Multicarrier Systems with Antenna Diversity for Wireless Communications

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ABSTRACT

Multicarrier Systems with Antenna Diversity for Wireless Communications

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Future wireless communications systems need a high quality of service coupled with high data rate transmission for multimedia services. Achieving this goal in the hostile wireless environment with its limited spectrum has several challenges and implies the necessity of a communication system that is able to increase the channel capacity and overcome the difficulties of the wireless transmission environment with reasonable system complexity. Two of the most enabling technologies for the next generation of wireless systems are orthogonal frequency division multiplexing (OFDM) and multiple-input multiple output (MIMO) systems. MIMO systems have been originally designed for known flat fading channels. In this research, some novel MIMO-OFDM schemes for broadband wireless applications are developed and presented. The objective of the proposed schemes is to enhance the performance of OFDM systems over multipath fading channels by using antenna diversity techniques, and also to make MIMO systems applicable to frequency selective multipath fading channels. For the performance evaluation, both bit error rate (BER) and channel capacity analysis are considered. The channel capacity of MIMO-OFDM systems is analytically evaluated and it is shown that the channel capacity of the these systems can be dramatically increased as a function of the number of antennas. The BER performance of the MIMO-OFDM systems is analytically evaluated.

New closed-form expressions for the BER performance of the MIMO-OFDM systems over frequency selective fading channels are derived. On the other hand, the growing popularity of both MIMO and OFDM systems creates the need for adaptive modulation to integrate temporal, spatial and spectral components together. The performance improvement offered by adaptive modulation over non-adaptive systems is remarkable. Furthermore, other dimensions such as frequency and space may yield further gains by providing additional degrees of freedom that can be exploited by adaptive modulation. In this research, a new adaptive modulation scheme for our MIMO-OFDM system (SFBC-OFDM) is presented. The proposed scheme exploits the benefits of space-frequency block codes (SFBC), OFDM and adaptive modulation to provide a high quality of transmission for wireless communications over frequency selective fading channels. It is shown that adaptive modulation can greatly improve the performance of the conventional SFBC-OFDM systems. Finally, a novel antenna selection algorithm is proposed for our MIMO-OFDM system. Three different forms of antenna selection are considered: transmit antenna selection, receive antenna selection, and joint transmit/receive antenna selection. The coding and diversity advantages of the MIMO-OFDM system with antenna selection are examined using average SNR gain, outage probability and BER analysis. The system performance of different forms of the proposed scheme is evaluated and compared. It is shown that the proposed scheme can greatly improve the performance of the conventional SFBC-OFDM systems.

This thesis is dedicated to my parents.

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LIST OF ABBREVIATIONS AND SYMBOLS

ADSL	Asymmetric Digital Subscriber Line
AOFDM	Adaptive Orthogonal Frequency Division Multiplexing
A-SFBC-OFDM	Adaptive Space-Frequency Block Coded Orthogonal Frequency Division Multiplexing
A/D	Analog to Digital Converter
ARQ	Automatic Repeat Request
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BWA	Broadband Wireless Access
CN	Complex Normal
CSI	Channel State Information
DAB	Digital Audio Broadcasting
D/A	Digital to Analog Converter
DFT	Discrete Fourier Transform
DVB	Digital Video Broadcasting
FEC	Forward Error Correction
FFT	Fast Fourier Transform
HIPERLAN	High-Performance Local Area Network
ICI	Inter-Channel Interference
IFFT	Inverse Fast Fourier Transform
ISI	Inter-Symbol Interference
LAN	Local Area Network
MIMO	Multiple-Input Multiple-Output
MIMO-EQ	Multiple-Input Multiple-Output EQualizer
MIMO-OFDM	Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing

ML	Maximum Likelihood
MLSE	Maximum Likelihood Sequence Estimation
MMSE	Minimum Mean Square Error
MPSK	M-ary Phase-Shift Keying
MPSK-SFBC-OFDM	M-ary Phase-Shift Keying
	Space-Frequency Block Coded
	Orthogonal Frequency Division Multiplexing
MQAM	M-ary Quadrature Amplitude Modulation
MQAM-SFBC-OFDM	M-ary Quadrature Amplitude Modulation
	Space-Frequency Block Coded
	Orthogonal Frequency Division Multiplexing
MRC	Maximum Ratio Combining
OFDM	Orthogonal Frequency Division Multiplexing
OTD	Orthogonal Transmit Diversity
OTD-OFDM	Orthogonal Transmit Diversity
	Orthogonal Frequency Division Multiplexing
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase-Shift Keying
RAS	Receive Antenna Selection
Rx	Receiver
SFBC	Space-Frequency Block Codes
SFBC-OFDM	Space-Frequency Block Coded
	Orthogonal Frequency Division Multiplexing
SNR	Signal to Noise Ratio
STBC	Space-Time Block Codes
STBC-OFDM	Space-Time Block Coded
	Orthogonal Frequency Division Multiplexing
STTC	Space-Time Trellis Coding

TAS	Transmit Antenna Selection
TCM	Trellis Coded Modulation
TRAS	Joint Transmit and Receive Antenna Selection
TTCM	Turbo Trellis Coded Modulation
Tx	Transmitter
UMTS	Universal Mobile Telecommunication Service
VA	Viterbi Algorithm
WLAN	Wireless Local Area Network

Chapter 1

Introduction

1.1 Motivation

Future wireless communications systems need a high quality of service coupled with high data rate transmission for multimedia services. Achieving this goal in the hostile wireless environment with its limited spectrum has several challenges and implies the necessity of a communication system that is able to increase the channel capacity and overcome the difficulties of the wireless transmission environment with reasonable system complexity. The major wireless channel impairments are the effects of multipath fading and interferences. Considerable research has been done in the past decades to overcome these difficulties by developing systems that can handle the demands of the wireless telecommunications industry for fixed and mobile systems.

Different modulation, coding and signal processing techniques including equalization techniques, spread spectrum techniques, and multicarrier modulation have been proposed to overcome the limitations caused by the wireless environment. Two of the most enabling technologies for the next generation of wireless systems are orthogonal frequency division multiplexing (OFDM) and multiple-input multiple output (MIMO) systems.

OFDM as a special case of multicarrier modulation, has gained much attention, since it is less complex than single carrier systems that require powerful equalizers, and is better in terms of spectral efficiency than the spread spectrum techniques [1]-[19]. OFDM was introduced for high data rate transmission many years ago; however, practical interest has only increased recently, due in part to advances in digital signal processing and microelectronics. It was later proposed and used for a wide variety of applications such as digital video broadcasting (DVB), digital audio broadcasting (DAB), and asymmetric digital subscriber line (ADSL) services. It has found its applications in the wireless local area networks (WLAN) and broadband wireless access (BWA) and has been chosen as the modulation technique for the new IEEE802.11a standard [20] as well as high-performance LAN (HIPERLAN) [20]-[22].

In this modulation technique the data stream to be transmitted is divided into several lower rate data streams (each being modulated on a subcarrier) making the symbol duration long enough to avoid complicated equalization. A small interval known as the cyclic prefix is inserted in the OFDM symbols to eliminate intersymbol interference (ISI) and interchannel interference (ICI) almost completely.

In a hostile wireless environment, the channel dispersion will extensively attenuate some subcarriers. Therefore, in the presence of multipath fading some subchannels will have high attenuation and low signal-to-noise ratio (SNR), resulting in a high error probability, hence decreased overall system performance. This problem can be mitigated by using the following techniques:

- Use of coding across subcarriers. This technique has been studied with different schemes such as Reed Solomon codes, convolutional codes, trellis coded modulation (TCM) and Turbo codes.

- Use of different signal constellations for different subcarriers, i.e., adaptive bit allocation to the subcarriers.

- Use of frequency-domain equalizers. This technique, however, leads to noise enhancement.

• Use of precoding. This technique can compensate fading by inverting subcarrier fading at the transmitter, but it requires knowledge about the channel at the transmitter.

• Use of antenna diversity. This technique is also an effective method that can provide extra diversity (spatial diversity) to the OFDM system and can greatly improve the overall system performance.

Antenna diversity technique is well known as an effective way to improve the signal quality and to increase the channel capacity. Antenna diversity techniques fall under two main categories: receiver diversity and transmit diversity. Receive diversity is a classical diversity technique used to improve the received signal quality by combining multiple independently received signals (corresponding to the same transmitted signal).

Transmit diversity techniques are very attractive for the downlink in wireless communication systems, because it is more affordable to add equipment to the base station rather than the mobile units. Transmit diversity has been recently studied in different forms of modulation and coding techniques. Some interesting approaches for transmit diversity have been proposed by Wittneben [23], and Seshadri and Winters [24] in which copies of the same symbol are transmitted at different times, creating an artificial multipath distortion. Then an equalizer is used to resolve the multipath distortion and to obtain the diversity gain. More recently, significant progress has been made by many other researchers [25], [26]. One of these techniques is Space-Time Trellis Coding (STTC) [27].

STTC integrates multiple antennas at the transmitter and receiver, with coding and modulation to combat fading and achieve higher data rates for wireless communications. In addition, it gives the best trade-off between spectral efficiency and power consumption. The decoding of these codes requires the use of a vector form of the Viterbi decoder. At a fixed number of transmit antennas, the decoding complexity of STTC exponentially increases with the number of states.

In order to reduce the complexity of the decoder, a simple transmit diversity scheme using two antennas was proposed by Alamouti [28]. This scheme has a simple maximum likelihood decoding algorithm based on linear processing at the receiver. Later, Tarokh *et al.* [29]-[30] introduced a generalized form of Alamouti's scheme named Space-Time Block Codes (STBC), or Orthogonal Transmit Diversity (OTD).

STTC and STBC were originally designed for known flat fading channels. As space-time coding does not require any form of interleaving, these systems are attractive for delay-sensitive applications.

On the other hand, recent information-theoretic results show that a multipath fading channel can provide a rich scattering channel for space-time systems, when the channel gain between pairs of transmitter and receiver antennas are uncorrelated.

However, in future broadband wireless communication systems where symbol duration can be smaller than the channel delay spread, the channel propagation can have frequency selective effects. In non-flat fading channels such as frequency selective multipath channels, convolution of the channel impulse response with STBC output destroys the orthogonality of the STBC. Consequently, these techniques are often only effective over frequency flat fading channels, such as indoor wireless systems or low data-rate networks.

The effects of frequency selectivity on space-time coding have been studied before. It was shown that in the presence of ISI induced by frequency selective multipath channels, the coding advantage decreases significantly, specially when delay spread becomes relatively high, resulting in intolerable performance degradation. In order to maintain the decoding simplicity and take advantage of existing space-time codes designed for flat fading channels, some additional processing is required to make space-time codes suitable for application over frequency selective fading channels. One approach for mitigating the ISI is to use an equalizer at the receiver. It was shown by Cioffi *et al.* [34],[35], that a multiple-input multiple-output equalizer

(MIMO-EQ) can equalize the frequency-selective fading channels and convert them into ISI-free channels where space-time coding is applicable. The main drawback of using an equalizer is high receiver complexity.

Another approach is to employ OFDM. It has been shown that OFDM can be used to transform frequency-selective fading channels into several flat fading subchannels. Hence, STBC with OFDM can be effectively used in non-flat fading channels [14],[15].

Several combinations of OFDM with different antenna diversity techniques have been considered, including transmitter diversity, receiver diversity and space-time coding [36]-[49]. For example, in [43], Ye Li *et al.*, studied a combination of OFDM with three transmitter diversity techniques: STTC, delay and permutation transmitter diversity. They showed that STTC-OFDM can provide better performance than the other mentioned methods.

1.2 Research Objectives and Contributions

In this research, we develop and propose some novel MIMO-OFDM schemes (OTD-OFDM or SFBC-OFDM) for broadband wireless applications. The main objective of the proposed schemes is to enhance the performance of OFDM systems over multipath fading channels by using antenna diversity techniques and also to make MIMO systems applicable for frequency selective multipath fading channels. Note that the terms OTD-OFDM and SFBC-OFDM will be used interchangeably.

For the performance evaluation we use both bit error rate and channel capacity analysis. The channel capacity of MIMO-OFDM systems will be analytically evaluated and it will be shown that by using MIMO systems the capacity of the channel can be dramatically increased as a function of the number of antennas. The bit error rate performance of MIMO-OFDM systems will be analytically evaluated. We will derive new closed-form expressions for BER performance of the MIMO-OFDM systems over frequency selective fading channels.

On the other hand, the growing popularity of both MIMO and OFDM systems creates the need for adaptive modulation to integrate temporal, spatial and spectral components together. The goal of adaptive modulation in an OFDM system is to allocate, according to the instantaneous condition of a subchannel, an appropriate number of bits and to choose the suitable modulation mode for transmission in each subcarrier, in order to improve the system performance or to keep the overall BER performance at a certain desired level.

The performance improvement offered by adaptive modulation over non-adaptive systems is remarkable. Furthermore, other dimensions such as frequency and space may yield further gains by providing additional degrees of freedom that can be exploited by adaptive modulation.

In this research, a new adaptive modulation scheme for our MIMO-OFDM system (SFBC-OFDM) is presented. The proposed scheme, exploits the benefits of space-frequency block codes (SFBC), OFDM and adaptive modulation to provide a high quality of transmission for wireless communications over frequency selective fading channels. Spectral efficiency advantage of the proposed system is examined. It is shown that adaptive modulation can greatly improve the performance of the conventional SFBC-OFDM systems.

Finally, a novel antenna selection algorithm is proposed for our MIMO-OFDM system. Three different forms of antenna selection are considered: transmit antenna selection, receive antenna selection, and, joint transmit/receive antenna selection. The coding and diversity advantages of the MIMO-OFDM system with antenna selection are examined using average SNR gain, outage probability and BER analysis validated by numerical simulation. It is shown that the proposed scheme can greatly improve the performance of the conventional SFBC-OFDM systems.

1.3 Dissertation Outline

This dissertation is organized as follows: In Chapter 2, a brief overview of diversity techniques is presented. In Chapter 2, also different schemes, such as time diversity, frequency diversity, antenna diversity, and the most popular schemes known as MIMO systems such as space-time coding techniques are discussed in detail.

In Chapter 3, the application of MIMO systems over frequency selective fading channels using an OFDM system is proposed. Also the main concept of OFDM is reviewed and our proposed MIMO-OFDM system is presented. The proposed scheme is explained in more detail and both encoding and decoding algorithms are explained by some examples.

In Chapter 4, our analytical evaluation for the channel capacity of MIMO-OFDM systems is presented.

In Chapter 5, the bit error rate (BER) performance of MIMO-OFDM systems is analytically evaluated. Some new closed-form expressions of BER performance of MIMO-OFDM systems over frequency selective fading channels are derived.

In Chapter 6, a new adaptive modulation scheme is presented for MIMO-OFDM systems. The proposed scheme exploits the benefits of space-frequency block codes, OFDM and adaptive modulation to provide high quality transmission for wireless communications over frequency selective fading channels. Spectral efficiency advantage of the proposed system is examined.

In Chapter 7, a novel antenna selection algorithm is proposed for MIMO-OFDM systems. Three different forms of antenna selection are considered: transmit antenna selection, receive antenna selection, and, joint transmit/receive antenna selection. The coding and diversity advantages of the MIMO-OFDM systems with antenna selection are examined using average SNR gain, outage probability and BER analysis.

Finally, Chapter 8 concludes this thesis and also future directions are presented.

Chapter 2

Diversity Techniques

In wireless communications systems, diversity techniques have been used to combat multipath effects of fading to improve the system performance and to increase the channel capacity. The principle idea of diversity is to send several replicas of the information signal to the receiver. Since the receiver, receives signals which come from different independent fading links, some of the received signals will not be affected by fading in a given time. Diversity techniques have been classified in the literature into the following categories.

1-*Temporal (Time) Diversity*

2-*Frequency Diversity*

3-*Polarization Diversity*

4-*Spatial (Antenna) Diversity*

The remainder of this chapter is organized as follows. An overview of time diversity, frequency diversity, polarization diversity, and antenna diversity techniques will be presented in Sections 2.1, 2.2, 2.3, and, 2.4, respectively. In Section 2.4, transmit antenna diversity, receive antenna diversity and joint transmit/receive antenna diversity schemes are explained. The orthogonal transmit diversity (OTD) technique is reviewed and both encoding and decoding algorithms for OTD system are described. Finally, Section 2.5 concludes this chapter.

2.1 Time Diversity

In time diversity technique, a transmitted signal is spread over a time period that is greater than the coherence time of the channel (the coherence time of the channel is the minimum time separation between independent channel fades). Hence, the received signals can be uncorrelated. Time interleaving, together with Forward Error Correction Coding (FEC), and Automatic Repeat Request (ARQ) can provide time diversity improvements. This technique is useful for time selective fading channels.

A drawback of time diversity is the delay caused by time spreading, specially in slow varying channels where time diversity introduces large delays.

2.2 Frequency Diversity

In frequency diversity techniques, transmitted signal energy is spread over different frequency intervals. The carrier frequencies should be separated enough so that the fading associated with the different frequencies is uncorrelated. Hence, frequency intervals should be less than the coherence bandwidth of the channel (the coherence bandwidth is the smallest frequency separation between independent fadings). This kind of diversity can be provided by spread spectrum techniques, FEC and OFDM. It is applicable for frequency selective fading channels.

2.3 Polarization Diversity

In this scheme, antennas with different polarizations are employed for reception and/or transmission. An antenna can transmit either a vertically polarized or a horizontally polarized wave. The received signals will exhibit uncorrelated fading statistics, when vertical polarized and horizontal polarized waves are transmitted simultaneously. Polarization diversity is a special case of space diversity because separate antennas are used. However, since only two orthogonal polarizations exist, there will be only two available diversity branches.

2.4 Antenna Diversity

In scattering environments, spatial (antenna) diversity is an effective, practical and promising technique for reducing the multipath fading effects.

Spatial diversity techniques fall under the following categories:

1-*Receive antenna diversity*

2-*Transmit antenna diversity*

3-*Transmit/Receive antenna diversity*

2.4.1 Receive Antenna Diversity

In a classical receive antenna diversity approach, multiple antennas are employed at the receiver that receive different copies of the transmitted information signal. The receiver performs selection, switching, or combining of the received signals in order to mitigate channel fading and improve the quality of the received signal.

The following types of receive antenna diversity can be considered:

2.4.1.1 Selection Diversity

In selection diversity technique, based on the M_R received signals, the received signal with the largest power, SNR is selected. Figure 2.1 shows a block diagram of the selection diversity technique.

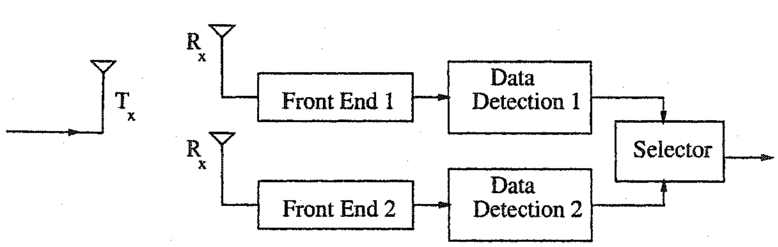

Figure 2.1: Selection Diversity.

2.4.1.2 Switched Diversity

Switched diversity technique is like selection diversity, but it switches to another antenna if the received signal falls below a certain threshold. Figure 2.2 shows a block diagram of the switch diversity technique.

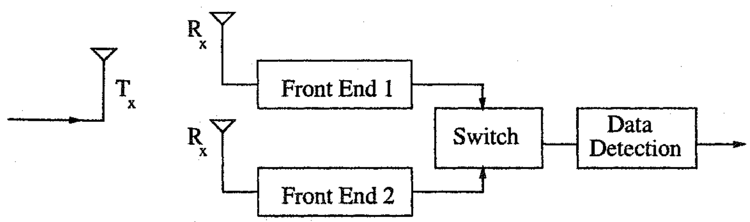

Figure 2.2: Switched Diversity.

2.4.1.3 Maximum Ratio Combining (MRC)

In this scheme, weighted replicas of the received signals are linearly combined. This method is very efficient and can improve the system performance linearly by the number of receive antennas but its complexity becomes prohibitive when the number of antennas is high. Figure 2.3 shows a block diagram of the maximum ratio combining technique.

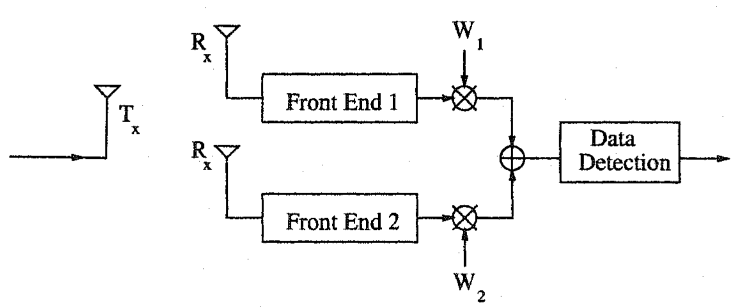

Figure 2.3: Maximum Ratio Combining.

As an example we can briefly review the maximum ratio combining method for a case with one transmit and two receive antennas ($M_T = 1$, $M_R = 2$).

The transmitter sends a signal s through the transmitter antenna and the receiver antennas, receive $r_1 = h_1\, s + v_1$ and $r_2 = h_2\, s + v_2$, respectively, where v_1 and v_2 denote the AWGN and h_1 and h_2 denote the channel gain between the transmitter antenna and the two receiver antennas. The receiver combines the received signals with coefficients $W_1 = h_1^*, W_2 = h_2^*$ where $*$ indicates complex conjugate. We have:

$$y = h_1^* r_1 + h_2^* r_2 = \left(\|h_1\|^2 + \|h_2\|^2\right) s + h_1^* v_1 + h_2^* v_2 \tag{2.1}$$

where $\|.\|^2$ is the norm operation.

After maximum-likelihood (ML) decision, the signal s will be detected from y. As can be seen, MRC can increase the SNR of the system and, therefore, it can significantly improve the system performance.

2.4.2 Transmit Antenna Diversity

Transmit antenna diversity is a technique that can provide spatial diversity gain by using multiple antennas at the transmitter. It has been studied extensively as an effective technique for combating detrimental effects in wireless fading channels. Its relative implementation simplicity and the feasibility of having multiple antennas at the base station makes it an attractive method. Several transmit diversity techniques have been proposed including:

2.4.2.1 Delay Diversity

Delay diversity was proposed by Wittneben[23], and later a similar scheme was suggested by Seshadri and Winters [24]. The idea of delay diversity techniques is to convert the spatial diversity into frequency diversity by transmitting the same symbols through multiple antennas at different times, hence creating a multipath distortion with independent path fades and the same path energy. Figure 2.4 shows a block diagram of the delay diversity technique.

For example in a system with $M_T = 2$, $M_R = 1$ the effective channel can be expressed as:

$$H(e^{j2\pi\tau}) = h_1 + h_2\, e^{-j2\pi\tau} \tag{2.2}$$

where h_1 and h_2 are the channel impulse responses between transmitter antenna-1, antenna-2 and the receiver antenna, respectively. Since h_1 and h_2 are random i.i.d (identical independent distributions), $H(e^{j2\pi\tau})$ is also random and the frequency correlation function can be expressed as:

$$R(e^{j2\pi\nu}) = \frac{1}{2}E\,\left(H(e^{j2\pi\tau})\,H^*(e^{j2\pi(\tau-\nu)})\right) = \frac{1}{2}\left(1 + e^{-j2\pi\nu}\right) \tag{2.3}$$

where E is the expectation operator.

Such a channel looks like a 2-path channel. At the receiver, an MLSE (Maximum Likelihood Sequence Estimator) or MMSE (Minimum Mean Square Error) equalizer is used to resolve multipath and capture the diversity gain in the system. It can provide a diversity order of M_T without loss of bandwidth efficiency.

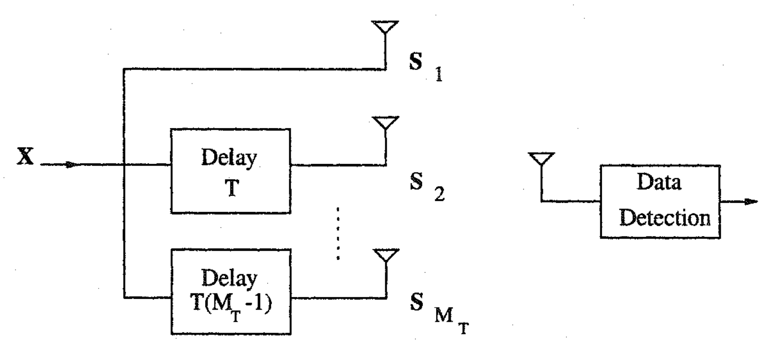

Figure 2.4: Delay Diversity.

2.4.2.2 Transmit diversity with antenna Hopping

In this method, assuming that channel state information is available at the transmitter side, at time i , $(1 < i < M_T)$ the signal will be transmitted from antenna i. This method can achieve a diversity order of M_T using ML detector or MRC at the receiver. Figure 2.5 shows a block diagram of the transmit diversity with antenna hopping technique.

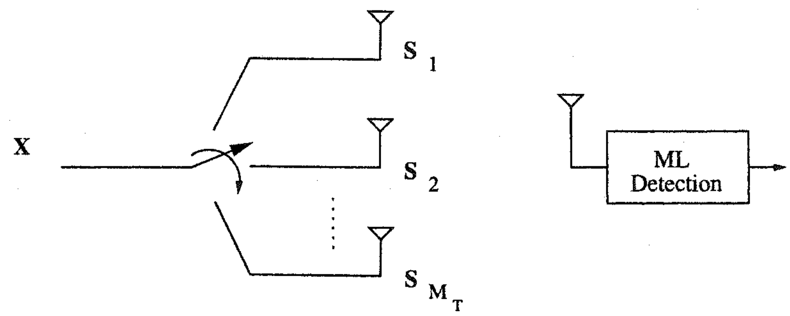

Figure 2.5: Transmit diversity with antenna Hopping.

2.4.2.3 Orthogonal Transmit Diversity

Orthogonal transmit diversity first was proposed by Alamouti [28] for flat fading channels using two antennas. Later, a generalized form of Alamouti's scheme called Space-Time Block Codes (STBC), or Orthogonal Transmit Diversity (OTD), was introduced by Tarokh *et al.* [29]. STBC has a simpler method for obtaining transmitter diversity without any sacrifice in bandwidth and without using complex decoding. STBC can provide full diversity gain from the channel with a simple signal processing (linear combining) and minimal encoder/decoder complexity.

Although STBC achieves maximum diversity, its performance is inferior compared to Space-Time Trellis Codes (STTC) [27]. On the other hand in STTC, when the number of transmit antennas is fixed, the decoding complexity that is measured by the number of trellis states in the decoder, increases exponentially with the transmission rate. STBC is appealing since it can provide full diversity gain from the channel while a very simple maximum-likelihood decoding algorithm is used at the decoder. This new method uses a theory of orthogonal design to design space-time block codes. While providing full diversity, the codes presented for STBC can provide a maximum possible transmission rate allowed by the theory for the real constellations, such as pulse amplitude modulation (PAM). A complex orthogonal design which can provide full diversity and full transmission rate is not possible for more than two antennas [29]. For complex orthogonal design, some possible codes are presented in [29] that provide 1/2 and 3/4 of a full transmission rate for 3 and 4 transmit antennas.

In this section, we review the theory of space-time block code. Consider a block of symbols that can be sent in T time slots through M_T transmit antennas, where the space-time block code can be represented by a $T \times M_T$ transmission matrix $\mathbf{G}$ given by:

$$\mathbf{G} = \begin{pmatrix} \mathcal{G}_{1,1} & \mathcal{G}_{1,2} & \cdots & \mathcal{G}_{1,M_T} \\ \mathcal{G}_{2,1} & \mathcal{G}_{2,2} & \cdots & \mathcal{G}_{2,M_T} \\ \vdots & \vdots & \cdots & \vdots \\ \mathcal{G}_{T,1} & \mathcal{G}_{T,2} & \cdots & \mathcal{G}_{T,M_T} \end{pmatrix} \tag{2.4}$$

The $(T \times M_T)$ transmission matrix $\mathbf{G}$ is based on a complex generalized orthogonal design, as defined in [29]-[30].

Each element $\mathcal{G}_{i,j}$ is a linear combination of a subset of indeterminate $s_1, s_2, \cdots, s_K$ and their conjugates as follow:

$$\mathbf{G} = \sum_{k=1}^{K} (A_k\, s_k + B_k\, s_k^*) \tag{2.5}$$

where A_k and B_k are $K \times K$ complex matrices and $s_1, s_2, \cdots, s_K$ are information symbols. The symbols s_k and s_k^* are modulated separately with A_k and B_k.

Orthogonal design requires that

$$\mathbf{G}^{\dagger}\mathbf{G} = \left(|s_1|^2 + |s_2|^2 + \cdots |s_K|^2\right) I_{M_T} \tag{2.6}$$

where $\mathbf{G}^{\dagger}$ is the Hermitian of $\mathbf{G}$ and I_{M_T} is the $M_T \times M_T$ identity matrix. Due to the fact that K symbols are transmitted in T time slots, the code rate of $\mathbf{G}$ is defined to be $R = \frac{K}{T}$. It is worth mentioning that the relative transmission rate to the maximum possible rate of a full-diversity code is called code rate, which is $R \leq 1$. Since the elements of $\mathbf{G}$ are linear combinations of symbols s_k and s_k^*, encoding only requires linear processing.

At the transmitter side, at the time slot t, $t = 1, 2, \cdots, T$, the t-th row of $\mathbf{G}$ is transmitted through transmit antennas at the same time, in other words the j-th element of t-th row of $\mathbf{G}$, $(\mathcal{G}_{t,j})$ is transmitted through antenna $j = 1, 2, \cdots, M_T$.

Consider a MIMO (Multiple Input/Multiple Output) system consisting of M_T multiple transmit antennas and M_R receive antennas. In a narrow-band, flat-fading channel, i-th receive antenna (Rx) receives the superposition of the M_T transmitted signals from j-th transmit antenna (Tx) passed through the channel $h_{i,j}$ that can be written as

$$r_{t,i} = \sum_{j=1}^{M_T} h_{i,j}\, \mathcal{G}_{t,j} + v_{t,i} \qquad i = 1, 2, ..., M_R \tag{2.7}$$

where $v_{t,i}$ is a complex additive white Gaussian noise (AWGN) with a zero mean and a variance of $\frac{\sigma^2}{2}$.

For decoding of the received signal, assume that the channel state information $(h_{i,j})$ is available or can be estimated at the receiver. Based on the minimization of the following metrics:

$$\sum_{i=1}^{M_R} \sum_{t=1}^{T} \left| r_{t,i} - \sum_{j=1}^{M_T} h_{i,j}\, \mathcal{G}_{t,j} \right|^2 \tag{2.8}$$

The receiver can detect the symbols $s_1, s_2, \cdots, s_K$. In an orthogonal design in the above metric all transmitted symbols can be separated from each other and simply decoded by ML detection. For example in orthogonal design given by:

$$\mathbf{G}_2 = \begin{bmatrix} +s_1 & s_2 \\ -s_2^* & +s_1^* \end{bmatrix} \tag{2.9}$$

the above metric can be expressed as:

$$\sum_{i=1}^{M_R} \left(\left| r_{1,i} - h_{i,1}\, s_1 - h_{i,2}\, s_2 \right|^2 + \left| r_{2,i} + h_{i,1}\, s_2^* - h_{i,2}\, s_1^* \right|^2 \right) \tag{2.10}$$

Minimizing the above metric is equivalent to minimizing the two separate following metrics:

$$\left| \left[\sum_{i=1}^{M_R} \left(h_{i,1}^*\, r_{1,i} + h_{i,2}\, r_{2,i}^* \right) \right] - s_1 \right| + |s_1|^2 \left(\sum_{i=1}^{M_R} \left(|h_{i,1}|^2 + |h_{i,2}|^2 \right) - 1 \right) \tag{2.11}$$

$$\left| \left[\sum_{i=1}^{M_R} \left(h_{i,2}^*\, r_{1,i} - h_{i,1}\, r_{2,i}^* \right) \right] - s_2 \right| + |s_2|^2 \left(\sum_{i=1}^{M_R} \left(|h_{i,1}|^2 + |h_{i,2}|^2 \right) - 1 \right) \tag{2.12}$$

which leads to an estimate of $\widetilde{s_1}$ and $\widetilde{s_2}$, which can be written as:

$$\begin{aligned} \widetilde{s_1} &= \textstyle\sum_{i=1}^{M_R} \left(|h_{i,1}|^2 + |h_{i,2}|^2 \right)\, s_1 + \sum_{i=1}^{M_R} \left(h_{i,1}^*\, v_{1,i} + h_{i,2}\, v_{2,i}^* \right) \\ \widetilde{s_2} &= \textstyle\sum_{i=1}^{M_R} \left(|h_{i,1}|^2 + |h_{i,2}|^2 \right)\, s_2 + \sum_{i=1}^{M_R} \left(h_{i,2}^*\, v_{1,i} - h_{i,1}\, v_{2,i}^* \right) \end{aligned} \tag{2.13}$$

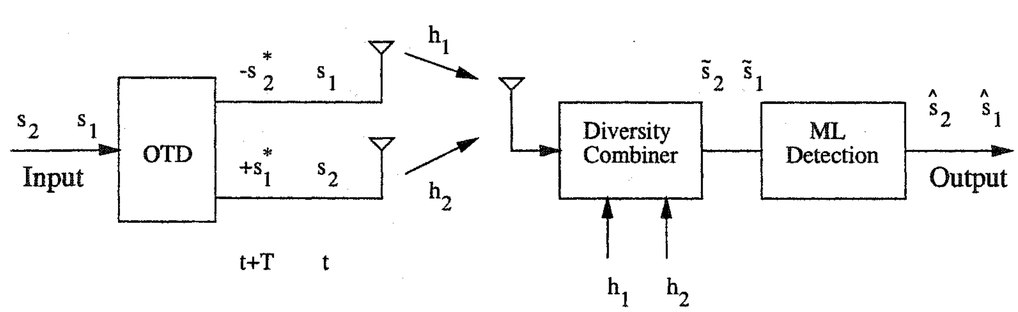

Figure 2.6: Orthogonal Transmit Diversity with $M_T = 2$, $M_R = 1$.

Here, we consider an OTD system with $M_T = 2$, $M_R = 1$. As shown in Figure 2.6, at a given time interval, two signals are simultaneously transmitted from two separate antennas. In the first interval time, the signal s_1 is transmitted through antenna 1, and the signal s_2 is transmitted through antenna 2. In the next time interval, the signal $-s_2^*$ is transmitted from antenna 1 and the signal s_1^* is transmitted from antenna 2. The received signals at the receiver antenna can be written as:

$$r_1 = r(t) = h_1(t)\, s_1(t) + h_2(t)\, s_2(t) + v_1(t)$$
$$r_2 = r(t+T) = h_2(t+T)\, s_1^*(t) - h_1(t+T)\, s_2^*(t) + v_2(t) \tag{2.14}$$

where v_1 and v_2 denote AWGN, and h_1 and h_2 denote the channel gain between the two transmitter antennas and the receiver antenna respectively. Assuming that the channel gains are constant over two symbols, the above equation can be written as:

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & +s_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{2.15}$$

$$\mathbf{r} = \mathbf{G}_2\, \mathbf{h} + \mathbf{v} \tag{2.16}$$

or

$$\mathbf{r}' = \mathbf{H}\, \mathbf{s} + \mathbf{v}' \tag{2.17}$$

where $\mathbf{r}' = \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix}$, $\mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$, $\mathbf{v}' = \begin{bmatrix} v_1 \\ v_2^* \end{bmatrix}$ and $\mathbf{H} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$ is an orthogonal matrix, i.e.,

$$\mathbf{H}^\dagger \mathbf{H} = \left(|h_1|^2 + |h_2|^2\right) \mathbf{I}_2 \tag{2.18}$$

where $\mathbf{H}^\dagger$ is the Hermitian of $\mathbf{H}$.

The combiner at the receiver side, knowing the channel state information, combines the two received signals r_1 and r_2 resulting in an estimate of the transmitted signal as follows:

$$\widetilde{\mathbf{s}} = \mathbf{H}^\dagger\, \mathbf{r} = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} \mathbf{s} + \mathbf{H}^\dagger\, \mathbf{v}' \tag{2.19}$$

which can be re-written as:

$$\begin{aligned}\widetilde{s_1} &= \left(|h_1|^2 + |h_2|^2\right)\, s_1 + h_1^* v_1 + h_2\, v_2^* \\ \widetilde{s_2} &= \left(|h_1|^2 + |h_2|^2\right)\, s_2 + h_2^* v_1 - h_1\, v_2^*\end{aligned} \tag{2.20}$$

At the end, the symbols of $\widetilde{s_1}$ and $\widetilde{s_2}$ are detected by an ML detector. As we can see, the above scheme can provide the same diversity order as MRC scheme. The above scheme can be used with more antennas at the receiver and can provide a diversity order of $2\,M_R$ which is full diversity gain of MIMO system.

2.5 Conclusion

In wireless communications systems, diversity techniques have been used to combat multipath effects of fading, to improve the system performance and increase the channel capacity. The principal idea of diversity is to send several replicas of the information signals to the receiver. Since the receiver, receives signals which come from different independent fading links, some of received signals will not be affected by fading at a given time.

In this chapter, several diversity techniques, including time diversity, frequency diversity, polarization diversity, and antenna diversity were described. Transmit antenna diversity, receive antenna diversity and joint transmit/receive antenna diversity schemes were explained. Finally, the orthogonal transmit diversity (OTD) technique was described. Also, both encoding and decoding algorithms for OTD systems were described.

Chapter 3

OTD-OFDM Systems

The focus of this chapter is two fold: to address the problem of multipath fading channel dispersion that extensively attenuates some subcarriers in the OFDM system, and to consider the possible application of the orthogonal transmit diversity (OTD) over non-flat fading channels by using an OFDM system. The objective of the proposed scheme is to enhance the performance of the OFDM system over multipath fading channels by using antenna transmit diversity techniques. We outline our proposed scheme and discuss our approach, plan and methodology in addressing the research issues.

The remainder of this chapter is organized as follows. An overview of the conventional OFDM systems is presented in Section 3.1. In Section 3.2, the system model for OTD-OFDM is presented. Both encoding and decoding algorithms for OTD-OFDM systems are described. Some examples are provided to review these algorithms for various OTD codes in more detail. Finally, Section 3.3 concludes this chapter.

3.1 Orthogonal Frequency Division Multiplexing

Orthogonal Frequency Division Multiplexing (OFDM) as a special case of multicarrier modulation, has gained much attention, since it is less complex than single carrier systems that require powerful equalizers, and is better in terms of spectral efficiency than the spread spectrum techniques [13], [14]. OFDM was introduced for high data rate transmission many years ago; however, practical interest has only increased recently, due in part to advances in digital signal processing and micro-electronics. It was later proposed and used for a wide variety of applications such as digital video broadcasting (DVB), digital audio broadcasting (DAB), and asymmetric digital subscriber line (ADSL) services. It has found its applications in the wireless LAN (WLAN) and broadband wireless access (BWA) and has been chosen as the modulation technique for the new IEEE802.11a standard [20] as well as high-performance LAN (HIPERLAN) [21].

In this modulation technique the data stream to be transmitted is divided into several lower rate data streams (each being modulated on a subcarrier), making the symbol duration long enough to avoid complicated equalization. In order to obtain a high spectral efficiency the frequency responses of the subchannels are overlapping and orthogonal. To keep this orthogonality, a guard time interval known as cyclic prefix is added within the OFDM symbol. The cyclic prefix is inserted in the OFDM symbols to eliminate inter-symbol interference (ISI) and inter-channel interference (ICI) almost completely [14].

The cyclic prefix is a copy of the last part of the OFDM symbol which is appended to the transmitted symbol, as shown in Figure 3.1.

