Supply Chain Reconfiguration and Inventory Integration in a Stochastic Environment

Hany Osman

A Thesis

in

The Department

of

Mechanical and Industrial Engineering

Presented in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy (Mechanical Engineering) at
Concordia University
Montreal, Quebec, Canada

February 2011

CONCORDIA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared

By: **Hany Osman**

Entitled: **Supply Chain Reconfiguration and Inventory Integration in a Stochastic Environment**

and submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy (Mechanical Engineering)

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the final examining committee:

Dr. S. Li ______________________ Chair

Prof. M.Y. Jaber ______________________ External Examiner

Prof. V. Chvatal ______________________ External to Program

Prof. M.Y. Chen ______________________ Examiner

Dr. O. Kuzgunkaya ______________________ Examiner

Dr. K. Demirli ______________________ Thesis Supervisor

Approved by

____
Chair of Department or Graduate Program Director

__________ ______________________
Dean of Faculty

Abstract

Supply Chain Reconfiguration and Inventory Integration in Stochastic Environment

Hany Osman, Ph.D.

Concordia University, 2011

The problem addressed in this thesis concerns a company that recently has been failing to fulfill its promised delivery dates to its end customers. The problem is analyzed from a supply chain perspective to investigate the reasons behind this failure. The first reason for this delivery lag is the existence of some unreliable suppliers that are not capable of delivering the raw and the machined components on time. The second reason is related to the inefficient inventory systems employed at the existing stockpoints in the chain. The inventory policy at each stockpoint not only fails to provide enough inventory levels to satisfy the downstream demand but it also ignores demand and lead time variations. Furthermore, the company expects a demand increase which will call for a long-term capacity reallocation throughout its supply network. This thesis proposes new methods for deciding on the updates that should take place at the strategic and tactical planning levels of this problem.

At the strategic decision level, the supply chain is reconfigured to reallocate the available capacities and distribute material among the reliable and coordinated suppliers. A bilinear goal programming model is developed to represent the strategic reconfiguration and supplier selection problem studied at this stage. Three goals are considered through this model: distributing material among highly reliable suppliers,

distributing material among well-coordinated suppliers, and distributing material in such a way as to minimize distribution and inventory costs. A modified Benders decomposition algorithm is proposed to handle the complexity of this model. The algorithm saves about 75% of the computation time compared to a generic linearization scheme.

At the tactical level, a joint inventory-production system is designed to decide on the cycle time, the shipping frequency, the order quantity, and the production sequence at each member of the supply chain. A novel formulation of the economic lot and delivery scheduling problem is established to determine the optimal inventory and production sequence policies. Common cycle time and integer-multiplier policies are applied to synchronize the supply chain. A hybrid algorithm integrating linearization, outer approximation, and Benders decomposition techniques is developed to solve the proposed joint inventory-production models. The integer multiplier mechanism attains cost savings up to 16.3% as compared to the common cycle time policy.

To deal with the underlying variations in demand and delivery lead time, two models representing decentralized and centralized safety stock placement approaches are developed. Order statistics distributions are consulted to determine the functional lead time at the multiple sourced stockpoint existing in the chain. Each strategy states the fill rates and the safety amounts required to satisfy the desired end customer service level. Nonlinearity and binary restrictions involved in the centralized model are handled through the Benders decomposition technique. Cost savings between 22.17% and 44.15% are achieved when safety amounts are placed using the centralized policy instead of the decentralized one.

In addition to contributing to the field of research through introducing new models and solution algorithms, the thesis provides the industry with viable supply chain strategies for handling problems such as supplier selection, distribution networks and inventory control.

The thesis is dedicated to the Egyptian revolution, 25^{th} of January 2011.

Acknowledgment

First and foremost, I sincerely offer praise to Allah, my God, who reconciled me to accomplish this thesis and facilitated everything to make it possible.

I am much indebted to Dr. Demirli for his supervision and recommendations during the last four years. At each stage of this thesis, his guidance contributed to the value of the research. I really appreciate the time he shared with me in brainstorming the ideas and methodologies introduced in this thesis. Dr. Demirli also improved my technical writing skills and taught me how to propose the research, define the problem, and describe the results. I gained great experience from working with him as a research and teaching assistant.

I would like to express my gratitude to Concordia University for accepting me as a Ph.D. student and supporting me financially and technically. In particular, I gratefully thank the staff in both the School of Graduate Studies and the Faculty of Engineering and Computer Science who granted me the Concordia Fee Remission Award. Also, I would like to thank the library and the IT-helpdesk staff for their assistance and cooperation.

I kindly offer my regards to all of my professors and colleagues in the Industrial Engineering department at Zagazig University, Egypt, where I have established my industrial engineering background and have been awarded the master degree.

I owe my deepest gratitude to my parents and my wife for their encouragement and moral support. Lastly, I present my sincere respect to all of my teachers at all the educational stages through which I have passed.

Hany Osman

Table of Contents

Appendix

List of Figures

List of Tables

List of Symbols

Supply Chain Reconfiguration and Supplier Selection Model

Chapter 3

Indices:

i: Index set of machined parts, $i = 1,2,\ldots I$

j: Index set of T1-suppliers, $j = 1,2,\ldots J$

k: Index set of T2-suppliers, $k = 1,2,\ldots K$

r: Index set of raw components, $r = 1,2,\ldots R$

t: Index set of time periods, $t = 1,2,\ldots T$

Parameters:

w_1, w_2, w_3: Associated weights assigned to each goal

Z_1, Z_2, Z_3: Specified target of each goal

T_j : Measure of the on-time delivery (OTD) performance of each T1-supplier

T_k : Measure of the OTD-performance of each T2-supplier

F_{jk}: Measure of the preference of T1-supplier j to T2-supplier k

C_{ij} : Cost of obtaining one unit of machined part i from T1-supplier j

C_{rjk}: Cost of delivering one unit of raw component r from T2-supplier k to T1-supplier j

H_{ijt}: Cost of holding one unit of part i at T1-supplier j in time period t

Q_{ijt}: Cost of shortage of one unit of part i at T1-supplier j in time period t

LT_1: Maximum number of T1-suppliers required to be in the network

LT_2: Maximum number of T2-suppliers required to be in the network

A_{jt} : Maximum capacity of the link connecting the company with T1-supplier j at time period t

A_{jkt} : Maximum capacity of the link connecting T1-suppliers j with T2-suppliers k at time period t

S_{rkt} : 0-1 matrix indicating the possibility of supplying the raw component r at T2-supplier k in time period t

U_{jt}: Machining capacity of T1-supplier j in time period t

U_{kt}: Supplying capacity of T2-supplier k in time period t

D_{it} : Company's demand of part i at time period t

S : Scrap ratio

P_{ir} : Usage of part i from component r

G_{jt}: Maximum inventory limit at T1-suppliers j in time period t

E_{it}: Maximum shortage allowed from part i at time period t

N: Minimum number of T1- and T2-suppliers allowed to share in machining or supplying one item respectively, $N > 1$

Pr: Percentage of demand specifying the maximum number of suppliers allowed to share in machining or supplying one item respectively

I_{ij}^{o} : Inventory level at the starting of the first period.

B_{ij}^{o} : Shortage level at the starting of the first period.

M: Very big number

Decision Variables:

X_{ijt}: Amount of part i shipped from T1-supplier j to the company at time period t

X_{rjkt}: Amount of component r shipped from T2-supplier k to T1-supplier j at time period t

R_{ijt}: Capacity allocated to part i from the total capacity of T1-supplier j at time period t

R_{rkt}: Capacity allocated to component r from the total capacity of T2-supplier k at time period t

I_{ijt}: Inventory amount of part i at T1-supplier j at time period t

B_{ijt}: Shortage amount of part i at T1-supplier j at time period t

Lj: Binary variable indicating whether or not T1-supplier j is selected

L_k: Binary variable indicating whether or not T2-supplier k is selected

L_{jk}: Binary variable indicating whether or not there is a relation between T1-supplier j and T2-supplier k

L_{ijt}: Binary variable indicating whether or not part i is assigned to T1-supplier j at time period t

L_{rkt}: Binary variable indicating whether or not component r is assigned to T2-supplier k at time period t

d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-: Deviational variables of the three goals respectively.

Joint Inventory-Production Model

Chapter 4

Indices:

k: Index set of T2-suppliers, $k = 1,2,\ldots. K$

j: Index set of T1-suppliers, $j = 1,2,\ldots. J$

n_k: Number of items at T2-supplier k, $k = 1,2,\ldots. K$

n_j: Number of items at T1-supplier j, $j = 1,2,\ldots. J$

n_a: Number of items at the assembly facility a

i: Stands for items at any facility, k, j or a

q: Stands for sequence positions at any facility, k or j

Parameters:

h_{ki}^{r} : Holding cost per unit of item i as a raw material at T2-supplier k per unit time

h_{ki}^{f} : Holding cost per unit of item i as a finished item at T2-supplier k per unit time

h_{ji}^{r} : Holding cost per unit of item i as a raw material at T1-supplier j per unit time

h_{ji}^{f} : Holding cost per unit of item i as a finished item at T1-supplier j per unit time

h_{ai} : Holding cost per unit of item i at the assembly facility per unit time

D_{ki} : Demand rate of item i at T2-supplier k

D_{ji} : Demand rate of item i at T1-supplier j

P_{ki} : Production rate of item i at T2-supplier k

P_{ji} : Production rate of item i at T1-supplier j

S_{ki} : Set-up time of item i at T2-supplier k

S_{ji} : Set-up time of item i at T1-supplier j

B_{ki} : Production set-up cost of item i at T2-supplier k

B_{ji} : Production set-up cost of item i at T1-supplier j

A_{k} : Transportation cost per delivery at T2-supplier k

A_{j} : Transportation cost per delivery at T1-supplier j

A_{a} : Transportation cost per delivery at the assembly facility

Decision variables:

T : Common cycle time

X_{kiq} : 0-1 variable represents whether or not item i is sequenced in position q at T2-supplier k

X_{jiq} : 0-1 variable represents whether or not item i is sequenced in position q at T1-supplier j

Safety Stock Placement Model

Chapter 5

Indices:

i: Index set of supply chain stages, $i = 1,2,\ldots I$

j: Index set of stockpoints at a given stage, $j = 1,2,\ldots.J_i$

k: Index set of items, $k = 1,2,\ldots.K$

Parameters:

h_{ijk} : Holding cost of item k at stockpoint j in stage i per unit time

sl_k : Service level required to be met of item k

q_{ijk} : Order quantity of item k at stockpoint j in stage i

d_{ijk} : Mean demand of item k at stockpoint j in stage i

l_{ijk} : Mean lead time of item k at stockpoint j in stage i

U_{krst}: 0-1 matrix that specifies whether or not item k passes through the stockpoint r at the most downstream stage and the stockpoint s at the intermediate stage, and the stockpoint t at the most upstream stage.

σd_{ijk} : Standard deviation of demand for item k at stockpoint j in stage i

σl_{ijk} : Standard deviation of lead time for item k at stockpoint j in stage i

σ_{ijk} : Standard deviation of lead time demand for item k at stockpoint j in stage i

Decision variables:

F_{ijk} : Fill rate of item k at stockpoint j in stage i

$E(Z)_{ijk}$: Standardized stockout quantity for a standard normal distribution for item k at stockpoint j in stage i

Z_{ijk} : Standard normal deviate for item k at stockpoint j in stage i

Abbreviations

Analytic Hierarchy Process	AHP
Analytic Network Process	ANP
Analytical Target Cascading	ATC
Benders Decomposition	BD
Bill of Material	BOM
Economic Lot and Delivery Scheduling Problem	ELDSP
Generalized Lambda Distribution	GLD
Goal Programming	GP
Mixed Integer Non-linear	MINL
Mixed Integer Programming	MIP
Normally Distributed Order Statistics	NDOS
Order Statistics	OS
Outer Approximation	OA
Quadratic Assignment	QA
Safety Stock Consolidation	SSC
Safety Stock Placement	SSP
Standard Loss Integral	SLI
Supply Chain Reconfiguration and Supplier Selection	SCRSS

Chapter 1

Introduction

1.1 Supply Chain Modeling and Optimization

One of the most important planning stages in supply chain management is modeling the processes running across the chain, especially those processes that show interactions between supply chain members. Given the fact that actions taken by one member can influence the profitability of others, policies that manage these shared activities should be devised from the supply chain global perspectives. Procurement, resource allocation, and demand management are some examples of joint processes in which deciding upon them from a supply chain standpoint returns benefits to the all the supply chain members.

Supply chain operations are managed through three planning levels, strategic, tactical and operational. A particular decision is categorized into one of these three levels based on how frequently the decision is taken. Table 1.1 classifies decisions considered through supply chain operations management.

Decisions deployed at these levels can be optimized in a hierarchical manner in which results of a parent level are considered fixed while deciding on a child level. Such an approach to planning is followed in Alebachew et al. (2009) who present two models for consecutively establishing strategic and tactical plans. Ahumada et al. (2009) review models that have been developed to plan agricultural supply chains strategically, tactically and operationally. Alternatively, two or more decision phases can be integrated to be planned together. Hammam et al. (2009) plan for strategic and tactical decision levels simultaneously through one model developed to design a supply chain.

Table 1.1 Hierarchy of supply chain decision levels

Planning level	Time horizon	Considered decisions
Strategic	Long term (2+ years)	▪ Number, size, location of facilities ▪ Information and equipment technology required ▪ Long-term raw material and energy contracts ▪ Labor skills needed
Tactical	Quarter to 2 years	▪ Operation hours and output rates ▪ Workforce size ▪ Production decisions ▪ Inventory decisions ▪ Transportation strategies ▪ Subcontracting levels
Operational	Short term	▪ Daily production level and distribution planning ▪ Production scheduling ▪ Material and order processing follow-up ▪ Shipping modes

1.2 Problem Definition

The problem stated in this thesis is a prototype of an industrial case defined at an assembly company. The company is looking to update the supplying strategies of one of the component families used in the final assemblies. The configuration of the supply chain of these components is depicted in Figure 1.1. The last stage of this chain is the company which assembles components into finished products. The intermediate stage is a set of Tier1 (T1) suppliers. The company depends on these suppliers as outsources to perform the machining processes for the components. The initial stage is the raw component suppliers, Tier2 (T2) suppliers. The procurement process starts from the company which sends the demand forecast to the outsourcing T1-suppliers. At this juncture, this forecast is updated based on the bill of material (BOM) to calculate the components forecast which is sent to the raw component T2-suppliers.

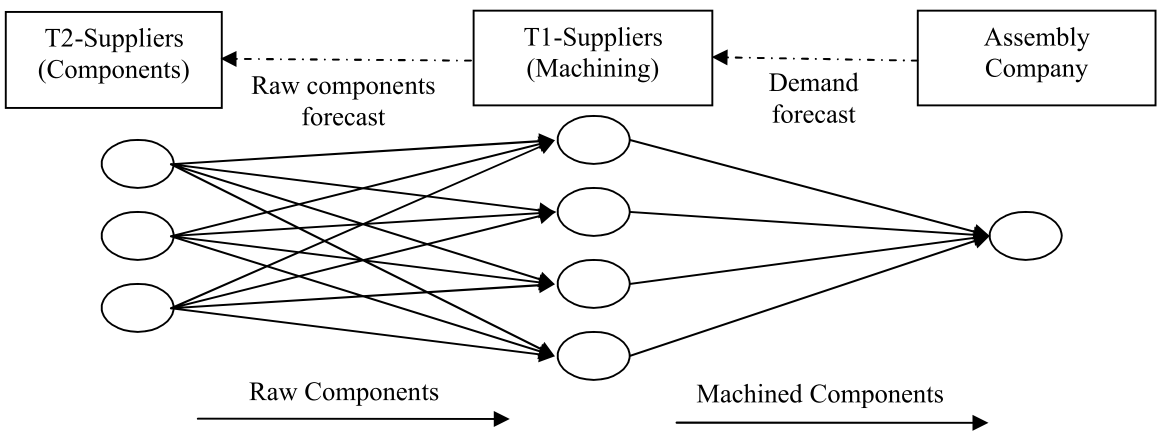

Figure1.1: Supply chain network with material and information flows

The company plans to design new supplying and inventory strategies throughout the supply chain for three reasons. The first reason is related to the delay occurring in receiving the raw components at the T1-stage, and the machined components at the last stage. This serious drawback stems from the poor delivery performance of the suppliers and the inefficient inventory systems employed at each of these stages. These inventory systems are established based on random procurement decisions because some suppliers, especially at T1-stage, do not know precisely when and how much to order from their predecessors. Undoubtedly, these unplanned decisions cause the company to be out of the machined components stock required to run its assembly schedules. The second reason is that the company wants to place sufficient safety amounts across the supply chain to cope with the variations occurring in upstream lead time and downstream demand. The third reason is related to the expected demand increase during the next three years which will call for the reallocation of the available capacities of the suppliers at each stage.

The problem being addressed incorporates different decisions that should be optimized at the strategic and tactical planning levels of the supply chain. Specifically, capacity reallocation and supplier selections require the strategic plans to be updated, while new tactical plans are required to determine the cycle and safety stock levels across the supply chain.

1.3 Research Objectives and Motivations

Throughout the thesis, the problem described in Section 1.2 is analyzed and resolved through the introduction of new strategies enabling the company to overcome the difficulties related to delayed deliveries and ordering decisions. In addition, the strategies introduced will plan for the expected demand increase by reallocating suppliers' available capacities. The problem is investigated from a supply chain perspective, therefore the overall objective of the new strategies is to establish a robust supply chain in terms of capacity utilization, material delivery and inventory control. The new tactics, developed through establishing mathematical models, are not limited to the problem studied in this thesis; they can be adapted for application to other supply chains encountering deficiencies in their decisions at the strategic and tactical levels.

To resolve the problem, three planning stages take place through which mathematical models are developed and new policies are established. The three stages are detailed below in hierarchical order.

First, to cope with the expected increase in demand and improve the delivery performance of the entire supply chain, changes in the current strategic plans should take place. In this regard, the thesis will introduce a strategic supply chain reconfiguration and supplier selection (SCRSS) model that aims at getting rid of those members who cause the supply chain to fail, and reallocating capacities of the selected suppliers. Primarily, the model redistributes material among the reliable and highly coordinated suppliers and secondly it keeps distribution costs at minimum. The new strategy attained at this level will specify material flow throughout the chain on a yearly basis.

Second, to optimize inventory and production decisions taken at the tactical level of planning, a joint inventory-production system will be designed. The system is formulated through the economic lot and delivery scheduling problem (ELDSP) representation. This representation will be employed to synchronize the supply chain so that it can respond rapidly to changes in demand and product designs. Also, synchronization will enhance the coordination among suppliers.

A new, efficient formulation of the ELDSP will be introduced based on the quadratic assignment (QA) representation. Through this formulation, two policies will be investigated to carry out the synchronization. The first policy is the just-in-time policy that restricts each member of the chain to employing the common cycle time policy, while the second is the integer multipliers mechanism that limits the cycle time of a member existing at a given stage to being an integer multiplier of the cycle time of its downstream stage. Each of these synchronization approaches will establish the new ordering and production strategies required at the tactical decision level. This new strategy will determine the cycle time, the order quantity, and the shipping frequency at each supplier and the assembly company. The strategy will also specify the production sequence at each supplier facility.

Third, to cope with the uncertainty of customer demand and supplier lead time, adequate safety amounts should be placed at the relevant stockpoints throughout the supply chain. Two safety stock placement (SSP) policies will be designed in order to enable the prescribed customer service level to be achieved.

- The first policy will be designed based on the decentralized approach of holding safety stocks, in which each stockpoint is responsible for coping with the variability of its successor's demand and predecessor's lead time. A supply chain SSP model will be developed to specify the fill rate along with the sufficient safety amounts that should be employed at each stockpoint to meet the underlying uncertainty. In addition to these two decision variables, the recommended strategy at this stage will specify the reorder point at each stockpoint to complete the identification of the (Q, r) inventory system that should be established.

- The second policy will be designed to set up safety stock consolidation centers throughout the supply chain. In this policy, the safety amounts required from all stockpoints placed in a given stage will be consolidated at the most relevant one among them. The purpose of such consolidation is to reduce these safety amounts, which will be reflected positively on the safety stock holding costs. Such a reduction can be accomplished through pooling the variability of lead time demand encountered by stockpoints of a given stage at one aggregation place. A supply chain consolidation (SSC) model will be proposed to select these aggregation centers based on capacity restrictions, holding costs, and credits given to each candidate center to encourage consolidation.

A framework of these planning stages is shown in Figure 1.2. Strategic decisions are taken first to reconfigure the supply chain. This includes decisions regarding supplier selection, material distribution and capacity utilization. In the intermediate and final stages, results of the material distributions obtained from the topmost planning level will be introduced into the inventory models to represent the demand at each stockpoint. In the same hierarchical manner, the order amounts resulting from the inventory policies devised at the second stage will be inserted into the safety stock models to establish adequate safety stock levels.

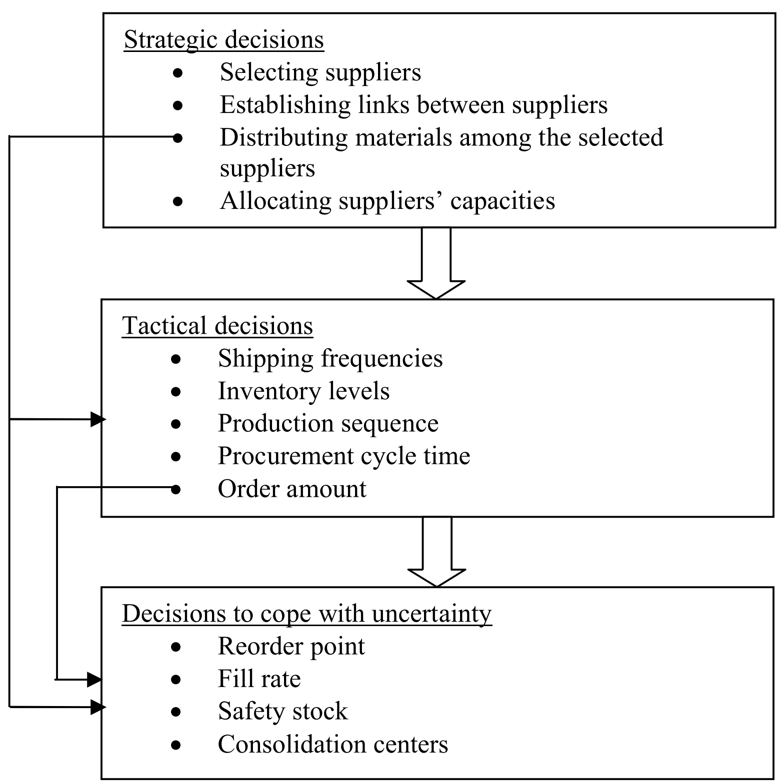

Figure 1.2 Outline of the decisions considered through the three planning stages designed to solve the underlying problem

1.4 Research Methodology

The research investigates a real life problem to ensure that the strategies proposed in this thesis can be applied in the industry. An industrial supply chain problem defined at a Canadian aerospace company is studied. Information about the on-time delivery performance of each member of the chain and coordination with other members is collected. Also, the uncertain environment in which the supply chain functions and the strategic and tactical restrictions imposed on the supply chain activities are defined. Following this, the new objectives that will identify the new supplying strategies are presented.

The decision-making process will take place at three integrated stages, namely the supply chain strategic reconfiguration phase, the inventory and production control phase and the safety stock optimization phase. The relevant literature is reviewed to discover how each of these problems has been addressed by other researchers and to provide a background that will help in developing efficient solution methodologies.

The strategic level combines decisions regarding supplier selection and material distribution. A bilinear mathematical model will be formulated to represent this problem. In this model, bilinearity appears in the formulation of the first objective of the model related to assigning the largest amount of material to the reliable and coordinated suppliers. This is because the amount of material assigned to a given supplier, which is a continuous variable, is multiplied by a binary variable representing the decision taken on this supplier. In order to deal with the difficulty associated with bilinear terms, two mathematical programming techniques can be employed. The first is the linearization approach that linearizes the bilinear model into an equivalent linear model. The second is to apply the Benders decomposition (BD) technique that handles the difficulty of bilinear terms through separating binary variables and continuous variables.

At the tactical planning level, inventory policies and production sequences at each member of the chain have to be established. Given that the sequence of production affects the inventory holding cost of the unprocessed and finished items, it is more economic to incorporate sequencing decisions while determining ordering decisions. The problem resulting from this incorporation along with seeking a synchronized supply chain is known as the ELDSP. The problem is handled in the literature through heuristic and metaheuristic approaches, while it can be handled more efficiently through mathematical programming techniques such as linearization, BD, and outer approximation (OA) techniques. These approaches provide a greater opportunity to reach optimal solutions as compared to those techniques applied in the literature.

Prior to design, the safety stock placement model, the lead times at each tier and the assembly facility have to be determined. Since all the tiers and the assembly facility receive materials from multiple sources having different lead times, the functional lead time at each supply chain member can be calculated by consulting order statistics probability distributions.

Safety stock strategies will be established through the development of mathematical models that tackle the safety stock placement problem from a supply chain perspective. By solving these models through the use of mathematical programming tools, optimal fill rates, safety amounts, and consolidation centers can be determined.

Computational experiments are designed to evaluate the ability of the proposed models to provide feasible solutions to different supply chain configurations. In addition, these experiments are devised to check the efficiency of the proposed methods in reaching optimal solutions to the tested problems. Comparative studies of the alternative policies introduced at the tactical planning level are conducted. For confidentiality reasons, the actual data related to the real case could not be acquired. Instead, different sets of hypothesized data are used to run the experiments.

The methodology used in conducting this doctoral research can be divided into three main stages. The first stage includes defining the problem, and establishing a mathematical and statistical background. The second stage involves developing mathematical models and solution algorithms. In the third stage the proposed models and algorithms are validated by conducting computational experiments. Figure 1.3 illustrates the outline of the applied research methodology.

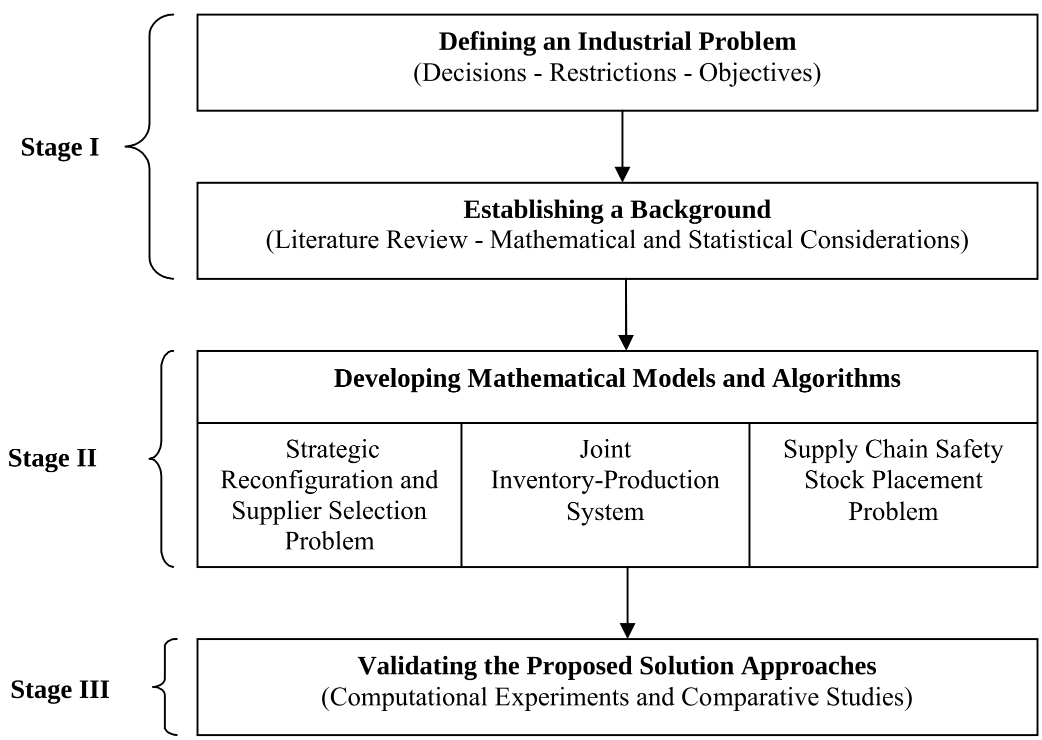

Figure 1.3 Stages of the applied research methodology

In this chapter, the supply chain problem under study has been defined. The problem will be solved in three stages in chapters 3, 4, and 5. The objectives and motivations of this thesis stated in details. The thesis will provide new models, solution approaches and polices that will assist supply chain researchers and practitioners while dealing with supply chain reconfiguration and inventory integration problems. The applied research methodology is discussed to show how the proposed models will be handled through decomposition and linearization techniques. The outline of the thesis is depicted in the following section.

1.5 Thesis Outline

The thesis is composed of seven chapters. The remaining chapters are organized as follows:

- Chapter 2 reviews the literature and summarizes the research that has been conducted on the three underlying sub-problems: the SCRSS problem, the ELDSP, and the SSP problem.
- Chapter 3 introduces the proposed research concerning the strategic part of the problem. The bilinear goal programming model developed to formulate the SCRSS problem is discussed. The chapter also explains the established decomposition algorithm that adapts the classical BD approach to handle goal programming models. The chapter ends with the numerical experiments implemented to validate the model and to compare the proposed algorithm to the generic linearization scheme used as an alternative approach to solve the proposed SCRSS model.

- Chapter 4 deals with the tactical part of the problem. It explains how the proposed supply chain joint inventory-production system is designed. The new formulation of the common ELDSP is explained. The hybrid algorithm developed to solve the two proposed supply chain inventory models is discussed. Experiments on different supply chain problem sizes are conducted to validate the proposed models and evaluate the computational efficiency of the hybrid algorithm. A comparative study is conducted to compare the applied synchronization strategies.
- Chapter 5 tackles the uncertain environment surrounding the supply chain. The proposed safety stock models representing centralization and decentralization strategies of placing safety stocks are demonstrated. The decomposition approach used to solve the proposed SSC model is illustrated. Finally, computational and comparative studies are conducted to assess the developed models and the proposed algorithm, and at the same time show how much saving can be attained through consolidating safety stocks based on results of the centralization policy.
- Chapter 6 concludes the thesis and states its contributions to the field of research and the supply chain community. In addition, some future extensions to the research are suggested.
- The appendix attached at the end of the thesis presents the theory applied in this research. The mathematical programming background is given through a discussion of linearization and decomposition techniques applied to solve the proposed models. Also, some statistical considerations about order statistics (OS) and approximation of the standard loss integral (SLI) are indicated.

Chapter 2

Literature Review

In this chapter, three independent literature reviews are presented to cover the related research that has been conducted on the supplier selection problem, the economic lot and delivery scheduling problem (ELDSP), and the safety stock placement (SSP) problem. Various solution approaches are applied to solve these three models such as mathematical programming techniques, simulation, heuristics, genetic algorithms and multi-agent systems. However, the literature shows that mathematical programming models and techniques are powerful tools that can enable supply chain decision makers to reach optimal supply chain policies for these three problems.

