Analysis of the Outage Probability for Wireless Communication Systems with Multiple Antennas

Hao Shen

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Abstract

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We present in this thesis comprehensive analysis of the outage probability for multiple-input multiple-output (MIMO) systems over quasi static fading channels with and without receive antenna selection. We consider two channel models in the analysis: independent Rayleigh fading and correlated Rayleigh fading. For the independent fading case, we assume that 1) for a given M receive antennas, the receiver selects the best L antennas that maximize the capacity; 2) the channel state information (CSI) is perfectly known at the receiver, but not at the transmitter; and 3) the fading coefficients change very slowly such that averaging with respect to these coefficients is not possible. Under these assumptions, we derive two upper bounds on the outage probability with receive antenna selection. The first bound is used to show that the diversity order is maintained with antenna selection. The second bound is used to quantify the degradation in signal-to-noise ratio (SNR) due to antenna selection. Furthermore, we analyze the asymptotic behavior of the outage probability for MIMO

systems as the number of transmit antennas tends to infinity. We extend our asymptotic results to the case with receive antenna selection. For all cases, we derive explicit expressions for the threshold for the outage probability.

For spatially correlated fading channels, in addition to the assumptions made for the independent fading case, it it assumed that the spatial correlation is present at both ends of the wireless communications link, and the transmit and receive correlation matrices may or may not be full rank. With these assumptions, we derive explicit bounds for the outage probability and show that the diversity order is simply the product of the rank of the transmit correlation matrix and the rank of the receive correlation matrix. We also derive an expression for the degradation in SNR due to the presence of spatial correlation. We extend our analysis to MIMO correlated fading channels with receive antenna selection, where selection is based on maximizing the channel capacity. We derive explicit upper bounds for the outage probability and show that the diversity order with antenna selection is the same as that of the full complexity system. We also derive an expression that quantifies the loss in SNR due to antenna selection. For both channel models, we present several numerical examples for both channels models that validate our analysis.

Dedicated to my wife, my mother and father......

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Contents

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List of Acronyms

AWGN	Additive white Gaussian noise
BER	Bit error rate
CDMA	Code division multiple access
CSI	Channel state information
i.i.d.	Independent and identically distributed
MIMO	Multiple-input-multiple-output
MISO	Multiple-input-single-output
MRC	Maximum ratio combining
pdf	Probability density function
PEP	Pairwise error probability
RF	Radio frequency
rv	Random variable
SIMO	Single-input-multiple-output
SNR	Signal-to-noise ratio
W-CDMA	Wideband CDMA

Chapter 1

Introduction

1.1 Diversity and MIMO Systems

Providing reliable wireless communication has been the center of attention of the communication community over the past few years. This stems from the fact that, in a wireless environment, unlike other applications, achieving reliable communication becomes much more challenging due to the possibility that received signals from multipaths may add destructively, which, consequently, results in serious performance degradation. It has been shown that a key technique for achieving reliable wireless communication is to introduce some sort of *diversity* into the system.

Diversity comes in different forms, including time, frequency, and space diversity. A combination of two or more types of diversity is also possible, depending on the application. *Time diversity* can be achieved through employing error-correcting coding schemes combined with interleaving such as turbo codes, which provides redundancy to the receiver in the time domain. *Frequency diversity* is the case when replicas of the transmitted signal are provided to the receiver in the form of redundancy in the frequency domain. This is achieved when signals are transmitted on different frequency bands separated by more than the coherence bandwidth of the channel. *Space diversity*, or *antenna diversity*, which is the focus of this thesis, is achieved by employing spatially separated antennas at the transmitter and/or at the receiver. The separation requirements vary with antenna height, propagation environment and frequency so that the subchannels are uncorrelated.

The basic idea of space diversity is that, if two or more independent subchannels are available, these subchannels will fade in an uncorrelated manner, e.g., some subchannels are severely faded while others are less attenuated. This means that the probability of all the subchannels simultaneously fade below a certain level is much lower than the probability of any individual subchannel fades below that level. Thus, a proper combination of the signals from different subchannels results in greatly reducing the severity of fading, thereby improving the reliability of data transmission. Another advantage of space diversity compared with time and frequency diversity is that it does not induce any loss in bandwidth efficiency. This property is very attractive for high data rate wireless communications applications.

The exploitation of space diversity is based on a multiple-input multiple-output (MIMO) channel model, which employs multiple antenna elements at both the transmitter and receiver. MIMO systems are now being used for third-generation cellular systems such as wideband code-division multiple access (W-CDMA) systems and are being considered for future high-performance modes of the IEEE 802.11 standard for wireless local area networks.

MIMO systems offer a great increase in the spectral efficiency, which is based on the utilization of space diversity. The data stream from a single user is demultiplexed into N separate sub-streams. Each sub-stream is then encoded into channel symbols. After modulation, the N parallel data streams are transmitted from the N transmit antennas. At the receiver, multiple receive antennas are used to separate the different data streams. Thus, a drastic increase in the channel capacity can be achieved through MIMO systems, as shown by Telatar [1], and Foschini and Gans [2].

A number of papers have been published recently on the MIMO systems for achieving reliable communication over wireless links. They include the early work done by Guey *et al.* [3], in which they consider signal design techniques that exploit the diversity provided by employing multiple antennas at the transmitter. Then Tarokh *et al.* introduced in 1998 [4] a new class of codes (referred to as *space-time codes*) suitable for systems equipped with multiple transmit antennas. In their paper, they develop design guidelines for space-time codes over Rayleigh and Rician channels. They show that the performance of space-time codes depends heavily on the number of transmit and receive antennas employed in the system, in addition, of course, to the underlying code.[1] Since their discovery, space-time codes have enjoyed a tremendous amount of attention from the coding community, where researchers have performed considerable work on designing codes, following Tarokh's guidelines, that achieve maximum coding

[1] The performance of space-time codes (over fading channels) is characterized by two parameters: coding and diversity gains.

and diversity gains [5]–[9].

From another perspective, channel capacity and the outage probability of multiple-antenna systems are studied in [10]–[12]. In [10], the authors study the limit phenomenon of the outage probability and outage capacity for multiple-input single-output (MISO) systems as the number of transmit antennas goes to infinity. They compare their results with the Shannon capacity for additive white Gaussian noise (AWGN) channels, and show that the outage probability had a threshold phenomenon similar to that of the AWGN case. However, the analytical approach used in [10] is based on the complex Wishart distribution of the channel matrix, which becomes cumbersome to extend to the more general case, i.e., an arbitrary number of receive antennas.

In [11], Wang, *et al.* derive analytical expressions for the probability density function (pdf) of the random mutual information for MIMO systems. They show that this pdf can be approximated well by a Gaussian distribution. These expressions, however, are presented in a complicated form and thus cannot be used to analyze the limit behavior of the outage probability. Approximating the pdf of the mutual information by a Gaussian distribution is also reported in [12] in simple forms, with the assumption that the number of transmit and/or receive antennas is large. In Chapter 2, we will show how these expressions can be used to give explicit thresholds for the outage probability.

1.2 Antenna Selection for MIMO Systems

One of the drawbacks of employing multiple antennas, however, is the associated complexity. That is, the complexity that arises from employing a separate radio frequency (RF) chain for every employed antenna, which results in a significant increase in the implementation cost. *Antenna selection* has been introduced recently as a means to alleviate this complexity, while exploiting the diversity provided by the transmit and receive antennas [13]–[21] (and references therein). The idea behind antenna selection centers around using only a subset of the available antennas in MIMO systems, thereby reducing the number of required RF chains to as few as the number of selected antennas.

In [13], Molisch, *et al.* consider receive antenna selection based on maximizing the channel capacity. They demonstrate that only a small loss in capacity is suffered due to antenna selection. Efficient algorithms for performing antenna selection based on maximizing the capacity are introduced in [14]–[16]. In [14], Gorokhov proposes a sub-optimal selection strategy by successively eliminating the "worst" receive antennas. Based on this, in [15], Alkhansari *et al.* start from an empty set of selected antennas and successively add the "best" antennas to this set, which further reduces the computational complexity.

In [16], the authors provide analytical results on the outage probability for single-input multiple-output (SIMO) systems where they show that the diversity order is maintained with antenna selection. The authors extend this conclusion to MIMO systems by using the following argument. They first show that the MIMO capacity with antenna selection is lower bounded by the capacity of a set of parallel independent SIMO channels, each with antenna selection. They use this result along with the fact that a MIMO channel can be viewed as a set of parallel independent SIMO sub-channels with maximum ratio combining (MRC) at high signal-to-noise ratio (SNR) to conclude that the diversity order for MIMO systems is maintained with antenna selection.

The authors show explicitly in [17] that the diversity order of the outage probability with antenna selection over independent fading channels is the same as that of the full complexity system. They also derive a tight upper bound on the loss in SNR incurred due to antenna selection. In [18] and [19], the authors consider receive antenna selection over quasi-static fading channels, where selection is based on maximizing the received instantaneous SNR. It is shown that the diversity order of the bit error rate (BER) with antenna selection is similar to that of the full complexity system provided that the underlying space-time code is full rank. When the space-time code is not full rank, the diversity order deteriorates with antenna selection and becomes dependent on the number of selected antennas. It is also observed in [20] that the diversity order deteriorates with antenna selection when the underlying channel is fast fading. Other work related to antenna selection for MIMO systems can be found in [21]–[30].

In all the above works, it is assumed that the sub-channels fade independently. This assumption, however, is not practical, especially for systems that have poor scattering conditions and/or insufficient spacing between adjacent antennas. In such cases, the subchannels become correlated and this significantly degrades the channel capacity [31]−[34]. Therefore, it is necessary to study the effect of antenna selection on the correlated fading channels.

A widely accepted mathematical model for the correlated fading channel is presented in [35]. In deriving their model, the authors assume that the fading correlation between two distinct transmit (receive) antennas to the same receive (transmit) antenna is independent of the receive (transmit) antenna. They also assume that the fading correlation of two distinct antenna pairs is the product of the corresponding transmit and receive correlation.

In [36], the authors study the outage probability of MISO systems in the presence of transmit correlation. They show that the outage probability depends on the transmission rate as well as SNR, and they devise efficient transmission strategies when correlation is present. The authors in [37] study the pairwise error probability (PEP) for MIMO systems in the presence of fading correlation. They show that the diversity order of the PEP is given by the product of the rank of the transmit correlation matrix and the rank of the receive correlation matrix. In [38], the authors study the impact of joint transmit-receive correlation on the BER performance of combined convolutional coding and orthogonal space time block coding with receive antenna selection. They show that the diversity order with antenna selection is the same as that of the full complexity system, i.e., no antenna selection.

1.3 Information Theory Preliminaries

In this section, we recall from information theory basic definitions that are related to the main theme of this thesis, that is, outage probability [39].

Definition 1 *The entropy* $\mathcal{H}(X)$ *of a discrete random variable* X *is defined by*

$$\mathcal{H}(X) = -\sum_{X} p(x) \log p(x). \tag{1.1}$$

where $p(x)$ *is the probability density function (pdf) of* X*.*

The entropy is a measure of the average uncertainty in the random variable. If log has base 2, the entropy is the average number of bits required to describe a random variable.

Definition 2 *The joint entropy* $\mathcal{H}(X, Y)$ *of a pair of discrete random variables* (X, Y) *with a joint distribution* $p(x, y)$ *is defined as*

$$\mathcal{H}(X, Y) = -\sum_{X}\sum_{Y} p(x, y) \log p(x, y). \tag{1.2}$$

Definition 3 *The conditional entropy* $\mathcal{H}(Y|X)$ *is defined as*

$$\mathcal{H}(Y|X) = -\sum_{X}\sum_{Y} p(x, y) \log p(y|x). \tag{1.3}$$

Theorem 1 *(Chain rule):*

$$\mathcal{H}(X,Y) = \mathcal{H}(X) + \mathcal{H}(Y|X).$$

Note that $\mathcal{H}(Y|X) \neq \mathcal{H}(X|Y)$. However, $\mathcal{H}(X) - \mathcal{H}(X|Y) = \mathcal{H}(Y) - \mathcal{H}(Y|X)$.

Definition 4 *The relative entropy between two probability density function $p(x)$ and $q(x)$ is defined as*

$$D(p||q) = \sum_{X} p(x) \log \frac{p(x)}{q(x)}.$$

The relative entropy is always non-negative and is zero if and only if $p = q$.

Definition 5 *The mutual information $\mathcal{I}(X;Y)$ is the relative entropy between the joint distribution and the product distribution $p(x)\,q(x)$, i.e.,*

$$\mathcal{I}(X;Y) = \sum_{X}\sum_{Y} p(x,y) \log \frac{p(x,y)}{p(x)\,p(y)}.$$

The relationship between entropy and mutual information is given by

$$\begin{aligned}
\mathcal{I}(X;Y) &= \mathcal{H}(X) - \mathcal{H}(X|Y) \\
&= \mathcal{H}(Y) - \mathcal{H}(Y|X) \\
&= \mathcal{H}(X) + \mathcal{H}(Y) - \mathcal{H}(X,Y) \\
&= \mathcal{I}(Y;X).
\end{aligned}$$

Definition 6 *The channel capacity of a discrete memoryless channel is defined as*

$$\mathcal{C} = \max_{p(x)} \mathcal{I}(X;Y),$$

where the maximum is taken over all possible input distributions $p(x)$.

Channel capacity is the highest rate in bits per channel use at which information can be sent with arbitrarily low probability of error.

In this thesis, we consider quasi-static fading channels, where the matrix of the fading coefficients, denoted by H, remains constant over the entire frame and changes independently from one frame to another. Thus, Shannon capacity does not exist in the ergodic sense. For each realization of H, the *instantaneous mutual information* between the input and output of a quasi-static fading channel depends on the fading coefficients and therefore, it is a random variable, which is also referred to as $\mathcal{I}(X;Y|H)$.

We assume in this thesis that the channel state information (CSI) is known at the receiver, but not at the transmitter. Thus, to maximize the channel capacity, the total transmit power, denoted by P, has to be distributed equally among all the available N transmit antennas. Consequently, the instantaneous mutual information is given by

$$\begin{aligned}\mathcal{I} &= \log_2 \det\left(I_M + \frac{\rho}{N} H H^H\right) \\ &= \log_2 \det\left(I_N + \frac{\rho}{N} H^H H\right),\end{aligned}$$

where ρ is the average SNR at each receive antenna.

When the information transmission rate, denoted by R, is higher than the instantaneous mutual information, an outage event occurs.

Definition 7 *The outage probability is defined as the probability of an outage event, i.e.,*

$$\mathcal{P}_{out} = \Pr\left(\mathcal{I} < R\right).$$

1.4 Thesis Outline

The rest of the thesis is outlined as follows.