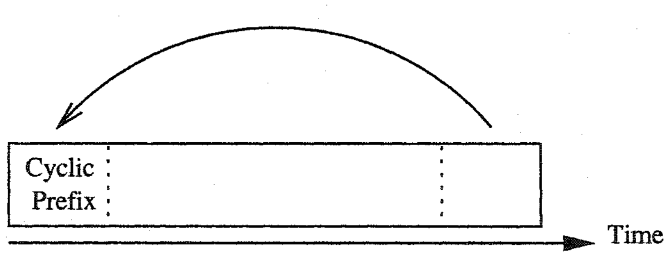

Figure 3.1: Cyclic prefix in OFDM symbol.

A high level block diagram of an OFDM system is shown in Figure 3.2. The input data stream is modulated using M-ary phase shift keying (MPSK) or M-ary quadrature amplitude modulation (MQAM), having the symbol period T_s. The sequence of MPSK/MQAM data is converted into parallel form, generating blocks of $\mathbf{S}_l = \{s(lN), s(lN+1), ..., s(lN+N-1)\}^T$ (superscript T in $(.)^T$ denotes the transpose operation) which has a block duration of NT_s where l is the index of blocks, and N is the block size. In order to take advantage of the fast Fourier transform (FFT) in the modulation and demodulation process, it is desirable to choose N as a power of two. The data vector $\mathbf{S}_l$ is modulated using the inverse discrete Fourier transform (IDFT) and generating a vector that can be written as:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s(k)\, e^{j2\pi kn/N} \qquad 0 \le n \le N-1 \tag{3.1}$$

After adding a cyclic prefix of length Γ as a guard time interval, a modulated block is generated:

$\mathbf{X}_l = \{x(lN+N-1-\Gamma), x(lN+N-\Gamma), ..., x(lN+N-1), x(lN), x(lN+1), ..., x(lN+N-1)\}$.

Then the block $\mathbf{X}_l$ after some general functions shown in Figure 3.2, is transmitted through a multipath channel.

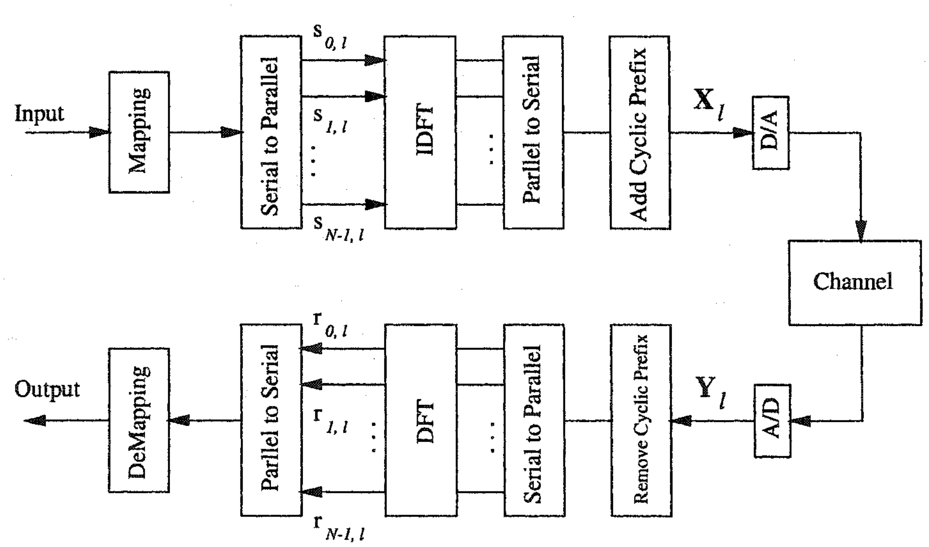

Figure 3.2: A digital implementation of a baseband OFDM system.

Assuming that the guard time interval is longer than the largest delay spread of a multipath channel, to avoid ISI, the received signal will be the convolution of the channel and the transmitted signal as follows:

$$\mathbf{Y}_l = \mathbf{X}_l \otimes \mathbf{h}_l + \mathbf{v}_l \tag{3.2}$$

Assuming that the channel is static during an OFDM block, after removing the cyclic prefix, the DFT output as the demodulated received signal can be expressed as:

$$\mathbf{r}_l = \mathbf{DFT}(\mathbf{Y}_l) \tag{3.3}$$

$$\mathbf{r}_l = \mathbf{DFT}(\mathbf{IDFT}(\mathbf{s}_l)) \, .\mathbf{DFT}(\mathbf{h}_l) + \mathbf{DFT}(\mathbf{v}_l) \tag{3.4}$$

$$\mathbf{r}_l = \mathbf{S}_l \, .\mathbf{H}_l + \mathbf{n}_l \tag{3.5}$$

where $\mathbf{r}_l$ contains the N received data samples, $\mathbf{S}_l$ contains the N transmitted MPSK/MQAM data symbols, $\mathbf{h}_l$ is the channel impulse response of the channel (padded with zeros to obtain a length of N), and $\mathbf{v}_l$ the channel noise. By discarding the block index of l for simplicity, (3.5) can be re-written as:

$$r(k) = H(k)\, s(k) + n(k) \qquad\qquad 0 \le k \le N-1 \tag{3.6}$$

where $r(k)$, $H(k)$, $s(k)$ and $n(k)$ represent the elements of $\mathbf{r}_l$, $\mathbf{S}_l$, $\mathbf{H}_l$, and $\mathbf{n}_l$ respectively ($\mathbf{H}_l$ is a diagonal matrix whose elements are $H(k)$, $k = 0, 1, \cdots, N-1$). As can be seen, the demodulated signal is the product of the input signal $s(k)$ with the channel gain $H(k)$. Thus OFDM can convert the frequency selective fading channel into N flat subchannels on which the OTD technique can be applied.

Note that (3.6) holds true if the length of the cyclic prefix (Γ) satisfies $\Gamma \ge L$ (L is the largest delay spread of a multipath channel). The loss in spectral efficiency due to the use of a cyclic prefix is given by $\frac{\Gamma}{\Gamma+N}$ and becomes negligible for $N \gg \Gamma \ge L$.

3.2 Orthogonal Transmit Diversity with OFDM

The orthogonal transmit diversity (OTD) technique, also called space-time block coding (STBC) was proposed for frequency flat fading channels [28]. However, in broadband wireless communication systems where symbol duration can be smaller than the channel delay spread, the channel propagation can have frequency selective effects. In non-flat fading channels such as frequency selective multipath channels, convolution of the channel impulse response with OTD output destroys the orthogonality of the OTD. Consequently, this technique is often only effective over flat fading channels, such as indoor wireless systems or low data-rate networks.

The effects of frequency selectivity on space-time coding have been studied in [51]-[52]. It was shown that in the presence of ISI induced by frequency selective multipath channels, the coding advantage decreases significantly, especially when delay spread becomes relatively high, resulting in intolerable performance degradation. In order to maintain the decoding simplicity and take advantage of the existing space-time codes designed for flat fading channels, some additional processing is required to make space-time codes suitable for application over frequency selective fading channels. One approach for mitigating the ISI is to use an equalizer at the receiver. It is shown that a multiple-input/multiple-output equalizer (MIMO-EQ) can equalize the frequency-selective fading channels and convert them into ISI-free channels where space-time coding is applicable [34],[35]. The main drawback of using an equalizer is high receiver complexity.

Another approach is to employ orthogonal frequency division multiplexing (OFDM). It is shown that OFDM can be used to transform frequency-selective fading channels into several flat fading subchannels. Hence, OTD combined with OFDM can be effectively used in non-flat fading channels. Here we extend and generalize the idea of OTD-OFDM to utilize space-frequency diversity to provide superior performance and higher channel capacity. The system model for OTD-OFDM is shown in Figure 3.3.

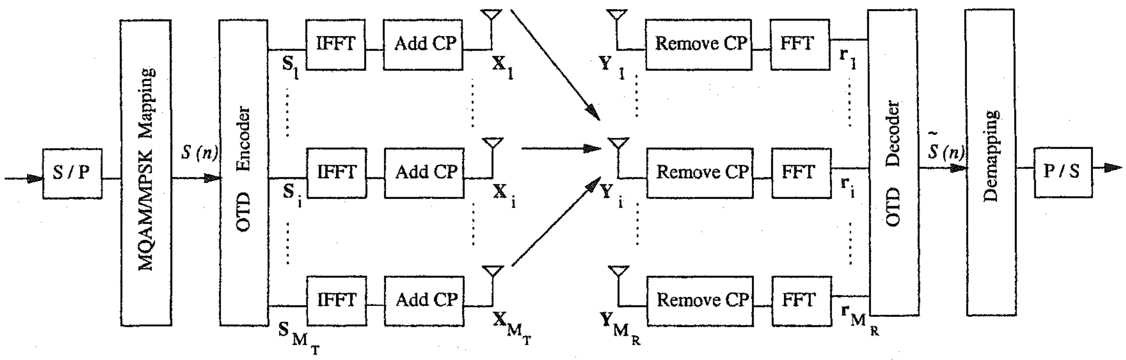

Figure 3.3: Block diagram of the OTD-OFDM system.

3.2.1 OTD-OFDM:

Figure 3.3 shows a block diagram of the OTD-OFDM system with M_T transmitter and M_R receiver antennas. A stream of information is first converted from serial to parallel.

Assuming an OFDM with N subcarriers, let N_s be the number of subbands, that has been chosen to be $N_s = N/T$, i.e, each subband includes T adjacent sub-channels (T is the symbol period of OTD system). Then, all subbands are modulated using MQAM/MPSK where M is determined by the number of the allocated bits. Therefore, a signal vector:

$$S(n) = \{s_0(n),\ s_1(n), s_2(n), ...,\ s_{N_t-1}(n)\}$$

is provided as the input for the OTD system, where N_t is chosen to be equal to N multiplied by the code rate of the OTD system (R_c).

A space-time block code is defined by a ($T \times M_T$) transmission matrix G given by:

$$\mathbf{G} = \begin{pmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,M_T} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,M_T} \\ \vdots & \vdots & \cdots & \vdots \\ g_{T,1} & g_{T,2} & \cdots & g_{T,M_T} \end{pmatrix} \tag{3.7}$$

where each of the elements $g_{i,j}$ is a linear combination of a subset of elements of $S(n)$ and their conjugates. The $(T \times M_T)$ transmission matrix $\mathbf{G}$ is based on a complex generalized orthogonal design, as defined in [29]-[30].

In order to utilize the space-frequency diversity, the input blocks for OFDM at each transmit antenna should have the length N.

OTD provides M_T blocks: $\mathbf{S}_1(n)$, $\mathbf{S}_2(n), \cdots, \mathbf{S}_{M_T}(n)$ of length N, each consisting of $\frac{N}{T}$ sub-blocks i.e.,

$$\mathbf{S}_i(n) = \left(\mathbf{s}_{i,0}(n) \quad \mathbf{s}_{i,1}(n) \quad \ldots \quad \mathbf{s}_{i,\frac{N}{T}-1}(n)\right)^T,$$

$$(i = 1, 2, \cdots, M_T).$$

where superscript T in $(.)^T$ denotes the transpose operation.

Then OFDM modulators, generate blocks of:

$$\mathbf{X}_1(n),\ \mathbf{X}_2(n), \cdots, \mathbf{X}_{M_T}(n)$$

where they will be transmitted by the first, second, ..., and M_T-th transmit antennas simultaneously. Given that the guard time interval is longer than the largest delay spread of a multipath channel (to avoid ISI), the received signal will be the convolution of the channel and the transmitted signal. Assuming that the channel is static during an OFDM block, at the receiver side after removing the cyclic prefix, the FFT output as the demodulated received signal at $j-$th receive antenna can be expressed as:

$$\mathbf{r}_j(n) = \sum_{i=1}^{M_T} \mathbf{H}_{j,i}(n)\, \mathbf{S}_i(n) + \mathbf{W}_j(n) \tag{3.8}$$

where

$$\mathbf{r}_j(n) = (r_{j,0}(n), \cdots, r_{j,N-1}(n))^T \quad j = 1, \cdots, M_R$$

and $\mathbf{W}_j(n) = (W_{j,0}(n), \cdots, W_{j,N-1}(n))^T$ denotes the AWGN and $\mathbf{H}_{j,i}(n)$ represents a diagonal matrix whose elements ($H_{j,i,k} \quad i = 1, 2, \cdots, M_T,\ j = 1, 2, \cdots, M_R,\ k = 0, 1, \cdots, N-1$) are the gains of the subchannels, i.e., the DFT of the impulse response of the channel $h_{j,i}$.

Knowing the channel information at the receiver, the Maximum Likelihood (ML) decoding can be used for the OTD decoding of the received signal, which is only a linear process. At the end, the elements of the block $\widetilde{S}(n) = \{\widetilde{s}_m(n)\}_{m=0}^{N_t-1}$ are demodulated to extract the information.

3.2.1.1 Example-1:

Consider an OTD-OFDM system with two transmitter antennas ($M_T = 2$), using the code $\mathbf{G}_2$. Like conventional OFDM, the sequence of input data is converted into parallel form, generating blocks of $S_l = \{s\,(lN),\ s\,(lN+1), ...,\ s\,(lN+N-1)\}$, where l is the index of block, and N is the block size which is chosen to be equal to the FFT size in the OFDM system. Let us assume that $s_n(l) = s\,(lN+n)$ for $n = 0, \cdots, N-1$.

The orthogonal block code for two transmitter antennas is defined by:

$$G_2 = \begin{pmatrix} s_{2k} & s_{2k+1} \\ -s_{2k+1}^* & s_{2k}^* \end{pmatrix} = \begin{pmatrix} \mathrm{s}_1 & \mathrm{s}_2 \end{pmatrix} \qquad k = 0, \cdots, \frac{N}{2} - 1 \tag{3.9}$$

where the implicit dependency on the block index l has been omitted for brevity.

Sub-blocks $\mathbf{s}_1$ and $\mathbf{s}_2$ are:

$$\mathbf{s}_1 = \begin{pmatrix} s_{2k} & -s_{2k+1}^* \end{pmatrix}^T$$
$$\mathbf{s}_2 = \begin{pmatrix} s_{2k+1} & s_{2k}^* \end{pmatrix}^T$$

OTD provides two blocks of $\mathbf{S}_1$ and $\mathbf{S}_2$ with the length of N, for OFDM systems at the transmitters. In order to utilize the space-frequency diversity, the input blocks are encoded as follows:

$$\begin{aligned} \mathbf{S}_1 &= \begin{pmatrix} s_0 & -s_1^* & s_2 & -s_3^* & \cdots & s_{N-2} & -s_{N-1}^* \end{pmatrix}^T \\ \mathbf{S}_2 &= \begin{pmatrix} s_1 & +s_0^* & s_3 & +s_2^* & \cdots & s_{N-1} & +s_{N-2}^* \end{pmatrix}^T \end{aligned} \tag{3.10}$$

OFDM modulators generate blocks of $\mathbf{X}_1$ and $\mathbf{X}_2$ where they are transmitted by the first and second transmitter antenna respectively.

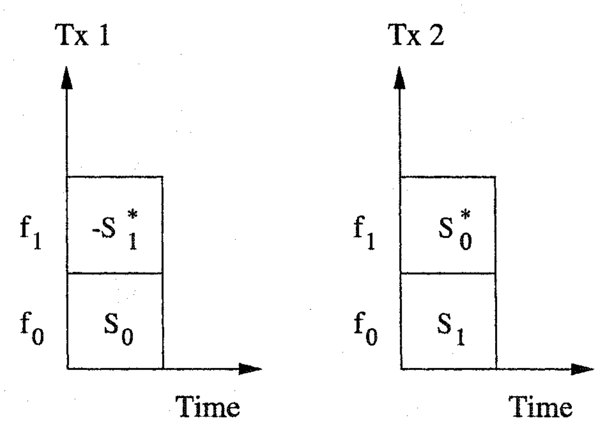

Figure 3.4: OTD-OFDM (2Tx-M_R Rx).

Assuming that the guard time interval is longer than the largest delay spread of a multipath channel, to avoid ISI, the received signal will be the convolution of the channel and the transmitted signal. Assuming that the channel is static during an OFDM block, at the receiver side after removing the cyclic prefix, the FFT output as the demodulated received signal can be expressed as:

$$\mathbf{r}_j = \mathbf{H}_{j,1}\,\mathbf{S}_1 + \mathbf{H}_{j,2}\,\mathbf{S}_2 + \mathbf{W}_j \tag{3.11}$$

where $\mathbf{r}_j = (r_{j,0}, \cdots, r_{j,N-1})^T$ and $\mathbf{H}_{j,1}$ and $\mathbf{H}_{j,2}$ represent diagonal matrices whose elements ($H_{j,i,k}$, $i = 1, 2,$ $j = 1, 2, \cdots, M_R,$ $k = 0, 1, \cdots, N-1$) are the DFT of the impulse response of the channels $h_{j,1}$ and $h_{j,1}$ and $\mathbf{W}_j = (W_{j,0}, \cdots, W_{j,N-1})^T$ denotes AWGN.

Assuming that the channel information is known at the receiver, the ML detection can be used for decoding of received signal, which can be written as:

$$\begin{aligned} \widetilde{s}_{2k} &= \textstyle\sum_{j=1}^{M_R} \left(H^*_{j,1,2k}\, r_{j,1,k} + H_{j,2,2k}\, r^*_{j,2,k}\right) \\ \widetilde{s}_{2k+1} &= \textstyle\sum_{j=1}^{M_R} \left(H^*_{j,2,2k+1}\, r_{j,1,k} - H_{j,1,2k+1}\, r^*_{j,2,k}\right) \end{aligned} \tag{3.12}$$

where

$$r_{j,1,k} = r_{j,2k}, \qquad r_{j,2,k} = r_{j,2k+1}, \qquad k = 0, \cdots, \frac{N}{2} - 1 \tag{3.13}$$

Assuming that the channel gains between the two adjacent subchannels are approximately equal, i.e., $H_{j,1,2k} = H_{j,1,2k+1}$, $H_{j,2,2k} = H_{j,2,2k+1}$, by substituting (3.11) and (3.13) into (3.12) the decoded signal can be expressed as:

$$\begin{aligned} \widetilde{s}_{2k} &= \textstyle\sum_{j=1}^{M_R} \left(|H_{j,1,2k}|^2 + |H_{j,2,2k}|^2\right)\, s_{2k} + \textstyle\sum_{j=1}^{M_R} \left(H^*_{j,1,2k}\, W_{j,2k} + H_{j,2,2k}\, W^*_{j,2k+1}\right) \\ \widetilde{s}_{2k+1} &= \textstyle\sum_{j=1}^{M_R} \left(|H_{j,1,2k}|^2 + |H_{j,2,2k}|^2\right)\, s_{2k+1} + \textstyle\sum_{j=1}^{M_R} \left(H^*_{j,2,2k}\, W_{j,2k} - H_{j,1,2k}\, W^*_{j,2k+1}\right) \end{aligned} \tag{3.14}$$

The above variables provide a diversity gain of order two for every s_{2k} and s_{2k+1}. As can be seen, the total channel gain is the sum of squares of two channel gains, which is more reliable than one single channel and obviously it can outperform conventional OFDM. It can be concluded that the proposed scheme can provide significant gains in performance over conventional OFDM. At the end, the elements of the block $\widetilde{S} = \{\widetilde{s}_m\}_{m=0}^{N_t-1}$ are demodulated to extract the information.

3.2.1.2 Example-2:

Consider an OTD for three transmitter antennas ($M_T = 3$) with a code of $\mathbf{G}_3$ given by:

$$\mathbf{G}_3 = \begin{pmatrix} +s_{4k} & +s_{4k+1} & +s_{4k+2} \\ -s_{4k+1} & +s_{4k} & -s_{4k+3} \\ -s_{4k+2} & +s_{4k+3} & +s_{4k} \\ -s_{4k+3} & -s_{4k+2} & +s_{4k+1} \\ +s^*_{4k} & +s^*_{4k+1} & +s^*_{4k+2} \\ -s^*_{4k+1} & +s^*_{4k} & -s^*_{4k+3} \\ -s^*_{4k+2} & +s^*_{4k+3} & +s^*_{4k} \\ -s^*_{4k+3} & -s^*_{4k+2} & +s^*_{4k+1} \end{pmatrix} \tag{3.15}$$

which can be written as:

$$\mathbf{G}_3 = \left[\begin{array}{ccc} \mathbf{s}_{1,k} & \mathbf{s}_{2,k} & \mathbf{s}_{3,k} \end{array} \right] \quad k = 0, \cdots, \frac{N}{8} - 1$$

where

$$\begin{aligned} \mathbf{s}_{1,k} = [&+s_{4k}\,, \quad -s_{4k+1}\,, \quad -s_{4k+2}\,, \quad -s_{4k+3} \\ &+s^*_{4k}\,, \quad -s^*_{4k+1}\,, \quad -s^*_{4k+2}\,, \quad -s^*_{4k+3}]^T \\ \mathbf{s}_{2,k} = [&+s_{4k+1}\,, \quad +s_{4k}\,, \quad +s_{4k+3}\,, \quad -s_{4k+2} \\ &+s^*_{4k+1}\,, \quad +s^*_{4k}\,, \quad +s^*_{4k+3}\,, \quad -s^*_{4k+2}]^T \\ \mathbf{s}_{3,k} = [&+s_{4k+2}, \quad -s_{4k+3}\,, \quad +s_{4k}\,, \quad +s_{4k+1} \\ &+s^*_{4k+2}\,, \quad -s^*_{4k+3}\,, \quad +s^*_{4k}\,, \quad +s^*_{4k+1}]^T \end{aligned} \tag{3.16}$$

where superscript T in $(.)^T$ denotes the transpose operation.

OTD provides three blocks of $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$ with the length of N as follows.

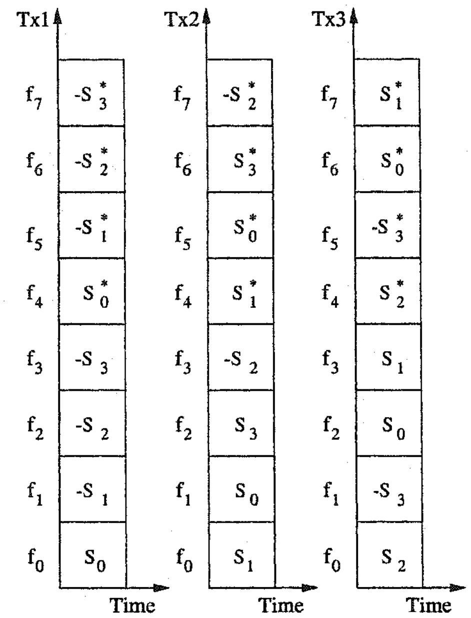

Figure 3.5: OTD-OFDM (3Tx-M_R Rx).

$$\begin{aligned} \mathbf{S}_1 &= \begin{bmatrix} s_{1,0} & s_{1,1} & \dots & s_{1,\frac{N}{8}-1} \end{bmatrix}^T \\ \mathbf{S}_2 &= \begin{bmatrix} s_{2,0} & s_{2,1} & \dots & s_{2,\frac{N}{8}-1} \end{bmatrix}^T \\ \mathbf{S}_3 &= \begin{bmatrix} s_{3,0} & s_{3,1} & \dots & s_{3,\frac{N}{8}-1} \end{bmatrix}^T \end{aligned} \tag{3.17}$$

OFDM modulators generate blocks $\mathbf{X}_1$, $\mathbf{X}_2$ and $\mathbf{X}_3$ that are transmitted by the first, second and third transmitter antenna, respectively.

At the receiver side, the received signal after OFDM demodulation can be written as:

$$\mathbf{r}_j = \mathbf{H}_{j,1}\,\mathbf{S}_1 + \mathbf{H}_{j,2}\,\mathbf{S}_2 + \mathbf{H}_{j,3}\,\mathbf{S}_3 + \mathbf{W}_j \tag{3.18}$$

where $\mathbf{r}_j$, $\mathbf{H}_{j,1}$, $\mathbf{H}_{j,2}$, $\mathbf{H}_{j,3}$ and $\mathbf{W}_j$ are defined in (3.8). ML detection can be used for decoding of the received signal, which can be written as:

$$\begin{aligned}
\widetilde{s}_{4k} &= \textstyle\sum_{j=1}^{M_R} \big(+H^*_{j,1,8k}\, r_{j,1,k} + H^*_{j,2,8k}\, r_{j,2,k} + H^*_{j,3,8k}\, r_{j,3,k} \\
&\qquad +H_{j,1,8k}\, r^*_{j,5,k} + H_{j,2,8k}\, r^*_{j,6,k} + H_{j,3,8k}\, r^*_{j,7,k}\big) \\
\widetilde{s}_{4k+1} &= \textstyle\sum_{j=1}^{M_R} \big(+H^*_{j,2,8k}\, r_{j,1,k} - H^*_{j,1,8k}\, r_{j,2,k} + H^*_{j,3,8k}\, r_{j,4,k} \\
&\qquad +H_{j,2,8k}\, r^*_{j,5,k} - H_{j,1,8k}\, r^*_{j,6,k} + H_{j,3,8k}\, r^*_{j,8,k}\big) \\
\widetilde{s}_{4k+2} &= \textstyle\sum_{j=1}^{M_R} \big(+H^*_{j,3,8k}\, r_{j,1,k} - H^*_{j,1,8k}\, r_{j,3,k} - H^*_{j,2,8k}\, r_{j,4,k} \\
&\qquad +H_{j,3,8k}\, r^*_{j,5,k} - H_{j,1,8k}\, r^*_{j,7,k} - H_{j,2,8k}\, r^*_{j,8,k}\big) \\
\widetilde{s}_{4k+3} &= \textstyle\sum_{j=1}^{M_R} \big(-H^*_{j,3,8k}\, r_{j,2,k} + H^*_{j,2,8k}\, r_{j,3,k} - H^*_{j,1,8k}\, r_{j,4,k} \\
&\qquad -H_{j,3,8k}\, r^*_{j,6,k} + H_{j,2,8k}\, r^*_{j,7,k} - H_{j,1,8k}\, r^*_{j,8,k}\big)
\end{aligned} \tag{3.19}$$

where

$$r_{j,i,k} = r_{j,8k+i-1} \tag{3.20}$$

$k = 0, \cdots, \frac{N}{8} - 1$, $i = 1, 2, \cdots, 8$ and, we assume that the channel information is known at the receiver, and that the channel gains of eight adjacent subchannels are approximately equal, i.e., $H_{j,1,8k+m} = H_{j,1,8k}$, $H_{j,2,8k+m} = H_{j,2,8k}$, $H_{j,3,8k+m} = H_{j,3,8k}$, for $m = 0, 1, \cdots, 7$.

Note that, the channel gain variation in OTD-OFDM over frequency selective multipath channels, depends on the channel delay, the number of paths and the block size. When the block size is large enough, the channel gain between the eight subcarriers is almost constant. Substituting (3.18) and (3.20) into (3.19), the decoded signal can be expressed as:

$$
\begin{aligned}
\widetilde{s}_{4k} &= 2 \sum_{j=1}^{M_R} \left(|H_{j,1,8k}|^2 + |H_{j,2,8k}|^2 + |H_{j,3,8k}|^2\right) s_{4k} \\
&\quad + \sum_{j=1}^{M_R} \left(H_{j,1,8k}^* W_{j,8k} + H_{j,2,8k}^* W_{j,8k+1} + H_{j,3,8k}^* W_{j,8k+2}\right. \\
&\qquad \left. + H_{j,1,8k} W_{j,8k+4}^* + H_{j,2,8k} W_{j,8k+5}^* + H_{j,3,8k} W_{j,8k+6}^*\right) \\
\widetilde{s}_{4k+1} &= 2 \sum_{j=1}^{M_R} \left(|H_{j,1,8k}|^2 + |H_{j,2,8k}|^2 + |H_{j,3,8k}|^2\right) s_{4k+1} \\
&\quad + \sum_{j=1}^{M_R} \left(H_{j,2,8k}^* W_{j,8k} - H_{j,1,8k}^* W_{j,8k+1} + H_{j,3,8k}^* W_{j,8k+3}\right. \\
&\qquad \left. + H_{j,2,8k} W_{j,8k+4}^* - H_{j,1,8k} W_{j,8k+5}^* + H_{j,3,8k} W_{j,8k+7}^*\right) \\
\widetilde{s}_{4k+2} &= 2 \sum_{j=1}^{M_R} \left(|H_{j,1,8k}|^2 + |H_{j,2,8k}|^2 + |H_{j,3,8k}|^2\right) s_{4k+2} \\
&\quad + \sum_{j=1}^{M_R} \left(H_{j,3,8k}^* W_{j,8k} - H_{j,1,8k}^* W_{j,8k+2} - H_{j,2,8k}^* W_{j,8k+3}\right. \\
&\qquad \left. + H_{j,3,8k} W_{j,8k+4}^* - H_{j,1,8k} W_{j,8k+6}^* - H_{j,2,8k} W_{j,8k+7}^*\right) \\
\widetilde{s}_{4k+3} &= 2 \sum_{j=1}^{M_R} \left(|H_{j,1,8k}|^2 + |H_{j,2,8k}|^2 + |H_{j,3,8k}|^2\right) s_{4k+3} \\
&\quad + \sum_{j=1}^{M_R} \left(-H_{j,3,8k}^* W_{j,8k+1} + H_{j,2,8k}^* W_{j,8k+2} - H_{j,1,8k}^* W_{j,8k+3}\right. \\
&\qquad \left. - H_{j,3,8k} W_{j,8k+5}^* + H_{j,2,8k} W_{j,8k+6}^* - H_{j,1,8k} W_{j,8k+7}^*\right)
\end{aligned}
\tag{3.21}
$$

At the end, $\{\widetilde{s}_m\}_{m=0}^{N_t-1}$ is sent to the decision part.

3.2.1.3 Example-3:

Consider an OTD system with four transmitter antennas ($M_T = 4$) with a $\mathbf{G}_4$ code given by:

$$\mathbf{G}_4 = \begin{pmatrix} +s_{4k} & +s_{4k+1} & +s_{4k+2} & +s_{4k+3} \\ -s_{4k+1} & +s_{4k} & -s_{4k+3} & +s_{4k+2} \\ -s_{4k+2} & +s_{4k+3} & +s_{4k} & -s_{4k+1} \\ -s_{4k+3} & -s_{4k+2} & +s_{4k+1} & +s_{4k} \\ +s^*_{4k} & +s^*_{4k+1} & +s^*_{4k+2} & +s^*_{4k+3} \\ -s^*_{4k+1} & +s^*_{4k} & -s^*_{4k+3} & +s^*_{4k+2} \\ -s^*_{4k+2} & +s^*_{4k+3} & +s^*_{4k} & -s^*_{4k+1} \\ -s^*_{4k+3} & -s^*_{4k+2} & +s^*_{4k+1} & +s^*_{4k} \end{pmatrix} \tag{3.22}$$

which can be written as:

$$\mathbf{G}_4 = \begin{bmatrix} \mathbf{s}_{1,k} & \mathbf{s}_{2,k} & \mathbf{s}_{3,k} & \mathbf{s}_{4,k} \end{bmatrix} \quad k = 0, \cdots, \frac{N}{8} - 1 \tag{3.23}$$

where

$$\begin{aligned} \mathbf{s}_{1,k} &= [+s_{4k}, \quad -s_{4k+1}, \quad -s_{4k+2}, \quad -s_{4k+3} \\ &\qquad +s^*_{4k}, \quad -s^*_{4k+1}, \quad -s^*_{4k+2}, \quad -s^*_{4k+3}]^T \\ \mathbf{s}_{2,k} &= [+s_{4k+1}, \quad +s_{4k}, \quad +s_{4k+3}, \quad -s_{4k+2} \\ &\qquad +s^*_{4k+1}, \quad +s^*_{4k}, \quad +s^*_{4k+3}, \quad -s^*_{4k+2}]^T \\ \mathbf{s}_{3,k} &= [+s_{4k+2}, \quad -s_{4k+3}, \quad +s_{4k}, \quad +s_{4k+1} \\ &\qquad +s^*_{4k+2}, \quad -s^*_{4k+3}, \quad +s^*_{4k}, \quad +s^*_{4k+1}]^T \\ \mathbf{s}_{4,k} &= [+s_{4k+3}, \quad +s_{4k+2}, \quad -s_{4k+1}, \quad +s_{4k} \\ &\qquad +s^*_{4k+3}, \quad +s^*_{4k+2}, \quad -s^*_{4k+1}, \quad +s^*_{4k}]^T \end{aligned} \tag{3.24}$$

OTD provides four blocks $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$ and $\mathbf{S}_4$ with the length of N as follows.

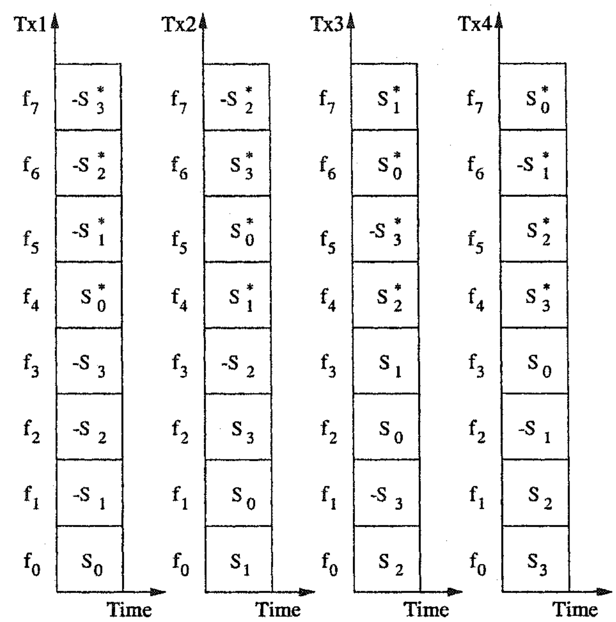

Figure 3.6: OTD-OFDM (4Tx-M_R Rx).

$$
\begin{aligned}
\mathbf{S}_1 &= \begin{bmatrix} \mathbf{s}_{1,0} & \mathbf{s}_{1,1} & \dots & \mathbf{s}_{1,\frac{N}{8}-1} \end{bmatrix}^T \\
\mathbf{S}_2 &= \begin{bmatrix} \mathbf{s}_{2,0} & \mathbf{s}_{2,1} & \dots & \mathbf{s}_{2,\frac{N}{8}-1} \end{bmatrix}^T \\
\mathbf{S}_3 &= \begin{bmatrix} \mathbf{s}_{3,0} & \mathbf{s}_{3,1} & \dots & \mathbf{s}_{3,\frac{N}{8}-1} \end{bmatrix}^T \\
\mathbf{S}_4 &= \begin{bmatrix} \mathbf{s}_{4,0} & \mathbf{s}_{4,1} & \dots & \mathbf{s}_{4,\frac{N}{8}-1} \end{bmatrix}^T
\end{aligned} \tag{3.25}
$$

OFDM modulators generate blocks $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$ and $\mathbf{X}_4$ that are transmitted by the first, second, third and fourth transmitter antenna, respectively. At the receiver side, the received signal after OFDM demodulation can be written as:

$$\mathbf{r}_j = \mathbf{H}_{j,1}\,\mathbf{S}_1 + \mathbf{H}_{j,2}\,\mathbf{S}_2 + \mathbf{H}_{j,3}\,\mathbf{S}_3 + \mathbf{H}_{j,4}\,\mathbf{S}_4 + \mathbf{W}_j \tag{3.26}$$

where $\mathbf{r}_j$, $\mathbf{H}_{j,1}$, $\mathbf{H}_{j,2}, \mathbf{H}_{j,3}$, $\mathbf{H}_{j,4}$ and $\mathbf{W}_j$ are defined in (3.8).

ML detection can be used for decoding of the received signal, which can be written as:

$$
\begin{aligned}
\widetilde{s}_{4k} &= \textstyle\sum_{j=1}^{M_R} \big(H^*_{j,1,8k}\, r_{j,1,k} + H^*_{j,2,8k}\, r_{j,2,k} + H^*_{j,3,8k}\, r_{j,3,k} \\
&\quad + H^*_{j,4,8k}\, r_{j,4,k} + H_{j,1,8k}\, r^*_{j,5,k} + H_{j,2,8k}\, r^*_{j,6,k}\big) \\
&\quad + H_{j,3,8k}\, r^*_{j,7,k} + H_{j,4,8k}\, r^*_{j,8,k}\big) \\[2ex]
\widetilde{s}_{4k+1} &= \textstyle\sum_{j=1}^{M_R} \big(H^*_{j,2,8k}\, r_{j,1,k} - H^*_{j,1,8k}\, r_{j,2,k} - H^*_{j,4,8k}\, r_{j,3,k} \\
&\quad + H^*_{j,3,8k}\, r_{j,4,k} + H_{j,2,8k}\, r^*_{j,5,k} - H_{j,1,8k}\, r^*_{j,6,k}\big) \\
&\quad - H_{j,4,8k}\, r^*_{j,7,k} + H_{j,3,8k}\, r^*_{j,8,k}\big) \\[2ex]
\widetilde{s}_{4k+2} &= \textstyle\sum_{j=1}^{M_R} \big(H^*_{j,3,8k}\, r_{j,1,k} + H^*_{j,4,8k}\, r_{j,2,k} - H^*_{j,1,8k}\, r_{j,3,k} \\
&\quad - H^*_{j,2,8k}\, r_{j,4,k} + H_{j,3,8k}\, r^*_{j,5,k} + H_{j,4,8k}\, r^*_{j,6,k}\big) \\
&\quad - H_{j,1,8k}\, r^*_{j,7,k} - H_{j,2,8k}\, r^*_{j,8,k}\big) \\[2ex]
\widetilde{s}_{4k+3} &= \textstyle\sum_{j=1}^{M_R} \big(-H^*_{j,4,8k}\, r_{j,1,k} - H^*_{j,3,4k}\, r_{j,2,k} + H^*_{j,2,8k}\, r_{j,3,k} \\
&\quad - H^*_{j,1,8k}\, r_{j,4,k} - H_{j,4,8k}\, r^*_{j,5,k} - H_{j,3,8k}\, r^*_{j,6,k}\big) \\
&\quad + H_{j,2,8k}\, r^*_{j,7,k} - H_{j,1,8k}\, r^*_{j,8,k}\big)
\end{aligned}
\tag{3.27}
$$

where

$$
r_{j,i,k} = r_{j,8k+i-1} \tag{3.28}
$$

$k = 0, \cdots, \frac{N}{8} - 1$, $i = 1, 2, \cdots, 8$ and, we assume that the channel information is known at the receiver, and that the channel gains of eight adjacent subchannels are approximately equal, i.e., $H_{j,1,8k+m} = H_{j,1,8k}$, $H_{j,2,8k+m} = H_{j,2,8k}$, $H_{j,3,8k+m} = H_{j,3,8k}$, and $H_{j,4,8k+m} = H_{j,4,8k}$ for $m = 0, 1, \cdots, 7$.

