of Neural Network Performance

For comparison, we chose MSE which is commonly used for this purpose. In Figure 7, the chart depicts the trend of changing the value of MSE based on the ratios of the training and validation & testing data sets. It can be seen from the Figure 7 that there is an obvious increasing trend when we decrease the ratio of the training data set in compare with the validation and testing. While 90:10 has the least MSE, 50:50 has the biggest MSE among all others.

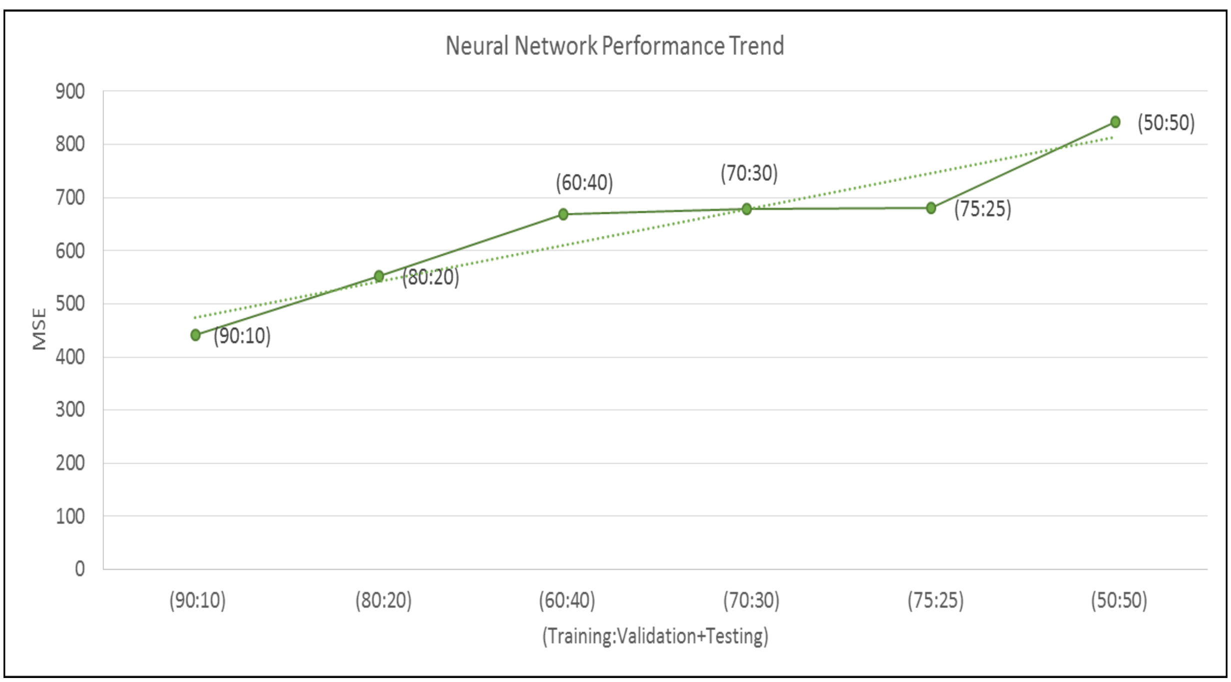


Figure 7: Neural Network Performance Evaluation

According to the evaluation results in Table 26, the total MSE (Final MSE) in neural network method is computed by finding the sum of the square errors for the whole data set and dividing it in to the total number of data. The result is the same as regular MSE we calculate for other data sets regardless of which data are used for training and which for validation & testing. The RMSE is also calculated using the MSEs. The 90:10 ratio gives the lowest MSE and is the best among other portions in neural network method.

Finally in comparing the 3 methods, the 90:10 neural network method gives the lowest MSE & RMSE, so it is the best among all others (Regression, Seasonally adjustment, Winter's model and other portions in NN). After that, the proposed seasonally adjusted method gives the lowest MSE & RMSE in compare with 80:20, 75:25, 70:30, 60:40 and 50:50, so it can be assumed that it is in the second position.

| **Forecasting Method** | **MSE** | **RMSE** | **MAD** |
|---|---|---|---|
| Neural Network (90:10) | 442.32 | 21.03 | - |
| Neural Network (80:20) | 552.64 | 23.51 | - |
| Regression Forecast With Seasonality Adjustment (II) | 575.95 | 24.00 | - |
| Neural Network (60:40) | 669.23 | 25.87 | - |
| Neural Network (70:30) | 677.97 | 26.04 | - |
| Neural Network (75:25) | 679.94 | 26.08 | - |
| Regression Forecast With Seasonality Adjustment (I) | 746.42 | 27.32 | - |
| Winter's Model (Optimized Version) | 835.71 | 28.91 | - |
| Neural Network (50:50) | 841.81 | 29.01 | - |
| Linear Regression | 1982.09 | 44.52 | - |

Table 26: Results Evaluation

# 4. Recharging Stations Planning Optimization

## 4.1. Assumptions

According to Wang & Lin (2013), following parameter values were assumed for implementing the proposed model. We used the same values when implementing the approaches proposed in this thesis in order to have comparable results. Table 27 shows the popular paths in this case and their associated passing flows per hour, as well as distances traveled in each path.