In chapter 2, we study the outage probability for MIMO systems over independent Rayleigh fading channels with receive antenna selection. We also study the asymptotic behavior of the outage probability with and without receive antenna selection as the number of transmit antennas tends to infinity.

In Chapter 3, we study the outage probability for MIMO systems over spatially correlated fading channels. We consider both the full complexity system and the system that employs receive antenna selection.

In Chapter 4, conclusions are made and directions for future work are suggested.

1.5 Thesis Contributions

The contributions of the thesis are summarized as follows.

- Two upper bounds on the outage probability for MIMO systems over independent Rayleigh channels with receive antenna selection are derived. The first bound shows that the diversity order is maintained with antenna selection. The second bound is used to accurately quantify the degradation in SNR with receive antenna selection.

- A thorough investigation of the asymptotic behavior of the outage probability as the number of transmit antennas tends to infinity is presented. Explicit expressions are derived for the threshold for the outage probability for any number of receive antennas and any number of selected antennas.

- An explicit upper bound on the outage probability over spatially correlated fading channels is derived. It is shown that the diversity order is the product of the rank of the transmit correlation matrix and the rank of the receive correlation matrix. Furthermore, a closed-form expression is derived to quantify the degradation in SNR that results from the presence of correlation.

- The outage probability over correlated fading channels with receive antenna selection is studied. To this end, an upper bound on the outage probability is derived where it is shown that the diversity order is maintained with antenna selection. Moreover, a closed-form expression for the loss in SNR due to antenna selection is derived.

Chapter 2

Analysis of the Outage Probability Over Independent Fading Channels

2.1 Introduction

In this chapter, we present a comprehensive performance analysis of the outage probability for MIMO systems with receive antenna selection. In our analysis, we assume that 1) for a given number of receive antennas M, the receiver uses L out of the available M antennas where the selected antennas are those that maximize the channel capacity; 2) the CSI is perfectly known at the receiver, but not at the transmitter; 3) the subchannel fade independently; and 4) the fading coefficients remain constant over the entire frame and change independently from one frame to another. Under these assumptions, we derive two upper bounds on the outage probability for any number of selected antennas. The first bound, albeit being loose, is used to show that the diversity order with antenna selection is the same as that of the full complexity system. The second bound is used to accurately quantify the loss in SNR due to antenna selection. Furthermore, we investigate the asymptotic behavior of the outage probability for MIMO systems as the number of transmit antennas tends to infinity. In particular, we derive explicit expressions for the threshold for these systems with and without receive antenna selection.[1] Several numerical examples are also given which validate our analysis.

The rest of the chapter is outlined as follows. In Section 2.2, we introduce the system model. The analysis of the outage probability for MIMO systems with antenna selection is presented in Section 2.3. The asymptotic phenomenon of the outage probability with and without antenna selection is analyzed in Section 2.4. Several numeral examples are discussed in Section 2.5. Finally, Section 2.6 concludes the chapter.

2.2 System Model and Some Existing Results

2.2.1 System Model

The system under consideration models a wireless communication system equipped with N transmit and M receive antennas. The received signal at time index k, in

[1] We remark that the threshold derived for MISO systems in [10] is a special case of the generalized threshold derived in this chapter.

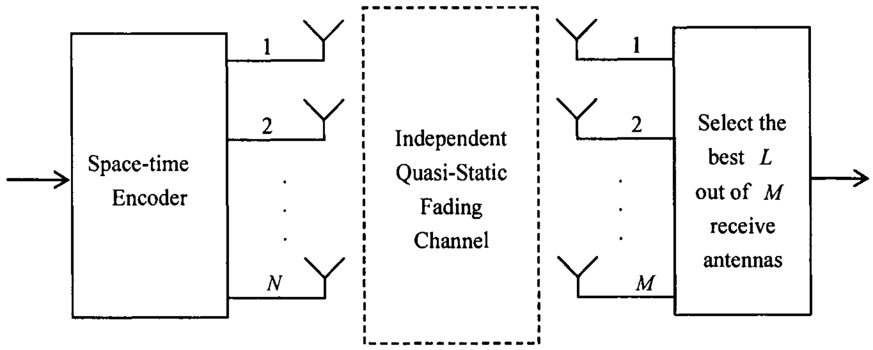

Figure 2.1: System Model. (Independent fading channel)

vector notation, is given as

$$\mathbf{y}[k] = H\mathbf{x}[k] + \mathbf{n}[k] \tag{2.1}$$

where H is an $M \times N$ matrix whose $(i, j)\,th$ entry, denoted by h_{ij}, models the fading between the ith transmit antenna and jth receive antenna, $\mathbf{x}[k]$ is a $N \times 1$ vector that represents the transmitted signal array at time k, and $\mathbf{n}[k]$ is a $M \times 1$ vector that represents the AWGN noise samples at time k. The entries of H are modeled as independent and identically distributed (iid) complex Gaussian random variables (rvs) with zero mean and variance 0.5 per dimension. The elements of $\mathbf{n}[k]$ are modeled as iid complex Gaussian rvs with zero-mean and variance $N_0/2$ per dimension. We also assume that the fading coefficients (elements of H) are constant over the entire frame and vary independently from one frame to another, which is essentially the case for quasi-static fading. Furthermore, we assume that the subchannels fade independently, and the CSI is known exactly at the receiver, but not at the transmitter.

Since the CSI is not available at the transmitter, the transmitted power has to be distributed equally among the transmit antennas to maximize the channel capacity. Hence, the average SNR at each receive antenna is given by $\rho \triangleq P/\left(N_0 B\right)$, where P is the total transmitted power and B is the transmission bandwidth. Let us define $\gamma \triangleq E_b/N_0$, where E_b denotes the average energy per transmitted bit. Thus, the relationship between ρ and γ is given by $\gamma = \rho/R$, where R is the information transmission rate in bits/sec/Hz.

2.2.2 Preliminaries

The mutual information of a MIMO channel is given by [1]

$$\mathcal{I} = \log_2 \det \left(I_M + \frac{\rho}{N} H H^H \right), \tag{2.2}$$

where $(\cdot)^H$ denotes Hermition transpose. In a full-rank system, (2.2) can be simplified by using singular value decomposition as

$$\mathcal{I} = \sum_{i=1}^{M} \log_2 \left(1 + \frac{\rho}{N} \lambda_i \left(H H^H \right) \right), \tag{2.3}$$

where $\lambda_i\left(HH^H\right)$ are the eigenvalues of HH^H. The joint pdf of these eigenvalues, after being ordered according to their amplitude, is given by [1]

$$p_{order}(\lambda_1,\ldots,\lambda_M) = K_{M,N}^{-1}\left(\prod_i \lambda_i^{N-M}\right)\left(\prod_{i>j}(\lambda_i-\lambda_j)^2\right)\exp\left(-\sum_i \lambda_i\right), \tag{2.4}$$

where $K_{M,N}$ is a normalizing factor.

When the information transmission rate is greater than the instantaneous mutual information, an outage event occurs. In quasi-static fading, since the fading coefficients are constant over the whole frame, we can not average them with an ergodic measure. In such an event, Shannon capacity does not exist in the ergodic sense [40]–[42]. The probability of such an event is normally referred to as *outage probability.* Owing to the recent work by Hochwald *et al.* in [12], the distribution of the random mutual information can be viewed as Gaussian when the number of transmit and/or receive antennas goes to infinity. (It is also a very good approximation for even small N and M, e.g. $N = M = 2$, see [12]). As such, for a sufficiently large N, the mutual information is approximated as [12][2]

$$\mathcal{I} \rightarrow \mathcal{N}\left(M\log_2(1+\rho), \frac{M\rho^2\left(\log_2 e\right)^2}{N(1+\rho)^2}\right). \tag{2.5}$$

This result will greatly simplify our analysis when we study the asymptotic behavior of the mutual information later in this chapter.

[2]It is also reported in [11] that the mutual information can be well approximated by a Gaussian distribution.

2.3 Analysis of the Outage Probability Over Independent Fading Channels with Receive Antenna Selection

In this section we study the impact of receive antenna selection on the diversity order of the outage probability, as well as the degradation in SNR due to antenna selection.

2.3.1 Diversity Order with Antenna Selection

Let H_{sel} denote a matrix of size $L \times N$ formed by selecting L rows of the matrix H. Clearly, there are $\binom{M}{L}$ subsets to choose from, but the selected subset is the one that results in maximizing the channel capacity described by (2.2) with H replaced by H_{sel}. Note that $H_{sel}H_{sel}^H$ is a submatrix of HH^H. However, the eigenvalues of $H_{sel}H_{sel}^H$ are not necessarily a subset of the eigenvalues of HH^H. For convenience, we shall assume for the rest of this chapter, unless otherwise stated, that the eigenvalues of any matrix that we encounter are ordered from smallest to largest according to their amplitude, e.g., $\lambda_1\left(H_{sel}H_{sel}^H\right) \leq \cdots \leq \lambda_L\left(H_{sel}H_{sel}^H\right)$.

When the receiver selects the best L antennas, the mutual information given by (2.3) becomes

$$\mathcal{I}_{sel} = \sum_{i=1}^{L} \log_2\left(1 + \frac{\rho}{N}\lambda_i\left(H_{sel}H_{sel}^H\right)\right). \tag{2.6}$$

Deriving a closed-form expression for the outage probability with antenna selection requires finding a closed-form expression for the joint pdf of the random variables $\lambda_i\left(H_{sel}H_{sel}^H\right)$ for $i = 1, \ldots, L$, which is unfortunately very cumbersome to obtain. As an alternative, we first derive an explicit upper bound on the outage probability from which one can easily see the impact of antenna selection on the diversity order. However, this bound is very loose and thus can not be used to quantify the degradation in SNR due to antenna selection. To this end, we later derive another tight upper bound to quantify the SNR degradation.

From [43, p. 189], we have

$$\begin{aligned}
\max_{1\le j\le n} \lambda_i\left(A_j\right) &\ge \frac{n-i}{n}\lambda_1\left(A\right) + \frac{i}{n}\lambda_{i+1}\left(A\right) \\
&\ge \frac{i}{n}\lambda_{i+1}\left(A\right) \qquad i = 1, \ldots, n-1,
\end{aligned} \tag{2.7}$$

where A_j is the $(n-1)\times(n-1)$ principal submatrix of A obtained by deleting the jth row and jth column from A. It is assumed in (2.7) that the eigenvalues λ_i for $i = 1, \ldots, n-1$ are nonnegative and placed in an increasing order. Applying (2.7) to $H_{sel}H_{sel}^H$ iteratively $M - L$ times yields

$$\begin{aligned}
\lambda_i\left(H_{sel}H_{sel}^H\right) &\ge \frac{L!\,(i+M-L-1)!}{M!\,(i-1)!}\lambda_{i+M-L}\left(HH^H\right) \\
&\ge \frac{L!\,(M-L)!}{M!}\lambda_{i+M-L}\left(HH^H\right) \\
&= \frac{\lambda_{i+M-L}\left(HH^H\right)}{\binom{M}{L}} \qquad i = 1, \ldots, L.
\end{aligned} \tag{2.8}$$

Thus, the outage probability, when the best L receive antennas are selected, is upper bounded by

$$
\begin{aligned}
\mathcal{P}_{sel} &= \Pr\left(\sum_{i=1}^{L}\log_2\left(1+\frac{\rho}{N}\lambda_i\left(H_{sel}H_{sel}^H\right)\right)<R\right)\\
&\leq \Pr\left(\sum_{i=1}^{L}\log_2\left(1+\frac{\rho}{N}\frac{\lambda_{i+M-L}\left(HH^H\right)}{\binom{M}{L}}\right)<R\right)\\
&\leq \Pr\left(\log_2\left(1+\frac{\rho}{N}\frac{\lambda_M\left(HH^H\right)}{\binom{M}{L}}\right)<R\right)\\
&\leq \Pr\left(\log_2\left(1+\frac{\rho}{N}\frac{1}{\binom{M}{L}}\frac{1}{M}\sum_{i=1}^{M}\lambda_i\left(HH^H\right)\right)<R\right)\\
&= \Pr\left(\log_2\left(1+\frac{\rho}{NM}\frac{1}{\binom{M}{L}}tr\left(HH^H\right)\right)<R\right)\\
&= \Pr\left(tr\left(HH^H\right)<\left(2^R-1\right)\binom{M}{L}\frac{NM}{\rho}\right)\\
&= P\left[\frac{2^R-1}{R}\binom{M}{L}\frac{NM}{\gamma},NM\right],
\end{aligned}
\tag{2.9}
$$

where $tr\left(A\right)$ denotes the trace of A, and $P(x,a)$ is the normalized incomplete gamma function defined as $P(x,a)=\frac{1}{\Gamma(a)}\int_0^x u^{a-1}e^{-u}du$, $x\geq 0$.

By using the power series expansion of $P(x,a)$ [44, p. 262]

$$
P(x,a)=x^a e^{-x}\sum_{n=0}^{\infty}\frac{x^n}{\Gamma(a+n+1)},
\tag{2.10}
$$

and letting $x\to 0$, we have $P(x,a)\to\frac{x^a}{\Gamma(a+1)}$. As such, the expression in (2.9) can be approximated at high SNR as

$$
P\left[\frac{2^R-1}{R}\binom{M}{L}\frac{NM}{\gamma},NM\right]\to k_0\gamma^{-NM}
\tag{2.11}
$$

where

$$k_0 = \frac{\left(\frac{2^R-1}{R}\binom{M}{L}NM\right)^{NM}}{(NM)!}.$$

Expression (2.11) suggests that the diversity order is maintained with antenna selection for any N, M, and L. However, this expression cannot be used to quantify the outage probability simply because the bound is too loose. This motivates us to derive another upper bound on the outage probability which is very tight.

2.3.2 Upper Bound on the SNR Degradation

Deriving an explicit upper bound on the SNR degradation seems very difficult to accomplish. As an alternative, we follow the following approach to derive an upper bound which can be evaluated numerically to accurately quantify the degradation in SNR. First, we derive a lower bound on the outage probability for any number of selected antennas, $1 \leq L \leq M$. Second, we show that the discrepancy between the actual outage probability and this lower bound increases with L. (This discrepancy is zero when $L = 1$ and reaches its maximum when $L = M$.) Lastly, the final upper bound is obtained by adding the maximum discrepancy to the lower bound, which is essentially equivalent to shifting the lower bound to the right by the maximum discrepancy (in dB). As we will show later, the resulting upper bound is quite tight for any number of selected antennas.

