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AN INVESTIGATION INTO PROPERTIES OF AN ESTIMATOR OF MEAN OF AN INVERSE GAUSSIAN POPULATION

Yogendra P. Chaubey and Debaraj Sen

An investigation into properties of an estimator of mean of an inverse Gaussian population*

Yogendra P. Chaubey and Debaraj Sen
Concordia University, Montreal, Canada

Abstract

This paper deals with preliminary test estimation for mean of an inverse Gaussian population. Preliminary test estimator has been shown to provide large gains in efficiency, especially around a neighbourhood of the prior guessed value of the parameter, for many distributions including exponential and normal, however, this has not been explored for the inverse Gaussian family of distributions. Owing to diverse applications of the inverse Gaussian model for non-negative and positively skewed data, the investigation considered here makes an important contribution in the area of preliminary test estimation. We consider both the cases of known and unknown dispersion parameters and demonstrate similar conclusions as obtained in the case of Gaussian populations in terms of the efficiency of the resulting estimator.

Key Words: Minimum mean square error, preliminary test estimator, inverse Gaussian population, relative bias, relative mean square error.

1 Introduction

Tests of hypothesis are often used to validate a given model. Such tests are referred to as preliminary tests of significance, where the word *preliminary* alludes to the notion of confirming tentatitively, the accepted value of a parameter under the null hypothesis. Bancroft (1944) proposed to use such prior guesses to be used in place of the usual estimator if the prior guess is ascertained using a test of hypothesis, otherwise the traditional estimator is to be

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used. The resulting estimator is termed as the *Preliminary Test Estimator* (PTE). To fix the basic idea behind this procedure, let us consider estimating the mean μ of some infinite population. Suppose, μ_0 is the prior guess of the parameter μ. For a given sample, let $\bar{X}$ be the sample mean, then under a variety of situations/models, it is a "good" estimator of μ. However, there may be a strong evidence in favor of μ_0, in that case, the statistician should choose μ_0 as the natural estimator. If the evidence is taken from the sample based on a test statistic T, such an estimator may be represented as

$$\begin{aligned} \hat{\mu}_{PTE} &= \bar{X} I_{\{T \in CR\}} + \mu_0 I_{\{T \notin CR\}} \\ &= \mu_0 + (\bar{X} - \mu_0) I_{\{T \in CR\}} \end{aligned}$$

where CR denotes the critical region for testing $H_0 : \mu = \mu_0$ vs. $H_0 : \mu \neq \mu_0$, based on a test statistic T, and I_A denotes the indicator of A.

Bancroft (1944) considered the case of Gaussian population and showed that such estimators may provide large gains in efficiency, especially, if the true value of the parameter is near the hypothesized value. They further provided guidelines for choosing the level of significance. This method has been adapted in various other situations by Bancroft (1964), Paul (1950), Huntsburger (1954), Arnold and Katti (1972), Bock *et al.* (1973), Han (1978), Ghosh and Sinha (1988), , Yancey, Judge and Bohrer (1989), Pandey and Malik (1990), Pandey, Malik and Dube (1995), Pandey (1997) and Pandey and Srivastava (2001) just to name a few. The paper by Pandey and Malik (1990) consists of some interesting work on estimation of mean from inverse Gaussian population based on adaptive estimation. The inverse Gaussian distribution with parameters μ and λ, denoted by $IG(\mu, \lambda)$ is described by the probability density function

$$f(x; \mu, \lambda) = \left(\frac{\lambda}{2\pi x^3} \right)^{1/2} \exp\left(-\frac{\lambda (x-\mu)^2}{2\mu^2 x} \right), \quad 0 < x < \infty. \tag{1}$$