Trip distributions along popular routes.

| No. | Type of routes | Distance (km) | Maximum flow (scooters/h) |
|---|---|---|---|
| 1 | Magong → Citou → A → Tongling → B → Chuwan → C → Siaomen → D → Erkan → E → Wai-An → S8 → S7 → S6 → Magong | 78.8 | 3 |
| 2 | Magong → Citou → A → Tongling → B → Chuwan → C → Siaomen → D → E → Erkan → S8 → S7 → S6 → Magong | 66.8 | 3 |
| 3 | Magong → Citou → A → Tongling → B → C → Chuwan → S8 → S7 → S6 → Magong | 59.6 | 3 |
| 4 | Magong → A → Tongling → B → C → Siaomen → D → E → Erkan → S8 → S7 → S6 → Magong | 66.4 | 3 |
| 5 | Magong → A → B → Chuwan → C → Siaomen → D → E → Wai-An → S8 → S7 → S6 → Magong | 78.2 | 3 |
| 6 | Magong → A → B → C → Siaomen → D → E → Wai-An → S8 → S7 → S6 → Magong | 78 | 4 |
| 7 | Magong → Citou → A → B → C → D → Siaomen → S8 → S7 → S6 → Magong | 63.2 | 2 |
| 8 | Magong → A → Tongling → B → C → Chuwan → S8 → S7 → S6 → Magong | 59.4 | 2 |
| 9 | Magong → A → Citou → B → C → D → E → Wai-An → S8 → S7 → S6 → Magong | 76.2 | 2 |
| 10 | Magong → Citou → A → Tongling → B → C → D → E → Wai-An → S8 → S7 → S6 → Magong | 76.4 | 2 |
| 11 | Magong → S6 → S5 → Beiliao → Guoye → Linto → S4 → Fongguei → S3 → S2 → S1 → Magong | 54.9 | 2 |
| 12 | Magong → Shanshuei → Shihli → Fongguei → S3 → S2 → S1 → S4 → Lintou → Guoye → Beiliao → S5 → S6 → Magong | 56.9 | 2 |

Table 27: Trip distribution along popular paths

**Sets:**

*P*= 1 - 12

*K*= 1 to 3 (slow recharging stations, fast recharging stations and battery exchange stations).

*i,j*= from 1 to 25

**Parameters:**

*W(p)*: [ 3 3 3 3 3 4 2 2 2 2 2 2]

*t(i,p)*: It is the same for all 12 paths and the unit of measurement is minutes.

| Nodes | Reharging Time |
|---|---|
| 1 | 56 |
| 2 | 19 |
| 3 | 22 |
| 4 | 28 |
| 5 | 33 |
| 6 | 30 |
| 7 | 30 |
| 8 | 25 |
| 9 | 20 |
| 10 | 32 |
| 11 | 28 |
| 12 | 28 |
| 13 | 20 |
| 14 | 20 |
| 15 | 20 |
| 16 | 20 |
| 17 | 20 |
| 18 | 10 |
| 19 | 10 |
| 20 | 10 |
| 21 | 10 |
| 22 | 10 |
| 23 | 10 |
| 24 | 10 |
| 25 | 10 |

Table 28: Recharging time in each node

$\beta(k)$: The rates (km/min) for slow-recharging, fast-recharging, and battery exchange stations are 0.133, 2, and 4, respectively.

$\gamma$: 40 (km)

$L$: Different budgetary levels

$U$(i,k): 2, 3 , …, 10

$C(i, k)$: for slow-recharging, fast-recharging, and battery exchange stations are thus 4000, 50000, and 20000 $ respectively.

$d(p, i, j)$: Appendix A

S(k): Vehicle sharing of a type k recharging station is set at 1.

- **Name of the nodes:**

| Nodes | Name |
|---|---|
| 1 | D1 |
| 2 | D2 |
| 3 | D3 |
| 4 | D4 |
| 5 | D5 |
| 6 | D6 |
| 7 | D7 |
| 8 | D8 |
| 9 | D9 |
| 10 | D10 |
| 11 | D11 |
| 12 | D12 |
| 13 | A |
| 14 | B |
| 15 | C |
| 16 | D |
| 17 | E |
| 18 | S1 |
| 19 | S2 |
| 20 | S3 |
| 21 | S4 |
| 22 | S5 |
| 23 | S6 |
| 24 | S7 |
| 25 | S8 |

Table 29: Name of the nodes

- **Installation constraints based on the location of the nodes**
    - The first 12 nodes are considered as attractions nodes called with D1, D2… D12. In the attraction centers, only slow recharging stations shall be installed.
    - The next 5 nodes are intersections and called with A, B… E. In these nodes, only fast recharging stations can be sited.

- The last 8 nodes are convenience stores called with S1, S2, …, S8. They can have any of the 3 recharging stations, including SRS, FRS and BES (battery exchange stations).

## 4.2. Implemented Approaches Results

Results for the MIP model solved using CPLEX approach through AIMMS software can be seen in the following figures. Tables 30 and 32 show the results by Wang & Lin (2013) through Lingo Branch and Bound whereas Tables 31 and 33 are results from AIMMS. Figures 8-9 are the output of the AIMMS which shows different characteristics of the models and their solvers.

