Improving Inventory Control through Narrowcasting Promotion Management

A Thesis

in

The Department

of

Concordia Institute for Information Systems Engineering (CIISE)

Presented in Partial Fulfillment of the Requirements

for the Degree of Master of Applied Science (Quality Systems Engineering) at

Concordia University

Montréal, Québec, Canada

August 2011

CONCORDIA UNIVERSITY
School of Graduate Studies

This is to certify that the thesis prepared

By: Payman Salimi

Entitled: Improving Inventory Control through Narrowcasting Promotion Management

and submitted in partial fulfillment of the requirements for the degree of

Master of Engineering (Quality System Engineering)

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the final examining committee:

Dr. Y. Zen ______ Chair

Dr. K.Schmitt ______ Examiner

Dr. X.Huang ______ Examiner

Dr. C.Wang ______ Supervisor

Approved by ______
Chair of Department or Graduate Program Director

______
Dean of Faculty

Date September 8, 2011

Abstract

Improving Inventory Control through Narrowcasting Promotion Management

Payman Salimi

Narrowcasting is a business-centric network management solution which integrates media devices such as digital screens, high-bandwidth communication channels, and media management systems for media content distribution. The real time integration of inventory management with the emerging narrowcasting promotion systems promises a new level of supply chain efficiency. We investigate system modeling and design aspects of narrowcasting-based inventory control, which are of importance in terms of developing software systems to materialize the potentials of narrowcasting as a highly responsive promotion media. We propose an extension to the classical news-vendor inventory model, which integrates the narrowcasting advertising component into the dynamic inventory management process. We also present a promotion scheduling model for narrowcasting-based inventory management. To demonstrate the application of the proposed models, we provide a complete case study in the context of quick service restaurant industry.

Acknowledgements

First, I would like to express my deepest gratitude to my supervisor, Dr. Chun Wang whose patience and kindness, as well as his academic experience, have been invaluable to me. It has been a great opportunity for the past two years being his student at Master's level.

The second person I owe my deepest gratitude to is Ali Nejati. Thank you for providing valuable input to my work. Also, I have to gratitude the pleasure of working with other colleagues in the friendly group of our lab at Concordia University. This thesis has benefited from their comments and our productive discussions.

I would also like to thank my family for the support they have provided me through my entire life and in particular, I must acknowledge my parents without their support and encouragement, I would not have finished this thesis. Also, a special thanks to my brother, sister, and their spouses for their never ending supports.

Last but not least I have to thank "Pizza Pizza" store 301 for sharing part of their inventory data and information with me in order to use them as part of my study.

Table of Contents

List of Figures viii

Litst or Tables ix

Chapter 1. Introduction 1

1.1 Inventory control and its challenges 3

1.2 Traditional inventory control policies 5

1.2.1 Demand forecasting 5

1.2.2 Flyers advertisement in SCM 6

1.2.3 Limitation of traditional methods 7

1.3 Narrowcasting based methods 8

1.4 Outline of the thesis 10

Chapter 2. Literature Review 12

2.1 Traditional inventory control 12

2.1.1 Continuous review policy 13

2.1.2 Periodic review policy 14

2.1.3 Time series based demand forecasting 14

2.1.3.1 Moving average 16

2.1.3.2 Exponential smoothing 17

2.1.3.3 Simple linear regression analysis 18

2.1.3.4 Double exponential smoothing using Holt's Method 19

2.1.3.5 Model for seasonal series 21

2.1.3.5.1 Seasonal factor for stationary series 22

2.1.3.5.2 Winter model for seasonal factor 22

2.1.3.6 Selecting an appropriate forecasting method 25

2.1.3.7 Compute forecasting constants 28

2.1.3.8 Computing initial value for forecasting method 29

2.1.3.9 Evaluating the forecasting method 31

2.1.3.10 Risk management in forecasting 32

2.2 Advertising based Inventory Control Strategies 32

2.2.1 Flyer advertising 33

2.2.2 Narrowcasting screens on shopping Cart 34

2.2.3 Portable shopping device and E-grocery channels 35

2.2.4 Audio broadcasting in retail stores 36

2.2.5 Narrowcasting screens 37

Chapter 3. Narrowcasting-Based Inventory Control Model 40

3.1 News-Vendor model with In-store advertising 40

3.2 Narrowcasting-Based inventory control procedure 42

3.3 Inventory forecasting 44

3.4 Predicting demand generated by narrowcasting advertisement 45

3.4.1 Probability of narrowcasting advertisement exposure 46

3.4.2 Expected exposure time for a narrowcasting advertisement....................... 48

3.5 A Pizza dough Case Study - Perishable product with seasonality.................... 50

Chapter 4. Promotion Scheduling in Narrowcasting-Based Inventory Control 60

4.1 The promotion scheduling problem.. 60

4.2 The ILOG OPL model 63

4.3 Worked example with sufficient screen time.. 65

4.4 Worked example with insufficient screen time 68

Chapter 5. Experimental results - Perishable product 71

5.1 Inventory forecasting for pizza dough 73

5.1.1 Inventory forecasting for first period form 11 AM to 3 PM....................... 73

5.1.2 Inventory forecasting for second period from 3 PM to 8 PM..................... 75

5.1.3 Inventory forecasting for third period from 8 PM to 1 AM........................ 76

5.2 Promotion scheduling for pizza dough 77

Chapter 6. Conclusion 81

Bibliography 83

List of Figures

Figure 2-1: Time series characteristics. (a) Linear trend; (b) Constant process; (c) Cyclic variation; (d) Random fluctuation. 15

Figure 2-2 : Flowchart for selecting appropriate forecasting model........ 27

Figure 2-3: Flowchart for method of computing the order of N........ 29

Figure 3-1: Flowchart for optimizing inventory model........ 43

Figure 3-2: Expected demand as a function of advertising........ 49

Figure 3-3: The average daily demand for medium pizza based on historical data........ 52

Figure 3-4: The inventory demand for last three weeks of pizza dough........ 52

Figure 3-5: The daily demand for medium pizza in three situations........ 59

Figure 4-1: Store level promotion scheduling environment........ 61

Figure 4-2 : Ads Distribution for each hour on each screen........ 68

Figure 5-1 :Sales difference without and after promoting the slices on screens in period I. 79

Figure 5-2: Sales difference without and after promoting the slices on screens in period II. 79

Figure 5-3: The spoilage comparison for pizza slices in period I........ 80

Figure 5-4: The spoilage comparison for pizza slices in period II........ 80

List of Tables

Table 2-1: Equivalent smoothing constants for k parameters (k<3) 25

Table 2-2: Selection appropriate forecasting method 26

Table 2-3: Table of computing initial value for different type of forecasting methods ... 30

Table 3-1: Demand for medium pizza (Read down from left) 51

Table 3-2: Three periods moving average for pizza dough example................................ 53

Table 3-3: Three period moving average forecasts for pizza dough example 54

Table 3-4: The forecasting result at the end of day 1 54

Table 3-5: The forecasting result at the end of day 2 55

Table 3-6: The forecasting result at the end of day 3 55

Table 3-7: The forecasting result at the end of day 4 56

Table 3-8: The forecasting result at the end of day 5 57

Table 3-9: The forecasting result at the end of day 6 57

Table 3-10: The forecasting result at the end of day 7 57

Table 3-11: The error result of medium pizza dough forecasting 58

Table 4-1 : Demand for outfit fashion product 65

Table 4-2: Week 6th forecasting for outfit fashion product 65

Table 4-3 : Cumulative holding cost for outfit fashion.. 66

Table 4-4: Percentage of people passes different sections... 66

Table 4-5: Per hour ad coverage for each screen 67

Table 4-6: Extra inventory for 4 products in four days 69

Table 4-7: Expiration Cost (holding cost + spoilage cost) for 4 products in four days.... 69

Table 4-8: Narrowcasting result at the end of day one 69

Table 4-9: Narrowcasting result at the end of day two 69

Table 4-10: Narrowcasting result at the end of day three 70

Table 4-11: Narrowcasting result at the end of day four 70

Table 5-1: Number of pizza slices cook for each type of pizza during one day 71

Table 5-2: Actual demand for different type of pizza slices 72

Table 5-3: Selecting appropriate α for pizza slices - form 11 AM to 3 PM 73

Table 5-4 : Forecasting and Narrocasting result for pizza slices - form 11 AM to 3 PM. 74

Table 5-5: The Error results of pizza slices' forecasting - form 11 AM to 3 PM 74

Table 5-6: Selecting appropriate α for pizaa slices - form 3 PM to 8 PM 75

Table 5-7: Forecasting and Narrocasting result for pizza slices - from 3 PM to 8 PM 75

Table 5-8: Selecting appropriate α for pizza slices - form 8 PM to 1 AM 76

Table 5-9 : Forecasting and Narrocasting result for pizza slices - from 8 PM to 1 AM... 76

Table 5-10: Pizza slices' price for different types 77

Table 5-11: Extra inventory for pizza slices in one day 77

Table 5-12: Narrowcasting Ad distribution for one day in pizza slices 78

Table 5-13: Pizza slices final sales after promoting the slices on screens. 78

Chapter 1. Introduction

In many industries and supply chains, inventory is one of the dominant costs. In the United States, for example, over a trillion dollars is invested in inventory [1]. For many managers, effective supply chain management is synonymous with reducing inventory levels in the supply chain. The goal of effective inventory management in the supply chain is to have the correct inventory at the right place at the right time to minimize system costs while satisfying customer service requirements. Traditionally, managers rely on various forecasting models and tools to help them in making production and inventory decisions. However, it is rarely the case that forecasting alone can match supply and demand. In today's highly dynamic and competitive markets, in addition to forecasting, companies need to use other tools such as advertising and promotion to manage the customer demand. For example, retail stores often use weekly flyers, TV and radio advertisements as their promotion tools to balance inventory. The advertisements through these traditional channels can change the demand to some extent. However, they cannot reflect the updated inventory policies in a real time or near real time manner. Usually, it takes two to three weeks to prepare and distribute the advertisements using traditional channels. In Quick-Service Restaurants (QSRs'), the long advertisement updating intervals imposed by traditional tools are not acceptable because the average shelf life of QSR items is short. Many QSRs have the policy of throwing everything away at the end of the day. In other QSRs, freshness is an important selling point. For example, Tim Hortons (a Canada based QSR franchise, http://www.timhortons.com) has an "always fresh" policy where coffee is served within 20 minutes of brewing or not at all. Another Canadian based QSR franchise, Pizza Pizza, has a policy not to keep the pizza slices longer than one hour on the shelf. In these cases, to effectively manage the inventory, an advertising channel with fast responsiveness is needed.

In recent years, retail stores and QSRs turned their attention to a newly emerged advertising channel: narrowcasting media. For example, many Tim Hortons stores in Canada and US have installed big screens which can be used to display their products. Narrowcasting is a business-centric network management solution which integrates media devices such as digital screens, media management systems (ranging from a single low-end computer to a grid of computers), a management and control software system for media content distribution, and high-bandwidth communications. It enables companies and organizations to target audiences in a wide range of settings with an unprecedented level of customization and timeliness. CAP Ventures recent market research [The North American Commercial Digital Display Market, 2005] reported that the newly emerged narrowcasting technology is a key solution for companies and organizations to deliver promotional contents and business messages more effectively compared with traditional approaches. The establishment of this narrowcasting infrastructure provides the opportunity of implementing highly responsive channels for delivering advertisements required by real time customer demand management.

However, existing promotion management solutions for narrowcasting primarily focus on the distribution of media contents for predefined promotions. The full benefits of the narrowcasting networks have not been harvested yet because the promotion media played on the screen is not synchronized with companies' core business processes, such as inventory management. For example, Pizza Pizza stores in Canada use narrowcasting screens to only repeat preset promotions which have been printed on flyers. The effective integration of SCM (supply chain management systems) with emerging narrowcasting networks is an important research topic of practical interests. It expands the existing SCM systems to incorporate an emerging promotion tool, which can be used to improve the efficiency of SCM. Before narrowcasting networks were introduced, companies could track customers' demand using point-of-sales scanners. However, they had very little power on influencing the demand in a timely manner. Narrowcasting networks provide companies with the flexibility of adjusting their promotion plan on the networks with a high level of responsiveness. Therefore, a company can precisely influence customers' demand according to its SCM situations, such as inventory levels.

The goal of this thesis is to develop narrowcasting promotion scheduling software systems that balance the customers' demands with the firm's supply capabilities. The demands are "adjusted" in a real time manner through effective scheduling of advertisements within the narrowcasting networks. This adjustment is made according to inventory management policies, distribution, and production processes to improve the overall supply chain efficiency.

1.1 Inventory control and its challenges

There are various reasons to hold an inventory and to use inventory control mechanisms in retail stores. Unexpected changes in customer demand; the presence of a significant uncertainty in the quantity and quality of supply, the delivery time, and economies of large scales offered by transportation companies are all justifications for holding an inventory. It is important for retailers to decide how much to stock in order to meet the demand, and it is also crucial not to over or under stock, so that there are no negative effects on sales and profits.

The major reason for inventory surplus is the difference between demand rate and supplies [2]. Because there is always lead-time delivery and the demand is random for different products, it is necessary for retail stores to have excess inventory, even if there is no fixed initial cost for ordering products [3]. Controlling and operating an inventory in a supply chain is complex because one must coordinate a products' inventory-level-control and the customer demand which is uncertain in nature. Holding the right amount at the right time and location is a challenging problem [1]. Poorly managed inventory incurs high holding costs among others, such as inventory maintenance, property taxes, insurance, and transportation.

In order to determine the correct inventory amount, the first question to answer is which ordering policy the retailer is to adopt. A retailer can have single or multiple ordering opportunities for a specific product during a selling season. Single opportunity models are usually used when the selling season is short and there is no second opportunity to reorder products based on realized customer demand. On the other hand, when the retailer needs to order products repeatedly during the year, multiple order opportunity models are used. There are two main policies in a multiple order opportunity model: continuous review policy and periodic review policy, I will discuss these models in detail in the next chapter.

A common challenge in various inventory control models is holding the right quantity at the right time in the appropriate place. Demand forecasting methods are used to determine the necessary quantity of the product to be ordered, and the right time to place the order for the product [4]. Demand forecasting can involve quality based methods, such as expert judgment, and quantitative methods, such as Time Series forecasting which mainly depend on historical sales data. However, as a general characteristic, forecasting is always wrong. Sometimes, forecasting errors can be substantial, which gives rise to applying correction actions through other channels such as advertising.

1.2 Traditional inventory control policies

The strategy and set of techniques used to determine how to control inventory is known as inventory policy [1]. In order to design an effective inventory policy, managers need to consider many characteristics of the supply chain. However, first and foremost is the customer demand forecasting. In addition to demand forecasting, advertising can be used as another channel to control the cost by reducing the inventory level.

1.2.1 Demand forecasting

Inventory demand forecasting is the process of predicting the customer demand for a certain period of time in order to improve the quality of inventory management decision making. Forecasting can be useful in inventory management, production planning, financial planning, staff scheduling, facilities planning, and process control [5]. A good illustration of inventory forecasting is spare parts of aircrafts, which needs to estimate the rate of usage for every part in order to determine procurement quantities. Also, the variability of forecast error over the procurement lead time is required to be forecasted [5]. Forecasting methods are categorized as either subjective or objective [6].

The subjective method uses human judgments and expert opinion. For example, Delphi-type method is known as subjective method. In this model a group of elected experts selected, as jury, to answer questionnaires in at least two or more rounds. At the end of each round the summary of the results is announced, and the experts are asked to revise their judgments again [7]. Objective forecasting methods are the results of data analysis. Causal models and Time series are two significant methods of objective forecasting. Causal forecasting methods are based on a known or perceived relationship between the factors to be forecast and other external or internal elements like weather conditions might improve the ability of a model to predict umbrella sales [8]. In fact, it is based on the relationship of times series and one or more independent variables. Also, compared to Times Series model, causal model needs a large amount of computation and data handling [4]. Time Series, on the other hand, is based on a previous series of observations at discrete time points which uses the past value and historical data [5]. Forecasting problems can also be classified in terms of forecast horizons, such as short term, long term, or intermediate term forecasting [4].

1.2.2 Flyers advertisement in SCM

In addition to forecasting, advertisements are also used as a method for controlling inventory. Among them, traditional paper print flyers are pretty common and still popular. This method is most often used by supply chain retailers in North America. Because its cost is low and it is easy to implement, many retailers prefer to use paper flyers instead of high tech intelligent methods. Flyers are an inexpensive and effective form of direct marketing [9]. Usually, retail stores which use flyers need to check their inventory monthly or weekly based on their promotion scheduling and the promotions received from producers help retailers design their flyers' content [10].

1.2.3 Limitation of traditional methods

Inventory forecasting and flyer advertisements are the most common traditional methods which many retailers still use. Most of the retailers use both methods in tandem to get the best result. However, there are limitations with these methods. As mentioned before, the forecasting results are not always accurate, which is one of the intrinsic properties of all forecasting methods. In addition, the longer forecast horizon, the less accurate the forecast will be.

There are also limitations on traditional advertising channels. Usually retailers have different policies regarding print flyers. Franchise stores need coordination of price, quality, and advertising is the task of marketing strategy and the determinant position of any product [11]. Franchise's headquarter might check the sales or inventory weekly, monthly or seasonally and based on the information they gain, scheduling the flyer advertisements. The main constraint of flyers is they cannot reflect the changes of inventory promptly. For example, if a retailer runs out of stock in the middle of the promotional period, there is no way to change the flyers and/or remove the item from flyers. On the other hand, if a retailer wants to increase the price discount given the slow sale of a product, it is hard for them to do it using flyers in a timely manner. Lack of real time response to the inventory changes is the most important deficiency of traditional advertisement channels.

