Spanning Tests for Small Cap Indexes as Separate Asset Classes International Evidence

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ABSTRACT

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Traditional spanning and step-down spanning tests are used to study whether or not small cap indexes in Asian and G7 countries could be separate asset classes of efficient portfolios for U.S. investors. Empirical tests on different index combinations show that the composition of a benchmark portfolio determines whether or not a small cap index could enlarge the original efficient frontier. The interaction among all assets in a portfolio is the key to the effectiveness and efficiency of a small cap index in efficient portfolios and constraints do not always reduce diversification benefits of a new asset. In addition, the time period the sample covered and length of holding time also influence the test results. Most small cap indexes of G7 countries are separate asset classes to the portfolio consisting of the popular indexes in G7 markets in our sample period. Moreover, pair-wise correlation is not an effective approach to search and study the diversification benefits. The step-down test results are consistent with empirical measures on risk and return of portfolios; this fact implies that the step-down approach, as an alternative method to correlation analysis, could be a powerful procedure to identify potential separate asset classes.

Keywords: portfolio diversification; small cap index; spanning test.

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1. Introduction

There is a growing interest amongst the financial community in the performance of small cap indexes and a great deal of research on the benefits of small caps for international diversification has been published. In finance theory, investors with monotonically increasing and strictly concave utility functions prefer higher expected return and lower variance. For arbitrary distributions of asset returns, investors will choose on the basis of mean and variance if the utility functions are quadratic; and for arbitrary preferences, investors will choose on the basis of mean and variance if asset returns are multivariate normally distributed. The restrictions required for mean-variance choice are quite strong. However, since mean-variance analysis can generate empirical predictions that may be testable, mean-variance analysis is widely used in finance. One of the important topics in the investment area is the existence and the magnitude of the benefits from diversifying over small cap stocks. From modern portfolio theory, if small cap returns do not perfectly correlate with returns of other classes of assets, investors could gain from size diversification.

A great deal of previous work shows that the return behavior of small cap stocks and large cap stocks are not the same and different conclusions are shown in literatures. Banz (1981) first reported the size anomaly: small cap stocks had substantially higher returns than that of large cap stocks even after controlling for risk over long investment horizons. Banz's paper leads to a number of subsequent studies on the small firm effect. However, some recent studies show different results on the validity of such an anomaly. For example, Berk (1996) finds that although the returns and firm sizes based on market value have an inverse relationship, there is no significant relationship between average returns and non-market based measures of firm size (book value of assets; book value of property, plant and equipment; total value of annual sales; total number of employees). Horowitz et al. (2000) find that returns of small cap portfolios did not exceed the returns of large cap portfolios for 1980–1996 data from NYSE, AMEX and NASDAQ, and that the returns of large firms are higher than that of small firms. Dimson and Marsh (1999) find that large cap stocks have higher returns than small cap stocks for the US and UK data. Moreover, Reilly and Wright (2002) also find large cap stocks outperform small cap stocks over the period 1984–2000.

The different results on the return of small caps and large caps lead people to study the cyclical behavior of the size premium. Brown et al. (1983) initially suggested such behavior and found that small caps outperformed large caps over some periods, but the size premium was reversed in other periods. In addition, based on US market data of 1926–1989 and return autocorrelations among different investment horizons, Reinganum (1992) finds that the changes of the size premium for small caps are not random but predictable since the premium reverses over long time horizons.

There are a lot of studies exploring the factors that result in the small firm effect. Chan et al. (1985) find that the risk premiums in a multi-factor pricing model contribute a large part of the size premium and because the changes in the risk premiums are related to business conditions, they argue that the economic expansions and contractions will have a large impact on the returns of small firms. Levis (2002) suggests that the cyclical pattern of the size premiums may result from changes in the economic fundamentals of small cap firms such as the cash flows and discount rates. In addition, Jensen et al. (1997) also find that significant positive premiums of small caps appears when U.S. monetary policy conditions are expansive and insignificant or negative premiums of small caps occurs when the monetary policy is restrictive. Moreover, Stoll and Whaley (1983) find that transaction costs and investors' holding-period are responsible for the higher returns of small caps.

Economic factors are commonly believed to be the causes for the different return behaviors between small and large caps. Chan and Chen (1991) argue that small cap firms are usually highly financially leveraged and face more cash flow problems, or have lower production efficiency than large firms. Small cap firms will react differently from large cap firms to the same macroeconomic impacts. Dimson and Marsh (1999) and Levis (2002) find that small firms rely more on short term financing so that small caps are more influenced by the condition of the credit markets. In addition, since there are more small companies in some industrial sectors than in others, there could be different performance among various industrial sectors. Dimson and Marsh (1999) find that differences in the weights of different industrial sectors between a small cap index and a large cap index are an important reason for the size premium. However, Levis (2002) concludes that although the difference in industrial sectors causes the small cap premium, industrial performance is not a critical factor. Moreover, Dimson and Marsh (1999) and Levis (2002) find that differences in dividend growth between small and large caps and superior earnings growth in some small firms are related to the size premium. Furthermore, Eun et al. (2004) state that the return behaviors of large cap and small cap stocks are quite different since returns on large-cap stocks are substantially influenced by common global factors while returns on small-cap stocks are primarily driven by local and idiosyncratic factors.

The benefits of international diversification on portfolio management are well documented in literatures and the mean-variance spanning tests have been used to study such benefits. Harvey (1995) uses the Emerging Market Data Base of the International Finance Corporation (IFC) (which contains eight Asian markets-India, Indonesia, Korea, Malaysia, Pakistan, Philippines, Taiwan, and Thailand) and calculates monthly value-weighted index returns with dividend reinvestment for the 20 countries. The evidence of the study for the period of March 1986 through June 1992 (75 observations) suggests that the emerging market returns are not spanned by the developed market returns (18 Morgan Stanley Capital International (MSCI) developed market returns). Bekaert and Urias (1996) use a sample consisting of forty-three U.S. funds and thirty-seven U.K. investment trusts for the period of January 1986 to August 1983. The closed-end funds are classified as emerging market funds following the emerging markets defined by the IFC. Mature market funds are defined by their inclusion in the FT-Actuaries World Index. Their spanning tests find significant diversification benefits for the U.K. country funds, but not for the U.S. funds. They conclude that the difference in diversification is related to differences in portfolio holdings. In addition, Errunza et al. (1999) examine whether portfolios of U.S. - traded securities (industry portfolios, multinational corporation stocks, closed-end country funds and ADRs) can mimic foreign indices so that investment in assets that trade only abroad is not necessary to gain the benefits from international diversification. They use monthly data from 1976 to 1993 for seven developed and nine emerging markets (Argentina, Brazil, Chile, Greece, India, Korea, Mexico, Thailand and Zimbabwe). Return correlations, mean-variance spanning, and Sharpe ratio test results show that direct investments in five out of nine emerging markets have significant gains beyond those attainable through home-made diversification. In addition, they also show that the incremental gains from international diversification beyond home-made diversification portfolios have diminished over time in a way consistent with changes in investment barriers. de Roon et al. (2001) use the same IFC indices as in Harvey (1995) and show the effect of transaction costs and short-selling constraints on the benefits of diversification and state that the benefits disappear when investors face short sales constraints or transaction costs. Moreover, Gerard et al. (2002) examine monthly returns and market value data on the DataStream aggregate equity market indices of the G7 countries and of the ten major industry sectors within each country for the period of December 1973 to November 1998. They conclude that that country specific factors rather than industrial structure drive international diversification benefits. Driessen et al. (2003) find that the benefits of international portfolio diversification differ across countries from the perspective of a local investor and that the benefits are largest for investors in developing countries. They also provide evidence that such diversification benefits vary over time as country risk changes. For the empirical analysis, they use data from several sources. The MSCI stock market indexes are used for developed countries and the S&P/IFC Global Indexes from Standard & Poor's (S&P) is used for developing countries. The total number of countries in their sample is 52 (23 developed markets and 29 developing markets) and monthly data from 1985 to 2002 are used.

The different data sets used by previous studies could be one of potential factors that result in the different conclusions. In fact, in the work of Fernandes (2002), emerging market equities are represented by the MSCI Emerging Markets Free (EMF) index and the U.S. market are represented the NYSE and the NASDAQ 100 indices. Monthly data on the indices from January 1988 (starting date of the EMF index) to December 2001 are examined. Fernandes (2002) shows that after allowing investment in several developed market asset classes, there are no significant gains from an aggregate investment in emerging market equities; emerging markets as a whole are spanned by developed market indices. This work provides evidence against indexing strategies in emerging markets.

Although the small abnormal is inclusive in the literatures, small-cap stocks could potentially be an effective vehicle for international diversification. Eun et al. (2004) argue that investors are more likely to invest in well-known, large foreign companies because these companies are highly visible and often multinational. In addition, institutional investors who track national market indices could reinforce this large-cap bias since large-cap stocks dominate market indices. Similarly, in documenting the gains from international diversification, academic studies tend to use large-cap stocks or national stock market indices dominated by the latter. However, the fact that large-cap stocks or stock market indices tend to co-move reduces the benefits from international diversification. Therefore, there could be potential benefits of small-cap stocks in international diversification. Eun et al. (2004) uses the data set that includes monthly stock prices and returns, number of shares outstanding for exchange-listed companies and MSCI stock market indices from the ten major countries (Australia, Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, the United Kingdom, and the United States) during the period of January 1980 to December 1999. The sample includes all U.S. firms listed on the New York Stock Exchange, American Stock Exchange and NASDAQ and all foreign firms from each of the ten countries for which DataStream has data available during the sample period. They create three value-weighted index funds for each country (a large-cap fund is the top 20 percent, a small cap fund is the bottom 20 percent, and a mid-cap fund is from the rest of stocks in each country). The monthly value-weighted returns for each fund in terms of U.S. dollars are calculated. The funds are updated once a year. The key findings of mean variance spanning tests by Eun et al. (2004) are as follows: first, international small-cap funds are not span-able by country stock market indices (except for Hong Kong). Second, the optimal international portfolio tends to comprise of the U.S. market index and foreign small-cap funds; neither foreign market indices nor mid-cap funds receive positive weights during the sample period. The extra gains from the augmented diversification with small-cap funds are statistically significant if transaction costs for small-cap funds are not excessive. In addition, Petrella (2005) uses data from December 1998 to December 2002 and performs regression-based tests for mean-variance spanning and finds that euro area small and mid cap stocks that are classified by size quartile and quintile rankings are independent asset classes.

Overall, the previous evidence indicates that small cap stocks behave differently from large caps and that small cap stocks could be potential asset classes to portfolio diversification. This paper will explore whether or not small cap stocks, represented by small cap indexes, could be separate asset classes and enlarge the efficient frontier of portfolios. In previous studies, either country indexes or size capped indexes (funds) are defined and composed by the authors in order to investigate diversification benefits of international investment or small cap effects. In fact, financial institutions and services provide a great deal of small cap indexes and large cap indexes for different countries. To avoid the problem that different methodologies used to form size-based portfolios could result in different results, we will use the indexes available in commercial databases to study if small caps could be separate asset classes.

Following the common practice of previous academic works on the international diversification, we will study the small cap benefits from the perspective of U.S. investors. To identify the behavior of different small cap indexes, we include emerging markets in a diversified portfolio because emerging market stocks could also give rise to large diversification opportunities and they make up a disproportionately small share of investors' equity holdings in U.S. investor portfolios. Data show that the emerging market share in US investors' portfolios is a little higher than 1% of foreign equities (Fernandes 2002). We want to investigate if the small cap indexes could be separate asset classes after the effects of investing in emerging markets and other international assets are considered.

This paper employs the approaches of the traditional spanning test and step-down spanning test on mean-variance frontiers; and the tests results are compared with empirical measures for portfolio optimization. Mean-variance spanning test and step-down test are econometric methods to test the hypothesis that adding new assets (a small cap index in our study) could enlarge the Markowitz efficient frontier of an investment portfolio. The spanning hypothesis is rejected when the efficiency of Markowitz frontier of a benchmark portfolio is not improved after new assets are included in the benchmark portfolio. Step-down spanning tests provide further statistical measures of the risk reduction of the global minimum variance portfolio on the frontier and the changes in the slope of the capital allocation line when the risk assets are combined with risk free assets.

Test results on indexes from U.S., Japan, emerging markets, Hong Kong, Taiwan, Singapore, Euro-Market and other members of G7 group show that the composition of benchmark portfolios, the time of sample period covered, the investors' holding period and the investment constraints all are factors that determine whether or not a small cap index is a separate asset class and that some small cap indexes are separate asset classes when the benchmark portfolio includes only popular market indexes in the news media for G7 country markets. Constraints do not necessarily reduce the diversification benefits of a small cap index relative to a benchmark portfolio. In addition, Pearson correlation and the linear relationship among assets in a portfolio cannot be used to explain the results of spanning tests and the correlation, which has been widely used to explore and explain diversification benefits of assets, is ineffective to identify and explain the benefits of portfolio diversification. Compared to the traditional spanning test, the step-down spanning test is more powerful and the results of our step-down tests are consistent with empirical measures on the portfolio efficiency. The step-down spanning test could be used as a standard approach in studying and building a Markowitz efficient frontier.

To best of our knowledge, our work is the first attempt using size-based indexes available in commercial databases to investigate the behavior of small caps in international markets. The rest of the paper is organized as follows: Section 2 describes the data. Section 3 shows the concept of mean-Variance Spanning and presents the results on small cap indexes in Asian markets and developed markets and provides evidence on the persistence of small caps as separate classes. Section 4 describes the economic significance of the step-down spanning test and presents results on some small indexes. Further tests show that the composition of benchmark assets influences the significance of the spanning tests by using market indexes of G7 countries is presented in this section . Section 5 compares the results of the traditional spanning test and the step-down spanning test with popular empirical measures. Finally, Section 6 offers the conclusion.

2. Data

Financial services in the world create and publish many small-cap stock benchmarks for different countries. Reilly et al. (2002) compare the six small-cap and micro-cap indexes to three large-cap stock indexes (the S&P 500 Index, the Russell

1000 Index, and the Wilshire Large-- Cap 750 Index) and a global stock and bond index from 1984 to 2000. They find that there are strong similarities among the small-cap stock indexes in term of risk and correlation between small-cap indexes or with large cap indexes. In this paper, the following indexes for U.S. and non- U.S. markets are used in our study.

2.1 Indexes in U.S. market

Reilly et al. (2002) find that large-cap stock series (S&P 500 Index, the Russell 1000 Index, and the Wilshire Large-Cap 750 Index) behave similar in risk, return and correlation. In our study, S&P 500 total return index (SP500) is used to represent large-cap stocks in U.S. market and the Russell 2000 Total Return (R2000) of the Frank Russell Company Indexes is used as the representative of the small-Cap Index for U.S. market. The Russell 2000 is the smallest 2,000 securities, based upon market cap size, of the Russell 3000 indexes. Russell 3000 is the 3,000 largest (by market cap) U.S.-domiciled stocks from the NYSE, the AMEX, and NASDAQ.

