

# Cooperative Diversity in CDMA over Nakagami- $m$ Fading Channels

Ali Moftah Ali Mehemed

A Thesis

in

The Department

of

Electrical and Computer Engineering

Presented in Partial Fulfillment of the Requirements  
for the Degree of Doctor of Philosophy at  
Concordia University  
Montreal, Quebec, Canada

April 2013

©Ali M. Ali Mehemed, 2013

# Concordia University

## School of Graduate Studies

This is to certify that the thesis prepared

By: **Ali Moftah Ali Mehemed**

Entitled: **Cooperative Diversity in CDMA over Nakagami- $m$  Fading Channels**

and submitted in partial fulfillment of the requirements for the degree of

**DOCTOR OF PHILOSOPHY (Electrical and Computer Engineering)**

complies with the regulations of this University and meets the accepted standards with respect to originality and quality.

Signed by the final examining committee:

\_\_\_\_\_ Chair

Dr. A. Ben Hamza

\_\_\_\_\_ External Examiner

Dr. C. D'Amours

\_\_\_\_\_ External to Program

Dr. A. Stancu

\_\_\_\_\_ Examiner

Dr. M. O. Ahmed

\_\_\_\_\_ Examiner

Dr. Y. R. Shayan

\_\_\_\_\_ Thesis Supervisor

Dr. W. Hamouda

Approved by \_\_\_\_\_

Dr. J. X. Zhang, Graduate Program Director

April 25, 2013

\_\_\_\_\_ Dr. Robin Drew, Dean

Faculty of Engineering and Computer Science

# Abstract

## Cooperative Diversity in CDMA over Nakagami- $m$ Fading Channels

Ali Moftah Ali Mehemed, PhD  
Concordia University, 2013

Spatial diversity can be employed by sending copies of the transmitted signal using multiple antennas at the transmitter/receiver, as implemented in multiple-input multiple-output (MIMO) systems. Spatial receive diversity has already been used in many applications with centralized systems where base station receivers are equipped with multiple antennas. However, due to the power constraints and the small size of the mobile terminal, it may not be feasible to deploy multiple transmit antennas. User cooperation diversity, a new form of space diversity, has been developed to address these limitations. Recently, user cooperative diversity has gained more attention as a less complex alternative to centralized MIMO wireless systems. It revealed the ability to improve wireless communications through reliable reception.

One common network of the user cooperation diversity is the direct sequence code division multiple access (DS-CDMA) in which the Rayleigh fading channels are adopted and the orthogonality between users is assumed. The Rayleigh fading channels are unrealistic since they cannot represent the statistical characteristics of the complex indoor environments. On the other hand, Nakagami- $m$  fading model is well known as a generalized distribution, where many fading environments can be modeled. It can be used to model fading conditions ranging from severe, light to no fading, by changing its fading

parameter  $m$ .

The bit-error-rate (BER) and outage probability of uplink cooperative DS-CDMA over Nakagami- $m$  has not been addressed in the literature. Thus, in this thesis, the performance of both decode-and-forward (DF) and amplify-and-forward (AF) cooperative asynchronous DS-CDMA system over Nakagami- $m$  fading channels is investigated. The Rake receiver is used to exploit the advantages of multipath propagation. Besides, multi-user detection (MUD) is used to mitigate the effect of multiple-access interference (MAI). We show that our proposed multi-user system achieves the full system diversity gain.

The first part of the thesis introduces a new closed-form expression for the outage probability and the error probability of the DF cooperative DS-CDMA over asynchronous transmission over independent non-identical Nakagami- $m$  fading channels. The underlying system employs MUD such as minimum mean square error (MMSE) and decorrelator detector (DD) to achieve the full diversity. The aforementioned closed-form expression is obtained through the moment generating function (MGF) for the total signal-to-noise ratio (SNR) at the base station where the cumulative density function (CDF) is obtained. Furthermore, we investigate the asymptotic behavior of the system at high SNR to calculate the achievable diversity gain. The results demonstrate that the system diversity gain is fulfilled when MUD is used to mitigate the effect of MAI.

In the second part of the thesis, we study the performance of cooperative CDMA system using AF relaying over independent non-identical distribution (i.n.i) Nakagami- $m$  fading channels. Using the MGF of the total SNR at the base station, we derive the outage probability of the system. This enables us to derive the asymptotic outage probability for any arbitrary value of the fading parameter  $m$ .

The last part of the thesis investigates the optimum power allocation and optimum relay location in AF cooperative CDMA systems over i.n.i Nakagami- $m$  fading channels. Moreover, we introduce the joint optimization of both power allocation and relay location under the transmit power constraint to minimize the outage probability of the system. The joint optimization of both power allocation and relay location is used to minimize the outage performance of the system, thereby achieving full diversity gain.

# Acknowledgments

Before anything, I must express my sincerest gratitude and thankfulness to Dr. Walaa Hamouda who has supervised the thesis. I cannot dictate in words the depth and extent of his immense contribution to this thesis through advices and valuable criticism. At the same time, under his supervision, I have been able to conduct my research with great flexibility and freedom that, in turn, has expanded the scope of the research. Constructive research that I have conducted throughout the research and through which I could interact with academics and researchers of various related fields would not have been possible without his guidance. His contribution to the thesis has been immeasurable, and my experience under his supervision reaches beyond my academic experience and has influenced me greatly throughout.

I am grateful to both Dr. Salama Ikki and Mr. Ali Afana for the many fruitful and friendly discussions.

Lastly, the thesis would not be what it is without invaluable feedback from the PhD committee, Prof. Omair Ahmad, Prof. Yousef R. Shayan, and Dr. Alina Stancu. I am very thankful to Prof. Claude D'Amours from Ottawa university. To all the committee members, I greatly appreciate their invaluable time reviewing the manuscript.

I cannot thank my family and parents enough for their unceasing support and unconditional love without which I would not have been able to pursue and engage in a lifetime experience as this.

ALI MOFTAH ALI MEHEMED

To my parents and my family for their love and patience

# Contents

<b>Abstract</b>	<b>iii</b>
<b>Acknowledgments</b>	<b>v</b>
<b>List of Tables</b>	<b>xi</b>
<b>List of Figures</b>	<b>xii</b>
List of Acronyms . . . . .	xv
<b>List of Symbols</b>	<b>xvii</b>
<b>Chapter 1 Introduction</b>	<b>1</b>
1.1 Motivation . . . . .	2
1.2 Thesis Contributions . . . . .	5
1.3 Outline of the Thesis . . . . .	6
<b>Chapter 2 Literature Review</b>	<b>8</b>
2.1 Multiple-Access Techniques . . . . .	8
2.2 MIMO Networks . . . . .	10
2.3 Cooperative Communications . . . . .	11
2.4 Multiuser Detection . . . . .	14
2.4.1 Conventional Detector . . . . .	14
2.4.2 Decorrelator Detector (DD) . . . . .	14

2.4.3	MMSE Detector . . . . .	16
2.5	Fading Channel Model . . . . .	17
2.5.1	Nakagami- $m$ Fading Simulator . . . . .	18
2.5.2	Simulation Results . . . . .	20
2.6	Conclusions . . . . .	20

## Chapter 3 Performance Analysis of Cooperative CDMA Systems using

<b>DF Relaying</b>	<b>22</b>
3.1 Introduction . . . . .	23
3.2 System Model . . . . .	24
3.3 Outage Probability Analysis . . . . .	27
3.3.1 Single-Relay Cooperation . . . . .	29
3.3.1.1 Perfect Inter-User Channel . . . . .	29
3.3.1.2 Imperfect Inter-User Channel . . . . .	31
3.3.1.3 Simulation and Numerical Results . . . . .	35
3.3.2 Multi-Relay Cooperation . . . . .	36
3.3.2.1 Simulation and Numerical Results . . . . .	41
3.4 BER Analysis . . . . .	44
3.4.1 System Model . . . . .	44
3.4.2 Error Performance Analysis . . . . .	49
3.4.2.1 PDF Evaluation . . . . .	50
3.4.2.2 probability of Bit error . . . . .	53
3.4.3 Simulation and Numerical Results . . . . .	56
3.4.4 Conclusion . . . . .	59

## Chapter 4 Performance Analysis of Cooperative CDMA Systems using

<b>AF Relaying</b>	<b>61</b>
4.1 Introduction . . . . .	61



4.2	System Model . . . . .	63
4.3	Outage Probability Analysis . . . . .	65
4.3.1	Numerical Bound . . . . .	67
4.3.2	Approximate Distribution of the Total SNR . . . . .	68
4.3.3	Simulation and Numerical Results . . . . .	70
4.4	BER Analysis . . . . .	74
4.4.1	System Model . . . . .	74
4.4.2	Total SNR of the system . . . . .	75
4.4.2.1	MUD at the base station only . . . . .	75
4.4.2.2	MUD at both relay and base station . . . . .	78
4.4.3	BER . . . . .	79
4.4.4	Numerical Results . . . . .	82
4.4.5	Conclusions . . . . .	85
<b>Chapter 5</b>	<b>Power Allocation and Relay Location Optimization</b>	<b>86</b>
5.1	Introduction . . . . .	87
5.2	Approximate Distributions of the Total SNR . . . . .	89
5.3	System Optimization . . . . .	90
5.3.1	Adaptive Power Allocation . . . . .	91
5.3.2	Relay Location Optimization . . . . .	92
5.3.3	Joint Optimization of the Relay Location and Power Allocation . . . . .	94
5.4	Numerical Results and Discussions . . . . .	96
5.5	Conclusions . . . . .	99
<b>Chapter 6</b>	<b>Conclusions and Future Works</b>	<b>100</b>
6.1	Summary and Conclusions . . . . .	100
6.2	Future works . . . . .	102
<b>Appendix A</b>		<b>104</b>

Appendix B	107
Bibliography	111

# List of Tables

2.1	Inverse Nakagami CDF Approximation Coefficients [66]. . . . .	20
-----	---	----

# List of Figures

2.1	Multiple access strategies: (a) FDMA, (b) TDMA, (c) CDMA . . . . .	9
2.2	Cooperation Methods. . . . .	12
2.3	Conventional detector. . . . .	14
2.4	Decorrelator Detector. . . . .	15
2.5	MMSE detector. . . . .	16
2.6	PDF of Nakagami- $m$ distribution as a function of different $m$ . . . . .	19
2.7	Block diagram of Nakagami- $m$ fading channel simulator [66]. . . . .	19
2.8	PDF of Nakagami- $m$ distribution with different fading parameter $m$ . . . .	21
2.9	CDF of Nakagami- $m$ distribution with different fading parameter $m$ . . . .	21
3.1	Cooperation module . . . . .	25
3.2	Cooperation module between users . . . . .	26
3.3	Outage probability for relay cooperation in DF DS-CDMA with different $m$ . .	34
3.4	Outage probability for cooperative DF DS-CDMA over i.i.d Nakagami- $m$ fading channel. . . . .	35
3.5	Outage probability for cooperative DF DS-CDMA over i.n.i channels with different $m$ . . . . .	36
3.6	Outage probability for cooperative DF DS-CDMA with different number of relays, $L$ , over i.i.d. Nakagami- $m$ fading channels. . . . .	42

3.7	Outage probability for cooperative DF DS-CDMA with different $L$ over independent nonidentical Nakagami- $m$ fading channels. . . . .	43
3.8	Outage probability of cooperative DF asynchronous DS-CDMA system using conventional detector and MMSE for $L = 1$ and $L = 3$ . . . . .	44
3.9	Transceiver structure for DF . . . . .	45
3.10	Cooperation module between users . . . . .	46
3.11	Cooperative DF DS-CDMA with $N=31$ , $K=16$ users, 1 path and different $m$ . . . . .	57
3.12	Cooperative DF DS-CDMA with $N=31$ , $K=16$ users, 2 paths and different $m$ . . . . .	57
3.13	BER as a function of inter-user channel with $m = 1$ . . . . .	58
3.14	BER as a function of inter-user channel with $m = 2$ . . . . .	58
3.15	BER performance of an asynchronous DS-CDMA system over 2 paths using DD and MF detectors. . . . .	59
4.1	Outage probability for cooperative AF DS-CDMA over i.i.d Nakagami- $m$ fading channel. . . . .	71
4.2	Outage probability for cooperative AF DS-CDMA with different fading figures $m$ and considering i.n.i Nakagami fading channels. . . . .	72
4.3	Performance of multi-relay cooperative system with different $L$ over independent non identical Nakagami- $m$ fading channels. . . . .	72
4.4	Outage probability for cooperative AF DS-CDMA with different $L$ over i.i.d. Nakagami- $m$ fading channels, analytical based on the approximate outage probability in (4.33). . . . .	73
4.5	Outage probability for cooperative AF DS-CDMA with different $L$ over independent non-identical Nakagami- $m$ fading channels. . . . .	73
4.6	Lower bound BER performance for cooperative AF DS-CDMA as a function of fading parameter $m$ over $P = 1$ frequency-selective Nakagami- $m$ fading channels. . . . .	83

4.7	Lower bound BER performance for cooperative AF DS-CDMA as a function of resolvable paths over Nakagami- $m$ fading channels with $m = 1$ .	84
4.8	Lower bound BER performance for cooperative AF DS-CDMA over one-path frequency-selective over Nakagami- $m$ fading channels with $m = 1.5$ where the MAI impact is investigated.	84
5.1	Relay optimization protocol, where $d$ is the optimum relay location	87
5.2	Asymptotic outage probability for cooperative AF DS-CDMA versus power allocation ratio at the source for different relay position for fixed $SNR = 15dB$ .	96
5.3	Outage probability performance of power allocations algorithm	97
5.4	Asymptotic outage probability for cooperative AF DS-CDMA versus relay location ratio for different source power allocation for fixed $SNR = 15dB$ and $m_{s,b} = 1$ , $m_{s,r} = 0.85$ , $m_{r,b} = 1$	98
5.5	Asymptotic outage probability for cooperative AF DS-CDMA versus power allocation ratio at the source for different relay position for fixed $SNR = 15dB$ .	98

## List of Acronyms

<b>AF</b>	amplify-and-forward
<b>AWGN</b>	additive white Gaussian noise
<b>BER</b>	bit-error-rate
<b>BLAST</b>	Bell laboratories layered space time
<b>BPSK</b>	binary phase shift keying
<b>CDF</b>	cumulative distribution function
<b>CDMA</b>	code-division multiple-access
<b>CRC</b>	cyclic redundancy check
<b>DD</b>	decorrelator detector
<b>DF</b>	decode-and-forward
<b>DS-CDMA</b>	direct sequence code division multiple access
<b>FDMA</b>	frequency division multiple access
<b>GSM</b>	Global System for Mobile Communications
<b>i.i.d</b>	independent identical distribution
<b>i.n.i</b>	independent non-identical distribution
<b>ISI</b>	inter symbol interference
<b>MA</b>	multiple access
<b>MAI</b>	multiple-access interference

<b>MC-CDMA</b>	multi-carrier CDMA
<b>MF</b>	matched-filter
<b>MIMO</b>	multiple-input multiple-output
<b>MMSE</b>	minimum mean square error
<b>MRC</b>	maximal-ratio-combiner
<b>MUD</b>	multi-user detection
<b>PDF</b>	probability distribution function
<b>RV</b>	random variables
<b>SIC</b>	successive interference cancellation
<b>SNR</b>	signal-to-noise ratio
<b>SSMA</b>	spread-spectrum multiple access
<b>STBC</b>	space-time blocking coding
<b>STC</b>	space time coding
<b>STTC</b>	space-time trellis coding
<b>TDMA</b>	time division multiple access
<b>TZF</b>	transmitter zero-forcing
<b>VBLAST</b>	Vertical-BLAST
<b>QoS</b>	quality-of-service



## List of symbols

$K$	number of users
$\mathbf{R}$	cross correlation matrix
$\mathbf{R}^{-1}$	inverse cross correlation matrix
$\rho_{i,j}$	the cross correlation between any two spreading codes
$f$	frame length
$C$	spreading code
$\mathbf{x}$	data vector
$\mathbf{n}$	additive white Gaussian noise vector
$\mathbf{H}$	channel matrix
$\mathbf{y}$	matched filter output vector
$m$	Nakagami fading figure
$\Gamma(.)$	Gamma function
$\Omega$	Nakagami average power
$N$	processing gain of spreading sequence
$r_b^I(t)$	Phase I received signal at the base station
$r_b^{II}(t)$	Phase II received signal at the base station
$r_r(t)$	received signal at relay ( $r$ )
$T_b$	bit period
$T_c$	chip period
$P$	number of resolvable paths
$\tau_s$	transmit delay of the $s^{th}$ user
$\tau_{s,p}$	delay of the path $p^{th}$ of user $s$
$h_{sb}$	complex fading channel coefficient from source to base station
$h_{sr}$	complex fading channel coefficient from source to relay

$h_{rb}$	complex fading channel coefficient from relay to base station
$\sigma_n^2$	noise variance
$\gamma_{sb}$	instantaneous SNR from source to base station
$\gamma_{sr}$	instantaneous SNR from source to relay
$\gamma_{rb}$	instantaneous SNR from relay to base station
$\bar{\gamma}_{sb}$	average SNR from source to base station
$\bar{\gamma}_{sr}$	average SNR from source to relay
$\bar{\gamma}_{rb}$	average SNR from relay to base station
$m_{sb}$	Nakagami fading figure from source to base station
$m_{sr}$	Nakagami fading figure from source to relay
$m_{rb}$	Nakagami fading figure from relay to base station
$E_s$	received signal energy
$N_o$	noise energy
$E\langle.\rangle$	statistical expectation
$\gamma(.,.)$	lower incomplete gamma function
$\Gamma(.,.)$	upper incomplete gamma function
$P_{out}$	outage probability
$\Re$	spectral efficiency
$L$	number of relays
$I_{sb}$	mutual information between source and base station
$I_{sr}$	mutual information between source and relay
$I_{rb}$	mutual information between relay and base station
$\tilde{x}$	estimate data of $x$

$\mathcal{K}$	cascaded path between $s \rightarrow r_{\mathcal{K}} \rightarrow b$
$M(s)$	moment generating function
$\mathcal{L}^{-1}\{.,.\}$	inverse Laplace transform
$F(.)$	cumulative density function
$f(.)$	probability density function
$\delta(.)$	Dirac delta function
$z$	output of the matched filter bank
$\mathbf{A}$	signal amplitude matrix
$D$	transmission delay during the second transmission period
$\mathbf{R}_b$	cross correlation matrix at the base station
$\mathbf{R}_r$	cross correlation matrix at the relay
$s$	source node
$r$	relay node
$b$	base station node
$\min(x, y)$	minimum of $x$ and $y$
$\max(x, y)$	maximum of $x$ and $y$
$D(s)$	relay decoding set
$\mathbf{H}_{s,b}$	channel matrix between the source and the base station
$\mathbf{H}_{s,r}$	inter-user channel matrix
$\mathbf{H}_{r,b}$	channel matrix between relay and base station
$[z_1]_{b1}$	decision statistics of bit $i$ for user 1 in the first transmission period
$[z_1]_{b2}$	decision statistics of bit $i$ for user 1 in the second transmission period
$Q(x)$	Gaussian Q-function

$\Phi_{A,B}(w_1, w_2)$	joint characteristics function of A and B
$\eta$	signal to interference and noise ratio
$f_{A,B}$	joint probability density function of A and B
$f_\eta$	probability density function of the output SNR
$P_{b_{error-free}}$	average BER for perfect inter-user channel case
$P_{b_{sr}}$	average BER for inter-user channel
$P_{b_{sb}}$	average BER for direct link channel
$P_{b_{total}}$	total average BER for imperfect inter-user channel case
$\beta$	amplification factor for AF scheme
$X_{total}$	total SNR at the base station for AF scheme
$I_{AF}$	total average mutual information for AF scheme
$I_{DF}$	total average mutual information for DF scheme
$P_b$	average BER for AF scheme
$d$	optimum relay location
$E_s$	source transmitted energy
$E_r$	relay transmitted energy
$F(.,.;.;.)$	Gauss hypergeometric series

# Chapter 1

## Introduction

Nowadays, the acceleration on wireless communication places demands on high data rate and high throughput requirements. In addition, wireless communication has become a part of our daily routine as in our homes, cars and computers. Cellular phones as an example of wireless systems, are important as they permit users to stay connected at any-place at any time with voice, multimedia, and high-speed Internet services. The common aspect of those services is that they require reliable link over different environments, and also require stable network in terms of spectral efficiency, system capacity, and transmission range. In order to fulfill the above goals, wireless systems' designers face many physical limitations such as signal fading occurring from multipath propagation, bandwidth limitation for each service provider, and transmitted power where wireless devices should offer long battery life and device size.

To improve spectral efficiency and utilize the available bandwidth in wireless systems, multiple access techniques are employed such that communication resources are shared among different users. The available communication resources can be shared among multiple users in many ways as in frequency division multiple access (FDMA), time division multiple access (TDMA), and code-division multiple-access (CDMA) where the signaling space is shared different dimensions (frequency, time, and code) respectively.

FDMA and TDMA are orthogonal multiplexing methods over frequency and time, respectively. In CDMA, the signal is modulated by a pseudo noise sequence and transmitted over the whole system bandwidth. Anti-multipath capabilities, soft capacity, soft hand off, and potential capacity increases over other multiple-access techniques are some of the characteristics of the direct sequence code division multiple access (DS-CDMA). Significant performance improvements are achieved from multi-user detection (MUD) techniques for DS-CDMA compared with the conventional receiver. The main advantage of the asynchronous CDMA over synchronous CDMA, TDMA and FDMA is its ability to use the spectrum more efficiently in mobile networks [1–4].

Diversity is one of the powerful communication techniques that mitigate the effect of fading resulting from the multipath propagation. Diversity techniques utilize the random nature of the radio propagation by using the independence of the faded version of the transmitted signal to enhance the system performance. Diversity can be achieved employing the following methods: *i*) time diversity where different copies of the same information are sent at different time slots. *ii*) frequency diversity where multiple copies of the same information are sent via channels of different frequency bands. *iii*) space diversity is a very popular diversity method where the same signal is transmitted/received over different spatial channels. Multi-input multi-output MIMO is one common method of space diversity where the diversity is achieved when multiple antennas are implemented to transmit and/or receive the users information [5–7].

## 1.1 Motivation

Multipath fading, interference, and scarcity of power and bandwidth are the main limitations in any wireless communication system. The multipath fading problem can be solved by applying spatial transmit/receive diversity techniques. Transmit diversity can be achieved by implementing multiple antennas at the transmitter as in MIMO systems.

However, implementing multiple antennas systems at the mobile terminal may be impractical due to cost, size, and power limitations. The potential solution for these limitations is to apply user cooperation diversity techniques by which mobile terminals share their physical resources to create virtual antennas and therefore, transmit diversity can be achieved [8], [9]. Cooperative diversity with multiple relays is an auspicious solution by which the reliability of wireless communication systems and the high data rate coverage can be improved [5], [10].

Cooperative DS-CDMA diversity was widely investigated using non-orthogonal spreading codes for synchronous networks over flat Rayleigh fading channels [11–14]. Furthermore, the authors in [15] have introduced an overview of joint iterative power allocation and linear interference cancellation for cooperative DS-CDMA where synchronous transmission is considered. In light of this, one of our objectives is to design a cooperative diversity scheme for asynchronous DS-CDMA networks over frequency-selective fading channels. The system under consideration is intended to improve the performance and provide full diversity gain. Full diversity gain is achieved using MUD to mitigate the multiple-access interference (MAI) [16]. In the literature, the outage probability over Rayleigh fading channel was extensively investigated in high signal-to-noise ratio (SNR) for cooperative asynchronous DS-CDMA networks [17]. In [18], the authors studied the effect of MAI on the performance of asynchronous DS-CDMA cooperative systems over multipath fading channels. In their work, the authors considered the multi-relay coded cooperation case, wherein the bit-error-rate (BER) performance of the system was analyzed for two cases: perfect and imperfect inter-user links over Rayleigh fading channels.

Cooperative CDMA techniques are of a major importance in improving the performance of multiuser networks. Due to this importance, much works have been conducted to study and analyze the performance of cooperative CDMA networks. In the literature, minor contributions have been introduced in analyzing the performance of these diversity techniques in general fading channels such as the Nakagami- $m$  model. With this

gap in mind, our aim in this thesis is to analyze selected performance metrics of CDMA cooperative diversity networks including error rate and outage probability performances over Nakagami- $m$  fading channel. In this context, we analyze the performance of several cooperative techniques. These techniques include a wide range of schemes that can be classified into classical techniques such as amplify-and-forward (AF) and decode-and-forward (DF). In that, we analyze the outage probability and the error performance for cooperative CDMA using DF and AF relaying. In the thesis, we consider MUD techniques in order to mitigate the effect of MAI to achieve full diversity, thus approaching the optimum performance for CDMA systems.

To avoid power disbursement under the constraint of satisfying the users quality-of-service (QoS), it is important to allocate users power and determine the optimal relay location. This leads to the joint power allocation and relay location problem which has been rarely studied in the literature on cooperative CDMA. The joint power allocation and relay selection for multiuser asynchronous DF cooperative communication was investigated in [19]. Along the same lines, the optimum power distribution for cooperative CDMA networks for both DF and AF relaying over Rayleigh fading channel was studied in [20]. Meanwhile, the authors in [15] have introduced an overview of joint iterative power allocation and linear interference cancellation for cooperative DS-CDMA where synchronous transmission was considered. It is worth mentioning that the above mentioned works have investigated systems over Rayleigh fading channels. In light of this, we study the optimization techniques for both power allocation and relay location in order to clarify the effect of the transmitted power and the relay positioning on the system performance over Nakagami- $m$  fading channel. Besides, the joint optimization is investigated to find the optimal relay positioning with an objective to minimize the total transmission power in order to reach the best performance.



## 1.2 Thesis Contributions

The main contributions of this thesis are as follows:

1. In Chapter 3, we study cooperative DS-CDMA over Nakagami- $m$  fading channels where the system model is investigated and analyzed using outage probability and BER metrics. Adopting DF relaying, as one of the main cooperative diversity techniques, we investigate the performance of the above mentioned system over asynchronous transmission and independent non-identical fading channels. We introduce new closed-form expressions for the outage and the error probabilities where the MUD is employed so as to achieve the full diversity gain for the system. Our method is based on the MGF and characteristic function for the total SNR at the base station where the cumulative distribution function (CDF) and probability distribution function (PDF) are obtained. We also examine the asymptotic performance of the system at high SNR from which we evaluate the achievable diversity gain for different system parameters. Our results show that the system diversity gain is achieved when MUD is used to cancel the effect of MAI. Moreover, analytical and simulation results are provided, where they show that the diversity advantage of the cooperative system under different parameter setting is achieved.
2. In Chapter 4, we address the performance of cooperative CDMA systems using AF relaying over independent non-identical distribution (i.n.i) Nakagami- $m$  fading channels. In the underlying AF relaying scheme, we analyze the outage probability of the system using the CDF of the total SNR at the base station. In that, we derive a simplified yet tight lower bound for AF relaying system. In addition, we investigate the outage probability of the system using the MGF of the total SNR at the base station. Since it is complicated to derive a closed-form expression for the outage probability, we derive an approximation for the PDF of the total SNR which in turn enables us to derive the asymptotic outage probability for any value of the

fading parameter  $m$ . As the second part of this chapter, we study the performance of asynchronous cooperative CDMA system using AF over frequency-selective over Nakagami- $m$  fading channels. We derive the BER of the system using the MGF of the total SNR at the base station. Since it is intractable to derive a closed-form expression for the PDF of the total SNR, we derive an upper bound for the PDF of the total SNR which enables us to derive a closed-form expression for the BER. Both analytical and simulation results show that the system diversity gain is enhanced when MUD is used to cancel the effect of MAI.

3. In Chapter 5, we study the optimum power allocation and the optimum relay location problems in AF cooperative CDMA systems. Furthermore, we investigate the joint optimization of both power allocation and relay location under the transmit power constraint to minimize the outage probability of the system. Simulation and analytical results illustrate the advantages obtained when power and relay location optimization are followed. Interestingly, our results show that the joint optimization of both power allocation and relay location brings the minimal outage probability performance of the system with full diversity gain.

The above contributions have resulted in following publications [21–25].

## 1.3 Outline of the Thesis

The remainder of this thesis is organized as follows.

Chapter 2 is a background chapter which introduces some of the essentials of cooperative CDMA systems. First, we start with a description of the diversity techniques and cooperative diversity. Then, we present a brief introduction on multi-user detection techniques. Later, we present the Nakagami- $m$  fading channel simulator that used in our research. Finally, we present a literature review of the relevant research work in cooperative CDMA.

In Chapter 3, we analyze the performance of cooperative CDMA systems using DF relaying over Nakagami- $m$  fading channels. Closed-form expression for the MGF of the total SNR at the base station is derived. This expression is then used to obtain the outage probability of the proposed scheme. Finally, the BER for the cooperative DF CDMA over frequency-selective fading channel is analyzed.

Chapter 4 considers the performance of cooperative CDMA systems using AF relaying over Nakagami- $m$  fading channels. In our analysis, we derive a simplified tight upper bound for the outage probability. Also, we derive an approximation for the PDF of the total SNR which enables us to derive the asymptotic outage probability of the system. We analyze the BER performance of asynchronous cooperative CDMA system using AF over frequency-selective over Nakagami fading channels.

In Chapter 5, we introduce the optimization problem for both power allocation and relay location for the cooperative AF CDMA systems over Nakagami- $m$  fading channel. In that, we study the joint optimization for both power allocation and relay location.

Chapter 6 provides a summary of the accomplished work throughout this thesis with valuable conclusions. Some possible future work based on the content of the thesis are also mentioned.

The mathematical notations used in this thesis are as follows: bold capital letters are used for matrices and bold lower-case letters are used for vectors;  $()^{-1}$  denote inverse of a matrix; and  $()^T$   $()^H$  are the transpose and Hermitian matrix operations respectively.

# Chapter 2

## Literature Review

This chapter summarizes recent works related to the problems investigated in the thesis. Our goal is to make the reader familiar with the many aspects involved, highlight the specific scenarios that we study through the thesis, and promote further work in the field.

### 2.1 Multiple-Access Techniques

Development of recent services and the continuous increase in the number of users in wireless networks have resulted in extensive research to offer large system throughput and reliable communication in wireless environments. To accommodate high throughput for the new services, the available spectrum should be utilized efficiently and flexibility in radio resource management should exist.

In order to ensure that all users connecting to a certain bandwidth receive equal access of its network resources, there is a need for multiple access (MA) schemes that allow such fair allocation possible. This section lists three main MA techniques: FDMA, TDMA, and CDMA as shown in Fig. 2.1 [1], [26].

In FDMA systems, an allocation technique that divides the frequency spectrum into a channel is employed. Simply, a specific portion of total bandwidth is allocated to an individual user. Users that are accessing the system at the same time have different

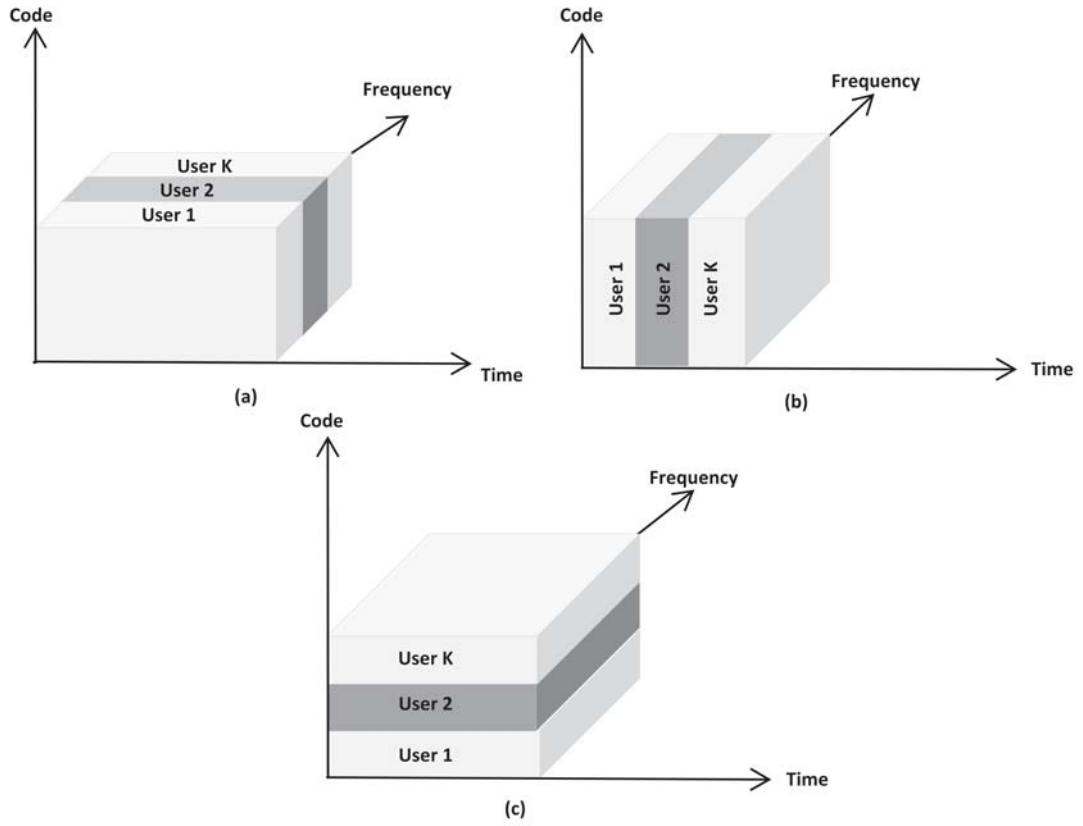


Figure 2.1: Multiple access strategies: (a) FDMA, (b) TDMA, (c) CDMA

frequency channels, and interference between these users are less prevalent. TDMA is utilized effectively in the second generation Global System for Mobile Communications (GSM) in which many users are allowed to share the same bandwidth by splitting the users data into different time-slot. The last technique is CDMA or spread-spectrum multiple access (SSMA) which is used in dividing and separating the users in spectrum technology [4]. A CDMA system is different from the other two in how it makes use of the available bandwidth Fig. 2.1; unlike the isolation technique, which separates different number of users into a single user communication link, what characterizes a CDMA system is the full utilization of the available bandwidth by each transmission.

## 2.2 MIMO Networks

MIMO networks achieve great gains in system throughput and spectral efficiency by employing different parallel transmissions of space-time signals in the same spectrum [27]. In addition, the real transmission rate of MIMO networks increases linearly proportional to the number of transmit and receive antennas. [28] , [29].

A mechanism behind the maximization of the system throughput using Bell laboratories layered space time (BLAST) technique and its simplified Vertical-BLAST (VBLAST) was originally proposed in [30]. In such schemes, independent signals are transmitted from multiple transmit antennas to improve the spectral efficiency. A BLAST scheme mainly depends on successive interference cancellation (SIC) to detect the signals at the receiver. MIMO systems can also be used to improve the quality of the transmission link (i.e., diversity) through what is known as space-time coding (STC). In these coding techniques, the same signal is transmitted from different antennas and hence achieving spatial diversity. STC utilizes optionally multiple receive antennas compounded with multiple transmit antennas [31], [32]. Two forms of this new STC approach are noteworthy: first, space-time trellis coding (STTC) is able to maximize diversity gain but sacrifices simplicity as the optimal decoding process at the receiver becomes complex; the second form is space-time blocking coding (STBC). There is no coding gain offered from using the latter, but it does not have coding complexity problem as STTC does. Other mechanisms of overcoming the STTCs complexity include multistage decoding [29] and antenna partitioning [33]. Given the same number of transmit and receive antennas, STTCs and STBCs achieve the same spacial diversity. However, STBCs are simpler to implement but are known to offer only diversity gain with no coding gain as STTCs.

Another category of MIMO systems targets an improvement in the link rate through spatial multiplexing and this can be achieved by using layered space time coding. A compromise between spatial diversity and spatial rate can also be gained by using multi-

layered or threaded space-time coding [34].

Recently, MIMO CDMA systems have gained considerable interest in literature. Sacramento *et al.* [35, 36] have studied the multi-user decorrelator detector in MIMO CDMA over i.i.d. Nakagami- $m$  fading channel. The authors in [35, 36] derived the BER where the linear decorrelator detector is used in order to mitigate the MAI. Along the same lines, the outage probability of MIMO transmit antenna selection was investigated in [37]. In [35, 36], the authors derived a closed-form expression for i.i.d Nakagami- $m$  fading channels. Nevertheless, employing multiple antennas at the transmitter and/or the receiver may increase the cost of the MIMO system.

## 2.3 Cooperative Communications

Spatial diversity can be employed by sending copies of the transmitted signal using multiple antennas at the transmitter/receiver, as in MIMO systems. However, due to both the small size of mobile terminals and power constraints, it may not be feasible to deploy multiple transmit antennas. User cooperation diversity, a new form of spatial diversity, has been developed to address this limitation. Recently, cooperative diversity has gained the attention of many researchers as a less complex alternative to centralized MIMO wireless systems [8], [9]. It has been shown that cooperative diversity can improve the capacity of wireless communications through reliable reception. Sendonaris *et al.* [8], [9], introduced the simplest case of two-user cooperation via orthogonal spreading codes. The cooperation among users can be classified into two basic cooperative schemes as shown in Fig.2.2: AF and DF. In AF, the relay (i.e., cooperative user) transmits an amplified version of the received partner's signal. On the other hand, in the DF mode the relay decodes the received signal followed by retransmission of the estimated data.