Lower Bound on the Outage Probability

When the best L out of M receive antennas are selected, the outage probability is given by

$$\mathcal{P}_{sel} = \Pr\left(\sum_{i=1}^{L}\log_2\left(1+\frac{\rho}{N}\lambda_i\left(H_{sel}H_{sel}^H\right)\right) < R\right). \tag{2.12}$$

For a concave function $\kappa(t)$, we have [43, Appendix B]

$$\kappa\left(\sum_i \alpha_i t_i\right) \geq \sum_i \alpha_i \kappa(t_i), \tag{2.13}$$

where $\alpha_i \geq 0$ and $\sum_i \alpha_i = 1$. Clearly, equality holds when all t_i are equal or when the sum has only one term. Note that $\log_2\left(1+\frac{\rho}{N}t\right)$ is a concave function in t. As such, let $\kappa(t) = \log_2\left(1+\frac{\rho}{N}t\right)$ and $\alpha_i = 1/L$. Consequently,

$$\begin{aligned}\sum_{i=1}^{L}\log_2\left(1+\frac{\rho}{N}\lambda_i\left(H_{sel}H_{sel}^H\right)\right) &\leq L\log_2\left(1+\frac{\rho}{N}\frac{1}{L}\sum_{i=1}^{L}\lambda_i\left(H_{sel}H_{sel}^H\right)\right)\\ &= L\log_2\left(1+\frac{\rho}{N}\frac{1}{L}tr\left(H_{sel}H_{sel}^H\right)\right).\end{aligned} \tag{2.14}$$

Substituting (2.14) into (2.12) yields

$$\begin{aligned}\mathcal{P}_{sel} &\geq \Pr\left(L\log_2\left(1+\frac{\rho}{N}\frac{1}{L}tr\left(H_{sel}H_{sel}^H\right)\right) < R\right)\\ &= \Pr\left(tr\left(H_{sel}H_{sel}^H\right) < \left(2^{R/L}-1\right)\frac{NL}{\rho}\right).\end{aligned} \tag{2.15}$$

Since $H_{sel}H_{sel}^H$ is a principal submatrix of HH^H, the trace of $H_{sel}H_{sel}^H$ will be less than (or equal to) the sum of the largest L diagonal entries in HH^H. Let us assume, without loss of generality, that the diagonal entries of HH^H are sorted from smallest to largest, i.e., $X_{1:M} \leq X_{2:M} \leq \cdots \leq X_{M:M}$, where $X_{i:M}$ denotes the ith diagonal entry of HH^H after sorting. (Note that $X_{i:M}$ for $i = 1, 2, \ldots, M$ are Chi-square rvs each with $2N$ degrees of freedom.) As such, (2.15) can be further lower bounded as

$$\mathcal{P}_{sel} \geq \Pr\left(\sum_{i=M-L+1}^{M} X_{i:M} < \left(2^{R/L} - 1\right)\frac{NL}{\rho}\right). \tag{2.16}$$

When $L = 1$, equality holds in (2.16), and thus the lower bound overlaps with the exact outage probability. Using [45, p. 9], we have

$$\begin{aligned} \mathcal{P}_{sel} &= \Pr\left(X_{M:M} < \left(2^R - 1\right)\frac{N}{\rho}\right) \\ &= \left[P\left(\left(2^R - 1\right)\frac{N}{\rho}, N\right)\right]^M. \end{aligned} \tag{2.17}$$

Maximum Discrepancy Between the Lower Bound and the Exact Outage Probability

It is clear that increasing the number of terms in (2.14) will lead to greater discrepancies. Thus, the maximum discrepancy occurs when all the M receive antennas are used. In this section, we derive an upper bound on this 'maximum' discrepancy. In [43, p. 466], it is shown that the function $f(A) = \log\det(A)$ is strictly concave on the convex set of positive definite Hermitian matrices A of square dimension. Armed with this result and that given by (2.13), we have

$$\log\det\left(\sum_i \alpha_i A_i\right) \geq \sum_i \alpha_i \log\det\left(A_i\right), \tag{2.18}$$

where $\alpha_i \geq 0$, $\sum_i \alpha_i = 1$, and the matrices A_i are all positive definite Hermitian.

Let $W = \frac{\rho}{N} H H^H$ and $A = I_M + W$ where I_M is the identity matrix of size $M \times M$. Now define

$$A_i \triangleq \begin{pmatrix} \frac{M}{M-1} & & & \frac{M}{2}w_{1i} & & & \\ & \ddots & & \vdots & & & \\ & & \frac{M}{M-1} & \frac{M}{2}w_{i-1,i} & & & \\ \frac{M}{2}w_{1i}^* & \cdots & \frac{M}{2}w_{i-1,i}^* & M w_{ii} & \frac{M}{2}w_{i,i+1} & \cdots & \frac{M}{2}w_{iM} \\ & & & \frac{M}{2}w_{i,i+1}^* & \frac{M}{M-1} & & \\ & & & \vdots & & \ddots & \\ & & & \frac{M}{2}w_{iM}^* & & & \frac{M}{M-1} \end{pmatrix}, \tag{2.19}$$

and let $\alpha_i = \frac{1}{M}$, where w_{ij} represents the (i,j)th entry of W. Consequently, $A = \sum_{i=1}^{M} \alpha_i A_i$. Using (2.18) and (2.19) yields

$$\begin{aligned} \log_2 \det\left(A\right) &\geq \sum_{i=1}^{M} \alpha_i \log_2 \det\left(A_i\right) \\ &= \frac{1}{M}\sum_{i=1}^{M} \log_2\left[\frac{M^M}{(M-1)^{M-1}} w_{ii} - \frac{1}{4}\frac{M^M}{(M-1)^{M-2}} \sum_{j\neq i} \left|w_{ij}\right|^2\right] \end{aligned} \tag{2.20}$$

When $N \geq M$, (2.20) is dominated by the first term inside the log function. In such cases, (2.20) can be simplified as

$$\begin{aligned}\log_2 \det(A) &\geq \frac{1}{M}\sum_{i=1}^{M}\log_2\left[\frac{M^M}{(M-1)^{M-1}}w_{ii}\right] \\ &= \log_2\left[\frac{M^M}{(M-1)^{M-1}}\right]+\log_2\left[\left(\prod_{i=1}^{M}w_{ii}\right)^{\frac{1}{M}}\right]. \end{aligned} \tag{2.21}$$

Furthermore, when $N > 3$, the entries w_{ii} will approach their arithmetic mean, and

$$\left(\prod_{i=1}^{M}w_{ii}\right)^{\frac{1}{M}} \approx \frac{1}{M}\sum_{i=1}^{M}w_{ii} = \frac{1}{M}tr\left(W\right). \tag{2.22}$$

Hence, (2.21) can be further simplified as

$$\log_2 \det(A) \geq (M-1)\log_2\left(\frac{M}{M-1}\right)+\log_2\left(tr\left(W\right)\right), \tag{2.23}$$

and thus the outage probability for the full complexity system can be upper bounded as[3]

$$
\begin{aligned}
\mathcal{P}_{full} &= \Pr\left(\log_2 \det(A) < R\right) \\
&\leq \Pr\left((M-1)\log_2\left(\frac{M}{M-1}\right) + \log_2\left(tr\left(W\right)\right) < R\right) \\
&= \Pr\left(tr\left(W\right) < 2^R \left(\frac{M-1}{M}\right)^{M-1}\right) \\
&= \Pr\left(tr\left(HH^H\right) < 2^R \left(\frac{M-1}{M}\right)^{M-1} \frac{N}{\rho}\right) \\
&= P\left(2^R \left(\frac{M-1}{M}\right)^{M-1} \frac{N}{\rho}, MN\right). \qquad (2.24)
\end{aligned}
$$

By combining the upper bound (given by (2.24)) and the lower bound (given by (2.16) when $L = M$), it is easy to see that the maximum discrepancy (when $L = M$) is given by

$$\zeta = 10\log_{10}\left[\frac{2^R}{\left(2^{R/M}-1\right)M}\left(\frac{M-1}{M}\right)^{M-1}\right] \text{ dB}. \qquad (2.25)$$

Upper Bound on the Outage Probability

Combining (2.16) and (2.25) yields an upper bound on the outage probability with receive antenna selection, which is given by

$$\mathcal{P}_{sel} \leq \Pr\left(\sum_{i=M-L+1}^{M} X_{i:M} \leq \left(2^{R/L}-1\right)\frac{NL}{\rho}\delta\right), \qquad (2.26)$$

[3]Expression (2.24) is a true upper bound when $N \geq M$. It also works very well when $N < M$ and for small values of N, as we will demonstrate later.

where $\delta = 10^{\varsigma/10}$. Although (2.26) is not in closed-form, it is easy to evaluate numerically and it yields a very tight upper bound on the SNR degradation for any number of selected antennas. We remark that this upper bound is a true upper bound for $N \geq M > 3$, as was assumed throughout the derivation that led to (2.25). However, this upper bound still yields very accurate estimates of the SNR degradation even when the constraints on the values of N and M are severely violated, e.g., $N = 2$ and $M = 8$.

2.4 Asymptotic Behavior of the Outage Probability with and without Antenna Selection

2.4.1 Full Complexity System

The limit phenomenon of the outage probability for MISO systems is analyzed in [10]. In this section, we extend the results in [10] to MIMO systems, i.e., to an arbitrary number of receive antennas, M. The analytical approach used in [10], however, is cumbersome to extend to the more general case. Alternatively, we use the Gaussian approximation result for the random mutual information given by (2.5) to carry out our analysis.

From (2.5), we can see that when $N \to \infty$, the ratio $\left(M\rho^2 \log_2^2 e\right) / \left(N(1+\rho)^2\right) \to 0$. As such, the pdf of the mutual information, denoted by $f_{\mathcal{I}}(x)$, will shrink to a spike at $x = M\log_2(1+\rho)$. If the information is transmitted at rate $R = M\log_2(1+\rho)$, the outage probability, $\mathcal{P}_{full} = \Pr(\mathcal{I} < R)$, will be recognized as the area under the left half-lobe of the curve $f_{\mathcal{I}}(x)$, which gives 0.5, as expected. In the cases when $R < M\log_2(1+\rho)$ or $R > M\log_2(1+\rho)$, $\mathcal{P}_{out}$ will be 0 or 1, respectively. In summary, when $N \to \infty$, we have

$$\mathcal{P}_{full} = \begin{cases} 0 & \gamma > \frac{2^{R/M}-1}{R} \\ \frac{1}{2} & \gamma = \frac{2^{R/M}-1}{R} \\ 1 & \gamma < \frac{2^{R/M}-1}{R} \end{cases} . \tag{2.27}$$

It is clear that the result in [10, Eq. (11)] is a special case of (2.27) when $M = 1$. Thus, (2.27) reveals the limit phenomenon of the outage probability for an arbitrary number of receive antennas, $M \geq 1$.

2.4.2 Antenna Selection System

$L = 1$

By expressing $P(x, n)$ for integer n as [44, pp. 262-263]

$$P(x, n) = 1 - e_{n-1}(x)e^{-x}, \tag{2.28}$$

where $e_{n-1}(x) = \sum_{k=0}^{n-1} x^k/k!$, and having

$$\frac{e_n(xn)}{e^{xn}} \overset{n\to\infty}{\longrightarrow} \begin{cases} 0 & x > 1 \\ \frac{1}{2} & x = 1 \\ 1 & 0 \le x < 1 \end{cases}, \tag{2.29}$$

the threshold in (2.17) can be expressed, when $N \to \infty$, as

$$\mathcal{P}_{sel} \to \begin{cases} 0 & \gamma > \frac{2^R-1}{R} \\ \left(\frac{1}{2}\right)^M & \gamma = \frac{2^R-1}{R} \\ 1 & \gamma < \frac{2^R-1}{R} \end{cases}. \tag{2.30}$$

Eq. (2.30) suggests that the threshold when $L = 1$ is the same as if the receiver is equipped with only one antenna ($M = 1$, see (2.27)). However, the significance of having multiple antennas and selecting the best one of them lies in the rapid decrease in the outage probability as a function of SNR.

$L > 1$

Due to the law of large numbers, when N is very large, all the entries of HH^H/N tend to 0 except for the M diagonal entries, resulting in somewhat a diagonal matrix. Furthermore, when $N \to \infty$, the diagonal entries of HH^H/N all converge to something very close to 1. In this case, all the largest L out of M diagonal entries in HH^H are approximately the same, and each one of them can be recognized as the largest one out of $M - L + 1$ Chi-square rvs. Hence, the outage probability, when

$N \to \infty$, can be written as

$$
\begin{aligned}
\mathcal{P}_{sel} &= \Pr\left(\sum_{i=1}^{L} \log_2\left(1 + \frac{\rho}{N}\lambda_i\left(H_{sel}H_{sel}^H\right)\right) < R\right) \\
&= \Pr\left(X_{M-L+1:M-L+1} < \left(2^{R/L} - 1\right)\frac{N}{\rho}\right) \\
&= \left[P\left(\frac{2^{R/L} - 1}{R}\frac{N}{\gamma}, N\right)\right]^{M-L+1}. \qquad (2.31)
\end{aligned}
$$

Consequently the threshold phenomenon for the outage probability is expressed as

$$
\mathcal{P}_{sel} = \begin{cases} 0 & \gamma > \frac{2^{R/L}-1}{R} \\ \left(\frac{1}{2}\right)^{M-L+1} & \gamma = \frac{2^{R/L}-1}{R} \\ 1 & \gamma < \frac{2^{R/L}-1}{R} \end{cases}. \qquad (2.32)
$$

When $L = M$, (2.32) becomes (2.27), and when $L = 1$, (2.32) becomes (2.30), which corroborates our result. Also, (2.32) suggests that the threshold for the case when the receiver selects the best L out of M antennas is the same as if the receiver is equipped with $M = L$ antennas and is using all of them. This phenomenon can be explained as follows. When $N \to \infty$, the fading effect is averaged out in the space dimension, and all the M receive antennas essentially receive the same signals. Thus, selection does not improve the performance. However, having infinite transmit antennas is hypothetical. For a realizable N, antenna selection gives a larger diversity order, which is NM rather than LM.