In addition to the aforementioned constraint, flyers consume paper and litter the environment. Furthermore, lead time for ordering print materials could be another issue which affects the efficiency of paper advertisements [10]. As a result, some retail companies have already started to use intelligent marketing methods such as narrowcasting to promote their products or service.

1.3 Narrowcasting based methods

One of the most popular methods of intelligent marketing is narrowcasting. Tim Horton's and Wal-Mart are two companies in North America that often use this method. Wal-Mart had spent over $10 million to test its narrowcasting promotion system between 2007-2009 [12]. They have started using their new network 'shopper-intelligent network at retail' as their first intelligent adverting method. This method encourages customers to purchase using screens in the store at any time of the day. In 2008, Wal-Mart had planned to assemble over 27,000 screens for 2,700 stores until the end of 2010 [13].

Narrowcasting is a business-centric network management solution which integrates media devices such as digital screens, media management systems (ranging from a single low-end computer to a grid of computers), a management and control software system for media content distribution, and high-bandwidth communications. It enables companies and organizations to target audiences in a wide range of settings with an unprecedented level of customization and timeliness. In retail stores, narrowcasting relies on a digital network screen system installed in the stores. The narrowcasting screens are also known as in-store product presentation optimizing tools [14]. This method of advertising started two decades ago when Wal-Mart installed an in-store television network in 1990s. The sandwich franchise Subway also implemented narrowcasting screens in 2006 [15]. One of the significant advantages of in-store networked screen systems was that consumers paid more attention to the in-store ads, because they were not distracted by alternatives outside the store [16].

The most significant advantage of narrowcasting screens is real time advertising. The promotion can change daily or even hourly based on the supply chain and inventory situations. Moreover, it is presumed that advertising a product in a retail store where customers are there to buy products will have more effect than other methods of advertising that took place outside of the retail store. Proctor and Gamble believes customers have a three to seven seconds glance to the items on a shelf [17]. This means these few seconds of marketing opportunity, if captured by narrowcasting screens installed close to the shelves, can be effective in terms of promoting the product. Customers usually visit a section of a store with the purpose of purchasing a specific category of products located in that area. Eighty one percent of people who experienced in-store video screens are drawn to the screens while they are shopping [18]. Based on this evidence, we could assume there is an increase in sales when we have video advertising is in the store. Analyzing the supply chain's present information, which helps to understand the potential customers market in a real time manner, can play a critical role on the success of a Supply Chain [19]. Intelligent Marketing such as Narrowcasting and Online marketing has the advantage of real time respond to the unexpected changes which have high value in today's supply chains marketing strategy.

In-store real-time advertising has done extremely well in various forms including ads on shopping carts, aisles and talking shelves, end-aisle displays, floor signs, and in-store audio broadcast [20]. However, one of the main limitations of these intelligent advertising methods is that the advertising schedule is not closely integrated with changes of the inventories. In fact, they are usually based on a manufacture's promotion plan, or have at least one period of delay.

1.4 Outline of the thesis

The objective of this study is to provide a real time promotion management system for reducing inventory costs by scheduling advertisements effectively in narrowcasting environments. We proposed a promotion management approach for narrowcasting networks, which provides companies with the flexibility of adjusting their promotion plan according to their inventory management needs, with a high level of responsiveness. From SCM systems design perspective, the proposed approach expands the existing SCM systems to incorporate an emerging promotion tool, which can be used to improve the efficiency of SCM. In terms of the scope of the research, we study the integration of narrowcasting networks and traditional forecasting methods in the context of news-vendor inventory management model. We assume that stores have the IT infrastructure to monitor the levels of inventories of various products in a real time manner.

The rest of the thesis is structured as follows. Chapter 2 presents a literature review of inventory management methods with an emphasis on time series forecasting methodology. In addition, literatures regarding the effectiveness of narrowcasting on sales will also be reviewed. In Chapter 3, we first set up the inventory control environment, which is an advertising extension to the news-vendor model. We then propose the in-store advertising adjustment model used to reduce the end period extra inventory. We present a narrowcasting scheduling framework which can be used to allocate screen times to advertisements in Chapter 4. Chapter 5 demonstrates the use of the proposed approach through a case study. Chapter 6 concludes the thesis, discusses limitations of this method and future research directions.

Chapter 2. Literature Review

For over 20 years, intelligent marketing has been combined with traditional methods of inventory control to improve supply chains' efficiency. As a method of inventory control, inventory forecasting has been the theme of considerable amount of research. Forecasting in inventory management is about predicting the future of items' quantity in inventory, based on sales, and customer demand [4]. Nonetheless, unpredicted changes in customers' behavior sometimes results in unreliable forecasting results. As a consequence, an intelligent marketing plan such as real time advertising could help the inventory against forecasting failures. In this chapter I first review different methods of forecasting and traditional inventory control. I will also summarize retail stores' advertising policies and relevant narrowcasting studies published in the literature.

2.1 Traditional inventory control

An important component of inventory management is controlling the inventory and finding the optimal ordering quantity for each replenishment cycle. There are a variety of approaches for controlling the inventory and the quantity of inventory depends on the characteristics of the products and its pattern of demand [21]. In this subsection, I briefly introduce two typical inventory control policies, namely continuous review policy and periodic review policy. I will also review time series based forecasting methods for inventory management.

2.1.1 Continuous review policy

In continuous review policy, the inventory is reviewed continuously, and an order is placed when the inventory reaches a particular level, called reorder point. It is normally assumed that daily demand is random and follows a normal distribution; every time the customer (e.g. a retail store) places an order from the manufacture, the customer pays a fixed cost, *K,* plus an amount proportional to the quantity ordered [22]. There will be an inventory holding cost per item per unit time and the inventory level is continuously reviewed. When an order is placed, the order will arrive after the appropriate lead time L. The reorder point R is calculated based on the following formula:

$$R = (L * AVG) + (z * STD * \sqrt{L})$$

where AVG is the average demand during a unit time; STD is the standard deviation of per unit time demand; and z is the safety factor. $L * AVG$ represents the average demand during the lead time period. $z * STD * \sqrt{L}$ is the safety stock, which is a level of extra stock, maintained below the cycle stock to buffer against stock out during the lead time. The optimal ordering quantity Q is calculated using the following formula:

$$Q = \sqrt{\frac{2K * AVG}{h}}$$

where K is known as a fixed cost and h is per-time-period holding cost.

In some cases the lead time is random or unknown. In this situation we assume the lead time is normally distributed with average lead time which denoted as *AVGL* and standard deviation as *STDL*. In this case, to calculate reorder points we will have:

$$R = (AVG * AVGL) + (z\sqrt{AVGL * STD^2 * AVG^2 * STDL^2})$$

Continuous review policy is most appropriate when inventory can be continuously reviewed, for example, when computerized inventory systems are used.

2.1.2 Periodic review policy

In periodic review policy, the inventory level is reviewed at the regular intervals, and an appropriate quantity is ordered after each review [1]. Assume that the inventory is reviewed every r period of time. Under periodic review policy, after each review, an appropriate quantity of product is ordered, such that the inventory position is raised to a target level, called the base-stock level, which is calculated using the following formula:

$$Base\ stock = (r + L) * AVG + z * STD * \sqrt{r + L}$$

In the formula, $(r + L) * AVG$ represents the average demand that the base-stock must cover. $z * STD * \sqrt{r + L}$ is the safety stock. Periodic review policy is appropriate for systems in which it is impossible or inconvenient to frequently review inventory and place orders if necessary.

2.1.3 Time series based demand forecasting

Time series based forecasting methods only require the previous data to predict the future values [23]. They are based on the assumption that historical data is an adequate indicator of future demand. Time series based methods are appropriate for the situations where the demand pattern of products does not change significantly from one year to the next. To select an appropriate time series forecasting model, we need to first establish a valid demand pattern model based on the characteristics of historical demand data.

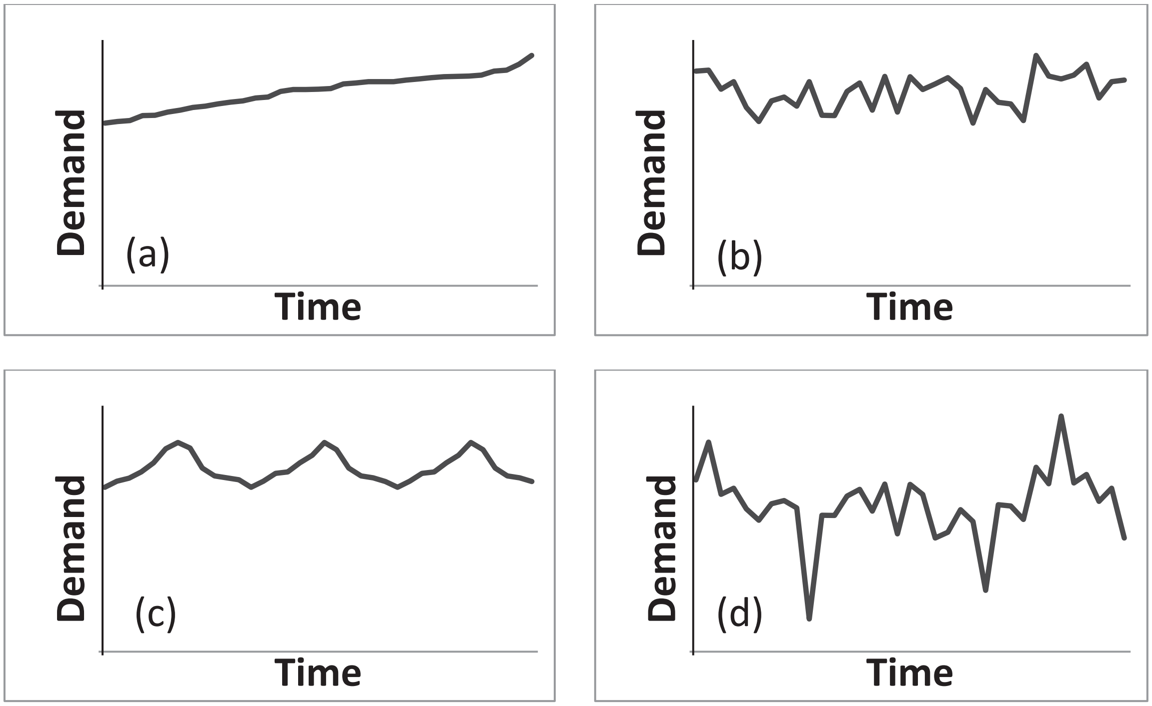


Figure 2-1: Time series characteristics. (a) Linear trend; (b) Constant process; (c) Cyclic variation; (d) Random fluctuation.

A few patterns of time series characteristics are shown in figure 2-3. Pattern (a) shows a trend pattern that can have increasing or decreasing tendency. Pattern (b) illustrates the process which remains constant over time. In fact, each observation in this pattern is a sum of a constant and a random fluctuation [4]. In pattern (c), the demand exhibits seasonality. The seasonal variations can be related to some causes like weather (e.g., soft drinks' demand), holiday periods, or policies. In addition to these three patterns, there is a random one which can result from combination of two other patterns or can be totally random and not recognizable. For example, stock market charts are mostly random patterns.

In time series based method, demand during the periods of $1,2,3,\ldots,t,\ldots$ is defined as $D_1, D_2, \ldots, D_t, \ldots$. When we are in the period of t, the values of $D_t, D_{t-1}, \ldots, D_1$ is known, but we have not observed D_{t+1}. Therefore, the forecasting period for *t+1* is defined as F_{t+1}. In fact, we forecast the value of D_{t+1} to $D_{t+\tau}$ after observing the value of D_t. The variable τ is called the forecast horizon which represents the number of periods into the future that we are forecasting [5].

As mentioned above, time series methods predict future values by using past observations. This means for time series method we can define a function as:

$$F_{t+\tau} = \sum_{n=0}^{t+\tau-1} a_n D_{t-n} \qquad \text{for some set of weights } a_0, a_1, \ldots$$

$a_0, a_1, \ldots$ are set of weights which apply to before t for each particular demand.

2.1.3.1 Moving average

Moving Average is a popular forecasting method which calculates the average of the most recent N observations. In this section we only study the simple moving average for constant process. Indeed, moving average will provide a good estimate of the mean of the time series when the mean value is changing slowly or we can assume it as a constant.

$$F_{t+1} = F_t + \frac{D_t - D_{t-N}}{N}$$

When new demands are observed, we remove the oldest observation from our calculation at each period. In fact, the previous forecast result is added to the difference of the last observed demand and the demand of the *N* period old divided to *N* in order to update the forecast.

The main defect of moving average is not having an accurate result when there is a trend in the series [4]. In addition, the moving average method creates lag in the computation indicator [23].

2.1.3.2 Exponential smoothing

Exponential smoothing is one of the most common methods of time series in terms of forecasting the immediate future [5]. The simple exponential smoothing method is appropriate when the demand has no trend or seasonality pattern [24]. This model is based on the weighted average of all previous data, with the current value of demand which has more weight [25]. The initial estimate of the level F_0 can assume to be the average of all historical data because demand has been assumed to have no clear trend or seasonality, as a result we have the following [4]:

$$F_0 = \frac{1}{n}\sum_{i=1}^{t-1} D_i$$

$$F_t = \alpha D_{t-1} + (1-\alpha)F_{t-1}$$

where $0 < \alpha \leq 1$ is the smoothing constant, which determines the relative weight placed on the current observation of demand. The reason this method is called exponential smoothing is because we fit the continuous exponential curve $g(i) = \alpha \exp(-\alpha i)$ to the weights which applied to the data of i periods old [4].

For finding the best value for the constant of α, it is important to know the operating characteristics of exponential smoothing. In general, the smoothing constant for exponential smoothing is between 0.01 and 0.3 [5].

Since it is easier to choose N, this may be a good suggestion to find a relationship between the number of periods N in a moving average and the smoothing constant in a simple exponential smoothing. By equating the average age of data for the exponential smoothing and moving average methods when the age of data for exponential smoothing system is equal to an N-period moving average we will have [5]:

$$N = \frac{2 - \alpha}{\alpha}$$

In this situation both methods will have the same forecast errors' distribution [4]. However, this does not mean that the forecasts result for both methods are the same. In order to use exponential smoothing, we only need to save the last forecast. In contrast, for moving averages it is necessary to save all *N* past data points. This is the most significant advantage of the exponential smoothing method compared with the moving average. It is important to mention both exponential smoothing and moving-average forecasts will lag behind a trend if one exists.

2.1.3.3 Simple linear regression analysis

Regression analysis is a forecasting method which fits a straight line to a set of data. The base form of regression is least squares, which was published by Legendre and by Gauss. This method is used when there are two or more variables [26]. Usually many forecasting methods take the advantage of regression to estimate initial parameters or constant values. When we assume $(D_1, t_1), (D_2, t_2), \dots, (D_n, t_n)$ are n pair of data for the two variables of *D* and *t,* the value of t_i is the observed value for *t* when D_i is the observed value of *D*. Then the relationship of *D* and *t* can represent as a straight line:

$$\widehat{D} = a + bt$$

where $\widehat{D}$ is the predicted value, which is usually the demand in forecasting the inventory level and t is time in the period. The objective to solve the problem is to compute the value of a and b.

$$b = \frac{S_{xy}}{S_{xx}}$$

$$a = \overline{D} - b(n+1)/2$$

$$S_{xy} = n\sum_{i=1}^{n} iD_i - \frac{n(n+1)}{2}\sum_{i=1}^{n} D_i$$

$$S_{xx} = \frac{n^2(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4}$$

and $\overline{D}$ is the average of observed demands during periods of 1, 2,…, n. For using this forecasting technique the estimated model parameters has to update at the end of each period. This method has time consuming calculations when we want to forecast a long period of time [5].

2.1.3.4 Double exponential smoothing using Holt's Method

One of the well-liked time series forecasting methods is double exponential smoothing, with linear trend tracking [26]. This method requires two smoothing constants of α and β which can be estimated via non-linear optimization techniques, such as the Marquardt Algorithm or smallest mean square error (MSE) [27].

There will be two smoothing equations: one refers to the value of the series and the other one for the trend (the slope) [4]:

$$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$$

$$G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$$

S_t is known as the value of the intercept at time *t* and G_t is the value of the slope at time *t*. D_t is the most recent demand which is observed. The First smoothing equation is the trend of the previous period, G_{t-1}, adding to the last smoothed value, S_{t-1}, to eliminate the lag and adjust S_t to the appropriate base of the current value [23]. The second smoothing equation is used to update the trend, which is expressed as a difference between the last two smoothed values. The forecasting equation is similar to the basic form of single smoothing, but here applied to the updating of the trend. The τ step ahead from *t*, $F_{t,t+\tau}$ is given by:

$$F_{t+\tau} = S_t + \tau G_t$$

There are a variety of schemes to compute the initial values for S_t and G_t in double exponential smoothing. The simple method is to set S_1 equal to D_1 and for G_1 we can use one of the suggested formulas [27]:

$$G_1 = D_2 - D_1$$

$$G_1 = [(D_2 - D_1) + (D_3 - D_2) + (D_4 - D_3)]/3$$

$$G_1 = (D_n - D_1)/(n-1)$$

Also, if we assume the forecast error's distribution for exponential smoothing and moving average is equal, we can use following formula to compute α [5]:

$$\alpha = \frac{2}{N+1}$$

If α is the smoothing constant for single smoothing, the equivalent value of the smoothing constant for a model with k parameters (for double exponential smoothing $k=2$) can be estimated from following equation [5]:

$$(1-\alpha_k)^k = 1-\alpha_1$$

As a result for double exponential smoothing the second constant is computed as follows:

$$(1-\beta)^2 = 1-\alpha$$

The double exponential smoothing using Holt's method performs better for series and trends because it is explicitly designed to track the trend in the data. In addition, it is easier to update forecasts when new observations become available. Nevertheless, historical data should be reviewed carefully with statistical tests in order to determine if obvious patterns exist [28].