2.2 Indexes in international markets

For European and emerging markets, we used the MSCI Europe Total Return (EUT), MSCI EU Value Total Return (EUV), MSCI Europe Small Cap Total Return (EUS) and S&P/IFCI Emerging Composite Total Return (EMG). For Hong Kong, MSCI Hong Kong Total Return (HKT) and MSCI Hong Kong Small Cap Total Return (HKS) are used. For the Japanese market, we use the MSCI Japan Total Return (JPT), BARRA/Nikko Large Cap TR Total Return (NIKL), BARRA/Nikko Small Cap Total Return index (NIKS). For Chinese markets, MSCI Zhong Hua Value Total Return

(CNV) and MSCI Zhong Hua Growth Total Return (CNG) are used. For Taiwan, MSCI Taiwan Value Total Return (TWV) and MSCI Taiwan Growth Total Return index (TWG) are used. For Singapore, MSCI Singapore Value Total Return (SINV) and MSCI Singapore Small Cap Total Return (SINS) are used. In addition, MSCI value and small cap total return indexes of different G7 countries are also collected.

The sample periods of most indexes are from January 1999 to December 2004; NIKL, NIKS, EMG and indexes in U.S. market are from January 1989 to December 2004.All the monthly returns of the above indexes are from the Ibbotson Database of Ibbotson Associates and converted to U.S. Dollar return. We use monthly data because monthly data are less influenced by bid-ask or thin trading effects (Ferson et al.1993).

Moreover, we also retrieve some popular indexes in the news media for G7 countries from DataStream for the period of December 1998 to December 2004. Specifically, we obtain the total return indexes in US dollar of S&P TSX (SPTSX) in Canada, S&P MIB (SPMIB) in Italy, CAC 40 (CAC40) in France, DAX 30 (DAX30) in Germany and FTSE 100 (FTSE100) in U.K. and the index of NIKKIE 300 (NIKI300) in Japan. We obtain monthly returns in U.S. dollar for these index series. The characteristics of different indexes in our study are presented in Appendix 1.

Since there is no small/large cap index available in some markets, we use value indexes to represent value segment and growth indexes to represent growth segment of that markets.

3. Regression based tests of spanning

3.1 Mean-variance spanning test

Huberman and Kandel (1987) first introduced the concept of mean-variance spanning. A set of K risky assets spans a larger set of N+K risky assets if the minimum-variance frontier of the K assets is the same as the minimum variance frontier of the K +N assets. In the previous literature, the first set of K risky assets are often called the benchmark assets and the second set of N risky assets are called the test assets. Following Huberman and Kandel (1987) and Kan and Zhou (2001), we check if the small-cap index of Hong Kong (HKS), for example, can be spanned by a benchmark portfolio consisting of several benchmark indices: for instance, SP500, R2000, EMG, EUV and EUG. The spanning test involves a procedure where the new assets (each small-cap index to be tested) are regressed on the benchmark assets as follows:

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * EUV + ... + \varepsilon_i \quad (1)$$

where R_i is the return on the small-cap index to be tested from the i-th country, and SP500 (R2000, EMG and EUV etc.) denotes the return on the benchmark index. α_i is the estimated regression intercept for the small-cap index i and β_j (j =1,2...) is the estimated regression coefficient associated with the benchmark assets j. The null hypothesis of spanning is equivalent to the joint hypothesis that α is equal to zero and the sum of β_j is equal to one:

$$H_0: \alpha_i = 0, \text{ and } \Sigma\, \beta_j = 1 \quad (2)$$

Several test statistics have been proposed to test this hypothesis.

3.1.1 Lagrange multiplier (LM) test

For notational brevity, we use the matrix form of model (1) in what follows:

$$R = XB + E, \tag{3}$$

When there is only one new asset, as is the case in our analysis, R is a T × 1 matrix of Rt (T is the length of the time-series); X is a T × (K + 1) matrix with its typical row as [1, SP500, R2000, EMG, EUV…].B = [α, β]', and E is a T ×1 matrix with ε_t as its typical row.

We assume T ≥ K+2 and X'X is nonsingular. For the purpose of obtaining exact distributions of the test statistics, we also assume that, conditional on the returns of benchmark assets, the disturbances ε_t are independent and identically distributed as multivariate normal with mean zero and variance Σ.

The unconstrained maximum likelihood estimator of Σ is

$$\Sigma_u = \frac{1}{T}(R - X\hat{B})'(R - X\hat{B}) \tag{4}$$

Let Σc be the constrained maximum likelihood estimator of Σ and

$$U = |\Sigma u| / |\Sigma c| \tag{5}$$

Huberman and Kandel (1987) proved that the likelihood ratios of unconstrained and constrained estimation of Eq.3 could be used to test the hypothesis of Eq.2. According to Kan and Zhou (2001), when N=1, the likelihood ratio test (LR) can then be written as

$$LR = T \ln(1 + \lambda 1) \tag{6}$$

where $$1/U = (1 + \lambda 1) \tag{7}$$

The Lagrange multiplier test (LM) is then given by

$$\text{LM} = \text{T}\lambda 1 / (1 + \lambda 1) \sim \chi_2^2 \qquad (8)$$

Substituting Eq.7 into Eq.8, we get the Lagrange multiplier

$$\text{LM=T (1-U)} \sim \chi_2^2 \qquad (9)$$

3.1.2 Small Sample Distribution of Spanning Test

Kan and Zhou (2001) state that asymptotic tests (like the Lagrange multiplier) are problematic to apply in finite samples. We will also use the F-test for small samples under the null hypothesis of spanning developed by Kan and Zhou (2001):

$$(\frac{1}{U} - 1)(\frac{T - K - 1}{2}) \sim F_{2,T-K-1} \qquad (10)$$

where T is the length of the time-series; U is the same as Eq.5, the ratio of unconstrained and constrained maximum likelihood estimator of variance.

In addition, according to Kan and Zhou (2001), the Lagrange multiplier test is better than the other two (Likelihood ratio LR and Wald test W) in the spanning test with small samples. In fact, when N=1, the relationships between any two of the three tests are simple. For example, $LM = W/(1 + W/T)$. We will use Lagrange multiplier in our spanning test.

To detect possible multicullinearity, we will calculate the Variance Inflation Factor (VIF) for all independent variables on OLS estimation of Eq.1.VIF is a formal method of detecting the presence of multicollinearity and it measures how much the variances of the estimated regression coefficients are inflated as compared to when the predictor variables are not linearly related.

$$(VIF)_i = (1 - R_i^2)^{-1} \qquad i = 1, 2 \ldots k \qquad (11)$$

where R_i^2 is the coefficient of multiple determination when Xi is regressed on the K-1 other X variables in the model. Mean VIF values denoted by $(\overline{VIF})$ are calculated by:

$$(\overline{VIF}) = \frac{\sum_{i=1}^{K} (VIF)_i}{K} \qquad (12)$$

VIF values of larger than 10 are often used as an indicator of serious multicollinearity problems.

3.2 Mean-variance spanning tests on small cap indexes in Asian markets

The economy and stock markets of Asian countries have grown quickly in recent years. Our attention is first focused on the small cap indexes in Asian markets. All the indexes selected and the parts of the market they represent are listed in table 1.

[Insert Table 1 about here]

Table 2 reports characteristics of monthly returns of different indexes in US and Asian markets. Through our sample period, the average returns (arithmetic or geometric) of the small-cap index are greater than that of large cap and the indexes with higher returns have higher standard deviations in each country or region. For the Zhong Hua index, the average return of MSCI Zhong Hua Growth TR is less than that of MSCI Zhong Hua Value TR.

[Insert Table 2 about here]

Table 3 presents the results of the analysis of the correlation structure of the index returns. First, the correlations between small-size portfolios and large-size portfolios are less than one. The imperfect correlation between small and large cap returns implies that small caps could potentially enhance portfolio diversification. Second, the correlation level between the US and European markets is greater than the correlation level between the US and Asian markets. For example, the correlation between the SP500 and the EUV is 0.80 and the correlation between R2000 and EUS is 0.80. However, the correlation between the SP500 and NIKL is 0.47 and the correlation between R2000 and NIKS is 0.20. Small cap indexes from Asia might bring great benefits for diversification because of their low correlation with the U.S. market. Finally, among the Asian markets, the correlation coefficients between indexes show some country specific characteristics. The correlation between HKL and NIKL is 0.46 and the correlation between HKS and NIKS is 0.42; while the correlation between HKL and CNV is 0.91 and the correlation between HKS and CNG is 0.76. Therefore, the imperfect correlation among Asian markets also may provide potential gains to portfolio diversification. In addition, although most of the correlations in table 3 are significant at the 5 percent level, the correlation between EUT and NIKS (0.1774) and the correlation between TWV and three indexes of Japan market (0.1461 to 0.1636) are not significant at 5 percent level (The critical value at 5 % significant level is 0.1780, see note on table 3).

[Insert Table 3 about here]

To provide a formal test of the hypothesis that investing in small caps of Asian markets could enlarge the efficient frontier for U.S. investors, we employ the spanning tests.

Results of LM test statistics and small sample F test statistics on small cap indexes in Asian markets are reported in Table 4. We form several benchmark portfolios in our tests. The small cap indexes from Asia have different spanning test results with these benchmark portfolios.

First, the spanning test results for Japanese small cap index (NIKS) are significant for all our benchmark portfolios and the spanning test results for other small cap indexes of Asian markets are not significant for all our benchmark portfolios. Specifically, we form the basic investment portfolio by four benchmark indexes of SP500, R2000, EMG and EUT or by five benchmark indexes of SP500, R2000, EMG, EUS and EUV. Both Japanese small cap and large cap indexes (NIKL) have significant spanning test. The spanning test for other small indexes from Chinese small cap indexes (HKS, TWG and CNG) and Singapore (SINS) are not significant. In addition, when we include NIKL in the basic portfolio, NIKS still has significant spanning test result although the correlation coefficient between NIKL and·NIKS is greater than 0.82.

Moreover, if we include both NIKL and HKS in our basic portfolio, NIKS is still significant. The correlation coefficient between HKS and NIKS is 0.42 and the correlation between HKS and NIKL is 0.54 (table 3). The situation that an index

(NIKS) could be a separate asset class and have significant spanning test even when this index is highly correlated with one of benchmark assets (NIKL) or when this index has a low correlation with one of benchmark assets (HKS) makes pair-wise correlation almost useless in identifying potential independent asset classes.

In fact, correlation is widely used among both academics and professionals to explain diversification benefits and there is an extensive literature trying to identify the factors driving the correlations. For example, Gerard et al. (2002) summarize the previous works on international diversification by the correlation method and reach the conclusion that diversifying across countries may yield higher benefits than diversifying across industries because the average correlation between the countries is noticeably lower than the average correlation between the industries. Eun et al. (2004) also use correlation as main tool to explain their findings. However, our results on Asian indexes show that pair-wise correlation is not a good indicator for studying the diversification of a portfolio.

[Insert Table 4 and Table 5 about here]

The estimated VIF, Mean VIF and R-square of OLS estimation on different index combinations are listed in tables 4 and 5. We find no evidence of serious multicollinearity problems from the results. In addition, the adjusted R-squares of OLS estimation of Japanese small index in different index combinations are from 12% to 69% when the spanning tests of NIKS are significant. However, the adjusted R-squares of other small index combinations are also in the range of 12% to 69%, but their spanning tests are insignificant. This fact means that linear relationship among indexes cannot explain spanning test result. The interactions among assets in a portfolio determine if a new asset could be a separate asset.

Our spanning test results of small cap indexes in Asian markets are interesting. Although Japan is a member of the G7 group, the small cap indexes in Japan still could be a separate asset class for U.S. investors. However, the other small cap indexes in Asian markets are not independent asset classes for diversification when indexes from major markets are available. In addition, Petrella (2005) also reports that small indexes of all European market combined are independent asset classes. To investigate diversification benefits of small cap indexes from different developed countries, we further conduct the spanning test on individual small cap indexes for the markets in G7 countries.

3.3 Mean-variance spanning tests on small cap indexes in G7 markets

There are numerous indexes for G7 markets. Considering the consistency of the methods used to develop the indexes, we focus on the small cap indexes in different countries from MSCI index family. MSCI value and MSCI small cap total return indexes in different countries are used in our study (table 6). The spanning test results for these small cap indexes are listed in table 7. We have the following findings from table 7.

[Insert Table 6 and Table 7 about here]

First, MSCI small cap total return indexes for each country are not independent asset classes when MSCI value total return indexes (used as proxies for large cap segment of individual markets) of that countries are included in the benchmark portfolio containing SP500, R2000 and EMG indexes. However, if we use NIKL as the proxy for Japanese large cap market, both NIKS and RJpS are significant in the spanning tests with the benchmark portfolio containing SP500, R2000, EMG and NIKL. Even in this situation, MSCI small cap index (JpS) is still not significant in the test.

The insignificance of the tests for MSCI small cap indexes of G7 countries in portfolios containing MSCI value indexes may be due to the composition of both MSCI value and MSCI small cap indexes. MSCI Small Cap Indexes select the most liquid securities relative to their market capitalization, and targets for index inclusion 40% of the full market capitalization of the eligible small cap universe within each industry group, within each country. Effective after the close of trading on September 28, 2001, MSCI broadened the eligible companies' full market capitalization range from USD 200 ~ 800 million to USD 200 ~1,500 million. However, the MSCI country value index is formed by dividing constituents of an underlying MSCI Standard Country Index into a value index and a growth index, each targeting 50% of the market capitalization of the underlying country index. Prior to May 30, 2003, the indices used Price/Book Value (P/BV) ratios to divide the standard MSCI country indices into value and growth indices. All securities were classified as either "value" securities (low P/BV securities) or "growth" securities (high P/BV securities), relative to each MSCI country index. Therefore, some companies may be included in both MSCI small cap and MSCI value (or NIKL) indexes at the same time.

Second, the U.S. and Japan are the two largest economies in the G7 and the world. When we build the benchmark portfolio by SP500 R2000 EMG NIKL and NIKS, the spanning test are significant for the small cap indexes of Canada and Italy but insignificant for the indexes of Germany, France and UK. These differences in the tests may be due to the characteristics of small companies listed on the stock markets of different countries. For example, there is a lot of natural resource companies listed in the Canadian market.

Moreover, when we compose the benchmark portfolio by nine assets of SP500, R2000, EMG, NIKL, NIKS and the MSCI small cap indexes of the other 4 countries, Canadian small cap and Italian small cap indexes (CaS and ItS) still have significant spanning tests. In addition, when we compose the benchmark portfolio by SP500, R2000, EMG, NIKL and the other five small indexes of G7 countries, the spanning tests for NIKS and RJpS are also significant respectively. These findings indicate that the small cap indexes in Canada, Italy and Japan could be independent asset classes in our sample period.

Most important and similar to our finding in the tests on indexes from Asian markets, even though the small index to be tested has high correlation with some of the benchmark indexes, this small cap index could still have a significant spanning test. Table 8 lists the results of correlation calculation. For example, the correlation coefficient between NIKS and NIKL is 0.82, but NIKS is still significant in most of our test. However, the spanning test of NIKS is not significant when JpV is in the benchmark portfolio. The correlation coefficient between NIKS and JpV is 0.85. On the other hand, UKS and GES do not have a correlation coefficient of more than 0.80 with any indexes, but the spanning tests on these indexes are insignificant. In addition, the adjusted R-squares of OLS estimations in table 7 also show no difference between the portfolios with significant spanning test and the combinations with insignificant spanning test. The results of spanning test on small cap indexes from Asian and G7 countries imply that mean-variance spanning test results depend on the interactions among the assets included in a portfolio. An index to be tested may be a separate asset class under some combination of benchmarks but not be a separate asset class in other benchmark portfolios. Pair-wise correlation is not a good indicator for designing the diversification portfolio and the spanning test could be used as an alternative for portfolio diversification.

