En-Route Flight Planning: A Mathematical Modeling Approach for Operating Cost Minimization, Dynamic Speed Control and Mid-air Collision Avoidance

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ABSTRACT

EN-ROUTE FLIGHT PLANNING: A MATHEMATICAL MODELING APPROACH FOR OPERATING COST MINIMIZATION, DYNAMIC SPEED CONTROL AND MID-AIR COLLISION AVOIDANCE

GOLBARG MOEINI

The presented study discusses the Air Traffic Flow Management Problem by introducing alternative routing options for aircrafts in a constrained airspace. This work aims to minimize the total cost while all safety constraints such as mid-air collision avoidance and separation distance between aircrafts are respected. In this regard, a mixed integer programming (MIP) model has been developed by using a non-time indexed modeling strategy that benefits from a 3-dimensional (3D) network. The model provides the flight-route with a list of consecutive nodes to be visited by an aircraft and calculations on speed changes, and exact arrival and departure times on each travelling arc. Therefore, the separation distance between aircrafts on the network will be guaranteed despite of high travelling speeds over the arcs to avoid mid-air collisions. Designed for a single airport arrival and departure instances, the NP-hard nature of the MIP model doesn't prevent large problems to be solved on a personal computer using CPLEX, but also provides real-time decision making possible through performing an iterative solution. To conclude the results for various air traffic capacities and verify mid-air collision avoidance, a simulation model has been developed using ARENA simulation software by Rockwell.

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1 Introduction

With considerable increase in demand for airline travelling services, the system faces consistent challenges globally. Statistics shows increase in demand for flight services despite the destructive impact of 2001 terrorist attacks and consequent financial crisis throughout the decade since then. International Air Transportation Association (IATA) has reported 125 billion dollars increase in the Revenue Passenger Kilometers (RPK) and 150 billion dollars rise in Available Seat Kilometers (ASK) from 2002 to 2012 for international scheduled passenger traffic [42]. Federal Aviation Administration (FAA) reported increase in air traffic by 3.5% in 2012 [39]. Moreover, it estimates that the number of passengers carried will increase from 731 million passengers in 2011 to 1.2 billion in 2032. Therefore, ever more industries rely on the air transportation systems to accelerate their commercial operations. Alongside long-haul flights which are still being dominated by major carriers such as United Airlines, Lufthansa or Singapore Airlines, such growth in demand will result in new relatively smaller airline companies to become a major player in short flight services such as inter-country flights in North America, China, and largely in Europe. This will brought the competition to the next level in which the airline travelling is becoming necessity rather than luxury.

Although these statistics exhibit a prosperous and growing air travelling business, the airline industry is facing serious challenges to answer the increasing demand. Congested airport terminals, frequent delays, limited airspace capacity around the airports particularly in North America, and crowded airspaces between airports in Europe are some of the challenges to tackle. Furthermore, instable and increasing fuel prices in recent years, alongside increasing labor costs are putting many airline companies in extreme financial difficulties. On the other hand, unpredictable weather conditions in most parts of the world result in terrific air travel delays. A report prepared by Airlines for America states 103 million system delay minutes for 2011 which has resulted to $7.7 billion in direct aircraft operating costs for scheduled U.S. passenger airlines. Among it, the cost of aircraft block (taxi plus airborne) time was estimated $75.27 per minute [43]. Moreover, FAA has announced that 24% of all the domestic flights in 2007 were 15 minutes late and 2% of the flights were cancelled. Studies show that these delays can be divided into 38% because of late arriving aircraft induced from air congestions, 29% because of airline managements, 28% affected by national aviation system, and only 6% of these distributions is caused by weather conditions [43].

According to statistics and reports congestion and air traffic control is the main dilemma for the day to day growing airline industry and the subsequent demanding services put serious pressure on air traffic controllers (ATC). Congestions occur when traffic load passes the arrival or departure capacity of the airports or the air sector capacities or an unforeseen event like a sudden change in weather condition. However, different geographies are facing different issues. US because of its vast land, possesses more air spaces that brings less congestion in its air sectors and more in its airports arrival and departure capacities. On the other hand, European countries have the opposite situation and face the problem with air sector congestion and interactions between airports rather than their airport runway capacities.

Controlling the air transportation traffic whether in ground phases in airport or in flight phase in air space is handled by the ATC (Air Traffic Controller). ATC should manage the traffic and assure the flights safety by preventing any incident from taxing to navigating in open skies. They instruct pilots with on-time service for upcoming route, resulting altitude, change of speed, and all other flight characteristics. FAA in its Long Range Forecasts of Workload Measures, have estimated the increase rate of tower operations 56.9% by 2030. On the other hand, the Air Route Traffic Control Center (ARTCC) is waiting for over 100% increase on control operations [41].

Therefore, increasing air traffic is becoming uncontrollable for ATC personnel to effectively determine safe flight plans for aircrafts. Each year, several airports in USA report that long operating hours, highly stressful working environment and continuously increasing air traffic conditions cause poor decisions from ATC personnel result in incidents which are frequently handled through sophisticated traffic alert and collision avoidance system (TCAS) such as mid-air collision avoidance system (MIDCAS) imbedded in airplanes [27]. Moreover, under high volume air traffic, ATC personnel frequently compromise economic objectives of airlines for the safety of flights and passengers. Therefore, the resulting issues such as delays, early arrivals, and fuel consumption costs are distributed and imposed unevenly among airlines.

As the statistics illustrate, the pressure on the ATC operations is a concerning issue that requires having a universal decision making support system to leverage the air traffic and guarantee the safety of the passengers and profitability of airlines. This supporting system should be able to introduce new strategies to overcome the capacity limitations and answer the demand growth. In recent years, many groups and studies have worked on this problem whether in academic section or industry level like NASA. Most of these studies are following the same objective, and developing new tactics to reduce the flights delay and subsequent congestions along reducing ATC work load.

Rescheduling and rerouting the flights and air traffics have been the focal key of this subject. However, transferring flight planning duty from ground based ATC to individual aircraft system and reducing the ATC task to only monitor and manage the system with much less workload is a new topic in this field that have been received lots of attentions in recent years. For instance, the new ongoing NASA air space program called Next Generation Air Transportation System (NextGen), is another emphasis on the importance of these convertible models. NextGen proposes to reallocate the ground based unit to the satellite based and use GPS monitoring to ensure a more accurate control over the network. This new precision concludes to tighter safety distances, better prevention of collision, more reasonable route selection, less congestion and delays that all together leads to have new capacities in the network.

Consequently, an en-route decision making control system, in theory, can assist the pilot to determine the optimal flight path in real-time with an objective to optimize the overall performance of aircrafts in a given airspace while ensuring all safety rules even when the airspace is heavily congested. This matter not only decreases the pressure on the ATC but also gives additional capacity for administering higher rates of traffics and sustaining upcoming growth in air travelling industry.

1.1 Contribution of This Research

This work studies the Air Traffic Flow Management (ATFM) problem to reduce the ATC workload. This research work particularly focuses on delays and congestions during flights' landings and take-offs in a micro scale level. This study aims to optimize the total cost of flight by assigning them the best route within the given schedule (arrival times to the airspace are known). The model developed in this work considers all the airport flight zone and flights characteristics to provide a real sense of the situation. Flights schedule, speed maneuvers, safety distance and conflict avoidance are some of the factors that are mentioned in this optimization model. This model is able to guide flights from their start point to the end in the given airspace by assigning them to the best available routes, adjusting their speed through the entire route and ensuring the flight's safety from air-collision with respect to other flying aircrafts. Consequently a minimum flight cost for all aircrafts is ensured without jeopardizing the safety.

The main contribution of this thesis stands on that the model which is designed on continuous time basis, therefore there is no discrete time windows used in its formulation. Moreover the rerouting option which allows flights to deviate from their original path is considered. Flight phases, maneuvers and speed changing are included in the model. Not only flights safety distances are respected but also three set of conflict constraints are designed to assure prevention of mid-air collision.

Respectively, the novelty of this study is in consideration of two aspects of ATFM problem: i) a decision support system to be used by ground based ATC; or ii) a decision support system for pilot to use as part of the free-flight concept. The designed model and its computational performance is suitable to be used by the ground based ATC to control the stream of air traffic in its desirable size. Alternatively, the proposed model can be used by a pilot to find the best flight plan for the given airspace conditions. In both cases the safety is assured within strong conflict constraints of the model. The model gives ATC the power to optimize the flight routes based on equal chances or to weight them by the first come first serve order. In both scenarios the downstream data will be used for the upcoming traffic. After optimizing the schedule for the set of flights, the matching flight schedule is announced by ATC. But if the model be used by the pilots directly, he will get the downstream traffic data from the ATC and implement it to find its own route and report back to the ATC. In this case, ATC duty is to transfer data between flights. Thus computation load is divided between flights instead of all be assigned to the ATC. Thereafter, the time that is saved from ATC can be used for a better communication between flights. In this case, since each flight determines its own schedule, the dependency of the flights is reduced to the safety criteria. Therefore, the system can adopt and recover itself faster to any new disturbances such as unpredicted weather situation, which is one of the most important issues in ATFM problem.

The presented thesis unfolds as following: chapter 2 provides the literature review. The mathematical formulation of the model accompanied with the explanation of its notation is stated in chapter 3. In chapter 4 the solution to the model is offered. Chapter 5 illustrates the results derived from the model implementation. Chapter 6 provides a simulation model for mathematical model verification. Finally, the conclusion and future works are presented in chapter 7.

2 Literature Review

2.1 Air Traffic Flow Management Problem

Studies on air traffic problem have been categorized into several sub-domains. The work has been started with a single airport ground holding problem with ground holding policy. To better handle the air traffic around the airport, assuring the safety, and decreasing the fuel consumption, FAA set a policy to prioritize the arrival flights to land, rather than departure flights to take off as long as their arrival to the destination is not jeopardized. Later, the research was extended to multi airport ground holding problem and the relation between different airports respecting their limitations has been studied. Afterwards, air traffic control problem has been introduced. Despite the previous works which were concentrated on the ground handling and flight phases around the airport (i.e. landing and takeoff the flights), air traffic control brought new approaches by taking account the air sectors and the traffic of the airspace. This new line of research work has been more beneficial for European industry that is dealing with congestion over the air, rather than overcrowded airports of North America. The fundamental of ATFM problem led to a new area called Air Traffic Flow Management Rerouting problem (ATFMR).

2.1.1 ATFM Studies

One of the first ATFM optimization model was introduced by Odoni [39]. He formulated a model and provided the algorithm for optimal ground holding pattern, Krozel et al. [28] presented a routing and scheduling algorithm for ATFM including ground delays, route selection and airborne holding, aligning with Collaborative Decision Making philosophy.

Lulli and Odoni [21] have developed a deterministic model mostly based on Odoni [39]. Their work focuses on ground and airborne delays and also the conflict between efficiency and equity in the air traffic problem in European countries. Therefore, the en-route capacities are more important and fairness of scheduling has been discussed thoroughly. Their results show that in some cases assigning an airborne delay rather than ground holding to a flight in terms of total delay can be considered. The model is built on discrete time basis, deterministic demand, deterministic capacity, known location of airborne delay, equal speed for all flights and no option for rerouting. These assumptions give the flexibility to apply two decision variables to each flight for controlling the ground holding delay at the origin airport and airborne delay before landing at the destination airport.

Mukherjee et al. [23] have developed a dynamic stochastic model in answer to the Single Airport Ground Holding problem. They have improved the work of Richetta and Odoni [36] by adding the revision placements. Their model not only has the ability to change the assigned ground delay to the flights based on the upcoming weather conditions, but also its methodology gives airlines the power to cancel or replace the flights and reconsider the delay costs. Moreover, efficiency and equity has been respected.

Gupta et al. [15] have developed a deterministic mixed integer model for scheduling flights departure at airport runways. They have tested different scenarios just as first in first out or simple parking area with no dependency. They have considered flights safety, efficiency, and the equality of the scheduling. The problem is designed within a double assignment problem for flights departure scheduling, so that firstly, flights are delivered in a sequenced queue, and secondly, they have to take off within a specific time window by obeying separations constraints. On the other hand, throughput, system delay and maximum delay are being defined as the model objective function.

Churchill et al. [11] have presented a research about the influence of flight propagated delay on the strategic air traffic management. Two models have been developed in their work. The first model is designed to distinguish the propagated delays that an aircraft faces during several flights. On the other hand, second model studies the relationship of the delays to the airports.

Bertsimas et al. [9] have studied the ATFM problem with the concept of fairness of the delay. They have focused to optimize the fairness of the common rule of first scheduled first served with the imposed conflicts. They have initiated fairness metric and have tried to minimize the deviation from it by designing an integer programming model based on the Bertsimas [35] and incorporated with an exponential penalty tactic. As a result of their work they have showed the inefficiencies of the current fairness procedures in use and moreover adaptability of their fairness metric with the current industry expectation. They have tested their model based on the historical derived data from 2007 and claimed to be applicable to nationwide scale.

Agustin et al. [2], Glover et al. [3],Yoon et al. [10], Sun et al. [6] and D' Ariano et al. [7] are the other important works in ATFM research.

2.1.2 Capacitated ATFM Studies

There are many studies that have considered airspace and airport capacities in their models since congestions and delays are result of dismissed available capacities in the system.

Lindsay et al. [37] has designed a binary IP model to minimize the total delay (ground and airborne) with respect to airport and airspace capacity constraint for individual flights.

Bertsimas and Patterson [39] have presented a new model that in compare to its previous works has a more reasonable computational time which makes it applicable to use on a real size network. In their work, by considering the en-route capacities, a set of flights have been introduced that is based on their origin and destinations, and the air sectors they pass through. The model intends to reduce the total cost by reducing the airborne and ground delays, and also taking care of the connectivity between air sector, airport and time.

Dell'Olmo et al. [32] have approached the ATFM problem by studying the trade-off between airport's landing and take-off and the airport's capacity restriction. They have incorporated a capacity envelope for the airport to have a more realistic analysis of the situation. The envelope is defined as a grouping of arrival and departure in a time window. The model is developed based of the dependency of the flights, unlike the other works that are based on acceptance arrival rate in single airport ground holding problems and added with acceptance departure rate in multi airport ground holding problems.