With CDMA being one of the generic multiple-access platforms for the second and third generation wireless systems, it is of great interest to study the application of co-

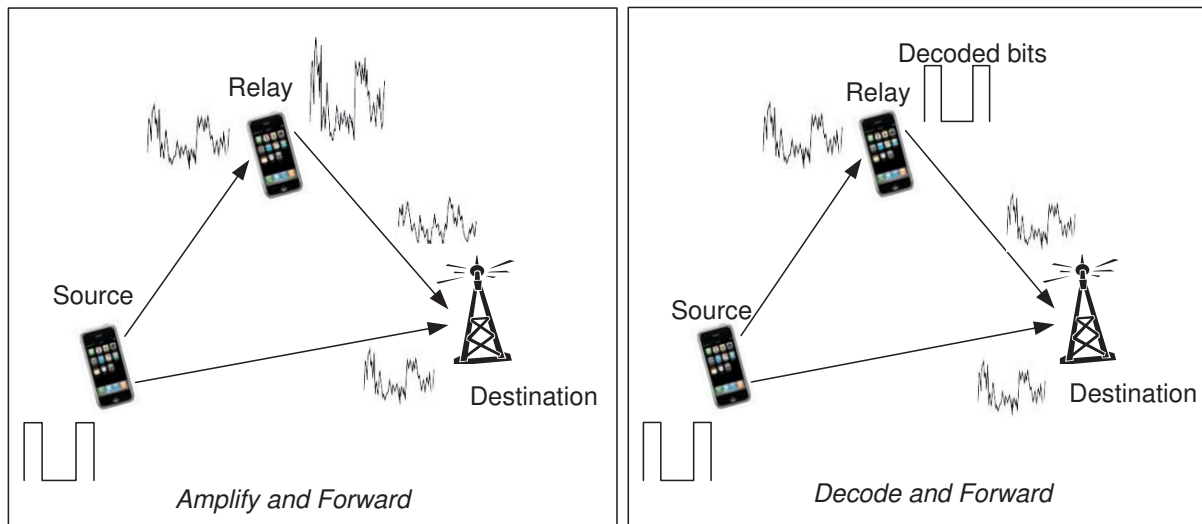


Figure 2.2: Cooperation Methods.

operative diversity when employed in these systems. One major problem with CDMA is due to the interference resulting from the asynchronous transmission of users' signals over the shared coded channel. Venturino *et. al.* [11] studied the performance of cooperative networks based on synchronous CDMA systems using non-orthogonal spreading codes. In [18], a study of the effect of MAI on the performance of asynchronous DS-CDMA cooperative systems over multipath fading channels was considered. In their work, the authors have considered the single cooperation relay case, wherein the BER performance of the system was analyzed for the two cases: perfect and imperfect inter-user link over Rayleigh fading channels. The performance of DF at high SNR was analyzed in [5]. Along the same direction, the authors in [17] investigated the outage probability of multi-user cooperative diversity in an asynchronous CDMA uplink over Rayleigh fading channel. The performance of multiuser AF relay networks has also been introduced in [12] where the authors derived an approximation for the outage probability over Rayleigh fading channels. The joint iterative power allocation and the interference cancellation for DS-CDMA systems using AF relaying was studied in [38]. In their work, the authors obtained the required power level for a multi-relay system. We remark that the conventional co-



operative systems without employing CDMA over Rayleigh fading channel have been intensively studied in the literature [10, 39–43]. For instance, the performance analysis of incremental-relaying cooperative-diversity networks over Rayleigh fading channels is investigated in [44] where a complete analytical method is presented to obtain closed-form expressions for the error rate, outage probability and the average achievable rate using both DF and AF over i.i.d Rayleigh fading channels. In [45], the outage probability of DF over Rayleigh fading channel was analyzed.

Nakagami- $m$  fading channels are well known as a generalized distribution, where many fading environments can be modeled. The exact BER of cooperative DS-CDMA systems using DF in the presence of multipath propagation has been derived in [14]. In this work, the author in [14], has considered a conventional DF protocol where the relays that offer the best instantaneous SNR are activated. Similarly, the authors in [46] investigated a cooperative downlink transmission scheme for DS-CDMA systems over Nakagami- $m$  fading channels to achieve relay diversity. They proposed transmitter zero-forcing (TZF) at the base station for suppressing the downlink multiuser interference. Furthermore, the authors in [47] have investigated the performance of downlink multi-user relay network employing single AF relay. They have derived a closed-form expression for the outage probability at high SNR.

The performance of cooperative diversity over Nakagami- $m$  fading channels have been widely studied in traditional cooperative communications [48–55]. As in [56], the outage probability of cooperative relay networks over i.i.d Nakagami- $m$  fading channels was evaluated. Along the same direction, the outage probability for DF over independent non identical flat Nakagami- $m$  using MGF was introduced in [57]– [58]. For instance, the authors in [59] have introduced a performance evaluation and optimization of dual-hop case over Nakagami- $m$  fading channels in the presence of channel interference. In their work, they investigated the effect of power allocation, relay position and joint optimization for AF schemes.

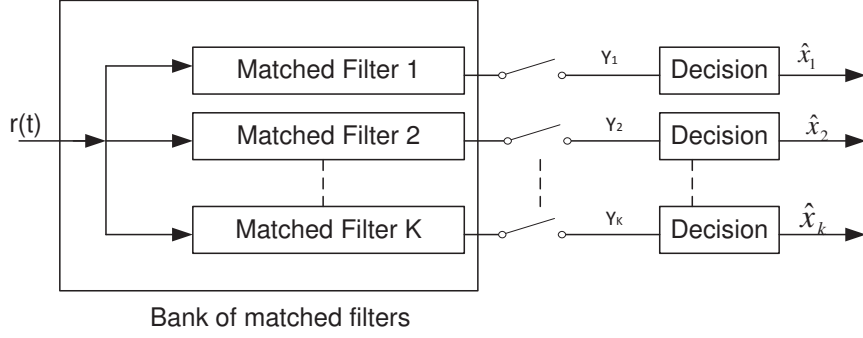


Figure 2.3: Conventional detector.

## 2.4 Multiuser Detection

MAI is considered as the major drawback in achieving the optimum performance for CDMA systems. Thus, intensive research has been conducted to develop MUD techniques to overcome the effect of MAI [11, 16–18, 60–65].

### 2.4.1 Conventional Detector

Fig. 2.3 shows the conventional detector, also known as the matched filter detector, which consists of a bank of  $K$  matched filters for a  $K$ -user system. The matched filters are followed by the samplers which are employed to sample the output at the bit intervals  $T_s$ . The main disadvantage of the conventional detector is due to the treatment of the MAI as noise, which has a severe effect on the system performance in a multi-user system.

### 2.4.2 Decorrelator Detector (DD)

The decorrelator detector (DD) proposed as a multiuser detector is known to cancel the effect of MAI at the cost of noise enhancement. As shown in Fig. 2.4, the DD applies the inverse of the correlation matrix of users spreading codes  $[\mathbf{R}^{-1}]$  to the output of the matched filter bank.

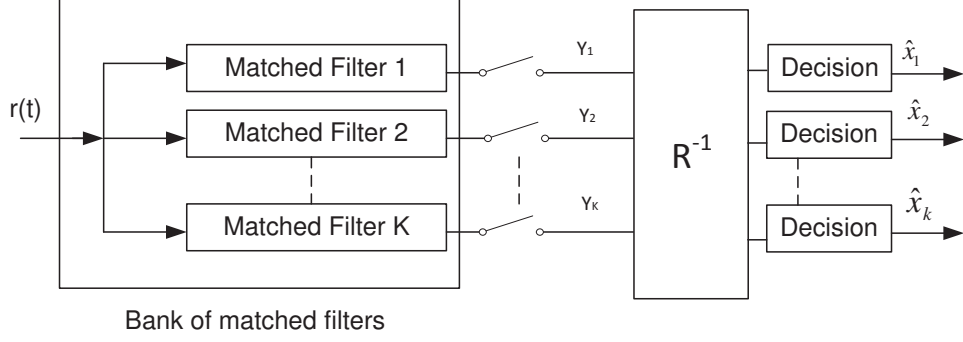


Figure 2.4: Decorrelator Detector.

The elements of the correlation matrix  $\mathbf{R}$  ( $K \times K$ ) are defined as

$$[\mathbf{R}]_{i,j} = \rho_{i,j} \quad (2.1)$$

where  $\rho_{i,j}$  denotes the cross correlation between the spreading codes  $C_i$  and  $C_j$ , in DS-CDMA system. Let the output of the conventional detector be expressed as

$$\mathbf{Y} = \mathbf{R}\mathbf{H}\mathbf{x} + \mathbf{n}, \quad (2.2)$$

where  $\mathbf{x} = [x_1 \ x_2 \ x_3 \ \cdots \ x_K]^T$  represents the data vector of the  $K$  users,  $\mathbf{n}$  is the additive white Gaussian noise (AWGN) vector ( $K \times 1$ ), and  $\mathbf{H}$  denotes the channel coefficient diagonal matrix. Applying the inverse of correlation matrix to the output of the conventional detector,

$$\mathbf{z} = (\mathbf{R})^{-1}\mathbf{Y} = \mathbf{H}\mathbf{x} + \mathbf{R}^{-1}\mathbf{n}. \quad (2.3)$$

From (2.3), we can easily notice that the effect of MAI is totally removed with only noise enhancement. That is the main advantage of DD compared with the conventional detector is its ability to remove all the MAI. However, it enhances the system noise dramatically [16].

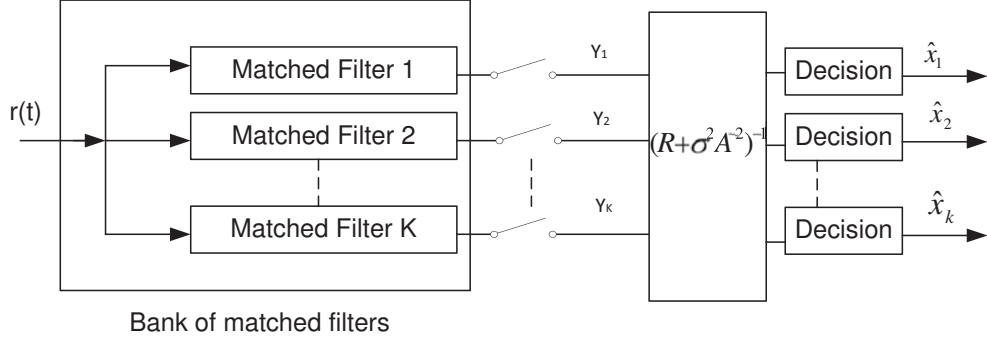


Figure 2.5: MMSE detector.

### 2.4.3 MMSE Detector

Different from the DD, the MMSE multiuser detector mitigates the effect of both noise and MAI as shown in Fig. 2.5. The MMSE detector is modified by the transformation,

$$\mathbf{M} = (\mathbf{R} + \bar{\gamma}\mathbf{I})^{-1} \quad (2.4)$$

where  $\bar{\gamma} = \mathbb{E}\langle |h|^2 \rangle E_s / N_o$  is the average SNR,  $h$  is the fading channel coefficient,  $\mathbf{I}$  is identity matrix,  $E_s$  is the transmitted signal energy and  $N_o/2$  is the two sided spectral energy of the noise.

The MMSE linear transformation  $\mathbf{M}$  is used to minimize the mean-squared-error between the transmitted data  $\mathbf{x}$  and the estimated data  $\hat{\mathbf{x}}$  at the output of the detector [16] as

$$\mathbf{M} = \arg \underbrace{\min}_{\mathbf{M}} E [|\mathbf{x} - \hat{\mathbf{x}}|^2]. \quad (2.5)$$

Besides the effect of the MAI in cooperative CDMA systems, received signals experience fading which severely affects the system performance. In the following section, we discuss the basic concepts of fading.

## 2.5 Fading Channel Model

The most predominant reasons of multipath propagation are reflection, diffraction, and scattering of transmitted signals, which result in the fading effect in the wireless channel. There are many statistical methods applied to describe signal variations in wireless mobile environments. Researchers have extensively investigated the statistical models for wireless fading channels. The Rayleigh fading model is widely used to model fading channels [1]. In the literature, there have been many works on the performance of different wireless communication systems operating over Nakagami- $m$  fading channels [44, 59]. Most of these works are theoretical and not based on accurate Nakagami simulator. The PDF as modeled in [67] for Nakagami- $m$  distribution can be written as

$$p_{R(r)} = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m r^{2m-1} e^{-\frac{mr^2}{\Omega}} \quad (2.6)$$

where  $\Omega$  is defined as

$$\Omega = E[R^2] = \overline{R^2} \quad (2.7)$$

$E[.]$  represents expectation and

$$m = \frac{(\overline{R^2})^2}{(R^2 - \overline{R^2})^2} \geq \frac{1}{2} \quad (2.8)$$

where  $m$  is a parameter that controls the severity of fading, the smaller the  $m$  the more severe the fading is, where the value of  $m = 1$  represents the case of Rayleigh fading. The  $\Omega$  parameter represents the average power of the fading path and  $\Gamma(.)$  is the gamma function.  $R$  is defined as the sum of squared independent Gaussian random variables (RV) each with zero mean and variance  $\sigma_X^2$ ,

$$R = \sqrt{X_1^2 + X_2^2 + \cdots + X_n^2} . \quad (2.9)$$

Fig. 2.6 shows the Nakagami- $m$  PDF distributions as a function of  $m$ .

### 2.5.1 Nakagami- $m$ Fading Simulator

The inverse transformation method is employed in [66] to obtain a Nakagami- $m$  phasor. The generation of Nakagami- $m$  amplitude and phase are performed as presented in Fig. 2.7. The Rayleigh fading envelope is generated from both white Gaussian inputs. Let the RV  $R_{Ray}$  denote a Rayleigh envelope sample and the RV  $\Theta$  denote the phase corresponding to the  $R_{Ray}$ . The authors in [66] perform the transformation as

$$u = F_{Ray}(r) = 1 - e^{\frac{-r^2}{2\sigma_{Ray}^2}} \quad (2.10)$$

where  $\sigma_{Ray}^2$  is the second moment of the RV  $R_{Ray}$ . As in (2.10), the RV  $R_{Ray}$  is transformed into uniform RV on  $[0, 1)$  where  $F_{Ray}(r)$  is the CDF of a Rayleigh distribution. The  $F_{Naka}^{-1}(u)$  is the inverse function of the Nakagami- $m$  CDF defined inclusively by

$$F_{Naka}(x) = \int_0^x \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m r^{2m-1} e^{\frac{-mr^2}{\Omega}} dt. \quad (2.11)$$

It is well known that transforming a uniform distribution  $u$  by the inverse CDF results in  $A = F_{Ray}^{-1}(u)$  which is a RV having Nakagami- $m$  distribution with PDF given by (2.6) and illustrated in Fig. 2.6.

There is no closed form solution for (2.11), however an approximation of the inverse Nakagami- $m$  CDF is considered in [66]. The authors in [66] also derive an accurate approximation given by

$$G(\eta) = \eta + \frac{a_1\eta + a_2\eta^2 + a_3\eta^3}{1 + b_1\eta + b_2\eta^2} \quad (2.12)$$

where  $\eta$  is a supplementary variable defined as

$$\eta = \left( \sqrt{\ln \frac{1}{1-u}} \right)^{\frac{1}{m}} \quad (2.13)$$

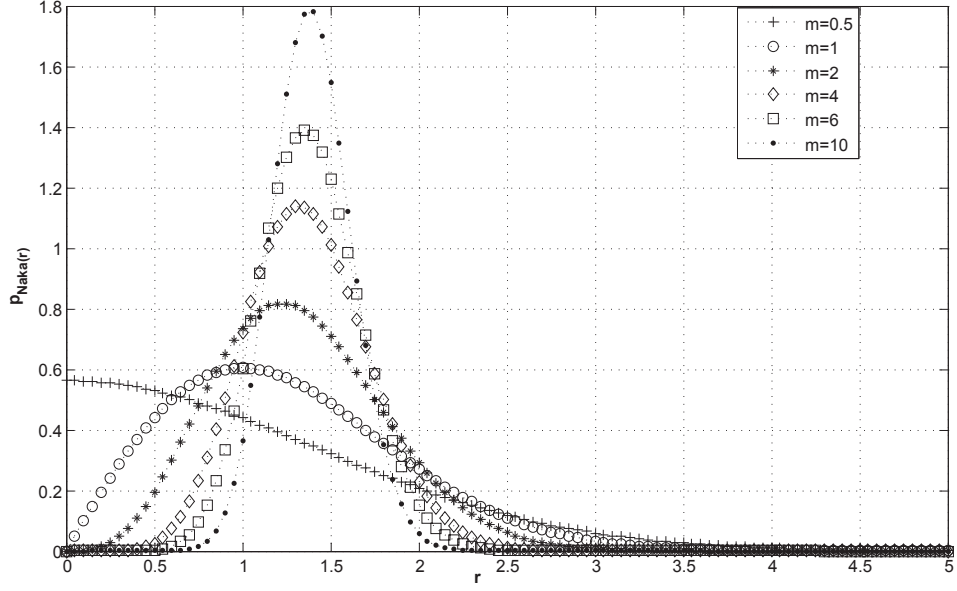


Figure 2.6: PDF of Nakagami- $m$  distribution as a function of different  $m$ .

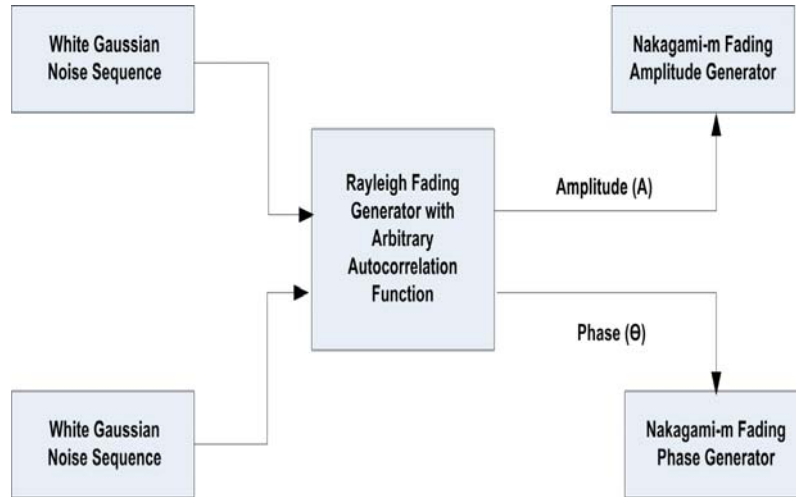


Figure 2.7: Block diagram of Nakagami- $m$  fading channel simulator [66].

with  $a_1$ ,  $a_2$ ,  $a_3$ ,  $b_1$  and  $b_2$  are coefficients used to minimize the approximation error and  $G(u) = F_{Naka}^{-1}(u)$ . Table 2.1 [66] shows the chosen coefficients for different values of  $m = 0.65, 0.75, 0.85, 1.5, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0$ , and 10.

Table 2.1: Inverse Nakagami CDF Approximation Coefficients [66].

m	$a_1$	$a_2$	$a_3$	$b_1$	$b_2$
0.65	-0.0828	-4.5634	-15.8819	63.1955	23.2981
0.75	-0.0547	-0.3697	-1.0336	6.2107	1.8533
0.85	-0.0336	-0.1543	-0.4733	4.9250	1.2082
1.5	0.0993	0.0560	0.2565	0.5276	0.0770
2.0	0.1890	-0.0128	0.2808	-0.0809	0.0638
3.0	0.3472	-0.2145	0.2626	-0.6779	0.1690
4.0	0.4846	-0.4231	0.2642	-0.9729	0.2727
5.0	0.6023	-0.6238	0.2789	-1.1798	0.3732
6.0	0.7139	-0.8305	0.3223	-1.3232	0.4558
7.0	0.8167	-1.0244	0.3761	-1.4233	0.5192
8.0	0.9260	-1.2350	0.4557	-1.4872	0.5628
10.0	1.1088	-1.6095	0.6015	-1.6046	0.6488

## 2.5.2 Simulation Results

The Nakagami- $m$  fading channel simulator in [66] is shown to be accurate, therefore we will use it through our work for simulation purposes. Setting  $\Omega = 2$  as in [66], Fig. 2.8 presents the perfect match between theoretical and simulation results for the Nakagami- $m$  PDF. In Fig. 2.9, the CDF of the Nakagami- $m$  distribution is shown for both theoretical and simulation models.

## 2.6 Conclusions

We have presented in this chapter a brief survey of existing works in the literature that are related to our work. We have reviewed the user cooperation and described the different MUD techniques. Moreover, we have shown the modeling of the Nakagami- $m$  Fading Channels simulator.



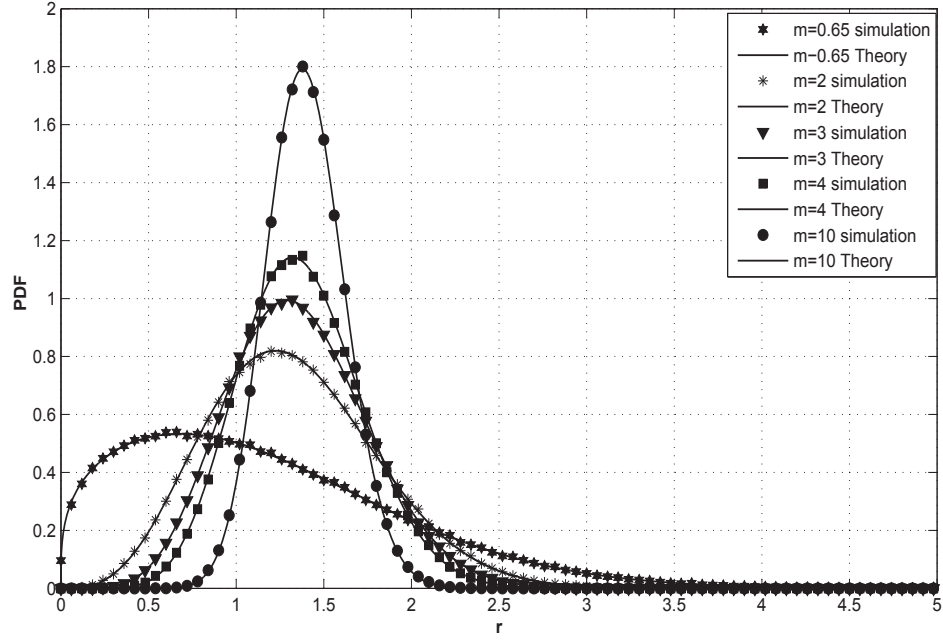


Figure 2.8: PDF of Nakagami- $m$  distribution with different fading parameter  $m$ .

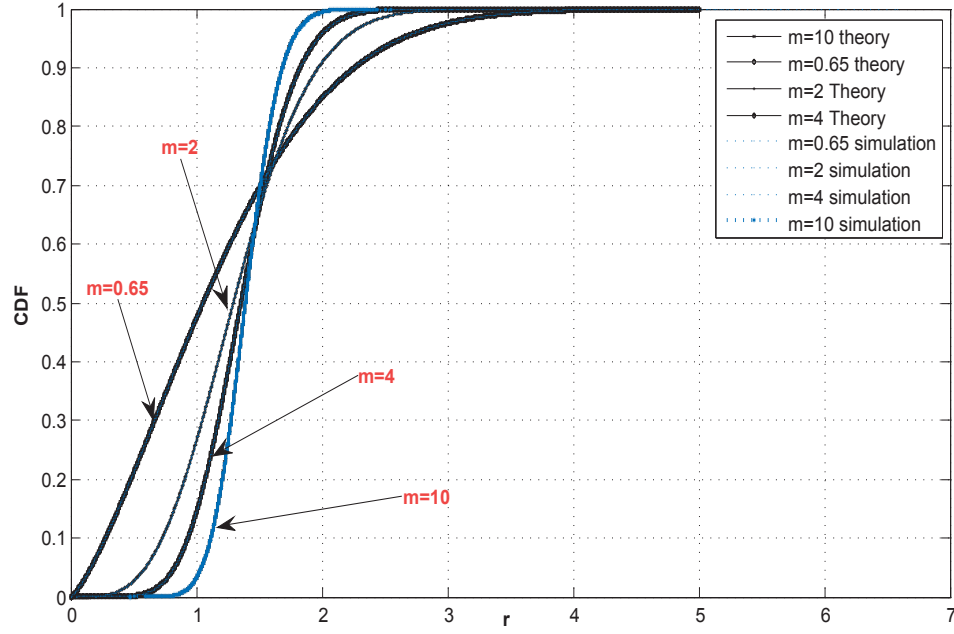


Figure 2.9: CDF of Nakagami- $m$  distribution with different fading parameter  $m$ .

## Chapter 3

# Performance Analysis of Cooperative CDMA Systems using DF Relaying

It is widely known that the Rayleigh fading model cannot characterize the channel of many communication systems. In that perspective, the Nakagami- $m$  fading model is known as a generalized distribution, where many fading environments can be represented. Nakagami- $m$  fading model has the benefits of involving the Rayleigh fading and one-sided Gaussian model as special cases. Moreover, it can be used to model fading conditions ranging from severe, light to no fading, by changing the fading parameter  $m$ . In the literature, there have been many works on the analysis of the DF relaying specially in the case of Rayleigh fading channels. However, to the best of our knowledge, few works [14,46] have been reported on the analysis of cooperative DF relaying in CDMA system when considering Nakagami- $m$  fading channels.

In this chapter, we study a cooperative DF DS-CDMA over Nakagami- $m$  fading channels where the system performance is analyzed using outage probability and BER metrics. We investigate the system model over asynchronous transmission wherein the MUD is employed to mitigate the MAI effects at both; the base station and the relay sides in order to pertain full diversity gain. For the outage probability analysis of the multi-relay

scenario, we derive a closed-form expression for the MGF of the received SNR at the base station. When multi-relays have the reliability to cooperate, single-relay selection may become necessary to reduce overhead and complexity. Moreover, the bandwidth efficiency will be improved. In that context, a closed-form expression for the CDF of single-relay cooperation is derived differently for the sum of two independent non-identical distributed gamma random variables.

In the second part of this chapter, we use the characteristic function of the total received SNR at the base station to find its PDF. From these statistics, we derive a closed-form expression for the BER for both perfect and imperfect inter-user channels.

### 3.1 Introduction

The reliability of wireless communication systems and the high data rate coverage can be improved using the user cooperation diversity techniques. Adopting these techniques, the mobile terminals can create virtual antennas in order to overcome the power constraints and their size limitations. This is carried out by sharing their physical resources, and thus the transmitter diversity can be achieved [8], [9].

One common network of user cooperation diversity is the DS-CDMA as it represents many of the existing technologies for communication systems such as multi-carrier CDMA (MC-CDMA) and optical CDMA. In these CDMA systems, Rayleigh fading channels are commonly adopted for study and orthogonality between users is assumed [11]. In CDMA systems, to overcome the effect of MAI arising from the non-orthogonality of users' spreading codes, multiuser detectors such as the MMSE and decorrelator have been presented. In [18], the authors studied the effect of MAI on the performance of asynchronous DS-CDMA cooperative systems over multipath fading channels. In their work, the authors considered the multi-relay coded cooperation case, wherein the BER performance of the system was analyzed for the two cases: the perfect and the imperfect

inter-user link over Rayleigh fading channels.

The outage probability over Rayleigh fading channels was investigated at high SNR in [17] for cooperative asynchronous DS-CDMA networks. The authors in [45] determined a closed-form expression for the outage probability in DF cooperative networks assuming an independent and non-identical Rayleigh fading channels. In a Nakagami- $m$  fading channel, the authors in [56] obtained a closed-form expression for the outage probability of cooperative relay networks assuming independent identical distribution (i.i.d) fading scenario. Also the authors in [46] evaluated the BER of a cooperative downlink transmission scheme for DS-CDMA systems over Nakagami- $m$  fading channels to achieve relay diversity. They proposed transmitter TZF at the base station for suppressing the downlink multiuser interference. Moreover in [47], the performance of downlink multi-user relay network using single AF relay has been studied where the outage probability at high SNR is derived. Furthermore, the authors in [68] have considered a multi-relay network in Nakagami- $m$  fading channels where the outage probability and average BER of an AF-based relaying are derived. The performance analysis for opportunistic DF with selection combining receiver at the destination has been evaluated in terms of the outage probability in [69]. Also in [69], the authors have derived a closed-form expression for the outage of the system over non identical Nakagami fading channels. Other works on the outage probability in DF over independent not identical flat Nakagami- $m$  channel using the MGF are introduced in [57, 70, 71].

## 3.2 System Model

We consider an uplink  $K$ -user asynchronous non-orthogonal DS-CDMA system, transmitting over Nakagami- $m$  fading channels. In our model, as shown in Fig. 3.1, we consider a set of available cooperating users  $\mathbf{s} \in \{1, \dots, K\}$  from which a set  $\ell \in \{1, \dots, L\}$  of decodable relays (i.e., cooperating users that have correctly decoded the source messages)

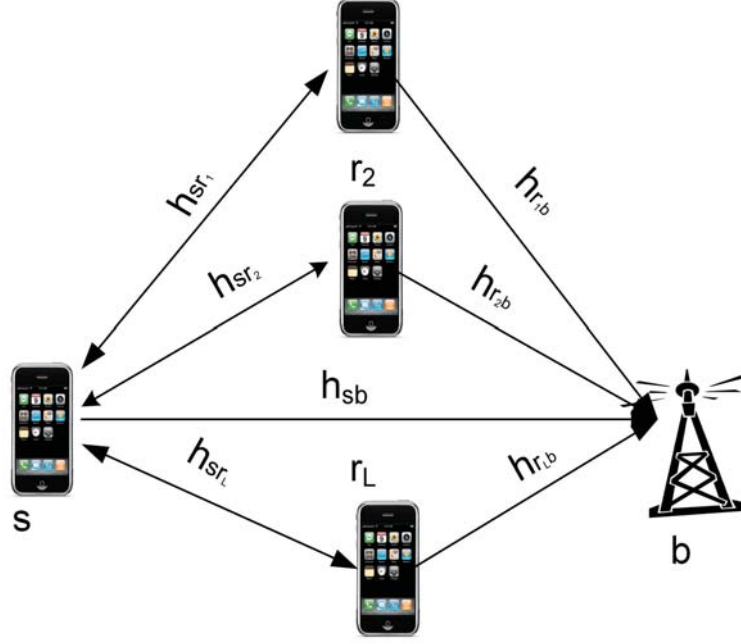


Figure 3.1: Cooperation module

are able to transmit to the base station,  $b$ , where  $L \in \{1, \dots, K - 1\}$ . A half-duplex system is assumed and each user is equipped with a single antenna. The independent fading channel coefficients between partners and base station are defined as follows, source-relay  $h_{sr}$ , source-base station  $h_{sb}$ , and relay-base station  $h_{rb}$ , all are modeled as independent non-identical Nakagami- $m$  random variables (RVs). The instantaneous SNRs of different links are denoted by  $\gamma_{sr} = |h_{sr}|^2 \frac{E_s}{N_o}$ ,  $\gamma_{sb} = |h_{sb}|^2 \frac{E_s}{N_o}$ ,  $\gamma_{rb} = |h_{rb}|^2 \frac{E_s}{N_o}$  where  $E_s$  is the transmitted signal energy,  $N_o$  is the noise spectral energy,  $|h_{sr}|^2$ ,  $|h_{sb}|^2$ , and  $|h_{rb}|^2$  are gamma distributed RVs with PDF given by,

$$p_{\gamma_{ij}}(x) = \frac{B_{ij}^{m_{ij}}}{\Gamma(m_{ij})} x^{m_{ij}-1} \exp(-x B_{ij}), \quad (3.1)$$

where  $m_{ij} > 0.5$  is the fading parameter,  $\Gamma(\cdot)$  is the gamma function [ [72], eq. (8.310,1)],  $B_{ij} = \frac{m_{ij}}{\bar{\gamma}_{ij}}$  with average SNR,  $\bar{\gamma}_{ij} = \mathbb{E}(|h_{ij}|^2) E_s / N_o$ , and  $\mathbb{E}(\cdot)$  denote expectation. The CDF of  $\gamma_{ij}$  is then given by

$$F_{\gamma_{ij}}(x) = \frac{\gamma(m_{ij}, x B_{ij})}{\Gamma(m_{ij})}, \quad (3.2)$$

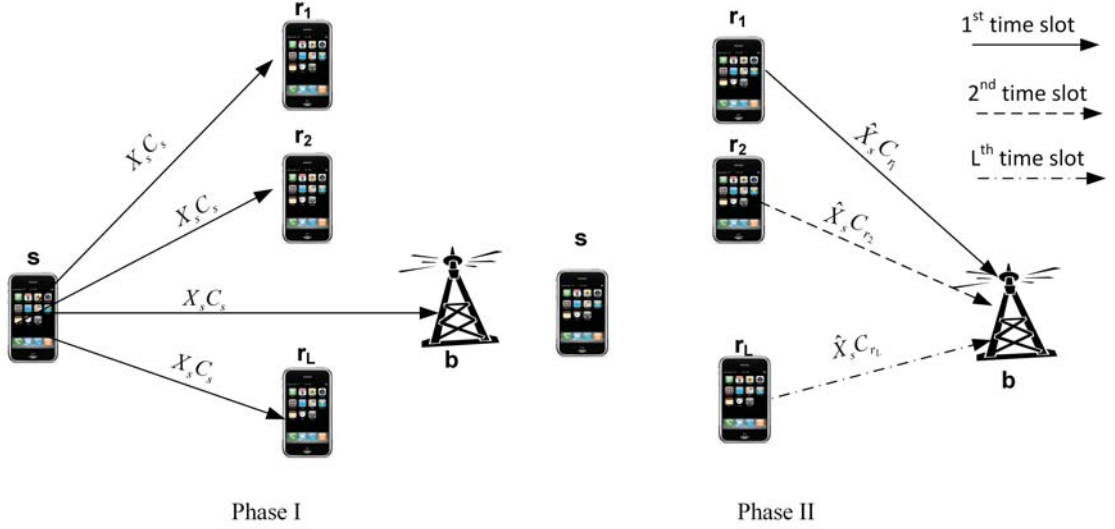


Figure 3.2: Cooperation module between users

where  $\gamma(.,.)$  is the lower incomplete gamma function [ [72], eq. (8.350,1)].

The cooperative communication process can be divided into two phases as shown in Fig. 3.2;

(i) Phase I: Each user transmits its own DS-CDMA modulated data to the base station and to  $L$ -relays. During this phase, the received signal at the base station can be written as

$$r_b^I(t) = \sum_{i=0}^{f-1} \sum_{s=1}^K x_s(i) C_s(t - \tau_s - iT_b) h_{sb} + n_b^I(t), \quad (3.3)$$

where  $f$  is the frame length,  $x_s(i) \in \{1, -1\}$  is the  $i^{th}$  data symbol of user  $s$ ,  $C_s(t)$  is the spreading code of user  $s$  with spreading gain  $N = \frac{T_b}{T_c}$ ,  $T_b$  is the bit period,  $T_c$  is the chip period, and  $\tau_s$  is the random transmit delay of the  $s^{th}$  user which is assumed to be uniformly distributed along the symbol period.  $n_b^I(t)$  is the AWGN with zero mean and variance  $\sigma_n^2 = N_o/2$ . In (3.3),  $h_{sb}$  denotes the channel coefficient between user  $s$  and the base station, drawn from Nakagami- $m$  fading channel with  $\Omega_{sb} = E(|h_{sb}|^2) = 1$  with parameter  $m_{sb}$ . The received signal at user ( $r$ ) can be written as

$$r_r(t) = \sum_{i=0}^{f-1} \sum_{s=1, s \neq r}^K x_s(i) C_s(t - \tau_s - iT_b) h_{sr} + n_r(t), \quad (3.4)$$

where  $h_{sr}$  is the channel coefficient between user  $s$  and partner ( $r$ ) and is drawn from Nakagami- $m$  fading channel with  $E(|h_{sr}|^2) = 1$  with parameters  $m_{sr}$  and  $\Omega_{sr}$ .  $n_r(t)$  is a Gaussian noise with zero mean and variance  $\sigma_n^2 = N_o/2$ .

(ii) Phase II: In this phase, each cooperating user transmits the received signal to the base station at different time slots, as shown in Fig. 3.2, which is expressed as

$$r_b^{II}(t) = \sum_{i=0}^{f-1} \sum_{s=1}^K \sum_{\ell=1}^L \tilde{x}_s(i) C_{r_\ell}(t - D_{s,r_\ell} - \tau_{r_\ell} - i\ell T_b) h_{r_\ell b} + n_b^{II}(t), \quad (3.5)$$

where  $r_\ell$  is the  $\ell^{th}$  relay cooperating with user  $s$ ,  $\tilde{x}_s(i)$  is estimate data for  $x_s(i)$ ,  $n_b^{II}(t)$  is a Gaussian noise with zero mean and variance  $\sigma_n^2 = N_o/2$ , and  $D_{k,\ell}$  is the transmission delay during the second transmission period.  $h_{r_\ell b}$  is the channel coefficient between user  $r_\ell$  and the base station, modeled as a Nakagami- $m$  RV with  $E(|h_{r_\ell b}|^2) = 1$  and parameters  $m_{r_\ell b}$  and  $\Omega_{r_\ell b}$  [18]. As one can see from (3.5) and Fig. 3.2, the second transmission phase spans  $L$  time slots in order to complete the cooperation process.