[Insert Table 8 about here]

3.4 Persistence of a small cap index as an asset class

The extant literature has well documented that the results of the spanning test varies with the time period of the sample. Our previous results show that benchmark portfolios could not span some small cap indexes in the period of January 1999 to December 2004.We are interested in whether these small assets could be a separate asset class for another or a longer period of time. Unfortunately, most small cap indexes available in databases are from MSCI index family and MSCI small cap indexes have data only back to January 1999.Two non-MSCI Small cap indexes of Japanese market have data back to 1989.Therefore, we use these two small cap indexes, NIKS and RJpS, to conduct our test. The benchmark asset is composed of SP500, R2000, EMG, EUT and NIKL. The spanning test results are listed in table 9.We divide the sixteen years from January 1989 to December 2004 into three sub-periods. We conduct tests for these sub-periods respectively and the 16 year period as a whole.

[Insert Table 9 about here]

The spanning tests on Japanese small cap indexes are insignificant for the period of January 1989 to December 1993 and the period of January 1994 to December 1998. However, the spanning tests on both small cap indexes are significant at 5 % for the period of January 1999 to December 2004 and the whole period of January 1989 to December 2004. In addition, the mean VIF does not exceed 10, so there is no serious multicollinearity among independent variables in our test. Moreover, there is no difference in adjusted R-squares between portfolios with significant or insignificant spanning test.

Since the spanning tests are significant for the whole sample period of January 1989 to December 2004, and insignificant in the first and second five-year period of January 1989 to December 2004, the results in table 9 imply that not only the time that the test data covered is important to the test results, but also that the length of the holding period influence the results. A small cap index (asset) cannot be a separate asset class forever.

Moreover, in previous sections we find that the pair-wise correlation is poor indicator to identify an asset class and the spanning test may be used as an alternative for portfolio diversification. However, the results that there is no permanent independent small asset class implies that the spanning test method may be not easy to apply in practice because this method requires people to predict the ex ante behavior and interaction of many indexes (small cap indexes and benchmark assets) in a proposed portfolio.

4. Step-down mean-variance spanning test

4.1 Economic significance of the step-down spanning test

Spanning tests also have an economic interpretation. Kan and Zhou (2001) decompose the spanning test in two parts: one is related to the tangency portfolio, and the other to the global minimum variance portfolio on the efficient frontier. To better assess the statistical evidence against the spanning hypothesis, Kan and Zhou (2001) suggest researchers should examine the two components of the spanning hypothesis ($\alpha = 0$ and $\Sigma\ \beta j = 1$) individually, instead of jointly. Following Kan and Zhou (2001), we use the step-down procedure to test the spanning hypothesis.

First, we test $\alpha = 0$.

$$F_1 = (T - k - 1)(\frac{1}{U} - 1) \qquad (13)$$

where U is the ratio of unconstrained estimate of variance and the constrained estimate of variance by imposing only the constraint of $\alpha = 0$ on Eq.5. Under the null hypothesis, F_1 has a central F-distribution with 1 and $T - K - 1$ degrees of freedom.

Second, we test Σ βj = 1 conditional on α = 0.

$$F_2 = (T - K)(\frac{1}{U} - 1) \qquad (14)$$

where U is the ratio of constrained estimate of variance by imposing only the constraint of α = 0 and the constrained estimate of variance by imposing both the constraints of α = 0 and Σ βj= 1 on Eq.5. F_2 has a central F-distribution with 1 and T −K degrees of freedom, and it is independent of F_1.

Let the level of significance of the first test be α1 and that of the second test be α2. Under the step-down procedure, Kan and Zhou (2001) suggest that we accept the spanning hypothesis if we accept both tests. The significance level of the step-down test overall is

$$1 - (1 - \alpha 1)(1 - \alpha 2) = \alpha 1 + \alpha 2 - \alpha 1 \alpha 2 \qquad (15)$$

The step-down test provides information on the causes of the rejection of the traditional spanning test. If the rejection is due to the first test (F_1 significant), then the two tangent portfolios on the efficient frontier are statistically different; and if the rejection is due to the second test, the two global minimum-variance portfolios are different statistically.

[Insert Table 10 about here]

The step-down mean-variance spanning tests on a subset of our sample indexes are listed in table 10.The behavior of these small cap indexes are different. Several of them could improve the tangency portfolio (F_1 test is significant), and several of them could improve the globe minimum variance portfolio (F_2 test is significant). Moreover, several of the small cap indexes could improve both the tangency portfolio and the global minimum variance portfolio while others bring no benefits to diversification. In addition, the behavior of the small cap indexes also changes with time. NIKS and RJpS, for example, have different results in the period of 1983 to 2004 when they are included in a benchmark portfolio. NIKS and RJpS improve both the tangency portfolio and the global minimum variance portfolio in the period of January 1999 to December 2004; however, NIKS and RJpS could not add benefits to both the tangency portfolio and the global minimum variance portfolio in the period of January 1989 to December1993.

When we conduct spanning test in the section 3.3, we find that GeS is insignificant and ItS is significant (table 7). However, the results in table 10 show that the small cap index from Germany (GeS) has a significant F_1 test; while the small cap index from Italy (ItS) only has a significant F_2 test. An asset with significant F_2 test reduces risk in the global minimum variance portfolios and this fact does not necessarily mean this asset is economically important; however, the difference in the tangency portfolio can have significant economic meaning (Kan et al. 2001). The significant F_1 test is very useful for investors with a low risk aversion because they will benefit from the improvement of the tangency portfolio's characteristics. Since an asset may have insignificant spanning test and at the same time, it may still have significant F_1 test (GeS, for example), the step-down test is better than the traditional spanning test. However, even though an index does have significant F_1 or F_2 results *ex post*, we still have to find ways to work on indexes *ex ante*.

4.2. Small cap indexes of G7 countries in popular index portfolios

Our previous results show that the results of spanning test vary with the perspectives of investors. A test portfolio may be an asset class under one combination of benchmarks but may not be an asset class under another combination of benchmark assets. The composition of the benchmark portfolio is an important factor that decides whether a new asset is a separate asset class or not.

To provide more evidence that the significance of the spanning test is influenced by the combination of benchmark assets, we use some popular indexes from different markets of G7 countries in the news media to form our benchmark portfolio. Specifically, we use SPMIB to represent Italian market, SPTSX for Canadian market, FTSE100 for U.K. market, CAC40 for France Market and DAX30 for Germany market. We combine these indexes with SP500, R2000 and EMG to form the benchmark portfolio of nine indexes. The results of spanning test and step-down test on small cap indexes in G7 countries with this benchmark portfolio are listed in table 11.

[Insert Table 11 about here]

First, all small cap indexes, except for UKS, have significant spanning tests. Among these indexes with a significant spanning test, Cas, FrS and GeS only have significant F_2 test and ItS, NIKS and RJpS are significant on both F_1 and F_2 tests. The Mean VIF shows no serious multicollinearity. Moreover, similar to previous tests, there are no differences on adjusted R-Squares between the significant and insignificant spanning tests and between step-down tests.

Second, the test results of small cap indexes in table 11 are different from the results of these small cap indexes in tables 7 and 10. For example, FrS has significant LM and F2 test in table 11, but all these tests are insignificant in tables 7 and 10. In addition, GeS has significant spanning test and F_2 test when it is tested against this popular indexes portfolio, but it has insignificant spanning test in table 7 and has significant F_1 test and insignificant F_2 test in table 10. These facts show that the combinations of benchmark portfolios influence the results of the spanning test and the results of the step-down test. Therefore, the combination of benchmark is a very important factor to determine whether or not a small cap asset could bring diversification benefits and the type of benefits (F_1 or F_2 test) it could result in.

Furthermore, because the benchmark portfolio in table 11 includes indexes that are often reported in the news media to represent the performance of different markets in the world, the significant spanning tests on the small indexes indicate that these popular indexes do not tell the whole story about these markets and there is the "large-cap bias"suggested by Eun et al. (2004). Some small cap assets could be separate asset classes if an investment portfolio includes only these popular indexes.

5. Assessment of diversification benefits

5.1 Three measures of portfolio efficiency for diversification

There are many empirical measures that are used by academic and professional works to evaluate effectiveness and efficiency in portfolio diversification or portfolio management. We employ three popular measures to assess the diversification benefits when a small cap index is added in a benchmark portfolio.

5.1.1 The reduction in the portfolio risk

First, we find the global minimum variance portfolios that are formed by the benchmark assets (GMVB) and benchmark assets plus the testing asset (GMVT). Then the difference between the standard deviations of both global minimum variance portfolios is used as the measure of reduction in the portfolio risk. Let GMV represent the decrease in the standard deviation of the global minimum variance portfolios and ST represent the standard deviation of the global minimum variance portfolios. We have

$$GMV = ST_{GMVB} - ST_{GMVT} \tag{16}$$

where the subscripts represent different portfolios.

5.1.2 Sharpe Ratio

When investors think only about the mean and variance of their portfolios and they can borrow or lend at risk-free rates, the tangency portfolio of risky assets is more important than the global minimum variance in determining the return on their investments. The Sharpe ratio is one of the popular measures for portfolio efficiency. Similar to the work of Bekaert and Urias, (1996), Eun et al. (2004) and Petrella (2005), we compute the changes in the Sharpe ratio of the tangency portfolio after adding the test asset to the benchmark assets.

Petrella (2005) states that both poor out-of-sample performance of the optimal portfolio and instability of the optimal portfolio's weights are two practical problems in the estimation of risk in mean variance optimization. In addition, positive risk-free rates will result in higher returns per unit of risk for the optimal portfolios than assuming zero risk free rates and will highlight undesirable characteristics of the tangency portfolio. Jorion (1985) points out that assuming zero for the risk-free rate could reduce bias of the positive risk-free rates. Therefore, we will assume risk-free rate to be zero in our analysis.

Let SP represent the change in the portfolio's Sharpe ratio, R and ST represent the return and standard deviation of the tangency portfolio, respectively.

$$SP = \frac{R_T}{ST_T} - \frac{R_B}{ST_B} \quad (17)$$

where the subscripts of T and B represent test portfolio and benchmark portfolio, respectively.

5.1.3 Expected return change from efficient portfolios

The third measure of the diversification benefit is the gain in expected return of the efficient portfolio by adding new assets into benchmark assets. Since the change of expected return or variance measure of the efficient portfolios will give us the same results to evaluate the diversification benefit, we only present the results on the reduction in the risk of efficient portfolios.

5.2 Investment policy constraints

Investors often face restrictions on their investments. Some restrictions limit the weights (w_i) of the capital that investors could invest in different asset classes. These limitations will have an impact on our analysis. Similar to the work of Petrella (2005), we use three sets of investment policy constraints in our analysis.

5.2.1 Unconstrained policy

In unconstrained policy, all assets can be long or short up to 100% of total capital and the sum of the weights invested in each asset adds to one.

$$\sum_{i=1}^{K+N} w_i = 1 \quad \text{and} \quad -1 \le w_i \le 1 \tag{18}$$

where K is the number of benchmark assets and N is the number of test assets. In our study, N=1.

5.2.2 No short sale policy

The condition of unconstrained weights in assets could result in large long and short positions in some assets. However, large short positions are difficult to implement in practice, particularly in small and mid cap stocks; and in addition, short sales could be too costly for most of the stocks (Petrella 2005). We use no short sale policy as that portfolio weights are non-negative.

$$\sum_{i=1}^{K+N} w_i = 1 \quad \text{and} \quad 0 \le w_i \le 1 \tag{19}$$

where K is the number of benchmark assets and N is the number of test assets. In our study, N=1.

5.2.3 Upper bound policy

Upper bound policy restricts the investment position in a single asset. In portfolio management, upper and/or lower limits are frequently imposed in solving optimal portfolio problems. We set 0.5 as upper bound in the optimization process.

$$\sum_{i=1}^{K+N} w_i = 1 \quad \text{and} \quad 0 \le w_i \le 0.5 \tag{20}$$

where K is the number of benchmark assets and N is the number of test assets. In our study, N=1.

5.3 Estimation of Markowitz mean-variance frontier

Markowitz mean-variance frontiers are used to find the global minimum variance portfolios and the tangent portfolios. To compute GMV and SP in Eq.16 and Eq.17, we use historical sample estimation of risk and return as inputs to the Optimizer of Ibbotson to get the efficient frontiers. The results of diversification gains, based on GMV and SP, under constraints of Eq.18, Eq.19 and Eq.20 are listed in table 12. The empirical measures show the following characteristics.

[Insert Table 12 about here]

First, most of the small cap indexes with significant spanning tests have bigger diversification gains on both empirical measures of GMV and SP, and several of them have a gain only on one of the two empirical measures. For example, the NIKS has a significant spanning test and SINS is insignificant in our previous spanning tests. In table 12, under the unconstrained policy, NIKS in case 18 has a gain of 0.066 on SP and 0.284 on GMV, and SINS in case 23 has a gain of 0.069 on SP and 0.003 on GMV. We notice that the SP of SINS is slightly higher than that of NIKS, but SINS still is insignificant in the spanning test because GMV of SINS is very small. GeS is another example. The spanning test of GeS is insignificant in table 7; however, the SP of GeS in the case 43 is similar to SP of other small cap indexes that are significant in spanning test. In fact, the step-down test (table 10) shows GeS could bring in diversification benefit in the tangent portfolio. The results clearly show that both the gains on GMV and SP contribute to the significance of spanning tests.

Second, consistent with our previous discussion, the behavior of a small cap index in diversification depends on the composition of the benchmark portfolio and changes with time. NIKS, for example, has GMV of 0.640 and SP of 0.071 in the case 52; and as the composition of benchmark changes, it has gains of 0.071 on GMV and 0.105 on SP in the case 17. Since the results in table 12 are consistent with the step-down tests of table 10, the step-down test could provide more information than traditional spanning tests to evaluate the diversification benefits.

Most important, constraints on the investments have impacts on the potential diversification gains that a small cap index could bring. In most of our cases, constraints reduce the gains of GMV and SP of a small cap index to a benchmark portfolio. However, constraints do not necessarily result in the decrease of GMV and SP simultaneously. For example, NIKS in case 28, the constraints increase the GMV while decrease the SP at the same time. RJPS in case 57, the constraints play in a different way; they increase the SP and decrease the GMV. Moreover, with the same constraints, the diversification gains with the same small cap indexes change differently when the benchmark portfolios are different. For instance, NIKS in case 2 and case17 is combined with different benchmark portfolios, the no short sale constraint reduces both GMV and SP in the case 2, but the no short sale constraint increases GMV in the case 17. Wang (1998) suggests that imposing constraints on portfolio weights and no short selling could reduce the estimation risk and improves the efficiency of optimal portfolios estimated by sample moments. However, the results in table 12 do not support his argument. The effect of upper bound constraints is mixed. For instance, NIKS in case 2 and 17, the upper bound constraint reduce both GMV and SP in case 2 but increase GMV and reduce SP in case 17.The only difference between case 2 and case17 is that the assets included in the benchmark portfolios are different. Therefore, we need to be careful when considering constraints on the diversification benefits of small caps because the effects of constraints depend on the asset combination of a portfolio and constraints could sometimes even increase the benefit on a specific economic measure.

Fig.1 clearly shows that small cap indexes with significant spanning tests add more benefits to diversification. The point of the cases with significant spanning tests are located on the right upper part in dimensions of GMV and SP in Fig1a to Fig1c. In addition, we can also find that the constraints overall reduce the diversification benefits arising from the small cap indexes (Fig.1d).

