

# **Modeling of Contention-free and Contention MAC Protocols for Broadband over Power Line Networks**

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# Abstract

## **Modeling of Contention-free and Contention MAC Protocols for Broadband over Power Line Networks**

Although the power lines were designed to deliver electric power, not for communications, a large volume of research on Broadband over Power line communications (BPLC) had appeared over the decades. The successful transmission of data over the power lines will make the BPL technology a cost-effective solution for the so-called “last miles” in communication networks [1]. Recently, this effort received a significant boost by the approval of the IEEE P1901 standard [20]. This standard specifies the regulations for the operation of the Physical and MAC layers of the BPL network. It proposes two major MAC protocols – Time Division Multiplexing Access (TDMA) for the contention-free access and Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) for the contention access. In this thesis, we present the performance models of the contention-free access and the contention access mechanisms. For the contention-free transmission mode, we determine analytically the probability distribution of the number of beacon periods that it takes to transmit a MAC frame over multi-hop communications subject to transmission errors, then, we developed an analysis that determines the network capacity, the number of users that the system may support. Also, we determine the MAC efficiency of the network when it’s operating under both contention-free and contention mode. These results may be helpful in the design of BPL networks.

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# List of Abbreviations

<b>ACK</b>	Acknowledgements
<b>AIFS</b>	Allocation Interframe Space
<b>AMR</b>	Automatic Meter Reading
<b>B2BIFS</b>	Beacon-to-beacon Interframe Space
<b>BC</b>	Back-off Counter
<b>BM</b>	Basic Service Set (BSS) Manager
<b>BPL</b>	Broadband over Power Line
<b>BPLC</b>	Broadband over Power Line Communications
<b>BSS</b>	Basic Service set
<b>CFIFS</b>	Contention-free Interframe Space
<b>CIFS</b>	Contention Interframe Space
<b>CPE</b>	Customer Premises Equipment
<b>CSMA</b>	Carrier Sense Multiple Access
<b>CSMA/CA</b>	Carrier Sense Multiple Access with Collision Avoidance
<b>CW</b>	Contention Window
<b>DC</b>	Deferral Counter
<b>DSL</b>	Digital Subscriber Line
<b>EHV</b>	Extremely High Voltage
<b>FFT</b>	Fast Fourier Transform
<b>HV</b>	High Voltage

<b>IFS</b>	Interframe Space
<b>IH</b>	In-home
<b>IP</b>	Internet Protocol
<b>LV</b>	Low Voltage
<b>MAC</b>	Medium Access Control
<b>MPDU</b>	Medium Access Control (MAC) Protocol Data Unit
<b>MSDU</b>	Medium Access Control (MAC) Service Data Unit
<b>MV</b>	Medium Voltage
<b>NEK</b>	Network Encryption Key
<b>NTU</b>	Network Termination Unit
<b>OPERA</b>	Open PLC European Research Alliance
<b>PB</b>	Physical Layer (PHY) Block
<b>PBB</b>	Physical Layer (PHY) Block Body
<b>PBCS</b>	Physical Layer (PHY) Check Sequence
<b>PBH</b>	Physical Layer (PHY) Header
<b>PBM</b>	Proxy Basic Service Set (BSS) Manager
<b>PDF</b>	Probability Density Function
<b>PGF</b>	Probability Generation Function
<b>PHY</b>	Physical Layer
<b>PHYNET</b>	Physical Network
<b>PLC</b>	Power Line Communication
<b>PRS</b>	Priority Resolution Slots
<b>QoS</b>	Quality of Service

<b>RIFS</b>	Response Interframe Space
<b>RP</b>	Repeaters
<b>SACK</b>	Selective Acknowledgement
<b>SARQ</b>	Selective Repeat Automatic Repeat Request
<b>SIFS</b>	Short Interframe Space
<b>STA</b>	Station
<b>TDMA</b>	Time Division Multiplexing Access
<b>WLAN</b>	Wireless Local Area Network

# List of Symbols

$T_{bp}$	Beacon period time
$S_{beacon}$	Bits in beacon
$T_{B2BIFS}$	Beacon-to-Beacon Interframe Space
$T_{RIFS\_default}$	Response Interframe Space
$T_{AIFS\_access}$	Allocation Interframe Space
$S_{sack}$	Bits in selectively ACK
$R^{PHY}$	PHY rate
$S_{1901-FFT}$	Bits in 1901 FFT frame control information
$S_{TIA-1133}$	Bits in TIA-1133 frame control information
$S_{PBH}$	Bits in PHY Block Header
$S_{PBCS}$	Bits in PHY Block Check Sequence
$S_{PBB}$	Bits in PHY Block body which carries the data
$m$	The number of segments in a MAC Frame
$n$	The number of CPEs in the network
$\psi$	Throughput in the MAC layer
$K_\ell$	The number of segments in a MPDU
$B_{i,x}$	Bernoulli random variable
$\tilde{L}_x$	The remaining number of segments in a long MPDU still waiting to be transmitted at the end of the $x'$ th trial
$W_x$	The successfully transmission of a long MPDU will be

	completed on the $x'th$ trial
$\bar{W}$	The average number of trials needed to transmit a long MPDU
$\bar{X}$	The average number of PBs that will be transmitted successfully in a single trial
$p$	The probability of receiving a segment in error in any hop
$R$	The number of hops between the source and destination
$i_r$	The number of trials needed to transmit one segment successfully in the $r'th$ hop
$I$	The number of trials needed to transmit a segment successfully from the source to the final destination
$Q_x$	The probability of transmitting a segment successfully from the source to the final destination by the end of $x'th$ trial
$B_{i,x}^*$	Bernoulli random variable
$L_x$	The number of segments which have reached its destination successfully by the end of $x'th$ trial
$\rho$	geometrically distributed parameter
$\bar{L}_x$	The average number of segments that will reach to its destination by the end of $x'th$ trial
$G_x$	A MAC frame has been transmitted successfully by the end of the $x'th$ trial
$a_r$	The distance between source and destination is $r$ hops
$\bar{F}_R$	The number of trials needed to transmit a MPDU over $R$

	hops
$F$	The number of trials needed for a CPE to transmit a MPDU successfully to its final destination
$p_r$	The probability of segment error in link $r$
$K_T$	The total number of segments in $n$ frames
$q_{r x}$	The conditional probability that a segment will be at the $r$ 'th repeater at the end of the $x$ 'th trial
$q_r$	The probability that a segment will be at the $r$ 'th repeater at the end of any randomly chosen trial
$S_r$	The number of segments which are at the $r$ 'th repeater by the end of any random trial
$\beta$	The threshold value that probability that the number of segments exceeding this value at the $r$ 'th repeater is less than $\alpha$
$p^*$	The probability that a station has a frame to transmit in the reserved region
$B$	The number of slots available for the contention traffic
$\Pi(i, j)$	The steady state probability that the node has a DC value of $i$ and BC value of $j$
$p_i$	The probability that the node under consideration finds the medium idle
$p_0$	The probability that a node will transmit in any randomly chosen slot of time



$T_I$	Time for a slot
$T_S$	Time for successful transmission
$T_C$	Time for a collision
$\tilde{n}$	Number of stations contending in the CSMA region
$N_{bp}$	Number of bits in a Beacon Period
$N_{CSMA}$	Number of bits for the CSMA traffic
$R'_b$	Transmission rate for the CSMA traffic
$T'$	The duration of a bit for the CSMA traffic
$N_{Data}$	The number of bits for the data payload in the contention traffic
$T_{Data}$	The duration of the data payload
$N_{ACK}$	The number of bits of the ACK
$T_{ACK}$	The duration of the ACK

# Chapter 1

## Introduction

### 1.1 Introduction of Power line communications networks

Although the delivery of voice, data and video at high speeds is the culmination of years of research and development that began with basic dial-up service since the early 1990's, statistics shows that only less than 4% people in this world have access to the broadband service [1]. A major hurdle is the high cost of deploying the last-mile access. The last-mile access can be defined as the final link from the backbone networks to the customers [1]. Thus, the usage of the existing infrastructure for realization of the access networks is a more attractive solution for network providers. In this case, there are three kinds of already existing wireline network which could be used to connect the subscribers to the transport telecommunications networks - digital subscriber line (DSL) through telephone company telephone lines, cable modem through cable company coaxial cable lines and electrical power supply lines.

However, the coverage of DSL and cable networks are not wide enough, when building the new DSL and cable lines, both of them are connected to a central node, so modems are needed on both the subscriber and the central node side. Additionally, the cable networks should be able to transmit bidirectional while the subscriber lines have to be modified to ensure application of DSL technology, which results in extra cost. [19]

Because of all these, power lines are attractive for communications purposes. From the service providers' point of view, because the power line has omnipresence that reaches most homes and businesses, even in the most rural areas, there is no need to purchase spectrum or to hang, dig, or lay new wires, there is also no need for the difficult, expensive, and time-consuming site acquisition, permission, and licensing tasks needed for a typical deployment [1]. For the electric utility companies, BPL can create new sources of revenue from an existing investment, and it can help create a smart grid for the utility companies that would enable enhanced utility applications. End users can also benefit from BPL, the competition will bring the service price down and this new technology will turn every single socket in the world into an interface to the World Wide Web.

## **1.2 History of Power line communications network**

The application of electrical supply networks in telecommunications has been known since the beginning of the twentieth century, but has only been used for narrowband communication for a long time, such as tele-remote relay applications, public lighting and home automation. These applications make possible the management of numerous electrical devices within a building or a private house from a central control position without the installation of an extra communications network. At that time, the data transmission speed was also very low.

The idea of using electricity distribution networks for broadband communications came out in the 1990's along with the development of Internet. The research of power-line channel characteristics, modulation techniques and communication protocols increased dramatically. Companies such as Nortel Networks and Siemens realized the potential of deploying broadband service over power-lines. However developing this technology faced lots of hurdles during that time [21]. The research on channel characteristics enhanced the transmission frequencies up to 30 MHz, because the system requires a wide bandwidth to support the high data rates. Several integrated circuits and power-line communications systems for broadband communications were presented. [2] The first large-scale field trial was carried out by Nor.Web (the North West Electricity Board, a British electricity supply and distribution company). In 1997, the negative results of the test led the company withdraw from the research of this project. According to Nor. Web, economically, the internet access through power-lines would not be feasible. However, the reasons for the withdrawal were probably the resistance of radio amateurs, regulations and technical problems, which made the business economically unfeasible [2]. Later on, power-line communications field trials in public low voltage distribution networks have been carried out by several distribution companies in Europe. But after nearly two decades of development, this technology has still not delivered on its promise to provide the products required for the proliferation of power line communication.

In late 2010, an IEEE project resulted in the definition of a standard for high-speed (>100 Mbps at the physical layer) communication devices via electric power lines, so-called broadband over power line (BPL) devices. Adoption of this standard is an

extraordinary step for the power line communications industry. Under this standard, this new network technology is now available to implementers and manufacturers [20].

### **1.3 Power distribution networks for data transmission**

As described in the previous sections, broadband over power line communication is based on the power distribution networks. In this section, the basic idea of power grid which relates to the BPL networks will be introduced.

A power grid basically consists of power plants or generators, transmission substations, transmission lines, power substations that operate at different voltage levels with transformers to switch between voltage levels. Those voltage categories are extremely high voltage (EHV typically 155 to 765 kV), high voltage (HV, 45 to 155 kV), medium voltage (MV, 2 to 45 kV), and low voltage (LV, 100 to 600 V) [1] resulting in the infrastructure of an electric utility consisting of high-, medium-, and low-voltage lines. Electrical power is transmitted over HV transmission lines, distributed over MV, and used inside buildings at LV. Because of the level of noise on the high voltage lines, these lines are not suitable for the broadband transmission, only MV and LV lines are used for BPL. MV lines are usually less branched than LV lines, making point-to-point connections possible.

The Telecommunication network (backhaul network) will be connected to the MV lines through an injector[3], once dropped on the medium-voltage lines, the signal can only travel so far before it degrades. To counter this, special devices are installed on the lines to act as repeaters. The repeaters repeat the received signal in a new transmission, amplifying it for the next leg of the journey[4]. Then, the BPL signal is transferred to the LV power line through the LV distribution transformer and delivered directly to the end-user[16]. The architecture of the BPL network is shown in Figure 1.1

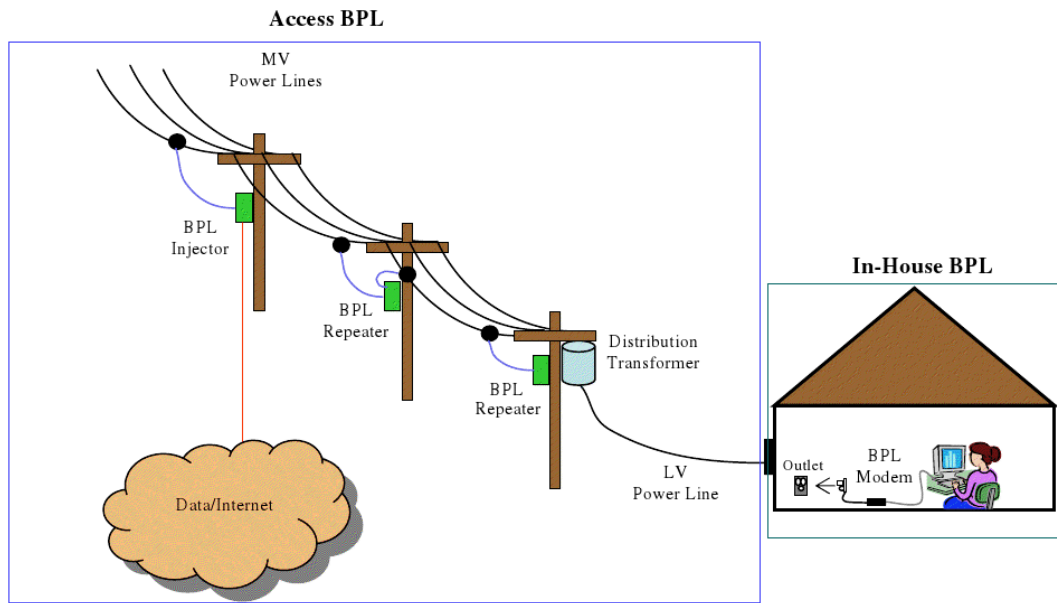


Figure 1.1 BPL architecture.

## **1.4 Problems specific to power line communication networks**

The fact that the power line networks are not designed for communication purpose, that they were originally developed to transmit the electric power from a small number of sources to a large number of end-users makes them face several challenges, such as a large and frequency-dependent attenuation, changing impedance and fading as well as unfavorable noise conditions.

In general, the power line channel is a very noisy transmission medium. Various noise sources, acting from the supply network, due to different kind of electric loads connected to the network, and from the network environment, can negatively influence a PLC system, causing disturbances in an error-free data transmission. Moreover the noise characteristics on MV and LV lines are different. LV lines are terminated at time-varying end users electrical appliances, so the noise is mostly generated by the end customers [1], for instance, the background noise, impulsive noise and colored noise. On the MV grid, the environmental noise caused by weather and the on/off switching of capacitor can also affect the data transmission on the MV lines [1]. On the other hand, to provide higher data rates, PLC networks have to operate in a frequency spectrum of up to 30 MHz, which is also used by various radio services [19]. Unfortunately, a PLC network acts as an antenna which produces electromagnetic radiation in its environment and disturbs other services operating in the same frequency range [22].

Another concern about the BPL networks is the liquidity of signal attenuation characteristics; reflection from the terminals of branch lines on the power line is one of the major factors in signal attenuation increases [5]. Signal attenuation occurs more easily on a MHz band than on a KHz band, as we introduced before, the frequency spectrum of the PLC networks can go up to 30 MHz, which may cause a serious problem. Also signal attenuation has more affection when the distance of power lines is long or there are more branches [5]. Due to the high attenuation that occurs during transmission, it is necessary to use repeaters to amplify the signal at regular intervals. Also, since the data signal cannot pass through transformers, they must be bypassed. Routing data around transformers can be costly [1].

What's more, since data is transmitted on a shared medium, like cable broadband, this makes it easier to snoop the line. The fact that the LV signal can potentially go to several hundred homes is not very secure [6]. The same line going into many homes means the same traffic going down that line. This provides an opportunity for hackers to acquire sensitive data.

In order to minimize the negative impacts of power line transmission medium, BPL access systems have to provide a very good network utilization of the shared transmission medium and, simultaneously, a satisfactory QoS [7]. Both requirements can be achieved by the application of efficient MAC layer techniques which will be studied in this thesis.



## 1.5 Related research

Before IEEE published the official P1901 standard for Broadband over power line networks, several organizations had already proposed specifications to regulate the PHY and MAC layers of the BPLC networks, such as OPERA (Open PLC European Research Alliance) and Homeplug Alliance. These specifications developed a new generation of PLC networks that led to research on the low cost and high performance of the BPLC technology [23]. Thus, large amount of research work have been published that investigate the protocols specified in those projects. In this section, we will introduce relevant work related to the applications of data transmission over power line networks.