Filar et al. [24] have presented an adaptive model to optimize the imposed cost in occurring situation that leads to fluctuation in the available capacity of the airspace. In their model all the participated parties including passengers, airline and airport corporations have been paid attention. They have not only considered ground holding policy but also have used airborne delay, flight cancellation and designed a model to adjust the aircrafts schedule both in air and in airport. Later, the model has been tested in Sydney airport of Australia.

Rios et al. [14] have applied Dantzig-Wolfe decomposition method to the Bertsimas [35] model. They have suggested that this approach overcomes the computational difficulties in their model and makes it practical to be used in everyday industry. They have introduced each flight as a sub-problem and coupled it with the capacity constraints as connecting constraints. Moreover they have applied two heuristic to conclude to an integral solution. Based on the results, it is been claimed the near optimal solution is obtained in a much faster runtime.

Other related literature on capacitated ATFM research are Agustin et al. [1] & [2], Churchill et al. [4], D' Ariano et al. [7], Mukherjee et al. [13], Weber et al. [25] and Ma et al. [30],

2.1.3 Stochastic ATFM Studies

Following is a review on studies focused on uncertainty conditions like weather condition that reflects in capacity uncertainties.

Ma et al. [30] have presented a model on multi-commodity dynamic network flow for short term air traffic flow management validated by the Beijing's ATC data. Their model is based on China's national airspace system including airports, sectors, air routes, transfer of control points, and navigation points. They have designed the model in a way that in case of sudden decrease of capacity, for example bad weather in 2-3 hours it can recover the flow and assign new delays to flights.

Weber et al. [25] have believed that absence of the models to measure the reduced capacities within a reasonable time which leads to defining strategies to overcome the situation is the main issue. Therefore, they have focused on the tools used to optimize the forced departure delays, and vast their core model by adding the en route capacities to it.

Wan et al. [19] have studied the uncertainties and the restrictions applicable to control the system, such as ground delay programs, and developing strategies based on them.

McCrea et al. [17] have designed a model that considers the probabilistic weather condition and attempts to reroutes the flights by avoiding the collision and respecting airline equity and other criteria within large scale networks.

Mukherjee et al. [13] have studied the AFTM problem with weather uncertainty based on Ball et al [31]. They have developed a linear integer dynamic rerouting model which focuses on a micro scale network including a single airport and its connected airspace, and also their capacity variations due to weather condition. The model considers ground delays to be static while the rerouting decisions are dynamic. In other word, the rerouting actions first place by the end of each fixed points, and after that, based on the current situation of the network and the updated weather condition, flights can take off.

Andreatta et al. [5] have focused on three aspects of airport arrival and departure trade-off, uncertainty in airport capacities, and interactions between different hubs all at once in an aggregate mathematical model. They have developed a model to optimize the mix of flight times and flight delays in a combination of different airports. Their model suggests that which flights should be delayed during each time window rather than assigning delay times to each single flight. They have studied the ATFM problem in a microscopic level and having the authority of decision making. In other word, they have designed a model that determines number of the flights that should be delayed and it would be airline's duty to select which flights to choose and delay.

Glover et al. [3] have designed a two stage stochastic integer model with two objectives of equity and efficiency for ground delay program. The model assigns arrival slots to flights with regards to Ground Delay Program. Based on weather conditions fluctuations, it uses different capacity scenarios. For model simplification, they have assumed just two weather conditions, clear and unclear, in order to reduce the model complexity from multi stochastic IP to two stochastic IP. Another assumption they have made in their model is that there is no airborne holding option rather that the ground holding.

Yoon et al. [10] have developed a geometric model in this regard. In their model they have incorporated a mixed of ground delay and rerouting to minimize the flights cost. Their works oppose to the traditional approaches which delay flights in the airport until the weather clears. They have included four factors of storm size, location, maximum duration, and also the ratio of ground verse airborne cost in their dual hybrid model.

Churchill et al. [4] have addressed a stochastic integer model, compatible with the current principles of collaborative decision making, to harmonize the resource allocation in ATFM problems while these resources whether airports or congested airspaces are being accessed with different authorities in independently processes. The model optimizes the coordination of the air traffic by assigning flights arrival time to congested resources. Afterward, a set of linking constraints between sequenced congested resources are applied to ensure each flight using multiple resources, has been assigned to a well-matched slot. Based on the uncertainty included in the model, a two stage stochastic formulation is used to develop an initial and conditional plan. This formulation is only manageable for single capacity condition change.

Agustin et al. [1] have developed a deterministic model based on the Bertsimas and Stock's work [35], and later designed a 0-1 stochastic version of it based on simple and full recourse policies.

2.1.4 Rerouting in ATFM Studies

The presented work suggests no predetermined route for flights and allows flights choose their best route within all the potential paths available. These works introduce a new domain in ATFM problem which allows flights the rerouting option.

One of the earliest rerouting algorithms was proposed by Helm [38] who presented a multi-commodity minimum cost flow on a time space network.

Bertsimas et al. [9] have presented an integer programming model which can be applied for large scaled network like United States or European instances. Their presented model uses base mathematical model of [35] and [34] to tackle problem efficiently in large size networks. They have covered all phases of flight, take off, cruise, and landing, and cooperate all the management options like ground holding, airborne holding, speed changes, and most important of all, rerouting into their model. For applying the rerouting option to their work, they have added new constraints that assure the local constraints perform for both means, routing and rerouting, without defining new rerouting variables. Unlike the previous work which considered flights routing as a set of predetermination of continuous air sectors, they defined sets of possible routes. And at last, in order to enhance the model computational time, they have applied three classes of valid inequalities to assure the relaxations.

Agustin et al. [2] have presented a mixed 0-1 deterministic model that considers flight cancelation and rerouting to study AFTM problem. They have developed a model that doesn't need branch and bound implementation for the optimality. The model is based on short term policy and is tested for large scale networks. As their objectives, different kinds of cost have been introduced like cost of travelling time, penalty of deviation from the scheduled time, penalty of deviation from the scheduled routes, and ground and airborne delay costs.

See also Yoon et al. [10], D' Ariano et al. [7], McCrea et al. [17], Sun et al. [6], Bertsimas et al. [34].

2.1.5 Mid-air Collision Avoidance in ATFM

One of the important aspects in ATFM modeling is assuring the safety and air collision avoidance. In this work three sets of safety and conflict constraints are introduced to the model which prevents mid-air collision through entire flight journey. These constraints also assign a separation distance between flights to avoid the wake turbulences.

Rathinam et al. [18] developed the model that was firstly introduced by Bertsimas et al. [35] to be used in optimizing the aircraft taxi times in airports. In their work the aircraft types are considered as a factor to determine the separation between them and also conflict constraints are being introduced to the model. Later the result is compared with the current used techniques of First Come First Serve.

D' Ariano et al. [7] have focused on the flight conflict detection in the airport terminal maneuvering area. They have studied safe airborne decisions in congested airports by considering rerouting and scheduling the flights. Later, they have added some realistic factors like speed changing and airspace capacities as well as conflict constraints to their work. They have tried to optimize the use of the runways and airways between flights and balance the flights delays. In their work, airport terminal features like runways, air segments and holdings have been modeled.

Alonso et al [8] have studied collision prevention concept in Air Traffic Management problem. They have designed a mixed integer linear model which adjusts flights speed and altitude for their best performance by avoiding the air collision. In each of their proposed configuration, they have tried to minimize the flight changes with regards to safety parameters and forced flights to return to their initial formation for a safe journey. Moreover, in their study various separation distances and wind effect are considered.

2.1.6 Network Design in ATFM Studies

Each study has proposed a different design for the airspace in their network. Dell'Olmo and Lulli [33] have proposed a mixed integer heuristic model with a free flight path scenario that considers a network with no fix routes for European air traffic network. They have designed a two level hierarchical architecture which the first level considers each flight as a single commodity with its own specifications such as departure and arrival time, and the second level stands for single airway that restricts the flight travel.

Grabbe et al. [29] have suggested a new coordination toward user-preferred paths. Traditionally, and to this very day, formation of the air routes are Central East Pacific, they have studied a new approach to serve the air users by providing them a minimum travel time and optimal wind dynamics with neglecting the formation of the Central East Pacific routes. As the result, their model has claimed to save users time and traveling distances. However, from the ATCs point of view the sectors work load has not changed a lot.

Geng et al. [22] have presented a new approach toward ATFM, with Dynamic Air Route Open Close Problem (DROP) definition. The main contribute of DROP is that it considers shortest occupancy of time over routes in a cost based objective which has not been included in ATFM studies. They have two main groups of military and civil users and criticized the traditional use of routes which allows users to occupy certain routes permanently. They suggested every user to have a temporary access based on its demand.

Following to their previous study, Geng et al. [16] proposed a new approach named Dynamic Air Route Adjustment. Alongside ATFM studies that aim to have the best use of the current airspace, they focus on introducing temporary new air sectors to the air network for civil and military users. This approach primarily has been conducted by Chinese air managements. The objective of this study is to minimize the delay cost of flights and also the cost of using extra space for congested flight zones.

Churchill et al. [12] have introduced an airspace volume containing airspace and airports. They have defined set of possibilities for each flight as entry and exit points for each volume. Their work is based on the latest work of Bertsimas et al. [9] and tries to improve the ensuing model with a three tiered framework of strategic, tactical and opportunistic. In this work the airspace is divided into a 3-D mesh network including nodes which are connected by arcs and are used as the flight possible paths. In the first layer, they generate an appropriate continent wide model, which can be run several times per day. This layer uses the recent updates of weather forecasts and weather history. In the next layer, the model answers to the need of a specific airport or a congested air sector, while the third layer controls the unexpected upcoming events such as weather conditions at a specific region.

See also Gupta et al. [15], McCrea et al. [17].

2.1.7 Speed Control in ATFM

One of the control tools in flights mission is the capability of speed adjustment. Bertsimas and Patterson [34] have introduced an integer, multi-commodity dynamic network flow model which is proposed for a single airline. This work is based on the European airspace condition and weather uncertainties aims to solve the problem with flexibility to change the flight route, adjusting the flight time and speed to reduce the delay cost.

Sun et al. [6] have introduced an integer linear aggregate air traffic model for the ATFM problem. The aim of their work is to optimize the total travel time in the scale of National Airspace System of United States based on a multi-commodity network. The optimal route and optimized delay are assigned to the flights based on their departure, destination and speed control. Afterward ground delay policies can be added to the model by including the airport in the network. For better computational time a dual decomposition method is applied to break the model into small linear sub problems. On the other hand, the provided iterative algorithm gives the flexibility of having local variables in each sub problems, for instance, different weather conditions.

Flener et al. [26] have conducted a study to minimize the air traffic complexity within each air sector. Air sector traffic is affected by the number of flights traveling within and near its boarders. Therefore, they have considered different factors to manage this complexity such as, changing the flights take off time and their arrival time to that specific air sector, optimizing speed and re-arranging their traveling point's altitude within the air sector.

See also Bertsimas et al. [9], Alonso et al [8], and D' Ariano et al. [7].

2.2 Critical Review of Literature with Respect to the Proposed Thesis

This work approaches the ATFM problem by designing a model standing on continuous time basis, unlike other works, which are mostly built based on discrete time windows. In this model, time is introduced as a decision variable. On the other hand, model has the strength to use the rerouting option all over the network, which means there is no predetermined route for flights. Therefore, based on the imposing situation to the network, such as current and upcoming traffic, airport and airspace congestion and weather condition the model can effectively optimize every flight path individually. Moreover, the model has the flexibility to change flight speed based on the necessity at every arc through flight journey, which has not been done explicitly in any other literature until now. The defined objective of this work is considering airborne delay cost, ground delay cost, and fuel usage cost sensitivity to speed change as the total cost. Most previous works modeled the fuel consumption cost as a function of delay times. The variation of the speed which inherently moves the fuel usage has not been taken in to account. The model is fixed so that the prioritizing based on the flights penalties and characteristics is fairly respected. Another bright point of this current research is introducing a set of conflict avoidance constraints that are based on flights separation requirements, which prevent any mid-air collision during flight phases.

There are main characteristics to this study that can be classified as followed:

- Problem size
    - Micro scale network
    - Considering airspace as nodes and connected arcs
    - Single airport
- Optimization model specification
    - Mathematical
        - Deterministic, MIP model
    - Theoretical
        - Optimizing the total cost including the ground delay, airborne delay, and fuel expenses
        - Rerouting
        - Speed adjustment
        - Continuous time basis

2.3 Literature Review Summary

In this chapter, different studies on Air Traffic Management topic were reviewed and for further improvement, this work is concluded. The contributions of this work are designing a non-time indexed formulation, incorporating the rerouting option, speed control and collision avoidance constraints. Hence, the goal of the model is to track the flight from its origin to destination, monitor speed and timing in each node and arc, and give the possibility of rerouting precisely while respecting the collision avoidance. Additionally, in view of the average flight speeds at each arc, estimating the real fuel consumption cost is possible. The proposed model enables us to optimize the aircraft routing by minimizing traveling times, delay cost, operating cost, fuel cost, air and sound pollution, whereas safety and technical constraints are respected. Moreover the elegant formulation enables the optimal usage of the airspace by reducing the safety distances between aircrafts. This model is designed for micro scale network sizes of the airport air controller zone. Although the primary execution time of the model has been quite satisfactory, an enhanced version of the model has been added to make it easier for an ordinary desktop computer to solve.

3 Formulation of Aircraft Routing Problem

This section is devoted to the presentation of mathematical formulation used to determine aircraft routing problem. The objective of the model is to decide flight schedule details over an airspace network for the shortest travel time with minimum cost incurred between two opposite extremities. While aircrafts enter 3D mesh network from one of the two dummy nodes $(N_D^1 \ and \ N_D^2)$, safety constraints ensure their minimum separation distance from other flights in progress to avoid mid-air collisions.

Aircraft's visiting nodes are labeled as N and their connecting arcs labeled as $\varDelta$. In order to navigate in the airspace, aircrafts strictly follow neighbor nodes using the connecting arcs. There are three types of nodes in the system. First, the arriving nodes which are nodes located outside of the mesh. An aircraft arriving to the airspace enters network through one of these outside nodes. Similarly, an aircraft, intended to leave the airspace has to reach one of these nodes. The second types of nodes are the ground nodes where an aircraft uses the node for landing and departure. Finally the internal nodes are the connecting nodes that enable an aircraft to establish the route between the ground nodes and the external nodes.