2.1.3.5 Model for seasonal series

Seasonal series are used for patterns that repeat every N period [4]. In order to use this method; we have to be able to specify the length of the season. To signify the seasonality we assume a set of seasonal factors c_t, when $1 \leq t \leq N$.

2.1.3.5.1 Seasonal factor for stationary series

If we want to compute the seasonal factor for items that have no trend, we can use the following model. First, we compute the mean value of all observed data. Secondly, we divide each observation to computed mean value to find the daily factors. We average the factors for periods within each season, which are N seasonal factors in order to have the average factor for each day in our seasonal period. When this factor is close to 1 it means the demand for that period of time is equal to average. On the other hand, when the factor's value is much lower than one or higher than one there is a computable percentage decrease or increase on demand.

For some group of items which are seasonal and repeat every N-period we have to do seasonal decomposition by using the moving average method to compute the N-period moving average. From the historical data we can find the period lengths and calculate the N-period moving average. Then, we have to "Centre" the moving average result. Afterward, the values for the upper period and the lower period are computed using the average center values for each N-period moving average. Finally the ratio is equal to the demand divided into the centered forecast data and our factors are equal to the average of each certain period's ratio [4]. The most significant limitation to this method is when new data becomes available, all seasonal factors need to be calculated again.

2.1.3.5.2 Winter model for seasonal factor

The winter method is a type of triple exponential smoothing, which is used for trending seasonal series. This method makes it easy to add new observations whenever they become available. We assume in this model the time series is represented by the following model [29]:

$$D_t = (\mu + Gt)c_t + \epsilon_t$$

where μ is the permanent component and G is a linear trend component. c_t is a multiplicative seasonal factor and ϵ_t is known as a random error. The length of the season is *N* periods, which is equal to a sum of seasonal factors. For each period we have to estimate three different constants: $\alpha, \beta,$ and γ. Each of these constants in sequence is related to series, trend, and seasonal factors [4]. These constants need to be estimated to minimize the error estimation of MSE. This method is the best way when it is done with a computer.

$$S_t = \alpha\left(\frac{D_t}{c_{t-N}}\right) + (1-\alpha)(S_{t-1} + G_{t-1})$$

$$G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$$

$$c_t = \gamma\left({D_t}/{S_t}\right) + (1-\gamma)c_{t-N}$$

where S_t is the current level of the data series G_t is the trend which is computed like Holt's method, and c_t is a seasonal factor. The forecast made in period *t* for any future period $t + \tau$ is equal to:

$$F_{t,t+\tau} = (S_t + \tau G_t)c_{t+\tau-N}$$

To obtain the initial values we use slope and seasonal factors. If we assume the last two seasons of data are available, and *t=0* is our assumption, so the past observations are recognized as $D_{-2N+1}, D_{-2N+2}, \ldots, D_0$. The following formulas will be used to compute the initial values [4]:

$$V_1 = \frac{1}{N} \sum_{j=-2N+1}^{-N} D_j$$

$$V_2 = \frac{1}{N} \sum_{j=-N+1}^{0} D_j$$

$$G_0 = (V_2 - V_1)/N$$

$$S_0 = V_2 + G_0(N-1)/2$$

$$c_t = \frac{D_t}{V_i - [\frac{N+1}{2} - j]G_0} \qquad for -2n+1 \le t \le 0$$

$$c_{-N+1} = \frac{c_{-2N+1} + c_{-N+1}}{2}, \ldots, c_0 = \frac{c_{-N} + c_0}{2}$$

The above initialization is only one method suggested by Winters [29]. However, seasonal factors can be computed from a previous method discussed in the moving average. In addition, we can use linear regression similar to Holt's method to compute S_0 and G_0, and for computing the seasonal factors we only needs to divide each demand observation to the baseline period [4].

Smoothing constants			Moving Average
α	β	γ	N
0.50	0.293	0.206	3
0.40	0.225	0.157	4
0.30	0.163	0.112	6
0.25	0.134	0.091	7
0.20	0.106	0.072	9
0.15	0.078	0.053	12
0.10	0.051	0.035	19
0.01	0.005	0.003	199

Table 2-1: Equivalent smoothing constants for k parameters (k<3)

To find the triple exponential equivalent smoothing constant, when the variance of forecast errors for single exponential smoothing and moving average are equal we can use the following table [23]. As shown in the table equivalent N is a simple moving average.

We also can consider when the demand had a significant autocorrelation we would expect a short smoothing interval that gives a more accurate result in forecasting with short lead time [30].

2.1.3.6 Selecting an appropriate forecasting method

In most cases the ordering quantity is known or when the optimal quantity calculated once, same policy will be followed for all periods until unexpected changes happened. A common thought about choosing appropriate forecasting is an incorrect opinion of choosing complex forecasting models always gives better results than simple ones [31].

Trend and seasonality are two main characteristics of historical data for selection the forecasting method. When mean value of observed data change rarely, we can assume a trend pattern exist for our items [32]. In contrast, the seasonality of a time series is known as a pattern that repeats during fixed intervals of time. In fact, when a time series is influenced by seasonal factors, such as month of the year or day of the week, there will be a seasonal factor in our data [32]. A Canadian survey from 2000 stated that most Canadian companies use a combination of subjective and objective forecasting methods [33]. However, in both methods the important parameter to notice is seasonality. Based on this survey approximately 65% of Canadian businesses check seasonality of their data first.

Forecasting Method	Most Suited Data Types
Moving Average	No Trend, No Seasonality
Exponential Smoothing	No Trend, No Seasonality
Simple Linear Regression Analysis	Varying Trends, No Seasonality
Double exponential smoothing using Holt's Method	Varying Trends, No Seasonality
Seasonal Factor for Stationary Series	No Trend, Varying Seasonality
Winter Model for Seasonal Factor	Varying Trends, and Seasonality

Table 2-2: Selection appropriate forecasting method

A common problem is choosing which forecasting method is the best for a particular product. There is no precise answer. One must ask what measurement is used for the "best", to minimize the average forecast error. Based on available historical data different forecasting models will work better for different situations, depending on data characteristics.

I have grouped the forecasting based on our data type. First I look at recent historical data and plot the observed data try to find a correlation between them. This correlation can be stationary, trend, or seasonality. For the type of products we are studying in this research an immediate forecast is usually necessary.

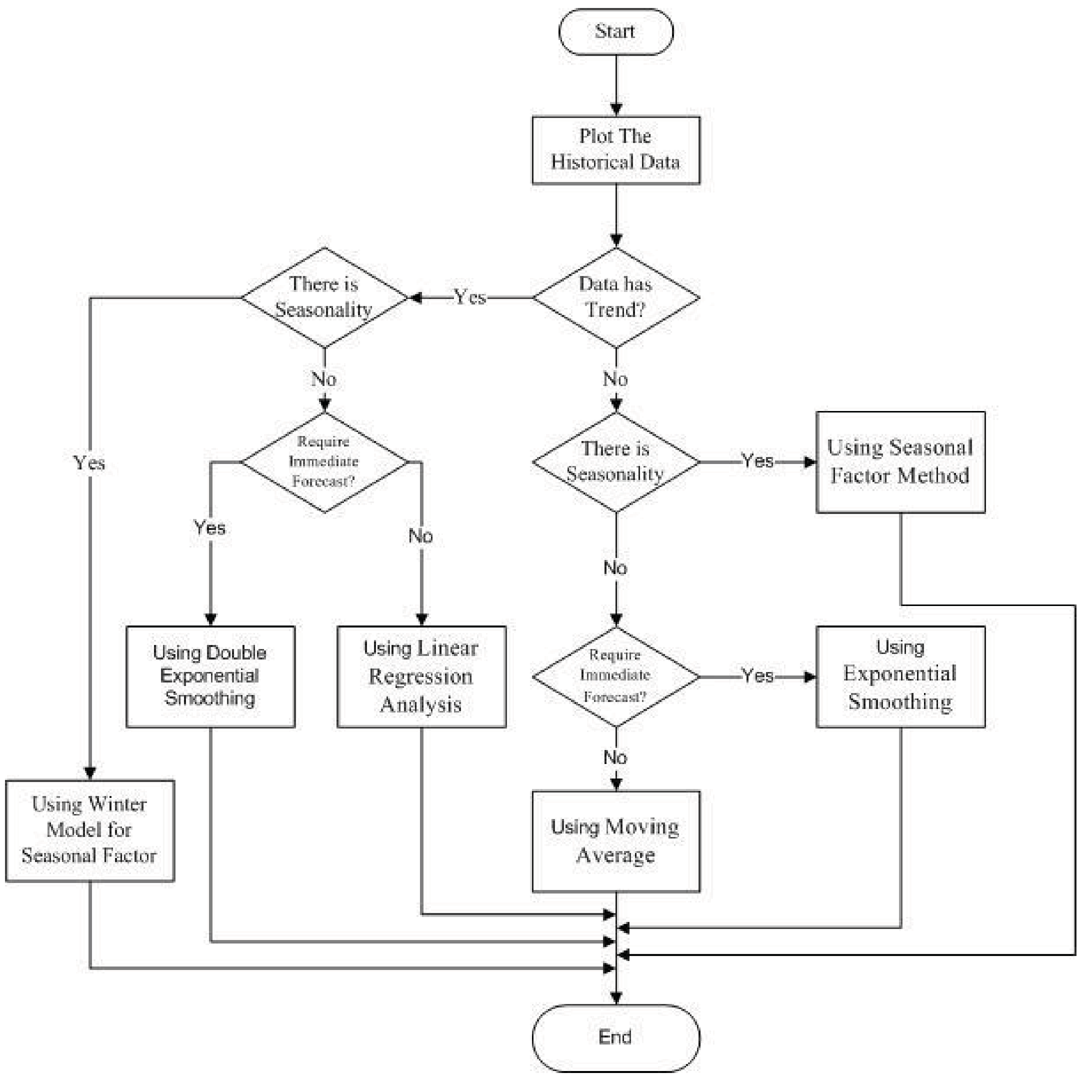


Figure 2-2 : Flowchart for selecting appropriate forecasting model

As a result, if there were two options to choose from, more value will be given to the exponential smoothing for the immediate forecasting [34]. For example, imagine a situation in which there is no trend or seasonality, but we need to do daily forecasting immediately in order to schedule the narrowcast ads. We could use Moving Average or Exponential Smoothing. However, for selecting the appropriate model from these two forecasting methods exponential smoothing will give us a better result. The main reason to choose the exponential smoothing is easier to update our forecasting when new demand becomes available; therefore, in short-time forecasting we can get result quickly.

This above algorithm is not coded on any computer program. Only the first part for plotting the data will be done on the computer, and the result will be analyzed manually and visually. With this method we take advantage of the objective and subjective method to forecast our data.

2.1.3.7 Compute forecasting constants

Usually for forecasting methods we need to estimate the constant value. The significant limitation is choosing the best value. For example in moving average *MA(1)*, a moving average of order 1, our result is often unreliable. In fact, taking the last known data as the next period's forecast is truly impractical. Using *MA(3)* or *MA(5)* would effectively smooth out seasonal effects for the additive seasonal data pattern. In this study we used a kind of trial and error method to select the most appropriate order of N for our forecasting.

We assign N, the value of inventory review period, r, and do forecasting for the latest observed data, and compare it with N when it is equal to $2r$ and $r/2$. From the three available results the best two will be selected and the new *N* assigned to average or those two will be selected. The new result will compare again, and the procedure continues until find the best value.

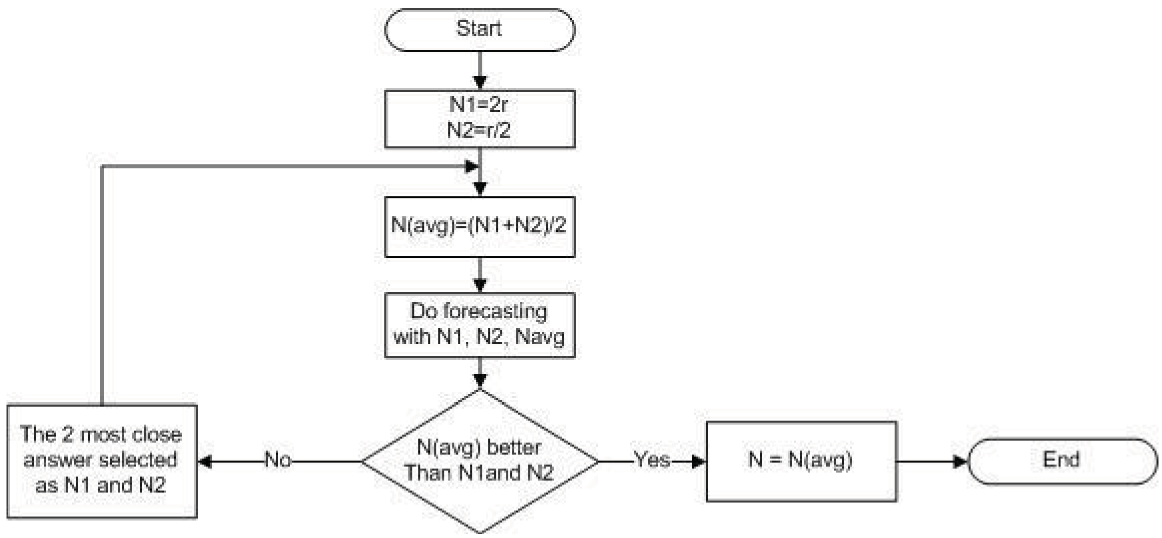


Figure 2-3: Flowchart for method of computing the order of N

2.1.3.8 Computing initial value for forecasting method

When the suitable value for constant or order value *N* is selected, the initial value is necessary to be computed in order to start forecasting. Based on the forecasting model, the method of computing initial data is variable. In some cases such as Holt's model, in first day demand is observed and rest of the week is forecasted in sequence by using the first day observed demand. In this situation when we are forecasting the second day, we need to adjust the inventory's difference for the first and second day. Indeed, the difference between the first day observation and the real demand has to be considered in our scheduling.

Table 2-3 shows the methods of computing the initial value that is used in this research. Some models have more than one method to calculate the initial value. In table 2-3 we listed those methods that are most common and used in the references.

Forecasting Model	Initial Value
Moving Average	$F_0 = \frac{1}{n}\sum_{i=1}^{n} D_i$
Exponential Smoothing	$F_0 = \frac{1}{n}\sum_{i=1}^{n} D_i$
Simple Linear Regression Analysis	No initial value needs to compute
Double exponential smoothing (Holt's Method)	S_0 is the intercept and G_0 is the slope for linear trend line of pervious observed demands.
Seasonal Factor for Stationary Series	No initial value needs to compute
Winter Model for Seasonal Factor	$V_1 = \frac{1}{N}\sum_{j=-2N+1}^{-N} D_j$ $V_2 = \frac{1}{N}\sum_{j=-N+1}^{0} D_j$ $G_0 = (V_2 - V_1)/N$ $S_0 = V_2 + G_0(N-1)/2$ $c_0 = \frac{c_{-N} + c_0}{2}$

Table 2-3: Table of computing initial value for different type of forecasting methods

2.1.3.9 Evaluating the forecasting method

To evaluate the accuracy of forecasting in period of *t*, e_t is defined as the difference between the forecasted value and the actual demand for that period.

$$e_t = F_t - D_t$$

Let $e_1, e_2, \ldots, e_n$ be the forecast errors observed over n periods, to measure the forecast accuracy during these n periods, three different methods exist. The mean absolute deviation (MAD), mean squared error (MSE), and mean absolute percentage error (MAPE) are the three common measures of forecast accuracy [34].

$$MAD = ({}^{1}/_{n}) \sum_{i=1}^{n} |e_i|$$

$$MSE = ({}^{1}/_{n}) \sum_{i=1}^{n} e_i^{\,2}$$

$$MAPE = \left[({}^{1}/_{n}) \sum_{i=1}^{n} \left| {}^{e_i}/_{D_i} \right| \right] * 100$$

To determine the reliability of the forecasting method we can use the sum of all the forecasting errors in order to evaluate if it is bias or not. Indeed, the best situation of forecasts is when they are unbiased which means $E(e_i) = 0$.

$$bias_n = \sum_{t=1}^{n} E_t$$

One way of tracking a forecasting method is to graph the values of the forecast error over the time. An Unbiased forecast error has a random fluctuation above and below zero. In addition, the straight line's slope should be zero. The tracking signal (TS) is the ratio of the bias and the MAD and it can be used to check the forecast result performance. [2]

$$TS_t = \frac{bias_t}{MAD_t}$$

If the TS is in the range of ± 6, it means the forecast result is biased. The result could be either under-forecasting $(TS < -6)$ or over-forecasting $(TS > +6)$. In this case, we may decide to change our forecasting method. In general, TS method checks forecasting constantly and detects underestimates demand.

2.1.3.10 Risk management in forecasting

The risk associated with the forecasting error has to be measured to plan for the future [2]. Under-forecasting or over-forecasting can cause significant problems in inventory, transportation, sourcing, pricing, advertising, and information management. Future plans are determined from forecasts, so the actual inventory, pricing, and advertisement plans depend on the accuracy of forecasting. On an operational level, forecasting plays a critical role in day-to-day activities such as scheduling the narrowcast advertisements.