There are several works regarding application of BPL networks to the AMR (automatic meter reading) systems, such as [8][9]. BPL technology can be used for monitoring of the energy consumption, control of energy losses and even for the automatic billing systems. In the future, home owners will be able to monitor the appliances, change temperature of their air conditioning, and pay their hydro bills at anytime and anywhere. Paper [10] suggests application of the BPL to systems other than in-home automation networks, such as its utilization in ship automation systems through their LAN (local area networks). This work found that BPL implemented through the SPS (shipboard power system) cables has a strong potential for use as a communications network for ship automation systems, because the hundreds of kilometers of cabling from

fiber and Ethernet networks add considerable weight and increase overall cost of the ship while the ship wireless LAN coverage is limited by the ship hull.

In [11], the efficiency of MAC protocols in the OPERA projects based on two scenarios has been presented – a network with and without repeaters, both scenarios ignore transmission errors in the links. The authors analyzed the TDMA protocols in the OPERA specification, they assume that the time allocations for the network elements are decided by a token delivered by one of the stations in the network. The data token contains all the necessary information for a station to access the network. The network architecture has a tree topology, there is one master station at the top and several slaves along the branches. They demonstrate two basic network models, one without repeaters and one with repeaters. Later on, they analyze the MAC efficiency of each model by calculating the ratio of the throughput in the MAC layer over the transmission speed in the PHY layer and the throughput in the MAC layer is defined as the ratio of the total successful data transmission over a transmission duration which includes data payload and the overhead. Their analytical and simulation results show that the MAC efficiency is reduced with the increasing of the number of stations in the network.

In [12] [13], researchers studied a MAC mechanism from HomePlug 1.0, which extend the random backoff algorithm of the conventional CSMA/CA used in 802.11 and takes transmission errors into consideration. In both works, a tri-dimensional discrete-time Markov chain is used to model and analyze the HomePlug system. The state of each station is represented by three variables. Papers [12, 13] maximize one of the three values

in each state and the throughput is still defined as the ratio of average payload size over the average time required for successful transmission. In [12], the MAC performance has been evaluated under the saturated traffic while in [13], both MAC throughput and MAC delay have been analyzed under both saturated and unsaturated conditions.

Meanwhile, [14] proposed a different homogenous bi-dimensional Markov chain for modeling the CSMA/CA protocol introduced in [12,13]. The state of each station is represented by two variables. Elimination of one of the variables has reduced the complexity of the analysis. We have used the same Markov chain model for the CSMA/CA protocol studied in Chapter 4. In [15], the authors increase the MAC efficiency by using the optimum parameters in [14]; this approach avoids the dropping of MAC efficiency when the network is operating under large number of users which may cause more collisions. We leave details of this model to Chapter 4 because of its significance to our own work.

## **1.6 Major contributions of the Thesis**

This thesis makes several contributions to the performance modeling of Broadband over Power Line Networks. In the following we summarize the major contributions of the thesis,

1). We derive analytically probability distribution of the number of beacon intervals that it will take to transmit a MAC frame using contention-free transmission model under different scenarios. In these derivations, we assume multihop communications between source and destination pairs and we consider both homogeneous and nonhomogeneous error probabilities over the hops as well as fixed and random number of hops between source and destination. Then using the preceding result, we develop a method that determines the network capacity under contention-free transmission mode. Network capacity determines the number of users that can be simultaneously supported in the contention-free traffic mode.

2). We determine the MAC efficiency of the network when the users are allowed to contend for the channel during a certain period of time of a beacon interval. The MAC efficiency shows the number of stations which are allowed to transmit simultaneously in a beacon interval.

## **1.7 Thesis Outline**

This thesis is concerned with the performance evaluation of contention-free and contention traffic in the Broadband over Power line Networks. The remainder of the thesis is organized as follows:

**Chapter 2: Power line communication network architecture.** In this chapter, the architecture of the BPLC networks and the basic components in the networks are introduced. We describe the functionality of all stations within the access network and in-home network and how they interact with each other.

**Chapter 3: Modeling of Broadband over Power Line Networks with contention-free traffic.** This chapter explains the TDMA access for contention-free traffic. We build several models for the network operating under different scenarios such as, with and without repeaters or transmission errors. Each model is analyzed in detail and we present numerical results to illustrate the analysis.

**Chapter 4: Modeling of Broadband over Power Line Networks with contention traffic.** In this chapter, the CSMA/CA mechanism used for the contention traffic is presented. We propose a model of bi-dimensional Markov chain to analyze the state of each station in the network. Finally, the MAC efficiency of the CSMA/CA protocol are shown based on the numerical results we obtained in the previous chapter.

**Chapter 5: Conclusion and future work.** We summarize the main contributions in this thesis and the future work.

# **Chapter 2**

## **Power line communications network architecture**

Power line communications networks consist of several components interacting with each other. In this chapter, we will introduce the architecture of the power line communications networks by explaining each component in details. There are two different kinds of BPL systems depending on modulation technique used in the PHY layer – FFT system and wavelet system, in this thesis, we only analyze the FFT system.

### **2.1 Access network**

As explained in the previous chapter, only the Medium-voltage and Low-voltage power distribution lines are suitable for communications due to the noise issue and the high signal attenuation. P1901 Access network is specified to operate on the MV and LV lines for communication with the backhaul network as shown in Figure 2.1.

Before introducing the access network, we need to define the related terminology. First, the “Basic Service Set” (BSS) is the basic building block of a network of P1901 stations. BSS consists of several stations and a single management station (BM) which

will manage and associate all the other stations within that BSS. In an access network, the BSS and BM are referred to as a “cell” and a “HE” respectively and there may be several cells in one access network. But there is typically only one access network on a given physical power line medium. Cells in an access network are logically isolated from each other, and the communication to and from the backhaul occurs within each cell at the HE.

## **2.2 Access stations**

Figure 2.1 depicts an example of the access cell and all the key components within it. The structure of the cell is a function of the physical topology of the locations of its stations. There are three topologies - ring, mesh and tree topology. The three topologies can also be combined together for different access installations; however the logical network structure should be more tree-oriented in order to avoid loops when forwarding a frame. In the access cells, HE controls the entire cell and acts as a component of a bridge between the cell and the backhaul network. As may be seen, access networks include RPs (Repeaters) and (Network Termination Units). The purpose of using RPs and NTUs is to regenerate the frames to enable them to reach further. Repeaters may also perform path (next hop) selection and QoS functions. CPEs (Customer Premises Equipments) are the “end-users” of the cell which will be installed in the customers’ premises. The detailed description of these components will be given in the following discussions.

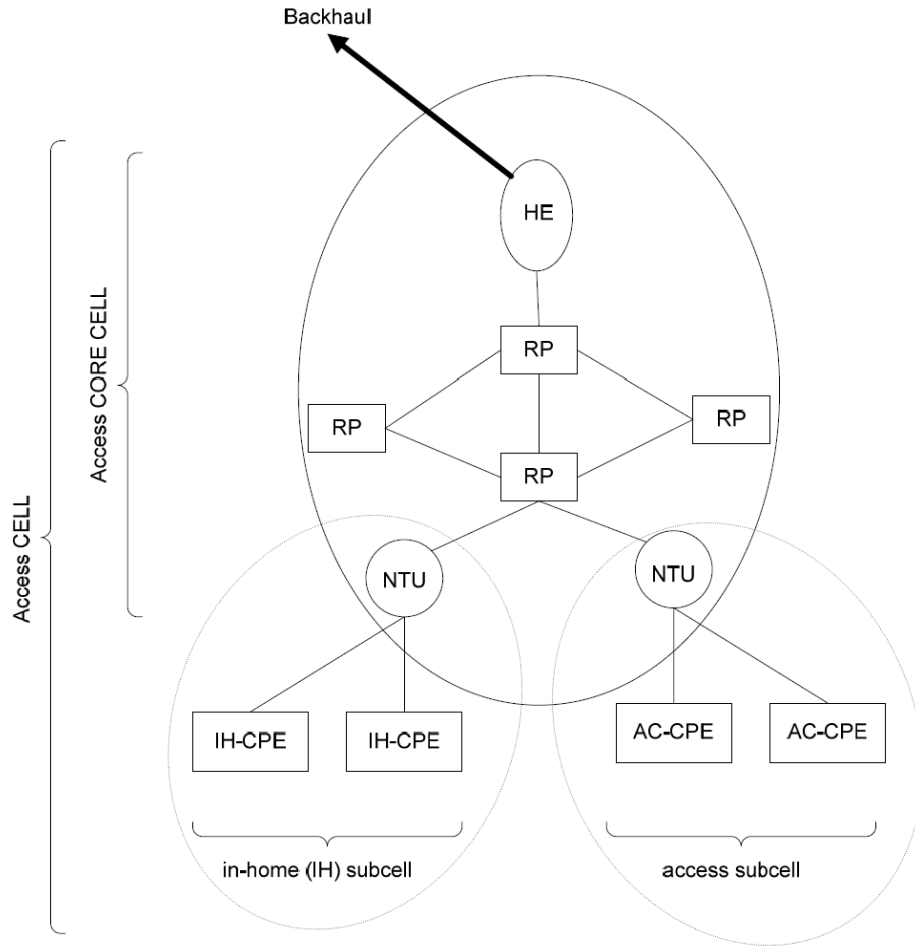


Figure 2.1 Access network cell components

From Figure 2.1, it may be seen that an access network may be divided into a core cell and several subcells. An access core cell consists of a group of stations within an access network that share the same network encryption key (NEK) and includes HE, Repeaters, and NTUs but excludes CPEs. In our example, there are two different subcells connected to the access core cell via two different active NTUs, which are in-home (IH) subcell and access subcell.



Although both subcells provide an extension of the core cell, the functionality of each subcell is not always the same. In the access subcell, NTU may connect several access CPEs and it may act as a proxy, a repeater, and/or an HE for the subcell, the access CPE is associated with the access subcell through access protocols and connect several in-home BSS. In the in-home subcell, the role of NTU is to act as a BM to all the associated in-home CPEs using in-home protocols and communicate with access core cell using access protocols, and also the in-home CPE has two identities too, it is actually a station associated with two or more BSSs. It has an in-home BSS association where it can be the BM or STA in this network, and it also is associated with the BSS between the NTU and the in-home CPEs. Figure 2.2 shows the access subcell and in-home cell.

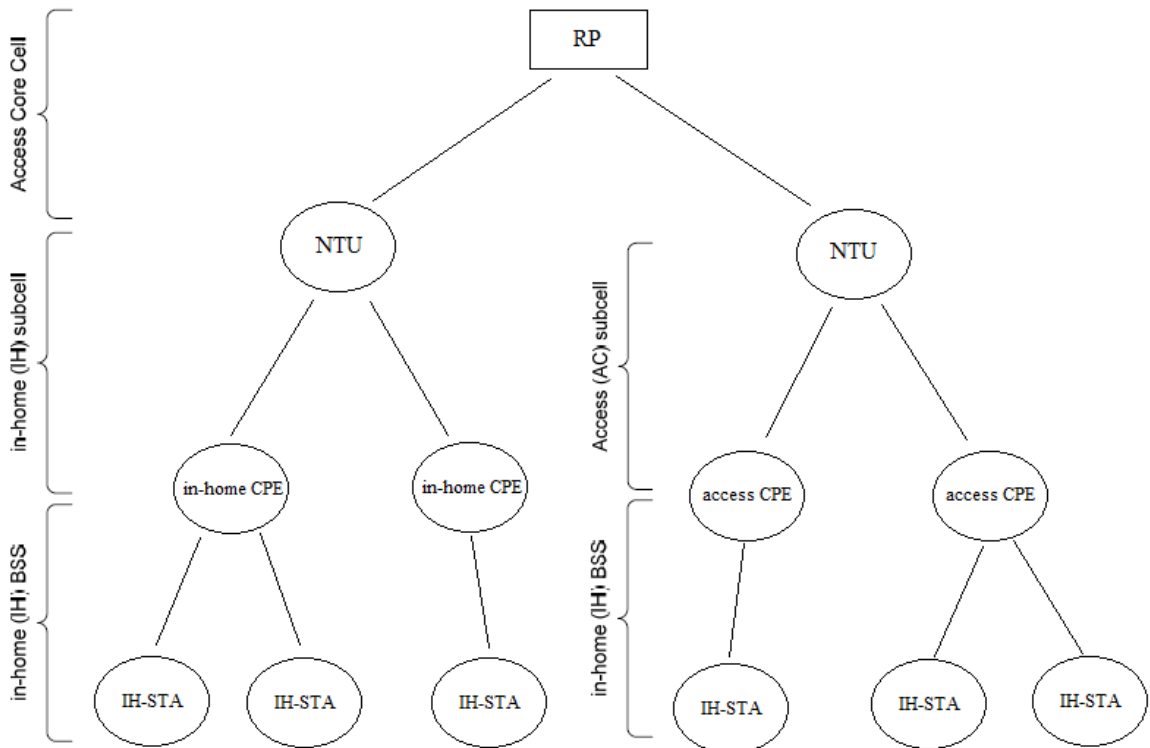


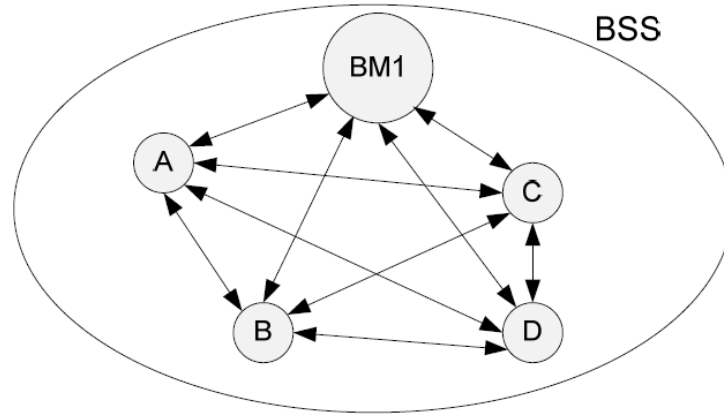
Figure 2.2 Subcells

## 2.3 In-home stations

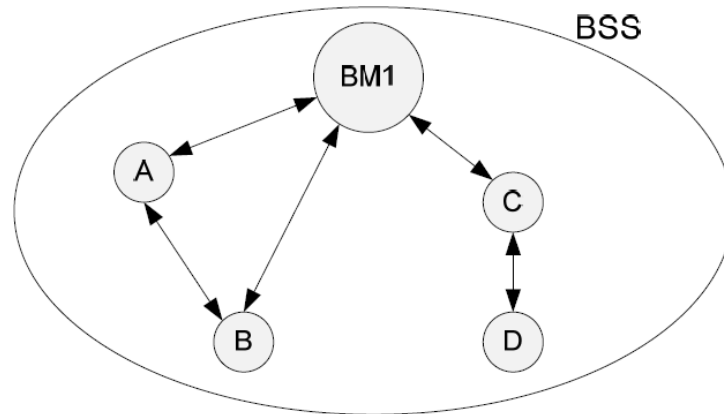
The in-home stations (STAs) are the main components within an in-home BSS. The in-home BSS is managed by a single STA called “BM” as we introduced before. Each STA in the in-home BSS should be capable to manage the BSS. There are three different kinds of BM depending on their different functionality in the BSS. Level-0 BM station is often referred to as “STA” in the in-home network, this BM can only operate in a CSMA-only mode, and usually doesn’t provide QoS support. Level-1 BM station can provide uncoordinated mode of operation (that is, operation when no other network is nearby) and schedule the TDMA allocations for all the requiring STAs within the BSS. Level-2 BM station has an additional functionality compared to the Level-1 BM station which is allowed to provide coordinated mode of operation with BMs of neighboring networks.

We need to introduce a concept here - the physical network (PHYNET). Physical network of a given STA is the set of STAs whose PHYs (physical layers) interoperate with the STA’s PHY (without use of relays or repeaters). All STAs in a PHYNET have the potential to interfere with each other. Figure 2.3 shows two examples of the PHYNET, in Figure 2.3(a), all the stations are within the same PHYNET, the lines between them show the ability to interfere. In Figure 2.3(b), as we can see that station A, B and BM1 have the same PHYNET set because each of them can communicate with the other two stations within the PHYNET. Station D is not in the PHYNET of BM1 which makes it a “hidden station” to BM1. When BM1 is trying to communicate with station D, it uses station C as a proxy to relay the frames. So in an in-home BSS, station C is playing a role

of proxy BSS manager (PBM) which relays all frames between the BSS manager and a station in the BSS that is hidden from the BM. The proxy STA can also be treated as a “repeater” in the network.



(a)



(b)

Figure 2.3 Examples of PHYNET

## 2.4 Differences between In-home BSS and access cell

Table 2.1 summarize the differences between in-home BSS and access cell.

Table 2.1 Comparison of in-home BSS and access cell.