The mathematical formulation provides a concrete flight plan including the traveling route of each aircraft $(\mathcal{R}^f)$ between nodes of the network along with the average speed over the connecting arcs. The breakthrough of this research is achieved by introducing non-time indexed formulation to calculate the exact and continuous arrival and departure times to each node in the network. It also helped to incorporate separation distances between flights throughout the 3D network. Furthermore, the application of the non-time indexing provides us with the exact travelling time over the length of each arc to determine the speed changes of aircraft and consequently to evaluate the better estimate over fuel cost.

To further signify the importance of non-time indexing approach, it is worth to mention that most literatures on the topic provide discrete time periods (time-indexed strategy) where the departure and arrival to each node over the network can only fall into calculated time periods. Since the aircraft flies at high speeds in the air, even small deviations from the intended flight times may lead to low safety clearance for mid-air collisions.

While the model is discussed as a planning tool for multi-aircraft routing, it can be used as a route planner for a single aircraft where the routes of all other aircrafts in the air-space are used as input parameters.

3.1 Input Parameters

Below, the definitions of the parameters, used in the model are given.

F	Set of flights, ($\forall f \in F$)
N	Set of nodes, ($\forall i \in N$)
Δ	Set of arcs, ($\forall \delta \in \Delta$)
Δ^f	Set of arcs makes the optimal route for flight f
S	Speed of an aircraft on arc δ, ($\forall v \in S$)
Λ	Predefined set of speed changes between two consecutive arcs, ($\forall \lambda \in \Lambda$)
ent^f	Entry point of flight f, ($ent^f \in \{N_D^1, N_D^2\}$)
ext^f	Exit point of flight f, ($ent^f \in \{N_D^1, N_D^2\}$)
e^f	Earliest possible arrival time of flight f to its destination point
l^f	Latest possible arrival time of flight f to its destination point
${P_E}^f$	Penalty cost of flight f for arriving or departing early
${P_L}^f$	Penalty cost of flight f for arriving or departing late
P_F	Cost of one gallon aircraft fuel
$SD^{ff'}$	Required separation distance (time) for flight f' following flight f
$\mathcal{D}_\delta$	Distance of arc δ
Sch_{ent}^f	Schedule time of leaving the airspace (either landing or departing)
Sch_{ext}^f	Scheduled time of arrival to the airspace (either landing or departing)
ω_i^-	The list of incoming nodes that can route aircrafts to node i.
ω_i^+	The list of outgoing nodes that node i can route aircrafts to.

δ_i^-	A flight leaving node i can only arrive to the node from a limited set of arcs due to the maneuvering capabilities of the aircraft.	
δ_i^+	A flight arriving to node i can only leave the node from a limited set of arcs due to the maneuvering capabilities of the aircraft.	
w, μ	Weights used to determine speed bounds $w = \{w^-, w^+\}$ *and* $\mu = \{\mu^-, \mu^+\}$	
ϕ	Time to travel a unit distance with minimum speed limit $\phi = \{\phi^-, \phi^+\}$	Since these terms are determined empirically, for maximum and minimum speed limits different values are used.
φ	Time to travel a unit distance with maximum speed limit $\varphi = \{\varphi^-, \varphi^+\}$	

For simplicity, we indexed δ^- and $\delta^+$by the dominant node, which is node i in above definitions as δ_i^- and δ_i^+.

3.2 Variables

In this section, the decision variables used in mathematical model are described:

$a_{\delta}^{f} \in \mathcal{R}^{+}$	Arrival time for flight f at node i from arc $\delta \in \delta_i^-$
$d_{\delta}^{f} \in \mathcal{R}^{+}$	Departure time for flight f from node i on arc $\delta \in \delta_i^+$
X_{δ}^{f}	$= \begin{cases} 1 & \text{if flight } f \text{ travels on arc } \delta \\ 0 & \text{Otherwise} \end{cases}$
$Z_{\delta v}^{f}$	$= \begin{cases} 1 & \text{if flight } f \text{ travels on arc } \delta \text{ with speed } v \\ 0 & \text{otherwise} \end{cases}$
$I_{\delta\lambda}^{f}$	$= \begin{cases} 1 & \text{if the speed of flight } f \text{ is increased by } \lambda \text{ in arc } \delta \\ 0 & \text{otherwise} \end{cases}$
$\beta_{\delta}^{ff'}$	$= \begin{cases} 1 & \text{if flight } f' \text{ follows flight } f \text{ on arc } \delta \\ 0 & \text{otherwise} \end{cases}$
$\theta_{\delta\delta'}^{ff'}$	$= \begin{cases} 1 & \text{if flight } f \text{ leaves node } i \text{ through } \delta \in \delta_i^- \\ & \text{prior to flight } f' \text{ arrives to node } i \text{ from } \delta' \in \delta_i^+ \\ 0 & \text{otherwise} \end{cases}$
$\theta_{i}^{ff'}$	$= \begin{cases} 1 & \text{if flight } f \text{ pass node } i \text{ prior to flight } f' \\ 0 & \text{otherwise} \end{cases}$
$\alpha_{\delta\delta'}^{ff'}$	$= \begin{cases} 1 & \text{if flight } f \text{ leaves the starting node of arc } \delta \text{ before flight } f' \text{the ending} \\ & \text{of travelling arc } \delta \ (\delta' \in \delta_i^-, \delta \in \delta_i^+) \\ 0 & \text{otherwise} \end{cases}$
v_{δ}^{f}	Speed of flight f at arc δ
t_{δ}^{f}	Required time for flight f to travel arc δ
$T_{\delta v}$	Required time for flight f to travel arc δ with speed v
ψ_{δ}^{f}	The fuel cost per unit of time at arc δ
Ω_{δ}^{f}	Total fuel consumption cost on arc δ

${T_E}^f$	Total earliness of flight f
${T_L}^f$	Total lateness of flight f
π_δ^f	Time to travel a unit distance on arc δ

3.3 Assumptions

Two sets of assumption have been incorporated in the model to strengthen the model accuracy.

3.1.1 Routing

It is assumed that an individual flight can pass a node only once. Subsequently, an aircraft can travel an arc only once.

3.1.2 Speed

It is assumed that average speed change of an aircraft from one arc to the other is bounded. Two methods have been developed for this matter.

- Either by the predefined speed change Λ. Let S_δ be the average speed of an aircraft on arc δ. The average speed of the aircraft on the consecutive arc, δ' can be $S_{\delta'} = S_\delta \pm \lambda_{\delta'}$ where λ is predefined discrete amount of speed change and $\forall \lambda \in \Lambda$.
- Or, by a continuous-variable τ where the time to travel a unit distance (π_δ) with a given speed (v_δ) on arc δ is bounded as $\pi_{\delta'} - \tau^- \leq \pi_\delta \leq \pi_{\delta'} - \tau^+$ where $\pi_{\delta'}$ is the unit travelling time on pervious arc. The terms π and τ are determined as:

$$\pi_\delta = {}^{1}/_{v_\delta} \tag{1}$$

$$\tau^- = w^-(\phi^- - \pi_{\delta'}) + \mu^-(\pi_{\delta'} - \varphi^-) \tag{2}$$

$$\tau^+ = w^+(\phi^+ - \pi_{\delta'}) + \mu^+(\pi_{\delta'} - \varphi^+) \tag{3}$$

As shown in Fig. 2, a near-linear relationship between the current speed $(v_{\delta'})$ and its upper $\left(v^{u}_{\delta'} = \frac{1}{\pi_{\delta'} - \tau^{-}}\right)$ and lower $\left(v^{l}_{\delta'} = \frac{1}{\pi_{\delta'} + \tau^{+}}\right)$ bounds are obtained for the following parameters.

- Speed Increase: $[w^{-}, \mu^{-}] = [0.01, 0.4]$ and $[\phi^{-}, \varphi^{-}] = [0.01,\ 0.001]$
- Speed Decrease: $[w^{+}, \mu^{+}] = [0, 0.85]$ and $[\phi^{+}, \varphi^{+}] = [0,\ 0.0009]$

Parameters, used for speed-bound control are determined empirically. Replacing actual speed v_{δ} by π_{δ} in the model enables us to avoid nonlinearity for computing travelling time. Instead of determining traveling time on an arc by a nonlinear term $t^{f}_{\delta} = \mathcal{D}_{\delta}/v^{f}_{\delta}$, a linear expression $\left(t^{f}_{\delta} = \mathcal{D}_{\delta}\pi^{f}_{\delta}\right)$ is used to compute travelling time which enormously improve the computation time.

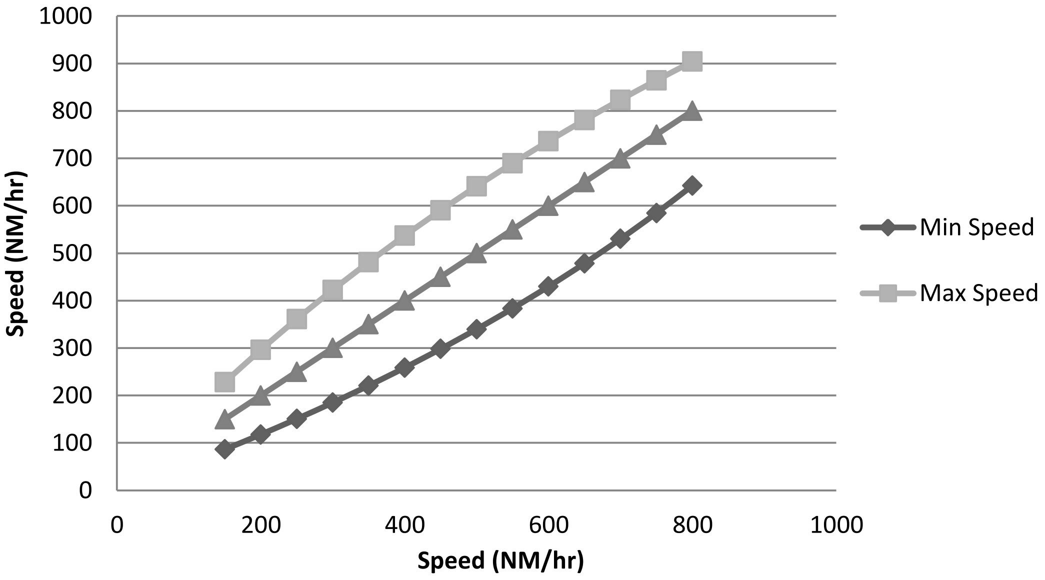

Figure 3-1: Speed Control between Consecutive Arcs (Speed in Nautical Mile per Hour – NM/Hour)

The details of both speed control strategies and performance comparison are discussed later in the thesis.

3.4 Problem Formulation

The main objective of the proposed mathematical model is to determine en-route flight plan for a set of flights on a network under ATC coverage before landing and after take-off. The network is consisting of N nodes and Δ arcs. While fuel consumption and flights delays are main criteria for reducing the total flights cost, the proposed mathematical model ensures all safety rules to avoid mid-air collision in the network.

3.4.1. Objectives

In order to solve a large scale optimization problem, it is important to obtain a strong formulation. The proposed formulation avoids nonlinearity under all circumstances, yet still archives all its objectives. Precise control of speed on each arc enables us to determine exact traveling times so the midair collision is avoided at all times. The proposed mathematical model considers the minimization of total cost that is incurred from delays, earliness and fuel consumption. Other strategies can easily be integrated into the formulation of the objective function. Constraints of the model are categorized in 5 distinct groups: objective function supporting constraints, the routing constraints; speed constraints; timing constraints, and safety and conflict constraints.

$$minimize \sum_{f\in F}\left({P_E}^f\,{T_E}^f + {P_L}^f{T_L}^f + \sum_{\delta\in\Delta}\Omega_\delta^f\right) \tag{4}$$

Delays cost are the penalties for being early or late based on the primary schedule of the aircraft. Therefore any deviation from the schedule is penalized. On the other hand, the fuel cost is determined as a function of traveling distances and fuel consumption coefficient for the given speed. In order not to discriminate any flight, an incremental delay cost is utilized so no fly can be delayed for an extended period of time.

The most important factor in determination of the travel cost is fuel cost. This portion of the cost is a function of traveling distance, and fuel consumption coefficient. The later depends on number of factors such as aircraft fleet type, weather condition, altitude, payload, and speed. The two factors of speed and altitude is set in the formulations of this work to provide an estimation of a fuel cost over an aircraft journey.

John-Paul Clarke 27] studied a relationship between speed and fuel consumption based on industry data validated for different aircraft fleet types (Figure 3-2). Constructed on his works, the following formulation is implemented to the model. Let $\psi_{\mathrm{v}}^{\mathrm{f}}$ be the cost of flying an aircraft for a nautical mile with a given speed v. Then, the cost of travelling on an arc is:

$$\Omega_{\delta}^{f} = \mathcal{D}_{\delta}\psi_{v}^{f} \quad (5)$$

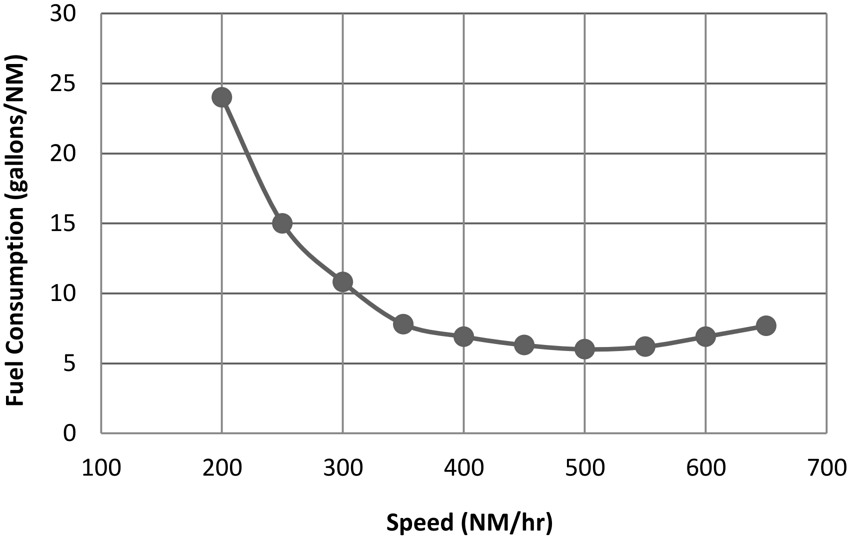

Figure 3-2: Relationship between Aircraft Speed and Fuel Consumption Extracted from Industry Data

Although the relationship between fuel consumption and speed is aircraft specific, the trend given in Figure 3-2 slightly varies for other aircraft types. In the present work, the fuel consumption rate for Boeing 777-200LR is implemented. In order to incorporate the fuel usage cost in to the mathematical model by a linear constraint, the fuel consumption rate in terms of π_δ^f is defined as:

$$\psi_v^f = \begin{cases} \psi_{v^*}^f\left(0.8x_\delta^f + 1000\left(\pi_\delta^f - \frac{x_\delta^f}{v^*}\right)\right) & if\ \pi_\delta^f \geq \frac{1}{v^*} \\ \psi_{v^*}^f\left(0.8x_\delta^f + 1000\left(\frac{x_\delta^f}{v^*} - \pi_\delta^f\right)\right) & otherwise \end{cases} \quad (6)$$

where P_F is the unit aircraft fuel cost. In Equation (6), v^* is the optimum cruse speed and $\psi^f_{v^*}$ is the fuel consumption rate at optimal speed. For $v^* = 500\text{NM/hr}$ and $\psi^f_{v^*} = 6$ gallons, the fuel consumption-speed relationship given in Figure 3-3 is obtained. It is clear from Figure 3-3 that Equation (6) fits well with the industry data within the typical aircraft speed limits.