Substituting (3.26) and (3.28) into (3.27), the decoded signal can be expressed as:

$$
\begin{aligned}
\widetilde{s}_{4k} &= 2\sum_{j=1}^{M_R}\left(|H_{j,1,8k}|^2+|H_{j,2,8k}|^2+|H_{j,3,8k}|^2+|H_{j,4,8k}|^2\right)s_{4k}\\
&+\sum_{j=1}^{M_R}\big(H^*_{j,1,8k}W_{j,8k}+H^*_{j,2,8k}W_{j,8k+1}+H^*_{j,3,8k}W_{j,8k+2}\\
&\qquad+H^*_{j,4,8k}W_{j,8k+3}+H_{j,1,8k}W^*_{j,8k+4}+H_{j,2,8k}W^*_{j,8k+5}\\
&\qquad+H_{j,3,8k}W^*_{j,8k+6}+H_{j,4,8k}W^*_{j,8k+7}\big)\\
\widetilde{s}_{4k+1} &= 2\sum_{j=1}^{M_R}\left(|H_{j,1,8k}|^2+|H_{j,2,8k}|^2+|H_{j,3,8k}|^2+|H_{j,4,8k}|^2\right)s_{4k+1}\\
&+\sum_{j=1}^{M_R}\big(+H^*_{j,2,8k}W_{j,8k}-H^*_{j,1,8k}W_{j,8k+1}+H^*_{j,4,8k}W_{j,8k+2}\\
&\qquad+H^*_{j,3,8k}W_{j,8k+3}+H_{j,2,8k}W^*_{j,8k+4}-H_{j,1,8k}W^*_{j,8k+5}\\
&\qquad+H_{j,4,8k}W^*_{j,8k+6}+H_{j,3,8k}W^*_{j,8k+7}\big)\\
\widetilde{s}_{4k+2} &= 2\sum_{j=1}^{M_R}\left(|H_{j,1,8k}|^2+|H_{j,2,8k}|^2+|H_{j,3,8k}|^2+|H_{j,4,8k}|^2\right)s_{4k+2}\\
&+\sum_{j=1}^{M_R}\big(+H^*_{j,3,8k}W_{j,8k}+H^*_{j,4,8k}W_{j,8k+1}-H^*_{j,1,8k}W_{j,8k+2}\\
&\qquad-H^*_{j,2,8k}W_{j,8k+3}+H_{j,3,8k}W^*_{j,8k+4}-H_{j,4,8k}W^*_{j,8k+5}\\
&\qquad-H_{j,1,8k}W^*_{j,8k+6}-H_{j,2,8k}W^*_{j,8k+7}\big)\\
\widetilde{s}_{4k+3} &= 2\sum_{j=1}^{M_R}\left(|H_{j,1,8k}|^2+|H_{j,2,8k}|^2+|H_{j,3,8k}|^2+|H_{j,4,8k}|^2\right)s_{4k+3}\\
&+\sum_{j=1}^{M_R}\big(-H^*_{j,4,8k}W_{j,8k}-H^*_{j,3,8k}W_{j,8k+1}+H^*_{j,2,8k}W_{j,8k+2}\\
&\qquad-H^*_{j,1,8k}W_{j,8k+3}-H_{j,4,8k}W^*_{j,8k+4}-H_{j,3,8k}W^*_{j,8k+5}\\
&\qquad+H_{j,2,8k}W^*_{j,8k+6}-H_{j,1,8k}W^*_{j,8k+7}\big)
\end{aligned}
\tag{3.29}
$$

At the end, the elements of block $\{\widetilde{s}_m(n)\}_{m=0}^{N_s-1}$ are demodulated to extract the information.

3.2.1.4 Example-4:

Consider an OTD system with three antennas ($M_T = 3$) with a rate 3/4 code of $\mathbf{H}_3$ given by:

$$\mathbf{H}_3 = \begin{pmatrix} +s_{3k} & +s_{3k+1} & +\frac{s_{3k+2}}{\sqrt{2}} \\ -s^*_{3k+1} & +s^*_{3k} & +\frac{s_{3k+2}}{\sqrt{2}} \\ +\frac{s^*_{3k+2}}{\sqrt{2}} & +\frac{s^*_{3k+2}}{\sqrt{2}} & \frac{\left(-s_{3k}-s^*_{3k}+s_{3k+1}-s^*_{3k+1}\right)}{2} \\ +\frac{s^*_{3k+2}}{\sqrt{2}} & -\frac{s^*_{3k+2}}{\sqrt{2}} & \frac{\left(+s_{3k+1}+s^*_{3k+1}+s_{3k}-s^*_{3k}\right)}{2} \end{pmatrix} \tag{3.30}$$

which can be written as:

$$\mathbf{H}_3 = \begin{bmatrix} \mathbf{s}_{1,k} & \mathbf{s}_{2,k} & \mathbf{s}_{3,k} \end{bmatrix} \quad k = 0, \cdots, \frac{N}{4} - 1 \tag{3.31}$$

where

$$\mathbf{s}_{1,k} = \left[+s_{3k}\,, \quad -s^*_{3k+1}\,, \quad +\frac{s^*_{3k+2}}{\sqrt{2}}, \quad +\frac{s^*_{3k+2}}{\sqrt{2}}\right]^T$$

$$\mathbf{s}_{2,k} = \left[+s_{3k+1}\,, \quad +s^*_{3k}\,, \quad +\frac{s^*_{3k+2}}{\sqrt{2}}, \quad -\frac{s^*_{3k+2}}{\sqrt{2}}\right]^T$$

$$\mathbf{s}_{3,k} = \left[+\frac{s_{3k+2}}{\sqrt{2}}, \quad +\frac{s_{3k+2}}{\sqrt{2}}, \frac{\left(-s_{3k}-s^*_{3k}+s_{3k+1}-s^*_{3k+1}\right)}{2}, \quad \frac{+\left(s_{3k+1}+s^*_{3k+1}+s_{3k}-s^*_{3k}\right)}{2}\right]^T \tag{3.32}$$

OTD provides three blocks $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$ with the length of N as follows.

$$\begin{aligned} \mathbf{S}_1 &= \begin{bmatrix} \mathbf{s}_{1,0} & \mathbf{s}_{1,1} & \ldots & \mathbf{s}_{1,\frac{N}{4}-1} \end{bmatrix}^T \\ \mathbf{S}_2 &= \begin{bmatrix} \mathbf{s}_{2,0} & \mathbf{s}_{2,1} & \ldots & \mathbf{s}_{2,\frac{N}{4}-1} \end{bmatrix}^T \\ \mathbf{S}_3 &= \begin{bmatrix} \mathbf{s}_{3,0} & \mathbf{s}_{3,1} & \ldots & \mathbf{s}_{3,\frac{N}{4}-1} \end{bmatrix}^T \end{aligned} \tag{3.33}$$

OFDM modulators generate blocks $\mathbf{X}_1$, $\mathbf{X}_2$ and $\mathbf{X}_3$ that are transmitted by the first, second, third transmit antenna, respectively.

At the receiver side, the received signal after OFDM demodulation can be written as:

$$\mathbf{r}_j = \mathbf{H}_{j,1}\,\mathbf{S}_1 + \mathbf{H}_{j,2}\,\mathbf{S}_2 + \mathbf{H}_{j,3}\,\mathbf{S}_3 + \mathbf{W}_j \tag{3.34}$$

where $\mathbf{r}_j$, $\mathbf{H}_{j,1}$, $\mathbf{H}_{j,2}, \mathbf{H}_{j,3}$ and $\mathbf{W}_j$ are defined in (3.8).

ML detection can be used for decoding of the received signal, which can be written as:

$$\begin{aligned}
\widetilde{s}_{3k} &= \textstyle\sum_{j=1}^{M_R} \big(H^*_{j,1,4k}\, r_{j,1,k} + H_{j,2,4k}\, r^*_{j,2,k} \\
&\quad + \tfrac{1}{2}\left(r_{j,4,k} - r_{j,3,k}\right)\, H^*_{j,3,4k} - \tfrac{1}{2}\left(r_{j,3,k} + r_{j,4,k}\right)^*\, H_{j,3,4k}\big) \\
\widetilde{s}_{3k+1} &= \textstyle\sum_{j=1}^{M_R} \big(H^*_{j,2,4k}\, r_{j,1,k} - H_{j,1,4k}\, r^*_{j,2,k} \\
&\quad + \tfrac{1}{2}\left(r_{j,4,k} + r_{j,3,k}\right)\, H^*_{j,3,4k} + \tfrac{1}{2}\left(-r_{j,3,k} + r_{j,4,k}\right)^*\, H_{j,3,4k}\big) \\
\widetilde{s}_{3k+2} &= \textstyle\sum_{j=1}^{M_R} \Big(\tfrac{1}{\sqrt{2}}\left(r_{j,1,k} + r_{j,2,k}\right)\, H^*_{j,3,4k} \\
&\quad + \tfrac{1}{\sqrt{2}}\, r^*_{j,3,k}\left(H_{j,1,4k} + H_{j,2,4k}\right) + \tfrac{1}{\sqrt{2}}\, r^*_{j,4,k}\left(H_{j,1,4k} - H_{j,2,4k}\right)\Big)
\end{aligned} \tag{3.35}$$

where

$$r_{j,i,k} = r_{j,4k+i-1} \tag{3.36}$$

$k = 0, \cdots, \frac{N}{4} - 1$, $i = 1, 2, 3{,}4$ and, we assume that the channel information is known at the receiver, and that the channel gains of four adjacent subchannels are approximately equal, i.e., $H_{j,1,4k+m} = H_{j,1,4k}$, $H_{j,2,4k+m} = H_{j,2,4k}$, $H_{j,3,4k+m} = H_{j,3,4k}$, for $m = 0, 1, 2, 3$.

Substituting (3.34) and (3.36) into (3.35), the decoded signal can be expressed as:

$$
\begin{aligned}
\widetilde{s}_{3k} &= \tfrac{4}{3} \textstyle\sum_{j=1}^{M_R} \left(|H_{j,1,4k}|^2 + |H_{j,2,4k}|^2 + |H_{j,3,4k}|^2\right) s_{4k} \\
&+ \textstyle\sum_{j=1}^{M_R} \Big(H^*_{j,1,4k}\, W_{j,4k} + H_{j,2,4k}\, W^*_{j,4k+1} \\
&\quad + \tfrac{1}{2}\left(W_{j,4k+3} - W_{j,4k+2}\right) H^*_{j,3,4k} - \tfrac{1}{2}\left(W_{j,4k+2} + W_{j,4k+3}\right)^* H_{j,3,4k}\Big) \\
\widetilde{s}_{3k+1} &= \tfrac{4}{3} \textstyle\sum_{j=1}^{M_R} \left(|H_{j,1,4k}|^2 + |H_{j,2,4k}|^2 + |H_{j,3,4k}|^2\right) s_{4k+1} \\
&+ \textstyle\sum_{j=1}^{M_R} \Big(H^*_{j,2,4k}\, W_{j,4k} - H_{j,1,4k}\, W^*_{j,4k+1} \\
&\quad + \tfrac{1}{2}\left(W_{j,4k+3} + W_{j,4k+2}\right) H^*_{j,3,4k} + \tfrac{1}{2}\left(-W_{j,4k+2} + W_{j,4k+3}\right)^* H_{j,3,4k}\Big) \\
\widetilde{s}_{3k+2} &= \tfrac{4}{3} \textstyle\sum_{j=1}^{M_R} \left(|H_{j,1,4k}|^2 + |H_{j,2,4k}|^2 + |H_{j,3,4k}|^2\right) s_{4k+2} \\
&+ \textstyle\sum_{j=1}^{M_R} \Big(\tfrac{1}{\sqrt{2}}\left(W_{j,4k} + W_{j,4k+1}\right) H^*_{j,3,4k} \\
&\quad + \tfrac{1}{\sqrt{2}}\, W^*_{j,4k+2}\left(H_{j,1,4k} + H_{j,2,4k}\right) + \tfrac{1}{\sqrt{2}}\, W^*_{j,4k+3}\left(H_{j,1,4k} - H_{j,2,4k}\right)\Big)
\end{aligned}
\tag{3.37}
$$

At the end, the elements of the block $\{\widetilde{s}_m(n)\}_{m=0}^{N_s-1}$ are demodulated to extract the information.

3.2.1.5 Example-5:

Consider an OTD system with four transmitter antennas ($M_T = 4$) with a rate 3/4 code of $\mathbf{H}_4$ given by:

$$\mathbf{H}_4 = \begin{pmatrix} +s_{3k} & +s_{3k+1} & +\frac{s_{3k+2}}{\sqrt{2}} & +\frac{s_{3k+2}}{\sqrt{2}} \\ -s^*_{3k+1} & +s^*_{3k} & +\frac{s_{3k+2}}{\sqrt{2}} & -\frac{s_{3k+2}}{\sqrt{2}} \\ +\frac{s^*_{3k+2}}{\sqrt{2}} & +\frac{s^*_{3k+2}}{\sqrt{2}} & \frac{\left(-s_{3k}-s^*_{3k}+s_{3k+1}-s^*_{3k+1}\right)}{2} & \frac{+\left(-s_{3k+1}-s^*_{3k+1}+s_{3k}-s^*_{3k}\right)}{2} \\ +\frac{s^*_{3k+2}}{\sqrt{2}} & -\frac{s^*_{3k+2}}{\sqrt{2}} & \frac{\left(+s_{3k+1}+s^*_{3k+1}+s_{3k}-s^*_{3k}\right)}{2} & \frac{-\left(s_{3k}+s^*_{3k}+s_{3k+1}-s^*_{3k+1}\right)}{2} \end{pmatrix} \qquad (3.38)$$

which can be written as:

$$\mathbf{H}_4 = \begin{bmatrix} \mathbf{s}_{1,k} & \mathbf{s}_{2,k} & \mathbf{s}_{3,k} & \mathbf{s}_{4,k} \end{bmatrix} \quad k = 0, \cdots, \frac{N}{4} - 1 \qquad (3.39)$$

where

$$\begin{aligned} \mathbf{s}_{1,k} &= \left[+s_{3k}\,, \quad -s^*_{3k+1}\,, \quad +\frac{s^*_{3k+2}}{\sqrt{2}}, \quad +\frac{s^*_{3k+2}}{\sqrt{2}}\right]^T \\ \mathbf{s}_{2,k} &= \left[+s_{3k+1}\,, \quad +s^*_{3k}\,, \quad +\frac{s^*_{3k+2}}{\sqrt{2}}, \quad -\frac{s^*_{3k+2}}{\sqrt{2}}\right]^T \\ \mathbf{s}_{3,k} &= \left[+\frac{s_{3k+2}}{\sqrt{2}}, \quad +\frac{s_{3k+2}}{\sqrt{2}}, \frac{\left(-s_{3k}-s^*_{3k}+s_{3k+1}-s^*_{3k+1}\right)}{2}, \quad \frac{+\left(s_{3k+1}+s^*_{3k+1}+s_{3k}-s^*_{3k}\right)}{2}\right]^T \\ \mathbf{s}_{4,k} &= \left[+\frac{s_{3k+2}}{\sqrt{2}}, \quad -\frac{s_{3k+2}}{\sqrt{2}}, \frac{\left(-s_{3k+1}-s^*_{3k+1}+s_{3k}-s^*_{3k}\right)}{2}, \quad \frac{-\left(s_{3k}+s^*_{3k}+s_{3k+1}-s^*_{3k+1}\right)}{2}\right]^T \end{aligned} \qquad (3.40)$$

OTD provides four blocks $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$ and $\mathbf{S}_4$ with the length of N as follows.

$$\begin{aligned} \mathbf{S}_1 &= \begin{bmatrix} \mathbf{s}_{1,0} & \mathbf{s}_{1,1} & \cdots & \mathbf{s}_{1,\frac{N}{4}-1} \end{bmatrix}^T \\ \mathbf{S}_2 &= \begin{bmatrix} \mathbf{s}_{2,0} & \mathbf{s}_{2,1} & \cdots & \mathbf{s}_{2,\frac{N}{4}-1} \end{bmatrix}^T \\ \mathbf{S}_3 &= \begin{bmatrix} \mathbf{s}_{3,0} & \mathbf{s}_{3,1} & \cdots & \mathbf{s}_{3,\frac{N}{4}-1} \end{bmatrix}^T \\ \mathbf{S}_4 &= \begin{bmatrix} \mathbf{s}_{4,0} & \mathbf{s}_{4,1} & \cdots & \mathbf{s}_{4,\frac{N}{4}-1} \end{bmatrix}^T \end{aligned} \qquad (3.41)$$

OFDM modulators generate blocks $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$ and $\mathbf{X}_4$ that are transmitted by the first, second, third and fourth transmit antenna, respectively.

At the receiver side, the received signal after OFDM demodulation can be written as:

$$\mathbf{r}_j = \mathbf{H}_{j,1}\,\mathbf{S}_1 + \mathbf{H}_{j,2}\,\mathbf{S}_2 + \mathbf{H}_{j,3}\,\mathbf{S}_3 + \mathbf{H}_{j,4}\,\mathbf{S}_4 + \mathbf{W}_j \tag{3.42}$$

where $\mathbf{r}_j$, $\mathbf{H}_{j,1}$, $\mathbf{H}_{j,2}, \mathbf{H}_{j,3}$, $\mathbf{H}_{j,4}$ and $\mathbf{W}_j$ are defined in (3.8).

ML detection can be used for decoding of the received signal, which can be written as:

$$\begin{aligned}
\widetilde{s}_{3k} =& \textstyle\sum_{j=1}^{M_R} \big(H^*_{j,1,4k}\, r_{j,1,k} + H^*_{j,2,4k}\, r_{j,2,k} \\
&+\tfrac{1}{2}\left(r_{j,4,k} - r_{j,3,k}\right)\,\left(H^*_{j,3,4k} - H^*_{j,4,4k}\right) \\
&-\tfrac{1}{2}\left(r_{j,3,k} + r_{j,4,k}\right)^*\,\left(H_{j,3,4k} + H_{j,4,4k}\right)\big) \\
\widetilde{s}_{3k+1} =& \textstyle\sum_{j=1}^{M_R} \big(H^*_{j,2,4k}\, r_{j,1,k} - H_{j,1,4k}\, r^*_{j,2,k} \\
&+\tfrac{1}{2}\left(r_{j,4,k} + r_{j,3,k}\right)\,\left(H^*_{j,3,4k} - H^*_{j,4,4k}\right) \\
&+\tfrac{1}{2}\left(-r_{j,3,k} + r_{j,4,k}\right)^*\,\left(H_{j,3,4k} + H_{j,4,4k}\right)\big) \\
\widetilde{s}_{3k+2} =& \textstyle\sum_{j=1}^{M_R} \Big(\tfrac{1}{\sqrt{2}}\left(r_{j,1,k} + r_{j,2,k}\right)\, H^*_{j,3,4k} + \tfrac{1}{\sqrt{2}}\left(r_{j,1,k} - r_{j,2,k}\right)\, H^*_{j,4,4k} \\
&+\tfrac{1}{\sqrt{2}}\, r^*_{j,3,k}\,\left(H_{j,1,4k} + H_{j,2,4k}\right) + \tfrac{1}{\sqrt{2}}\, r^*_{j,4,k}\,\left(H_{j,1,4k} - H_{j,2,4k}\right)\Big)
\end{aligned} \tag{3.43}$$

where

$$r_{j,i,k} = r_{j,4k+i-1} \tag{3.44}$$

$k = 0, \cdots, \frac{N}{4} - 1$, $i = 1, 2, \cdots, 4$ and, we assume that the channel information is known at the receiver, and that the channel gains of four adjacent subchannels are approximately equal, i.e., $H_{j,1,4k+m} = H_{j,1,4k}$, $H_{j,2,4k+m} = H_{j,2,4k}$, $H_{j,3,4k+m} = H_{j,3,4k}$, and $H_{j,4,4k+m} = H_{j,4,4k}$, for $m = 0, 1, 2, 3$.

Substituting (3.42) and (3.44) into (3.43), the decoded signal can be expressed as:

$$
\begin{aligned}
\widetilde{s}_{3k} \;&= \tfrac{4}{3} \textstyle\sum_{j=1}^{M_R} \left(|H_{j,1,4k}|^2 + |H_{j,2,4k}|^2 + |H_{j,3,4k}|^2 + |H_{j,4,4k}|^2\right) s_{4k} \\
&+ \textstyle\sum_{j=1}^{M_R} \Big(H^*_{j,1,4k} W_{j,4k} + H_{j,2,4k} W^*_{j,4k+1} \\
&\qquad + \tfrac{1}{2} \left(W_{j,4k+3} - W_{j,4k+2}\right) \left(H^*_{j,3,4k} - H^*_{j,4,4k}\right) \\
&\qquad - \tfrac{1}{2} \left(W_{j,4k+2} + W_{j,4k+3}\right)^* \left(H_{j,3,4k} + H_{j,4,4k}\right)\Big)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{s}_{3k+1} &= \tfrac{4}{3} \textstyle\sum_{j=1}^{M_R} \left(|H_{j,1,4k}|^2 + |H_{j,2,4k}|^2 + |H_{j,3,4k}|^2 + |H_{j,4,4k}|^2\right) s_{4k+1} \\
&+ \textstyle\sum_{j=1}^{M_R} \Big(H^*_{j,2,4k} W_{j,4k} - H_{j,1,4k} W^*_{j,4k+1} \\
&\qquad + \tfrac{1}{2} \left(W_{j,4k+3} + W_{j,4k+2}\right) \left(H^*_{j,3,4k} - H^*_{j,4,4k}\right) \\
&\qquad + \tfrac{1}{2} \left(-W_{j,4k+2} + W_{j,4k+3}\right)^* \left(H_{j,3,4k} + H_{j,4,4k}\right)\Big)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{s}_{3k+2} &= \tfrac{4}{3} \textstyle\sum_{j=1}^{M_R} \left(|H_{j,1,4k}|^2 + |H_{j,2,4k}|^2 + |H_{j,3,4k}|^2 + |H_{j,4,4k}|^2\right) s_{4k+2} \\
&+ \textstyle\sum_{j=1}^{M_R} \Big(\tfrac{1}{\sqrt{2}} \left(W_{j,4k} + W_{j,4k+1}\right) H^*_{j,3,4k} + \tfrac{1}{\sqrt{2}} \left(W_{j,4k} - W_{j,4k+1}\right) H^*_{j,4,4k} \\
&\qquad + \tfrac{1}{\sqrt{2}} W^*_{j,4k+2} \left(H_{j,1,4k} + H_{j,2,4k}\right) + \tfrac{1}{\sqrt{2}} W^*_{j,4k+3} \left(H_{j,1,4k} - H_{j,2,4k}\right)\Big)
\end{aligned}
\tag{3.45}
$$

At the end, the elements $\{\widetilde{s}_m(n)\}_{m=0}^{N_s-1}$ are demodulated to extract the data.

3.3 Conclusion

In this chapter, a new scheme consisting of a combination of orthogonal frequency division multiplexing (OFDM), high order orthogonal transmit diversity (OTD) is presented. We described both encoding and decoding algorithms for the OTD-OFDM systems. Some examples were provided to review these algorithms for various OTD codes in more detail. It is shown that using the proposed scheme, space-time coding can be applied for high data-rate transmission in frequency selective fading channels. In the next, it will be shown that OTD can greatly improve the performance of the conventional OFDM systems.

Chapter 4

Channel Capacity of the MIMO-OFDM and OTD-OFDM Systems

The use of multiple input/multiple output (MIMO) systems has recently been shown to have the potential of achieving high bit rates [25]-[33]. The theoretical information capacity of MIMO systems for Rayleigh fading channels was derived in [54]-[57]. Following these studies, several researchers have designed some practical codes, such as space-time trellis codes [27] to approach these high rates. In addition to these codes, orthogonal transmit diversity (OTD) was proposed for the application over flat fading channels, having less complexity compared to other schemes [28]-[30].

So far, most of the research in theoretical information analysis of MIMO systems has focused on the narrow-band flat fading case [54]-[58]. In this chapter, we present an information theoretical analysis of OTD and MIMO systems over frequency-flat fading channels. Then, we provide an analytical expression for channel capacity of MIMO-OFDM and OTD-OFDM systems, where the channel is a multipath frequency selective fading channel, assuming that the channel is unknown at the transmitter and is perfectly known at the receiver. These expressions are then numerically evaluated and OTD-OFDM and MIMO-OFDM systems are studied and compared.

4.1 System model for flat fading MIMO Channels

Consider a MIMO (multiple input/multiple output) system including M_T transmitter antennas and M_R receiver antennas as shown in Figure 4.1. In a narrow-band flat-fading channel, the i-th receiver (RX) antenna receives the superposition of the M_T transmitted signals.

$$r_i\,(t) = \sum_{j=1}^{M_T} h_{i,j}(t)\, s_j(t) + v_i(t) \qquad\qquad i = 1, 2, ..., M_R \qquad (4.1)$$

where $v_i(t)$ is a complex additive white Gaussian noise (AWGN) with zero mean and variance of $\frac{\sigma^2}{2}$ and $h_{i,j}(t)$ is the channel gain between the j-th transmitter antenna and i-th receiver antenna.

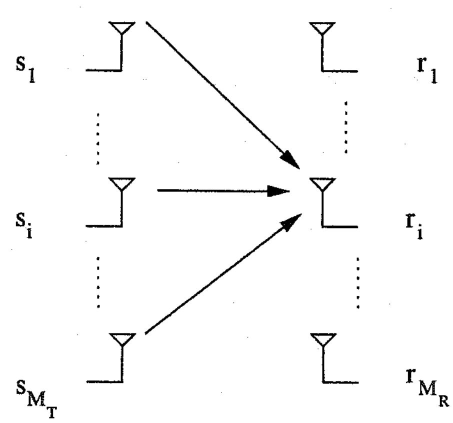

Figure 4.1: MIMO system model.

Equivalently (4.1) can be expressed in the form of a matrix as follows:

$$\mathbf{r}(t) = \mathbf{H}(t)\,\mathbf{s}(t) + \mathbf{v}(t) \qquad (4.2)$$

where

- $\mathbf{H}(t) = [\,h_{i,j}(t)\,]$ is an $M_R \times M_T$ matrix that represents the overall channel impulse response.
- $\mathbf{s}(t) = [\,s_1(t)\,, s_2(t)\,, ..., s_{M_T}(t)\,]^T$ is the transmitted signals vector.
- $\mathbf{r}(t) = [\,r_1(t)\,, r_2(t)\,, ..., r_{M_R}(t)\,]^T$ is the received signals vector.
- $\mathbf{v}(t) = [\,v_1(t)\,, v_2(t)\,, ..., v_{M_R}(t)\,]^T$ is the AWGN vector.

and superscript T in $(.)^T$ denotes the transpose operation.

4.2 Capacity of flat fading MIMO Channels

The concept of capacity of the communication channel was introduced by Shannon in 1948 [96]. The Shannon capacity is the maximum asymptotically (in the block size) error-free transmission rate supported by the channel. Perhaps the most famous illustration of this idea was the capacity formula derived in [96] for the capacity C of the additive white Gaussian noise channel (with variance N_0) with an input power constraint P, given by

$$C = \frac{1}{2}\log_2\left(1 + \frac{P}{N_0}\right) \tag{4.3}$$

For single-antenna flat fading channels and a coherent receiver (where the receiver uses perfect channel state information), the capacity was shown to be [97]:

$$C = \frac{1}{2}E_{\mathrm{h}}\left(\log_2\left[1 + \frac{P\,|h|^2}{N_0}\right]\right) \tag{4.4}$$

where E is the expectation operation and is taken over the fading channel $\{h(n)\}$ and the channel is assumed to be stationary and ergodic. This is called the *ergodic channel capacity* [98]. This is the transmission rate of information when the channel state information (CSI) is not available at the transmitter. If the channel is perfectly known at the transmitter (and at the receiver), one can do slightly better through optimizing the allocation of transmitted power according to the "water-filling" solution [99],[100].

Note that when we are dealing with complex channel (as is usual in communication with in-phase and quadrature-phase transmissions), the factor $\frac{1}{2}$ in the above equation disappears [54]. From this point on, we assume complex channels.

In the following, we will find the capacity of frequency-flat fading MIMO channels. The capacity of the MIMO channels, where the channel $\mathbf{H}(t)$ is perfectly known at the receiver, defined as the maximum mutual information between received signal $\mathbf{r}(t)$ and the transmitted signal $\mathbf{s}(t)$, can be expressed as follows:

$$C = \max_{trace(\mathbf{Q}) \leq P} \{ I(\mathbf{s}; \mathbf{r}, \mathbf{H}) \} \tag{4.5}$$

where the mutual information can be written as:

$$I(\mathbf{s}; \mathbf{r}, \mathbf{H}) = I(\mathbf{s}; \mathbf{H}) + I(\mathbf{s}; \mathbf{r} \mid \mathbf{H}) \tag{4.6}$$

If the channel state information (CSI) is not available at the transmitter, mutual information between transmitted signal $\mathbf{s}(t)$ and the channel $\mathbf{H}(t)$ will be equal to zero or $I(\mathbf{s}; \mathbf{H}) = 0$ and therefore,

$$I(\mathbf{s}; \mathbf{r}, \mathbf{H}) = I(\mathbf{s}; \mathbf{r} \mid \mathbf{H}) \tag{4.7}$$

When the channel $\mathbf{H}(t)$ is fixed, (4.7) can be expressed as [54]:

$$I(\mathbf{s}; \mathbf{r}, \mathbf{H}) = \log_2 \left\{ \det \left[\mathbf{I}_{M_R} + \frac{\mathbf{HQH}^{\dagger}}{\sigma^2} \right] \right\} \tag{4.8}$$

and in ergodic conditions when $\mathbf{H}(t)$ is random, (4.8) can be expressed as:

$$I(\mathbf{s}; \mathbf{r}, \mathbf{H}) = E_{\mathbf{H}} \left(\log_2 \left\{ \det \left[\mathbf{I}_{M_R} + \frac{\mathbf{HQH}^{\dagger}}{\sigma^2} \right] \right\} \right) \tag{4.9}$$

where,

- $\mathbf{I}_{M_R}$ is $M_R \times M_R$ identity matrix.
- $\mathbf{Q} = E_t(\mathbf{ss}^{\star})$ is the $M_T \times M_T$ autocorrelation matrix of the transmitted signal, that is a non-negative matrix with

$$trace(\mathbf{Q}) = \sum_{i=1}^{M_T} E_t \left(\|s_i\|^2 \right) \leq P \tag{4.10}$$

where P is the total transmitted power.

Channel capacity in the ergodic case can be defined as:

$$C = \max_{trace(\mathbf{Q}) \leq P} E_{\mathbf{H}} \left(\log_2 \left\{ \det \left[\mathbf{I}_{M_R} + \frac{\mathbf{H}\mathbf{Q}\mathbf{H}^{\dagger}}{\sigma^2} \right] \right\} \right) \tag{4.11}$$

It is shown [54]-[57] that it can be expressed as:

$$C = E_{\mathbf{H}} \left(\log_2 \left\{ \det \left[\mathbf{I}_{M_R} + \frac{P}{M_T \sigma^2} (\mathbf{H}\mathbf{H}^{\dagger}) \right] \right\} \right) \tag{4.12}$$

that represents the channel capacity of the MIMO (M_T , M_R) system over frequency flat fading channels.

When the channel is assumed to be fixed, (4.12) can be expressed as:

$$C = \log_2 \left\{ \det \left[\mathbf{I}_{M_R} + \frac{P}{M_T \sigma^2} (\mathbf{H}\mathbf{H}^{\dagger}) \right] \right\} \tag{4.13}$$

Example: MIMO system with $M_T = 2$, $M_R = 1$

Consider a MIMO system with two transmitted antennas and one receiver antenna ($M_T = 2$, $M_R = 1$). In this case the received signal can be rewritten as:

$$\mathbf{r}(t) = \mathbf{H}(t)\, \mathbf{s}(t) + \mathbf{v}(t) \tag{4.14}$$

where $\mathbf{H} = [\, h_1 \, , h_2]$, $\mathbf{s}(t) = [\, s_1(t) \, , s_2(t) \,]^T$, $\mathbf{r}(t) = [\, r_1(t)]$ and $\mathbf{v}(t) = [\, v_1(t)]$.

It can be easily seen that

$$\mathbf{H}\mathbf{H}^{\dagger} = \|h_1\|^2 + \|h_2\|^2 \tag{4.15}$$

Therefore, the channel capacity using (4.12) can be written as:

$$C_{MIMO\,(M_T=2,\, M_R=1)} = E_{\mathbf{H}} \left(\log_2 \left[1 + \frac{P}{2\sigma^2} (\|h_1\|^2 + \|h_2\|^2) \right] \right) \tag{4.16}$$

4.3 Channel capacity of the OTD systems

Here we will consider an OTD system with $M_T = 2$, $M_R = 1$ and calculate the channel capacity and then we will compare it with the conventional MIMO system.

OTD system with $M_T = 2,\ M_R = 1$:

Consider an OTD system with $M_T = 2$, $M_R = 1$. In this case the received signal can be written as:

$$\begin{aligned} r_1 &= r(t) = h_1(t)\, s_1(t) + h_2(t)\, s_2(t) + v_1(t) \\ r_2 &= r(t+T) = h_2(t+T)\, s_1^*(t) - h_1(t+T)\, s_2^*(t) + v_2(t) \end{aligned} \tag{4.17}$$

As assumed in [28] channel impulse response is fixed over two consecutive symbols, the above equation can be simplified to:

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2^* \end{bmatrix} \tag{4.18}$$

equivalent to (4.2) we can easily see that $\mathbf{H} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$ and therefore:

$$\mathbf{H}\mathbf{H}^\dagger = \mathbf{H}^\dagger\mathbf{H} = \left(\|h_1\|^2 + \|h_2\|^2\right) \mathbf{I}_2 \tag{4.19}$$

And with the same approach as the MIMO system, the channel capacity for the OTD system in this case can be expressed as:

$$C_{OTD\,(M_T=2,\,M_R=1)} = \max_{trace(\mathbf{Q})\leq P} \frac{1}{2} E_{\mathbf{H}} \left(\log_2 \left\{\det\left[\mathbf{I}_2 + \frac{\mathbf{HQH}^\dagger}{\sigma^2}\right]\right\}\right) \tag{4.20}$$

or

$$C_{OTD\,(M_T=2,\,M_R=1)} = \max_{trace(\mathbf{Q})\leq P} \frac{1}{2} E_{\mathbf{H}} \left(\log_2 \left\{\det\left[\mathbf{I}_2 + \frac{\mathbf{H}^\dagger\mathbf{HQ}}{\sigma^2}\right]\right\}\right) \tag{4.21}$$

where the factor $\frac{1}{2}$ is associated with the use of two time intervals in the OTD system.

The above expression can be written as:

$$C_{OTD\,(M_T=2,\,M_R=1)} = \max_{trace(\mathbf{Q})\leq P} \frac{1}{2} E_{\mathbf{H}} \left(\log_2 \left\{ \det \left[\mathbf{I}_2 + \frac{1}{\sigma^2} (\|h_1\|^2 + \|h_2\|^2)\, \mathbf{Q} \right] \right\} \right) \tag{4.22}$$

It was shown in [56] that the function $\log_2 \{\det(.)\}$ is concave on the set of non-negative definite matrices and it was concluded that $\mathbf{Q}$ proportional to the identity is optimal [56]. It implies that matrix $\mathbf{Q} = \frac{P}{2}\mathbf{I}_2$. Therefore, the channel capacity (4.22) can be written as:

$$C_{OTD\,(M_T=2,\,M_R=1)} = E_{\mathbf{H}} \left(\log_2 \left[1 + \frac{P}{2\sigma^2} (\|h_1\|^2 + \|h_2\|^2) \right] \right) \tag{4.23}$$

that is equal to the channel capacity of a MIMO system with $M_T = 2$, $M_R = 1$ as given in (4.16).

Conclusion:

It can be concluded that OTD ($M_T = 2$, $M_R = 1$) can achieve the full channel capacity of the MIMO ($M_T = 2$, $M_R = 1$) system and there is no loss of achievable data rate.

4.4 Channel Capacity of MIMO-OFDM and OTD-OFDM over frequency selective fading channels

4.4.1 Channel Capacity of the MIMO-OFDM Systems

So far, we have restricted our discussion to frequency-flat fading MIMO channels. In the following, we shall discuss frequency selective fading MIMO channels. We begin by introducing the system model of a MIMO-OFDM system shown in Figure 4.2.

At the input of the system for the j-th transmitter antenna, the m-th block of input symbol data $\mathbf{x}_{mj} = [\, x_{mj,0}\,, x_{mj,1}\,, \ldots, x_{mj,N-1}\,]^T$ is modulated by OFDM and generates a block $\mathbf{s}_{mj} = [\, s_{mj,0}\,, s_{mj,1}\,, \ldots, s_{mj,N-1+\Gamma}\,]^T$ and then is sent to all of the receiver antennas. N is the size of FFT, Γ is the length of the cyclic prefix.

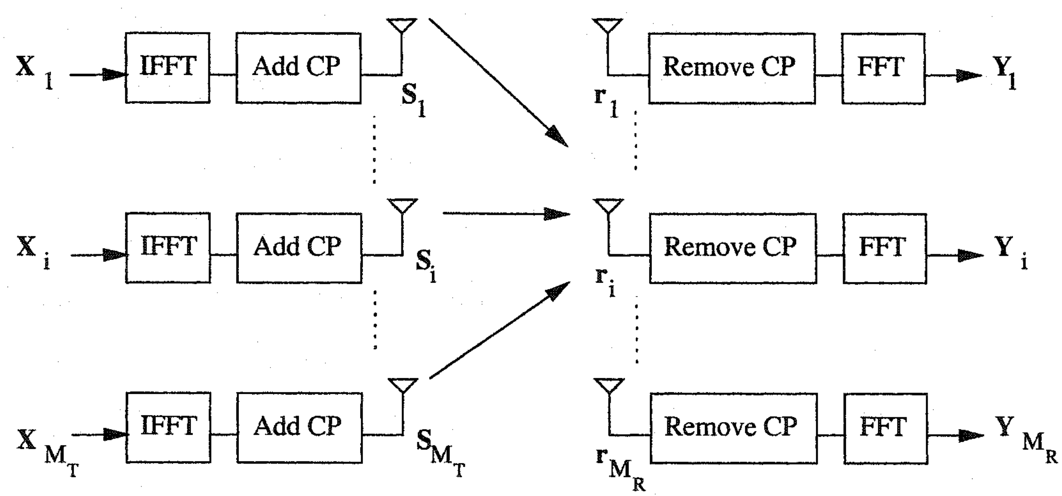

Figure 4.2: MIMO-OFDM system model.

The received signals at the i-th receiver antenna can be expressed as:

$$r_{mi,k} = h_{i,1} * s_{m1,k} + h_{i,2} * s_{m2,k} + \cdots + h_{i,j} * s_{mj,k} + \cdots + h_{i,M_T} * s_{mM_T,k} + v_{mi,k} \tag{4.24}$$

where $*$ is the convolution operation and

- $i = 1, 2, \cdots, M_R, \quad j = 1, 2, \cdots, M_T, \quad k = 0, 1, \cdots, N-1+\Gamma$ are the indices.
- $h_{i,j}$ is the channel impulse response from the j-th transmitter antenna to the i-th receiver antenna.
- $v_{mi,k}$ is the AWGN.

The signal demodulated at the output of the FFT can be written as:

$$y_{mi,k} = H_{i,1,k}\, x_{m1,k} + H_{i,2,k}\, x_{m2,k} + \cdots + H_{i,j,k}\, x_{mj,k} + \cdots + H_{i,M_T,k}\, x_{mM_T,k} + w_{mi,k} \tag{4.25}$$

where

- $i = 1, 2, \cdots, M_R, \quad j = 1, 2, \cdots, M_T, \quad k = 0, 1, \cdots, N-1$ are the indices.
- $[H_{i,j,0}, H_{i,j,1}, ..., H_{i,j,N-1}]^T$ are the DFT of the $h_{i,j}$.
- $y_{mi,k}$ is the received signal at the i-th receiver antenna.
- $v_{mi,k}$ is the AWGN.

This can be written as:

$$\underline{Y_k} = \underline{H_k}\,\underline{X_k} + \underline{W_k} \tag{4.26}$$

where

- $\underline{H_k} = \begin{bmatrix} H_{1,1,k} & H_{1,2,k} & \cdots & H_{1,M_T,k} \\ H_{2,1,k} & H_{2,2,k} & \cdots & H_{2,M_T,k} \\ \vdots & \vdots & \cdots & \vdots \\ H_{M_R,1,k} & H_{M_R,2,k} & \cdots & H_{M_R,M_T,k} \end{bmatrix}$ is a $(M_R \times M_T)$ matrix.
- $\underline{Y_k} = [\, y_{m1,k}, y_{m2,k}, ..., y_{mM_R,k}\,]^T$ is a $(M_R \times 1)$ vector.
- $\underline{X_k} = [\, x_{m1,k}, x_{m2,k}, ..., x_{mM_T,k}\,]^T$ is a $(M_T \times 1)$ vector.
- $\underline{W_k} = [\, w_{m1,k}, w_{m2,k}, ..., w_{mM_R,k}\,]^T$ is a $(M_R \times 1)$ vector.