<b>Feature</b>	<b>IN-home BSS</b>	<b>Access cell</b>
<b>Proxy management</b>	Setting is required for BM to assign PBMs in case of hidden nodes in BSS	Not required since each STA is synchronized with HE using multi-hop beacon
<b>Maximum hops between STA and BM/HE</b>	1 Proxy level (PBM) for BM to STA	Up to 63 repeaters between HE and STA
<b>Time management</b>	Centralized by the BM	Either Centralized by the HE or distributed – each STA negotiates with its direct and in-direct neighbour
<b>Repeating</b>	Supported with single level addressing, with Ethernet layer bridging in each hop	Supported with two levels addressing, with TEI layer routing without requiring Ethernet layer bridge in each Repeater
<b>Smoothing</b>	Supported in STA-to-STA single-hop transmission	Support in edge-STA to edge-STA multi-hop transmission
<b>Maximum STAs in network</b>	253	4095
<b>Protocol</b>	1901 in-home protocol and frame structures	1901 FFT access protocol and frame structures
<b>BM/HE assignment</b>	Predefined or elected	Predefined only
<b>BM/HE de-assignment</b>	Allowed (if not predefined)	Not allowed, HE may not change its role

**Proxy management:** BM and HE provide scheduling information using beacons as to be explained in the following chapters. In the in-home BSS, there is only one PBM from BM to STAs which means the distance of each branch in the tree topology is limited. BM transmits central beacons to all the STAs connected directly including PBM and end-users, after receiving the central beacons, PBM relays all the related scheduling information to the hidden STAs by using the proxy beacon. In the access cell, there could be up to 63 repeaters between HE and STAs, the repeaters relay the scheduling information to the end-users according to the routing table, so the assignment of PBM is unnecessary.

**Time management:** In the FFT in-home BSS, BM is coordinating all the other STAs in a centralized manner. BM is the only station which has the right to decide the transmission schedule of the requiring STAs during the contention-free period. However, the access cells may be centralized and distributed networks. Allocations in access networks are managed in a centralized manner near the head end and in a distributed manner in the remainder of the access network. So, in the centralized scheduling scheme, the HE determines the allocations and provides them to the STAs. In a distributed scheduling scheme, each station independently determines the allocation from itself to the next hop STA.

**BM and HE assignment:** Each FFT STA in an in-home BSS shall be capable of managing the BSS, a single station could be selected to become the BM according to a set of predefined rules. The main difference between the access HE and the in-home BM

is that the HE is a dedicated station, while the BM is dynamically determined by the BSS STAs. The role of the HE cannot be changed between stations unless a specific backup HE has been set up.

## **2.5 MAC layer functional description**

The MAC protocol provides two fundamental services in the P1901 standard, a contention-free service to support the QoS requirements of demanding audio-video and voice applications and a contention service to allow STAs contend for the channel. The MAC protocol for the contention-free traffic is TDMA (Time Division Multiple Access) while CSMA/CA (Carrier Sense Multiple Access with Collision Avoidance) is used for the contention traffic. Under the CSMA/CA protocol, before a STA may transmit, it will sense the medium to determine whether another STA is transmitting. If the medium is not determined to be busy, the transmission may proceed. If the medium is determined to be busy, the STA will defer until the end of the current transmission. Next, we will give general description of information transfer through the MAC protocol.

### **2.5.1 Beacon Interval**

In the access cell, the transmission over time is structured into a repeating synchronization process called the “Beacon Interval” (also referred as Beacon Cycle and Beacon Period). The basic service set (BSS) manager (BM) transmits a beacon frame in

the beacon region that signals the end of the current beacon interval and the beginning of the next one. Beacon interval is the duration of the period from the beginning of the current beacon frame to the beginning of the next beacon frame. At the beginning of each beacon period, the HE or the BM of the cell determines the schedule of the channel access which is broadcasted to all the stations in the beacon frame. When the network is operating in an uncoordinated mode, each beacon frame is transmitted in a single beacon slot.

In an uncoordinated mode, a CSMA region (called “minimum CSMA region”) shall immediately follow the beacon region, together with other CSMA regions located elsewhere in the beacon period, are used by STAs in the cell to contend for the channel through CSMA/CA protocol. After the minimum CSMA region, the reserved region follows which allows the transmission of data in the reserved allocations. The details of all the reserved contention-free allocations and CSMA allocations are provided by the HE during the beacon frame.

In chapter 3, we will present performance modeling of the contention-free access scheme in the reserved region. As may be seen in Figure 2.4, the reserved region may consist of several allocations, each one reserved to one station. An STA transmitting in contention-free mode may begin its transmission in the medium immediately at the start of its allocation.

For example, in allocation #2, the transmitting STA may transmit one long MPDU (MAC Protocol Data Unit) frame during the assigned time. After completion of the data frame transmission, the recipient STA may transmit an ACK frame to indicate whether the data has been received correctly or not. Actually, the transmitting STA is allowed to transmit one or more times in the assigned time which is called MPDU bursting as shown in Figure 2.5. The MPDU burst may consist of multiple data MPDUs, when the ACK response is only expected at the end of the MPDU burst, the burst can be treated as a long MPDU. Next we explain the generation of the long MPDU.

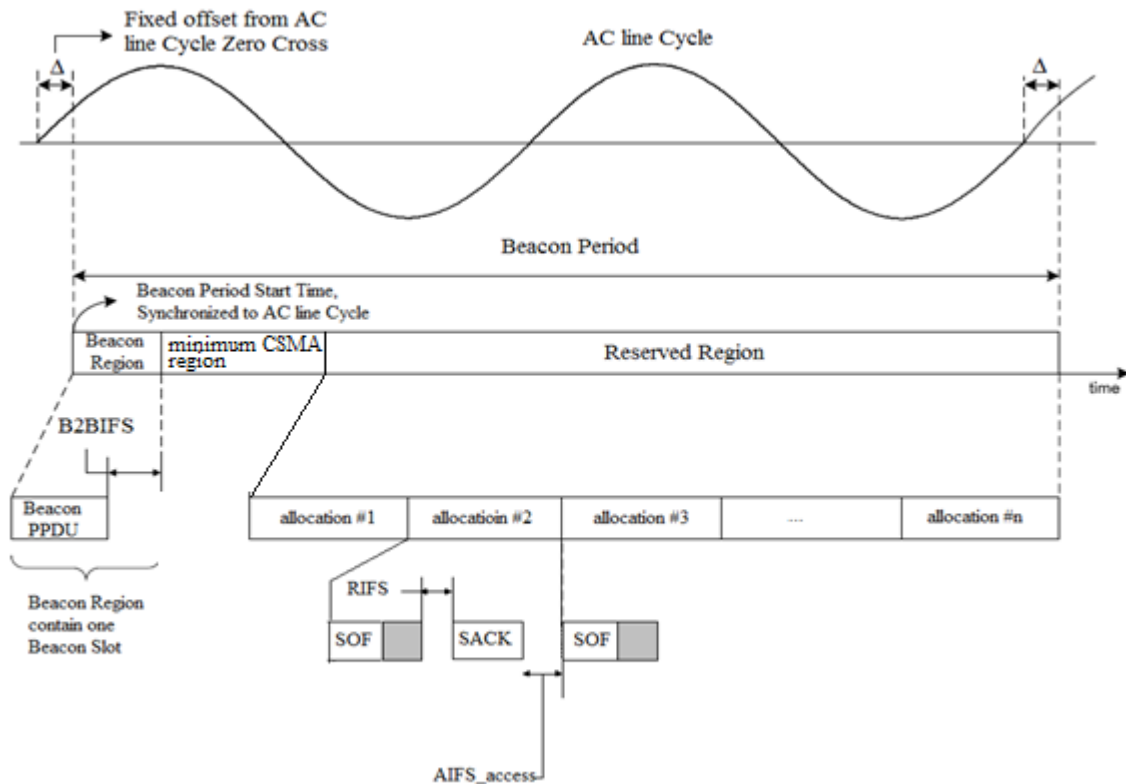


Figure 2.4 Beacon interval structure with beacon and reserved regions



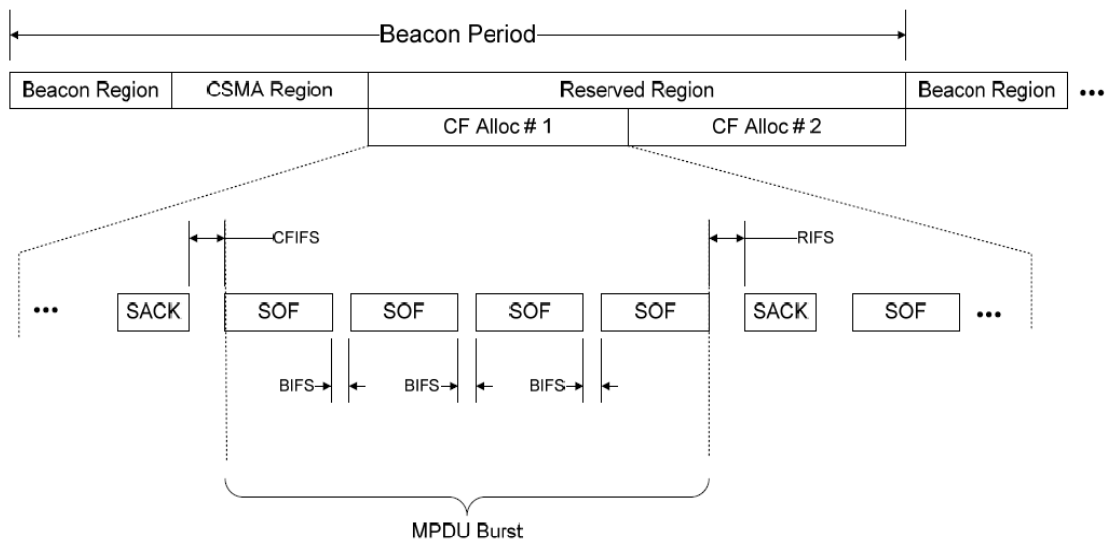


Figure 2.5 MPDU bursting

## 2.5.2 Long MPDUs

The purpose of using the long MPDU is to improve the protocol efficiency. When the MPDUs are being generated by MAC from MSDUs (MAC Service Data Units), several MPDUs that belong to the same stream can be concatenated together into an MPDU stream. The process of generating an MPDU stream from MSDUs is referred to as “MAC framing.” Later on, each MAC frame stream is segmented into 512-octet segments for transportation as part of an MPDU payload. A segment can contain a fraction of a MAC frame and/or multiple MAC frames depending on their sizes. Figure 2.6 shows the MAC frame fragmentation and long MPDU generation. Each segment is encrypted and then inserted into a PBB (Physical Layer (PHY) Block Body). Then the PB (Physical Layer (PHY) Block) header and PBCS (Physical Layer (PHY) Check Sequence) are added to

the PBB comprise a PB. After that, a long MPDU can be generated by combining frame control information by one or more PBs.

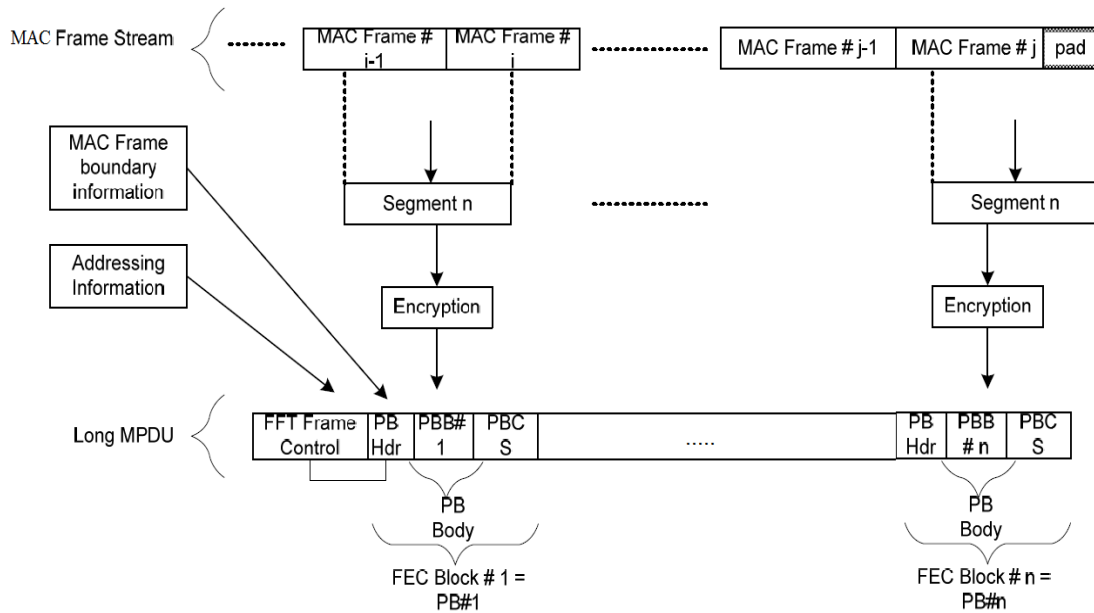


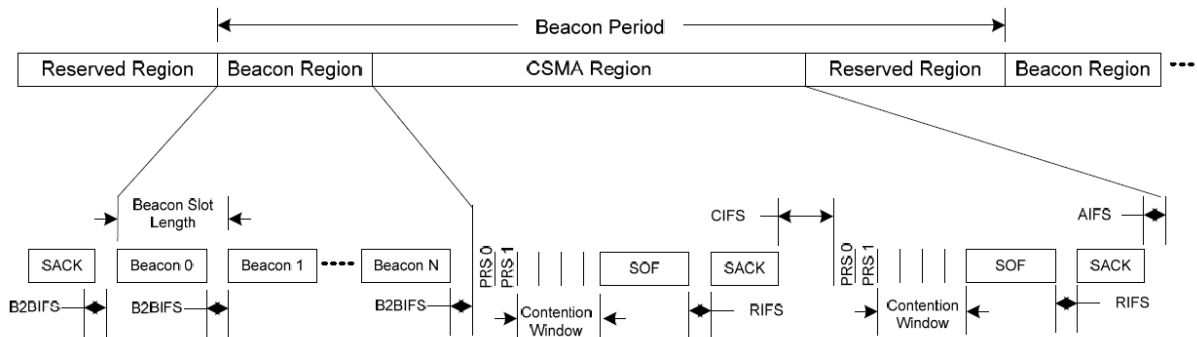
Figure 2.6 MAC frame fragmentation and long MPDU generation

When the transmission of a long MPDU is finished by the transmitting STA, an acknowledgement (SACK) is expected from the receiving STA (Figures 2.4 and 2.5). The segments which are not received correctly are going to be retransmitted. The MAC uses Selective Repeat Automatic Repeat Request (SARQ) at the segment level. A retransmitted segment is packaged in a new PB and is retransmitted with that MPDU, which may also contain new segments.

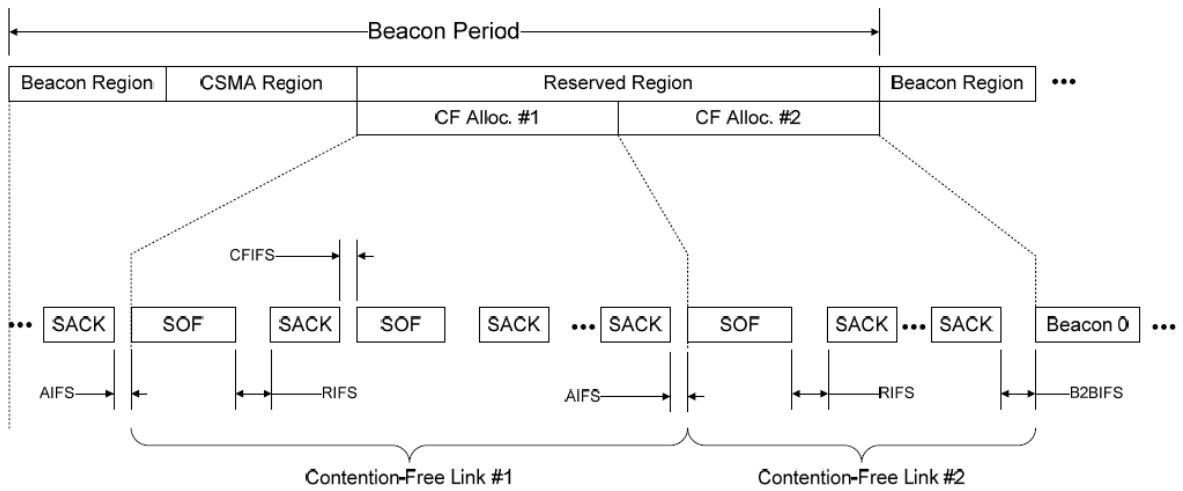
### 2.5.3 Interframe spaces

There are several interframe spaces in a beacon interval. These interframe spaces differ from region to region.

Figure 2.7(a) shows the interframe spaces in a beacon and CSMA regions while Figure 2.7(b) shows the interframe spaces in reserved region. These interframe spaces are described in detail below.



(a) IFS in the Beacon region and CSMA region



(b) IFS in the Reserved region

Figure 2.7 IFS in the Beacon Interval

**B2BIFS (Beacon-to-Beacon Interframe Space):** The beacon region will have a number of beacon slots when the network is operating in the coordinated mode. Each beacon slot will include a Beacon PPDU and an associated B2BIFS. As may be seen from the figure, each Beacon PPDU is followed by a B2BIFS.

**AIFS (Allocation Interframe Space):** An AIFS is inserted to mark the termination of a region and the start of the next region. The only exception is if a CSMA region is followed by a reserved region starting with a CSMA allocation, in this case, the two CSMA regions can be combined into a single region, so the AIFS is not necessary.