2.5 Numerical Results

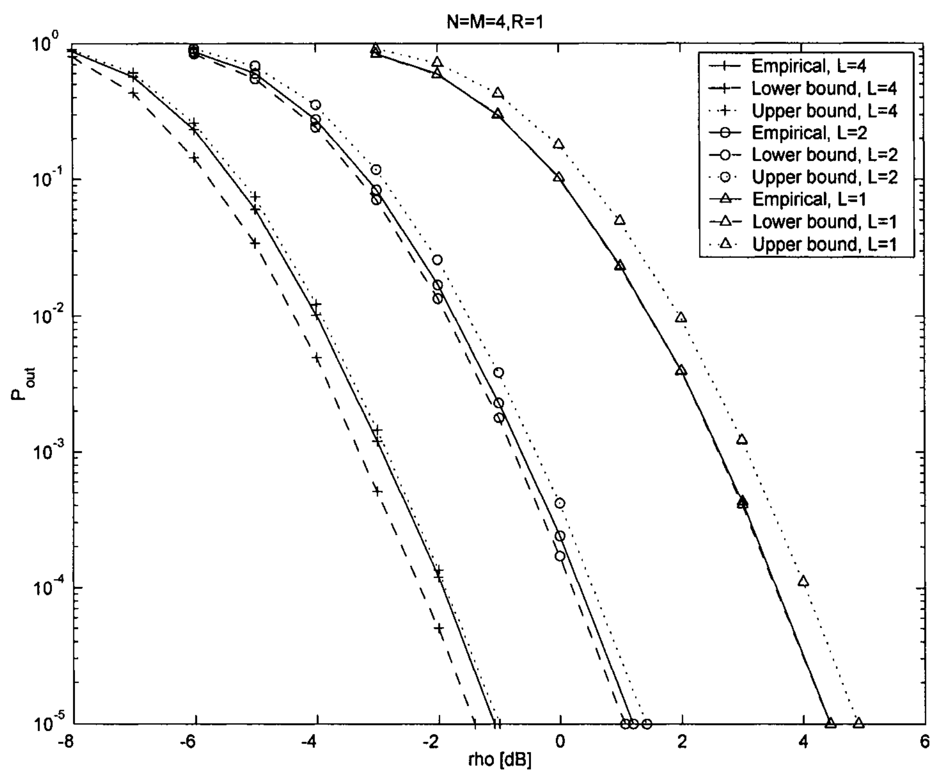

Figure 2.2: Lower bound (2.16) and upper bound (2.26) on the outage probability, for $N = M = 4$, $L = 4$ (plus sign), $L = 2$ (circle) and $L = 1$ (triangle-upward).

In Fig. 2.2 ,we plot the outage probability as a function of ρ in dB for three schemes: full-complexity scheme with $N = M = 4$ (solid), antenna selection scheme with $N = M = 4$, $L = 2$ (dashed) and $N = M = 4$, $L = 1$ (dotted). For each of these schemes, we also plot the lower and upper bounds presented in (2.16) and (2.26), respectively. We observe from the figure that all curves corresponding to the same N have the same slope, suggesting that they have the same diversity order. We also observe from the figure that the discrepancy between the lower bound and the exact outage probability increases with L. When $L = 1$, the lower bound overlaps with the empirical curve, which is explained by (2.17). When $L = M$, the discrepancy reaches its maximum, which is upper bounded by ζ given by (2.25). Specifically, when $L = 1$, the discrepancy is 0 dB. It increases to 0.1 dB when $L = 2$. When $L = M = 4$, the discrepancy is the maximum, which is 0.36 dB. Using (2.25), we find that ζ is 0.47 dB. This is clearly a very tight upper bound on the discrepancy in SNR. Based on ζ, we plot the upper bound on the outage probability by shifting the lower bound by 0.47 dB to the right. We see that the resulting upper bound is very close to the empirical.

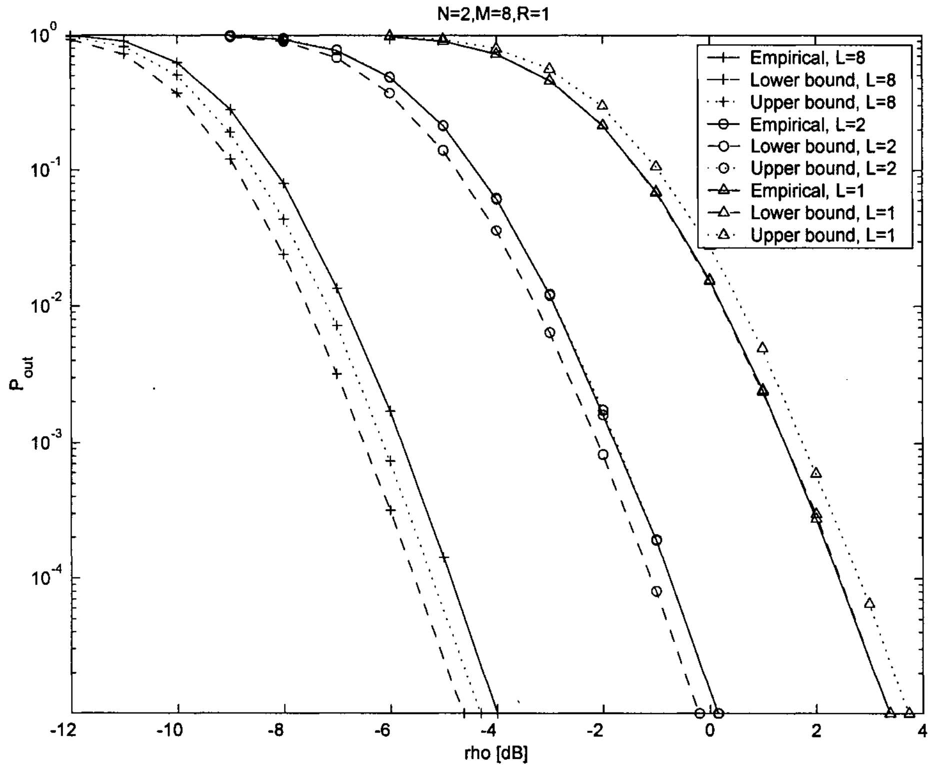

Figure 2.3: Lower bound (2.16) and upper bound (2.26) on the outage probability, for $N = 2, M = 8$, $L = 8$ (plus sign), $L = 2$ (circle) and $L = 1$ (triangle-upward).

In Fig. 2.3, we plot the lower and upper bounds on the outage probability when $N = 2$ and $M = 8$, which severely violates the assumptions we used in deriving (2.21) and (2.22). When $L = 8$, which is the worst case, we see that the discrepancy between the upper bound and the empirical curve is about -0.2 dB. When $L = 1$, the discrepancy increases to about 0.25 dB, which is also very close to the empirical value. This clearly suggests that the derived upper bound yields, even for extreme cases, very close approximations for the loss in SNR due to antenna selection.

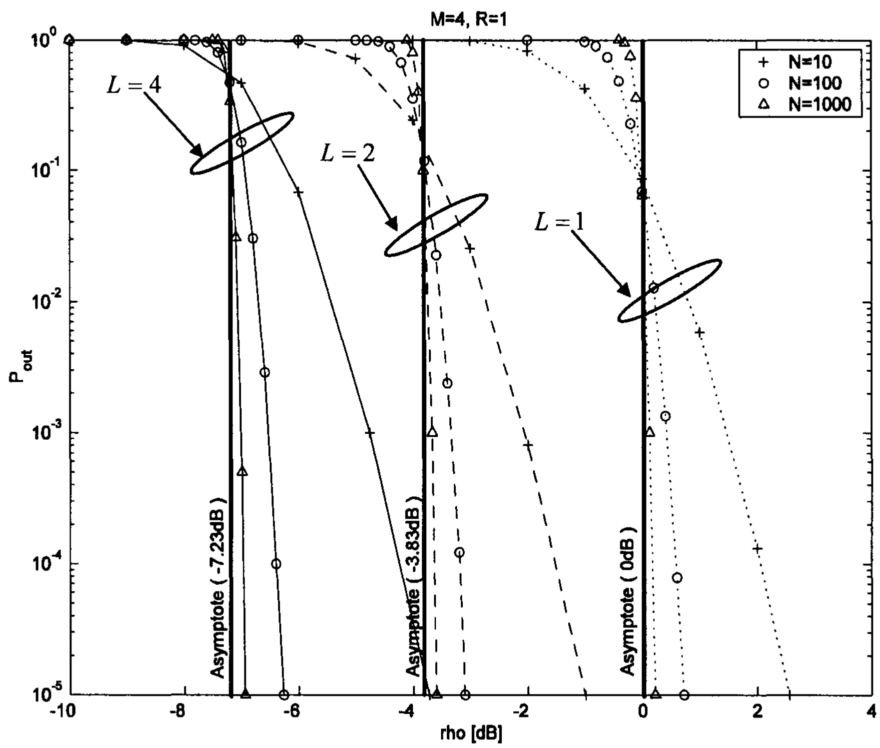

Figure 2.4: Outage probability comparison for a full-complexity system (solid) and antenna-selection systems: $L = 2$ (dashed) and $L = 1$ (dotted).

In Fig. 2.4, we plot the outage probability as a function of ρ in dB for $M = 4$

(solid), $M = 4$, $L = 2$ (dashed) and $M = 4$, $L = 1$ (dotted). For each of these schemes, we consider three values of N, namely, $N = 10$, 100, and 1000. We observe from the figure that all curves corresponding to the same N have the same slope, suggesting that they have the same diversity order. We also observe from the figure that the asymptotes for the above mentioned schemes are placed at $\rho = -7.23$ dB, -3.83 dB and 0 dB, respectively, which are given by (2.27), (2.32) and (2.30), respectively.

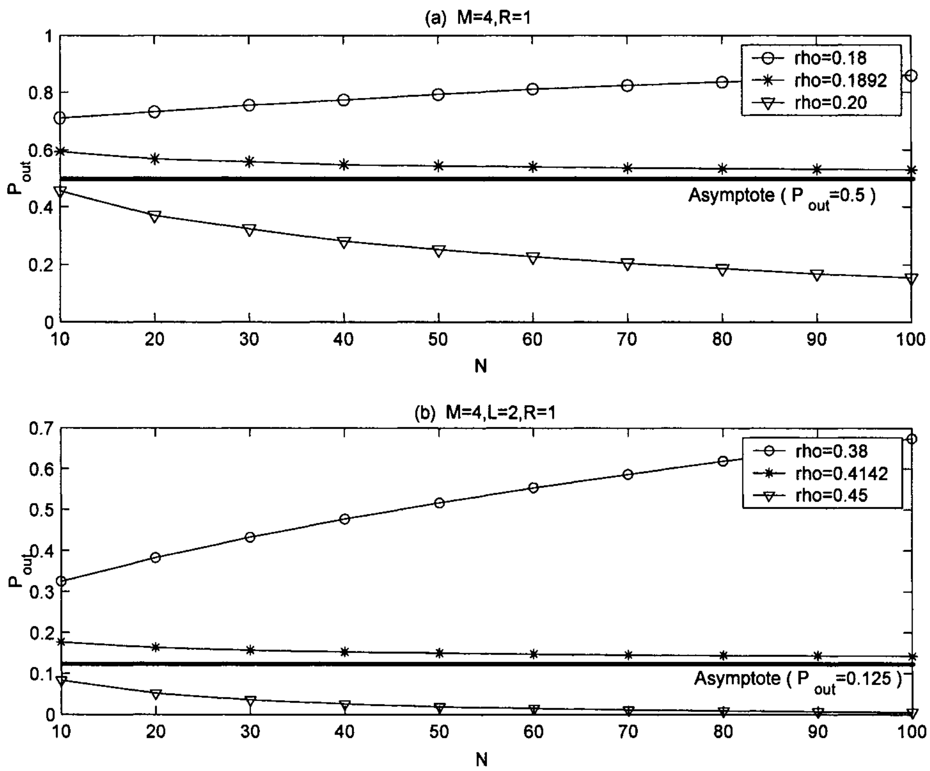

Figure 2.5: Limit phenomenon of P_{out} versus N for (a) $R = 1, M = 4$. (b) $R = 1, M = 4, L = 2$.

In Fig. 2.5, we plot the outage probability as a function of the number of transmit antennas. We observe from Fig. 2.5(a) that if $\rho > 0.1892$, the outage probability goes to 0 as $N \to \infty$; if $\rho < 0.1892$, the outage probability goes to 1; and if $\rho = 0.1892$, the outage probability will be 0.5, which confirms our result given by (2.27). Note that the limit 0.1892 is equivalent to -7.23 dB on the log scale, which is the same asymptote shown in Fig. 2.4 (solid line). Fig. 2.5(b) manifests a similar threshold behavior as that presented in Fig. 2.5(a) but with antenna selection. The limit in this figure appears at $\rho = 0.4142$, and the corresponding outage probability moves down to 0.125, which is given by (2.32). Note that the limit 0.4142 in Fig. 2.5(b) is equivalent to -3.83 dB on a log scale, which is consistent with the asymptote given in Fig. 2.4 (dashed line).

Figs. 2.6 compares the limit behavior of the outage probability for the cases $M = 1$ and 2 when $L = 1$. We observe that both cases have the same asymptote, which is placed at $\rho = 0$ dB. However, the outage probability for the $M = 2, L = 1$ case has a sharper slope compared to that of the $M = 1, L = 1$ case, as expected. We also observe from the figure that the gap between the outage probabilities for the two schemes decreases as the number of transmit antennas increases. Both curves essentially overlap when $N \to \infty$. This suggests that having multiple antennas at the receiver and selecting a subset of them does not help much when $N \to \infty$. It does help, however, for practical values of N.

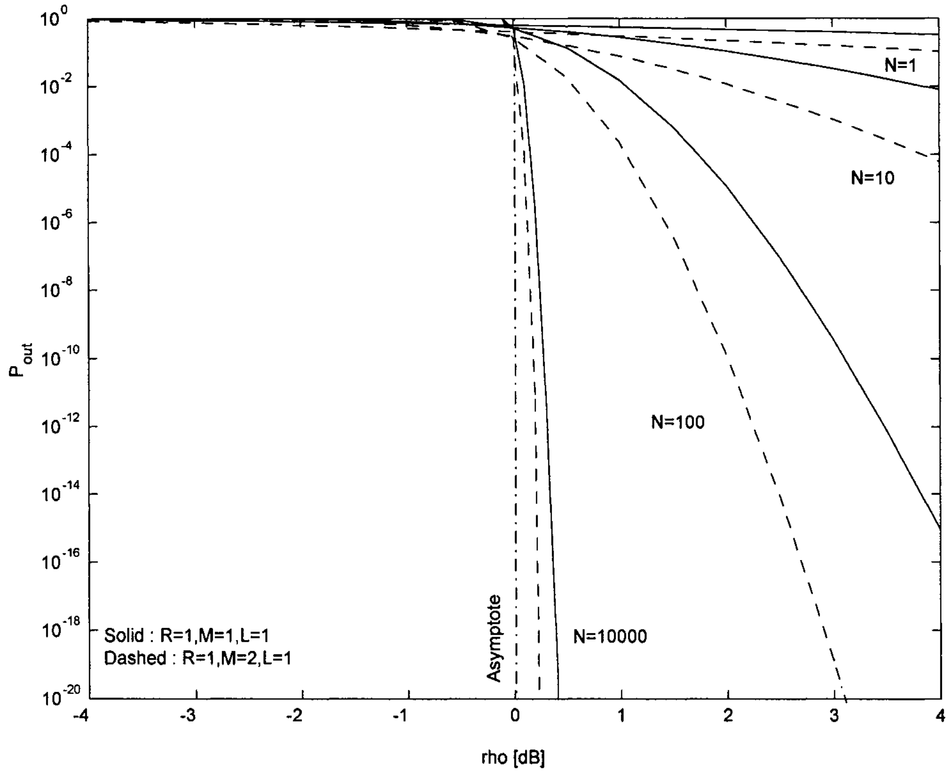

Figure 2.6: Limit phenomenon of outage probability when the best antenna is selected at the receiver, for $R = 1$, $M = 1$, $L = 1$ (solid), and $R = 1$, $M = 2$, $L = 1$ (dashed) as a function of N.