This distribution was extensively studied by Tweedie (1957a, b) but it is popularised by the review article by Folks and Chhikara (1978). It is enthusiastically recommended as an alternative to the usual Gaussian distribution in Chhikara and Folks (1989) and Seshadri (1998) for modelling positive and/or positively skewed data. The parameter μ is the mean and λ is known as the dispersion parameter as the variance of this distribution is given by $\sigma^2 = \frac{\mu^3}{\lambda}$. For a random sample, a minimal sufficient statistic for (μ, λ) is given by $(\bar{X}, \sum_{i=1}^{n} \frac{1}{X_i})$. It is also interesting to note that

$$\bar{X} \sim IG(\mu, n\lambda) \quad \text{and} \quad \lambda \sum_{i=1}^{n} \left(\frac{1}{X_i} - \frac{1}{\bar{X}} \right) \sim \chi^2_{n-1}. \tag{2}$$

As such, $\bar{X}$ provides the best unbiased estimator of μ and

$$U = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{1}{X_i} - \frac{1}{\bar{X}} \right) \tag{3}$$

provides that for $\frac{1}{\lambda}$, and moreover they are independent. The reader is referred to the excellent texts by Chhikara and Folks (1989) and Seshadri (1998) for other details concerning theory and applications of the inverse Gaussian distribution.

Here, we investigate the performance of PTE of mean μ in a single sample setup. Section 2 considers the case with the known λ and section 3 considers the unknown case. Section four presents a numerical study on the relative bias and relative MSE properties of the resulting estimator.

2 Preliminary Test Estimation of Mean with a Prior Guess on Mean

2.1 Known λ Case

As explained above, the PTE requires testing about the prior guess. In this case, we first test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. In this case, the UMP-unbiased test is given in the form of the critical region:

$$CR = \{\bar{X} : \bar{X} < k_1 \text{ } or \text{ } \bar{X} > k_2\}$$

where k_1, k_2 are determined from the conditions

$$\int_{k_1}^{k_2} g(t)dt = 1 - \alpha \quad \text{and} \quad \int_{k_1}^{k_2} tg(t)dt = \mu_0(1 - \alpha)$$

and g is the pdf of $\bar{x}$. Chhikara and Folks (1989) show that this is equivalent to considering the test statistic

$$Z = \frac{\sqrt{n\lambda}(\bar{X} - \mu_0)}{\mu_0\sqrt{\bar{X}}}.$$

and corresponding critical region,

$$|Z| > z_{1-\alpha/2}$$

where $z_{1-\alpha/2}$, is the $100(1 - \alpha/2)\%$ percentiles of the standard normal distribution. Using the above critical region, the constants k_1 and k_2 can be found as,

$$k_1 = \left[\frac{\mu_0 c_1 + \sqrt{\mu_0^2 c_1^2 + 4\mu_0 n\lambda}}{2\sqrt{n\lambda}} \right]^2$$

and

$$k_2 = \left[\frac{\mu_0 c_2 + \sqrt{\mu_0^2 c_2^2 + 4\mu_0 n\lambda}}{2\sqrt{n\lambda}} \right]^2$$

where $c_1 = -z_{1-\alpha/2}$ and $c_2 = z_{1-\alpha/2}$. Now the computation of a preliminary test estimator of the mean $\hat{\mu}$ is given by

$$\begin{aligned} \hat{\mu}_z &= \bar{X} I_{\bar{X} \in CR} + \mu_0 I_{\bar{X} \notin CR} \\ &= \bar{X} - (\bar{X} - \mu_0) I_{[k_1 < \bar{X} < k_2]} \end{aligned} \quad (4)$$

2.2 Unknown λ Case

For unknown λ, the UMP-unbiased test for $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ is given in the form of the critical region:

$$CR = \{\bar{X} < k_3 \; or \; \bar{X} > k_4\}$$

where k_3 and k_4 are determined by

$$\int_{k_3}^{k_4} h(u|v) du = 1 - \alpha \quad \text{and} \quad \int_{k_3}^{k_4} u h(u|v) du = (1-\alpha) \int_{-\infty}^{\infty} u h(u|t) du$$

and $h(u|v)$ denotes the conditional density function of $\bar{X}$ given V. Chhikara and Folks (1989) show that it is equivalent to consider the statistic,