### 3.3 Outage Probability Analysis

The outage probability is an important performance analysis that measures and characterizes the system quality. The outage probability,  $P_{out}$ , is the probability that the mutual information between the source and the base station falls below a certain spectral efficiency  $\mathfrak{R}$ . In this section, the outage probability of the system in the case of single relay cooperation using the total CDF of the sum of two gamma RV is investigated for both the error-free inter-user channel and imperfect inter-user channel cases. Multi-relay cooperation is studied in depth and a closed-form expression for the outage probability using the moment generating function is also derived.

In this section, the repetition-based cooperative diversity protocol is considered [5] where the partners (relays) fully decode the received signals and repeat the information to the base station in the second time slot. The available degrees of freedom  $\frac{K}{2N}$  [17] is a

function of the  $K$  total number of users,  $N$  the length of the available spreading code, and  $1/2$  which stands for the bandwidth expansion needed for relaying due to the half-duplex constraint.

Here, the performance of the cooperative diversity protocol under diversity combining is studied. Let us consider the direct link between the source  $s$  and the base station  $b$  where the mutual information is given by [17]

$$I_{sb} = \frac{K}{2N} \log \left( 1 + \frac{2N\gamma_{sb}}{K^2[\mathbf{M}]_{s,s}} \right), \quad (3.6)$$

with  $[\mathbf{M}]_{s,s}$  in the case of MMSE detection is given by  $[\mathbf{M}]_{s,s} = [(\mathbf{R}_b + \text{SNR}^{-1}\mathbf{I})^{-1}]_{s,s}$  as in (2.4) where it represents the  $s^{th}$  row and  $s^{th}$  column element of the  $[\mathbf{M}]$  ( $Kf \times Kf$ ) matrix.  $\text{SNR} \triangleq \frac{E_s}{N_o}$  is the signal-to-noise-ratio in the absence of fading,  $\mathbf{R}$  is a function of cross correlation between delayed signature waveforms where  $\mathbf{R}_b$  is the ( $Kf \times Kf$ ) cross-correlation matrix at the base-station receiver and  $\mathbf{R}_{r,s \neq r}$  is the  $((Kf - 1) \times (Kf - 1))$  cross-correlation matrix at the relay, defined as [18]

$$\mathbf{R}_b = \begin{bmatrix} \rho_{1,1} & \cdots & \rho_{1,2} & \cdots & \rho_{1,K} \\ \rho_{2,1} & \cdots & \rho_{2,2} & \cdots & \rho_{2,K} \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ \rho_{K,1} & \cdots & \rho_{K,2} & \cdots & \rho_{K,K} \end{bmatrix} \quad (3.7)$$

where  $\rho_{ij}$  is the cross-correlation value between any two spreading codes  $C_i$  and  $C_j$ . In (3.6),  $\frac{2N}{K^2}$  is the normalized discrete-power constraint [17]. Then, the outage probability occurs when  $I_{sb}$  fails to achieve a target rate of  $\mathfrak{R}$ , which can be written as

$$P_{out} = P_r[I_{sb} < \mathfrak{R}]. \quad (3.8)$$



From (3.6) and (3.8) we have

$$P_{out} = P_r \left[ \frac{K}{2N} \log \left( 1 + \frac{2N\gamma_{sb}}{K^2[\mathbf{M}]_{s,s}} \right) < \Re \right] = P_r [\gamma_{sb} < \gamma_{th-sb}] \quad (3.9)$$

where

$$\gamma_{th-sb} = \frac{2^{\frac{2N\Re}{K}} - 1}{\frac{2N}{K^2[\mathbf{M}]_{s,s}}}. \quad (3.10)$$

From (3.9), we can notice that the  $P[\gamma_{sb} < \gamma_{th-sb}]$  is the CDF of  $\gamma_{sb}$  which is given as (3.2) and hence,

$$P_{out-noncoop} = P_r [\gamma_{sb} < \gamma_{th-sb}] = \frac{\gamma(m_{sb}, B_{sb}\gamma_{th-sb})}{\Gamma(m_{sb})}. \quad (3.11)$$

To have a better insight into the performance of the system, we first examine the single-relay scenario followed by the general multi-relay scenario.

### 3.3.1 Single-Relay Cooperation

In this section a single relay cooperation system is studied, where the group of users  $\{1, 2, \dots, K\}$  are arranged in groups each of two cooperating partners. Furthermore, a closed-form expression for the CDF is derived based on of the sum of two independent non-identical distributed gamma RV. This expression is then used to obtain the outage probability. In our analysis we consider both perfect and imperfect inter-user channels. In both cases, the repetition-based cooperative diversity scenario is applied where the relays successfully decode the received signals and then resend the received information to the base station [5].

#### 3.3.1.1 Perfect Inter-User Channel

In this section, we consider perfect inter-user channel between users. This assumption is ideal, however, it serves as a bench mark for the optimum performance. Later we will

study the more practical case where errors exist.

The total mutual information  $s \rightarrow b$  and  $r \rightarrow b$  for the error-free case (i.e., inter-user channel) is given by [17]

$$I_{total} = \frac{K}{2N} \log \left( 1 + \frac{2N\gamma_{sb}}{K^2[\mathbf{M}]_{s,s}} + \frac{2N\gamma_{rb}}{K^2[\mathbf{M}]_{r,r}} \right). \quad (3.12)$$

From (3.12), with the help of [72], [41] and after algebraic manipulations, the CDF of the sum of the two gamma RVs is derived as shown in Appendix A and expressed as

$$\begin{aligned} F_{total}(y) &= F_{X_{sb}}(x) - \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} \exp(-yB_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} \\ &\times \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} \int_0^y x^{k+m_{sb}-1} \exp(-x(B_{sb} - B_{rb})) dx. \end{aligned} \quad (3.13)$$

- **The case of independent and identical channels**

In this case,  $B_{sb} = B_{rb} = B$  and  $m_{sb} = m_{rb} = m$ . Hence, (3.13) is reduced to

$$F_{total}(y) = F_{X_{sb}}(x) - \frac{B^m}{\Gamma(m)} \exp(-yB) \sum_{n=0}^{m-1} \frac{(B)^n}{n!} \sum_{k=0}^n \binom{n}{k} \frac{(-1)^k}{k+m} y^{m+n}. \quad (3.14)$$

Finally, the outage probability in the case of i.i.d and using (3.2) is give by

$$P_{out} = \frac{\gamma(m, \gamma_{sb}B)}{\Gamma(m)} - \frac{B^m}{\Gamma(m)} \exp(-yB) \sum_{n=0}^{m-1} \frac{(B)^n}{n!} \sum_{k=0}^n \binom{n}{k} \frac{(-1)^k}{k+m} y^{m+n}. \quad (3.15)$$

- **The case of independent but non identical channels**

In this case,  $B_{sb} \neq B_{rb}$  and  $m_{sb} \neq m_{rb}$ . Let us define  $\nu = k + m_{sb}$  and  $\mu = B_{sb} - B_{rb}$  and using [ [72], eq.(3.381,1)], (3.13) in this case is given by

$$\begin{aligned} F_{total}(y) &= \frac{\gamma(m_{ij}, \gamma B_{ij})}{\Gamma(m_{ij})} - \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} \exp(-yB_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} \\ &\times \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} \frac{1}{\mu^\nu} \gamma(\nu, \mu y). \end{aligned} \quad (3.16)$$

The outage probability when  $B_{sb} > B_{rb}$  is then given by

$$\begin{aligned}
P_{out} &= \frac{\gamma(m_{sb}, \gamma B_{sb})}{\Gamma(m_{sb})} - \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} \exp(-y B_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} \\
&\times \frac{1}{(B_{sb} - B_{rb})^{k+m_{sb}}} \gamma(k + m_{sb}, (B_{sb} - B_{rb})y). \tag{3.17}
\end{aligned}$$

### 3.3.1.2 Imperfect Inter-User Channel

As mentioned earlier, assuming perfect inter-user channel is unrealistic and can only use as a lower bound for the optimum performance. Therefore, we here consider the effect of the inter-user channel errors subject to decoding errors at relays. If errors exist at a relay, no cooperation will take place. Let us define the event  $P_{sr}$  as the outage probability for the source-relay link as in (3.9)

$$P_{sr} = P_r \left[ \frac{K}{2N} \log \left( 1 + \frac{2N\gamma_{sr}}{K^2[\mathbf{M}]_{r,r}} \right) < \mathfrak{R} \right]. \tag{3.18}$$

Note that  $P_{out}$  is defined in (3.17) as the accumulated outage probability from source and relay links at the base station. The system is in outage if one of these links (source-base station or source-relay links) is in outage, or if the relay can successfully decode the source data, but the mutual information at the output of the maximal-ratio-combiner (MRC) in the base station falls below certain spectral efficiency  $\mathfrak{R}$ .

Let us define  $L_1$  as the event that the link between the source-base station is in outage, i.e., we have

$$L_1 := \{I_{sb} : I_{sb} < \mathfrak{R}\},$$

in similar way, we define

$$L_2 := \{I_{sr} : I_{sr} < \mathfrak{R}\}$$

$$L_3 := \{(I_{sb}, I_{rb}) : I_{sr} + I_{rb} < \mathfrak{R}\}.$$

The system declares an outage either when both the source-base station link as well as the source-relay link are in outage, or when the relay succeed to decode but the received mutual information at the output of the MRC falls below the spectral efficiency. Then the system outage probability can be written as

$$P_{out\_total} = P_r(L_1)P_r(L_2)P_r(L_3|L_1L_2) + P_r(L_1)P_r(L_2^c)P_r(L_3|L_1L_2^c) \quad (3.19)$$

where  $L_2^c$  is the complement of  $L_2$  and  $P_r(L_3|L_1L_2) = 1$ . Moreover, from the conditional probability we have  $P_r(L_1)P_r(L_3|L_1L_2^c) = P_r(L_3)$  since  $L_3 \subseteq L_1$ . Thus (3.19) can be written as

$$P_{out\_total} = P_r(L_1)P_r(L_2) + P_r(L_2^c)P_r(L_3). \quad (3.20)$$

Then we get

$$P_{out\_total} = Pr[\gamma_{sb} < \gamma_{th-sb}]Pr[\gamma_{sr} < \gamma_{th-sr}] + Pr[\gamma_{sr} \geq \gamma_{th-sr}]Pr[\gamma_{total} < \gamma_{th-total}]$$

$$= P_{sb}P_{sr} + (1 - P_{sr})P_{out}, \quad (3.21)$$

where  $P_{sb}$  and  $P_{sr}$  are defined as in (3.11).

- **The case of independent and identical channels**

In this case,  $B_{sb} = B_{sr} = B_{rb} = B$  and  $m_{sb} = m_{sr} = m_{rb} = m$ . By applying (3.11)

and (3.15) in (3.21), the outage probability is given by

$$P_{out} = \frac{\gamma(m, B\gamma_{th-sb})}{\Gamma(m)} \cdot \frac{\gamma(m, B\gamma_{th-sr})}{\Gamma(m)} + \left(1 - \frac{\gamma(m, B\gamma_{th-sr})}{\Gamma(m)}\right) \times \left(\frac{\gamma(m, B\gamma_{sb})}{\Gamma(m)} - \frac{B^m}{\Gamma(m)} \exp(-yB) \sum_{n=0}^{m-1} \frac{(B)^n}{n!} \sum_{k=0}^n \binom{n}{k} \frac{(-1)^k}{k+m} y^{m+n}\right). \quad (3.22)$$

- **The case of independent but non identical channels**

In this case,  $B_{sb} \neq B_{sr} \neq B_{rb}$  and  $m_{sb} \neq m_{rs} \neq m_{rb}$ . By applying (3.11) and (3.17) in (3.21), the outage probability of the system is given by

$$P_{out} = \frac{\gamma(m_{sb}, B_{sb}\gamma_{th-sb})}{\Gamma(m_{sb})} \cdot \frac{\gamma(m_{sr}, B_{sr}\gamma_{th-sr})}{\Gamma(m_{sr})} + \left(1 - \frac{\gamma(m_{sr}, B_{sr}\gamma_{th-sr})}{\Gamma(m_{sr})}\right) \times \left(\frac{\gamma(m_{sb}, \gamma B_{sb})}{\Gamma(m_{sb})} - \frac{(B_{sb})^{m_{sb}}}{\Gamma(m_{sb})} \exp(-yB_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} \times \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} \frac{1}{(B_{sb} - B_{rb})^{k+m_{sb}}} \gamma(k + m_{sb}, (B_{sb} - B_{rb})y)\right). \quad (3.23)$$

When  $\gamma_{sr}$  is high (i.e.,  $\gamma_{sr} \rightarrow \infty$ ), the source-relay link will be error-free, and (3.23) reduces to (3.17). Since  $B_{ij} = \frac{m_{ij}}{\bar{\gamma}_{ij}}$  with average SNR  $\bar{\gamma}_{ij} = E\langle h_{ij}^2 \rangle E_s / N_o$ , using (3.23) at high SNR ( $\bar{\gamma}_{ij} \rightarrow \infty$ ) and according to [ [72], eq. (8.351,2)]

$$\frac{\gamma(m_{sb}, B_{sb}\gamma_{th})}{\Gamma(m_{sb})} \approx \frac{(m_{sb})^{m_{sb}-1} (\gamma_{th})^{m_{sb}}}{\Gamma(m_{sb})} (\bar{\gamma}_{sb})^{-m_{sb}}. \quad (3.24)$$

From [ [72], eq. (8.350,5)], we get  $\frac{\gamma(m_{sr}, m_{sr}\gamma_{th-sr})}{\Gamma(m_{sr})} = 0$ , and using [ [72], eq. (8.352,2)] we get

$$\sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} = \frac{\exp(B_{rb}) \Gamma(m_{rb}, B_{rb})}{\Gamma(m_{rb})}. \quad (3.25)$$

Based on [ [72], eq. (8.351,4)], at high SNR, (3.25) can be approximated as  $\approx \frac{(B_{rb})^{m_{rb}}}{\Gamma(m_{rb})}$ . By applying (3.24) we get  $\frac{1}{(B_{sb}-B_{rb})^{k+m_{sb}}} \cdot \gamma(k + m_{sb}, (B_{sb} - B_{rb})y) \approx (k + m_{sb})^{(k+m_{sb})} y^{(k+m_{sb})}$ .

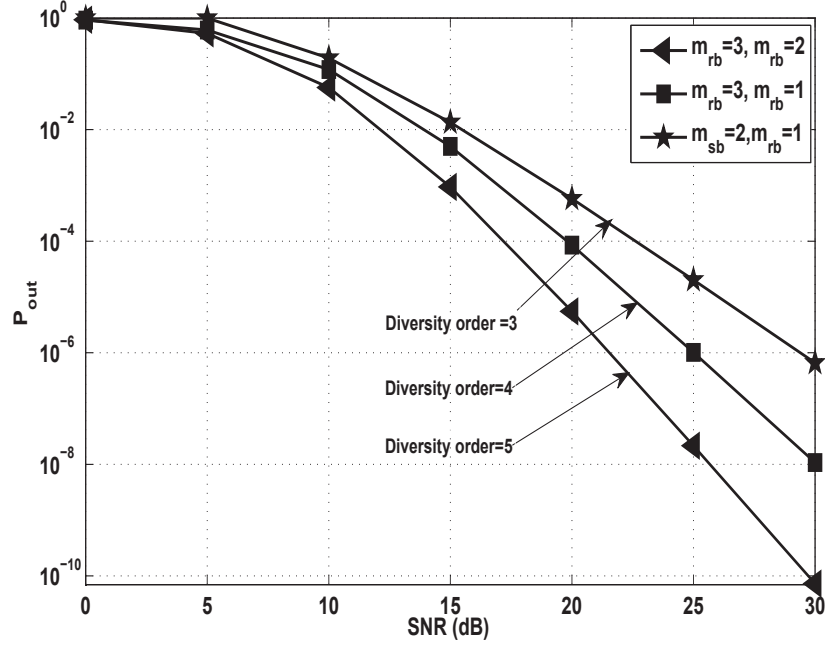


Figure 3.3: Outage probability for relay cooperation in DF DS-CDMA with different  $m$ .

Then, the outage probability in (3.23) can be asymptotically expressed as

$$\begin{aligned}
 P_{out\_total} &\approx \frac{(m_{sb})^{m_{sb}}}{\Gamma(m_{sb})} \frac{(m_{rb})^{m_{rb}}}{\Gamma(m_{rb})} (\bar{\gamma}_{sb})^{-m_{sb}} (\bar{\gamma}_{rb})^{-m_{rb}} \\
 &\times \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} (k + m_{sb})^{(k+m_{sb})} y^{(k+m_{sb})}. \quad (3.26)
 \end{aligned}$$

Now assume that  $(\bar{\gamma}_{sb}) = (\bar{\gamma}_{rb}) = \Delta$ . Then, the outage probability in (3.26) is reduced to

$$\begin{aligned}
 P_{out\_total} &\approx \frac{(m_{sb})^{m_{sb}}}{\Gamma(m_{sb})} \frac{(m_{rb})^{m_{rb}}}{\Gamma(m_{rb})} (\Delta)^{-m_{sb}-m_{rb}} \\
 &\times \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} (k + m_{sb})^{(k+m_{sb})} y^{(k+m_{sb})}. \quad (3.27)
 \end{aligned}$$

Finally, the expression in (3.27) shows that the achievable diversity order is  $m_{sb} + m_{rb}$  as shown in Fig. 3.3.

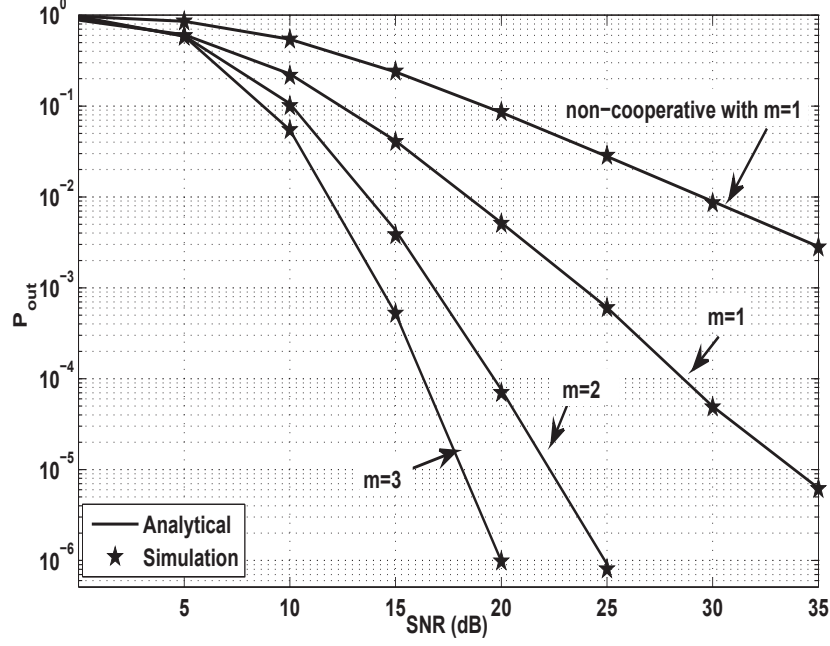


Figure 3.4: Outage probability for cooperative DF DS-CDMA over i.i.d Nakagami- $m$  fading channel.

### 3.3.1.3 Simulation and Numerical Results

In what follows, we present some numerical results for the outage probability. In all figures, we set the spreading gain to  $N = 31$  for a system with number of users  $K = 16$ . Without loss of generality, we assume that the spectral efficiency  $\mathfrak{R} = 1 \text{ bit/sec/Hz}$ . An MMSE detector is used to mitigate the effect of MAI.

Fig. 3.4 shows the outage probability for the cooperative system in the case of perfect inter-user channel, where we assume i.i.d Nakagami- $m$  fading channel with  $\gamma_{sb} = \gamma_{rb} = \gamma$  and  $m_{sb} = m_{rb} = m$ . The results Fig. 3.5 shows the outage probability of the system with different fading severity  $m_{ij}$  where we assume independent not identical Nakagami- $m$  fading channel (i.e.,  $m_{sb} \neq m_{rb}$ ). Clearly, from both Fig. 3.4 and Fig. 3.5, the outage probability of the underlying system is improved where the diversity order is achieved with increasing  $m_{sb}$  and/or  $m_{rb}$ .

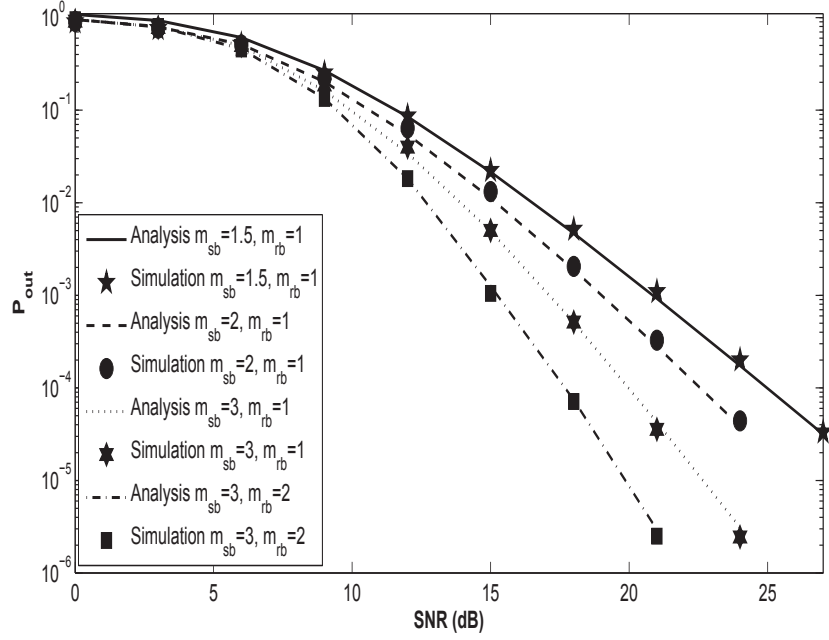


Figure 3.5: Outage probability for cooperative DF DS-CDMA over i.n.i channels with different  $m$

### 3.3.2 Multi-Relay Cooperation

In the previous subsection, we analyzed the system performance for the single-relay cooperation based on the sum of the two i.n.i gamma RVs. Here, we generalize for the multi-relay case using different approach other than the one used in the single-relay scenario due to the mathematical tractability in deriving closed-form expressions.

In this section, the effect of asynchronous transmission on the outage probability of DF CDMA system over independent non-identical Nakagami- $m$  fading channels is studied. We consider repetition-based cooperative diversity. In this scheme, the number of available relays (cooperating users or partners) and the decoding set are denoted, respectively by  $L$  and  $D(s)$  where  $D(s) \subset L$ . The decoding set  $D(s)$  is defined as the set of relays that have the ability to fully decode the source information (i.e., no decoding errors). Because of the half-duplex constraint, cooperation is performed in two time slots. In the first time slot, each user  $s$  transmits its signal to the set of relays  $L$  and to the base station  $b$ , and keeps silent in the second time slot. In the second time slot, only relays from the decoding



set  $D(s)$  forward the source signal to the base station. The outage probability  $P_{out}$  can be defined as the probability that the mutual information  $I_{DF}$  falls below certain rate  $\mathfrak{R}$ . Note that the partners' set  $r_{\mathcal{K}} \in D(s)$  is a random set. Now, the outage probability of the system between user  $s$  and the base station  $b$  is given by

$$P_{out} = P_r[I_{DF} < \mathfrak{R}] = \sum_{D(s)} P_r[D(s)] P_r[I_{DF} < \mathfrak{R}|D(s)]. \quad (3.28)$$

We noted that it is difficult to find a closed-form expression for the outage probability of the DF cooperative network, specially for the case of independent but not identical Nakagami- $m$  fading. This is due to the difficulty of finding the pdf of the sum of gamma random variables given in (3.28). Similar to [45], we simplify the computation of the outage probability by indicating that the cooperation diversity can be visualized as a system that has effectively  $L + 1$  paths between source  $s$  and base station  $b$ . Let us denote path (0) as the direct path between  $s$  and  $b$  and the  $\mathcal{K}$ th cascaded path between  $s \rightarrow r_{\mathcal{K}} \rightarrow b$  where  $\mathcal{K} = 1, 2, \dots, L$ . Let us also define  $\gamma_{\mathcal{K}}$  as the received SNR for the link  $s \rightarrow r_{\mathcal{K}}$  and the link  $r_{\mathcal{K}} \rightarrow b$ . The PDF of  $\gamma_{\mathcal{K}}$  is then defined as

$$f_{\gamma_{\mathcal{K}}}(\gamma) = f_{\gamma_{\mathcal{K}}|link \ down}(\gamma)Pr[link \ down] + f_{\gamma_{\mathcal{K}}|link \ active}(\gamma)Pr[link \ active], \quad (3.29)$$

where  $f_{\gamma_{\mathcal{K}}|link \ down}$  and  $f_{\gamma_{\mathcal{K}}|link \ active}$  are conditional PDFs, and  $Pr[link \ down]$  and  $Pr[link \ active]$  are the probabilities of inactive and active relays (i.e., ones that fully decode without errors). If the link is down, then the PDF  $f_{\gamma_{\mathcal{K}}|link \ down}(\gamma) = \delta(\gamma)$ , where  $\delta(\cdot)$  is the Dirac delta function. The probability of occurrence of this event is defined as in (3.11),

$$U_{\mathcal{K}} = P_r[\gamma_{\mathcal{K}} < \gamma_{th}] = F_{\gamma_{\mathcal{K}}}(\gamma_{th}), \quad (3.30)$$

where  $F_{\gamma_{\mathcal{K}}}(\cdot)$  is the CDF of  $\gamma_{\mathcal{K}}$ . Note that (3.30) represents the probability that the  $\mathcal{K}$ th relay (partner) does not belong to the decoding set  $D(s)$ . Note also that the link  $s \rightarrow b$

is not connected to any relay, i.e.,  $U_0 = 0$ . The probability that the link is active equals  $1 - U_{\mathcal{K}}$ , with the conditional PDF given by

$$f_{\gamma_{\mathcal{K}}|link \text{ active}}(\gamma) = \frac{B_{\mathcal{K}}^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})} \gamma^{m_{\mathcal{K}}-1} \exp(-\gamma B_{\mathcal{K}}). \quad (3.31)$$

Using (3.31) in (3.29) we have

$$f_{\gamma_{\mathcal{K}}}(\gamma) = U_{\mathcal{K}} \delta(\gamma) + (1 - U_{\mathcal{K}}) \frac{B_{\mathcal{K}}^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})} \gamma^{m_{\mathcal{K}}-1} \exp(-\gamma B_{\mathcal{K}}). \quad (3.32)$$

Eq. (3.32) represents the PDF of the  $\mathcal{K}$ th cascaded path from source to base station. Therefore the total outage probability can be written as

$$P_{out}(\gamma_{th}) = P_r \left[ \left( \gamma_{sb} + \sum_{\mathcal{K}=1}^L \gamma_{\mathcal{K}} \right) < \gamma_{th} \right]. \quad (3.33)$$

From (3.33), we can notice that the mathematical probability model for the CDMA system is the CDF of the sum of  $U_{\mathcal{K}}$ 's. Using the MGF approach, the CDF of the sum of RVs can be extracted, i.e.  $M_{\gamma_{\mathcal{K}}}(s) = E\{e^{-s\gamma_{\mathcal{K}}}\}$ . Using (3.32), the MGF of the cascaded link is given by

$$M_{\gamma_{\mathcal{K}}}(s) = U_{\mathcal{K}} + (1 - U_{\mathcal{K}}) \left( 1 + \frac{s}{B_{\mathcal{K}}} \right)^{-m_{\mathcal{K}}}. \quad (3.34)$$

Since  $\gamma_{\mathcal{K}}$ 's are assumed to be independent, then the total MGF of the sum is expressed as the multiplication of MGFs

$$M_{\gamma_{total}}(s) = M_{\gamma_{sb}}(s) \prod_{\mathcal{K}=1}^L M_{\gamma_{\mathcal{K}}}(s), \quad (3.35)$$

where  $M_{\gamma_{sb}}(s)$  is the MGF for the direct link  $\mathbf{s} \rightarrow \mathbf{b}$ , given by

$$M_{\gamma_{sb}}(s) = \left( 1 + \frac{s}{B_{sb}} \right)^{-m_{sb}}. \quad (3.36)$$

By applying (3.34) and (3.36) into (3.35), a product of  $L + 1$  terms  $\prod_{\mathcal{K}=1}^L (1 + U_{\mathcal{K}})$  can be expanded as in [70],

$$\prod_{\mathcal{K}=1}^L (1 + U_{\mathcal{K}}) = 1 + \sum_{\mathcal{K}=1}^L \sum_{\lambda_0=0}^{L-\mathcal{K}} \sum_{\lambda_1=\lambda_0+1}^{L-\mathcal{K}+1} \cdots \sum_{\lambda_{\mathcal{K}}=\lambda_{\mathcal{K}-1}+1}^L \prod_{n=0}^{\mathcal{K}} U_{\lambda_n}. \quad (3.37)$$

Using (3.37),  $M_{\gamma_{total}}(s)$  can be written as

$$\begin{aligned} M_{\gamma_{total}}(s) &= \left(1 + \frac{s}{B_{sb}}\right)^{-m_{sb}} \left(\prod_{\mathcal{K}=1}^L U_{\mathcal{K}}\right) \prod_{\mathcal{K}=1}^L \left(1 + \frac{1 - U_{\mathcal{K}}}{U_{\mathcal{K}}} \left(1 + \frac{s}{B_{\mathcal{K}}}\right)^{-m_{\mathcal{K}}}\right) \\ &= \left(\prod_{\mathcal{K}=1}^L U_{\mathcal{K}}\right) \left(1 + \frac{s}{B_{sb}}\right)^{-m_{sb}} + \left(\prod_{\mathcal{K}=1}^L U_{\mathcal{K}}\right) \sum_{\mathcal{K}=1}^L \sum_{\lambda_1=1}^{L-\mathcal{K}+1} \sum_{\lambda_2=\lambda_1+1}^{L-\mathcal{K}+2} \cdots \\ &\quad \sum_{\lambda_{\mathcal{K}}=\lambda_{\mathcal{K}-1}+1}^L \left(\prod_{n=1}^{\mathcal{K}} \frac{1 - U_{\lambda_n}}{U_{\lambda_n}}\right) \prod_{n=0}^{\mathcal{K}} \left(1 + \frac{s}{B_{\mathcal{K}_{\lambda_n}}}\right)^{-m_{\mathcal{K}_{\lambda_n}}}. \end{aligned} \quad (3.38)$$

Finally, the outage probability is expressed as

$$P_{out(CDMA)} = \mathcal{L}^{-1}\{M_{\gamma_{total}}(s)/s; t\}_{|t=\gamma_{th}}, \quad (3.39)$$

where  $\mathcal{L}^{-1}\{.;.\}$  is the inverse Laplace transform. Using [ [73], page 223 eq. (21)] the inverse Laplace of the first term in (3.38) is given by

$$\mathcal{L}^{-1}\left\{\frac{\left(1 + \frac{s}{B_{sb}}\right)^{-m_{sb}}}{s}; t\right\}_{|t=\gamma_{th}} = \frac{\gamma(m_{sb}, B_{sb}\gamma_{th})}{\Gamma(m_{sb})}. \quad (3.40)$$

Using [74], one can get the PDF of the sum of independent but non identically distributed

gamma variables  $X_n \sim \mathcal{G}(m_n, B_n)$ , as the PDF of  $Y = \sum_{n=1}^N X_n$  can be expressed as

$$p_Y(\gamma) = \prod_{n=1}^N \left( \frac{B_1}{B_n} \right)^{m_n} \sum_{k=0}^{\infty} \frac{\delta_k \gamma^{\sum_{n=1}^N m_n + k - 1} e^{-\gamma/B_1}}{B_1^{\sum_{n=1}^N m_n + k} \Gamma\left(\sum_{n=1}^N m_n + k\right)} U(\gamma), \quad (3.41)$$

where  $B_1 = \min_n \{B_n\}$  and the coefficients  $\delta_y$  are obtained as

$$\begin{cases} \delta_0 &= 1 \\ \delta_{k+1} &= \frac{1}{k+1} \sum_{i=1}^{k+1} \left[ \sum_{j=1}^N m_j \left(1 - \frac{B_1}{B_n}\right)^i \right] \delta_{k+1-i}. \\ k &= 0, 1, 2, \dots \end{cases} \quad (3.42)$$

Finally, the outage probability  $P_{out}$  of the cooperative DF DS-CDMA over Nakagami- $m$  fading channels with arbitrary  $m_{\mathcal{K}}$  can be obtained after many algebraic manipulation, with the help of [72]- [75] as

$$\begin{aligned} P_{out(CDMA)} &= \left( \prod_{\mathcal{K}=1}^L U_{\mathcal{K}} \right) \frac{\gamma(m_{sb}, B_{sb} \gamma_{th})}{\Gamma(m_{sb})} + \left( \prod_{\mathcal{K}=1}^L U_{\mathcal{K}} \right) \sum_{\mathcal{K}=1}^L \sum_{\lambda_1=1}^{L-\mathcal{K}+1} \sum_{\lambda_2=\lambda_1+1}^{L-\mathcal{K}+2} \cdots \sum_{\lambda_{\mathcal{K}}=\lambda_{\mathcal{K}-1}+1}^L \\ &\times \left( \prod_{n=1}^{\mathcal{K}} \frac{1 - U_{\lambda_n}}{U_{\lambda_n}} \right) \left[ \prod_{i=0}^{\mathcal{K}} \left( \frac{B_1}{B_i} \right)^{m_{\mathcal{K}\lambda_i}} \right] \sum_{k=0}^{\infty} \delta_k \frac{\gamma\left(\sum_{i=0}^{\mathcal{K}} m_{\mathcal{K}\lambda_i} + k, B_1 \gamma_{th}\right)}{\Gamma\left(\sum_{i=0}^{\mathcal{K}} m_{\mathcal{K}\lambda_i} + k\right)}. \end{aligned} \quad (3.43)$$

It is worth mentioning that for the special case of integer Nakagami- $m$  fading channel, the infinite sum in (3.43) is removed as in [76]. Let us now investigate the diversity behavior of (3.43) when the SNR is sufficiently large, i.e.,  $(\bar{\gamma}_{ij} \rightarrow \infty)$ . According to [ [72], eq. (8.351,2)] we have

$$\frac{\gamma(m_{sb}, B_{sb} \gamma_{th})}{\Gamma(m_{sb})} \approx \frac{(m_{sb})^{m_{sb}-1} (\gamma_{th})^{m_{sb}}}{\Gamma(m_{sb})} (\bar{\gamma}_{sb})^{-m_{sb}},$$

and

$$U_{\mathcal{K}} = \frac{\gamma(m_{\mathcal{K}}, B_{\mathcal{K}}\gamma_{th})}{\Gamma(m_{\mathcal{K}})} \approx \frac{(m_{\mathcal{K}})^{m_{\mathcal{K}}-1}(\gamma_{th})^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})}(\bar{\gamma}_{\mathcal{K}})^{-m_{\mathcal{K}}}.$$

Using [ [72], eq. (8.350,5)]

$$\frac{\gamma\left(\sum_{i=0}^{\mathcal{K}} m_{\mathcal{K}_{\lambda_i}} + k, B_1\gamma_{th}\right)}{\Gamma\left(\sum_{i=0}^{\mathcal{K}} m_{\mathcal{K}_{\lambda_i}} + k\right)} = 0.$$

Finally, the outage probability in (3.43) can be asymptotically expressed as

$$P_{out(CDMA)} \approx \left( \prod_{\mathcal{K}=1}^L \frac{(m_{\mathcal{K}})^{m_{\mathcal{K}}-1}(\gamma_{th})^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})}(\bar{\gamma}_{\mathcal{K}})^{-m_{\mathcal{K}}} \right) \frac{(m_{sb})^{m_{sb}-1}(\gamma_{th})^{m_{sb}}}{\Gamma(m_{sb})}(\bar{\gamma}_{sb})^{-m_{sb}}. \quad (3.44)$$

Without loss of generality, let  $\bar{\gamma}_{sb} = \bar{\gamma}_{\mathcal{K}} = \Delta$ . Then, the outage probability in (3.44) is reduced to

$$P_{out(CDMA)} \approx \prod_{\mathcal{K}=1}^L \frac{(m_{\mathcal{K}})^{m_{\mathcal{K}}-1}(\gamma_{th})^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})} \cdot \frac{(m_{sb})^{m_{sb}-1}(\gamma_{th})^{m_{sb}}}{\Gamma(m_{sb})}(\Delta)^{-m_{sb}-\sum_{\mathcal{K}=1}^L m_{\mathcal{K}}}. \quad (3.45)$$

The expression in (3.45) shows that the achievable diversity order of the system is

$$m_{sb} + \sum_{\mathcal{K}=1}^L m_{\mathcal{K}}.$$

### 3.3.2.1 Simulation and Numerical Results

In what follows, we present some numerical results for the outage probability for both single and multi-relay cooperation. In order to generate the Nakagami- $m$  fading channel coefficients, we used the simulator described in Section 2.5.1. We build a Mont-Carlo link-level simulation to assess the accuracy of the derived analytical results. We assume asynchronous cooperative DS-CDMA with fully loaded system where  $N = K$  over independent non-identical Nakagami- $m$  fading channels. All simulations are based on the Nakagami simulator presented in [66]. The channels are modeled as block fading channels where the fading coefficients are considered fixed for the duration of one frame and change

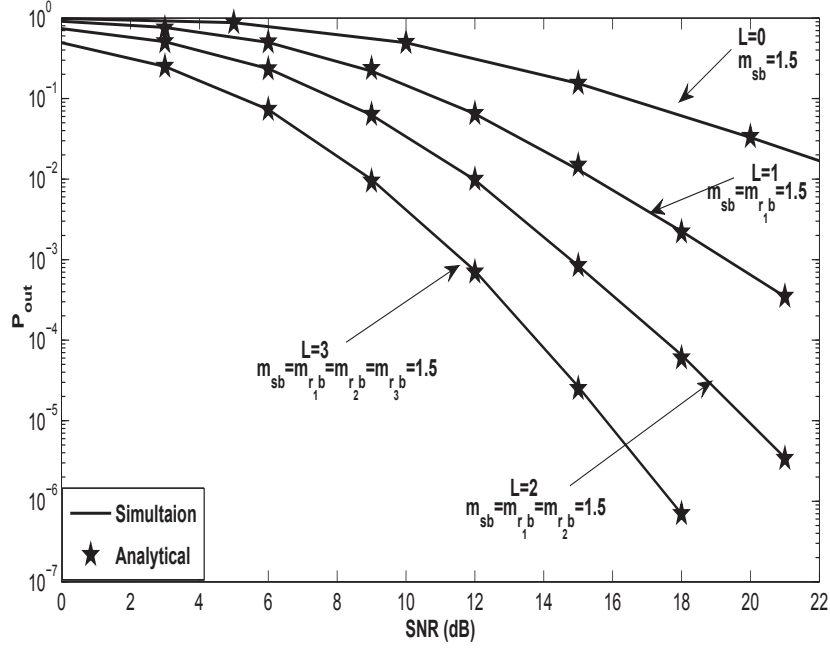


Figure 3.6: Outage probability for cooperative DF DS-CDMA with different number of relays,  $L$ , over i.i.d. Nakagami- $m$  fading channels.

independently from one frame to another. Without loss of generality, we assume that the spectral efficiency  $\Re = 1$  bit/sec/Hz and that the fading parameters  $m_{sr_\ell} = m_{r_\ell b}$ . A MMSE detector is used to mitigate the effect of MAI.