2.6 Concluding Remarks

In this chapter, we studied the outage probability for MIMO systems with receive antenna selection. We derived an upper bound on the outage probability with antenna selection and showed that the resulting diversity order is the same as that of the full complexity system. Motivated by the fact this upper bound is loose, we derived another upper bound on the outage probability which can be used to accurately quantify the loss in SNR due to antenna selection. Furthermore, we presented a thorough investigation of the limit behavior of the outage probability for MIMO systems with and without receive antenna selection. In particular, we derived closed-form expressions for the threshold for the outage probability as the number of transmit antennas tends to infinity.

Chapter 3

Analysis of the Outage Probability Over Correlated Fading Channels

3.1 Introduction

In this chapter, we study the outage probability for MIMO systems in the presence of spatial correlation at the transmitter and receiver. In our study, we adopt the correlation channel model presented in [35]. We also assume that the channel state information (CSI) is perfectly known at the receiver, but not at the transmitter. Thus the transmitted power is distributed equally among the transmit antennas to maximize the channel capacity. We derive explicit bounds for the outage probability and show that the diversity order is the product of the ranks of the transmit and receive correlation matrices. We also derive a closed-form expression for quantifying the degradation in SNR due to the presence of correlation. We extend our study of the outage probability to MIMO correlated fading channels with receive antenna selection where selection is based on maximizing the channel capacity. That is, for a given M receive antennas, we assume that the receiver selects the best L antennas that maximize the channel capacity. We derive explicit upper bounds for the outage probability with antenna selection and show that the diversity order is the same as that of the full complexity system. We also derive an expression that accurately quantifies the degradation in SNR due to antenna selection. We lastly present several numerical examples that validate our analysis.

The remainder of the chapter is outlined as follows. In Section 3.2, we introduce the system model and review some existing results. The outage probability for MIMO correlated fading channels is studied in Section 3.3. The outage probability for MIMO correlated fading channels with receive antenna selection is analyzed in Section 3.4. Several numeral examples are discussed in Section 3.5. Finally, Section 3.6 concludes this chapter.

3.2 System Model and Preliminaries

3.2.1 System Model

The system model is depicted in Fig. 3.1, which models a wireless communication system equipped with N transmit antennas and M receive antennas. The received

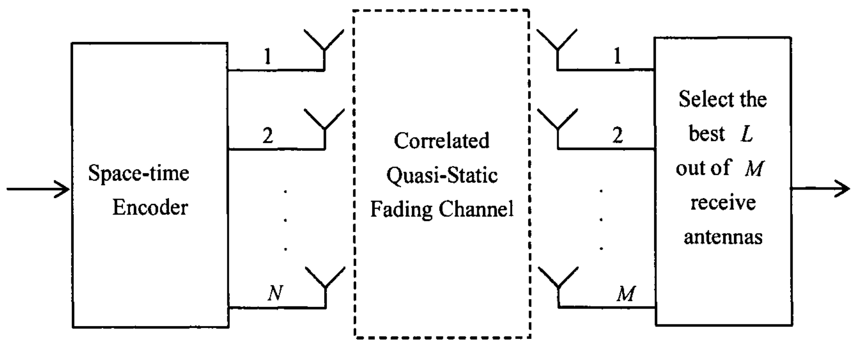

Figure 3.1: System Model. (Correlated fading channel)

signal at time index k, in vector notation, is given as

$$\mathbf{y}[k] = H\mathbf{x}[k] + \mathbf{n}[k], \tag{3.1}$$

where H is defined as an $M \times N$ matrix, with the $(i,j)^{th}$ entry, denoted by h_{ij}, modeling the fading between the i^{th} transmit antenna and j^{th} receive antenna, $\mathbf{x}[k]$ is a $N \times 1$ vector representing the transmitted signal array at time k, and $\mathbf{n}[k]$ is a $M \times 1$ vector representing the AWGN noise samples at time k. The entries of H are modeled as complex Gaussian random variables with zero mean and unit variance. The elements of $\mathbf{n}[k]$ are independent and identically distributed (iid) complex Gaussian random variables with zero mean and variance $N_0/2$ per dimension. We also assume that the fading coefficients (elements of H) are constant over the entire frame and vary independently from one frame to another, which is essentially the case for quasi-static fading. Furthermore, we assume that the CSI is known exactly at the receiver, but not at the transmitter.

When spatial correlation is present at both ends of the wireless communications link, matrix H can be expressed as [35]

$$H = R_r^{1/2} G R_t^{1/2}, \tag{3.2}$$

where R_t and R_t are the transmit and receive correlation matrices, respectively, and the entries of G are iid complex circular symmetric Gaussian random variables with zero mean and unit variance. Let w_t denote the rank of R_t and w_r denote the rank of R_r. It is clear that R_t and R_r are Hermitian matrices.

3.2.2 Mutual Information and Outage Probability

Since the CSI is not available at the transmitter, the total transmit power, denoted by P, has to be distributed equally among the transmit antennas to maximize the channel capacity. In this case, the mutual information of a MIMO channel is given by [1]

$$\mathcal{I} = \log_2 \det(I_M + \frac{\rho}{N} H H^H) \tag{3.3}$$

where $\rho = P/(N_0 B)$ is the SNR at each receive antenna, where B is transmission bandwidth. Note that the mutual information is a random variable that is a function of H. In quasi-static fading channels, the fading coefficients remain constant over the whole frame. Thus, the mutual information can not be averaged using an ergodic measure. In this case, when the information transmission rate, denoted by R, is greater than the instantaneous mutual information, an outage event occurs. The probability of such an event is normally referred to as *outage probability*, which is given by

$$\mathcal{P}_{out} = \Pr\left(\mathcal{I} < R\right). \tag{3.4}$$

3.3 Outage Probability for the Full Complexity System

3.3.1 Definitions and Simple Results

In this section, we introduce some simple results that we will need in subsequent sections.

Definition 8 *Let $A = [a_{ij}] \in R_{n\times n}$. Define $A_I = [\mu_{ij}] \in \mathbb{R}_{n\times n}$ as the indication matrix of A where*

$$\mu_{ij} = \begin{cases} 0, & a_{ij} = 0 \\ 1, & a_{ij} \neq 0 \end{cases}.$$

Lemma 2 *Let $D_k = [d_{ij}] \in \mathbb{R}_{n\times n}$ such that $d_{ij} \geq 0$ when $i = j = k$ and 0 otherwise* *for* $k \in \{1, \ldots, n\}$. *Also, let* $W = [w_{ij}] \in \mathbb{C}_{n \times n}$. *Then*

$$\det(W + D_k) \geq \det(W).$$

Proof. Using Laplace expansion by minors, the determinant of A, denoted by $\det(A)$, can be expressed as

$$\det(A) = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \det(A_{ij}),$$

where A_{ij} is formed by eliminating row i and column j from A. Then we have

$$\begin{aligned} \det(W + D_k) &= \det(W) + (-1)^{k+k} d_{kk} \det(W_{kk}) \\ &= \det(W) + d_{kk} \det(W_{kk}). \end{aligned}$$

Since W is Hermitian, W_{kk} is also Hermitian. Thus, $d_{kk} \det(W_{kk}) \geq 0$ and $\det(W + D_k) \geq \det(W)$. ■

Lemma 3 *Let* $D = [d_{ij}] \in \mathbb{R}_{n \times n}$ *be a diagonal matrix with non-negative diagonal entries, and let* $W = [w_{ij}] \in \mathbb{C}_{n \times n}$ *be a Hermitian matrix. Then*

$$\det(W + D) \geq \det(W).$$

Proof. Using Lemma 1 above, we have

$$\begin{aligned}
\det(W+D) &= \det\left(W+\sum_{k=1}^{n} D_k\right) \\
&\geq \det\left(W+\sum_{k=1}^{n-1} D_k\right) \\
&\;\;\vdots \\
&\geq \det(W).
\end{aligned}$$

■

Theorem 4 *Let $D = [d_{ij}] \in \mathbb{R}_{n\times n}$ be a diagonal matrix with non-negative diagonal entries, and $G = [g_{ij}] \in \mathbb{C}_{m\times n}$ be a full rank matrix. Then*

$$\det\left(I_m + m_D G D_I G^H\right) \leq \det\left(I_m + GDG^H\right) \leq \det\left(I_m + n_D G D_I G^H\right), \tag{3.5}$$

where m_D and n_D are the smallest and largest positive diagonal entries in D, respectively, I_m is the $m\times m$ identity matrix and D_I is the indication matrix of D.

Proof. We first prove the inequality $\det\left(I_m + m_D G D_I G^H\right) \leq \det\left(I_m + GDG^H\right)$. Without loss of generality, assume that $rank(G) = m$. Define $W = G^H G$. Using the determinant identity $\det(I+AB) = \det(I+BA)$ and $\det(AB) = \det(A)\cdot\det(B)$, we have

$$\begin{aligned}
\det\left(I_m + GDG^H\right) &= \det\left(I_n + DW\right) \\
&= \det\left(W\right) \cdot \det\left(W^{-1} + D\right) \\
&= \det\left(W\right) \cdot \det\left(W^{-1} + m_D D_I + D^*\right),
\end{aligned}$$

where $D^* = D - m_D D_I$ is a diagonal matrix with non-negative diagonal entries. Obviously, $W^{-1} + m_D D_I$ is Hermitian. From Lemma 2, we know that $\det\left(W^{-1} + m_D D_I + D^*\right) \geq \det\left(W^{-1} + m_D D_I\right)$. Since W is Hermitian, we have $\det\left(W\right) > 0$. Hence,

$$\begin{aligned}
\det\left(I_m + GDG^H\right) &\geq \det\left(W\right) \cdot \det\left(W^{-1} + m_D D_I\right) \\
&= \det\left(I_m + m_D G D_I G^H\right).
\end{aligned}$$

Following the same approach, one can easily show that $\det\left(I_m + GDG^H\right) \leq \det\left(I_m + n_D G D_I G^H\right)$. ■

Theorem 5 *Let $D = [d_{ij}] \in \mathbb{R}_{n\times n}$ be a diagonal matrix with non-negative diagonal entries, and let $G = [g_{ij}] \in \mathbb{C}_{m\times n}$ be a full rank matrix. Then*

$$\begin{aligned}
\det\left(I_m + G D_I G^H\right) &= \det\left(I_m + G_D G_D^H\right) \\
&= \det\left(I_{rank(D)} + G_D^H G_D\right), \qquad (3.6)
\end{aligned}$$

where G_D is an $m \times rank\,(D)$ matrix formed by eliminating the $n - rank\,(D)$ all-zero columns from GD_I.

Proof. Without loss of generality, assume that $d_{ii} > 0$ for $1 \leq i \leq rank\,(D)$ and 0 otherwise, i.e., all positive values in D appear at the first $rank\,(D)$ diagonal entries and the rest are zero.[1] Consequently, we have

$$\begin{aligned} I_m + GD_I G^H &= I_m + \begin{bmatrix} G_D & 0 \end{bmatrix} \begin{bmatrix} G_D^H \\ 0 \end{bmatrix} \\ &= I_m + G_D G_D^H, \end{aligned}$$

and

$$\begin{aligned} \det\left(I_m + GD_I G^H\right) &= \det\left(I_m + G_D G_D^H\right) \\ &= \det\left(I_{rank(D)} + G_D^H G_D\right). \end{aligned}$$

■

3.3.2 Transmit Correlation Model

To study the outage probability for MIMO correlated fading channels, we start with a simple case, that is, when the correlation is present only at one end of the communication link, i.e., $R_r = I_M$ or $R_t = I_N$. Since they have the same effect on the

[1]If the positive values in D appear randomly along the diagonal, we can interchange rows and columns to put them at the first $rank\,(D)$ diagonal entries, which does not change the determinant.

mutual information, we consider $R_r = I_M$. Using singular value decomposition, we have $R_t = CD_tC^H$, where C is unitary and D_t is a diagonal matrix with non-negative diagonal entries. Consequently, the outage probability can be expressed as

$$\begin{aligned}
\mathcal{P}_{out} &= \Pr\left(\log_2 \det\left(I_M + \frac{\rho}{N}HH^H\right) < R\right) \\
&= \Pr\left(\log_2 \det\left(I_M + \frac{\rho}{N}GR_tG^H\right) < R\right) \\
&= \Pr\left(\log_2 \det\left(I_M + \frac{\rho}{N}GCD_tC^HG^H\right) < R\right) \\
&= \Pr\left(\log_2 \det\left(I_M + \frac{\rho}{N}GD_tG^H\right) < R\right). \qquad (3.7)
\end{aligned}$$

The last line of (3.7) follows immediately from the one before it because GC and G have the same eigenvalue distribution since C is unitary. Denote the smallest and largest positive diagonal entries of D_t by m_t and n_t, respectively, and denote D_{I_t} as the indication matrix of D_t. (Note that D_t is deterministic.) Using Theorem 4 above, we can bound the outage probability in (3.7) as

$$\begin{aligned}
&\Pr\left(\log_2 \det\left(I_M + \frac{\rho n_t}{N}GD_{I_t}G^H\right) < R\right) \leq \mathcal{P}_{out} \\
&\qquad \leq \Pr\left(\log_2 \det\left(I_M + \frac{\rho m_t}{N}GD_{I_t}G^H\right) < R\right). \qquad (3.8)
\end{aligned}$$

Following Theorem 5, (3.8) can be simplified as

$$\begin{aligned}
&\Pr\left(\log_2 \det\left(I_{w_t} + \frac{\rho n_t}{N}G_t^HG_t\right) < R\right) \leq \mathcal{P}_{out} \\
&\qquad \leq \Pr\left(\log_2 \det\left(I_{w_t} + \frac{\rho m_t}{N}G_t^HG_t\right) < R\right), \qquad (3.9)
\end{aligned}$$

where G_t is an $M \times w_t$ submatrix of GD_{I_t} formed by eliminating the $N - w_t$ all-zero columns from GD_{I_t}.