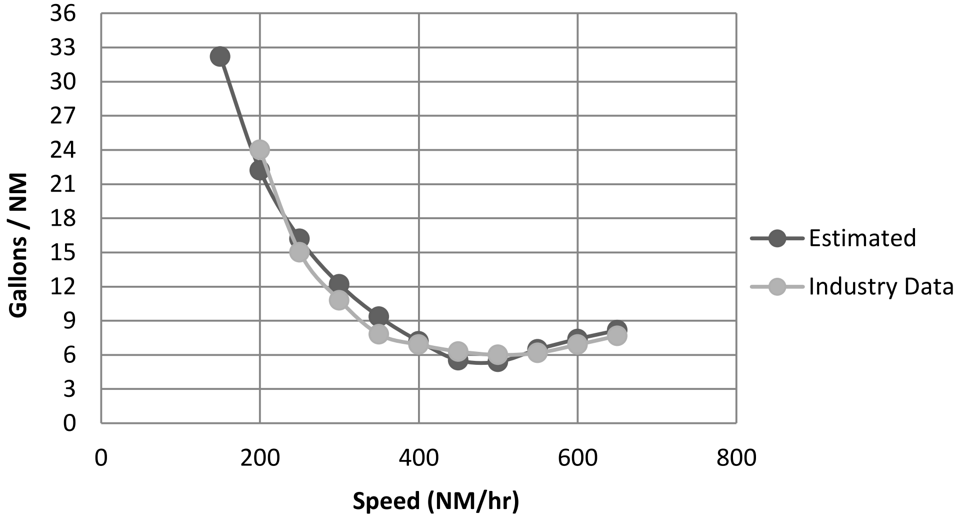

Figure 3-3: Estimating Fuel Consumption

Consequently, Equation (5) and (6) result in a mathematical model that approximates the fuel cost of flying an aircraft based on its traveling speed over the connected path of arcs.

3.5 Constraints

3.5.1 Objective Function Supporting Constraint

The Equation (7) is an actual constraint in the model that determines the delay or earliness based on the difference between actual arrival of an aircraft to the exit node and also, the schedule arrival to the exit node.

$$\sum_{\delta \in \delta^{-}_{ext^f}} a^f_\delta = Sch^f_{ext} + {T_L}^f - {T_E}^f \qquad \forall f \in F \qquad (7)$$

3.5.2 Routing Constraints

After the objective function and the constraints associated with it are introduced, the second group of constraints, the routing and rerouting, is presented as follows:

$$\sum_{\delta \in \delta^{+}_{\mathrm{ent}^{\mathrm{f}}}} \mathrm{x}^{\mathrm{f}}_{\delta} = 1 \qquad \forall \mathrm{f} \in \mathrm{F} \qquad (8)$$

$$\sum_{\delta \in \delta^{-}_{\mathrm{ext(f)}}} \mathrm{x}^{\mathrm{f}}_{\delta} = 1 \qquad \forall \mathrm{f} \in \mathrm{F} \qquad (9)$$

$$\sum_{\delta \in \delta^{-}_{\mathrm{i}}} \mathrm{x}^{\mathrm{f}}_{\delta} = \sum_{\delta \in \delta^{+}_{\mathrm{i}}} \mathrm{x}^{\mathrm{f}}_{\delta} \qquad \forall \mathrm{i} \in \mathrm{N}, \forall \mathrm{f} \in \mathrm{F} \qquad (10)$$

$$\mathrm{x}^{\mathrm{f}}_{\delta(\mathrm{ki})} + \mathrm{x}^{\mathrm{f}}_{\delta(\mathrm{ik})} \leq 1 \qquad \forall \mathrm{i} \in \mathrm{N}, \forall \mathrm{k} \in \omega_i, \forall \mathrm{f} \in \mathrm{F} \qquad (11)$$

$$\sum_{\delta \in \delta^{-}_{\mathrm{i}}} \mathrm{x}^{\mathrm{f}}_{\delta} + \sum_{\delta \in \delta^{+}_{\mathrm{i}}} \mathrm{x}^{\mathrm{f}}_{\delta} \leq 2 \qquad \forall \mathrm{i} \in \mathrm{N}, \forall \mathrm{f} \in \mathrm{F} \qquad (12)$$

Equation (8) declares that every flight has to take one and only one entering arc connected to its origin to enter the network. And in the same way, in equation (9), each exiting flight in the network has to take one and only one arc connected to its scheduled destination to leave the network. The entering and exiting points have been defined on runways and the outer bound zone of ATC respectively. Based on flight mission (whether arrival or departure), following to its origin and destination, these points will be assigned to the flight. Equation (10) states that each flight after passing each arc has to take only one of the consecutive arcs that are connected to its current arc. This way the flight routing continuity will be maintained. Equation (11) notes that each flight can only take an arc once in every direction, so that it cannot take one arc in both directions. Equation (12), states that each flight can only pass each crossing node through different arcs at most once in the entire journey. The reason we imbedded constraints (11) and (12) is to reduce the complexity of the problem. If aircrafts are allowed to visit the same arc or node more than once, then in order to differentiate each arrival and departure to same node, it is needed to introduce an additional index (visiting index).

3.5.3 Speed Constraints

Duration of flight is directly dependent of the aircraft speed on its route. Since speed is a decision variable, computation of flight time using $t = \mathcal{D}/_{v}$ expression leads to a nonlinear mathematical model. As pointed before, for avoiding the nonlinearity, two separate approaches are proposed for speed control during a flight: Discrete and Continues which are explained in details below:

3.1.1.1 Discrete Speed Control:

In this method, a predefined set of speed change is used:

$\lambda \in \Lambda$ where $\Lambda = \{-\lambda_{\max}, \cdots, -\lambda_1, 0, \lambda_1, \cdots, \lambda_{\max}\}$

Discrete speed changing value results in discrete speed value as well:

$v_\delta^{\mathrm{f}} \in \mathrm{S}$ where $\mathrm{S} = \{\mathrm{v}_{\min}, \mathrm{v}_1, \cdots, \mathrm{v}_{\max}\}$

On the next step, based on the distances of the arcs in the network and the possible speed of flights, a time table is derived:

Table 3-1: Speed and Distance Relationship

Arc Length Speed	$\mathcal{D}_1$	$\mathcal{D}_2$	$\cdots$	$\mathcal{D}_n$
v_1	T_{11}	T_{21}	$\cdots$	T_{n1}
v_2	T_{12}	T_{22}	$\cdots$	T_{n2}
$\vdots$	$\vdots$			
v_m	T_{1m}	T_{2m}	$\cdots$	T_{nm}

By picking the time values from this table, the non-linearity of $v_\delta^f = \mathcal{D}_\delta / t_\delta^f$ is skipped. Constraints holding the discrete speed method are introduced as followed:

$$\sum_{\delta \in \delta_i^+} \left(\sum_{v \in S} v\, z_{\delta v}^f - \sum_{\lambda \in \Lambda} \lambda I_{\delta,\lambda}^f \right) = \sum_{\delta \in \delta_i^-} \sum_{v \in S} v\, z_{\delta v}^f \qquad \forall i \in N, \forall f \in F \qquad (13)$$

$$\sum_{v \in S} z_{\delta v}^f = x_\delta^f \qquad \forall i \in N, \forall \delta \in \delta_i, \forall f \in F \qquad (14)$$

$$\sum_{\lambda\in\Lambda} I^{f}_{\delta\lambda} = x^{f}_{\delta} \qquad \forall i \in N, \forall\delta \in \delta_{i}, \forall f \in F \tag{15}$$

$$a^{f}_{\delta} = d^{f}_{\delta} + \sum_{v\in S} z^{f}_{\delta v}\, T_{\delta v} \qquad \forall i \in N, \forall\delta \in \delta_{i}, \forall f \in F \tag{16}$$

$$x^{f}_{\delta}\, v^{f}_{ent} = \sum_{v\in S} v z^{f}_{\delta v} \qquad \delta \in \delta^{+}_{ent^{f}}, \forall f \in F \tag{17}$$

$$x^{f}_{\delta}\, v^{f}_{ext} = \sum_{v\in S} v z^{f}_{\delta v} \qquad \delta \in \delta^{-}_{ext^{f}}, \forall f \in F \tag{18}$$

In Equation (13) speed can be changed within the range of allowed speed changes between two consecutive arcs. Equations (14) and (15) assure that if a flight travels an arc, a speed and a speed change limit will be assigned to it for that arc. Equation (16) calculates the arrival time of the flight to the end of an arc based on its speed on that arc. Moreover, as each flight has a primary schedule declaring its mission (whether it is a departure flight or arrival flight), its desired arrival and departure time and speed to the system. In this regards, equations (17) and (18) command their pre-scheduled initial and final speed to the flights.

3.1.1.2 Continuous Speed Control:

Another approach to implement speed changes over traveling arcs is to define a formulation to continuously control the speed of an aircraft for more realistic results. Although the discrete speed control is easier to model and gives stronger control on the limits for speed change from one arc to the next one, it may lead to unexpected mid-air conflict situations. The benefit of having such control over the speed is to alleviate the complexity of the model by removing binary variables of $z^{f}_{\delta v}$ and $I^{f}_{\delta\lambda}$. Another important advantage of freely choosing continuous speed values rather than discrete ones is that the cost of travel is not compromised by forcing aircraft to choose a specific speed value. Hence, a second speed control approach is proposed. In this approach, speed is defined as time to travel a unit distance, $\pi_\delta^f = 1/v_\delta^f$. Consequently, the travelling time on a given arc is determined by:

$$\mathrm{t}_\delta^\mathrm{f} = \mathcal{D}_\delta \pi_\delta^\mathrm{f} \qquad \forall \delta \in \delta_i, \forall f \in F \qquad (19)$$

$$\mathrm{a}_\delta^\mathrm{f} = \mathrm{d}_\delta^\mathrm{f} + \mathcal{D}_\delta \pi_\delta^\mathrm{f} \qquad \forall i \in N, \forall \delta \in \delta_i, \forall f \in F \qquad (20)$$

In this new approach, the problem raises when defining the speed change limits between two consecutive arcs. Assume that aircraft is flying over arc (δ) with speed of 450 NM per hour, and the next visiting arc will be (δ'). Depending on aircraft capabilities and the length of traveling arc, the speed may change to a lower or higher value. To determine upper and lower bound of this change, actual speed could be implemented as $(450 - \%\ \text{of speed decrease} \leq \mathrm{v} \leq 450 + \%\ \text{of speed increase})$. However, due to restrictions of operation research concepts, nonlinearity of flight time would be introduced to the system $(\mathrm{t}_\delta^\mathrm{f} = \mathcal{D}_\delta / \mathrm{v}_\delta^\mathrm{f})$. Nonlinearity constraint can be avoided by defining speed as time to travel a unit distance (π_δ) as show in Equation. Consequently speed limits can be bounded as $(\pi_{\delta'} - \tau^- \leq \pi_\delta \leq \pi_{\delta'} - \tau^+)$. The values of τ is a function of current π_δ as shown in Equation 2. Finally, by converting the π_δ^f back to actual speed and configuring speed bounds on Figure 3-1 the nonlinearity problem is removed. As discussed later in results, continues speed control approach significantly reduces the computation time and still provide realistic speed control limits during flight.

3.5.4 Timing Constraints

The bellow constraints declare the time adjustment of flights in their route as the third group.

$$d_\delta^f \geq x_\delta^f Sch_{ent}^f \qquad \delta \in \delta_{ent^f}^+, \forall f \in F \qquad (21)$$

$$a_\delta^f \leq l^f \qquad \forall \delta \in \delta_{ext(f)}^-, \forall f \in F \qquad (22)$$

$$a_\delta^f \geq e^f \qquad \forall \delta \in \delta_{ext(f)}^-, \forall f \in F \qquad (23)$$

$$a_\delta^f \leq M x_\delta^f \qquad \forall i \in N, \forall \delta \in \delta_i, \forall f \in F \qquad (24)$$

$$d_\delta^f \leq M x_\delta^f \qquad \forall i \in N, \forall \delta \in \delta_i, \forall f \in F \qquad (25)$$

$$\sum_{\delta \in \delta_i^-} a_\delta^f = \sum_{\delta \in \delta_i^+} d_\delta^f \qquad \forall i \in N, \forall f \in F \qquad (26)$$

Inequality (21) commands flights their pre-shredded arrival time to the network to start their journey. Inequalities (22) and (23) force flights to finish their journey within the maximum and minimum available time horizon. Inequalities (24) and (25) define a travelling time for a flight on an arc if and only if that flight passes that specific arc. Constraint (26) guarantees the flights time connectivity between two arcs, which means they cannot stop at any node on their journey. (Figure 3-4).