$$T = \frac{\sqrt{n-1}(\bar{X} - \mu_0)}{\mu_0 \sqrt{V\bar{X}}}$$

where

$$v = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{X_i} - \frac{1}{\bar{X}}\right)$$

and corresponding critical region,

$$\left| \frac{\sqrt{(}n-1)(\bar{X} - \mu_0)}{\mu_0 \sqrt{(}\bar{X}).V} \right| > t_{1-\frac{\alpha}{2}},$$

were $t_{1-\frac{\alpha}{2}}$, is the $100(1-\frac{\alpha}{2})\%$ percentiles of the student's t distribution with $(n-1)$ degrees of freedom. This gives k_3 and k_4 in terms $t_{1-\frac{\alpha}{2}}$, as

$$k_3 = \left[\frac{\mu_0 c_1 \sqrt{V} + \sqrt{\mu_0^2 c_1^2 V + 4\mu_0 (n-1)}}{2\sqrt{n-1}} \right]^2$$

and

$$k_4 = \left[\frac{\mu_0 c_2 \sqrt{V} + \sqrt{\mu_0^2 c_2^2 V + 4\mu_0(n-1)}}{2\sqrt{n-1}} \right]^2 .$$

And hence, the PTE of μ in this case is given by

$$\begin{aligned} \hat{\mu}_t &= \bar{X} I_{\{\bar{X} \in CR\}} + \mu_0 I_{\{\bar{X} \notin CR\}} \\ &= \bar{X} - (\bar{X} - \mu_0) I_{\{k_3 < \bar{X} < k_4\}} \end{aligned} \quad (5)$$

In order to judge the performance of PTE, we need compute its bias and MSE. This is explained in the following section.

3 Bias and MSE of PTE's

3.1 Known λ Case

The moments of PTE with known λ depend only on the distribution of $\bar{X}$. The following propositions will be used in computing the bias and MSE for known λ case.

Proposition 3.1 *The power function for the test* $H_0 : \mu = \mu_0$ *against* $H_1 : \mu \neq \mu_0$ *is*

$$\pi(\mu) = 1 - \Pr[k_1 < \bar{X} < k_2 | \mu]$$

which may be written as

$$\pi(\mu) = 1 - F(k_2; \mu, n\lambda) + F(k_1; \mu, n\lambda), \quad (6)$$

where $F(x; \mu, \lambda)$ *denotes the cumulative distribution function of an* $IG(\mu, \lambda)$ *distribution.*

Computation of the above power function may be easily computed using the distribution function of a standard normal variate using the following formula,

$$F(x; \mu, \lambda) = \Phi \left\{ \sqrt{\frac{\lambda}{x}} \left(\frac{x}{\mu} - 1 \right) \right\} + \mathrm{e}^{2\lambda/\mu} \Phi \left\{ -\sqrt{\frac{\lambda}{\mathrm{x}}} \left(\frac{\mathrm{x}}{\mu} + 1 \right) \right\},$$

where $\Phi(.)$ denotes the *c.d.f.* of the standard normal variable. Figure 1 provides the power function for $n = 16$, $\mu_0 = 1$, $\lambda = 1$, using the above formula. Computation of bias and MSE of the PTE is facilitated by the use of the above formula as shown in the following proposition.