The lower and upper bounds in (3.9) suggest that the outage probability for an $M \times N$ fading channel with transmit correlation can be recognized as the outage probability for an $M \times w_t$ independent fading channel with a scaling of the SNR that ranges between ρm_t and ρn_t, i.e.,

$$\mathcal{P}_{out} = \Pr\left(\log_2 \det\left(I_{w_t} + \frac{\widehat{\rho}}{N} G_t^H G_t\right) < R\right), \tag{3.10}$$

where $\rho m_t \leq \widehat{\rho} \leq \rho n_t$. Obviously, the diversity order is $w_t M$. Similarly, when only receive correlation is present, one can easily show that the diversity order of the outage probability is $N w_r$.

3.3.3 Combined Transmit and Receive Correlation

Let $R_t = C_t D_t C_t^H$ and $R_r = C_r D_r C_r^H$, where C_t and C_r are unitary matrices, and D_t and D_r are diagonal matrices with non-negative eigenvalues. The outage probability is then given by

$$\begin{aligned}
\mathcal{P}_{out} &= \Pr\left(\log_2 \det\left(I_M + \frac{\rho}{N} H H^H\right) < R\right) \\
&= \Pr\left(\log_2 \det\left(I_M + \frac{\rho}{N}\left(R_r^{1/2} G\right) R_t \left(R_r^{1/2} G\right)^H\right) < R\right) \\
&= \Pr\left(\log_2 \det\left(I_M + \frac{\rho}{N}\left(D_r^{1/2} G\right) D_t \left(D_r^{1/2} G\right)^H\right) < R\right). \qquad (3.11)
\end{aligned}$$

Denote D_{I_t} and D_{I_r} as the indication matrices of D_t and D_r, respectively. Using Theorem 4, the outage probability in (3.11) can be bounded as

$$\Pr\left(\log_2 \det\left(I_M + \frac{\rho n_t}{N}\left(D_r^{1/2}G\right) D_{I_t} \left(D_r^{1/2}G\right)^H\right) < R\right) \leq \mathcal{P}_{out}$$
$$\leq \Pr\left(\log_2 \det\left(I_M + \frac{\rho m_t}{N}\left(D_r^{1/2}G\right) D_{I_t} \left(D_r^{1/2}G\right)^H\right) < R\right) \quad (3.12)$$

where m_t and n_t are the smallest and the largest positive diagonal entries in D_t, respectively.

Applying Theorem 4 to (3.12) again but now with respect to D_r yields

$$\Pr\left(\log_2 \det\left(I_M + \frac{\rho n_t n_r}{N} D_{I_r} G D_{I_t} G^H D_{I_r}\right) < R\right) \leq \mathcal{P}_{out}$$
$$\leq \Pr\left(\log_2 \det\left(I_M + \frac{\rho m_t m_r}{N} D_{I_r} G D_{I_t} G^H D_{I_r}\right) < R\right), \quad (3.13)$$

where m_r and n_r are the smallest and the largest positive diagonal entries in D_r, respectively.

Following Theorem 5, (3.13) can be simplified as

$$\Pr\left(\log_2 \det\left(I_{w_r} + \frac{\rho n_t n_r}{N} G_{t,r} G_{t,r}^H\right) < R\right) \leq \mathcal{P}_{out}$$
$$\leq \Pr\left(\log_2 \det\left(I_{w_r} + \frac{\rho m_t m_r}{N} G_{t,r} G_{t,r}^H\right) < R\right), \quad (3.14)$$

where $G_{t,r}$ is a $w_r \times w_t$ submatrix of $D_{I_r} G D_{I_t}$ formed by eliminating the $M - w_r$ all-zero rows and the $N - w_t$ all-zero columns from $D_{I_r} G D_{I_t}$. It is clear from (3.14) that diversity order of the outage probability with transmit and receive correlation is

$w_t w_r$.

3.3.4 Degradation in SNR due to Correlation

In this section, we derive an expression for the loss in SNR due to the presence of correlation. At sufficiently high SNR, the instantaneous mutual information can be approximated as

$$\begin{aligned}
&\log_2 \det\left(I_M + \frac{\rho}{N} R_r G R_t G^H\right) \\
\approx\ &\log_2 \det\left(I_{w_r} + \frac{\rho}{N}\left[\prod_{i=1}^{w_r} \lambda_i\left(R_r\right)\right]^{\frac{1}{w_r}}\left[\prod_{j=1}^{w_t} \lambda_j\left(R_t\right)\right]^{\frac{1}{w_t}} G_{t,r} G_{t,r}^H\right), \qquad (3.15)
\end{aligned}$$

where $\lambda_t\left(A\right)$, $t = 1, 2, \ldots, rank\left(A\right)$ are the eigenvalues of A. By comparing the last line of (3.15) with the instantaneous mutual information for a $w_r \times w_t$ independent fading channel, which is given by

$$\log_2 \det\left(I_{w_r} + \frac{\rho}{w_t} G_{t,r} G_{t,r}^H\right)$$

it is easy to see that the loss in SNR due to the presence of correlation is approximated by

$$10 \log_{10}\left(\frac{N}{w_t}\left[\prod_{i=1}^{w_r} \lambda_i\left(R_r\right)\right]^{-\frac{1}{w_r}}\left[\prod_{j=1}^{w_t} \lambda_j\left(R_t\right)\right]^{-\frac{1}{w_t}}\right) \quad \text{dB}. \qquad (3.16)$$

As we will demonstrate later, this expression is accurate for all correlation levels and it holds for full rank and rank deficient correlation matrices.

3.4 Outage Probability with Receive Antenna Selection

In this section, we study the outage probability for MIMO correlated fading channels with antenna selection at the receiver. That is, the case when the receiver uses only L out of the available M receive antennas, where $1 \leq L \leq M$. Clearly, there are $\binom{M}{L}$ subsets to choose from, but what we are interested in the one that results in maximizing the channel capacity. The selected channel matrix is denoted by H_{sel}, which is formed by selecting the L rows from H that maximize $\det\left(I_L + \frac{\rho}{N} H_{sel} H_{sel}^H\right)$. We first start with the case $L = 1$, i.e., when the receiver selects the best antenna, and then generalize our analysis to an arbitrary number of selected antennas.

3.4.1 When the Receiver Selects the Best Antenna: $L = 1$

Receive Correlation Model

Suppose $R_t = I_N$, i.e., receive correlation only. Thus,

$$H = R_r^{1/2} G. \tag{3.17}$$

Let $G = [g_{ij}] \in \mathbb{C}_{\mathrm{M}\times\mathrm{N}}$ and $R_r^{1/2} = [r_{ij}] \in \mathbb{R}_{\mathrm{M}\times\mathrm{M}}$. Then

$$H = \begin{bmatrix} \sum_{k=1}^{M} r_{1k}g_{k1} & \sum_{k=1}^{M} r_{1k}g_{k2} & \cdots & \sum_{k=1}^{M} r_{1k}g_{kN} \\ \sum_{k=1}^{M} r_{2k}g_{k1} & \sum_{k=1}^{M} r_{2k}g_{k2} & \cdots & \sum_{k=1}^{M} r_{2k}g_{kN} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{k=1}^{M} r_{Mk}g_{k1} & \sum_{k=1}^{M} r_{Mk}g_{k2} & \cdots & \sum_{k=1}^{M} r_{Mk}g_{kN} \end{bmatrix}. \tag{3.18}$$

Denote the i^{th} row in H by S_i, i.e.,

$$S_i = \begin{bmatrix} \sum_{k=1}^{M} r_{ik}g_{k1} & \sum_{k=1}^{M} r_{ik}g_{k2} & \cdots & \sum_{k=1}^{M} r_{ik}g_{kN} \end{bmatrix}. \tag{3.19}$$

Without loss of generality, let us assume that

$$\|S_1\|^2 \geq \|S_2\|^2 \geq \cdots \geq \|S_M\|^2. \tag{3.20}$$

Then the outage probability when the best receive antenna is selected can be written as

$$\begin{aligned} \mathcal{P}_{sel} &= \Pr\left(\log_2\left(1 + \frac{\rho}{N}\|S_1\|^2\right) < R\right) \\ &= \Pr\left(\bigcap_{i=1}^{M}\left(\|S_i\|^2 < \left(2^R - 1\right)\frac{N}{\rho}\right)\right). \end{aligned} \tag{3.21}$$

Now we derive an expression for $\|S_i\|^2$. Denote the $(i,j)^{th}$ entry in H by s_{ij} where

$$s_{ij} = \sum_{k=1}^{M} r_{ik} g_{kj}. \tag{3.22}$$

Accordingly, $\|S_i\|^2$ is given as

$$\begin{aligned}
\|S_i\|^2 &= \sum_{j=1}^{N} |s_{ij}|^2 \\
&\approx \sum_{j=1}^{N} \sum_{k=1}^{M} |r_{ik}|^2 |g_{kj}|^2 \\
&= \sum_{k=1}^{M} |r_{ik}|^2 \sum_{j=1}^{N} |g_{kj}|^2 \\
&= \sum_{k=1}^{M} |r_{ik}|^2 X_k,
\end{aligned} \tag{3.23}$$

where $X_k = \sum_{j=1}^{N} |g_{kj}|^2$, $k = 1, \ldots, M$ are iid Chi-square random variables (rvs), each with $2N$ degrees of freedom. The approximation used to arrive at the second line of (3.23) is needed to simplify the analysis without which it is not possible to proceed further in the analysis. To justify this approximation, we plot in Fig. 3.2 the cumulative distribution function (cdf) of the random variable $\|S_i\|^2$ with and without the approximation. It is evident from the figure that the difference between the cdfs is very small, which justifies the approximation.

For notation convenience, denote $\|S_i\|^2$ by Y_i, for $i = 1, \ldots, M$. From (3.21), we

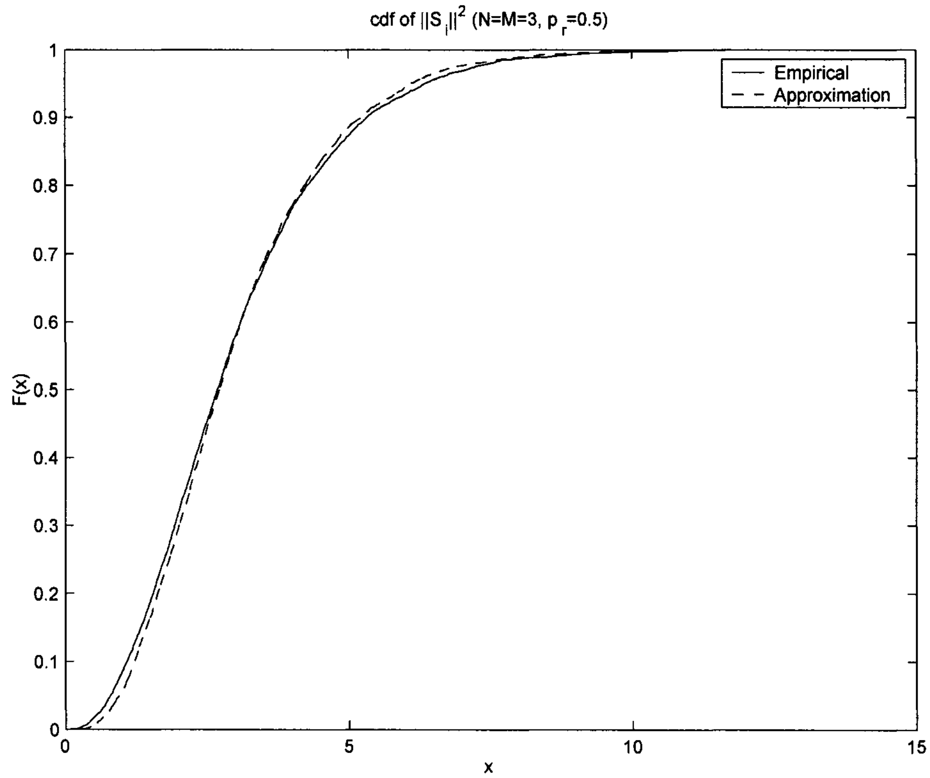

Figure 3.2: Cumulative distribution function of $\|S_i\|^2$ with and without the approximation (see Eq. (3.23)).

have

$$\begin{aligned} \mathcal{P}_{sel} &= \Pr\left(\bigcap_{i=1}^{M}\left(Y_i < \left(2^R - 1\right)\frac{N}{\rho}\right)\right) \\ &= \int \cdots \int_{\Phi} f_{Y_1,\ldots,Y_M}\left(y_1,\ldots,y_M\right) dy_1 \cdots dy_M, \end{aligned} \tag{3.24}$$

where the integration support region, denoted by Φ, is given by

$$\Phi = \begin{cases} Y_1 < \left(2^R - 1\right) \frac{N}{\rho} \\ \vdots \\ Y_M < \left(2^R - 1\right) \frac{N}{\rho} \end{cases} . \tag{3.25}$$

Denote the set of $\{Y_i \mid i = 1, \dots, M\}$ by Ω. Since Ω is of rank w_r, we may find a rank w_r subset of Ω, denoted by $\Theta = \left\{Y_{a_j} \mid j = 1, \dots, w_r\right\}$, such that Y_i, $i = 1, \dots, M$ can be represented by the linear combinations of Y_{a_j}, $j = 1, \dots, w_r$. Thus, Φ is contained in the region Φ' where

$$\begin{aligned} \Phi' &= \begin{cases} Y_{a_1} < \left(2^R - 1\right) \frac{N}{\rho} \\ \vdots \\ Y_{a_{w_r}} < \left(2^R - 1\right) \frac{N}{\rho} \end{cases} \\ &= \begin{cases} \sum_{k=1}^{M} \left|r_{a_1 k}\right|^2 X_k < \left(2^R - 1\right) \frac{N}{\rho} \\ \vdots \\ \sum_{k=1}^{M} \left|r_{a_{w_r} k}\right|^2 X_k < \left(2^R - 1\right) \frac{N}{\rho} \end{cases} . \end{aligned} \tag{3.26}$$

Obviously, Φ' is simply an expansion of Φ which results from dropping off $M - w_r$ restrictions from (3.25).