2.1 Supplier Selection Problem

This part of the review focuses on the recent research conducted on the supplier selection problem. The relevant articles that have been published since 2000 are reviewed to show the preferred criteria for selecting suppliers as well as the applied techniques for solving this problem. This section also includes the application of the Benders decomposition (BD) technique to supply chain related problems.

Supplier selection criteria may differ from one problem to another. Among these criteria, cost minimization is the most common one involved in deciding on the selected suppliers. For example, in Yan et al. (2003) cost is the single criterion used in the supplier selection decision process. Logical constraints representing the relationships among products, producers and suppliers are incorporated with system constraints in a mixed integer model that minimizes production, transportation, distribution and procurement supply chain costs. A simplified representation of these logical constraints is developed to obtain a reduced number of inequalities that replace the logical constraints in the proposed model. In Tanonkou et al. (2006) suppliers are selected based on shipment and ordering costs, and safety stock cost at the supplied distribution centres. The work integrates the facility location problem with the supplier selection problem in a nonlinear programming model that is solved using the Lagrangean Relaxation method.

Akanle and Zhang (2008) tackle the problem of satisfying customer orders by choosing the optimal set of resources, suppliers of various components, assembly plants, and transportation options at minimal cost. The time involved in delivering and manufacturing components and assembling final products is restricted by the due date of the order. A multi-agent system is developed to model the resource options existing in the supply chain. An iterative agent bidding process is proposed to allow the agent-based supply-chain model to interact with customer orders representing the future demand.

Other criteria such as delivery performance, customer satisfaction, quality, flexibility and environmental performance are also used to decide which suppliers are selected. Ehap and Benita (2000) develop an iterative method to solve a multi-objective model handling both strategic and operational decisions of the supply chain. Each level of decisions is represented by a sub-model. The strategic sub-model aims at optimizing supply chain configuration and material flow decisions, while the operational model aims at achieving a trade-off among cost, service level and flexibility measures, and at the same time accommodates for anticipated demand.

Altiparmak et al. (2006) propose a mixed integer nonlinear model with three conflicting objectives: cost, service level, and capacity utilization balance. The set of Pareto optimal solutions is obtained using a genetic algorithm. The model offers different alternatives to the decision makers by applying two different weighting approaches to the conflicting objectives. Cost, delivery performance and environmental performance are the three multi-objectives of the genetic algorithm proposed by Komoto et al. (2005). The proposed algorithm selects a suitable reconfiguration rule that governs distribution of orders among suppliers. A discrete event simulation technique is used to evaluate these objectives. The reconfigured chain is examined to check whether or not it is capable of satisfying environmental and delivery requirements.

In Dotoli et al. (2005), a hierarchical decision system is proposed to design an integrated e-supply chain. At the first level, candidates for each stage of the chain are ranked based on their financial return and cost, risk management, flexibility, service quality, service time, and environmental performance. Then a network design module that represents the integrated e-supply chain with a digraph describing partners, material and information links, determines the configuration of the network. The selected network configuration is evaluated through a validation module by comparing tactical and operational performance indices.

Zhiying and Jens (2007) propose a multi-objective supplier selection model solved using a genetic algorithm to recover the nonlinearity of the model. A trade-off among four criteria, cost, quality, delivery and flexibility, is used to support the decision for selecting suppliers. Huang and Keskar (2007) integrate strategic thinking with quantitative optimization in order to make the optimal decisions on supplier selection that match the targeted business strategy. They propose a set of comprehensive metrics classified under seven categories: reliability, responsiveness, flexibility, cost and financial, assets and infrastructure, safety, and environmental metrics.

Other researchers use special techniques to assign weights to the criteria controlling the supplier selection decisions. Williams (2007) combines analytic hierarchy process (AHP) and goal programming (GP) to model a multi-objective decision-making problem that aims at selecting the best warehouses among the possible candidates. The AHP is used to give weight or priority to warehouses based on two conflicting criteria: customer satisfaction level and operational cost. These priorities are incorporated in a GP model that considers system and goal constraints. In Liao and Kao (2010), the AHP approach is incorporated with the Taguchi loss function and multi-choice goal programming model in an integrated approach to identify the selected suppliers among the candidates. The Taguchi loss function is applied to estimate the total deviations from the targets specified for the five criteria used to select suppliers. These criteria are product quality, offering price, delivery lead time, service, and warranty degree.

Demirtas and Üstün (2008) integrate analytic network process (ANP) and a multi-objective mixed integer linear programming model to solve the supplier selection and order allocation problem. Weights are assigned to the multi-criteria using the ANP approach that extends the concepts of the AHP. The set of the efficient solutions of the model is obtained using the ε-constraint and reservation level driven Tchebycheff procedure methods. The quality of the solutions obtained by these two algorithms is compared using an additive utility function. The suppliers are evaluated according to 14 criteria that are involved in four control hierarchies: benefits, opportunities, cost and risks. In Lin et al. (2011), the ANP approach is integrated with the TOPSIS technique to evaluate candidate suppliers based on their unit price, quality defect rate and delayed delivery rate. The resulted score of each supplier represents the coefficient of the objective function of the linear model formulated to represent the supplying process at the motherboard manufacturer, Asus Tech., in Taiwan.

Huang and Qu (2008) deal with a specific type of supply chain in which the alternative suppliers of a stage have the right to decide autonomously on the configuration of their respective upstream stages. The methodology applied to configure this kind of chain is analytical target cascading (ATC) in which each alternative enterprise existing in a stage should be modeled as an individual ATC element. They introduce new kinds of elements, ‘‘OR‘‘ elements, to the ATC in order to represent alternative enterprises at each stage. Each ‘‘OR‘‘ element will select its best alternative element based on predefined internal working logic and evaluation criteria. Table 3.1 presents a classification of the reviewed articles that investigate the supplier selection problem. The table shows fourteen different criteria that can be used individually or simultaneously to select a supplier. A detailed review of such a supply chain problem is presented in Ho et al. (2010). This review provides more criteria to select suppliers than those appearing in Table 2.1.

Table 2.1 Taxonomy of the reviewed articles investigating the supplier selection problem

	Cost	Service level	Flexibility	Capacity utilization	Delivery performance	Environmental performance	Risk management	Quality	Safety	Responsiveness	Assets and infrastructure	Warranty degree	Lead time	Coordinated suppliers
Ehap and Benita (2000)	*	*	*											
Yan et al. (2003)	*													
Komoto et al. (2005)	*				*	*								
Dotoli et al. (2005)	*		*		*	*	*	*						
Altiparmak et al. (2006)	*	*	*	*										
Tanonkou et al. (2006)	*													
Zhiying and Jens (2007)	*		*		*			*						
Huang and Keskar (2007)	*		*		*	*			*	*	*			
Williams (2007)	*	*												
Demirtas and Üstün (2008)	*			*				*						
Akanle and Zhang (2008)	*													
Liao and Kao (2010)	*	*						*				*	*	
Lin et al. (2011)	*				*			*						
This Thesis	*				*									*

The BD technique has been applied to solve supply chain network design problems after the generalization of the technique by Geoffrion (1972) in order to handle nonlinearity using nonlinear duality theory. In Geoffrion and Graves (1974), a multi-commodity capacitated distribution system design problem is solved to optimality using a small number of Benders cuts.

Van Roy (1986) introduces a unified framework that combines BD and Lagrangian relaxation to solve the capacitated facility location problem. For the same problem, Wentges (1996) strengthens Benders cuts through two heuristics that modify a new cut in order to accelerate upper and lower bounds convergence. Üster et al. (2007) also accelerate the classical BD technique by introducing three different approaches that add multiple cuts using dual problem disaggregation.

Dogan and Goetschalckx (1999) have developed an integrated design method based on BD to solve a multi-period production-distribution system with seasonal customer demand and multiple network configurations. For the supply chain design problem with multiple transportation modes, the BD technique outperforms a proposed simplex-based branch-and-bound method when applied to complex problems (Cordeau et al., 2006).

Cakir (2008) reformulates the multi-commodity, multi-mode distribution planning problem using the BD approach. The algorithm reaches the optimal solution showing the validity of using Benders' cuts on such problems. Costa (2005) provides a detailed survey of the application of the BD approach to the fixed charge network design problem.

In this thesis, the BD approach is adapted to deal with bilinear goal programming models. The model proposed in chapter 3 to select suppliers and distribute materials among them is decomposed into two goal programming models. The first model which is represented by the master problem is formulated to select suppliers based on their on time delivery performance and the level of coordination among them. The second model that is represented by the sub-problem assigns materials to the selected suppliers showing the highest on time delivery performance and level of coordination, and at the same time to minimize the supply chain transportation and inventory costs.

2.2 Economic Lot and Delivery Scheduling Problem

The main concern of the ELDSP is how to decide on cycle time at each stage of a supply chain as well as production sequence at each supplier node. The objective is to fulfill end customer demand with minimum transportation, and inventory holding and setup costs. This problem is critical because it not only integrates supply chain stages but also incorporates inventory and production decisions. An optimal solution has been found for a single supplier and single assembler running on the common cycle time policy (Jensen and Khouja, 2004; Ju and Clausen, 2004; and Torabi et al., 2006). For the same policy, the optimal solution is also reached by Nikandish et al. (2009) but for small and medium problem instances of a supply chain comprising one supplier at the initial stage, multiple manufacturers at the intermediate stage and multiple retailers at the final stage. For larger and more complex supply chains, the problem has not yet been tackled, neither for the common cycle policy nor for other synchronization policies like the integer multiplier mechanism.

The ELDSP was the motivation for research by Hahm and Yano (1992). The strategy introduced in their work determines the production and delivery intervals of a single item through a supply chain including one supplier, and one assembly facility. They prove that the optimal solution of such a problem must have an integer ratio between the production interval and the delivery interval. This leads to the development of a simple procedure that provides optimal values of these intervals by examining different cases for this ratio.

For the multiple items case, Hahm and Yano (1995-a) apply the common cycle time policy at supplier and assembler sites. The objective is to find the production cycle time at the supplier site that is followed by a single shipment to the assembler at the end of each cycle. A heuristic procedure is proposed to find the cycle time and production sequence that minimize inventory and transportation costs. An error bound procedure is developed to evaluate the quality of the proposed heuristic. Hahm and Yano (1995-b) relax the assumption by Hahm and Yano (1995-a) regarding the single shipment at the end of each cycle. They allow for multiple shipments through a production cycle which requires the multiple items to be partitioned into groups. A lower bounding approach is proposed to assess the quality of the heuristic used in sequencing the items. The heuristic is found to be very close to the lower bound and also surpasses the single shipment strategy.

The power of two multipliers policy is proposed by Hahm and Yano (1995-c). In this policy, each item is produced 2^m times, where m is an integer that may differ from one item to another. The delivery policy in Hahm and Yano (1995-c) differs from that proposed by Hahm and Yano (1992-a), in which multiple equally-spaced shipments take place at each cycle. The experimental study conducted in this work shows two distinguished benefits of the power of two multipliers mechanism. First, it outperforms the common cycle time policy. Second, it results in cost savings as compared to other approaches that sequentially decide on production and delivery decisions.

Khouja (2000) extends the work of Hahm and Yano (1995-a) to consider quality issues that require a rework cost to be added, and volume flexibility that requires the production rate to be adjusted. The solution obtained using a proposed heuristic is compared to the global optimal solution obtained by enumerating all the possible sequences. The algorithm reaches the optimal solution in 68% of the problems tested and provides a very close objective value for the rest of the problems. Khouja (2003) proposes some incentive alignment mechanisms to encourage members of a supply chain to accept the synchronized policy of running on equal cycle time. The work focuses on the simple serial supply chain with one facility at each stage. A heuristic method is proposed to sequence the items involved at each stage of the chain. The method employs the RAND algorithm proposed by Kaspi and Rosenblatt (1991) and the sequencing rules proposed by Hahm and Yano (1995-a).

Jensen and Khouja (2004) develop an algorithm for solving the ELDSP of a single supplier and one assembler inventory system to optimality in polynomial time. The algorithm starts with finding upper and lower bounds of the cycle time. Then it partitions this feasible range of the cycle time into a number of intervals in which each one has a unique optimal sequence. The algorithm guarantees optimality because it enumerates the associated cost of each interval, then selects the minimum among them. This algorithm is computationally studied in Ju and Clausen (2004). Problems containing a small number of items are handled in efficient solution times, but the algorithm takes a longer time for large-sized problems. Consequently, Ju and Clausen (2004) introduce a hybrid algorithm that mixes the heuristic proposed by Hahm and Yano (1995-a) with the Jensen and Khouja (2004) method. The hybrid algorithm reaches the optimal solution for large-sized problems in a shorter time as compared to the heuristic proposed by Jensen and Khouja (2004) which performs better for small-sized problems.

Torabi et al. (2006) develop a new mixed binary nonlinear model to represent the ELDSP for a simple supply chain. The chain is composed of single supplier manufacturing multiple items on a flexible flow line and an assembly facility running on the same cycle time as the supplier. An enumeration method that is capable of reaching an optimal solution to small scale problems is proposed. For medium and large scale problems, a hybrid genetic algorithm is developed. The algorithm not only provides near optimal solutions for large scale chains, but also it outperforms the enumeration method applied to small-sized chains.

Another mixed binary nonlinear programming model to represent the ELDSP of a two-echelon supply chain is proposed by Torabi and Jenabi (2009). The model allows for lot streaming at the supplier site to minimize the manufacturing lead time. The proposed model could not be solved to optimality. Instead, two hybrid genetic algorithms are proposed to solve this complex model under two different strategies. The first algorithm assumes that each production cycle time is a power of two multiplier of a basic period, while the second assumes production cycle time of each product is an integer multiplier of this basic period. Computational experiments show that the latter policy gives better solution quality than the former, while the former has less computation time than the latter.

A more complex inventory system is studied by Kim et al. (2006) in which a single manufacturer supplies multiple retailers. Each retailer receives its particular item on a number of equal deliveries and all items are rotating on the same cycle time. An efficient heuristic is used to decide on the raw material procurement policy, the production sequence, and the delivery quantity and frequency of the finished items.

In chapter 4, the ELDSP for a three stages supply chain is addressed. The most upstream and intermediate stages are composed of two sets of T2 and T1 suppliers respectively. The most downstream stage is an assembler. The problem is formulated in the quadratic assignment representation. A hybrid algorithm that combines linearization, Benders decomposition and outer approximation approaches is developed to solve large scale instances of this problem to optimality. Two synchronization policies are investigated to synchronize the supply chain based on the common cycle time approach and the integer multipliers mechanism.

2.3 Safety Stock Placement in Supply Chains

In the safety stock literature, two concepts can be applied while solving the SSP problem using analytical models. The first one, proposed by Simpson (1958), is called the local stock concept, which means that each stockpoint can control its inventory decisions autonomously. Alternatively, Clark and Scarf (1960) introduce the echelon stock concept wherein each stockpoint has to consider the inventory of its successors.

Simpson's (1958) model can be considered as one of the initial works that dealt with demand uncertainty in multi-stage production and inventory systems. The model determines the combination of service times which refer to the safety stock that should be offered by each stage to satisfy customer orders at a predetermined service level. Inderfurth (1991) extends this work and establishes the optimal policy for divergent supply chains taking into account the impact of demand correlation on SSP using risk pooling effects in divergent systems.

The work introduced by Clark and Scarf (1960) started a new research problem of safety stock in supply chains by introducing the echelon stock concept. Their model focuses on a single-item serial system undergoing stochastic demand and constant lead time. The solution algorithm, which is based on discounted cost dynamic programming, can reach the optimal inventory policy. A similar study is carried out by Schmidt and Nahmias (1985) but for a supply chain of one item being assembled from two different components. Rosling (1989) generalizes the Clark and Scarf (1960) model by considering a general assembly supply chain. This general assembly system can be represented as a serial system in which the optimal policy can be found using the Clark and Scarf model.

Inderfurth and Minner (1998) deal with different service measures that restrict the amount of safety stock. These measures are probability of stockout occurrence and quantity. Their proposed model seeks to find the safety amount that guarantees covering demand fluctuations during a time period called the coverage time. Minner (1997) derives the forward and backward recursive formulas to find the optimal policy of these coverage times. Dynamic programming algorithms that need little computation are used to find the solution for serial, convergent and divergent supply chains.

Graves and Willems (2000) simplify the SSP problem from its stochastic nature to a deterministic optimization by imposing some key assumptions. Each stage in the chain is assumed to work with a base-stock inventory policy having some guaranteed service time or stock to satisfy the stationary demand of its downstream stages. The source of uncertainty of the problem is the variability of customer demand while the replenishment lead time is assumed to be deterministic. The stochastic lead time demand is bounded by a maximum value that can be obtained from its mean and standard deviation for a given customer service level. They consider only supply chains that can be represented as a spanning tree while general supply chains under the same assumptions are handled by Graves and Lesnaia (2004). The optimal solution of such a general case is found using a branch and bound algorithm.

Sitompul and Aghezzaf (2006) extend the problem addressed in Graves and Willems (2000), but for serial supply chains, to consider the capacity limitation constraints. They state that safety stock amounts have to be updated by a tabulated correction factor that relates safety stock with a measure representing degree of capacity to cover demand variations. Further relationships between demand variability, capacity, delivery lead time and safety stocks are investigated in Sitompul et al. (2007). These relationships provide a deep understanding of the SSP problem in capacitated supply chains. Relying on these relationships, a solution approach is proposed and tested using Mont Carlo simulation.

Kim et al. (2005) deal with two echelon supply chains comprising a single supplier and multiple retailers under non-stationary demand pattern. Two adaptive inventory control models are proposed, namely centralized and decentralized models. In the centralized model, the supplier who manages the inventory system for all retailers should have a safety lead time to deal with demand uncertainty faced by the retailers. Conversely, in the decentralized model, each retailer should hold sufficient safety stock amounts to deal with demand variations. Boulaksil et al. (2009) apply a simulation-based approach to determine the safety stocks of a multi-stage, multi-item supply chain. A mathematical model representing the supply chain planning problem is solved several times under a rolling horizon setting. The model allows backordering at each stage and does not state any assumptions about the demand distribution that is generated before solving the model in the form of a series of forecasts. The backorder amounts resulting from the model represent the safety stock size that should be kept in the chain to prevent backordering.

Jung et al. (2008) propose a linear programming model that determines the base stock level under dependency of service measures at different stages of a supply chain. The inventory level at production facilities and the base stock level at warehouses are also constrained by the safety production capacity limit. Louly and Dolgui (2009) consider an assembly system facing constant demand and discrete distributed random lead time of components delivery. The model is solved using a branch and bound algorithm and is valid for any discrete probability distribution. The model also provides significant savings for assembly systems undergoing unreliable component delivery time. Persona et al. (2007) tackle the safety stock determination problem for assemble-to-order and make-to-order systems. Their cost-based analytical models consider demand as a normally distributed random variable while they consider constant lead time for delivering the sub-assemblies.

The case of stochastic lead time and customer demand for a single stock is handled by Eppen and Martin (1988). An exponential smoothing model is proposed to estimate the unknown distribution parameters of lead time and demand. Using regression relationships implemented through simulation and factorial experiments, Hayya et al. (2009) obtain a regression equation that represents optimal cost, order quantity, and the safety stock factor in terms of cost parameters, standard deviation of demand, and standard deviation of lead time. The proposed model considers order crossover occurrence by working on the parameters of the effective lead time demand distribution. Ettl et al. (2000) develop an inventory-queue model generating the base stock level at each store of a supply network. The network consists of a collection of stockpoints stocking only one item. The nominal lead time at each store is assumed to be independent and identically distributed while demand is considered to be non-stationary. The optimal solution is found by driving the gradients in explicit form then using a conjugate gradient routine that searches for the solution.

Using Markove chain queue models, Saharidis et al. (2009) analyze two control policies for two echelon supply chains: base stock control and echelon base stock control. Each echelon has a subcontractor that is required to supply the echelon during stockout periods. The underlying demand follows Poisson distribution while the production time is exponentially distributed. Numerical results show that jointly deciding on safety stock, subcontracting, and backordering is more profitable than independent control policies for these three decision problems.

Simchi-Levi et al. (2005) propose a unified framework that integrates stages employing continuous review base stock inventory control for tree structured supply chains. The underlying lead time is assumed to be stochastic, sequential and exogenously determined with known probability distribution while the customer demand follows an independent Poisson process. Based on the stochastic service model approach, a recursive equation that shows the backlog at each stage and characterizes the dependencies among stages is developed.

The safety stock placement strategies proposed in this thesis consider the lead time and demand as two independent normally distributed random variable. The service level applied in the proposed models is the fill rate which represents a specified percentage of demand that will be satisfied from stock. The decentralized and centralized strategies proposed in chapter 5 establish the safety amounts required to be placed at each of the multi-sourced stockpoint, and at each given stage existing in the supply chain respectively.

Chapter 3

Supply Chain Reconfiguration and Supplier Selection Problem

3.1 Introduction

This chapter discusses the solution methodology used to resolve the strategic part of the underlying problem. The objectives at this strategic level are to provide the future capacity utilization strategy required to cope with the expected demand increase, and to maximize the on-time delivery performance of the supply chain. In this regard, the supply chain has to be reconfigured in order to get rid of the unreliable suppliers, and to utilize the available capacities of the reliable and highly coordinated suppliers in satisfying the future demand.

Some practical cases of reconfigured supply chains are summarized in Section 3.2. Section 3.3 identifies the decisions considered at the strategic planning level along with the objectives of the future supplying strategies. Section 3.4 discusses the proposed bilinear supply chain reconfiguration and supplier selection (SCRSS) model. The generic linearization scheme applicable to linearize this model is illustrated in Section 3.5. The modified Benders decomposition (BD) method proposed to handle the SCRSS model is explained in Section 3.6. This adapted BD algorithm handles bilinear goal programming models in which the complicating binary variables affect the values of the deviational variables of goal attainment. Computational experiments recorded in Section 3.7 show that the modified BD algorithm outperforms a generic linearization scheme by reaching the optimal solution for large-sized problems with about 75% reduction in the computation time.

3.2 Supply chain Reconfiguration: Real Cases

Several industrial examples are summarized in this section to show the practical reasons that call for reconfiguring a supply chain, and also to emphasize the significant role of reconfiguration in saving budgets and utilizing resources. For instance, the rapid advancement of technologies in the computer industry was the main driver behind reconfiguring the Digital Equipment Corporation supply chain (Arntzen et al., 1995). The new strategy reduced the cumulative cost by $1 billion and the assets by $400 million and increased unit utilization by 500%. P&G's supply chain has been reconfigured to optimize product sourcing problems (Camm et al., 1997). After two years of implementing modelling recommendations, 12 sites have been closed and annual savings have reached $250 million per year.

The BASF North American distribution system is also a good example of a company that realized great benefits from reconfiguring its supply network (Sery et al., 2001). In 1995, the firm placed the objectives of reconfiguring this network. The objectives aim at reducing distribution costs and providing a sufficient level of customer service. The proposed model outcomes resulted in cost savings of $10 millions and increasing the volume delivered within one day from 77% to 90%. Hewlett-Packard (HP) achieved cost savings of $10 million by reducing the number of contract manufacturers (Laval et al., 2005). For a divergent supply chain reconfiguration, Vila et al. (2006) analyze raw material processing when there is a limited and regulated availability of raw material. The study was applied in a partnership with three large Canadian lumber companies and a 15.4% increase in after tax profits was attained.

3.3 Strategic Planning Level

As the company in this study will encounter a demand increase during the next three years, plans at the strategic level should have an emphasis on providing the adequate capacity to face this anticipated demand. Also, the established policies at this level should be able to retrieve the company's competitive position that has been affected by the unsatisfactory on-time delivery performance of the unreliable suppliers. Accordingly, the company is planning to rely on those suppliers who are capable of fulfilling their promised delivery dates and to discard the unreliable suppliers from any future plan.

To establish the strategic plans, the supply chain depicted in Figure (1.1) has to be reconfigured in order to select the suppliers, reallocate their capacities and distribute the raw and machined components among the selected suppliers. T2-suppliers at the initial stage and T1-suppliers at the intermediate stage are chosen based on their ability to deliver the raw and machined components on time. The raw components are distributed among tiers at these two stages based on their on-time delivery performance and the coordination recognized between them. A minimum distribution cost is also considered at this level but at a lower priority.

The strategic reconfiguration problem is handled as a multiple-objective optimization problem, in which the company plans to assign the largest amount of material to the reliable and coordinated suppliers and at the same time seeks to achieve minimum distribution cost. This multi-objective optimization problem is handled through the goal programming (GP) approach. The proposed GP model considers strategic constraints that are imposed on supply chain reconfigurations. These include constraints specifying the size of each stage, the available capacity of each link and node existing in the chain, material balance at each node, and inventory constraints. The output of this strategic model is the reconfigured chain described by number of selected tiers and material distribution among them on an annual basis.

3.4 Supply Chain Reconfiguration and Supplier Selection Model

A bilinear GP model is developed to establish the future supplying strategy of the company. The model incorporates the three goals concerned with the on-time delivery performance of each supplier, coordination among T1- and T2-suppliers, and distribution and inventory costs. Based on the priority assigned to each goal, a compromised solution is attained that minimizes the deviation from each target planned for each goal.

$$Min\ G = w_1 d_1^- + w_2 d_2^- + w_3 d_3^+ \tag{3.1}$$

Subject to

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} T_j L_j X_{ijt} + \sum_{r=1}^{R} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{t=1}^{T} T_k L_k X_{rjkt} - d_1^+ + d_1^- = Z_1 \tag{3.2}$$

$$\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T} F_{jk} L_{jk} X_{rjkt} - d_2^+ + d_2^- = Z_2 \tag{3.3}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} C_{ijt} X_{ijt} + \sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T} C_{rjkt} X_{rjkt}$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} H_{ijt} I_{ijt} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} Q_{ijt} B_{ijt} - d_3^+ + d_3^- = Z_3 \tag{3.4}$$

Objective function (3.1) aims at minimizing the deviations d_1^-, d_2^-, d_3^+ from the three values Z_1, Z_2 and Z_3 representing the target of each goal. The values of these targets can be found by solving the SCRSS model with a single objective that represents each goal individually. The first goal (3.2) seeks to assign as much material as possible to those reliable suppliers having the highest on-time delivery performance while the second goal (3.3) tries to dispense the raw components among the T1-suppliers preferred by T2-suppliers.

The on-time delivery measures T_j and T_k reflect the recognized delivery performance of each supplier during the last year. Based on the percentage of the orders delivered on time, a given supplier is assigned a score between 0 - 100. The preference measure F_{jk} represents how far T1-supplier j is favored by T2-supplier k. Preference is measured on a scale between 0 -100 based on the ability of T1-suppliers to forecast the dependent demand of the company and the stability of their ordering policy. If the management desires to decide on these parameters T_k , T_k , F_{jk} subjectively, values of these parameters can be represented in the model by fuzzy numbers. Distribution and inventory costs are kept at a minimum through the achievement of the third goal (3.4). Transportation costs are represented as a percentage of unit price at T2-suppliers or unit machining cost at T1-suppliers and are embedded in the cost parameters C_{rjkt} and C_{ijt} respectively.

The first goal (3.2) has the highest priority as it assigns the material to those suppliers having the best on-time delivery performance. At a lower level, distributing the raw components among the T1-suppliers preferred by T2-suppliers can help in enhancing the material on-time delivery performance. Hence, the highest weight is assigned to the on-time delivery goal followed by the second goal. Cost minimization has to be at the lowest priority because distributing the material among reliable and well coordinated suppliers might be accompanied by a high distribution cost. Hence, w_1, w_2, w_3, is introduced in the model to represent the penalty associated with each unit of deviation from the target of each goal. The analytic hierarchy process or the analytic network process could be utilized to find values of these weights. Altering the values of these weights results in different scenarios among which the decision maker can select the preferred one.