**RIFS (response interframe space):** A RIFS marks the end of a long MPDU and the start of its associated response SACK.

**CIFS (Contention Interframe Space):** CIFS time is the duration that a STA shall wait within a contention period after the medium enters the idle state before the STA starts transmission.

**CFIFS (Contention Free Interframe Space):** The time interval between the end of the SACK PPDU and a subsequent SOF PPDU in a reserved region.

**SIFS (short interframe space):** Within a CFP, a STA will wait for SIFS time in the idle state following the STA's own transmission before the STA starts a new transmission.

# **Chapter 3**

## **Modeling of Broadband over Power Line Networks with Contention-free Traffic**

### **3.1 Introduction**

As we explained before, in P1901, MAC protocol offers two modes of data transfer, – contention-free and contention based access. Contention-free access is based on time division multiple access (TDMA) and supports real-time services with quality of service (QoS) requirements. In this chapter, we build up several models for the contention-free access network and analyze each of them in details. Then, we provide numerical results that enables the performance evaluation of the contention-free access mechanism.

### **3.2 Modeling of a single cell network with a single hop and error free transmission**

In this section, we will assume a single cell network with no repeaters and error free transmission. Thus all the stations will be within one hop distance of the HE, and the

network will be operating in uncoordinated mode. Figure 3.1 shows the network topology under consideration. In the following we will determine the throughput of this system.

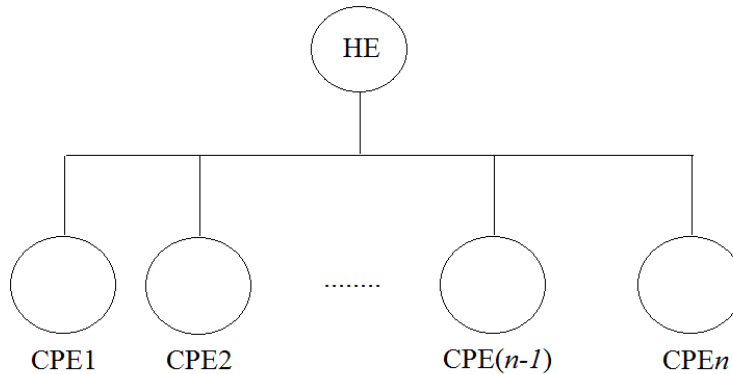


Figure 3.1 Power line network without repeaters

It will be assumed that there are  $n$  CPEs in this network, each one will transmit a long MPDU to the HE during the reserved region, it will be assumed that there is equal number of segments in each long MPDU which is given by  $m$ . The beacon interval only consists of beacon region and a reserved region. Figure 2.4 gives us the beacon interval structure along with contention-free allocations.

As explained before, each CPE will transmit a long MPDU during its assigned time to it in the beacon cycle and the transmission is successfully completed only when the transmitter CPE receives the SACK of all the segments correctly. Here, we first assume that no error happens during the transmission.

The throughput in the MAC layer is given by the ratio of the amount of data load carried in a beacon interval to the total duration of that beacon interval, as given below,

$$\psi = \frac{S_{data}}{S_{\text{beacon interval}}} = \frac{S_{data}}{R^{PHY} * T_{bp}} \quad (3.1)$$

where the system parameters are presented in Table 3.1

We note that  $S_{\text{beacon interval}} = S_{\text{overhead}} + S_{data}$ , and the overhead is given by,

$$\begin{aligned} S_{\text{overhead}} = & S_{\text{beacon}} + R^{PHY} * T_{\text{min CSMA}} + R^{PHY} * T_{B2BIFS} + n * R^{PHY} * T_{RIFS\_default} \\ & + n * R^{PHY} * T_{AIFS\_Access} + n * S_{\text{sack}} + n * m * (S_{PBH} + S_{PBCS}) \\ & + n * (S_{1901FC} + S_{TIA-1113}) \end{aligned} \quad (3.2)$$

So the data payload is given by,

$$S_{data} = R^{PHY} * T_{bp} - S_{\text{overhead}} \quad (3.3)$$

Therefore, the throughput in the MAC layer is given by,

$$\psi = \frac{R^{PHY} * T_{bp} - S_{\text{overhead}}}{R^{PHY} * T_{bp}} \quad (3.4)$$



Table 3.1 System parameters

$T_{bp}$	Beacon period time	33.33 ms
$S_{beacon}$	Bits in beacon	136 octets
$T_{B2BIFS}$	Beacon-to-Beacon Interframe Space	90 $\mu$ s $\pm$ 0.5 $\mu$ s
$T_{RIFS\_default}$	Response Interframe Space	140 $\mu$ s $\pm$ 0.5 $\mu$ s
$T_{AIFS\_access}$	Allocation Interframe Space	50 $\mu$ s
$S_{sack}$	Bits in selectively ACK	16 bits
$R^{PHY}$	PHY rate	4.9226 Mbps
$S_{1901-FFT}$	Bits in 1901 FFT frame control information	128 bits
$S_{TIA-1133}$	Bits in TIA-1113 frame control information	25 bits (1901 FFT hybrid mode)
$S_{PBH}$	Bits in PHY Block Header	4 octets
$S_{PBCS}$	Bits in PHY Block Check Sequence	4 octets
$S_{PBB}$	Bits in PHY Block body which carries the data	512 octets
$S_{PB}$	Bits in PHY Block	520 octets
$T_{minCSMA}$	Duration of the minimum CSMA region	1500 $\mu$ s

Next, we will determine the relationship between the number of CPEs which can be supported in a beacon interval simultaneously and the number of segments in a long MPDU. We assume that there are  $n$  CPEs in the network and each CPE will transmit a long MPDU which consists of  $m$  segments (also referred as PBs). Thus, the total number of segments in a beacon interval is given by  $n * m$ , and the total number of bits for the data payload is given by  $n * m * S_{PBB}$ . Meanwhile, the data payload has been defined in (3.3) which gives us the following equation,

$$R^{PHY} * T_{bp} - S_{overhead} = n * m * S_{PBB} \quad (3.5)$$

From the above we determine the relationship between  $n$  and  $m$  by solving (3.5) which is given by,

$$n = \frac{R^{PHY} * T_{bp} - S_{beacon} - R^{PHY} * T_{minCSMA} - R^{PHY} * T_{B2BIFS}}{[m] * S_{PB} + R^{PHY} * T_{RIFS\_default} + R^{PHY} * T_{AIFS\_Access} + S_{1901FC} + S_{TIA-1113}} \quad (3.6)$$

where  $[m]$  means the floor value of  $m$ , which is the largest integer not greater than  $m$ .

### 3.3 Modeling of a single cell network with a single hop and transmission errors

In this section, we modify the analysis of the previous section to allow transmission errors. We will determine number of beacon intervals (trials) that it will take to transmit a long MPDU. We assume that each segment in the long MPDU is mapped into a single PB and each PB is carried in a single symbol.

As we explained before, when a PB is received in error, a retransmission is required in the next beacon interval, and the transmission will be repeated until all the segments in a long MPDU are received error free.

#### 3.3.1 Modeling with infinite resources

In this subsection, we consider a single hop network, and we assume that all the segments of a long MPDU may be transmitted simultaneously in a beacon period. It is assumed that the number of segments in a MPDU ( $K_\ell$ ) is geometrically distributed with parameter  $\rho$ . Thus the probability that there are  $j$  segments in a MPDU frame is given by,

$$\Pr(K_\ell = j \text{ segments}) = (1 - \rho)\rho^{j-1} \quad (3.7)$$

Then, the PGF of the number of segments in a MPDU is given by,

$$K_\ell(z) = E[z^{K_\ell}] = \frac{(1 - \rho)z}{1 - \rho z} \quad (3.8)$$

We assume that the probability of receiving a segment in error in each transmission is  $p$  and it is independent of other transmissions. Let  $i_1$  denote the number of trials that it will take to transmit a segment successfully, then,

$$\Pr(i_1 = x \text{ trials}) = (1 - p)p^{x-1}, \quad x = 1, 2, 3, \dots \quad (3.9)$$

Let us define  $B_{i,x}$  as the following Bernoulli random variable,

$$B_{i,x} = \begin{cases} 1 & \text{if the } i\text{'th segment has not reached its destination} \\ & \text{successfully by the end of } x\text{'th trial} \\ 0 & \text{otherwise} \end{cases}$$

Let  $B_{i,x}(z)$  denote the PGF of the random variable  $B_{i,x}$ ,

$$B_{i,x}(z) = p^x z + 1 - p^x \quad (3.10)$$

Let  $\tilde{L}_x$  denote the remaining number of segments in a long MPDU still waiting to be transmitted successfully at the end of the  $x$ 'th trial, then,

$$\tilde{L}_x = \sum_{i=1}^{K_\ell} B_{i,x} \quad (3.11)$$

Let us define the PGF of  $\tilde{L}_x$  as,

$$\tilde{L}_x(z) = E[z^{\tilde{L}_x}] \quad (3.12)$$

Since,  $\tilde{L}_x$  is given by a random sum of independent identically distributed random variables, its PGF is given by,

$$\begin{aligned}
\tilde{L}_x(z) &= K_\ell(z)|_{z=B_{i,x}(z)} = \frac{(1-\rho)z}{1-\rho z} \Big|_{z=B_{i,x}(z)} \\
&= \frac{(1-\rho)(p^x z + 1 - p^x)}{1-\rho(p^x z + 1 - p^x)} \\
&= \frac{(1-\rho)p^x}{(1-\rho + \rho p^x)} \frac{z}{1 - \frac{\rho p^x}{(1-\rho + \rho p^x)} z} \\
&\quad + \frac{(1-\rho)(1-p^x)}{(1-\rho + \rho p^x)} \frac{1}{1 - \frac{\rho p^x}{(1-\rho + \rho p^x)} z}
\end{aligned} \tag{3.13}$$

The inversion of the above PGF gives us the probability distribution of the number of segments still waiting to be transmitted successfully by the end of the  $x'$ th trial,

$$\begin{aligned}
\Pr(\tilde{L}_x = n) &= \frac{(1-\rho)p^x}{(1-\rho + \rho p^x)} \left( \frac{\rho p^x}{(1-\rho + \rho p^x)} \right)^{n-1} U[n-1] \\
&\quad + \frac{(1-\rho)[(1-p)^x]}{(1-\rho + \rho p^x)} \left( \frac{\rho p^x}{(1-\rho + \rho p^x)} \right)^n U[n]
\end{aligned} \tag{3.14}$$

$$\text{where } U[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Let us define  $W_x$  as the random variable that the transmission of a long MPDU will be completed on the  $x'$ th trial, then probability distribution of  $W_x$  is given by,

$$\Pr(W_x) = \Pr(\tilde{L}_x = 0) - \Pr(\tilde{L}_{x-1} = 0) \tag{3.15}$$

So the average number of trials needed to transmit a long MPDU is:

$$\bar{W} = \sum_{x=1}^{\infty} x \Pr(W_x) \quad (3.16)$$

### 3.3.2 Modeling with finite resources

In this subsection, we still consider a single hop network but we assume that each MPDU has been allocated fixed number of PBs to transmit its segments in any single trial. Let us assume that the number of PBs assigned to an MPDU is  $\zeta$ . Defining  $X$  as the number of segments that will be received error free among  $\zeta$  segments.

The probability of transmitting  $k$  segments error free in a long MPDU which consists of  $m$  segments in a trial is given by,

$$\Pr(X = k | K_\ell = \zeta) = \binom{\zeta}{k} (1-p)^k p^{\zeta-k}, \quad k = 0, 1, \dots, \zeta \quad (3.17)$$

We can obtain approximately, the average number of trials needed to transmit a long MPDU frame as

$$\bar{W} = \frac{\bar{K}_\ell}{\bar{X}} \quad (3.18)$$

From (3.17), the average number of PBs that will be transmitted successfully in a trial is  $\bar{X} = \zeta(1-p)$ ,  $0 < p < 1$ , Thus,

$$\bar{W} = \frac{\bar{K}_\ell}{\bar{X}} = \frac{1}{\zeta(1-p)} = \frac{1}{(1-\rho)(1-p)\zeta} \quad (3.19)$$

### 3.4 Modeling of a single cell network with multiple hops and transmission errors

When the distances between HE and the end-users are too long, the signal may be attenuated too much by the time it reaches to the destination, to solve this problem, repeaters and NTUs are placed between the HE and the CPEs to extend the coverage of a cell. Figure 3.2 shows an example of the network topology we will analyze in this section.

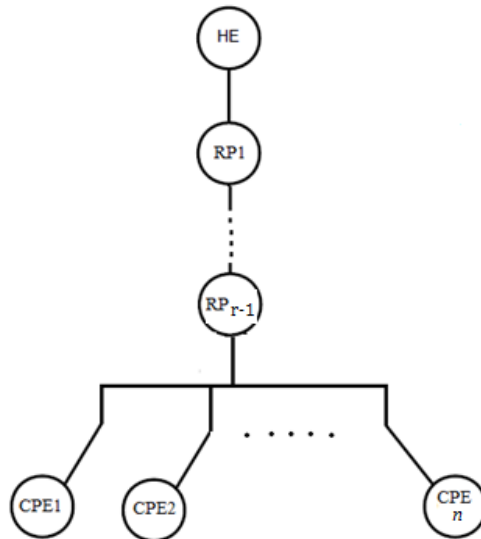


Figure 3.2 Network topology with repeaters

We will assume that the repeaters and NTUs have the same functionality and will not distinguish between two of them. The path between a CPE and HE may contain several repeaters, thus we will have multi-hop communications between a source and destination pair.

As in the standard, we assume that the segments of a MPDU will be transmitted hop by hop and according to the selective-repeat-ARQ mechanism. Thus a node can transmit a segment to the next node after it has been received error free from the previous node along the path. The segments which have been received in error need to be retransmitted in the next beacon cycle. On the other hand, the segments which have been received error free will be transmitted to the next node along the path during the next beacon cycle.

As a result, the segments will advance along the path towards the final destination independent of each other. The service time of a MPDU will begin by the transmission of all its segments in a beacon cycle by the CPE to the next node along the path, and it will be completed when all its segments reach to the destination error free.

An important performance measure is the number of beacon cycles needed to transmit a MPDU from the source to destination. This measure will be referred to as number of trials in the following analysis.



### 3.4.1 Fixed number of hops between the source and destination and fixed number of segments in a MPDU frame

We will determine the probability distribution of the number of trials that it will take to transmit a frame error free from source to destination. Initially, we will assume that the number of segments in a MPDU has a constant value of  $K_\ell$ , and it will also be assumed that there are  $R - 1$  repeaters in the network,  $r = 1 \dots R - 1$ , where  $R \geq 1$ . In addition, source and destination will be considered as repeaters with  $r = 0$  and  $R$  respectively. We note that the destination will be a virtual repeater since it only receives but does not transmit information. The hop between repeaters  $r - 1$  and  $r$  will be referred as the  $r$ 'th hop. Further, the probability of receiving a segment in error in any hop is  $p$  for all the transmissions. So the probability distribution of the number of trials needed to transmit one segment successfully in the  $r$ 'th hop is given by,

$$\Pr(i_r = x \text{ trials}) = (1-p)p^{x-1}, \quad x=1,2,\dots, \quad r=1\dots R \quad (3.20)$$

Let us define  $I_r(z)$  as the PGF of  $i_r$ , then,

$$I_r(z) = E[z^{i_r}] = \frac{(1-p)z}{1-pz} \quad (3.21)$$

The number of trials needed to transmit a segment successfully from the source to the final destination (for example from CPE to HE) is given by,

$$I = \sum_{r=1}^R i_r \quad (3.22)$$

Since  $I$  is given by the sum of the independent identically distributed random variables, thus, the PGF of  $I$  is given by,

$$I(z) = [I_r(z)]^R = \left[ \frac{(1-p)z}{1-pz} \right]^R \quad (3.23)$$

We note that the above PGF corresponds to the negative binomial distribution [17]. So, the inversion of the above PGF gives us the probability distribution of  $I$  as,

$$\Pr(I = x \text{ trials}) = \binom{x-1}{R-1} (1-p)^R p^{x-R}, \quad x = R, R+1, \dots \quad (3.24)$$

Let  $Q_x$  denote the probability of transmitting a segment successfully from the source to the final destination by the end of  $x$ 'th trial,

$$Q_x = \sum_{n=R}^x \Pr(I = n) = \sum_{n=R}^x \binom{n-1}{R-1} (1-p)^R p^{n-R}, \quad x \geq R \quad (3.25)$$

We note that given a MPDU with  $K_\ell$  segments, then the probability that the whole MPDU has been transmitted successfully to the final destination by the end of  $x$ 'th trial is  $Q_x^{K_\ell}$ .