Since the index $k = 0, 1, \cdots, N-1$, therefore (4.26) can be extended to:

$$\underline{\mathbf{Y}} = \underline{\mathbf{HX}} + \underline{\mathbf{W}} \tag{4.27}$$

where

- $\underline{\mathbf{H}} = \begin{bmatrix} \underline{H_0} & \underline{0} & \cdots & \underline{0} \\ \underline{0} & \underline{H_1} & \cdots & \underline{0} \\ \vdots & \vdots & \cdots & \vdots \\ \underline{0} & \underline{0} & \cdots & \underline{H_{N-1}} \end{bmatrix}$ is a $(NM_R \times NM_T)$ matrix, that can be

considered as

$$\underline{\mathbf{H}} = diag\left\{\underline{H_k}\right\}_{k=0}^{N-1}$$

- $\underline{0}$ is an $(M_R \times M_T)$ matrix with all zero elements.
- $\underline{\mathbf{Y}} = [\underline{Y_0}, \underline{Y_1}, ..., \underline{Y_{N-1}}]^T$ is a $(NM_R \times 1)$ vector.
- $\underline{\mathbf{X}} = [\underline{X_0}, \underline{X_1}, ..., \underline{X_{N-1}}]^T$ is a $(NM_T \times 1)$ vector.
- $\underline{\mathbf{W}} = [\underline{W_0}, \underline{W_1}, ..., \underline{W_{N-1}}]^T$ is a $(NM_R \times 1)$ vector.

The channel capacity of the MIMO-OFDM system in the ergodic case can be expressed as:

$$C = \max_{trace(\mathbf{Q}) \leq P} \frac{1}{N} E_{\mathbf{H}} \left(\log_2 \left\{ \det \left[\mathbf{I}_{N\,M_R} + \frac{\underline{\mathbf{H}}\underline{\mathbf{Q}}\underline{\mathbf{H}}^{\dagger}}{\sigma^2} \right] \right\} \right) \tag{4.28}$$

where,

- $\mathbf{I}_{N\,M_R}$ is $(N\,M_R \times N\,M_R)$ identity matrix.
- $\underline{\mathbf{Q}} = E_t(\underline{\mathbf{X}}\,\underline{\mathbf{X}}^{\dagger})$ is the $N\,M_T \times N\,M_T$ covariance matrix of the transmitted

signal, that is a non-negative matrix given by $\underline{\mathbf{Q}} = diag\left\{Q_k\right\}_{k=0}^{N-1}$.

The factor $\frac{1}{N}$ is related to the fact that each OFDM symbol includes N data symbols.

In MIMO-OFDM systems, the total available power is allocated uniformly across all space-frequency subchannels. In the following we set $Q_k = \frac{P}{N\,M_T}\mathbf{I}_{M_T}$, $(k = 0, 1, ..., N-1)$ Therefore, (4.28) can be written as:

$$C = \frac{1}{N} E_{\mathbf{H}} \left(\log_2 \left\{ \det \left[\mathbf{I}_{N\,M_R} + \frac{P}{N\,M_T\,\sigma^2} (\underline{\mathbf{H}\mathbf{H}}^{\dagger}) \right] \right\} \right) \tag{4.29}$$

that represents the channel capacity of the MIMO-OFDM $(M_T\,, M_R)$ system over frequency selective multipath fading channels.

It is good to show that the channel capacity of the MIMO-OFDM $(M_T\,, M_R)$ system, can also be expressed as:

$$C = \frac{1}{N} \sum_{k=0}^{N-1} C_k \tag{4.30}$$

where

$$C_k = E_{H_k} \left(\log_2 \left\{ \det \left[\mathbf{I}_{M_R} + \frac{P}{N\,M_T\,\sigma^2} (\underline{H_k}\,\underline{H_k}^{\dagger}) \right] \right\} \right) \tag{4.31}$$

which can be expressed as:

$$C = \frac{1}{N} \sum_{k=0}^{N-1} E_{H_k} \left(\log_2 \left\{ \det \left[\mathbf{I}_{M_R} + \frac{P}{N\,M_T\,\sigma^2} (\underline{H_k}\,\underline{H_k}^{\dagger}) \right] \right\} \right) \tag{4.32}$$

that is another expression for the channel capacity of the MIMO-OFDM $(M_T\,, M_R)$ system over frequency selective multipath fading channels.

4.4.2 Channel Capacity of the OTD-OFDM Systems

Here we will consider an OTD-OFDM system and calculate the channel capacity and then we will compare it with the conventional MIMO-OFDM system.

OTD-OFDM system with $M_T = 2$, $M_R = 1$:

Consider an OTD system with $M_T = 2$, $M_R = 1$, using the code $\mathbf{G}_2$ (code rate $R_c = 1$). In this case, the received signal for the k-th subchannel can be written as:

$$\begin{bmatrix} y_1[n,k] \\ y_2[n,k] \end{bmatrix} = \begin{bmatrix} +x_1[n,k] & +x_2[n,k] \\ -x_2^*[n,k] & +x_1^*[n,k] \end{bmatrix} \begin{bmatrix} H_1[n,k] \\ H_2[n,k] \end{bmatrix} + \begin{bmatrix} W_1[n,k] \\ W_2[n,k] \end{bmatrix} \tag{4.33}$$

$$\mathbf{Y} = \mathbf{G}_2 \mathbf{H}' + \mathbf{W} \tag{4.34}$$

This can be written as

$$\mathbf{Y}' = \underline{H}_k \mathbf{X} + \mathbf{W}' \tag{4.35}$$

$$\begin{bmatrix} y_1[n,k] \\ y_2^*[n,k] \end{bmatrix} = \begin{bmatrix} H_1[n,k] & H_2[n,k] \\ H_2^*[n,k] & -H_1^*[n,k] \end{bmatrix} \begin{bmatrix} x_1[n,k] \\ x_2[n,k] \end{bmatrix} + \begin{bmatrix} W_1[n,k] \\ W_2^*[n,k] \end{bmatrix} \tag{4.36}$$

where

$$\underline{H}_k = \begin{bmatrix} +H_1[n,k] & +H_2[n,k] \\ +H_2^*[n,k] & -H_1^*[n,k] \end{bmatrix}, \quad k = 0, 1, \cdots, N-1 \tag{4.37}$$

is the equivalent k-th subchannel, so that

$$\underline{H}_k \, \underline{H}_k^\dagger = \underline{H}_k^\dagger \, \underline{H}_k = \left(|H_1[n,k]|^2 + |H_2[n,k]|^2\right) \mathbf{I}_2 \tag{4.38}$$

where $\mathbf{I}_2$ is a identity matrix and $H_1[n,k]$ and $H_2[n,k]$ are the DFTs of the h_1 and h_2 respectively.

Therefore, similar to MIMO-OFDM and OTD, the channel capacity of OTD-OFDM in this case will be:

$$C_{OTD} = \frac{1}{2N} \sum_{k=0}^{N-1} E_{H_k} \left(\log_2 \left\{ \det \left[\mathbf{I}_2 + \frac{P}{2N\,\sigma^2} (\underline{H_k}^{\dagger} \underline{H_k}) \right] \right\} \right) \tag{4.39}$$

Substituting (4.38) into (4.39) we can obtain

$$C_{OTD} = \frac{1}{2N} \sum_{k=0}^{N-1} E_{H_k} \left(\log_2 \left\{ \det \left[\mathbf{I}_2 + \frac{P}{2N\,\sigma^2} \left(|H_1[n,k]|^2 + |H_2[n,k]|^2 \right) \mathbf{I}_2 \right] \right\} \right) \tag{4.40}$$

therefore,

$$C_{OTD} = \frac{1}{N} \sum_{k=0}^{N-1} E_{H_k} \left(\log_2 \left\{ \det \left[1 + \frac{P}{2N\,\sigma^2} \left(|H_1[n,k]|^2 + |H_2[n,k]|^2 \right) \right] \right\} \right) \tag{4.41}$$

that is equal to the channel capacity of MIMO-OFDM with $M_T = 2$, $M_R = 1$, over frequency selective fading channels.

4.5 Simulation Results

In this section, we present the numerical results for the channel capacity of MIMO and OTD over Rayleigh flat fading channels and for the channel capacity of MIMO-OFDM and OTD-OFDM, over a non-flat frequency selective fading channel.

The considered non-flat fading channel is a multipath fading channel with coherence bandwidth smaller than the total bandwidth of the multicarrier system and thus seen as frequency selective fading. The fading process is assumed to be stationary and slowly varying compared to the symbol duration of the multicarrier signal, such that it is approximately constant during one OFDM block length and varies from one OFDM block to another. The fading process impulse response at the antenna i can be expressed as [108]

$$h_i(t) = \sum_{m=0}^{L-1} \alpha_{m,i}(t)\, \delta(t - m\,T_s) \tag{4.42}$$

where the tap weight $\alpha_{m,i}(t)$ is a complex Gaussian random process with zero mean and variance $\frac{1}{L}$ (equal power), and $m\,T_s$ is the time delay of the m-th path and L is the total number of resolvable paths ($L = 8$). With this model we have assumed that the path delays are multiples of the symbol duration T_s. The OFDM system includes $N = 64$ subcarriers and a cyclic prefix which is longer than the channel delay. The channel state information is assumed to be known at the receiver but unknown at the transmitter. Finally, SNR is defined as $SNR = \frac{P}{\sigma^2}$ in the MIMO systems and $SNR = \frac{P}{N\sigma^2}$ in the MIMO-OFDM systems, where P is the total transmit power per symbol time.

Here, we consider the average capacity (ergodic capacity). Using Monte Carlo simulation, the channel capacity results are averaged over 10000 channel realizations.

First, the capacity of a MIMO system over flat fading channels and later the capacity of the MIMO-OFDM system over frequency selective fading channels will be considered. Figures 4.3, 4.4 and 4.5 show the channel capacity of various forms of the MIMO, OTD, MIMO-OFDM and OTD-OFDM systems. Figure 4.3 shows that OTD and MIMO systems have the same channel capacity, for flat fading channels. In the case of multipath fading channel, Figures 4.4 and 4.5 are given to compare MIMO systems with MIMO-OFDM and OTD systems with OTD-OFDM. Note that comparison is made between MIMO (OTD) over flat fading channel with MIMO-OFDM (OTD-OFDM) over frequency selective fading channel.

It can be seen that the capacity of the OTD and OTD-OFDM systems are approximately the same. The difference is essentially equal to the transmitted power penalty associated with the cyclic prefix. These results indicate that the use of OFDM transforms a frequency selective fading channel into several flat fading subchannels.

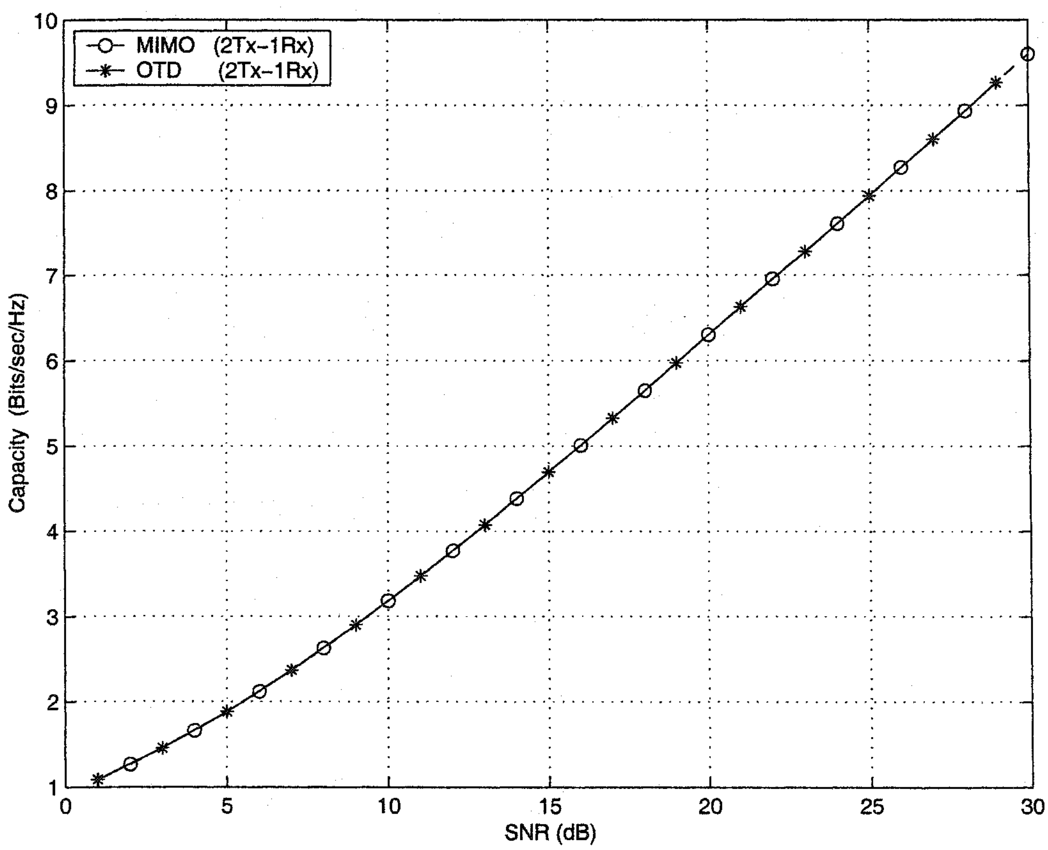

Figure 4.3: Channel Capacity of MIMO and OTD systems over flat fading channel.

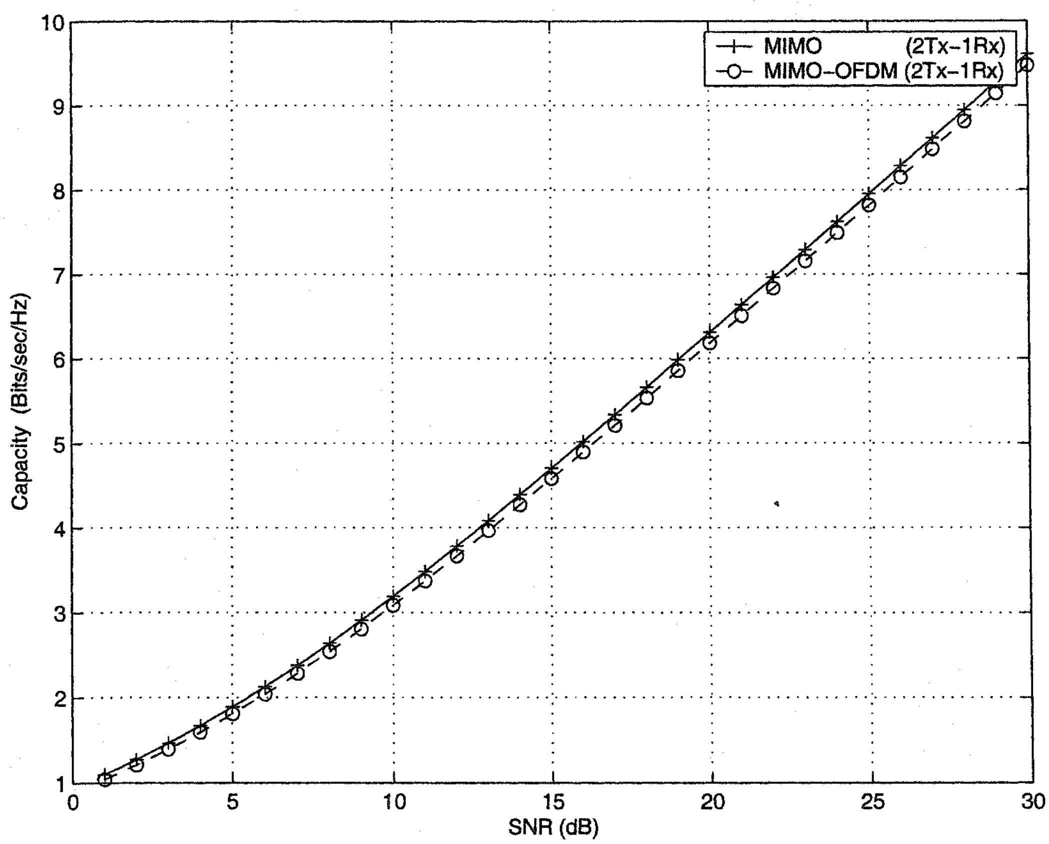

Figure 4.4: Channel Capacity of MIMO over flat fading channel and MIMO-OFDM over multipath fading.

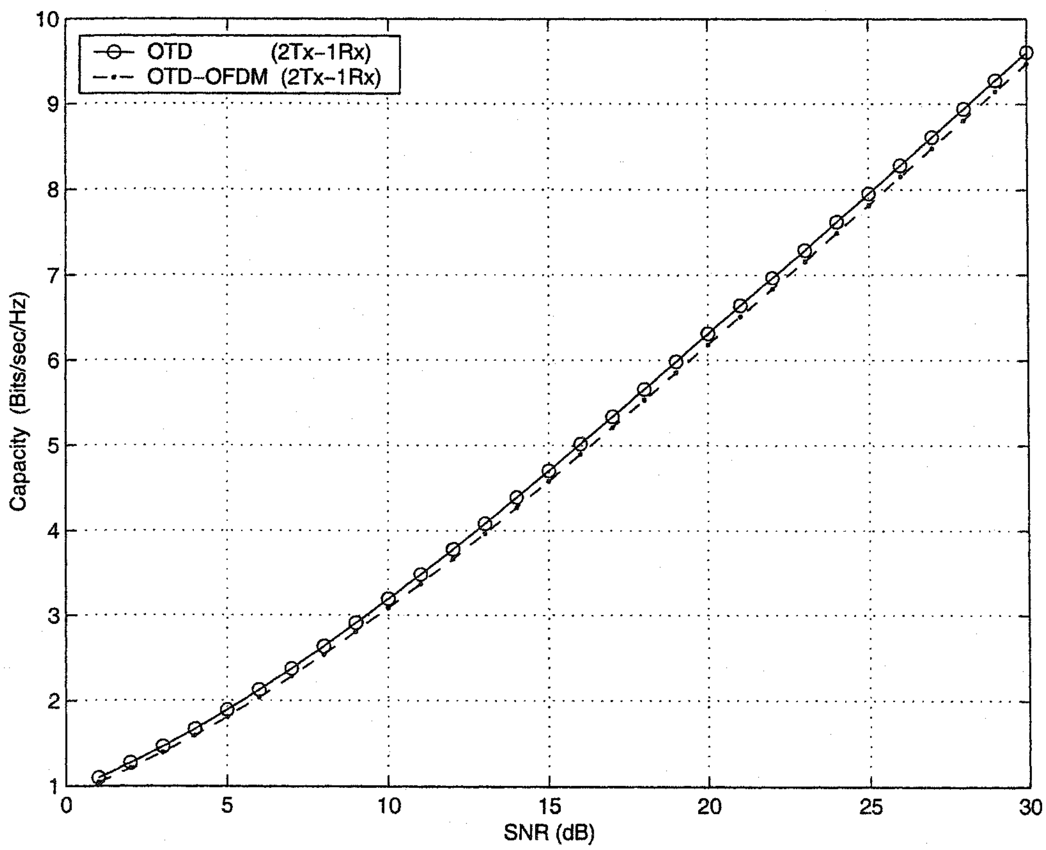

Figure 4.5: Channel Capacity of OTD over flat fading channel and OTD-OFDM over multipath fading channel.

Chapter 5

Performance Analysis of Space-Frequency Coded OFDM Systems

In this chapter, new closed-form expressions for the bit error rate (BER) performance of space-frequency block coded OFDM (SFBC-OFDM) systems are derived and evaluated over frequency selective fading channels. In the performance evaluation both M-ary phase shift keying (MPSK) and M-ary quadrature amplitude modulation (MQAM) are considered. Numerical results are provided for analysis and simulations. In addition, the performances of several forms of SFBC-OFDM system are also evaluated and compared. It is shown that the results from the closed-form formula are very close to the analytical formula and simulation results.

5.1 Introduction

There is a growing demand for high-speed, spectrally efficient and reliable communication. Providing high-quality services in a wireless environment has several challenges. Recently, the use of multiple-input multiple output (MIMO) systems, employing space-time block codes (STBC) has been proposed as an efficient solution for future wireless systems, since they can greatly improve the system performance over flat fading channels with a reasonable level of complexity [28]-[30].

For non-flat fading channels such as frequency selective multipath channels, OFDM can be used to convert the non-flat fading channel into several flat subchannels. Hence, STBC with OFDM can be effectively used in non-flat fading channels. Several combinations of STBC with OFDM, have been considered in the past for two transmitter antennas and one receiver antenna [28]. In order to utilize the diversities in frequency and space, the so called SFBC-OFDM systems have been proposed [44]-[48]. Numerical simulation has been used for performance evaluation of these systems.

The closed-form BER expression would serve as an attractive alternative to the commonly used bounds for evaluating performance. In this chapter, we derive some expressions for general SFBC-OFDM systems including accurate formulas and approximated formulas. Then we present very interesting closed-form expressions for average BER performance. In the analysis, both M-ary phase shift keying (MPSK) and M-ary quadrature amplitude modulation (MQAM) are considered. Numerical results are provided for analysis and simulations. Furthermore, performance of several forms of the SFBC-OFDM system are also evaluated and compared.

In addition to presenting the accurate expressions of BER performance for general SFBC-OFDM systems, we also provide approximation formulas for the following reasons. First, we can use the approximation formulas to calculate the average BER expressions and to be able to present some new closed-form formulas. Second, in Chapter 6, we would like to express the numbers of bits in terms of BER by inverting the BER formulas. The inverse function for accurate BER formulas is difficult to obtain, where, we can easily invert the approximation formulas.

The remainder of this chapter is organized as follows: The system model is presented in Section 5.2. In Section 5.3, the BER performance of the conventional

OFDM is reviewed. For the SFBC-OFDM, BER performance is analytically evaluated in Section 5.4. Numerical results of analysis and simulations are presented in Section 5.5. Finally, Section 5.6 concludes this chapter.

5.2 System Model

Figure 5.1 shows a block diagram of the SFBC-OFDM system. A stream of information is first converted from serial to parallel.

Assuming an OFDM with N subcarriers, let N_s be the number of subbands that has been chosen to be $N_s = N/T$, i.e, each subband includes T adjacent subchannels (T is the symbol period of SFBC system). Then, all subbands are modulated using MQAM/MPSK where M is determined by the number of the allocated bits. Therefore, a signal vector:

$$S(n) = \{s_0(n),\, s_1(n), s_2(n), ...,\, s_{N_t-1}(n)\}$$

is provided as the input for the SFBC system, where N_t is chosen to be equal to N multiplied by the code rate of SFBC system (R_c). A space-time block code is defined by a $(T \times M_T)$ transmission matrix $\mathbf{G}$ given by:

$$\mathbf{G} = \begin{pmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,M_T} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,M_T} \\ \vdots & \vdots & \cdots & \vdots \\ g_{T,1} & g_{T,2} & \cdots & g_{T,M_T} \end{pmatrix} \tag{5.1}$$

where each of the elements $g_{i,j}$ is a linear combination of a subset of elements of $S(n)$ and their conjugates.

In order to utilize the space-frequency diversity, the input blocks for the OFDM at each transmitter antenna should have the length N. SFBC provides M_T blocks:

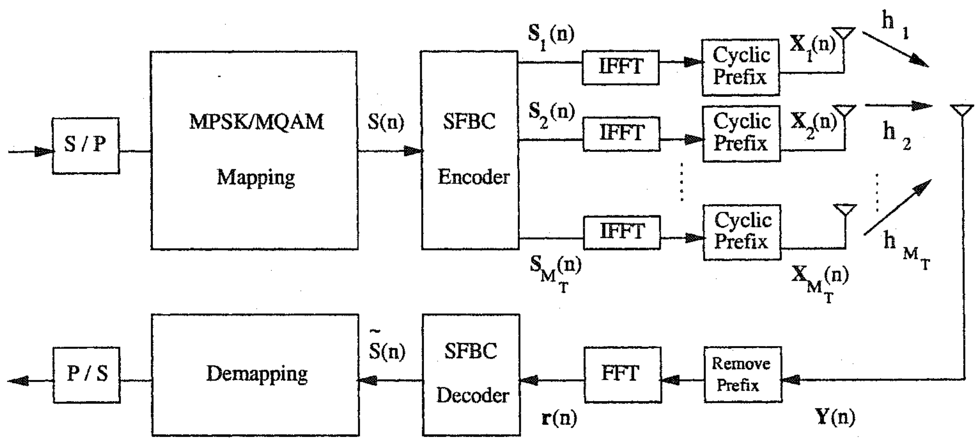

Figure 5.1: Block diagram of the SFBC-OFDM system.

$$\mathbf{S}_1(n),\ \mathbf{S}_2(n), \cdots, \mathbf{S}_{M_T}(n)$$

of the length N, each consisting of $\frac{N}{T}$ sub-blocks, i.e.,

$$\mathbf{S}_i(n) = \begin{pmatrix} \mathbf{s}_{i,0}(n) & \mathbf{s}_{i,1}(n) & \ldots & \mathbf{s}_{i,\frac{N}{T}-1}(n) \end{pmatrix}^T,$$

$$(i = 1, 2, \cdots, M_T).$$

Then OFDM modulators generate blocks of:

$$\mathbf{X}_1(n),\ \mathbf{X}_2(n), \cdots, \mathbf{X}_{M_T}(n)$$

where they will be transmitted by the first, second, ..., and M_T-th transmitter antennas simultaneously. Given that the guard time interval is longer than the largest delay spread of a multipath channel (to avoid ISI), the received signal will be the convolution of the channel and the transmitted signal. Assuming that the channel is static during an OFDM block, at the receiver side, after removing the cyclic prefix, the FFT output as the demodulated received signal can be expressed as:

$$\mathbf{r}(n) = \sum_{i=1}^{M_T} \mathbf{H}_i(n)\, \mathbf{S}_i(n) + \mathbf{W}(n) \tag{5.2}$$

where

$$\mathbf{r}(n) = (r_0(n), \cdots, r_{N-1}(n))^T$$

and $\mathbf{W}(n) = (W_0(n), \cdots, W_{N-1}(n))^T$ denotes the AWGN and $\mathbf{H}_i(n)$ represents a diagonal matrix whose elements $(H_i[j]\,, \quad i = 1, 2, \cdots, M_T,\ j = 0, 1, \cdots, N-1)$ are the subchannels.

Knowing the channel information at the receiver, Maximum Likelihood (ML) decoding can be used for SFBC decoding of the received signal. This simplifies the decoding since it involves only linear processing. At the end, the elements of block $\{\tilde{s}_j(n)\}_{j=0}^{N_t-1}$ are demodulated to extract the information data. We have illustrated the encoding and decoding of the SFBC-OFDM systems in more detail with some examples in Chapter 4.

5.3 Review of the uncoded OFDM systems

In this section, the bit-error rate (BER) performance analysis of the conventional OFDM systems is reviewed. Let $s(n)$ be the MQAM/MPSK signal and N be the size of the inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT) in the OFDM system, i.e., the number of subcarriers in the OFDM system. The multipath fading channel can be expressed as

$$H(z) = \sum_{l=0}^{L-1} h(l)\, z^{-l} \tag{5.3}$$

where $h(l)$ is the impulse response of the multipath fading channel. In order to remove the ISI, the length of the cyclic prefix (Γ) should be larger than the channel delay spread (L) (i.e., $\Gamma \geq L$). Let $y(n)$ be the received signal at the transmitter and $r(n)$ be the signal after removing the cyclic prefix and FFT of the remaining signal. The received signal can be expressed as

$$\mathbf{r}(n) = \mathbf{H}(n)\ \mathbf{S}(n) + \mathbf{W}(n) \tag{5.4}$$

where $\mathbf{r} = (r_0, \cdots, r_{N-1})^T$ and $\mathbf{W} = (W_0, \cdots, W_{N-1})^T$ denotes AWGN and $\mathbf{H}$ represents a diagonal matrix whose elements ($H[k]$, $k = 0, 1, \cdots, N-1$) are DFT of the impulse response of the multipath channel.

It can be seen that a non-flat multipath fading channel is converted into N ISI-free subchannels. At the end the information data can be detected and extracted through (5.4).

The BER expression of the OFDM system can be written as

$$BER = \frac{1}{N}\sum_{k=0}^{N-1} BER[k] \tag{5.5}$$

where $BER[k]$ is the instantaneous BER of the k-th subchannel in the block of the OFDM system.

5.3.1 BER expression for the MQAM-OFDM system

Assume a square MQAM with Gray bit mapping is employed for each subchannel and β bits/symbol is assigned for each subchannel where $M = 2^{\beta}$. Also the negligible degradation due to the cyclic prefix in the OFDM is not considered. The expression for the instantaneous BER of the k-th subchannel in the block of the OFDM over a frequency selective fading channel can be written as [53],[108].

$$BER_{MQAM}[k] = \frac{2}{\beta}\left(1 - \frac{1}{\sqrt{2^{\beta}}}\right) \times erfc\left(\sqrt{\frac{1.5\,\gamma_s\ |H[k]|^2}{2^{\beta}-1}}\right) \tag{5.6}$$

where $\gamma_s = \frac{E_s}{N_0}$, E_s is the symbol energy at the transmitter and $\frac{N_0}{2}$ is the variance of the real/imaginary part of the AWGN and $erfc(x)$ is the *complementary error function*, defined as

$$erfc(x) = \int_x^{\infty} \exp(-t^2)\, dt. \tag{5.7}$$

We can approximately express (5.6) as [72],[73]:

$$BER_{MQAM}[k] \approx 0.2 \exp\left(-\frac{1.6\,\gamma_s\,|H[k]|^2}{2^\beta - 1}\right) \tag{5.8}$$

Therefore, the closed-form BER expression for the OFDM system can be written as

$$BER_{MQAM} = \frac{2}{N\beta}\left(1 - \frac{1}{\sqrt{2^\beta}}\right)\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\,\gamma_s\,|H[k]|^2}{2^\beta - 1}}\right) \tag{5.9}$$

or approximately,

$$BER_{MQAM} \approx \frac{0.2}{N}\sum_{k=0}^{N-1} \exp\left(-\frac{1.6\,\gamma_s\,|H[k]|^2}{2^\beta - 1}\right) \tag{5.10}$$

The average BER can be represented as

$$\overline{BER}_{MQAM} = \int_0^{\infty} BER_{MQAM}\, p(\gamma)\, d\gamma \tag{5.11}$$

where $p(\gamma)$ is the probability density function of $\gamma = \gamma_s\,|H[k]|^2$.

Since $|H[k]|$ is Rayleigh-distributed, $|H[k]|^2$ has a chi-square probability distribution with two degrees of freedom. Consequently, γ is also chi-square-distributed as follows.

$$p(\gamma) = \frac{1}{\overline{\gamma}} \exp\left(-\frac{\gamma}{\overline{\gamma}}\right) \qquad \gamma \geq 0 \tag{5.12}$$

where $\overline{\gamma}$ is defined as

$$\overline{\gamma} = \gamma_s E \left\{ |H[k]|^2 \right\}$$

The term $E \left\{ |H[k]|^2 \right\}$ is simply the average of $|H[k]|^2$.

Substituting (5.10) and (5.12) into (5.11) we can obtain

$$\overline{BER}_{MQAM} = \frac{0.2}{1 + \frac{1.6\,\gamma_s}{2^{\beta}-1}} \tag{5.13}$$

5.3.2 BER expression for the MPSK-OFDM system

Assume MPSK with Gray bit mapping is employed for each subchannel of the OFDM. We can express the instantaneous BER of the k-th subchannel in the block of the OFDM as

$$BER_{MPSK}[k] = \frac{1}{\beta}\, erfc\left(\sqrt{\gamma_s\, |H[k]|^2} \sin\left(\frac{\pi}{2^{\beta}}\right)\right) \tag{5.14}$$

It can be approximated as [73]

$$BER_{MPSK}[k] \approx 0.2 \exp\left(-\frac{7\,\gamma_s\,|H[k]|^2}{2^{1.9\beta}+1}\right) \tag{5.15}$$

Therefore, the closed-form BER expression of the OFDM system can be written as

$$BER_{MPSK} = \frac{1}{N\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\gamma_s\,|H[k]|^2}\,\sin\left(\frac{\pi}{2^\beta}\right)\right) \tag{5.16}$$

or approximately,

$$BER_{MPSK} \approx \frac{1}{N}\sum_{k=0}^{N-1} 0.2\exp\left(-\frac{7\,\gamma_s\,|H[k]|^2}{2^{1.9\beta}+1}\right) \tag{5.17}$$

Similar to the MQAM-OFDM case, we can easily obtain

$$\overline{BER}_{MPSK} = \frac{0.2}{1+\frac{7\,\gamma_s}{2^{1.9\beta}+1}} \tag{5.18}$$

5.4 BER Performance of the SFBC-OFDM system

In this section we present the closed-form BER expression for the SFBC-OFDM system employing M_T transmitter and one receiver antennas.

5.4.1 BER expression for the MQAM-SFBC-OFDM system

Consider an SFBC system employing M_T transmitter antennas and one receiver antenna. The decoder minimizes the decision metric

$$|\tilde{s}_n - s_k|^2 \tag{5.19}$$

Similar to (3.14) we can show that

$$\tilde{s}_k = \frac{1}{M_T\,R_c}\sum_{i=1}^{M_T} |H_i[k]|^2\,s_k + \eta_k \tag{5.20}$$

where $H_i[k]$ is the normalized k-th subchannel associated to the i-th transmitter antenna. R_c is the code rate, and, η_k is the noise component.

We can express instantaneous SNR as

$$\gamma = \frac{1}{M_T R_c} \sum_{i=1}^{M_T} |H_i[k]|^2 \gamma_s \tag{5.21}$$

Similar to the conventional OFDM, we can express the BER of the MQAM-SFBC-OFDM over a frequency selective fading channel as

$$BER_{MQAM} = \frac{2}{N\beta}\left(1 - \frac{1}{\sqrt{2^\beta}}\right) \times \sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{1.5\,\gamma_s \sum_{i=1}^{M_T} |H_i[k]|^2}{R_c\, M_T\,(2^\beta - 1)}}\right) \tag{5.22}$$

It can be approximated as

$$BER_{MQAM} \approx \frac{0.2}{N} \sum_{k=0}^{N-1} \exp\left(-\frac{1.6\,\gamma_s \sum_{i=1}^{M_T} |H_i[k]|^2}{R_c\, M_T\, (2^\beta - 1)}\right) \tag{5.23}$$

The average BER can be represented as

$$\overline{BER}_{MQAM} = \int_0^\infty ... \int_0^\infty BER_{MQAM}\, p(\gamma_1)...p(\gamma_{M_T})\, d\gamma_1...d\gamma_{M_T} \tag{5.24}$$

where, $\gamma_i = \gamma_s\, |H_i[k]|^2, \quad i = 1, 2, \cdots, M_T$ and $p(\gamma_i)$ is the probability density function:

$$p(\gamma_i) = \frac{1}{\overline{\gamma_i}} \exp\left(-\frac{\gamma_i}{\overline{\gamma_i}}\right) \qquad \gamma_i \geq 0 \tag{5.25}$$

Substituting (5.23) and (5.25) into (5.24) we can obtain

$$\overline{BER}_{MQAM} = \frac{0.2}{\left(1 + \frac{1.6\,\gamma_s}{R_c\, M_T\, (2^\beta - 1)}\right)^{M_T}} \tag{5.26}$$

5.4.2 BER expression for the MPSK-SFBC-OFDM system

With the same approach as (5.22), the expression for the BER of the MPSK-SFBC-OFDM can be written as

$$BER_{MPSK} = \frac{1}{N\,\beta}\sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\gamma_s \sum_{i=1}^{M_T} |H_i[k]|^2}{R_c\, M_T}}\,\sin\left(\frac{\pi}{2^{\beta}}\right)\right) \tag{5.27}$$

or approximately,

$$BER_{MPSK} \approx \frac{1}{N}\sum_{k=0}^{N-1} 0.2\exp\left(-\frac{7\,\gamma_s \sum_{i=1}^{M_T} |H_i[k]|^2}{R_c\, M_T\,(2^{1.9\,\beta}+1)}\right) \tag{5.28}$$

and similar to the MQAM-SFBC-OFDM case, we can obtain

$$\overline{BER}_{MPSK} = \frac{0.2}{\left(1+\frac{7\,\gamma_s}{R_c\,M_T\,(2^{1.9\,\beta}+1)}\right)^{M_T}} \tag{5.29}$$

5.5 Simulation Results

The performance of the SFBC-OFDM systems is evaluated by computer simulation for frequency selective fading channels. The considered fading channels are multi-path fading channels with coherence bandwidth smaller than the total bandwidth of the multicarrier system and thus seen as frequency selective fading. The fading process is assumed to be stationary and slowly varying compared to the symbol duration of the multicarrier signal, such that it is approximately constant during one OFDM block length. The fading process impulse response at the antenna i ($i = 1, 2, ..., M_T$) can be expressed as [108]

$$h_i(t) = \sum_{m=0}^{L-1} \alpha_{m,i}(t)\,\delta(t-\tau_m(t)) \tag{5.30}$$

where the tap weight $\alpha_{m,i}(t)$ is a complex Gaussian random process with zero mean and variance $\frac{1}{L}$ (equal power), and $\tau_m(t)$ is the time delay of the m-th path and L is the total number of resolvable paths ($L = 4$). With this model we have assumed that the path delays, $\tau_m(t)$, are multiples of the symbol duration T_s. The OFDM system includes $N = 512$ subcarriers and a cyclic prefix which is longer than the channel delay. It is assumed that the channel state information is available at the receiver.

Note that the term simulation in the figures is the simulation of the proposed system (Figure 5.1), otherwise, the results obtained from Monte Carlo simulation of the formulas using several realizations of the channel.

The average BER performance evaluation for SFBC-OFDM is shown in Figure 5.2. The results are presented for an uncoded BPSK-OFDM and SFBC-OFDM. SFBC system with 2Tx-1Rx (code $\mathbf{G}_2$, BPSK on each subcarrier), 3Tx-1Rx (code $\mathbf{G}_3$, QPSK on each subcarrier), 4Tx-1Rx (code $\mathbf{G}_4$, QPSK on each subcarrier) are considered, where the $\mathbf{G}_2$, $\mathbf{G}_3$ and $\mathbf{G}_4$ are the given space-time codes in Chapter 3. The total rate in each case is 1 bits/s/Hz because the code rate of $\mathbf{G}_2$ is equal to 1, where as $\mathbf{G}_3$ and $\mathbf{G}_4$ are rate $\frac{1}{2}$ codes. The first set of curves is obtained from simulation results of the proposed system (Figure 5.1). The second set of curves is provided by calculating the average of BER form Equation (5.26). It can be seen that SFBC (2Tx-1Rx) provides a remarkable BER performance gain for OFDM and significantly outperforms uncoded OFDM; that is a gain of about 15 dB at a BER of 10^{-4}. More gain is achieved by higher order SFBC with more antennas at the transmitter. It can be seen that the results from the closed-form formula are very close to the simulation results.

The average BER performance evaluation for MQAM-SFBC-OFDM, is shown in Figure 5.3. It can be seen that the results from the closed-form formula (Equation (5.26)) (approximation) are very close to the analytical formula (Equation (5.22)) and the simulation results.

Figure 5.4 shows the average BER for transmission of 3 bits/s/Hz. The results are presented for an uncoded 8-PSK-OFDM and SFBC-OFDM using two, three, and four transmitter antennas. The transmission using two transmitter antennas employs the 8-PSK constellation and the code $\mathbf{G}_2$. For three and four transmitter antennas, 16-QAM constellations and codes $\mathbf{H}_3$ and $\mathbf{H}_4$ are employed. The total rate in each case is 3 bits/s/Hz because $\mathbf{H}_3$ and $\mathbf{H}_4$ are rate 3/4 codes. It can be seen that at the BER of 10^{-5} the code $\mathbf{H}_4$ gives about 7 dB gain over the use of code $\mathbf{G}_2$. The $\mathbf{G}_2$, $\mathbf{G}_3$, $\mathbf{G}_4$, $\mathbf{H}_3$ and $\mathbf{H}_4$ are the given space-time codes in [29] and Chapter 3. It is also shown that the results from the closed-form formula are very close to the analytical formula (accurate formula) results.