[Insert Fig. 1 about here]

The economic significance of step-down spanning is related to the empirical measures on portfolio efficiency on the efficient frontier. In table 10, we find that both F_1 test and F_2 test are significant for CaS. Fig.2 and results in table 12 show that adding CaS to the benchmark portfolio decreases the risk and increase the Sharpe ratio. The efficient frontier is enlarged. Moreover, we find that both step-down F tests on FrS in table 10 are insignificant.The results in table 12 and Fig.3 confirm our previous tests. It is clear that FrS does not bring in the diversification benefit. Both the frontiers with/without FrS are identical.

[Insert Fig. 2 and Fig. 3 about here]

In previous tests, we also find insignificant spanning test for GeS and significant spanning test for ItS in table 7. The step- down test results of the table 10 show GeS could result in some diversification benefits in the tangent portfolio (the F_1 test is significant) but cannot reduce the risk of the global minimum variance portfolio on the efficient frontier portfolio.Fig.4 and the results in table 12 clearly reflect the result of step-down spanning test and empirical measures. In addition, the results in table 10 and 12 and Fig.5 show that ItS could decrease the risk of the global minimum variance portfolio, but brings little benefit to the tangency portfolio (F_1 is insignificant and F_2 is significant in the step-down test). Moreover, Fig. 6 and table 12 also show that UKS is not a single asset class.

[Insert Fig 4, Fig.5 and Fig.6 about here]

In the step-down mean-variance spanning test on NIKS and RJpS, we find that the diversification benefits resulting from adding the small cap indexes to benchmark portfolios change over time. In the period of 1989 to 1993, NIKS brings in almost no benefit of diversification to the tangency portfolio and little reduction in the risk of the global minimum variance portfolio. The p-value of F_1 is 0.96 and p-value of F_2 test is 0.56 (table 10). The reduction of standard deviation is 0.256 and the gain of Sharpe ratio is 0.001(table 12). But in 1994-1998, the F_2 test of NIKS is significant and GMV is 0.316. Furthermore, in the period of 1999 to 2004, NIKS are significant on both F_1 and F_2 tests (table 10), the GMV is 0.640 and Sharpe ratio increases by 0.071.The empirical measure on RJpS in table 12 are also consistent with the results of the step-down spanning test of table10. Fig.7 to Fig.9 illustrates this historical change of diversification benefits that NIKS could bring . In Fig.7 NIKS has almost no effect on the benchmark frontier; and Fig.8 shows the effect of NIKS on reducing the risk of the benchmark frontier. The effects of NIKS are the most significant in Fig.9; NIKS enlarges the frontier of the benchmark portfolio on both directions: reducing the standard deviation of the global minimum variance portfolio and increasing the Sharpe ratio of the tangent portfolio on the efficient frontier.

[Insert Fig. 7 Fig. 8 and Fig. 9 about here]

In fact, all our results on empirical measures are consistent with statistical significant analyses in the previous sections. There are clear advantages of the step-down spanning test over traditional spanning tests. By separating the sources of diversification benefits rather than combining the test statistics in the traditional spanning test, we could find more potential separate small cap assets that are relevant for the tangency portfolios or global minimum variance portfolios on Markowitz mean-variance frontier.

6. Conclusion

The spanning test is a useful approach to identify a separate asset class for portfolio diversification. The results of spanning test on small cap indexes in Asian markets and G7 countries show that the composition of the benchmark portfolio determines whether a small cap index could be a separate asset class or not. In addition, even though a small cap index is an independent asset class in one period, this small cap index may not be a separate asset class in another period with the same benchmark portfolio. The lengths of holding period and interaction among the assets in a portfolio influence the test results.

Popular indexes in the news media for major stock markets in the G7 countries do not reflect all movements of the markets. Most small cap indexes in the G7 countries could bring diversification benefits to a portfolio that consists of these popular indexes.

In general, constraints reduce the diversification benefits of small cap indexes on benchmark portfolios. However, constraints do not always reduce the usefulness of a small cap index; in some cases, constraints could even result in additional benefits. In addition, the effect of constraints is influenced by the composition of assets in a portfolio.

The commonly used pair-wise correlation between two assets is a poor indictor to analyze potential diversification benefits of adding a new asset into a portfolio. It is the interaction among all assets in a portfolio that determines whether or not adding a new asset could enlarge the original efficient frontier.

The step-down mean-variance spanning test is a powerful tool that can be used to search for a potential separate asset class given a benchmark portfolio and its results are consistent with empirical measures on variance and Sharpe ratio for portfolio efficiency. Two F tests of step-down mean-variance spanning test provide information on the effect of a new asset on the tangency portfolio and the global minimum variance portfolio on Markowitz mean-variance frontier, respectively. Therefore, based on the needs and characteristics of an investment portfolio, the step-down mean-variance spanning test could be used, as an alternative to correlation analysis, to identify a separate asset class. However, the problem of how to predict the behavior of assets in a portfolio and the interaction among these assets *ex ante* needs further study.

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Table 1

Representative indexes for different markets

Different countries/regions and their corresponding market indexes used in this paper are presented. The market segment in the table is the part of the market the index used in our study to describe that part of the market. The variable names are the names used for abbreviation in our paper. All index data are from Ibbotson Database and in U.S. dollar.

Country/Region	Index	Market Segment	Variable Name
U.S.	S&P 500 TR	Large Cap	SP500
	Russell 2000 TR	Small Cap	R2000
Emerging countries	S&P/IFCI Emerging Composite TR	Whole market	EMG
Europe	MSCI Europe TR	Whole market	EUT
	MSCI EU Value TR	Value	EUV
	MSCI Europe Small Cap TR	Small Cap	EUS
Japan	MSCI Japan TR USD	Whole market	JPT
	BARRA/Nikko Large Cap TR USD	Large Cap	NIKL
	BARRA/Nikko Small Cap TR USD	Small Cap	NIKS
Singapore	MSCI Singapore Value TR USD	Value	SINV
	MSCI Singapore Small Cap TR USD	Value	SINS
China	MSCI Zhong Hua Value TR	Value	CNV
	MSCI Zhong Hua Growth TR	Growth	CNG
Hong Kong	MSCI Hong Kong TR USD	Whole market	HKT
	MSCI Hong Kong Small Cap TR USD	Small Cap	HKS
Taiwan	MSCI Taiwan Value TR USD	Value	TWV
	MSCI Taiwan Growth TR USD	Growth	TWG

Note: Since there is no small/large cap index available in some Asian markets, we use value indexes and growth indexes of those markets in our tests.

Table 2

Summary statistics for different indexes

This table shows descriptive statistics for the monthly percentage returns for different indexes in our study. All return data are in U.S. dollar. The holding period used in the calculation is one month. The sample period is January 1999 through December 2004.

Index	T Periods	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
S&P 500 TR	72	0.10	0.21	4.60
Russell 2000 TR	72	0.71	0.90	6.15
S&P/IFCI Emerging Composite TR	72	1.13	1.33	6.42
MSCI Hong Kong TR USD	72	0.63	0.86	6.94
MSCI Hong Kong Small Cap TR USD	72	0.91	1.18	7.49
MSCI Japan TR USD	72	0.22	0.39	5.84
BARRA/Nikko Large Cap TR USD	72	0.29	0.48	6.21
BARRA/Nikko Small Cap TR USD	72	1.04	1.25	6.67
MSCI Zhong Hua Value TR	72	0.69	0.96	7.44
MSCI Zhong Hua Growth TR	72	0.48	0.73	7.18
MSCI Taiwan Value TR USD	72	-0.02	0.45	9.85
MSCI Taiwan Growth TR USD	72	0.19	0.71	10.39
MSCI Singapore Value TR USD	72	1.40	1.70	7.75
MSCI Singapore Small Cap TR USD	72	1.39	1.68	7.69
MSCI Europe TR	72	0.24	0.36	4.97
MSCI EU Value TR	72	0.59	0.73	5.30
MSCI Europe Small Cap TR	72	0.88	1.01	5.23

Table 3

Return correlation matrix of sample indexes

Pearson correlations between the series of indexes are reported. The sample period is January 1999 through December 2004.

	SP500	R2000	EMG	HKT	HKL	HKS	NIKL	NIKS	JPT	CNV	CNG	TWV	TWG	SINV	SINS	EUV	EUS	EUT
SP500	1.00																	
R2000	0.69	1.00																
EMG	0.76	0.75	1.00															
HKT	0.59	0.64	0.77	1.00														
HKL	0.66	0.67	0.79	0.96	1.00													
HKS	0.52	0.61	0.75	0.81	0.78	1.00												
NIKL	0.47	0.42	0.54	0.44	0.46	0.54	1.00											
NIKS	0.28	0.20	0.37	0.26	0.27	0.42	0.82	1.00										
JPT	0.49	0.42	0.56	0.45	0.47	0.56	0.98	0.84	1.00									
CNV	0.59	0.62	0.75	0.94	0.91	0.81	0.39	0.25	0.43	1.00								
CNG	0.61	0.65	0.79	0.90	0.92	0.76	0.52	0.29	0.50	0.80	1.00							
TWV	0.32	0.31	0.66	0.48	0.46	0.47	0.15	0.15	0.16	0.49	0.44	1.00						
TWG	0.57	0.49	0.75	0.54	0.52	0.55	0.32	0.26	0.33	0.52	0.49	0.78	1.00					
SINV	0.54	0.46	0.62	0.69	0.64	0.61	0.35	0.23	0.36	0.71	0.58	0.35	0.42	1.00				
SINS	0.54	0.51	0.61	0.70	0.67	0.65	0.36	0.32	0.37	0.71	0.64	0.34	0.37	0.88	1.00			
EUV	0.80	0.68	0.71	0.59	0.63	0.49	0.37	0.22	0.41	0.59	0.58	0.31	0.39	0.47	0.47	1.00		
EUS	0.68	0.80	0.79	0.65	0.66	0.63	0.42	0.31	0.45	0.64	0.62	0.42	0.54	0.44	0.45	0.84	1.00	
EUT	0.82	0.74	0.72	0.59	0.63	0.50	0.40	0.18	0.42	0.56	0.62	0.28	0.41	0.45	0.44	0.96	0.86	1.00

Note: The t-stats for correlation $t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{n-2}$. With n=72 and at significant level 5%, the correlation between return series is significant when their correlation coefficient is greater than 0.1780.

Table 4

Mean-variance spanning tests for Asian small cap indexes

The results of mean-variance spanning tests on different index portfolios are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to be test if it could enlarge the efficient frontier formed by *Benchmark assets*. Alpha, Sum of Beta and Adj-R^2 are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * EUV + ... + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H_0: $\alpha = 0$ and $\Sigma\ \beta j = 1$.The sample period is from January 1999 to December 2004.

Test asset	Benchmark assets	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test p-value	F Test p-value
Panel A: Japan Market							
NIKS	SP500 R2000 EMG and EUT	0.75	0.33	0.12	3.34	**14.65** **0.00**	**8.56** **0.00**
	SP500 R2000 EMG EUS and EUV	0.78	0.45	0.12	4.53	**10.18** **0.01**	**5.43** **0.01**
	SP500 R2000 EMG EUS EUV and NIKL	0.86	0.82	0.69	4.09	**6.09** **0.05**	**3.01** **0.06**
	SP500 R2000 EMG EUS EUV NIKL and HKS	0.86	0.83	0.69	4.07	**6.07** **0.05**	**2.95** **0.06**
NIKL	SP500 R2000 EMG EUS and EUV	-0.09	0.60	0.26	4.53	**7.11** **0.03**	**3.45** **0.04**
Panel B: Hong Kong Market							
HKS	SP500 R2000 EMG and EUT	-0.13	0.79	0.56	3.34	2.72 0.26	1.32 0.27
	SP500 R2000 EMG EUS and EUV	-0.10	0.85	0.56	4.53	1.34 0.51	0.63 0.54
	SP500 R2000 EMG EUS EUV and NIKL	-0.08	0.94	0.58	4.09	0.22 0.90	0.10 0.91
	SP500 R2000 EMG EUS EUV NIKL and NIKS	-0.14	0.95	0.58	4.56	0.19 0.91	0.09 0.92

Table 4 (cont.)

Mean-variance spanning tests for Asian small cap indexes

The results of mean-variance spanning tests on different index portfolios are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by *Benchmark assets.* Alpha, Sum of Beta and Adj-R^2 are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * EUV + ... + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H_0: $\alpha = 0$ and $\Sigma\ \beta j = 1$.The sample period is from January 1999 to December 2004.

Test asset	Benchmark assets	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test p-value	F Test p-value
Panel C: Taiwan Market							
	SP500 R2000 EMG and EUT	-0.90	1.10	0.59	3.34	1.61 0.45	0.77 0.47
	SP500 R2000 EMG EUS and EUV	-0.66	1.24	0.61	4.53	2.49 0.29	1.18 0.31
TWG	SP500 R2000 EMG EUS EUV and NIKL	-0.68	1.14	0.63	4.09	1.36 0.51	0.62 0.54
	SP500 R2000 EMG EUS EUV NIKL and NIKS	-0.81	1.16	0.62	4.56	1.76 0.41	0.80 0.45
Panel D: Chinese Market							
	SP500 R2000 EMG and EUT	-0.43	0.93	0.61	3.34	1.02 0.60	0.48 0.62
	SP500 R2000 EMG EUS and EUV	-0.42	0.88	0.61	4.53	1.74 0.42	0.82 0.45
CNG	SP500 R2000 EMG EUS EUV and NIKL	-0.41	0.94	0.61	4.09	0.89 0.64	0.41 0.67
	SP500 R2000 EMG EUS EUV NIKL and NIKS	-0.23	0.90	0.62	4.56	0.87 0.65	0.39 0.68
Panel E: Singapore Market							
	SP500 R2000 EMG and EUT	0.83	0.85	0.37	3.34	2.15 0.34	1.03 0.36
	SP500 R2000 EMG EUS and EUV	0.90	0.81	0.36	4.53	2.68 0.26	1.28 0.29
SINS	SP500 R2000 EMG EUS EUV and NIKL	0.90	0.82	0.35	4.09	2.45 0.29	1.15 0.32
	SP500 R2000 EMG EUS EUV NIKL and NIKS	0.55	0.89	0.38	4.56	0.89 0.64	0.40 0.67

Note: the test values in bold are significant at 10% significant level.

Table 5

VIF Results

VIF results for different combination of test assets are listed in the table. $(VIF)_i = (1 - R_i^2)^{-1}$ i = 1, 2,..., k where R_i^2 is the coefficient of multiple determination when Xi is regressed on the K - 1 other X variables in the model. Mean VIF values denoted by $(\overline{VIF})$ are calculated by:

$$(\overline{VIF}) = \frac{\sum_{i=1}^{K} (VIF)_i}{K}$$

Test Asset	Benchmark Assets									Mean
NIKS	SP500	R2000	EMG	EUT	EUS	EUV	NIKL	NIKS	HKS	VIF
	3.66	2.85	3.14	3.72						3.34
	3.93	3.37	3.77		6.29	5.31				4.53
	4.00	3.37	4.08		6.29	5.35	1.45			4.09
	4.03	3.39	5.07		6.42	5.44	1.54		2.61	4.07
NIKL	3.93	3.37	3.77		6.29	5.31				4.53
HKS	3.66	2.85	3.14	3.72						3.34
	3.93	3.37	3.77		6.29	5.31				4.53
	4.00	3.37	4.08		6.29	5.35	1.45			4.09
	4.00	3.72	4.09		6.84	5.50	4.22	3.57		4.56
TWG	3.66	2.85	3.14	3.72						3.34
	3.93	3.37	3.77		6.29	5.31				4.53
	4.00	3.37	4.08		6.29	5.35	1.45			4.09
	4.00	3.72	4.09		6.84	5.50	4.22	3.57		4.56
CNG	3.66	2.85	3.14	3.72						3.34
	3.93	3.37	3.77		6.29	5.31				4.53
	4.00	3.37	4.08		6.29	5.35	1.45			4.09
	4.00	3.72	4.09		6.84	5.50	4.22	3.57		4.56
SINS	3.66	2.85	3.14	3.72						3.34
	3.93	3.37	3.77		6.29	5.31				4.53
	4.00	3.37	4.08		6.29	5.35	1.45			4.09
	4.00	3.72	4.09		6.84	5.50	4.22	3.57		4.56

Table 6

Representative indexes for different markets in G7 countries

Different regions and countries in G7 and their corresponding market indexes used in this paper are presented. The market segment in the table is the part of the market the index used in our study to describe that part of the market. The variable names are the names used for abbreviation in our paper. All index data are from Ibbotson Database and in U.S. dollar.