2.2 Advertising based Inventory Control Strategies

If we treat advertising as a causal factor, forecasting demand under the presence of advertisement can be seen as a type of casual forecasting. Causal forecasting assumes that the demand is highly correlated with certain factors in the environment. In fact, in causal method forecast is a function of some other pieces other than historical data. For example, advertising, the unemployment rate, the weather, or anything besides the sales in this quarter. In this study we have just focused on adverting factor as a causal factor.

Nowadays all products in a store have to have acceptable quality to place on the store's shelf. Additionally, some group on people doesn't look for special brand and just pay attention to the economic matters of product [35]. They shop based on stores' flyers, or intelligent advertisement's methods such as narrowcasting screens in the stores. As a result, having an efficient marketing plan can play a critical role in supply chain management. In general, supply chain stores have various marketing strategies for promoting their products and services. These marketing plans are not only to attract the customers but to control the inventory sales.

2.2.1 Flyer advertising

Flyers and ad promotions are an important aspect of retailer activities. Usually large supply chain retailers spend about half of their promotional budgets on printing flyers [36]. The volume of flyers, the printed location of ads, and the promotion of the flyers may affect a flyer's effectiveness on a store performance [37]. Based on Els Gijsbrechts research in 2003 the main effect of traffic costumer depends on the type and size of the discount and specials on the first page [37]. In fact, many consumers only pay attention to the first page of flyers and many other promotions get lost in various pages of flyers. Another significant limitation of this traditional advertising channel is its time delay. Using only flyer promotion cannot be the only solution to control the inventory level, especially when products are perishable.

Often franchise supply chains such as Pizza Pizza or Tim-Hortons have seasonal promotions, and based on each season condition they print out flyers to send to local customers and inform them of their upcoming promotions. Retail stores such as IGA, Metro, and Wal-mart plan weekly or monthly flyers. Often these stores promote perishable products which cannot be kept in the inventory long term.

In today's highly competitive retail supply chains, promotion ads play an important role to attract customers to the store and affect their in-store spending. However, consumers do not always react to individual promotions, but rather to the store's overall promotional offer. Little research has been done on the effectiveness of store flyers, which is still one of the most important media to advertise retail promotions. One of the other significant limitations of this model is that flyers are effective whenever there is a noticeable price reduction on products. This model may help to reduce inventory costs, however, it will increase the planned sales and the markdown policy may decrease the final profit.

2.2.2 Narrowcasting screens on shopping Cart

Narrowcast ads are a method of broadcasting advertisement for a narrow range of customers. Screens on shopping cart are a good illustration of narrowcasting advertising [38]. Screens can predict shopping lists and combine knowledge-based techniques with statistical and learning algorithms to build individual consumer models that capture different aspects of shopping behavior. This model was used to offer individual promotions to customers. A proposed model used data mining to use a customer's purchase history to create an individual model to forecast a shopper's behaviors.

The goal of this method is to deliver promotions which have been received from product manufacturers or planned by a retailer. In addition, the goals become much wider when each screen is individualized for a consumer. For example, a manufacturer can categorize ads based on revenue levels, brand loyalty, or their market share with the manufacturer. This system allows the user to weigh each of these goals relative to each other and assign a weight to each individual goal within a group. These weightings can be used in an optimization procedure to map goal weights to the appropriate promotion parameters that maximize the results for each goal.

The shopping list also helps the system narrow down the list of products for the shopper, this being the case, the cart screen may give personalized promotions and discounts. The main limitation of this model is for large scale stores that have individual promotion; the large store must connect their screens to inventory management system and have real time communication with inventory which makes the complexity of this method [39]. Moreover, there was not direct connection between inventory level control and the promotion plans in this research.

2.2.3 Portable shopping device and E-grocery channels

E-grocery is a new method of grocery shopping through the internet using e-Commerce technology. In this area an E-grocery application has been proposed that introduces a handheld device for customers to make it possible for them to order or purchase goods when they are not in the store. Each customer had a device similar to a smart phone that was connected to a store's inventory, and he/she was able to order the necessary products through that device. One of the advantages of this device was its real-time response and its ability to promote a product by sending a message to the device.

This method was not efficient in that, it was not user friendly for all generations and worked in just certain areas like: stores that have young generation customers [40]. Indeed, this method becomes popular when e-Commerce websites proliferated. In addition, this device was able to work only in some area close to that store and it was not have services everywhere.

Other method of intelligent advertising similar to this model was proposed in 2006 which sent the ads via mobile communications [41]. The main goal of this project was to schedule the ads based on priority and item's need. Furthermore, this system was organized to schedule based on day, time slot, and shoppers' age and gender. Nonetheless, many customers ignored receiving this kind of message on their cell-phones and few others did not notice the message during their shopping.

2.2.4 Audio broadcasting in retail stores

New forms of broadcast media have emerged over the past decade. The radio/audio advertising method is new and exciting way to reach the target consumer. These ads compete with paper flyer consumer's attention [42]. Supermarkets and retail stores have been advertising on radio for over 50 years. The radio advertising-plans were similar for all stores in the past. Now, supermarket chains have developed an advanced technology that allows individual grocery stores to function as their own radio stations. National supermarket chains broadcast their own networks.

Retail audio networks are similar to traditional radio networks in that they offer a new source of advertising inventory. Based on Arbitron's market research [42] 25% of shoppers in the United States think that retail audio advertising would influence on their shopping decisions. As a result, this method of intelligent marketing can influence the final sales of the supply chain.

The main advantage of this model is its affiliation to each market individually. In addition, retail audio allows advertisers to reach consumers when they are ready to buy. In this model there are sensors beside the shelves and when any customer gets close to that area the radio starts to play. There is a very strong connection between purchase thought, and actual purchases and the time that shoppers spend at the grocery store [42]. As a result scheduling an adequate time slot to play the advertisement is important limitation of this model since only one radio ad can play in the store at each time. However, this problem is not issue in narrowcasting screens.

2.2.5 Narrowcasting screens

Retail stores and Quick-Service Restaurant (QSRs) turned their attention to a newly emerged advertising channel: narrowcasting media. For example, many Tim-Hortons stores in Canada and US have installed big screens which can be used to display their products. Narrowcasting is a business-centric network management solution which integrates media devices such as digital displays, media management systems (ranging from a single low-end computer to a grid of computers), a management and control software system for media content distribution, and high-bandwidth communications. Narrowcasting enables companies and organizations to target audiences in a wide range of settings with an unprecedented level of customization and timeliness [43]. CAP Ventures recent market research reported that the newly emerged narrowcasting technology is a key solution for companies and organizations to deliver promotional content and business messages more effectively compared to traditional approaches [44].

Based on a 2005 study by Williams et al. over one-third of Americans have watched narrowcasting videos that are broadcast in stores [18]. The study was on over 200,000 U.S. consumers aged 18+. Based on this report 30% of people, who notice the narrowcasting screens, had made an unplanned purchase. More market research had been done on the effectiveness of screens in the stores in 2009 [45]. Based on this research, when the screens were installed in-store and they asked customers about the screens, 58% of customers had noticed the screens. In addition, 30% of shoppers decided which brand they had to buy inside the product aisle [46]. The real-time in-store advertisement is useful when it is placed close to the product's in-store location [47].

A recent study in the Philippines addressing store displays and their impact on sales was done on powdered milk sales [35]. The study found a 20% increase on sales when the screens were installed close to the product. However, if the screen was placed along the aisle and island display the sale increase was 8%. As well, the combination of displays was also evaluated with four types of displays that generated a sales increase of 72%. If a three promotion combination was used the sale increased 56%, while a two promotion combination had a 40% sales increase. These numbers were calculated when it was in combination with a price reduction.

Scheduling plays an important role in manufacturing, real time advertising and production systems, as well as in information-processing environments. A scheduled advertisement plan can have significant effect on total sales as well as inventory management. The main target is to schedule advertisements on in-store screens based on the inventory level, and producer's promotions. Media narrowcasting screens in the store connect to the inventory via a Promotion Management System (PMS). The main function of the PMS is to check the inventory level and forecast the potential sales until end of the period. Based on the forecasted sales, and expected sales the system assigns a set time for advertising on the screen. From forecasted data the demand will be estimated for the end of the period to ensure the cost will be minimized, after considering advertising costs. One of the main advantages of PMS is using this method for perishable products to avoid extra spoilage cost.

The PMS is designed to make the shopping experience for a customer more attractive and efficient; on the other hand, it helps the marketplace reduce its inventory size, and increase its Return on Investment (ROI). There is evidence that the stores with promotion screens have greater sales, even for products that are not promoted on-screen compared to similar stores without screens [45]. People's reaction to a narrowcasting system can be used to dynamically select the correct sales price for product advertisement based on location. The main goal of this research is to reduce the same message delivery conflict for various advertising plans for each branding channels. As an illustration most retail stores and QSRs that use narrowcasting have flyer promotions that use the same advertising plan for both channels. Our objective is to separate the intelligent advertising plan from traditional channels to be most efficient.

Chapter 3. Narrowcasting-Based Inventory Control Model

Narrowcasting is emerging as a new type of media through which advertisements can be delivered to the targeted audience in a wide range of settings with an unprecedented level of customization and timeliness. Combining narrowcasting with traditional inventory control methods represents an interesting research direction with important practical implications. In this chapter, we propose an inventory control framework which integrates traditional forecasting methods with narrowcasting-based advertising. We design the framework in the context of news-vendor model. We will also provide case studies to demonstrate the application of the framework to a textile store setting.

3.1 News-Vendor model with In-store advertising

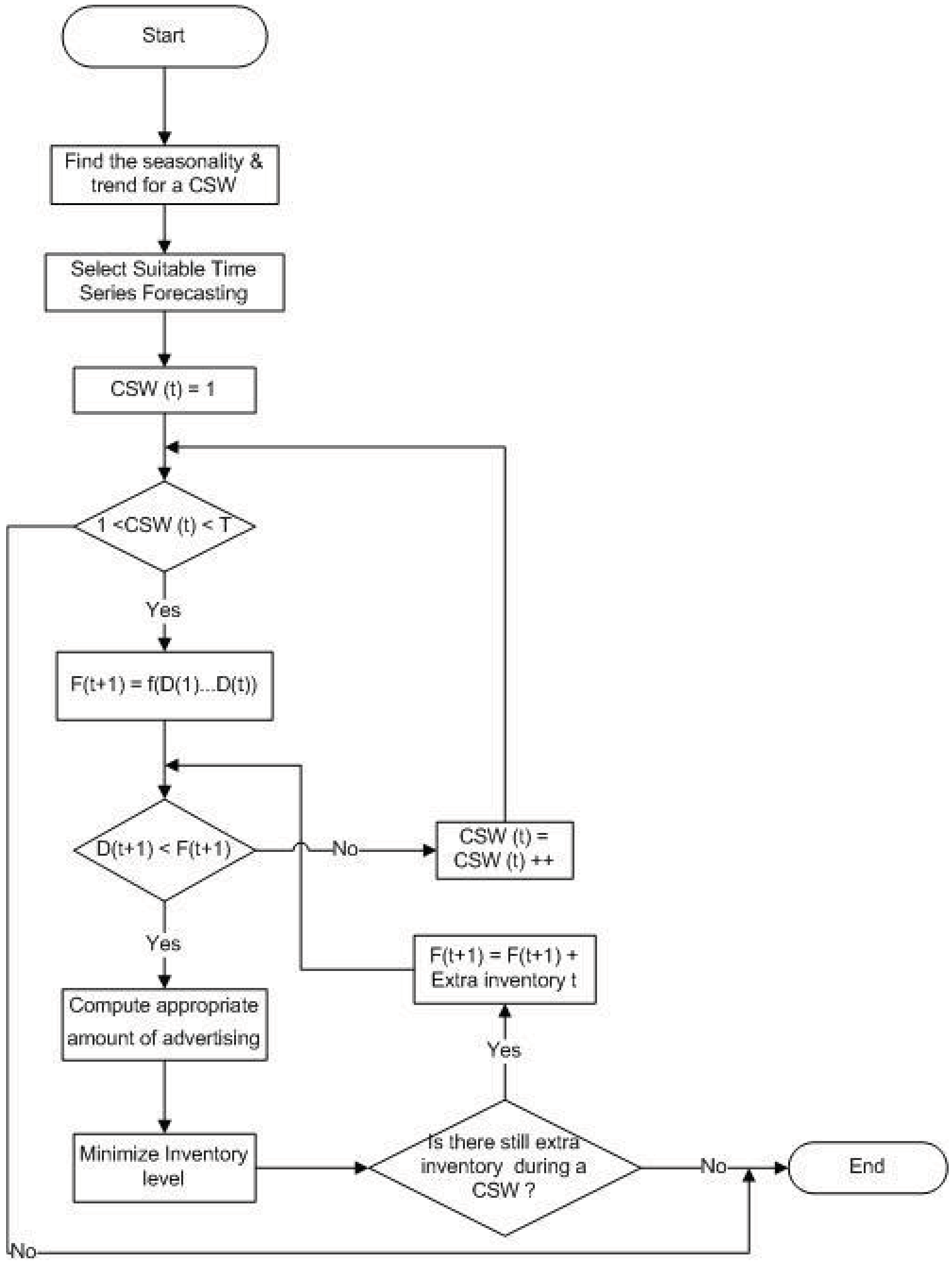
Newsboy or news-vendor problem is a classic single-period inventory management problem (SPP). The SPP model assumes that if any inventory remains at the end of the period, a discount is used to sell it or it is disposed [4]. If the order quantity is smaller than the realized demand, the news-vendor forgoes some profit. The goal is to find a product's order quantity that minimizes the expected spoilage cost under probabilistic demand. The newsvendor problem is a crucial building block of the stochastic inventory theory because of its simple and elegant structure as well as its rich managerial insights. The newsvendor model reflects many real situations and is often used to help make decision in many industries, such as fashion and sporting, both at the manufacturing and sale level.

An important aspect of the news vendor problem is the effect of advertising on sales. In most of the cases, advertising has positive impact on sales. It, therefore, should be considered as a causal factor when forecasting the demand for the next period. While there are not many well-documented cases of advertising response functions of any type [48], they are assumed to be either S-shaped or concave. In considering the S-shaped response function, increasing doses of advertising creates an increasing increment of sales up to the inflection point where the response pattern then changes [48]. On the other hand, the concave response function indicates that as advertising expenditures increase so do sales but at a diminishing rate from thebe ginning [48]. In other words, there is no inflection point. The debate about the shape continues [49] although there is evidence that the S-shaped function does not exist [50] or is so weak that it should not be used to justify advertising expenditures [51]. According to Jones's experiments on the short term advertising effect, the market share of the brand name that exposed to a group of customers within one week before buying will increase 11% if the customers are exposed to the advertisement once and an increment of 14% if customers are exposed twice to the advertisement [48].

In this thesis, we restrict ourselves to a specific extension of the classical news-vendor problem in which in-store advertising, narrowcasting in particular, is integrated into inventory control strategies. We assume that order quantity has been fixed at the beginning using some forecasting methods. We use in-store advertising to adjust the stochastic demand in a way that, at the end of the period, the unsold inventory is minimized. We use the market share increment rate presented in Jones's short term advertising experiment results to predict the responses of in-store advertising.

3.2 Narrowcasting-Based inventory control procedure

To predict the demand of a period of time, it is necessary to use historical data as the base of the forecasting, in the mean time, causal factors need also be considered. Causal factors such as advertising can actually generate demands. Unlike many causal factors, like weather or economics, a unique character that advertising has is it is controllable. In the context of news-vendor problem, if the demand of each day of the period is adjusted to the expected level, the end-period unsold inventory may be minimized. We present an inventory control procedure which combines time-series forecasting and advertising in a way that forecasted shortage of demand is compensated by putting in advertisement through in-store narrowcasting network. The procedure works in an iterative fashion. We first divide the selling period into equal *customer shopping windows* (CSW). A CSW is the average length of time that regular customers stay in the store. As shown in Figure 3-1, at the beginning of the selling period, we first break down the whole inventory for the selling period into chunks for each CSW based on the demand trend and seasonality during the period derived from historical data. That is, if we have T CSWs in the selling period, calculate the planned demands for each CSW, denoted as $\overline{D}(1), \overline{D}(2), \ldots, \overline{D}(T)$. When the selling period starts, we use the demand in previous CSWs to forecast the future demands. Suppose we are currently at CSW t. We use the demands from CSW 1 to t that is $D(1), D(2), \ldots, D(t)$ to predict $D(t+1)$. If $D(t+1) < \overline{D}(t+1)$, an extra inventory for $t+1$ is likely to be resulted. We then try to reduce the extra inventory by using appropriate amount of advertising to increase $D(t+1)$. Extra inventory unresolved during a CSW will be carried over to next CSW.



3-1: Flowchart for optimizing inventory model

To ensure the effectiveness of this procedure, we need to accurately forecast the demand for a CSW. If an extra inventory resulted, we need to compute appropriate dose of advertisement in order to reduce the extra inventory as much as possible. We describe these two important components in the next two sections of this Chapter.

3.3 Inventory forecasting

In the narrowcasting based inventory control model, demand forecasting for the next CSW is needed. We apply a time series method to forecast the demand of a CSW based on historical data. We assume that there is no seasonality or trend in demand. According to the forecasting method selection procedure depicted in Figure 2-4, we choose the single exponential smoothing as the forecasting method for the time being. Note that other methods could be selected if the demand demonstrates trend and seasonality.