The average BER performance evaluation for the MQAM-SFBC-OFDM (3Tx-1Rx) and the MQAM-SFBC-OFDM (4Tx-1Rx), using codes of $\mathbf{G}_3$, $\mathbf{G}_4$, $\mathbf{H}_3$, and $\mathbf{H}_4$, are shown in Figures 5.5, and 5.6. It can be seen that the results from the closed-form formula are very close to the simulation results.

5.6 Conclusion

In this chapter, new closed-form expressions for the bit error rate (BER) performance of space-frequency block coded OFDM (SFBC-OFDM) systems were derived and evaluated over frequency selective fading channels. In the performance evaluation both M-ary phase shift keying (MPSK) and M-ary quadrature amplitude modulation (MQAM) were considered. Numerical results were provided for analysis and simulations. In addition, performances of several forms of the SFBC-OFDM system were also evaluated and compared. It was shown that the results from the closed-form and analytical formulas are very close to the simulation results.

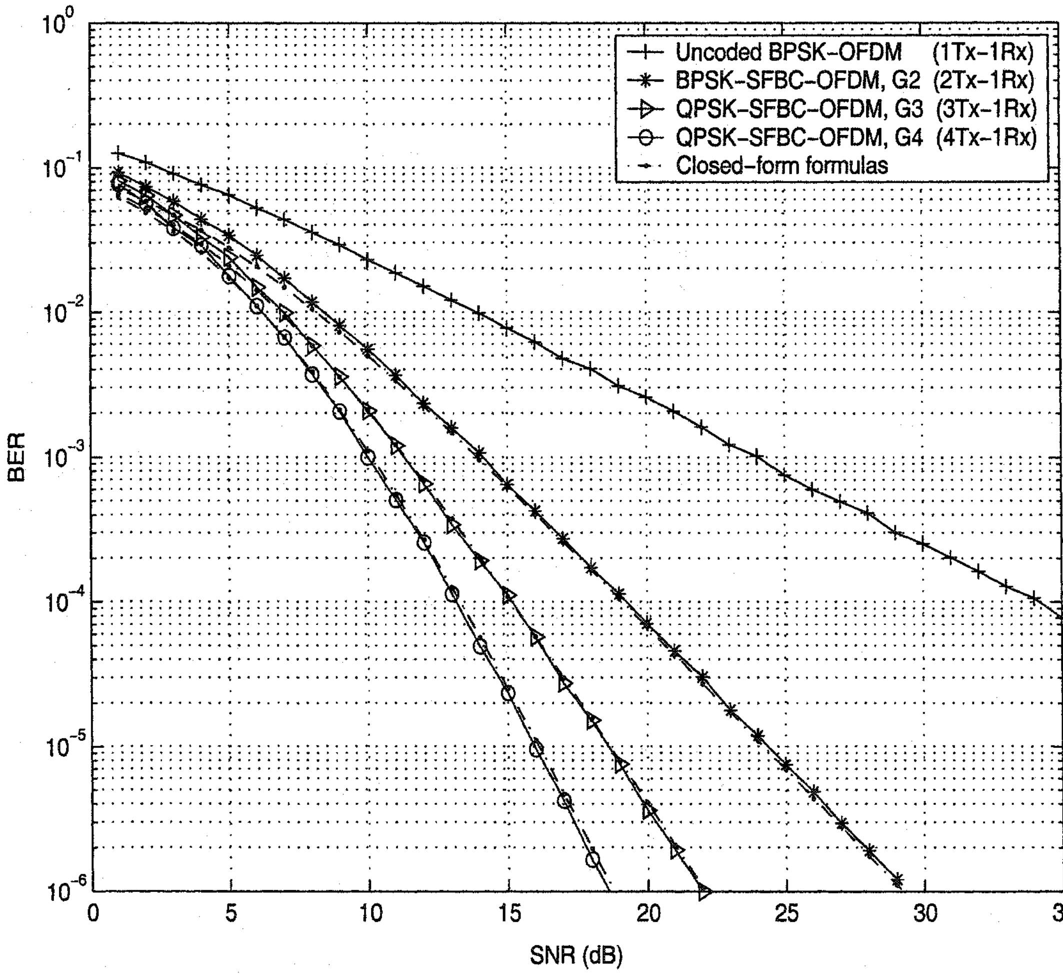

Figure 5.2: Average BER of MQAM-SFBC-OFDM.

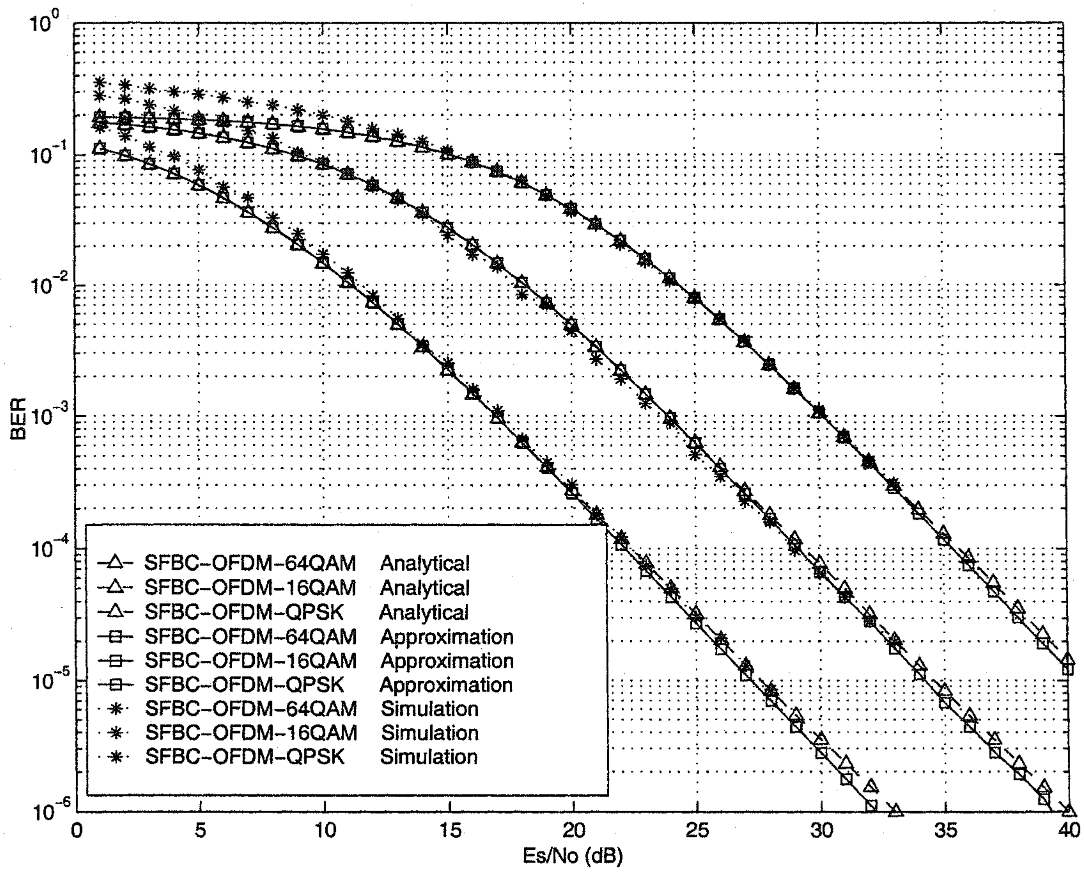

Figure 5.3: Average BER of MQAM-SFBC-OFDM (2Tx-1Rx).

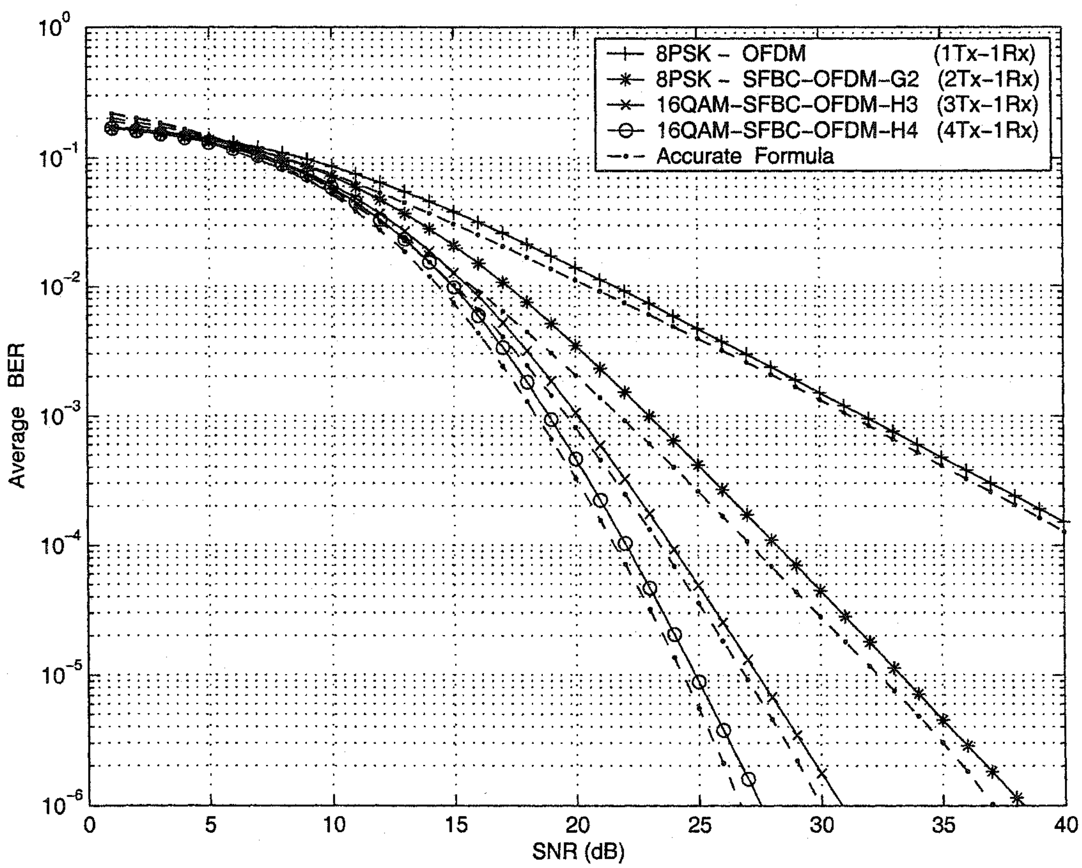

Figure 5.4: Average BER of MPSK-SFBC-OFDM (2Tx-1Rx).

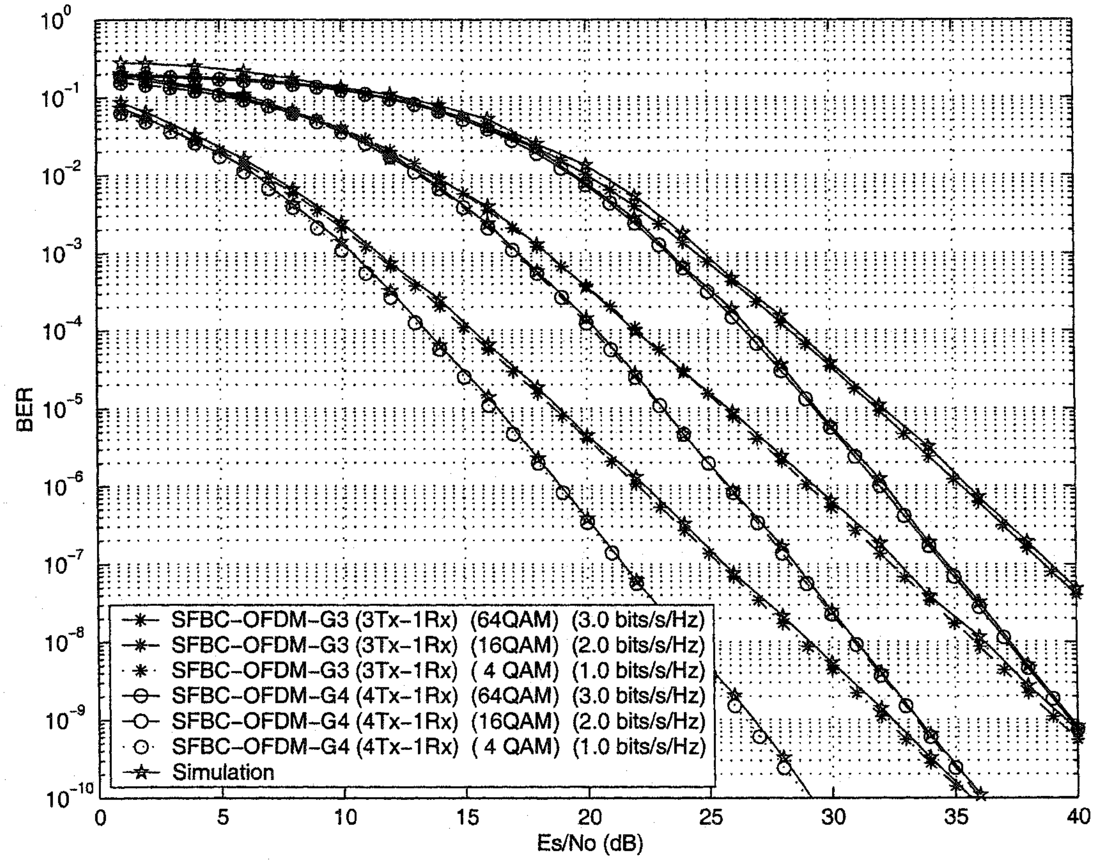

Figure 5.5: Average BER of MQAM-SFBC-OFDM (3Tx-1Rx), (4Tx-1Rx) using the codes G3 and G4.

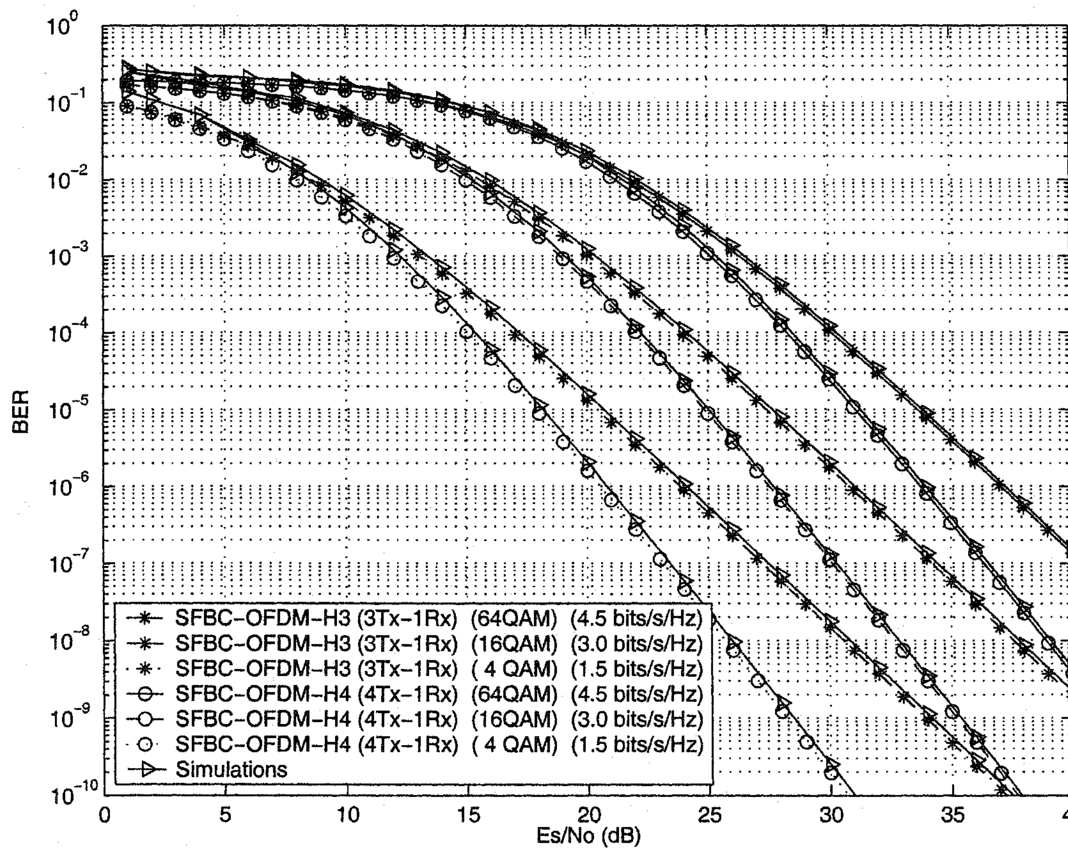

Figure 5.6: Average BER of MQAM-SFBC-OFDM (3Tx-1Rx), (4Tx-1Rx) using the codes H3 and H4.

Chapter 6

Adaptive Modulation for Space Frequency Block Coded OFDM System

The growing popularity of both MIMO and multicarrier systems such as OFDM creates the need for adaptive modulation to integrate temporal, spatial and spectral components together. In this chapter, a new scheme consisting of a combination of adaptive modulation, orthogonal frequency division multiplexing (OFDM), high order space-frequency block codes (SFBC), and antenna selection is presented. The proposed scheme exploits the benefits of space-frequency block codes, OFDM and adaptive modulation to provide high quality transmission for wireless communications over frequency selective fading channels. Spectral efficiency advantage of the proposed system is examined. It is shown that antenna selection with adaptive modulation can greatly improve the performance of the conventional SFBC-OFDM systems.

6.1 Introduction

A major limitation of wireless communication systems is the result of the channel fading. Conventional fixed-mode modulation schemes suffer from errors caused by deep fades. "An effective approach to mitigating these detrimental effects is to adaptively adjust the modulation and/or channel coding format as well as a range of other system parameters based on the near-instantaneous channel quality information perceived by the receiver, which is fed back to the transmitter with the aid of a feedback channel." This plausible principle was recognized by Hayes [59] as early as 1968. Hayes [59], proposed an adaptive modulation scheme based on power adaptation. Several years later, Goldsmith *et al.* [60]-[62] proposed variable-rate, variable-power adaptive schemes. They found that "The extra throughput achieved by the additional variable-power assisted adaptation over the constant-power, variable-rate scheme is marginal" for most types of fading channels [60]-[62].

In [63] Czylwik showed that "from the optimum power distribution only a gain in order of 1 dB is obtained. Therefore, it is recommended, not to optimize the power distribution and to use a constant power spectrum in order to save computational complexity".

Hanzo *et al.* [64]-[66] proposed an adaptive scheme based on a set of mode switching levels designed for achieving a high average BPS throughput, while keeping the target BER of the system at a desired level. They showed that adaptive modulation provides promising advantages compared to the fixed-mode schemes in terms of BER performance, spectral efficiency, etc.

The associated principles can also be applied in multicarrier systems such as DMT and OFDM systems. The adaptive modulation scheme for multicarrier systems, the so called Adaptive OFDM (AOFDM), was first proposed by Kalet [1] and was developed by Czylwik [63] and by Cioffi *et al.* [2] and some other schemes have been studied in [70]-[73]. AOFDM exploits the variation of the signal quality due to the variation of the channel in the time and frequency domains.

The goal of adaptive modulation in an OFDM system is to allocate (according to the instantaneous condition of a subchannel) an appropriate number of bits and to choose the suitable modulation mode for transmission in each subcarrier, in order to improve the system performance or to keep the overall bit error rate (BER) performance at a certain desired level.

The performance improvement offered by the adaptive modulation over non-adaptive systems is remarkable. Furthermore, other dimensions such as frequency and space may yield further gains by providing additional degrees of freedom that can be exploited by adaptive modulation.

To this end, a combination of space-time block coding (STBC) and AOFDM has been considered in [74]. In [74], it is shown that the full benefit of AOFDM and STBC cannot be exploited at the same time. In [75]-[78], we presented some new space-frequency block coded OFDM schemes in conjunction with transmitter antenna selection and adaptive modulation that can improve the overall system performance.

In this chapter, we review these schemes with high-order space-time block codes such as $\mathbf{G}_2$, $\mathbf{G}_3$, $\mathbf{G}_4$, $\mathbf{H}_3$ and $\mathbf{H}_4$ and propose a subcarrier-by-subcarrier basis antenna selection scheme for non-flat fading channels that can provide superior system performance. In this study, the term adaptive modulation refers to bit rate adaptation, and therefore, AOFDM is the variable-rate case.

The performance of the proposed system called A-SFBC-OFDM, has been analytically evaluated and has been compared with that of the non-adaptive SFBC-OFDM system. It is shown that using the antenna selection scheme at the transmitter in conjunction with adaptive modulation can improve the performance of the SFBC-OFDM system.

The rest of this chapter is organized as follows. Section 6.2 presents an overview of adaptive modulation and AOFDM systems. The proposed scheme is presented in Section 6.3. In Section 6.3, both BER and spectral efficiency of the adaptive and non-adaptive SFBC-OFDM systems are analytically evaluated. Numerical results for the performance evaluation are presented in Section 6.4. Finally, Section 6.5 concludes this chapter.

6.2 Adaptive OFDM

The allocation of power and bits to the OFDM subchannels can be uniform or non-uniform. In the uniform case, each subchannel carries the same number of bits with equal power. On the other hand, in the non-uniform case, bits and power can be allocated in a way to maximize the number of transmitted bits (spectral efficiency) or to minimize the overall probability of error.

In Section 5.2, it was shown that the bit error probability of OFDM subcarriers over frequency selective fading channels depends on the frequency response of the channel (see (5.9)). In a frequency selective fading channel, some subchannels have a deep fade while others have relatively negligible attenuation. The occurrence of bit errors is mostly caused by a set of severely faded subcarriers, while in the rest of subcarriers bit errors are observed less often. If the subcarriers having a high BER to be transmitted can be identified and excluded from transmission, the overall BER can be improved at the price of a slight loss of throughput. Since the fading only deteriorates the SNR of certain subcarriers but improves others', the potential loss of throughput due to the exclusion of faded subcarriers can be mitigated by employing higher-order modulation modes on the subcarriers having high SNR values. This is the principle idea of AOFDM. Several AOFDM schemes have been proposed in the past. In this section, we review some of the most effective schemes, including AOFDM based on *water-filling* and AOFDM based on *mode switching levels* [68].

6.2.1 Adaptation based on water-filling

The concept of *water-filling* can be used for power allocation and then for bit allocation. It says that the optimal power allocation maximizes the channel capacity and it is given by:

$$P(f) = \begin{cases} K - \frac{\Phi_n(f)}{|H(f)|^2} & f \in W \\ 0 & f \notin W \end{cases} \tag{6.1}$$

where $P(f)$ is the signal power density function, $H(f)$ is the channel frequency response with bandwidth W, $\Phi_n(f)$ is the power spectral density function of the AWGN, and K is a constant, determining the total power to be allocated [108]. From (6.1), more power is allocated to the subchannels with greater channel magnitude or lower noise power. Several bit and power allocation algorithms have been studied in [69]-[70].

A sample output of the bit and power allocation algorithm is shown in Figure 6.1 for a frequency selective fading channel. As can be observed, no bit is allocated to the subchannels corresponding to a deep fade, while more bits are allocated to the subchannels with strong channel gain. Also, it can be seen that subchannel power varies as a function of channel gain and bit allocation.

The principle idea behind adaptive modulation is to adapt the transmission parameters to take advantage of prevailing channel conditions. It aims to exploit the variation of the wireless channel by dynamically adjusting certain transmission parameters to changing environmental conditions.

6.2.2 Adaptation based on mode switching level

The general idea of adaptive modulation is to choose a set of suitable modulation parameters based on the channel state information (CSI) known at the transmitter. The signal-to-noise ratio (SNR) can be considered as a proper metric that can be obtained from CSI. In this case, a possible algorithm is as follows:

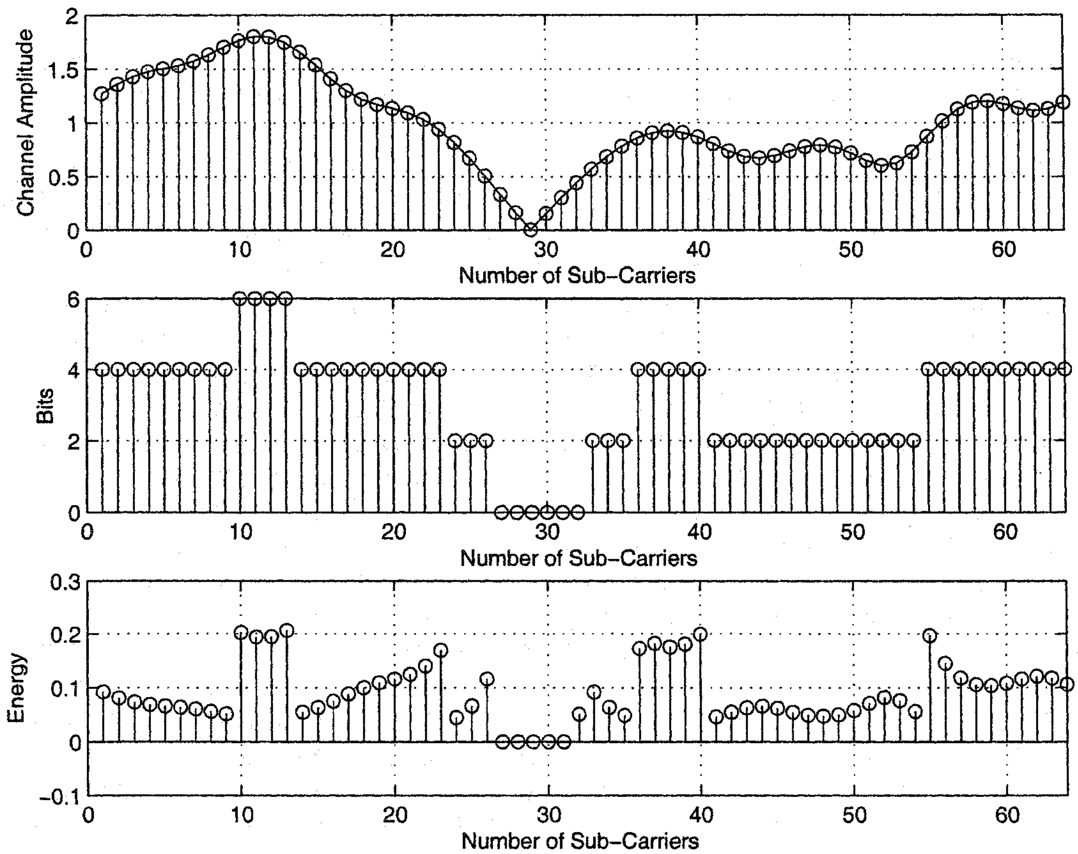

Figure 6.1: An example of bit and power allocation.

1-Find the SNR.

2-Find the BER associated with the SNR for each mode. Then, determine the switching level, which is the lowest required SNR for a given mode corresponding to a given target-BER.

3-Based on a given target-BER, select the modulation mode. This provides the largest throughput while maintaining the target-BER.

For example, consider Figure 6.2, that represents a set of BER curves for BPSK, QPSK, 16-QAM, and 64-QAM constellations. The set of adaptation/ switching thresholds can be obtained from the closed-form BER expression as a function of SNR or simply by reading the SNR points corresponding to a target-BER. For example, if the target-BER is 10^{-4} , the thresholds are 8.4, 11.4, 18.2, and 24.3 dB,

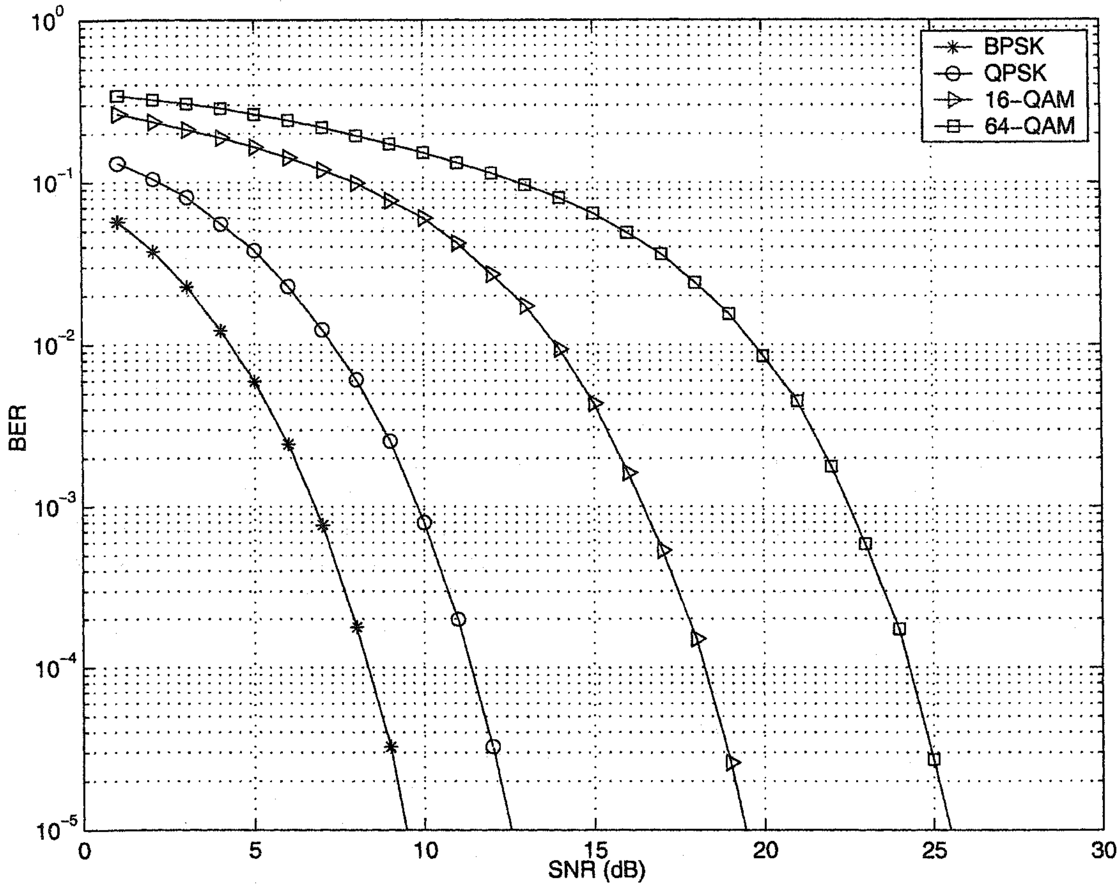

Figure 6.2: BER for various modulation schemes.

respectively.

The goal of an adaptive modulation algorithm is to ensure that the most efficient mode is always used over varying channel conditions based on a mode selection criterion. In contrast, systems with fixed-mode modulation are designed for the worst-case channel conditions, resulting in insufficient utilization of the full channel capacity.

The capacity improvement offered by adaptive modulation over non-adaptive systems can be remarkable. In Figure 6.3, we present the spectral efficiency performance (Bits/S/Hz) vs. SNR (dB) for four different uncoded modulation levels referred to as BPSK, QPSK, 16-QAM, and 64-QAM. The spectral efficiency was

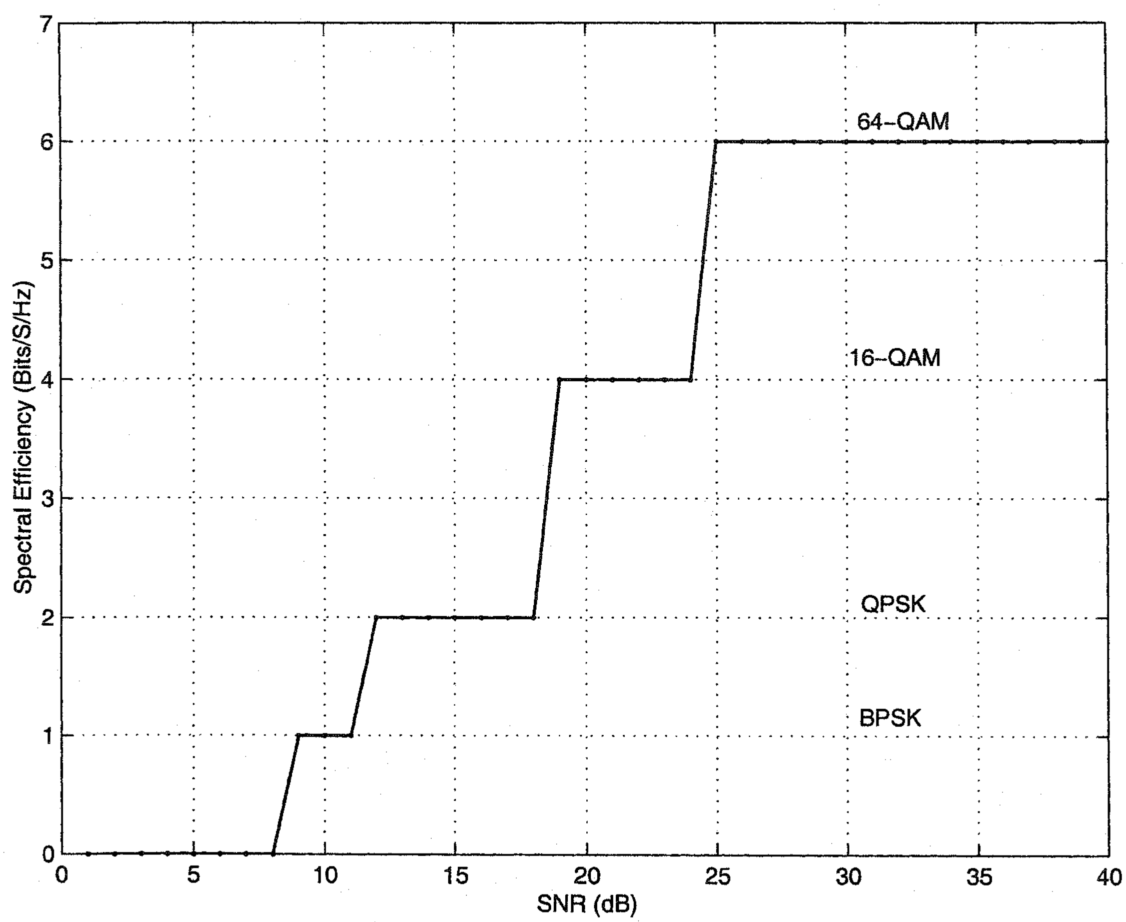

Figure 6.3: Spectral efficiency for various modulation schemes.

obtained for each modulation scheme by taking into account maintaining the target-BER of 10^{-4}. It can bee seen that each modulation is optimal for use in different quality regions, and adaptive modulation selects the mode with the highest spectral efficiency for each link.

6.2.3 Adaptive OFDM

In this section, we determine adaptive bit allocation for maximizing spectral efficiency. We start with bit allocation for the conventional OFDM system. An MQAM is employed for each subchannel and $\beta[n,k]$ bits/symbol is assigned for the k-th subchannel in the n-th block, where $M = 2^{\beta[n,k]}$. Also the negligible degradation due to the cyclic prefix in the OFDM has not been considered.

The expression for the instantaneous BER of the k-th subchannel in the n-th block of the OFDM (square MQAM with Gray bit mapping on each subcarrier) over a frequency selective fading channel can be written as:

$$BER[n,k] = \frac{2}{\beta[n,k]}\left(1 - \frac{1}{\sqrt{2^{\beta[n,k]}}}\right) erfc\left\{\sqrt{\frac{1.5\,\gamma_s\,|H[n,k]|^2}{2^{\beta[n,k]}-1}}\right\} \tag{6.2}$$

where the $erfc(x)$ function can be represented by.

$$erfc(x) = \int_x^{\infty} \exp(-t^2)\,dt \tag{6.3}$$

According to [72]-[73], (6.2) can be approximated as:

$$BER[n,k] = 0.2\exp\left\{-\frac{1.6\,\gamma_s\,|H[n,k]|^2}{2^{\beta[n,k]}-1}\right\} \tag{6.4}$$

where $\gamma_s = \frac{E_s}{N_0}$, E_s is the symbol energy at the transmitter and $\frac{N_0}{2}$ is the variance of the real/imaginary part of the AWGN. $H[n,k]$ is the frequency response of the fading channel.

By inverting (6.4), the maximum instantaneous data rate $\beta[n,k]$ that can be transmitted under a target-BER (BER_t) constraint for a given instantaneous SNR can be represented as:

$$\beta[n,k] = \log_2\left\{1 - \frac{1.6\,\gamma_s\,|H[n,k]|^2}{\ln\left(\frac{BER_t}{0.2}\right)}\right\} \tag{6.5}$$

The average spectral efficiency (number of bits per second per Hz) can be written as:

$$R = E_{H[n,k]}\left\{\beta[n,k]\right\} \tag{6.6}$$

6.2.4 Non-adaptive OFDM

In order to compare the spectral efficiency of AOFDM with that of non-adaptive OFDM, in this section, we evaluate the spectral efficiency of the non-adaptive OFDM systems. In the case of non-adaptive modulation for OFDM, the same number of bits is allocated to each subband, i.e., $\beta[n,k] = \beta$. The average BER can be written as:

$$\overline{BER} = E_{H[n,k]}\left\{BER[n,k]\right\} \tag{6.7}$$

$$\overline{BER} = \int_0^\infty BER[n,k]\, p(\gamma)\, d\gamma \tag{6.8}$$

where $p(\gamma)$ is the probability density function of $\gamma = \gamma_s\,|H[n,k]|^2$. Since $|H[n,k]|$ is Rayleigh-distributed, $|H[n,k]|^2$ has a chi-square probability distribution with two degrees of freedom. Consequently, γ is also chi-square-distributed as follows.

$$p(\gamma) = \frac{1}{\overline{\gamma}} \exp\left(-\frac{\gamma}{\overline{\gamma}}\right) \qquad \gamma \geq 0 \tag{6.9}$$

where $\overline{\gamma}$ is defined as

$$\overline{\gamma} = \gamma_s\, E_{H[n,k]}\left\{|H[n,k]|^2\right\}$$

The term $E_{H[n,k]}\left\{|H[n,k]|^2\right\}$ is simply the average of $|H[n,k]|^2$. Therefore, by substituting (6.4) and (6.9) into (6.8) we can obtain [72]

$$\overline{BER} = \frac{0.2}{1 + \frac{1.6\,\gamma_s}{2^\beta - 1}} \tag{6.10}$$

Then we can invert (6.10) to express β as a function of γ_s and the $\overline{BER_t}$ as:

$$\beta = \log_2\left\{1 + \frac{1.6\,\gamma_s}{\left(\frac{0.2}{\overline{BER_t}} - 1\right)}\right\} \tag{6.11}$$

The average spectral efficiency, in this case is equal to β.

6.3 Adaptive Modulation for SFBC-OFDM

As stated earlier, adaptive modulation is a well known and powerful technique for increasing the spectral efficiency and improving the system performance of the OFDM systems. The performance improvement offered by adaptive modulation over non-adaptive systems is remarkable. Furthermore, other dimensions such as frequency and space may yield further gains by providing additional degrees of freedom that can be exploited by adaptive modulation.

Antenna selection has been considered and studied as an efficient technique for improving performance and for reducing the complexity and cost due to the RF/chains. Recently, there has been a growing interest in applying the antenna selection technique to multiple input/multiple output (MIMO) systems [87]-[94]. In [93],[94] an interesting antenna selection technique is presented for the application of MIMO systems over flat fading channels, when space-time coding is used.

In this chapter, we propose some new space-frequency block coded OFDM schemes in conjunction with transmitter antenna selection and adaptive modulation that can improve the overall system performance. We review these schemes with high-order space-time block codes such as $\mathbf{G}_2$, $\mathbf{G}_3$, $\mathbf{G}_4$, $\mathbf{H}_3$ and $\mathbf{H}_4$.

The performance of the proposed system called A-SFBC-OFDM, has been analytically evaluated and has been compared with non-adaptive SFBC-OFDM.

6.3.1 System Model

Figure 6.4 shows a block diagram of the proposed system. Assuming that the channel state information is available at the transmitter, antenna selection maximizes the signal to noise ratio and therefore, minimizes the probability of error.