Country/Region	Index	Market Segment	Variable Name
U.S.	S&P 500 TR	Large Cap	SP500
	Russell 2000 TR	Small Cap	R2000
Emerging countries	S&P/IFCI Emerging Composite TR	whole market	EMG
Canada	MSCI Canada Small Cap TR	Small Cap	CaS
	MSCI Canada Value TR	Value	CaV
France	MSCI France Small Cap TR	Small Cap	FrS
	MSCI France Value TR	Value	FrV
Germany	MSCI Germany Small Cap TR	Small Cap	GeS
	MSCI Germany Value TR	Value	GeV
Italy	MSCI Italy Small Cap TR	Small Cap	ItS
	MSCI Italy Value TR	Value	ItV
Japan	BARRA/Nikko Large Cap TR USD	Large Cap	NIKL
	BARRA/Nikko Small Cap TR USD	Small Cap	NIKS
	MSCI Japan Small Cap TR	Small Cap	JpS
	MSCI Japan Value TR	Value	JpV
	Russell/NOMURA Japan Small Cap TR	Small Cap	RJpS
U.K.	MSCI U.K. Small Cap TR	Small Cap	UKS
	MSCI U.K. Value TR	Value	UKV

Table 7

Mean-variance spanning tests for small cap indexes of G7 Countries

The results of mean-variance spanning tests on different index portfolios are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by *Benchmark assets*. Alpha, Sum of Beta and Adj-R^2 are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * NIKL + ... + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H_0: $\alpha = 0$ and $\Sigma\ \beta j = 1$.The sample period is from January 1999 to December 2004.

Test asset	Benchmark assets	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test p-value	F Test p-value
Panel A: Canada Market							
CaS	SP500 R2000 EMG and CaV	0.22	0.91	0.74	2.97	1.67 0.43	0.80 0.45
	SP500 R2000 EMG NIKL and NIKS	0.74	0.76	0.60	3.18	**8.62** **0.01**	**4.49** **0.01**
	SP500 R2000 EMG NIKL NIKS FrS GeS ItS UKS	0.72	0.79	0.58	3.74	**6.41** **0.04**	**3.03** **0.06**
Panel B: France Market							
FrS	SP500 R2000 EMG and FrV	-0.45	0.83	0.69	2.95	3.67 0.16	1.80 0.17
	SP500 R2000 EMG NIKL and NIKS	-0.37	0.93	0.57	2.77	0.88 0.65	0.41 0.67
Panel C: Germany Market							
GeS	SP500 R2000 EMG and GeV	-0.93	0.88	0.56	2.86	3.54 0.17	1.73 0.19
	SP500 R2000 EMG NIKL and NIKS	-1.17	0.96	0.54	3.18	3.98 0.14	1.93 0.15
Panel D: Italy Market							
ItS	SP500 R2000 EMG and ItV	0.42	0.95	0.70	2.46	1.62 0.44	0.77 0.47
	SP500 R2000 EMG NIKL and NIKS	0.57	0.68	0.37	3.18	**6.52** **0.04**	**3.29** **0.04**
	SP500 R2000 EMG NIKL NIKS FrS GeS CaS UKS	0.69	0.76	0.60	3.83	**6.41** **0.04**	**3.03** **0.06**

Table 7 (cont.)

Mean-variance spanning tests for small cap indexes of G7 Countries

The results of mean-variance spanning tests on different index portfolios are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by *Benchmark assets.* Alpha, Sum of Beta and Adj-R^2 are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * NIKL + ... + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H_0: $\alpha = 0$ and $\Sigma\ \beta j = 1$.The sample period is from January 1999 to December 2004.

Test asset	Benchmark assets	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test p-value	F Test p-value
Panel E: Japan Market							
NIKS	SP500 R2000 EMG and JpV	0.45	0.89	0.72	2.44	2.23 0.33	1.07 0.35
	SP500 R2000 EMG and NIKL	0.90	0.74	0.68	2.48	**9.14** **0.01**	**4.87** **0.01**
	SP500 R2000 EMG NIKL FrS GeS ItS CaS UKS	0.96	0.78	0.68	3.53	**7.06** **0.03**	**3.37** **0.04**
JpS	SP500 R2000 EMG and JpV	0.17	1.01	0.74	2.95	0.17 0.92	0.08 0.92
	SP500 R2000 EMG and NIKL	0.66	0.85	0.68	2.48	3.74 0.15	1.84 0.17
RJpS	SP500 R2000 EMG and JpV	0.35	0.92	0.77	2.44	1.68 0.43	0.80 0.45
	SP500 R2000 EMG and NIKL	0.81	0.78	0.75	2.48	**9.43** **0.01**	**5.05** **0.01**
	SP500 R2000 EMG NIKL FrS GeS ItS CaS UKS	0.76	0.82	0.75	3.53	**6.18** **0.05**	**2.91** **0.06**
Panel F: UK Market							
UKS	SP500 R2000 EMG and UKV	0.16	0.95	0.76	2.90	0.68 0.71	0.32 0.73
	SP500 R2000 EMG NIKL and NIKS	0.13	0.94	0.73	3.18	0.51 0.78	0.23 0.79

Note: the test values in bold are significant at 10% significant level.

Table 8

Return correlation matrix of indexes in G7 countries

Pearson correlations between the series of indexes are reported. The sample period is January 1999 through December 2004.

	SP500	R2000	EMG	CaS	CaV	FrS	FrV	GeS	GeV	ItS	ItV	NIKL	NIKS	JpS	JpV	RJpS	UKS	UKV
SP500	1.00																	
R2000	0.69	1.00																
EMG	0.76	0.75	1.00															
CaS	0.61	0.74	0.73	1.00														
CaV	0.69	0.57	0.59	0.75	1.00													
FrS	0.58	0.73	0.68	0.63	0.54	1.00												
FrV	0.76	0.62	0.62	0.59	0.65	0.74	1.00											
GeS	0.61	0.67	0.69	0.60	0.53	0.78	0.65	1.00										
GeV	0.71	0.69	0.63	0.57	0.61	0.75	0.83	0.67	1.00									
ItS	0.54	0.60	0.59	0.56	0.52	0.79	0.74	0.63	0.70	1.00								
ItV	0.47	0.53	0.46	0.50	0.51	0.68	0.71	0.50	0.61	0.81	1.00							
NIKL	0.47	0.42	0.54	0.47	0.31	0.29	0.32	0.27	0.25	0.30	0.26	1.00						
NIKS	0.28	0.20	0.37	0.34	0.30	0.22	0.20	0.21	0.10	0.19	0.22	0.82	1.00					
JpS	0.30	0.24	0.37	0.37	0.31	0.24	0.22	0.20	0.11	0.20	0.24	0.83	0.97	1.00				
JpV	0.41	0.27	0.43	0.42	0.44	0.21	0.32	0.21	0.20	0.26	0.28	0.89	0.85	0.86	1.00			
RJpS	0.31	0.24	0.39	0.37	0.30	0.23	0.22	0.19	0.11	0.21	0.22	0.86	0.99	0.98	0.88	1.00		
UKS	0.65	0.80	0.80	0.69	0.58	0.79	0.66	0.69	0.67	0.68	0.62	0.52	0.39	0.40	0.44	0.41	1.00	
UKV	0.74	0.49	0.63	0.56	0.62	0.55	0.79	0.56	0.70	0.64	0.64	0.35	0.22	0.21	0.38	0.24	0.65	1.00

Table 9

Mean-variance spanning tests for small cap indexes in different periods

The results of mean-variance spanning tests on the small cap index in different periods are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by *Benchmark assets*. Alpha, Sum of Beta and Adj-R^2 are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * EUT + ... + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H_0: $\alpha = 0$ and $\Sigma \beta j = 1$.The sample period is from January 1989 to December 2004 and monthly total return data in US dollar are used.

Test asset period	Benchmark assets	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test p-value	F Test p-value
Panel A: NIKS							
Jan.1989 - Dec.1993	SP500 R2000 EMG NKIL and EUT	-0.03	1.09	0.80	2.50	0.38 0.83	0.17 0.84
Jan.1994 - Dec.1998	SP500 R2000 EMG NKIL and EUT	0.08	0.77	0.77	2.46	3.18 0.20	1.51 0.23
Jan.1999 - Dec.2004	SP500 R2000 EMG NKIL and EUT	0.88	0.71	0.68	3.03	**10.07** **0.01**	**5.37** **0.01**
Jan.1989 - Dec.2004	SP500 R2000 EMG NKIL and EUT	0.48	0.80	0.75	2.23	**8.81** **0.01**	**4.47** **0.01**
Panel B: RJPS							
Jan.1989 - Dec.1993	SP500 R2000 EMG NKIL and EUT	-0.01	1.12	0.84	2.50	0.93 0.63	0.42 0.66
Jan.1994 - Dec.1998	SP500 R2000 EMG NKIL and EUT	-0.03	0.84	0.79	2.46	2.09 0.35	0.97 0.38
Jan.1999 - Dec.2004	SP500 R2000 EMG NKIL and EUT	0.79	0.76	0.75	3.03	**10.27** **0.01**	**5.49** **0.01**
Jan.1989 - Dec.2004	SP500 R2000 EMG NKIL and EUT	0.44	0.84	0.79	2.23	**7.07** **0.03**	**3.56** **0.03**

Note: the test values in bold are significant at 5 % level

Table 10

Step-down mean variance spanning test on small cap indexes

Step-down spanning tests for small cap indexes are reported. The first step (F1) is an F-test of $H_0: \alpha = 0$, and the second step(F2) is a F-test of $H_0: \Sigma \beta j = 1$ conditional on $\alpha = 0$. Significant level for both tests are calculated by $1 - (1 - \alpha 1)(1 - \alpha 2) = \alpha 1 + \alpha 2 - \alpha 1 \alpha 2$. The sample periods are the same as table 4, table 7 and table 9.

Test asset	Benchmark assets	F1-test Statistic	F1-test p-value	F2-test Statistic	F2-test p-value	Significant level
Panel A: Asian Market						
NIKS	SP500 R2000 EMG EUS and EUV	1.03	0.31	**9.84**	**0.00**	0.32
NIKL	SP500 R2000 EMG EUS and EUV	0.02	0.89	**7.33**	**0.01**	0.90
TWG	SP500 R2000 EMG EUS and EUV	0.69	0.41	1.68	0.20	0.53
HKS	SP500 R2000 EMG EUS and EUV	0.03	0.86	1.24	0.27	0.90
CNG	SP500 R2000 EMG EUS and EUV	0.59	0.44	1.05	0.31	0.62
SINS	SP500 R2000 EMG EUS and EUV	1.43	0.24	1.12	0.29	0.46
Panel B: G7 Market						
CaS	SP500 R2000 EMG NIKL and NIKS	**3.19**	**0.08**	**5.59**	**0.02**	**0.10**
FrS	SP500 R2000 EMG NIKL and NIKS	0.42	0.52	0.40	0.53	0.77
GeS	SP500 R2000 EMG NIKL and NIKS	**3.66**	**0.06**	0.19	0.66	0.68
ItS	SP500 R2000 EMG NIKL and NIKS	1.02	0.32	**5.55**	**0.02**	0.33
UKS	SP500 R2000 EMG NIKL and NIKS	0.11	0.74	0.36	0.55	0.88
NIKS	SP500 R2000 EMG and NIKL	**3.75**	**0.06**	**5.76**	**0.02**	**0.08**
RJpS	SP500 R2000 EMG and NIKL	**4.13**	**0.05**	**5.71**	**0.02**	**0.06**
Panel D:	Index history					
A: NIKS vs. SP500 R2000 EMG NKIL and EUT						
Jan.1989 - Dec.1993		0.00	0.96	0.35	0.56	0.98
Jan.1994 - Dec.1998		0.02	0.88	**3.06**	**0.09**	0.89
Jan.1999 - Dec.2004		**3.66**	**0.06**	**6.81**	**0.01**	**0.07**
Jan.1989 - Dec.2004		2.64	0.11	**6.24**	**0.01**	0.12
B: RJPS vs. SP500 R2000 EMG NKIL and EUT						
Jan.1989 - Dec.1993		0.00	0.99	0.86	0.36	0.99
Jan.1994 - Dec.1998		0.00	0.95	1.98	0.17	0.96
Jan.1999 - Dec.2004		**4.03**	**0.05**	**6.65**	**0.01**	**0.06**
Jan.1989 - Dec.2004		**2.74**	**0.10**	**4.33**	**0.04**	0.13

Note: the test values in bold are significant at 10 % level

Table 11

Mean-variance spanning tests of small cap indexes of G7 in popular benchmarks

The results of mean-variance spanning tests and step-down spanning tests for small cap indexes on different small cap indexes in G7 countries are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by popular *Benchmark assets*. Alpha, Sum of Beta and Adj-R^2 are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * NIKI300 + ... + \varepsilon_i$$

The benchmark assets are combination of popular indexes of SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30.Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H_0: $\alpha = 0$ and $\Sigma\ \beta j = 1$ The first step (F1) is an F-test of H0: α=0, and the second step (F2) is a F-test of H0: $\Sigma\ \beta j = 1$ conditional on α=0. The sample period is from January 1999 to December 2004.

Test asset	Alpha	Sum of Beta	Adj-R2	LM Test p-value	F Test p-value	F1 test p-value	F2 test p-value
CaS	0.53	0.77	0.66	**6.71** **0.03**	**3.19** **0.05**	1.88 0.18	**4.43** **0.04**
FrS	0.28	0.73	0.71	**4.91** **0.09**	2.27 0.11	0.35 0.55	**4.23** **0.04**
GeS	-0.50	0.66	0.64	**6.59** **0.04**	**3.12** **0.05**	0.84 0.36	**5.41** **0.02**
ItS	0.89	0.76	0.66	**8.33** **0.02**	**4.05** **0.02**	**4.43** **0.04**	**3.49** **0.07**
NIKS	1.06	0.69	0.72	**11.38** **0.00**	**5.82** **0.00**	**5.68** **0.02**	**5.55** **0.02**
JpS	0.73	0.80	0.75	**5.24** **0.07**	**2.43** **0.10**	2.58 0.11	2.23 0.14
RJpS	0.89	0.74	0.79	**10.82** **0.00**	**5.49** **0.01**	**5.49** **0.02**	**5.12** **0.03**
UKS	0.37	1.07	0.79	1.96 0.38	0.87 0.42	1.09 0.30	0.65 0.42

Note: 1.Mean VIF (the average Variance Inflation Factor for all independent variables) is 5.62.
2. The test values in bold are significant at 10 % level

Table 12

Diversification benefits with small indexes

Two popular empirical measures for diversification benefits on different cases are reported. GMV is the decrease in the standard deviation on the globe minimum variance portfolio after a small cap index is included in the benchmark portfolio, and SP is the increase in the Sharpe ratio on the tangent portfolio when the small cap index is included. The formula used to calculate GMV and SP are Eq.16 and Eq.17.The policy constraints are defined in Eq.18 to Eq.20. Span test indicates the results of spanning test, 1 significant and 0 insignificant. For panel A to panel C, the sample period is from January 1999 to December 2004. The sample period of panel D is from January 1989 to December 2004.