The outage probability for the cooperative DS-CDMA system with different number of relays  $L = 0, 1, 2, 3$  over i.i.d channels where  $m_{sb} = m_{r_\ell b} = m_{sr_\ell}$  is shown in Fig. 3.6. The results show the perfect matching between the analysis and simulations. The outage probability of the system over different fading channel parameters  $m_{sb} \neq m_{r_\ell b} \neq m_{sr_\ell}$  is also shown in Fig. 3.7. Clearly, from Fig. 3.7,  $P_{out}$  of the underlying system is improved and the full diversity gain is achieved with increasing  $L$  and/or  $m_{ij}$ . Fig. 3.7 also shows that the derived closed-form expression of the outage probability in (3.43) can be applied for an arbitrary value of  $m$ , both single and multi-relay scenarios and different channel environments (i.e., i.i.d. or i.n.i channels). The effect of the fading parameter on the  $P_{out}$  performance is evident from these results where larger  $m$  improves the system performance. In Fig. 3.6, we consider a multi-relay case where the fading statistics of the

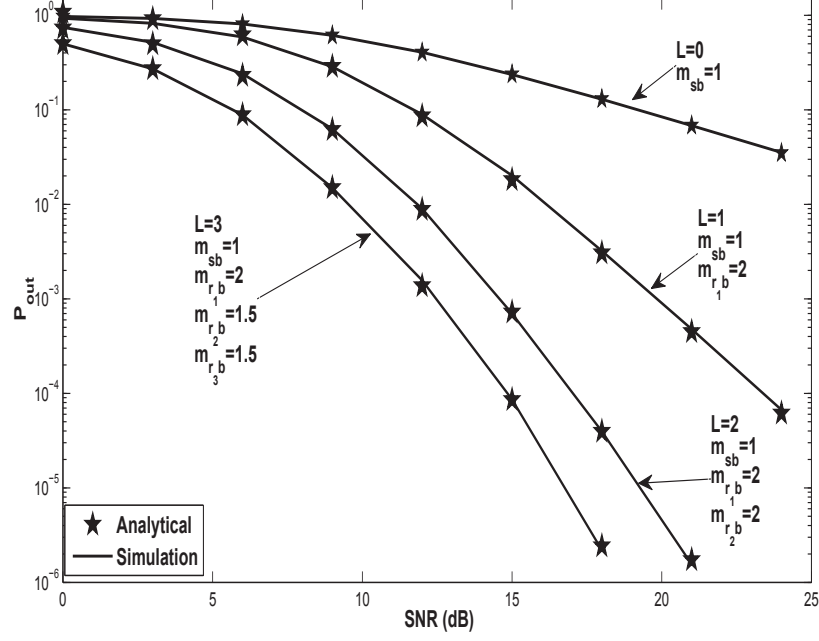


Figure 3.7: Outage probability for cooperative DF DS-CDMA with different  $L$  over independent nonidentical Nakagami- $m$  fading channels.

relay-base station and source-base station are identical. The results show the diversity gain achieved for different fading environments and number of relays. Finally, Fig. 3.7 shows the same results as in Fig. 3.6 but with non-identical Nakagami channels.

Fig. 3.8 shows a comparison of the outage probability of the cooperative DF asynchronous DS-CDMA system when using the conventional matched-filter detector and MMSE. The results are shown for  $L = 1$  and  $L = 3$  relays. The results in this figure show that the MMSE is able to achieve the full system diversity while the conventional detector exhibits an error floor due to the multiuser interference. That is, the diversity advantage of the cooperative system cannot be reached without mitigating the effect of multiuser interference.

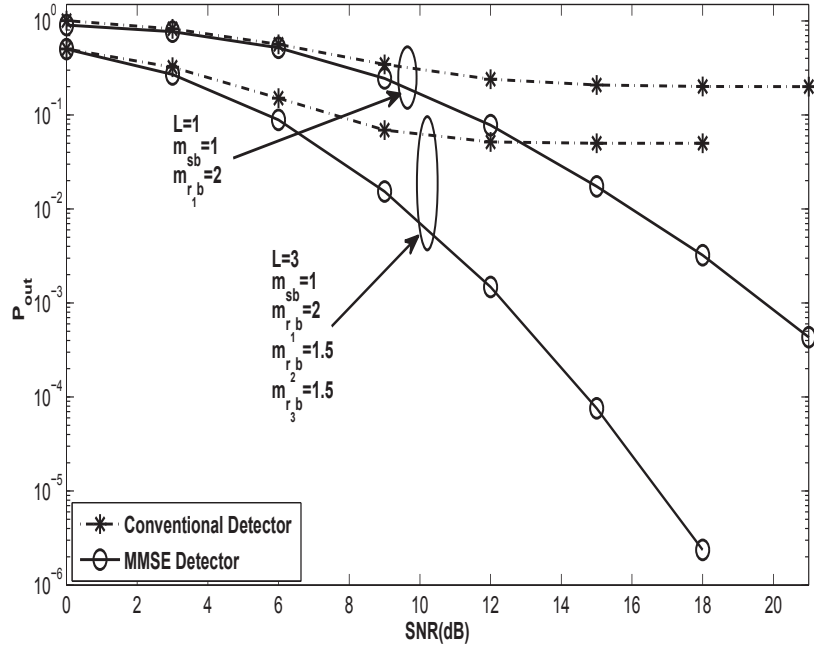


Figure 3.8: Outage probability of cooperative DF asynchronous DS-CDMA system using conventional detector and MMSE for  $L = 1$  and  $L = 3$ .

### 3.4 BER Analysis

In this section, we investigate the BER performance of the cooperative asynchronous DS-CDMA systems in frequency-selective Nakagami- $m$  fading channels for a single-relay scenario. It is noteworthy that if multi-relay scenario ( $L$ -relays) is used then the transmission protocol needs  $(L+1)$  time-slot in order to send a message from source to destination. Meanwhile, in single relay scenario, two time-slot are only needed. This leads to better bandwidth efficiency utilization in cooperative systems.

#### 3.4.1 System Model

We consider an uplink  $K$ -user asynchronous non-orthogonal DS-CDMA system, transmitting over frequency-selective Nakagami- $m$  fading channel. The Cooperation among users takes place as in Section 3.3.2. Fig. 3.9 shows the transceiver structure for DF, where  $W_i$  branch serves the user data itself including spreading and modulation, while  $Y_i$  branch



serves the partner's data including MUD. We assume every relay has an error detection capability (i.e., cyclic redundancy check (CRC)), to decide whether the cooperation will take place or not. Fig. 3.10 shows the cooperative scheme among users at two consecu-

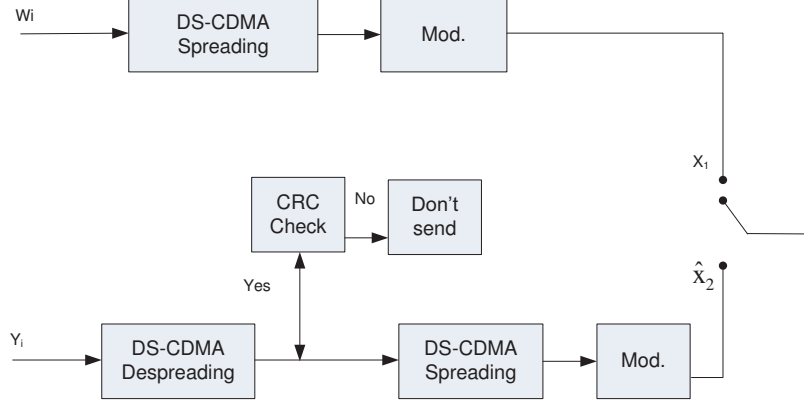


Figure 3.9: Transceiver structure for DF .

tive phases as in Section 3.2.

Assuming a multipath channel with  $P$  paths, the received signal at the base station and the cooperating partner in the first transmission phase can be written as

$$r_b^I(t) = \sum_{i=0}^{f-1} \sum_{s=1}^K \sum_{p=1}^P \sqrt{E_s} x_s(i) C_s(t - \tau_s - \tau_{s,p} - iT_b) h_{sb}^p + n_b^I(t), \quad (3.46)$$

$$r_r(t) = \sum_{i=0}^{f-1} \sum_{s=1, s \neq r}^K \sum_{p=1}^P \sqrt{E_s} x_s(i) C_s(t - \tau_s - \tau_{s,p} - iT_b) h_{sr}^p + n_r(t), \quad (3.47)$$

where  $x_s$ ,  $C_s$ ,  $\tau_s$ ,  $T_b$ ,  $h_{sb}^p$ ,  $n_b^I$  are defined as in Section 3.2.. While the received signal at the second transmission phase at the base station can be defined as

$$r_b^{II}(t) = \sum_{i=0}^{f-1} \sum_{s=1}^K \sum_{p=1}^P \sqrt{E_s} \tilde{x}_s(i) C_r(t - D_{s,r} - \tau_r - \tau_{r,p} - iT_b) h_{rb}^p + n_b^{II}(t). \quad (3.48)$$

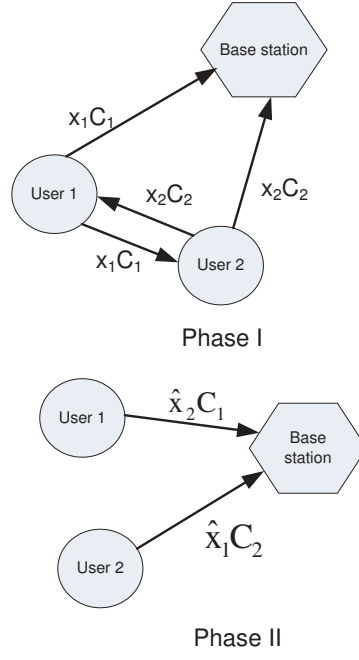


Figure 3.10: Cooperation module between users

The output of the bank of matched filters at the base station and relay, after stacking all matched-filtered outputs in the base station and relay ( $r$ ), and by dropping the time index from the module (3.46) and (3.47), are defined as in (2.2) and repeated here as

$$\mathbf{y}_b^I = \mathbf{R}_b \mathbf{H}_{s,b} \mathbf{x} + \mathbf{n}_b^I \quad (3.49)$$

$$\mathbf{y}_r = \mathbf{R}_r \mathbf{H}_{s,r} \mathbf{x}_r + \mathbf{n}_r, \quad (3.50)$$

where  $\mathbf{R}_b$  is the  $(KfP \times KfP)$  cross-correlation at the base station receiver, defined as [18]

$$\mathbf{R}_b = \begin{bmatrix} \rho_{1,1}^p & \cdots & \rho_{1,2}^p & \cdots & \rho_{1,K}^p \\ \rho_{2,1}^p & \cdots & \rho_{2,2}^p & \cdots & \rho_{2,K}^p \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ \rho_{K,1}^p & \cdots & \rho_{K,2}^p & \cdots & \rho_{K,K}^p \end{bmatrix}, \quad (3.51)$$

with  $\mathbf{H}_{s,b}$  being an  $(KfP \times KfP)$  channel coefficient matrix between the source and the base station, defined as [18]

$$\mathbf{H}_{s,b} = \begin{bmatrix} h_{1,b}^p & 0 & \dots & \dots & 0 \\ 0 & \dots & h_{2,b}^p & \dots & 0 \\ \vdots & \dots & \dots & \dots & \vdots \\ 0 & \dots & \dots & \dots & h_{K,b}^p \end{bmatrix}, \quad (3.52)$$

$\mathbf{x}$  is the  $(KfP \times 1)$  user data vector of the  $K$  total users, defined as  $\mathbf{x} = [x_1(1), \dots, x_1(f), \dots, x_K(1), \dots, x_K(f)]^T$ , and  $\mathbf{n}_b^I$  is the  $(KfP \times 1)$  complex Gaussian random noise vector with elements being zero mean and variance  $\sigma_n^2$ . Similarly at the relay,  $\mathbf{x} = [x_1(1), \dots, x_1(f), \dots, x_K(1), \dots, x_K(f)]_{s \neq r}^T$ ,  $\mathbf{n}_r$  is  $((K-1)fP \times 1)$  Gaussian noise vector,  $\mathbf{H}_{s,r}$  is the  $((K-1)fP \times ((K-1)fP))$  inter-user channel matrix [18], and  $\mathbf{R}_r$  is the  $((K-1)fP \times ((K-1)fP))$  cross correlation matrix at the relay receiver, defined respectively as [18]

$$\mathbf{H}_{s,r} = \begin{bmatrix} h_{1,r}^p & 0 & \dots & \dots & 0 \\ 0 & \dots & h_{2,r}^p & \dots & 0 \\ \vdots & \dots & \dots & \dots & \vdots \\ 0 & \dots & \dots & \dots & h_{K,r}^p \end{bmatrix}_{s \neq r} \quad (3.53)$$

$$\mathbf{R}_r = \begin{bmatrix} \rho_{1,1}^p & \dots & \rho_{1,2}^p & \dots & \rho_{1,K}^p \\ \rho_{2,1}^p & \dots & \rho_{2,2}^p & \dots & \rho_{2,K}^p \\ \vdots & \dots & \dots & \dots & \vdots \\ \rho_{K,1}^p & \dots & \rho_{K,2}^p & \dots & \rho_{K,K}^p \end{bmatrix}_{s \neq r}. \quad (3.54)$$

We remark that both of MMSE and decorrelator detector as MUDs converge at high SNR region and result in the same performance. Therefore, we tune our investigation to the use of decorrelator detector as MUD which results in a significant performance gain over the conventional detector as detailed in Section 2.4. Employing a decorrelator

detector at the base station [77], we have

$$\mathbf{z}_b^I = (\mathbf{R}_b^{-1})\mathbf{y}_b^I = \mathbf{H}_{s,b}\mathbf{x} + \mathbf{R}_b^{-1}\mathbf{n}_b^I. \quad (3.55)$$

Also, considering MUD at the relay side, the soft data estimate at the decorrelator output is given by

$$\mathbf{z}_r = (\mathbf{R}_r^{-1})\mathbf{y}_r = \mathbf{H}_{s,r}\mathbf{x}_r + \mathbf{R}_r^{-1}\mathbf{n}_r. \quad (3.56)$$

Taking into account the  $i^{th}$  data bit of the  $s^{th}$  user,  $s \in \{1, \dots, K\}$ , the decorrelator output at the base station for the first and second transmission phases of (4.38) and (3.56) can be rewritten subsequently as

$$[\mathbf{z}_1]_b^I = \mathbf{H}_{s,b}[\mathbf{x}_1]_b^I + \mathbf{R}_b^{-1}[\mathbf{n}_1]_b^I, \quad (3.57)$$

$$[\mathbf{z}_1]_b^{II} = \mathbf{H}_{r,b}[\mathbf{x}_1]_b^{II} + \mathbf{R}_b^{-1}[\mathbf{n}_1]_b^{II}, \quad (3.58)$$

where  $[\mathbf{z}_1]_b^I$  and  $[\mathbf{z}_1]_b^{II}$  are the decision statistics correspond of the  $i$  bit for user  $s$  and  $[\mathbf{x}_1]_b^I, [\mathbf{x}_1]_b^{II}$  identify the data vector transmitted during the first and second transmission periods as  $[\mathbf{x}_1]_b^I = [\mathbf{x}_1]_b^{II} = [\sqrt{E_s}x_1(1), \dots, \sqrt{E_s}x_1(1), \dots, \sqrt{E_s}x_1(f), \dots, \sqrt{E_s}x_K(1), \dots, \sqrt{E_s}x_K(f)]^T$

$$\begin{aligned} [z_1^1(i)]_{b1} &= \sqrt{E_s}h_{s,b}^1(i)x_1(i) + [\mathbf{R}_b^{-1}\mathbf{n}_b^I]^1 \\ &\vdots \\ [z_1^P(i)]_{b1} &= \sqrt{E_s}h_{s,b}^P(i)x_1(i) + [\mathbf{R}_b^{-1}\mathbf{n}_b^I]^P. \end{aligned} \quad (3.59)$$

The  $P$  components of  $[\mathbf{z}_1^p]_b^I$  represents the decision statistics of bit  $i$  for user 1 before RAKE combining over the  $p$  path, during the first transmission period

$$\begin{aligned}
[z_1^1(i)]_b^{II} &= \sqrt{E_s} h_{r,b}^1(i) \hat{x}_1(i) + [\mathbf{R}_b^{-1} \mathbf{n}_b^{II}]^1 \\
&\vdots \\
[z_1^P(i)]_b^{II} &= \sqrt{E_s} h_{r,b}^P(i) \hat{x}_1(i) + [\mathbf{R}_b^{-1} \mathbf{n}_b^{II}]^P,
\end{aligned} \tag{3.60}$$

while the  $P$  components of  $[\mathbf{z}_1^p]^{II}$  is the decision statistics of bit  $i$  for user 1 before RAKE combining, in the second transmission period. By gathering (3.59) and (3.60), the decision statistic of the desired user signal can be written as

$$\hat{x}_1(i) = h_{s,b}^{1*}(i)[z_1^1(i)]_b^I + \dots + h_{s,b}^{P*}(i)[z_1^P(i)]_b^I + h_{r,b}^{1*}(i)[z_1^1(i)]_b^{II} + \dots + h_{r,b}^{P*}(i)[z_1^P(i)]_b^{II}. \tag{3.61}$$

Assessing the  $m^{th}$  bit of the user 1, the  $\hat{x}_1(i)$  can be expressed as

$$\begin{aligned}
\hat{x}_1(i) &= \sqrt{E_s} \left[ |h_{s,b}^1(i)|^2 + \dots + |h_{s,b}^P(i)|^2 + |h_{r,b}^1(i)|^2 + \dots + |h_{r,b}^P(i)|^2 \right] x_1(i) \\
&+ \operatorname{Re} \left\{ h_{s,b}^{1*}(i) [\mathbf{R}_b^{-1} \mathbf{n}_b^I]^1 + \dots + h_{s,b}^{P*}(i) [\mathbf{R}_b^{-1} \mathbf{n}_b^I]^P + h_{r,b}^{1*}(i) [\mathbf{R}_b^{-1} \mathbf{n}_b^{II}]^1 \right. \\
&+ \left. \dots + h_{r,b}^{P*}(i) [\mathbf{R}_b^{-1} \mathbf{n}_b^{II}]^P \right\}.
\end{aligned} \tag{3.62}$$

### 3.4.2 Error Performance Analysis

The probability of error at the base station given that the Nakagami- $m$  channel coefficient using (3.62) in the case of perfect inter-user channel can be written as [18, 78, 79]

$$P_{\text{error-free}} = Q \left( \frac{\sum_{p=1}^P \sqrt{E_s} (|h_{s,b}^p(i)|^2 + |h_{r,b}^p(i)|^2)}{\sqrt{\sigma_x^2}} \right), \tag{3.63}$$

where  $Q(\cdot)$  is the Gaussian Q-function and  $\sigma_x^2$  is the variance of the noise in (3.62) and defined as

$$\sigma_x^2 = \sigma_n^2 \sum_{p=1}^P |h_{s,b}^p(i)|^2 (R_b)_{s,s}^{-2} + |h_{r,b}^p(i)|^2 (R_b)_{r,r}^{-2}, \tag{3.64}$$

where  $(R_b)_{s,s}^{-2}$  is the square of the sum of the  $s^{th}$  row in the inverse cross correlation matrix [18]. Let us denote  $C_s = (R_b)_{s,s}^{-2}$ , and  $C_r = (R_b)_{r,r}^{-2}$ . Also  $\nu_{sb}^p = |h_{s,b}^p(i)|^2$  and  $\nu_{rb}^p = |h_{r,b}^p(i)|^2$ . Thus  $\nu_{sb}^p$  and  $\nu_{rb}^p$  are gamma distributed with characteristic function defined as

$$\Phi(jw) = \frac{1}{(1 - jw\beta)^m}, \quad (3.65)$$

from the definition of gamma distribution, we have  $\beta_{ij} = \frac{m_{ij}}{\gamma_{ij}(R_{ij})^{-2}}$  with average SNR,  $\gamma_{ij} = \frac{\mathbb{E}\langle |h_{ij}|^2 \rangle E_s}{N_o}$ . Let us designate the variable  $\eta$  as

$$\eta = \frac{A}{\sqrt{B}}, \quad (3.66)$$

where  $A = \sum_{p=1}^P \nu_{sb}^p + \nu_{rb}^p$  and  $B = \sum_{p=1}^P \nu_{sb}^p C_s + \nu_{rb}^p C_r$ . Then  $P_{bp} = Q(\sqrt{\frac{E_s \eta^2}{\sigma_n^2}})$

### 3.4.2.1 PDF Evaluation

To find the pdf of  $\eta$ , we need to evaluate the joint characteristic function of  $A$  and  $B$  as

$$\Phi_{A,B}(w_1, w_2) = \frac{1}{\beta^{2mP}} \prod_{p=1}^P \frac{1}{(y - jC_s w_2)^m} \cdot \frac{1}{(y - jC_r w_2)^m}, \quad (3.67)$$

where  $y = \frac{1}{\beta} - jw_1$ , then we can rewrite (3.67) as

$$\Phi_{A,B}(w_1, w_2) = \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{\beta^{2mP}} \prod_{g=1}^{2P} \frac{1}{(w_2 - \frac{y}{jC_g})^m}. \quad (3.68)$$

In order to simplify (3.68), we use a partial fraction expansion method of a rational function with high order poles as in [80]. Therefore, the characteristic function in (3.68) can be simplified to

$$\Phi_{A,B}(w_1, w_2) = \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{\beta^{2mP}} \sum_{g=1}^{2P} \sum_{i=1}^{m-1} \frac{C_{gi}}{(w_2 - \frac{y}{jC_g})^{m-i}}, \quad (3.69)$$

where

$$C_{gi} = \frac{K_{gi}}{y^{2mP-m+i}} \quad g = 1, \dots, 2P, i = 0, \dots, m-1,$$

and  $K_{gi}$  denotes a function of the cross-correlations between users. Then, the PDF of  $f_{A,B}$  can be obtained as in [81]

$$\begin{aligned} f_{A,B} &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{A,B}(w_1, w_2) \exp(-jw_1 A) \exp(-jw_1 B) dw_1 dw_2 \\ &= \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{4\pi^2 \beta^{2mP}} \sum_{g=1}^{2P} \sum_{i=1}^{m-1} \lambda_{gi} B^{m-i-1} \left(A - \frac{B}{C_g}\right)^{2mP-m+i-1} \exp\left(-\frac{A}{\beta}\right), \end{aligned} \quad (3.70)$$

where  $\lambda_{gi} = \frac{4\pi^2 K_{gi}}{\Gamma(m-i)\Gamma(2mP-m+i)}$ .

Letting  $W = B$ , the PDF of  $\eta$  and  $W$  as defined (3.66) can be obtained using the variable transformation method from the following relation as

$$f(\eta, W) = f(A, B) |\Omega(\eta, W)|, \quad (3.71)$$

where  $|\Omega(\eta, W)| = \sqrt{W}$  represents the Jacobian of the transformation.

By substituting (3.70) into (3.71), we get

$$f_{\eta,W} = \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{4\pi^2 \beta^{2mP}} \sum_{g=1}^{2P} \sum_{i=1}^{m-1} \lambda_{gi} W^{m-i-\frac{1}{2}} \left(\eta \sqrt{W} - \frac{W}{C_g}\right)^{2mP-m+i-1} \exp\left(-\frac{\eta \sqrt{W}}{\beta}\right) \quad (3.72)$$

where  $\eta = \frac{A}{\sqrt{B}}$  or  $\eta = \frac{A}{\sqrt{W}}$

$$f_{\eta} = \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{4\pi^2 \beta^{2mP}} \sum_{g=1}^{2P} \sum_{i=1}^{m-1} \lambda_{gi} \Lambda_{gi} \quad (3.73)$$

where

$$\Lambda_{gi} = \int_0^{C_g^2 \eta^2} W^{m-i-\frac{1}{2}} \left( \eta \sqrt{W} - \frac{W}{C_g} \right)^{2mP-m+i-1} \exp\left(-\frac{\eta \sqrt{W}}{\beta}\right) dW. \quad (3.74)$$

Applying the binomial series expansion, the integration in (3.74) is reduced to

$$\begin{aligned} \Lambda_{gi} &= \sum_{d=0}^{2mP-m+i-1} \binom{2mP-m+i-1}{d} \frac{(-1)^{2mP-m+i-1-d} \eta^d}{(C_g)^{2mP-m+i-1-d}} \int_0^{C_g^2 \eta^2} (\sqrt{W})^{4mP-3-d} \\ &\times \exp\left(-\frac{\eta \sqrt{W}}{\beta}\right) dW. \end{aligned} \quad (3.75)$$

In what follows, we represent the integration (3.75) by  $I_{ud}$  and with help [72]

$$\int_0^y x^{v-1} e^{-\mu x} dx = \frac{1}{\mu^v} \gamma(v, \mu y) \quad (3.76)$$

we can rewrite (3.75) as

$$I_{gd} = 2 \int_0^{C_g \eta} (\sqrt{W})^{4mP-3-d} \exp\left(-\frac{\eta \sqrt{W}}{\beta}\right) dW. \quad (3.77)$$

Using  $\sqrt{W} = t$  we get

$$I_{gd} = 2 \int_0^{C_g \eta} (t)^{4mP-2-d} \exp\left(-\frac{\eta t}{\beta}\right) dt \quad (3.78)$$

where  $dw = 2t dt$ . Using (3.76) we get

$$I_{gd} = \frac{2^{4mP-d}}{\left(\frac{\eta}{\beta}\right)^{4mP-1-d}} \gamma\left(4mP-1-d, \frac{C_g \eta^2}{2\beta}\right). \quad (3.79)$$

With respect to the confluent hypergeometric function [ [82], eq.(13.1.32)] we get

$$I_{gd} = \frac{2C_g^{4mP-1-d} \eta^{4mP-1-d}}{(4mP-1-d)} \exp\left(-\frac{C_g \eta^2}{2\beta}\right) {}_1F_1\left(1; 4mP-d; \frac{C_g \eta^2}{2\beta}\right). \quad (3.80)$$



Then,

$$\begin{aligned}\Lambda_{gi} &= \sum_{d=0}^{2mP-m+i-1} \binom{2mP-m+i-1}{d} \frac{(-1)^{2mP-m+i-1-d}}{(C_g)^{2mP-m+i-1-d}} \cdot \frac{2C_u^{4mP-1-d}\eta^{4mP-1}}{4mP-1-d} \\ &\times \exp\left(-\frac{C_g\eta^2}{2\beta}\right) {}_1F_1\left(1; 4mP-d; \frac{C_g\eta^2}{2\beta}\right)\end{aligned}\quad (3.81)$$

$$\begin{aligned}\Lambda_{gi} &= 2C_g^{2mP+m-i}\eta^{4mP-1} \sum_{d=0}^{2mP-m+i-1} \binom{2mP-m+i-1}{d} \frac{(-1)^{2mP-m+i-1-d}}{4mP-1-d} \\ &\times \exp\left(-\frac{C_g\eta^2}{2\beta}\right) {}_1F_1\left(1; 4mP-d; \frac{C_g\eta^2}{2\beta}\right).\end{aligned}\quad (3.82)$$

By substituting (3.82) into (3.73) we have

$$\begin{aligned}f_\eta &= \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{2\pi^2\beta^{2mP}} \sum_{g=1}^{2P} \sum_{i=1}^{m-1} \lambda_{gi} C_g^{2mP+m-i}\eta^{4mP-1} \sum_{d=0}^{2mP-m+i-1} \binom{2mP-m+i-1}{d} \\ &\times \frac{(-1)^{2mP-m+i-1-d}}{4mP-1-d} \exp\left(-\frac{C_g\eta^2}{2\beta}\right) {}_1F_1\left(1; 4mP-d; \frac{C_g\eta^2}{2\beta}\right).\end{aligned}\quad (3.83)$$

### 3.4.2.2 probability of Bit error

Here, we evaluate the probability of bit error by averaging (3.63) over the derived pdf in (3.83)

$$P_{\text{error-free}} = \int_0^\infty Q(\sqrt{\zeta\eta^2}) f_\eta d\eta, \quad (3.84)$$

where  $\zeta = \frac{\beta}{N_0} = \frac{\mathbb{E}[|h_{rb}^P(i)|^2]E_s}{m\sigma_n^2}$  is the normalized average SNR. Using the preferred form of the Gaussian  $Q$ -function, the analysis is simplified as [83]

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp^{-\frac{x^2}{2\sin^2\theta}} d\theta \quad (3.85)$$

$$P_{b_{error-free}} = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^\infty Q(\sqrt{\zeta\eta^2}) f_\eta d\eta d\theta. \quad (3.86)$$

By substituting (3.84) and (3.85) into (3.86) we get

$$\begin{aligned} P_{b_{error-free}} &= \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{2\pi^3 \beta^{2mp}} \sum_{u=1}^{2P} \sum_{i=1}^{m-1} \lambda_{gi} C_g^{2mP+m-i} \sum_{d=0}^{2mP-m+i-1} \binom{2mP-m+i-1}{d} \\ &\times \frac{(-1)^{2mP-m+i-1-d}}{4mP-1-d} \\ &\times \int_0^{\frac{\pi}{2}} \int_0^\infty \exp^{-\eta^2(\frac{\zeta}{2\sin^2\theta} + \frac{C_g}{2\beta})} \eta^{4mP-1} {}_1F_1(1; 4mP-d; \frac{C_g\eta^2}{2\beta}) d\eta d\theta. \end{aligned} \quad (3.87)$$

Using [ [72], eq. (7.621,4)], we can solve the integral  $\int_0^\infty \exp^{-\eta^2(\frac{\zeta}{2\sin^2\theta} + \frac{C_g}{2\beta})} \eta^{4mP-1} {}_1F_1(1; 4mP-d; \frac{C_g\eta^2}{2\beta}) d\eta$  as

$$\begin{aligned} P_{b_{error-free}} &= \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{2\pi^3 \beta^{2mp}} \sum_{g=1}^{2P} \sum_{i=1}^{m-1} \lambda_{gi} C_g^{2mP+m-i} \sum_{d=0}^{2mP-m+i-1} \binom{2mP-m+i-1}{d} \\ &\times \frac{(-1)^{2mP-m+i-1-d}}{4mP-1-d} \\ &\times \int_0^{\frac{\pi}{2}} \Gamma(2mP) \left(\frac{\zeta}{2\sin^2\theta}\right)^{-2mP} {}_2F_1(4mP-d-1, 2mP; 4mP-d; -\frac{C_g\sin^2\theta}{\zeta}) d\theta \end{aligned} \quad (3.88)$$

or simply as

$$\begin{aligned} P_{b_{error-free}} &= \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{2\pi^3 \beta^{2mp}} \sum_{g=1}^{2P} \sum_{i=1}^{m-1} \lambda_{gi} C_g^{2mP+m-i} \sum_{d=0}^{2mP-m+i-1} \binom{2mP-m+i-1}{d} \\ &\times \frac{(-1)^{2mP-m+i-1-d}}{4mP-1-d} \\ &\times \frac{\Gamma(2mP)}{2\zeta^{2mP}} \int_0^{\frac{\pi}{2}} (\sin^2\theta)^{2mP} {}_2F_1(4mP-d-1, 2mP; 4mP-d; -\frac{C_g\sin^2\theta}{\zeta}) d\theta. \end{aligned} \quad (3.89)$$

Using [ [72], eq. (7.512,12)], we can solve the integral in (3.88) where  $x = \sin^2\theta$  and  $2\sin\theta\cos\theta d\theta = dx$  so  $d\theta = \frac{1}{2\sqrt{x}\sqrt{1-x}}dx$  to get

$$\begin{aligned}
P_{b_{error-free}} &= \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{2\pi^3 \beta^{2mp}} \sum_{g=1}^{2P} \sum_{i=1}^{m-1} \lambda_{gi} C_g^{2mP+m-i} \sum_{d=0}^{2mP-m+i-1} \binom{2mP-m+i-1}{d} \\
&\times \frac{(-1)^{2mP-m+i-1-d} \Gamma(2mP) \Gamma(2mP + \frac{1}{2}) \Gamma(\frac{1}{2})}{4mP-1-d} \frac{1}{4\zeta^{2mP}} \frac{\Gamma(2mP+1)}{\Gamma(2mP+1)} \\
&\times {}_3F_2(2mP + \frac{1}{2}, 4mP-d-1, 2mP; 2mP+1, 4mP-d; -\frac{C_{gs}}{\zeta}). \quad (3.90)
\end{aligned}$$

Finally,

$$\begin{aligned}
P_{b_{error-free}} &= \frac{\prod_{g=1}^{2P} \frac{j}{C_g^{2mP}}}{2\pi^3 \beta^{2mp}} \sum_{g=1}^{2P} \sum_{i=1}^{m-1} \lambda_{gi} C_g^{2mP+m-i} \sum_{d=0}^{2mP-m+i-1} \binom{2mP-m+i-1}{d} \\
&\times \left( \frac{(-1)^{2mP-m+i-1-d}}{4mP-1-d} \right) \left( \frac{\Gamma(2mP + \frac{1}{2}) \Gamma(\frac{1}{2})}{8mP\zeta^{2mP}} \right) \\
&\times {}_3F_2(2mP + \frac{1}{2}, 4mP-d-1, 2mP; 2mP+1, 4mP-d; -\frac{C_{gs}}{\zeta}). \quad (3.91)
\end{aligned}$$

The BER in (3.91) can be asymptotically investigated when the SNR is sufficiently high. Knowing that  ${}_3F_2(2mP + \frac{1}{2}, 4mP-d-1, 2mP; 2mP+1, 4mP-d; -\frac{C_{gs}}{\zeta}) \rightarrow 1$  as  $\zeta \rightarrow \infty$ . Clearly one can see that  $P_{b_{error-free}} \equiv \frac{1}{\zeta^{2mP}}$ . Therefore the achievable diversity order of the system is  $2mP$ .

In the same way as in (3.91), the average probability of bit error of the inter-user channel can be written as

$$\begin{aligned}
P_{b_{sr}} &= \frac{\prod_{r=1}^P \frac{j}{C_r^{m_{sr}P}}}{2\pi^3 \beta^{2m_{sr}P}} \sum_{r=1}^P \sum_{i=1}^{m_{sr}-1} \lambda_{ri} C_r^{m_{sr}P+m_{sr}-i} \sum_{d=0}^{m_{sr}P-m_{sr}+i-1} \binom{m_{sr}P-m_{sr}+i-1}{d} \\
&\times \left( \frac{(-1)^{m_{sr}P-m_{sr}+i-1-d}}{2m_{sr}P-1-d} \right) \left( \frac{\Gamma(m_{sr}P + \frac{1}{2}) \Gamma(\frac{1}{2})}{4m_{sr}P\zeta_r^{m_{sr}P}} \right) \\
&\times {}_3F_2(m_{sr}P + \frac{1}{2}, 2m_{sr}P-d-1, m_{sr}P; m_{sr}P+1, 2m_{sr}P-d; -\frac{C_{rs}}{\zeta_r}), \quad (3.92)
\end{aligned}$$

where  $\lambda_{ri} = \frac{4\pi^2 K_{ri}}{\Gamma(m-i)\Gamma(mP-m+i)}$  and  $\zeta_r = \frac{\mathbb{E}[|h_{sr}^P(i)|^2]E_s}{m_{sr}\sigma_n^2}$ . Finally the average probability of bit error at the base station can be written as

$$P_{b_{total}} = P_{b_{sr}}P_{b_{sb}} + (1 - P_{b_{sr}})P_{b_{error-free}}, \quad (3.93)$$

where  $P_{b_{sr}}$  is the average BER at the relay side,  $P_{b_{sb}}$  stands for the average BER for the direct link between the user  $s$  and the base station, and  $P_{b_{error-free}}$  represents the average BER for the perfect inter-user channel.