### **3.4.2 Fixed number of hops between the source and destination with variable number of segments in a MPDU frame**

In this case, we assume that the length of a MPDU in number of segments is geometrically distributed with parameter  $\rho$  as in (3.7)

Let us define  $B_{i,x}^*$  as the following Bernoulli random variable,

$$B_{i,x}^* = \begin{cases} 1 & \text{if the } i\text{'th segment has reached its destination} \\ & \text{successfully by the end of } x\text{'th trial} \\ 0 & \text{otherwise} \end{cases}$$

Let  $B_{i,x}^*(z)$  denote the PGF of the random variable  $B_{i,x}^*$ , then,

$$B_{i,x}^*(z) = Q_x z + 1 - Q_x \quad (3.26)$$

Then, the number of segments, which have reached its destination successfully by the end of  $x$ 'th trial is,

$$L_x = \sum_{i=1}^{K_\ell} B_{i,x}^* \quad (3.27)$$

Since  $L_x$  is random sum of the independent identically distributed random variables,  $L_x(z) = K_\ell(z) |_{B_{i,x}^*(z)}$ . The PGF of  $L_x$  is given by,

$$L_x(z) = \frac{(1 - \rho)(Q_x z + 1 - Q_x)}{1 - \rho(Q_x z + 1 - Q_x)} \quad (3.28)$$

The probability distribution of  $L_x$  can be found out,

$$\begin{aligned} \Pr(L_x = m) &= \frac{(1-\rho)Q_x}{(1-\rho+\rho Q_x)} \left( \frac{\rho Q_x}{(1-\rho+\rho Q_x)} \right)^{m-1} U[m-1] \\ &+ \frac{(1-\rho)(1-Q_x)}{(1-\rho+\rho Q_x)} \left( \frac{\rho Q_x}{(1-\rho+\rho Q_x)} \right)^m U[m] \end{aligned} \quad (3.29)$$

$$\text{where } U[m] = \begin{cases} 1 & m \geq 0 \\ 0 & m < 0 \end{cases}$$

From (3.28), average number of segments that will reach to its destination by the end of  $x'$ th trial is given by,

$$\bar{L}_x = \left. \frac{dL_x(z)}{dz} \right|_{z=1} = \frac{Q_x}{1-\rho} \quad (3.30)$$

Thus, probability that a MAC frame has been transmitted successfully by the end of the  $x'$ th trial is given by,

$$\Pr(G_x) = \sum_{j=1}^{\infty} Q_x^j \Pr(K_\ell = j) = \sum_{j=1}^{\infty} Q_x^j (1-\rho) \rho^{j-1} = \frac{(1-\rho)Q_x}{1-\rho Q_x} \quad (3.31)$$

where  $Q_x$  has been defined in (3.25).

Probability that the transmission of a MAC frame will be completed on the  $x'$ th trial is given by,

$$\begin{aligned} \Pr(W_x) &= \Pr(G_x) - \Pr(G_{x-1}) = \frac{(1-\rho)Q_x}{1-\rho Q_x} - \frac{(1-\rho)Q_{x-1}}{1-\rho Q_{x-1}} \\ &= \frac{(1-\rho)(Q_x - Q_{x-1})}{(1-\rho Q_x)(1-\rho Q_{x-1})} \end{aligned} \quad (3.32)$$

Finally, average number of trials which will take to transmit a MAC frame to its final destination is:

$$\bar{W} = \sum_{x=R}^{\infty} x \Pr(W_x) = \sum_{x=R}^{\infty} x \left( \frac{(1-\rho)(Q_x - Q_{x-1})}{(1-\rho Q_x)(1-\rho Q_{x-1})} \right) \quad (3.33)$$

### 3.4.3 Variable number of hops between the source and destination with variable number of segments in a MPDU frame

In this subsection, we will assume that the number of hops between source and destination may have any value between 1 to  $\tilde{R}$  with equal probability. Then, probability that the distance between source and destination is  $R$  hops is given by,

$$\Pr(a_R) = \frac{1}{\tilde{R}}, \quad 1 \leq R \leq \tilde{R} \quad (3.34)$$

Let us define  $\bar{F}_R$  as the number of trials needed to transmit a MPDU over  $R$  hops, then, its average is given by,

$$\bar{F}_R = \sum_{x=R}^{\infty} x \Pr(W_x) \quad (3.35)$$

where  $\Pr(W_x)$  is given by (3.32)

Next, let us define  $F$  as the number of trials needed to transmit a MPDU from source to destination which are separated by  $R$  hops, where  $1 \leq R \leq \tilde{R}$ ,

$$\bar{F} = \sum_{R=1}^{\bar{R}} \bar{F}_R \Pr(a_R) \quad (3.36)$$

### 3.4.4 Fixed number of hops between the source and destination with variable number of segments in a MPDU frame and nonhomogeneous segment's transmission error probability over the hops

In the analysis up to now, we assumed that the error probability for the transmission of a segment over any hop is the same. Here, we assume that the probability of segment error over hop  $r$  is given by  $p_r$ . Hence, (3.20) is given by,

$$\Pr(i_r = x \text{ trials}) = (1 - p_r) p_r^{x-1}, \quad x = 1, 2, \dots \quad (3.37)$$

Then, the number of trials needed to transmit a segment successfully from the source to the final destination is

$$I = \sum_{r=1}^R i_r \quad (3.38)$$

The PGF of variable  $i_r$  is given by,

$$I_r(z) = \frac{(1 - p_r)z}{1 - p_r z} \quad (3.39)$$

Thus, the PGF of  $I$  is given by,

$$\begin{aligned}
I(z) &= \prod_{r=1}^R I_r(z) = \prod_{r=1}^R \frac{(1-p_r)z}{1-p_r z} = z^R \prod_{r=1}^R \frac{(1-p_r)}{(1-p_r z)} \\
&= z^R \left( \prod_{r=1}^R (1-p_r) \right) \left( \prod_{r=1}^R \frac{1}{(1-p_r z)} \right)
\end{aligned} \tag{3.40}$$

Application of partial fraction expansion in the above leads to

$$I(z) = z^R \prod_{r=1}^R (1-p_r) \left( \sum_{r=1}^R \frac{A_r}{1-p_r z} \right) \tag{3.41}$$

$$\text{Where } A_r = \prod_{\substack{j=1 \\ j \neq r}}^R \frac{1}{1-p_j z} \Bigg|_{z=\frac{1}{p_j}}$$

The inversion of the above PGF with the application of the Z-transform pair  $A\alpha^n \Leftrightarrow \frac{A}{1-\alpha z}$  and the property of Z-transform [18]:  $(n-k) \Leftrightarrow z^k F(z)$  gives the probability distribution of  $I$  as,

$$\Pr(I = x \text{ trials}) = \prod_{r=1}^R (1-p_r) \sum_{r=1}^R A_r p_r^{x-R}, \quad x = R, R+1, \dots \tag{3.42}$$

Then,  $Q_x$  defined in (3.25) as the probability that the transmission of a segment will be completed by the end of  $x$ 'th trial is given by,

$$Q_x = \sum_{k=R}^x \left( \prod_{r=1}^R (1-p_r) \sum_{r=1}^R A_r p_r^{k-R} \right) \tag{3.43}$$

Thus, probability that a MAC frame with geometrically distributed number of segments has been transmitted successfully by the end of the  $x$ 'th trial is given by,

$$\Pr(G_x) = \sum_{j=1}^{\infty} Q_x^j (1-\rho)\rho^{j-1} = \frac{(1-\rho)Q_x}{1-\rho Q_x} \quad (3.44)$$

Probability that the transmission of a MAC frame will be completed on the  $x'$ th trial is given by,

$$\begin{aligned} \Pr(W_x) = \Pr(G_x) - \Pr(G_{x-1}) &= \frac{(1-\rho)Q_x}{1-\rho Q_x} - \frac{(1-\rho)Q_{x-1}}{1-\rho Q_{x-1}} \\ &= \frac{(1-\rho)(Q_x - Q_{x-1})}{(1-\rho Q_x)(1-\rho Q_{x-1})} \end{aligned} \quad (3.45)$$

Finally, average number of trials which will take to transmit a MAC frame to its final destination is:

$$\bar{W} = \sum_{x=R}^{\infty} x \Pr(W_x) = \sum_{x=R}^{\infty} x \left( \frac{(1-\rho)(Q_x - Q_{x-1})}{(1-\rho Q_x)(1-\rho Q_{x-1})} \right) \quad (3.46)$$

### 3.5 Derivation of network capacity

In this section, we will determine the number of MAC frame transmissions that may be supported simultaneously in a beacon period. Let us assume that there are  $n$  users and each of them attempting to transmit a single MAC frame at any time. As soon as a user completes the transmission of a frame, it immediately begins the transmission of another frame.



Let us consider a randomly chosen trial. The service of frames by each user forms a renewal process. In the language of renewal theory, the amount of service given to a frame until the random slot is known as age of the service. A frame will be on its  $x$ 'th transmission during the randomly chosen trial if its age is  $X = x$ . From the pdf of age given by [18],

$$\Pr(X = x) = \frac{1}{\bar{W}} [1 - \Pr(G_x)] \quad (3.47)$$

where  $\Pr(G_x)$  is given by (3.31) and  $\bar{W}$  is from (3.33).

A user will start transmission of a frame by transmitting all its segments in the first hop. Since the segments may experience errors independent of each other, they will spread out as they travel from source to destination. To determine this spread, let us define  $q_{r|x}$  as the conditional probability that a segment will be at the  $r$ 'th repeater at the end of the  $x$ 'th trial, which will be given by,

$$q_{r|x} = \binom{x}{r} (1-p)^r p^{x-r}, \quad x \geq r, \quad r = 0, \dots, R-1 \quad (3.48)$$

Thus probability that a segment will be at the  $r$ 'th repeater at the end of any randomly chosen trial is given by,

$$q_r = \sum_{x=1}^M q_{r|x} \Pr(X = x) \quad (3.49)$$

with  $\Pr(X = x)$  and  $q_{r|x}$  given by (3.47) and (3.48) respectively.

Next, we will determine the probability distribution of the total number of segments in the  $n$  frames. Let us define  $K_T$  as the total number of segments in  $n$  frames, then,

$$K_T = \sum_{\ell=1}^n K_\ell \quad (3.50)$$

where  $K_\ell$  has the geometric distribution.

Since  $K_T$  is given by the sum of the independent identically distributed random variables, thus, the PGF of  $K_T$  is given by  $K_T(z) = [K_\ell(z)]^n$ . We note that this is the PGF of the negative binomial distribution as before, so, the inversion of this PGF gives the probability distribution of  $K_T$  as,

$$\Pr(K_T = j \text{ segments}) = \binom{j-1}{n-1} (1-p)^n p^{j-n}, \quad j \geq n \quad (3.51)$$

Since all the frames are i.i.d. in number of segments, let us define  $S_r$  as the number of segments which are at the  $r$ 'th repeater by the end of any random trial, then,

$$\Pr(S_r = k | K_T = j) = \binom{j}{k} q_r^k (1-q_r)^{j-k}, \quad 0 \leq k \leq j \quad (3.52)$$

Unconditioning the above with respect to the total number of segments,

$$\Pr(S_r = k) = \sum_{j=\max(n,k)}^{\infty} \binom{j}{k} q_r^k (1-q_r)^{j-k} \Pr(K_T = j) \quad (3.53)$$

Let us define  $\beta$  as the number of segments that a beacon interval can transmit. Then, the network capacity is determined by the maximum value of  $n$  satisfying the inequality below at the  $r$ 'th repeater,

$$\Pr(S_r \geq \beta) \leq \alpha \quad (3.54)$$

Where  $\alpha$  is given a small value.

Next, we will determine the number of segments that a beacon interval can carry. We note that there may not be segments belonging to all the frames at each repeater. The number of frames that may have segments in a beacon interval will be bounded by  $\min(n, \gamma)$  where  $\gamma$  is the beacon interval capacity when there is a single segment belonging to each frame in a beacon interval. Then, the beacon interval overhead is given by,

$$\begin{aligned} S_{overhead} = & S_{beacon} + R^{PHY} T_{B2BIFS} + R^{PHY} T_{\min CSMA} \\ & + \min(n, \gamma) R^{PHY} T_{RIFS\_default} \\ & + \min(n, \gamma) R^{PHY} T_{AIFS\_Access} + \min(n, \gamma) S_{sack} \\ & + \gamma(S_{PBH} + S_{PBCS}) + \min(n, \gamma)(S_{1901FC} + S_{TIA-1113}) \end{aligned} \quad (3.55)$$

$$S_{data} = \gamma S_{PBB} \quad (3.56)$$

where system parameters have been defined in Table 3.1. The total number of bits needed to support  $n$  frames is given by,

$$S_{Total} = S_{overhead} + S_{data} \quad (3.57)$$

The total number of bits that can be carried by a beacon interval is given by,

$$S_{\text{beacon interval}} = R^{PHY} T_{bp} \quad (3.58)$$

The solution of (3.55, 3.56, 3.57 and 3.58) gives us the number of segments that may be carried in a beacon interval as a function of the number of users in the network.

## 3.6 Numerical Results

In this section, we present some numerical results regarding the analysis in this chapter.

### 3.6.1 Single cell network with single hop and error free transmission

In this subsection, we consider the scenario that a single cell network with single hop and error free transmission.

Figure 3.3 presents the number of segments in a long MPDU ( $m$ ) as a function of the number of CPEs that can be supported in a beacon interval simultaneously ( $n$ ) from (3.6).

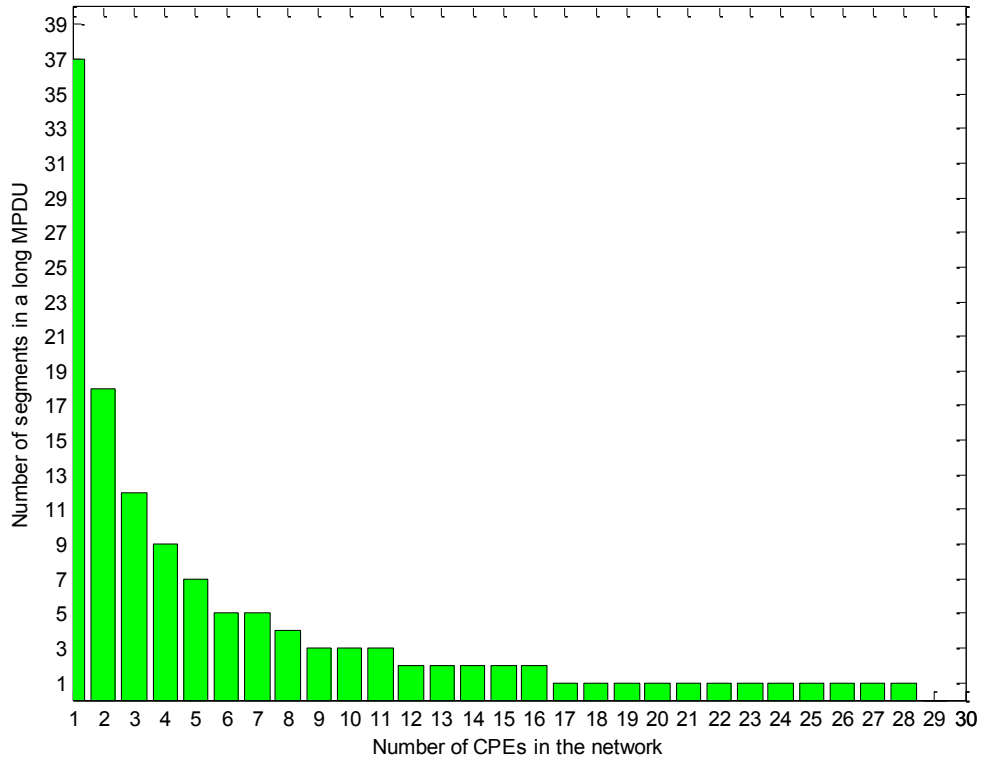


Figure 3.3 The number of segments in a long MPDU as a function of  $n$ .

As may be seen, when the number of CPEs in the network increases from 1 to 28, the number of PBs which can be supported in a long MPDU decreases from 37 to 1. The result shows that the beacon interval can only support 28 segments from 28 users simultaneously.

### 3.6.2 Homogenous segment transmission error probability

In this subsection, we consider the scenario that the hops are homogenous with identical segment transmission error probability. The numerical results are based on the following values of network parameters.

- Segment transmission error probability  $p = 0.2$
- Duration of the beacon period  $T_{bp} = 33.33 \text{ ms}$
- The number of segments in a MPDU is geometrically distributed with parameter  $\rho = 0.75$
- When repeaters are taking into account, the number of hops between source and destination is  $R = 5$

The results are expressed in number of trials which may be converted to time durations.

First, we give the results when there are no repeaters in the model. Figure 3.4 shows the probability of transmitting all the segments in a long MPDU error free on the  $x'$ th trial according to (3.15) for different values of parameter  $\rho$ . We notice that for all the four cases, the probability is high in the first two trials. It drops fast after the third trial, and the probability goes to zero after the 7th trial, which means, the transmission of a long MPDU being completed after the 7th trial is negligible. On the first trial, the green line has the highest probability because of the less number of segments in the long MPDU.