Since the set $\left\{Y_{a_j} \mid j = 1, \dots, w_r\right\}$ is of rank w_r, there exists a subset of $\{X_k \mid k = 1, \dots, M\}$, denoted by $\left\{X_{b_j} \mid j = 1, \dots, w_r\right\}$, such that $\left|r_{a_j b_j}\right|^2 > 0$ for $j = 1, \dots, w_r$. Note that $X_k \geq 0$. Thus, by dropping off $M - 1$ summation terms from (3.26), Φ' is further expanded into the region Φ'', which is given by

$$\Phi'' = \begin{cases} \left|r_{a_1 b_1}\right|^2 X_{b_1} < \left(2^R - 1\right) \frac{N}{\rho} \\ \vdots \\ \left|r_{a_{w_r} b_{w_r}}\right|^2 X_{b_{w_r}} < \left(2^R - 1\right) \frac{N}{\rho} \end{cases}$$

$$= \begin{cases} X_{b_1} < \frac{\left(2^R - 1\right)\frac{N}{\rho}}{\left|r_{a_1 b_1}\right|^2} \\ \vdots \\ X_{b_{w_r}} < \frac{\left(2^R - 1\right)\frac{N}{\rho}}{\left|r_{a_{w_r} b_{w_r}}\right|^2} \end{cases}. \tag{3.27}$$

Since $\Phi \subseteq \Phi' \subseteq \Phi''$, the outage probability is upper bounded as

$$\begin{aligned} \mathcal{P}_{sel} &= \int_{\Phi} \cdots \int f_{Y_1, \ldots, Y_M} \left(y_1, \ldots, y_M\right) dy_1 \cdots dy_M \\ &\leq \int_{\Phi''} \cdots \int f_{X_1, \ldots, X_{w_r}} \left(x_1, \ldots, x_{w_r}\right) dx_1 \cdots dx_{w_r}. \end{aligned} \tag{3.28}$$

Note that X_{b_j}, $j = 1, \ldots, w_r$ are iid Chi-square rvs each with $2N$ degrees of freedom. Thus, (3.28) can be simplified as

$$\begin{aligned} \mathcal{P}_{sel} &\leq \prod_{j=1}^{w_r} P\left(\frac{\left(2^R - 1\right) N}{\rho \left|r_{a_j b_j}\right|^2}, N\right) \\ &\leq \left[P\left(\frac{\left(2^R - 1\right) N}{\rho \zeta}, N\right)\right]^{w_r} \end{aligned} \tag{3.29}$$

where $\zeta = \min\limits_{i,j=1}^{M} \left(\left|r_{ij}\right|^2\right)$ and $P(x, a)$ is the normalized incomplete gamma function defined by

$$P(x,a) = \frac{1}{\Gamma(a)} \int\limits_0^x u^{a-1} e^{-u} du, \qquad x \geq 0. \tag{3.30}$$

Using the power series expansion of $P(x,a)$

$$P(x,a) = x^a e^{-x} \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(a+n+1)}, \tag{3.31}$$

we have at high SNR

$$\left[P\left(\frac{(2^R - 1)\, N}{\rho\zeta}, N \right) \right]^{w_r} \rightarrow \alpha^{w_r} \rho^{-N w_r}, \tag{3.32}$$

where

$$\alpha = \frac{\left(\frac{(2^R-1)N}{\zeta} \right)^N}{N!}.$$

It is clear from (3.32) that the diversity order of the outage probability with receive correlation when the best antenna is selected is $N w_r$, which is the same diversity order of the full complexity system with receive correlation.

Combined Transmit and Receive Correlation

We have shown in (3.12) that for the full complexity system with correlation being present at both ends, the outage probability is upper bounded as

$$\mathcal{P}_{out} \leq \Pr\left(\log_2 \det\left(I_M + \frac{\rho m_t}{N}\left(D_r^{1/2}G\right) D_{I_t}\left(D_r^{1/2}G\right)^H\right) < R\right). \tag{3.33}$$

Using Theorem 5, (3.33) can be simplified as

$$\mathcal{P}_{out} \leq \Pr\left(\log_2 \det\left(I_M + \frac{\rho m_t}{N} H_r H_r^H\right) < R\right), \tag{3.34}$$

where H_r is an $M \times w_t$ matrix formed by eliminating the $N - w_t$ all-zero columns from $D_r^{1/2} G D_{I_t}$.

Obviously, H_r models an $M \times w_t$ fading channel with receive correlation. Since we only perform column operations on (3.33) to arrive at (3.34), this does not influence the receive antenna selection. Hence, selecting the best receive antenna from H would be equivalent to selecting the best receive antenna from H_r. Using the result given by (3.29), we can upper bound the outage probability as

$$\mathcal{P}_{sel} \leq \left[P\left(\frac{\left(2^R - 1\right) N}{\rho m_t \zeta}, w_t\right)\right]^{w_r}, \tag{3.35}$$

where $\zeta = \min_{i,k=1}^{M}\left(\left|r_{ik}\right|^2\right)$ is a constant. At high SNR, the term on the right hand side of (3.35) can be approximated as

$$\left[P\left(\frac{\left(2^R-1\right)N}{\rho m_t \zeta}, w_t\right)\right]^{w_r} \to \left(\frac{\left(\frac{\left(2^R-1\right)N}{\zeta m_t}\right)^{w_t}}{w_t!}\right)^{w_r} \rho^{-w_t w_r}. \tag{3.36}$$

The expression in (3.36) suggests that the diversity order of the outage probability for MIMO systems with joint transmit-receive correlation when the best receive antenna selected is $w_t w_r$, which is the same result obtained for the full complexity system.

3.4.2 When the Receiver Selects the Best L Antennas:

Obviously, the diversity of the outage probability with receive antenna selection is lower bounded by that when only the best receive antenna is selected and upper bounded by the performance of the full complexity system, i.e., $L = M$. We have already shown that a diversity order of $w_t w_r$ is achieved by selecting the best receive antenna. Since the diversity order can not be higher than $w_t w_r$, we claim that the full diversity is maintained with antenna selection for an arbitrary number of selected antennas, $L \leq M$.

3.4.3 Degradation in SNR due to Antenna Selection

In this section we derive an expression for approximating the loss in SNR due to antenna selection. For a full complexity system, the instantaneous mutual information can be approximated as (see (3.15))

$$\begin{aligned}
\mathcal{I} &= \log_2 \det\left(I_M + \frac{\rho_1}{N} R_r G R_t G^H\right) \\
&\approx \log_2 \det\left(I_{w_r} + \frac{\rho_1}{N}\left[\prod_{i_1=1}^{w_r} \lambda_{i_1}(R_r)\right]^{\frac{1}{w_r}}\left[\prod_{j_1=1}^{w_t} \lambda_{j_1}(R_t)\right]^{\frac{1}{w_t}} G_{t,r} G_{t,r}^H\right) \\
&= \sum_{k_1=1}^{w_r} \log_2\left(1 + \frac{\rho_1}{N}\left[\prod_{i_1=1}^{w_r} \lambda_i(R_r)\right]^{\frac{1}{w_r}}\left[\prod_{j_1=1}^{w_t} \lambda_j(R_t)\right]^{\frac{1}{w_t}} \lambda_{k_1}\left(G_{t,r} G_{t,r}^H\right)\right) \quad (3.37)
\end{aligned}$$

where ρ_1 is the SNR and $\lambda_u(A)$, $u = 1, 2, \ldots, rank(A)$ are the eigenvalues of A. When the best L receive antennas are selected (in terms of maximizing capacity), the instantaneous mutual information can be approximated as

$$\begin{aligned}
\mathcal{I}_{sel} &= \log_2 \det\left(I_L + \frac{\rho_2}{N}\left(R_r^{1/2}\right)_{sel} G R_t G^H \left(R_r^{1/2}\right)_{sel}^H\right) \\
&\approx \log_2 \det\left(I_{w_{r,sel}} + \frac{\rho_2}{N}\left[\prod_{i_2=1}^{w_{r,sel}} \lambda_{i_2}(R_{r,sel})\right]^{\frac{1}{w_{r,sel}}}\left[\prod_{j_2=1}^{w_t} \lambda_{j_2}(R_t)\right]^{\frac{1}{w_t}} G_{sel} G_{sel}^H\right) \\
&= \sum_{k_2=1}^{w_{r,sel}} \log_2\left(1 + \frac{\rho_2}{N}\left[\prod_{i_2=1}^{w_{r,sel}} \lambda_{i_2}(R_{r,sel})\right]^{\frac{1}{w_{r,sel}}}\left[\prod_{j_2=1}^{w_t} \lambda_{j_2}(R_t)\right]^{\frac{1}{w_t}} \lambda_{k_2}\left(G_{sel} G_{sel}^H\right)\right) \quad (3.38)
\end{aligned}$$

where ρ_2 is the SNR, $R_{r,sel} = \left(R_r^{1/2}\right)_{sel}\left(R_r^{1/2}\right)_{sel}^H$ is the best Hermitian submatrix of R_r that has the largest determinant, and $w_{r,sel}$ is the rank of $R_{r,sel}$. Note that

$$\prod_{k_2=1}^{w_{r,sel}} \lambda_{k_2}\left(G_{sel} G_{sel}^H\right) \geq \left(\prod_{k_1=1}^{w_r} \lambda_{k_1}\left(G_{t,r} G_{t,r}^H\right)\right)^{w_{r,sel}/w_r} . \qquad (3.39)$$

Thus, by comparing (3.37) and (3.38), we have

$$\frac{\rho_2}{\rho_1} \lesssim \frac{w_r}{w_{r,sel}} \frac{\left[\prod_{i_1=1}^{w_r} \lambda_{i_1}(R_r)\right]^{\frac{1}{w_r}}}{\left[\prod_{i_2=1}^{w_{r,sel}} \lambda_{i_2}(R_{r,sel})\right]^{\frac{1}{w_{r,sel}}}},$$

and, consequently,

$$\rho_2 \lesssim \rho_1 + 10\log_{10}\left(\frac{w_r}{w_{r,sel}} \frac{\left[\prod_{i_1=1}^{w_r} \lambda_{i_1}(R_r)\right]^{\frac{1}{w_r}}}{\left[\prod_{i_2=1}^{w_{r,sel}} \lambda_{i_2}(R_{r,sel})\right]^{\frac{1}{w_{r,sel}}}}\right) \quad \text{(all in dB)}. \tag{3.40}$$

From (3.40), we can see that the loss in SNR due to receive antenna selection in correlated fading channels can be approximated by

$$10\log_{10}\left(\frac{w_r}{w_{r,sel}} \frac{\left[\prod_{i_1=1}^{w_r} \lambda_{i_1}(R_r)\right]^{\frac{1}{w_r}}}{\left[\prod_{i_2=1}^{w_{r,sel}} \lambda_{i_2}(R_{r,sel})\right]^{\frac{1}{w_{r,sel}}}}\right) \quad \text{dB}. \tag{3.41}$$

3.5 Numerical Examples

Example 1: In this example, we consider full-rank R_t and R_r where these matrices are drawn from the exponential correlation model given by [34]

$$R_t = \begin{bmatrix} 1 & \rho_t^* & \cdots & \rho_t^{N-1^*} \\ \rho_t & 1 & \cdots & \rho_t^{N-2^*} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_t^{N-1} & \rho_t^{N-2} & \cdots & 1 \end{bmatrix}$$

and

$$R_r = \begin{bmatrix} 1 & \rho_r^* & \cdots & \rho_r^{M-1^*} \\ \rho_r & 1 & \cdots & \rho_r^{M-2^*} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_r^{M-1} & \rho_r^{M-2} & \cdots & 1 \end{bmatrix}$$

with $0 \leq \rho_t, \rho_r < 1$. We assume $N = M = 3$, $R = 1$, and we use two values of ρ_t and ρ_r, namely, 0.1 and 0.5.

We plot in Fig. 3.3 the outage probability versus ρ in dB for the cases $N = 3$, $M = 3, L = 1, 2, 3$ with $\rho_t = \rho_r = 0.1$. We also plot in the same figure as a baseline the outage probability for the $N = M = 3$ case with $\rho_t = \rho_r = 0$, i.e., independent fading. We observe from the figure that all curves have the same slope, suggesting that they have the same diversity order, which is 9 in this case. Note also that the degradation in SNR due to correlation for the $L = 3$ case is 0.06 dB at $P_{out} = 10^{-5}$, whereas this degradation as predicted (3.16) is 0.0582, which are very close. Moreover, the loss in SNR due to antenna selection is 1.2 and 4.0 dB for the $L = 2$ and $L = 1$ cases, respectively, whereas these losses, as predicted by (3.41), are 1.73 and 4.74 dB,

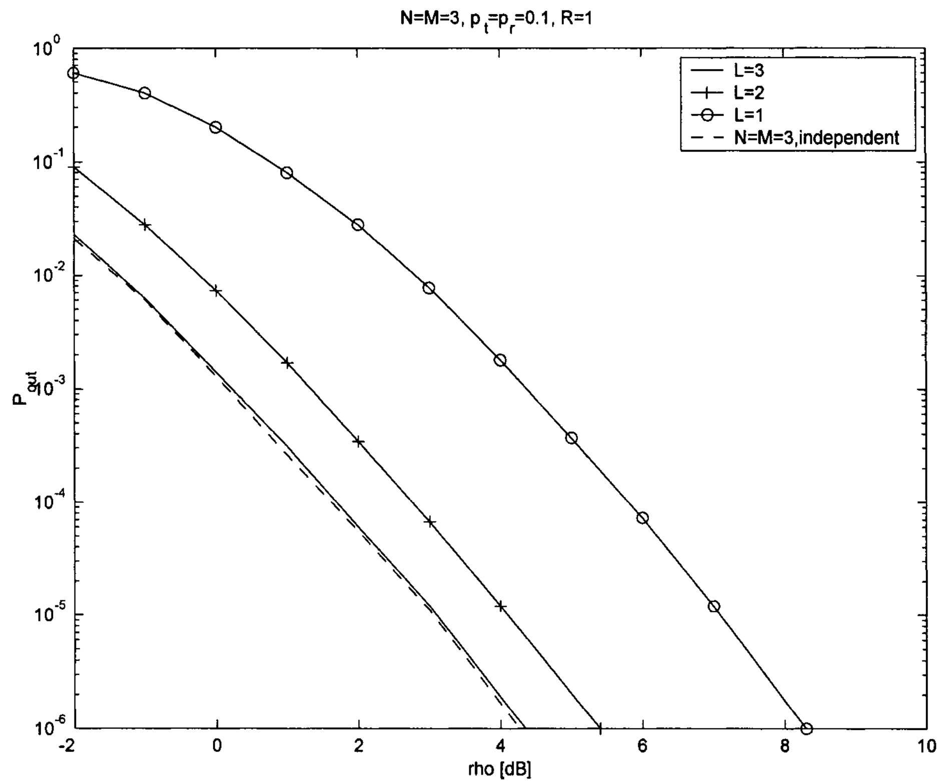

Figure 3.3: Outage probability for MIMO correlated fading channels with receive antenna selection. (Exponential correlation model with $M = N = 3$, $\rho_t = \rho_r = 0.1$.)

respectively (all at $P_{out} = 10^{-5}$.)