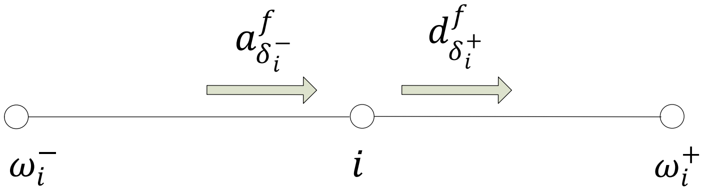

Figure 3-4: **Time Connectivity**

3.5.5 Safety and Conflict Constraints

Here, the last group of constraints, the safety and conflict constraints are introduced as followed:

$$\mathrm{d}_\delta^{f'} - \mathrm{d}_\delta^{f} \geq \mathrm{ST} - \mathrm{M}\left(1 - \beta_\delta^{ff'}\right) - \mathrm{M}(2 - \mathrm{x}_\delta^{f} - \mathrm{x}_\delta^{f'}) \qquad \forall i \in N, \forall \delta \in \delta_i^+, \forall f, f' \in F \mid f < f' \tag{27}$$

$$\mathrm{d}_\delta^{f} - \mathrm{d}_\delta^{f'} \geq \mathrm{ST} - \mathrm{M}\beta_\delta^{ff'} - \mathrm{M}(2 - \mathrm{x}_\delta^{f} - \mathrm{x}_\delta^{f'}) \qquad \forall i \in N, \forall \delta \in \delta_i^+, \forall f, f' \in F \mid f < f' \tag{28}$$

$$\mathrm{a}_\delta^{f'} - \mathrm{a}_\delta^{f} \geq \mathrm{ST} - \mathrm{M}\left(1 - \beta_\delta^{ff'}\right) - \mathrm{M}(2 - \mathrm{x}_\delta^{f} - \mathrm{x}_\delta^{f'}) \qquad \forall i \in N, \forall \delta \in \delta_i^-, \forall f, f' \in F \mid f < f' \tag{29}$$

$$\mathrm{a}_\delta^{f} - \mathrm{a}_\delta^{f'} \geq \mathrm{ST} - \mathrm{M}\beta_\delta^{ff'} - \mathrm{M}(2 - \mathrm{x}_\delta^{f} - \mathrm{x}_\delta^{f'}) \qquad \forall i \in N, \forall \delta \in \delta_i^-, \forall f, f' \in F \mid f < f' \tag{30}$$

Inequalities (27) to (30) mandate the minimum safety distance of SD between each two flights that are traveling the same arc in the same direction at the arc's starting and ending node. In other words they have to leave the starting node with the minimum SD and arrive to the ending node with the minimum SD as well. The binary variable of $\beta_\delta^{ff'}$ is used to take the control of determining leading flight. On the other hand x_δ^{f} and $\mathrm{x}_\delta^{f'}$ are applied to keep the constraints valid if and only if the flights are passing the same arcs. The situation is showed on Figure 3-5.

Figure 3-5: Safety Distance between Two Consecutive Flights

$$\mathrm{d}_{\overleftarrow{\delta}}^{\mathrm{f'}} - \mathrm{a}_{\vec{\delta}}^{\mathrm{f}} \geq \mathrm{SD} - \mathrm{M}\left(1 - \alpha_{\vec{\delta}\overleftarrow{\delta}}^{\mathrm{ff'}}\right) - \mathrm{M}\left(2 - \mathrm{x}_{\vec{\delta}}^{\mathrm{f}} - \mathrm{x}_{\overleftarrow{\delta}}^{\mathrm{f'}}\right) \qquad \forall i \in N, \forall \vec{\delta} \in \delta_i^-, \forall \overleftarrow{\delta} \in \delta_i^+, \forall f, f' \in F \mid f < f' \qquad (31)$$

$$\mathrm{d}_{\vec{\delta}}^{\mathrm{f}} - \mathrm{a}_{\overleftarrow{\delta}}^{\mathrm{f'}} \geq \mathrm{SD} - \mathrm{M}\alpha_{\vec{\delta}\overleftarrow{\delta}}^{\mathrm{ff'}} - \mathrm{M}\left(2 - \mathrm{x}_{\vec{\delta}}^{\mathrm{f}} - \mathrm{x}_{\overleftarrow{\delta}}^{\mathrm{f'}}\right) \qquad \forall i \in N, \forall \vec{\delta} \in \delta_i^+, \forall \overleftarrow{\delta} \in \delta_i^-, \forall f, f' \in F \mid f < f' \qquad (32)$$

Inequalities (31) to (32) are designed to prevent each two flight collide. These constraints assure that flights are allowed to enter an arc if and only if that arc has been cleared with the minimum SD time. Therefore, any mid-air collision is prevented. The binary variable of $\alpha_{\vec{\delta}\overleftarrow{\delta}}^{ff'}$ control the flights order. In Figure 3-6 this scenario is demonstrated.

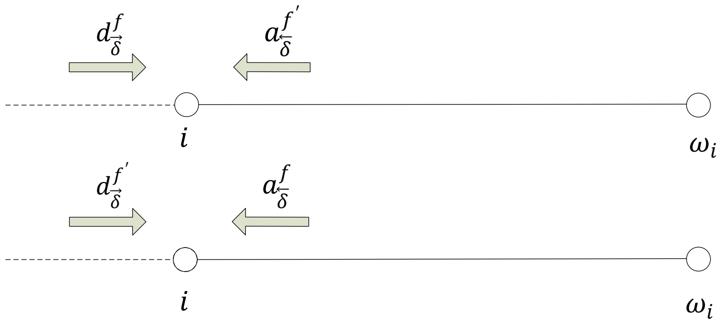

Figure 3-6: **Flight from Opposite Directions**

$$\mathrm{a}_{\delta'}^{\mathrm{f'}} - \mathrm{a}_{\delta}^{\mathrm{f}} \geq \mathrm{SD} - \mathrm{M}\left(1 - \theta_{\delta'\delta}^{\mathrm{f'f}}\right) - \mathrm{M}(2 - \mathrm{x}_{\delta}^{\mathrm{f}} - \mathrm{x}_{\delta'}^{\mathrm{f'}}) \qquad \forall i \in N, \forall \delta \in \delta_i^-, \delta' \in \delta_i^-, \delta \neq \delta', \ \forall f, f' \in F \mid f < f' \tag{32}$$

$$\mathrm{a}_{\delta}^{\mathrm{f}} - \mathrm{a}_{\delta'}^{\mathrm{f'}} \geq \mathrm{SD} - \mathrm{M}\theta_{\delta'\delta}^{\mathrm{f'f}} - \mathrm{M}(2 - \mathrm{x}_{\delta}^{\mathrm{f}} - \mathrm{x}_{\delta'}^{\mathrm{f'}}) \qquad \forall i \in N, \forall \delta \in \delta_i^-, \delta' \in \delta_i^-, \delta \neq \delta', \ \forall f, f' \in F \mid f < f' \tag{33}$$

The final set of inequalities (32) and (33) constrain the flights to encounter on the crossing nodes. Such constraints seem to be similar to inequalities (27) - (30). However, they are different in a manner to avoid conflicts for aircrafts using different arcs yet flying through same node. To implement such restriction, a four dimensional binary variable $\theta_{\delta'\delta}^{\mathrm{f'f}} = 1$ is defined as an arc specific variable to check if aircraft f passes through node i before aircraft f'. To reduce the array size of this variable, a new binary variable $\vartheta_{\mathrm{i}}^{\mathrm{ff'}} = 1$ is defined to be node specific. It checks if aircraft f passes through node i prior to the arrival of aircraft $\mathrm{f'}$. Finally, a minimum SD is included in inequalities (34) and (35) to ensure safety between two flying aircrafts.

$$\sum_{\delta' \in \delta_i^-} a_{\delta'}^{f'} - \sum_{\delta \in \delta_i^-} a_{\delta}^{f} \geq SD - M\left(1 - \vartheta_i^{f'f}\right) - M\left(2 - \sum_{\delta \in \delta_i^-} X_{\delta}^{f} - \sum_{\delta' \in \delta_i^-} X_{\delta'}^{f'}\right) \qquad \forall i \in N, \forall ff' \in F \mid f < f' \qquad (34)$$

$$\sum_{\mathrm{\delta} \in \mathrm{\delta}_\mathrm{i}^-} \mathrm{a}_{\mathrm{\delta}}^{\mathrm{f}} - \sum_{\mathrm{\delta}' \in \mathrm{\delta}_\mathrm{i}^-} \mathrm{a}_{\mathrm{\delta}'}^{\mathrm{f}'} \geq \mathrm{SD} - \mathrm{M}\vartheta_\mathrm{i}^{\mathrm{f}'\mathrm{f}} - \mathrm{M}\left(2 - \sum_{\mathrm{\delta} \in \mathrm{\delta}_\mathrm{i}^-} \mathrm{X}_{\mathrm{\delta}}^{\mathrm{f}} - \sum_{\mathrm{\delta}' \in \mathrm{\delta}_\mathrm{i}^-} \mathrm{X}_{\mathrm{\delta}'}^{\mathrm{f}'}\right) \qquad \forall i \in N, \forall ff' \in F \mid f < f' \qquad (35)$$

The cross-passing on a node is illustrated in Figure 3-7.

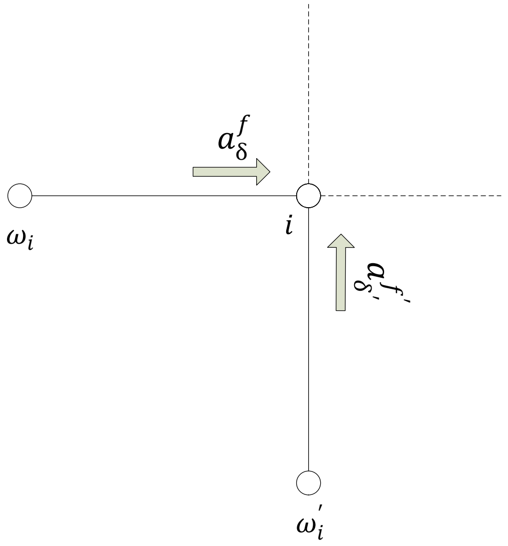

Figure 3-7: Collision Avoidance at Node

4 Solutions and Results

The en-route flight planning model discussed in section 3 is designed to serve both current ATC centered and Free Flight Concept (FFC) based fair traffic management philosophies. FFC aims at transferring en route flight planning decision to individual aircrafts. The FFC philosophy enables a pilot to determine the best available route for his/her aircraft in order to optimize its objectives.

In this section, the proposed en-route flight planning model is tested on both scenarios (namely centralized and decentralized). In both cases, the objective is to determine conflict free en route plans for aircrafts with minimum delay/earliness and fuel consumption costs. One of the important characteristics of the given mathematical model is its capability of ensuring safety under all circumstances. Such assurance is achieved through a non-time indexed modeling strategy.

Both centralized and decentralized scenarios were tested on various traffic conditions. Corresponding mathematical models were solved in IBM ILOG CPLEX Optimization Studio 12.2, using Optimization Programming Language (OPL) on a personnel computer with 64 bit operating system, 3.40 GHz Intel Core i7-2600 CPU and 16.0 GB RAM. Airspace around an airport is considered. A 3D mesh that consists of N nodes and Δ arcs is used to model the Aircrafts enter/exit to/from airspace through two dummy nodes (N_D^1, N_D^2). Aircrafts enter the airspace through one of the dummy nodes for either departure or landing purpose. The direction of aircraft is determined randomly (50% of the aircrafts enter the airspace to departure). Times between Arrivals (TBA) are also random following Exponential Distribution with varying average. The length of each arc is determined based on their locations in the airspace. Arcs near the center are shorter than the arcs near the exterior of the mesh. For aircrafts approaching to the airspace from outside, entry speeds are set greater than 400 NM/hr. Minimum speed on the ground is assumed to be 50 NM/hr. Between two consecutive arcs, aircrafts are allowed to change their speeds up to approximately $\pm 150\ NM/hr$. Aircrafts' speeds vary between $[50 - 400]\ NM/hr$. The travelling time on an arc is determined based on the average speed and the distance. Finally aircrafts are separated from each other by SD. Models were tested for various SD and average TBA values. The impacts of SD and TBA on the given objectives (average flight time in airspace, average cost and program execution times) are studied for both scenarios.

4.1 Centralized Model

In the centralized model, all flights are solved during a time window with a non-time indexed network flow. This network consists of F flights, N nodes and Δ arcs. The objective that is assigned to the model includes earliness or lateness cost, fuel consumption cost or flight time in airspace. The resulted plan declares the flights routing by the arcs they travel and the arrival and departure time on the arcs.

4.1.1 Results and Verification of Centralized Model

The proposed Centralized model has been tested on various instances. In one scenario the model is tested for traffic size of 8 flights with the objective function of minimization of total cost including fuel consumption cost and delay cost (earliness and lateness). In this sample, flights' time between arrivals is randomly distributed with average of 30 seconds. The safety distance between flights during their journey whether they are following each other on the same arc or they are passing each other on the same node is set to 1 minute. Flights which are planned to land, arrive to the network with the speed of 300 NM/hr and finish their journey in the airport by 50 NM/hr speed. In the same way for departure flights, the starting speed is set to 50 NM/hr and they have to leave the network by the speed of 300 NM/hr. In this case the network that is applied to the model has 34 nodes and 89 bidirectional arcs. The node number 0 resembles the inner bound of the network on the ground and the node number 33 resembles the outer border of the network in airspace. Table 4-1 shows the flights schedule resulted from the Centralized model for this sample. In this table, flights routes including arcs and nodes and the timing at each node are demonstrated. Therefore, tracing flights through each step of their journey is clearly applicable. The execution time for this traffic stream is 1099 seconds.

Table 4-1: Sample Traffic Schedule Resulted from Centralized Model

Flight Number	Departure Node	Arrival Node	Departure Time (Minutes)	Arrival Time (Minutes)
1	0	5	0.000	0.120
1	5	14	0.120	3.659
1	14	22	3.659	6.493
1	22	31	6.493	7.813
1	31	32	7.813	9.613
1	32	25	9.613	11.713
1	25	33	11.713	12.313
2	0	5	1.000	1.120
2	5	14	1.120	4.660
2	14	22	4.660	7.493
2	22	31	7.493	8.813
2	31	32	8.813	10.613
2	32	25	10.613	12.713
2	25	33	12.713	13.313
3	0	5	2.000	2.120
3	5	14	2.120	5.660
3	14	22	5.660	8.493
3	22	31	8.493	9.813
3	31	32	9.813	11.613
3	32	25	11.613	13.713
3	25	33	13.713	14.313
4	33	29	0.790	1.390
4	29	21	1.390	4.069
4	21	12	4.069	8.202
4	12	3	8.202	12.429
4	3	0	12.429	12.549
5	33	27	1.577	2.177
5	27	19	2.177	4.280
5	19	10	4.280	8.155
5	10	1	8.155	12.890
5	1	0	12.890	13.009
6	33	29	1.790	2.390
6	29	21	2.390	5.069
6	21	12	5.069	9.202
6	12	3	9.202	13.429
6	3	0	13.429	13.549
7	33	27	4.179	4.779
7	27	19	4.779	6.564
7	19	10	6.564	9.914
7	10	1	9.914	14.110
7	1	0	14.110	14.230
8	33	29	2.790	3.390
8	29	21	3.390	6.069
8	21	12	6.069	10.202
8	12	3	10.202	14.429
8	3	0	14.429	14.549

Based on the result shown in Table 4-1, Figure 4-1 is concluded to show the flights travelling time in system.