### 3.4.3 Simulation and Numerical Results

In what follows, we present simulation results to study the BER performance of the cooperative system over Nakagami- $m$  channel considering different scenarios.  $K = 16$  users asynchronous DS-CDMA system with binary phase shift keying (BPSK) transmission is assumed where each user data is spreaded using non-orthogonal Gold codes of length  $N = 31$  chips. A frame length of  $\ell = 100$  bits is considered. The delay between users is uniformly distributed with mean  $D$  chips. To neglect the effect of inter symbol interference (ISI), the delay of each path,  $\tau_{k,p}$ , is taken as multiple of chip periods of length less than the symbol period. Fading coefficients are fixed for the two cooperative phases [11]. Since we neglect the path loss, we consider  $\Omega_{kb}^p = \Omega_{kr}^p = \Omega_{rb}^p = 2$  and for the identical channels case we assume  $m_{kb}^p = m_{rb}^p = m_{kr}^p$ . Considering perfect inter-user channels between users in Fig. 3.11 and Fig. 3.12, the performance of cooperative and non-cooperative schemes for different multipath environments is evaluated. For fair comparison between non-cooperative and cooperative schemes, the total transmitted power for non-cooperative case per user data is doubled.

In Fig. 3.13 and Fig. 3.14, we show BER for non-perfect inter-user channel. It is clear from these results that as inter-user SNR gets smaller, the system loses diversity,

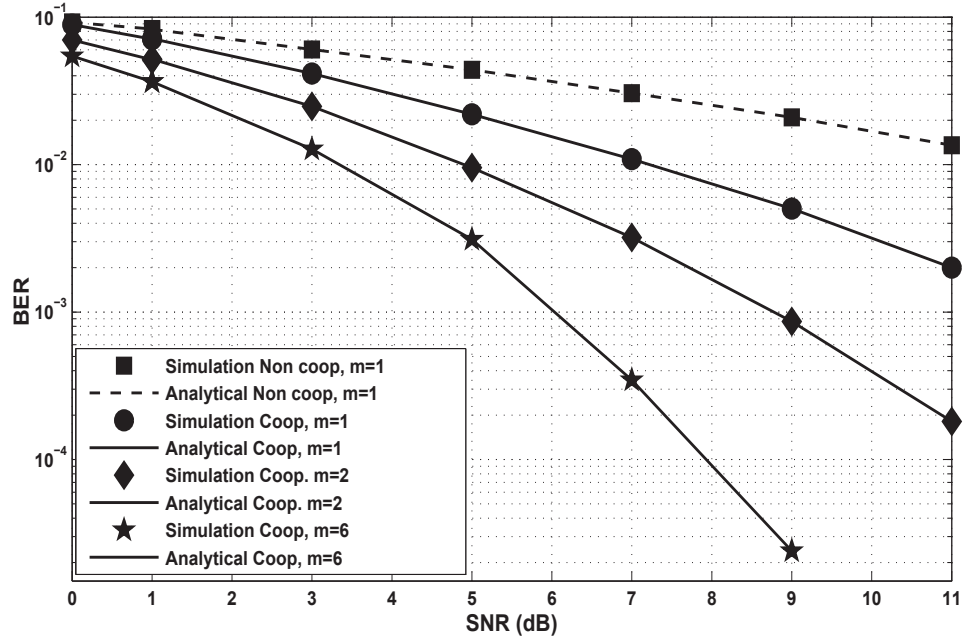


Figure 3.11: Cooperative DF DS-CDMA with  $N=31$ ,  $K=16$  users, 1 path and different  $m$ .

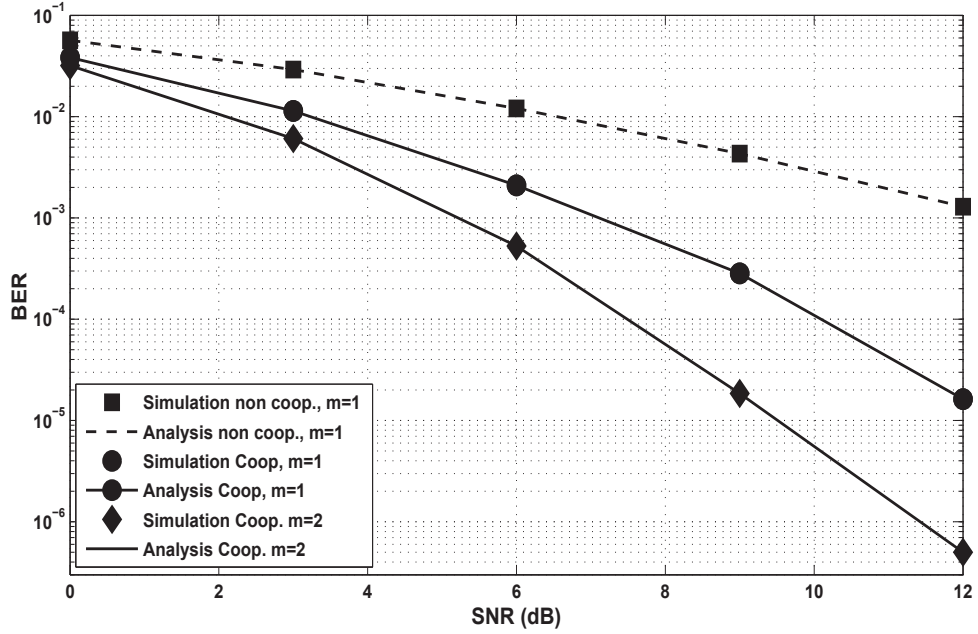


Figure 3.12: Cooperative DF DS-CDMA with  $N=31$ ,  $K=16$  users, 2 paths and different  $m$ .

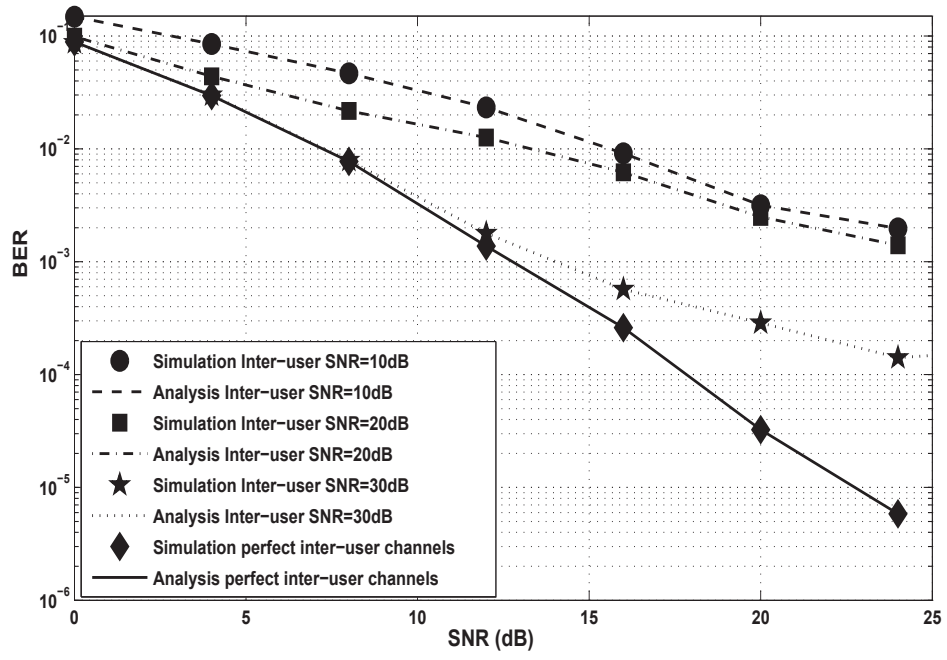


Figure 3.13: BER as a function of inter-user channel with  $m = 1$ .

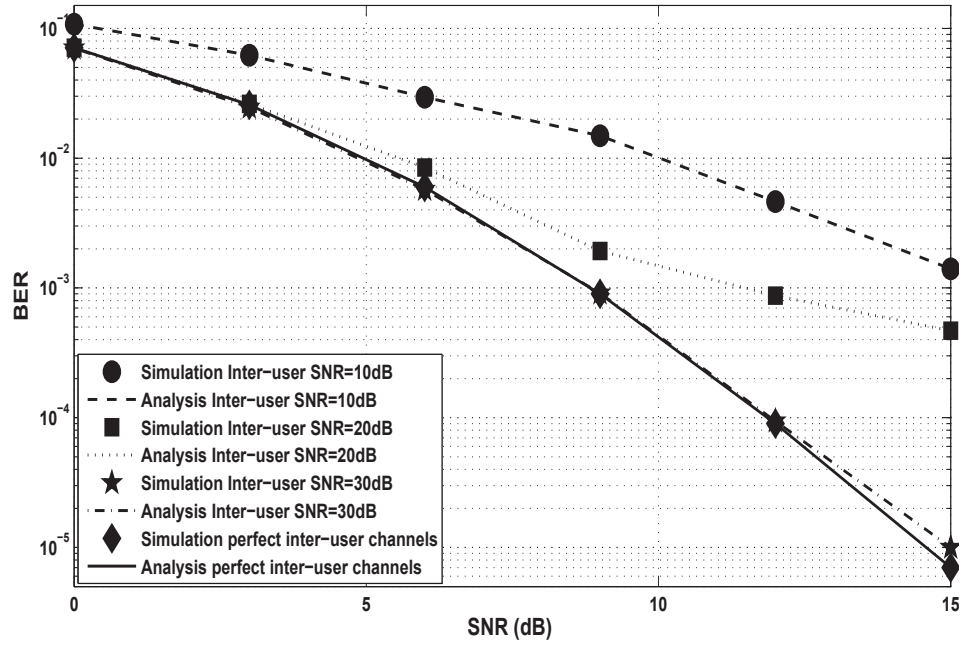


Figure 3.14: BER as a function of inter-user channel with  $m = 2$ .

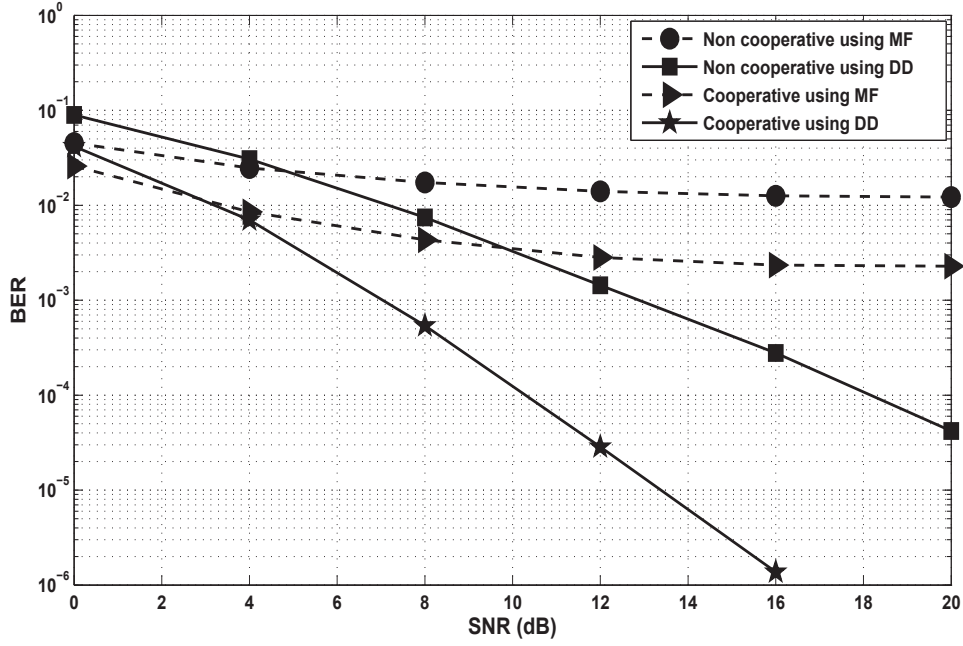


Figure 3.15: BER performance of an asynchronous DS-CDMA system over 2 paths using DD and MF detectors.

while in the case of high inter-user SNR, the performance converges to the case of perfect inter-user channel. Also, Figs. 3.11-3.14 show the perfect matching between the analysis and the simulation results. As a special case, we tested our system in the case of  $m_{kb}^p = m_{rb}^p = m_{kr}^p = 1$  which results in the Rayleigh fading channel case. Fig. 3.15 shows the performance of both cooperative and non-cooperative schemes using matched filter and decorrelator detector. The results in this figure show that the using MUD achieves full diversity while the matched filter saturates after a certain SNR. These results also show the effect of multi-user interference on the system performance. That is, the diversity advantage of the cooperative system cannot be reached without mitigating the effect of multiuser interference.

### 3.4.4 Conclusion

We have investigated the performance of the cooperative diversity in a DS-CDMA system under the diversity combining of the relayed information at the base station over

Nakagami- $m$  fading channels. The cooperative system studied employ MUD to combat the effect of multi-user interference at both the base station and the relay sides. An exact closed-form expression for the outage probability of the DF cooperative DS-CDMA has been derived for both multi-relay and single-relay scenario and different fading parameters. Also, an exact probability of bit error has been derived under different system setting, such as the effect of different fading parameter, inter-user channels error, different multipath channels. From the outage probability and bit error rate we evaluated the overall system diversity under different system settings. We showed that the system is able to achieve the full diversity gain by combating the effect of multi-user interference.



## Chapter 4

# Performance Analysis of Cooperative CDMA Systems using AF Relaying

In the previous chapter, we conducted performance analysis of cooperative CDMA systems employing DF relaying strategy. In this chapter we continue our investigation for the case of AF relaying as a simpler scheme. No decoding of the received signal is performed in this case which leads to less complexity at the relay side. Therefore, in the underlying AF relaying scheme, we analyze the outage probability and the BER of the system using the MGF approach. We assess our numerical results with the Mont-Carlo simulations.

### 4.1 Introduction

The performance of multiuser AF relay networks has been studied in [12] where the authors derived an approximation for the outage probability over Rayleigh fading channels. The joint iterative power allocation and the interference cancellation for DS-CDMA systems using AF relaying was analyzed in [38]. In their work, the authors determined the required power level for a multi-relay system. In [44], performance analysis of incremental-relaying cooperative-diversity networks over Rayleigh fading channels is presented where a complete analytical method is introduced to obtain closed-form expressions for the er-

ror rate, outage probability and the average achievable rate using DF and AF over i.i.d Rayleigh fading channels.

In [84], a performance analysis of TDMA relay protocols over i.i.d Nakagami- $m$  fading channels is presented. The authors in [84] considered Alamouti-coded system with two-stage protocols and fixed gain AF. The distributed power control and opportunistic power control was investigated in [85]. Wherein multi-relays, multi-sources, and one destination over AF cooperative relaying are considered. Along the same lines, the performance of differential Chaos shift keying cooperative CDMA was analyzed in [86]. In [86], a single-relay cooperation with DF relaying over Rayleigh fading channels was considered.

Here and different from previous works, we analyze the outage probability of the system using the CDF of the total SNR at the base station. In that, we derive a simplified yet tight lower bound for the relaying system. In addition, we investigate the outage probability of the system using the MGF of the total SNR at the base station. Since it is complicated to derive a closed-form expression for the outage probability, we derive an approximation for the PDF of the total SNR which in turn enables us to find the asymptotic outage probability for any fading parameter  $m$ . As a second part of this chapter, we study the error performance of asynchronous cooperative CDMA system using AF Nakagami- $m$  fading channels. In that, we derive the BER of the system using the MGF of the total SNR at the base station. Since it is intractable to derive a closed-form expression of the PDF of the total SNR, we derive an upper bound for the PDF of the total SNR which enables us to derive a closed-form expression for the BER. Our results show that the system diversity gain is achieved when MUD is used to combat the effect of MAI.

## 4.2 System Model

The system model employed here is the same as in Section 3.2, where the cooperation involves two phases; in the first phase, each user transmits its own modulated data to the base station and to  $L$  relays. During this phase, the received signal at both base station and relay are defined as in (3.3) and (3.4). While in the second phase, each cooperating user transmits the amplified version of the received signal to the base station, expressed as

$$r_b^{II}(t) = \sum_{i=0}^{f-1} \sum_{s=1}^K \sum_{\ell=1}^L r_{r_\ell}(t - D_{s,r_\ell} - \tau_{r_\ell} - i\ell T_b) h_{r_\ell b} \cdot \beta + n_b^{II}(t) \quad (4.1)$$

where  $r_\ell$  is the  $\ell^{th}$  relay cooperating with user  $s$ ,  $n_b^{II}(t)$ ,  $D_{s,\ell}$  are defined in Section 2.3.  $h_{r_\ell b}$  is the channel coefficient between user  $r_\ell$  and the base station, modeled as a Nakagami- $m$  random variable (RV),  $\beta$  is the amplification factor for the AF scheme at the relay node, expressed as [5]

$$\beta = \sqrt{\frac{E_s}{E_s |h_{sr_\ell}|^2 + N_o}}, \quad (4.2)$$

where  $E_s$  is the transmitted signal energy and fixed gain AF is assumed. The independent fading channel coefficients between users and base station are characterized the same as in Section 2.3.

The output of the matched filters at the base station receiver and relays, can be defined in vector form for the first phase I as in (3.49). Then, the output of the bank of matched filters at the relay and base station are respectively expressed as,

$$\mathbf{y}_{r_\ell} = \mathbf{R}_r \mathbf{H}_{s,r_\ell} \mathbf{x} + \mathbf{n}_{r_\ell}, \quad (4.3)$$

$$\mathbf{y}_{b2} = \mathbf{R}_b \mathbf{H}_{r_\ell,b}(\mathbf{y}_{r_\ell}) \beta + \mathbf{n}_{b2}, \quad (4.4)$$

where  $\mathbf{R}_r$ ,  $H_{s,r_\ell}$ ,  $\mathbf{H}_{r_\ell,b}$  are defined as in (3.54), (3.53) and (3.53) respectively. Applying a

decorrelator multi-user detector at the base station, we get

$$\mathbf{z}_{b1} = (\mathbf{R}_b^{-1})\mathbf{y}_{b1} = \mathbf{H}_{s,b}\mathbf{x} + \mathbf{R}_b^{-1}\mathbf{n}_{b1}, \quad (4.5)$$

Note that, we consider AF scheme in the sense that no hard decisions are made at the relay. However, to mitigate the effect of interference at the relay, we employ MUD using the decorrelator detector where the soft output of the detector is amplified before retransmission. Therefore, the soft output of the decorrelator detector, before amplification, is given by

$$\mathbf{z}_{r\ell} = (\mathbf{R}_r^{-1})\mathbf{y}_{r\ell} = \mathbf{H}_{s,r\ell}\mathbf{x} + \mathbf{R}_r^{-1}\mathbf{n}_{r\ell}. \quad (4.6)$$

Thus, the received signal at the base station in the second phase of relaying is given by

$$\mathbf{z}_{b2} = (\mathbf{R}_b^{-1})\mathbf{y}_{b2} = \mathbf{H}_{r\ell,b}\mathbf{H}_{s,r\ell}\beta\mathbf{x} + \mathbf{H}_{r\ell,b}\beta\mathbf{R}_r^{-1}\mathbf{n}_{r\ell} + \mathbf{R}_b^{-1}\mathbf{n}_{b2}. \quad (4.7)$$

Considering the  $i^{th}$  data bit of the  $s^{th}$  user,  $s \in \{1, \dots, K\}$ , the decorrelator output at the base station for the first and second transmission phases of (4.5) and (4.7) can be rewritten respectively as

$$[z_s(i)]_{b1} = h_{s,b}x_s(i) + N_{b1}, \quad (4.8)$$

$$[z_s(i)]_{b2} = h_{r\ell,b}h_{s,r\ell}\beta x_s(i) + h_{r\ell,b}\beta N_r + N_{b2}, \quad (4.9)$$

where  $[z_s(i)]_{b1}$  and  $[z_s(i)]_{b2}$  are the decision statistics correspond to the  $i^{th}$  bit for user  $s$ ,  $N_{b1} = [[\mathbf{R}_b^{-1}]_{s,s}\mathbf{n}_r]_s$ ,  $N_r = [[\mathbf{R}_r^{-1}]_{r,r}\mathbf{n}_r]_{r\ell}$ , and  $N_{b2} = [[\mathbf{R}_b^{-1}]_{s,s}\mathbf{n}_{b2}]_s$  are the  $s^{th}$  element in the corresponding vector.

### 4.3 Outage Probability Analysis

In AF, the relay (i.e., cooperative user) transmits an amplified version of the received partner's signal after linear filtering using a decorrelator detector as a multi-user detector. In this section, the average mutual information of AF relaying in DS-CDMA for the multi-relay cooperative system is derived.

Now, the average mutual information for AF CDMA transmission can be derived using a vector representation of (4.8) and (4.9) as [5]

$$\underbrace{\begin{bmatrix} [z_s(i)]_{b1} \\ [z_s(i)]_{b2} \end{bmatrix}}_{\mathbf{z}} = \underbrace{\begin{bmatrix} h_{s,b} \\ h_{r_\ell,b}\beta h_{s,r_\ell} \end{bmatrix}}_{\mathbf{h}} x_s(i) + \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ h_{r_\ell,b}\beta & 0 & 1 \end{bmatrix}}_{\mathbf{G}} \cdot \underbrace{\begin{bmatrix} N_r \\ N_{b1} \\ N_{b2} \end{bmatrix}}_{\mathbf{N}}. \quad (4.10)$$

Since the channel is assumed to be memoryless, the average mutual information  $I_{AF}$  can be defined as [5]

$$I_{AF} \leq I(x, z) \leq \log_2 \det \left( I + (E_s h h^H) (G \mathbb{E} \langle N N^H \rangle G^H)^{-1} \right), \quad (4.11)$$

where the covariance of the noise  $\mathbb{E} \langle N N^H \rangle = \text{diag} \left( N_o [\mathbf{R}_r^{-1}]_{r,r}, N_o [\mathbf{R}_b^{-1}]_{s,s}, N_o [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell} \right)$  [17]. Then,

$$h h^H = \begin{bmatrix} |h_{s,b}|^2 & h_{s,b} h_{r_\ell,b}^H \beta h_{s,r_\ell}^H \\ h_{s,b}^H h_{r_\ell,b} \beta h_{s,r_\ell} & |h_{r_\ell,b}|^2 \beta^2 |h_{s,r_\ell}|^2 \end{bmatrix}$$

$$(G \mathbb{E} \langle N N^H \rangle G^H)^{-1} =$$

$$= \begin{bmatrix} \frac{1}{N_o [\mathbf{R}_b^{-1}]_{s,s}} & 0 \\ 0 & \frac{1}{|h_{r_\ell,b}|^2 \beta^2 N_o [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell} + N_o [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}} \end{bmatrix},$$

and  $\det$  of (4.11) calculated as

$$\begin{aligned} & \det \left( I + (E_s h h^H) (G \mathbb{E} \langle N N^H \rangle G^H)^{-1} \right) = \\ & = 1 + \frac{E_s |h_{s,b}|^2}{N_o [\mathbf{R}_b^{-1}]_{s,s}} \frac{E_s |h_{r_\ell,b}|^2 \beta^2 |h_{s,r_\ell}|^2}{|h_{r_\ell,b}|^2 \beta^2 N_o [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell} + N_o [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}}. \end{aligned} \quad (4.12)$$

Substituting (4.2) into (4.12), and after many algebraic manipulations, we have

$$\begin{aligned} & \det \left( I + (E_s h h^H) (G \mathbb{E} \langle N N^H \rangle G^H)^{-1} \right) \\ & = 1 + \frac{E_s |h_{s,b}|^2}{N_o [\mathbf{R}_b^{-1}]_{s,s}} + \frac{\frac{E_s |h_{r_\ell,b}|^2}{N_o [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}} \cdot \frac{E_s |h_{s,r_\ell}|^2}{N_o [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}}}{\frac{E_s |h_{r_\ell,b}|^2}{N_o [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}} + \frac{E_s |h_{s,r_\ell}|^2}{N_o [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}} + 1}. \end{aligned} \quad (4.13)$$

It is shown that (4.13) is increasing in  $\beta$ , so the amplification factor for the AF in (4.2) should be active [5]. After substitution and algebraic manipulations, using  $\gamma_{ij} = |h_{ij}|^2 \frac{E_s}{N_o}$ , and the normalized discrete-power constraint as in [17], the multi-relay scenario  $I_{AF}$  finally can be written as

$$I_{AF} = \frac{K}{2N} \log_2 \left( 1 + \frac{2N\gamma_{s,b}}{K^2 [\mathbf{R}_b^{-1}]_{s,s}} + \sum_{\ell=1}^L \frac{\frac{2N\gamma_{s,r_\ell}}{K^2 [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}} \cdot \frac{2N\gamma_{r_\ell,b}}{K^2 [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}}}{\frac{2N\gamma_{s,r_\ell}}{K^2 [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}} + \frac{2N\gamma_{r_\ell,b}}{K^2 [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}} + 1} \right). \quad (4.14)$$

For simplicity, we define  $X_{sb} = \frac{2N\gamma_{sb}}{K^2 [\mathbf{R}_b^{-1}]_{s,s}}$ ,  $X_{sr_\ell} = \frac{2N\gamma_{s,r_\ell}}{K^2 [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}}$ , and  $X_{r_\ell b} = \frac{2N\gamma_{r_\ell b}}{K^2 [\mathbf{R}_r^{-1}]_{r_\ell,r_\ell}}$ , where the total SNR at the base station

$$X_{total} = X_{sb} + \sum_{\ell=1}^L \frac{X_{sr_\ell} X_{r_\ell b}}{X_{sr_\ell} + X_{r_\ell b} + 1}. \quad (4.15)$$

As one can see from (4.14) that in AF the mutual information depends on the weakest link, while in the case of DF the mutual information depends on the relay-destination link as in (3.12).

### 4.3.1 Numerical Bound

Eq. (4.15) should be reformulated in a more mathematical form in order to ease the computation of the PDF of the total SNR. Using the upper bound introduced in [58], we have

$$X_{total} \leq X_{up} = X_{sb} + \sum_{\ell=1}^L \min(X_{sr_\ell}, X_{rb_\ell}) = X_{sb} + \sum_{\ell=1}^L X_\ell, \quad (4.16)$$

where  $X_\ell = \min(X_{sr_\ell}, X_{rb_\ell})$ , and the approximated SNR value  $X_{up}$  is analytically more tractable than the exact expression in (4.15). This approximation, introduced in [58], is shown to be accurate in medium and high SNRs. Also it shows to simplify the derivation of the outage probability. Accordingly, we start by deriving the CDF of the indirect link in the case of single-relay. Given the CDF of  $X_{rb}$  defined in (3.2), the CDF of  $X_\ell$  is given by

$$\begin{aligned} F_{\min(X_{sr}, X_{rb})}(\gamma_{th}) &= 1 - P[X_{sr} > \gamma_{th} \text{ and } X_{rb} > \gamma_{th}] \\ &= 1 - \frac{\Gamma(m_{sr}, B_{sr}\gamma_{th})}{\Gamma(m_{sr})} \frac{\Gamma(m_{rb}, B_{rb}\gamma_{th})}{\Gamma(m_{rb})}. \end{aligned} \quad (4.17)$$

After some manipulation, the CDF of the sum of gamma RVs is derived as shown in Appendix B and expressed as

$$\begin{aligned} F_{X_{total}}(y) &= \frac{\gamma(m_{sb}, \gamma B_{sb})}{\Gamma(m_{sb})} - \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} \exp(-y(B_{sr} + B_{rb})) \int_0^y \left( x^{(m_{sb}-1)} \right. \\ &\times \exp(-x(B_{sb} - B_{sr} - B_{rb})) \sum_{n=0}^{m_{sr}-1} \frac{(B_{sr})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} x^k \\ &\times \left. \sum_{i=0}^{m_{rb}-1} \frac{(B_{rb})^i}{i!} \sum_{f=0}^i \binom{i}{f} (-1)^f y^{i-f} x^f \right) dx. \end{aligned} \quad (4.18)$$

Finally, the  $P_{out}$  for the AF scheme can be determined numerically by solving the integral in (4.18) using mathematical software such as Maple and Mathematica. In the case of Nakagami- $m$  fading channels, the closed-form expression for the PDF of the total SNR

is intractable to derive. In what follows, we provide a closed-form expression for the asymptotic outage probability.

### 4.3.2 Approximate Distribution of the Total SNR

The asymptotic outage probability is conventionally evaluated using the PDF of the  $s \rightarrow b$  and  $s \rightarrow r \rightarrow b$  links. Due to the intractability of the PDF of SNR,  $X_{total}$ , given in (4.15), it becomes difficult to obtain a closed-form expression for the outage probability in the AF case. Alternatively, we introduce an approximation for the PDF of  $X_{total}$  at high SNRs. This approximation will enable us to find a closed-form expression for the outage probability of the underlying system. In the following section, the accuracy of our analytical results will be assessed through some simulation examples.

At high SNR, (3.1) can be approximated as

$$p_{X_{ij}}(\gamma) \approx \frac{B_{ij}^{m_{ij}}}{\Gamma(m_{ij})} \gamma^{m_{ij}-1} + H.O., \quad (4.19)$$

where H.O. stands for high order terms. Applying Laplace transform to (4.19) and with the help of [73], we have

$$M_{X_{ij}}(s) = \frac{B_{ij}^{m_{ij}}}{s^{m_{ij}}}. \quad (4.20)$$

In the case of indirect link, let  $\Psi_\ell, \ell = 1, 2, \dots, L$  be non-negative RV and let the PDF of  $\Psi_\ell$  be approximated as in (4.19). Let us also define  $Y = \sum_{\ell=1}^L \Psi_\ell$ , then

$$M_Y(s) = \prod_{\ell=1}^L M_{\Psi_\ell}(s). \quad (4.21)$$

Using (4.21), we have

$$M_Y(s) = \frac{\prod_{\ell=1}^L B_\ell^{m_\ell}}{s^{\sum_{\ell=1}^L m_\ell}} \quad (4.22)$$

and by applying the inverse Laplace transform,  $\mathcal{L}^{-1}\{M_Y(s); y\}$ , the PDF of  $Y$  can be



written as

$$f_Y(y) = \frac{y^{(\sum_{\ell=1}^L m_{\ell}-1)}}{\Gamma(\sum_{\ell=1}^L m_{\ell})} \prod_{\ell=1}^L B_{\ell}^{m_{\ell}}. \quad (4.23)$$

Using the method in [58], the PDF of the approximate distribution for the  $s \rightarrow r_{\ell} \rightarrow b$  link can be written as

$$f_{X_{s,r_{\ell},b}}(\gamma) \approx f_{X_{sr_{\ell}}}(\gamma) + f_{X_{r_{\ell}b}}(\gamma), \quad (4.24)$$

where  $f_{X_{sr_{\ell}}}(\gamma)$ , and  $f_{X_{r_{\ell}b}}(\gamma)$  can be approximated at high SNR as in (5.1), yielding to

$$f_{X_{sr_{\ell}}}(\gamma) \approx \frac{B_{sr_{\ell}}^{m_{sr_{\ell}}}}{\Gamma(m_{sr_{\ell}})} \gamma^{m_{sr_{\ell}}-1} + H.O., \quad (4.25)$$

$$f_{X_{r_{\ell}b}}(\gamma) \approx \frac{B_{r_{\ell}b}^{m_{r_{\ell}b}}}{\Gamma(m_{r_{\ell}b})} \gamma^{m_{r_{\ell}b}-1} + H.O. \quad (4.26)$$

Examining (4.24) when  $E_s/N_o \rightarrow \infty$ ,  $f_{X_{s,r,b}}(\gamma)$  can be simplified to

$$f_{X_{s,r_{\ell},b}}(\gamma) \approx \Omega_{\ell} \gamma^{m_{r_{\ell}}-1} + H, \ell = 1, 2, \dots, L, \quad (4.27)$$

where  $m_{r_{\ell}} = \min(m_{sr_{\ell}}, m_{r_{\ell}b})$  and

$$\Omega_{\ell} = \begin{cases} \frac{B_{sr_{\ell}}^{m_{sr_{\ell}}}}{\Gamma(m_{r_{\ell}b})} & m_{sr_{\ell}} < m_{r_{\ell}b} \\ \frac{B_{r_{\ell}b}^{m_{r_{\ell}b}}}{\Gamma(m_{r_{\ell}b})} & m_{sr_{\ell}} > m_{r_{\ell}b} \\ \frac{1}{\Gamma(m_{r_{\ell}})} (B_{sr_{\ell}}^{m_{sr_{\ell}}} + B_{r_{\ell}b}^{m_{r_{\ell}}}) & m_{sr_{\ell}} = m_{r_{\ell}b} = m_{r_{\ell}}. \end{cases} \quad (4.28)$$

Thus, the total PDF of the approximated SNR is given by

$$f_{X_{AF}}(\gamma) \approx f_{X_{s,b}}(\gamma) + f_{X_{s,r_{\ell},b}}(\gamma), \quad (4.29)$$

and the corresponding MGF is given by

$$M_{X_{AF}}(s) = M_{X_{sb}}(s) \cdot M_{X_{s,r_\ell,b}}(s). \quad (4.30)$$

Now, using (4.20) and (4.22), we have

$$M_{X_{AF}}(s) = \frac{B_{sb}^{m_{sb}} \prod_{\ell=1}^L \Omega_\ell \Gamma(m_{r_\ell})}{s^{(m_{sb} + \sum_{\ell=1}^L m_{r_\ell})}}. \quad (4.31)$$

Finally, the asymptotic outage probability  $P_{out}$  of the cooperative CDMA system using AF relaying is given by

$$P_{out} = \mathcal{L}^{-1} \left\{ \frac{M_{X_{AF}}(s)}{s}; t \right\}_{|t=\gamma_{th}} \quad (4.32)$$

$$P_{out} = B_{sb}^{m_{sb}} \prod_{\ell=1}^L \Omega_\ell \Gamma(m_{r_\ell}) \cdot \frac{\gamma_{th}^{(m_{sb} + \sum_{\ell=1}^L m_{r_\ell})}}{\Gamma(m_{sb} + \sum_{\ell=1}^L m_{r_\ell} + 1)}. \quad (4.33)$$

Examining (4.33), it is easy to see that the achieved diversity order is  $(m_{sb} + \sum_{\ell=1}^L m_{r_\ell})$ .

### 4.3.3 Simulation and Numerical Results

We present some numerical results for the outage probability of multi-relay cooperation system. We build a Mont-Carlo link-level simulation to verify these results with the analytical model derived. We assume asynchronous cooperative DS-CDMA where every user data are spread using non-orthogonal gold codes of length  $N = 31$  chips and number of users  $K = 16$  with different number of relays  $L = 0, 1, 2, 3$  over independent non-identical Nakagami- $m$  fading channels and BPSK transmission. The channels are modeled as block fading channels where the fading coefficients are considered fixed for the duration of one frame  $f = 100$  and change independently from one frame to another. Unless otherwise mentioned, a decorrelator detector is used to mitigate the effect of MAI at both the relay and base station receivers.

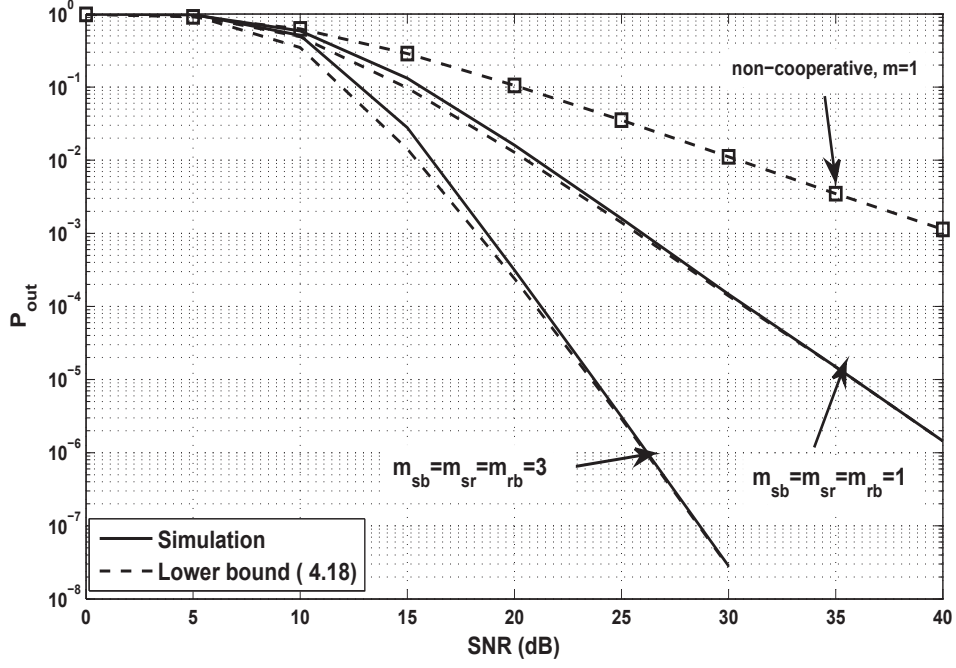


Figure 4.1: Outage probability for cooperative AF DS-CDMA over i.i.d Nakagami- $m$  fading channel.