Equations (3.5) and (3.6) restrict the number of selected T1- and T2-suppliers by the given parameters LT_1 and LT_2 respectively. Equations (3.7) and (3.8) ensure that a link is established only among the selected suppliers, while equations (3.9) and (3.10) assign machined and raw components to those selected suppliers.

$$\sum_{j=1}^{J} L_j \leq LT_1 \tag{3.5}$$

$$\sum_{k=1}^{K} L_k \leq LT_2 \tag{3.6}$$

$$L_{jk} \leq L_j \qquad j = 1, 2..., J \ , \ k = 1, 2....T \tag{3.7}$$

$$L_{jk} \leq L_k \qquad j = 1, 2..., J \ , \ k = 1, 2....T \tag{3.8}$$

$$L_{ijt} \leq L_j \qquad i = 1, 2..., I \, , \ j = 1, 2..., J, \ t = 1, 2...T \tag{3.9}$$

$$L_{rkt} \leq L_k \qquad r = 1, 2..., R \, , \ k = 1, 2..., K, \ t = 1, 2...T \tag{3.10}$$

Relying on a single source to supply the material is very risky. Also, having many sources for material procurement may not lead to a highly coordinated supply chain. The purpose of constraints (3.11) and (3.12) is to assign a given item to at least N number of suppliers. This parameter should be set to a value greater than one to avoid relying on a single source. Equations (3.13) and (3.14) prevent the assigning of any specific item to too many sources through the parameter Pr that forces the assigned material to be greater than a fraction of that item's demand. For example, if it is required to share the item among at most five suppliers, Pr should be set equal to 0.2. Moreover, the fraction Pr restricts the shipping amounts to satisfy a minimum allowed limit assuring the practicality of assigning a product to a supplier.

$$\sum_{j=1}^{J} L_{ijt} \geq N \qquad i=1,2...,I\,,\ t=1,2...T \qquad (3.11)$$

$$\sum_{k=1}^{K} L_{rkt} \geq N \qquad r=1,2...,R\,,\ t=1,2...T \qquad (3.12)$$

$$X_{ijt} \geq Pr\, D_{it}\, L_{ijt} \qquad i=1,2....I,\ j=1,2...J,\ t=1,2....T \qquad (3.13)$$

$$\sum_{j=1}^{J} X_{rjkt} \geq Pr \sum_{i=1}^{I} D_{it}\, P_{ir}\, L_{rkt} \qquad r=1,2....R,\ k=1,2...K,\ t=1,2....T \qquad (3.14)$$

Equations (3.15) and (3.16) restrict the total shipping amounts through each link to satisfy the maximum capacity of that link at any time period, whereas equations (3.17) and (3.18) limit the shipped amount from each item to the allocated capacity to that item at each node.

$$\sum_{i=1}^{I} X_{ijt} \leq A_{jt} \times L_{j} \qquad j=1,2....J\,,\ t=1,2....T \qquad (3.15)$$

$$\sum_{r=1}^{R} X_{rjkt} \leq A_{jkt} \times L_{jk} \qquad j=1,2...,J,\ k=1,2....K\,, t=1,2....T \qquad (3.16)$$

$$X_{ijt} \leq R_{ijt} \qquad i=1,2....I\,,\ j=1,2....J\,,\ t=1,2....T \qquad (3.17)$$

$$\sum_{j=1}^{J} X_{rjkt} \leq R_{rkt} \qquad r=1,2....R,\ k=1,2....K\,,\ t=1,2....T \qquad (3.18)$$

In the underlying supply chain, raw material suppliers cannot supply all varieties of raw material, which is not the case for T1-suppliers that can perform all required machining operations. Equations (3.19) and (3.20) reserve capacity to the items assigned to a given supplier. In equation (3.19), if the binary variable L_{ijt} is equal to zero, the reserved capacity R_{ijt} will be zero, otherwise this reserved capacity will be positive. Similarly in equation (3.20), capacity of a given T2-supplier k is reserved to the assigned raw component r if and only if the binary variable L_{rkt} is equal to 1 and the parameter S_{rkt}, that represents the ability of the supplier to provide the raw component r, is equal to 1. Equations (3.21) and (3.22) ensure that the summation of the quotas assigned to each item should not exceed the capacity of each supplier.

$$R_{ijt} \le M\, L_{ijt} \qquad i = 1, 2....I,\ j = 1, 2...J,\ t = 1, 2...T \qquad (3.19)$$

$$R_{rkt} \le M\, S_{rkt}\ L_{rkt} \qquad r = 1, 2....R,\ \ k = 1, 2...K,\ \ t = 1, 2...T \qquad (3.20)$$

$$\sum_{i=1}^{I} R_{ijt} \le U_{jt}\ \ L_j \qquad j = 1, 2....J\ ,\ \ t = 1, 2....T \qquad (3.21)$$

$$\sum_{r=1}^{R} R_{rkt} \le U_{kt}\ \ L_k \qquad k = 1, 2....K\ ,\ \ t = 1, 2....T \qquad (3.22)$$

The company's demand is met through equation (3.23) while equations (3.24), (3.25) and (3.26) are the material balance, inventory and shortage limits at T1-suppliers respectively. For some parts, the relation to their raw components is not one to one. Parameter p_{ir} in equation (3.24) obtained from the bill of material of T1-suppliers represents the relation between the machined component i and the raw part r. This parameter gives more generality to the model to be used in case of considering T1-suppliers as assemblers. As a coordinated supply chain, a shortage policy is placed to limit the shortage amount of each item from all the suppliers. Initial inventory and shortage amounts are specified through equations (3.27) and (3.28), respectively. The rest of the equations (3.29)-(3.36) are the non-negativity and binary restrictions on the decision variables.

$$\sum_{j=1}^{n} X_{ijt} = D_{it} \qquad i = 1, 2....I, \quad t = 1, 2....T \tag{3.23}$$

$$(1+s)\sum_{i=1}^{I} p_{ir}\left(X_{ijt} - I_{ijt-1} + I_{ijt} + B_{ijt-1} - B_{ijt}\right) = \sum_{k=1}^{K} X_{rjkt} \quad r = 1, 2...R,\ j = 1, 2...J,\ t = 1, 2...T \tag{3.24}$$

$$\sum_{i=1}^{I} I_{ijt} \le G_{jt} \qquad j = 1, 2..., J,\ t = 1, 2...T \tag{3.25}$$

$$\sum_{j=1}^{J} B_{ijt} \le E_{it} \qquad i = 1, 2...I,\ t = 1, 2...T \tag{3.26}$$

$$I_{ijt} = I_{ij}^{o} \qquad i = 1, 2...I,\ j = 1, 2...J,\ t = 0 \tag{3.27}$$

$$B_{ijt} = B_{ij}^{o} \qquad i = 1, 2...I,\ j = 1, 2...J,\ t = 0 \tag{3.28}$$

$$X_{ijt}, R_{ijt}, I_{ijt}, B_{ijt} \ge 0 \qquad i = 1, 2....I,\ j = 1, 2...J,\ t = 1, 2...T \tag{3.29}$$

$$X_{rjkt} \ge 0 \qquad r = 1, 2....R,\ j = 1, 2...J,\ k = 1, 2...K,\ t = 1, 2...T \tag{3.30}$$

$$R_{rkt} \ge 0 \qquad r = 1, 2....R,\ k = 1, 2...K,\ t = 1, 2...T \tag{3.31}$$

$$L_j \textit{ is binary} \qquad j = 1, 2, ...J \tag{3.32}$$

$$L_k \textit{ is binary} \qquad k = 1, 2, ...K \tag{3.33}$$

$$L_{jk} \textit{ is binary} \qquad j = 1, 2, ...J,\ k = 1, 2, ...K \tag{3.34}$$

$$L_{ijt} \textit{ is binary} \qquad i = 1, 2, ...I,\ j = 1, 2, ...J,\ t = 1, 2, ...T \tag{3.35}$$

$$L_{rkt} \textit{ is binary} \qquad r = 1, 2, ...r,\ k = 1, 2, ...K,\ t = 1, 2, ...T \tag{3.36}$$

$$d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^- \ge 0$$

The first two goals are the sources of bilinearity in the model, because they comprise binary variables multiplied by continuous variables. The binary variables represent the selected suppliers and the established links among them, whereas the continuous variables represent material distribution through the network. Although the model is basically proposed to handle a reconfiguration problem, it can be applied to configure new supply chains by distributing material among the best candidate suppliers.

3.5 Generic Linearization Approach

While dealing with mixed integer bilinear models, one possible way to resolve this bilinearity is to linearize the model using linearization schemes (Peterson, 1971; Glover, 1975, 1984; Adams and Sherali, 1990; and Adams and Forrester, 2007). Peterson (1971) transforms a bilinear model to an equivalent linear one by applying equations (3.37) and (3.38) to the model. Section 2.1.1 provides a background of this linearization approach.

$$lL \leq Y \leq uL \tag{3.37}$$

$$X - u(1-L) \leq Y \leq X - l(1-L) \tag{3.38}$$

Where:

l: is the lower bound on the continuous variable *X*,

u: is the upper bound on the continuous variable *X*,

Y: is the new variable replacing the bilinear term *LX.*

If the binary variable *L* equals zero, *Y* will be equal to zero as well because the first constraint is binding in this case, while the second constraint is the binding one if the value of the binary variable is equal to one. In that case, *Y* is equal to *X*.

Glover's (1975) scheme is the same as the one proposed in Peterson (1971) except that it handles the multiplication of a binary variable by a function *F(w)* in a discrete or continuous variable *w*. Glover (1984) provides a scheme where the relation between the continuous variables and the proposed auxiliary variables replacing the bilinear term is not established and hence it does not accommodate for terms including the continuous variable only. A more recent scheme introduced by Adams and Sherali (1993) is not applicable to the proposed SCRSS model since it restricts each constraint to be a function of either the binary or the continuous variable. The SCRSS model, however, involves many constraints in which both variables appear together. Because the auxiliary variable

Y that replaces LX will appear in equations (3.2) and (3.3), the linearization approach of Adams and Forester (2005) is not applicable here. Their reduced scheme is built given that the auxiliary variable Y should not appear in the functional constraints.

The price being paid for these linearization methods is an increase in problem size through addition of new variables and constraints. For large scale models, increased problem size has a detrimental effect on the computation time because the mixed integer programming (MIP) solver will account for these added variables and constraints at all branching nodes. Below, the modified BD technique to alleviate this problem and reduce the solution time is presented.

3.6 Modified Benders Decomposition Algorithm

A different method for handling bilinear models is to apply the generalized BD technique (Geoffrion, 1972) which can be used when there are complicating variables that prevent the application of a straightforward method to the problem. The BD approach transforms a bilinear model to a linear one that optimizes the values of the continuous (non-complicating) variables for given values of the binary (complicating) variables. The master problem, comprising the constraints on these complicating variables and the added cuts obtained from the sub-problem, optimizes the values of the complicating variables then passes on these values to the sub-problem. After each iteration, a step that checks the upper and lower bounds obtained from both problems should take place. The background of the BD technique is given in Section A.2.1.

Codato and Fischetti (2006) propose an algorithm to solve MIP problems using combinatorial Benders cuts when the master problem is represented by a pure binary model. The algorithm can handle two cases. In the first case, the original objective function depends only on the continuous (non-complicating) variables. Consequently, the master problem is solved with no objective. Instead, an updated bounding constraint is added to the sub-problem to guarantee better values for its objective in the next iteration. Whether or not the sub-problem is feasible, a combinatorial Benders cut is added to the master problem to avoid reaching a previously obtained solution (0-1 combination). In this case, the added constraint acts as a feasibility and optimality cut. The algorithm stops when the master problem becomes infeasible which means that all the basic feasible points are evaluated. In the second case, the original objective has zero-coefficient for the non-complicating variables. Thus, the objective considered in the master problem is the original objective, while the sub-problem has no objectives in this case. To solve this, the algorithm iterates only with the combinatorial feasibility cut in order to update the values of the complicating variables that are optimized based on the objective function of master problem.

In the proposed SCRSS goal programming model, the variables appearing in the objective function are only the deviational variables d_1^- , d_2^- , d_3^+ which can be considered as non-complicating variables. These deviational variables are directly affected by the complicating binary variables. Consequently, the master problem in this particular case can include an objective function that guides the search for the best 0-1 combination of the binary variables toward faster convergence. For example, if the deviational variable of the first goal, shown in equation (3.2), which assigns material to reliable suppliers, is required to be minimized, the model should select suppliers having the highest delivery performance. This necessitates contemplating a new goal (3.40) that considers some parts, $\sum T_j L_j$ and $\sum T_k L_k$, from equation (3.2) in the master problem to select those suppliers. Similarly, another new goal that links each T2-supplier with its preferred T1-suppliers (3.41) should be considered which results in formulating the master problem as a GP model. The target values of each goal Y_1 and Y_2 can be obtained by maximizing each goal independently subject to the constraints given by equations (3.5) - (3.8) and (3.32) - (3.34).

In the modified BD algorithm, both the master problem and the sub-problem are represented as GP models. The objective of the master problem is to select reliable suppliers, while the sub-problem optimally distributes material among them. The combinatorial Benders cut, given by equation (3.42), is added to the master problem when the sub-problem could not reach a feasible solution for the given values of complicating variables. On the other hand, if the sub-problem has a feasible solution for those given values, the classical Benders cut, shown in equation (3.43), is added to the master problem to find better values of the binary variables using duality theory. The following subsections show how the BD approach can be adapted to solve the proposed SCRSS model.

3.6.1 Master Problem

In this problem, suppliers are primarily selected and links among them are established through optimizing the binary variables L_j, L_k, L_{jk} considered to be the complicating variables. The optimality cut (3.43) updates the values assigned to these variables if they are not optimal to the original problem. The other binary variables, L_{ijt} and L_{rkt}, are considered as non-complicating variables since they do not appear in any bilinear terms. Hence, their values will be determined from the sub-problem. The branch-and-cut algorithm that starts with relaxing the integrality condition on the binary variables is applied to solve the master problem. The objective function of the master problem is shown in equation (3.39). In addition to the constraints given by equations (3.5)-(3.8) and (3.32)-(3.34), the master problem includes the following equations (3.40)-(3.43). If the sub-problem is infeasible, a feasibility cut (3.42) is added (Codato and Fischetti, 2006). Conversely if it is feasible, an optimality cut (3.43) is added (Geoffrion, 1972).

$$Min\ g = w_1 d_4^- + w_2 d_5^- + \alpha \tag{3.39}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} T_j L_j + \sum_{r=1}^{R} \sum_{j=1}^{J} \sum_{k=1}^{K} T_k L_k - d_4^+ + d_4^- = Y_1 \tag{3.40}$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} F_{jk} L_{jk} - d_5^+ + d_5^- = Y_2 \tag{3.41}$$

$$\sum_{j:L_j^p=0} L_j + \sum_{j:L_j^p=1} (1-L_j) + \sum_{k:L_k^p=0} L_k + \sum_{k:L_k^p=1} (1-L_k) \geq 1 \qquad p = 1, 2 .....f \tag{3.42}$$

$$\alpha \geq w_1 d_1^{-h} + w_2 d_2^{-h} + w_3 d_3^{+h} + \sum_{j=1}^{J} \lambda_j (L_j - L_j^h) + \sum_{k=1}^{K} \mu_k (L_k - L_k^h)$$

$$+ \sum_{j=1}^{J} \sum_{k=1}^{K} \gamma_{jk} (L_{jk} - L_{jk}^h) \qquad h = 1, 2, ...q \tag{3.43}$$

$$d_4^+,\ d_4^-,\ d_5^+,\ d_5^- \geq 0$$

In the optimality cut (3.43), λ_j, μ_k, and γ_{jk} are the dual variables associated with the constraints that assign values to each of the binary variables L_j, L_k and L_{jk} respectively in the sub-problem. h and p are indices of the feasible and infeasible solutions of the sub-problem respectively. Based on the values of these dual multipliers, the optimality cut determines which binary variables should keep their values, which should be leveled up to one, and which should be reduced down to zero. Function α provides a lower estimate of optimal value of the objective function of the sub-problem for given values of the binary variables L_j, L_k and L_{jk}. In order to ensure the boundedness of this problem, α should have a minimum limit.

3.6.2 Sub-Problem

In the modified BD algorithm, the objective function of the sub-problem is identical to the original objective (3.1). The problem is solved subject to equations (3.2)-(3.4), (3.9)-(3.31), (3.35) and (3.36) plus the following equations (3.44)-(3.46) equating the values of the complicating variables to those obtained from the master problem.

$$L_j = L_j^h \quad : \quad \lambda_j \qquad j = 1, 2....n \tag{3.44}$$

$$L_k = L_k^h \quad : \quad \mu_k \qquad k = 1, 2....t \tag{3.45}$$

$$L_{jk} = L_{jk}^h \quad : \quad \gamma_{jk} \qquad j = 1, 2....n \quad , k = 1, 2....t \tag{3.46}$$

3.6.3 Optimality Check

To show the difference between the classical Benders bounds and the applied bounds in the modified BD algorithm, consider equation (3.47) that shows a typical objective function optimizing complicating variables *x* and non-complicating variables *y*. Based on the BD approach, this objective can be decomposed into two objectives. The first one, which optimizes the complicating variables (3.48), belongs to the master problem, while the second one, optimizing the non-complicating variables for given values of the complicating variables (3.49), belongs to the sub-problem (Conejo et al., 2006).

$$Min \sum_{i=1}^{n} c_i x_i + \sum_{j=1}^{m} d_j y_j \tag{3.47}$$

$$Min \sum_{i=1}^{n} c_i x_i + \alpha \tag{3.48}$$

$$Min \sum_{j=1}^{m} d_j y_j \tag{3.49}$$

The lower and upper bounds applied in the classical BD approach are shown in equations (3.50) and (3.51). The lower bound is obtained from the relaxed master problem while the upper bound is obtained from the restricted sub-problem.

$$Z_L^h = \sum_{i=1}^{n} c_i \, x_i^h \; + \alpha^h \tag{3.50}$$

$$Z_U^h = \sum_{i=1}^{n} c_i \, x_i^h \; + \sum_{j=1}^{m} d_j \, y_j^h \tag{3.51}$$

The modified BD algorithm adapts the classical BD technique in order to handle the special case of formulating the objective functions considered by the master problem and the sub-problem. The deviational variables d_1^-, d_2^-, and d_3^+ appearing in the original objective function (3.1) depend on both the complicating variables L_j, L_k, and L_{jk} and the non-complicating variables X_{ijt}, X_{rjkt}, I_{ijt}, and B_{ijt}, while the deviational variables d_4^- and d_5^- appearing in the objective function of the master problem (3.39) depend only on the complicating variables L_j, L_k and L_{jk}. So, if the deviational variables d_1^-, d_2^-, d_3^+, d_4^- and d_5^- are replaced by complicating and non-complicating variables in both objectives (3.1) and (3.39), terms including complicating variables in the master problem and the original problem objectives are different. Moreover, the objective function of the sub-problem is the original objective that involves contribution of all the variables. This formulation differs from that of the classical BD technique, in which the contribution of the complicating variable to the original objective is considered in the master problem and the contribution of the non-complicating variables is considered in the sub-problem. So, the formulation of the objective functions used in the modified BD algorithm is not the same as that used in the classical BD approach (3.47)-(3.49). Consequently, the upper and lower bounds (3.50) and (3.51) used in the classical BD technique that consider the contributions of the complicating variables to both bounds can not be applied in the modified BD algorithm.

Function α appearing in equation (3.43) aims at finding better values of the complicating variables L_j, L_k and L_{jk} with respect to the original objective function represented in the right-hand side of the equation. So, if the value of this original objective function obtained by solving the sub-problem is equal to that value of α determined by solving the master problem, it means that complicating variables have reached their optimal values. Equation (3.52) shows the optimality condition used to recognize the convergence between master and sub-problem formulated in the modified BD algorithm. The left-hand side of the equation is the upper bound obtained from the sub-problem while the right-hand side represents the lower bound resulting from the master problem. This condition can be recognized in the lower and upper bounds used in the classical BD approach (3.50) and (3.51). At the optimal iteration both bounds are equal which means that the second term in the right-hand sides of equations (3.50) and (3.51) should be equal, justifying the condition.

$$w_1 d_1^{-h} + w_2 d_2^{-h} + w_3 d_3^{+h} = \alpha^h \tag{3.52}$$

3.7 Computational Experiments

Experiments were performed using a computer with 4-2.2 GHz AMD Opteron 64-bit processors and 8 GB RAM. Both algorithms were coded using AMPL (Fourer et al., 2003), and solved using CPLEX 11. The solver option was set to solve integer problems using a branch-and-cut algorithm and to apply the dual-simplex method to solve the primal problem of linear models. Other options were also tried but the difference in solution time is not found to be significant.

The proposed algorithm is compared to Peterson's (1971) generic linearization approach (3.37) and (3.38) mentioned in Section 3.5. The linearization approach adds auxiliary variables and constraints to the bilinear SCRSS model in order to develop the equivalent linear model. Table 3.1 demonstrates a comparison between solving the SCRSS model through its equivalent model and solving it by the modified BD algorithm.

Table 3.1 Comparison between the proposed algorithm and the generic linearization scheme proposed by Peterson (1971)

Problem No.	Number of products	Num. of raw materials	Num. of T1-supplier	Num. of T2-supplier	Modified Benders algorithm				Linearization			%Time saving
					Const.	Var.	Gen. cuts	Time min.	Const.	Var.	Time	
1	4	6	7	6	1213	1458	70	0.1	6032	3049	2.3 sec.	--
2	4	6	7	6	1213	1458	155	0.9	6032	3049	3 sec.	--
3	7	10	12	8	2960	4747	221	1.6	21074	10754	1.2 min.	--
4	7	10	12	8	2960	4747	496	8.3	21074	10754	4.9 min.	--
5	10	13	14	10	4658	8515	92	2.2	38796	19850	11.4 min.	81
6	10	13	14	10	4658	8515	365	5.7	38796	19850	27.9 min.	80
7	14	18	17	11	15082	7295	145	2.3	70186	35987	20.8 min.	89
8	14	18	17	11	15082	7295	681	27.3	70186	35987	93.1 min.	71
9	15	20	20	12	9086	20623	191	9.1	98396	50318	25.7 min.	65
10	15	20	20	12	9086	20623	1143	36.5	98396	50318	83.2 min.	56
11	19	22	23	13	11912	28351	248	12.2	134514	69125	55.2 min	78
12	19	22	23	13	11912	28351	532	49.9	134514	69125	5.1 hr.	84
13	23	25	25	15	15370	39426	301	12.9	189632	97396	60.1 min.	79
14	23	25	25	15	15370	39426	1252	72.1	189632	97396	6.6 hr.	82

For a given problem instance, each algorithm results in a different problem size. To illustrate that, consider a supply chain that includes 23 machined components, 25 raw components, 25 T1-suppliers, 15 T2-suppliers, and three time periods. The linearization approach will add about 58,000 new variables and 174,300 new constraints to the original model. On the other hand, the generated cuts through applying the BD approach will not yield a model having such a huge size.

The tested problems shown in Table 3.1 are all solved to optimality. Seven different problem sizes are tested; each problem size is tested twice with two different sets of input parameters to study the effect of changing values of these parameters on solution time. This influence is noticeable in Table 3.1 through the 10th and the 13th problems. The 10th problem, which has smaller dimensions than the 13th problem, has a longer solution time. This illustrates that input data can increase the solution time and the number of generated cuts required to reach the optimal solution.

The linearization approach outperforms the modified BD method in solving small-sized problems, whereas the modified BD algorithm shows its computational efficiency if it is applied to larger problem configurations. To clarify that, consider the linearization of the first problem comprising 1,213 constraints and 1,458 variables using Peterson's (1971) approach. An equivalent problem including 6,032 constraints and 3,049 variables is solved in 2.3 seconds, while the number of generated cuts required to solve this problem instance using the modified BD algorithm is 70 and the solution time is 6.7 seconds. On the other hand, the number of generated cuts required to solve the last problem including 15,370 constraints and 39,426 variables is 1,252 and the solution time is 72.1min, whereas linearizing this problem instance results in an equivalent problem solved in 6.6 hr. Except for the first four problems, the computational results prove the efficiency of the modified BD algorithm as compared to the linearization approach for solving larger models with the same input parameters. For those large models tested, solution time saving varies from 56% to 89% as shown in the last column of the table.

Table 3.2 shows distribution of the solution time between the master problem and the sub-problem. By comparing the total solution time, the master-problem solution time and the sub-problem solution time of two problems with the same size, it is noticeable that as the number of generated cuts increases, most of the increase occurring in the total solution time is consumed in solving the master problem. To clarify that, consider the last two experiments that belong to the same supply chain size. In the 13^{th} experiment implemented in 12.9 min, there are 302 master problems solved in 5.8 min and the 302 sub-problems are solved in 7.1 min, while the last experiment, which includes 951 more master problems and 951 more sub-problems than the 13^{th} one, is executed in 72.1 min. This conclusion is expected given that the number of constraints of the binary master-problem is increased by one with each added cut, while the size of the sub-problem remains constant. Such accumulation of added cuts results in consuming 42 min of the 59.3 min increase in the total solution time to the master problem.

Table 3.2 Master and sub-problem solution time through applying the proposed modified Benders algorithm

Problem No.	Number of constraints	Number of variables	Generated cuts	Solution time		
				Master problem	Sub-problem	Total time
1	1213	1458	70	4.1 sec.	2.6 sec.	6.7 sec.
2	1213	1458	155	43.1 sec.	6.9 sec.	49 sec.
3	2960	4747	221	1.1 min.	0.5 min.	1.6min.
4	2960	4747	496	7.32 min.	59 sec.	8.3 min.
5	4658	8515	92	0.7 min.	1.5 min.	2.2 min.
6	4658	8515	365	2.8 min.	2.9 min.	5.7 min.
7	15082	7295	145	0.7 min.	1.6 min.	2.3 min.
8	15082	7295	681	20.5 min.	6.8 min.	27.3 min.
9	9086	20623	191	4.7 min.	4.4 min.	9.1 min.
10	9086	20623	1143	24.2 min.	12.3 min.	36.5 min.
11	11912	28351	248	4.9 min.	7.3 min.	12.2 min.
12	11912	28351	532	33.4 min.	16.5 min.	49.9 min.
13	15370	39426	301	5.8 min.	7.1 min.	12.9 min
14	15370	39426	1252	47.8 min.	24.3 min.	72.1 min.

The decisions obtained through solving the SCRSS model establish the future supplying strategy of the company. The strategy is characterized by fulfilling the company's requirements regarding demand and delivery performance through utilizing the capacities of the reliable and coordinated suppliers. Moreover, results of the SCRSS model provide the amounts of raw and machined components distributed among the supply chain members on a yearly basis. Consequently, the contribution of this model is that it is not only selects suppliers based on three criteria but also it distributes the material to the selected highly reliable and well coordinated suppliers at minimum inventory and transportation costs. Other criteria such as, quality, service level, flexibility, and environmental performance could be also considered in the model by formulating the goal equation of each of these criteria. Having the results of the strategic level on hand, the next step is to design the inventory system at each member to establish the ordering policy from upstream stages.

Chapter 4

Supply Chain Integrated Production-Inventory System

4.1 Introduction

Combining inventory with production sequence decisions is a common problem in the literature known as the economic lot and delivery scheduling problem (ELDSP). In this problem, it is required to establish the synchronization policy that coordinates between inbound production scheduling and the outbound deliveries. The overall objective is to minimize the transportation, inventory holding and setup costs across the entire supply chain. The literature on the ELDSP, reviewed in Section 3.2, shows that a supply chain can be synchronized based on a common cycle time policy, an integer multipliers policy, and an integer powers of two multipliers policy. The common cycle time policy, which fully synchronizes the supply chain, forces all inventory systems existing in the chain to run on equal cycle time T. Such an equal cycle time policy has failed to guarantee optimal schedules for the ELDSP problem (Hahm and Yano, 1995).

In the integer multipliers policy, which partially synchronizes the supply chain, the cycle time at each stage is an integer multiplier of that time at its adjacent downstream stage. The cycle time of each firm given by the third policy is an integer power of two multiplier of a basic cycle time.

A new formulation of the ELDSP is proposed in this thesis based on the quadratic assignment (QA) representation. The developed nonlinear mixed integer model is solved using a hybrid algorithm through linearization, outer approximation (OA) and Benders decomposition (BD) techniques. Two cases are studied, the common cycle time policy and the integer-multiplier policy. Computational experiments show the efficiency of the conducted approach to reach the optimal solution for both cases in a short time. Furthermore, experiments demonstrate that a cost saving up to 16.29% can be achieved by synchronizing the supply chain inventory system using the integer-multiplier mechanism instead of the common cycle time policy.

The following section identifies the decisions under consideration at the tactical planning level of the problem defined in Section 1.2. The section also shows the hierarchical link between this level and the results previously obtained in Chapter 4. Section 4.3 illustrates advantages of supply chain synchronization policies as compared to the independent inventory policies. The proposed joint inventory production model is discussed in Section 4.4. The model involves bilinear and polynomial terms, and nonlinear terms representing the inventory setup cost. Section 4.5 shows the linearization schemes used to transform bilinear and polynomial terms into equivalent linear terms. The OA approach is applied to linearly approximate the nonlinear setup cost terms, and the BD technique is deployed to handle the complex binary variables existing in the OA master problem. The hybrid OA-BD algorithm used to solve the equivalent model obtained through the linearization stage is explained in Section 4.6. Section 4.7 shows changes that have to be made in the proposed model to represent the ELDSP when the supplier stages apply the integer-multipliers policy. Computational experiments performed on the proposed model and algorithm are presented in Section 4.8.

4.2 Tactical Planning Level

The strategic reconfiguration model proposed in Chapter 3 specifies material flow through the network on a yearly basis. However, some questions regarding inventory and production management have not yet been answered. For example, if the first T2-supplier is required to provide the first T1-supplier with a specific amount per year, what is the inventory policy that states the number of orders spreading this amount through the year? What is the production sequence at each tier? Should all tiers have to be synchronized at the same cycle time?

In this chapter, an integrated production-inventory policy is proposed to answer the above questions from a supply chain perspective. Results of the proposed model determine the tactical inventory and production decisions that should be taken to solve the problem addressed in this thesis. These include decisions regarding shipping frequency, replenishment cycle time, order quantity, and production sequence at each node of the chain. Other tactical decisions required to cope with the uncertainty of demand and lead time will be studied in the following chapter.

The material distribution strategy proposed at the strategic level is considered while designing the joint inventory-production system at the tactical level. Specifically, the value of the decision variable X_{ijt} that represents the amount of part i shipped from T1-

supplier j to the company at time period t is handled here as the input value of the annual demand D_{ji} of item i at T1-supplier j. Similarly, the annual demand of each raw component at each T2-supplier can be found from the value of decision variable X_{rjkt} by summing the amounts of component r shipped from T2-supplier k to all T1-suppliers.

4.3 Synchronization versus Independent Policies

Independent inventory policies which apply a different cycle time at each stockpoint of the chain do not allow for synchronization. Synchronization enhances the coordination among stockpoints and allows for better vision of material movement. Moreover, the full synchronization of a supply chain leads to a better response to changes in demand and product designs as compared to the partial synchronization approach and the independent policies (Khouja, 2003).

Synchronizing a supply chain either through full or partial synchronization strategy often represents an advantage for some members of the chain and a disadvantage for others. This is because members at different stages have conflicting objectives. For example, stockpoints having low holding cost and high setup cost seek to employ their independent longer cycle time policies while shorter cycle time policies are favored by stockpoints having high holding cost and low setup cost. Khouja (2003) proposes three incentive alignments to allow for supply chain synchronization. These incentives call for altering the unit holding cost, the ordering cost, and the unit cost in such a way that members are encouraged to accept synchronization.

4.4 Common Cycle Time Policy

With a common cycle time policy, all stages of the supply chain are fully synchronized at an equal cycle time T. Assumptions of the ELDSP for this just-in-time policy are given below:

1. All stages are running on an equal cycle time T, which is a decision variable.
2. Components at each supplier node are produced on a single production line.
3. All the components produced are shipped in one shipment at the end of the production cycle.
4. Shipping amounts are equal to the units demanded at the subsequent stage.
5. Production and demand rates are deterministic and constant.
6. At downstream stages, the holding cost per unit increases because of the added value to the components.
7. At each supplier node, the setup cost is sequence independent.
8. Delivery charges are constant.
9. The production setup costs and times at the assembly facility are negligible, so they do not affect the ordering policy.

To derive the chain wide inventory cost, consider an inventory system of raw material and finished items at a given tier, T1 or T2, handing out three items. The profile of this inventory system is depicted in parts (a) and (b) of Figure 4.1 respectively. Each of the three items has an annual demand D, a production setup time S and a production rate P. Part (c) shows the inventory levels of four machined components being assembled in the final products at the company site. Each of these items could go into more than one final assembly. Using this figure, the chain-wide inventory cost can be easily established.