The basic concept of antenna selection is to transmit each subband using the antenna which has the smallest attenuation for that subband. Given that K_T transmitter antennas are available, we select M_T out of K_T antennas ($M_T \leq K_T$),

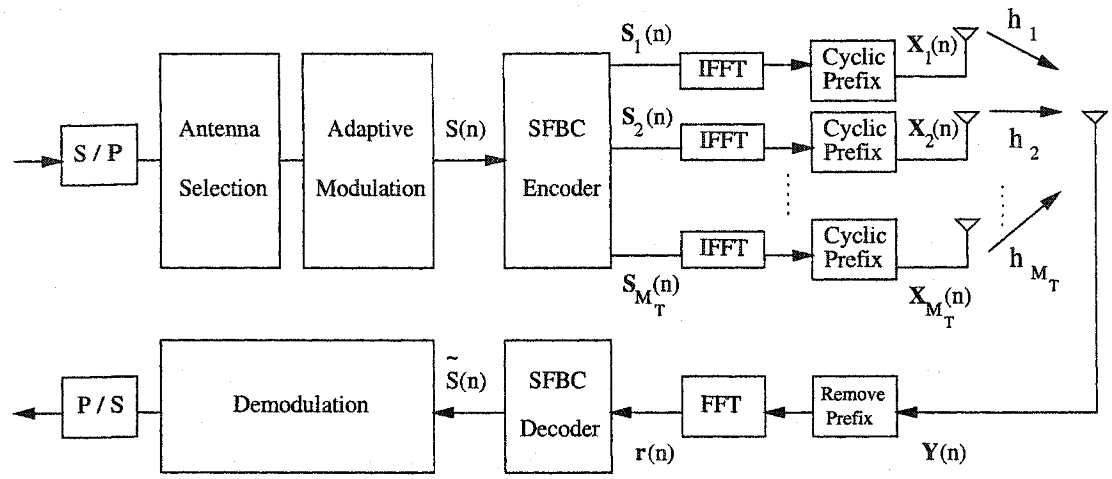

Figure 6.4: Block diagram of the proposed system.

whose subband amplitudes are larger than the rest.

Considering a multiple input/single output (MISO) system with K_T transmitter antennas and one receive antenna, the k-th subchannel response in the n-th block, i.e., $\mathcal{H}[n,k]$ is a $(K_T \times 1)$ vector, given by:

$$\mathcal{H}[n,k] = [H_1[n,k] \quad H_2[n,k] \quad \cdots \quad H_{K_T}[n,k]]$$

In order to minimize the average BER, we can maximize the average SNR that is equivalent to selecting the M_T out of K_T columns of $\mathcal{H}[n,k]$. Therefore, the optimal transmission is to transmit on the transmitter antennas corresponding to the M_T columns with the highest Frobenius norms [93],[94]. Therefore, the equivalent selected subchannel vector will be a $(M_T \times 1)$ vector, given by:

$$\mathcal{H}^{'}[n,k] = \left[H_1^{'}[n,k] \quad H_2^{'}[n,k] \quad \cdots \quad H_{M_T}^{'}[n,k]\right]$$

Note that $H_i^{'}[n,k]$ is equal to some $H_j^{'}[n,k]$, for each $i \in \{1,2,...,M_T\}$ and for some $j \in \{1,2,...,K_T\}$.

Subchannels assigned to the i-th antenna are represented as

$$\left(H_i^{'}[n,0],\ H_i^{'}[n,1], \cdots,\ H_i^{'}[n,N_s-1]\right) \qquad for \qquad i = 1,2,\cdots,M_T$$

Assuming an OFDM with N subcarriers, N_s is the number of subbands. N_s has been chosen to be N/T, i.e, each subband includes T adjacent subchannels (T is the symbol period of the SFBC system).

Based on the characteristics of the selected subbands, i.e., $H_i'[n,k]$, the number of allocated bits will be calculated. This bit allocation can be done using water-filling. Here, we consider a simpler method of adaptive bit allocation where, an average BER target is set. The size of the constellation for each subband is determined based on the target BER. Therefore, a signal vector:

$$S(n) = \{s[n,0],\, s[n,1], s[n,2], ...,\, s[n+N_t-1]\} \tag{6.12}$$

is provided as the input for the SFBC system, where N_t is equal to N multiplied by the code rate of SFBC system (R_c).

A space-time block code is defined by a $(T \times M_T)$ transmission matrix $\mathbf{G}$ given by

$$\mathbf{G} = \begin{pmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,M_T} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,M_T} \\ \vdots & \vdots & \cdots & \vdots \\ g_{T,1} & g_{T,2} & \cdots & g_{T,M_T} \end{pmatrix} \tag{6.13}$$

where each element $g_{i,j}$ is a proper linear combination of a subset of elements of $S(n)$ and their conjugates, as $\mathbf{G}$ is an orthogonal design.

In order to utilize the space-frequency diversity, the input blocks for OFDM at each transmit antenna should be of length N. SFBC provides M_T blocks: $\mathbf{S}_1(n)$, $\mathbf{S}_2(n), \cdots, \mathbf{S}_{M_T}(n)$ of length N, each consisting of $\frac{N}{T}$ sub-blocks, i.e.,

$$\mathbf{S}_i(n) = \begin{pmatrix} \mathbf{s}_i[n,0] & \mathbf{s}_i[n,1] & ... & \mathbf{s}_i[n, \frac{N}{T}-1] \end{pmatrix}^T \qquad for \qquad i = 1, 2, \cdots, M_T$$

where superscript T in $(.)^T$ denotes the transpose operation.

Then OFDM modulators generate blocks:

$$\mathbf{X}_1(n),\ \mathbf{X}_2(n), \cdots, \mathbf{X}_{M_T}(n)$$

that are transmitted by the first, second, ..., and M_T-th transmit antennas simultaneously. In order to avoid ISI, the guard time interval is chosen to be longer than the largest delay spread of the multipath channel. Then received signal will be the convolution of the channel and the transmitted signal. Assuming that the channel is static during an OFDM block, at the receiver side after removing the cyclic prefix, the FFT output as the demodulated received signal can be expressed as:

$$\mathbf{r}(n) = \sum_{i=1}^{M_T} \mathbf{H}_i'(n)\, \mathbf{S}_i(n) + \mathbf{W}(n) \tag{6.14}$$

where $\mathbf{r}(n) = (r[n,0], \cdots, r[n, N-1])^T$. $\mathbf{W}(n) = (W[n,0], \cdots, W[n, N-1])^T$ denotes the AWGN, and $\mathbf{H}_i'(n)$ represents a diagonal matrix whose elements $(H_i'[n,j],\ i = 1, 2, \cdots, M_T,\ j = 0, 1, \cdots, N-1)$ are the gains of the selected subchannels. Knowing the channel information at the receiver, the Maximum Likelihood (ML) decoding can be used for SFBC decoding of the received signal. At the end, the elements of block $\{\widetilde{s}[n,j]\}_{j=0}^{N_t-1}$ are demodulated to extract the data. We have illustrated the encoding and decoding of the SFBC-OFDM system with some examples in Chapter 3.

6.3.2 Adaptive Modulation for SFBC-OFDM

In this section, we determine adaptive bit allocation for maximizing spectral efficiency of SFBC-OFDM employing antenna selection (selecting M_T out of K_T transmitter antennas). Assume that MQAM is employed for each subchannel and $\beta[n,k]$ bits/symbol is assigned for the kth subchannel in the nth block, and $M = 2^{\beta[n,k]}$. The negligible degradation due to the cyclic prefix in OFDM is not being considered.

6.3.2.1 Adaptive SFBC-OFDM with subband antenna selection

Consider an SFBC system employing M_T transmitter antennas and one receiver antenna. The decoder minimizes the decision metric

$$\left|\tilde{s}[n,k] - s[n,k]\right|^2 \tag{6.15}$$

Similar to (3.14), we can show that

$$\tilde{s}[n,k] = \frac{1}{M_T R_c} \sum_{i=1}^{M_T} \left|H_i'[n,k]\right|^2 s[n,k] + \eta[n,k] \tag{6.16}$$

where $H_i'[n,k]$ is the selected subchannel associated with i-th transmitter antenna, R_c is the code rate, and, $\eta[n,k]$ is the noise component as stated in (3.14).

From (6.16), we can express instantaneous SNR as:

$$\gamma = \frac{1}{M_T R_c} \sum_{i=1}^{M_T} \left|H_i'[n,k]\right|^2 \gamma_s \tag{6.17}$$

Similar to the conventional OFDM (6.2), we can express the instantaneous BER of each subcarrier of MQAM-SFBC-OFDM over a frequency selective fading channel as:

$$BER[n,k] = \frac{2}{\beta[n,k]}\left(1 - \frac{1}{\sqrt{2^{\beta[n,k]}}}\right) \times erfc\left\{\sqrt{\frac{1.5\,\gamma_s \sum_{i=1}^{M_T} \left|H_i'[n,k]\right|^2}{M_T R_c \left(2^{\beta[n,k]}-1\right)}}\right\} \tag{6.18}$$

This can be approximated as:

$$BER[n,k] = 0.2 \exp\left\{-\frac{1.6\,\gamma_s \sum_{i=1}^{M_T} \left|H_i'[n,k]\right|^2}{M_T R_c \left(2^{\beta[n,k]}-1\right)}\right\} \tag{6.19}$$

By inverting (6.19), the suitable modulation scheme and the corresponding number of bits can be calculated from:

$$\beta[n,k] = \log_2 \left\{ 1 - \frac{1.6\,\gamma_s \left(\sum_{i=1}^{M_T} \left| H_i'[n,k] \right|^2 \right)}{M_T\, R_c\, \ln\left(\frac{BER_t}{0.2}\right)} \right\} \tag{6.20}$$

The average spectral efficiency can be written as:

$$R = E\left\{ \beta[n,k] \right\} \tag{6.21}$$

6.3.2.2 Non-adaptive SFBC-OFDM (without antenna selection)

In the case of non-adaptive SFBC-OFDM and without antenna selection the average BER can be written as:

$$\overline{BER} = E_{H_1[n,k],\ldots,H_{M_T}[n,k]} \left\{ BER[n,k] \right\} \tag{6.22}$$

Since $|H_i[n,k]|$, $i = (1, \ldots, M_T)$ is i.i.d Rayleigh-distributed, $|H_i[n,k]|^2$ has a chi-square probability distribution with two degrees of freedom. Consequently, $\gamma_i = \gamma_s \left| H_i[n,k] \right|^2$ is also chi-square-distributed with the probability density function as follows.

$$p(\gamma_i) = \frac{1}{\overline{\gamma_i}} \exp\left(-\frac{\gamma_i}{\overline{\gamma_i}} \right) \qquad \gamma_i \geq 0 \tag{6.23}$$

Since,

$$\overline{BER} = \int_0^\infty \ldots \int_0^\infty BER[n,k]\, p(\gamma_1) \ldots p(\gamma_{M_T})\, d\gamma_1 \ldots d\gamma_{M_T} \tag{6.24}$$

Substituting (6.19) and (6.23) into (6.24) we obtain:

$$\overline{BER} = \frac{0.2}{\left(1 + \frac{1.6\,\gamma_s}{(M_T\, R_c)\,(2^\beta - 1)} \right)^{M_T}} \tag{6.25}$$

We can invert (6.25) to express β as a function of γ_s and the target $\overline{BER}$:

$$\beta = \log_2 \left\{ 1 + \frac{1.6\,\gamma_s}{R_c\,M_T \left(\left(\frac{0.2}{\overline{BER}_t} \right)^{1/M_T} - 1 \right)} \right\} \tag{6.26}$$

6.4 Simulation Results

The performance of the proposed system is evaluated by computer simulation over a frequency selective fading channel. The considered fading channel is a multipath fading channel with coherence bandwidth smaller than the total bandwidth of the multicarrier system and thus seen as frequency selective fading. The fading process is assumed to be stationary and slowly varying compared to the symbol duration of the multicarrier signal, such that it is approximately constant during one OFDM block length. The fading process impulse response at the antenna i ($i = 1, 2, ..., M_T$) can be expressed as [108]

$$h_i(t) = \sum_{m=0}^{L-1} \alpha_{m,i}(t)\,\delta(t - \tau_m(t)) \tag{6.27}$$

where the tap weight $\alpha_{m,i}(t)$ is a complex Gaussian random process with zero mean and variance $\frac{1}{L}$ (equal power) and $\tau_m(t)$ is the time delay of the m-th path and L is the total number of resolvable paths ($L = 4$). With this model we have assumed that the path delays, $\tau_m(t)$, are multiples of the symbol duration T_s. The OFDM system includes $N = 512$ subcarriers and a cyclic prefix which is longer than the channel delay. It is assumed that the channel state information is available at the receiver and transmitter. Finally, SNR is defined as $SNR = \gamma_s$.

6.4.1 AOFDM System

The average spectral efficiencies of AOFDM and non-adaptive OFDM were expressed in (6.6) and (6.11). The average spectral efficiencies of non-adaptive OFDM and adaptive OFDM are compared in Figure 6.5 for different target BERs. It can be seen that adaptive OFDM can greatly improve the performance of OFDM. For example, at a target-BER=10^{-3} and a spectral efficiency of 6 bits/sec/Hz about 14 dB gain can be obtained, and at a target-BER=10^{-5} and a spectral efficiency of 4 bits/sec/Hz about 30 dB gain can be obtained.

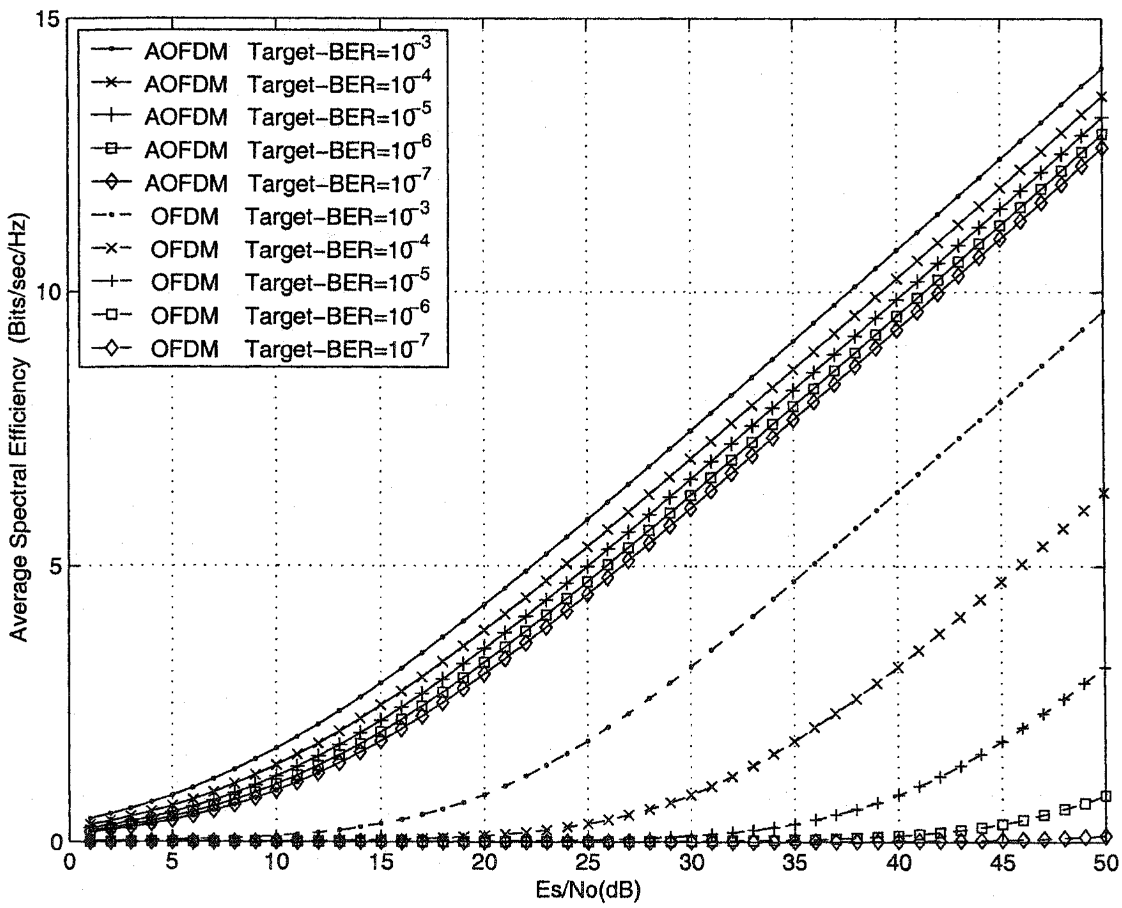

Figure 6.5: Average spectral efficiency of non-adaptive OFDM and AOFDM.

6.4.2 A-SFBC-OFDM (2Tx-1Rx) system using the code $\mathbf{G}_2$

In the case of A-SFBC-OFDM with two transmitter antennas and one receiver antenna (2Tx-1Rx), using the code $\mathbf{G}_2$ (code rate $R_c = 1$), (6.19) and (6.20) can be expressed as:

$$BER[n,k] = 0.2\exp\left\{ -\frac{0.8\,\gamma_s\left(\left|H_1'[n,k]\right|^2 + \left|H_2'[n,k]\right|^2\right)}{2^{\beta[n,k]}-1} \right\} \tag{6.28}$$

$$\beta[n,k] = \log_2\left\{ 1 - \frac{0.8\,\gamma_s\left(\left|H_1'[n,k]\right|^2 + \left|H_1'[n,k]\right|^2\right)}{\ln\left(\frac{BER_t}{0.2}\right)} \right\} \tag{6.29}$$

For non-adaptive MQAM-SFBC-OFDM (2Tx-1Rx), (6.25) and (6.26) can be written as:

$$\overline{BER} = \frac{0.2}{\left(1 + \frac{0.8\,\gamma_s}{(2^{\beta}-1)}\right)^2} \tag{6.30}$$

and

$$\beta = \log_2\left\{ 1 + \frac{0.8\,\gamma_s}{\left(\sqrt{\frac{0.2}{BER_t}} - 1\right)} \right\} \tag{6.31}$$

The average BER performance for MQAM-SFBC-OFDM is shown in Chapter 5. It was shown that the results from the closed-form formula are very close to the simulation results. Also, the benefits of using antenna selection is described in Chapter 7.

Figure 6.6 compares the spectral efficiency of the SFBC-OFDM (2Tx-1Rx) and A-SFBC-OFDM (2Tx-1Rx). The results indicate that A-SFBC-OFDM (2Tx-1Rx) is better than SFBC-OFDM in terms of spectral efficiency for the given target BER. For example, at a target-BER of 10^{-6} and a spectral efficiency of 4 bits/sec/Hz about 18.5 dB, 20 dB, and 22 dB gain can be obtained when K_T is 2, 4 and 8, respectively.

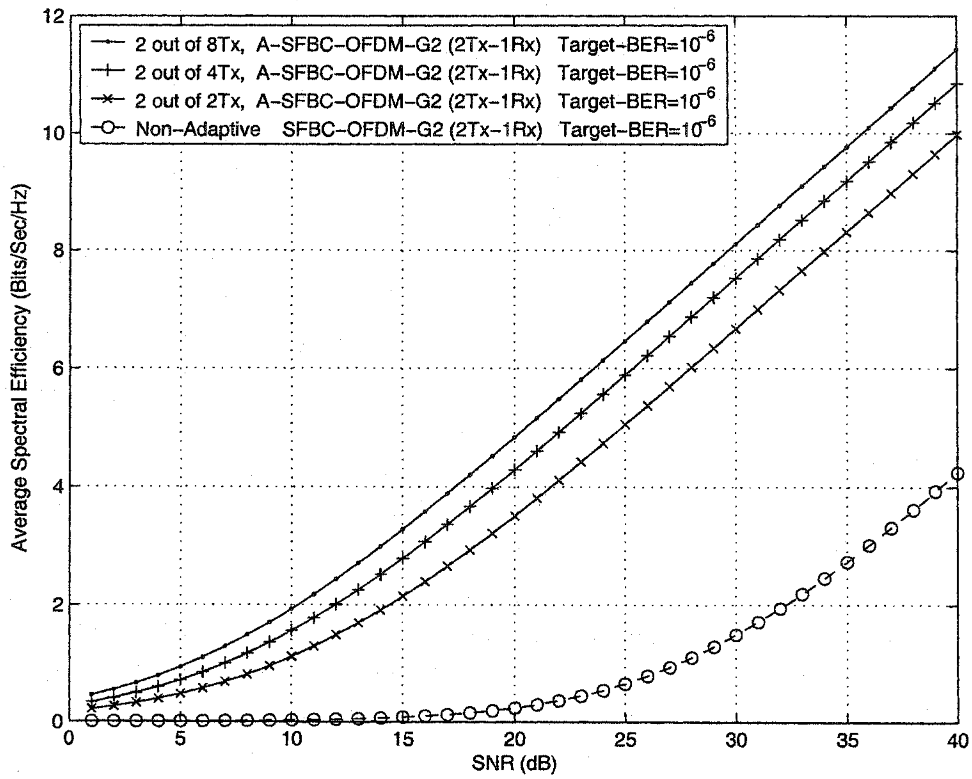

Figure 6.6: Average spectral efficiency of non-adaptive SFBC-OFDM (2Tx-1Rx) and A-SFBC-OFDM (2Tx-1Rx).

6.4.3 A-SFBC-OFDM (3Tx-1Rx) system using the code $\mathbf{G}_3$

In the case of A-SFBC-OFDM (3Tx-1Rx) ($M_T = 3$) using the code $\mathbf{G}_3$ ($R_c = 0.5$), (6.19) and (6.20) can be expressed as:

$$BER[n,k] = 0.2 \exp\left\{ -\frac{1.6\,\gamma_s \sum_{i=1}^{3} \left|H_i'[n,k]\right|^2)}{1.5\left(2^{\beta[n,k]}-1\right)} \right\} \tag{6.32}$$

$$\beta[n,k] = \log_2\left\{ 1 - \frac{1.6\,\gamma_s \left(\sum_{i=1}^{3}\left|H_i'[n,k]\right|^2\right)}{1.5\,\ln\left(\frac{BER_t}{0.2}\right)} \right\} \tag{6.33}$$

For the non-adaptive MQAM-SFBC-OFDM (3Tx-1Rx) using the code $\mathbf{G}_3$, (6.25) and (6.26) can be written as

$$\overline{BER} = \frac{0.2}{\left(1 + \frac{1.6\,\gamma_s}{1.5\,(2^{\beta}-1)}\right)^3} \tag{6.34}$$

and

$$\beta = \log_2 \left\{ 1 + \frac{1.6\,\gamma_s}{1.5\left(\left(\frac{0.2}{\overline{BER_t}}\right)^{1/3} - 1\right)} \right\} \tag{6.35}$$

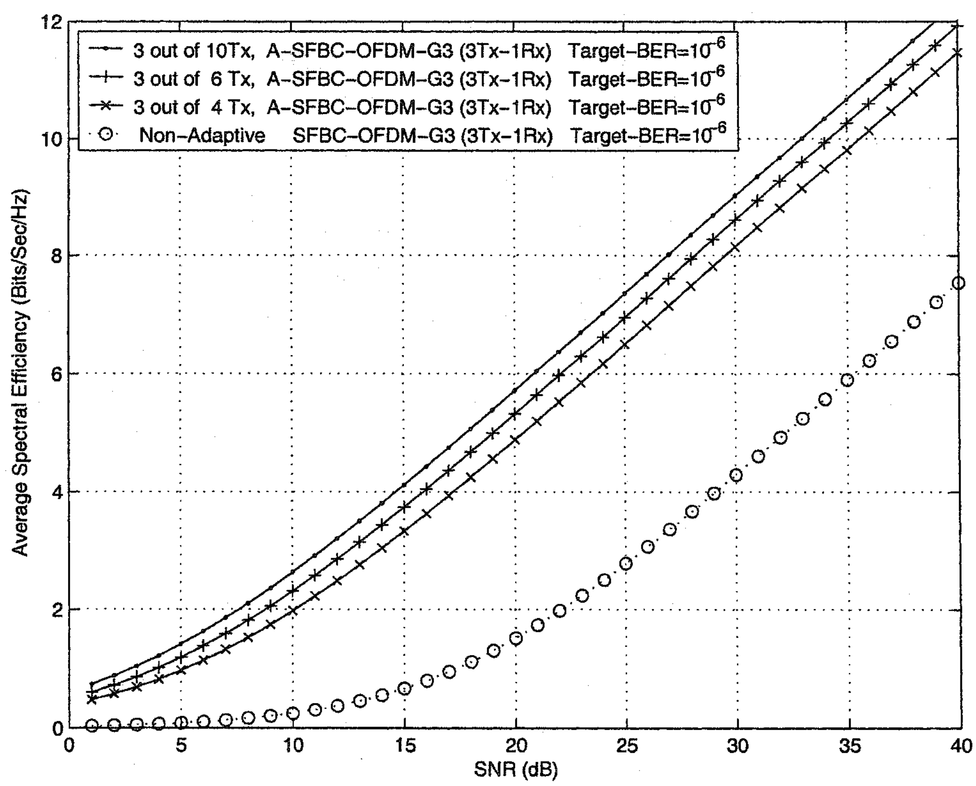

Figure 6.7: Average spectral efficiency of non-adaptive SFBC-OFDM (3Tx-1Rx) and A-SFBC-OFDM (3Tx-1Rx) using the code G3.

Figure 6.7 compares the spectral efficiency of the SFBC-OFDM with that of A-SFBC-OFDM, both employing code of $\mathbf{G}_3$. As shown, A-SFBC-OFDM is better than SFBC-OFDM in terms of spectral efficiency for the given target-BER. For example, at a spectral efficiency of 6 bits/sec/Hz, gains of 13 dB, 14 dB and 16 dB can be obtained when K_T is 4, 6 and 10, respectively.

6.4.4 A-SFBC-OFDM (4Tx-1Rx) system using the code $\mathbf{G}_4$

In the case of A-SFBC-OFDM (4Tx-1Rx) ($M_T = 4$) using the code $\mathbf{G}_4$ ($R_c = 0.5$), (6.19) and (6.20) can be expressed as:

$$BER[n,k] = 0.2 \exp\left\{ -\frac{1.6\,\gamma_s \sum_{i=1}^{4} \left|H_i'[n,k]\right|^2)}{2\left(2^{\beta[n,k]}-1\right)} \right\} \tag{6.36}$$

$$\beta[n,k] = \log_2 \left\{ 1 - \frac{1.6\,\gamma_s \left(\sum_{i=1}^{4} \left|H_i'[n,k]\right|^2\right)}{2\,\ln\left(\frac{BER_t}{0.2}\right)} \right\} \tag{6.37}$$

For non-adaptive MQAM-SFBC-OFDM (4Tx-1Rx) using the code $\mathbf{G}_4$, (6.25) and (6.26) can be written as

$$\overline{BER} = \frac{0.2}{\left(1 + \frac{1.6\,\gamma_s}{2\,(2^{\beta}-1)}\right)^4} \tag{6.38}$$

and

$$\beta = \log_2 \left\{ 1 + \frac{1.6\,\gamma_s}{2\left(\left(\frac{0.2}{\overline{BER}_t}\right)^{1/4} - 1\right)} \right\} \tag{6.39}$$

Figure 6.8 compares, the spectral efficiency of the SFBC-OFDM with that of A-SFBC-OFDM, both employing the code $\mathbf{G}_4$. As shown, A-SFBC-OFDM is better than SFBC-OFDM in terms of spectral efficiency for the given target BER. For example, at a spectral efficiency of 6 bits/sec/Hz, gains of about 8 dB, 10.5 dB and 11 dB can be obtained when K_T is 4, 6 and 10, respectively.

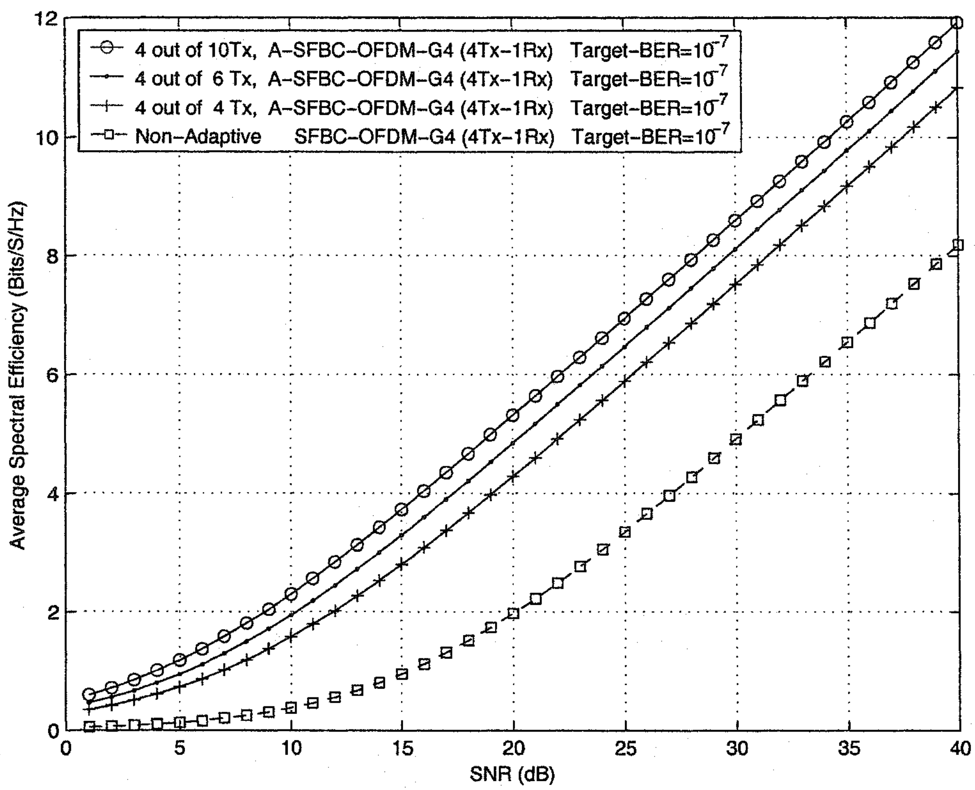

Figure 6.8: Average spectral efficiency of non-adaptive SFBC-OFDM (4Tx-1Rx) and A-SFBC-OFDM (4Tx-1Rx) using the code G4.

6.4.5 A-SFBC-OFDM (3Tx-1Rx) system using the code $\mathbf{H}_3$

In the case of A-SFBC-OFDM (3Tx-1Rx) ($M_T = 3$) using the code $\mathbf{H}_3$ ($R_c = 3/4$), (6.19) and (6.20) can be expressed as

$$BER[n,k] = 0.2\exp\left\{-\frac{1.6\,\gamma_s \sum_{i=1}^{3}\left|H_i'[n,k]\right|^2)}{2.25\left(2^{\beta[n,k]}-1\right)}\right\} \tag{6.40}$$

$$\beta[n,k] = \log_2\left\{1 - \frac{1.6\,\gamma_s\left(\sum_{i=1}^{3}\left|H_i'[n,k]\right|^2\right)}{2.25\,\ln\left(\frac{BER_t}{0.2}\right)}\right\} \tag{6.41}$$

For the non-adaptive MQAM-SFBC-OFDM (3Tx-1Rx) using the code $\mathbf{H}_3$, (6.25) and (6.26) can be written as

$$\overline{BER} = \frac{0.2}{\left(1 + \frac{1.6\,\gamma_s}{2.25\,(2^{\beta}-1)}\right)^3} \tag{6.42}$$

and

$$\beta = \log_2 \left\{ 1 + \frac{1.6\,\gamma_s}{2.25\left(\left(\frac{0.2}{\overline{BER}_t}\right)^{1/3} - 1\right)} \right\} \tag{6.43}$$

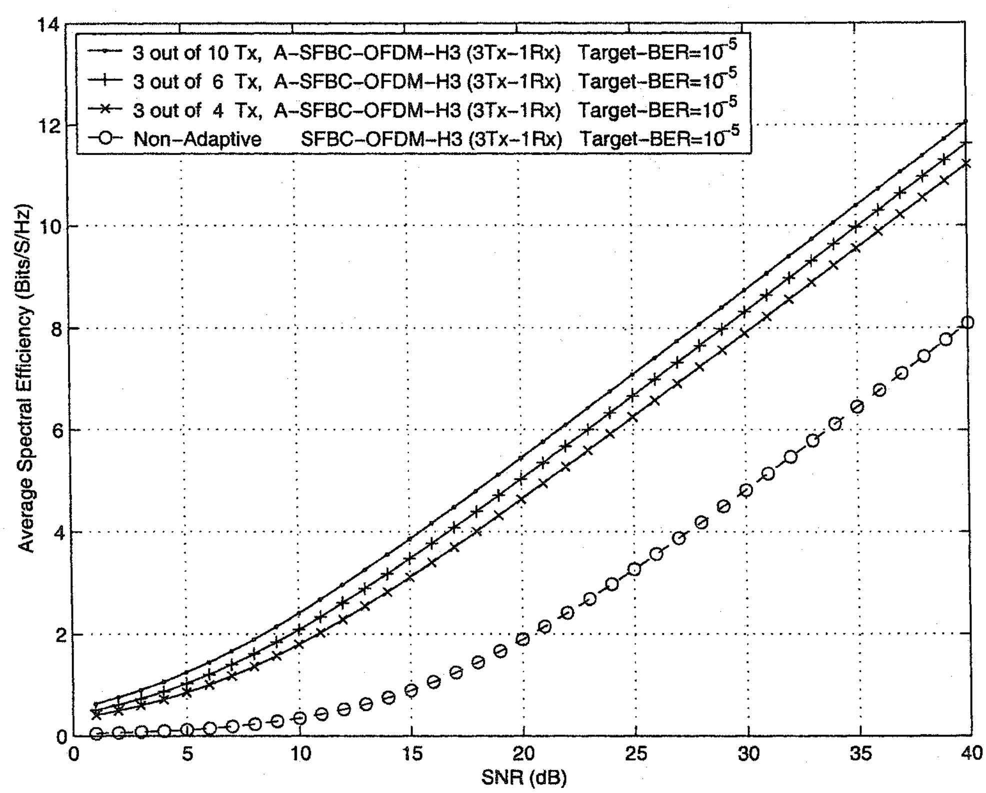

Figure 6.9: Average spectral efficiency of non-adaptive SFBC-OFDM (3Tx-1Rx) and A-SFBC-OFDM (3Tx-1Rx) using the code H3.

Figure 6.9 compares the spectral efficiency of the SFBC-OFDM with that of A-SFBC-OFDM, both employing the code $\mathbf{H}_3$. As shown, A-SFBC-OFDM is better than SFBC-OFDM in terms of spectral efficiency for the given target BER. For example, at a spectral efficiency of 6 bits/sec/Hz, gains of 9.5 dB, 10.5 dB and 12 dB can be obtained when K_T is 4, 6 and 10, respectively.

6.4.6 A-SFBC-OFDM (4Tx-1Rx) system using the code $\mathbf{H}_4$

In the case of A-SFBC-OFDM (4Tx-1Rx) ($M_T = 4$) using the code $\mathbf{H}_4$ ($R_c = 3/4$), (6.19) and (6.20) can be expressed as

$$BER[n,k] = 0.2 \exp\left\{-\frac{1.6\,\gamma_s \sum_{i=1}^{4}\left|H_i'[n,k]\right|^2)}{3\left(2^{\beta[n,k]}-1\right)}\right\} \tag{6.44}$$

$$\beta[n,k] = \log_2\left\{1 - \frac{1.6\,\gamma_s\left(\sum_{i=1}^{4}\left|H_i'[n,k]\right|^2\right)}{3\,\ln\left(\frac{BER_t}{0.2}\right)}\right\} \tag{6.45}$$

For the non-adaptive MQAM-SFBC-OFDM (4Tx-1Rx) using the code $\mathbf{H}_4$, (6.25) and (6.26) can be written as:

$$\overline{BER} = \frac{0.2}{\left(1 + \frac{1.6\,\gamma_s}{3\,(2^{\beta}-1)}\right)^4} \tag{6.46}$$

and

$$\beta = \log_2\left\{1 + \frac{1.6\,\gamma_s}{3\left(\left(\frac{0.2}{BER_t}\right)^{1/4} - 1\right)}\right\} \tag{6.47}$$

Figure 6.10 compares, the spectral efficiency of the SFBC-OFDM with that of A-SFBC-OFDM, both employing the code $\mathbf{H}_4$. As shown, A-SFBC-OFDM is better than SFBC-OFDM in terms of spectral efficiency for the given target BER. For example, at a spectral efficiency of 6 bits/sec/Hz, gains of 9.5 dB, 11.5 dB and 12.5 dB can be obtained when K_T is 4, 6 and 10, respectively.

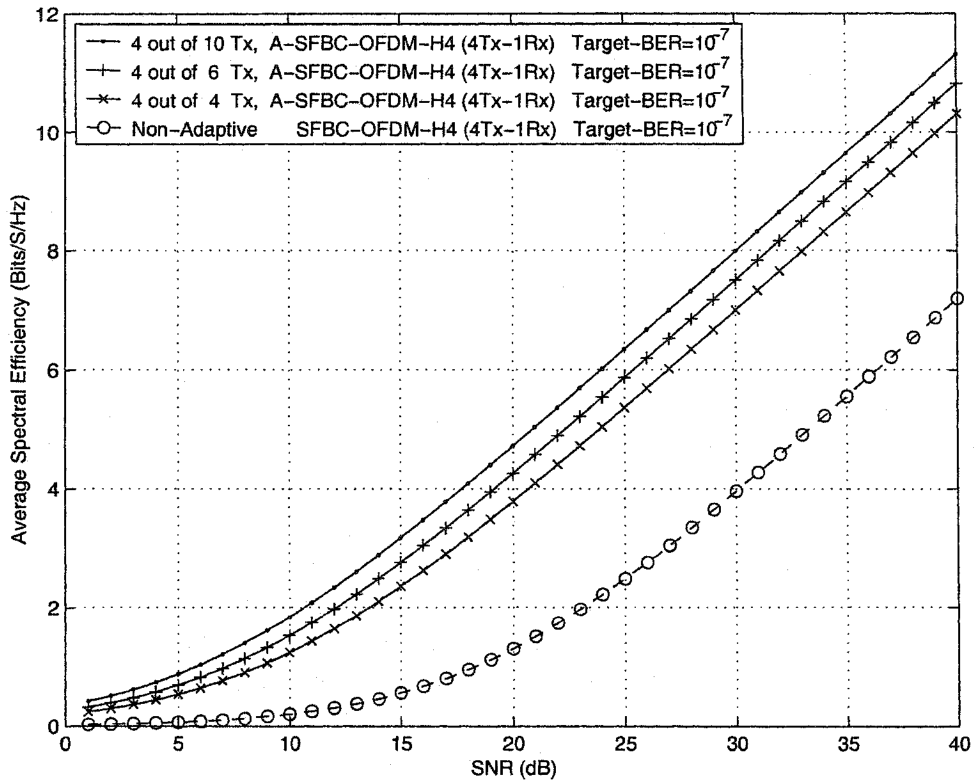

Figure 6.10: Average spectral efficiency of non-adaptive SFBC-OFDM (4Tx-1Rx) and A-SFBC-OFDM (4Tx-1Rx) using the code H4.

6.5 Conclusion

In this chapter, a new scheme consisting of a combination of adaptive modulation, orthogonal frequency division multiplexing (OFDM), high order space-frequency block coding (SFBC) and antenna selection is presented. Spectral efficiency advantage of the proposed system is examined. It is shown that antenna selection with adaptive modulation can greatly improve the performance of conventional SFBC-OFDM systems.

Chapter 7

MIMO-OFDM Systems with Antenna Selection Over Frequency Selective Fading Channels

In this chapter, a new scheme consisting of a combination of a multiple-input/multiple-output (MIMO) system, orthogonal frequency division multiplexing (OFDM) and antenna selection is presented. The proposed scheme exploits the benefits of the MIMO systems, OFDM and antenna selection to provide high quality transmission for wireless communications over frequency selective multipath fading channels. We examine the coding and diversity advantages of the MIMO-OFDM system with antenna selection using average SNR gain, outage probability and BER analysis validated by numerical simulation. The system performances of different forms of the proposed scheme are evaluated and compared. It is shown that the proposed scheme can greatly improve the performance of the conventional MIMO-OFDM systems.