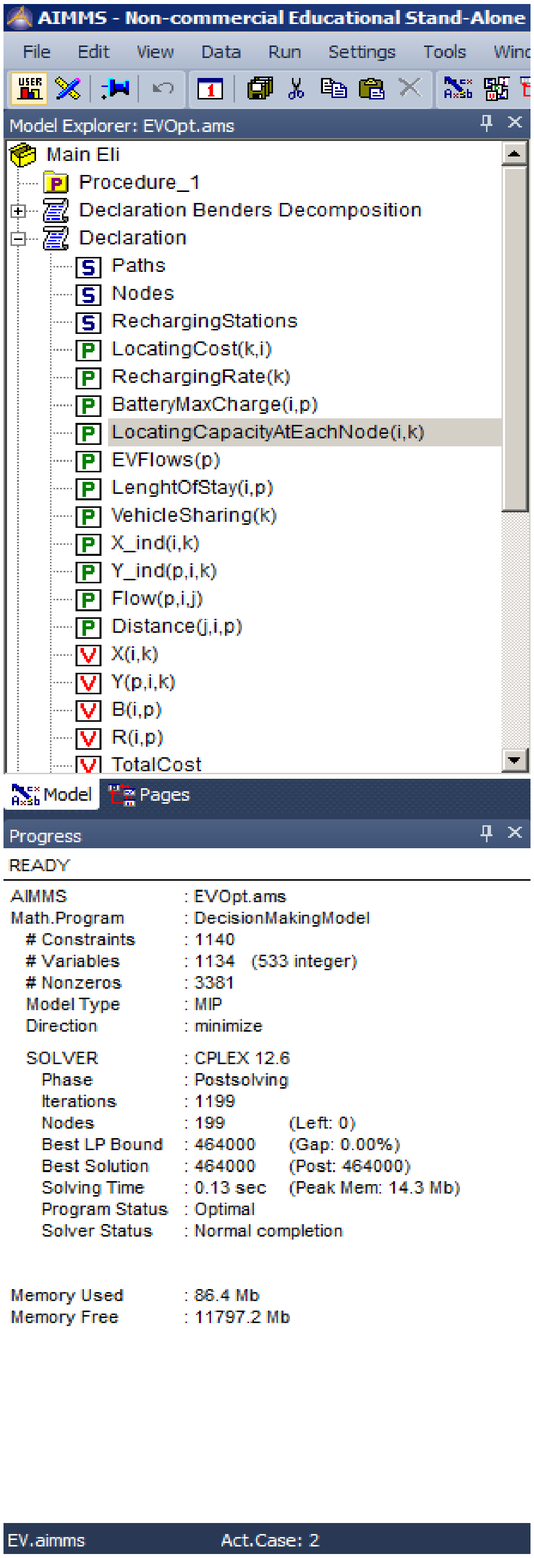


Figure 8: AIMMS result for U=5 constraint

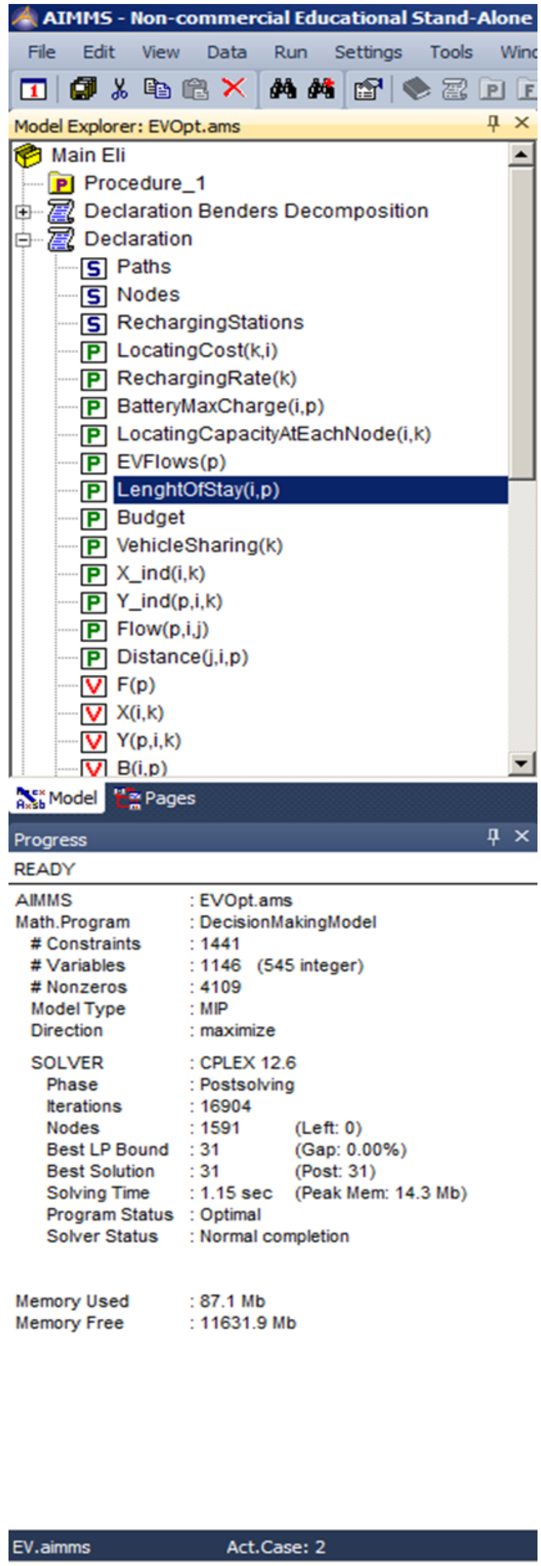


Figure 9: AIMMS result for budgetary 464000 $ and U=5

with a vehicle range of 40 km

| Location capacity | Location (number of SRS) | Location (number of FRS) | Location (number of BES) | Total number of stations (SRS, FRS, BES) | Total location cost (NT$) |
|---|---|---|---|---|---|
| 5 | D1(5), D2(4), D3(3), D5(3), D6(4), D7(2), D10(4), D11(4), D12(4), S4(2), S6(5), S7(4), S8(5) | S8(5), C(3), D(4), E(5) | S7(5), S8(5) | 76(49, 17, 10) | 1,246,000 |
| 6 | D1(5), D2(5), D3(5), D4(6), D5(6), D6(6), D7(2), D10(4), D11(4), D12(4), S4(2), S6(6), S7(6), S8(6) | S8(4), D(6) E(3) | S6(3), S7(5), S8(6) | 94(67, 13, 14) | 1,198,000 |
| 7 | D1(6), D2(5), D3(2), D6(6), D7(2), D10(4), D11(4), D12(4), S1(2), S6(3), S7(6), S8(7) | D(7), E(6) | S7(7)S7(7), S8(8), S8(7) | 78(51, 13, 14) | 1,134,000 |
| 8 | D1(8), D2(5), D3(3), D4(5), D5(6), D6(8), D7(2), D10(4), D11(4), D12(4), S1(2), S6(7), S7(8), S8(8) | D(2), E(7) | S7(7), S8(8) | 98(74, 9, 15) | 1,046,000 |
| 9 | D1(7), D2(5), D3(3), D4(9), D5(9), D6(9), D7(2), D10(4), D11(4), D12(4), S4(2), S6(6), S7(6), S8(9) | E(7) | S7(8), S8(9) | 103(79, 7, 17) | 1,006,000 |
| 10 | D1(9), D2(2), D3(3), D4(3), D5(6), D6(6), D7(2), D10(4), D11(4), D12(4), S2(2), S6(4), S7(3), S8(5) | D(4), E(3) | S7(10), S8(10) | 84(57, 7, 20) | 978,000 |

Table 30: Results of Wang & Lin (2013) Approach for cost minimization

| Location Capacity | Location (number of SRS) | Location (number of FRS) | Location (number of BES) | Total number of stations (SRS, FRS, BES) | Total Location Cost ( $) |
|---|---|---|---|---|---|
| 2 | D2(2), D3(2), D4(2), D5(2), D6(2) s7(1), S8(2) | D (2), E(2), C(1), s8(2), B(1) | S2(1), S4(1), S8(2), S7(1) | 27(13,9,5) | 602,000.00 |
| 3 | D2(1), D3(2), D4(2), D5(2), D6(1)s7(1), S8(1) | D (3), E(3) | S2(1), S4(1), S8(3), S7(1) | 23 (10, 7, 6) | 510,000.00 |
| 4 | D2(2), D3(2), D4(4), D5(3), D6(1), s7(1), S8(1) | D (2), E(4) | S2(1), S4(1), S8(4) | 26 (14,6, 6) | 476,000.00 |