The forecasting is a short term forecasting on a CSW bases. Based on the suggestions in [26], the constant of α is configured between the range of 0.01 and 0.3 such that we give more weight to our previous forecast rather than last observed demand. To determine the smoothing constant α, we apply three values 0.01, 0.3, and $\frac{0.01+0.3}{2}$ as α to historical data. We select the best two of them and use the average between those two as our new α and continue this procedure until the differences between the best two is less than a predefined threshold. Next, we determine the initial forecast which is computed from the following equation for exponential smoothing method:

$$\mathrm{F_0} = \frac{1}{\mathrm{n}}\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{D_i}$$

Forecasts for any other CSW t is computed using the following equation:

$$F_t = \alpha D_{t-1} + (1-\alpha)F_{t-1}$$

At the end of each CSW, we forecast the demand for the next CSW. By comparing the planned sale and the forecasting, we find the extra inventory we need to deal with through advertising. We advertise to increase our sales for the next CSW taking consideration of the extra inventory needed to be got rid of.

3.4 Predicting demand generated by narrowcasting advertisement

One of the most frequent questions about the advertising is, “How effective would advertising be?” The first experiential generalizations of advertising effectiveness was derived by Clarke (1976), who showed 90% impact of advertising on sales which occurred within months of the advertisement [52]. Assmuss et al. (1984) analyzed 128 studies assessing the impact of advertising on sales. They found the short-run advertising elasticity to have a grand mean of 0.221 [53]. Whereas an individual marketing manager might settle for an answer specific for his/her own products, potential market with regulation of advertising and its effect on competitors. Advertising usually increases sales when it influences consumer purchasing behavior [48]. Several factors have contributed to the growing popularity of in-store narrowcasting screens. The main advantage is when time is short, when advertising should occur close to the product location to avoid the confusion [54]. Once inside the store people are shopping the best place to impact the consumer is when the consumer is in the aisle reaching for the product [55].

3.4.1 Probability of narrowcasting advertisement exposure

Based on Jones' study [48], we can infer that the possibility that a customer will buy a specific brand name increases 11% if she is exposed to the ad once and 14% if twice during a short period before buying. Once we know the number of customers in the store during a specific CSW, the increment of sale depends on the probability that a customer see the ads. If we know how many customers in store will see the advertisement once and how many of them will see two more times, based on Jones response rate, we can easily calculate the sale increment generated by advertising. To accomplish this, we start with computing the probability that a customer sees an advertisement once or two and more times. We define the following notations first:

m	Number of screens in the store
w	Number of time units in a customer shopping window (CSW)
i	Index of screens
j	Index of the advertisement for a product
$c_{i,j}$	Coverage of advertisement j on screen i (the number of time units assigned to product j on screen i)
a_i	Possibility of a customer seeing screen i during a CSW
$p\left(Ad_j^0\right)$	Possibility of ad j not being seen by a customer during her CSW
$p\left(Ad_j^1\right)$	Possibility of ad j being seen once by a customer during her CSW

$p\left(Ad_j^{2+}\right)$ Possibility of ad j being seen twice or more by a customer during her CSW

We also assume that

- A customer does not revisit a location in the store.
- A customer who passes the location where a screen is installed will pay attention to the screen.
- A customer will not watch more than one advertisement when she passes a screen.
- A costumer who had an unplanned purchase due to seeing the screens' ads, purchases that item once.

Using the defined notation the probabilities can be expressed as the following formula.

$$p\left(Ad_j^0\right) = \prod_{i=1}^{m}\left(1 - \frac{a_i c_{i,j}}{w}\right)$$

$$p\left(Ad_j^1\right) = \sum_{i=1}^{m}\left[\left(\prod_{k=1}^{i-1}\left(1 - \frac{a_k c_{k,j}}{w}\right)\right) * \left(a_i * \frac{c_{i,j}}{w}\right) * \left(\prod_{l=i+1}^{m}\left(1 - \frac{a_l c_{l,j}}{w}\right)\right)\right]$$

$$p\left(Ad_j^{2+}\right) = 1 - p\left(Ad_j^0\right) - p\left(Ad_j^1\right)$$

It is clear that for each customer who walked in the store during the CSW three situations will happen in term of seeing advertisement j on screen i. First, the advertisement j is not being seen by the customer during her CSW $p\left(Ad_j^0\right)$. Second, the ad j is being seen once $p\left(Ad_j^1\right)$. Third, it is being seen twice or more by the customer during CSW $p(Ad_j^{2+})$. Simply we know $p(Ad_j^0) + p(Ad_j^1) + p(Ad_j^{2+}) = 1$ and $p(Ad_j^0 \cap Ad_j^1 \cap Ad_j^{2+}) = 0$.

Suppose that a_i is the probability of viewing the i^{th} screen by a customer and $c_{i,j}$ is the probability seeing the number(s) of time units assigned to product *j* on screen *i*. $c_{i,j}$ and a_i are independent; therefore, the probability of seeing the advertisement in one time unit of CSW is equal to $\frac{a_i c_{i,j}}{w}$ and the possibility of not seeing that advertisement is $\left(1 - \frac{a_i c_{i,j}}{w}\right)$.

Given $p(Ad_j^1)$ and $p(Ad_j^{2+})$, the demand generated for a product j by an adverting schedule with a specific coverage of product *j* on different screens when *n* is number of customers visiting the store during a CSW, it can be calculated by the following formula:

$$Generated\ Demand = n * p(Ad_j^1) * 11\% + n * p(Ad_j^{2+}) * 14\%$$

3.4.2 Expected exposure time for a narrowcasting advertisement

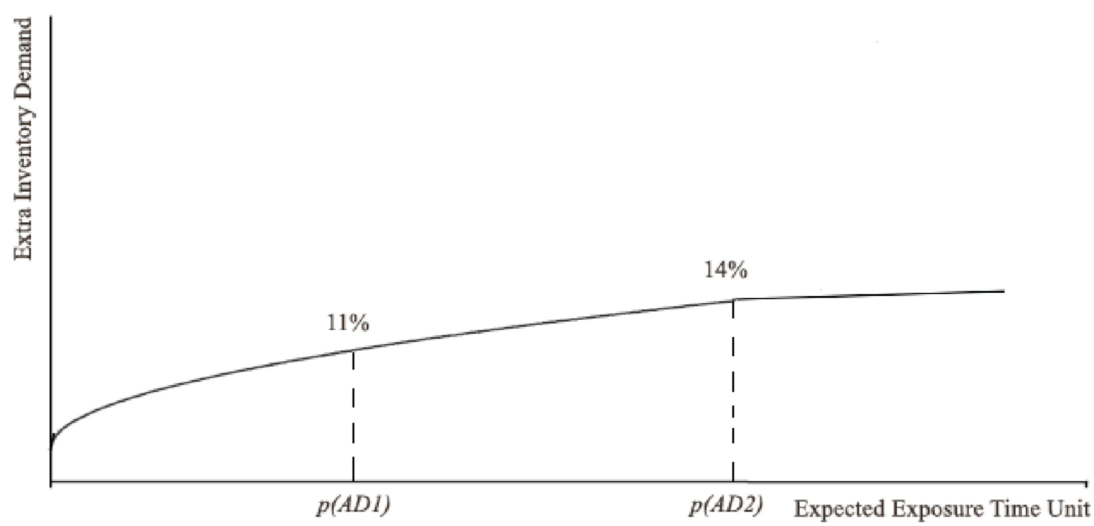
Instead of calculating the possibility of seeing an ad once or twice, we can also predict the demand generated by advertising using the concept of *expected exposure time*. For an advertisement, the expected exposure time EE_j is defined using the following formula:

$$EE_j = \sum_{i=1}^{m} \frac{a_j c_{ij}}{w}$$

Since demand is a random variable which depends on advertising expenditure [48], we can simply assume the expected sales is a function of expected exposure of a narrowcasting advertisement on screens. Based on the construction of the response function in [56], we calculate the sales generated by an advertisement using the following formula:

$$Generated\ Sales = n * (\omega EE_j{}^{\alpha})$$

where ω and α are determined as constants which indicate the effectiveness of advertising, $0 \leq \alpha \leq 1$. When $\omega = 0$, it indicates that demand is independent of advertisement and for all $\omega > 0$, the larger the value of α, the more effective is the advertising. We pick the values of ω and α in a way that the function matches Jones' results. Figure 3-2 shows the response function. Based on this figure at first exposure the sales increase 11% and on second exposure it becomes 14%. For more than two exposures we will have still increase but the slop of this increase drops and we can consider as constant.



3-2: Expected demand as a function of advertising

3.5 A Pizza dough Case Study - Perishable product with seasonality

We presented real data for a perishable product which have been gotten from "Pizza Pizza" franchise store in Montreal. This data is for sales of their medium pizza from January 17th 2011 to February 13th 2011. We built our case study on pizza dough which is one the perishable items in their inventory.

The restaurant, "Pizza Pizza", is one the biggest Supply chain franchises in Canada with over 600 stores since 1967. This franchise currently has 8 stores in Montreal and they are all using the narrowcasting advertising method in parallel with their traditional monthly flyers. Each store has at least one narrowcasting screen installed in the store beside the cash register. However, this screen schedule is from one center in Ottawa, and it is based on a monthly advertisement plan. We did a case study on one of the Pizza Pizza stores in Montreal regarding their inventory and tried to find out the result of when this narrowcasting screen connects to the store's inventory. All data in this case are real data which was obtained from this store from January 17th, 2011 to February 13th, 2011. We asked them to provide us the first three weeks of data and based on those three weeks we built our model. At the end we asked them for 4th week's sales to compare with our results.

One of the most significant reasons of Pizza Pizza's success is using the fresh pizza dough. Different sizes of pizza dough are prepared in special trays and kept in the fridge at a special temperature for no more than 7 days. The main constraint is pizza dough takes half of the store's fridge and there is no option to keep it outside or in the freezers.

In this study there is only one screen in the store which runs 10 hours. Ad length is around 10 seconds and we define one hour time-window and repeat it for 10 hours. The following figures were reported to us. There is a periodic inventory policy, with r = 7 days (review period), Thus, we will have the following table for medium pizza sales:

	r_1	r_2	r_3
Friday	85	104	89
Saturday	67	60	75
Sunday	55	59	63
Monday	38	27	31
Tuesday	44	47	40
Wednesday	38	60	41
Thursday	68	60	45

Table 3-1: Demand for medium pizza (Read down from left)

The average weekly quantity is 15 piles of dough (each pile includes 30 dough units) which are equal to 450 pizza dough. The actual pizza demand for every day is variable based on historical data, but the average of these variable demands is approximately 58 units.

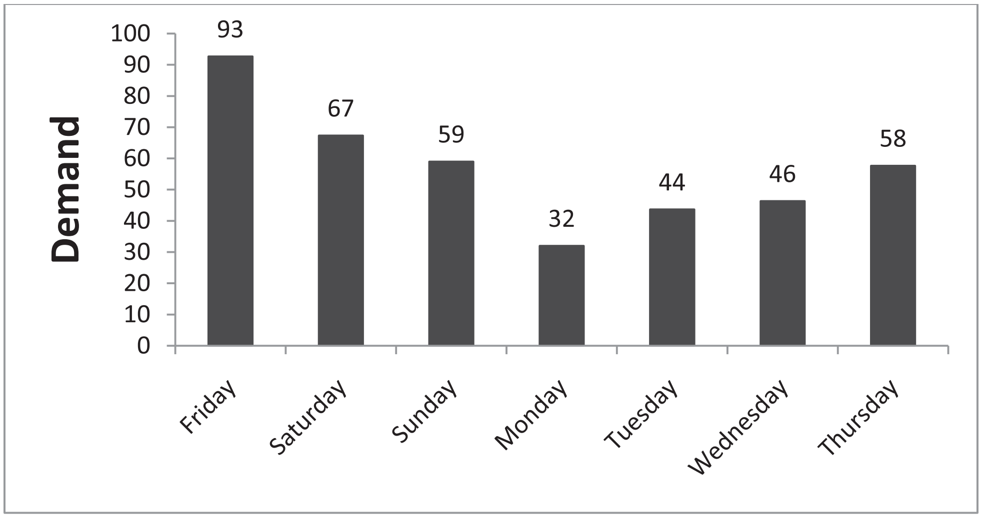


Figure 3-3: The average daily demand for medium pizza based on historical data

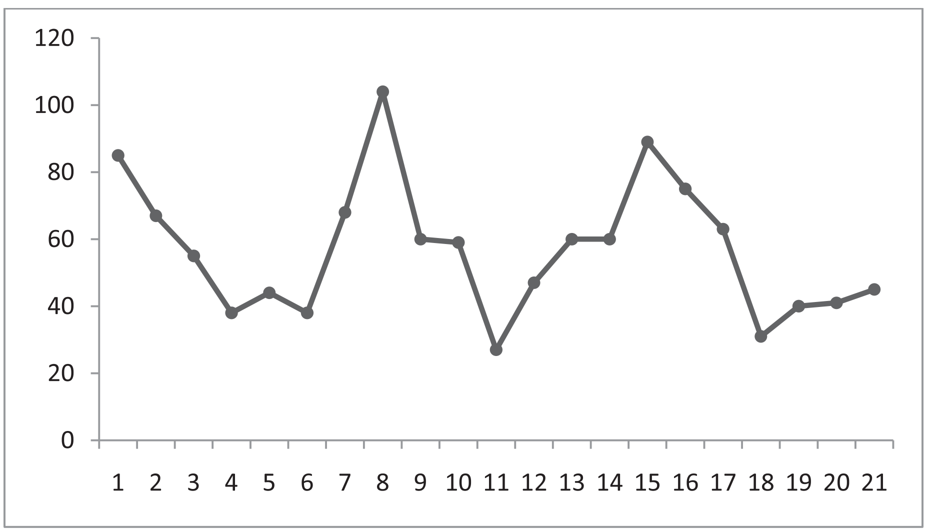


Figure 3-4: The inventory demand for last three weeks of pizza dough

Figure 3-4 proof there is pattern weekly trend for the current data. When we studied the data we have observed there is an unusual fluctuation on week two of the observation. As a consequence, we used the subjective forecasting beside the objective method and asked the store manager. He admitted the data at this period of time is stationary until the end of March. After this period there will be a decreasing trend which will be followed by increasing trend from September. For week two he mentioned the super bowl which is once a year effect on data and that week is one of the exceptions.

To do the forecasting, we selected seasonal factors method; first thing in this method is computing the average of all observation and divide each observation by this mean value. The mean value of all data is 56.95

	r_1	r_2	r_3
Friday	1.49	1.83	1.56
Saturday	1.18	1.05	1.32
Sunday	0.97	1.04	1.11
Monday	0.67	0.47	0.54
Tuesday	0.77	0.83	0.70
Wednesday	0.67	1.05	0.72
Thursday	1.19	1.05	0.79

Table 3-2: Three periods moving average for pizza dough example

Next, the average factors which corresponding to the same day of the season will be calculated. For example, the average of all factors for Mondays, and so on. As it is obvious on table 3-3 on Friday there is 63 percent sales increase; however, Thursday is an average day because the average for this day is very close to 1 and so on. Forecasting for number of medium pizza on any day of the week would be obtained by multiplying the sample mean of 56.95 by the appropriate seasonal factor. Therefore, the forecasting result would be:

	Factor	X_f
Friday	1.63	93
Saturday	1.18	67
Sunday	1.04	59
Monday	0.56	32
Tuesday	0.77	44
Wednesday	0.81	46
Thursday	1.01	58

Table 3-3: Three period moving average forecasts for pizza dough example

Based on table 3-4 the day one the forecasted sales are higher than average sales which shows there is no need to advertise. At the end of day one the real sales was 95 which was 2 pizzas even more than our forecasted result. It is shown on table 3-4 there is no need to do advertising for next day for medium pizza.

Real Planned Sales	Forecast result	Extra Demand	Forecast + Ads	Real Data + Ads	Forecasted - Real Demand
95	93	0	93	95	-2
	67	0	67		
	59				
	32				
	44				
	46				
	58				

Table 3-4: The forecasting result at the end of day 1

Real Planned Sales	Forecast result	Extra Demand	Forecast + Ads	Real Data + Ads	Forecasted - Real Demand
95	93	0	93	95	-2
65	67	0	67	65	2
	59	0	59		
	32				
	44				
	46				
	58				

Table 3-5: The forecasting result at the end of day 2

Real Planned Sales	Forecast result	Extra Demand	Forecast + Ads	Real Data + Ads	Forecasted - Real Demand
95	93	0	93	95	-2
65	67	2	67	65	2
49	59	10	59	49	10
	32				
	44				
	46				
	58				

Table 3-6: The forecasting result at the end of day 3

At the end of day 3 we can observe the forecasted result was not accurate and our sales dropped. It means we need to advertise the current item for next day to increase the sales for 10 items. It means for per hour it is necessary to sell one medium pizza more. In addition, if we assumed per hour 20 people walk in to the store. We need to find the probability of buying one pizza in an hour by looking at the narrowcasting advertisement.

$$p\left(Ad_j^0\right) = (1 - c_1)$$

$$p\left(Ad_j^1\right) = (c_1)$$

$$p\left(Ad_j^{2+}\right) = 1 - (c_1) - (1 - c_1) = 0$$

$$Generated\ Demand = n * p\left(Ad_j^1\right) * 11\% + n * p\left(Ad_j^{2+}\right) * 14\%$$

$$59 - 49 = 20 * p\left(Ad_j^1\right) * 11\% + 20 * p\left(Ad_j^{2+}\right) * 14\%$$

$$0.5 = p\left(Ad_j^1\right) * 0.11 + p\left(Ad_j^{2+}\right) 0.14 = p\left(Ad_j^1\right) * 0.11 \Rightarrow c_1 = 4.5 \cong 4$$

Based on frequency of ads if the ad is shown 4 times per hour we will expect to sell 10 pizzas extra for next day.