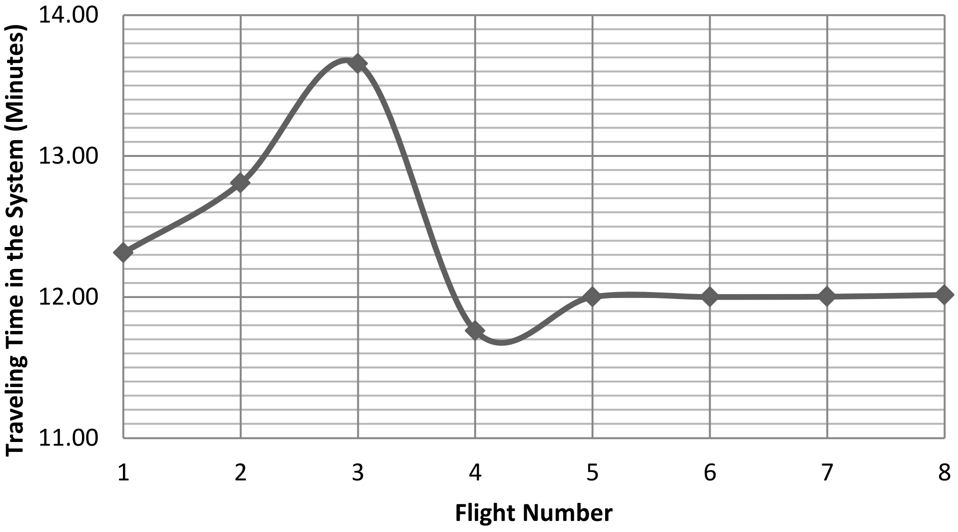

Figure 4-1: Travelling Time in the System of the Sample Traffic Resulted from Centralized Model

In Figure 4-2 based on the data provided at Table 4-1 each flight route versus time is drawn to explore the accuracy of conflict constraints. Two flights collide if they pass the same node or arc at the same time which reflects in this figure by two overlapped nodes. As shown on this figure, no collision is occurred. Therefore, the 3 sets of developed constraints to protect flights safety and also avoid every kind of air collision strongly assure the system. For that reason, it can be stated that the developed Centralized model can be used as a powerful decision making tool to help air traffic controllers.

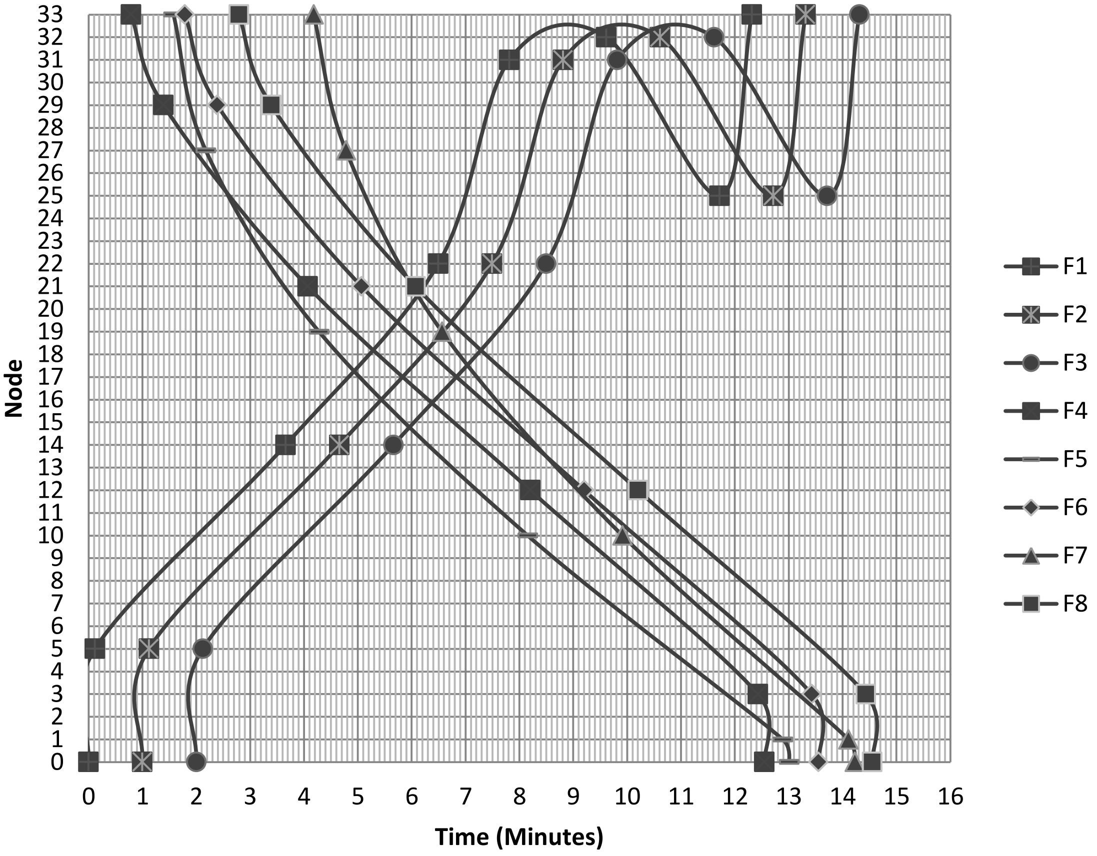

Figure 4-2: Collision Free Flight Route Resulted from Centralized Model

4.1.2 Impact of Time between Arrival and Safety Distance Factor on the System Behavior for Centralized Model

In the next step, the impact of different TBAs and SDs on the average flight time in the airspace, travelling cost of the flights and also the execution time of the solver are studied. For this matter, 4 different traffic streams with different combination of safety distance and time between arrivals and in each combination 5 different traffic sizes are tested. Table 4-2 illustrates the result of these testes.

Table 4-2: Impact of TBA and SD on Flight Time in Airspace and Travelling Cost

Problem Size	TBA (Seconds)	SD (Seconds)	Execution Time (Seconds)	Average Flight Time in Airspace ($)	Average Travelling Cost ($)	Delay Cost ($)	Fuel Cost ($)
4	30	30	0.33	6.4245	33038.5	38.5	33000.0
8	30	30	1.63	6.4245	33038.5	38.5	33000.0
12	30	30	4.44	6.4869	33039.8	39.8	33000.0
16	30	30	13.43	6.7113	33044.2	44.2	33000.0
20	30	30	413.93	7.1043	33052.1	52.1	33000.0
4	30	60	0.35	6.4250	33038.5	38.5	33000.0
8	30	60	2.04	6.4245	33038.5	38.5	33000.0
12	30	60	6.88	7.0740	33051.5	51.5	33000.0
16	30	60	273.60	8.1496	33073.0	73.0	33000.0
20	30	60	Out of Memory				
4	15	30	0.25	6.4250	33038.5	19.3	33000.0
8	15	30	1.46	6.4245	33038.5	38.5	33000.0
12	15	30	4.92	6.4248	33038.5	38.5	33000.0
16	15	30	13.24	6.4250	33038.5	38.5	33000.0
20	15	30	60.45	6.6994	33044.0	44.0	33000.0
4	15	60	0.37	6.6748	33043.5	43.5	33000.0
8	15	60	2.64	7.1650	33053.4	53.4	33000.0
12	15	60	144.85	8.4490	33079.0	79.0	33000.0
16	15	60	Out of Memory				
20	15	60	Out of Memory				

For better analysis of this table Figure 4-3 and Figure 4-4 are being derived. Figure 4-3 shows the average flight time in the system versus the traffic size. As shown in this figure the higher SDs and lower TBAs causes denser traffic which forces longer travelling time. Similarly,

Figure 4-4 shows the higher traffic rates result in higher costs for the system while the objective in this scenario is minimization of total cost. By differentiating between fuel costs and delay costs **Figure 4-5** demonstrates that the system is more sensitive to the taken SD rather than the TBA. While the minimum SD should be respected through the entire network for all flights, TBA is just an initial state which can be recovered in the network later. In other word, the dominant element to control the system in the terms like cost and time is the safety distance between aircraft. The other fact that can be extracted from **Figure 4-5** is that the main contribution of cost sensitivity to the changes in SD and TBA is the delay (earliness/ lateness) cost. Therefore, the fuel cost (that is in the order of 30000 and highly dependent of the flight path) is insensitive to values of SD and TBA. Careful observation of the cases with higher SDs in **Figure 4-5** reveals that the incomplete use of airspace capacity results in more delay costs for shorter TBAs. Consequently, more powerful models, similar to the current work that can guarantee flight safeties with more accurate timings (Hence, shorter SDs) can accept higher levels of air traffic with moderate increase in delay-associated costs.

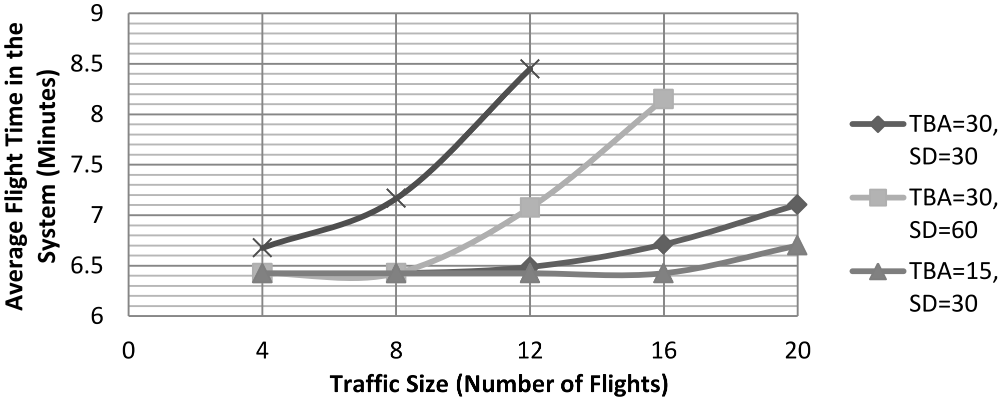

Figure 4-3: Impact of TBA and SD on Average Flight Time

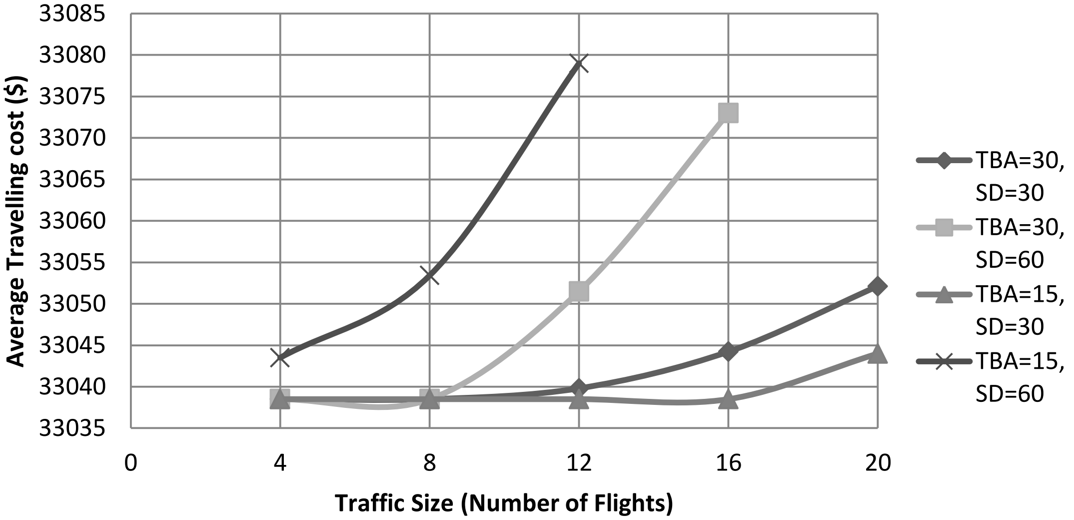

Figure 4-4: Impact of TBA and SD on Average Travelling Cost

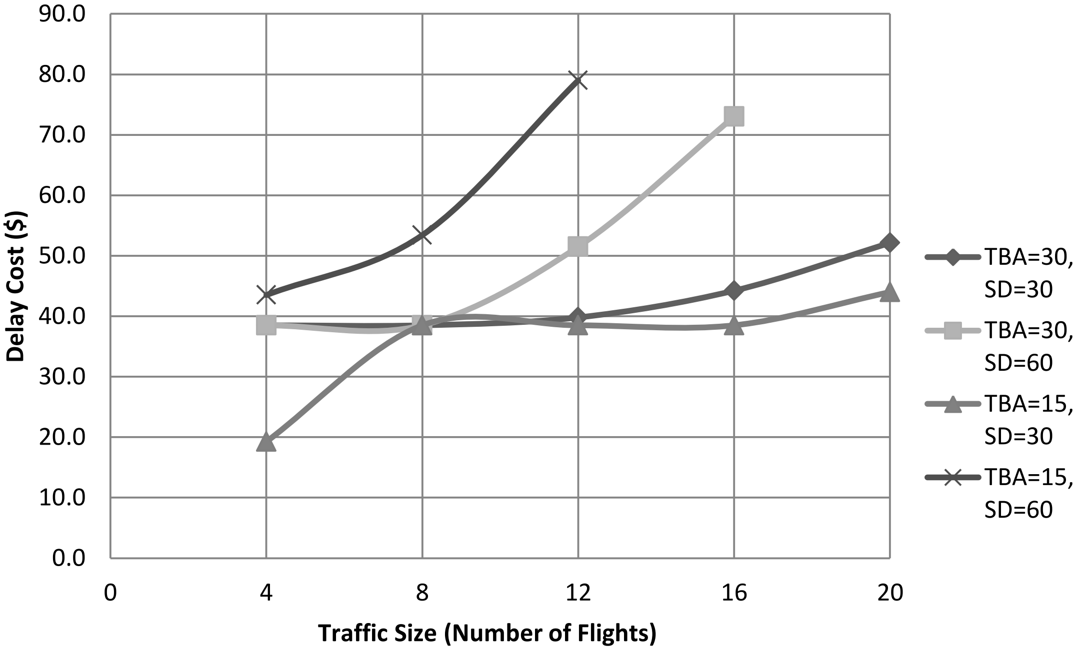

Figure 4-5: Impact of TBA and SD on Average Delay Cost

4.2 Decentralized Model

The decentralized model handles the problem by considering flights independent of each other and only based on their pre-scheduled plan. The only common point between flights is the conflict and safety constraints. Each flight checks itself within set of constraints against the previously solved flights in the system. In case of conflict, that specific flight is forced to adapt itself by the other flights in the system (Figure 4-6). Alike the centralized model, objective of earliness and lateness cost, fuel consumption cost or traveling time in airspace is applied. With respect to what discussed before, developing a method that gives the best time performance to handle the high rate of today's air traffic is crucially essential. It should be noted that this study is conducted on a personal computer, therefore the time result can be valid for every other user as well. Based on the tests and the results collected from the decentralized model, it is claimed that the model provides a reasonable time performance with respects to all safety and conflict constraints and it can be used as an effective tool.