| 5 | D2(1), D3(4), D4(3), D5(3) | D(1), E(5) | S2(1), S4(1), S8(4) | 23 (11,6,6) | 464,000.00 |
| 6 | D2(1), D3(4), D4(3), D5(3) | D(1), E(5) | S2(1), S4(1), S8(4) | 23 (11,6,6) | 464,000.00 |
| 7 | D2(1), D3(4), D4(3), D5(3) | D(1), E(5) | S2(1), S4(1), S8(4) | 23 (11,6,6) | 464,000.00 |

Table 31: Results of AIMMS for cost minimization; with a vehicle range of 40 km

As you can see from Table (32) and Table(33), the main difference between two versions is how the second improved version perfoms smarter distributions in compare with the initial approach.

(Note: SRS: Slow-Recharging Station, FRS: Fast-Recharging Station, BES: Battery Excahnge Station)

For instance, take a look at the budgetary constraint of 100,000.00 $ in Table (32) and Table (33). According to the assumptions of the case study, the locating cost for SRS, FRS and BES are 4000, 50,000 and 20,000 $ respectively. While initial model has used all the 100,000 $ to install 5 stations of type 3 (BES type , which costs 20,000 for each) at only 1 node called S8, the second model has used this budget to install 12 stations ( 9 from type SRS which costs 4000 $ for each and 3 from type BES which costs 20,000 $ for each) at 8 different nodes including D2, D3, D4, D5, D8, D10, S4 and S8. Although in both cases almost the same budget has used:

Initial Model: 5*20,000=100,000 $

Improved Model: 9*4000 + 3*20,000=96,000 $

Second model (improved version) has covered 4 paths instead of only 2 paths and in other terms, 10 flows instead of only 5 flows with station capacity of 5 instead of station capacity of 10. There is a significant decrease in the required budget to cover all the paths from 978,000 $ in initial MIP model to 464,000 $ in improved MIP model which is solved through branch and cut algorithm. This is due to the better and smarter distribution of facilities and budget. It is obvious that the required budget has been decreased to less than the half of the Wang & Lin (2013) approach. One can find similar logic for other budgetary constraints as well.