7.1 Introduction

There is a growing demand for high-speed, spectrally efficient and reliable communication. Providing high-quality services in a wireless environment has several challenges. Recently, the use of multiple-input multiple output (MIMO) systems, employing space-time block codes (STBC) has been proposed as an efficient solution for future wireless systems, since they can greatly improve the system performance over flat fading channels with a reasonable level of complexity [28]-[30]. However, multiple antenna deployment requires multiple RF chains that are expensive. Antenna selection, which is much cheaper than RF chain, has been proposed for the application of MIMO systems over flat fading channels [87]-[95].

In Chapter 3, a new scheme for the MIMO-OFDM systems called SFBC-OFDM was proposed as an efficient solution for broadband wireless communication systems over non-flat fading channels.

In this chapter, we propose an adaptive subcarrier-by-subcarrier basis antenna selection scheme for the SFBC-OFDM systems. Assuming that the channel state information (CSI) is available, antenna selection at the transmitter (or receiver) maximizes the signal to noise ratio (SNR). This minimizes the probability of error.

The performance of the proposed scheme is analytically evaluated over frequency selective fading channels. It is shown that adaptive antenna selection leads to an increase in effective instantaneous SNR visible through a parallel shift in the average bit error rate (BER) curves. We derive an analytical expression for the improvement in average SNR that serves as an indicator of coding gain. In addition, it is shown that antenna selection leads to a remarkable improvement in diversity. This is shown, using outage probability analysis, as an indicator of diversity gain. Here, we use the SFBC-OFDM system for two transmitter antennas ($N_T = 2$). Generalization to higher order SFBC-OFDMs is straightforward.

The rest of this chapter is organized as follows. The proposed scheme is presented in Section 7.2. In Section 7.3, the proposed antenna selection at the transmitter and/or receiver is described. Then for the SFBC-OFDM and the proposed scheme, both SNR gain and the outage probability are analytically evaluated. Numerical analysis for the performance evaluation is presented in Section 7.4. Finally, Section 7.5 concludes this chapter.

7.2 System Model

Figure 7.1 depicts a high level block diagram of the proposed system. A block of data is serial-to parallel converted. Assuming an OFDM with N subcarriers, N_s is the number of subbands, that has been chosen to be $N_s = N/2$, i.e, each subband includes two adjacent subchannels.

All subbands are modulated using MQAM where M is determined by the number of the allocated bits. Therefore, a signal vector

$$\{s[0]\,,\, s[1]\,, ...,\, s[N-1]\,\}$$

is provided as the input for the SFBC system.

Considering an $(M_T\,,\, M_R)$ MIMO system, the goal of antenna selection is to select N_T out of M_T transmitter antennas and/or N_R out of M_R receiver antennas. We use the SFBC system for two transmitter antennas ($N_T = 2$); therefore the effective MIMO system will be $(2,\, N_R)$. SFBC provides two blocks of $\mathbf{S}_1$ and $\mathbf{S}_2$ of length N, for the OFDM systems at the transmitters. In order to utilize the space-frequency diversity, the input blocks are encoded as follows.

$$\mathbf{S}_1 = (\; s[0] \quad -s^*[1] \quad s[2] \quad -s^*[3] \;\cdots\; s[N-2] \quad -s^*[N-1]\,)^T$$

$$\mathbf{S}_2 = (\; s[1] \quad +s^*[0] \quad s[3] \quad +s^*[2] \;\cdots\; s[N-1] \quad +s^*[N-2]\,)^T$$

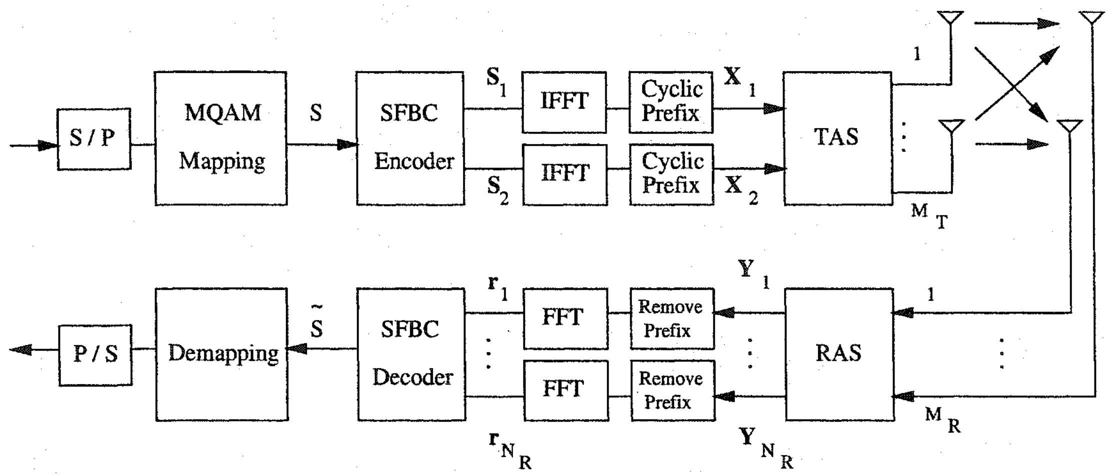

Figure 7.1: Block diagram of the proposed system.

OFDM modulators generate blocks $\mathbf{X}_1$ and $\mathbf{X}_2$ that are transmitted by the first and second selected transmitter antennas, respectively.

Assuming that the guard time interval is longer than the largest delay spread of a multipath channel, the received signal will be the convolution of the channel and the transmitted signal. Assuming that the channel is static during an OFDM block, at the receiver side after removing the cyclic prefix, the FFT output as the demodulated received signal can be expressed as:

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{H}^{'}_{1,m}\,\mathbf{S}_1 + \mathbf{H}^{'}_{1,n}\,\mathbf{S}_2 + \mathbf{W}_1 \\ &\vdots \\ \mathbf{r}_{N_R} &= \mathbf{H}^{'}_{N_R,m}\,\mathbf{S}_1 + \mathbf{H}^{'}_{N_R,n}\,\mathbf{S}_2 + \mathbf{W}_{N_R} \end{aligned} \tag{7.1}$$

where $\mathbf{r}_i = (r_i[0], \cdots, r_i[N-1])^T$; $\mathbf{H}^{'}_{i,m}$ and $\mathbf{H}^{'}_{i,n}$ represent diagonal matrices whose elements $\left(H^{'}_{i,j}[l]\,, i = 1, \cdots, N_R,\ j = 1, 2\,, l = 0, \cdots, N-1\right)$ are the selected subchannels and $\mathbf{W}_i = (W_i[0], \cdots, W_i[N-1])^T$ denotes AWGN. Also, assume that m-th and n-th transmitter antenna elements are chosen on each subchannel.

Knowing the channel information at the receiver, ML detection can be used for decoding of received signals, which can be written as:

$$\widetilde{s}[2k] = \sum_{i=1}^{N_R} H'^{*}_{i,m}[2k]\, r_i[2k] + H'_{i,n}[2k]\, r_i^*[2k+1]$$

$$\widetilde{s}[2k+1] = \sum_{i=1}^{N_R} H'^{*}_{i,n}[2k+1]\, r_i[2k] - H'_{i,m}[2k+1]\, r_i^*[2k+1] \tag{7.2}$$

$k = 0, \cdots, \frac{N}{2} - 1.$

Assuming that the channel gains between two adjacent subchannels are approximately equal, i.e., $H'_{i,m}[2k] = H'_{i,m}[2k+1]$ and $H'_{i,n}[2k] = H'_{i,n}[2k+1]$, then by substituting (7.1) into (7.2) the decoded signal can be expressed as:

$$\widetilde{s}[2k] = \sum_{i=1}^{N_R} \left(\left| H'_{i,m}[2k] \right|^2 + \left| H'_{i,n}[2k] \right|^2 \right) s[2k]$$

$$+ \sum_{i=1}^{N_R} H'^{*}_{i,m}[2k]\, W_i[2k] + \sum_{i=1}^{N_R} H'_{i,n}[2k]\, W_i^*[2k+1]$$

$$\widetilde{s}[2k+1] = \sum_{i=1}^{N_R} \left(\left| H'_{i,m}[2k] \right|^2 + \left| H'_{i,n}[2k] \right|^2 \right) s[2k+1]$$

$$+ \sum_{i=1}^{N_R} H'^{*}_{i,n}[2k]\, W_i[2k] - \sum_{i=1}^{N_R} H'_{i,m}[2k]\, W_i^*[2k+1] \tag{7.3}$$

The above variables provide a diversity gain of order $2N_R$ for every $s[2k]$ and $s[2k+1]$ where proper antenna selection, as can be seen, can increase the SNR at the receiver and improve the system performance. It can be concluded that the proposed scheme can provide significant gains in performance over conventional OFDM and conventional SFBC-OFDM systems. At the end, the elements of block $\{\widetilde{s}[j]\}_{j=0}^{N-1}$ are demodulated to extract the information data.

7.3 Antenna Selection

In this section, an adaptive subcarrier-by-subcarrier basis antenna selection scheme will be presented. We consider antenna selection at the transmitter or at the receiver side or jointly in both transmitter and receiver sides. We present the selection algorithm that maximizes the average SNR and, therefore, minimizes the average BER.

Consider a MIMO-OFDM system that employs M_T transmitter and M_R receiver antennas. The received signal for $k-$th subchannel can be written as:

$$\mathbf{r}[k] = \mathbf{H}[k]\,\mathbf{S}[k] + \mathbf{W}[k] \tag{7.4}$$

where, $\mathbf{X}[k]$ and $\mathbf{W}[k]$ are the transmitted signal and AWGN respectively and $\mathbf{H}[k]$ is the subchannel response given by:

$$\mathbf{H}[k] = \begin{bmatrix} H_{1,1}[k] & H_{1,2}[k] & \cdots & H_{1,M_T}[k] \\ H_{2,1}[k] & H_{2,2}[k] & \cdots & H_{2,M_T}[k] \\ \vdots & \vdots & \cdots & \vdots \\ H_{M_R,1}[k] & H_{M_R,2}[k] & \cdots & H_{M_R,M_T}[k] \end{bmatrix} \tag{7.5}$$

The goal of antenna selection is to select N_T transmitter (N_R receiver) antennas out of M_T transmitter (M_R receiver) antennas.

7.3.1 Transmitter antenna selection (TAS)

Consider a (M_T, N_R) MIMO system, i.e., there are M_T available transmitter antennas and N_R available receiver antennas. In this case, subchannel matrix $\mathbf{H}[k]$ is a ($M_T \times N_R$) matrix. In the TAS scheme, N_T antennas are selected out of M_T transmitter antennas. Therefore, there is a total of $\begin{pmatrix} M_T \\ N_T \end{pmatrix}$ possible selections of transmitter antennas. In order to minimize the average BER_k we can maximize the average SNR_k that is equivalent to selecting the best N_T out of M_T columns of $\mathbf{H}[k]$.

The optimal transmission is to transmit on the transmitter antennas corresponding to the N_T columns with the highest Frobenius norms.

In general, the received SNR at each subchannel can be written as:

$$SNR_k = \gamma_s \sum_{i=1}^{N_R} \sum_{j=1}^{N_T} \left| H'_{i,j}[k] \right|^2 \tag{7.6}$$

where $\gamma_s = \frac{E_s}{N_0}$, E_s is the symbol energy at the transmitter and $\frac{N_0}{2}$ is the variance of the real/imaginary part of the AWGN and $H'_{i,j}[k]$ is the selected subchannel. Note that $H'_{i,j}[k]$ is equal to the element of $\mathbf{H}[k]$, i.e., $H_{p,q}[k]$. As can be seen, the above antenna selection ensures that the resulting SNR_k at each subchannel is maximized.

Since the average SNR_k in conventional SFBC-OFDM is:

$$E\{SNR_k\} = \gamma_s \sum_{i=1}^{N_R} \sum_{j=1}^{N_T} E\left\{ \left| H'_{i,j}[k] \right|^2 \right\} = N_T N_R \gamma_s \tag{7.7}$$

The average SNR gain obtained from the antenna selection can be defined by:

$$Gain_{SNR} = \frac{E\left\{\gamma_s \sum_{i=1}^{N_R} \sum_{j=1}^{N_T} \left|H'_{i,j}[k]\right|^2\right\}}{N_T N_R \gamma_s} = \frac{E\left\{\sum_{i=1}^{N_R} \sum_{j=1}^{N_T} \left|H'_{i,j}[k]\right|^2\right\}}{N_T N_R} \tag{7.8}$$

In the case of SFBC with $N_T = 2$ and $M_R = N_R$, we simply select the first and second largest Frobenius norms. Assume that the m-th and n-th transmitter antenna elements are chosen on each subchannel. The average SNR gain obtained from the antenna selection can be written as:

$$\begin{aligned} Gain_{SNR} &= \frac{E\left\{\gamma_s \sum_{i=1}^{N_R} \left(\left|H'_{i,m}[k]\right|^2 + \left|H'_{i,n}[k]\right|^2 \right)\right\}}{2 N_R \gamma_s} \\ &= \frac{E\left\{\sum_{i=1}^{N_R} \left(\left|H'_{i,m}[k]\right|^2 + \left|H'_{i,n}[k]\right|^2 \right)\right\}}{2 N_R} \end{aligned} \tag{7.9}$$

Figure 7.2, shows the average SNR gain for transmitter antenna selection for various selection configurations. We select N_T transmitter antennas out of M_T transmitter antennas, where $N_R = M_R$ is the number of receiver antennas. It can be seen that, in each case, the average SNR gain is increased as the number of transmitter antennas increases.

Probability density function of the selected subchannels:

Since the squared column norms of subchannels are i.i.d chi-square distributed, let x_i be the square of the i-th column norm. Then,

$$f_X(x_i) = \frac{1}{(N_R-1)!} x_i^{(N_R-1)} e^{-x_i} \tag{7.10}$$

and

$$F_X(x_i) = 1 - \sum_{l=0}^{N_R-1} \frac{x_i^l}{l!} e^{-x_i} \tag{7.11}$$

are the probability density function (pdf) and the cumulative distribution function (cdf), respectively.

The joint probability density function of the first and second largest column Frobenius norms (x and y) when two out of M_T transmitter antennas are selected ($N_T = 2$), can be represented by [93], [94]:

$$f_{XY}(x,y) = M_T(M_T-1)\left[F_Y(y)\right]^{M_T-2} f_X(x) f_Y(y) \tag{7.12}$$

Outage Probability:

The outage probability can be considered as a good indicator of diversity gain increase due to the use of antenna selection. The outage probability for each subchannel is defined as:

$$P_{rob}(C[k] < C_{outage}[k]) = P_{outage} \tag{7.13}$$

where $C_{outage}[k]$ is the outage capacity for each subchannel. The channel capacity of SFBC-OFDM with transmitter antenna selection is given by:

$$C[k] = \log_2\{1 + \gamma_s(x[k] + y[k])\} \tag{7.14}$$

where $x[k]$ and $y[k]$ are the first and second largest column Frobenius norms for the k -th subchannel.

Therefore,

$$P_{outage} = P_{rob}\left(\log_2\left\{1 + \gamma_s\,(x[k] + y[k])\right\} < C_{outage}[k]\right) = \tag{7.15}$$

$$P_{rob}\left(x[k] + y[k] < \frac{2^{C_{outage}[k]} - 1}{\gamma_s} = \xi\right) = \int\int f_{XY}(x, y)\, dxdy \tag{7.16}$$

It can be approximately expressed as [93], [94]:

$$P_{rob}\left(C[k] < C_{outage}[k]\right) = 2\left[F_X\left(\frac{\xi}{2}\right)\right]^{M_T} \tag{7.17}$$

for $N_R = 1$ it can be written as:

$$P_{rob}\left(C[k] < C_{outage}[k]\right) = 2\left[1 - e^{-\xi/2}\right]^{M_T} \tag{7.18}$$

and for $N_R = 2$ it can be written as:

$$P_{rob}\left(C[k] < C_{outage}[k]\right) = 2\left[1 - e^{-\xi/2}\left(1 + \frac{\xi}{2}\right)\right]^{M_T} \tag{7.19}$$

Figures 7.3 and 7.4 compares the outage probability of the proposed system obtained from the above formulas with those from numerical simulations. It can be seen that the results are very close.

Bit error probability analysis:

The average bit error rate (BER) for the proposed system with MQAM signals is:

$$\overline{BER_k} = \int_{x=0}^{\infty}\int_{y=0}^{x} BER_k(x, y)\, f_{XY}(x, y)\, dy\, dx \tag{7.20}$$

where $f_{XY}(x, y)$ is stated in (7.12).

The BER of each subcarrier of the proposed TAS scheme ($N_T = 2$) with MQAM signals over a frequency selective fading channel can be expressed as:

$$BER_k = \frac{2}{\beta}\left(1 - \frac{1}{\sqrt{2^\beta}}\right) \times erfc\left\{\sqrt{\frac{1.5\gamma_s\,\sum_{i=1}^{N_R}\left(\left|H'_{i,m}[k]\right|^2 + \left|H'_{i,n}[k]\right|^2\right)}{2\,(2^\beta - 1)}}\right\} \tag{7.21}$$

where $\gamma_s = \frac{E_s}{N_0}$, E_s is the symbol energy at the transmitter and β is the bits/symbols assigned for each subcarrier and $\beta = \log_2 M$.

It can be approximated as

$$BER_k = 0.2 \exp \left\{ \frac{-1.6\gamma_s \sum_{i=1}^{N_R} \left(\left| H'_{i,m}[k] \right|^2 + \left| H'_{i,n}[k] \right|^2 \right)}{2 \left(2^{\beta} - 1 \right)} \right\} \tag{7.22}$$

Note that in the BER performance analysis of the transmitter antenna selection only two approximations have been assumed, as stated in (7.22) and (7.17).

Here, we consider some examples of transmitter antenna selection for analysis of the BER of the proposed system with 4QAM (QPSK) signals ($\beta = 2$), when there are two effective transmitter antennas and one receive antenna, i.e. $N_T = 2$ and $N_R = M_R = 1$. Since,

$$BER_k(x, y) = 0.2 e^{\frac{-0.8 E_s}{3 N_o}(x+y)} \tag{7.23}$$

$$BER_k(x, y) = 0.2 e^{-\mu(x+y)} \tag{7.24}$$

where $x = \left| H'_{1,m}[k] \right|^2$ and $y = \left| H'_{1,n}[k] \right|^2$ are the norms of the selected subchannels, and $\mu = \left(\frac{-0.8 E_s}{3 N_o} \right)$.

Also, from (7.10) and (7.11) for $N_R = 1$ we can find that:

$$f_X(x) = e^{-x} \qquad \textit{and} \qquad f_Y(y) = e^{-y} \tag{7.25}$$

$$F_X(x) = 1 - e^{-x} \qquad \textit{and} \qquad F_Y(y) = 1 - e^{-y} \tag{7.26}$$

Example-1: ***No Selection***

In this case, we have $N_T = M_T = 2$. From (7.12) we can find that:

$$f_{XY}(x, y) = 2 f_X(x) f_Y(y) = 2 e^{-(x+y)} \tag{7.27}$$

Therefore,

$$\overline{BER_k} = \int_{x=0}^{\infty} \int_{y=0}^{x} 0.4\, e^{-\mu(x+y)}\, e^{-(x+y)} dy\, dx \tag{7.28}$$

We can obtain:

$$\overline{BER_k} = \frac{0.2}{(\mu+1)^2} \tag{7.29}$$

Example-2: ***Selection of 2 out of 3 transmitter antennas***

In this case, we have $N_T = 2$, $M_T = 3$. From (7.12) we can find that:

$$f_{XY}(x, y) = 6\left[1 - e^{-y}\right] f_X(x)\, f_Y(y) = 6\left[1 - e^{-y}\right] e^{-(x+y)} \tag{7.30}$$

Therefore,

$$\overline{BER_k} = \int_{x=0}^{\infty} \int_{y=0}^{x} 1.2\, e^{-\mu(x+y)} \left[1 - e^{-y}\right] e^{-(x+y)} dy\, dx \tag{7.31}$$

We can obtain:

$$\overline{BER_k} = \frac{0.6}{(2\mu+3)\,(\mu+1)^2} \tag{7.32}$$

Example-3: ***Selection of 2 out of 4 transmitter antennas***

In this case, we have $N_T = 2$, $M_T = 4$. From (7.12) we can find that:

$$f_{XY}(x, y) = 12\left[1 - e^{-y}\right]^2 f_X(x)\, f_Y(y) = 12\left[1 - e^{-y}\right]^2 e^{-(x+y)} \tag{7.33}$$

Therefore,

$$\overline{BER_k} = \int_{x=0}^{\infty} \int_{y=0}^{x} 2.4\, e^{-\mu(x+y)} \left[1 - e^{-y}\right]^2 e^{-(x+y)} dy\, dx \tag{7.34}$$

We can obtain:

$$\overline{BER_k} = \frac{2.4}{(2\mu+3)\,(2\mu+4)\,(\mu+1)^2} \tag{7.35}$$

Example-4: ***Selection of 2 out of 8 transmit antennas***

In this case, we have $N_T = 2$, $M_T = 8$. From (7.12) we can find that:

$$f_{XY}(x,y) = 56\left[1-e^{-y}\right]^6 f_X(x)\, f_Y(y) = 65\left[1-e^{-y}\right]^6 e^{-(x+y)} \tag{7.36}$$

Therefore,

$$\overline{BER_k} = \int_{x=0}^{\infty}\int_{y=0}^{x} 11.2\, e^{-\mu(x+y)}\left[1-e^{-y}\right]^6 e^{-(x+y)} dy\, dx \tag{7.37}$$

We can obtain

$$\begin{aligned}\overline{BER_k} = 11.2\Bigg\{&\left[\frac{1}{2\,(\mu+1)^2} - \frac{6}{(\mu+2)\,(2\mu+3)} + \frac{15}{(\mu+3)\,(2\mu+4)}\right. \\ &- \frac{20}{(\mu+4)\,(2\mu+5)} + \frac{15}{(\mu+5)\,(2\mu+6)} - \frac{6}{(\mu+6)\,(2\mu+7)} \\ &\left.+ \frac{1}{(\mu+7)\,(2\mu+8)}\right] - \frac{1}{(\mu+1)}\left[\frac{1}{(\mu+1)} - \frac{6}{(\mu+2)}\right. \\ &\left.+ \frac{15}{(\mu+3)} - \frac{20}{(\mu+4)} + \frac{15}{(\mu+5)} - \frac{6}{(\mu+6)} + \frac{1}{(\mu+7)}\right]\Bigg\}\end{aligned} \tag{7.38}$$

Figures 7.5 and 7.6 depict the average BER of the proposed system with transmitter antenna selection. By observing the results, we find that TAS provides both coding gain (parallel shift) and diversity gain (increased slope). It is also shown that the simulation results are very close to the analytical formulas.

7.3.2 Receiver antenna selection (RAS)

Consider an (N_T, M_R) MIMO system, i.e., there are N_T available transmitter antennas and M_R available receiver antennas. In this case subchannels matrix $\mathbf{H}[k]$ is an $(N_T \times M_R)$ matrix. In the RAS scheme, N_R antennas are selected out of M_R receiver antennas. Therefore, there is a total of $\begin{pmatrix} M_R \\ N_R \end{pmatrix}$ possible selections of receiver antennas. Maximizing the SNR_k, minimizes the $\overline{BER_k}$. This is equivalent to the selection of the N_R out of M_R rows of $\mathbf{H}[k]$. The optimal RAS is to receive signals on the N_R receiver antennas corresponding to the N_R rows with the highest Frobenius norms.

In the case of SFBC with $N_T = 2$ and $N_R = 2$, we simply select the 2 rows with the first and second largest Frobenius norms.

Similar to TAS, the average SNR gain obtained from RAS can be defined by:

$$Gain_{SNR} = \frac{E\left\{\gamma_s \sum_{i=1}^{N_R} \left(\left|H'_{i,1}[k]\right|^2 + \left|H'_{i,2}[k]\right|^2\right)\right\}}{2\, N_R\, \gamma_s}$$

$$= \frac{E\left\{\sum_{i=1}^{N_R} \left(\left|H'_{i,1}[k]\right|^2 + \left|H'_{i,2}[k]\right|^2\right)\right\}}{2\, N_R} \quad (7.39)$$

where, $H'_{i,1}[k]$ and $H'_{i,2}[k]$ are the selected subchannels.

Figure 7.7 shows the average SNR gain for a RAS scheme for various selection configurations. We select N_R out of M_R receiver antennas, where $N_T = M_T$ is the number of transmitter antennas. It can be seen that, in each case, the average SNR gain is increased by the increase in the number of receive antennas.

7.3.3 Joint transmitter/receiver antenna selection (TRAS)

Consider an $(M_T\,, M_R)$ MIMO system. In this case subchannels matrix $\mathbf{H}[k]$ is an $(M_T \times M_R)$ matrix. In the TRAS scheme, N_R antennas are selected out of M_R receiver antennas and N_T antennas are selected out of M_T transmitter antennas. Therefore, there is a total of $\begin{pmatrix} M_T \\ N_T \end{pmatrix}\begin{pmatrix} M_R \\ N_R \end{pmatrix}$ possible selections of transmitter/receiver antennas. The optimal selection is to select a $(N_T \times N_R)$ sub-matrix of the $(M_T \times M_R)$ subchannel with the highest Frobenius norm.

In general, the SNR gain obtained from the antenna selection can be defined by:

$$Gain_{SNR} = \frac{E\left\{\gamma_s \sum_{i=1}^{N_R} \sum_{j=1}^{N_T} \left|H'_{i,j}[k]\right|^2\right\}}{N_T\, N_R\, \gamma_s} = \frac{E\left\{\sum_{i=1}^{N_R} \sum_{j=1}^{N_T} \left|H'_{i,j}[k]\right|^2\right\}}{N_T\, N_R} \quad (7.40)$$

Figure 7.8 shows the average SNR gain for TRAS for various selection configurations. It can be seen that, in each case, the average SNR gain is increased as the number of transmitter and receiver antennas increases.

7.4 Simulation Results

The performance of the proposed system is evaluated by computer simulation over a frequency selective fading channel. The considered fading channel is a multipath fading channel with coherence bandwidth smaller than the total bandwidth of the multicarrier system and thus seen as frequency selective fading. The fading process is assumed to be stationary and slowly varying compared to the symbol duration of the multicarrier signal, such that it is approximately constant during one OFDM block length. The fading process impulse response of the path between the i-th transmitter antenna and the j-th receiver antenna, $(i = 1, 2, \cdots, M_R \quad, j = 1, 2, \cdots, M_T)$ can be expressed as:

$$h_{i,j}(t) = \sum_{m=0}^{L-1} \alpha_{m,i,j}(t)\, \delta(t - \tau_m(t)) \tag{7.41}$$

where $\alpha_{m,i,j}(t)$ is a tap weight, $\tau_m(t)$ is the time delay of the m-th path and L is the total number of resolvable paths ($L = 4$). It is assumed that the channel state information (CSI) is available at the receiver and transmitter. The channel taps are complex Gaussian random processes with zero mean and variance $\frac{1}{L}$ (equal power). With this model we have assumed that the path delays, $\tau_m(t)$, are multiples of the symbol duration T_s. The OFDM system includes $N = 512$ subcarriers and a cyclic prefix which is longer than the channel delay.

The performances of the proposed scheme are analytically evaluated over frequency selective fading channels and compared to each other. It is shown that adaptive antenna selection leads to an increase in the effective instantaneous SNR visible through a parallel shift in the average bit error rate (BER) curves. We derived an analytical expression for the improvement in the average SNR that serves as an indicator of the coding gain. In addition, it is shown that the antenna selection leads to a remarkable improvement in the diversity gain. This is shown, using outage probability analysis, as an indicator of diversity gain.

Figure 7.2 shows the average SNR gain for transmitter antenna selection for various selection configurations. We select N_T out of M_T transmitter antennas, where $M_R = N_R$ is the number of the receiver antennas. It can be seen that, in each case, the average SNR gain is increased with the number of transmitter antennas.

Figure 7.3 shows the outage probability of the proposed system for $N_T = 2$, $M_R = N_R = 1$ and $\gamma_s = 10$ dB. It can be seen that the theoretical results (Equation 7.18) are very close to the simulation results and a considerable improvement in outage probability can be obtained using antenna selection. For example, for a capacity of 2.5 Bits/Hz, the outage probability of 0.09 with no selection ($M_T = N_T = 2$) reduces to about 3×10^{-5} with selection of $N_T = 2$ out of $M_T = 7$ transmitter antennas.

Figure 7.4 shows the outage probability of the proposed system for $N_T = 2$, $M_R = N_R = 2$ and $\gamma_s = 10$ dB. It can be seen that the theoretical results (Equation 7.19) are very close to the simulation results and a considerable improvement in outage probability can be obtained using antenna selection. For example, for a capacity of 3.25 Bits/Hz, the outage probability of 0.01 with no selection ($M_T = N_T = 2$) reduces to about 4×10^{-5} with selection of $N_T = 2$ transmitter antennas out of $M_T = 4$ transmitter antennas. We can obtain the behavior of the receiver antenna selection in the same manner.

Figure 7.5 depicts the average BER of the proposed system with transmitter antenna selection. By observing the results, we find that TAS provides both coding gain (parallel shift) and diversity gain (increased slope). It is also shown that the simulation results are very close to the analytical formulas. The first set of curves is provided by calculating the average of BER form Equation (7.21) using several realizations of MIMO channel and selection of 2 out of M_T transmitter antennas.

The second set of curves is obtained from simulation results of the proposed system (Figure 7.1). Finally other curves are obtained from the analytical formulas of Equations (7.29), (7.32), (7.35) and (7.38).

Figure 7.6 depicts the simulation results for the average BER of the proposed system (Figure 7.1) with transmitter antenna selection for the case of $N_T = 2$, $M_R = N_R = 2$. By observing the results, again we can find that TAS provides both coding gain and diversity gain.

Figure 7.7 shows the average SNR gain for a RAS scheme for various selection configurations. We select N_R out of M_R receiver antennas, where $N_T = M_T$ is the number of transmitter antennas. It can be seen that, in each case, the average SNR gain will be increased by the number of receive antennas.

Figure 7.8 shows the average SNR gain for a TRAS for various selection configurations. It can be seen that, in each case, the average SNR gain, will be increased by the number of transmitter and receiver antennas.

7.5 Conclusion

A scheme consisting of SFBC-OFDM with antenna selection is proposed in this chapter. The performance of the proposed scheme is analytically evaluated for different configurations over frequency selective fading channels. It is shown that adaptive antenna selection leads to an increase in the effective instantaneous SNR visible through a parallel shift in the average bit error rate (BER) curves. We derived an analytical expression for the improvement in the average SNR that serves as an indicator of the coding gain. In addition, it is shown that antenna selection leads to a remarkable improvement in the diversity gain. This is shown, using outage probability analysis, as an indicator of diversity gain. We used the SFBC-OFDM system for two transmitter antennas. Generalization to higher order SFBC-OFDMs is straightforward. The proposed scheme can be considered as a promising technique for high data rate wireless transmission over frequency selective multipath fading channels and for broadband wireless communications.

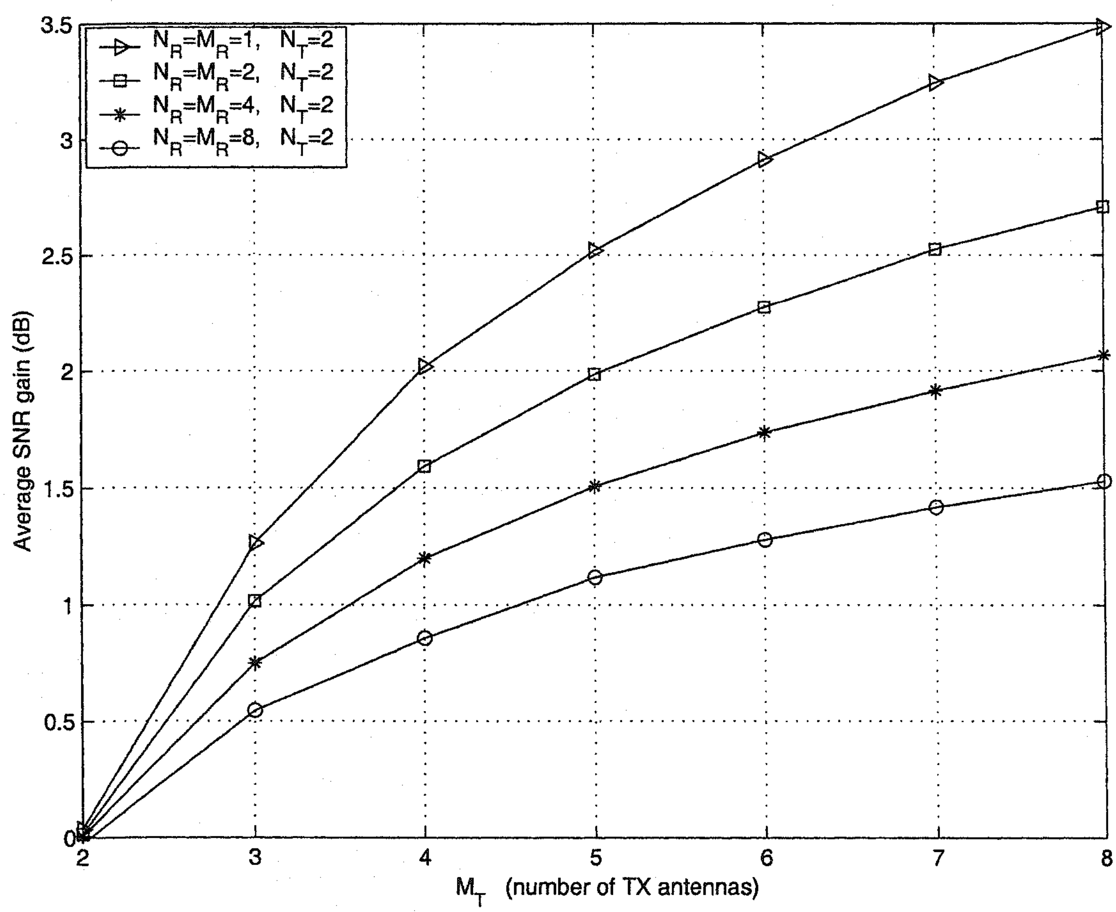

Figure 7.2: Average SNR gain for SFBC-OFDM with transmit antenna selection.

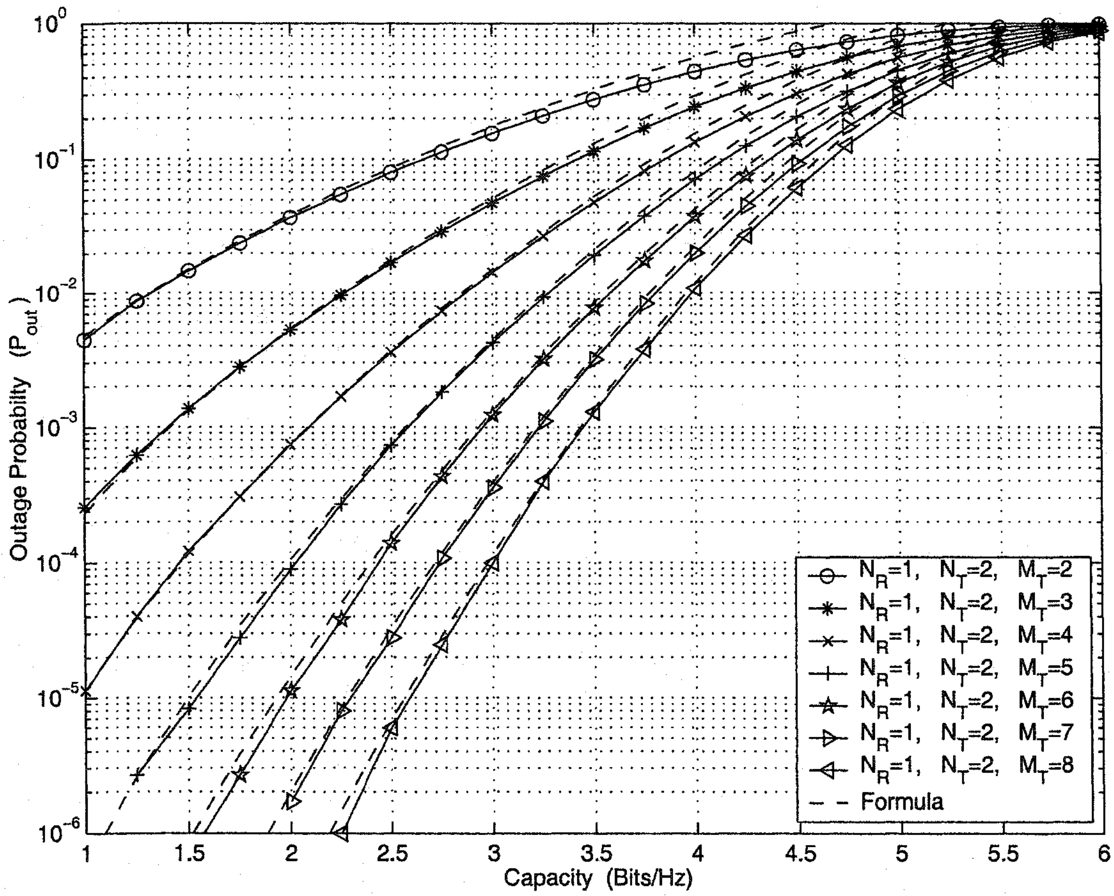

Figure 7.3: Outage probability for SFBC-OFDM with transmit antenna selection.

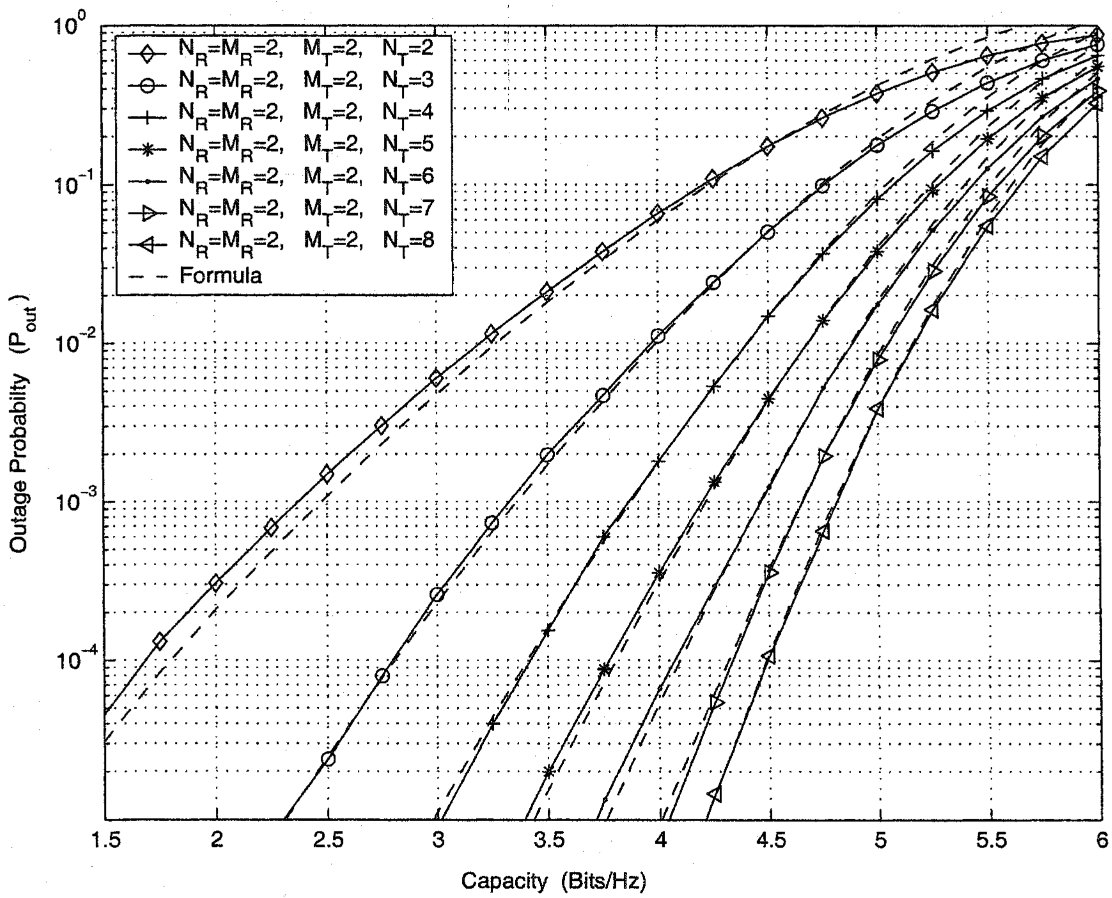

Figure 7.4: Outage probability for SFBC-OFDM with transmit antenna selection.