Case	*Test asset*	*Benchmark assets*	*Span test*	*Unconstrained*		*No short*		*Upper bound*	
				GMV	*SP*	*GMV*	*SP*	*GMV*	*SP*
Panel A: Small cap indexes in Asia									
3	HKS	SP500 R2000 EMG and EUT	0	0.083	0.006	0.015	0.000	0.027	0.008
4	TWG	SP500 R2000 EMG and EUT	0	0.011	0.007	0.000	0.000	0.000	0.000
5	CNG	SP500 R2000 EMG and EUT	0	0.011	0.001	0.000	0.000	0.000	0.000
6	SINS	SP500 R2000 EMG and EUT	0	0.029	0.053	0.010	0.030	0.028	0.047
2	NIKS	SP500 R2000 EMG and EUT	1	0.463	0.056	0.380	0.032	0.418	0.050
17	NIKS	SP500 R2000 EMG EUS EUV and NIKL	1	0.071	0.105	0.135	0.031	0.134	0.031
20	HKS	SP500 R2000 EMG EUS and EUV	0	0.015	0.034	0.000	0.000	0.002	0.000
21	TWG	SP500 R2000 EMG EUS and EUV	0	0.006	0.048	0.000	0.000	0.000	0.000
22	CNG	SP500 R2000 EMG EUS and EUV	0	0.003	0.039	0.000	0.000	0.000	0.000
23	SINS	SP500 R2000 EMG EUS and EUV	0	0.003	0.069	0.002	0.031	0.019	0.031
18	NIKS	SP500 R2000 EMG EUS and EUV	1	0.284	0.066	0.281	0.031	0.339	0.031
19	NIKL	SP500 R2000 EMG EUS and EUV	1	0.199	0.034	0.146	0.000	0.205	0.000
24	HKS	SP500 R2000 EMG EUS EUV and NIKL	0	-0.008	0.035	0.000	0.000	0.000	0.000
25	TWG	SP500 R2000 EMG EUS EUV and NIKL	0	-0.006	0.049	0.000	0.000	0.000	0.000
26	CNG	SP500 R2000 EMG EUS EUV and NIKL	0	-0.008	0.042	0.000	0.000	0.000	0.000
27	SINS	SP500 R2000 EMG EUS EUV and NIKL	0	0.011	0.069	0.000	0.031	0.000	0.031
28	NIKS	SP500 R2000 EMG EUS EUV NIKL and HKS	1	0.070	0.106	0.135	0.031	0.134	0.031
29	HKS	SP500 R2000 EMG EUS EUV NIKL and NIKS	0	-0.004	0.020	0.000	0.000	0.000	0.000

Table 12 (Cont.)

Diversification benefits with small indexes

Two popular empirical measures for diversification benefits on different cases are reported. GMV is the decrease in the standard deviation on the globe minimum variance portfolio after a small cap index is included in the benchmark portfolio, and SP is the increase in the Sharpe ratio on the tangent portfolio when the small cap index is included. The formula used to calculate GMV and SP are Eq.16 and Eq.17.The policy constraints are defined in Eq.18 to Eq.20. Span test indicates the results of spanning test, 1 significant and 0 insignificant. For panel A to panel C, the sample period is from January 1999 to December 2004. The sample period of panel D is from January 1989 to December 2004.

Case	*Test asset*	*Benchmark assets*	*Span test*	*Unconstrained*		*No short*		*Upper bound*	
				GMV	*SP*	*GMV*	*SP*	*GMV*	*SP*
Panel A: Small cap indexes in Asia									
31	CNG	SP500 R2000 EMG EUS EUV NIKL and NIKS	0	0.007	0.022	0.000	0.000	0.000	0.000
32	SINS	SP500 R2000 EMG EUS EUV NIKL and NIKS	0	0.000	0.037	0.000	0.021	0.000	0.021
Panel B: Small cap indexes in G 7									
1	CaS	SP500 R2000 EMG and CaV	0	0.041	0.042	0.009	0.027	0.018	0.047
7	FrS	SP500 R2000 EMG and FrV	0	0.065	0.007	0.010	0.000	0.013	0.000
8	GeS	SP500 R2000 EMG and GeV	0	0.032	0.045	-0.001	0.060	0.025	0.000
9	ItS	SP500 R2000 EMG and ItV	0	0.012	0.041	0.003	0.027	0.017	0.034
10	NIKS	SP500 R2000 EMG and JpV	0	0.036	0.038	0.024	0.030	0.019	0.035
11	JpS	SP500 R2000 EMG and JpV	0	0.000	0.005	0.000	0.009	0.000	0.011
12	RJpS	SP500 R2000 EMG and JpV	0	0.025	0.031	0.016	0.024	0.011	0.029
14	JpS	SP500 R2000 EMG and NIKL	0	0.061	0.057	0.069	0.012	0.074	0.025
13	NIKS	SP500 R2000 EMG and NIKL	1	0.186	0.101	0.170	0.032	0.204	0.050
15	RJpS	SP500 R2000 EMG and NIKL	1	0.185	0.097	0.162	0.027	0.196	0.044
16	UKS	SP500 R2000 EMG and UKV	0	0.015	0.016	0.000	0.011	0.000	0.027
42	FrS	SP500 R2000 EMG NIKL and NIKS	0	0.010	0.003	0.002	0.000	0.007	0.000
43	GeS	SP500 R2000 EMG NIKL and NIKS	0	0.002	0.056	0.000	0.000	0.002	0.000
45	UKS	SP500 R2000 EMG NIKL and NIKS	0	0.012	0.009	0.000	0.005	0.000	0.005
41	CaS	SP500 R2000 EMG NIKL and NIKS	1	0.188	0.085	0.109	0.077	0.158	0.061
44	ItS	SP500 R2000 EMG NIKL and NIKS	1	0.173	0.040	0.140	0.029	0.189	0.029

Table 12 (Cont.)

Diversification benefits with small indexes

Two popular empirical measures for diversification benefits on different cases are reported. GMV is the decrease in the standard deviation on the globe minimum variance portfolio after a small cap index is included in the benchmark portfolio, and SP is the increase in the Sharpe ratio on the tangent portfolio when the small cap index is included. The formula used to calculate GMV and SP are Eq.16 and Eq.17.The policy constraints are defined in Eq.18 to Eq.20. Span test indicates the results of spanning test, 1 significant and 0 insignificant. For panel A to panel C, the sample period is from January 1999 to December 2004. The sample period of panel D is from January 1989 to December 2004.

Case	*Test asset*	*Benchmark assets*	*Span test*	*Unconstrained*		*No short*		*Upper bound*	
				GMV	*SP*	*GMV*	*SP*	*GMV*	*SP*
Panel B: Small cap indexes in G 7									
46	NIKS	SP500 R2000 EMG NIKL FrS GeS ItS CaS UKS	1	0.115	0.078	0.128	0.013	0.128	0.025
47	RJpS	SP500 R2000 EMG NIKL FrS GeS ItS CaS UKS	1	0.097	0.065	0.118	0.009	0.118	0.020
48	ItS	SP500 R2000 EMG NIKL NIKS FrS GeS CaS UKS	1	0.142	0.050	0.077	0.005	0.082	0.016
49	CaS	SP500 R2000 EMG NIKL NIKS FrS GeS ItS UKS	1	0.127	0.062	0.047	0.053	0.050	0.050
Panel C: Small cap indexes in G 7 popular benchmark									
40	UKS	SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30	0	0.015	0.023	0.000	0.008	0.000	0.008
33	CaS	SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30	1	0.127	0.064	0.064	0.085	0.068	0.055
34	FrS	SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30	1	0.114	0.007	0.000	0.000	0.000	0.000
35	GeS	SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30	1	0.133	0.021	0.000	0.000	0.000	0.000
36	ItS	SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30	1	0.110	0.095	0.034	0.024	0.038	0.024
37	NIKS	SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30	1	0.171	0.118	0.168	0.030	0.168	0.030
38	JpS	SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30	1	0.070	0.062	0.104	0.009	0.097	0.009

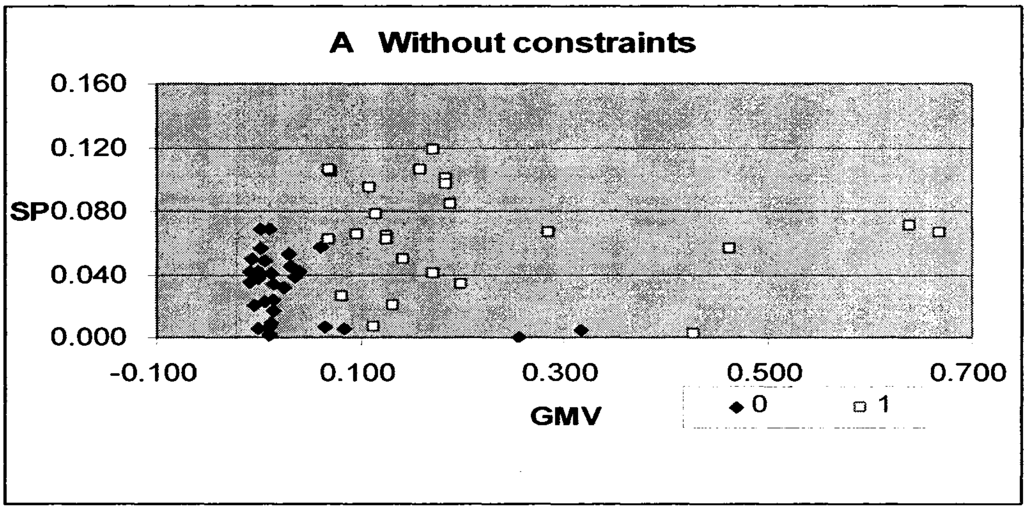
Table 12 (Cont.)

Diversification benefits with small indexes

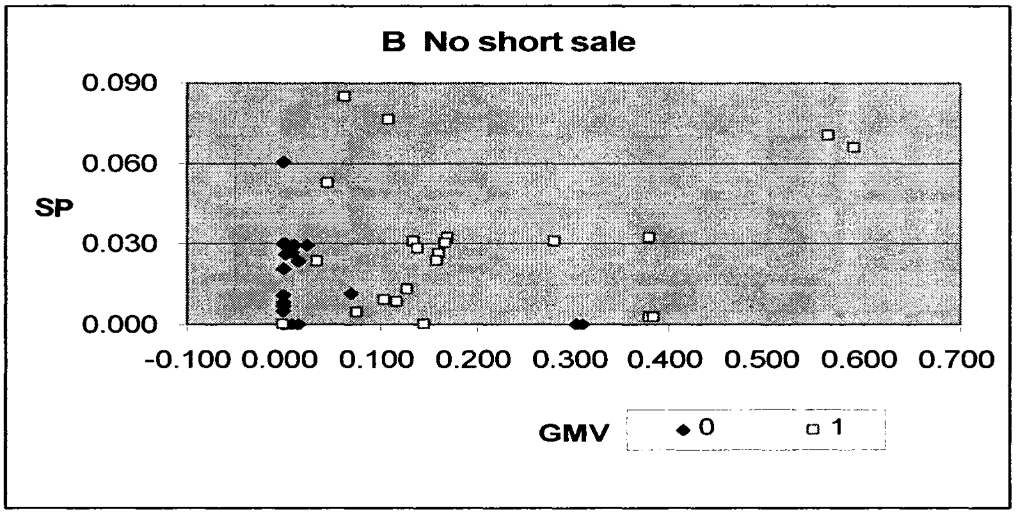
Two popular empirical measures for diversification benefits on different cases are reported. GMV is the decrease in the standard deviation on the globe minimum variance portfolio after a small cap index is included in the benchmark portfolio, and SP is the increase in the Sharpe ratio on the tangent portfolio when the small cap index is included. The formula used to calculate GMV and SP are Eq.16 and Eq.17.The policy constraints are defined in Eq.18 to Eq.20. Span test indicates the results of spanning test, 1 significant and 0 insignificant. For panel A to panel C, the sample period is from January 1999 to December 2004. The sample period of panel D is from January 1989 to December 2004.

Case	*Test asset*	*Benchmark assets*	*Span test*	*Unconstrained*		*No short*		*Upper bound*	
				GMV	*SP*	*GMV*	*SP*	*GMV*	*SP*
Panel C: Small cap indexes in G 7 popular benchmark									
39	RJPS	SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30	1	0.159	0.106	0.159	0.024	0.158	0.024
Panel D: History performance of Japanese small cap indexes									
50	NIKS 89-93	SP500 R2000 EMG NKIL and EUT	0	0.256	0.001	0.000	0.000	0.000	0.000
51	NIKS 94-98	SP500 R2000 EMG NKIL and EUT	0	0.316	0.005	0.304	0.000	0.304	0.000
54	RJPS 89-93	SP500 R2000 EMG NKIL and EUT	0	0.255	0.000	0.000	0.000	0.000	0.000
55	RJPS 94-98	SP500 R2000 EMG NKIL and EUT	0	0.317	0.004	0.309	0.000	0.000	0.000
52	NIKS 99-04	SP500 R2000 EMG NKIL and EUT	1	0.640	0.071	0.565	0.071	0.567	0.074
53	NIKS 89-04	SP500 R2000 EMG NKIL and EUT	1	0.082	0.026	0.380	0.003	0.389	0.003
56	RJPS 99-04	SP500 R2000 EMG NKIL and EUT	1	0.668	0.067	0.590	0.066	0.592	0.069
57	RJPS 89-04	SP500 R2000 EMG NKIL and EUT	1	0.428	0.002	0.385	0.002	0.394	0.003

Fig.1 Gains on reduce in the risk of the global minimum variance portfolio and increase in the Sharp ratio on the tangent portfolio. This figure provides graphical evidence of the increase of GMV and SP when spanning test is significant.Fig.1a refers to the unconstrained optimization process, Fig.1b to the no short sale constrained optimization process, Fig.1c to the upper bound constrained optimization process and Fig.1d to the unconstrained and two constrained optimization processes on the cases with significant spanning test. All the data are from table 12. The SP and GMV are estimated by Optimizer of Ibbotson Association.