Real Planned Sales	Forecast result	Extra Demand	Forecast + Ads	Real Data + Ads	Forecasted - Real Demand
95	93	0	93	95	-2
65	67	2	67	65	2
49	59	10	69	59	10
30	32	2	44	42	2
	44				
	46				
	58				

Table 3-7: The forecasting result at the end of day 4

At the end of fourth day 2 items sold less which can schedule screens for 20% of day four to increase the sales for two more pizza. It means approximately it needs to show the ad once per hour.

Real Planned Sales	Forecast result	Extra Demand	Forecast + Ads	Real Data + Ads	Forecasted - Real Demand
95	93	0	93	95	-2
65	67	2	67	65	2
49	59	10	69	49	10
30	32	2	44	42	2
44	44	0	46	46	0
	46				
	58				

Table 3-8: The forecasting result at the end of day 5

Real Planned Sales	Forecast result	Extra Demand	Forecast + Ads	Real Data + Ads	Forecasted - Real Demand
95	93	0	93	95	-2
65	67	2	67	65	2
49	59	10	59	49	10
30	32	2	44	42	2
44	44	0	46	46	0
50	46	0	46	50	0
	58				

Table 3-9: The forecasting result at the end of day 6

Real Planned Sales	Forecast result	Extra Demand	Forecast + Ads	Real Data + Ads	Forecasted - Real Demand
95	93	0	93	95	-2
65	67	2	67	65	2
49	59	10	59	49	10
30	32	2	44	42	2
44	44	0	46	46	0
50	46	0	46	50	-4
45	58	0	58	45	13

Table 3-10: The forecasting result at the end of day 7

At the end of the period, Day 7, we can observe the summary of forecasting result and real data. As it is showing on table 3-10 at the end of the period $13 - 4 = 9$ units of pizza did not sell. The advertising helped to sell 12 items more which optimizes our sales by 49%. If we compute the *TS* error for validation of our forecasting result we will have:

Real Planned Sales - W6	Forecast result	E_t	$\lvert e_i \rvert$
95	93	2	2
65	67	-2	2
49	59	10	10
30	32	0	0
44	44	0	0
50	46	-4	4
45	58	13	13
378	**399**	**21**	**33**

Table 3-11: The error result of medium pizza dough forecasting

$$\mathrm{MAD} = \left({}^{1}/_{\mathrm{n}}\right) \sum_{\mathrm{i}=1}^{\mathrm{n}} \lvert \mathrm{e_i} \rvert = \left({}^{33}/_{7}\right) = 4.71$$

$$Bias_n = \sum_{t=1}^{n} E_t = 21$$

$$TS_t = \frac{21}{4.71} \cong 4.45$$

While the $TS = 4.45$, it is in the range of ± 6, which is a signal that forecast is acceptable.

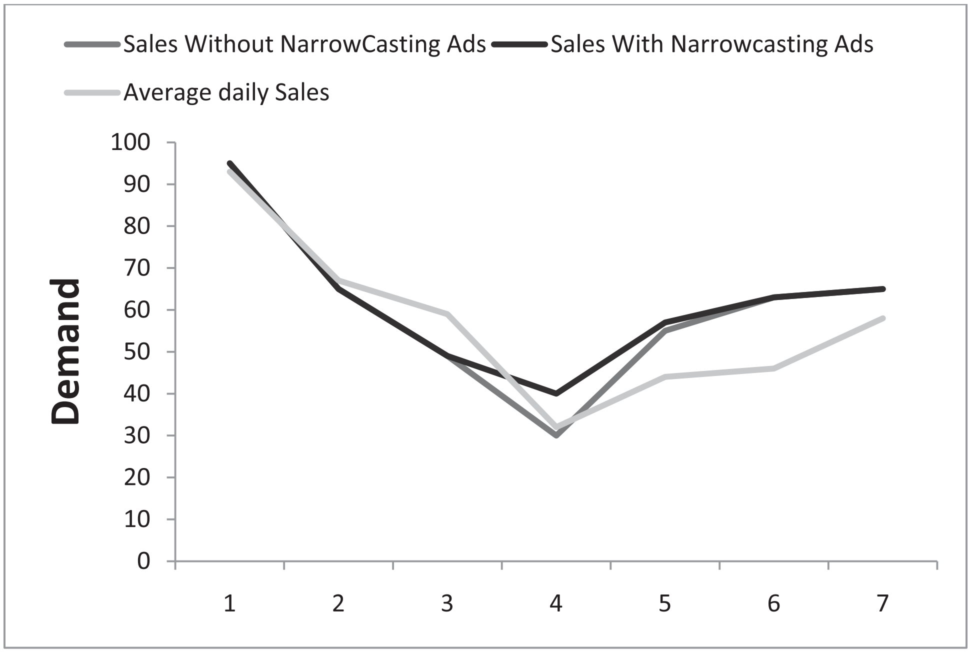


Figure 3-5: The daily demand for medium pizza in three situations

Chapter 4. Promotion Scheduling in Narrowcasting-Based Inventory Control

The full benefits of the narrowcasting networks cannot be realized if they are not integrated with companies' core business processes. Existing narrowcasting scheduling solutions focus primarily on the distribution of media contents for predefined promotions. Given a set of ads that needs to be played during a period of time, an important task which makes the promotion effective is to find out when and on which screens those ads should be played. In this chapter we develop a narrowcasting promotion scheduling system that allocates screen times to advertisements in a way that maximal advertising effectiveness is achieved. The promotion scheduling model proposed in this chapter is an integral component of the narrowcasting based inventory control framework presented in Chapter 3.

4.1 The promotion scheduling problem

A narrowcasting promotion schedule can be driven by many factors in a supply chain. For example, in the case of a franchise QSRs' store, the store needs to promote its products based on freshness levels, expiration times, and/or other parameters of its inventory; the corporate (franchisor) needs to promote, at strategic level, for new products, brand names etc; a vendor of a product or a category of products needs to promote his/her own brand name or products. In this research, we restrict ourselves to the extension of news-vendor model in which a promotion schedule is driven by the product expiration costs.

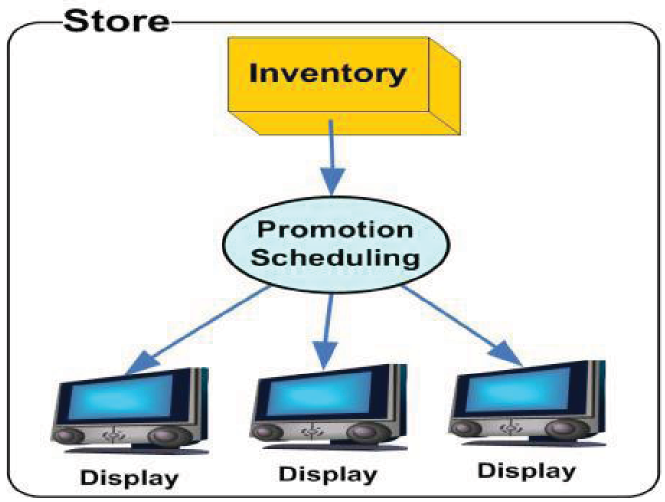


Figure 4-1: Store level promotion scheduling environment

As shown in Figure 4-1, the promotion scheduling system consists of an inventory management component, narrowcasting displays, and a scheduling component. Taken the store inventory situations during each CSW as inputs, the task of the scheduling component is to allocate screen times to the advertisements of a group of products which need to be promoted such that the end period expiration costs are minimized. In addition to the notations defined for the narrowcasting-based inventory control model in Chapter 3, section 3.4.1, we need the following additional notations to define the promotion scheduling problem.

u_j Unit expiration cost of product j. This is the cost incurred when a product is not sold and has to be disposed.

e_j Forecasted extra inventory for product j during a CSW (calculated by forecasting algorithms)

q Number of products to be advertised

n Number of customers visiting the store during a CSW

Since the demand generated by an advertising schedule within a CSW can be calculated by $n * p\left(Ad_j^1\right) * 11\% - n * p\left(Ad_j^{2+}\right) * 14\%$ and the objective of promotion scheduling is to minimize the end period expiration costs across all products, the objective function of the promotion scheduling can be expressed by the following formula:

$$\min \sum_{j=1}^{q} u_j * \left[e_j - n * p\left(Ad_j^1\right) * 11\% - n * p\left(Ad_j^{2+}\right) * 14\%\right]$$

Three set of constraints apply. First, for each screen the allocated promotion time units cannot exceed the CSW window w, that is $\sum_{j=1}^{q} c_{i,j} \leq w$, for $i = 1 \ldots m$. In addition, the demand generated by an advertising schedule should not greater than the extra inventory, that is $e_j - N * p\left(Ad_j^1\right) * 11\% - N * p\left(Ad_j^{2+}\right) * 14\% \geq 0$, for $j = 1 \ldots q$. Also, for $i = 1 \ldots m$ and $j = 1 \ldots q$, the coverage $c_{i,j}$ should be a positive integer.

If we use expected exposure as input, using the advertising response function proposed in Chapter 3, the demand generated by an advertising schedule within a CSW can be calculated by $n * 0.11 * {EE_j}^{0.35}$ and the objective of promotion scheduling is to minimize the end period expiration costs across all products, the objective function of the promotion scheduling can be expressed by the following formula:

$$EE_j = \sum_{i=1}^{m} \frac{a_i * c_{i,j}}{w}$$

$$\min \sum_{j=1}^{q} u_j * \left[e_j - \left(n * 0.11 * {EE_j}^{0.35}\right)\right]$$

Three set of constraints apply which the first two are exactly same as district model. Each allocated promotion time units cannot exceed the CSW window w. In addition, the demand generated by an advertising schedule should not greater than the extra inventory, that is $e_j - (n * 0.11 * {EE_j}^{0.35})$, for $j = 1 \dots q$. Also, for $i = 1 \dots m$ and $j = 1 \dots q$, the coverage $c_{i,j}$ should be a positive integer.

In the above integer programming models for the promotion scheduling problem, the decision variable is the coverage $c_{i,j}$. Once $c_{i,j}$ for all $i = 1 \dots m$ and $j = 1 \dots q$ are decided, the demand generated by the schedules that implement the set of $c_{i,j}$ is determined. In fact, a specific set of $c_{i,j}$ can be implemented by many different schedules. Based on our assumptions on advertising response function, those schedules are indifferent as long as they provide the same expected exposure during a CSW.

4.2 The ILOG OPL model

We decided to solve the optimization problem using the commercial optimization package ILOG CPLEX. The following is the model description using ILOG's

Optimization Programming Language (OPL). This OPL model can be used as the input of ILOG CPLEX optimization engine.

```
// Input variables
int m=...;                    // #of screens
int q=...;                    // #of products
int n=...;                    // #of customers
int w=...;                    // #of time units is CSW

int u[1..q]=...;              // unit expiration cost
float a[1..m]=...;                  // probability of seeing each
screen
int exInv[1..q]=...;     // extra inventory for each unit



dvar int c[i in 1..m][j in 1..q];

dexpr float EE[j in 1..q]= sum(i in 1..m)(a[i]*c[i][j]/w);


minimize sum(j in 1..q)(u[j]*(exInv[j]-(n*0.11*EE[j]^0.35)));
constraints
{
      forall(i in 1..m)sum(j in 1..q)(c[i][j])<=w;

      forall(j in 1..q)(exInv[j]-(n*0.11*EE[j]^0.35))>=0;
      forall(j in 1..q, i in 1..m)(c[i][j])>=0;
}

execute output
{
      var out1 = new IloOplOutputFile("C://Documents and
Settings//Administrator//opl//Narrowcasting//Narrowcasting-V2//Result-
2.txt")
      for(var i=1; i<=m; i++)
      {
            for(var j=1; j<=q; j++)
            {
                  out1.write(" "+c[i][j]);
            }
            out1.write("\n");
      }
}
```

4.3 Worked example with sufficient screen time

We used the data set from forecasting and time series analysis text book which is for nonperishable products [26]. The last 5 week demand for a product is given in table 4-1 and week 6 is forecasted based on last five weeks.

	Week 1	Week 2	Week 3	Week 4	Week 5
Mon	656	577	537	603	641
Tue	659	549	640	497	632
Wed	601	624	531	600	644
Thu	624	521	639	561	677
Fri	545	520	600	556	574
Sat	502	594	617	505	624
Sun	565	620	636	704	629

Table 4-1 : Demand for outfit fashion product

The extra demand based on forecasting result is showing on table 4-2. Based on this study usually the extra demand is equal to forecasting difference and average sales plus the unsold items from previous day.

Forecasting	Average Sales	Real Planned Sales -W6	Extra Demand	Narrowcasting Time slot $c_{i,j}$	Extra Demand Items' Sold	Carryon Difference
582	595	580	13	104	13	0+2
572	595	57	23+2	696	25	0-4
573	595	567	22-4	224	18	0+6
569	595	561	26+6	1688	32	0+8
568	595	592	27+8	2176	35	0-24
572	595	581	23-24	-	-	0-1-9
573	595	574	22-10	88	12	0-1

Table 4-2: Week 6th forecasting for outfit fashion product

As product is not perishable the unit expiation cost for outfit fashion product is considered as its holding cost. The cumulative holding cost for per unit is a function of time in table 4-3.

Day	Cumulative holding cost (Cent)
1	3
2	6
3	9
4	12
5	15
6	18
7	21

Table 4-3 : Cumulative holding cost for outfit fashion

We used Nielsen Media Research source for one week average of in-store audience retailer "X" in September 2007 and screens' impressions on Female 25-54 which is over half of the shoppers [35]. Bases on this data the retailer store divided to different zones and percentage of female who pass each zone is calculated. In this study this percentage is known as value of a_i.

Retail Store Zone	**Female 25-54 who pass Aisle**
Lobby Zone	**100 %**
Runway – Rear Wall	**75 %**
Runway – Front Wall	**63%**
Dairy Zone	58 %
Meat & Poultry Zone	44 %
Fashion clothing Zone	**42 %**
Frozen food Zone	32 %
Bakery Zone	23 %
Pharmacy Zone	9 %

Table 4-4: Percentage of people passes different sections

We assume each zone has one screen and per product can be advertised in its own zone, lobby zone, rear wall, and front wall. As a result the Outfit fashion product ad can show on global screens (Lobby, Runway Rear, and Runway Front) and house product zone. Furthermore, we assumed hourly 50 people come to the store and average spending shopping time for each customer is one hour. Retail store is open from 10 AM to 6 PM and screens are on all day long and each ad's length is 15 seconds, which means our $w = 240$.

Extra Demand / Per Hour	Lobby Zone	Runway – Rear Wall	Runway – Front Wall	Fashion clothing Zone	Sales Increase/ Per Hour	Carryon Difference
13/8	1	4	0	8	13/8	0
25/8	17	4	0	66	25/8	0
18/8	9	0	8	11	18/8	0
32/8	4	0	27	180	32/8	0
35/8	1	29	2	240	35/8	0
0	0	0	0	0	0	0
12/8	1	2	0	8	12/8	0

Table 4-5: Per hour ad coverage for each screen

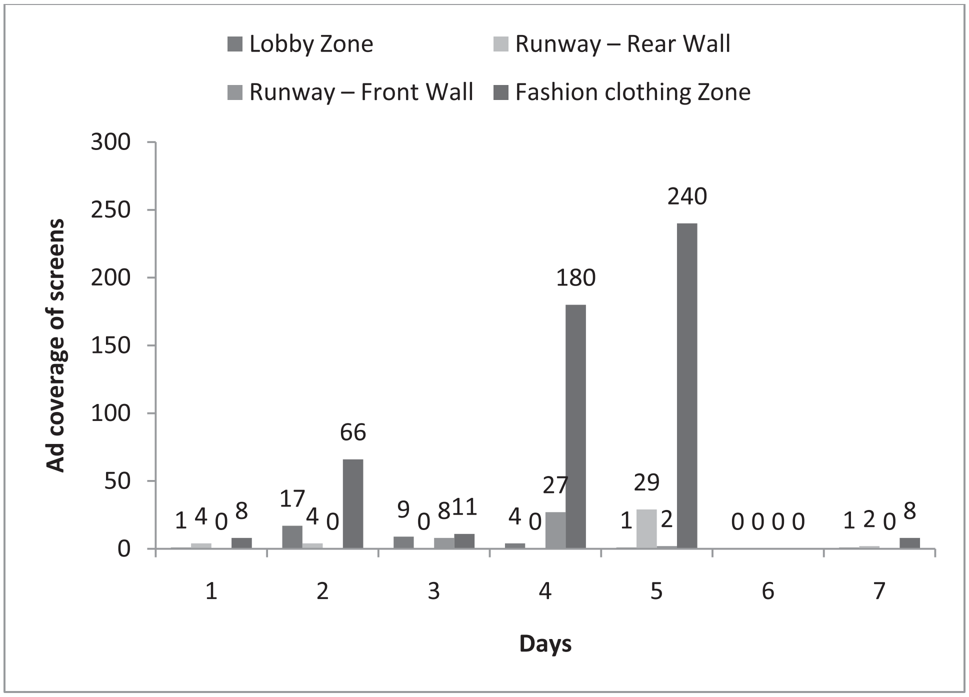


Figure 4-2 : Ads Distribution for each hour on each screen

4.4 Worked example with insufficient screen time

Since the hours we have for each time slot is limited, sometimes it happen that all necessary advertisements are not scheduled for a product. This case usually happens when there is more than one product which needs to advertised. In this case products will bid for each time slot base on extra inventory value and the expiration date. Table 4-5 shows the extra inventory for 3 different products which belong to one category and they promote on three similar screens. In addition, it is assumed 200 people pass the store hourly, and store is open for 10 hours. The average time spent in store is one hour and we schedule screens for one hour. There are two screens which running 20 minutes every hour which is 120 slots in per hour available time for advertisement. Screen one is seen by 100% of customers and screen 2 is seen by 63% of customers.