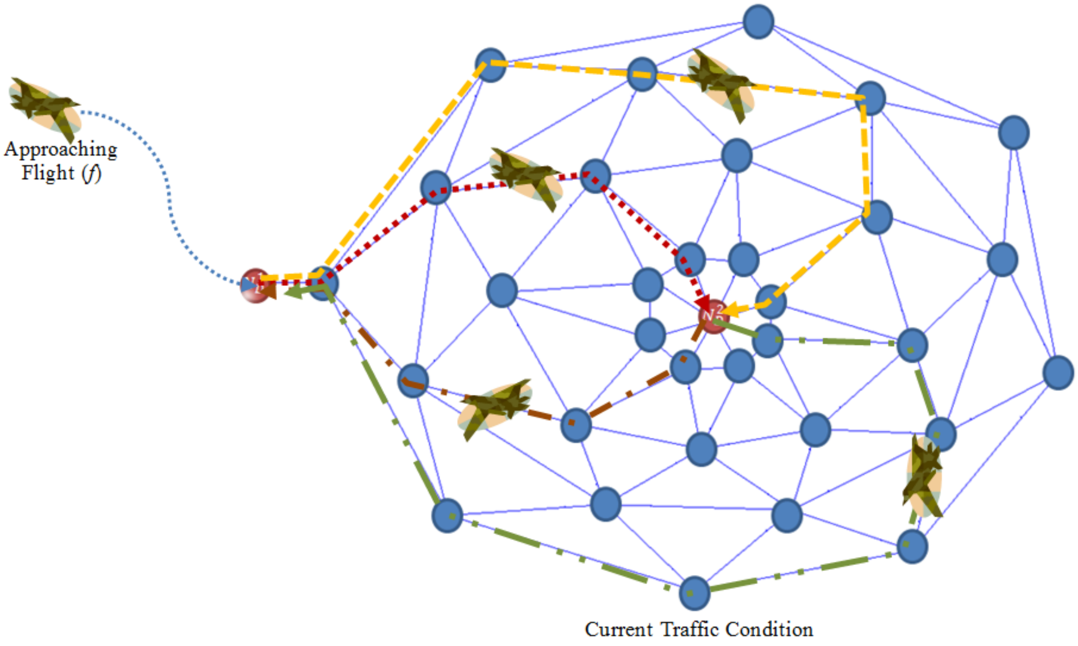

Figure 4-6: Traffic Conditions at the Time of New Arrival

4.2.1 Results and Verification of Decentralized Model

In this section, the Decentralized model is tested for a sample stream of 100 flights with the objective function of minimization of total cost including fuel consumption cost and delay cost (earliness and lateness). Sample flights' time between arrivals are randomly distributed with average of 12 seconds. The safety distance between flights during their journey whether they are following each other on the same arc or they are passing each other on the same node is set to 0.8 minute. Flights which are planned to land, arrive to the network with the speed of 400 NM/hr and finish their journey in the airport by 100 NM/hr speed. In the same way for departure flights the starting speed is set to 100 NM/hr and they have to leave the network by the speed of 400 NM/hr. In this case the network that has been applied to the model has 34 nodes and 89 bidirectional arcs.

Similar to the previous case, the node number 0 resembles the inner bound of the network on the ground and the node number 33 resembles the outer border of the network in airspace. Table 4-1 shows the schedule resulted from the Decentralized model for this sample for flights 50 to 60. In this table, flights routes including arcs and nodes and the timing at each node are demonstrated. Therefore, tracing flights through each step of their journey is clearly applicable. The execution time for this traffic stream is 28 seconds.

Table 4-3: Sample Traffic Schedule Resulted from Decentralized Model

Flight Number	From Node	To Node	From Time (Minutes)	To Time (Minutes)
51	33	29	13.85	14.15
51	29	21	14.15	15.56
51	21	12	15.56	17.81
51	12	3	17.81	20.40
51	3	0	20.40	20.50
52	0	1	16.50	16.60
52	1	10	16.60	19.20
52	10	19	19.20	21.09
52	19	27	21.09	22.02
52	27	33	22.02	22.32
53	33	29	14.85	15.15
53	29	21	15.15	16.56
53	21	12	16.56	18.81
53	12	3	18.81	21.40
53	3	0	21.40	21.50
54	33	29	15.85	16.15
54	29	21	16.15	17.56
54	21	12	17.56	19.81
54	12	3	19.81	22.40
54	3	0	22.40	22.50
55	33	29	16.85	17.15
55	29	21	17.15	18.56
55	21	12	18.56	20.81
55	12	3	20.81	23.40
55	3	0	23.40	23.50
56	0	1	17.50	17.60
56	1	10	17.60	20.20
56	10	19	20.20	22.09
56	19	27	22.09	23.02
56	27	33	23.02	23.32
57	0	1	18.50	18.60
57	1	10	18.60	21.20
57	10	19	21.20	23.09
57	19	27	23.09	24.02
57	27	33	24.02	24.32
58	33	29	17.85	18.15
58	29	21	18.15	19.56
58	21	12	19.56	21.81
58	12	3	21.81	24.40
58	3	0	24.40	24.50
59	33	29	18.85	19.15
59	29	21	19.15	20.56
59	21	12	20.56	22.81
59	12	3	22.81	25.40
59	3	0	25.40	25.50
60	0	1	19.50	19.60
60	1	10	19.60	22.20
60	10	19	22.20	24.09
60	19	27	24.09	25.02
60	27	33	25.02	25.32

Based on the result shown in Table 4-3, Figure 4-7 is concluded to show the flights travelling time in the system. By assuming the empty airspace at the starting point, the first flights that enter to the system face a lower traffic rate as the system has not yet reached to its steady state rate. Consequently, by adding flights to the system the traffic gets denser and it causes longer traveling time for the flights. On the other hand, the initial parameter of the sample like low time between arrival (12 seconds) and high safety distance (0.8 minutes) result in additional traffic. Therefore as shown in Figure 4-7 an increasing trend in travelling time in the system for flights can be observed.

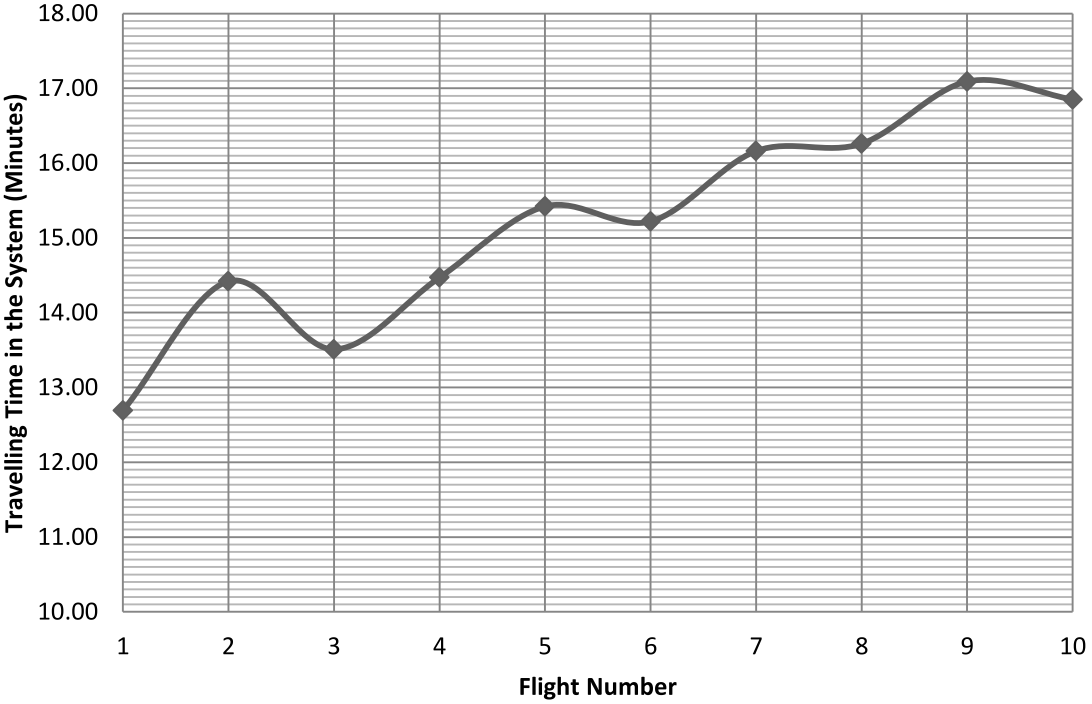

Figure 4-7: Travelling Time in the System of the Sample Traffic Resulted from Decentralized Model

Alike to the Centralized model, in Figure 4-8 based on the data provided at Table 4-3 every flight route versus time is drawn to explore the accuracy of conflict constraints. Two flights collide if they pass the same node or arc at the same time which reflects in this figure by two overlapped nodes. As shown on this figure, no collision is occurred. Therefore, the 3 sets of developed constraints to protect flights safety and also avoid every kind of air collision strongly assure the system. For that reason, it can be stated that the developed Decentralized model can be used as a powerful decision making tool to help air traffic controllers.

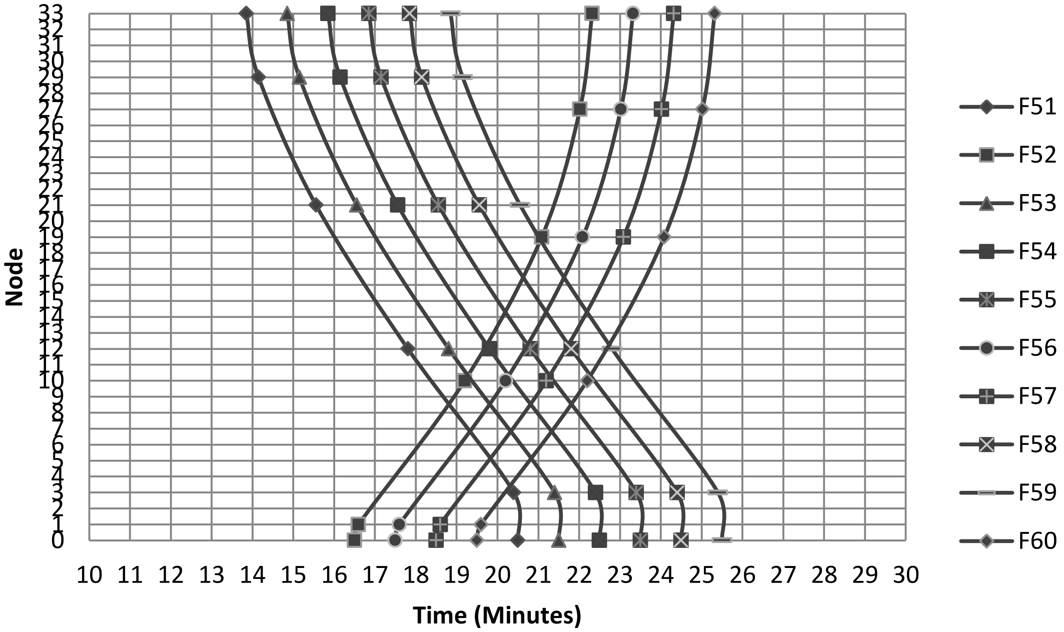

Figure 4-8: Collision Free Flight Route Resulted from Decentralized Model

4.2.2 Impact of Time between Arrival and Safety Distance Factor on the System Behavior for Decentralized Model

On the next step, the impact of TBA and SD is being observed on the system behavior. In this experiment two different objectives (Time in the System and Total Cost) separately have been applied to the model. As noted before, the earlier flights are more on ease to travel as the system is less congested. As time passes by, the system reaches to its steady state phase where the traffic becomes stable itself. However, this transition period directly depends on the values of TBA and SD applied to the system (as plotted in the Figure 4-9 and Figure 4-10). In these figures the SD is fixed to 1 minute. Flights arrival in this scenario is randomly generated with the TBA varied from 0.15 minutes to 1.4 minutes.