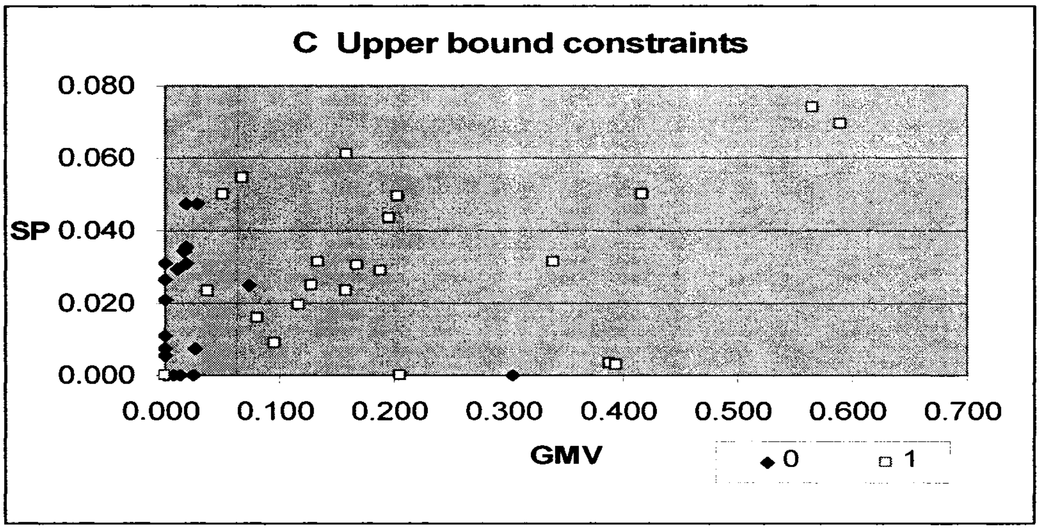


Note: 1 represents the cases with significant spanning test and 0 represents the cases with insignificant spanning test.

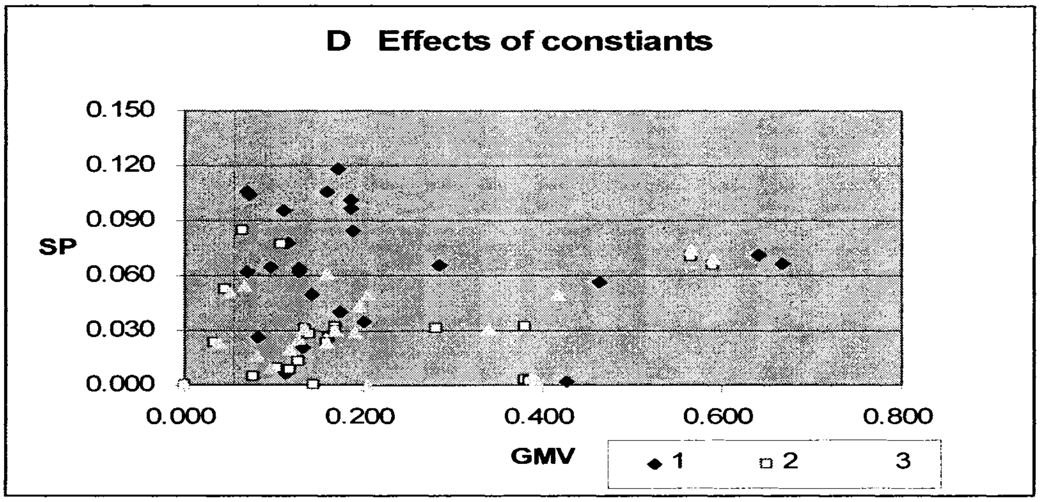


Note: 1 represents the cases with significant spanning test and 0 represents the cases with insignificant spanning test.

Fig. 1(cont.) Gains on reduce in the risk of the global minimum variance portfolio and increase in the Sharp ratio on the tangent portfolio. This figure provides graphical evidence of the increase of GMV and SP when spanning test is significant.Fig.1a refers to the unconstrained optimization process, Fig.1b to the no short sale constrained optimization process, Fig.1c to the upper bound constrained optimization process and Fig.1d to the unconstrained and two constrained optimization processes on the cases with significant spanning test. All the data are from table 12. The SP and GMV are estimated by Optimizer of Ibbotson Association.



Note: 1 represents the cases with significant spanning test and 0 represents the cases with insignificant spanning test.



Note: 1 represents the cases under constraints, 2 is under the condition of no short sale and 3 represents the cases under upper bound condition.

Fig.2 Efficient Frontier – case 41 CaS. This figure provides graphical evidence of the shift of the frontier when CaS is included in a benchmark portfolio. Case 41 refers to the case No.41 in table 12.No constraints are applied to estimate the frontier by Optimizer of Ibbotson Association.

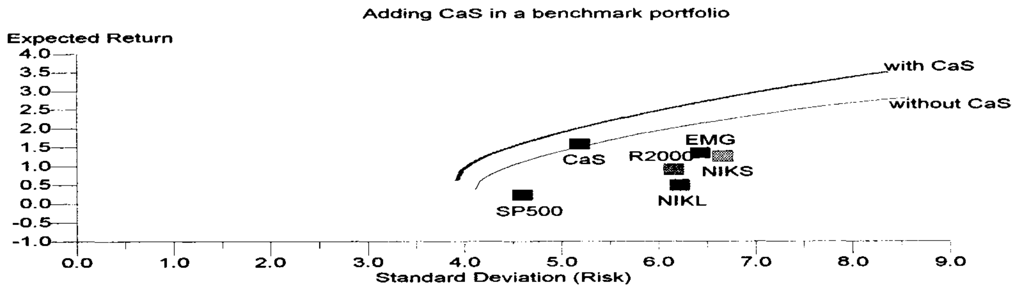


Fig.3 Efficient Frontier – case 42 FrS. This figure provides graphical evidence of the shift of the frontier when FrS is included in a benchmark portfolio. Case 42 refers to the case No.42 in table 12.No constraints are applied to estimate the frontier by Optimizer of Ibbotson Association.

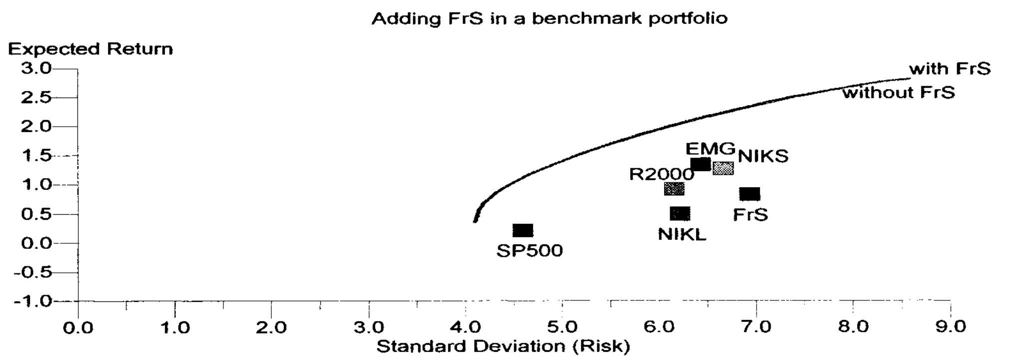


Fig.4 Efficient Frontier – case 43 GeS. This figure provides graphical evidence of the shift of the frontier when GeS is included in a benchmark portfolio. Case 43 refers to the case No.43 in table 12.No constraints are applied to estimate the frontier by Optimizer of Ibbotson Association.

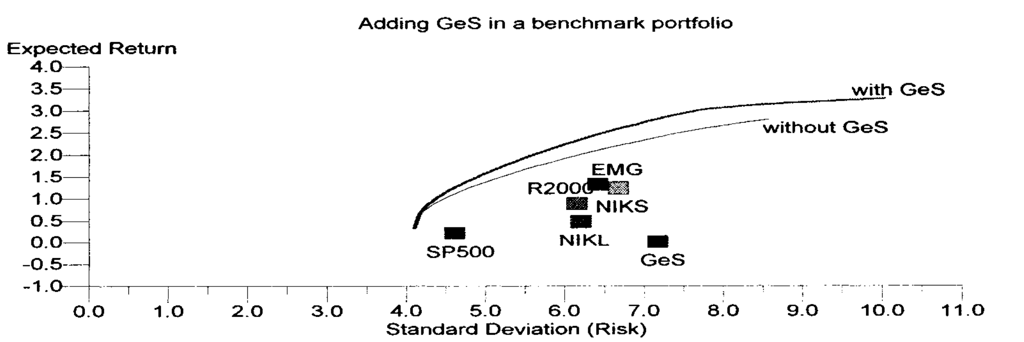


Fig.5 Efficient Frontier – case44 ItS. This figure provides graphical evidence of the shift of the frontier when ItS is included in a benchmark portfolio. Case 44 refers to the case No.44 in table 12.No constraints are applied to estimate the frontier by Optimizer of Ibbotson Association.

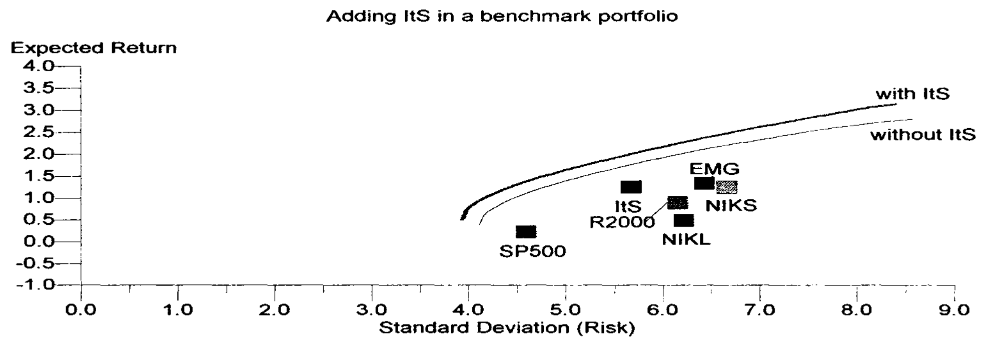


Fig.6 Efficient Frontier – case45 UKS. This figure provides graphical evidence of the shift of the frontier when UKS is included in a benchmark portfolio. Case 45 refers to the case No.45 in table 12.No constraints are applied to estimate the frontier by Optimizer of Ibbotson Association.

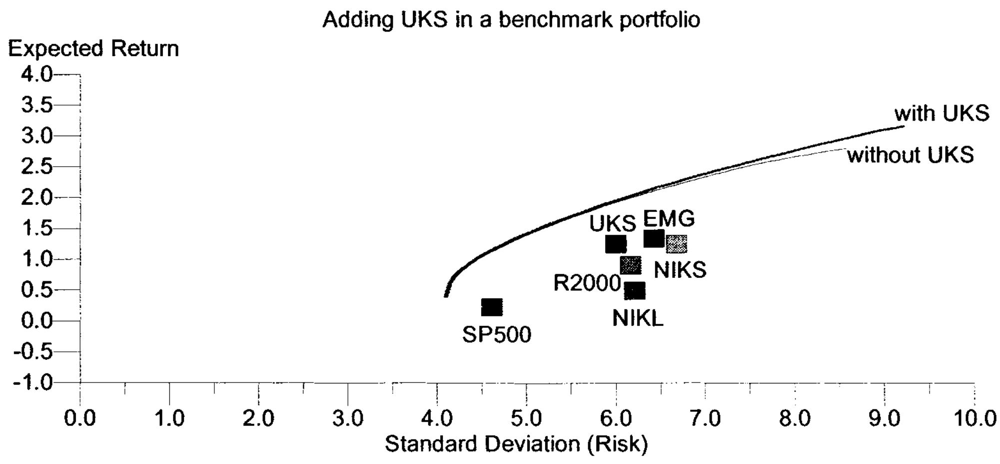


Fig.7 Efficient Frontier - case 50 NIKS 1989-1993. This figure provides graphical evidence of the shift of the frontier when NIKS of 1989-1993 is included in a benchmark portfolio. Case 50 refers to the case No.50 in table 12.No constraints are applied to estimate the frontier by Optimizer of Ibbotson Association.

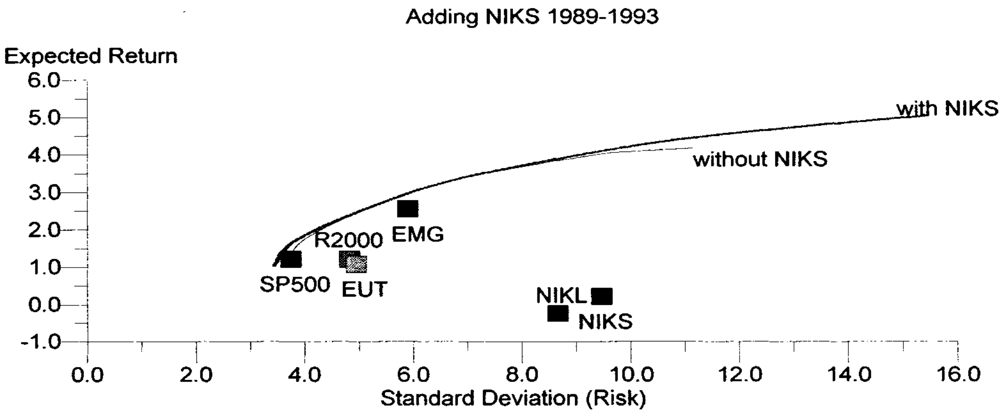


Fig.8 Efficient Frontier - case 51 NIKS 1994-1998. This figure provides graphical evidence of the shift of the frontier when NIKS of 1994-1998 is included in a benchmark portfolio. Case51 refers to the case No51 in table 12.No constraints are applied to estimate the frontier by Optimizer of Ibbotson Association.

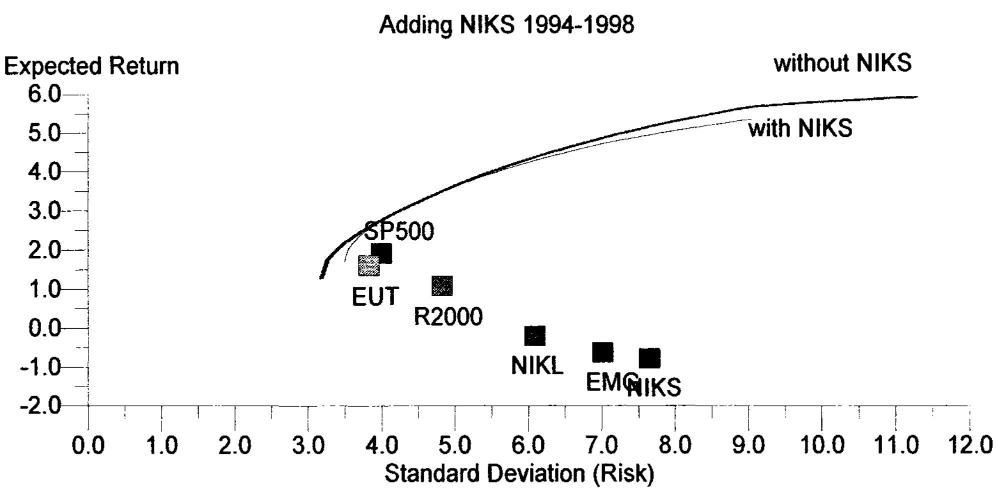
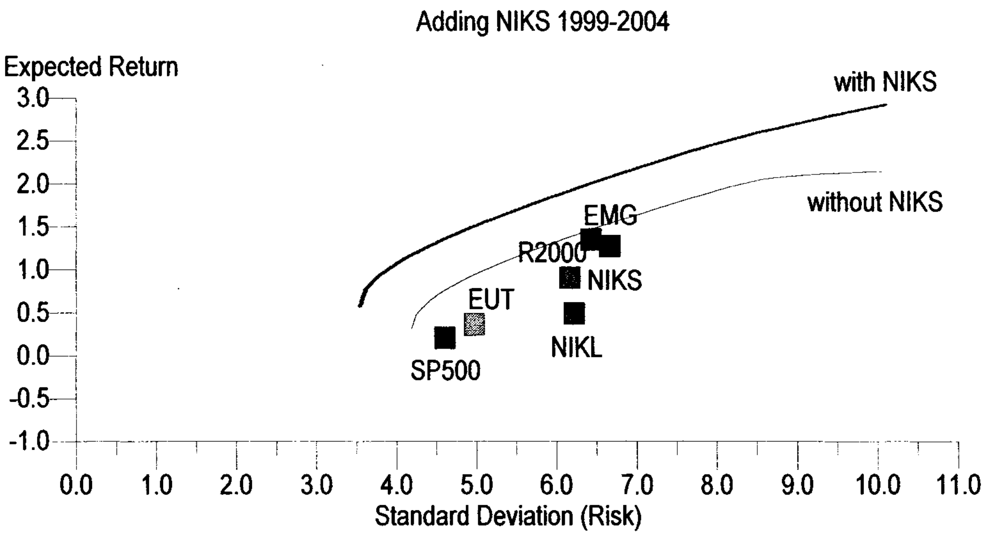


Fig.9 Efficient Frontier - case 52 NIKS 1999-2004. This figure provides graphical evidence of the shift of the frontier when NIKS of 1999-2004 is included in a benchmark portfolio. Case 52 refers to the case No.52 in table 12.No constraints are applied to estimate the frontier by Optimizer of Ibbotson Association.