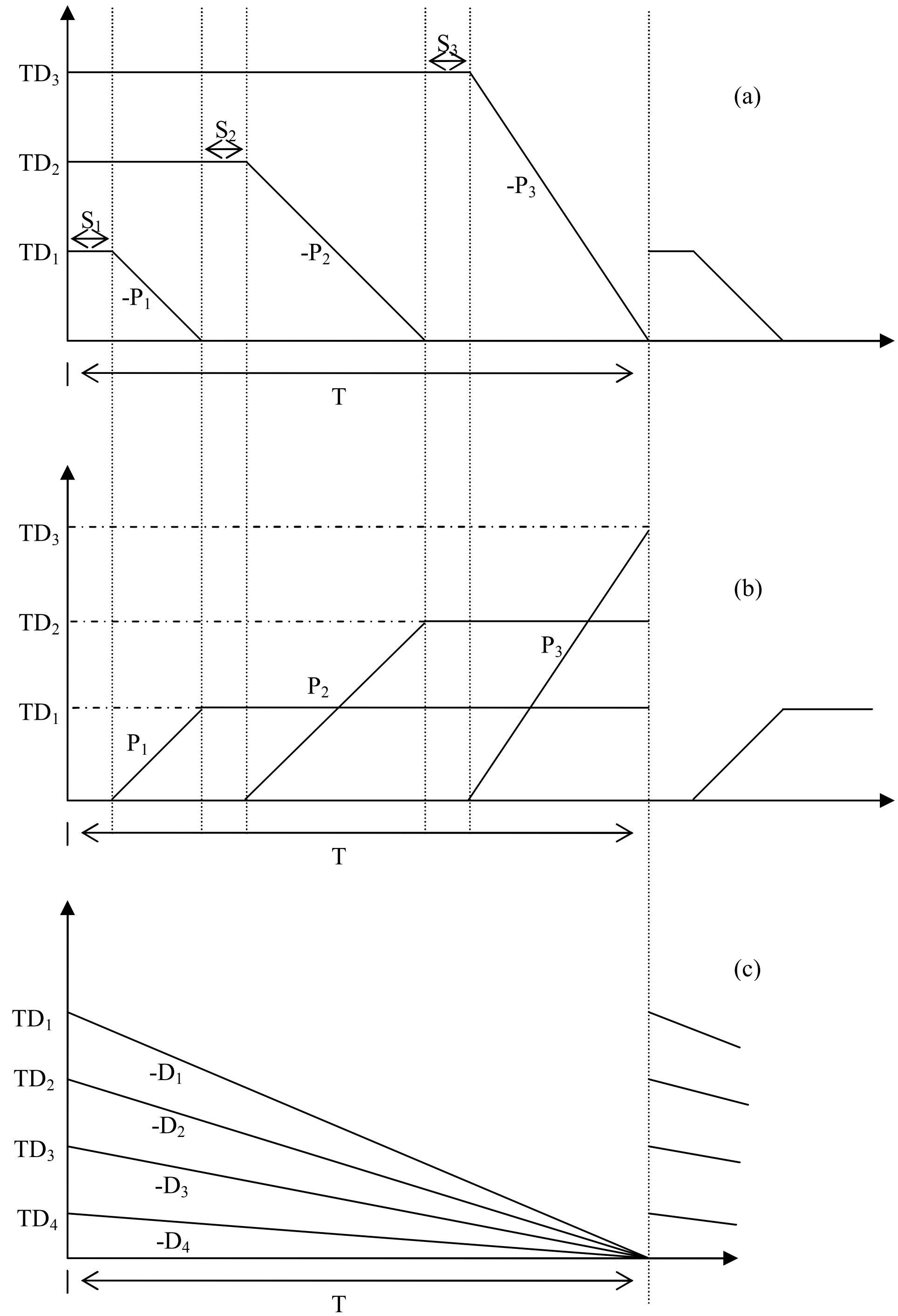

Figure 4.1: Inventory profile at a supplier site; (a) raw material, (b) processed items, and at the assembly facility (c).

For T1- and T2-suppliers, the model represents an integrated inventory-production system. The cycle time T, which stands for the time of replenishing the inventory, covers holding the item as a raw material, processing the item, and holding the item as a final product until the shipping date. A given supplier ships the accumulated products after all the sequenced items have been produced. For the company which is considered as an assembly facility, the cycle time is the same cycle time defined by the economic order quantity (EOQ) inventory model.

As shown in Figure 4.1-(a), the raw material of a certain item i is kept in stock until the start of its processing. This includes setup time of the production line to process this item. If the production sequence of the three items at a given tier is 1-2-3, it means that the binary variables, X_{11}, X_{22}, and X_{33}, are equal to 1 and the other variables, X_{12}, X_{23}, X_{21}, X_{23}, X_{31}, and X_{32}, are equal to zero, where the first subscript represents the item and the second subscript represents the sequence. Given this sequence of production, the average annual inventory of raw material $(\mathrm{RMI})_{123}$ is given by equation (4.1). Taking T as common, and by considering other possibilities of production sequence, 1-3-2, 2-1-3…3-2-1, the average annual raw material inventory holding cost for n items can be represented by equation (4.2).

$$RMI_{123} = \frac{1}{T}\left[\begin{array}{l} \left(S_1 TD_1 + \frac{TD_1}{2P_1} TD_1\right) X_{11} + \left(\left(S_2 + \left(S_1 + \frac{TD_1}{P_1}\right) X_{11}\right) TD_2 + \frac{TD_2}{2P_2} TD_2\right) X_{22} + \\ \left(\left(S_3 + \left(S_1 + \frac{TD_1}{P_1}\right) X_{11} + \left(S_2 + \frac{TD_2}{P_2}\right) X_{22}\right) TD_3 + \frac{TD_3}{2P_3} TD_3\right) X_{33} \end{array}\right] \tag{4.1}$$

$$RMI = \sum_{i=1}^{n} h_i^r \sum_{q=1}^{n} \left(\frac{TD_i^2}{2P_i} X_{iq} + D_i X_{iq} \left(S_i + \sum_{\substack{b=1 \\ b \neq i}}^{n} \sum_{l=1}^{q-1} \left(S_b + \frac{T D_b}{P_b} \right) X_{bl} \right) \right) \tag{4.2}$$

As depicted in Figure 4.1-(b), a machined item is kept in stock until end of production of the last sequenced item. For a given sequence 1-2-3, the annual average inventory of machined components $(\mathrm{MCI})_{123}$ is given by equation (4.3). Taking T as common, considering other possibilities of production sequence, 1-3-2, 2-1-3…3-2-1, the n machined components inventory holding cost can be represented by equation (4.4).

$$MCI_{123} = \frac{1}{T}\left[\begin{array}{l} \frac{TD_1}{2P_1} TD_1 X_{11} + \left(\left(S_2 + \frac{TD_2}{P_2}\right) X_{22} + \left(S_3 + \frac{TD_3}{P_3}\right) X_{33}\right) X_{11} TD_1 + \\ \frac{TD_2}{2P_2} TD_2 X_{22} + \left(S_3 + \frac{TD_3}{P_3}\right) X_{33} X_{22} TD_2 + \frac{TD_3}{2P_3} TD_3 X_{33} \end{array}\right] \tag{4.3}$$

$$MCI = \sum_{i=1}^{n} h_i^f \sum_{q=1}^{n} \left(\frac{TD_i^2}{2P_i} X_{iq} + D_i X_{iq} \sum_{\substack{b=1 \\ b \neq i}}^{n} \sum_{l=q+1}^{n} \left(S_b + \frac{T D_b}{P_b} \right) X_{bl} \right) \tag{4.4}$$

Inventory holding costs (IHC) for all tiers at both stages can be summed as shown in equation (4.5). The transportation and setup costs at both types of tier (TSC) are given by equation (4.6). At the assembly facility, the inventory profile is depicted in Figure 4.2-(c). The transportation and inventory cost $(TIC)_a$ at this stage, is shown in equation (4.7).

$$\begin{aligned} IHC = {} & \sum_{k=1}^{K} \left[\begin{array}{l} \sum_{i=1}^{n_k} h_{ki}^r \sum_{q=1}^{n_k} \left(\frac{TD_{ki}^2}{2P_{ki}} X_{kiq} + D_{ki} X_{kiq} \left(S_{ki} + \sum_{\substack{b=1 \\ b \neq i}}^{n_k} \sum_{l=1}^{q-1} X_{kbl} \left(S_{kb} + \frac{T D_{kb}}{P_{kb}} \right) \right) \right) + \\ \sum_{i=1}^{n_k} h_{ki}^f \sum_{q=1}^{n_k} \left(\frac{TD_{ki}^2}{2P_{ki}} X_{kiq} + D_{ki} X_{kiq} \sum_{\substack{b=1 \\ b \neq i}}^{n_k} \sum_{l=q+1}^{n_k} X_{kbl} \left(S_{kb} + \frac{T D_{kb}}{P_{kb}} \right) \right) \end{array}\right] + \\ & \sum_{j=1}^{J} \left[\begin{array}{l} \sum_{i=1}^{n_j} h_{ji}^r \sum_{q=1}^{n_j} \left(\frac{TD_{ji}^2}{2P_{ji}} X_{jiq} + D_{ji} X_{jiq} \left(S_{ji} + \sum_{\substack{b=1 \\ b \neq i}}^{n_j} \sum_{l=1}^{q-1} X_{jbl} \left(S_{jb} + \frac{T D_{jb}}{P_{jb}} \right) \right) \right) + \\ \sum_{i=1}^{n_j} h_{ji}^f \sum_{q=1}^{n_j} \left(\frac{TD_{ji}^2}{2P_{ki}} X_{jiq} + D_{ji} X_{jiq} \sum_{\substack{b=1 \\ b \neq i}}^{n_j} \sum_{l=q+1}^{n_j} X_{jbl} \left(S_{jb} + \frac{T D_{jb}}{P_{jb}} \right) \right) \end{array}\right] \end{aligned} \tag{4.5}$$

$$TSC = \frac{1}{T} \left(\sum_{k=1}^{K} \left(A_k + \sum_{i=1}^{n_k} B_{ki} \right) + \sum_{j=1}^{J} \left(A_j + \sum_{i=1}^{n_j} B_{ji} \right) \right) \tag{4.6}$$

$$TIC_a = \frac{A_a}{T} + \sum_{i=1}^{n_a} D_{ai}\, h_{ai} \frac{T}{2} \tag{4.7}$$

The proposed joint inventory-production model is given by equations (4.8)-(4.16). The chain-wide inventory costs shown in equation (4.8) are the summation of equations (4.5), (4.6), and (4.7).

$$TC = \sum_{k=1}^{K}\left[\begin{array}{l} \sum_{i=1}^{n_k} h_{ki}^{r} \sum_{q=1}^{n_k}\left(\frac{TD_{ki}^2}{2P_{ki}} X_{kiq} + D_{ki}\, X_{kiq}\left(S_{ki} + \sum_{\substack{b=1\\ b\neq i}}^{n_k}\sum_{l=1}^{q-1} X_{kbl}\left(S_{kb} + \frac{T\,D_{kb}}{P_{kb}}\right)\right)\right) + \\ \sum_{i=1}^{n_k} h_{ki}^{f} \sum_{q=1}^{n_k}\left(\frac{TD_{ki}^2}{2P_{ki}} X_{kiq} + D_{ki}\, X_{kiq} \sum_{\substack{b=1\\ b\neq i}}^{n_k}\sum_{l=q+1}^{n_k} X_{kbl}\left(S_{kb} + \frac{T\,D_{kb}}{P_{kb}}\right)\right) \end{array}\right] +$$

$$\sum_{j=1}^{J}\left[\begin{array}{l} \sum_{i=1}^{n_j} h_{ji}^{r} \sum_{q=1}^{n_j}\left(\frac{TD_{ji}^2}{2P_{ji}} X_{jiq} + D_{ji}\, X_{jiq}\left(S_{ji} + \sum_{\substack{b=1\\ b\neq i}}^{n_j}\sum_{l=1}^{q-1} X_{jbl}\left(S_{jb} + \frac{T\,D_{jb}}{P_{jb}}\right)\right)\right) + \\ \sum_{i=1}^{n_j} h_{ji}^{f} \sum_{q=1}^{n_j}\left(\frac{TD_{ji}^2}{2P_{ki}} X_{jiq} + D_{ji}\, X_{jiq} \sum_{\substack{b=1\\ b\neq i}}^{n_j}\sum_{l=q+1}^{n_j} X_{jbl}\left(S_{jb} + \frac{T\,D_{jb}}{P_{jb}}\right)\right) \end{array}\right]$$

$$+ \sum_{i=1}^{n_a} D_{ai}\, h_{ai}\frac{T}{2} + \frac{1}{T}\left(\sum_{k=1}^{K}\left(A_k + \sum_{i=1}^{n_k} B_{ki}\right) + \sum_{j=1}^{J}\left(A_j + \sum_{i=1}^{n_j} B_{ji}\right) + A_a\right) \qquad (4.8)$$

Subject to

$$\sum_{i=1}^{n_k}\left(S_{ki} + \frac{D_{ki}}{P_{ki}}\, T\right) \leq T \qquad k = 1,2,....,K \qquad (4.9)$$

$$\sum_{i=1}^{n_j}\left(S_{ji} + \frac{D_{ji}}{P_{ji}}\, T\right) \leq T \qquad j = 1,2,....,J \qquad (4.10)$$

$$\sum_{q=1}^{n_k} X_{kiq} = 1 \qquad k = 1,2,....,K, \quad i = 1,2,....,n_k \qquad (4.11)$$

$$\sum_{i=1}^{nk} X_{kiq} = 1 \qquad k = 1,2,....,K, \quad q = 1,2,....,n_k \qquad (4.12)$$

$$\sum_{q=1}^{n_j} X_{jiq} = 1 \qquad j = 1,2,....,J, \quad i = 1,2,....,n_j \qquad (4.13)$$

$$\sum_{i=1}^{n_j} X_{jiq} = 1 \qquad j = 1,2,....,J, \quad q = 1,2,....,n_j \qquad (4.14)$$

$$X_{jiq} \text{ is binary} \qquad j = 1,2,....,J,\; i = 1,2,....,n_j \quad q = 1,2,....,n_j \qquad (4.15)$$

$$X_{kiq} \text{ is binary} \qquad k = 1,2,....,K, \quad i = 1,2,....,n_k \quad q = 1,2,....,n_k \qquad (4.16)$$

The objective function is minimized subject to two sets of constraints. The first set is the cycle feasibility constraints, shown in equations (4.9) and (4.10). These constraints ensure that the resulting cycle time T is sufficient to set up the equipment used in production, and to process the units demanded at any given tier. The second set of constraints (4.11), (4.12), (4.13) and (4.14) is the QA constraints that guarantee assigning only one item to only one position in the production sequence at each tier. Constraints (4.15) and (4.16) are the binary restrictions imposed on the QA variables.

Results of this model answer two of the three questions mentioned in Section 4.2. First, by knowing the optimal value of the cycle time T and the annual demand at each member of the chain, the inventory policy can be easily established at each of member through determining the order amount and the number of orders. Second, results of the binary variables X_{jiq}, and X_{kiq} specify the production sequence at each supplier.

The developed joint inventory-production model given by equations (4.8)-(4.16) is built for a three-stage supply chain including an assembly facility and two stages of suppliers. The model can be easily extended to include a retailer stage by adding the inventory setup and holding costs at this stage to the objective function (4.8). Inventory costs at a given retailer are similar to those shown in equation (4.7) representing the EOQ model. Likewise, one or more supplier stages can be easily entered into this inventory production system. In such a case, the cycle feasibility and QA constraints as well as the inventory cost at the added stages should be added to the model.

Results of this model specify the production sequence that should be implemented at any given tier and the cycle time that should be employed at each stage. The ordering policy, which states the order amount and ordering frequency at each member of the chain, can be easily determined using the obtained value of the cycle time and the given value of the demand.

4.5 Linearizing Bilinear and 0-1 Polynomial Terms

In the model proposed in Section (4.4), objective function (4.8) includes bilinear terms including binary variables X_{kiq}, X_{jiq} multiplied by T which is a continuous variable. The objective function also involves two kinds of polynomial terms: pure binary polynomial terms and mixed binary polynomial terms. The former type of term multiplies two binary variables together as is the case in $X_{kiq} \times X_{kib}$ and $X_{jiq} \times X_{jib}$, while the latter kind is represented in terms containing the QA binary variables $X_{kiq} \times X_{kib}$ and $X_{jiq} \times X_{jib}$ multiplied by the cycle time T. These nonlinear terms should be linearized in order to be handled by a solver.

Different linearization schemes have been introduced in the literature to overcome this difficulty, among which the scheme introduced by Adams and Forrester (2005) is applied to linearize bilinear terms, and the one introduced by Hahn et al., (2008) to transform the two QA variables into a single binary variable. Section A.1 gives a background of the linearization techniques applied in this chapter.

Hahn et al. (2008) introduce a linearization approach to handle the binary polynomial terms existing in the generalized QA problem. This approach, discussed in Section (A.1.2), is applied to the proposed joint inventory-production model in order to linearize the binary polynomial terms including $X_{kiq} \times X_{kib}$ and $X_{jiq} \times X_{jib}$. Equations (4.17)-(4.22) show the new constraints added to the model, where Y_{kiq}^{bl} and Y_{jiq}^{bl} are two auxiliary binary variables introduced to replace these binary polynomial terms, respectively. Other generic linearization schemes, such as those of Zangwill (1965) and Glover and Woolsey

(1974), can be used to linearize this part of the model. These generic schemes are tested at the initial stage of the conducted numerical experiments, but the applied scheme is found to have better computational efficiency.

$$X_{kiq} = \sum_{\substack{l=1 \\ l \neq q}}^{n_k} Y_{kiq}^{bl} \qquad k = 1,2,....K,\ i = 1,2,....n_k,\ q = 1,2,....,n_k,\ b = 1,2,....n_k : b \neq i \tag{4.17}$$

$$X_{jiq} = \sum_{\substack{l=1 \\ l \neq q}}^{n_j} Y_{jiq}^{bl} \qquad j = 1,2,....J,\ i = 1,2,....n_j,\ q = 1,2,....n_j,\ b = 1,2,....n_j : b \neq i \tag{4.18}$$

$$Y_{kiq}^{bl} = Y_{kbl}^{iq} \qquad k = 1,2,....,K,\ i = 1,2,....,n_k,\ q = 1,2,....,n_k,\ b = 1,2,....,n_k, l = 1,2,....,n_k : i < b,\ l \neq q \tag{4.19}$$

$$Y_{jiq}^{bl} = Y_{jbl}^{iq} \qquad j = 1,2,....,J,\ i = 1,2,....,n_j,\ q = 1,2,....,n_j,\ b = 1,2,....,n_j, l = 1,2,....,n_j : i < b,\ l \neq q \tag{4.20}$$

$$Y_{kiq}^{bl}\ \textit{is binary} \qquad k = 1,2,....,K,\ i = 1,2,....,n_k,\ q = 1,2,....,n_k,\ b = 1,2,....,n_k, l = 1,2,....,n_k \tag{4.21}$$

$$Y_{jiq}^{bl}\ \textit{is binary} \qquad j = 1,2,....,J,\ i = 1,2,....,n_j,\ q = 1,2,....,n_j,\ b = 1,2,....,n_j, l = 1,2,....,n_j \tag{4.22}$$

Once this replacement goes into the joint inventory-production model, a new set of bilinear terms will appear by multiplying Y_{kiq}^{bl} and Y_{jiq}^{bl} each by *T*. The objective function, given by equation (4.8), comprises other bilinear terms including $X_{kiq} \times T$ and $X_{jiq} \times T$. The linearization approach of Adams and Forrester (2005), explained in Section (A.1.1), is applied to transform these bilinear terms into linear terms. Equations (4.23)-(4.30) show the added constraints to the joint inventory-production model, where the auxiliary variables V_{kiq}, V_{jiq}, Z_{kiq}^{bl} and Z_{jiq}^{bl} are introduced to handle the bilinearity appearing in $X_{kiq} \times T$, $X_{jiq} \times T$, $Y_{kiq}^{bl} \times T$, and $Y_{jiq}^{bl} \times T$, respectively.

$$Z_{kiq}^{bl} \geq T - U\left(1 - Y_{kiq}^{bl}\right) - L\,Y_{kiq}^{bl} \qquad k = 1,2,....,K,\ i = 1,2,....,n_k,\ q = 1,2,....,n_k,\ b = 1,2,....,n_k, l = 1,2,....,n_k : i \neq b,\ l \neq q \tag{4.23}$$

$$Z_{jiq}^{bl} \geq T - U\left(1 - Y_{jiq}^{bl}\right) - L\,Y_{jiq}^{bl} \qquad j = 1,2,....,J,\; i = 1,2,....,n_j,\; q = 1,2,....,n_j,$$
$$b = 1,2,....,n_j, l = 1,2,....,n_j : i \neq b,\; l \neq q \qquad (4.24)$$
$$V_{kiq} \geq T - U\left(1 - X_{kiq}\right) - L\,X_{kiq} \qquad k = 1,2,....K,\; i = 1,2,....n_k,\; q = 1,2,....n_k \qquad (4.25)$$
$$V_{jiq} \geq T - U\left(1 - X_{jiq}\right) - L\,X_{jiq} \qquad j = 1,2,....,J,\; i = 1,2,....,n_j,\; q = 1,2,....,n_j \qquad (4.26)$$
$$V_{jiq} \geq 0 \qquad j = 1,2,....,J,\; i = 1,2,....,n_j,\; q = 1,2,....,n_j \qquad (4.27)$$
$$V_{kiq} \geq 0 \qquad k = 1,2,....K,\; i = 1,2,....,n_k,\; q = 1,2,....,n_k \qquad (4.28)$$
$$Z_{kiq}^{bl} \geq 0 \qquad k = 1,2,....,K,\; i = 1,2,....,n_k,\; q = 1,2,....,n_k,$$
$$b = 1,2,....,n_k, l = 1,2,....,n_k \qquad (4.29)$$
$$Z_{jiq}^{bl} \geq 0 \qquad j = 1,2,....,J,\; i = 1,2,....,n_j,\; q = 1,2,....,n_j,$$
$$b = 1,2,....,n_j, l = 1,2,....,n_j \qquad (4.30)$$

The lower bound L imposed on the cycle time T represents the minimum value of the cycle time satisfying the cycle feasibility constraints given by equations (4.9) and (4.10). The upper bound U imposed on the cycle time T should be equal to the time horizon of the model. The model is developed to establish the inventory policy that will be implemented annually. Thus, through the conducted experiments this bound is set equal to one year.

The equivalent model includes the two sets of equations (4.17)-(4.22) and (4.23)-(4.30) in addition to the constraints of the inventory model given by equations (4.9)-(4.16). The new objective function resulting from linearizing the polynomial and bilinear terms existing in the original objective function (4.8) is given by equation (4.31). This function still has nonlinearity owing to the nonlinear terms of the setup cost. The following section illustrates how this nonlinearity is resolved by decomposing the model using the OA approach.

$$Min\ TC =$$

$$\sum_{k=1}^{K}\left[\begin{array}{l}\sum_{i=1}^{n_k} h_{ki}^{r} \sum_{q=1}^{n_k}\left(\frac{D_{ki}^2}{2P_{ki}}\left(V_{kiq}+LX_{kiq}\right)+D_{ki}\,S_{ki}\,X_{kiq}+D_{ki}\sum_{\substack{b=1\\ b\neq i}}^{n_k}\sum_{l=1}^{q-1}\left(S_{kb}Y_{kiq}^{bl}+\frac{D_{kb}}{P_{kb}}\left(Z_{kiq}^{bl}+LY_{kiq}^{bl}\right)\right)\right)+\\ \sum_{i=1}^{n_k} h_{ki}^{f} \sum_{q=1}^{n_k}\left(\frac{D_{ki}^2}{2P_{ki}}\left(V_{kiq}+LX_{kiq}\right)+D_{ki}\sum_{\substack{b=1\\ b\neq i}}^{n_k}\sum_{l=q+1}^{n_k}\left(S_{kb}Y_{kiq}^{bl}+\frac{D_{kb}}{P_{kb}}\left(Z_{kiq}^{bl}+LY_{kiq}^{bl}\right)\right)\right)\end{array}\right]+$$

$$\sum_{j=1}^{J}\left[\begin{array}{l}\sum_{i=1}^{n_j} h_{ji}^{r} \sum_{q=1}^{n_j}\left(\frac{D_{ji}^2}{2P_{ji}}\left(V_{jiq}+LX_{jiq}\right)+D_{ji}\,S_{ji}\,X_{jiq}+D_{ji}\sum_{\substack{b=1\\ b\neq i}}^{n_j}\sum_{l=1}^{q-1}\left(S_{jb}Y_{jiq}^{bl}+\frac{D_{jb}}{P_{jb}}\left(Z_{jiq}^{bl}+LY_{jiq}^{bl}\right)\right)\right)+\\ \sum_{i=1}^{n_j} h_{ji}^{f} \sum_{q=1}^{n_j}\left(\frac{D_{ji}^2}{2P_{ji}}\left(V_{jiq}+LX_{jiq}\right)+D_{ji}\sum_{\substack{b=1\\ b\neq i}}^{n_j}\sum_{l=q+1}^{n_j}\left(S_{jb}Y_{jiq}^{bl}+\frac{D_{jb}}{P_{jb}}\left(Z_{jiq}^{bl}+LY_{jiq}^{bl}\right)\right)\right)\end{array}\right]$$

$$+\sum_{i=1}^{n_a} D_{ai}\,h_{ai}\frac{T}{2}+\frac{1}{T}\left(\sum_{k=1}^{K}\left(A_k+\sum_{i=1}^{n_k}B_{ki}\right)+\sum_{j=1}^{J}\left(A_j+\sum_{i=1}^{n_j}B_{ji}\right)+A_a\right) \tag{4.31}$$

4.6 Decomposition of the Equivalent Model

After linearizing the bilinear and polynomial terms of the objective function (4.8), a decomposition stage has to be established in order to handle other difficulties resulting from nonlinear setup cost terms and binary restrictions on the QA variables. This decomposition stage takes place in two steps. The first stage is to decompose the equivalent model given by equations (4.9)-(4.31) into a mixed integer master problem and a nonlinear sub-problem using the OA decomposition approach. The theory of the OA method is discussed in Section A.2.2. The purpose of this decomposition is to linearly approximate the nonlinear setup cost terms. In the second step, the difficulty resulting from the binary variables incorporated in the master problem is resolved through decomposing it further by the BD technique. A background of the BD approach is explained in Section A.2.1.

4.6.1 Applying Outer Approximation Decomposition Technique

Decomposing the equivalent model using the OA approach yields an integer linear master problem and a nonlinear sub-problem. The master problem is basically formulated to find feasible values of binary variables X_{jiq} , X_{kiq} , Y_{kiq}^{bl} and Y_{jiq}^{bl} . The master problem objective function (OAM) is depicted in equation (4.32) which is the original objective except that nonlinear terms represent setup costs. Constraints of the master problem are all the constraints considered by the equivalent model represented by equations (4.9)-(4.31) in addition to the linear constraint (4.33). This linear constraint is generated at each iteration g to approximate the nonlinear terms embedded in the objective function (4.27). Also, this constraint represents the connection between the master problem and the sub-problem.

$$Min\ OAM =$$

$$\sum_{k=1}^{K}\left[\begin{array}{l}\sum_{i=1}^{n_k} h_{ki}^{r}\sum_{q=1}^{n_k}\left(\frac{D_{ki}^2}{2P_{ki}}\left(V_{kiq}+LX_{kiq}\right)+D_{ki}\,S_{ki}\,X_{kiq}+D_{ki}\sum_{\substack{b=1\\b\neq i}}^{n_k}\sum_{l=1}^{q-1}\left(S_{kb}Y_{kiq}^{bl}+\frac{D_{kb}}{P_{kb}}\left(Z_{kiq}^{bl}+LY_{kiq}^{bl}\right)\right)\right)+\\ \sum_{i=1}^{n_k} h_{ki}^{f}\sum_{q=1}^{n_k}\left(\frac{D_{ki}^2}{2P_{ki}}\left(V_{kiq}+LX_{kiq}\right)+D_{ki}\sum_{\substack{b=1\\b\neq i}}^{n_k}\sum_{l=q+1}^{n_k}\left(S_{kb}Y_{kiq}^{bl}+\frac{D_{kb}}{P_{kb}}\left(Z_{kiq}^{bl}+LY_{kiq}^{bl}\right)\right)\right)\end{array}\right]+$$

$$\sum_{j=1}^{J}\left[\begin{array}{l}\sum_{i=1}^{n_j} h_{ji}^{r}\sum_{q=1}^{n_j}\left(\frac{D_{ji}^2}{2P_{ji}}\left(V_{jiq}+LX_{jiq}\right)+D_{ji}\,S_{ji}\,X_{jiq}+D_{ji}\sum_{\substack{b=1\\b\neq i}}^{n_j}\sum_{l=1}^{q-1}\left(S_{jb}Y_{jiq}^{bl}+\frac{D_{jb}}{P_{jb}}\left(Z_{jiq}^{bl}+LY_{jiq}^{bl}\right)\right)\right)+\\ \sum_{i=1}^{n_j} h_{ji}^{f}\sum_{q=1}^{n_j}\left(\frac{D_{ji}^2}{2P_{ji}}\left(V_{jiq}+LX_{jiq}\right)+D_{ji}\sum_{\substack{b=1\\b\neq i}}^{n_j}\sum_{l=q+1}^{n_j}\left(S_{jb}Y_{jiq}^{bl}+\frac{D_{jb}}{P_{jb}}\left(Z_{jiq}^{bl}+LY_{jiq}^{bl}\right)\right)\right)\end{array}\right]$$

$$+\sum_{i=1}^{n_a} D_{ai}\,h_{ai}\,\frac{T}{2}+\alpha \qquad (4.32)$$

Subject to

Equations (4.9)-(4.30)

$$\alpha \geq \frac{1}{T_g}\left(\sum_{k=1}^{K}\left(A_k + \sum_{i=1}^{n_k} B_{ki} \right) + \sum_{j=1}^{J}\left(A_j + \sum_{i=1}^{n_j} B_{ji} \right) + A_a \right)$$

$$- \left(T - T_g\right)\frac{1}{T_g^2}\left(\sum_{k=1}^{K}\left(A_k + \sum_{i=1}^{n_k} B_{ki} \right) + \sum_{j=1}^{J}\left(A_j + \sum_{i=1}^{n_j} B_{ji} \right) + A_a \right) \qquad g = 1, 2, ...G \qquad (4.33)$$

The sub-problem finds optimal values of the cycle time T and the auxiliary variables V_{jiq} ,V_{kiq} ,Z_{kiq}^{bl} and Z_{jiq}^{bl} based on values of the other decision variables X_{jiq} , X_{kiq} , Y_{kiq}^{bl} and Y_{jiq}^{bl} , obtained from the master problem. This can be done through minimizing the sub-problem objective function (OAS) shown in equation (4.34) and satisfying constraints (4.9), (4.10), and (4.35)-(4.38).

$$Min\ OAS =$$

$$\sum_{k=1}^{K}\left[\begin{array}{l} \sum_{i=1}^{n_k} h_{ki}^{r} \sum_{q=1}^{n_k}\left(\frac{D_{ki}^2}{2P_{ki}}\left(V_{kiq} + LX_{kiq}^{*}\right) + D_{ki}\, S_{ki}\, X_{kiq}^{*} + D_{ki}\sum_{\substack{b=1 \\ b\neq i}}^{n_k}\sum_{l=1}^{q-1}\left(S_{kb}Y_{kiq}^{bl*} + \frac{D_{kb}}{P_{kb}}\left(Z_{kiq}^{bl} + LY_{kiq}^{bl*}\right)\right)\right) + \\ \sum_{i=1}^{n_k} h_{ki}^{f} \sum_{q=1}^{n_k}\left(\frac{D_{ki}^2}{2P_{ki}}\left(V_{kiq} + LX_{kiq}^{*}\right) + D_{ki}\sum_{\substack{b=1 \\ b\neq i}}^{n_k}\sum_{l=q+1}^{n_k}\left(S_{kb}Y_{kiq}^{bl*} + \frac{D_{kb}}{P_{kb}}\left(Z_{kiq}^{bl} + LY_{kiq}^{bl*}\right)\right)\right) \end{array} \right] +$$

$$\sum_{j=1}^{J}\left[\begin{array}{l} \sum_{i=1}^{n_j} h_{ji}^{r} \sum_{q=1}^{n_j}\left(\frac{D_{ji}^2}{2P_{ji}}\left(V_{kiq} + LX_{kiq}^{*}\right) + D_{ji}\, S_{ji}\, X_{jiq}^{*} + D_{ji}\sum_{\substack{b=1 \\ b\neq i}}^{n_j}\sum_{l=1}^{q-1}\left(S_{jb}Y_{jiq}^{bl*} + \frac{D_{jb}}{P_{jb}}\left(Z_{jiq}^{bl} + LY_{jiq}^{bl*}\right)\right)\right) + \\ \sum_{i=1}^{n_j} h_{ji}^{f} \sum_{q=1}^{n_j}\left(\frac{D_{ji}^2}{2P_{ji}}\left(V_{jiq} + LX_{jiq}^{*}\right) + D_{ji}\sum_{\substack{b=1 \\ b\neq i}}^{n_j}\sum_{l=q+1}^{n_j}\left(S_{jb}Y_{jiq}^{bl*} + \frac{D_{jb}}{P_{jb}}\left(Z_{jiq}^{bl} + LY_{jiq}^{bl*}\right)\right)\right) \end{array} \right] +$$

$$\sum_{i=1}^{n_a} D_{ai}\, h_{ai}\, \frac{T}{2} + \frac{1}{T}\left(\sum_{k=1}^{K}\left(A_k + \sum_{i=1}^{n_k} B_{ki} \right) + \sum_{j=1}^{J}\left(A_j + \sum_{i=1}^{n_j} B_{ji} \right) + A_a \right) \qquad (4.34)$$

Subject to

Equations (4.9) and (4.10)

$$Z_{kiq}^{bl} \geq T - U\left(1 - Y_{kiq}^{bl*}\right) - L\,Y_{kiq}^{bl*} \qquad k = 1,2,....,K,\ i = 1,2,....,n_k,\ q = 1,2,....,n_k,$$
$$b = 1,2,....,n_k, l = 1,2,....,n_k : i \neq b,\ l \neq q \qquad (4.35)$$

$$Z_{jiq}^{bl} \geq T - U\left(1 - Y_{jiq}^{bl*}\right) - L\,Y_{jiq}^{bl*} \qquad j = 1,2,....,J,\ i = 1,2,....,n_j,\ q = 1,2,....,n_j,$$
$$b = 1,2,....,n_j, l = 1,2,....,n_j : i \neq b,\ l \neq q \qquad (4.36)$$
$$V_{kiq} \geq T - U\left(1 - X_{kiq}^{*}\right) - L\,X_{kiq}^{*} \qquad k = 1,2,....,K,\ i = 1,2,....,n_k,\ q = 1,2,....,n_k \qquad (4.37)$$
$$V_{jiq} \geq T - U\left(1 - X_{jiq}^{*}\right) - L\,X_{jiq}^{*} \qquad j = 1,2,....,J,\ i = 1,2,....,n_j,\ q = 1,2,....,n_j \qquad (4.38)$$

In order to run the iterations between the master problem and the sub-problem there should be an initialization stage that provides a feasible solution of the decision variables. An initial problem is solved in this stage that minimizes only the inventory holding cost terms given in objective function (4.27), and considers all the constraints appearing in the equivalent model. The optimal solution is reached when the lower bound resulting from the relaxed master problem (OAM) equals the upper bound resulting from the restricted sub-problem (OAS).