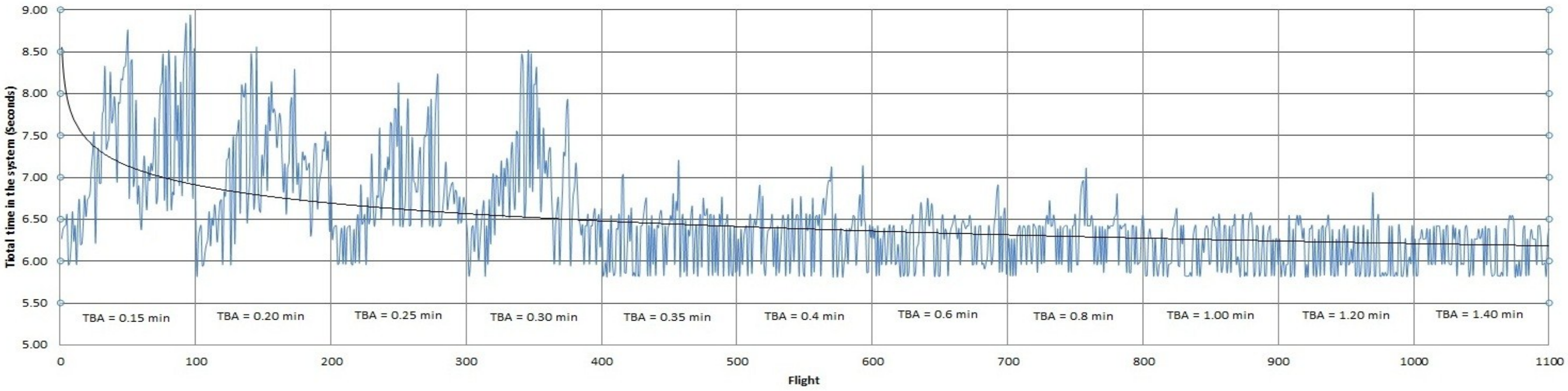

Figure 4-9: Impact of Time between Arrival on Time in the System (Objective of Total Time) for Decentralized Model

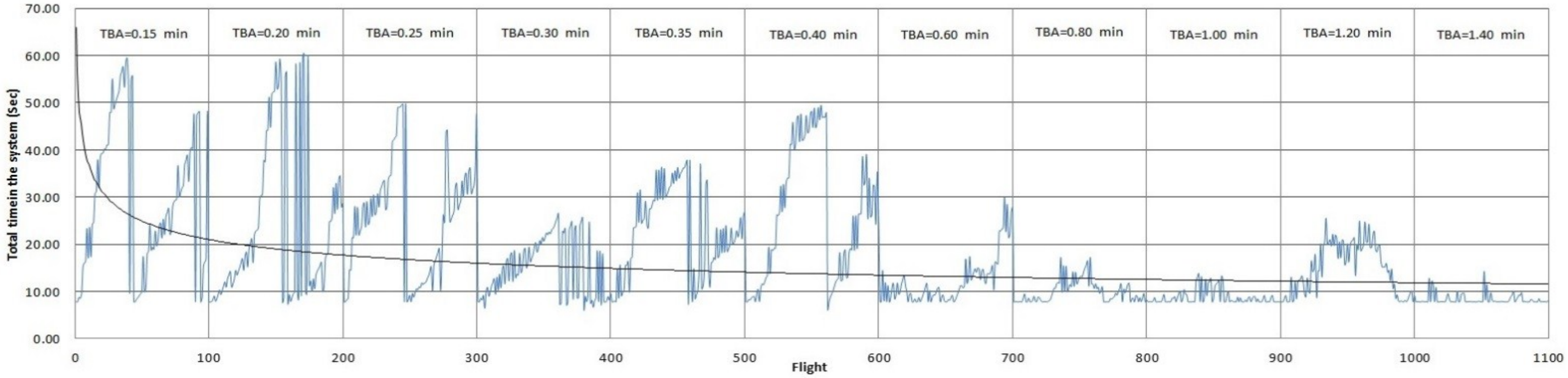

Figure 4-10: Impact of Time between Arrival on Time in the System (Objective of Total Cost) for Decentralized Model

Based on Figure 4-9 and Figure 4-10, as expected, by increasing the TBA, traffic in the system decreases. In other word, higher TBA causes less dense traffic and therefore system can reach to its steady state earlier. But it should be noted that after a certain amount of increase in TBA the system turn out to be insensitive. The reason is that the available capacity of the airspace becomes high enough to accommodate added flights. For example in Figure 4-9, when the airspace is tightly congested$(0.15\ min. \leq TBA \leq 0.30\ min.)$, the system hardly can reach steady state and be balanced. But with $(TBA > 0.8\ min)$ no traffic is observed as the lowest travelling time in the system remains steady at around 6.25 minutes. As a result, within the $TBA \sim 0.60\ min$ there is a balance in the rate of flights in the system, which can be called the Optimum Arrival Rate. In this period, airspace is fairly congested with a reasonable traffic rate and the safety of the flights in air is fully assured. Therefore it can be specified that based on the network characteristics and also the flights introductory schedule, there is an optimal combination of SD and TBA for this system. Obviously, the applied objective is the main parameter to define the system behavior. While minimization of total travelling time is targeted, system tries to speed up the flight journey. In this case as shown in Figure 4-9, the lowest travelling time is calculated around 6.25 minutes. But when the objective is set to total cost minimization including fuel and delay (earliness/lateness) cost, system is forced to decrease delay penalties and therefore, traveling time has to be compromised for minimum delay costs. Consequently, unlike the previous case (objective function of total traveling time), for an identical traffic stream and network, as shown in the lowest travelling time is around 10 minutes.

4.3 Comparison of Centralized and Decentralized Models

As explained before, the execution time of the solver relies on the problem size. By increasing the network size or traffic size, the execution time increases exponentially. Therefore, a sample test is developed to compare models performances in answer to an identical problem. In this test, 10 traffic streams with different sizes of 1 to 10 flights are chosen. They have been tested on both Centralized and Decentralized models with the objective of minimizing the total cost. (Figure 4-11)

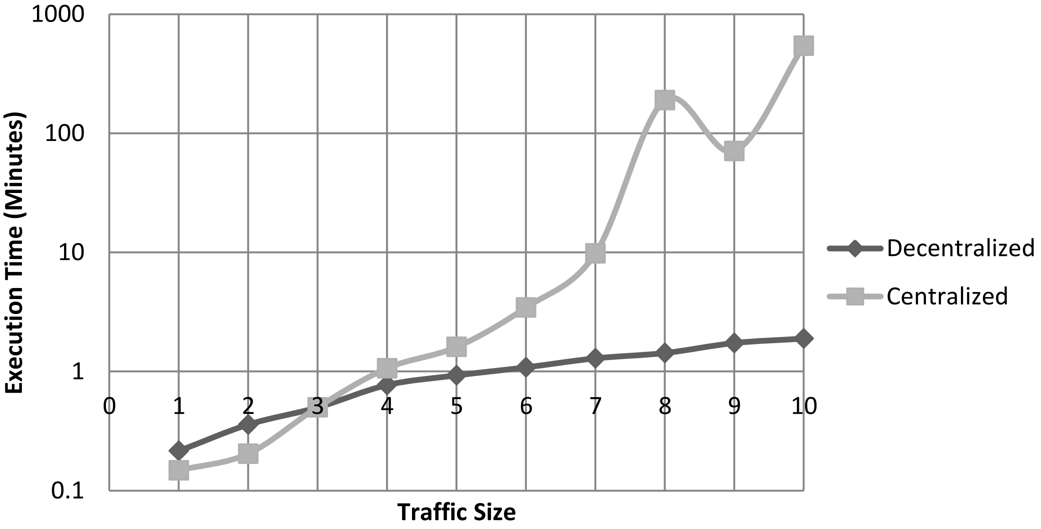

Figure 4-11: Comparing the Computational Time of Centralized and Decentralized Models

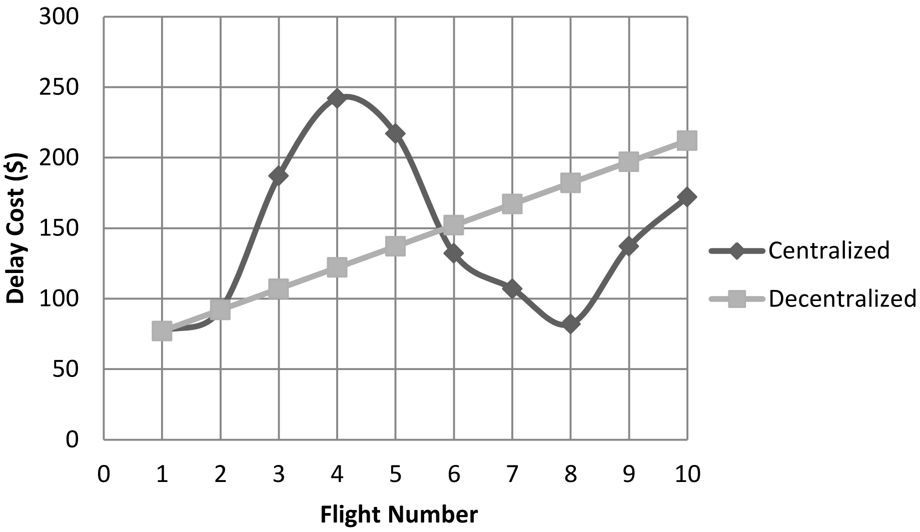

Figure 4-12: Comparing the Delay Cost of Centralized and Decentralized Models

Although the executing time sensitivity to the problem size exists for both Centralized and Decentralized model proposed in this work, the Centralized model is more sensitive and tends to grow faster for larger traffic sizes rather than Decentralized model. As shown in Figure 4-11 there is considerable difference in terms of execution time between the two models. Figure 4-12 represents delay cost for each flight number when the model is executed for a traffic size of 10 flights on Centralized and Decentralized formulation. It is observed that delay time can be propagated incrementally depending on the network capacity in Decentralized model toward the latest flight scheduled in the system. Therefore, fairness is not respected and flights scheduled earlier in the system can be favored with less delay costs. On the other hand, Centralized model, which naturally considers all flights together in the optimization process, does not propagate delay. However, Centralized model may favor some flights in order to use arcs and airspace more effectively. Such policy may lead to excessive delays of some flights. While such problems may be eliminated by various modeling techniques such as minimax or incremental cost for delays, complexity of the problem may further be increased. For this specific case of 10 flights traffic size, with Centralized formulation almost 70% of flights have less or equal delay as compared to Decentralized approach. Yet, for Centralized model the maximum delay occurred in the system is more than the maximum delay in Decentralized model. Accordingly, Decentralized model is preferred based on the near optimality of its results and superior execution time performance.

5 Simulation

Simulation aims to provide a visual formation of the system for an advanced insight over the system behavior. Therefore, in this work, a discrete system simulation model is designed to help understanding the developed optimization model's tactics and verify the accuracy of its result. This model is developed in ARENA 13.9 by Rockwell environment.

The simulation model is built by the network used in the optimization model contains nodes and arcs by using the Route option. Next, the optimization model result is imported to the simulation model as flights schedule. This schedule navigates flights based on their arrival order to the airspace through nodes and arcs with their related timing under the module of Advance Transportation. Accuracy of the results was verified both visually and through control logics built in the simulation model. Figure 5-1 illustrates a snapshot of the simulation model.

Figure 5-1: Snapshot from Simulation Model – Current Traffic around Montréal-Pierre Elliott Trudeau International Airport

6 Conclusions and Future Works

With considerable increase in demand for airline travelling services, the system faces consistent challenges globally to answer its customers. Congested airport terminals, frequent delays, limited airspace capacity around the airports, and particularly, crowded airspaces between airports are only some of the challenges to tackle. Furthermore, instable fuel prices hiking to an unknown value in recent years, alongside increasing labor costs have caused extreme difficulties. On the other hand, unpredictable weather conditions in most parts of the world result in terrific air travel delays. ATC should manage this uncontrollable traffic, handle congestions and delays, and assure flight safeties by preventing any incident from taxing to navigating in open skies.

This work studies the ATFM problem to reduce the ATC workload by proposing new algorithm to handle traffic and better using of the airspace. The main contribution of the presented mathematical model is its capability of determining the exact times for an aircraft passing through nodes in 3-D space by benefiting from a non-time indexed formulation strategy. Therefore, this work is able to provide the rerouting option for the user. The minimization of fuel consumption cost and the costs associated to flights' delays (earliness or lateness) are considered.

The model developed in this work considers airport flight zone and flights characteristics to provide a real sense of the situation. Flights schedule, speed maneuvers (controlled by discrete or continuous speed method), controllable safety distance and firm conflict avoidance are some of the factors that are mentioned in this work. The models (Centralized and Decentralized) are able to guide flights from their start point in the system, assign them to the best available shortest route, adjust their speed through the entire route, assure the flights safety to avoid mid-air collision with other flying aircrafts, and set their timing to minimize traveling time and associated costs to the system.

Keeping in mind that aircrafts are fastest transportation vehicles, the speeds of aircrafts at these points (nodes) are determined precisely. Therefore, the discretization of time forces aircrafts to move from one air-segment to another only when the time interval is changed. Clearly, the discretized aircraft motion is not reflecting the reality. Since the whereabouts of the aircraft between two consecutive time intervals cannot be known. Large safety distances between aircrafts are imposed in order to assure the safety of aircrafts. Consequently, the discrete-time based models cannot guarantee the optimal usage of the airspace due to large safety distance. Although reducing the time interval would increase the utilization of the airspace, the computational complexity would be significantly increased as a result.

Benefiting from a 3-dimensional (3D) network, the model provides a list of consecutive nodes to be visited by an aircraft. The non-time indexed approach treats the time-space relationship in a manner that exact data on the travelling time of each aircraft on flight-route will be achieved. Therefore, the separation distance between aircrafts on the network will be guaranteed despite of high travelling speeds over the arcs. Designed for a single airport arrival and departure instances, the NP-hard nature of the MIP model doesn't prevent large problems to be solved on a personal computer using CPLEX, but also provides real-time decision making possible through performing an iterative solution (Decentralized model).

Hence, by incorporating exact speed and time in the aircraft routing problem discussed in this work, following advantages are achieved: i) collision avoidance is ensured; ii) airspace is more effectively used by allowing large number of aircrafts in the region; iii) finally, the fuel consumption cost is formulated more precisely.

Correspondingly, the uniqueness of this study is considering two aspects of ATFM problem that asks for systems to be used by ground based ATC or in cabin pilots. The designed model and its implementation is so practical that could be used by the ground based ATC to control the stream of traffic in its desirable size or by the pilot to find the best flight available route and maneuver for itself. Benefiting from the Decentralized model, system can also adapt and recover itself faster to any new disturbances such as unpredicted weather situation. Not only the model provides the user with reasonable execution time, but also the fairness between flights is maintained so that no flight may delayed excessively comparing to others.

To conclude the results for various air traffic capacities and verify mid-air collision avoidance, a simulation model has been developed using ARENA simulation software by Rockwell.

To illustrate a vision for future works, execution time enhancement is suggested to result in better performance of Centralized model for larger aircraft traffic size. Furthermore, flight phases (Taxi, Take off, Climb, Cruise, Descend, and Landing) can be developed mathematically to be integrated with the current model to elaborate a real time full flight traffic simulation (FFTS). Finally, model can be tailored to accommodate for different aircraft types and their specific characteristics such as specific fuel consumption (SFC) rate, weight, endurance, and passenger capacity to more accurately estimate flight-dependent air traffic costs.

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