with a vehicle range of 40 km, and the station capacity of 10

| Budgetary constraint (NT$) | Location (number of SRS) | Location (number of FRS) | Location (number of BES) | Total number of stations (SRS, FRS, BES) | Path covered | Number of flows covered (percent) |
|---|---|---|---|---|---|---|
| 100,000 | – | – | S8(5) | 5(0, 0, 5) | P3, P7 | 5 (16.1%) |
| 200,000 | D10(2), D11(2), D12(4), S3(2) | – | S8(8) | 18(10, 0, 8) | P2,P3,P7,P11 | 10 (32.3%) |
| 300,000 | D1(3), D2(3), D3(3), D7(2), D10(4), D11(4), D12(4), S3(2), S6(3)S7(3), S8(3) | – | S8(8) | 42(34, 0, 8) | P2,P3,P4,P8,P11,P12 | 15 (48.4%) |
| 400,000 | D1(7), D6(2), D7(2), D10(4), D11(4), D12(4), S3(2) | – | S7(5), S8(10) | 40(25, 0, 15) | P2, P3, P4, P7, P8, P10, P11, P12 | 19 (61.3%) |
| 500,000 | D1(6), D3(3), D5(3), D6(4), D7(2), D9(2), D10(4), D11(4), D12(4), S8(6) | – | S7(8), S8(9) | 55(38, 0, 17) | P2, P3, P4, P7, P8, P9, P10, P11, P12 | 21 (67.7%) |
| 600,000 | D1(9), D2(5), D4(3), D5(6), D6(2), D7(2), D9(2), D10(4), D11(2), D12(2), S1(2), S3(2), S5(2), S6(2), S7(3), S8(2) | – | S7(10), S8(10) | 70(50, 0, 20) | P1, P2, P3, P4, P7, P8, P9, P10, P11, P12 | 24 (77.4%) |
| 700,000 | D1(10), D4(3), D5(6), D6(7), D7(2), D10(4), D11(4), D12(4), S1(2), S8(5) | E(3) | S7(8), S8(10) | 68(47, 3, 18) | P1, P2, P3, P4, P5, P8, P9, P10, P11, P12 | 25 (80.6%) |
| 800,000 | D1(9), D2(5), D3(6), D5(6), D6(8), D7(2), D10(4), D11(2), D12(4), S3(2), S5(2), S6(2), S7(3), S8(5) | E(3) | S7(10), S8(10) | 83(60, 3, 20) | P1, P2, P3, P4, P5, P7, P8, P9, P10, P11, P12 | 27 (87.1%) |
| 900,000 | D1(7), D2(5), D3(3), D4(3), D5(3), D6(8), D7(2), D10(4), D11(2), D12(4), S1(2), S3(2), S5(2), S6(6), S7(10), S8(3) | E(7) | S7(5),S8(9) | 87(66, 7, 14) | P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, P12 | 28 (90.3%) |
| 974,000 | D1(5), D4(3), D5(6), D6(6), D10(2), D11(2), D12(2), S3(2), S7(10), S8(8) | D(4), E(3) | S2(2), S4(2), S7(8), S8(10) | 75(46, 7, 22) | P1, P2, P3, P4, P5, P6, P7, P8, P9, P11, P12 | 29 (93.5%) |
| 978,000 | D1(9), D2(2), D4(3), D5(6), D6(5), D7(2), D10(4), D11(4), D12(4), S4(2), S6(4), S7(4), S8(8) | E(7) | S7(10), S8(10) | 84(57, 7, 20) | P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, P12 | 31 (100%) |

Table 32: Results of Wang & Lin (2013) Approach for path maximiziation

| Budgetary Constraint | Location (number of SRS) | Location (number of FRS) | Location (number of BES) | Total number of stations (SRS, FRS, BES) | Path Covered | Number of Flows Covered (percent) |
|---|---|---|---|---|---|---|
| 100,000.00 | D2(2), D3(2), D4(2), D5(2), D8(1), D10(2) | - | S4(1), S8(2) | 12 (9, 0, 3) | P3, P4, P11, P12 | 10 (32.3%) |
| 200,000.00 | D2(2), D3(2), D4(3), D5(2), D10(1), D11(1), D12(1) | E(1) | S8(4), S4(1) | 18 (12,1,5) | P2, P3, P4,P6, P8, P11 | 17 (54.8%) |
| 300,000.00 | D2(1), D3(3), D4(4), D5(2), D8(1), D9(2), D10(2), D11(2), D12(2), S2(1) | E(3) | S8(3) | 26 (20, 3, 3) | P1, P2, P3, P4, P5, P6, P11, P12 | 23 (74.2%) |
| 400,000.00 | D2(1), D3(3), D4(4), D5(1), D6(1), D8 (1), D10(1) | E(5) | S1(1), S3(1), S8(3) | 22 (12,5,5) | P1,P2, P3, P4, P5, P6, P8, P10, P11, P12 | 27(87.1%) |
| 414,000.00 | D2(4), D4(4), D5(3) | E(5) | S2(1), S4(1), S8(4) | 22 ( 11, 5, 6) | P1,P2, P3, P4, P5, P6, P8, P9, P10, P11, P12 | 29(93.5%) |
| 464,000.00 | D2(1), D3(4), D4(3), D5(3) | D(1), E(5) | S2(1), S4(1), S8(4) | 23 (11,6,6) | P1,P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, P12 | 31 (100%) |