The same experiment mentioned above is repeated for the case $\rho_t = \rho_r = 0.5$ and the results are plotted in Fig. 3.4. It is clear from the figure that the diversity order is maintained with antenna selection. Furthermore, the degradation in SNR at $P_{out} = 10^{-5}$ due to correlation for the $L = 3$ case is 1.4 dB, and its corresponding predicted value is 1.665 dB. As for the loss in SNR due to antenna selection, the losses are 1.09 and 4.02 dB for the $L = 2$ and $L = 1$ cases, respectively, which are very close to their predicted values 1.07 and 3.94 dB, respectively.

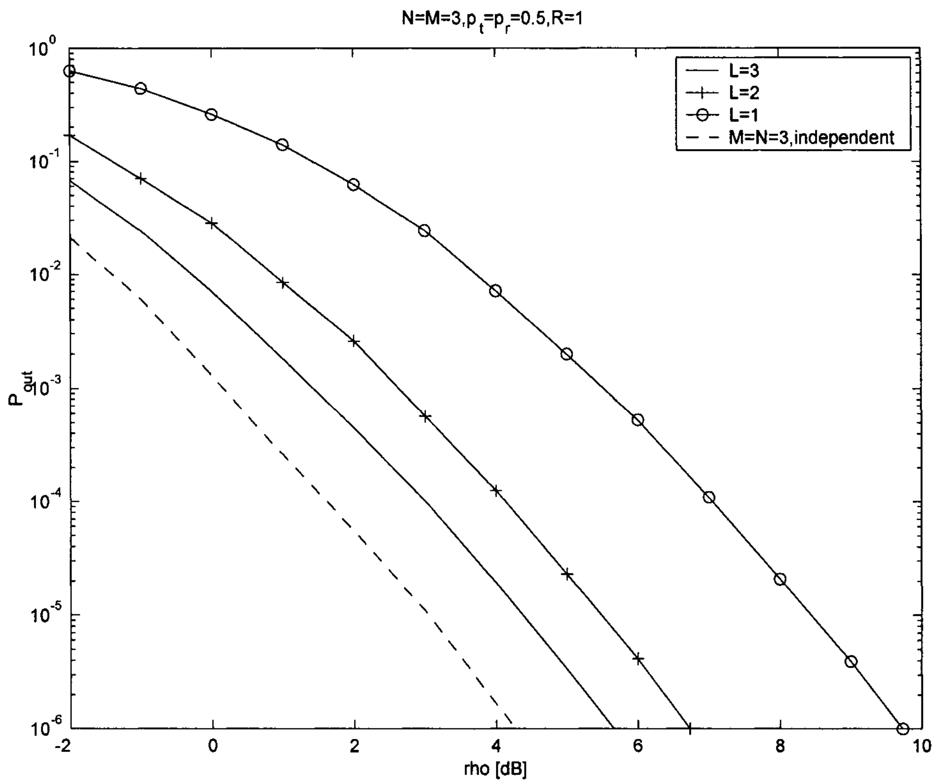

Figure 3.4: Outage probability for MIMO correlated fading channels with receive antenna selection. (Exponential correlation model with $M = N = 3$, $\rho_t = \rho_r = 0.5$.)

Example 2:

In this example, we consider the matrices used in the previous example with $\rho_t = \rho_r = 1$. Clearly, $w_t = w_r = 1$, which means they are rank deficient. The simulation results are plotted in Fig. 3.5. We also plot in the same figure the results for independent fading with $N = M = 1$, which we use as a baseline. It is clear from the figure that the diversity order is maintained with antenna selection. Furthermore, the degradation is SNR due to correlation for the $L = 3$ case is -4.82 dB, whereas its predicted value is -4.77 dB (both at $P_{out} = 10^{-3}$). This result suggests that

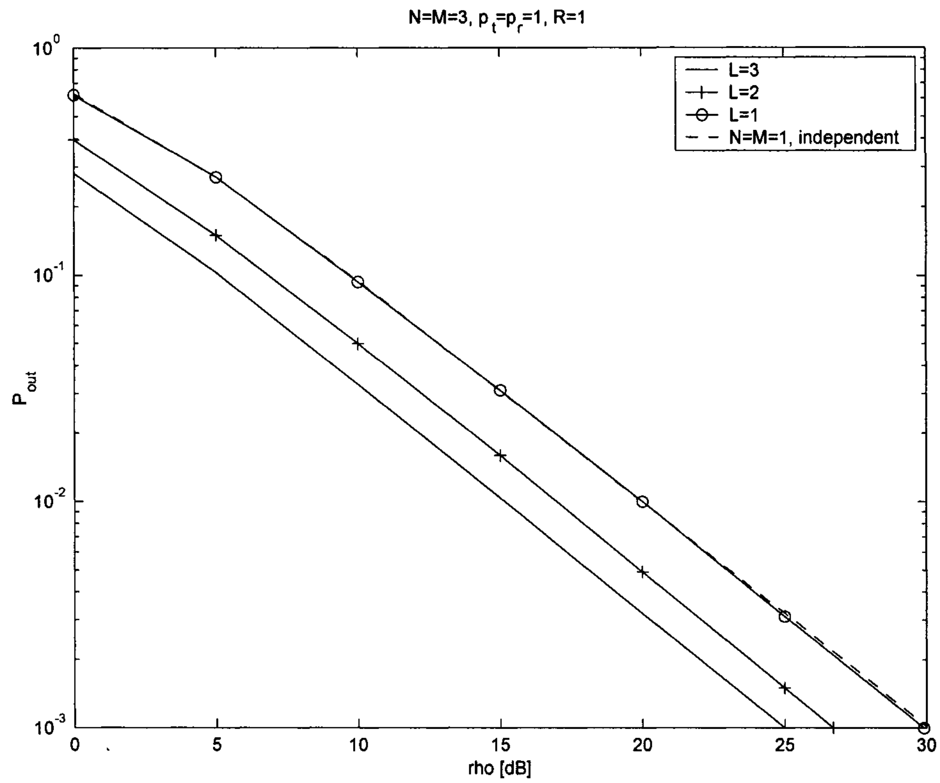

Figure 3.5: Outage probability for MIMO correlated fading channels with receive antenna selection. (Exponential correlation model with $M = N = 3$, $\rho_t = \rho_r = 1.0$.)

the system with correlated fading is superior to that with independent fading. This makes sense because in the correlated fading case there are three receive antennas, whereas there is only one receive antenna in the independent fading case. With antenna selection, the losses at $P_{out} = 10^{-3}$ are 1.7 and 4.8 dB for the cases $L = 2$, and 1, respectively. The predicted values for these losses are 1.76 and 4.77 dB.

Example 3:

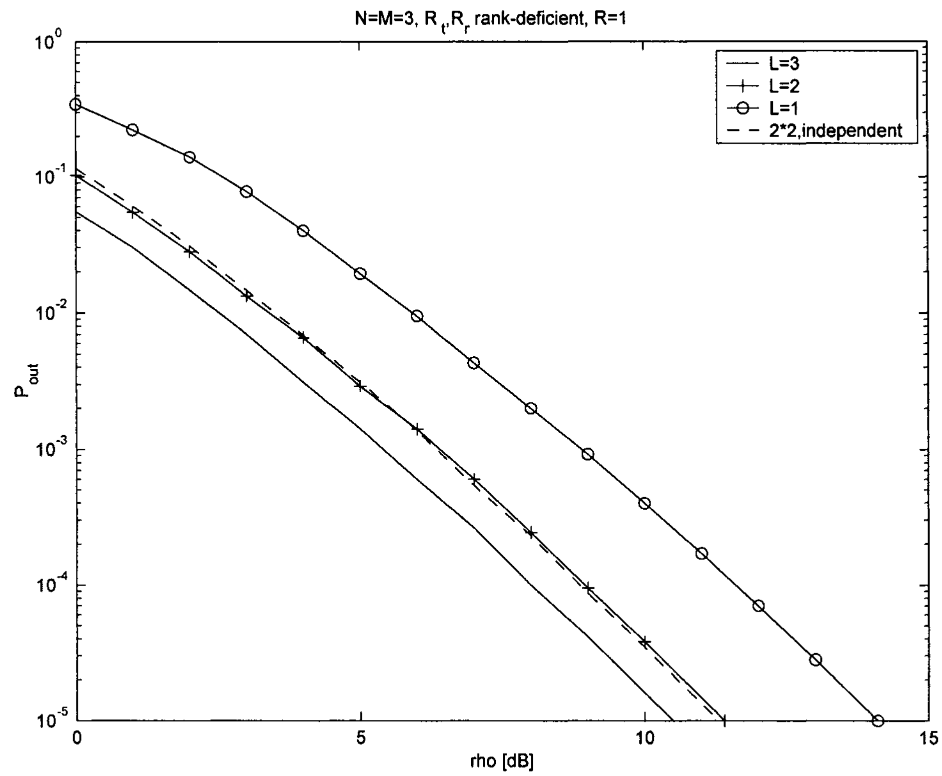

Figure 3.6: Outage probability for MIMO correlated fading channels with receive antenna selection. (Rank-deficient fading correlation)

In this example, we use

$$R_t = R_r = \begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{8} \\ \frac{3}{4} & 1 & \frac{3}{4} \\ \frac{1}{8} & \frac{3}{4} & 1 \end{bmatrix}.$$

Clearly, $w_t = w_r = 2$. The simulation results for this case for various antenna selection scenarios are plotted in Fig. 3.6 along with the independent fading case with $N = M = 2$. The same observation is made here; that is, the diversity order is maintained with antenna selection. Moreover, the loss in SNR due to correlation for the $L = 3$ case is -0.9 dB, whereas it predicted value is -0.93 dB. For antenna selection, for the cases $L = 1$, and 2, the losses are 3.8 and 0.9 dB, which are comparable to their predicted values 4.36 and 1.38 dB, respectively.

3.6 Concluding Remarks

We studied in this chapter the outage probability for MIMO systems over spatially correlated Rayleigh fading channels. We showed that the diversity order of the outage probability is simply the product of the rank of the transmit correlation matrix and the rank of the receive correlation matrix. We also studied the diversity order of the outage probability in the face of performing antenna selection at the receiver, where selection is based on maximizing the channel capacity. We showed that the diversity order with antenna selection is the same as that of the full complexity system. We also derived expressions that can be used to accurately predict the degradation in SNR due to the presence of correlation as well as due to antenna selection.

Chapter 4

Conclusions and Future Work

4.1 Conclusions

For independent Rayleigh fading channels, we studied the outage probability for MIMO systems with receive antenna selection. We derived an upper bound on the outage probability with antenna selection and showed that the resulting diversity order is the same as that of the full complexity system. We also derived another upper bound on the outage probability which is tighter. Furthermore, we presented a thorough investigation of the limit behavior of the outage probability for MIMO systems with and without receive antenna selection. In particular, we derived closed-form expressions for the threshold for the outage probability as the number of transmit antennas tends to infinity.

We further extended our results to MIMO systems over spatially correlated Rayleigh fading channels. We showed that the diversity order of the outage probability is simply the product of the rank of the transmit correlation matrix and the rank of the receive correlation matrix. We also studied the diversity order of the outage probability in the face of performing antenna selection at the receiver, where selection is based on maximizing the channel capacity. We showed that the diversity order with antenna selection is the same as that of the full complexity system. We also derived expressions that can be used to accurately predict the degradation in SNR due to the presence of correlation as well as due to antenna selection.

4.2 Future Work

Wireless communication is one of the most practical disciplines that works with applied science. From the beginning of our research, we bear in mind that our target is to investigate the wireless communications for real-life applications. Based on this thought, we started our work from the simplest case - independent fading model, and later extended to the more realistic correlated fading model. Here, we list several immediate extensions of our research that look especially attractive for future exploration.

4.2.1 Asymptotic Behavior of the Outage Probability for Correlated Fading Channels

In Chapter 2, we studied the asymptotic behavior of the outage probability over independent fading channels. Due to the limited research time, we did not investigate the corresponding results over correlated fading channels in Chapter 3, which is very interesting for future research. In this case, both the transmit and the receive correlation matrices should be taken into account and the resulting threshold is supposed to be a function of the correlation coefficients. In particular, if the transmit correlation matrix has finite rank, even if the number of transmit antennas tends to infinity, threshold does not exist, since in this case the influence of fading can not be averaged either in space or in time domain.

4.2.2 Antenna Selection at the Transmitter

When transmit antenna selection is considered, full or limited CSI will be fed back from the receiver to the transmitter for selection purpose. If the transmitter is armed with full CSI, it selects the best transmit antennas and waterfills the total transmit power to these antennas according to the feedback information. However, this imposes higher demands on the bandwidth and increases the corresponding implementation cost. In practice, the feedback channel has a very limited capacity such that the limited feedback information is especially attractive. In particular, the only information fed back is the selected subset of antennas to be employed. Consequently, the transmitter can effectively distribute the total power on the best transmit antennas at negligible bandwidth lost.

4.2.3 Channel Estimation Error

In real-life applications, the channel fading coefficients are estimated by inserting pilot sequences into the transmitted signals. Assume the fading coefficients remain constant over the duration of a whole frame and vary independently from one frame to another. At the beginning of each frame, orthogonal pilot sequences are sent from transmit antennas. After receiving the pilot sequences, one technique for estimating the CSI is the minimum mean square error (MMSE) algorithm. It has been shown that with MMSE, the estimation error resulted from channel noise can be modeled as a zero mean complex Gaussian random variable. It would be interesting to investigate the sensitivity of antenna selection to channel estimation error. The error may also happen to the control bits of the feedback channel. By knowing the error probability of each control bit, quantifying the impact of feedback channel error on the transmit antenna selection case is of interest as well.

4.2.4 Frequency Selective Fading Channels

In our work, we only investigated the antenna selection over flat fading channels. However, for high data rate wireless communication systems, such as W-CDMA, the signal duration may be small compared to the multipath spread of the channel, resulting in a frequency-selective fading channel or equivalently, a temporal ISI channel.

Hence, it is attractive, but also challenging, to study the effect of antenna selection over frequency-selective fading channels. In this case, selection will take into account the influence of multipath propagation, which greatly complicates the analysis.

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