The OA master problem is a mixed integer model that can be solved using Cplex, while the sub-problem is a nonlinear model that needs a nonlinear solver like Minos or Snopt. In the conducted experiments, the sub-problem is solved using Cplex by considering some properties of this problem. The first property is the convexity of objective function (4.34) in the continuous variable T for given values of other variables X_{jiq}, X_{kiq}, Y_{kiq}^{bl}, Y_{jiq}^{bl}, V_{jiq}, V_{kiq}, Z_{kiq}^{bl} and Z_{jiq}^{bl}. Equation (4.39) shows that the second derivative of this function with respect to cycle time T is positive. Secondly, equations (4.35)-(4.38) find feasible values of the variables V_{jiq}, V_{kiq}, Z_{kiq}^{bl} and Z_{jiq}^{bl} from the given values of variables X_{jiq}, X_{kiq}, Y_{kiq}^{bl} and Y_{jiq}^{bl} obtained from the master problem.

So, the continuous variables V_{jiq} ,V_{kiq} ,Z_{kiq}^{bl} and Z_{jiq}^{bl} can be replaced in equation (4.34) by T if their corresponding variables X_{jiq} ,X_{kiq} ,Y_{kiq}^{bl} ,and Y_{jiq}^{bl} equal one, while they equal zero if their corresponding binary variables equal zero.

$$\frac{\partial^2 OAS}{\partial T^2} = \frac{1}{T^3}\left(\sum_{k=1}^{K}\left(A_k + \sum_{i=1}^{n_k} B_{ki}\right) + \sum_{j=1}^{J}\left(A_j + \sum_{i=1}^{n_j} B_{ji}\right) + A_a\right) \tag{4.39}$$

Equation (4.40) shows the value of T that minimizes the total cost equation (4.34) derived by differentiating equation (4.34) with respect to T. By considering the constraints given by equations (4.9) and (4.10) that ensure the feasibility of T to cover setup and production times of all products, the optimal value of T can be found as the maximum between feasible T resulting from the constraints shown in equations (4.9) and (4.10) and T obtained from equation (4.40).

$$T = \sqrt{\left(\sum_{k=1}^{K}\left(A_k + \sum_{i=1}^{n_k} B_{ki}\right) + \sum_{j=1}^{J}\left(A_j + \sum_{i=1}^{n_j} B_{ji}\right) + A_a\right) \Big/ \left(0.5\sum_{i=1}^{n_a} D_{ai}\, h_{ai} + C_1 + C_2 + C_3 + C_4 + C_5 + C_6\right)} \tag{4.40}$$

Where

$$C_1 = \sum_{k=1}^{K}\sum_{i=1}^{nk}\sum_{\substack{q=1 \\ Xkiq=1}}^{nk} (h_{ki}^{r} + h_{ki}^{f})\frac{D_{ki}^2}{2P_{ki}} \tag{4.41}$$

$$C_2 = \sum_{k=1}^{K}\sum_{i=1}^{nk}\sum_{q=1}^{nk}\sum_{\substack{b=1 \\ b\neq i}}^{nk}\sum_{\substack{l=1 \\ Y_{kiq}^{bl}=1}}^{q-1} h_{ki}^{r} D_{ki}\frac{D_{kb}}{P_{kb}} \tag{4.42}$$

$$C_3 = \sum_{k=1}^{K}\sum_{i=1}^{nk}\sum_{q=1}^{nk}\sum_{\substack{b=1 \\ b\neq i}}^{nk}\sum_{\substack{l=q+1 \\ Y_{kiq}^{bl}=1}}^{nk} h_{ki}^{f} D_{ki}\frac{D_{kb}}{P_{kb}} \tag{4.43}$$

$$C_4 = \sum_{j=1}^{j}\sum_{i=1}^{nj}\sum_{\substack{q=1 \\ Xjiq=1}}^{nj} (h_{ji}^{r} + h_{ji}^{f})\frac{D_{ji}^2}{2P_{ji}} \tag{4.44}$$

$$C_5 = \sum_{j=1}^{J}\sum_{i=1}^{nj}\sum_{q=1}^{nj}\sum_{\substack{b=1 \\ b\neq i}}^{nj}\sum_{\substack{l=1 \\ Y_{jiq}^{bl}=1}}^{q-1} h_{ji}^{r} D_{ji}\frac{D_{jb}}{P_{jb}} \tag{4.45}$$

$$C_6 = \sum_{j=1}^{J} \sum_{i=1}^{nj} \sum_{q=1}^{nj} \sum_{\substack{b=1 \\ b \neq i}}^{nj} \sum_{\substack{l=q+1 \\ Y_{jiq}^{bl}=1}}^{nj} h_{ji}^{f} D_{ji} \frac{D_{jb}}{P_{jb}} \tag{4.46}$$

4.6.2 Decomposing the Outer Approximation Master Problem

Experiments were conducted by solving the linearized version of the proposed model using the OA decomposition approach. The solver reached the optimal solution for the first four problems as shown in Table 4.2. None of the other problems could be solved in twelve hours. The algorithm is interrupted while solving the fifth problem instance and it is found that the solver stalls in solving the master problem. The master problem of this problem instance, which is lager than the first four problems, is solvable without considering the optimality cut given by equation (4.33) into its formulation. This implies that large-sized master problems have to be decomposed further in order to be solved. Therefore, the generalized BD technique is deployed to decompose this master problem into a master problem and a sub-problem.

What calls for applying BD technique here is that it separates between the constraints imposed on the binary variables to be considered in the master problem and the constraints imposed on the continuous variables, including equation (4.33), to be considered in the sub-problem. The binary variables, considered as complicating variables, are optimized through a master problem while the continuous variables, considered as non-complicating variables, are optimized through a sub problem. Another complexity of the OA master problem is related to equations (4.23) and (4.24). Initial results of the tested problem instances indicate that the master problem is solved faster if these two constraints are not considered in the formulation. Consequently, these constraints should be separated from the pure binary constraints while decomposing this problem using the BD technique. Equations (4.11)-(4.16), (4.17)-(4.22), (4.47) and (4.48) build the Benders master problem. The objective function (BM) considers those terms of the objective function (OAM), equation (4.32), related to the binary variables X_{jiq} , X_{kiq} , Y_{kiq}^{bl} and Y_{jiq}^{bl} . Equation (4.48), called the optimality cut, is used to adjust values of the binary variables Y_{kiq}^{bl} and Y_{jiq}^{bl} based on the results of the sub-problem. At each iteration *f*, the master problem finds the values X_{jiq}^{f} , X_{kiq}^{f} , Y_{kiq}^{blf} and Y_{jiq}^{blf} of binary variables, while the sub-problem finds values of continuous variables V_{jiq} , V_{kiq} , Z_{kiq}^{bl} and Z_{jiq}^{bl} . Also, the sub-problem finds dual variables γ_{kiq}^{blf} and ε_{jiq}^{blf} associated with equations (4.54) and (4.55) respectively in order to generate the optimality cut. Function β provides a lower estimate of optimal value of the sub-problem objective function (BS) for the given values of the binary variables Y_{kiq}^{bl} and Y_{jiq}^{bl} .

$$
\begin{aligned}
Min\ BM = & \sum_{k=1}^{K}\left[\begin{array}{l}
\sum_{i=1}^{n_k} h_{ki}^{r} \sum_{q=1}^{n_k}\left(\frac{D_{ki}^{2}}{2P_{ki}} LX_{kiq} + D_{ki}\, S_{ki}\, X_{kiq} + D_{ki} \sum_{\substack{b=1\\ b\neq i}}^{n_k} \sum_{l=1}^{q-1} \left(S_{kb} Y_{kiq}^{bl} + \frac{D_{kb}}{P_{kb}} L\, Y_{kiq}^{bl} \right)\right) + \\
\sum_{i=1}^{n_k} h_{ki}^{f} \sum_{q=1}^{n_k}\left(\frac{D_{ki}^{2}}{2P_{ki}} LX_{kiq} + D_{ki} \sum_{\substack{b=1\\ b\neq i}}^{n_k} \sum_{l=q+1}^{n_k} \left(S_{kb} Y_{kiq}^{bl} + \frac{D_{kb}}{P_{kb}} L\, Y_{kiq}^{bl} \right)\right)
\end{array}\right] \\
& + \sum_{j=1}^{J}\left[\begin{array}{l}
\sum_{i=1}^{n_j} h_{ji}^{r} \sum_{q=1}^{n_j}\left(\frac{D_{ji}^{2}}{2P_{ji}} LX_{jiq} + D_{ji}\, S_{ji}\, X_{jiq} + D_{ji} \sum_{\substack{b=1\\ b\neq i}}^{n_j} \sum_{l=1}^{q-1} \left(S_{jb} Y_{jiq}^{bl} + \frac{D_{jb}}{P_{jb}} L\, Y_{jiq}^{bl} \right)\right) + \\
\sum_{i=1}^{n_j} h_{ji}^{f} \sum_{q=1}^{n_j}\left(\frac{D_{ji}^{2}}{2P_{ji}} LX_{jiq} + D_{ji} \sum_{\substack{b=1\\ b\neq i}}^{n_j} \sum_{l=q+1}^{n_j} \left(S_{jb} Y_{jiq}^{bl} + \frac{D_{jb}}{P_{jb}} L\, Y_{jiq}^{bl} \right)\right)
\end{array}\right] \\
& + \beta \qquad (4.47)
\end{aligned}
$$

Subject to

Equations (4.11 - 4.16, 4.17 - 4.22)

$$\beta \geq BS^f + \sum_{k=1}^{K}\sum_{i=1}^{n_k}\sum_{q=1}^{n_k}\sum_{\substack{b=1\\b\neq i}}^{n_k}\sum_{\substack{l=1\\l\neq q}}^{n_k}\gamma_{kiq}^{blf}\left(Y_{kiq}^{bl} - Y_{kiq}^{blf}\right) + \sum_{j=1}^{J}\sum_{i=1}^{n_j}\sum_{q=1}^{n_j}\sum_{\substack{b=1\\b\neq i}}^{n_j}\sum_{\substack{l=1\\l\neq q}}^{n_j}\varepsilon_{jiq}^{blf}\left(Y_{jiq}^{bl} - Y_{jiq}^{blf}\right) \quad f = 1,2,....F \tag{4.48}$$

The sub-problem objective function (BS) considers those terms of the objective function (OAM), equation (4.32), related to the non-complicating variables V_{jiq} , V_{kiq} , Z_{kiq}^{bl} , Z_{jiq}^{bl} , T and α. The problem tries to satisfy all the constraints on these non-complicating variables for those given values of the complicating variables. Equations (4.9), (4.10), (4.27)-(4.30), (4.33) and (4.49 - 4.55) represent the formulation of this sub-problem.

$Min\ BS =$

$$\sum_{k=1}^{K}\left[\sum_{i=1}^{n_k}h_{ki}^{r}\sum_{q=1}^{n_k}\left(\frac{D_{ki}^2}{2P_{ki}}V_{kiq} + D_{ki}\sum_{\substack{b=1\\b\neq i}}^{n_k}\sum_{l=1}^{q-1}\frac{D_{kb}}{P_{kb}}Z_{kiq}^{bl}\right) + \sum_{i=1}^{n_k}h_{ki}^{f}\sum_{q=1}^{n_k}\left(\frac{D_{ki}^2}{2P_{ki}}V_{kiq} + D_{ki}\sum_{\substack{b=1\\b\neq i}}^{n_k}\sum_{l=q+1}^{n_k}\frac{D_{kb}}{P_{kb}}Z_{kiq}^{bl}\right)\right] +$$
$$\sum_{j=1}^{J}\left[\sum_{i=1}^{n_j}h_{ji}^{r}\sum_{q=1}^{n_j}\left(\frac{D_{ji}^2}{2P_{ji}}V_{jiq} + D_{ji}\sum_{\substack{b=1\\b\neq i}}^{n_j}\sum_{l=1}^{q-1}\frac{D_{jb}}{P_{jb}}Z_{jiq}^{bl}\right) + \sum_{i=1}^{n_j}h_{ji}^{f}\sum_{q=1}^{n_j}\left(\frac{D_{ji}^2}{2P_{ji}}V_{jiq} + D_{ji}\sum_{\substack{b=1\\b\neq i}}^{n_j}\sum_{l=q+1}^{n_j}\frac{D_{jb}}{P_{jb}}Z_{jiq}^{bl}\right)\right]$$
$$+ \sum_{i=1}^{n_a}D_{ai}\,h_{ai}\frac{T}{2} + \alpha \tag{4.49}$$

Subject to

Equations (4.9, 4.10, 4.27 - 4.30, 4.33)

$$Z_{kiq}^{bl} \geq T - U\left(1 - Y_{kiq}^{bl}\right) - L\,Y_{kiq}^{bl} \qquad k = 1,2,....,K,\ i = 1,2,....,n_k,\ q = 1,2,....,n_k,\ b = 1,2,....,n_k, l = 1,2,....,n_k : i \neq b,\ l \neq q \tag{4.50}$$

$$Z_{jiq}^{bl} \geq T - U\left(1 - Y_{jiq}^{bl}\right) - L\,Y_{jiq}^{bl} \qquad j = 1,2,....,J,\ i = 1,2,....,n_j,\ q = 1,2,....,n_j,\ b = 1,2,....,n_j, l = 1,2,....,n_j : i \neq b,\ l \neq q \tag{4.51}$$

$$V_{kiq} \geq T - U\left(1 - X_{kiq}^{*}\right) - L\,X_{kiq}^{*} \qquad k = 1,2,....K,\ i = 1,2,....n_k,\ q = 1,2,....n_k \tag{4.52}$$

$$V_{jiq} \geq T - U\left(1 - X_{jiq}^{*}\right) - L\,X_{jiq}^{*} \qquad j = 1,2,....J,\ i = 1,2,....n_j,\ q = 1,2,....n_j \tag{4.53}$$

$$Y_{kiq}^{bl} = Y_{kiq}^{bl*} \quad : \gamma_{kiq}^{blf} \qquad k = 1,2,....,K,\ i = 1,2,....,n_j,\ q = 1,2,....,n_j,\ b = 1,2,....,n_j, l = 1,2,....,n_j : i \neq b,\ l \neq q \tag{4.54}$$

$$Y_{jiq}^{bl} = Y_{jiq}^{bl*} \qquad : \varepsilon_{jiq}^{blf} \qquad j = 1,2,....,J,\ i = 1,2,....,n_j,\ q = 1,2,....,n_j,$$
$$b = 1,2,....,n_j, l = 1,2,....,n_j : i \neq b,\ l \neq q \qquad (4.55)$$

The combinatorial feasibility cut can be discarded here because the Benders sub-problem gives feasible solutions of the continuous variables V_{jiq} ,V_{kiq} ,Z_{kiq}^{bl} ,Z_{jiq}^{bl} and T for any 0-1 combination, X_{jiq}^{*} ,X_{kiq}^{*} ,Y_{kiq}^{bl*} , and Y_{jiq}^{bl*} , of the binary variables passed from the Benders master problem. Furthermore, the Benders sub-problem has two important characteristics that should be utilized in formulating the optimality cut (4.48). First, for a given tier *j*, if item *i* is sequenced on any position *q*, this makes the corresponding X_{jiq} variable equals one and V_{jiq} equals *T*. The same relation applies to variables X_{kiq} and V_{kiq} for a given tier *k*. So, the impact of changing the sequence of an item on the Benders sub-problem objective function is always *T*. Consequently, the binary variables X_{jiq} and X_{kiq} do not affect the objective function of the Benders sub-problem and can be excluded from the optimality cut (4.49).

The second characteristic of the Benders sub-problem is related to the dual variables associated with constraints (4.54) and (4.55). Each of these dual variables is fixed at one value at any feasible iteration. To explain this characteristic, two features of the continuous variables Z_{jiq}^{bl} and Z_{kiq}^{bl} have to be demonstrated. First, for Z_{jiq}^{bl} , its value is determined by the given value of Y_{jiq}^{bl} through the applied linearization scheme. This linearization scheme forces Z_{jiq}^{bl} to be equal to *T* when Y_{jiq}^{bl} equals one, and to be equal to zero when Y_{jiq}^{bl} equals zero. The second feature is that the variables Z_{jiq}^{bl} do not appear in any other constraints that have influence on the objective function (4.49). This implies that if the binary variable Y_{jiq}^{bl} is equal to one, the continuous variable Z_{jiq}^{bl} will appear in the sub-problem objective function with the value of T. The variable Z_{jiq}^{bl} will not contribute to the sub-problem objective function if the binary variable Y_{jiq}^{bl} equals zero. Consequently, the unit price of variable Y_{jiq}^{bl} is the coefficient of Z_{jiq}^{bl} in the objective function (4.49) multiplied by T. Similarly, the unit price of variable Y_{kiq}^{bl} is the coefficient of Z_{kiq}^{bl} in the objective function (4.49) multiplied by T. These two characteristics of the optimality cut and the value of the dual variables have great influence on the convergence between the Benders master problem and sub-problem. Such an effect is recognized in the very small number of Benders iterations, between four and eight, required to reach optimal solutions of the tested problems in Section 4.7.

The lower bound, obtained from the relaxed master problem, is equal to the value BM given by equation (4.47), while the upper bound, shown in equation (4.56), is the value BS obtained from the restricted sub-problem plus the contribution of the binary variables to objective function (4.32).

$$\begin{aligned}
\text{Upper bound} = \text{BS} + \sum_{k=1}^{K} & \left[\begin{array}{l} \displaystyle\sum_{i=1}^{n_k} h_{ki}^{r} \sum_{q=1}^{n_k} \left(\frac{D_{ki}^2}{2P_{ki}} LX_{kiq} + D_{ki}\, S_{ki}\, X_{kiq} + D_{ki} \sum_{\substack{b=1\\ b\neq i}}^{n_k} \sum_{l=1}^{q-1} \left(S_{kb} Y_{kiq}^{bl} + \frac{D_{kb}}{P_{kb}} L Y_{kiq}^{bl} \right) \right) + \\ \displaystyle\sum_{i=1}^{n_k} h_{ki}^{f} \sum_{q=1}^{n_k} \left(\frac{D_{ki}^2}{2P_{ki}} LX_{kiq} + D_{ki} \sum_{\substack{b=1\\ b\neq i}}^{n_k} \sum_{l=q+1}^{n_k} \left(S_{kb} Y_{kiq}^{bl} + \frac{D_{kb}}{P_{kb}} L Y_{kiq}^{bl} \right) \right) \end{array} \right] \\
+ \sum_{j=1}^{J} & \left[\begin{array}{l} \displaystyle\sum_{i=1}^{n_j} h_{ji}^{r} \sum_{q=1}^{n_j} \left(\frac{D_{ji}^2}{2P_{ji}} LX_{jiq} + D_{ji}\, S_{ji}\, X_{jiq} + D_{ji} \sum_{\substack{b=1\\ b\neq i}}^{n_j} \sum_{l=1}^{q-1} \left(S_{jb} Y_{jiq}^{bl} + \frac{D_{jb}}{P_{jb}} L Y_{jiq}^{bl} \right) \right) + \\ \displaystyle\sum_{i=1}^{n_j} h_{ji}^{f} \sum_{q=1}^{n_j} \left(\frac{D_{ji}^2}{2P_{ji}} LX_{jiq} + D_{ji} \sum_{\substack{b=1\\ b\neq i}}^{n_j} \sum_{l=q+1}^{n_j} \left(S_{jb} Y_{jiq}^{bl} + \frac{D_{jb}}{P_{jb}} L Y_{jiq}^{bl} \right) \right) \end{array} \right]
\end{aligned} \tag{4.56}$$

4.7 Integer Multiplier Policy

The second policy investigated in this thesis that partially synchronizes the multi-stage inventory system is the integer-multiplier mechanism. In this policy, the cycle time at each stage is an integer multiplier of the cycle time at its successor stage. In the three-stage inventory model, the cycle time at the company is T and m_1T at any T1-supplier while it equals m_1m_2T at any T2-supplier. Except for updating the cycle times at all stages, the annual inventory cost function can be derived using Figure 5.1 and the analysis followed in Section 4.2. The inventory model representing this strategy is shown in equations (4.57)-(4.66).

$$MIN\ TC = \sum_{k=1}^{K}\left[\begin{array}{l}\displaystyle\sum_{i=1}^{n_k} h_{ki}^{r}\sum_{q=1}^{n_k}\left(\frac{m_2m_1TD_{ki}^2}{2P_{ki}}X_{kiq} + D_{ki}X_{kiq}\left(S_{ki}+\sum_{\substack{b=1\\b\neq i}}^{n_k}\sum_{l=1}^{q-1}X_{klb}\left(S_{kb}+\frac{m_2m_1T\,D_{kb}}{P_{kb}}\right)\right)\right)+\\ \displaystyle\sum_{i=1}^{n_k} h_{ki}^{f}\sum_{q=1}^{n_k}\left(\frac{m_2m_1TD_{ki}^2}{2P_{ki}}X_{kiq} + D_{ki}X_{kiq}\sum_{\substack{b=1\\b\neq i}}^{n_k}\sum_{l=q+1}^{n_k}X_{klb}\left(S_{kb}+\frac{m_2m_1T\,D_{kb}}{P_{kb}}\right)\right)\end{array}\right]$$

$$+\sum_{j=1}^{J}\left[\begin{array}{l}\displaystyle\sum_{i=1}^{n_j} h_{ji}^{r}\sum_{q=1}^{n_j}\left(\frac{m_1TD_{ji}^2}{2P_{ji}}X_{jiq} + D_{ji}X_{jiq}\left(S_{ji}+\sum_{\substack{b=1\\b\neq i}}^{n_j}\sum_{l=1}^{q-1}X_{jlb}\left(S_{jb}+\frac{m_1T\,D_{jb}}{P_{jb}}\right)\right)\right)+\\ \displaystyle\sum_{i=1}^{n_j} h_{ji}^{f}\sum_{q=1}^{n_j}\left(\frac{m_1TD_{ji}^2}{2P_{ki}}X_{jiq} + D_{ji}X_{jiq}\sum_{\substack{b=1\\b\neq i}}^{n_j}\sum_{l=q+1}^{n_j}X_{jlb}\left(S_{jb}+\frac{m_1T\,D_{jb}}{P_{jb}}\right)\right)\end{array}\right]$$

$$+\sum_{i=1}^{n_a}D_{ai}\,h_{ai}\frac{T}{2}+\frac{1}{T}\left(\frac{1}{m_2m_1}\sum_{k=1}^{K}\left(A_k+\sum_{i=1}^{n_k}B_{ki}\right)+\frac{1}{m_1}\sum_{j=1}^{J}\left(A_j+\sum_{i=1}^{n_j}B_{ji}\right)+A_a\right) \qquad (4.57)$$

Subject to

$$\sum_{i=1}^{n_k}\left(S_{ki}+\frac{D_{ki}}{P_{ki}}\,m_2m_1T\right) \le \ m_2m_1T \qquad k=1,2,....,K \qquad (4.58)$$

$$\sum_{i=1}^{n_j}\left(S_{ji}+\frac{D_{ji}}{P_{ji}}m_1T\right)\le m_1T \qquad j=1,2,....,J \tag{4.59}$$

$$\sum_{q=1}^{n_k}X_{kiq}=1 \qquad k=1,2,....K,\quad i=1,2,....n_k \tag{4.60}$$

$$\sum_{i=1}^{nk}X_{kiq}=1 \qquad k=1,2,....K,\quad q=1,2,....n_k \tag{4.61}$$

$$\sum_{q=1}^{n_j}X_{jiq}=1 \qquad j=1,2,....,J,\quad i=1,2,....,n_j \tag{4.62}$$

$$\sum_{i=1}^{n_j}X_{jiq}=1 \qquad j=1,2,....,J,\quad q=1,2,....,n_j \tag{4.63}$$

$$X_{jiq}\ is\ binary \qquad j=1,2,....,J,\ i=1,2,....,n_j\quad q=1,2,....,n_j \tag{4.64}$$

$$X_{kiq}\ is\ binary \qquad k=1,2,....,K,\quad i=1,2,....,n_k\quad q=1,2,....,n_k \tag{4.65}$$

$$m_1,m_2\ge 0\ \ and\ \text{int}\,eger \tag{4.66}$$

For fixed values of the multipliers m_1 and m_2, the model can be solved using the hybrid algorithm discussed in Sections 4.3 and 4.4. An optimal solution can be found for a specified range of each multiplier. This can be done through running the algorithm over a nested loop that alters the combination of multipliers m_1 and m_2. The optimal combination is the one that results in minimum chain-wide inventory cost.

4.8 Computational Results

Experiments were performed using a computer with 4-2.2 GHz AMD Opteron 64-bit processors and 8 GB RAM. The hybrid algorithm was coded using AMPL (Fourer et al., 2003), and solved using CPLEX 11.0. Sixteen problems representing different supply chain configurations are tested to evaluate the performance of the hybrid method. The configuration of these problems is depicted in Table 4.1.

Table 4.1: Supply chain structure of the 16 tested instances of the ELDSP
J: number of T1-suppliers, n_j: number of items at each of the j^{th} T1-supplier
K: number of T2-suppliers, n_k: number of items at each of the k^{th} T2-supplier
n_a: number of items at the assembly facility

Problem number	J	n_j	K	n_k	n_a
1	1	2	1	3	2
2	1	3	1	3	3
3	2	2,2	2	2,2	3
4	2	3,3	2	3,3	4
5	3	5,4,4	3	5,4,5	7
6	4	5,6,4,6	3	5,6,4	9
7	6	3,5,6,4,5,4	4	3,4,5,3	8
8	3	8,8,8	3	8,8,8	14
9	5	8,9,6,7,5	4	5,7,8,6	10
10	12	9,7,5,8,10,5,6,8,9,7,6,10	10	6,8,5,9,5,7,8,9,8,7	15
11	10	7,9,8,10,8,9,8,9,10,9	5	7,8,5,8,6	19
12	4	7, 10, 10, 9	11	6,5,8,7,5,7,8,9,7,10,8	18
13	6	8,11,10,11,9,10	10	8,7,9,7,8,11,7,9,8,6	17
14	10	7,10,9,10,8,9,10,9,10,9	11	7,5,8,9,5,7,8,10,7,6,8	18
15	9	8,10,12,10,11,12,13,9,10	13	9,8,10,10,8,9,12,9,8,10,9,7	16
16	15	8,11,10,10,9,10,11,10,11,11,9,10,7,10,8	12	9,7,10,11,7,9,10,11,9,8,10,9	20

A comparison between solving the equivalent model given by equations (4.9)-(4.31) using the OA approach and the proposed hybrid OA-BD algorithm is shown in Table 4.2. Results shown in the second and third columns conclude that the OA method outperforms the hybrid OA-BD in the first two problems, while the next two problems show the opposite. Starting from the fifth problem, the OA approach could not provide an optimal solution in 12 hours. Throughout these 16 problems, the number of iterations required to reach the optimal solution is at most two OA iterations. Each OA iteration includes at most four Benders iterations.

Table 4.2 details the solution time elapsed in solving a given problem using the proposed hybrid OA-BD method. The fourth column gives the time taken to provide a feasible solution required to start the iterations. Looking to the fifth and sixth columns, it is clear that as the problem size increases the Benders master problem consumes most of the elapsed solution time, while the Benders sub-problem takes a longer time than the

Benders master problem for smaller supply chains. The last two columns depict the number of variables and constraints resulting from decomposing the equivalent non-linear model through the proposed OA-BD method.