Table 33: Results of AIMMS for path maximiziation; Vehicle range=40 km, station capacity = 5

- **Genetic Algorithm**

According to results of Table 34 (Genetic Algorithm), the location capacity of 5 has the best result among others. In locations capacity of 6 and 7 trials, the algorithm has not used capacity of more than 5 which shows that it has stopped increasing the number of locations at this step. It is obvious from the table that location capacity of 5 is the optimum capacity for this case. These results are achieved after 100 iterations and with population size of 200 which makes it more reliable.

| Location Capacity | Location (number of SRS) | Location (number of FRS) | Location (number of BES) | Total number of stations (SRS, FRS, BES) | Total Location Cost ( $) |
|---|---|---|---|---|---|
| 2 | D2(2), D3(2), D4(2), D5(2), D6(2), D9(1), D10(1), D11(1), S7(1), S8(2) | A(1), B(2), C(2), D (2), E(2), S8(1) | S2(1), S4(2), S8(2), S7(1) | 32(16,10,6) | 684,000.00 |
| 3 | D3(2), D4(2), D5(2), D6(3), D9(3), D10(3), D11(1) | C(3), D (3), E(3) | S4(2), S8(3) | 29 (15, 9, 5) | 610,000.00 |
| 4 | D2(2), D3(2), D4(4), D5(3), D6(1), s7(1), S8(1) | D (4), E(4) | S4(2), S8(4) | 23 (9,8, 6) | 556,000.00 |
| 5 | D3(2), D4(1), D5(1), D9(1), D10(1), D11(1) | D(3), E(5) | S4(2), S8(4) | 21 (7,8,6) | 548,000.00 |
| 6 | D3(2), D4(1), D5(1), D6(1), D9(1), D10(1), D11(1), S7(1) | D(3), E(5) | S4(2), S8(4) | 23 (9,8,6) | 556,000.00 |
| 7 | D3(2), D4(1), D5(1), D6(1), D9(1), D10(1), D11(1) | D(3), E(5) | S4(2), S8(4) | 22 (8,8,6) | 552,000.00 |

Table 34: GA Results for cost minimization; with a vehicle range of 40 km

According to Table 35, we have used the capacity location 5 to maximize the number of covered flows with different budgetary constraints. With different levels of budget, we can compare the results of branch-and-cut algorithm in AIMMS for second model and Lingo Branch and Bound for initial model in the paper. According to the table 35, this project can be finished by 548,000 $ through installing 21 stations in 10 different nodes.

| Budgetary Constraint | Location (number of SRS) | Location (number of FRS) | Location (number of BES) | Total number of stations (SRS, FRS, BES) | Path Covered | Number of Flows Covered (percent) |
|---|---|---|---|---|---|---|
| 100,000.00 | D3(2), D9(1), D10(1), D11(1) | - | S8(2), S4(2) | 9 (5, 0, 4) | P3, P8, P11, P12 | 9 (29.0%) |
| 200,000.00 | D3(2), D9(1), D10(1), D11(1) | E(2) | S8(2), S4(2) | 11 (5,2,4) | P2, P3, P4, P8, P11, P12 | 15 (48.4%) |
| 250,000.00 | D3(2), D9(1), D10(1), D11(1) | E(3) | S4(2), S8(2) | 12(5,3,4) | P2, P3, P4, P8, P9, P11, P12 | 17(54.8%) |
| 300,000.00 | D3(2), D9(1), D10(1), D11(1) | E(4) | S4(2), S8(2) | 13 (5, 4, 4) | P1, P2, P3, P4, P8, P9, P10, P11, P12 | 19 (61.3%) |
| 400,000.00 | D3(2), D5(1), D9(1), D10 (1), D11(1) | E(5) | S4(2), S8(2) | 15 (6,5,4) | P1,P2, P3, P4, P8, P9, P10, P11, P12 | 22(71.0%) |
| 500,000.00 | D3(2), D4(1), D5(1), D6(1), D9(1), D10(1), D11(1), S7(1) | D(2), E(5) | S4(2), S8(3) | 20 ( 8,7,5) | P1,P2, P3, P4, P6, P7, P8, P9, P10, P11, P12 | 28(90.3%) |
| 548,000.00 | D3(2), D4(1), D5(1), D9(1), D10(1), D11(1) | D(3), E(5) | S4(2), S8(4) | 21 (7,8,6) | P1,P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, P12 | 31 (100%) |

Table 35: GA Results for path maximiziation; Vehicle range=40 km, station capacity = 5

## 4.3. Location Planning Results Comparison

According to Table 36, while branch-and-cut algorithm has the best performance, genetic algorithm's response is fairly close to the mentioned exact algorithm. So in some cases when there is no exact solution for the problem or for large scale projects with huge solving time, genetic algorithm can be a substitute for branch-and-cut algorithm. Improving the model through adding one more constraint to limit station installation to the required cases only besides using a widely used algorithm for solving MIP problems, are the main factors which lead to significant decrease in the locating cost in this case.

| Results | Minimum Locating Cost ($) | Number of Installed Recharging Stations | Run Time |
|---|---|---|---|
| Initial Problem in Lingo Branch and Bound | 978,000.00 | 84 | less than 2 min, in some cases the time can be up to 60 min |
| Improved Problem in Barnch and Cut Algorithm | 464,000.00 | 23 | 1.15 seconds |
| Improved Problem in Genetic Algorithm | 548,000.00 | 21 | 9.43 min in average |

Table 36, Approaches' Results Comparison

In Figure 10 below, the difference in the performance of the three methods is illustrated clearly. It is obvious that the exact algorithm has the best performance among others, however genetic algorithm performs almost competitively with this algorithm.