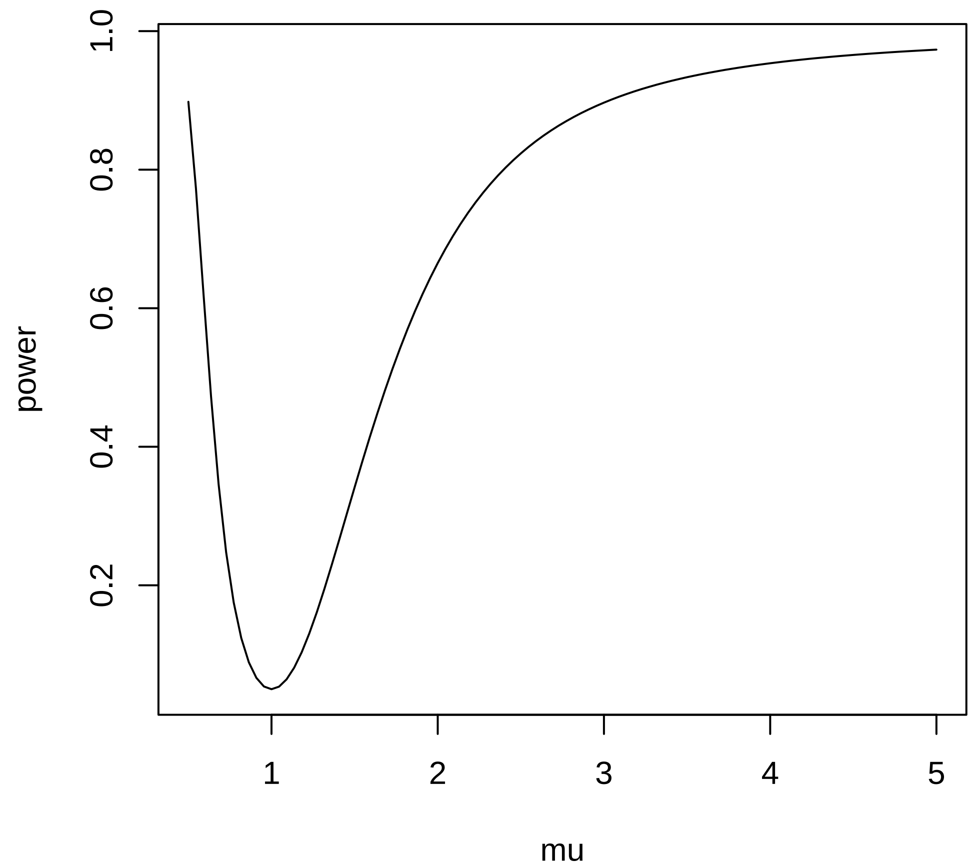

Figure 3.1: Power function for the UMP Unbiased Test in IG Case, $n = 16, H_0 : \mu = 1$

Proposition 3.2 *The expressions for bias and MSE of $\hat{\mu}_z$, are respectively given by*

$$Bias(\hat{\mu}_z) = \mu_0[1 - \pi(\mu)] - \int_{k_1}^{k_2} w f_{\bar{X}}(w) dw, \tag{7}$$

and

$$MSE(\hat{\mu}) = \frac{\mu^3}{n\lambda} + 2\mu B(\hat{\mu}_z) + \mu_0^2(1 - \pi(\mu)) - \int_{k_1}^{k_2} w^2 f_{\bar{X}}(w) dw \tag{8}$$

*where $\pi(\mu)$ is the power function of the UMP-unbiased test for testing $H_0 : \mu = \mu_0$ vs. $H_0 : \mu \neq \mu_0$, and $B(\hat{\mu}_z)$ denotes the bias of $\hat{\mu}_z$ given in Eq. (**??**).*

Proof: Using the expression in Eq. (??), straight forward calculation provides,

$$E(\hat{\mu}_z) = \mu - \int_{k_1}^{k_2} (w - \mu_0) f_{\bar{X}}(w) dw,$$

and the expression for the bias follows, noting that

$$\pi(\mu) = 1 - \Pr[k_1 < \bar{X} < k_2 | \mu] = \int_{k_1}^{k_2} f_{\bar{X}}(w) dw.$$

For simplifying the MSE expression, we note that

$$(\hat{\mu}_z - \mu)^2 = (\bar{X} - \mu)^2 + [\mu_0^2 - \bar{X}^2 + 2\mu(\bar{X} - \mu_0)] I_{k_1 \leq \bar{X} \leq k_2}$$

and hence, the result follows. □

3.2 Unknown λ Case

We note in this case that the critical region depends on the values of V. Hence, for computing the moments of $\hat{\mu}_t$, first we compute the conditional moments $b(v) = E[(\hat{\mu}_t - \mu)|V = v]$, and $m(v) = E[(\hat{\mu}_t - \mu)^2|V = v]$ using the Prop. (??) replacing k_1, k_2 by $k_3(v) \simeq k_3, k_4(v) \simeq k_4$. Hence, we obtain the following expressions for bias and MSE in the unknown case.