Table 4.2: Computational results of the OA approach and the proposed hybrid OA-BD algorithm applied to the common cycle time policy
N.S: Not Solvable

Number	OA solution time (sec.)	OA-BD solution time (sec.)					OA-BD problem size	
		Total	Initial problem	Benders master problem	Benders sub-problem	OA-Sub-problem	Number of variables	Number of constraints
1	0.09	0.12	0.015	0.03	0.06	0.02	223	107
2	0.11	0.13	0.015	0.03	0.06	0.02	363	178
3	0.21	0.13	0.016	0.02	0.06	0.02	163	70
4	3.53	0.15	0.018	0.04	0.07	0.02	723	354
5	N.S.	0.36	0.03	0.09	0.21	0.02	5535	3449
6	N.S.	0.69	0.08	0.19	0.39	0.02	11683	7847
7	N.S.	0.51	0.06	0.14	0.28	0.02	8739	5562
8	N.S.	4.20	0.55	1.55	2.05	0.03	49923	38120
9	N.S.	4.11	0.62	1.41	2.05	0.02	47655	35730
10	N.S.	18.9	3.99	7.22	7.70	0.04	167931	129540
11	N.S.	19.82	3.85	8.33	7.60	0.03	172663	136086
12	N.S.	16.83	3.13	6.38	7.27	0.04	136995	106849
13	N.S.	27.42	5.60	12.20	9.58	0.04	219471	175117
14	N.S.	27.63	5.19	11.60	10.79	0.04	225567	177248
15	N.S.	46.62	10.30	20.06	16.20	0.05	345843	276832
16	N.S.	82.35	20.82	38.18	23.25	0.08	478311	385598

Experiments related to the integer multipliers policy are shown in Table 4.3. The same 16 problem instances used in testing the common cycle policy are exercised here. Each of the multipliers, m_1 and m_2, is considered to be an input parameter that takes a value from 1 to 6. So for each problem, the proposed model given by equations (4.57) - (4.66) is linearized first then solved 36 times using the proposed OA-BD algorithm to search for the optimal multiplier values at each stage. The proposed algorithm shows a reasonable solution time for solving large scale supply chains, such as the last three problem instances.

Table 4.3: Computational results of the hybrid OA-BD applied to the integer multiplier policy over a specified range of m_1, m_2: $m_1 \leq 6$, $m_2 \leq 6$
m_1: multiplier at T1-stage m_2: multiplier at T2-stage

Problem	Multipliers		Solution time					% cost saving
	m_1	m_2	Total	Initial problem (sec.)	Benders master problem	Benders sub-problem	Sub-problem (sec.)	
1	2	1	5.59 sec	0.02	1.39 sec	2.69 sec	0.80	2.99
2	4	1	5.03 sec	0.02	1.22 sec	2.51 sec	0.72	16.30
3	2	1	4.27 sec	0.01	0.96 sec	2.11 sec	0.71	1.29
4	2	1	5.37 sec	0.02	1.28 sec	2.68 sec	0.73	1.69
5	1	1	12.63 sec	0.04	3.07 sec	7.2 sec	0.73	0
6	1	1	12.29 sec	0.04	3.11 sec	6.88 sec	0.74	0
7	2	2	18.37 sec	0.06	5.01 sec	10.14 sec	0.75	5.94
8	1	2	2.49 min	0.64	53.60 sec	73.62 sec	0.99	8.50
9	2	1	2.37 min	0.66	47.73 sec	69.81	0.94	1.69
10	3	1	11.37 min	4.55	4.39 min	4.48 min	1.45	4.27
11	1	1	12.17 min	3.93	5.10 min	4.59 min	1.42	0
12	3	2	16.18 min	5.93	7.32 min	5.99 min	1.64	12.95
13	3	2	16.70 min	5.80	7.22 min	5.94 min	1.72	15.01
14	1	2	16.52 min	5.75	7.03 min	6.11 min	1.72	9.33
15	2	1	20.09 min	10.21	7.83 min	5.78 min	1.36	1.91
16	2	2	48.13 min	18.71	22.55 min	13.50 min	2.23	6.75

The last column of Table 4.3 demonstrates the percentage of cost savings by applying the integer multipliers policy instead of the common cycle approach to synchronize the supply chain. As shown in this column, the integer multipliers policy gives the same results of the common cycle time policy for only three problems, the fifth, sixth and eleventh problems. Results of the other problems show that synchronizing the supply chain at the integer multipliers policy results in a cost reduction that can reach 16.3% compared to the common cycle time policy. This implies that the common cycle time strategy is not guaranteed to be the optimal tactic to synchronize the supply chain. Consequently, the integer multipliers mechanism should be investigated to answer the third question mentioned in Section 4.2 regarding the synchronization of all tiers at the same cycle time. On the other hand, the common cycle time approach could be a better choice to synchronize a supply chain if the products under consideration undergo changes in their design.

Results of the experiments demonstrate the efficiency of the methodology introduced in this chapter in designing the supply chain joint inventory-production system. Optimal inventory policies are obtained for multiple-stage supply chains under deterministic demand and lead time assumptions. The proposed synchronization strategies state the cycle time and the production sequence at each stage of the supply chain. Given the cycle time on hand, ordering frequency and the order size can be determined. The next chapter continues planning the tactical level by establishing the safety stock strategy required to cope with the uncertainty of demand and lead time.

Chapter 5

Safety Stock Placement Optimization

5.1 Introduction

In today's competitive environment, uncertainty is considered to be an inherent part of most supply chain inventory systems. This uncertainty is caused by several factors among which are customer demand and supplier lead time. To cope with fluctuations occurring in these two random variables, safety amounts should be placed at the relevant supply chain stocking nodes. From the economic aspect, the safety stock placement (SSP) problem should be given more attention by supply chain management researchers and practitioners, since understocking leads to customer dissatisfaction and overstocking results in high investment in inventory holding costs.

In this chapter, the SSP problem of the underlying multi-stage supply chain is tackled. The supply chain includes multiple-sourced stockpoints, in which each stock point undergoes demand and lead time fluctuations. Concepts of order statistics (OS) are incorporated in the proposed methodology to find the parameters of the lead time probability distribution at each stockpoint. Two safety stock positioning models are proposed to establish the fill rates along with the safety amounts across the chain. The recommended fill rates and safety amounts should lead the entire supply chain to meet a pre-specified end customer service level that represents a prescribed percentage of satisfied demand. The decentralized policy, characterizing the first model, allows each stockpoint to individually handle changes in its downstream demand and upstream lead time. On the other hand, the centralized policy proposed by the second model seeks to achieve cost savings through pooling the variability of lead time demand occurring at each stage at one aggregation center. The decentralized model is solvable to optimality using the nonlinear commercial solver Minos, whereas a decomposition technique based on the Benders decomposition (BD) technique is developed to solve the safety stock consolidation (SSC) model.

The establishment of the complete inventory system at each stockpoint in the supply chain is described in the following section. The specific underlying problem is defined in Section 5.3. Next, Section 5.4 discusses the application of normal OS to obtain the parameters of the lead time probability distribution at the multiple-sourced stockpoints. The SSP and the models are presented in Sections 5.5 and 5.6, respectively. The BD algorithm proposed to handle the difficulty embedded in the mixed integer nonlinear SSC model is explained in Section 5.7. A comparison between the two models is conducted in Section 5.8. This section also shows the computational efficiency of the decomposition method used to solve the SSC model.

5.2 Establishing the (*Q, r*) Inventory System

The inventory-production model developed in Chapter 4 ignores the stochastic environment surrounding the supply chain. Cycle time T and order quantities Q at each member of that chain have been determined either by the common cycle time policy or by the integer multipliers mechanism based on the deterministic assumption of customer demand. Moreover, lead time variability has not yet been accounted for.

To establish the (*Q, r*) system, in which *r* is the reorder point, one of two approaches can be followed. The first one is to simultaneously decide on the two decision variables *Q* and *r*. In this case the uncertainty of the random variables (e.g., lead time and demand) is considered through establishing the inventory system. The difficulty of this approach belongs to the complexity of the model that jointly finds optimal values of *Q* and *r*, and the computations required to find this optimal solution. The other method finds *Q* first, based on the average value of the demand, and then a subsequent safety stock model is established. The purpose of the safety stock model is to specify the reorder point *r* based on the predetermined value of the order amount *Q*. In this research the second approach is followed to decide sequentially on *Q* and *r*.

To set up the (*Q, r*) inventory system at each stockpoint existing in the supply chain under consideration in this thesis, the order amount *Q* at each stockpoint of the chain is simply determined from the deterministic models proposed in Chapter 4. This can be done through multiplying the obtained cycle time *T* by the given value of the average demand *D*. Subsequently, the reorder point *r* is determined by summing the safety stock amount to the average lead time demand. Establishing the (*Q, r*) system this way does not guarantee reaching the optimal policy of such a stochastic inventory system. A future extension to the research conducted in this part of the thesis is to develop the mathematical model and solution approach required to establish the optimal (*Q, r*) system of the stochastic inventory problem under study.

5.3 Safety Stock Problem Description and Assumptions

At each member of the supply chain depicted in Figure (1.1), there is a stockpoint facing a stochastic environment resulting from volatile downstream demand and variable predecessor lead time. The lead time is defined here as the time elapsed in shipping material from one stockpoint to its successor. The underlying stochastic environment is characterized by independent and normally distributed demand and lead time.

The current inventory strategies applied at each member of the supply chain ignore the uncertainty of downstream demand and upstream lead time. This represents an incentive to set up a safety stock strategy that can fulfill the end customer demand at a certain service level. The service level employed represents the percentage of demand satisfied from the shelf (i.e., the fill rate). The new strategy is expected to establish the fill rates and allocate the adequate safety amounts at the relevant places throughout the network. The overall objective is to fulfill the company's end customer demand at minimum safety stock holding cost across the entire chain. Such a strategy should respect the material distribution results proposed by the strategic reconfiguration and supplier selection model developed in Chapter 3. In addition, the strategy should consider the underlying assumptions of the economic lot delivery and scheduling models proposed in Chapter 4, that are used to specify replenishment intervals and order amounts at each stockpoint. These assumptions are:

1. A single item can be replenished from multiple locations.
2. Replenishment cycle time is common among suppliers of a given stage.
3. Production can start upon receiving shipments ordered from the multiple sources.

The first and third assumptions call for applying OS concepts in order to figure out the delivery time used in calculating safety stock at each stockpoint. Since each stockpoint receives material from multiple sources, the lead time for receiving an item is considered as the maximum among these multiple-source lead times. Consequently, this maximum is considered as a random variable having a probability distribution. To determine the maximum of a set of random variables, OS theory should be consulted. Section 6.4 illustrates the application of OS to find parameters of the maximum delivery time at each stockpoint.

To solve the above-mentioned safety stock problem, two models are developed in this chapter. The first model, the SSP model, is established based on the decentralized approach of allocating safety stocks. Following this approach, each stockpoint is required to keep sufficient safety amounts from each item at its site to meet both kinds of variability mentioned above. The second model, the SSC model, is formulated based on the centralization principles of safety stock distribution. Centralization principles require safety pooling to be applied at each stage. This can be attained by consolidating the safety amounts of each single item required at all stockpoints of a given stage at that which has the lowest inventory holding cost and enough capacity. Stockpoints preferred to be consolidation centers will be given an amount of credits to cover their responsibilities for holding the consolidated safety amounts. In return, a consolidation center is required to cope with the variations of lead time demand of other stockpoints by shipping a sufficient amount of stock to their downstream stage. The impact of such consolidation is that smaller overall amounts of safety stock are kept as a consequence of the resulting decrease in variability due to SSC at each stage.

5.4 Variable Lead Time of Multiple-Sourced Stockpoints

The problem of the multiple sourced inventory system has been widely investigated by researchers to figure out the effect of order splitting on the lead time distribution, in which the entire order is distributed among multiple sources instead of being replenished from a sole source (Sculli and Wu, 1981; Pan, 1987; Ramasesh, 1988; and Pan et al., 1991). Another problem relating to a multiple-sourced stochastic inventory system arises when the lead time is considered as the maximum among the multiple-sourced lead times. An assembly system is a common example that exhibits this way of calculating lead time, where assembly cannot start until all the required components being assembled have been received. Also, a production batch of a single item may require all the raw material supplied from multiple vendors to be processed in one production run. In that case, production of this item starts once all the required material is on hand.

In the problem being studied, a multiple-sourced stockpoint faces the stochastic problem of determining delivery lead time of its input material, specifically the elapsed time to transport these materials from upstream stages to the stockpoint. If a stockpoint receives material from n sources considering their lead time as independent and identically normally distributed random variables, the maximum among these n variables equals the maximum of a random sample of size n taken from a normally distributed population (Clark, 1961). Consequently, if the delivery time is a normally distributed random variable, the maximum among them is also a normally distributed random variable. The determination of this maximum can be found by consulting OS distributions and moments (David and Nagaraja, 2003). The mean of this random variable is the expected lead time that will be used along with its variance in calculating the safety amounts required to cope with the stochastic environment of the supply chain.

The expected value of the i^{th} order statistics for a set of independent standard normal random variables $X_1, X_2 \ldots X_n$ is given by equation (5.1) where i represents the order. If i equals n, it represents the maximum of this OS.

$$E(X_i) = \frac{n!}{(i-1)!(n-i)!} \int_{-\infty}^{\infty} x\{\phi(x)\}^{i-1}\{1-\phi(x)\}^{n-i} f(x)dx \qquad (5.1)$$

Godwin (1949) establishes tables of mean, variance and covariance of NDOS of size 10 or less. For samples of 20 or less, tables of the expected value of the i^{th} order statistics are established by Teichrow (1956). For larger sample sizes of 2(1) 100(25) 250(50) 400, Harter (1961) presents the expected values of NDOS. Federer (1951), Blom (1958), Wescott (1977), and Royston (1982) introduce algorithms to approximate the expected values of OS. These algorithms apply numerical methods and do not provide closed form solutions to find moments of OS.

Simchi-Levi et al. (2005) consider such a case of lead time representation in their model and apply the approximation method introduced by Clark (1961) to determine the lead time at assembly facilities. Clark (1961) finds the maximum among a finite set of random variables through successive iterations that require searching in the standard normal table each time. However, searching in the normal table is time-consuming and is found to be difficult to put into a computer code. Further inventory models that incorporate explicit forms for determining the maximum lead time at assembly facilities have to be introduced to facilitate handling the difficulty of variable lead time.

The algorithm introduced by Ozturk and Aly (1991) is applied here to approximate parameters of the normally distributed lead time random variable at each stockpoint. The algorithm approximates the expected value and variance of NDOS using the generalized lambda distribution (GLD). In this case, the moments of GLD order statistics are used as an approximation to the moments of standard NDOS. In addition to providing results with small errors, the algorithm proposed by Ozturk and Aly (1991) is straightforward and needs less computational efforts compared to Clark's (1961). Table 5.1 shows a comparison between these two approximation methods. The second column of the table represents the upper bound of the error in estimating the maximum among a set of random variables using the GLD algorithm proposed by Ozturk and Aly (1991), and Clark's (1961) error values are depicted in the third column. Clark's (1961) error results are subjected to increase if an approximation method is used instead of searching in the normal table.

Table 5.1: Comparison between the absolute error in estimating the mean of the maximum among *n* standard normal random variables using Ozturk and Aly (1991) and Clark (1961).

n	Ozturk and Aly (1991)	Clark (1961)
2	0.00019	0.00000
3	0.00029	0.00130
4	0.00074	0.00260
5	0.00122	0.00130
6	0.00146	0.00070
7	0.00151	0.00000
8	0.00171	0.00060
9	0.00195	0.00130
10	0.00208	0.00210

The inverse distribution function of the GLD proposed by Ramberg and Schemeiser (1972) is shown in equation (5.2) where λ_1, λ_2, λ_3 and λ_4 are the parameters of the distribution. For 0, 0.1975, 0.1349 and 0.1349 given values of these parameters, Schemeiser (1977) shows that the maximum absolute error through approximating the standard normal distribution by the GLD is 0.001. Equations (5.3) and (5.5) represent the closed form given by Ozturk and Aly (1991) to approximate mean and variance of standard NDOS using the GLD. The β function used to calculate the variance is shown in equation (5.7).

$$F^{-1}(p) = \lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2} \tag{5.2}$$

$$m_i = \frac{C_i - C_{n-i+1}}{\lambda_2 C_{n+1}} \tag{5.3}$$

Where,

$$C_r = r\prod_{k=1}^{r}\left(1 + \frac{\lambda_3 - 1}{k}\right) \tag{5.4}$$

$$v_i = \frac{\beta(2\lambda_3 + i, t) - 2\beta(\lambda_3 + i, t + \lambda_4) + \beta(i, t + 2\lambda_4)}{\lambda_2^2 \beta(i, t)} - (m_i - \lambda_1)^2 \tag{5.5}$$

Where $t = n - i + 1$ (5.6)

and $\beta(x, y) = \frac{(x-1)!(y-1)!}{(x+y-1)!}$ (5.7)

The parameters m_i and v_i of the standard NDOS are used to drive mean $E(X_i)$ and variance $Var(X_i)$ of the original OS. If the n lead times at a given stockpoint are represented by identical normal distributions having mean μ and variance σ^2, parameters of the maximum lead time distribution are given by equations (5.8) and (5.9) where i equals n.

$$E(X_i) = \mu + \sigma m_i \tag{5.8}$$

$$Var(X_i) = \sigma^2 v_i \tag{5.9}$$

5.5 Decentralized Safety Stock Placement Model

In this section, the proposed SSP model that is based on the decentralized approach is discussed. The contribution of this model is the incorporation of the service per units demanded (i.e., fill rate) as a measure of service in a multi-stage supply chain. The supply chain, which comprises multiple-sourced stockpoints, faces customer demand and supplier lead time variability. Another original aspect of this model concerns the relation between the fill rates required to be established from a supply chain perspective. Each item moves throughout the network on a path that starts from the T2-suppliers stage until it reaches the company. The expected fill rates at the stockpoints placed on a given path should satisfy the end customer service level. This is ensured through satisfying the constraint setting the service level as a lower bound on the multiplication of these fill rates.

The mean l_{ijk} and variance σl_{ijk} of the delivery time of item k at stockpoint j placed in stage i are calculated using equations (5.8) and (5.9), respectively. Equation (5.10) represents the standard deviation of lead time demand in the case of variable demand and variable lead time. Equation (5.11) shows the relationship between standardized stockout quantity per order cycle and the fill rate of a single item at a given stockpoint (Tersine, 1988). This equation will be extended in the model to be applied in a supply chain context.

$$\sigma_{ijk} = \sqrt{l_{ijk}\sigma d_{ijk}^2 + d_{ijk}^2\, \sigma l_{ijk}^2} \tag{5.10}$$

$$sl = 1 - \frac{\sigma\, E(Z)}{q} \tag{5.11}$$

The SSP model proposed to establish the safety stock decisions regarding the fill rates and the safety amounts is given by equations (5.12)-(5.16).

$$Min\ SSHC \ = \ \sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{k=1}^{K} h_{ijk}\sigma_{ijk}\, Z_{ijk} \tag{5.12}$$

Subject to

$$F_{1rk}\times F_{2sk}\times F_{3tk}\geq sl_k \qquad k=1,2,...K,\ r=1,\ s=1,2,...J_2,\ t=1,2,...,J_3 : U_{krst}=1 \tag{5.13}$$

$$E(Z_{ijk})=\frac{(1-F_{ijk})\,q_{ijk}}{\sigma_{ijk}} \qquad i=1,2,...I \quad j=1,2,...J_i\ \ k=1,2,...K \tag{5.14}$$

$$Z_{ijk}=\left[E(Z)_{ijk}\text{ - }0.38984228\right]\times\left[\begin{array}{c}\text{-}1.75294+0.4442135E(Z)_{ijk}\text{ - }0.07061455E(Z)_{ijk}^{2}\\ \text{-}\dfrac{0.17592241}{E(Z)_{ijk}+0.044212641}\text{-}\dfrac{0.0012267386}{E(Z)_{ijk}+0.00030570313}\end{array}\right]$$

$$i=1,2,...I\ \ j=1,2,...J_i\ \ k=1,2,...K \tag{5.15}$$

$$Z_{ijk}, E(Z)_{ijk}, F_{ijk}\ \geq 0 \qquad i=1,2,...I \quad j=1,2,...J_i\ \ k=1,2,...K \tag{5.16}$$

Objective function (5.12) minimizes the safety stock holding cost at each stockpoint. The amount $\sigma_{ijk}\,Z_{ijk}$ is the safety stock of item *k* that should be kept at stockpoint *j* in stage *i* per cycle. The desired service level of item *k* is satisfied through equation (5.13) where U_{krst} is a four dimensional binary matrix. For the case handled in this thesis in which the supply chain composed of three stages, the entries U_{krst} specify whether or not item *k* passes through the stockpoint *r* located at the most downstream stage where *i* =1, and the stockpoint *s* located at the intermediate stage where *i* = 2, and the stockpoint *t* located at the most upstream stage where *i* = 3. This equation ensures that each path of a given item *k* on the network will yield a service level greater than or equal to the desired one. The multiplication of the fill rates on a given path gives the model the flexibility to assign high fill rate to the stockpoints having low holding cost and assign lower fill rate at the stockpoints incurring high holding cost. Equation (5.14), which is driven from equation (5.11), calculates the standard stockout quantity $E(Z)_{ijk}$ of item *k* at stockpoint *j* in stage i. Brown's (1967) nonlinear approximation is shown in equation (5.15). This convex nonlinear approximation is used instead of searching in statistical tables to find value of Z_{ijk} for a given value of $E(Z)_{ijk}$. The drawback of this function is that it does not provide a reasonable approximation when it is applied to large absolute values of $E(Z)$ close to 4.5. The non-negative restriction on the decision variables is insured by the last constraint.

The reorder point of each item k at stockpoint j in stage i can be directly determined by adding the safety stock $\sigma_{ijk} Z_{ijk}$ to the average lead time demand $l_{ijk} d_{ijk}$. The model is coded using Ample (Fourer et al., 2003), and is solvable directly to optimality using Minos.

When a stockpoint undergoes order crossover effects resulting from receiving a recently placed order before the order placed earlier, μ and σ^2 appearing in equations (5.8) and (5.9) refer to the effective lead time normal distribution. Effective lead time can be obtained from the original lead time by considering the time elapsed between placing the first order and receiving the first delivery (Hayya et al., 2009). Another approach to considering order crossover while setting up safety stock plans is to design the safety levels with regard to the shortfall distribution instead of the lead time demand distribution (Bradley et al., 2005; Robinson et al., 2008). Shortfall and lead time demand distributions have the same mean, while the variance of the shortfall distribution is affected by parameters of the number of outstanding orders. Robinson (2001) demonstrates how these parameters can be derived from probability distribution of the lead time demand. To apply this approach in the proposed model, the standard deviation of lead time demand σ_{ijk} in equation (5.14) is replaced by the standard deviation of the shortfall distribution. So, whether the effective lead time or the shortfall distribution is used, the proposed SSP model is still valid for handling stochastic lead times with order crossover.

5.6 Safety Stock Consolidation Model

SSC is recommended to reduce variability, especially when a supply chain faces high demand and lead time variations. The inventory consolidation problem has been studied in the literature to examine the effect of consolidation on inventory cycle stock and safety stock savings. Wanke (2009) classifies major papers handling this problem. The consolidation models proposed by these papers are built given that both cycle stock and safety stock of decentralized locations are consolidated in one or more centralized locations.

Because the consolidation model proposed in this thesis is designed to handle variability in delivery lead time and customer demand in the context of an integrated production inventory system, it is preferred to keep the cycle stock close enough to the production line. This will facilitate shipping on the promised delivery dates. In such cases, consolidation takes place in safety stock only. In addition to this unique way of consolidating safety stock, the proposed model differs significantly in three aspects from those appearing in Maister (1976), Zinn et al. (1989), Mahmoud (1992), Evers and Beier (1993), Tallon (1993), Evers (1995), Caron and Marchet (1996), Evers and Beier (1998), Tyagi and Das (1998), Das and Tyagi (1999), Ballou and Burnetas (2003), Ballou (2005), and Wanke (2009).

First, the service level considered in the proposed model represents the probability of stockout amount while that employed in the literature is the probability of stockout occurrence. Probability of stockout amount service level is more informative than probability of stockout occurrence since it shows how many of the demanded units are not satisfied.

Second, the safety factor associated with the underlying service level in the proposed SSC model is a decision variable while that used in other models appearing in the above cited papers is a known parameter. Moreover, in the proposed SSC model, the value of that factor is decided upon from the viewpoint of the whole supply chain while its value is set by each consolidation center prior to solving those models.

Third, previous research consolidates cycle stock and safety stock of decentralized locations placed at one stage at the chosen independent consolidation centers. The complexity of the proposed model lies in the constraint imposed on the fill rates at the selected consolidation centers at each stage of a multi-stage supply chain. This relationship, which has not been considered before in a consolidation model, implies that the fill rates of these centers should yield a customer service level greater than or equal to the desired one. In addition to the capacity of a candidate center and its holding cost per unit, the amount of credit that is given to each center affects the decision of selecting the consolidation centers. This credit is considered as a motivation to accept the responsibility of holding such consolidated safety stock.

The operational analysis given by Evers and Beier (1998) recommends pooling variability of demand instead of pooling variability of lead time demand at each candidate consolidation center. Their model considers only the variability of lead time at each candidate consolidation center to go through safety stock calculations. In contrast to the proposed model, variability of lead time at decentralized locations is discarded because no inventory is kept there. In the proposed consolidation model, given the fact that each facility is responsible for meeting its cycle demand, equation (5.17) shows the pooled variability that sums the independent lead time demand variances at each stockpoint of a given stage where each entry σ_{ijk}^2 is obtained from equation (5.10).

$$\sigma_{ik} = \sqrt{\sum_{j=1}^{J} \sigma_{ijk}^2} \tag{5.17}$$

In the SSP model proposed in Section 5.5, each stockpoint is responsible for meeting its ongoing fluctuations in demand and lead time by keeping sufficient safety amounts. The concept of consolidation is applied here to centralize the safety amount of each item to be located in one place at each stage. As such, if any stockpoint faces demand or lead time positive variations, the consolidation center is required to ship an amount sufficient to meet such variations to the downstream stage. The impact of this concept is a reduction in the total safety amounts of each item stored at each stage.

Throughout the SSC model given by equations (5.18)-(5.26), the previously defined decision variables are used to represent each stage instead of each stockpoint. For example, the decision variable F_{ijk} appears here as F_{ik} to symbolize the fill rate of item k required from stage i to meet the service level s_{lk} for this item k. A binary decision variable X_{ijk} is defined to decide on which stockpoint j is used to hold the consolidated safety amount of each item k at stage i.

Two more parameters are introduced, c_{ij} that represents the total capacity of stockpoint j in stage i, and the motivation cost w_{ijk} that indicates the amount of money paid by the supply chain partners to stockpoint j in stage i as an incentive to take the burden of handling the consolidated safety stock of item k. Through objective function (5.18), these motivating dollars along with the holding cost are used to select the most relevant stockpoint among the feasible candidates to be the safety consolidation center of item k at stage i. Constraint, given by equation (5.19), ensures that the consolidated safety amount of item k is assigned to only one stockpoint among the available J_i points at stage i. Capacity restriction of stockpoint j at stage i to hold one or more items is satisfied through equation (5.20).

$$Min\ ASC = \sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{k=1}^{K} w_{ijk}X_{ijk} + h_{ijk}\sigma_{ik} Z_{ik}X_{ijk} \tag{5.18}$$

Subject to

$$\sum_{j=1}^{J_j} X_{ijk} = 1 \qquad i=1,2,...I \quad k=1,2,...K \tag{5.19}$$

$$\sum_{k=1}^{K} \sigma_{ik} Z_{ik}X_{ijk} \le c_{ij} \qquad i=1,2,...I \quad j=1,2,...J_i \tag{5.20}$$

$$\prod_{i=1}^{I} F_{ik} \ge sl_k \qquad k=1,2,...K \tag{5.21}$$

$$E(Z_{ik}) = \frac{(1-F_{ik})\sum_{j=1}^{J_i} q_{ijk}}{\sqrt{\sum_{j=1}^{J_i}\sigma_{jik}^2}} \qquad i=1,2,...I \quad k=1,2,...K \tag{5.22}$$

$$Z_{ik} = \left[E(Z)_{ik}\text{-}0.38984228\right]\times\begin{bmatrix}\text{-}1.75294+0.4442135E(Z)_{ik}\text{-}0.07061455E(Z)_{ik}^2 \\ \text{-}\dfrac{0.17592241}{E(Z)_{ik}+0.044212641}\text{-}\dfrac{0.0012267386}{E(Z)_{ik}+.00030570313}\end{bmatrix}$$

$$i=1,2,...I \quad k=1,2,...K \tag{5.23}$$

$$X_{ijk}\quad is\,binary \qquad i=1,2,...I \quad j=1,2,...J_i \quad k=1,2,...K \tag{5.24}$$

$$Z_{ik}, E(Z)_{ik}, F_{ik} \ge 0 \qquad i=1,2,...I \quad k=1,2,...K \tag{5.25}$$

5.7 Decomposition of the Consolidation Model

The difficulty with the SSC model lies in the binary variable X_{ijk}, and the nonlinear approximation (5.23). The binary variable hinders Minos from solving this model since it is a nonlinear solver, while the nonlinear constraint, given by equation (5.23), prevents

Cplex from solving the model directly. Consequently, the model is decomposed into a binary master problem that is solved using Cplex and a continuous nonlinear sub-problem that is solved using Minos. This kind of decomposition follows the generalized BD technique discussed in Section 2.2.1, in which the complicating variable takes its value from the master problem then the sub-problem finds the solution of other variables for these given values of the binary variable.

5.7.1 Master Problem

The master problem is solved to find the optimal values of the complicating binary variables X_{ijk}, where these values are then sent to the sub-problem. If the sub-problem is infeasible to those given values of the complicating variables X_{ijk}, it adds the feasibility cut (5.27) to the master problem. The feasibility cut used here is the combinatorial cut proposed by Codato et al. (2006). According to the model constraints, it is better to apply the cut only to those stockpoints that show insufficient capacity in a previous infeasible iteration s. This accelerates the master problem toward reaching a feasible 0-1 combination by minimizing the number of X_{ijk} candidates included in the cut. The parameters X^{s}_{ijk} and Z^{s}_{ik} are the recorded values of the binary variable X_{ijk} and the standard normal Z_{ik} at the infeasible iteration s. The cut searches for new combinations of 0-1 value of the complicating variables that were not considered infeasible before.