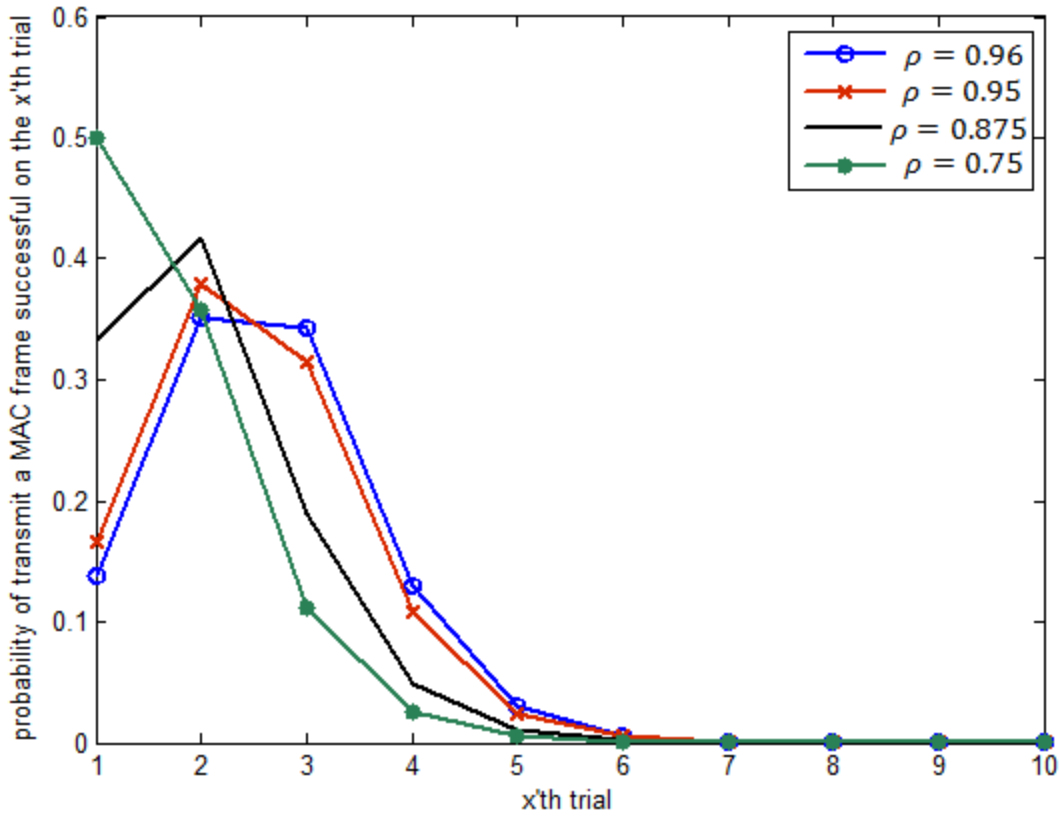


Figure 3.4 Probability that the transmission of all the segments in a long MPDU will be completed on the  $x'th$  trial within a single hop for different value of parameter  $\rho$ .

Figures 3.5, 3.6 and 3.7 plot the results for the model when there are 5 hops between the source and the destination. Figure 3.5 presents the average number of segments that will reach to the destination by the end of  $x'th$  trial from (3.30). Since we are assuming  $\rho = 0.75$ , the average number of segments in a frame is 4. It may be seen that by 12'th trial the average of the number of segments reaches to 4. In Figure 3.6, we plot the probability that the transmission of a MPDU to its final destination will be completed on the  $x'th$  trial from (3.32). It may be seen that probability that the transmission of a MPDU will be completed beyond the 14'th trial is negligible. In Figure

3.7, we present the average number of trials needed for a CPE to transmit a MPDU successfully to its final destination as a function of the maximum number of hops,  $\tilde{R}$ . The number of hops between source and destination is equally likely to have any value between  $1 \dots \tilde{R}$ .

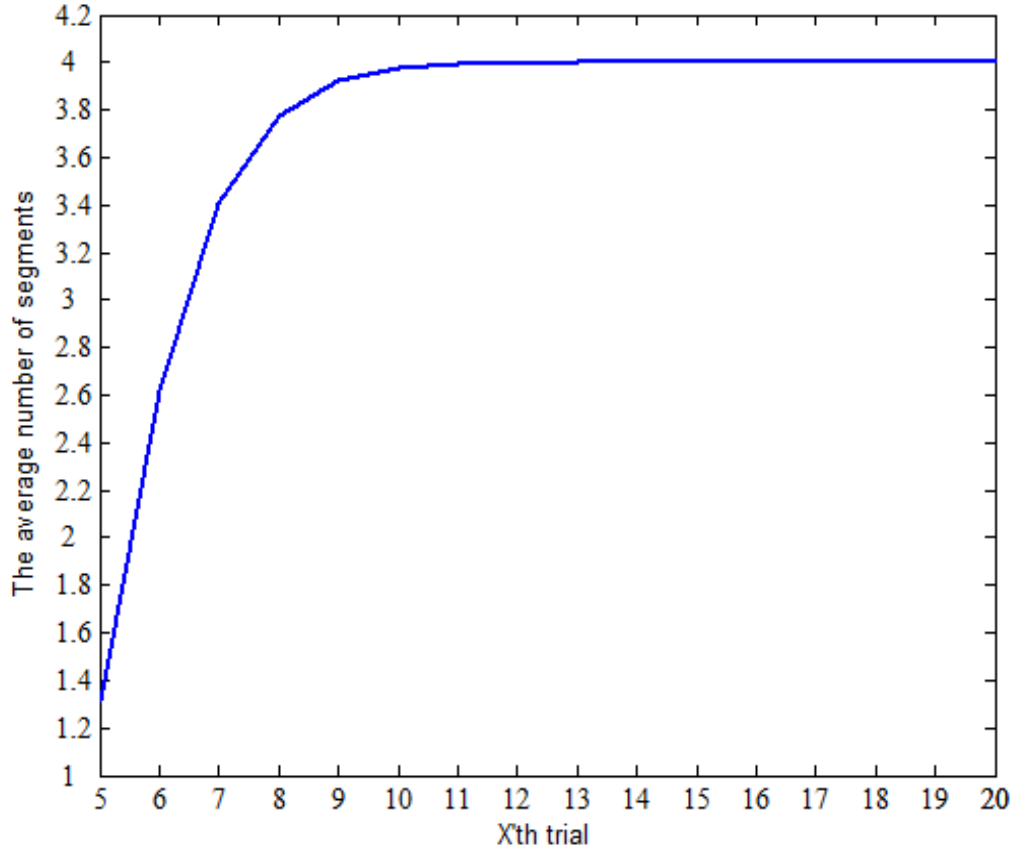


Figure 3.5 The average number of segments which have reached its final destination successfully by the end of  $x$ 'th trial with homogenous segment error probably,  $p = 0.2$ , parameter for the number of segments in a frame,  $\rho = 0.75$  and constant number of hops,  $R = 5$ .



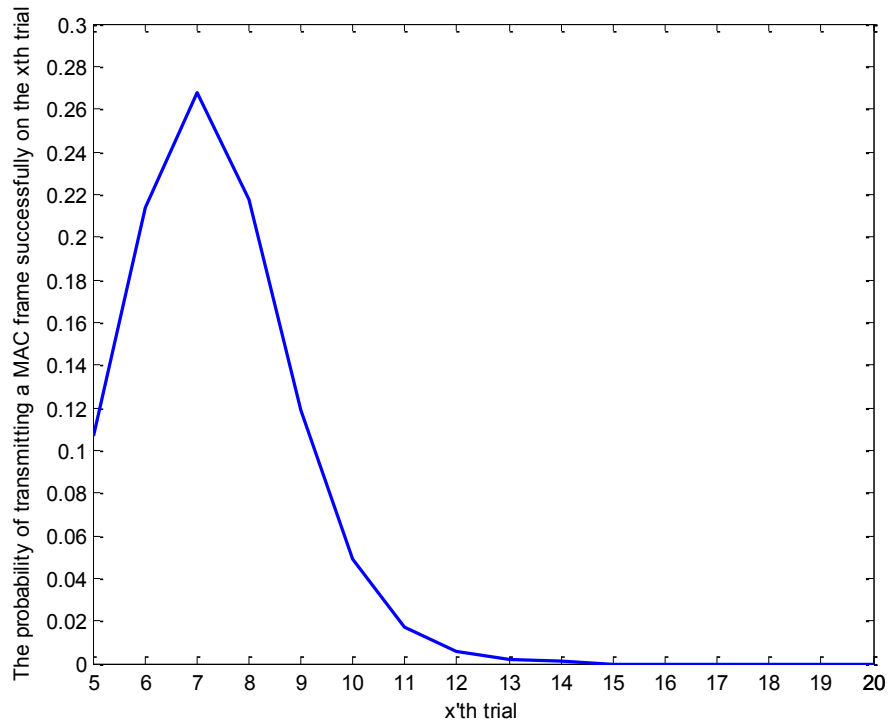


Figure 3.6 The probability that the transmission of a MPDU to its final destination will be completed on the  $x$ 'th trial with homogenous segment error probabilities,  $p = 0.2$  and constant number of hops,  $R = 5$ .

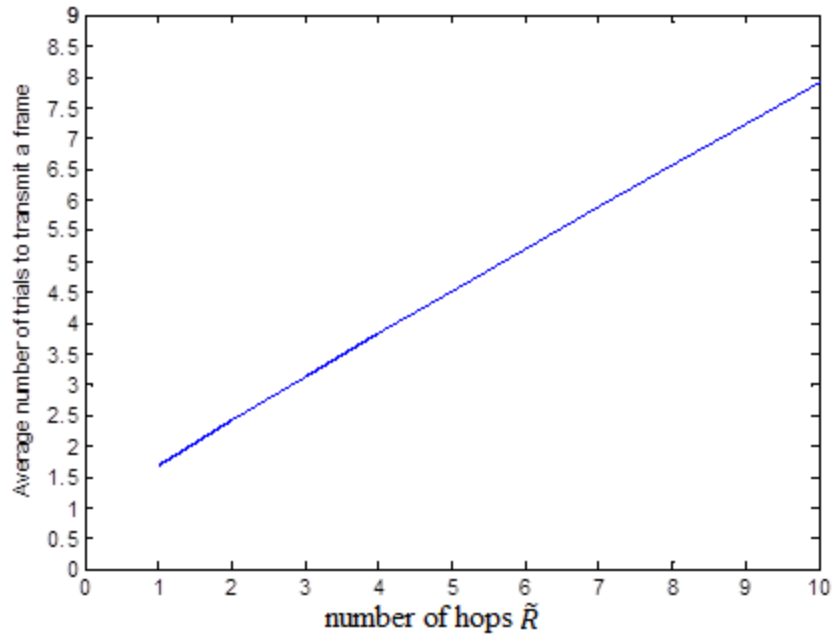


Figure 3.7 Average number of trials to transmit a frame with different number of hops with homogenous segment error probabilities,  $p = 0.2$ .

### 3.6.3 Nonhomogenous segment transmission error probabilities

Next, we consider the scenario of nonhomogenous segment transmission error probabilities over different hops. We assume that  $R = 5$  hops between source and destination with segment error probabilities of values 0.1, 0.2, 0.3, 0.4 and 0.5 respectively. The probability of completing the transmission of a MPDU will be completed to its final destination on the  $x$ 'th trial is given by (3.32), the results are shown in Figure 3.8. It may be seen that this probability is negligible for number of trials larger than 20.

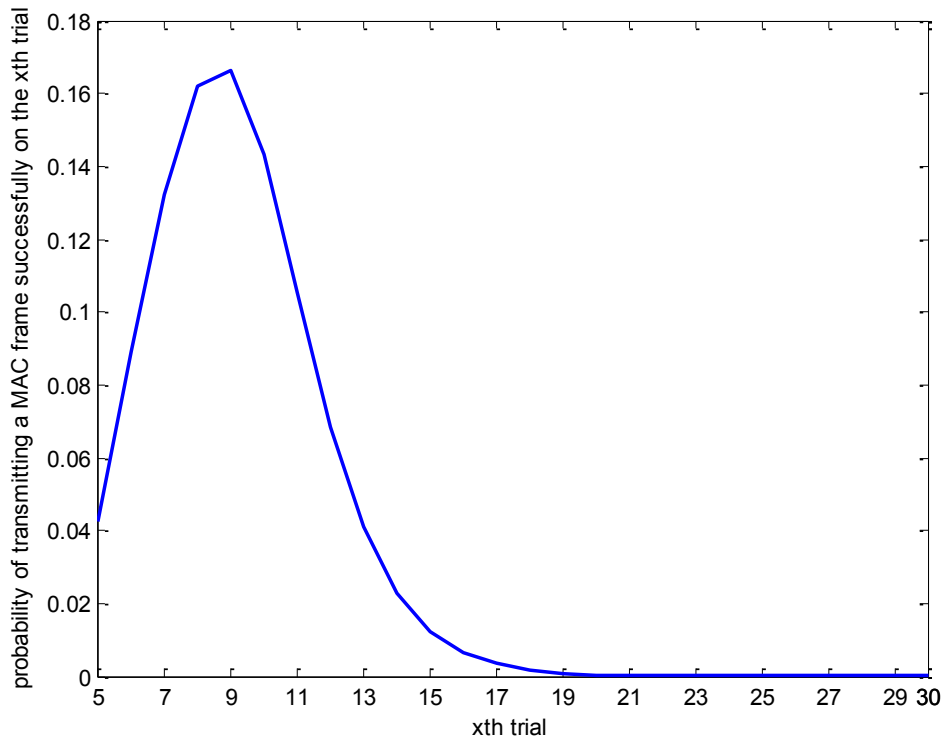


Figure 3.8 The probability that the transmission of a MPDU will be completed to its final destination on the  $x$ 'th trial with nonhomogenous segment error probabilities and constant number of hops,  $R = 5$ .

### 3.6.4 Network Capacity

In this section, we present an example of network capacity determination, with homogenous segment transmission error probability of  $p = 0.2$ .

From Figure 3.6, it may be seen that probability that the transmission of a MPDU will take more than 14 trials is negligible. Figure 3.9 presents the probability that a frame will be on its  $x$ 'th transmission during the randomly chosen trial based on (3.47), and the probability drops to zero after the 14'th trial. Table 3.2 gives us the value of  $q_{r|x}$  for different values of  $x$  and  $r$ . The results show the probability that a segment will be at the  $r$ 'th hop by the end of the  $x$ 'th trial.

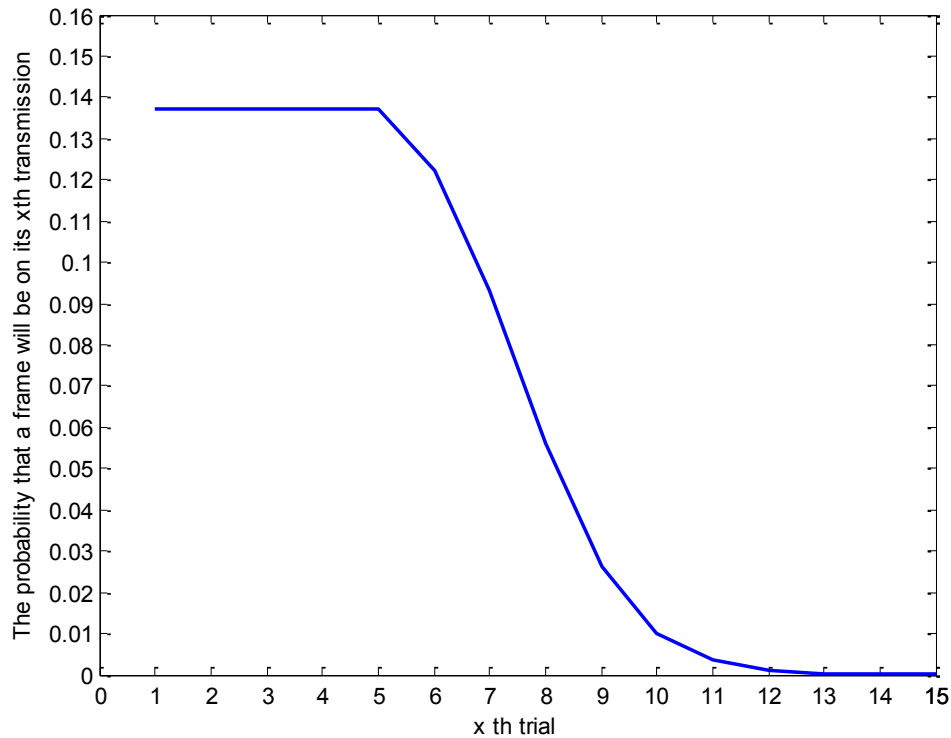


Figure 3.9 Probability that a long MPDU will be on its  $x$ 'th transmission during the randomly chosen trial

Table 3.2 Probability that a segment will be on the  $r$ 'th hop by the end of the  $x$ 'th trial,

$q_{r|x}$ .