Proposition 3.3 *The expressions for bias and MSE of $\hat{\mu}_t$, are respectively given by*

$$Bias(\hat{\mu}_t) = \int_0^\infty b(v) f_V(v) dv$$

and

$$MSE(\hat{\mu}_t) = \int_0^\infty m(v) f_V(v) dv$$

where $f_V(v)$ is the probability density function of the Chi-square distribution with $(n-1)$ degrees of freedom.

4 A Numerical Comparison of the Estimators

The above formulae are used to compute the bias and MSE for various sample sizes and different values of μ. The integrals involved were computed using Splus `integrate` function. The value of λ was fixed at 1. Figures 4.1–4.4 summarize these computations, however, some representative values are also given in Tables 4.1-4.6.

Table 4.1 gives the bias of $\hat{\mu}_z$ for $n = 16$ and Table 4.2 presents that for $n = 20$ for two different values of α, namely, 1% and 5%. Similarly Table 4.3 and Table 4.4 present those for unknown λ case. Table 4.5 presents the relative MSE's for $n = 16$ and $n = 20$ respectively for the same values of α where as Table 4.6 present the relative efficiencies of the PTE's $\hat{\mu}_z$ and $\hat{\mu}_t$ with respect to the a sample mean for the $n = 16$ and $n = 20$. To visualize the effect of preliminary test on the bias and to assess the gain in efficiency, we plot the relative bias and relative efficiency (with respect to $\bar{X}$) for different sets of parameters, sample sizes and significance levels.

Based on these graphs and tables, we draw the following conclusions:

(1) Bias decreases as n increases.

(2) When $\mu \leq 1$ then bias increases, for $1 < \mu \leq 1.5$ then bias decreases but when $1.5 < \mu$ then again bias increases. As α increases bias also increases.

(3) For fixed μ, bias increases as μ_0 increases but for fixed μ_0 bias decreases as μ increases.

(4) The maximum possible loss of efficiency increases for $\mu = 1$ and $\mu \geq 1.5$ but when $\mu = 1.5$ efficiency decreases.

(4) The effective difference of efficiency is greater when α increases.

(5) The result indicate that in the case of IG estimators is effective in reducing the maximum loss of efficiency and increasing the effective difference.

(6) By examination of the values in graphs and tables, it will be seen that when λ is known, the preliminary test of significance controls the bias well for larger values of μ, resulting in substantial gains in relative efficiency.

Table 4.1: Bias of $\hat{\mu}_z$ for $n = 16$

μ	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$
		$\alpha = 1\%$			$\alpha = 5\%$	
0.5	0.135169	0.001663	0.000017	0.033591	0.000014	0.000000
1.0	0.000000	0.666944	0.701347	0.000000	0.33155	0.165811
1.5	-0.274586	0.459349	1.209603	-0.157975	0.361778	0.755871
2.0	-0.231339	0.000000	0.918142	-0.106140	0.000000	0.720122
2.5	-0.151435	-0.404113	0.469159	-0.061421	-0.284178	0.382874
3.0	-0.099347	-0.651850	0.000000	-0.037523	-0.409260	0.000000
3.5	-0.068564	-0.756061	-0.429436	-0.024774	-0.434245	-0.316430
4.0	-0.049948	-0.530559	-0.552975	-0.017527	-0.415125	-0.535160
4.5	-0.038148	-0.745209	-1.032303	-0.013118	-0.381317	-0.668068
5.0	-0.030303	-0.700645	-1.202236	-0.010269	-0.345466	-0.738977