Let us first assess the accuracy of the outage probability bound obtained in (4.18). Fig. 4.1 shows  $P_{out}$  for both non-cooperative and single-relay,  $L = 1$ , cooperative AF relaying over i.i.d Nakagami- $m$  fading channel. It is clear that the outage probability derived is tight at medium and high SNR. As shown in Fig. 4.1, the AF cooperative CDMA system offers a considerable outage probability gain compared with the non-cooperative system. In Fig. 4.2, we present the outage probability of the AF scheme over i.n.i Nakagami- $m$  fading channel with  $L = 1$ . As noted, when  $m_{ij}$  increases the system performance is improved. In Fig. 4.1 and Fig. 4.2, one can see the improved accuracy of the derived lower bound given by (4.15) for the outage probability as the SNR increases.

Fig. 4.3 shows the outage probability for the cooperative DS-CDMA system using AF relaying over i.n.i Nakagami- $m$  fading channel with different number of relays  $L = 0, 1, 2, 3$ . The matching between the analytical (eq. (4.18)) and simulation results at medium and high SNR is obvious from this figure. At low SNRs, the derived outage probability tends to deviate from the exact results when the number of relays is larger than one. Now

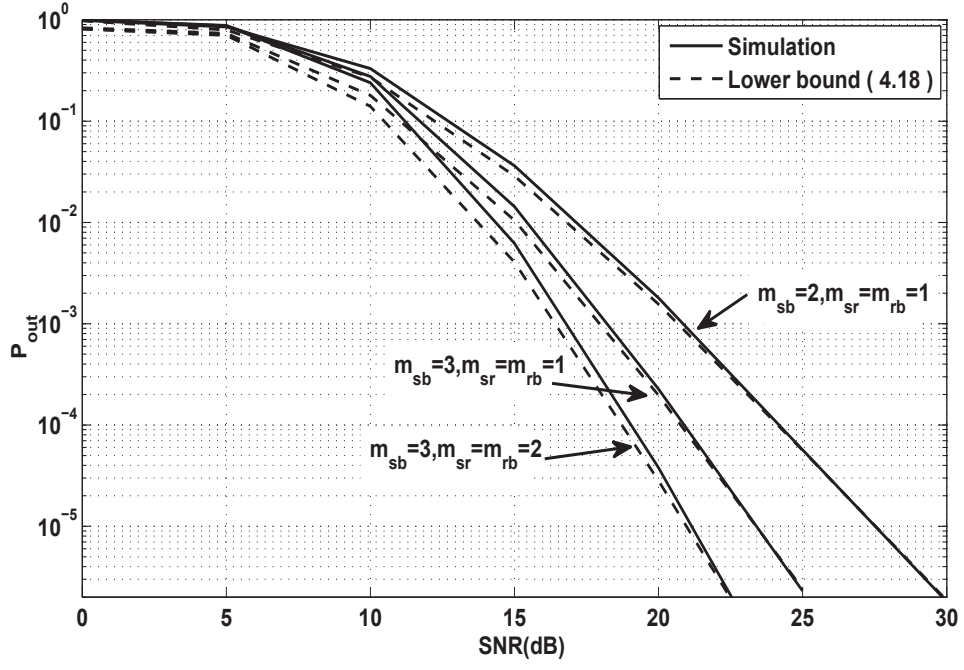


Figure 4.2: Outage probability for cooperative AF DS-CDMA with different fading figures  $m$  and considering i.n.i Nakagami fading channels.

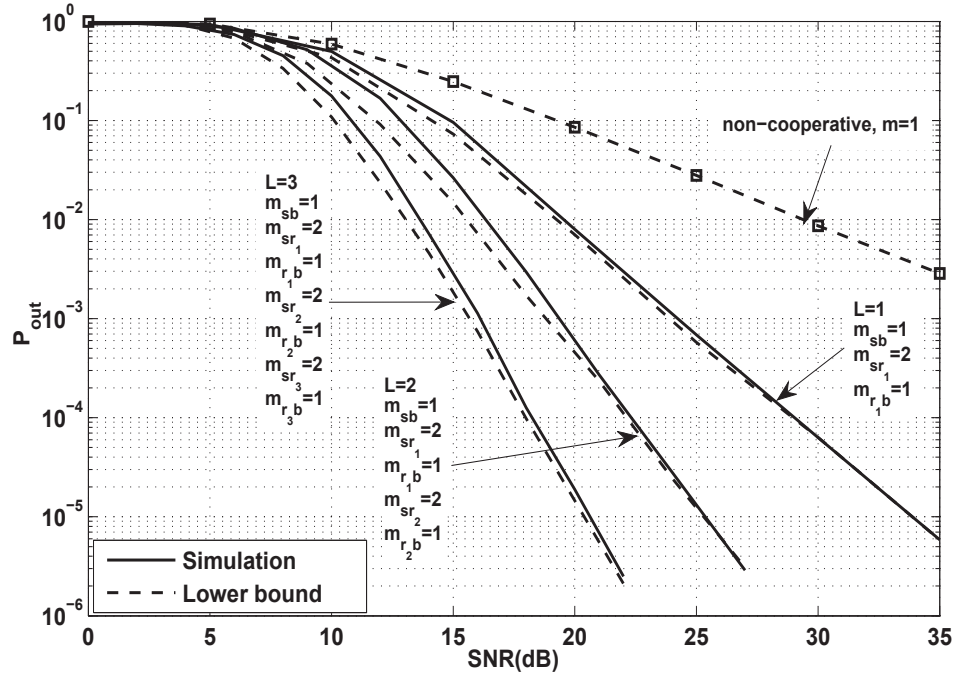


Figure 4.3: Performance of multi-relay cooperative system with different  $L$  over independent non identical Nakagami- $m$  fading channels.

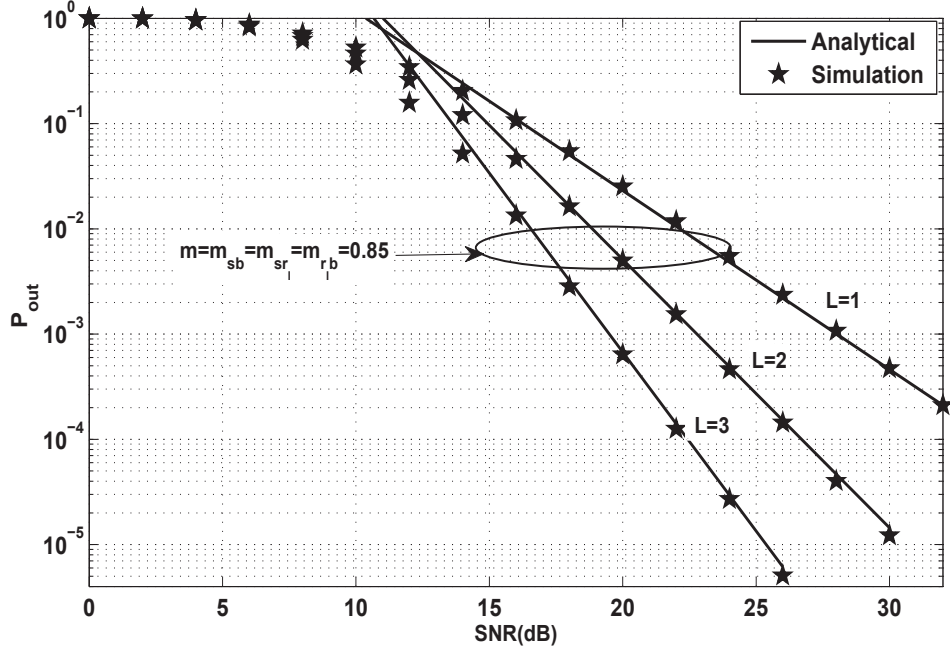


Figure 4.4: Outage probability for cooperative AF DS-CDMA with different  $L$  over i.i.d. Nakagami- $m$  fading channels, analytical based on the approximate outage probability in (4.33).

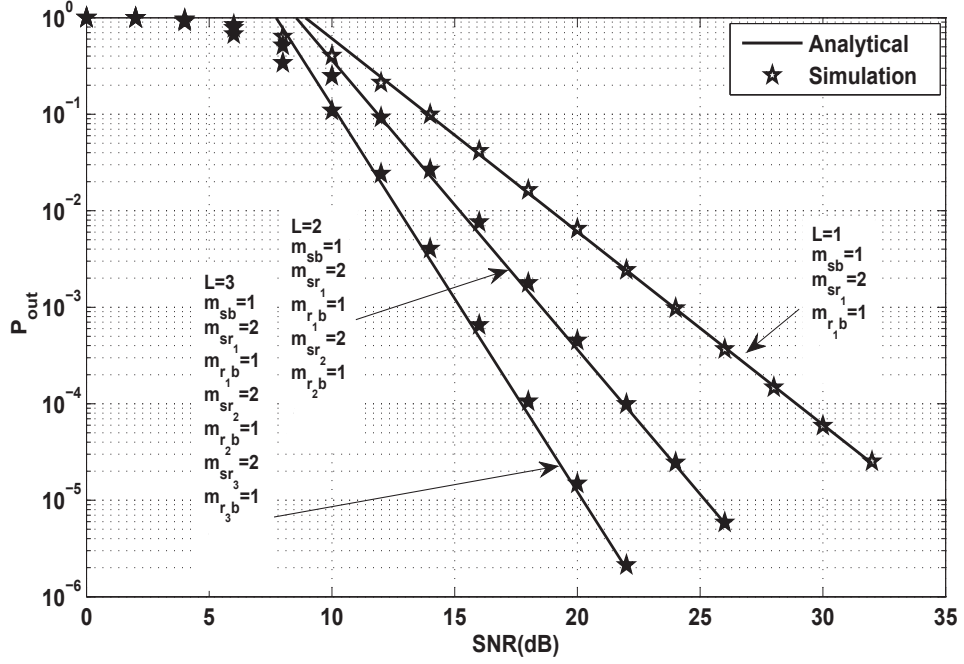


Figure 4.5: Outage probability for cooperative AF DS-CDMA with different  $L$  over independent non-identical Nakagami- $m$  fading channels.

we present results for our approximate closed-form expression of the outage probability derived in (4.33). Figs. 4.4 and 4.5 show the asymptotic outage probability of the AF system over i.i.d. (i.e.,  $m_{sb} = m_{sr_\ell} = m_{r_\ell b}$ ) and i.in.d. (i.e.,  $m_{sb} \neq m_{sr_\ell} \neq m_{r_\ell b}$ ) Nakagami- $m$  fading channels, respectively. The system performance is examined as a function of the fading parameter  $m_{ij}$  and number of cooperating relays  $L$ . It is noticeable from the results that the asymptotic  $P_{out}$  and simulation results are in excellent agreement at medium and high SNRs. Also from these results one can notice the improvement in diversity order as a function of the number of relays  $L$  and fading parameter  $m_{ij}$ . We also notice that the cooperative system guarantees large diversity advantages provided that interference cancellation is performed at both the relay and base station sides.

## 4.4 BER Analysis

Here, we analyze the BER performance of the AF relaying system over Nakagami- $m$  fading channels. Following the DF case in Chapter 3, here we investigate the BER performance for the AF case. To mitigate the effect of MAI we use a decorrelator detector where three scenarios are considered: (i) in the first system, namely multi-user detection at base station *MUD-b*, where MUD is only applied at the base station, (ii) *MUD-r-b* scenario, where MUD is applied at both the relay and the base station, and (iii) Conventional detection, referred as *conventional r-b*, where only matched filter detection is employed at the relay and base station.

### 4.4.1 System Model

The system model is described as in Section 3.4.1, wherein the cooperation among any two cooperating users occurs at two consecutive phases; in the first phase, each user transmits its own DS-CDMA data to the base station and to its cooperating partner. Through this phase, the received signal at the base station and the relay are defined in (3.46)

and (3.47) respectively. During the second phase, each cooperating user transmits the amplified version of the received signal to the base station, which is expressed as

$$r_{b2}(t) = \sum_{i=0}^{f-1} \sum_{s=1}^K \sum_{p=1}^P r_r(t - \tau_r - D_s - \tau_{r,p} - iT_b) h_{rb}^p \cdot \beta + n_{b2}(t). \quad (4.34)$$

#### 4.4.2 Total SNR of the system

In this section, we derive the total SNR of the AF relaying system where we investigate the impact of MAI. The total SNR is derived for two scenarios:

##### 4.4.2.1 MUD at the base station only

For the purpose of comparison, we study the effect of employing a simple relay receiver in order to reduce the complexity at the relay side. Therefore, in this subsection, we use MUD at the base station only.

The output of the matched filters bank at the base station receiver and relay is defined in vector form for the first phase as

$$\mathbf{y}_{b1} = \mathbf{R}_b \mathbf{H}_{s,b} \mathbf{x} + \mathbf{n}_{b1}, \quad (4.35)$$

$$\mathbf{y}_r = \mathbf{H}_{s,r} \mathbf{x} + \mathbf{n}_r, \quad (4.36)$$

$$\mathbf{y}_{b2} = \mathbf{R}_b \mathbf{H}_{r,b} (\mathbf{y}_r) \beta + \mathbf{n}_{b2}, \quad (4.37)$$

Employing a decorrelator detector at the base station, we have

$$\mathbf{z}_{b1} = (\mathbf{R}_b^{-1}) \mathbf{y}_{b1} = \mathbf{H}_{s,b} \mathbf{x} + \mathbf{R}_b^{-1} \mathbf{n}_{b1}. \quad (4.38)$$

Thus, from (4.37), as one can see that at the relay side, the received signal is amplified only then retransmitted to the base station. The output at the base station in the second

phase is also given by

$$\mathbf{z}_{b2} = (\mathbf{R}_b^{-1})\mathbf{y}_{b2} = \mathbf{H}_{r,b}\mathbf{H}_{s,r}\beta\mathbf{x} + \mathbf{H}_{r,b}\beta\mathbf{n}_r + \mathbf{R}_b^{-1}\mathbf{n}_{b2}. \quad (4.39)$$

Considering the  $i^{th}$  data bit of the  $s^{th}$  user,  $s \in \{1, \dots, K\}$ , the decorrelator output at the base station for the first and second transmission phases of (4.38) and (3.56) can be rewritten respectively as

$$[\mathbf{z}_1]_{b1} = \mathbf{H}_{s,b}[\mathbf{x}_1]_{b1} + \mathbf{R}_b^{-1}[\mathbf{n}_1]_{b1}, \quad (4.40)$$

$$[\mathbf{z}_1]_{b2} = \mathbf{H}_{r,b}\mathbf{H}_{s,r}\beta[\mathbf{x}_1]_{b2} + \mathbf{H}_{r,b}\beta[\mathbf{n}_1]_r + \mathbf{R}_b^{-1}[\mathbf{n}_1]_{b2}. \quad (4.41)$$

The  $P$  elements of  $[\mathbf{z}_1]_{b1}$  belonging to the decision statistics of bit  $i$  for user 1 before RAKE combining, during the first phase are given by

$$\begin{aligned} [z_1^1(i)]_{b1} &= \sqrt{E_s}h_{s,b}^1(i)x_1(i) + [\mathbf{R}_b^{-1}\mathbf{n}_{b1}]^1 \\ &\vdots \\ [z_1^P(i)]_{b1} &= \sqrt{E_s}h_{s,b}^P(i)x_1(i) + [\mathbf{R}_b^{-1}\mathbf{n}_{b1}]^P, \end{aligned} \quad (4.42)$$

while the  $P$  elements of  $[\mathbf{z}_1]_{b2}$  is the decision statistics of bit  $i$  for user 1 before RAKE combining, in the second phase

$$\begin{aligned} [z_1^1(i)]_{b2} &= \sqrt{E_s}h_{r,b}^1(i)h_{s,r}^1(i)x_1(i)\beta + [h_{r,b}^1(i)\beta\mathbf{n}_r]^1 + [\mathbf{R}_b^{-1}\mathbf{n}_{b2}]^1 \\ &\vdots \\ [z_1^P(i)]_{b2} &= \sqrt{E_s}h_{r,b}^P(i)h_{s,r}^P(i)x_1(i)\beta + [h_{r,b}^P(i)\beta\mathbf{n}_r]^P + [\mathbf{R}_b^{-1}\mathbf{n}_{b2}]^P. \end{aligned} \quad (4.43)$$



By merging (4.42) and (4.43), the decision statistic of the desired user signal is given by

$$x_1(i) = h_{s,b}^{1*}(i)[z_1^1(i)]_{b1} + \dots + h_{s,b}^{P*}(i)[z_1^P(i)]_{b1} + h_{r,b}^{1*}(i)[z_1^1(i)]_{b2} + \dots + h_{r,b}^{P*}(i)[z_1^P(i)]_{b2}. \quad (4.44)$$

In order to obtain the error probability, we have to find the total SNR at the base station output. Since we assume that the MRC technique is used at the base station, the instantaneous output SNR,  $\gamma_{AF}$ , is the sum of the instantaneous SNRs of the direct and indirect links which can be expressed as

$$\gamma_{AF} = \text{Direct SNR} + \text{Indirect SNR}$$

$$\gamma_{AF} = \sum_{p=1}^P \frac{E_s |h_{sb}^p|^2}{N_o (R_b)_{s,s}^{-2}} + \sum_{p=1}^P \frac{E_s |h_{rb}^p|^2 \beta^2 |h_{sr}^p|^2}{|h_{rb}^p|^2 \beta^2 N_o + N_o (R_b)_{s,s}^{-2}}. \quad (4.45)$$

Thus

$$\begin{aligned} \gamma_{AF} &= \sum_{p=1}^P \frac{E_s |h_{sb}^p|^2}{N_o (R_b)_{s,s}^{-2}} + \sum_{p=1}^P \frac{E_s^2 |h_{rb}^p|^2 |h_{sr}^p|^2}{E_s |h_{rb}^p|^2 N_o + N_o (R_b)_{s,s}^{-2} (E_s |h_{sr}^p|^2 + N_o)} \\ &= \sum_{p=1}^P \frac{E_s |h_{sb}^p|^2}{N_o (R_b)_{s,s}^{-2}} + \sum_{p=1}^P \frac{E_s^2 |h_{rb}^p|^2 |h_{sr}^p|^2}{E_s |h_{rb}^p|^2 N_o + E_s |h_{sr}^p|^2 N_o (R_b)_{s,s}^{-2} + N_o^2 (R_b)_{s,s}^{-2}} \end{aligned} \quad (4.46)$$

where  $(R_b)_{s,s}^{-2}$  represents the square of the sum of the  $s^{th}$  row in the inverse of the cross-correlation matrix [18]. By dividing (4.46) by  $N_o^2 (R_b)_{s,s}^{-2}$ , we get

$$\gamma_{AF} = \sum_{p=1}^P \frac{E_s |h_{sb}^p|^2}{N_o (R_b)_{s,s}^{-2}} + \sum_{p=1}^P \frac{\frac{E_s |h_{rb}^p|^2}{N_o (R_b)_{s,s}^{-2}} \frac{E_s |h_{sr}^p|^2}{N_o}}{\frac{E_s |h_{rb}^p|^2}{N_o (R_b)_{s,s}^{-2}} + \frac{E_s |h_{sr}^p|^2}{N_o} + 1}. \quad (4.47)$$

For simplicity, we define  $\gamma_{sb}^p = \frac{E_s |h_{sb}^p|^2}{N_o (R_b)_{s,s}^{-2}}$ ,  $\gamma_{sr}^p = \frac{E_s |h_{sr}^p|^2}{N_o}$ , and  $\gamma_{rb}^p = \frac{E_s |h_{rb}^p|^2}{N_o (R_b)_{s,s}^{-2}}$ , using

MUD at the base station only, the total SNR of the signal can be written as

$$\gamma_{total} = \sum_{p=1}^P \gamma_{sb}^p + \sum_{p=1}^P \frac{\gamma_{sr}^p \gamma_{rb}^p}{\gamma_{sr}^p + \gamma_{rb}^p + 1}. \quad (4.48)$$

Due to the intractability of (4.48), the approximated upper bound of the total SNR is employed and defined as [58],

$$\gamma_{total} \leq \gamma_{up} = \sum_{p=1}^P \gamma_{sb}^p + \sum_{p=1}^P \min(\gamma_{sr}^p, \gamma_{rb}^p). \quad (4.49)$$

Finally, the total SNR is defined simply as

$$\gamma_{up} \leq \sum_{p=1}^P \gamma_{sb}^p + \sum_{p=1}^P \gamma_{min}^p, \quad (4.50)$$

where  $\gamma_{min}^p = \min(\gamma_{sr}^p, \gamma_{rb}^p)$ .

#### 4.4.2.2 MUD at both relay and base station

The output of the bank of matched filters at the base station receiver and relay are represented in vector form by (3.49) and (3.50). Also, considering MUD at the relay side (i.e., no decoding is done), the soft data estimate at the decorrelator output is given by (3.56). Then, the relay will amplify and resend the resulting signal to the base station, where the amplification factor  $\beta$  can be defined as

$$\beta = \sqrt{\frac{E_s}{E_s |h_{sr}|^2 + N_o(R_r)^{-2}}}. \quad (4.51)$$

Thus, the second phase received signal at the base station can be represented as

$$\mathbf{y}_{b2} = \mathbf{R}_b \mathbf{H}_{r,b}(\mathbf{z}_r) \beta + \mathbf{n}_{b2} = \mathbf{R}_b \mathbf{H}_{r,b}(\mathbf{H}_{s,r} \mathbf{x} + \mathbf{R}_r^{-1} \mathbf{n}_r) \beta + \mathbf{n}_{b2}. \quad (4.52)$$

Considering the  $i^{th}$  data bit of the  $s^{th}$  user,  $s \in \{1, \dots, K\}$ , the decorrelator output at the base station for the first and second transmission phases of (4.38) and (3.56) can be rewritten respectively as

$$[\mathbf{z}_1]_{b1} = \mathbf{H}_{s,b}[\mathbf{x}_1]_{b1} + \mathbf{R}_b^{-1}[\mathbf{n}_1]_{b1}, \quad (4.53)$$

$$[\mathbf{z}_1]_{b2} = \mathbf{H}_{r,b}\mathbf{H}_{s,r}\beta[\mathbf{x}_1]_{b2} + \mathbf{H}_{r,b}\beta\mathbf{R}_r^{-1}[\mathbf{n}_1]_r + \mathbf{R}_b^{-1}[\mathbf{n}_1]_{b2}. \quad (4.54)$$

The total SNR is then defined as

$$\gamma_{AF} = \sum_{p=1}^P \frac{E_s |h_{sb}^p|^2}{N_o(R_b)_{s,s}^{-2}} + \sum_{p=1}^P \frac{\frac{E_s |h_{rb}^p|^2}{N_o(R_b)_{s,s}^{-2}} \frac{E_s |h_{sr}^p|^2}{N_o(R_r)_{r,r}^{-2}}}{\frac{E_s |h_{rb}^p|^2}{N_o(R_b)_{s,s}^{-2}} + \frac{E_s |h_{sr}^p|^2}{N_o(R_r)_{r,r}^{-2}} + 1} \quad (4.55)$$

For simplicity, we define  $\gamma_{sb}^p = \frac{E_s |h_{sb}^p|^2}{N_o(R_b)_{s,s}^{-2}}$ ,  $\gamma_{sr}^p = \frac{E_s |h_{sr}^p|^2}{N_o(R_r)_{r,r}^{-2}}$ , and  $\gamma_{rb}^p = \frac{E_s |h_{rb}^p|^2}{N_o(R_b)_{s,s}^{-2}}$ .

Finally, the total SNR at the base station

$$\gamma_{total} = \sum_{p=1}^P \gamma_{sb}^p + \sum_{p=1}^P \frac{\gamma_{sr}^p \gamma_{rb}^p}{\gamma_{sr}^p + \gamma_{rb}^p + 1}. \quad (4.56)$$

Using the approximated upper bound of the total SNR as in (4.49), we get

$$\gamma_{total} \leq \gamma_{up} = \sum_{p=1}^P \gamma_{sb}^p + \sum_{p=1}^P \min(\gamma_{sr}^p, \gamma_{rb}^p). \quad (4.57)$$

Then

$$\gamma_{up} = \sum_{p=1}^P \gamma_{sb}^p + \sum_{p=1}^P \gamma_{min}^p. \quad (4.58)$$

#### 4.4.3 BER

From the definition of gamma distribution, we have  $B_{ij} = \frac{m_{ij}}{\gamma_{ij}(R_{ij})^{-2}}$  with average SNR,  $\gamma_{ij} = \frac{\mathbb{E}\langle |h_{ij}|^2 \rangle E_s}{N_o}$ , and  $\mathbb{E}\langle \cdot \rangle$  denote expectation. Assuming independence between  $\gamma_{sb}$  and

$\gamma_{min}$ , the MGF of  $\gamma_{up}$  for (4.58) can be written as

$$M_{up}(s) = \prod_{p=1}^P M_{sb}^p(s) \prod_{p=1}^P M_{min}^p(s) = \prod_{p=1}^P M_{sb}^p(s) M_{min}^p(s), \quad (4.59)$$

where  $M_{sb}^p(s)$  is the MGF for the direct link and is given by

$$M_{sb}^p(s) = \left(1 + \frac{s}{B_{sb}^p}\right)^{-m_{sb}}. \quad (4.60)$$

Applying (4.60) into (4.59) we get

$$M_{up}(s) = \prod_{p=1}^P \left(1 + \frac{s}{B_{sb}^p}\right)^{-m_{sb}} \left(1 + \frac{s}{B_{min}^p}\right)^{-m_{min}}. \quad (4.61)$$

Assuming iid Nakagami- $m$  fading channels between the users themselves and the base station, (4.61) can be written as

$$M_{up}(s) = \prod_{p=1}^P \left(1 + \frac{s}{B_p}\right)^{-2m}. \quad (4.62)$$

Using [87], the partial fraction decomposition of the total MGF can be shown as

$$M_{up}(s) = \prod_{p=1}^P (B_p)^{2m} \sum_{p=1}^P \sum_{q=1}^{2m} \Lambda_{p,q} (s + B_p)^{-q}, \quad (4.63)$$

where

$$\Lambda_{p,q} = \frac{1}{(2m-q)!} \frac{d^{(2m-q)}}{ds^{(2m-q)}} \left[ \prod_{j=1, j \neq p}^P (s + B_j)^{-2m} \right]_{s=-B_p}. \quad (4.64)$$

Let us define  $A(x) = \prod_{j=1}^N a_j(x)$ , using [87], the  $g^{th}$  derivative of  $A(x)$  with respect to  $x$  is given by

$$\frac{d^g}{dx^g} A(x) = g! \sum_{\mathbb{O}} \prod_{j=1}^N \frac{1}{i_j!} \frac{d^{i_j}}{dx^{i_j}} a_j(x) \quad (4.65)$$

where  $\mathbb{O}$  is the set of nonnegative integers  $\{i_1, i_2, \dots, i_N\}$  such that  $\sum_{j=1}^N i_j = g$ . Thus, we can show that (4.64) can be written in closed-form as

$$\Lambda_{p,q} = (-1)^{2m-q} \sum_{\Upsilon} \prod_{j=1, j \neq p}^P \binom{2m_j + i_j - 1}{i_j} (B_j - B_p)^{-(2m_j + i_j)}, \quad (4.66)$$

where  $\Upsilon$  is the set of nonnegative integers  $\{i_1, i_2, \dots, i_{p-1}, i_{p+1}, \dots, i_P\}$  such that  $\sum_{p=1}^P i_p = 2m - q$ . Then, by taking the inverse Laplace transformation of (4.64), the PDF of the total SNR can be written with the help of [73] as

$$f(\gamma) = \prod_{p=1}^P (B_p)^{2m} \sum_{p=1}^P \sum_{q=1}^{2m} \frac{\Lambda_{p,q} \gamma^{(q-1)}}{(q-1)!} e^{-(B_p \gamma)}. \quad (4.67)$$

In what follows, the lower bound of the average BER is derived as

$$P_b = \int_0^\infty Q(\sqrt{\zeta \gamma}) f(\gamma) d\gamma \quad (4.68)$$

where  $\zeta$  is a constant that depends on the specific modulation type (i.e., for binary phase shift keying (BPSK)  $\zeta = 2$ ).

$$P_b = \prod_{p=1}^P (B_p)^{2m} \sum_{p=1}^P \sum_{q=1}^{2m} \frac{\Lambda_{p,q}}{(q-1)!} \int_0^\infty e^{-\gamma(B_p)} \gamma^{(q-1)} Q(\sqrt{\zeta \gamma}) d\gamma d\theta. \quad (4.69)$$

Using [88], we can simply solve (4.69) as

$$\mathbb{H}(a, b, c) = \int_0^\infty x^{a-1} e^{(-bx)} Q(\sqrt{cx}) dx, \quad (4.70)$$

where  $\mathbb{H}(a, b, c)$  has been derived in [88] for two cases as:

1. For a non integer case

$$\mathbb{H}(a, b, c) = \frac{\Gamma(a + \frac{1}{2}) 2^{a-1} \sqrt{c}}{a(c + 2b)^{a+\frac{1}{2}} \sqrt{\Pi}} {}_2F_1(1, a + \frac{1}{2}; a + 1; \frac{2b}{2b + c}) \quad (4.71)$$

2. for integer case

$$\mathbb{H}(a, b, c) = \frac{\Gamma(a)}{2b^a} \left[ 1 - \xi(b, c) \sum_{i=0}^{a-1} \binom{2i}{i} \left( \frac{1 - \xi(b, c)^2}{4} \right)^i \right], \quad (4.72)$$

where  $\xi(b, c)$  is defined as  $\xi(b, c) = \sqrt{\frac{c}{c+2b}}$ .

Thus, applying (4.71) in (4.69) for non integer  $m$  we get

$$P_b = \frac{1}{2\sqrt{\Pi}} \prod_{p=1}^P (B_p)^{2m} \sum_{p=1}^P \sum_{q=1}^{2m} \frac{\Lambda_{p,q} \Gamma(q + \frac{1}{2})}{q(q-1)!(1+B_p)^{q+\frac{1}{2}}} {}_2F_1(1, q + \frac{1}{2}; q+1; \frac{2B_p}{2B_p+2}) \quad (4.73)$$

Finally, the  $P_b$  for integer  $m$  and using (4.72) is given by

$$P_b = \prod_{p=1}^P (B_p)^{2m} \sum_{p=1}^P \sum_{q=1}^{2m} \frac{\Lambda_{p,q}}{(q-1)!} \frac{\Gamma(q)}{2B_p^q} \left[ 1 - \xi(B_p, \zeta) \sum_{i=0}^{q-1} \binom{2i}{i} \left( \frac{1 - \xi(B_p, \zeta)^2}{4} \right)^i \right] \quad (4.74)$$

where  $\xi(B_p, \zeta) = \sqrt{\frac{\zeta}{\zeta+2B_p}}$ .

From (4.73), we can investigate the behavior of the BER lower bound at high SNR ( $\gamma_{ij} \rightarrow \infty$ ). Noting that  ${}_2F_1(1, q + \frac{1}{2}; q+1; \frac{2B_p}{2B_p+2}) \rightarrow 1$  as  $\gamma_{ij} \rightarrow \infty$ , as one can see  $P_b \equiv \frac{1}{2\sqrt{\Pi}} \prod_{p=1}^P (B_p)^{2m}$  where the resulted diversity gain is  $(2mP)$

#### 4.4.4 Numerical Results

Here we use the analytical results presented in the previous section to assess the BER performance of the asynchronous cooperative CDMA network using AF relaying with BPSK transmission. In all results, we consider the same system parameters as in Section 3.4.3.

Fig.4.6 shows the BER performance for both cooperative and non-cooperative AF relaying as a function of the number of resolvable paths. As shown in Fig.4.6, the AF cooperative CDMA system offers significant diversity gain compared with non-cooperative system. As one can see from Fig.4.7, when the  $P$  goes high the system performance is

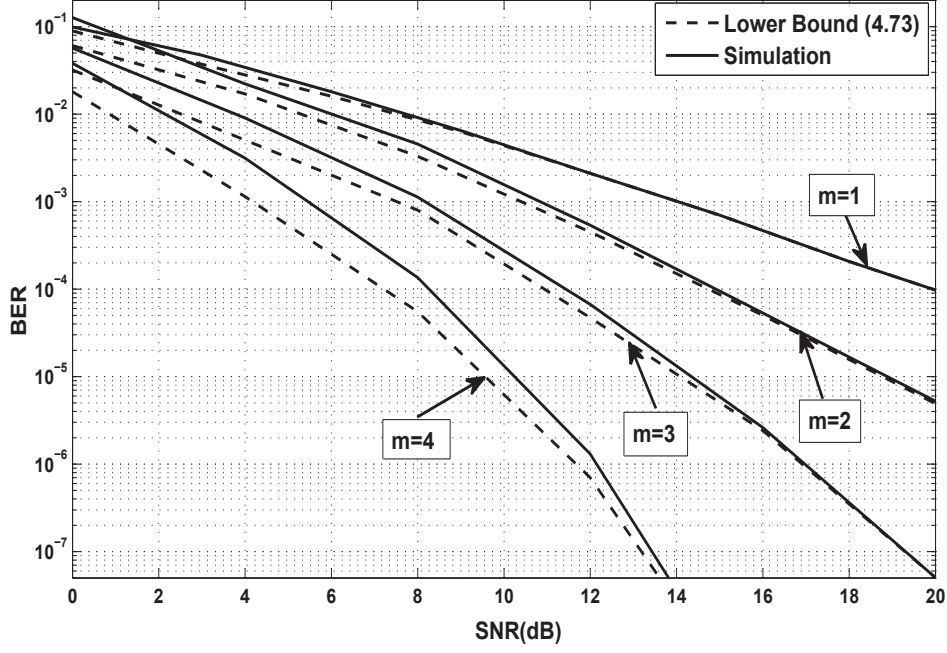


Figure 4.6: Lower bound BER performance for cooperative AF DS-CDMA as a function of fading parameter  $m$  over  $P = 1$  frequency-selective Nakagami- $m$  fading channels.

improved by benefiting from the multipath diversity. Fig.4.7 also shows the tightness between the simulation and analytical results given by (4.73) at medium and high SNR.

Fig. 4.8 examines the effect of MAI on the AF scheme, where we consider the three following scenarios: *MUD-b*, *MUD*, *MUD-r-b*, and *conventional r-b*. Fig. 4.8 illustrates the average BER when considering the three mentioned scenarios. It is clear that, the most advantageous performance will be achieved when employing MAI mitigation at both the relay and base station while the case of *MUD-b* only suffers from SNR loss due to the amplification of unwanted interfering signals. In both systems, full diversity is achieved for the AF scheme. On the other hand, the BER of the conventional matched filter (*conventional r-b*) exhibits an error floor at a certain SNR level due to the combined effect of multiuser interference at the relay and base station. These results reveal that the cancellation of MAI is necessary in order to reach the diversity advantage of the cooperative system.

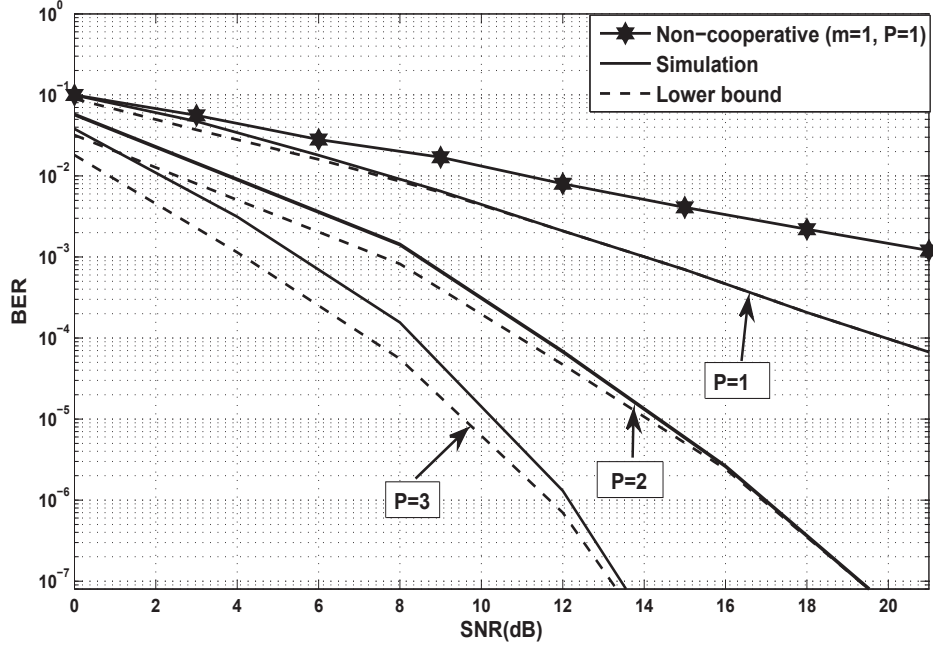


Figure 4.7: Lower bound BER performance for cooperative AF DS-CDMA as a function of resolvable paths over Nakagami- $m$  fading channels with  $m = 1$ .