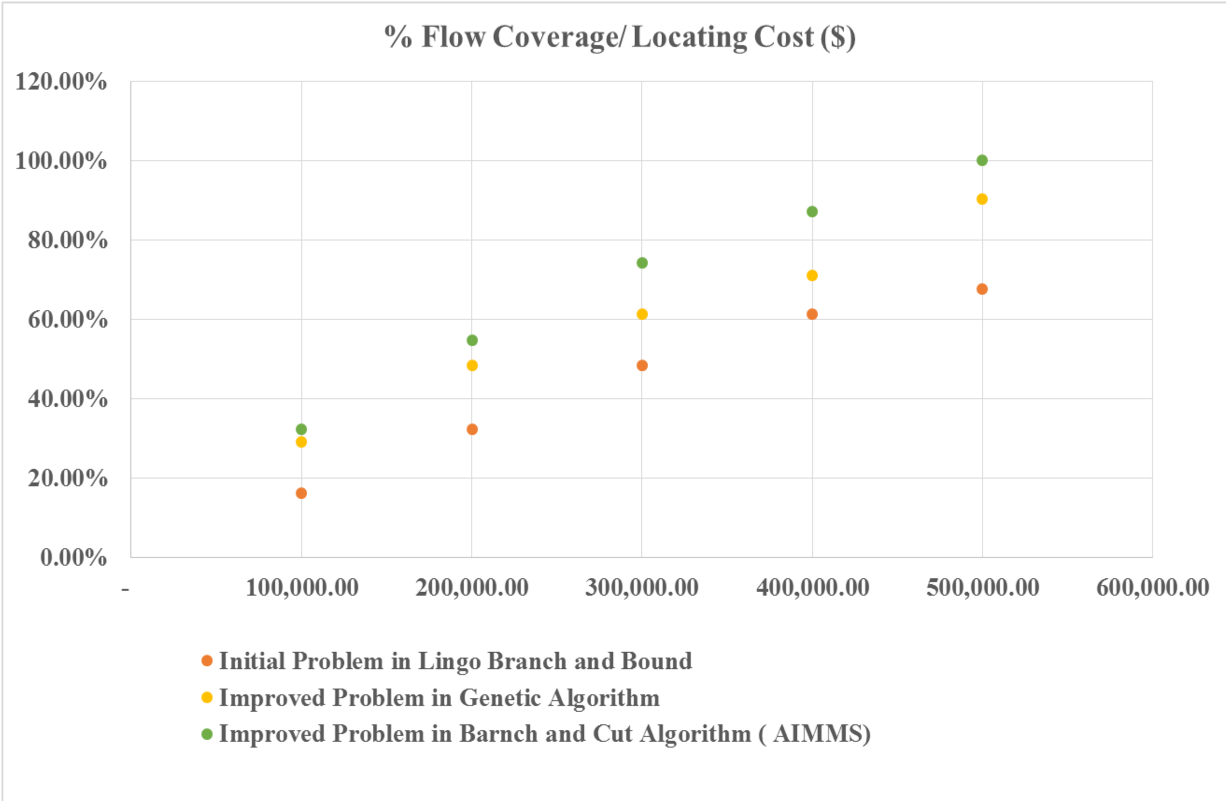


Figure 10: % Flow coverage for different methods

# Chapter 5:

# Conclusions and Future works

In this thesis, we address two core problems related to electric vehicles carsharing namely customer demand forecasting, and location planning problem of recharging stations. Four methods (regression forecast, regression forecast with seasonality adjustments, Winter's model and neural networks) are used for demand forecasting. An improved MIP model was optimized through branch-and-cut algorithm and genetic algorithm for the location planning problem of recharging stations. To analyze the strengths, weaknesses, opportunities, and threats of the proposed approaches, we conducted the SWOT analysis. The results of SWOT analysis for customer demand forecasting can be seen in Figure 11 below.

**S**

**Strengths**

- Proposing a comparative study among four different approaches in demand forecasting and finding the most optimal approach based on their MSE
- Performing an neural network performance analysis between different ratios which gives a more clear view of this method

**W**

**Weaknesses**

- There is a need for more research on customer tendency to use electric vehicles in the future. Environmental concerns, economic changes and population growth will effect on the pattern of the future demand and its trend during time.
- There is a need to consider the effects of peak hours, weekdays, holidays and weekends in predicting the future demand

**O**

**Opportunities**

- The results from this performance comparison study can be a base for other researches in the forecasting studies in order to find the best approaches

**T**

**Threats**

- Unpredicted changes like economic crises and climate changes may affect the predicted data, in such cases the results may not be reliable any more.

Figure 11: SWOT for demand forecasting

The output of first part of the thesis is of interest to many companies and industries. Having an accurate forecast of the customers' demands in different times helps these companies to satisfy their customers and become a benchmark in their business. Especially in the case of electric vehicles usage in carsharing companies, it gets more attention since it requires special infrastructures. Without an accurate demand forecasting, they cannot succeed in this kind of industry at all.