	Product A	Product B	Product C
Extra Inventory D1	10	48	27
Extra Inventory D2	8	6	33
Extra Inventory D3	4	6	5
Extra Inventory D4	47	39	17

Table 4-6: Extra inventory for 4 products in four days

	Product A	Product B	Product C
Expiration Cost D1	3	1	1
Expiration Cost D2	6	12	2
Expiration Cost D3	9	28	8
Expiration Cost D4	21	50	25

Table 4-7: Expiration Cost (holding cost + spoilage cost) for 4 products in four days

	Screen 1	Screen 2	Sales increase	Carryon Difference
Product A	0	20	10	0
Product B	71	32	20	28
Product C	49	68	20	7

Table 4-8: Narrowcasting result at the end of day one

At the end on day one we can observe the total available time slot is not enough to promote all three products. The observed result is the optimal result to reduce the expiration cost. The remaining unsold products added to next day extra inventory and the narrowcasting result for second day is shown on table 4-9.

	Screen 1	Screen 2	Sales increase	Carryon Difference
Product A	6	1	8	0
Product B	114	101	25	9
Product C	0	18	10	30

Table 4-9: Narrowcasting result at the end of day two

	Screen 1	Screen 2	Sales increase	Carryon Difference
Product A	0	1	3	1
Product B	3	59	15	0
Product C	117	60	24	11

Table 4-10: Narrowcasting result at the end of day three

	Screen 1	Screen 2	Sales increase	Carryon Difference
Product A	27	8	14	34
Product B	53	109	22	17
Product C	40	3	19	9

Table 4-11: Narrowcasting result at the end of day four

As we can see at the end of each day some of the products were not promoted as expected because of Insufficient screen time.

Chapter 5. Experimental results - Perishable product

We based this case study on real data for perishable products which we collected from "Pizza Pizza" franchise store in Montreal. This data is for sales of their pizza slices from July 15th 2011 to July 21th 2011. Based on these historical data we forecast July 22th and schedule advertisement for each slice. Their policy is to divide their 14 hours working day to three periods. First period is from 11 AM to 3 PM, second period is from 3 PM to 8 PM, and last period is from 8 PM to 1 AM. At the end of each period the remaining pizzas on the shelf are thrown away and fresh pizza is cooked. Based on their summer schedules for each type they schedule defined number of slices to cook. Table 5-1 shows the number of slices they cook for each type of pizza during one day.

	Pepperoni	Cheese	Vegetarian	All Dress	Bacon
11 AM	8	10	5	5	2
3 PM	6	7	5	4	2
8 PM	3	3	2	2	2

Table 5-1: Number of pizza slices cook for each type of pizza during one day

This store has two narrowcasting screens installed in the store, one beside the cash register, and the other one is hallway. The screen beside the cash register is seen with all customers, but the hallway screen is seen by 65% of the customer. The store is open 14 hours every day, and screens are working all day long. Approximately 15 people walk-in the store per hour. We used 7 days of data and based on them built our forecasting model.

Table 5-2 shows pizza demand during different periods for different types of pizzas from walk-in customers. The objective is using narrowcasting screens to increase the sales for walk-in customers in order to eliminate the spoilage cost of remaining slices at the end of each period.

	Pepperoni	Cheese	Vegetarian	All Dress	Bacon
15-Jul-11	**18**	**20**	**17**	**14**	**4**
11 AM	7	8	7	6	2
3 PM	6	7	6	5	1
8 PM	5	5	4	4	1
16-Jul-11	**9**	**10**	**8**	**7**	**2**
11 AM	4	4	3	3	1
3 PM	3	4	3	2	1
8 PM	2	3	2	2	1
17-Jul-11	**11**	**13**	**11**	**9**	**3**
11 AM	4	5	4	4	1
3 PM	4	5	4	3	1
8 PM	3	3	3	2	1
18-Jul-11	**9**	**10**	**9**	**8**	**2**
11 AM	4	4	4	3	1
3 PM	3	4	3	3	1
8 PM	2	3	2	2	1
19-Jul-11	**11**	**12**	**10**	**9**	**3**
11 AM	4	5	4	4	1
3 PM	4	4	4	3	1
8 PM	3	3	3	2	1
20-Jul-11	**15**	**16**	**14**	**12**	**4**
11 AM	6	6	6	5	2
3 PM	5	6	5	4	1
8 PM	4	4	4	3	1
21-Jul-11	**11**	**9**	**9**	**8**	**4**
11 AM	5	4	4	2	2
3 PM	4	3	3	3	1
8 PM	2	2	2	3	1

Table 5-2: Actual demand for different type of pizza slices

5.1 Inventory forecasting for pizza dough

By observing the data on table 5-2 we can assume the pizza demand for all slices are stationary and there is no noticeable trend or seasonality, and the appropriate forecasting method is using exponential smoothing for each type of pizza slices during each period.

5.1.1 Inventory forecasting for first period form 11 AM to 3 PM

The first step for forecasting using exponential smoothing is to compute a constant of α for all type of slices. We used day 7 to find out α for 5 types of pizza. In fact, the closest answer to the function below identifies the value of α:

$$D_7 \cong F_7 = \alpha D_6 + (1-\alpha)AVG$$

Slice Type	Day 6	Day 7	AVG Sales	$\boldsymbol{\alpha = 0.01}$	$\alpha = 0.3$	$\alpha = 0.16$
Pepperoni	6	5	5	**5**	5	5
Cheese	6	4	5	**5**	5	5
Vegetarian	6	4	4	**4**	5	4
All Dress	5	2	4	**4**	4	4
Bacon	2	2	1	**1**	1	1

Table 5-3: Selecting appropriate $\boldsymbol{\alpha}$ for pizza slices - form 11 AM to 3 PM

To compute the forecasting result for pizza slices which are sold from 11 AM to 3 PM the following formula is used:

$$F_8 = 0.01D_6 + (1-0.01)AVG$$

Type	F_8	Optimal Demand	D_8	**ExtInv = Optimal Demand - F_8**
Pepperoni	5	8	5	**3**
Cheese	5	10	4	**5**
Vegetarian	4	5	4	**1**
All Dress	4	5	2	**1**
Bacon	1	2	1	**1**

Table 5-4 : Forecasting and Narrowcasting result for pizza slices - form 11 AM to 3 PM

The D_8 observed after period is finished. To validate if the forecasting result was acceptable we have to check *TS.* We did the forecasting only for one period after (day 8) for each slices and in each time window, so our $n = 1$.

	D_8	F_8	$E_8 = F_8 - D_8$	$\lvert e_8 \rvert$	MAD	$bias_7$	TS_8
Pepperoni	5	5	0	0	0	0	0
Cheese	4	5	1	1	1	1	1
Vegetarian	4	4	0	0	0	0	0
All Dress	2	4	2	2	2	2	1
Bacon	1	1	0	0	0	0	0

Table 5-5: The Error results of pizza slices' forecasting - form 11 AM to 3 PM

The error calculation for pepperoni slices on day 8 from 11 AM to 3 PM calculated from following formula:

$$\mathrm{MAD} = \left({}^{1}\!/_{\mathrm{n}}\right) \sum_{\mathrm{i}=1}^{\mathrm{n}} |\mathrm{e_i}| = \left({}^{1}\!/_{1}\right) = 1$$

$$bias_n = \sum_{t=1}^{n} E_t = 1$$

$$TS_t = \frac{bias_t}{MAD_t} = \frac{1}{1} = 1$$

while $TS_t = 1$, and it is in the range of ± 6, the forecast result is biased for pepperoni slices and for rest of the slices' type the TS_t shows in the table 5-5.

5.1.2 Inventory forecasting for second period from 3 PM to 8 PM

In order to forecast our next day sales for second period we repeat the process we went in section 5.1.1. We used day 7 to find out best α for all 5 type of pizza slices.

Slice Type	Day 6	Day 7	AVG Sales	$\boldsymbol{\alpha = 0.01}$	$\alpha = 0.3$	$\alpha = 0.16$
Pepperoni	5	4	4	**4**	4	4
Cheese	6	3	5	**5**	5	5
Vegetarian	5	3	4	**4**	4	4
All Dress	4	3	3	**3**	3	3
Bacon	1	1	1	**1**	1	1

Table 5-6: Selecting appropriate $\boldsymbol{\alpha}$ for pizaa slices - form 3 PM to 8 PM

To compute the forecasting result for pizza slices which are sold from 11 AM to 3 PM the following formula is used:

$$F_8 = 0.01D_6 + (1 - 0.01)AVG$$

Type	F_8	Optimal Demand	D_8	**ExtInv = Optimal Demand -** $\boldsymbol{F_8}$
Pepperoni	4	6	4	**2**
Cheese	5	7	4	**2**
Vegetarian	4	5	3	**1**
All Dress	3	4	3	**1**
Bacon	1	2	1	**1**

Table 5-7: Forecasting and Narrowcasting result for pizza slices - form 3 PM to 8 PM

5.1.3 Inventory forecasting for third period from 8 PM to 1 AM

In order to forecast our next day sales for second period we repeat the process we went in section 5.1.1. We used day 7 to find out best α for all 5 types of pizza slices.

Slice Type	Day 6	Day 7	AVG Sales	$\boldsymbol{\alpha = 0.01}$	$\alpha = 0.3$	$\alpha = 0.16$
Pepperoni	4	2	3	**4**	4	4
Cheese	4	2	3	**4**	4	4
Vegetarian	4	2	3	**4**	4	4
All Dress	3	3	2	**3**	3	3
Bacon	1	1	1	**2**	2	2

Table 5-8: Selecting appropriate $\boldsymbol{\alpha}$ for pizza slices - form 8 PM to 1 AM

To compute the forecasting result for pizza slices which are sold from 11 AM to 3 PM the following formula is used:

$$F_8 = 0.01D_6 + (1 - 0.01)AVG$$

Type	F_8	Optimal Demand	D_8	**ExtInv = Optimal Demand -** $\boldsymbol{F_8}$
Pepperoni	4	3	2	**-1**
Cheese	4	3	2	**-1**
Vegetarian	4	2	2	**-2**
All Dress	3	2	3	**-1**
Bacon	2	2	1	**0**

Table 5-9 : Forecasting and Narrowcasting result for pizza slices - form 8 PM to 1 AM

As we can observe in table 5-9 the extra inventory value is not positive which means we forecast to sell more slices that the optimal demand. As a result, there is not necessary to promote these 5 types of slices on the screen at this period of time.

5.2 Promotion scheduling for pizza dough

In this study there are only two screens in the store which runs 14 hours. Ad length is around 15 second and the time-window for each period is equal to the length of the period. The slices price are various, table 5-10 show each pizza's price which we consider as our expiration cost in this model.

Pizza Type	Price
Pepperoni	$ 2.59
Cheese	$ 2.59
Vegetarian	$ 2.92
All Dress	$ 2.92
Bacon	$ 3.19

Table 5-10: Pizza slices' price for different types

At the end of each period we notice the ad distribution for each slice depends to forecasted result. The screen A is located on top of the slices showcase which is seen by 100% of walk-in customers. Screen B is located in hall way and it is seen by 65% of walk-in customers.

	Period I	Period II	Period III
	ExtInv = F_8-Optimal Demand	ExtInv = F_8-Optimal Demand	ExtInv = F_8-Optimal Demand
Pepperoni	3	2	-1
Cheese	5	2	-1
Vegetarian	1	1	-2
All Dress	1	1	-1
Bacon	1	1	0

Table 5-11: Extra inventory for pizza slices in one day

	Period I		Period II		Period III	
	Screen A	Screen B	Screen A	Screen B	Screen A	Screen B
Pepperoni	19	126	17	6	0	0
Cheese	431	5	4	26	0	0
Vegetarian	3	2	2	1	0	0
All Dress	3	2	2	1	0	0
Bacon	3	2	2	1	0	0

Table 5-12: Narrowcasting Ad distribution for one day in pizza slices

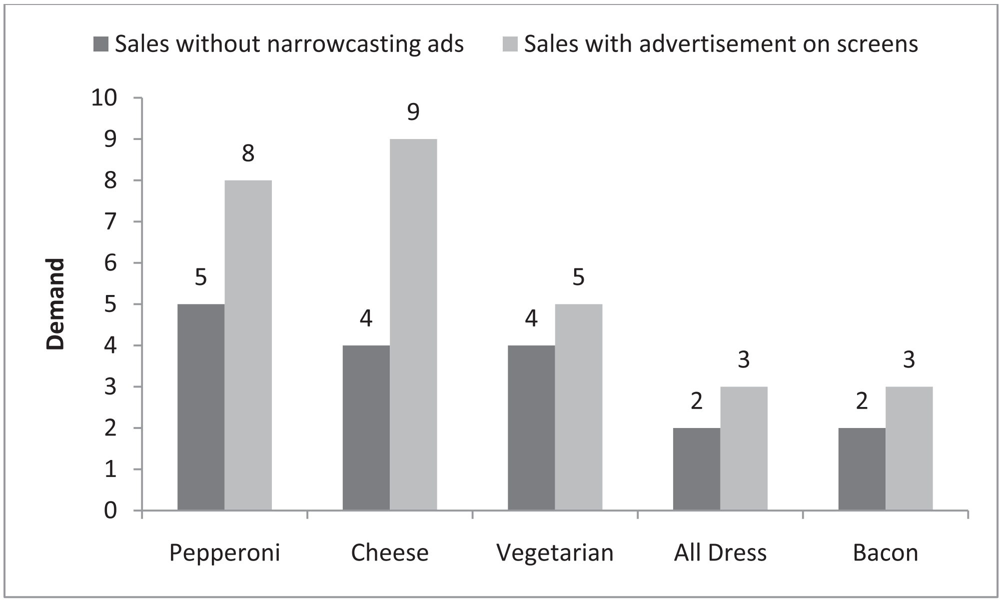
With the figure in table 5-12 and with using expected exposure formula we calculate the sales increase to verify our model and the result is shown in table 5-13. The calculation for pepperoni slices in period one is shown below, for the remaining slices we did the same calculation and add the final result on table 5-13.

$$EE(pep) = \sum_{i=1}^{2} \frac{a_i * c_{i,pep}}{960} = \left(\frac{19}{960} + \frac{0.65 * 126}{960}\right) = 0.105$$

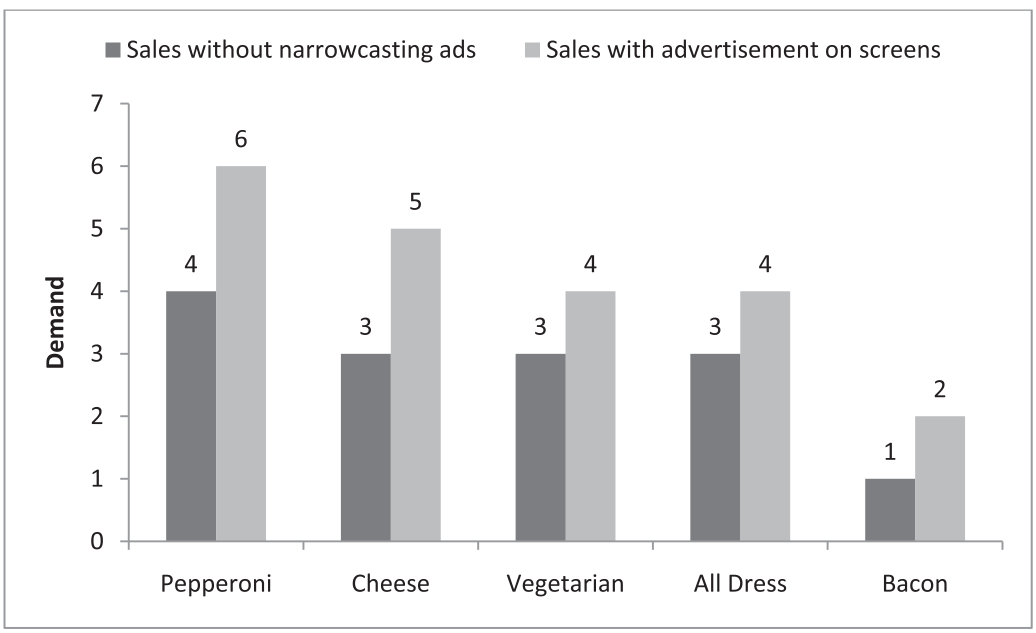
$$Sales\ increas = n * 0.11 * EE(j)^{0.35} = 60 * 0.11 * 0.105^{0.35} = 2.999 \cong 3$$

	Period I				Period II			
	Extra Sales	D_8	**Optimal Sales**	**Spoilage**	**Extra Sales**	D_8	**Optimal Sales**	**Spoilage**
Pepperoni	3	5	8	0	2	4	6	0
Cheese	5	4	10	1	2	3	7	2
Vegetarian	1	4	5	1	1	3	5	1
All Dress	1	2	5	3	1	3	4	0
Bacon	1	2	2	0	1	1	2	0

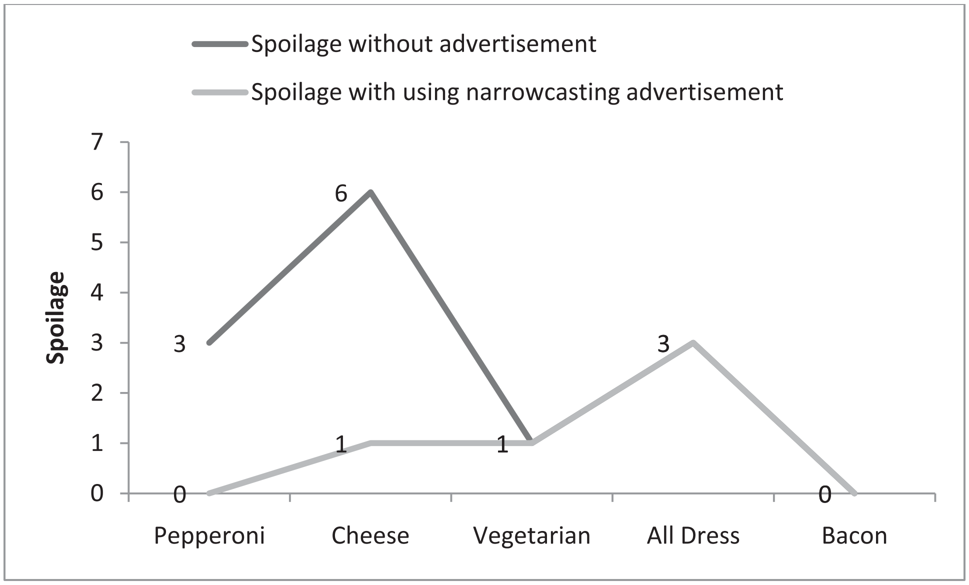
5-13: Pizza slices final sales after promoting the slices on screens.