$q_{r x}$	r=0	r=1	r=2	r=3	r=4
x=1	0.2	0.8	0	0	0
2	0.04	0.32	0.64	0	0
3	0.008	0.096	0.384	0.512	0
4	0.0016	0.0256	0.1536	0.4096	0.4096
5	0.00032	0.0064	0.0512	0.2048	0.4096
6	0.000064	0.001536	0.01536	0.08192	0.24576
7	1.28E-05	0.000358	0.004301	0.028672	0.114688
8	2.56E-06	8.19E-05	0.001147	0.009175	0.045875
9	5.12E-07	1.84E-05	0.000295	0.002753	0.016515
10	1.02E-07	4.1E-06	7.37E-05	0.000786	0.005505
11	2.05E-08	9.01E-07	1.8E-05	0.000216	0.00173
12	4.1E-09	1.97E-07	4.33E-06	5.77E-05	0.000519
13	8.19E-10	4.26E-08	1.02E-06	1.5E-05	0.00015
14	10.18E-11	8.39E-09	3.82E-07	4.2E-06	0.0000336

From (3.33), we find out that  $\bar{W} = 7.2818$ . First, we calculate the probability that a segment will be at the  $r$ 'th repeater by the end of any randomly chosen trial,  $q_r$ , from (3.49). The results are shown in Table 3.3. We determine the capacity with the source,  $r = 0$ , since it has the highest load.

Figure 3.10 presents the number of segments which could be supported by a beacon interval when the number of users in the network increases from 1 to 28. The results will be used to determine the network capacity shown in Table 3.4 .

Figure 3.11 presents the probability that the number of segments will be greater than  $\beta$  as a function of number of users,  $n$ , for  $q_0 = 0.1717$  with  $\rho$  as parameter. Assuming the value of  $\alpha = 0.05$ , we determined the probability for each value of  $n$ . For instance, we note that when  $\rho = 0.75$  , the probability that the number of segments in a beacon interval at the source is greater than 31 is above 0.05 for  $n = 19$ , thus we conclude that the maximum number of users that the network can support will be 18 under the chosen parameter values. Table 3.4. presents the results as well as the other three values of parameter  $\rho$  .

However in Figure 3.3, when the number of STAs in the network is 18, each STA is allowed to transmit only one segment in a long MPDU. Table 3.4 shows a result that when the average number of segments in a long MPDU is four, the network is still able to

support 18 users. The reason is the transmissions of the long MPDU from each user are not synchronized, as soon as a STA completes the transmission of a frame, it immediately begins the transmission of another frame. The transmission of long MPDU could occur during any time.

Table 3.3 Probability that a segment will be on the  $r$ 'th hop,  $q_r$

The repeater number ( $r$ )	Probability $q_r$
0 (source)	0.1717
1	0.0429
2	0.0107
3	0.0026
4	0.0006

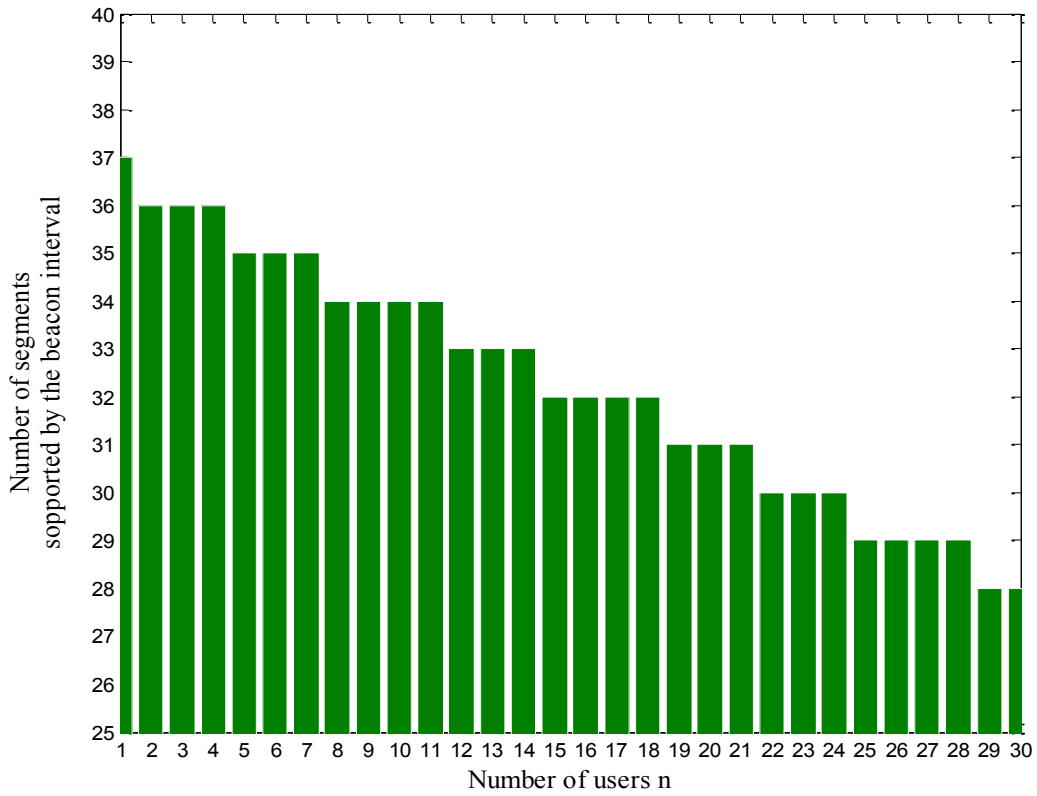


Figure 3.10 Number of segments which could be supported by a beacon interval as a function of number of users ( $n$ )



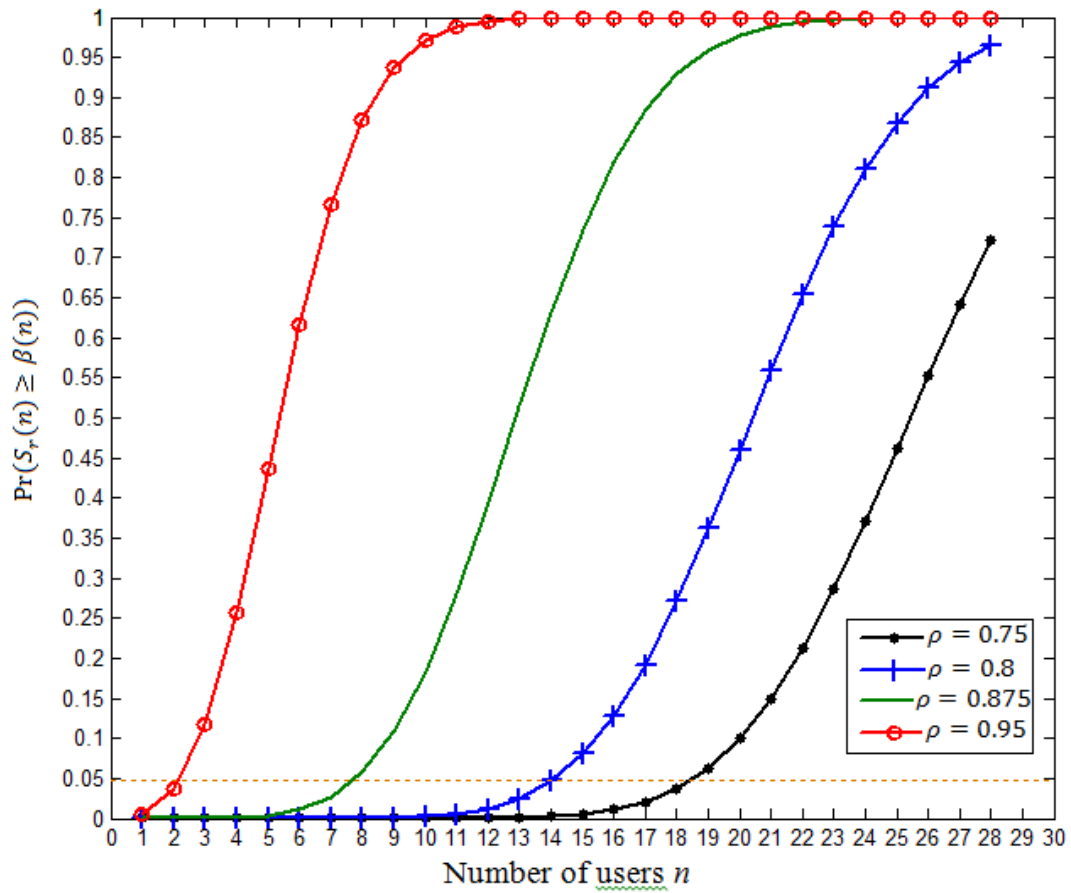


Figure 3.11 Probability that the number of segments will be greater than  $\beta$  as a function of number of users ( $n$ ) for  $p = 0.2, r = 0$  with  $\rho$  as a parameter.

Table 3.4 Network capacity with different values of parameter  $\rho$

$\rho$	Number of users ( $n$ )	$\Pr(S_r \geq \beta)$	$\beta$
0.75	18	0.0376	32
	19	0.0630	31
0.8	14	0.0469	33
	15	0.0804	32
0.875	7	0.0267	35
	8	0.0576	34
0.95	2	0.0371	36
	3	0.1182	36

# Chapter 4

## Modeling of Broadband over Power Line Networks with Contention Traffic

### 4.1 Introduction

In this chapter, we will introduce the contention access mechanism. The contention periods in the beacon interval are used by all the STAs in the network to contend for the medium. During the contention period the STA is permitted to transmit only if it wins the contention. CSMA/CA protocol is designed to reduce the collision probability between multiple STAs. First, we analyze the two major mechanisms supported by the MAC protocol and then present the numerical results that enable performance evaluations of the contention access mechanism. The expression of the MAC efficiency is obtain from [14], however, we will improve their work by analyzing the contention-free transmission and contention transmission together in a beacon interval.

## **4.2 Mechanisms description**

In the contention period, STAs are requested to contend for the channel. In order to eliminate the collisions, two procedures are used to resolve the conflicts – PRS (Priority Resolution Slots) based prioritized CSMA/CA and random backoff algorithm. In this section, these two mechanisms will be presented in details. A contention period begins with a priority resolution period which may be followed by a random back off period, then frame transmission takes place. This process repeats itself until contention period terminates.

### **4.2.1 PRS based prioritized CSMA/CA**

The priority resolution period is provided by MAC function to support the Quality of Service (QoS). When the medium is idle after a transmission has been completed, two Priority Resolution Slots (PRS) are transmitted to resolve priority among contending STAs -- PRS0 and PRS1. Four levels of priorities (CA3, CA2, CA1 and CA0) may be distinguished using these two slots. Each type of frame is assigned a priority level. Table 4.1 shows the states of PRS0 and PRS1 corresponding to the four levels of priorities, we note that CA3 = 0b11 is the highest and CA0 = 0b00 is the lowest priority level. All the stations are required to transmit in the priority resolution period even if they have no MPDU at this moment. The process is executed by assuming the priority to be 0b00 in order to maintain knowledge of the highest priority contending for the medium.

Each station senses the medium to determine which one wins the channel by having the highest priority. With this step, all the stations are informed of the priority level of the other STAs. Stations at the highest priority level contend for the medium in the contention state using random backoff algorithm as described in the next section. The ones with lower priority will defer their transmissions and wait for the start of the next priority resolution period to try to gain access to the channel.

Table 4.1 Channel access priority levels

Channel access priority	PRS1 state	PRS0 state
CA3	1	1
CA2	1	0
CA1	0	1
CA0	0	0

Figure 4.1 presents an example, where stations A, B and C have priority levels CA3, CA2 and CA0 respectively. Each station transmits a signal from the beginning of the priority resolution period. First, station C learns that A and B have higher priority by sensing that the signals they send in the PRS1 slot are 0b1. Then, by the end of the priority resolution period, station B learns that its priority level is lower than that of

station A. Thus, station A wins the medium and starts the transmission after the random backoff period.

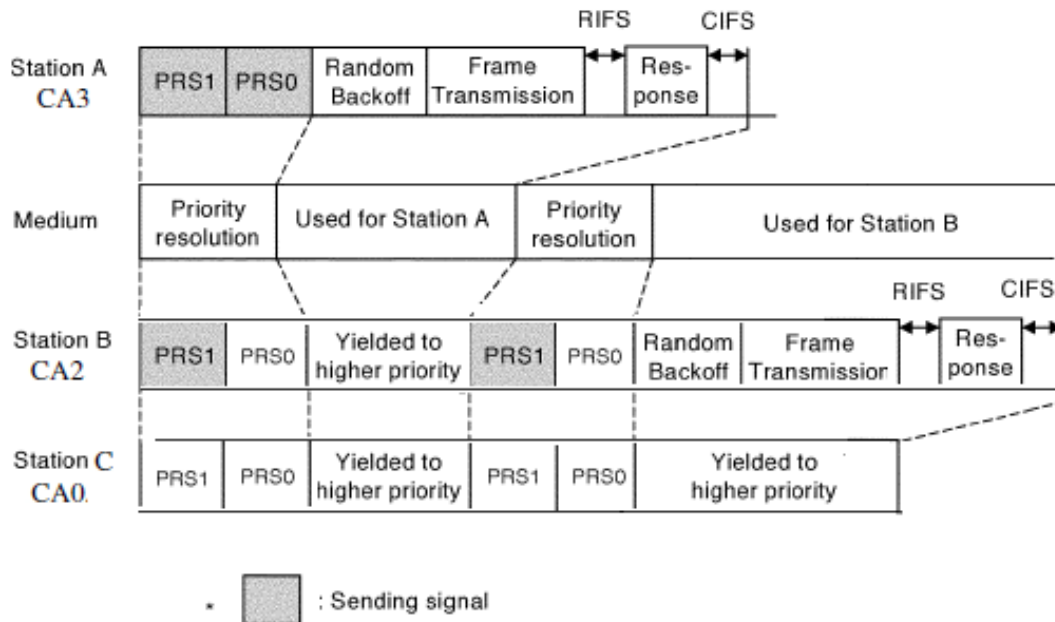


Figure 4.1 Example of Priority Resolution

## 4.2.2 Random Backoff

Random backoff algorithm is used to avoid the collisions when there are multiple stations at the highest priority level. STAs are required to contend for the channel during the contention state using the random backoff algorithm if they have the same highest

priority during the priority resolution period, for example, all of them have priority CA3. The random backoff procedure is introduced below.

It uses four counters to facilitate the operation. These are the Backoff Procedure Event Counter (BPC), the Backoff Counter (BC), the Deferral Counter (DC) and the Contention Window (CW). Next, we describe this procedure,

1). At the beginning of each contention period, all the stations set the values of BPC, BC and DC to zero.

2). In the contention state, the station shall set CW and DC based on the value of BPC and the priority level according to Table 4.2, and then set  $BC = \text{Random}(CW)$  (The function  $\text{Random}(CW)$  will return an integer uniformly distributed between zero and CW).

3). At each subsequent slot time, if the medium state is sensed idle and BC is not zero yet, BC is decreased by one while DC and BPC are fixed, once BC becomes zero, the station transmits. Otherwise, if the medium state is busy, proceed to step 5.

4). If collision happens, stations should stop transmitting and wait until the next priority resolution period, otherwise, the backoff procedure are completed and BPC is set to zero.

5). If a slot is sensed busy, both BC and DC are decremented by one at the end of that busy slot.

6). If DC becomes less than zero, the BPC is updated to the next higher value, and both DC and BC are reinitialized according to the new BPC value.

Figure 4.2 shows an example, stations A, B and C all have the highest priority level CA3. At the beginning of the contention state, stations A, B and C initialize the value of BPC, BC and DC to zero, and then set the value of CW and DC based on BPC and priority level in Table 4.2, in our case, since all the stations have the same priority level, so  $BPC_A = BPC_B = BPC_C = 0$ ,  $CW_A = CW_B = CW_C = 7$ ,  $DC_A = DC_B = DC_C = 0$ , then, each station sets  $BC = \text{Random}(CW)$ , we assume  $BC_A = 5$ ,  $BC_B = 3$ ,  $BC_C = 6$ . In slot 1, the medium is sensed idle, BC is decremented by one, so,  $BC_A = 4$ ,  $BC_B = 2$ ,  $BC_C = 5$ . In slot 2, the medium is sensed busy, both BC and DC are decremented by one, then the value of DC is less than zero, BPC is set to a higher value which is 1 and the value of CW and DC is chosen based on Table 4.2, and  $BC = \text{Random}(CW)$  which gives us  $BPC_A = BPC_B = BPC_C = 1$ ,  $CW_A = CW_B = CW_C = 15$ ,  $DC_A = DC_B = DC_C = 1$ , and  $BC_A = 10$ ,  $BC_B = 13$ ,  $BC_C = 3$ . In slot n, the value of BC of station C becomes zero, then station C starts to transmit, since there is no collision, the backoff procedure is completed.