To guide the master problem to the best X_{ijk} combination for the sub-problem, the optimality cut (5.28) is added after each feasible iteration of the sub-problem. The multiplier λ_{ijk} appearing in this cut reflects the change in the sub-problem objective function when the associated X_{ijk} changes from 0 to 1. This multiplier λ_{ijk} is calculated as follows: since each stage i accepts only one X_{ijk} to be 1 over the subscript j, the sub-problem is solved *ik(j-1)* times to evaluate the change λ_{ijk} due to replacing the X_{ijk} leveled at 1 by the other binary variables X_{ijk} leveled at 0 individually. Based on the passed values λ_{ijk} from the sub-problem, the optimality cut (5.28) gives more opportunity to assign 1 to the X_{ijk} variable that has minimum value of the multiplier λ_{ijk}. In the classical BD approach, this multiplier is the dual variable associated with constraint (5.28). The dual variable cannot be used in the optimality cut (5.28) because its returned values by the sub-problem are non-negative. This non-negativity is a result of the positive increase $h_{ijk}\, X_{ijk}$ in objective function (5.29) associated with increasing X_{ijk} from 0 to 1. The master problem is thus stated as follows:

$$Min\ MP = \sum_{i=1}^{I}\sum_{j=1}^{Ji}\sum_{k=1}^{K} w_{ijk} X_{ijk} + \alpha \tag{5.26}$$

Subject to

Equations (5.19), (5.24)

$$\sum_{i=1}^{I}\sum_{j=1}^{Ji}\sum_{k=1:X_{ijk}^{s}=0}^{K} X_{ijk} + \sum_{i=1}^{I}\sum_{j=1}^{Ji}\sum_{k=1:X_{ijk}^{s}=1}^{K} 1 - X_{ijk} \geq 1$$

$$s = 1,2,..,S \quad i = 1,2,...,I \quad j = 1,2,...J \ : \sum_{k=1}^{K} \sigma_{ik} Z_{ik}^{s} X_{ijk}^{s} > c_{ij} \tag{5.27}$$

$$\alpha \geq SP^{t} + \sum_{i=1}^{I}\sum_{j=1}^{Ji}\sum_{k=1}^{K} \lambda_{ijk}^{t} (X_{ijk} - X_{ijk}^{t}) \qquad t = 1,2,..,T \tag{5.28}$$

5.7.2 Sub-Problem

The sub-problem finds optimal values of $E(Z)_{ik}$, Z_{ik} *and* F_{ik} for the given values X_{ijk}^{t} of the complicating variables X_{ijk} at the feasible iteration t. The sub-problem is given by the following equations:

$$Min\ SP\ =\ \sum_{i=1}^{I}\sum_{j=1}^{Ji}\sum_{k=1}^{K} h_{ijk}\sigma_{ik}\, Z_{ik} X_{ijk} \tag{5.29}$$

Subject to

Equations (5.20)-(5.23), (5.25)

$$X_{ijk} = X_{ijk}^{t} \quad : \lambda_{ijk} \qquad i = 1,2,...I \quad j = 1,2,...J_i \quad k = 1,2,...K \tag{5.30}$$

The algorithm iterates between both problems until an optimal solution is obtained. This can be attained when the lower bound (5.31) obtained from the relaxed master problem equals the upper bound (5.32) resulting from the restricted sub-problem. Equality of both bounds implies that no more improvement in the values of the complicating variables can be achieved.

$$LB = \sum_{i=1}^{I}\sum_{j=1}^{Ji}\sum_{k=1}^{K} w_{ijk} X_{ijk}^{t} + \alpha \tag{5.31}$$

$$UB = \sum_{i=1}^{I}\sum_{j=1}^{Ji}\sum_{k=1}^{K} w_{ijk} X_{ijk}^{t} + \sum_{i=1}^{I}\sum_{j=1}^{Ji}\sum_{k=1}^{K} h_{ijk}\sigma_{ik}\, Z_{ik} X_{ijk}^{t} \tag{5.32}$$

5.8 Computational Experiments

The computational efficiency of solving the consolidation model through decomposition as well as the savings that can be obtained from distributing safety stock based on the obtained results of the SSC model are evaluated in this section. Experiments were conducted on AMD Sempron™ processor 3400+ 1.8GHz and 1 GB of RAM.

A comparison between the SSP model and the SSC model is shown in Table 5.2. Results are tabulated for seven different supply chain sizes. The first stage of the chain is a single stockpoint while the number of stockpoints at Tier-1 and Tier-2 stages is shown in the second and third columns.

Table 5.2: Comparison between the SSP model and the SSC model

Problem number	Number of Tier-1	Number of Tier-2	Number of Items	SSP model		SSC model				Cost savings ($)	% Saving	
				Safety stock amount	Holding cost	Safety stock amount	Holding cost ($)	Motivation cost ($)	Total cost ($)		Cost	Safety stock amount
1	3	4	7	17213	1,612,850	8536	820,520	423,100	1,243,620	369,230	22.89	50.40957
2	5	4	6	27381	2,553,660	11905	1,180,046	406,000	1,586,046	967,614	37.89	56.52095
3	4	5	10	32252	2,885,072	17603	1,667,517	578,020	2,245,537	639,535	22.17	45.42044
4	5	5	8	38302	3,414,066	17423	1,623,960	524,200	2,148,160	1,265,906	37.08	54.51151
5	7	3	9	32632	2,707,316	15600	1,261,791	580,860	1,842,651	864,665	31.94	52.19417
6	8	6	5	28811	2,512,972	11040	1,066,621	373,000	1,439,621	1,073,351	42.71	61.6813
7	9	7	4	25702	1,446,542	9753	511,836	296,000	807,836	638,706	44.15	62.05354

The SSC model gives less safety amount than the SSP model as a result of introducing the safety pooling concept into the consolidation model. Consequently, it has less investment in the holding cost. The motivation cost is the first term of the objective function (5.18) which represents the amount of money paid to the selected centers to entice them to handle the consolidated safety stock.

The assumed range in which the annual holding cost per unit takes its input values is \$50-\$150, while the assumed range of the motivation cost per class of item per year is \$30,000-\$50,000. The third-to-last column shows cost savings that can be attained by applying the SSC model to optimize the safety stock positioning throughout the chain. Based on the assigned ranges of cost parameters, cost savings that range from 22.17% - 44.15% can be achieved annually as shown in the second-to-last column. The last column shows the percentage reduction in safety stock size attainable through applying the centralization model. This percentage reflects the portfolio effect of SSC, introduced by

Zinn et al. (1989), shown in equation (5.33). Up to 62% of the safety amounts resulting from the decentralized model can be saved if the consolidation model is employed.

$$\text{Portfolio Effect} = 1 - \frac{\text{Sum of safety stock at consolidation centers}}{\text{Sum of safety stock at decentralized locations}} \tag{5.33}$$

Each of the seven problem instances of the multi-item model is solved directly using Minos in less than a second. Table 6.3 illustrates the computational experiments regarding the SSC model. Ten different problems with different values of input parameters are tested to check the computational efficiency of the proposed consolidation model and solution methodology. These different figures of the input parameters lead to solving the problems in different numbers of added cuts to the master problem. For example, in the sixth problem all the stockpoints have enough capacity to handle any number of products. Thus, no feasibility cuts are added to the master problem as depicted in the fifth column. In contrast, in the seventh problem, 219 feasibility cuts are added to the master problem in order to provide feasible X_{ijk} solutions to the sub-problem.

Table 5.3: Computational efficiency of the decomposition method used to solve the SSC model

Problem Number	Number of Tier-1	Number of Tier-2	Number of Items	Added cuts		Solution time (Sec.)		
				Feasibility	Optimality	Total	Master-problem	Sub-problem
1	3	4	7	52	17	21	3	2.5
2	6	8	11	4	6	29	0.46	0.35
3	4	5	10	25	18	40	2	2.5
4	8	6	5	31	30	46	1.4	2.5
5	9	7	4	5	7	10	0.4	0.3
6	12	9	12	0	4	26	0.15	0.12
7	5	5	8	219	18	59	20	9
8	7	13	8	10	4	16	0.6	0.5
9	10	11	7	28	23	83	2.5	2
10	10	12	9	31	5	17	1.5	0.8

Because of the efficiency of the λ_{ijk} multiplier in building the optimality cuts, the number of these cuts throughout the experiments is considered to be low, ranging from 4-30 cuts as shown in the sixth column. The drawback of the multiplier λ_{ijk} is the time it takes to be calculated. As shown in the last three columns of the table, the solution time of the master and sub-problems is very short compared to the total time of solving a problem. This indicates that most of the solution time of a given problem is consumed to calculate the multipliers at each iteration. The third-to-last column demonstrates the efficiency of the proposed BD method to reach the optimal solution of the SSC model. The solution time elapsed to solve any of the 10 different problems is very short, between 10 and 83 seconds.

The safety stock strategies introduced in this chapter handle the SSP problem from a supply chain perceptive, in which the overall objective is to minimize the placement costs of safety amounts across the chain. The first strategy identifies the optimal fill rate and safety amounts that should be placed at each stockpoint to face the uncertainty surrounding the supply chain. The second strategy aims at aggregating the safety stock placed at each stockpoint in a given stage at one aggregation center. The developed BD technique reaches the optimal consolidation policy that minimizes the safety stock placement cost through the entire supply chain. The resulting cost savings favor the consolidation approach to establish the safety stock policy required to cope with the variations in supplier lead time and customer demand.

Chapter 6

Conclusions and Future Work

6.1 Summary and Conclusion

Supply chain reconfiguration, inventory control, and safety stock placement problems are tackled in this thesis. These problems are investigated in order to make strategic and tactical decisions such as supplier selection, material distribution, capacity utilization, shipping frequency, order quantity, production sequence, safety amounts and fill rate, from a supply chain standpoint.

The problem under study replicates a real-life problem faced by an assembly company. This company has decided to resolve two problems existing in its current supplying strategies. The first problem is related to the delivery performance of the suppliers providing the company with raw and machined components. Some of these suppliers are unable to deliver these components on time to their downstream stage. This directly affects the promised delivery dates of the final assembly to the end customer. The second problem is related to the inefficient inventory systems employed at the stockpoints existing in the supply chain. First, these inventory systems do not provide adequate stocks due to random ordering from upstream stages. Second, no safety stock strategy is being applied at these stockpoints to cope with the uncertainty of downstream demand and upstream lead time. Moreover, the company expects a demand increase that requires suppliers' available capacity to be reallocated. The company seeks to establish new supplying and inventory strategies to fulfill this demand increase and handle the deficiencies of the current strategies.

The thesis solves the underlying problem in three hierarchical decision stages. In the first stage, strategic decisions such as supplier selection, material distribution and capacity utilization are established. These strategic decisions are respected while designing the integrated inventory-production system in the second stage. At the final stage that handles the uncertainty present in the chain, the proposed safety stock strategy also respects the results obtained in the previous strategic and tactical stages.

Through these decision stages, mathematical models are developed to formulate the underlying problem. These models are not limited to the problem studied in this thesis but they can be applied to handle vital industrial supply chain problems. For example, the strategic supplier selection model can represent a supply chain that has to be reconfigured in order to increase the amount of material delivered on time.

The second model characterizes a joint inventory-production system that aims at minimizing the supply chain inventory cost. The system is represented by the economic lot and delivery scheduling problem (ELDSP) formulation which is a common problem in the literature. The resulting optimal strategies guide supply chain partners to jointly decide on cycle time, order amounts and production sequence at each member of the chain from a supply chain perspective.

The third model determines the safety stock strategy required to face the fluctuations of demand and lead time in order to meet a predefined customer service level. Details of these models along with the proposed algorithms to solve them are discussed separately below.

The research is presented in three integrated parts. In the first part, the supply chain reconfiguration and multi-criteria supplier selection problem is addressed to select the best suppliers and reallocate their capacities. The problem is formulated as a bilinear goal programming model which aims at achieving three objectives. The first and second objectives are to maximize the amount of material assigned to the highly reliable and well coordinated suppliers, respectively. The third objective is to minimize the distribution and inventory costs. Achieving these objectives leads the new reconfigured supply chain to meet the expected demand increase, overcome the customer dissatisfaction caused by late deliveries.

Improvements in the on-time delivery performance of the supply chain may lead to an increase in the distribution cost as compared to the current strategy that aims at minimizing the cost as a single criterion. The distribution cost increase is expected as it conflicts with the other two goals that aim at improving the on-time delivery performance of the supply chain. Although the proposed model reconfigures an existing supply chain, it can be applied to configure new chains considering the suppliers as candidates; among them the model selects the best.

The strategic model is decomposed into a master problem and a sub-problem to resolve the bilinearity resulting from multiplying a binary variable by a continuous variable. The master problem and the sub-problem are formulated as two interrelated goal programming models. The sub-problem minimizes the deviations from the three goals considered in the proposed model, while the master problem selects the highly reliable and well coordinated suppliers. Following the Benders decomposition (BD) approach, the master problem finds the optimal values of the complicating binary variables representing supplier selection decisions, while the sub-problem optimizes values of the non-complicating variables, representing the material distribution decisions, for those given values of the complicating variables. A modified BD technique is developed to solve these goal programming models.

Experiments conducted on large-sized supply chains demonstrate that the modified BD technique efficiently outperforms the classical linearization approach used to linearize the proposed model. The proposed method reaches optimal solutions with a reduction in solution time ranging from 56% to 89% as compared to the classical linearization approach.

The economic lot and delivery scheduling problem for a multi-stage supply chain is investigated in the second part of the research. The problem is formulated in a new context through a quadratic assignment representation. The proposed constrained nonlinear mixed integer model is handled through a hybrid algorithm. The algorithm combines a linearization technique and outer approximation (OA) and BD techniques. The linearization scheme is applied to linearize bilinear and 0-1 polynomial terms existing in the chain-wide inventory cost function. The nonlinearity of terms representing the setup costs are handled by the OA method that decomposes the model into a master problem and a sub-problem. Since the OA master problem includes complex binary variables, it is found to be intractable by the commercial solver used in the experiments.

BD is applied to overcome this obstacle by decomposing the master problem into two problems.

The computational experiments detailed in Table 4.2 show a solution time of 0.15 seconds for a small scale supply chain including a combination of two-three suppliers at each stage. For a large scale supply chain including 15 suppliers at the initial stage and 12 suppliers at the intermediate one, the solution time reaches 82.35 seconds. Moreover, an optimal solution is attained for the case of the integer-multiplier policy over a specified range for each multiplier in a relatively short time. As shown in Table 5.3, a cost reduction up to 16.3% can be accomplished by applying the integer-multiplier policy rather than the common cycle time policy to synchronize the supply chain.

In the third part of the research, two safety stock placement models are proposed to allocate optimal safety amounts to the existing stockpoints in the supply chain. The supply chain faces variable demand and lead times among its stocking nodes. An explicit form is applied to determine the characteristics of the lead time at the multiple-sourced stockpoints by following order statistics concepts. The first model is developed based on the decentralized approach of safety stock allocation. The objective is to place minimum safety stock amounts at each stockpoint of the chain in order to achieve the desired end customer service level. The second model takes advantage of variability reduction resulting from safety pooling. At each stage, the amount of safety stock required from a given item is placed at the most appropriate stockpoint. This stockpoint behaves as a safety consolidation center that handles any fluctuations of upstream delivery time and downstream demand encountered at that stage.

The safety consolidation model includes nonlinear and integrality constraints that inhibit commercial solvers from handling it directly. A BD method is established to decompose this model into two problems. The resulting master and sub-problem problems are solvable directly by Cplex and Minos respectively. Benders optimality cuts and combinatorial Benders cuts are generated to converge between the master problem and the sub-problem. Computational experiments recorded in Table 5.3 show that the proposed decomposition method solves the nonlinear mixed integer safety consolidation model in a very short time, between 10 and 83 seconds.

The comparison made between the decentralized and the safety consolidation models illustrated in Table 5.2 shows that cost savings between 22.17% and 44.15% can be accomplished by employing the safety consolidation model. Also, a reduction up to 62% in safety amounts can be achieved by applying the consolidation policy.

6.2 Originality of the Thesis

The thesis contributes to the field of supply chain modeling and optimization research by developing new mathematical models and efficient solution techniques. The proposed techniques reach the optimal solutions of the developed nonlinear mixed integer models in reasonable time. From the industrial point of view, the research presents strategies that assist supply chain practitioners to establish their strategic and tactical level plans by specifying which suppliers are selected, how material is distributed among them, and how to control the inventory in both deterministic and stochastic environments. The achieved contributions can be stated as follows:

- The strategic model that reconfigures the supply chain selects suppliers based on a new combination of objectives. These objectives aim at selecting suppliers and allocating their capacities so as to dispense material among the highly reliable and well coordinated suppliers at the lowest possible distribution and inventory costs. The optimal strategy resulting from solving this model satisfies strategic constraints imposed on configuring a supply chain and designs the distribution network over a given number of time periods.

- A novel formulation of the common ELDSP based on the quadratic assignment representation is introduced. This new approach to modeling the ELDSP, along with the developed algorithm, makes it easier to attain the optimal design of multiple stages joint inventory-production systems.
- The thesis provides a comparative study that evaluates the synchronization of a multi-stage supply chain using the common cycle time approach and the integer multiplier mechanism. The comparison shows the computational time and the inventory costs of each case by examining different supply chain configurations.
- Two new supply chain safety stock placement models are proposed. The supply chain comprises multiple stages in which each stage involves multiple-sourced stockpoints. Each stockpoint faces variations in customer demand and supplier lead time. Order statistics theory is applied to decide on the functional lead time at each stockpoint. The two models are developed based on the centralized and decentralized approaches of placing the required safety amounts. Connected stockpoints are optimized simultaneously to establish the economic fill rates that satisfy the end customer service level.
- The safety stock consolidation model that represents the centralization approach of holding safety stocks differs from those models proposed in the literature in

two main ways. First, it consolidates safety stock but does not consolidate cycle stock. Secondly, fill rates employed at the consolidation centers are optimized simultaneously in order to provide a service level greater than or equal to the desired one. This gives the flexibility of establishing higher fill rates at the centers that have low placements costs and lower fill rates at the centers that have higher placement costs. The placement costs include the inventory holding cost and the amount of credits paid to a candidate center to handle the uncertainty of lead time and demand of an entire stage.

- Integrating decisions resulting from establishing the joint inventory-production policy with those obtained from the decentralized safety stock policy introduces a new (Q, r) control system. This system forms the inventory strategy at multiple-sourced stockpoints of a multiple-stage supply chain facing demand and lead time fluctuations.
- A modified BD method is established to overcome the difficulties associated with applying the classical BD approach to solve goal programming models. This can be done through adapting the algorithm to cope with a master problem and a sub-problem formulated as two goal programming models having different objective function structures compared to the traditional Benders method. The modified algorithm can be generally applied to bilinear goal programming models in which the complicating variables directly affect the minimization of the deviational variable associated with each goal.
- The hybrid algorithm developed to solve the proposed inventory models shows that linearization and decomposition approaches can be integrated to solve

complex nonlinear models. The algorithm incorporates two linearization schemes and two decomposition approaches to solve a model including nonlinear, bilinear, and polynomial terms in addition to binary restrictions on decision variables.

6.3 Future Work

Supply chain modeling and optimization is an attractive field of research that calls for more researchers to add their contributions. Extensions to the research conducted in this thesis can be viewed from two different angles: extensions to the entire work which covers the three decision stages, or extensions to each stage individually.

It would be interesting to further investigate the second part of the research that decides on the order amounts at each stockpoint and the third part that determines the reorder point, safety stock amounts and fill rate. The order quantity at a given stockpoint influences the reorder point and thus the safety stock level required at this stockpoint. Therefore, developing a new model that decides simultaneously on the optimal values of order quantity and reorder point could result in cost savings. The negative side of such a model lies in its complexity that may not allow for reaching optimal solutions. However, if a suboptimal solution is attained, it could provide lower inventory costs as compared to results obtained from deciding separately on order amount and reorder point.

The strategic reconfiguration and supplier selection model is established to redistribute material among reliable and coordinated suppliers at minimum cost. The model could be extended to consider other important criteria in selecting suppliers such as quality, agility and financial stability. Changing the approach used to represent the model parameters could be an interest for further research. For instance, instead of reconfiguring the supply chain based on a deterministic demand assumption, this demand could be represented as a random variable. In such a case, stochastic programming techniques would be used to solve the model. Other parameters of the model such as delivery cost, unit price cost, and suppliers' capacities could be introduced in the model in the uncertain or ambiguous representations.

Several extensions to the research related to the ELDSP could be conducted. The integer power of two multipliers mechanism could be investigated to synchronize the supply chain. The proposed joint inventory production model could be extended to handle the case when setup cost and time are sequence dependent. Another extension could be to allow suppliers to employ volume flexible production rate instead of the assumed fixed rate. Other issues such as considering imperfect quality equipments, incurring variable delivery charge per shipments, and delivering on multiple shipments could add more value to the model.

The proposed safety stock placement models could be extended by considering other probability distributions rather than the normal distribution to represent lead time and demand variability. The proposed consolidation model assumes that stockpoints placed at a given stage employ the same cycle time. By relaxing this assumption the problem can be investigated with different cycle times at the same stage. Also, the consolidation model only considers the cost savings from setting up consolidation centers at each stage; the problem could also be investigated to consider the effect of safety consolidation on the delivery time.

References

Adams, W.P., Forrester, R.J., 2005. A simple recipe for concise mixed 0-1 linearizations. *Operations Research Letters,* 33(1), 55-61.

Adams, W.P., Forrester, R.J., 2007. Linear forms of nonlinear expressions: New insights on old ideas. *Operations Research Letters,* 35, 510 – 518.

Adams, W.P., Sherali, H.D., 1990 Linearization strategies for a class of zero-One mixed integer programming problems. *Operations Research*, 38 (2), 217-226.

Yimer, A.D., Demirli, K., 2010. A genetic approach to two-phase optimization of dynamic supply chain scheduling. *Computers and Industrial Engineering,* 58, 411–422.

Ahumada, O., Villalobos, J.R., 2009. Application of planning models in the agri-food supply chain: A review. *European Journal of Operational Research*, 195, 1–20.

Akanle, O.M., Zhang, D.Z., 2008. Agent-based model for optimising supply-chain configurations. *International Journal of Production Economics*, 115(2), 444– 460.

Altiparmak, F., Gen, M., Lin, L., Paksoy, T., 2006. A genetic algorithm approach for multi-objective optimization of supply chain networks. *Computers and Industrial Engineering*, 51 (1) 196-215.

Arntzen, B.C., Brown, G.G., Harrison, T.P., Trafton, L. L., 1995. Global supply chain management at Digital Equipment Corporation*. Interfaces,* 25, 69-93.

Ballou, R.H., Burnetas, A., 2003. Planning multiple location inventories. *Journal of Business Logistics,* 24 (2), 65–89.

Ballou, R.H., 2005. Expressing inventory control policy in the turnover curve. *Journal of Business Logistics,* 26 (2), 143–164.

Blom, G., 1958. Statistical Estimates and Transformed Beta-Variables. John Wiley and Sons, Inc. New York.

Boulaksil, Y., Fransoo, J.C., Ernico, N. G., 2009. Setting safety stocks in multi-stage inventory systems under rolling horizon mathematical programming models. *OR Spectrum*, 31 (1) 121-40.

Bradley, J.R., Robinson, L.W., 2005. Improved base-stock approximations for independent stochastic lead times with order crossover. *Manufacturing and Service Operations Management,* 7 (4) 319-29.

Brown, R.G., 1967. Decision rules for inventory management. Holt, Rinehart and Winston. New York.

Çakır, O., 2009. Benders decomposition applied to multi-commodity, multi-mode distribution planning. *Expert Systems with Applications,* 36(4), 8212-8217.

Caron, F., Marchet, G., 1996. The impact of inventory centralization/decentralization on safety stock for two-echelon systems. *Journal of Business Logistics* 17 (1), 233–257.

Camm, J.D., Chorman, T., Sill, F., Evans, J., Sweeney, D., Wegryn, G., 1997. Blending OR/MS judgment, and GIS: Restructuring P&G's supply chain. *Interfaces* 27,128-142.

Clark, A.J., Scarf, H., 1960. Optimal policies for a multi echelon inventory problem. *Management Science*, 6, 475- 490.

Clausen, J., Ju, S., 2006. A hybrid algorithm for solving the economic lot and delivery scheduling problem in the common cycle case. *European Journal of Operational Research,* 175(2), 1141-1150.

Codato, G., Fischetti, M., 2006. Combinatorial Benders' Cuts for Mixed-Integer Linear Programming. *Operations Research*, 54 (4), 756-766.

Conejo, A., Castillo, E., Mínguez, R., Bertrand, R., 2006. Decomposition Techniques in Mathematical Programming. Springer-Verlag Berlin Heidelberg.

Cordeau, C.F., Pasin, F., Solomon, M.M., 2006. An integrated model for logistics network design. *Annals of Operations Research*, 144(1), 59–82.

Costa, A.M., 2005. A survey on benders decomposition applied to fixed-charge network design problems. *Computers and Operations Research*, 32(6), 1429–1450.

Das, C., Tyagi, R., 1999. Effect of correlated demands on safety stock centralization: patterns of correlation versus degree of centralization. *Journal of Business Logistics,* 20 (1), 205–213.

David, H. A., H. N. Nagaraja. 2003. Order statistics. Third edition. Wiley InterScience, NJ.

Demirtas, E.A., Üstün, Ö. 2008. An integrated multiobjective decision making process for supplier selection and order allocation. *Omega,* 36, 76 – 90.

Dogan, K., Goetschalckx, M., 1999. A primal decomposition method for the integrated design of multi-period production–distribution systems. *IIE Transactions,* 31(11), 1027-1036.

Dotoli, M., Fanti, M. P., Meloni, C., Zhou, M. C., 2005. A multi-level approach for network design of integrated supply chains. *International Journal of Production Research*, 43(20) 4267–4287.

Duran, M.A., Crossmann, I.E., 1986. An outer-approximation algorithm for a class of mixed-integer nonlinear programs. *Mathematical Programming,* 36(3), 307-339.

Edurado L., Barron C., 2007. Optimizing inventory decisions in a multi-stage multi-customer supply chain: A note. *Transportation Research Part E,* 43, 647-654.

Ehap, H.S., Benita, M.B., 2000. A multi-objective approach to simultaneous strategic and operational planning in supply chain design. *Omega,* 28, 581-598.

Eppen, G.D., R.K. Martin. 1988. Determining safety stock in the presence of stochastic lead time and demand. *Management Science*, 34 (11) 1380-90.

Evers, P.T., 1995. Expanding the square root law: an analysis of both safety and cycle stocks. *The Logistics and Transportation Review,* 31 (1), 1–20.

Evers, P.T., Beier, F.J., 1993. The portfolio effect and multiple consolidation points: a critical assessment of the square root law. *Journal of Business Logistics,* 14 (2), 109–125.

Ettl, M., Feigin, G.E., Lin, G.Y., Yao, D.D., 2000. A supply network model with base-stock control and service requirements. *Operations Research,* 48(2) 216-32.

Ezgi, A.D., Özden, Ü., 2008. An integrated multi-objective decision making process for supplier selection and order allocation. *Omega,* 36, 76 – 90.

Federer, W.T. 1951. Evaluation of Variance Components from a Group of Experiments with Multiple Classifications. Iowa Agricultural Experiment Station Research Bulletin No. 380.

Fourer, R., Gay, D.M., Kernighan, B.W., 2002. AMPL: A Modelling Language for Mathematical Programming. 2nd ed. California: Thomson Learning.

Geoffrion, A.M., 1972. Generalized Benders decomposition. *Journal of Optimization Theory and Applications,* 10(4), 237-60.

Geoffrion, A.M., Graves, G.W., 1974. Multi-commodity distribution system design by Benders decomposition. *Management Science*, 20(5), 822–845.

Ghomi, S.M.T.F., Torabi, S.A., Karimi, B., 2006. A hybrid genetic algorithm for the finite horizon economic lot and delivery scheduling in supply chains. *European Journal of Operational Research,* 173(1), 173-89.

Glover, F., 1975. Improved Linear Integer Programming Formulations of Nonlinear Integer Problems. *Management Science*, 22, 455-460.

Glover, F. 1984. An improved MIP formulation for products of discrete and continuous variables. *Journal of Information and Optimization Sciences*, 5(1), 69-71.

Glover, F., Woolsey, F., 1974. Converting the 0-1 polynominal programming problem to a 0-1 linear program. *Operations Research,* 22(1), 180-182.

Godwin, H.J., 1949. Some low moments of order statistics. *Annals of Mathematical Statistics,* 20, 279-85.

Graves, S.C., Lesnaia., E., 2004. Optimizing Safety Stock Placement in General Network Supply Chains. Innovation in Manufacturing Systems and Technology (IMST). 1, http://hdl.handle.net/1721.1/3915

Graves, S.C., Willems, S. P., 2000. Optimizing strategic safety stock placement in supply chains. *Manufacturing and Service Operations Managemen,t* 2(1), 68–83.

Hahm, J., Yano, C.A., 1992. The economic lot and delivery scheduling problem: the single item case. *International Journal of Production Economics,* 28 (2), 235-52.

Hahm, J., Yano, A.C., 1995-a. Economic lot and delivery scheduling problem: the common cycle case. *IIE Transactions,* 27 (2), 113-125.

Hahm, J., Yano, A.C., 1995-b. Economic lot and delivery scheduling problem: models for nested schedules. *IIE Transactions,* 27 (2), 126-139.

Hahm, J., Yano, A.C., 1995-c. Economic lot and delivery scheduling problem: powers of two policies. *Transportation Science,* 29 (3), 222-241.

Hahn, P.M., Kim, B.J., Guignard, M., Smith, J.M., Zhu, Y.R., 2008. An algorithm for the generalized quadratic assignment problem. *Computational Optimization and Applications,* 40 (3), 351-372.

Hammami, R., Friend, Y., Hadj-Alouane, A.B, 2009. A strategic-tactical model for the supply chain design in the delocalization context: Mathematical formulation and a case study. *International Journal of Production Economics,* 122, 351–365.

Harter, H.L. 1961. Expected values of normal order statistics. *Biometrika* 48 (1 and 2), 151-165.

Hayya, J.C., Harrison, T.P., Chatfield, D.C, 2009. A solution for the intractable inventory model when both demand and lead time are stochastic. *International Journal of Production Economics,* 122, 595–605.

Ho, W., Xu, X., Dey, P.K., 2010. Multi-criteria decision making approaches for supplier evaluation and selection: A literature review. *European Journal of Operational Research,* 202, 16–24.

Huang, G.Q., Qu, T. 2008. Extending analytical target cascading for optimal configuration of supply chains with alternative autonomous suppliers. *International Journal of Production Economics, 115(1*), 39– 54.

Huang, S.H., Keskar, H., 2007. Comprehensive and configurable metrics for supplier selection. *International Journal of Production Economics,* 105 (2), 510–523.

Inderfurth, K., 1991. Safety stock optimization in multi-stage inventory systems. *International Journal of Production Economics,* 24, 103-113.

Inderfurth, K., Minner, S., 1998. Safety stocks in multi-stage inventory systems under different service measures. *European Journal of Operational Research*, 106, 57- 73.