Table 4.2: Bias of $\hat{\mu}_z$ for $n = 20$

μ	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$
		$\alpha = 1\%$			$\alpha = 5\%$	
0.5	.073691	.000545	.000004	0.013155	0.000001	0.000000
1.0	.000000	.556286	.394361	0.000000	0.232133	0.060848
1.5	-.025092	.453104	1.010851	-0.138439	0.349072	0.618549
2.0	-.017663	.000000	.901902	-0.075309	0.000000	0.685998
2.5	-.097573	-.399575	.467579	-0.036206	-0.278504	0.379441
3.0	-.055609	-.625242	-.00000	-0.019052	-0.383887	0.000000
3.5	-.034233	-.696693	-.428009	-0.011162	-0.387064	-0.314403
4.0	-.022715	-.682034	.767887	-0.007169	-0.351793	-0.523547
4.5	-.060613	-.631524	-1.00486	-0.004956	-0.308290	-0.640477
5.0	-.011960	-.571377	-1.15-792	-0.003631	-0.267625	-0.692917

Table 4.3: Bias of $\hat{\mu}_t$ for $n = 16$

μ	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$
		$\alpha = 1\%$			$\alpha = 5\%$	
0.5	0.220419	0.056914	0.0194914	0.063415	0.001149	0.000075
1.0	0.000000	0.754592	1.051109	0.000000	0.412151	0.321653
1.5	-0.320748	0.465899	1.277217	-0.185189	0.375385	0.848951
2.0	-0.322579	0.000000	0.931265	-0.139115	0.000000	0.747992
2.5	-0.238804	-0.425646	0.473027	-0.086274	-0.306782	0.393048
3.0	-0.169686	-0.726898	0.000000	-0.055011	-0.462131	0.000000
3.5	-0.123476	-0.893645	-0.444393	-0.037340	-0.509779	-0.335851
4.0	-0.093282	-0.960589	-0.827869	-0.026917	-0.502651	-0.582566
4.5	-0.073113	-0.966792	-1.134207	-0.020412	-0.472982	-0.745113
5.0	-0.059190	-0.940561	-1.363285	-0.016134	-0.436640	-0.842185

Table 4.4: Bias of $\hat{\mu}_t$ for $n = 20$

μ	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$
		$\alpha = 1\%$			$\alpha = 5\%$	
0.5	0.154391	0.009589	0.001245	0.027164	0.000031	0.000002
1.0	0.000000	0.686583	0.776284	0.000000	0.298308	0.138373
1.5	-0.303374	0.465515	1.225797	-0.159564	0.361638	0.706735
2.0	-0.261062	0.000000	0.928041	-0.096314	0.000000	0.712842
2.5	-0.16503	-0.426797	0.475417	-0.049367	-0.296960	0.388071
3.0	-0.102495	-0.712976	0.000001	-0.027020	-0.426232	0.000000
3.5	-0.066754	-0.848150	-0.447743	-0.016238	-0.445135	-0.330139
4.0	-0.046032	-0.878382	-0.829270	-0.010609	-0.416043	-0.562023
4.5	-0.033447	-0.851539	-1.124489	-0.007423	-0.372609	-0.702414
5.0	-0.025408	-0.799343	-1.333576	-0.005486	-0.328976	-0.774574

Table 4.5: Relative MSE of $\hat{\mu}_z$ and $\hat{\mu}_t$ for $n = 16$, $\alpha = 5\%$

μ	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$
		$\alpha = 1\%$			$\alpha = 5\%$	
0.5	0.120303	0.031362	0.031250	0.186201	0.039450	0.032097
1.0	0.017443	0.454858	0.471961	0.015725	0.523701	0.800750
1.5	0.131528	0.119756	0.647558	0.133363	0.118039	0.689564
2.0	0.165866	0.034885	0.242931	.175505	0.031451	0.241816
2.5	0.181158	0.116823	0.065896	0.189792	0.109330	0.063769
3.0	0.202221	0.209199	0.052328	0.208383	0.204601	0.047176
3.5	0.227908	0.273657	0.114466	0.232186	0.275439	0.104433
4.0	0.256054	0.316686	0.197398	0.259088	0.323892	0.185390
4.5	0.285477	0.348612	0.275665	0.287699	0.359163	0.265356
5.0	0.315591	0.375782	0.341917	0.317272	0.387956	0.335461