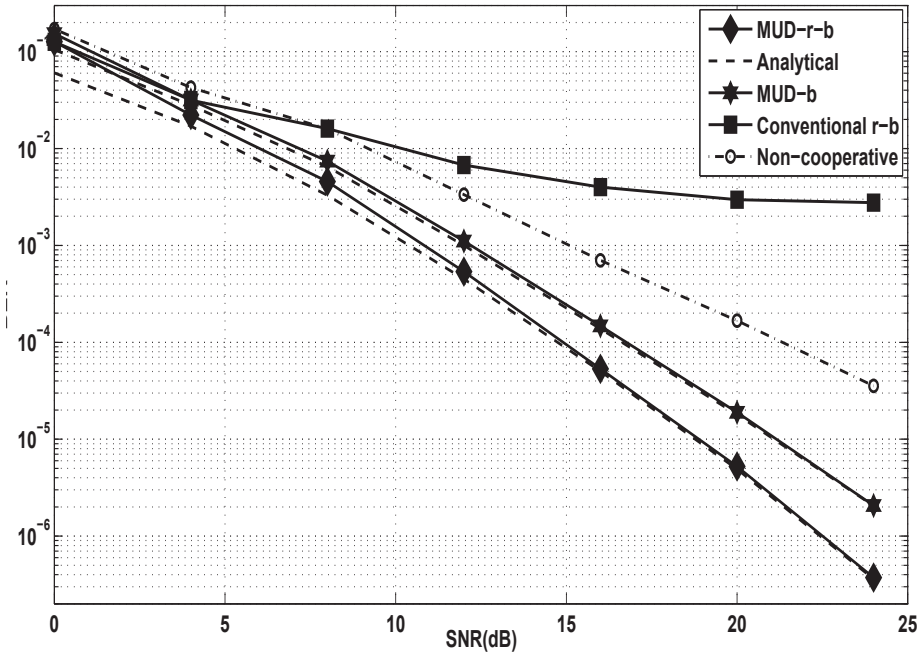


Figure 4.8: Lower bound BER performance for cooperative AF DS-CDMA over one-path frequency-selective over Nakagami- $m$  fading channels with  $m = 1.5$  where the MAI impact is investigated.



#### 4.4.5 Conclusions

We have investigated the performance of the cooperative diversity in a DS-CDMA system under MRC of the relayed data at the base station. For the underlying system, we have derived the asymptotic outage probability expression. Moreover, we have also derived a lower bound for average BER of the system. The derived expressions are tractable and generally can be used for different channel conditions and different modulation schemes. Our cooperative system employed MUD to suppress the multi-user interference at both the base station and the relay sides.

## Chapter 5

# Power Allocation and Relay Location Optimization

In the previous chapters, we conducted performance evaluation of conventional DF and AF cooperative relaying schemes where we considered relays have equal powers. In conventional relay systems, the overall power is allocated equally among relays. Since relays see different channels and hence, variable transmission and reception signal quality, it is important to distribute the overall power based on some performance metric. Particularly, in CDMA networks when a user is in deep fading or at the edge of the boundary of the base station coverage area, the users require to transmit with higher power. This leads to not only more power consumption from the users, but also to the interference increase with other base stations in the system coverage which affect the overall system throughput. Besides, the users transmission power are limited in which it may prohibit users from establishing the connection to the base station. Therefore, optimizing the power allocation becomes as a necessity to enhance the system performance.

Here, we also investigate the optimum relay location taking into account that the relays are considered as users. As such there is no control on users location. Therefore, we aim to find the optimum distance range in order to select the users that can act as

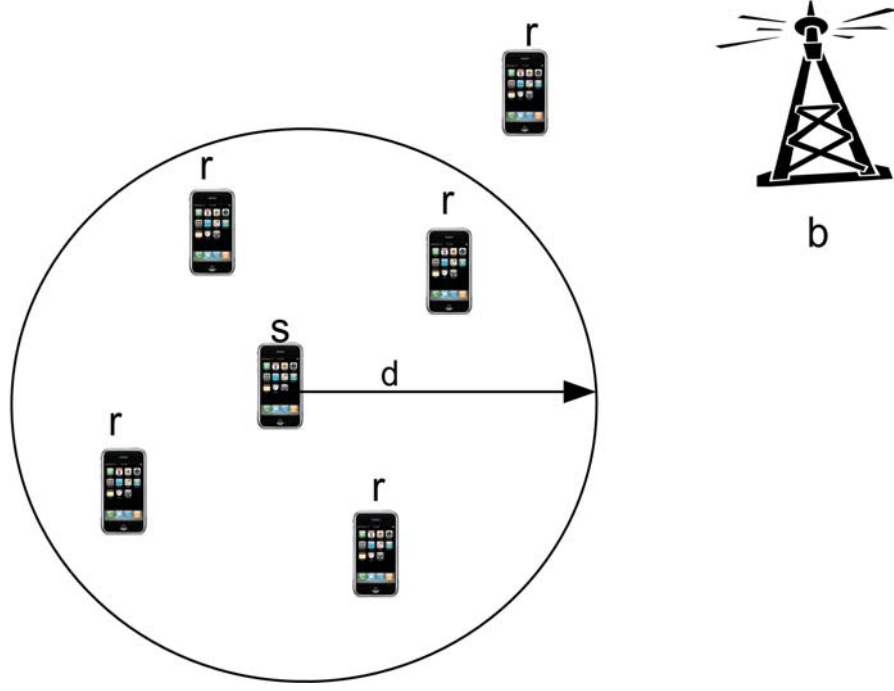


Figure 5.1: Relay optimization protocol, where  $d$  is the optimum relay location

best relays within this optimal distance range as shown in Fig. 5.1.

In the previous chapter, we noticed that the overall diversity order is  $m_{s,b} + \min(m_{s,r}, m_{r,b})$ , which represent the  $\min(m_{s,r}, m_{r,b})$  when no power allocation and relay location optimization are considered. This motivates us to utilize power and distance optimization to enhance the diversity and coding gain as represented in [59]. Interestingly, our results also show that the joint optimization of both power allocation and relay location brings the minimal outage probability performance of the system with diversity gain equal  $m_{s,b} + \max(m_{s,r}, m_{r,b})$

## 5.1 Introduction

Recently, there has been intensive interest in cooperative networks as they take advantage of broadcast transmission to create cooperation through distributed nodes. This technique, well-known as cooperative diversity, has been proven to significantly improve the quality of service by providing redundant copies of the source node packet. For various

cooperative networks, bandwidth efficiency and diversity gain are two main performance aspects.

The joint power allocation and relay selection for multiuser DF cooperative communication was studied in [19]. Along the same direction, the authors in [15] have introduced an overview of joint iterative power allocation and linear interference cancellation for cooperative DS-CDMA over synchronous transmission. In addition, the optimum power distribution for cooperative CDMA networks for both DF and AF relaying over Rayleigh fading channel was investigated in [20]. On the other hand, the authors in [89] have considered the rate-allocation problem, which has been subjected to minimizing the total transmit power in cooperative uplink system. In the downlink, optimization and performance of relay-assisted cellular network are considered in a multi-cell system in [90]. The authors in [90] studied the optimization of system parameters including relay location, path selection, and power allocation. It is worth mentioning that the above mentioned works have investigated systems over Rayleigh fading channels. In that perspective, the Nakagami- $m$  fading model is known as a generalized fading model can be represented. For instance, the authors in [59] have presented a performance evaluation and optimization of dual-hop over Nakagami- $m$  fading channels in the presence of channel interference. In their work, the authors investigated the effect of power allocation, relay position and joint optimization for AF schemes.

Considering a Cooperative AF CDMA system, in this chapter, we evaluate the optimum power allocation of the system over Nakagami- $m$  fading channels in order to minimize the outage probability. Also, we obtain the optimum relay location for the system over fixed transmission power to minimize the outage probability. In addition, we generalize the joint optimization problem for both power allocation and relay location in order to achieve the optimum outage probability performance and to reach full diversity.

## 5.2 Approximate Distributions of the Total SNR

The system model is the same as in Section 4.2 where single-relay is presented. As in Section 4.3.2 and by considering single-relay, the asymptotic outage probability is traditionally obtained by using the PDF of the indirect link between  $s \rightarrow r$  and  $r \rightarrow b$ . Due to the intractability of the PDF of  $X_{AF}$ , it becomes difficult to obtain a closed-form expression for the outage probability in the AF case. Instead, we employ an approximation of this distribution at high SNR. This approximation, as will be shown, will enable us to derive a closed-form expression for the outage probability of the underlying system.

In subsection 4.3.2, we approximated the PDF of the total SNR for the multi-relay cooperation. Here, we introduce a special case for single-relay scenario, therefore, at high SNR, (3.1) can be approximated as sentence

$$p_{\gamma_{ij}}(\gamma) \approx \frac{B_{ij}^{m_{ij}}}{\Gamma(m_{ij})} \gamma^{m_{ij}-1} + H.O., \quad (5.1)$$

where  $H.O.$  stands for high order terms. Applying Laplace transform to (5.1), and with the help of [73], the moment generating function (MGF) is given by

$$M_{\gamma_{ij}}(s) = \frac{B_{ij}^{m_{ij}}}{s^{m_{ij}}}. \quad (5.2)$$

Using the method in [58], the PDF of the approximate distribution for the link  $s \rightarrow r \rightarrow b$  can be written as

$$f_{\gamma_{s,r,b}}(\gamma) \approx f_{\gamma_{s,r}}(\gamma) + f_{\gamma_{r,b}}(\gamma), \quad (5.3)$$

where  $f_{\gamma_{s,r}}(\gamma)$  and  $f_{\gamma_{r,b}}(\gamma)$  can be approximated at high SNR as in (5.1)

$$f_{\gamma_{s,r}}(\gamma) \approx \frac{B_{sr}^{m_{sr}}}{\Gamma(m_{sr})} \gamma^{m_{sr}-1} + H.O., \quad (5.4)$$

$$f_{\gamma_{r,b}}(\gamma) \approx \frac{B_{rb}^{m_{rb}}}{\Gamma(m_{rb})} \gamma^{m_{rb}-1} + H.O. \quad (5.5)$$

Examining (5.3) when  $E_s/N_o \rightarrow \infty$ , a simplified expression for the  $f_{\gamma_{AF}}(\gamma)$  is given by

$$f_{\gamma_{AF}}(\gamma) \approx f_{\gamma_{s,b}}(\gamma) * f_{\gamma_{s,r,b}}(\gamma), \quad (5.6)$$

and the corresponding MGF is given by

$$M_{AF}(s) = M_{sb}(s) \cdot M_{s,r,b}(s). \quad (5.7)$$

Finally,  $P_{out}$  is defined as  $\mathcal{L}^{-1} \left\{ \frac{M_{AF}(s)}{s}; t \right\}_{|t=\gamma_{th}}$ ,

$$P_{out} = B_{sb}^{m_{sb}} \left( B_{sr}^{m_{sr}} \frac{\gamma_{th}^{(m_{sb}+m_{sr})}}{\Gamma(m_{sb} + m_{sr} + 1)} + B_{rb}^{m_{rb}} \frac{\gamma_{th}^{(m_{sb}+m_{rb})}}{\Gamma(m_{sb} + m_{rb} + 1)} \right). \quad (5.8)$$

### 5.3 System Optimization

In this section, we discuss an optimal power allocation strategy for the AF cooperative CDMA system considered last section. The asymptotic  $P_{out}$  derived in (5.8) can be written as a function of the power  $P$  and a distance  $d$  between the users themselves,

$$\begin{aligned} P_{out} &= B_{sb}^{m_{sb}} \left( B_{sr}^{m_{sr}} \frac{\gamma_{th}^{(m_{sb}+m_{sr})}}{\Gamma(m_{sb} + m_{sr} + 1)} + B_{rb}^{m_{rb}} \frac{\gamma_{th}^{(m_{sb}+m_{rb})}}{\Gamma(m_{sb} + m_{rb} + 1)} \right) \\ &= \frac{K^{2m_{sb}} ([\mathbf{R}_b]_{s,s}^{-1})^{m_{sb}} (m_{sb})^{m_{sb}} d_{sb}^{\alpha m_{sb}}}{(2N)^{m_{sb}} P_s^{m_{sb}}} \left( \frac{K^{2m_{sr}} ([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}} (m_{sr})^{m_{sr}} d_{sr}^{\alpha m_{sr}}}{(2N)^{m_{sr}} P_s^{m_{sr}}} \right. \\ &\quad \times \left. \frac{\gamma_{th}^{(m_{sb}+m_{sr})}}{\Gamma(m_{sb} + m_{sr} + 1)} + \frac{K^{2m_{rb}} ([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}} (m_{rb})^{m_{rb}} d_{rb}^{\alpha m_{rb}}}{(2N)^{m_{rb}} P_r^{m_{rb}}} \frac{\gamma_{th}^{(m_{sb}+m_{rb})}}{\Gamma(m_{sb} + m_{rb} + 1)} \right) \end{aligned} \quad (5.9)$$

where  $P_s \triangleq E_s$ ,  $E_s$  the source output energy,  $P_r \triangleq E_r$ ,  $E_r$  the relay output energy and  $P_T = P_s + P_r$  the total energy. Without loss of generality, we normalize the distance between the source and the base station  $d_{sb}^{\alpha m_{sb}} = 1$ , thus (5.9) becomes

$$\begin{aligned}
P_{out} = & \frac{K^{2m_{sb}} ([\mathbf{R}_b]_{s,s}^{-1})^{m_{sb}} (m_{sb})^{m_{sb}}}{(2N)^{m_{sb}}} \left( \frac{d_{sr}^{\alpha m_{sr}}}{P_s^{m_{sb}+m_{sr}}} \cdot \frac{K^{2m_{sr}} ([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}} (m_{sr})^{m_{sr}} \gamma_{th}^{(m_{sb}+m_{sr})}}{(2N)^{m_{sr}} \Gamma(m_{sb} + m_{sr} + 1)} \right. \\
& \left. + \frac{d_{rb}^{\alpha m_{rb}}}{P_s^{m_{sb}} (P_T - P_s)^{m_{rb}}} \cdot \frac{K^{2m_{rb}} ([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}} (m_{rb})^{m_{rb}} \gamma_{th}^{(m_{sb}+m_{rb})}}{(2N)^{m_{rb}} \Gamma(m_{sb} + m_{rb} + 1)} \right). \quad (5.10)
\end{aligned}$$

### 5.3.1 Adaptive Power Allocation

In this case, the power allocation optimization problem can be expressed as:

$$\begin{aligned}
(P_s^*, P_r^*) &= \arg \min_{(P_s, P_r)} P_{out}, \\
\text{subject to } & : P_s + P_r = P_T; \ P_s, P_r > 0, \quad (5.11)
\end{aligned}$$

where  $P_T = E_T$  and  $E_T$  is the total energy given by  $E_T = E_s + E_r$ . By doing the second-order derivative of  $P_{out}$  introduced in (5.10) with respect to  $P_s$ , it is clear that  $\partial^2 P_{out} / \partial P_s^2$  is positive in the period  $[0, P_T]$ . This shows that the target function is a strictly convex function of  $P_s$  in  $[0, P_T]$ . Thus, obtaining the first-order derivative of  $P_{out}$  with respect to

$P_s$ , the optimal power allocation can be expressed as

$$\begin{aligned}
\frac{\partial}{\partial P_s}(P_{out}) &= \frac{\partial}{\partial P_s} \left( \frac{K^{2m_{sb}}([\mathbf{R}_b]_{s,s}^{-1})^{m_{sb}}(m_{sb})^{m_{sb}}}{(2N)^{m_{sb}}} \left( \frac{d_{sr}^{\alpha m_{sr}}}{P_s^{m_{sb}+m_{sr}}} \right. \right. \\
&\times \frac{K^{2m_{sr}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}}(m_{sr})^{m_{sr}}\gamma_{th}^{(m_{sb}+m_{sr})}}{(2N)^{m_{sr}}\Gamma(m_{sb}+m_{sr}+1)} + \frac{d_{rb}^{\alpha m_{rb}}}{P_s^{m_{sb}}(P_T-P_s)^{m_{rb}}} \cdot \\
&\times \left. \left. \frac{K^{2m_{rb}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}}(m_{rb})^{m_{rb}}\gamma_{th}^{(m_{sb}+m_{rb})}}{(2N)^{m_{rb}}\Gamma(m_{sb}+m_{rb}+1)} \right) \right) \\
&= \frac{K^{2m_{sb}}([\mathbf{R}_b]_{s,s}^{-1})^{m_{sb}}(m_{sb})^{m_{sb}}}{(2N)^{m_{sb}}} \left( \frac{-(m_{sr}+m_{sb})d_{sr}^{\alpha m_{sr}}}{P_s^{m_{sr}+m_{sb}+1}} \right. \\
&\times \frac{K^{2m_{sr}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}}(m_{sr})^{m_{sr}}\gamma_{th}^{(m_{sb}+m_{sr})}}{(2N)^{m_{sr}}\Gamma(m_{sb}+m_{sr}+1)} + \frac{K^{2m_{rb}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}}(m_{rb})^{m_{rb}}\gamma_{th}^{(m_{sb}+m_{rb})}}{(2N)^{m_{rb}}\Gamma(m_{sb}+m_{rb}+1)} \\
&\times \left. \left( \frac{m_{rb}d_{rb}^{\alpha m_{rb}}}{P_s^{m_{sb}}(P_T-P_s)^{m_{rb}+1}} - \frac{m_{sb}d_{rb}^{\alpha m_{rb}}}{P_s^{m_{sb}+1}(P_T-P_s)^{m_{rb}}} \right) \right). \tag{5.12}
\end{aligned}$$

After many manipulations, the optimal source transmit power,  $P_s^*$  is the root of the following equation:

$$P_s = (P_T - P_s)^{\frac{m_r+1}{m_r}} \left( \frac{(m_r+m_{sb})d_{sr}^{\alpha m_r}}{P_s m_r d_{rb}^{\alpha m_r} - m_{sb} d_{rb}^{\alpha m_r} (P_T - P_s)} \right)^{\frac{1}{m_r}}. \tag{5.13}$$

The optimal transmit relay power is defined by  $P_r^* = P_T - P_s^*$ . It is difficult to get a closed-form expression for the source optimal power,  $P_s^*$ . Nevertheless, numerical solution using Mathematica or Maple can be determined by standard iterative root finding algorithms, such as the Newton's method and the Bisection method, with high efficiency.

### 5.3.2 Relay Location Optimization

In this section, we consider that the source, relay, and the base station are on straight line, i.e.,  $d_{sr} = d$ ,  $d_{rb} = 1 - d$  and  $d_{sb} = 1$ . This consideration is fair since the optimal location for the relay must lie on the line joining the source with the base station in order to minimize the effect of the path loss. Under a predefined power allocation, given by



$(P_s, P_r)$ , the problem of obtaining the optimal relay position that minimizes the system outage probability can be formulated as follows:

$$d^* = \arg \min_d P_{out} \quad \text{subject to } 0 < d < 1. \quad (5.14)$$

It is easy to show that

$$\begin{aligned} \frac{\partial^2}{\partial d^2}(P_{out}) &= \frac{K^{2m_{sb}}([\mathbf{R}_b]_{s,s}^{-1})^{m_{sb}}(m_{sb})^{m_{sb}}}{(2N)^{m_{sb}}} \left( \frac{\alpha m_{sr}(\alpha m_{sr} - 1)d^{\alpha m_{sr}-2}}{P_s^{m_{sb}+m_{sr}}} \right. \\ &\times \frac{K^{2m_{sr}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}}(m_{sr})^{m_{sr}}\gamma_{th}^{(m_{sb}+m_{sr})}}{(2N)^{m_{sr}}\Gamma(m_{sb} + m_{sr} + 1)} + \frac{\alpha m_{rb}(\alpha m_{rb} - 1)(1-d)^{\alpha m_{rb}-2}}{P_s^{m_{sb}}(P_T - P_s)^{m_{rb}}} \\ &\times \left. \frac{K^{2m_{rb}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}}(m_{rb})^{m_{rb}}\gamma_{th}^{(m_{sb}+m_{rb})}}{(2N)^{m_{rb}}\Gamma(m_{sb} + m_{rb} + 1)} \right) \end{aligned} \quad (5.15)$$

It can be seen that if  $\alpha m_{sr} - 1 > 0$  and  $\alpha m_{rb} - 1 > 0$ , then  $\frac{\partial^2}{\partial d^2}(P_{out}) > 0$ , which implies that the objective function is a strictly convex function of  $d$  in the interval  $d \in \{0, 1\}$ . After many manipulations the optimal relay position  $d^*$  is the root of the following equation:

$$\begin{aligned} d &= (1-d)^{\frac{\alpha m_{rb}-1}{\alpha m_{sr}-1}} \left( \frac{P_s^{m_{sr}}}{(P_T - P_s)^{m_{rb}}} \cdot \frac{K^{2m_{rb}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}}(m_{rb})^{m_{rb}+1}\gamma_{th}^{(m_{sb}+m_{rb})}}{(2N)^{m_{rb}}\Gamma(m_{sb} + m_{rb} + 1)} \right. \\ &\times \left. \frac{(2N)^{m_{sr}}\Gamma(m_{sb} + m_{sr} + 1)}{K^{2m_{sr}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}}(m_{sr})^{m_{sr}+1}\gamma_{th}^{(m_{sb}+m_{sr})}} \right)^{\frac{1}{\alpha m_{sr}-1}}. \end{aligned} \quad (5.16)$$

From (5.16), it can be shown that as the source energy decreases, the relay should be closer to the source in order to compensate for the degradation of the transmission quality of this link and vice versa. As a special case when  $m_{sb} = m_{sr} = m_{rb} = 1$ , i.e., Rayleigh fading channels, the closed form expression for the optimum relay location can be written as

$$d^* = \frac{P_s^{\frac{1}{\alpha-1}}}{P_s^{\frac{1}{\alpha-1}} + P_r^{\frac{1}{\alpha-1}}}. \quad (5.17)$$

### 5.3.3 Joint Optimization of the Relay Location and Power Allocation

The system performance can be further improved if we jointly optimize power allocation and relay location. The optimization problem can be expressed as:

$$\begin{aligned} (P_s^*, P_r^*, d^*) &= \arg \min_{(P_s, P_r, d)} P_{out}, \\ \text{subject to: } &P_s + P_r = P_T > 0; \quad 0 < d < 1. \end{aligned} \quad (5.18)$$

The matrix of the second partial derivative (Hessian matrix) of the objective function declared in (5.18) can be seen to be positive definite in the interval  $P_T > 0$  and  $0 < d < 1$ . The optimal power allocation and relay location can be determined by setting the first derivatives of the objective function with respect to  $P_s$  and  $d$  to zero as follows:

$$\begin{aligned} \frac{\partial}{\partial P_s}(P_{out}) &= \frac{K^{2m_{sb}}([\mathbf{R}_b]_{s,s}^{-1})^{m_{sb}}(m_{sb})^{m_{sb}}}{(2N)^{m_{sb}}} \left( \frac{-(m_{sr} + m_{sb})d_{sr}^{\alpha m_{sr}}}{P_s^{m_{sr}+m_{sb}+1}} \right. \\ &\quad \frac{K^{2m_{sr}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}}(m_{sr})^{m_{sr}}\gamma_{th}^{(m_{sb}+m_{sr})}}{(2N)^{m_{sr}}\Gamma(m_{sb} + m_{sr} + 1)} + \frac{K^{2m_{rb}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}}(m_{rb})^{m_{rb}}\gamma_{th}^{(m_{sb}+m_{rb})}}{(2N)^{m_{rb}}\Gamma(m_{sb} + m_{rb} + 1)} \\ &\quad \left. \left( \frac{m_{rb}d_{rb}^{\alpha m_{rb}}}{P_s^{m_{sb}}(P_T - P_s)^{m_{rb}+1}} - \frac{m_{sb}d_{rb}^{\alpha m_{rb}}}{P_s^{m_{sb}+1}(P_T - P_s)^{m_{rb}}} \right) \right) = 0 \end{aligned} \quad (5.19)$$

$$\begin{aligned} \frac{\partial}{\partial d}(P_{out}) &= \frac{K^{2m_{sb}}([\mathbf{R}_b]_{s,s}^{-1})^{m_{sb}}(m_{sb})^{m_{sb}}}{(2N)^{m_{sb}}} \left( \frac{\alpha d^{\alpha m_{sr}-1}}{P_s^{m_{sb}+m_{sr}}} \cdot \frac{K^{2m_{sr}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}}(m_{sr})^{m_{sr}+1}\gamma_{th}^{(m_{sb}+m_{sr})}}{(2N)^{m_{sr}}\Gamma(m_{sb} + m_{sr} + 1)} \right. \\ &\quad \left. - \frac{\alpha(1-d)^{\alpha m_{rb}-1}}{P_s^{m_{sb}}(P_T - P_s)^{m_{rb}}} \cdot \frac{K^{2m_{rb}}([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}}(m_{rb})^{m_{rb}+1}\gamma_{th}^{(m_{sb}+m_{rb})}}{(2N)^{m_{rb}}\Gamma(m_{sb} + m_{rb} + 1)} \right) = 0. \end{aligned} \quad (5.20)$$

It is difficult to find the solution of the nonlinear equations (5.19) and (5.20). Therefore, after many manipulations we can simplify the solution to

$$\begin{aligned}
P_s^{m_{sr}} &= \frac{(P_T - P_s)^{m_{rb}}}{(1-d)^{\alpha m_{rb}-1}} d^{\alpha m_{sr}-1} \cdot \frac{K^{2m_{sr}} ([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}} (m_{sr})^{m_{sr}+1} \gamma_{th}^{(m_{sb}+m_{sr})}}{(2N)^{m_{sr}} \Gamma(m_{sb} + m_{sr} + 1)} \\
&\times \frac{(2N)^{m_{rb}} \Gamma(m_{sb} + m_{rb} + 1)}{K^{2m_{rb}} ([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}} (m_{rb})^{m_{rb}+1} \gamma_{th}^{(m_{sb}+m_{rb})}}. \tag{5.21}
\end{aligned}$$

By substituting (5.21) into (5.19),

$$\frac{(P_s m_{rb} - P_r m_{sb}) m_{sr}}{P_r m_{rb} (m_{sr} + m_{sb})} = \frac{d}{1-d}. \tag{5.22}$$

Thus,

$$P_s = \frac{P_T (m_{sr} m_{rb} d + m_{sb} m_{rb} d - m_{sb} m_{sr} d + m_{sb} m_{sr})}{m_{sb} m_{rb} d + m_{sr} m_{rb} + m_{sb} m_{sr} - m_{sb} m_{sr} d} \tag{5.23}$$

$$P_r = \frac{P_T m_{sr} m_{rb} (1-d)}{m_{sb} m_{rb} d + m_{sb} m_{sr} + m_{sb} m_{rb} - m_{sb} m_{sr} d} \tag{5.24}$$

$d^*$  can be found by substituting (5.23) and (5.26) into (5.20). Therefore, the optimum relay location can be found by using the iterative standard root finding method.

$$\begin{aligned}
d &= (1-d)^{\frac{\alpha m_{rb}-1}{\alpha m_{sr}-1}} \left( \frac{\left( \frac{P_T (m_{sr} m_{rb} d + m_{sb} m_{rb} d - m_{sb} m_{sr} d + m_{sb} m_{sr})}{m_{sb} m_{rb} d + m_{sr} m_{rb} + m_{sb} m_{sr} - m_{sb} m_{sr} d} \right)^{m_{sr}}}{\left( \frac{P_T m_{sr} m_{rb} (1-d)}{m_{sb} m_{rb} d + m_{sb} m_{sr} + m_{sb} m_{rb} - m_{sb} m_{sr} d} \right)^{m_{rb}}} \right. \\
&\times \frac{(2N)^{m_{sr}} \Gamma(m_{sb} + m_{sr} + 1)}{K^{2m_{sr}} ([\mathbf{R}_r]_{r,r}^{-1})^{m_{sr}} (m_{sr})^{m_{sr}+1} \gamma_{th}^{(m_{sb}+m_{sr})}} \cdot \\
&\times \left. \frac{K^{2m_{rb}} ([\mathbf{R}_r]_{r,r}^{-1})^{m_{rb}} (m_{rb})^{m_{rb}+1} \gamma_{th}^{(m_{sb}+m_{rb})}}{(2N)^{m_{rb}} \Gamma(m_{sb} + m_{rb} + 1)} \right)^{\frac{1}{\alpha m_{sr}-1}}. \tag{5.25}
\end{aligned}$$

For the Rayleigh fading case  $d^*$  can be expressed as

$$d = (1-d) \left( \frac{d+1}{1-d} \right)^{\frac{1}{\alpha-1}}. \tag{5.26}$$

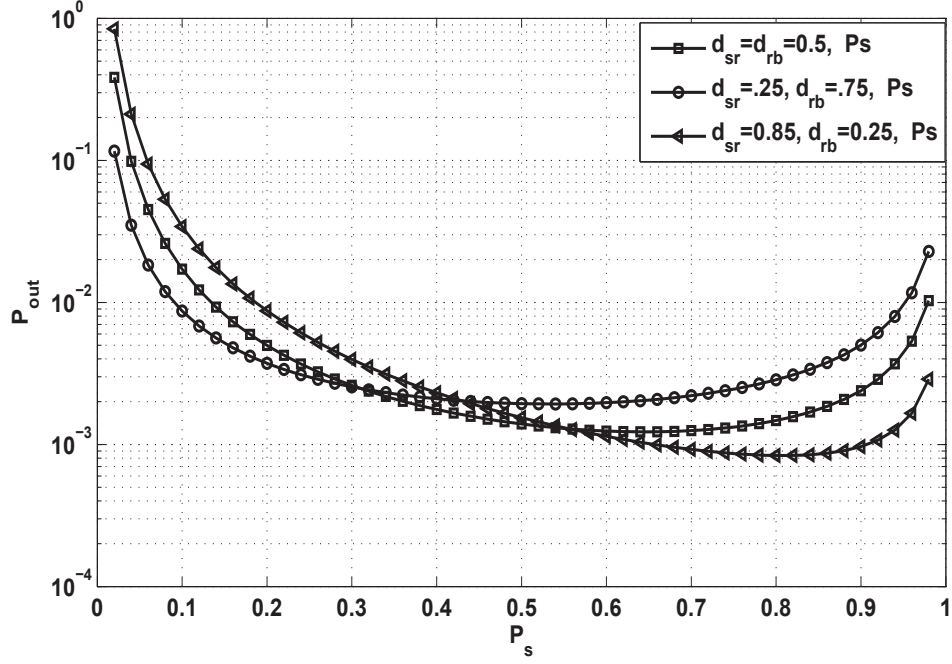


Figure 5.2: Asymptotic outage probability for cooperative AF DS-CDMA versus power allocation ratio at the source for different relay position for fixed  $SNR = 15dB$ .

## 5.4 Numerical Results and Discussions

In what follows, we present our analytical and simulation results for the outage probability. In all results, we set the spreading gain to  $N = 31$  for a system with number of users  $K = 16$ . Without loss of generality, we assume that the spectral efficiency  $\mathfrak{R} = 1bit/sec/Hz$ . A decorrelator detector is used to mitigate the effect of MAI.

Fig. 5.2 shows the asymptotic  $P_{out}$  performance for the cooperative AF relaying over i.n.i Nakagami- $m$  fading channel versus the power allocation at the source  $s$  for different relay location. As one can see that the outage performance is relatively fixed around same power allocation when the relay is located in the middle of the source and the base station. On the other hand, when the relay becomes closer to the base station, the optimum power allocation at the source node  $P_s$  approaches 1. This indicates that as the source-relay link becomes less reliable, more power is allocated to the source node, and the system converges to non-cooperative one.

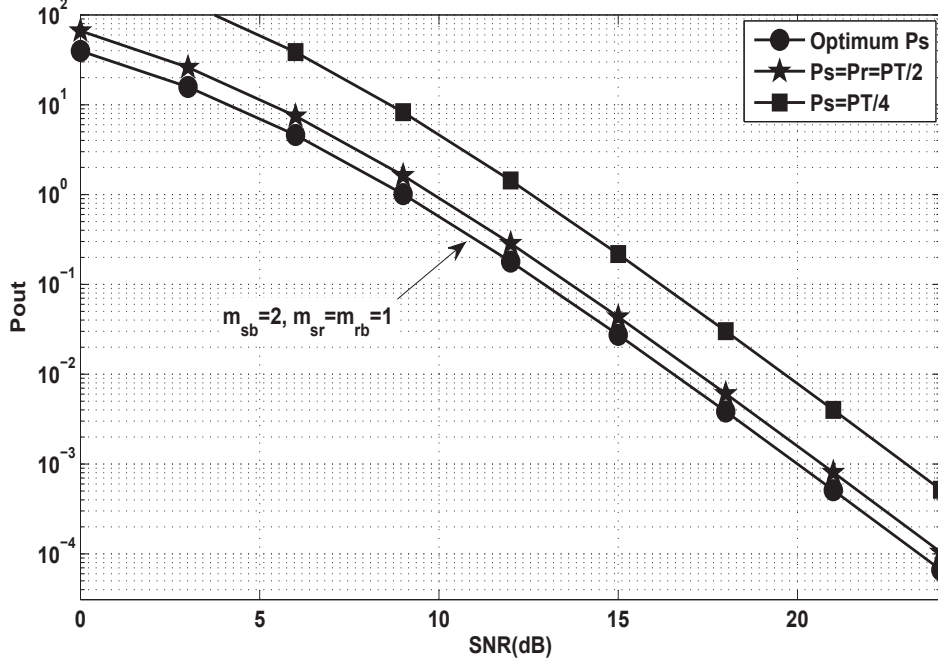


Figure 5.3: Outage probability performance of power allocations algorithm

Fig. 5.3 represents the impact of optimizing power allocation on the asymptotic outage performance of AF scheme over the different fading channels  $m_{sb} = 2$ ,  $m_{sr} = m_{rb} = 1$  when the relay is located in the middle of the source and base station. In Fig. 5.3, we notice that the power allocation brings only coding gain to the system. Fig. 5.4 shows the asymptotic  $P_{out}$  performance for the cooperative AF versus relay location for different power allocations at the source. When the power allocation is 0.5 which indicates that the relay location is the middle of the source and base station. On the other hand, when the relay becomes closer to the base station, the optimum power allocation at the source node  $P_s$  approaches 1.

In Fig. 5.5, we compare the outage performance for different optimization schemes with different fading parameters, i.e.,  $m_{s,b}$ ,  $m_{s,r}$  and  $m_{r,b}$ . It can be shown that under fixed relay location (i.e.,  $d = 0.5$ ), adaptive power allocation slightly outperforms the fixed power allocation (i.e.,  $P_s = P_r$ ) and brings only coding gain to the system. Otherwise, under the equal power allocation, selecting the optimal relay position significantly minimizes

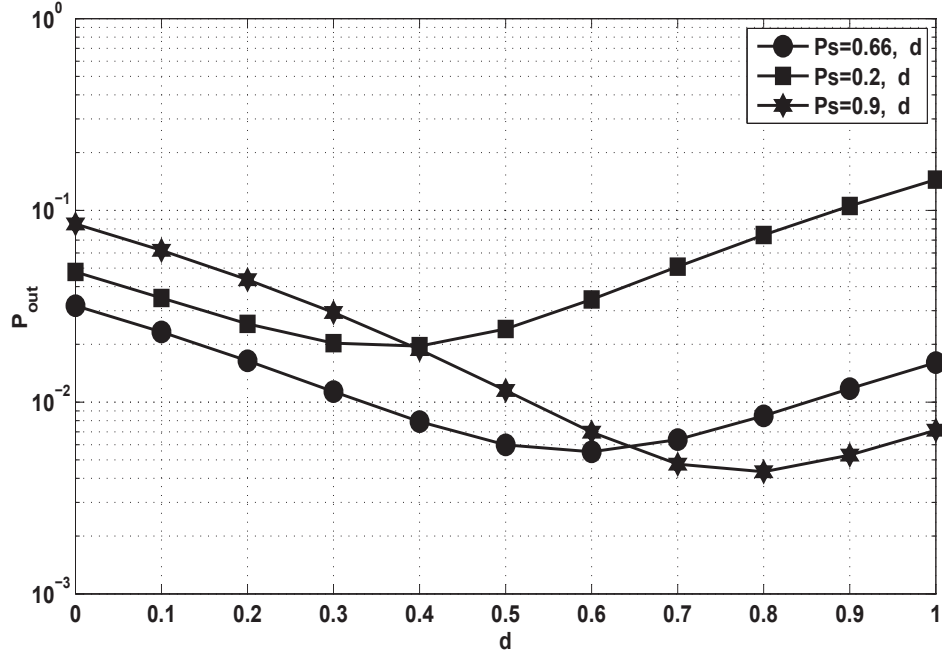


Figure 5.4: Asymptotic outage probability for cooperative AF DS-CDMA versus relay location ratio for different source power allocation for fixed  $SNR = 15dB$  and  $m_{s,b} = 1$ ,  $m_{s,r} = 0.85$ ,  $m_{r,b} = 1$ .