Jensen, M.T., Khouja, M., 2004. An optimal polynomial time algorithm for the common cycle economic lot and delivery scheduling problem. *European Journal of Operational Research,* 156 (2), 305-311.

Jung, Y.J., Blau, G., Pekny, J.F., Reklaitis,G.V., Eversdyk, D., 2008. Integrated safety stock management for multi-stage supply chains under production capacity constraints. *Computer and Chemical Engineering,* 32 (11), 2570-2581.

Liao, C.N., Kao, H.P., 2010. Supplier selection model using Taguchi loss function, analytical hierarchy process and multi-choice goal programming. *Computers & Industrial Engineering,* 58, 571–577.

Kaspi, M., Rosenblatt, M.J., 1991. On the economic ordering quantity for jointly replenishment items. *International Journal of Production Research,* 29, 107-114.

Keaton, M.H., 1994. A new functional approximation to the standard normal loss integral. *Production and Inventory Management Journal*, Second quarter, 58-62.

Khouja, M., 2003. Synchronization in supply chains: Implications for design and management. *Journal of the Operational Research Society,* 54 (9), 984-994.

Khouja, M., 2000. The economic lot and delivery scheduling problem: common cycle, rework, and variable production rate. *IIE Transactions,* 32(8), 715 – 725.

Khouja, M., Michalewicz, Z., Vijayaragavan, P., 1998. Evolutionary algorithm for economic lot and delivery scheduling problem. *Fundamental Informaticae,* 35 (1-4), 113-123.

Kim,C.O., Jun, J., Baek, J.K., Smith, R.L., Kim, Y.D., 2005. Adaptive inventory control models for supply chain management. *International Journal of Advanced Manufacturing Technology,* 26, 1184–1192.

Kim, T., Hong, Y., Chang, S.Y., 2006. Joint economic procurement-production-delivery policy for multiple items in a single-manufacturer, multiple-retailer system. *International Journal of Production Economics,* 103 (1), 199-208.

Komoto, H., Tomiyama, T., Nagel, M., Silvester, S., Brezet, H., 2005. A Multi-Objective Reconfiguration Method of Supply Chains through Discrete Event Simulation. *Fourth International Symposium on Environmentally Conscious Design and Inverse Manufacturing,* 320- 325.

Laval, C., Feyhl, M., Kakouros, S. 2005. Hewlett-Packard Combined OR and Expert Knowledge to Design Its Supply Chains. *Interfaces,* 35, 238-247.

Li, D., Sun., X., 2006. Nonlinear Integer Programming. International Series in Operations Research and Management Science, 84, Springer.

Lin, C.T., Chen, C.B., Ting Y.C., 2011. An ERP model for supplier selection in electronics industry. *Expert Systems with Applications,* 38, 1760–1765.

Louly, M.A., Dolgui, A., 2009. Calculating safety stocks for assembly systems with random component procurement lead times: A branch and bound algorithm. *European Journal of Operational Research*, 199, 723–731.

Mahmoud, M.M., 1992. Optimal inventory consolidation schemes: a portfolio effect analysis. *Journal of Business Logistics,* 13 (1), 193–214.

Maister, D.H., 1976. Centralization of inventories and the 'square root law'. *International Journal of Physical Distribution and Materials Management,* 6 (3), 124–134.

Minner, S. 1997. Dynamic programming algorithms for multi-stage safety stock optimization. *OR Spektrum*, 19, 261-271.

Nikandish, N., Eshghi, K., Torabi, S.A., 2009. Integrated Procurement, Production and Delivery Scheduling in a Generalized three Stage Supply Chain. *Journal of Industrial and Systems Engineering,* 3 (3), 189-212.

Ozturk, A., Aly, E.A.A., 1991. Simple approximation for the moments of normal order statistics. The Frontiers of Statistical Computation, Simulation and Modeling. *The First International Conference on Statistical Computing,* 1, 151-170.

Persona, A., Battini, D., Manzini, R., Pareschi, A., 2007. Optimal safety stock levels of subassemblies and manufacturing components. *International Journal of Production Economics*, 110 (1-2), 147-59.

Peterson, C.C, 1971. A note on transforming the product of variables to linear form in linear programms, working paper, Purdue University, Purdue.

Ramberg, J.S., Schmeiser, B.W., 1972. An approximation method for generating symmetric random variables. *Communications of the Association for Computing Machinery*, Inc. 17, 78-82.

Robinson, L.W., Bradley, J.R., 2001. Consequences of order crossover under order-up-to polices. *Manufacturing and Service Operations Management,* 3, 175-188.

Robinson, L.W., Bradley, J.R., 2008. Further improvements on base-stock approximations for independent stochastic lead times with order crossover. *Manufacturing and Service Operations Management,* 10 (2), 325-327.

Rose, C., Smith, M. D., 2002. Mathematical statistics with mathematica. New York: Springer-Verlag.

Rosling, K. 1989. Optimal inventory policies for assembly systems under random demands. *Operations Research,* 37 (4), 565-579.

Royston, J.P. 1982. Algorithms AS 177. Expected normal order statistics; exact and approximate. *Journal of the royal statistical society*. *Series C (Applied statistics)*, 31(2), 161-165.

Saharidis, G.K.D., Kouikoglou, V.S., Dallery, Y., 2009. Centralized and decentralized control polices for a two-stage stochastic supply chain with subcontracting. *International Journal of Production Economics*, 117(1), 117-126.

Sery, S., Presti, V., Shobrys, D.E., 2001. Optimization models for restructuring BASF North America's distribution system. *Interfaces,* 31, 55-65.

Schmeiser, B.W., 1977. Methods for modeling and generating probabilistic components in digital computer simulation when the standard distributions are not adequate: A survey. *IEEE Proceeding of the 1977 Winter Simulation Conference*, 51-57.

Schmidt, C.P., Nahmias, S., 1985. Optimal policy for a two-stage assembly system under random demand. *Operations Research,* 33 (5), 1130-1145.

Shore, H., 1982. Simple Approximations for the Inverse Cumulative Function, the Density Function and the Loss Integral of the Normal Distribution. *Journal of the Royal Statistical Society, Series C,* 31 (2), 108-114.

Simchi-Levi, D., Yao, Z., 2005. Safety stock positioning in supply chains with stochastic lead times. *Manufacturing and Service Operations Management,* 7(4), 295-318.

Simpson, K. 1958. In-process inventories. *Operations Research*, 6, 863-873.

Sitompul, C., Aghezzaf, E., 2006. Designing of Robust Supply Networks: The Safety Stock Placement Problem in Capacitated Supply Chains. *International Conference on Service Systems and Service Management,* 1, 203-209.

Sitompul, C., Aghezzaf, E., Dullaert, W., Van Landeghem, H., 2008. Safety stock placement problem in capacitated supply chains. *International Journal of Production Research,* 46 (17), 4709-4727.

Tallon, W.J., 1993. The impact of inventory centralization on consolidate safety stock: the variable supply lead time case. *Journal of Business Logistics*, 14 (1), 87–100.

Tanonkou, G.A., Benyoucef, L., Xiaolan, X., 2006. Integrated facility location and supplier selection decisions in a distribution network design. *IEEE International Conference on Service Operations and Logistics, and Informatics*, 399-404.

Teichroew, D., 1956. Tables of expected Values of Order Statistics and Products of Order Statistics for Samples of Size Twenty and Less from the Normal Distribution. *Annals of Mathematical Statistics,* 27(2), 410-426.

Tersine, R.J. 1988. Principles of inventory and materials management. North-Holland, Third edition.

Torabi, S.A., Jenabi, M., 2009. Multiple cycle economic lot and delivery-scheduling problem in a two-echelon supply chain. *International Journal of Advanced Manufacturing Technology,* 43 (7-8), 785-798.

Tyagi, R., Das, C., 1998. Extension of the square root law for safety stock to demands with unequal variances. *Journal of Business Logistics,* 19 (2), 197–203.

Üster, H., Easwaran, G., Akçali, E., Çetinkaya, S., 2007. Benders decomposition with alternative multiple cuts for a multi-product closed-loop supply chain network design mode. *Naval Research Logistics*, 54(8), 890 – 907.

Van Roy, T. J., 1986. A cross decomposition algorithm for capacitated facility location. *Operations Research*, 34 (1), 145–163.

Vila, D., Martel, A., Beauregard, R., 2006. Designing logistics networks in divergent process industries: A methodology and its application to the lumber industry. *International Journal of Production Economics,* 102, 358-378.

Wanke, P.F., 2009. Consolidation effects and inventory portfolios. *Transportation Research Part E,* 45, 107–124.

Wentges, P., 1996. Accelerating Benders' decomposition for the capacitated facility location problem. *Mathematcial Methods of Operations Research*, 44, 267-290.

Westcott, B., 1977. Algorithm AS 118; Approximate rankits. *Applied Statistics,* 26, 362-364.

William, H., 2007. Combining analytic hierarchy process and goal programming for logistics distribution network design. *IEEE International Conference on Systems, Man, and Cybernetics*. 714-719.

Yan, H., Yu, Z., Cheng, E., 2003. A strategic model for supply chain design with logical constraints: formulation and solution. *Computers and Operations Research,* 30, 2135–2155.

Zangwill, W.L., 1965. Media selection by decision programming. *Journal of Advertising Res*., 5 (3), 30-36.

Zhiying, L., Jens, R., 2007. A multi-objective supplier selection model under stochastic demand conditions. *International Journal of Production Economics*, 105 (1), 150-159.

Zinn, W., Levy, M., Bowersox, D.J., 1989. Measuring the effect of inventory centralization / decentralization on consolidate safety stock: the ‘square root law’ revisited. *Journal of Business Logistics,* 10 (2), 1–14.

Appendix

Mathematical and Statistical Considerations

A brief background to the theory applied in the research is discussed in this chapter. The mathematical programming part includes linearization and decomposition techniques applied to handle the proposed mixed integer nonlinear (MINL) models. The second part is related to some statistical considerations that enable better understanding of the research concerning the safety stock placement (SSP) problem presented in Chapter 6.

A.1 Linearization

MINL models are often intractable to be handled directly by commercial solvers. A common way to overcome this characteristic of MINL models is to approximate the convex hull of feasible integer linear solutions through applying linearization schemes. A linearization scheme is considered to be efficient if it closely approximates this convex hull by providing a tight linear programming relaxation and simultaneously keeps the model computationally tractable (Adams and Sherali, 1990).

In this section, common linearization techniques employed to handle MINL models that comprise bilinear and polynomial terms are explained. To distinguish between these two terms, a bilinear term is that term that includes a binary variable multiplied by a continuous one, while a polynomial term includes two or more binary variables multiplied together.

A.1.1 Linearization of Bilinear Terms

Consider the bilinear term xy where x is a binary variable and y is a continuous one. Consider also L and U as lower and upper bounds imposed on the continuous variable. A variable z can replace the bilinear term as shown in equation (A.1) if the four inequalities depicted in equations (A.2) and (A.3) are satisfied (Peterson, 1971).

$$z = xy \qquad (A.1)$$
$$Lx \le z \le Ux \qquad (A.2)$$
$$y - U(1-x) \le z \le y - L(1-x) \qquad (A.3)$$

Through these four inequalities, if the binary variable x equals zero, z will equal zero as well because equation (A.2) is active in this case. On the contrary, equation (A.3) is the active one when the value of the binary variable is equal to one which forces z to be equal one.

Given that z does not appear in any other constraints of a minimization problem, the right inequalities of equations (A.2) and (A.3) can be ignored if the objective function coefficient of variable z is non-negative while a non-positive coefficient leads to discarding the left inequalities. The same reduction can take place if there are constraints in the form of equations (A.4) and (A.5) correspondingly, where a_j and k are non-negative scalars (Adams and Forrester, 2007).

$$\sum_{j=1}^{n} a_j z_j \le k \qquad (A.4)$$
$$\sum_{j=1}^{n} a_j z_j \ge k \qquad (A.5)$$

If this generic scheme demonstrated by equations (A.1), (A.2) and (A.3) is applied to n bilinear terms existing in an MINL model, n continuous variables (z) and $4n$ inequalities will be added to the model. Adams and Forester (2005) modify this scheme to omit $3n$ inequalities and keep only n structural inequalities. Derivation of the modified scheme can be reached as follows: if z has a non-negative objective coefficient, the slack variable v that should be added to the left inequality of equation (A.2) can appear in the objective function and all constraints instead of z as shown in equation (A.6). In this case the four inequalities of equations (A.2) and (A.3) are replaced by the non-negativity restriction on v shown in equation (A.7) and the constraint represented in equation (A.8).

$$z = v + Lx \tag{A.6}$$
$$v \geq 0 \tag{A.7}$$
$$v \geq y - U(1-x) - Lx \tag{A.8}$$

A.1.2 Polynomial Linearization

Given a polynomial term that includes the product of binary variables, this multiplication can be replaced by one binary variable and two inequalities as shown in equations (A.9) and (A.10).

$$x = \prod_{j=1}^{J} x_j \tag{A.9}$$
$$\sum_{j=1}^{J} xj - (|J| - 1) \leq x \leq \frac{1}{|J|} \sum_{j=1}^{J} xj \tag{A.10}$$

The two inequalities (A.10) obligate x to be equal one if all the J binary variables x_j equal one, while the binary variable x will lie between a non-positive and a non-negative number if at least one of the J binary variables x_j equals zero. In such a case, x will be zero and the two inequalities are redundant. This linearization technique is introduced by

Zangwill (1965) for a multiplication of two binary variables. Watters (1967) extends this technique for polynomial terms including the product of two or more binary variables.

Glover and Woolsey (1974) establish another method to linearize polynomial terms. Their approach adds $|J|$ more constraints than the two inequalities (A.10) but the introduced variable x is a non-negative continuous variable. If at least one x_j equals zero, set of equations (A.11) and equation (A.12) will combine together to form an equality constraint forcing x to be zero. If all x_j equal one, equation (A.13), which is the left inequality introduced in equation (A.10) jointly with set of equations (A.11), will constrain x to be equal to one.

$$x \le xj \qquad j = 1, \ldots.., J \qquad (A.11)$$
$$x \ge 0 \qquad (A.12)$$
$$x \ge \sum_{j=1}^{J} xj - (|J| - 1) \qquad (A.13)$$

Hahn et al. (2008) introduce a special linearization technique to linearize polynomial terms appearing in the quadratic assignment (QA) problem. This special class of polynomial terms takes into account the pair-wise interactions between binary variables. Each polynomial term $x_{ij}x_{kn}$, which represents assigning job i, machine j and job k to machine n, is replaced by a non-negative continuous variable v_{ijkn} as shown in equation (A.14). The binary variable v_{ijkn} equals one if and only if job i is assigned to machine j and job k is assigned to machine n. Since $x_{ij}x_{kn}$ equals $x_{kn}x_{ij}$, the equality constraint, shown in equation (A.15), is added to the model to insure such equivalence. Another equality constraint, shown in equation (A.16), is added to assign job k to machine n given that job i is assigned to machine j.

$$v_{ijkn} = x_{ij} x_{kn} \qquad \forall i, j, k, n \qquad (A.14)$$

$$v_{ijkn} = v_{knij} \qquad \forall i, j, k, n\ ,\ \ i < k \qquad (A.15)$$

$$\sum_n v_{ijkn} = x_{ij} \qquad \forall i, j, k, n\ ,\ \ i \neq k \qquad (A.16)$$

$$v_{ijkn} \geq 0 \qquad \forall i, j, k, n \qquad (A.17)$$

A.2 Decomposition Techniques

In practice, the size of mathematical models can be large, consisting of a huge number of variables and constraints. Commercial solvers cannot handle these models directly if the models include complicating variables or constraints. Moreover, mathematical models may show another kind of difficulty represented in the form of non-linear, bilinear, or polynomial terms.

Decomposition techniques are useful tools to deal with intractable models including such different kinds of complexity. Among these techniques, the generalized Benders decomposition (BD) technique introduced by Geoffrion (1972), and the outer approximation (OA) approach developed by Duran and Grossman (1986) are discussed in this section.

A.2.1 Benders Decomposition Technique

The BD technique is suitably applied to mathematical models including complicating variables. A variable is considered to be complicating if it appears in all the constraints of a given model and prevents solving it by blocks. Also, if relaxing integrality or binary restrictions imposed on a variable will lead to solving the model easily, such an integer variable is considered to be complicating.

Equations (A.18), (A.19), (A.20) and (A.21) represent a mathematical model including two sets of variables. The complicating variable is considered to be x_i, while the non-complicating variable is y_j (Conejo et al., 2006).

$$Min \ \sum_{i=1}^{I} c_i x_i + \sum_{j=1}^{J} d_j y_j \qquad (A.18)$$

Subject to

$$\sum_{i=1}^{I} a_{li} x_i + \sum_{j=1}^{J} e_{lj} y_j \le b_l \qquad l = 1, \ldots.., L \qquad (A.19)$$

$$0 \le x_i \le x_i^{up} \qquad i = 1, \ldots.., I \qquad (A.20)$$

$$0 \le y_j \le y_j^{up} \qquad j = 1, \ldots.., J \qquad (A.21)$$

Using the BD technique, this problem is decomposed into a master and a sub-problem. The master problem presented by equations (A.22), (A.23), (A.24) and (A.25) determines the optimal value of the complicating variables x_i while the sub-problem illustrated in equations (A.26), (A.27), (A.28) and (A.29) finds the optimal values of the non-complicating variables y_j given those optimal values of the complicating variables x_i^k at iteration k. At each feasible iteration, an optimality cut (A.23) is added to the master problem. This cut, also called Benders cut, is built based on duality theory by considering the dual variables associated with constraint (A.29). Benders cut is used to drive the objective function of the master problem to move toward the objective function of the sub-problem through minimizing function α.

$$Min \ \sum_{i=1}^{I} c_i x_i + \alpha \qquad (A.22)$$

Subject to

$$\sum_{j=1}^{J} d_j y_j^k + \sum_{i=1}^{I} \lambda_i^k (xi - x_i^k) \le \alpha \qquad k = 1, \ldots.., v-1 \qquad (A.23)$$

$$0 \le x_i \le x_i^{up} \qquad i = 1, \ldots.., I \qquad (A.24)$$

$$\alpha \geq \alpha^{down} \qquad (A.25)$$

$$Min \ \sum_{j=1}^{J} d_j y_j \qquad (A.26)$$

Subject to

$$\sum_{i=1}^{I} a_{li} x_i + \sum_{j=1}^{J} e_{lj} y_j \quad \leq b_l \qquad l = 1, \ldots.., L \qquad (A.27)$$

$$0 \leq y_j \leq y_j^{up} \qquad j = 1, \ldots.., J \qquad (A.28)$$

$$x_i = x_i^v \ : \lambda_i \qquad i = 1, \ldots.., I \qquad (A.29)$$

An optimal solution is found when the lower bound (A.30) obtained from the relaxed master problem equals the upper bound (A.31) resulting from the restricted sub-problem.

$$z_{down}^v = \sum_{i=1}^{I} c_i x_i^v + \alpha^v \qquad (A.30)$$

$$z_{up}^v = \sum_{i=1}^{I} c_i x_i^v + \sum_{j=1}^{J} d_j y_j^v \qquad (A.31)$$

In this thesis, the BD technique is applied to deal with the complexity of binary variables. The master problem, which is a pure binary model, provides 0-1 combination to those binary variables. If the sub-problem is infeasible to those values, a combinatorial feasibility cut (A.32), introduced by Codato and Fischetti (2004), is added to the master problem after each infeasible iteration t to look for a feasible binary combination. This can be done through leading the master problem to generate a sum of 0-1 combination that differs from any previous infeasible combination by at least one. The feasibility cut may take different forms depending on the constraints included in the sub-problem.

$$\sum_{i:x_i^t=0}^{I} x_i + \sum_{i:x_i^t=1}^{I} (1 - x_i) \quad \geq \ 1 \qquad t = 1, \ldots.., T \qquad (A.32)$$

A.2.2 Outer Approximation Approach

Duran and Grossman (1986) establish an OA approach to solve a particular class of mixed integer nonlinear models. The characteristics of this class of models are the linearity of the integer variables and the convexity of the nonlinear function with respect to the continuous variables. The algorithm iterates between solving a relaxed mixed integer master problem and a nonlinear sub-problem.

A mixed integer nonlinear mathematical model that belongs to the specific class mentioned above is shown in equations (A.33), (A.34) and (A.35) (Li and Sun, 2006). Functions *f* and *g* are convex in *y* and linear in the integer variable *x*. It is assumed that *Y* is a non-empty convex set and *X* is a finite integer set.

$$Min\ f(x,y) \tag{A.33}$$

Subject to

$$g_i(x,y) \le 0 \qquad i = 1,.....,I \tag{A.34}$$

$$x \in X \subset Z^m,\ y \in Y \subseteq \Re^n \tag{A.35}$$

This model can be decomposed using the OA method into a master problem and a sub-problem. The master problem, which is depicted in equations (A.36), (A.37), (A.38) and (A.39), solves a mixed integer linear model considering the linear estimation of nonlinear functions *f* and *g*. For given values of the integer variable *x* obtained from the master problem, the sub-problem finds optimal value of the continuous variable *y* by solving the nonlinear model given by equations (A.33), (A.34) and (A.35). After each iteration a convergence check is taking place to recognize lower and upper bounds obtained from the master problem and the sub-problem respectively.

$$Min\ \alpha \tag{A.36}$$

Subject to

$$\alpha \geq f(x^k, y^k) + \Delta^K f(x^k, y^k)\begin{pmatrix} x - x^k \\ y - y^k \end{pmatrix} \qquad k = 1,.....,K \qquad (A.37)$$

$$0 \geq g(x^k, y^k) + \Delta^K g(x^k, y^k)\begin{pmatrix} x - x^k \\ y - y^k \end{pmatrix} \qquad k = 1,.....,K \qquad (A.38)$$

$$x \in X \subset Z^m,\ y \in Y \subseteq \Re^n,\ \alpha \in \Re^1 \qquad (A.39)$$

If the OA is applied to a 0-1 mixed integer nonlinear model, the combinatorial feasibility cut (A.32) is added to the master problem after each infeasible solution of the sub-problem.

A.3 Order Statistics

Order statistics (OS) deals with ordered random variables and studies their properties and applications. If the random variables $X_1, X_2, \ldots, X_n$ are arranged in an ascending order where $X_{(1)} \leq X_{(2)} \leq \ldots\ldots \leq X_{(n)}$, then $X_{(r)}$ is called the r^{th} OS. Assuming that these n random variables are independent and identically distributed with the probability density function (pdf) $f(x)$ and the cumulative distribution function (cdf) $F(x)$, the pdf of the r^{th} order statistic is given by equation(A.40) where r = 1, 2,..., n (Rose and smith 2002).

$$f_r(x) = \frac{n!}{(r-1)!(n-r)!} F^{r-1}(x)[1 - F(x)]^{n-r} f(x) \qquad (A.40)$$

In the safety stock positioning models proposed in this thesis, the delivery times of input materials coming from multiple sources are assumed to be independent and normally distributed random variables. So, for a given stockpoint, determining parameters of the probability distribution representing the maximum delivery time random variable is an example of OS. In particular, mean and variance of a standard normally distributed order statistics (NDOS) have to be calculated before starting to solve the safety stock analytical models. For NDOS, the pdf of the r^{th} OS is shown in equation (A.41) while equation (A.42) demonstrates the mean of that particular OS.

$$f_r(x) = \frac{n!}{(r-1)!(n-r)!}\int_{-\infty}^{\infty}\{\varphi(x)\}^{r-1}\{1-\varphi(x)\}^{n-r} f(x)dx \qquad (A.41)$$

$$E(X_r) = \frac{n!}{(r-1)!(n-r)!}\int_{-\infty}^{\infty}x\{\varphi(x)\}^{r-1}\{1-\varphi(x)\}^{n-r} f(x)dx \qquad (A.42)$$

Godwin (1949) establishes tables of mean, variance, and covariance of NDOS of size 10 or less. For samples of 20 or less, tables of the expected value of the r^{th} OS was established by Teichrow (1956). For larger sample sizes of 2(1) 100(25) 250(50) 400, Harter (1961) present the expected values of NDOS. Federer (1951), Blom (1958), Wescott (1977), and Royston (1982) introduce algorithms to approximate the expected values of OS. These algorithms apply numerical methods and do not provide any simple explicit form to find moments of OS.

Ozturk and Aly (1991) introduce an algorithm to approximate parameters of NDOS. The algorithm approximates the expected value and variance of NDOS using the generalized lambda distribution (GLD). In such cases, the moments of GLD OS are used as an approximation to the moments of standard NDOS.

The inverse distribution function of the GLD proposed by Ramberg and Schemeiser (1972) is shown in equation (A.43) where λ_1, λ_2, λ_3 and λ_4 are parameters of the distribution. For 0, 0.1975, 0.1349 and 0.1349 given values of these parameters, Schemeiser (1977) showed that the maximum absolute error through approximating the NDOS by the GLD is 0.001. Equations (A.44) and (A.46) show the closed form given by

Ozturk and Aly (1991) to approximate mean m_r and variance v_r of NDOS using GLD. The β function used to calculate the variance is shown in equation (A.48).

$$F^{-1}(p) = \lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2} \tag{A.43}$$

$$m_r = \frac{C_r - C_{n-r+1}}{\lambda_2 \, C_{n+1}} \tag{A.44}$$

Where $$C_r = r\prod_{k=1}^{r}\left(1 + \frac{\lambda_3 - 1}{k}\right) \tag{A.45}$$

$$v_r = \frac{\beta(2\lambda_3 + r,\, t) - 2\beta(\lambda_3 + r,\, t + \lambda_4) + \beta(r,\, t + 2\lambda_4)}{\lambda_2^2 \, \beta(r,\, t)} - (m_r - \lambda_1)^2 \tag{A.46}$$

Where $$t = n - r + 1 \tag{A.47}$$

and $$\beta(x, y) = \frac{(x-1)!(y-1)!}{(x+y-1)!} \tag{A.48}$$

Parameters m_r and v_r of the standard NDOS are used to drive mean $E(X_r)$ and variance $Var(X_r)$ of the original OS. If n OS are represented by identical normal distributions having mean μ and variance σ^2, parameters of the maximum OS distribution are given by equations (A.49) and (A.50) where r equals n.

$$E(X_r) = \mu + \sigma \, m_r \tag{A.49}$$

$$Var(X_r) = \sigma^2 \, v_r \tag{A.50}$$

A.4 Approximation to the Standard Loss Integral

In a stochastic inventory system, the lead time demand is handled as a random variable. To deal with such variability of lead time demand, an adequate safety amount should be kept in stock so as to fulfill a specific customer service. If the desired service level refers to a given fill rate, the standard normal deviate Z should be determined from the partial expectation $E(Z)$. The partial expectation stands for the expected number of stockouts during one cycle. The relation between Z and $E(Z)$ can be established using the standard loss integral (SLI) shown in equation (A.51) (Tersine, 1988). Fig A.1 depicts such a relationship between *Z* and *E(Z)*.

$$E(Z) = \int_{z}^{\infty} (t - Z)\, f(t)\, dt \qquad (A.51)$$

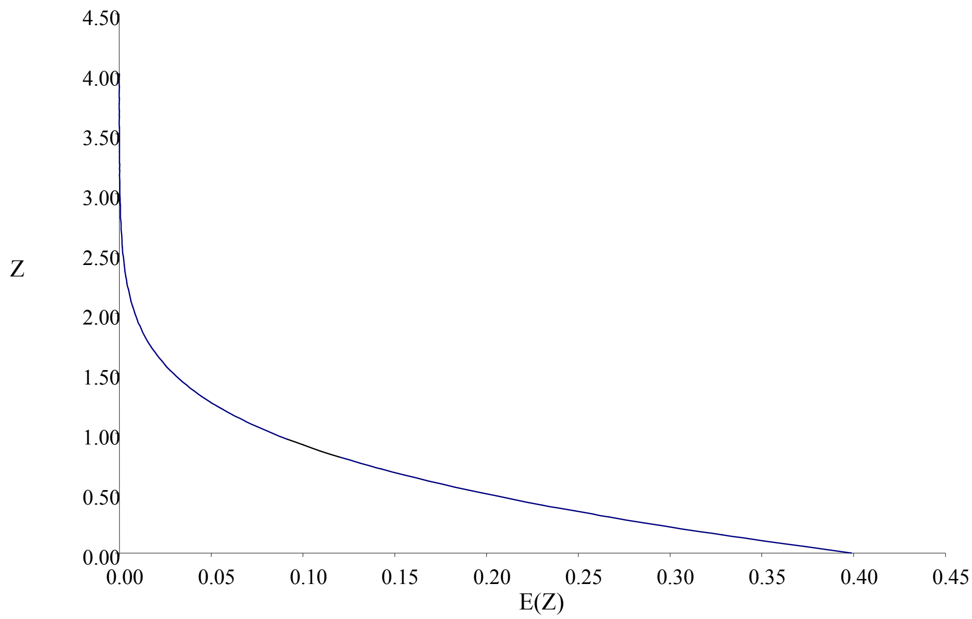

Fig A.1: Standard Normal Loss Integral

Brown (1967) established the original table of SLI using the functional approximation illustrated in equation (A.52). This approximation is widely applied in the literature to find the standard normal deviate *Z* from the partial expectation *E(Z)*.

$$Z = \left[E(Z)\text{-}0.38984228\right] \times \left[\begin{array}{l} \text{-}1.75294\text{+}0.4442135E(Z)\text{-}0.07061455E(Z)^2 \\ \text{-}\dfrac{0.17592241}{E(Z)\text{+}0.044212641}\text{-}\dfrac{0.0012267386}{E(Z)\text{+}.00030570313} \end{array}\right] \qquad (A.52)$$

Shore (1982) provides equation (A.53) to approximate *E(Z)* in terms of the distribution function *F(Z)*. In addition, other approximations are proposed to find *Z* using the inverse cumulative function. So, *Z* can be found from *E(Z)* in two steps.

$$E(Z)=\begin{bmatrix} 0.4115\{1-\ln[F(Z)/(1-F(Z)]\}, & if & F(Z)\le 0.5 \\ 0.4115[F(Z)/(1-F(Z)], & if & F(Z)\ge 0.5 \end{bmatrix} \qquad (A.53)$$

Keaton (1994) introduces three alternative exponential approximations to find standard normal deviate *Z* from the partial expectation *E(Z)*. The function is depicted in equation (A.54) while the alternative values of the parameters of this function are shown in Table A.1.

$$Z=\alpha E(Z)^{\beta}\exp\{\gamma E(Z)^{\delta}\}-E(Z) \qquad (A.54)$$

Table A.1: Coefficients of alternative loss integral approximations introduced by Keaton (1994)

Parameters	Approximation		
	1	2	3
α	1.94519891	1.83513389	1.82268153
β	-0.06100591	-0.06567952	-0.06609373
γ	-2.70426869	-2.62970236	-2.65829265
δ	0.50840810	0.54505649	0.56235517