Appendix 1 Description of sample indexes

1. BARRA/Nikko Style Index

Development: BARRA, INC. and the Nikko Securities Co., LTD.

Universe: All Japanese stocks listed on all the Stock Exchanges and traded on the OTC in Japan.

Base Date: The end of 1979 (=100)

Rebalance: Twice a year. End of June (The constituents of styles have been decided on the end of May) End of December (The constituents of styles have been decided on the end of November)

Dividends: Individual issue rates of return are adjusted for dividends and rights. It is assumed that dividend payments are reinvested.

Style: Large: Upper 85% of the market capitalization.

Small: Lower 15% of the market capitalization.

Source: http://www.nikko.jp/NRC/Index/style/manual.html

2. MSCI Index Serials

Development: Morgan Stanley Capital International Inc.

Source: http://www.msci.com/equity/index2.html

2.1 MSCI Small Cap Indices

Universe: 40% of the full market capitalization of the eligible small cap universe within each industry group, within each country. All listed equity securities of companies that have a company full market capitalization in the range of USD 200 – 1,500 million and a minimum free float-adjusted security market capitalization of USD 100 million comprise the small cap equity universe in each country.

Date: Most began January 1999 in our sample.

Maintenance: Semi-annual index reviews, intended to reconstitute the Small Cap Index Series on the basis of a new eligible small cap.

Quarterly index reviews, aimed at promptly reflecting significant market.

Ongoing event-related changes are generally implemented in the indices as the events occur.

Dividends: The Monthly Total Return methodologies continue to form the official index series until December 29, 2000. The Daily Total Return is re-based to the Monthly Total Return index levels of December 29, 2000 (the last trading day of 2000). MSCI's Daily Total Return methodology reinvests dividends in indices the day the security is quoted ex-dividend (ex-date).

2.2 MSCI Standard Index Series

Universe: MSCI Standard Index Series adjusts the market capitalization of index constituents for free float and targets for index inclusion 85% of free float-adjusted market capitalization in each industry group, in each country.

Maintenance: Annual full country index reviews that systematically re-assess the various dimensions of the equity universe for all countries and are conducted on a fixed annual timetable.

Quarterly index reviews, aimed at promptly reflecting other significant market events.

Ongoing event-related changes, such as mergers and acquisitions, are generally implemented in the indices promptly as they occur.

Dividends: The Monthly Total Return methodologies continue to form the official index series until December 29, 2000. The Daily Total Return is re-based to the Monthly Total Return index levels of December 29, 2000 (the last trading day of 2000). MSCI's Daily Total Return methodology reinvests dividends in indices the day the security is quoted ex-dividend (ex-date).

2.3 MSCI Growth and Value Indices

Universe: MSCI Value and Growth Index Series design is to divide constituents of an underlying MSCI Standard Country Index, into a value index and a growth index, each targeting 50% of the free float-adjusted market capitalization of the underlying index. The market capitalization of each constituent should be fully represented in the combination of the value index and the growth index, and, at the same time, should not be double-counted. A security may, however, be represented in both the value index and the growth index at a partial weight. From 1997 to May 2003, the value and growth indices have been constructed based on a single-dimensional framework that allocates securities in a MSCI Standard Country Index into either value or growth based on their Price to Book Value ratios (P/BV). Effective as of the close of May 30, 2003, MSCI applies a two-dimensional framework for style segmentation in which value and growth securities are categorized using different attributes. In addition, multiple factors are used to identify value and growth characteristics.

Maintenance: Semi-annual style index reviews and style review outside of the semi-annual style index reviews.

2.4 MSCI Zhong Hua

Universe: Aggregate of the MSCI Hong Kong Index and the MSCI China Free Index. The MSCI China Free Index represents the universe of opportunities for investment in the China equities market available to non-domestic investors. The index contains 31 stocks and has a market capitalization of USD 76.7 billion, as of November 23, 2000. MSCI has made changes to the MSCI China Free Index with effect from 1 June 2000. The new index is expanded to include Hong Kong listed companies owned by the People's Republic of China (PRC) or by companies incorporated in the PRC.

2.5 MSCI Europe Index.

Universe: As of May 2005, the MSCI Europe Index consisted of the following 16 developed market country indices: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom. MSCI captures approximately 85% of the market cap of each country.

2.6 MSCI Euro

Universe: The construction of the MSCI Euro indices begins with the securities included in the MSCI Europe Index. The less liquid securities in this broad benchmark are screened-out based on the following liquidity screen: for each security within a country, daily traded-value statistics for the latest quarter are calculated and the securities are ranked in order of decreasing liquidity. The bottom 5% of the least liquid securities, in terms of total market capitalization of the country in the MSCI Europe Index, is screened-out. This screening is performed on a country-by-country basis, since liquidity figures are not directly comparable across markets. From the list of securities that have passed the liquidity screen the largest securities in each country are selected until approximately 90% of the total market capitalization of the country in the MSCI Europe Index is captured.

Starting January 1, 1999, ten are currently eligible for the MSCI Euro Index: Austria France Netherlands Belgium Germany Portugal Finland Ireland Spain and Italy.

Maintenance: The MSCI Euro indices are reviewed annually in November to coincide with the November quarterly structural changes of the broader MSCI Europe Index. In addition, the MSCI Euro Indices are monitored continually and may be reviewed outside of the annual review in response to market-driven changes such as new issues, mergers, acquisitions, bankruptcies, and other similar corporate events.

3. Russell Indexes

Development: Frank Russell Company.

Source: http://www.russell.com/US/default.asp

3.1 Russell 3000 Index

Universe: The 3,000 largest U.S. companies based on total market capitalization, these companies represent approximately 98% of the invest-able U.S. equity market. As of the latest reconstitution, the average market capitalization was approximately $4.8 billion; the median market capitalization was approximately $944.7 million. The index had a total market capitalization range of approximately \$386.9 billion to \$182.6 million.

3.2 Russell 2000 Index

Universe: The 2,000 smallest companies in the Russell 3000 Index, which represents approximately 8% of the total market capitalization of the Russell 3000 Index. As of the latest reconstitution, the average market capitalization was approximately \$664.9 million; the median market capitalization was approximately \$539.5 million. The largest company in the index had an approximate market capitalization of \$1.8 billion.

3.3 Russell/Nomura Indexes

Russell Investment Group and Nomura Securities Co., Ltd. produce Russell/Nomura Japan Equity Indexes that serve to measure performance based on various investment policies. The indexes are value weighted and include only common stocks domiciled in Japan.

3.3.1 Russell/Nomura Total Market Index

Universe: Represents approximately 98% of the invest-able Japan equity market. As of December 2004, this index consisted of approximately 1,700 of the largest securities in Japan based on available (float adjusted) market capitalization. Over historical periods, the number of stocks included in the Total Market Index has varied from approximately 1,100 stocks in 1979, to 1,854 stocks in 2000. As of December 2004, the adjusted market capitalization range of companies in this index ranged from approximately 8.95 trillion yen to 6.2 billion yen. All other Russell/Nomura indexes are subsets of this universe of stocks.

3.3.2 Russell/Nomura Small Cap Index

Universe: Represents approximately the smallest 15% of companies ranked market value of the firms in the Russell/Nomura Total Market Index. Currently 1300 securities make up this index but this number will vary from year to year. The largest company in the index had an approximate adjusted market capitalization of 79.4 billion yen and the smallest company 6.2 billion yen as of December 2004.

4. S&P Indexes

Development: Standard & Poor's, a division of The McGraw-Hill Companies, Inc.

Source:

http://www2.standardandpoors.com/servlet/Satellite?pagename=sp/Page/HomePg&r=1&l=EN&b=10

4.1 S&P 500

Universe: 500 leading companies in leading industries of the U.S. economy. Although the S&P 500 focuses on the large-cap segment of the market, with over 80% coverage of U.S. equities it is also an ideal proxy for the total market. The index includes U.S. companies with market cap in excess of $4 billion. This market cap minimum is reviewed from time to time to ensure consistency with market conditions. The Index Committee strives to maintain a balance for the S&P 500 in line with the sector balance of the universe of eligible companies greater than $4 billion. The company in the index must be an operating company. Closed-end funds, holding companies, partnerships, investment vehicles and royalty trusts are not eligible. Real Estate Investment Trusts (REITs) are eligible for inclusion.

Maintenance: The S&P Index Committee on a regular basis, following a set of published guidelines for maintaining the index, maintains the S&P 500.

4.2 S&P/IFCI

Universe: Subsets of S&P/IFCG indices, measure the returns of stocks that are legally and practically available to foreign investors. Indices in S&P/IFCG target an aggregate market capitalization of 70-80% of the total capitalization of all exchange-listed shares. S&P/IFCI indices typically cover a high percentage of the stocks in the S&P/IFCG indices. To qualify for S&P/IFCG, a company typically must be domiciled in an emerging market and among the most actively traded securities in that market. To qualify for inclusion in S&P/IFCI, a company must have a minimum average invest-able market capitalization of US $125 million and trade at least US $50 million in the12 months prior to addition. All stocks in the S&P Emerging Markets are mapped according to the Global Industry Classification System (GICS®), which was implemented in 1999.

S&P/IFCI Composite now includes: Czech Republic Egypt Hungary China Poland Taiwan Russia India Turkey Indonesia Argentina Korea Brazil Malaysia Chile Philippines Mexico Thailand Peru Morocco Israel and South Africa.

Reconstitution: Once each year on November 1.Index constituents of S&P/IFCG and S&P/IFCI are reconstituted based on the index inclusion criteria. Share changes greater than 10% of a company's market capitalization or changes impacting a constituent's weight in the index by more than 20 basis points are made with two weeks notice. Changes that do not meet these thresholds are made on a daily basis.

4.3 S&P/TSX Composite Index

Universe: Issuers of Index Securities must be incorporated under Canadian federal, provincial, or territorial jurisdictions and listed on TSX. Securities issued by Limited Partnerships, Royalty Trusts, Real Estate Investment Trusts, and Mutual Fund Corporations, and preferred shares, exchangeable shares, warrants, installment receipts and other securities deemed inappropriate by the Committee from time to time are not eligible for inclusion in the Index. The security must represent a minimum weight of 0.025% of the Index, after including the QMV for that security in the total Float capitalization for the Index. In the event that any Index Security has a weight of more than 10% at any month-end, the minimum weights for the purpose of inclusion will be based on the S&P/TSX Capped Composite Index. Moreover, the index comprises approximately 71% of market capitalization for Canadian-based, Toronto Stock Exchange listed companies. The size of the S&P/TSX Composite (C$913.3 Billion in float market capitalization as of October, 2000) and its broad economic sector coverage has made the S&P/TSX Composite the premier indicator of market activity for Canadian equity markets since its launch on January 1, 1977.

Maintenance: The S&P/TSX Canadian Index Policy Committee maintains the Index. Meetings are held on a monthly basis and from time to time, as needed.

4.4 S&P /MIB

Development: Standard & Poor's and Borsa Italiana

Universe: The S&P/MIB currently measures the performance of 40 equities in Italy and seeks to replicate the broad sector weights of the Italian stock market.

The index is derived from the universe of stocks trading on Borsa Italiana exchanges. The S&P/MIB is market cap-weighted after adjusting constituents for free float, capturing approximately 80% of the domestic market capitalization. All stocks traded on Borsa Italiana exchanges are eligible for inclusion except savings shares (azioni di risparmio) and preferred shares. Stocks from Nuovo Mercato and foreign listed stocks are also eligible. The Index Committee strives to include the most liquid and sector-representative stocks in the Italian market. As a result, it is possible that not all 10 GICS® sectors will always be represented. GICS® methodology classifies a company according to its primary line of business as measured by revenues, earnings and/or the market perception of the stock The S&P/MIB index is based on free float market capitalization, and other factors including liquidity. The Mib30 is based on total market capitalization, as well as liquidity considerations.

Maintenance: The Index Committee maintains the S&P/MIB. The Committee meets quarterly and on an as-needed basis.

5. Nikkei 300

Development: Nihon Keizai Shimbun, Inc.

Source: http://www.nni.nikkei.co.jp/FR/SERV/nikkei_indexes/nifaq300.html

Universe: Market value-weighted index of the 300 major issues selected to represent listed stocks on the first section of the Tokyo Stock Exchange. The index is comprised of stocks with the largest market value in 36 industrial sectors. The 36 industrial sectors are chosen by Nihon Keizai Shimbun, Inc..

Maintenance: The index is reviewed every September. Changes, if any, become effective from early October. A review does not necessarily result in changes.

6. CAC 40

Source: http://www.euronext.com/editorial/wide/0,5371,1732_1203647,00.html

Universe: This index is made up of 40 shares, selected from the one hundred biggest companies listed on Euronext Paris, measured in terms of market capitalization. As the CAC40 is the benchmark for Euronext Paris, changes in the index are closely correlated to changes in the market as a whole. The index is widely used by portfolio managers to measure performance.

7. FTSE 100

Development: FTSE Group.

Universe: 100 most highly capitalized blue chip companies by full market value, representing approximately 80% of the UK market. Used extensively as a basis for investment products, such as derivatives and exchange-traded funds. This index is recognized as the measure of the UK financial markets.

Rebalance: A security will be inserted at the periodic review if it rises to 90th or above the position ranked by market value. A security will be deleted at the periodic review if it falls to 111th or below the position ranked by market value. The indices are reviewed using data from the close of the index calculation on the Tuesday after the first Friday of December for those indices reviewed annually; and the Tuesday after the first Friday of March, June, September and December for those reviewed quarterly.

Dividends: The Total Returns Indices are calculated daily.

Source: http://www.ftse.com/index.jsp

8. DAX 30

Development: Deutsche Boerse Group

Universe: A total return index of 30 selected German Prime Standard's 30 largest German companies, in terms of order book volume and market capitalization, traded on the Frankfurt Stock Exchange. The equities use free float shares in the index calculation. As of June 18, 1999 only XETRA equity prices are used to calculate all DAX indices.

Maintenance: The index is based on prices generated in the electronic trading system Xetra®. Its calculation starts at 9.00 a.m. and ends with the prices from the Xetra closing auction at 5.30 p.m. The percentages of the individual shares in the index, together with the computation factors for the current and the next trading day, are published every evening for the next trading day.

Sources: http://deutsche-boerse.com/dbag/dispatch/en/kir/gdb_navigation/home

http://www.bloomberg.com/markets/stocks/movers_index_dax.html