Table 4.2 BPC, CW and DC value for different priority levels

	Priority CA3 & CA2	Priority CA1 & CA0
BPC = 0	CW = 7.....DC = 0	CW = 7.....DC = 0
BPC = 1	CW = 15.....DC = 1	CW = 15.....DC = 1
BPC = 2	CW = 15.....DC = 3	CW = 31.....DC = 3
BPC > 2	CW = 31.....DC = 15	CW = 63.....DC = 15

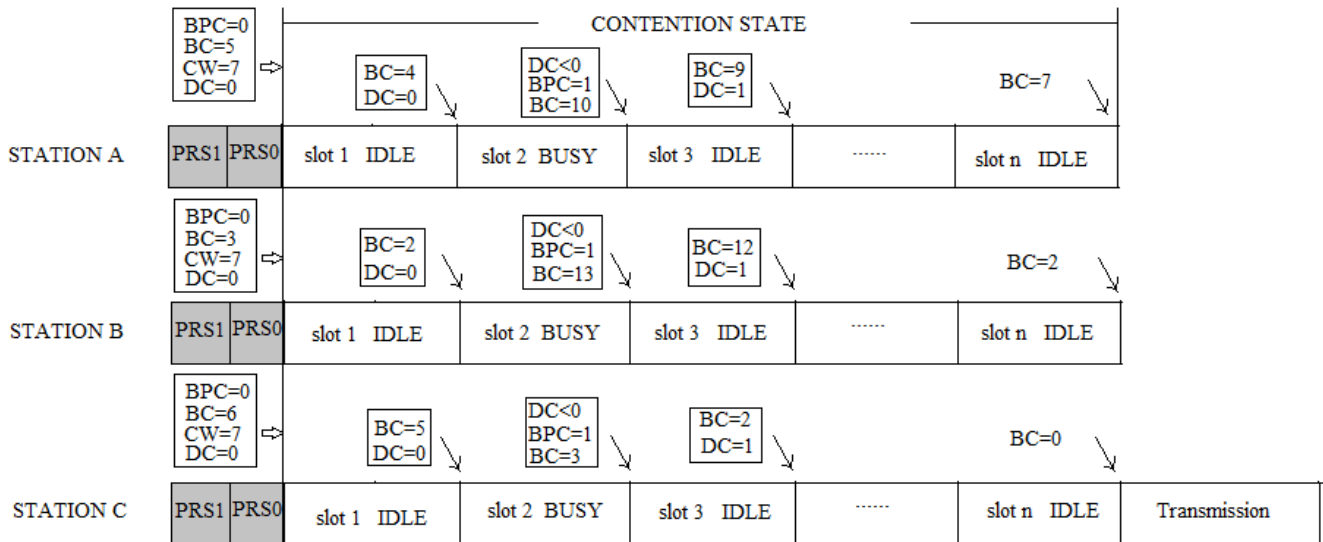


Figure 4.2 Example of the random backoff procedure

### 4.3 Performance Analysis

In this section, we will analyze the MAC efficiency of the contention traffic access. First, we will assume that there are  $n$  stations in the network. We let  $\gamma$  denote the allocations dedicated to the contention traffic in the minimum CSMA region and  $m$  allocations dedicated to the reserved region. We note that any of the allocations in the reserved region which are unused by the contention-free traffic may be used by the contention traffic.

We assume that each station alternates between the generation of a contention-free traffic frame and then the transmission of that frame. Let us define two states, as shown in Figure 4.3, state 0 means the station is waiting to generate a new frame, while state1 shows that the station is trying to transmit its frame. The state of the station alternates between states 0 and 1, and we define a transmission cycle time  $c = \bar{x} + y$ , where  $\bar{x}$  denotes the average waiting time that a station spends in state 0 and  $y$  denotes the time spent in state 1.

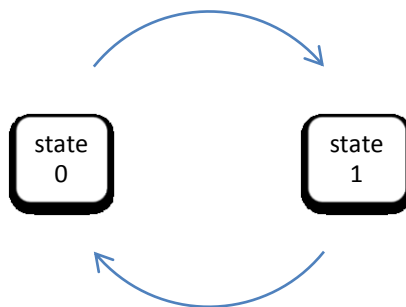


Figure 4.3 States of a station generating contention-free traffic

We assume that the time that a station spends in state 0 is exponentially distributed which is given by,

$$f(x) = \mu e^{-\mu x} \quad (4.1)$$

Let us define  $\tilde{p}$  as the probability that a station has a contention-free traffic frame to transmit in the reserved region, then,

$$\tilde{p} = \frac{y}{c} = \frac{y}{\frac{1}{\mu} + y} \quad (4.2)$$

Assuming that there are  $n$  stations in the network, the probability that  $k$  stations have contention-free traffic frames is given by,

$$\Pr(a = k) = \binom{n}{k} \tilde{p}^k (1 - \tilde{p})^{n-k} \quad (4.3)$$

where  $n$  is total number of stations generating contention-free traffic.

Let's define  $B$  as the number of slots available for the contention traffic, so the value of  $B$  is given by,

$$B = \gamma + m - a, \quad 0 \leq a \leq m \text{ and } \gamma \leq B \leq \gamma + m \quad (4.4)$$

Since the probability that there are  $\gamma$  slots available for the contention traffic is equal to the probability that all the  $m$  slots have been used by the contention-free allocations, which is given by  $\Pr(B = \gamma) = \Pr(a = m)$ . So the average value of  $B$  is given by,

$$\bar{B} = \sum_{b=\gamma}^{m+\gamma} b * \Pr(B = b) = \sum_{b=\gamma}^{m+\gamma} b * \Pr(a = \gamma + m - b) \quad (4.5)$$

Substituting from (4.3) and define  $k$  as  $k = \gamma + m - b$ ,

$$\bar{B} = \sum_{k=0}^m (\gamma + m - k) * \binom{n}{k} \tilde{p}^k (1 - \tilde{p})^{n-k}, \quad n \geq m \quad (4.6)$$

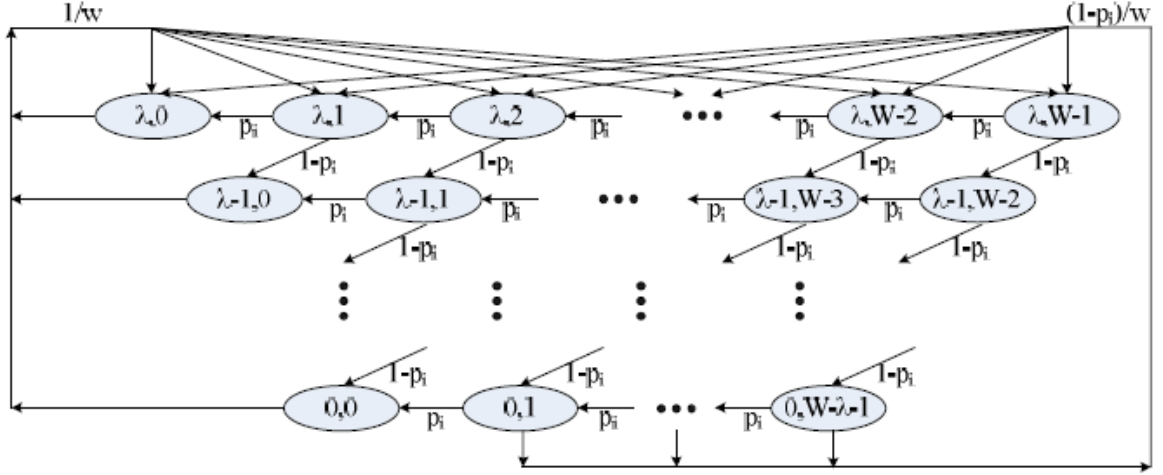


Figure 4.4 Bi-dimensional Markov chain

Next, we describe the bi-dimensional Markov chain model of the contention traffic developed in [14]. The following analysis is based on the research work from [14], however, we improve their work by bringing the contention-free traffic in to consideration.  $\Pi(i, j)$  is used to represent the steady state probability that a node has a DC value of  $i$  and BC value of  $j$  as shown in Figure 4.4.  $p_i$  represents the probability that a node finds the medium idle at current time slot. For instance, as introduced before, the value of BC will be decremented by one every time the station senses the medium is idle in the contention state. Thus, state  $(\lambda, 1)$  becomes  $(\lambda, 0)$  with probability  $p_i$ . Also

both BC and DC will be decremented by one if the slot are busy at this moment, which means that state  $(\lambda, 1)$  becomes  $(\lambda - 1, 0)$  with probability  $(1-p_i)$ .

From the given Markov Chain, the stationary distribution can be found from the following two recursive equations:

$$\Pi(\beta, W-\lambda-i_\beta) = (1-p_i) \sum_{j_\beta=1}^{\beta+i_\beta} p_i^{j_\beta-1} \Pi(\beta+1, W-\lambda-i_\beta+j_\beta) \quad (4.7)$$

For  $i_\beta = 1 - \beta, 2 - \beta, \dots, W - \lambda$ , and  $\beta = 0, 1, \dots, \lambda - 1$

$$\Pi(\lambda, W - \lambda - i_\lambda) = \Pi(\lambda, W - 1) \sum_{j_\lambda=0}^{\lambda+i_\lambda-1} p_i^{j_\lambda} \quad (4.8)$$

For  $i_\lambda = 1 - \lambda, 2 - \lambda, \dots, W - \lambda$

The probability that a node will transmit in any randomly chosen slot of time is given by

$$p_0 = \sum_{i=0}^{\lambda} \Pi(i, 0) \quad (4.9)$$

Since  $p_i$  is the probability that the node under consideration finds the medium idle, it is related to  $p_0$  through the following equation:

$$p_i = (1 - p_0)^{\tilde{n}-1} \quad (4.10)$$

In the above,  $\tilde{n}$  is the number of stations contending in the CSMA region.

Solving these equations numerically, the value of  $p_0$  can be found. Then, the probabilities of a successful transmission, an idle time and a collision are given by,

$$P_S = \tilde{n}p_0(1 - p_0)^{\tilde{n}-1} \quad (4.11)$$

$$P_I = (1 - p_0)^{\tilde{n}} \quad (4.12)$$

$$P_C = 1 - P_S - P_I \quad (4.13)$$

The MAC efficiency  $\eta$  can be expressed of  $P_S$ ,  $P_I$ ,  $P_C$  and time for a collision  $T_C$ , slot time  $T_I$  and time for successful transmission  $T_S$ . If the time of priority resolution is not considered [14], the relation is given by,

$$\eta = \frac{P_S T_{Data}}{P_S T_S + P_C T_C + P_I T_I} \quad (4.14)$$

This expression of the MAC efficiency is obtained from [14], in their research, the authors assumed that all the stations in the network are only allowed to contend for the channel. However, in the beacon interval, apart from the minimum CSMA region, stations are able to transmit in the allocations assigned by HE by using TDMA protocol, and then stations can only use the remaining time in the reserved region to contend for the medium. As a result, the duration time of the data payload, successful transmission and collision in the contention period should be modified.

In order to obtain the numerical results of the MAC efficiency, the following equations are needed for determining  $T_S$ ,  $T_{DATA}$  and  $T_C$ .

Number of bits in a Beacon Period is given by,

$$N_{bp} = R^{PHY} * T_{bp} \quad (4.15)$$

Number of bits available for the CSMA traffic depends on the percentage of the slots which are assigned for the contention traffic,

$$N_{CSMA} = \frac{\bar{B}}{m+\gamma} * R^{PHY} T_{bp} \quad (4.16)$$

So, the transmission rate for the CSMA traffic is given by,

$$R'_b = \frac{N_{CSMA}}{T_{bp}} = \frac{\frac{\bar{B}}{m+\gamma} * R^{PHY} T_{bp}}{T_{bp}} = \frac{\bar{B}}{m+\gamma} * R^{PHY} \quad (4.17)$$

The number of bits available for the data payload in the contention traffic is,

$$N_{Data} = N_{PB} = S_{PB} * 8 \quad (4.18)$$

where  $S_{PB}$  is defined in Table 3.1

So the duration of the data payload (PB) is given by,

$$T_{Data} = N_{Data}/R'_b \quad (4.19)$$

Also the number of bits of the ACK is given by,

$$N_{ACK} = S_{sack} * 8 \quad (4.20)$$

where  $S_{sack}$  is also defined in Table 3.1

So, the duration of the ACK is given by,

$$T_{ACK} = N_{ACK}/R'_b \quad (4.21)$$

The duration of the collision traffic is given by,

$$T_C = T_{PRS0} + T_{PRS1} + T_{PB} + T_{CIFS} \quad (4.22)$$

Finally, the duration of the successful transmission time is given by,

$$T_S = T_{PB} + T_{ACK} + T_{PRS0} + T_{PRS1} + T_{RIFS} + T_{CIFS} \quad (4.23)$$

## 4.4 Numerical Results

Next, we present some numerical results regarding the analysis in this chapter. According to (3.5) and Figure 3.3, if we assume that every station has one MPDU to transmit in the TDMA allocation and a MPDU consists of only a single segment, the maximum number of long MPDU that can be supported in the reserved region is 28. However, when the CSMA/CA scheme is taken into consideration, stations are allowed to contend for the access. We assume that the number of STAs in the network is  $n = 35$ , and in a beacon interval, the number of slots allocated to the minimum CSMA region and reserved region are  $\gamma = 2$  and  $m = 28$  respectively. The results are based on the following values of network parameters.

- The probability that a station has a frame to transmit in the reserved region is  $\tilde{p} = 0.8$
- The number of allocations dedicated to the contention traffic in the minimum CSMA region is  $\gamma = 2$
- The number of STAs in the network is  $n = 35$
- The number of STAs contending for the medium is  $\tilde{n} = 35$
- The number of slots in the reserved region is  $m = 28$



Figure 4.5 gives us the MAC efficiency of the CSMA region when the numbers of stations changes from 10 to 90. ( $\tilde{p} = 0.8, \gamma = 5$ ). Results are shown for 2 different cases of values of  $W$ , while  $\lambda$  has a constant value of 7.

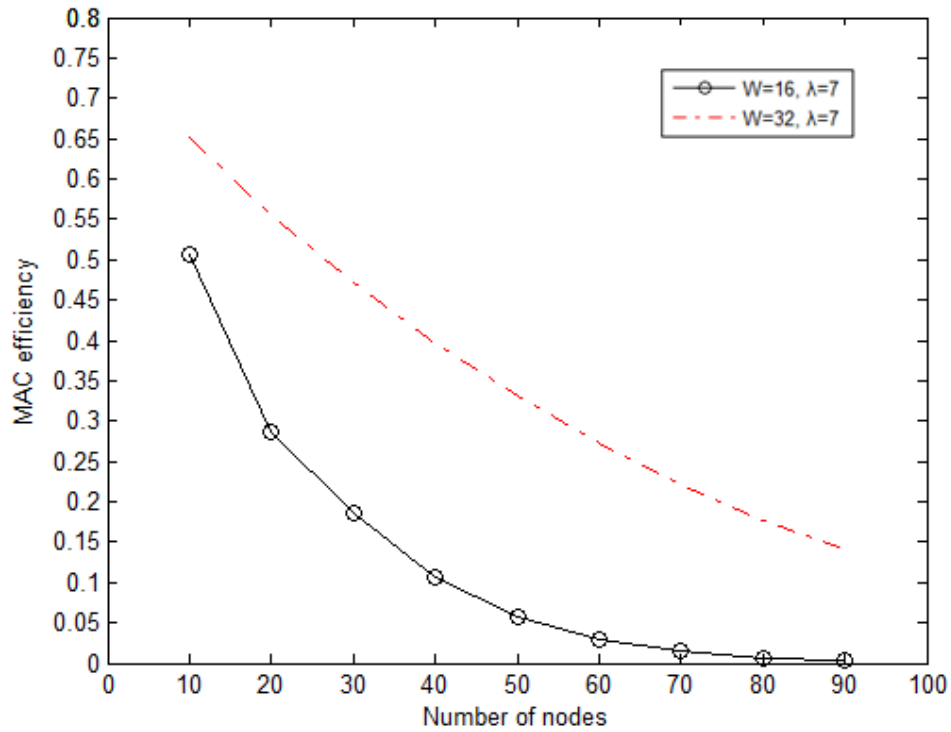


Figure 4.5 The MAC efficiency of the CSMA region when the numbers of stations changes from 10 to 90 for different values of  $W$  when  $\lambda$  is fixed at 7.

# Chapter 5

## Conclusions and Future Work

Broadband power line (BPL) network is one of the most promising access networks of the future due to their low cost and availability of the infrastructure. The recent boost to this effort came from the adoption and publication of the IEEE standard for Broadband over Power Line Networks: Medium Access Control and Physical Layer Specifications (referred to as “IEEE 1901”). This standard specifies the regulations for the operation of the Physical and MAC layers of the BPL network.

In this thesis, we consider the MAC protocols of contention-free and contention access schemes of the IEEE P1901 standard, which are TDMA and CSMA/CA protocols respectively. We developed performance models of both protocols.

In the contention-free traffic mode, we assume multi-hop communications between source and destination pairs and we consider both homogeneous and non-homogeneous error probabilities over the hops as well as fixed and random number of hops between source and destination. The main contribution of this thesis is the determination of the probability distribution of the number of beacon intervals that it will take to transmit a long MPDU and the network capacity of contention-free traffic.

Network capacity determines the number of users that can be simultaneously supported in the contention-free traffic mode.

Then, we determine the MAC efficiency when the network is operating under both contention-free and contention transmission modes with different number of users. The major contribution in the contention traffic analysis is that we improve the work from [14], we managed to combine the contention-free and contention transmission together, thus, the MAC efficiency is considered within the scope of the whole beacon interval.

We believe that the results of the thesis are valuable contribution to the design of power line communication networks. However, in the future, the work can be extended in several directions in order to make this new technology more feasible. In this work, we considered only unicast communications, between HE and a CPE. We would like to study communication between the CPEs. It will also be interesting to analyze the performance of the network with multicasting communications and broadcasting traffic, for instance, the broadcast transmission between HE and all the STAs associated within the cell.

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