The results of SWOT analysis for location planning of recharging stations problem can be seen in Figure 12 below.

| S — Strengths | W — Weaknesses |
|---|---|
| • Improved model brings a wiser distribution of the charging stations based on the demand<br>• Deducting the locating cost of the recharging stations installation to almost half of the initial achieved price through implementing the improved model in two different approaches | • There is not enough managerial studies to consider other limitations in the real-life such as the traffic volume in different nodes, population, climate and ergonomical effects and so on.<br>• There should be more research studies to identify other types of electric vehicles recharging and considering them in the model which may lower the cost and bring more demand satisfaction |
| **O — Opportunities** | **T — Threats** |
| • Comparative study between two popularly used methods in optimization can be a strong base for the future studies in fleet management field<br>• Due to the flexibility of the selected model for this study, the model can be adopted and implemented for so many other cases in other companies according to the results of this study | • While it is a very good idea to use the results of this study for other cases, there is the probability that with changing the type of optimization problem from MILP to other types like non-linear or quadratic programming, the performance results for the taken approaches changes due to different applications of the used methods<br>• While this case study was a real project, in some case when the size of the projects increase enormously, there may be a change in the performance of the methods, so one should take this in to account when implementing for other cases |

Figure 12: SWOT for Recharging Stations Planning

The main strength of the second part is the comparison study which shows the competency between an exact method namely branch and cut and a heuristic method namely genetic algorithm. We also improved the model which led to better distribution of the recharging stations based on the users’ real need.

# Future works

In forecasting section, we can consider the effects of other factors like customer behavior and/or car rental costs as well as climate impacts on the future customer demand in carsharing industry. Also, applying other methods like heuristics, meta-heuristics, and NARX method in neural networks for the forecasting problem can bring more in depth results.

In the optimization part, there are much more heuristics methods which are of interest for solving MIP problems. Particle swarm optimization, ant colony optimization algorithm and tabu search are other heuristic algorithms that can be used. Also conducting case studies with large datasets and variables is a good practice to examine the performance of mentioned approaches in real life situations. One can perform more in-depth studies for comparing the performance of exact methods and approximate algorithm in these cases as well.

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# Appendix A : Distance Matrix

```
d(:,:,1)= [0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0.2 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,2)= [0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0

0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
7.4 7.4 7.4 7.4 0 0 0 7.4 0 7.4 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,3)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
1.8 1.8 0 0 1.8 0 0 0 0 0 0 0
0 0 7.6 0 0 0 0 7.6 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
```

```
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,4)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
6.8 6.8 0 6.8 6.8 6.8 0 0 0 0 0 0
0 0 0 0 0 0 0.3 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,5)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
7 0 0 0 0 0 0 0 0 0 0 0
```

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0 6 0 6 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,6)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
14.7 0 0 0 14.7 14.7 0 0 14.7 14.7 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,7)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,8)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 4.2
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];

d(:,:,9)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
```

```
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   4.2
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   10.6 0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
   ];

d(:,:,10)=[0  0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   5.886 0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   0
    0 0   0   0   0   0   0   0   0   0   0   5.9
    0 0   0   0   0   0   0   0   0   0   0   0
```

```
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];

d(:,:,11)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 5.886
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 5.9 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];

d(:,:,12)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 5.9
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0

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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 5.9 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];

d(:,:,13)=[0.2 0.2 0.2 0 0 0 0.2 0 0 0.2 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,14)=[0 0 0 0 0 0 0 0 11 0 0 0
3.8 3.8 3.8 3.8 0 0 0 3.8 0 3.8 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
```

```
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 15 15 15 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,15)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
7.6 7.6 0 0 7.6 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 9.2 9.2 0 9.2 9.2 9.2 9.2 9.2 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,16)=[0 0 0 0 0 0 0 0 0 0 0 0


0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0.3 0.3 0 0.3 0.3 0.3 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 9.1 0 9.1 9.1 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,17)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
6 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 8.8 0 8.8 8.8 8.8 0 0 8.8 8.8 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
```

```
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,18)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,19)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 5.9 5.9
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,20)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 5.9 5.9
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,21)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
```

```
0 0 0 0 0 0 0 0 0 0 5.9 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 4.2
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0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,22)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 5.9
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 3.1 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,23)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
```

```
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 3.1
0 0 0 0 0 0 0 0 0 0 0 0
8.8 8.8 8.8 8.8 8.8 8.8 8.8 8.8 8.8 8.8 0 0
0 0 0 0 0 0 0 0 0 0 0 0
];
d(:,:,24)=[0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0
```

```
    0 0     0     0     0     0     0     0     0     0     0     0
    8.8     8.8   8.8   8.8   8.8   8.8   8.8   8.8   8.8   8.8   0     0
    ];
d(:,:,25)=[0    0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     13.9  0     0     0     0     13.9  0     0     0     0
    0 0     0     0     0     0     11.8  0     0     0     0     0
    0 6.5   0     6.5   0     0     0     0     0     0     0     0
    5.6     0     0     0     5.6   5.6   0     0     5.6   5.6   0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    0 0     0     0     0     0     0     0     0     0     0     0
    ];
```