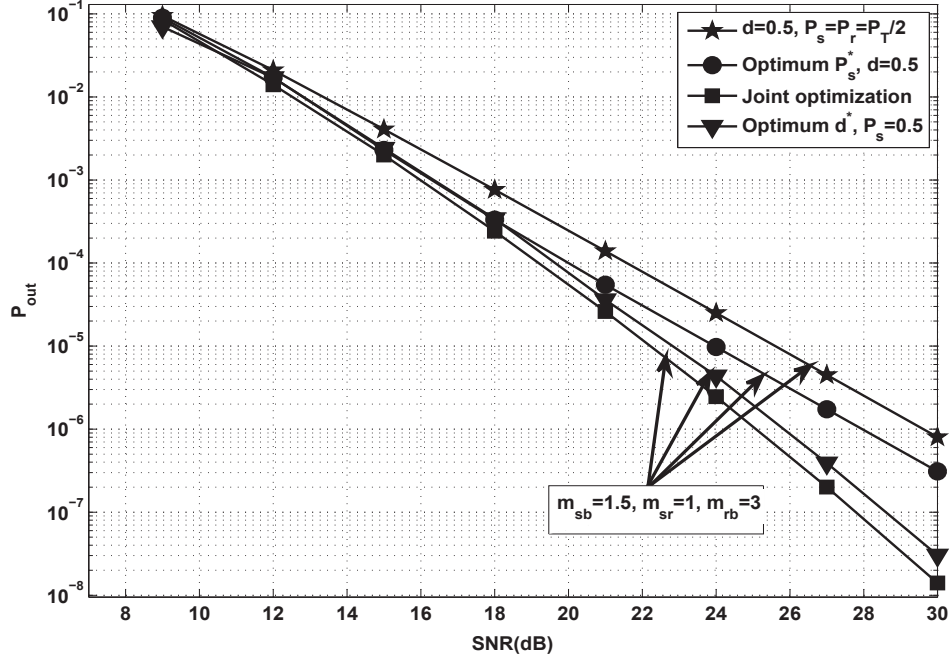


Figure 5.5: Asymptotic outage probability for cooperative AF DS-CDMA versus power allocation ratio at the source for different relay position for fixed  $SNR = 15dB$ .

the system's outage probability. Furthermore, we can find that joint optimization of the power allocation and relay location improves the diversity gain. From Fig. 5.5, we can find that the joint optimization improves the diversity gain to  $m_{s,b} + \max(m_{s,r}, m_{r,b})$  instead of  $m_{s,b} + \min(m_{s,r}, m_{r,b})$  as in Chapter 4. However, since we cannot find a closed form solution for the joint optimization, this feature cannot be proven mathematically. Note that for the case of Rayleigh fading channel, the optimization analysis will not improve the diversity gain. Finally, we remark that similar results were shown in [59] where no the direct link between the source and the base station not exists.

## 5.5 Conclusions

In this chapter, we have formulated the optimum power allocation problem over fixed relay location and optimum relay location over fixed power allocation in AF cooperative CDMA systems. Furthermore, we evaluated the joint optimization of both power allocation and relay location under the transmit power constraint to minimize the outage probability of the system. Our simulation and analytical results illustrated the outage probability performance of the system for different optimization schemes. Interestingly, our results shown that the joint optimization of both power allocation and relay location brings the minimal outage probability performance of the system with full diversity gain.

# Chapter 6

## Conclusions and Future Works

In this chapter, we highlight the main contributions of the accomplished work introduced in this thesis. Cooperative CDMA technique has been proposed to enable single antenna nodes in a multi-user scenario to share their antennas in a manner that creates a virtual MIMO system. This allows single-antenna mobiles to reap some of the benefits of MIMO systems. The main goal of this research was to analyze the performance of cooperative CDMA over Nakagami- $m$  fading channels using DF and AF relaying strategies.

### 6.1 Summary and Conclusions

This section briefly summarizes the accomplished work and the main contributions in this thesis.

In the Chapter 1,2, the necessary literature review of cooperative diversity techniques for CDMA systems were introduced. First, we revised the standard multiple-access techniques. These techniques have the advantage of improving the spectral efficiency and utilizing the available bandwidth in wireless systems. We have also reviewed the user cooperation and described the different MUD techniques. Moreover, we introduced the fading channels which can be implemented in the cooperative CDMA and presented the Nakagami- $m$  fading channel simulator that used in our research.



In Chapter 3, we have analyzed the performance of the cooperative diversity in a DS-CDMA system over Nakagami- $m$  fading channels using DF relaying, where the diversity combining of the relayed information at the base station is considered. Furthermore, in our work, we considered asynchronous transmission and non-orthogonal spreading codes. Asynchronous uplink transmission scenario for DF is considered where the MMSE is used as MUD at both the base station and the relay sides in order to achieve the full diversity of the system. We derived an exact closed-form expressions for the outage probability for both multi-relay and single-relay scenarios. In our proposed scenario, the performance analysis of the cooperative DS-CDMA system over DF relaying was investigated over frequency-selective Nakagami- $m$  fading channels where the BER was derived for the prescribed system. Moreover, the derived expressions have been investigated under different system settings, such as the effect of different fading parameter, inter-user channel errors, different number of relays. Finally, we showed through simulation and analytical results that the system is able to achieve the full diversity gain by mitigating the effect of multi-user interference using MUD such as MMSE and DD. Also, we showed that the performance of the matched filter exhibits an error floor after a certain SNR.

In Chapter 4, the performance of cooperative diversity in a DS-CDMA system using AF relaying was studied. In our research and due to intractability of obtaining a closed-form expression for the outage probability, we derived a tight lower bound for outage probability for AF relaying system. Moreover, the PDF of the total SNR was approximated which enabled us to derive the asymptotic outage probability for arbitrary value of the fading parameter  $m$ . In our work, and different from other existing works, we derived a lower bound for the BER. The derived expression was obtained using the MGF approach where the PDF of the total SNR at the base station was evaluated. Our cooperative system employed MUD to suppress the MAI at both the base station and the relay sides. Furthermore, the impact of MAI on the AF system performance was investigated where we showed that AF relying can achieve an acceptable diversity order

without using MUD at the relay station. Finally, both simulation and analytical results illustrated that the full diversity gain of the system is achieved.

In all above works, we considered uniform distance between users and with equal transmitted power. Therefore, to make our system more realistic, in Chapter 5, we proposed and formulated an optimization problem for both power allocation and relay location with an objective to optimize the total transmission power subject to minimizing the outage probability of the system. Furthermore, we investigated the joint optimization of both power allocation and relay location under the transmit power constraint. Our, simulation and analytical results presented the outage probability performance of the system where we demonstrated that the joint optimization of both power allocation and relay location brings the minimal outage probability performance of the system with full diversity gain.

## 6.2 Future works

In what follows we introduce possible directions of interest for future extension of this thesis.

- Assuming a perfect channel state information (CSI) at the relay side is an ideal assumption and not fully practical. Therefore, the effect of imperfect CSI and partial CSI should be investigated.
- Throughout the thesis, random relay selection strategy was assumed. Advanced strategies that consider the reliability of the (user-relay) and (relay-base station) links could be included in future works.
- We studied the performance of cooperative CDMA over uncorrelated fading channels. Studying the effect of channel correlation on the system performance is an interesting future work.

- In this thesis, little attention has been taken on the complexity of the transmitter/receiver needed to design the different protocols. Future works shall address this issue with much more depth to guarantee that the practical system achieves the optimal performance.

# Appendix A

In this appendix, we derive  $F_{total}(y)$  of (3.12). We have

$$I_{total} = \frac{K}{2N} \log \left( 1 + \frac{2N\gamma_{sb}}{K^2[M]_{s,s}} + \frac{2N\gamma_{rb}}{K^2[M]_{r,r}} \right) \quad (\text{A.1})$$

as in [41] we can rewrite (A.1) as

$$I_{total} = b \log (1 + a(X_{sb} + X_{rb})) \quad (\text{A.2})$$

where  $b = \frac{K}{2N}$ ,  $a = \frac{2N}{K^2}$ ,  $X_{sb} = \frac{\gamma_{sb}}{[M]_{s,s}}$ , and  $X_{rb} = \frac{\gamma_{rb}}{[M]_{r,r}}$ .

The outage probability is then written as

$$P_{out} = P_r [I_{total} < \Re] = P_r [b \log (1 + a(X_{sb} + X_{rb})) < \Re] \quad (\text{A.3})$$

which yields

$$P_{out} = P_r [X_{sb} + X_{rb} < y] \quad (\text{A.4})$$

where  $y = \frac{2^{\frac{\Re}{b}} - 1}{a}$ . Let  $X_{sb} \sim \mathcal{G}(m_{sb}, \bar{\gamma}_{sb})$  and  $X_{rb} \sim \mathcal{G}(m_{rb}, \bar{\gamma}_{rb})$  are two gamma RVs, we derived the CDF of the sum of two gamma RVs to obtain the outage probability as

$$P_{out} = F_{total}(y) = P_r [X_{sb} + X_{rb} < y] \quad (\text{A.5})$$

$$\begin{aligned}
&= \int_0^\infty P_r [X_{sb} + X_{rb} < y | X_{sb} = x] \cdot f_{X_{sb}}(x) dx \\
&= \int_0^\infty P_r [X_{rb} < y - x | X_{sb} = x] \cdot f_{X_{sb}}(x) dx
\end{aligned}$$

which yield

$$= \int_0^\infty F_{X_{rb}}(y - x) f_{X_{sb}}(x) dx.$$

Then, one can get

$$F_{total}(y) = \int_0^y F_{X_{rb}}(y - x) f_{X_{sb}}(x) dx, \quad \text{for } y \geq 0. \quad (\text{A.6})$$

As in (3.1), the PDF of  $X_{sb}$  is

$$f_{X_{sb}}(x) = \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} x^{m_{sb}-1} \exp(-x B_{sb}) \quad (\text{A.7})$$

and as in (3.2), the CDF of  $X_{rb}$  is

$$F_{X_{rb}}(y - x) = \frac{\gamma(m_{rb}, (y - x) B_{rb})}{\Gamma(m_{rb})}. \quad (\text{A.8})$$

From [ [72], eq. (8.352,6)], we have

$$\gamma(n, x) = (n - 1)! \left[ 1 - \exp(-x) \sum_{m=0}^{n-1} \frac{x^m}{m!} \right] \quad (\text{A.9})$$

Then using (A.9), we can rewrite (A.8) as

$$F_{X_{rb}}(y - x) = \frac{1}{\Gamma(m_{rb})} \cdot (m_{rb} - 1)! \left[ 1 - \exp(-(y - x) B_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{((y - x) B_{rb})^n}{n!} \right]. \quad (\text{A.10})$$

From [ [72], eq. (8.339,1)], we have  $\Gamma(m_{rb}) = (m_{rb} - 1)!$ . Therefore, we can rewrite (A.10)

as

$$F_{X_{rb}}(y - x) = 1 - \exp(-y B_{rb}) \exp(x B_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} (y - x)^n. \quad (\text{A.11})$$

Using the binomial expansion as in [ [72], eq.(1.111)], where  $(a+x)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$ , then (A.11) can be written as

$$F_{X_{rb}}(y-x) = 1 - \exp(-yB_{rb}) \exp(xB_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k x^k y^{n-k}. \quad (\text{A.12})$$

By substituting (A.12) into (A.6) and using (A.7) we have

$$F_{total}(y) = \int_0^y \left( 1 - \exp(-yB_{rb}) \exp(xB_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k x^k y^{n-k} \right) f_{X_{sb}}(x) dx \quad (\text{A.13})$$

$$\begin{aligned} &= \int_0^y f_{X_{sb}}(x) dx - \int_0^y f_{X_{sb}}(x) \exp(-yB_{rb}) \exp(xB_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k x^k y^{n-k} dx \\ &= F_{X_{sb}}(x) - \int_0^y \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} x^{m_{sb}-1} \exp(-xB_{sb}) \exp(-yB_{rb}) \exp(xB_{rb}) \end{aligned}$$

$$\times \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k x^k y^{n-k} dx$$

$$\begin{aligned} F_{total}(y) &= F_{X_{sb}}(x) - \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} \exp(-yB_{rb}) \sum_{n=0}^{m_{rb}-1} \frac{(B_{rb})^n}{n!} \\ &\quad \times \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} \int_0^y x^{k+m_{sb}-1} \exp(-x(B_{sb} - B_{rb})) dx. \end{aligned} \quad (\text{A.14})$$

# Appendix B

In this appendix, we derive  $F_{X_{total}}(y)$  of (4.18). We have

$$I_{AF} = \frac{K}{2N} \log \left( 1 + X_{sb} + \frac{X_{sr}X_{rb}}{X_{sr} + X_{rb} + 1} \right) \quad (\text{B.1})$$

we can rewrite (B.1) as

$$I_{AF} = b \log (1 + (X_{sb} + X_{min})), \quad (\text{B.2})$$

where  $b = \frac{K}{2N}$  and  $X_{min} = \min(X_{sr}, X_{rb})$ . The outage probability is then written as

$$P_{out} = P_r [I_{AF} < \Re] = P_r [b \log (1 + (X_{sb} + X_{min})) < \Re] \quad (\text{B.3})$$

which yields

$$P_{out} = P_r [X_{sb} + X_{min} < y] \quad (\text{B.4})$$

where  $y = 2^{\frac{\Re}{b}} - 1$ . Now, let  $X_{sb} \sim \mathcal{G}(m_{sb}, \gamma_{sb}^-)$  and  $X_{min} \sim \mathcal{G}(m_{min}, \gamma_{min}^-)$  be two gamma RVs. The CDF of the sum of two gamma RVs can be derived to find the outage probability as

$$P_{out} = F_{X_{total}}(y) = P_r [X_{sb} + X_{min} < y] \quad (\text{B.5})$$

$$\begin{aligned}
&= \int_0^\infty P_r [X_{sb} + X_{min} < y | X_{sb} = x] \cdot f_{X_{sb}}(x) dx \\
&= \int_0^\infty P_r [X_{min} < y - x | X_{sb} = x] \cdot f_{X_{sb}}(x) dx \\
&= \int_0^\infty F_{X_{rb}}(y - x) f_{X_{sb}}(x) dx.
\end{aligned}$$

Then,

$$F_{X_{total}}(y) = \int_0^y F_{X_{min}}(y - x) f_{X_{sb}}(x) dx, \quad \text{for } y \geq 0. \quad (\text{B.6})$$

As in (3.1), the PDF of  $X_{sb}$ ,

$$f_{X_{sb}}(x) = \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} x^{m_{sb}-1} \exp(-x B_{sb}) \quad (\text{B.7})$$

and as in (3.2), the CDF of  $X_{min}$  is

$$\begin{aligned}
F_{X_{min}}(y) &= 1 - P[X_{sr} > y \text{ and } X_{rb} > y] \\
&= 1 - \frac{\Gamma(m_{sr}, B_{sr}y)}{\Gamma(m_{sr})} \cdot \frac{\Gamma(m_{rb}, B_{rb}y)}{\Gamma(m_{rb})}.
\end{aligned} \quad (\text{B.8})$$

$$F_{X_{min}}(y - x) = 1 - \frac{\Gamma(m_{sr}, B_{sr}(y - x))}{\Gamma(m_{sr})} \cdot \frac{\Gamma(m_{rb}, B_{rb}(y - x))}{\Gamma(m_{rb})}. \quad (\text{B.9})$$

From [ [72], eq. (8.352,6)], we have

$$\Gamma(n, x) = (n - 1)! \exp(-x) \sum_{m=0}^{n-1} \frac{x^m}{m!}. \quad (\text{B.10})$$

Then using (B.10), we can rewrite (B.9) as

$$\begin{aligned}
F_{X_{min}}(y - x) &= 1 - \frac{1}{\Gamma(m_{sr})} \cdot (m_{sr} - 1)! \cdot \frac{1}{\Gamma(m_{rb})} (m_{rb} - 1)! \exp(-(y - x) B_{sr}) \\
&\times \exp(-(y - x) B_{rb}) \sum_{n=0}^{m_{sr}-1} \frac{(B_{sr}(y - x))^n}{n!} \sum_{i=0}^{m_{rb}-1} \frac{(B_{rb}(y - x))^i}{i!}.
\end{aligned} \quad (\text{B.11})$$



From [ [72], eq. (8.339,1)], we have  $\Gamma(m_{sr}) = (m_{sr} - 1)!$  and  $\Gamma(m_{rb}) = (m_{rb} - 1)!$ . Then, we can rewrite (B.11) as

$$F_{X_{min}}(y - x) = 1 - \exp(-y(B_{sr} + B_{rb})) \exp(x(B_{sr} + B_{rb})) \sum_{n=0}^{m_{sr}-1} \frac{(B_{sr})^n}{n!} (y - x)^n \\ \times \sum_{i=0}^{m_{rb}-1} \frac{(B_{rb})^i}{i!} (y - x)^i. \quad (\text{B.12})$$

Using the binomial expansion as in [ [72], eq.(1.111)], where  $(a + x)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$ , (B.12) can be written as

$$F_{X_{min}}(y - x) = 1 - \exp(-y(B_{sr} + B_{rb})) \exp(x(B_{sr} + B_{rb})) \sum_{n=0}^{m_{sr}-1} \frac{(B_{sr})^n}{n!} \sum_{k=0}^n \binom{n}{k} \\ \times (-1)^k x^k y^{n-k} \sum_{i=0}^{m_{rb}-1} \frac{(B_{rb})^i}{i!} \sum_{f=0}^i \binom{i}{f} (-1)^f x^f y^{i-f}. \quad (\text{B.13})$$

By substituting (B.13) into (B.6) and using (B.7) we have

$$F_{X_{total}}(y) = \int_0^y \left( 1 - \exp(-y(B_{sr} + B_{rb})) \exp(x(B_{sr} + B_{rb})) \sum_{n=0}^{m_{sr}-1} \frac{(B_{sr})^n}{n!} \right. \\ \times \sum_{k=0}^n \binom{n}{k} (-1)^k x^k y^{n-k} \sum_{i=0}^{m_{rb}-1} \frac{(B_{rb})^i}{i!} \sum_{f=0}^i \binom{i}{f} (-1)^f x^f y^{i-f} \left. \right) f_{X_{sb}}(x) dx \\ = \int_0^y f_{X_{sb}}(x) dx - \int_0^y f_{X_{sb}}(x) \exp(-y(B_{sr} + B_{rb})) \\ \times \exp(x(B_{sr} + B_{rb})) \sum_{n=0}^{m_{sr}-1} \frac{(B_{sr})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k \\ \times x^k y^{n-k} \sum_{i=0}^{m_{rb}-1} \frac{(B_{rb})^i}{i!} \sum_{f=0}^i \binom{i}{f} (-1)^f x^f y^{i-f} dx. \quad (\text{B.14})$$

Finally, the total CDF can be written after some manipulation as

$$\begin{aligned}
F_{X_{total}}(y) &= \frac{\gamma(m_{sb}, \gamma B_{sb})}{\Gamma(m_{sb})} - \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} \\
&\times \exp(-y(B_{sr} + B_{rb})) \int_0^y \left( x^{(m_{sb}-1)} \exp(-x(B_{sb} - B_{sr} - B_{rb})) \sum_{n=0}^{m_{sr}-1} \frac{(B_{sr})^n}{n!} \right. \\
&\times \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} x^k \sum_{i=0}^{m_{rb}-1} \frac{(B_{rb})^i}{i!} \sum_{f=0}^i \binom{i}{f} (-1)^f y^{i-f} x^f \Big) dx. \quad (\text{B.15})
\end{aligned}$$

# Bibliography

- [1] J. G. Proakis, *Digital Communication 4th Ed.* New York: McGraw Hill, 2001.
- [2] T. S. Rappaport, *Wireless Communication: Principles and Practice 2nd Ed.* Prentice Hall, 2002.
- [3] H. Schulze and L. C., *Theory and Applications of OFDMA and CDMA Wideband Wireless Communications.* Wiley, 2005.
- [4] C. Hsiao-Hwa, *The Next Generation CDMA Technologies.* Wiley, 2007.
- [5] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062 – 3080, Dec. 2004.
- [6] M. Uysal, *Cooperative Communications for improved wireless network transmission: frame for virtual antenna array applications.* IGI Global, 2010.
- [7] M. Dohler and Y. Li, *Cooperative Communications hardware, channel and PHY.* Wiley, 2010.
- [8] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity. part I. system description,” *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927 – 1938, Nov. 2003.
- [9] —, “User cooperation diversity. part II. implementation aspects and performance analysis,” *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939 – 1948, Nov. 2003.

- [10] Y. Zhao, R. Adve, and T. J. Lim, "Outage probability at arbitrary SNR with cooperative diversity," *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 700 – 702, Aug. 2005.
- [11] L. Venturino, X. Wang, and M. Lops, "Multiuser detection for cooperative networks and performance analysis," *IEEE Trans. Signal Processing*, vol. 54, no. 9, pp. 3315 – 3329, Sep. 2006.
- [12] X. Liu, X. Zhang, and D. Yang, "Outage probability analysis of multiuser Amplify-and-Forward relay network with the source-to-destination links," *IEEE Commun. Lett.*, vol. 15, no. 2, pp. 202 – 204, Feb. 2011.
- [13] J. Cheng and N. Beaulieu, "Accurate DS-CDMA bit-error probability calculation in rayleigh fading," *IEEE Trans. Wireless Commun.*, vol. 1, no. 1, pp. 3 – 15, Jan. 2002.
- [14] H. Boujemaa, "Exact and asymptotic BEP of cooperative DS-CDMA systems using decode and forward relaying in the presence of multipath propagation," *IEEE Trans. Wireless Commun.*, vol. 8, no. 9, pp. 4464 – 4469, Oct. 2009.
- [15] R. Lamare, "Joint iterative power allocation and linear interference suppression algorithms for cooperative DS-CDMA networks," *IET, Commun.*, vol. 6, no. 13, pp. 1930 – 1942, May 2012.
- [16] S. Verdu, *Multiuser Detection*. Cambridge, U.K.: Cambridge Univ. Press., 1998.
- [17] K. Vardhe, D. Reynolds, and M. C. Valenti, "The performance of multi-user cooperative diversity in an asynchronous CDMA uplink," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1930 – 1940, May 2008.
- [18] A. Eid, W. Hamouda, and I. Dayoub, "Performance of multi-relay coded cooperative diversity in asynchronous code-division multiple-access over fading channels," *IET, Commun.*, vol. 5, no. 5, pp. 683 – 692, Mar. 2011.

- [19] K. Vardhe, D. Reynolds, and B. D. Woerner, "Joint power allocation and relay selection for multiuser cooperative communication," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1255–1260, april 2010.
- [20] B. Wang and D. Zhao, "Optimum power distribution for uplink channel in a cooperative wireless CDMA network," in *Proc. IEEE International Conference on Communications (ICC '08)*, May 2008, pp. 4795–4801.
- [21] A. Mehemed and W. Hamouda, "Outage performance in cooperative CDMA systems over Nakagami- $m$  fading channels," in *Proc IEEE PIMRC*, 2011, pp. 1884–1888.
- [22] —, "Outage analysis of cooperative CDMA systems in Nakagami- $m$  fading channels," *IEEE Trans. Veh. Technol.*, vol. 61, no. 2, pp. 618–623, Feb. 2012.
- [23] —, "Af cooperative CDMA outage probability analysis in Nakagami- $m$  fading channels," *IEEE Trans. Veh. Technol.*, vol. 62, no. 3, pp. 1169–1176, 2013.
- [24] —, "Asymptotic outage probability for Amplify-and-Forward CDMA systems over Nakagami- $m$  fading channels," in *Proc. IEEE VTC*, Sept. 2012, pp. 1–5.
- [25] —, "Performance of turbo coded cooperative asynchronous DS-CDMA in frequency-selective fading channels," in *IEEE 25th Biennial Symposium on Communications*, May 2010, pp. 444–447.
- [26] P. Pahlavan, K. and Krishnamurthy, *Principles of Wireless Networks*. Pearson Education, Singapore, 2004.
- [27] D. Gesbert, M. Shafi, D. shan Shiu, P. Smith, and A. Naguib, "From theory to practice: an overview of MIMO space-time coded wireless systems," *IEEE J. Select. Areas Commun.*, vol. 21, no. 3, pp. 281–302, Apr. 2003.
- [28] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. on Telecom.*, vol. 10, no. 6, pp. 585–595, 1998.

- [29] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.
- [30] G. J. Foschini and M. J. Gans, "Layered space-time architecture for wireless communications in a fading environment when using multi-element antennas," *Bell Labs Tech J.*, vol. 1, no. 2, pp. 41–59, 1996.
- [31] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct 1998.
- [32] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.
- [33] V. Tarokh, A. Naguib, N. Seshadri, and A. R. Calderbank, "Combined array processing and space-time coding," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1121–1128, May 1999.
- [34] D. Gesbert, M. Shafi, D. Shiu, P. J. Smith, and A. Naguib, "From theory to practice: An overview of MIMO space-time coded wireless systems," *IEEE J. Select. Areas Commun.*, vol. 21, no. 3, pp. 281–302, Apr. 2003.
- [35] A. Sacramento and W. Hamouda, "Multiuser decorrelator detectors in MIMO CDMA systems over nakagami fading channels," *Wireless Communications, IEEE Transactions on*, vol. 8, no. 4, pp. 1944–1952, Apr. 2009.
- [36] —, "Performance of multiuser-coded CDMA systems with transmit diversity over nakagami- $m$  fading channels," *Vehicular Technology, IEEE Transactions on*, vol. 58, no. 5, pp. 2279–2287, Jun. 2009.

- [37] C.-C. Hung, C.-T. Chiang, N.-Y. Yen, and R.-C. Wu, "Outage probability of multiuser transmit antenna selection/maximal-ratio combining systems over arbitrary Nakagami- $m$  fading channels," *IET, Commun.*, vol. 4, no. 1, pp. 63–68, May 2010.
- [38] R. C. de Lamare and S. Li, "Joint iterative power allocation and interference suppression algorithms for cooperative DS-CDMA networks," in *Proc IEEE VTC*, 2010, pp. 1–5.
- [39] O. Amin, S. Ikki, and M. Uysal, "On the performance analysis of multirelay cooperative diversity systems with channel estimation errors," *IEEE Trans. Veh. Technol.*, vol. 60, no. 5, pp. 2050–2059, Jun. 2011.
- [40] S. Ikki and M. Ahmed, "Performance analysis of adaptive decode-and-forward cooperative diversity networks with best-relay selection," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 68–72, Jan. 2010.
- [41] E. Beres and R. Adve, "Selection cooperation in multi-source cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 1, pp. 118–127, Jan 2008.
- [42] T. Hunter, S. Sanayei, and A. Nosratinia, "Outage analysis of coded cooperation," *Information Theory, IEEE Transactions on*, vol. 52, no. 2, pp. 375–391, Feb. 2006.
- [43] A. Adinoyi, Y. Fan, H. Yanikomeroglu, H. Poor, and F. Al-Shaalan, "Performance of selection relaying and cooperative diversity," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 5790–5795, Dec. 2009.
- [44] S. S. Ikki and M. H. Ahmed, "Performance analysis of incremental-relaying cooperative-diversity networks over Rayleigh fading channels," *IET, Commun.*, vol. 5, no. 3, pp. 337–349, Feb. 2011.
- [45] N. C. Beaulieu and J. Hu, "A closed-form expression for the outage probability of

- decode-and-forward relaying in dissimilar Rayleigh fading channels,” *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 813–815, Dec. 2006.
- [46] W. Fang, L.-L. Yang, and L. Hanzo, “Performance of DS-CDMA downlink using transmitter preprocessing and relay diversity over Nakagami- $m$  fading channels,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 678–682, Feb. 2009.
- [47] N. Yang, M. El Kashlan, and J. Yuan, “Outage probability of multiuser relay networks in Nakagami- $m$  fading channels,” *IEEE Trans. Veh. Technol.*, vol. 59, no. 5, pp. 2120–2132, Jun. 2010.
- [48] J. Romero-Jerez, J. Martin, and A. Goldsmith, “Outage probability of MRC with arbitrary power cochannel interferers in Nakagami fading,” *IEEE Trans. Commun.*, vol. 55, no. 7, pp. 1283–1286, Jul. 2007.
- [49] S. I. Hussain, M.-S. Alouini, and M. O. Hasna, “A signal combining technique based on channel shortening for cooperative sensor networks,” in *Proc. IEEE World of Wireless Mobile and Multimedia Networks (WoWMoM)*, Jun. 2010, pp. 1–6.
- [50] H. Yu, I.-H. Lee, and G. Stuber, “Outage probability of decode-and-forward cooperative relaying systems with co-channel interference,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 266–274, Jan. 2012.
- [51] G. Femenias, “MGF-based performance analysis of selection diversity with switching constraints in Nakagami fading,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 9, pp. 2328–2333, Sept. 2006.
- [52] K. Yan, J. Jiang, Y. G. Wang, and H. T. Liu, “Outage probability of selection cooperation with MRC in Nakagami- $m$  fading channels,” *IEEE Signal Processing Lett.*, vol. 16, no. 12, pp. 1031–1034, DEC. 2009.



- [53] K.-S. Hwang, Y.-C. Ko, and M.-S. Alouini, "Performance analysis of incremental relaying transmission with adaptive QAM over nakagami- $m$  fading channels," in *Proc. IEEE Wireless Comm. and Mobile Computing Conf.*, Aug. 2008, pp. 458–463.
- [54] M. Benjillali and M. Alouini, "Outage performance of reactive cooperation in Nakagami- $m$  fading channels," in *Proc. IEEE SPAWC*, Jun. 2010, pp. 1–5.
- [55] G. Alexandropoulos, A. Papadogiannis, and K. Berberidis, "Relay selection vs. repetitive transmission cooperation: Analysis under Nakagami- $m$  fading," in *Proc. IEEE PIMRC*, Sept. 2010, pp. 140–144.
- [56] H. A. Suraweera, P. J. Smith, and J. Armstrong, "Outage probability of cooperative relay networks in Nakagami- $m$  fading channels," *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 834–836, Dec. 2006.
- [57] S. S. Ikki and M. H. Ahmed, "Performance analysis of adaptive decode-and-forward cooperative diversity networks with best-relay selection," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 68–72, Jan. 2010.
- [58] S. Ikki and M. H. Ahmed, "Performance analysis of cooperative diversity wireless networks over Nakagami- $m$  fading channel," *IEEE Commun. Lett.*, vol. 11, no. 4, pp. 334–336, Apr. 2007.
- [59] S. S. Ikki and S. Aissa, "Performance evaluation and optimization of dual-hop communication over Nakagami- $m$  fading channels in the presence of co-channel interferences," *IEEE Commun. Lett.*, vol. 16, no. 8, pp. 1149–1152, august 2012.
- [60] T. Pham and H. Nguyen, "Decorrelate-and-forward relaying scheme for multiuser wireless code division multiple access networks," *IET, Commun.*, vol. 4, no. 4, pp. 443–451, May 2010.

- [61] W.-J. Huang, Y.-W. Hong, and C.-C. Kuo, "Relay-assisted decorrelating multiuser detector (RAD-MUD) for cooperative CDMA networks," *IEEE J. Select. Areas Commun.*, vol. 26, no. 3, pp. 550 –560, april 2008.
- [62] E. Al-Hussaini and I. Sayed, "Performance of the decorrelator receiver for DS-CDMA mobile radio system employing RAKE and diversity through Nakagami fading channel," *IEEE Trans. Commun.*, vol. 50, no. 10, pp. 1566 – 1570, Oct. 2002.
- [63] X. Zhang, Y. Gong, and G. Xiao, "Multiuser detection for decode-and-forward cooperative relaying in DS-CDMA systems," in *Vehicular Technology Conference, 2009. VTC Spring 2009. IEEE 69th*, Apr. 2009, pp. 1 –5.
- [64] Y. Cao and B. Vojcic, "MMSE multiuser detection for cooperative diversity CDMA systems," in *Wireless Communications and Networking Conference, 2004. WCNC. 2004 IEEE*, vol. 1, Mar. 2004, pp. 42 – 47 Vol.1.
- [65] W. Chen and M. A. Do, "Performance analysis of adaptive MMSE reception for DS-CDMA in flat Nakagami fading channels," *IEEE Trans. Veh. Technol.*, vol. 49, no. 2, pp. 561 –564, Mar. 2000.
- [66] N. C. Beaulieu and C. Cheng, "Efficient Nakagami- $m$  fading channel simulation," *IEEE Trans. Veh. Technol.*, vol. 54, no. 2, pp. 413 – 424, Apr. 2005.
- [67] M. Nakagami, "The  $m$ -distribution, a general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*, 1960.
- [68] M. Soysa, H. A. Suraweera, C. Tellambura, and H. K. Garg, "Multiuser amplify-and-forward relaying with delayed feedback in Nakagami- $m$  fading," in *Proc. IEEE Wireless Comm. and Networking Conf. (WCNC)*, Mar 2011, pp. 1724 –1729.
- [69] F. Xu, F. Lau, Q. Zhou, and D.-W. Yue, "Outage performance of cooperative com-

- munication systems using opportunistic Relaying and selection combining receiver,” *IEEE Signal Processing Lett.*, vol. 16, no. 4, pp. 237–240, Jan. 2009.
- [70] N. C. Sagias, F. I. Lazarakis, G. S. Tombras, and C. K. Datsikas, “Outage analysis of decode-and-forward relaying over Nakagami- $m$  fading channels,” *IEEE Signal Processing Lett.*, vol. 15, pp. 41–44, Jan 2008.
- [71] G. C. Alexandropoulos, A. Papadogiannis, and K. Berberidis, “Performance analysis of cooperative networks with relay selection over Nakagami- $m$  fading channels,” *IEEE Signal Processing Lett.*, vol. 17, no. 5, pp. 441–444, May 2010.
- [72] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. San Diego, CA: Academic, 1994.
- [73] G. E. Roberts and H. Kaufman, *Table of Laplace Transformers*. W. B. Sannders company, 1966.
- [74] P. G. Moschopoulos, “The distribution of the sum of independent gamma random variables,” in *Annals Instit. Statist. Math*, vol. 37, no. 3, 1985, pp. 541–544.
- [75] M. S. Alouini, A. Abdi, and M. Kaveh, “Sum of gamma variates and performance of wireless communication systems over Nakagami- $m$  fading channels,” *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1471–1480, Nov. 2001.
- [76] G. K. Karagiannidis, N. C. Sagias, and T. A. Tsiftsis, “Closed-form statistics for the sum of squared Nakagami- $m$  variates and its applications,” *IEEE Trans. Commun.*, vol. 54, no. 8, pp. 1353–1359, Aug. 2006.
- [77] R. Lupas and S. Verdu, “Near-far resistance of multiuser detectors in asynchronous channels,” *IEEE Trans. Commun.*, vol. 38, no. 4, pp. 496–508, Apr. 1990.
- [78] A. Assra, W. Hamouda, and A. Youssef, “BER analysis of space-time diversity in

- CDMA systems over frequency-selective fading channels,” *IET, Commun.*, vol. 3, no. 7, pp. 1216–1226, Jul. 2009.
- [79] M. AlJerjawi and W. Hamouda, “Performance analysis of multiuser DS-CDMA in MIMO systems over Rayleigh fading channels,” *IEEE Trans. Veh. Technol.*, vol. 57, no. 3, pp. 1480–1493, May 2008.
- [80] O. Brugia, “A noniterative method for the partial fraction expansion of a rational function with high order poles,” *SIAM Review*, vol. 7, no. 3, pp. 381–387, Jul. 1965.
- [81] A. Papoulis and S. U. Pillai, *Probability, Random Variable and Stochastic Processes*. McGraw Hill, 2002.
- [82] M. Abramowitz and A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*. New York: Dover, 1964.
- [83] M. K. Simon and M. S. Alouini, *Digital Communication over Fading Channels*. New York: Wiley, 2005.
- [84] S. Atapattu, N. Rajatheva, and C. Tellambura, “Performance analysis of TDMA relay protocols over Nakagami- $m$  fading,” *IEEE Trans. Veh. Technol.*, vol. 59, no. 1, pp. 93–104, Jan. 2010.
- [85] Y.-K. Song and D. Kim, “Convergence and performance of distributed power control algorithms for cooperative relaying in cellular uplink networks,” *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4645–4651, Nov. 2010.
- [86] W. Xu, L. Wang, and G. Chen, “Performance of DCSK cooperative communication systems over multipath fading channels,” *IEEE Trans. Circuits Syst.*, vol. 58, no. 1, pp. 196–204, Jan. 2011.
- [87] S. V. Amari and R. B. Misra, “Closed-form expressions for distribution of sum of

- exponential random variables,” *IEEE Trans. Rel.*, vol. 46, no. 4, pp. 519 –522, Dec. 1997.
- [88] M. K. Simon and M. S. Alouini, *Digital Communication Over Fading Channels*. New Jersey: Wiley, 2005.
- [89] W. Tam, T. Lok, and T. Wong, “Power-minimizing rate allocation in cooperative uplink systems,” *IEEE Trans. Veh. Technol.*, vol. 58, no. 9, pp. 4919 –4929, Nov. 2009.
- [90] W.-H. Sheen, S.-J. Lin, and C.-C. Huang, “Downlink optimization and performance of relay-assisted cellular networks in multicell environments,” *IEEE Trans. Veh. Technol.*, vol. 59, no. 5, pp. 2529 –2542, Jun. 2010.