5-1 :Sales difference without and after promoting the slices on screens in period I.



5-2: Sales difference without and after promoting the slices
on screens in period II.



5-3: The spoilage comparison for pizza slices in period I

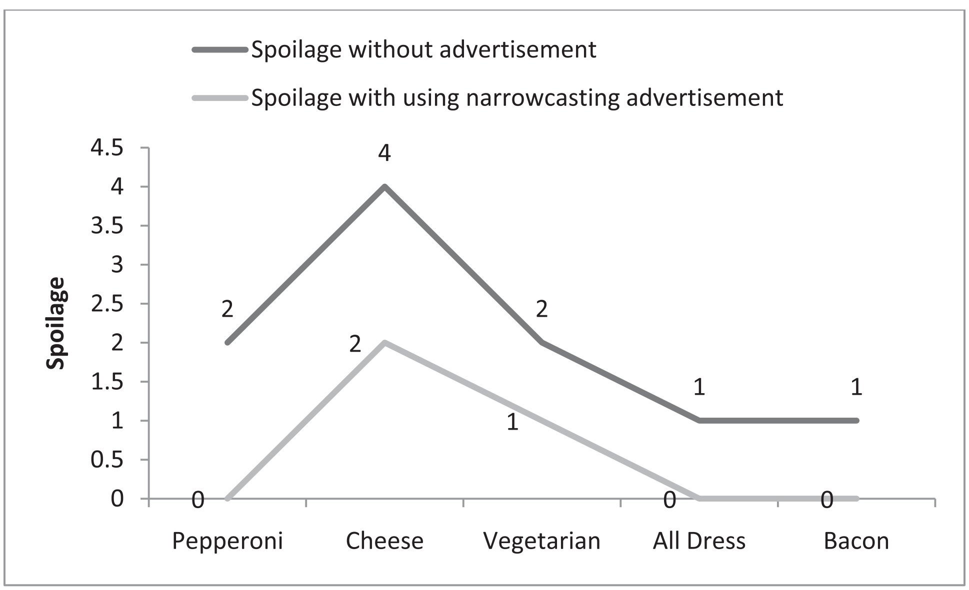


Figure 5-4: The spoilage comparison for pizza slices in period II

Chapter 6. Conclusion

Narrowcasting network has found its applications in retail stores, quick service restaurants, and many other public spaces such as airports, schools, and hospitals. The establishment of the narrowcasting infrastructure provides the opportunity of implementing highly responsive channels for delivering advertisements required by real time customer demand management. The purpose of this research is to propose the integration of existing inventory control models and the narrowcasting promotion systems with the objective of reducing inventory costs. In many industries and supply chains, inventory is one of the dominant costs. For many managers, effective supply chain management is synonymous with reducing inventory costs in the supply chain. We propose narrow-casting based inventory control aiming at reducing inventory costs through effective promotion management. We propose an extension to the classical news-vendor inventory model, which integrates the narrowcasting advertising component into the dynamic inventory management process. We also present a promotion scheduling model for narrowcasting-based inventory management. To demonstrate the application of the proposed models, we provide a complete case study in the context of quick service restaurant industry.

A limitation of the proposed approach is that the accuracy of forecasting has considerable impacts on the performance of the approach. In fact, investing in advertisement has a diminishing return in sales increment. Correcting a bigger forecasting error through advertising is usually costly and inefficient. Furthermore, we have assumed all customers have the same advertisement response function and their buying behaviors are also identical, which is rather an ideal situation.

As narrowcasting systems are rapidly adopted by quick service restaurants and retails stores, the need for dynamic integrated promotion management systems is becoming increasingly pressing. We will continue working along this direction. One of our future research topics is to develop an agent-based simulation platform, which can serve as a testing environment for evaluating the performance of various forecasting models and promotion management policies. To make the model more realistic, we will also consider different buying habits and advertising response rates of customers. For example, to obtain customer reaction data, we may deploy a small narrowcasting system in a store and use cameras on screens to capture customers' reactions. For large scale narrowcasting networks, the currently used general optimization package is not sufficiently responsive. Another future work is to develop heuristic based promotion scheduling algorithms and system architectures for large scale narrowcasting-based inventory management problems.

Bibliography

1. **David Simchi-Levi, Philip Kaminsky, Edith Simichi-Levi.** *Designing and managing the supply chain.* s.l. : Irwin/McGraw-Hill, 2003.

2. **S. Chopra, P.Meindl.** *Supply Chian Management Strategy, Planning, and Operation.* 4th Edition. s.l. : Pearson, 2010.

3. **A.Silver, David F.Pyke, Rein Peterson.** *Inventory Management and Production Planning and Scheduling.* 3rd Edition. 1998.

4. **Nahmias, Steve.** *Production and Operations Analysis.* 3rd Edition. s.l. : Irwin, 1997.

5. **Douglas C.Montgomery, Lynwood A.Johnson, John S.Gardiner.** *Forecasting & Time Series Analysis.* 2nd Edition. s.l. : McGraw-Hill, 1989.

6. **G.Brown, Robert.** *Statistical forecasting for inventory control.* s.l. : McGraw-Hill, 1959.

7. **Henk A. Akkermans, Paul Bogerd, Enver Yücesan, and Luk N. van Wassenhove.** *The impact of ERP on supply chain management:Exploratory findings from a European Delphi study.* 2, 2003, European Journal of Operational Research, Vol. 146, pp. 284-301.

8. **Bongju Jeong, , Ho-Sang Jung, and Nam-Kyu Park.** *A computerized causal forecasting system using genetic algorithms in supply chain management.* 3, s.l. : Elsevier, 2002, Journal of Systems and Software, Vol. 60, pp. 223-237.

9. **Els Gijsbrechts, Katia Campo, Tom Goossens.** *The impact of store flyers on store traffic and store sales: a geo-marketing approach.* s.l. : Journal of Retailing, 2003, pp. 1-16.

10. **Aîda Mimouni Chaabane, Ouidade Sabri, Béatrice Parguel.** *Competitive advertising within store flyers: A win–win strategy?* s.l. : Journal of Retailing and Consumer Services, 2010.

11. **Michael, Steven C.** *Can a franchise chain coordinate?* 4, s.l. : Journal of Business Venturing, 2002, Vol. 17, pp. 325-341 .

12. **Weier, Mary Hayes.** *Information Week.* [Online] 09 09, 2007. http://www.informationweek.com/news/software/bi/showArticle.jhtml?articleID=202400416&cid=RSSfeed_IWK_News.

13. **Bentonville, AR.** NEWS. *Walmart Stores.* [Online] September 3, 2008. http://walmartstores.com/pressroom/news/8566.aspx.

14. **Gerba, W. F.** Five visual merchandising tips for your in-store network. [Online] http://www.wirespring.com/dynamic_digital_signage_and_interactive.

15. **Manners, T.** Cool news of the day. [Online] March 12, 2006. http://www.reveries.com/ coolnews/2006/march/mar_21a.html.

16. **Breen, P.** Changing channels. [Online] May 2004. http://www.instoremarketer.org/?q=node/ 3729.

17. **Nelson, E. & Ellison, S.** *In a shift, marketers beef up ad spending inside.* s.l. : Wall Street Journal, 2005.

18. **Williams, D.** *Consumer interest and acceptance of video displays in retail environments.* s.l. : The Arbitron retail media study, 2005.

19. **Cornish, Susan L.** *Product Innovation and the Spatial Dynamics of Market Intelligence: Does Proximity to Markets Matter?* 2, 2008, Vol. 73, pp. 143-165.

20. **Kotler, P. & Keller, K.** *Marketing management.* s.l. : Prentice-Hall, 2009.

21. **Satyaveer S. Chauhan, Alexandre Dolgui, Jean-Marie Prot.** *A continuous model for supply planning of assembly systems with stochastic component procurement times.* s.l. : Int. J. Production Economics, 2009, pp. 411–417.

22. **Sidney Browne and Paul Zipkin.** *Inventory Models with Continuous, Stochastic Demands.* s.l. : Institute of Mathematical Statistics, 1991, Vol. I, pp. 419-435.

23. **Brown, Robert Goodell.** *Smoothing Forecasting and Prediction of Discrete Time Series.* s.l. : Prentice-Hall inc., 1963.

24. **Everette S. Gardner Jr. Commander.** *Exponential smoothing: The state of the art.* 1, s.l. : Journal of Forecasting, 1985, Vol. 4.

25. **Muth, John F.** *Optimal Properties of Exponentially Weighted Forecasts.* 1960, Journal of the American Statistical Association, Vol. 55, pp. 299-306.

26. **Douglas C.Montgomery, Cheryl L. Jennings, Murat Kulachi.** *Introduction to Time Series Analysis and Forecasting.* s.l. : A john Wiley & Sons,, 2008.

27. Engineering Statistics Handbook. [Online] 06 01, 2003. [Cited: 06 23, 2010.] http://www.itl.nist.gov/div898/handbook/index.htm.

28. **Rob J. Hyndman, Anne B.Koehler, J.Keith Ord, Ralph D.Snyder.** *Forecasting with exponential smoothing: the state space approach.* s.l. : Springer, 2008.

29. **Winters, Peter R.** *Forecasting Sales by Exponentially Weighted Moving Averages.* 1960, Informs, pp. 324-342.

30. **Weiss, H.** *Economic order quantity models with nonlinear holding cost.* 1989, European Journal of Operation Research, Vol. 9, pp. 56-60.

31. DecisionCraft Inc. [Online] 12 2001, 05. http://www.decisioncraft.com/dmdirect/forecastingtechnique.htm.

32. **Xiaozhe Wang, KateSmith-Miles, Rob Hyndmanc.** *Rule induction for forecasting method selection: Meta-learning the characteristics of univariate time series.* 2009, Neurocomputing, pp. 2581-2594.

33. **Robert D.Klassen, Benito E.Flores.** *Forecasting practices of Canadian firms: Survey result and comparisions.* 2001, Elsever, pp. 163-174.

34. **Gwilym M. Jenkins, George Edward Pelham Box.** *Time Series Analysis: Forecasting and Control.* New Jersey : Prentice Hall PTR, 1994.

35. **Gutierrez, Ben Paul B.** *In-Store Media: How Effective are they? Evidence from the Philippines.* 2006, Philippine Management Review, pp. 65-82.

36. **Bodapati, Anand V.** *The impact of out-of-store advertising on store sales.* s.l. : Stanford University, 1999.

37. **Els Gijsbrechts, Katia Campo, Tom Goossen.** *The impact of store flyers on store traffic and store sales: a geo-marketing approach.* s.l. : Elsevier Science, 2003, pp. 1-16.

38. **Rayid Ghani, Chad Cumby, Andrew Fano.** *Data Mining for Individual Consumer Models and Personalized Retail Promotions.* 2008, Auerbach Pub, pp. 201-223.

39. **Uta Juttner, Martin Christopher, Susan Baker.** *Demand chain management-integrating marketing and supply chain management.* 2005, Industrial Marketing Management, pp. 377-392.

40. **Rachel Bellamy Calvin Swart, Wendy A. Kellogg, John Richards, and Jonathan Brezin.** *Designing an E-Grocery Application for a Palm Computer: Usability and Interface Issues.* 2001, Personal Communications, IEEE, pp. 60-64.

41. **Arvind K. Tripathi, Suresh K. Nair.** *Narrowcasting of wireless advertising in malls.* 2006, European Journal of Operationa, pp. 1023–1038.

42. **Williams, Diane.** *The Arbitron Retail Media Study: Volume I: The Impact of Retail Audio Broadcasting in Grocery and Drugstores.* s.l. : Arbitron, 2005.

43. **Payman Salimi, Chun Wang.** *Improving inventory management through automated dynamic promotion scheduling.* s.l. : IEEEXplore, 2010, pp. 586 - 590.

44. **Weymouth.** InfoTrends/CAP Ventures Releases New Research on North American Wide Format Professional Digital Graphics Market . [Online] November 2006. http://www.infotrends-rgi.com/public/Content/Press/pr2005.html.

45. **G.Russell, Martha.** *Narrowcast Pricebook-Driven Precaution, Engagment at point of Influence, Purchase and consumption in distributed retail environments.* 2009, Journal of Software, pp. 365-373.

46. **Cooke, L.** Shopper marketing delivers ROI for CPG manufacturers. [Online] November 2008. http://www.cpgmatters.com/ShopperInsights1108.html.

47. **Breen, Peter.** In Store Marketer. [Online] May 2004. http://www.instoremarketer.org/?q=node/%203729.

48. **Jones, John Philip.** *When Ads Work.* New York : MESharp, 2007. 13:978-0-7656-1739-2.

49. **Bronnenberg, B.J.** *Advertising frequency decision in a discrete markov process under a budget constraint.* s.l. : Journal of Marketing Research, 1998, Vol. XXXV, pp. 399–406.

50. **Simon, J.L., Arndt, J.** *The shape of the advertising response function.* 20, s.l. : Journal of Advertising Research, 1980, pp. 11–28.

51. **Hanssens, D.M., Parson, L.J.** *Econometric and time-series market response models.* Boston : s.n., 1993, Operations Research and Management Science, pp. 409–464.

52. **Clarke, D.G.** *Econometric Measurement of the Duration of Advertising Effect on Sales.* s.l. : Journal of Marketing Research, Vol. 13, pp. 345-357.

53. **Assmus, G., Farley, J.U. & Lehmann, D.R.** *How advertising affects sales: meta-analysis.* s.l. : Journal of Marketing Research, 1984, Vol. 21, pp. 65-74.

54. **Morrison, G.** *Excerpts from workshop on the Effectiveness of high-tech in-store media:Canadian Advertising Research Foundation.* 2008.

55. **Harris, B.** *If not TV…?* s.l. : The Hub, 2006.

56. **Moutaz Khouja, Stephanie S. Robbins.** *Linking advertising and quantity decisions in the single-period inventory model.* s.l. : Elsevier, 2002, Vol. 86, pp. 93-105.

57. **Rudberg M, Olhager J.** *Manufacturing networks and.* 1, s.l. : OMEGA, 2003, Vol. 31, pp. 29–39.

58. **Suhong Lia, Bhanu Ragu-Nathanb, T.S. Ragu-Nathanb and S. Subba Raob.** *The impact of supply chain management practices on competitive advantage and organizational performance.* s.l. : Elsevier , 2006, OMEGA, pp. 107-124.

59. **Barfield, Jesse T., Raiborn Cecily A.** *Cost Accounting: Traditions and and Innovations.* 5th Edition. s.l. : Lavoisier, 2002.

60. **Suhaiza Zailani and Premkumar Rajagopal.** *Supply chain integration and performance: US versus East Asian companies.* 2005, Emerald Group Publishing Limited, pp. 379–393.

61. **Hesham K. Alfares.** *Inventory model with stock-level dependent demand rate and variable holding cost.* 2007, International Journal of Production Economics, pp. 259-265.

62. **Giri, B.C., Chauduri, K.S.** *Deterministic models of perishable inventory with stock-dependent demand rate and nonlinear holding cost.* 1998, European Journal of Operational Research, pp. 467–474.

63. **Rardin, R. L.** *Optimization in Operations Research.* 1998, Prentice Hall.

64. **I. K. Hui and H. C. W. Lau.** *A dynamic inventory-production scheduling model for small scale organisations,.* 2003, The International Journal of Advanced Manufacturing Technology, pp. 89-100.

65. **V. Suresh and D. Chaudhuri.** *Dynamic scheduling--a survey of research,.* 1993, International Journal of Production Economics,, pp. 53-63.

66. **Ramasesh, R.** *Dynamic job shop scheduling: a survey of simulation research,.* 43-57, Omega, Vol. 18, p. 1990.

67. **S. Minton, M. D. Johnston, A. B. Philips, and P. Laird,.** *Minimizing conflicts: a heuristic repair method for constraint satisfaction and scheduling problems.* 1992, Artificial Intelligence, Vol. 58, pp. 161-205.

68. **Gérard P. Cachon and Marshall Fisher.** *Supply Chain Inventory Management and the Value of Shared Information.* s.l. : Management Science, 2000, Vol. 46, pp. 1032-1048.

69. **Stevenson, William J.** *Operations Management .* s.l. : McGraw-Hill Irwin, 2008.

70. **Mark Fergusona, Vaidy Jayaraman, and Gilvan C. Souza.** *Note: An application of the EOQ model with nonlinear holding cost to inventory management of perishables.* 1, s.l. : Elsevier, 2006, European Journal of Operational Research, Vol. 180, pp. 485-490.