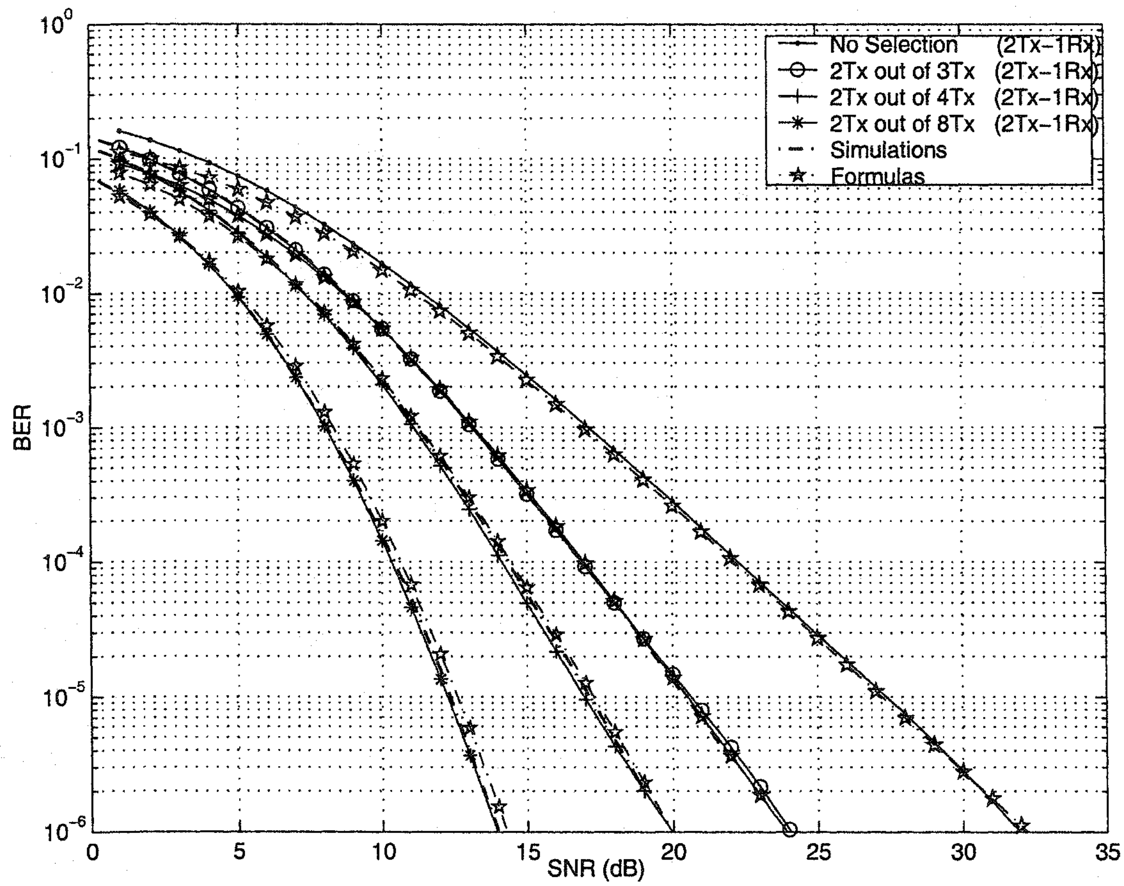

Figure 7.5: Average BER for SFBC-OFDM with transmit antenna selection (selecting N_T out of M_T).

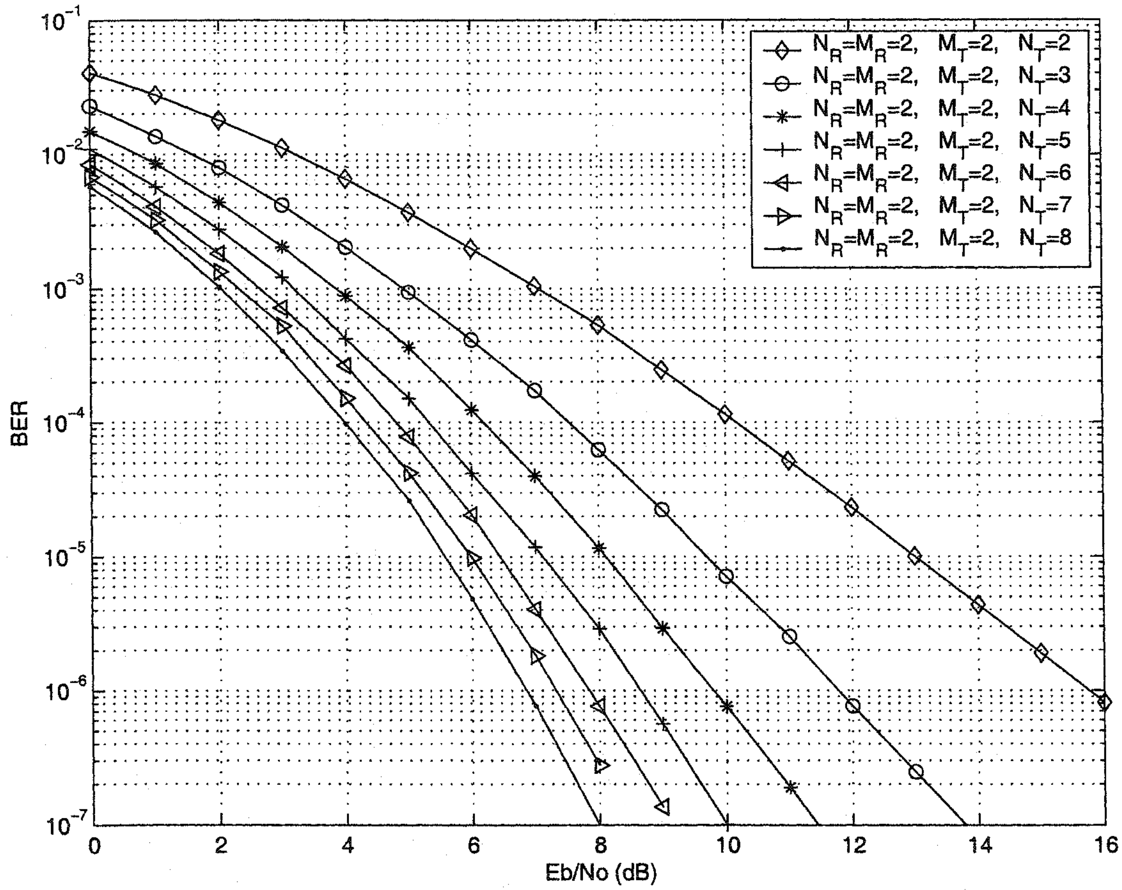

Figure 7.6: Average BER for SFBC-OFDM with transmit antenna selection (selecting N_T out of M_T).

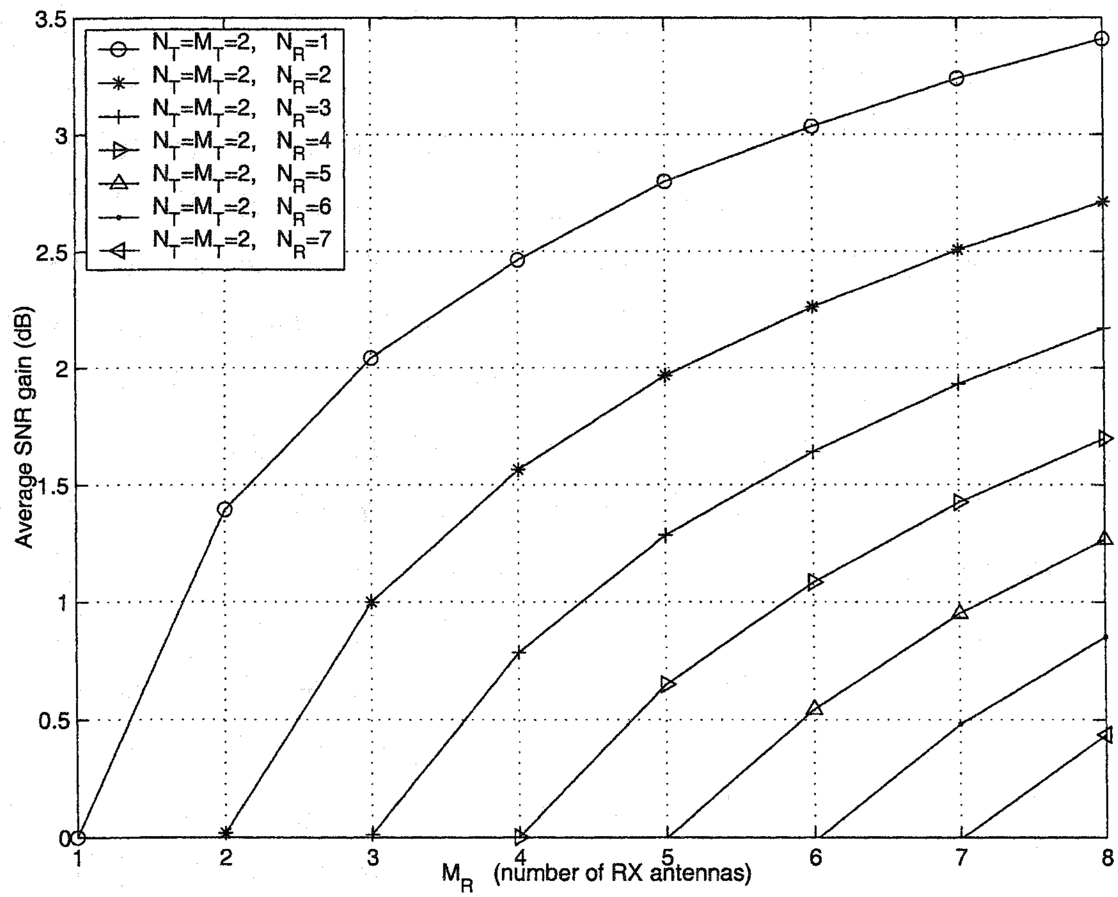

Figure 7.7: Average SNR gain for SFBC-OFDM with receive antenna selection.

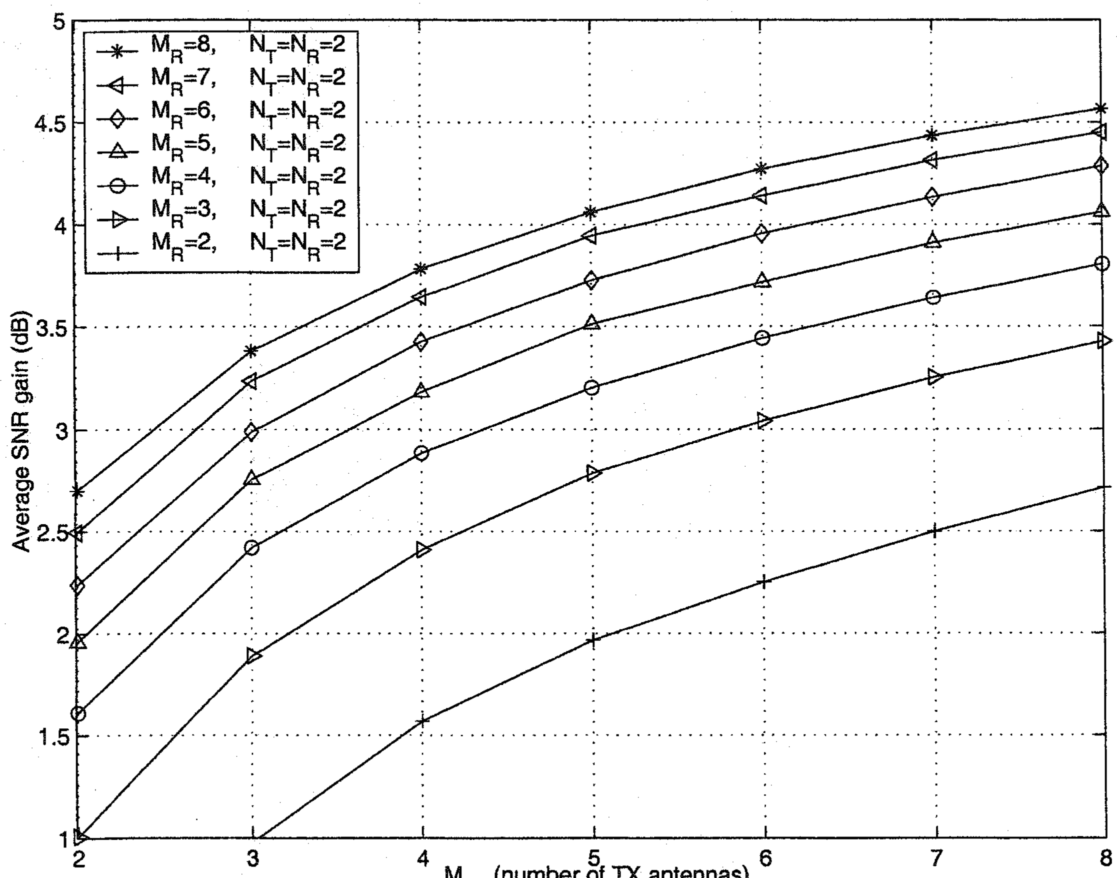

Figure 7.8: Average SNR gain for SFBC-OFDM with joint transmit/receive antenna selection.

Chapter 8

Conclusions and Future Work

In this dissertation we have developed and presented novel MIMO-OFDM schemes for broadband wireless applications. The focus of this research was on addressing the problem of multipath fading channel dispersion that extensively attenuates some subcarriers in an OFDM system and also to consider the possible application of multiple transmit and/or receive antennas for the applications over frequency selective fading channels.

8.1 Summary

In Chapter 2, a brief overview of diversity techniques was presented. In Chapter 2, also different schemes, such as time diversity, frequency diversity, antenna diversity, and the most popular schemes known as MIMO systems such as space-time coding techniques were discussed in more detail. It was shown that MIMO systems can improve the system performance and can be considered for the application in future wireless transmission systems.

In Chapter 3, the application of MIMO systems over frequency selective fading channel using an OFDM system was proposed. Also the main concept of OFDM was reviewed and later our proposed MIMO-OFDM system (OTD-OFDM or SFBC-OFDM) was presented. The proposed scheme was then explained in more detail and both encoding and decoding algorithms were explained by some examples. The objective of the proposed scheme was to enhance the performance of OFDM systems over multipath fading channels by using the antenna diversity techniques compared to the conventional OFDM and, on the other hand, to make MIMO systems applicable to frequency selective multipath fading channels.

In Chapter 4, the channel capacity of MIMO-OFDM systems were analytically evaluated and it was shown that by using MIMO systems the capacity of the channel can be dramatically increased as a function of the number of antennas. Some numerical examples were also provided.

In Chapter 5, the bit error rate (BER) performance of the proposed MIMO-OFDM systems were analytically evaluated. We could derive some new closed-form expressions of BER performance of MIMO-OFDM systems over frequency selective fading channels.

In the performance evaluation both M-ary phase shift keying (MPSK) and M-ary quadrature amplitude modulation (MQAM) were considered. Numerical results were also provided for analysis and simulations. In addition, performance of several forms of MIMO-OFDM systems were also evaluated and compared. It was shown that the results from the closed-form formulas were very close to the analytical formula results and simulation results.

In Chapter 6, a new adaptive modulation scheme was presented for the proposed MIMO-OFDM systems. The proposed scheme exploits the benefits of space-frequency block codes, OFDM and adaptive modulation to provide high quality transmission for wireless communications over frequency selective fading channels. We examined the spectral efficiency advantage of the proposed system. It was shown that antenna selection with adaptive modulation can greatly improve the performance of the conventional SFBC-OFDM systems.

In Chapter 7, a novel antenna selection algorithm was proposed for our MIMO-OFDM system (SFBC-OFDM). Three different forms of antenna selection were considered including, transmit antennas selection, receive antenna selection, and, joint transmit/receive antenna selection. The coding and diversity advantages of the MIMO-OFDM system with antenna selection were examined using average SNR gain, outage probability and BER analysis validated by numerical simulation. The system performance of different forms of the proposed scheme were evaluated and compared. It was shown that the proposed scheme can greatly improve the performance of the conventional SFBC-OFDM systems.

8.2 Contributions

• A new MIMO-OFDM system called OTD-OFDM was proposed. We extended and generalized the idea of OTD-OFDM to utilize space-frequency diversity to provide superior performance and higher channel capacity. The encoding and decoding algorithms for OTD-OFDM were explained by using some examples.

• We derived expressions for the ergodic channel capacity of the MIMO-OFDM and OTD-OFDM systems. We assumed that the channel is perfectly known at the receiver but is unknown at the transmitter.

• The bit error rate (BER) performance of the proposed MIMO-OFDM systems were analytically evaluated. We could also derive some new closed-form expressions for BER performance of the SFBC-OFDM systems over frequency selective fading channels. It was shown that the results from the closed-form formulas were very close to the analytical formula results and simulation results.

• A new scheme consisting of a combination of adaptive modulation and SFBC-OFDM aided antenna selection was presented. The proposed scheme exploits the benefits of space-frequency block codes, OFDM and adaptive modulation to provide high quality transmission for broadband wireless communications. It was shown that antenna selection with adaptive modulation can greatly improve the performance of the conventional SFBC-OFDM systems.

• A novel subcarrier by subcarrier basis antenna selection algorithm was proposed for our MIMO-OFDM system (SFBC-OFDM). It was shown that the proposed scheme can greatly improve the performance of the conventional SFBC-OFDM systems.

It is also noted that, the addition of Turbo Codes as the outer code to our proposed MIMO-OFDM systems provides a system with extra diversity and greatly improves the system performance. This addition and related works that we have performed during this thesis can be found in [75]-[86].

8.3 Future Work

Although this dissertation has provided theoretical and numerical results to demonstrate the superior performance of the proposed MIMO-OFDM systems for the application on future broadband wireless communication systems, there are some issues that require further study.

• The effects of channel estimation error on the overall system performance. There are expected to be some forms of performance degradation due to the channel estimation error.

• The effects of channel feedback delay on the overall system performance, where the transmitter is required to have the channel state information.

• Application of antenna beam-forming in conjunction with the proposed MIMO-OFDM systems.

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Appendix A

Statistical Methods for the Measurement of the Bit Error Rate (BER)

In this Appendix, we develop a procedure for Bit Error Rate (BER) estimation through the application of statistical methods.

Bit error rate is the measure for evaluating the performance of a communication system. It may be interpreted as the average rate at which error would occur in a long sequence of transmitted bits. To make a reliable measurement, the number of information bits should be large enough to be transmitted , or enough errors should be observed, so that a reasonable conclusion can be inferred from the test and simulation results.

Confidence Interval

The statistical method of confidence intervals will be used to establish a lower bound on the estimate of μ (or p). A confidence interval is a range of values that is likely to contain the actual value of some parameter of interest. The interval is derived from the measured value of the parameter, referred to as the point estimate, and the confidence level γ, the probability that the parameter's actual value lies within the interval.

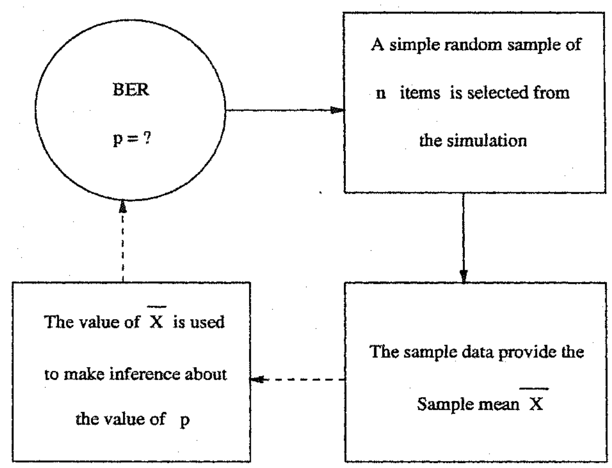

Figure A.1: The statistical process of using a sample mean to make inference about the value of p.

Mean:

We wish to estimate the mean μ of a random variable X. We use as the point estimate of μ the value

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

of the sample mean $\overline{X}$ of X. To find an interval estimate, we must determine the distribution of $\overline{X}$. In general, this is a difficult problem involving multiple convolutions. To simplify it we assume that $\overline{X}$ is normal. This is true if X is normal and it is approximately true for any X if n is large (Central Limit Theorem).

Known Variance:

First, suppose that the variance σ^2 of X is known. The normality assumption leads to the conclusion that the point estimator $\overline{x}$ of μ is $N(\mu_{\overline{x}}, \sigma_{\overline{x}})$.

It can be shown that:

$$\boxed{E\{\overline{x}\} = \mu \qquad\qquad \sigma_{\overline{x}}^2 = \frac{\sigma^2}{n}}$$

Proof:

$$\mu_{\overline{x}} = E\{\overline{x}\} = E\{\frac{1}{n}\sum_{i=1}^{n} x_i\} = \frac{1}{n}\sum_{i=1}^{n} E\{x_i\} = \frac{1}{n}\sum_{i=1}^{n} \mu = \mu$$

and

$$\sigma_{\overline{x}}^2 = E\{(\overline{x} - \mu_{\overline{x}})^2\} = E\{(\frac{1}{n}\sum_{i=1}^{n} x_i - \mu)^2\} = E\{(\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n}\sum_{i=1}^{n} \mu)^2\}$$

$$= \frac{1}{n^2}E\{[\sum_{i=1}^{n}(x_i - \mu)]^2\} = \frac{1}{n^2}E\{\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - \mu)\,(x_j - \mu)\}$$

$$= \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\{E(x_i\,x_j) - \mu E(x_j) - \mu E(x_i) + \mu^2\} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\{E(x_i\,x_j) - \mu^2\}$$

$$= \frac{1}{n^2}\sum_{i=1}^{n} E(x_i^2) + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n} E(x_i\,x_j) - \mu^2$$

$$= \frac{1}{n^2}\sum_{i=1}^{n}(\sigma^2 + \mu^2) + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n} E(x_i)E(x_j) - \mu^2$$

$$= \frac{n}{n^2}(\sigma^2 + \mu^2) + \frac{n(n-1)}{n^2}\mu^2 - \mu^2 = \frac{n}{n^2}\sigma^2$$

$$\sigma_{\overline{x}}^2 = \frac{1}{n}\sigma^2$$

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

Now consider

$$P_r(-z_{\alpha/2} < Z < z_{\alpha/2}) = \int_{-z_{\alpha/2}}^{z_{\alpha/2}} \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} dZ = 1 - \alpha$$

where Z is a normally distributed random variable with $N(0,\ 1)$.

Now, let

$$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

therefore, we conclude that

$$P_r(-z_{\alpha/2} < \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} < z_{\alpha/2}) = P_r(\mu - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \overline{X} < \mu + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}) = 1 - \alpha$$

This yields

$$P_r(\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}) = 1 - \alpha = \gamma$$

We can thus state with *Confidence Level* γ that μ is in the interval $\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.

It can be seen that for a certain *Confidence Level*, larger sizes of n provide a smaller interval estimate.

The standard values for $z_{\alpha/2}$ are given in the following table.

$1-\alpha$	$z_{\alpha/2}$
0.900	1.645
0.950	1.960
0.990	2.576
0.999	3.291

Tail Probabilities for the Gaussian distribution

Scheme-I: For Unknown Variance:

If the variance σ^2 of X is unknown, we cannot use the above expression. To estimate μ, we form the sample variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

This is an unbiased estimate of σ^2 (see the given proof) and it tends to σ^2 as $n \to \infty$. Hence, for large n, we can use the approximation $s \simeq \sigma$ in (6). This yields

$$P_r(\overline{x} - z_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \overline{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}) = 1 - \alpha = \gamma$$

We can thus state with confidence coefficient γ that μ is in the interval $\overline{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$.

As we stated earlier, the sample variance is an unbiased estimate of σ^2.

Proof:

$$E\{s^2\} = E\{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2\} = \frac{1}{n-1}\sum_{i=1}^{n} E\{(x_i - \overline{x})^2\}$$

$$= \frac{1}{n-1}\sum_{i=1}^{n} E\{[(x_i - \mu) - (\overline{x} - \mu)]^2\}$$

$$= \frac{1}{n-1}\sum_{i=1}^{n} E\{(x_i - \mu)^2\} + \frac{1}{n-1}\sum_{i=1}^{n} E\{(\overline{x} - \mu)^2\} - \frac{2}{n-1}E\{(\overline{x} - \mu)\sum_{i=1}^{n}(x_i - \mu)\}$$

recalling

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$n\overline{x} = \sum_{i=1}^{n} x_i$$

$$n\overline{x} = \sum_{i=1}^{n}(x_i - \mu) + n\mu$$

$$\sum_{i=1}^{n}(x_i - \mu) = n(\overline{x} - \mu)$$

therefore,we can continue,

$$E\{s^2\} = \frac{1}{n-1}\sum_{i=1}^{n}\sigma^2 + \frac{1}{n-1}nE\{(\overline{x}-\mu)^2\} - \frac{2}{n-1}E\{(\overline{x}-\mu)(\overline{x}-\mu)n\}$$

$$= \frac{1}{n-1}(n\sigma^2) + \frac{1}{n-1}n\sigma_{\overline{x}}^2 - \frac{2}{n-1}nE\{(\overline{x}-\mu)^2\}$$

$$= \frac{1}{n-1}(n\sigma^2) + \frac{1}{n-1}n(\frac{\sigma^2}{n}) - \frac{2}{n-1}n(\frac{\sigma^2}{n})$$

$$= \frac{1}{n-1}(n\sigma^2 + \sigma^2 - 2\sigma^2) = \sigma^2$$

$$\boxed{E\{s^2\} = \sigma^2}$$

Therefore, the sample variance is an unbiased estimate of σ^2.

In order to calculate sample variance, we can simplify it as follows.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}(x_i^2 - 2x_i\overline{x} + \overline{x}^2) = \frac{1}{n-1}\left\{\sum_{i=1}^{n}x_i^2 - 2\overline{x}\sum_{i=1}^{n}x_i + n\overline{x}^2\right\}$$

$$= \frac{1}{n-1}\left\{\sum_{i=1}^{n}x_i^2 - 2n\overline{x}^2 + n\overline{x}^2\right\} = \frac{1}{n-1}\left\{\sum_{i=1}^{n}x_i^2 - n\overline{x}^2\right\}$$

Since $x_i = \{0\ or\ 1\}$,

$$\sum_{i=1}^{n}x_i^2 = \sum_{i=1}^{n}x_i = n\overline{x}$$

therefore,

$$s^2 = \frac{1}{n-1}\left\{n\overline{x} - n\overline{x}^2\right\}$$

therefore,

$$\boxed{s^2 = \tfrac{n}{n-1}\overline{x}(1-\overline{x})}$$

Scheme-II: For Unknown Variance:

We would like to estimate the probability p of an event A. To do so, we form the zero-one random variable (R.V.) X associated with this event.

We know that $E[x] = p$ and $\sigma_x^2 = p(1-p)$. Therefore, the estimate of p is equivalent to the estimation of the mean of the R.V. X.

We repeat the experiment n times and we denote the number of successes of event A by k. The ratio $\overline{x} = \frac{k}{n}$ is the point estimate of p.

To find its interval estimate, we can form sample mean $\overline{X}$ of X. As shown before, for a large n, $\overline{X}$ approaches a Gaussian distribution with $N(\mu_{\overline{x}}, \sigma_{\overline{x}})$ where here it is $N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$ (central limit theorem).

Hence,

$$P_r\left\{|\overline{X} - p| < z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}\right\} = 1 - \alpha = \gamma$$

However, note that in using the above equation to develop an interval estimate of p, the value of p has to be *known*. Since the value of p is what we are trying to estimate, and is thus *unknown,* we use the following approximation:

The points of the $\overline{x}p$ plane that satisfy the inequality $|\overline{x} - p| < z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$ are in the interior of the ellipse

$$(\overline{x} - p)^2 = z_{\alpha/2}^2 \frac{p(1-p)}{n}$$

From this it follows that the γ confidence interval of p is the vertical segment (p_1, p_2) of Fig. A.2. The endpoints p_1 and p_2 of this segment are the roots of the above equation.

For $n \gg 1$, the following approximation can be used (see the given proof):

Consider $\widetilde{\sigma}_x^2 \simeq \overline{x}(1-\overline{x})$ as an approximation of the $\sigma_x^2 = p(1-p)$ therefore, we can write

$$p_1,\, p_2 \simeq \overline{x} \pm z_{\alpha/2}\sqrt{\frac{\overline{x}(1-\overline{x})}{n}}$$

Thus, the general expression for a confidence interval estimate of p can be expressed as follows:

$$\boxed{\overline{x} \pm z_{\alpha/2}\sqrt{\frac{\overline{x}(1-\overline{x})}{n}}}$$

where $1-\alpha$ is the confidence factor and $z_{\alpha/2}$ is the z value providing an area of $\alpha/2$ in the upper tail of the standard normal probability distribution.

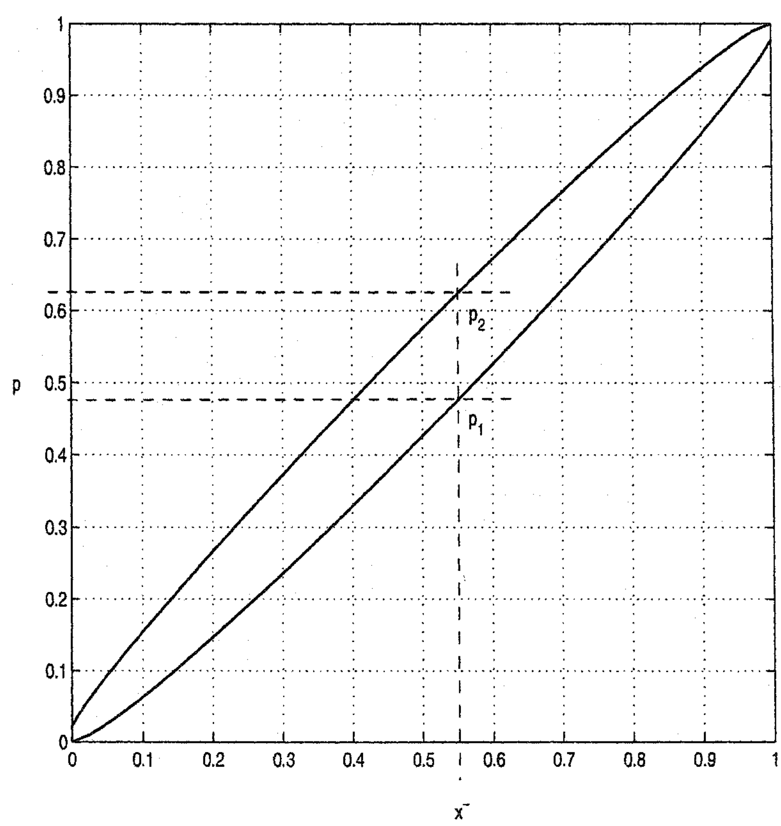

Figure A.2: The roots of the $(\overline{x}-p)^2 = z^2_{\alpha/2}\frac{p(1-p)}{n}$.

Proof-I:

Consider

$$(\overline{x}-p)^2 = z_{\alpha/2}^2 \frac{p(1-p)}{n}$$

$$\left(\overline{x}^2 - 2p\overline{x} + p^2\right) = z_{\alpha/2}^2 \frac{p-p^2}{n}$$

$$\left(1+\frac{z_{\alpha/2}^2}{n}\right)p^2 - \left(2\overline{x}+\frac{z_{\alpha/2}^2}{n}\right)p + \overline{x}^2 = 0$$

$$p_1,\, p_2 = \frac{1}{2\left(1+\frac{z_{\alpha/2}^2}{n}\right)}\left\{\left(2\overline{x}+\frac{z_{\alpha/2}^2}{n}\right) \pm \sqrt{\left(2\overline{x}+\frac{z_{\alpha/2}^2}{n}\right)^2 - 4\overline{x}^2\left(2\overline{x}+\frac{z_{\alpha/2}^2}{n}\right)^2}\right\}$$

Consider

$$\left(2\overline{x}+\frac{z_{\alpha/2}^2}{n}\right)^2 - 4\overline{x}^2\left(2\overline{x}+\frac{z_{\alpha/2}^2}{n}\right) = \frac{z_{\alpha/2}^4}{n^2} + 4\frac{z_{\alpha/2}^2}{n}\left(\overline{x}-\overline{x}^2\right)$$

For $n \gg 1$, it can be written as:

$$\simeq 4\frac{z_{\alpha/2}^2}{n}\left(\overline{x}-\overline{x}^2\right) = 4\frac{z_{\alpha/2}^2}{n}\overline{x}\left(1-\overline{x}\right)$$

therefore,

$$p_1,\, p_2 \simeq= \frac{1}{2\left(1+\frac{z_{\alpha/2}^2}{n}\right)}\left\{\left(2\overline{x}+\frac{z_{\alpha/2}^2}{n}\right) \pm \sqrt{4\frac{z_{\alpha/2}^2}{n}\overline{x}\left(1-\overline{x}\right)}\right\}$$

For $n \gg 1$, it can be written as:

$$p_1,\, p_2 \simeq \frac{1}{2}\left\{(2\overline{x}) \pm 2z_{\alpha/2}\sqrt{\frac{\overline{x}\left(1-\overline{x}\right)}{n}}\right\}$$

$$\boxed{p_1,\, p_2 \simeq \overline{x} \pm z_{\alpha/2}\sqrt{\frac{\overline{x}(1-\overline{x})}{n}}}$$

Proof-II: We would like to use this approximation as the variance:

$$\boxed{\widetilde{\sigma}_x^2 \simeq \overline{x}(1-\overline{x}) \qquad \textit{as an approximation of} \qquad \sigma_x^2 = p(1-p)}$$

Is $\widetilde{\sigma}_x^2$ an unbiased estimate of variance?

$$E\left\{\widetilde{\sigma}_x^2\right\} = E\left\{\overline{x}(1-\overline{x})\right\} = E\left\{\overline{x}-\overline{x}^2\right\} = E\left\{\frac{1}{n}\sum_{i=1}^{n} x_i - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2\right\}$$

$$E\left\{\widetilde{\sigma}_x^2\right\} = \frac{1}{n}\sum_{i=1}^{n} E\left\{x_i\right\} - \frac{1}{n^2}E\left\{\left(\sum_{i=1}^{n} x_i\right)^2\right\} = \frac{1}{n}\sum_{i=1}^{n} p - \frac{1}{n^2}E\left\{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i\, x_j\right\}$$

$$= \frac{n}{n}p - \frac{1}{n^2}E\left\{\sum_{j=1}^{n} x_i^2 + \sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n} x_i\, x_j\right\}$$

$$= p - \frac{1}{n^2}\sum_{i=1}^{n} E\left\{x_i^2\right\} + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n} E\left\{x_i\, x_j\right\}$$

$$= p - \frac{1}{n^2}\sum_{i=1}^{n} E\left\{x_i^2\right\} + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n} E\left\{x_i\right\} E\left\{x_j\right\}$$

$$= p - \frac{1}{n^2}\sum_{i=1}^{n}\left(p^2+\sigma_x^2\right) + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n} p.p = p - \frac{n}{n^2}\left(p^2+\sigma_x^2\right) + \frac{1}{n^2}n(n-1)\,p^2$$

$$= p - \frac{1}{n}p^2 - \frac{1}{n}\sigma_x^2 + p^2 - \frac{1}{n}p^2 = p - p^2 - \frac{1}{n}\sigma_x^2$$

$$= p(1-p) - \frac{1}{n}\sigma_x^2 = \sigma_x^2 - \frac{1}{n}\sigma_x^2 = \sigma_x^2(1-\frac{1}{n})$$

$$\boxed{E\left\{\widetilde{\sigma}_x^2\right\} = \sigma_x^2(1-\tfrac{1}{n})}$$

therefore, $\widetilde{\sigma}_x^2$ is not an unbiased estimate of variance. Note that for $n \gg 1$ it can be written as $E\left\{\widetilde{\sigma}_x^2\right\} \simeq \sigma_x^2$.

Now consider

$$\widehat{\sigma}_x^2 = \frac{n}{n-1}\widetilde{\sigma}_x^2$$

We can find that

$$E\left\{\widehat{\sigma}_x^2\right\} = E\left\{\frac{n}{n-1}\widetilde{\sigma}_x^2\right\} = \sigma_x^2$$

It can be seen that $\widehat{\sigma}_x^2$ results in an unbiased estimate of variance.

Example:

In order to determine the confidence interval for BER estimation consider the following case:

$\overline{x} = 5.46897 \times 10^{-4}$

$n = 18288640$ and $SNR = 14$ dB

Consider $\gamma = 0.99$ and therefore $z_{\alpha/2} = 2.576$.

Since the variance is unknown, we should use scheme-I or scheme-II, to find the confidence interval.

In the following we consider both schemes and compare the results.

Scheme-I:

In order to estimate p (BER), we form the sample variance, and from the information taken from simulation results we find:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2 = \frac{n}{n-1}\overline{x}(1-\overline{x})$$

$$s^2 = 5.465979335586835 \times 10^{-4}$$

$$P_r(\overline{x} - z_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \overline{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}) = 1 - \alpha = \gamma$$

We can thus state with confidence coefficient γ that μ is in the interval $\overline{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$.

$$\Delta = z_{\alpha/2}\frac{s}{\sqrt{n}} = 1.408280435283734 \times 10^{-5}$$

And finally we can calculate:

$$P_r\left\{\overline{x} - z_{\alpha/2}\tfrac{s}{\sqrt{n}} < p < \overline{x} + z_{\alpha/2}\tfrac{s}{\sqrt{n}}\right\} =$$
$$P_r\left\{5.328141956471626 \times 10^{-4} < p < 5.609798043528373 \times 10^{-4}\right\} = 0.99$$

showing that the confidence interval of this BER is within

$$\left\{5.328141956471626 \times 10^{-4},\ 5.609798043528373 \times 10^{-4}\right\}$$

with the probability of 0.99. Note that:

$$\frac{\Delta}{\overline{x}} = 0.02575037777$$

Scheme-II:

In order to estimate p (BER), we use this approximation as the variance:

$$\sigma_x^2 \simeq \overline{x}(1 - \overline{x}).$$

$$\sigma_x^2 \simeq 5.4660090039 \times 10^{-4}$$

And therefore, we can express the confidence interval as $\overline{x} \pm z_{\alpha/2}\sqrt{\frac{\overline{x}(1-\overline{x})}{n}}$.

$$\Delta = z_{\alpha/2}\sqrt{\frac{\overline{x}(1 - \overline{x})}{n}} = 1.408280396782226 \times 10^{-5}$$

And finally we can calculate:

$$P_r\left\{\overline{x} - z_{\alpha/2}\sqrt{\frac{\overline{x}(1-\overline{x})}{n}}z_{\alpha/2} < p < \overline{x} + z_{\alpha/2}\sqrt{\frac{\overline{x}(1-\overline{x})}{n}}\right\} =$$

$$P_r\left\{5.328141960321777 \times 10^{-4} < p < 5.609798039678222 \times 10^{-4}\right\} = 0.99$$

showing that the confidence interval of this BER is within

$$\left\{5.328141960321777 \times 10^{-4},\ 5.609798039678222 \times 10^{-4}\right\}$$

with the probability of 0.99. Note that:

$$\frac{\Delta}{\overline{x}} = 0.02575037707$$

It can be concluded that both schemes provide almost the same results in terms of the confidence interval. Although Scheme-I is more accurate.