Table 4.6: Relative Efficiency of $\hat{\mu}_z$ and $\hat{\mu}_t$ for $n = 16$, $\alpha = 5\%$

μ	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$	$\mu_0 = 1$	$\mu_0 = 2$	$\mu_0 = 3$
		$\alpha = 1\%$			$\alpha = 5\%$	
0.5	0.259759	0.996401	0.999997	0.167829	0.792141	0.973609
1.0	3.583147	0.137405	0.132426	3.974425	0.119343	0.078052
1.5	0.712775	0.782836	0.144774	0.702970	0.794223	0.135955
2.0	0.753619	3.583147	0.514549	0.712229	3.974425	0.516921
2.5	0.862507	1.337489	2.371157	0.823269	1.429157	2.450233
3.0	0.927205	0.896272	3.583147	0.899786	0.916418	3.974425
3.5	0.959816	0.799356	1.911042	0.942130	0.794186	2.094635
4.0	0.976356	0.789425	1.266476	0.964922	0.771862	1.348508
4.5	0.985193	0.806770	1.020260	0.977581	0.783069	1.059895
5.0	0.990206	0.831599	0.913964	0.984956	0.805503	0.931552

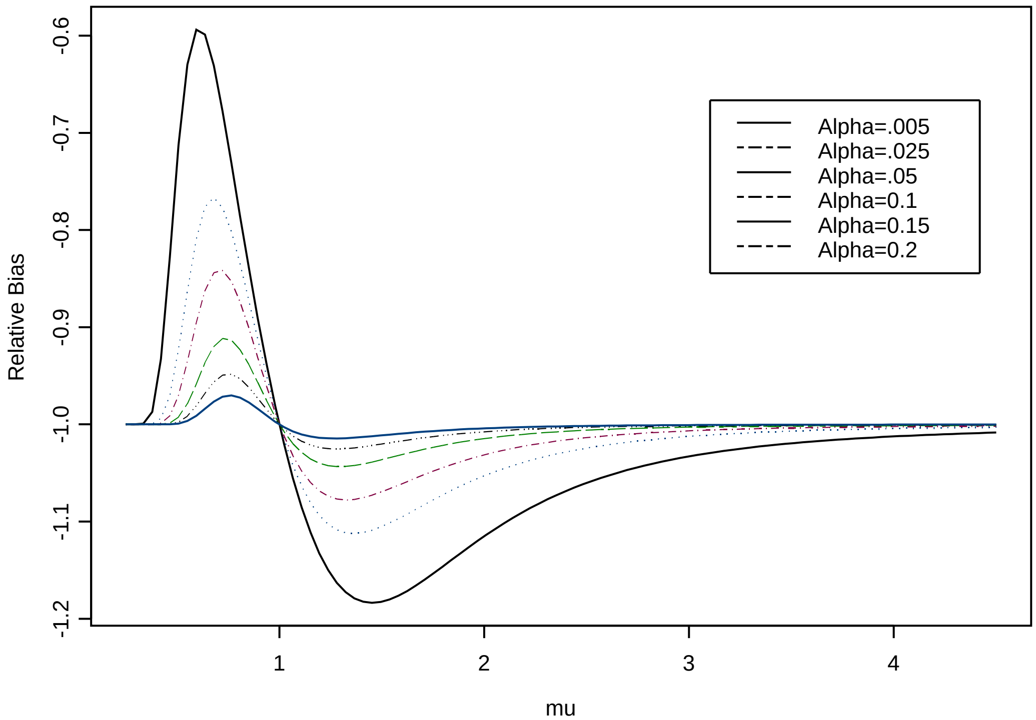

Figure 4.1: Relative Bias of $\hat{\mu}_z$ for $n = 16$

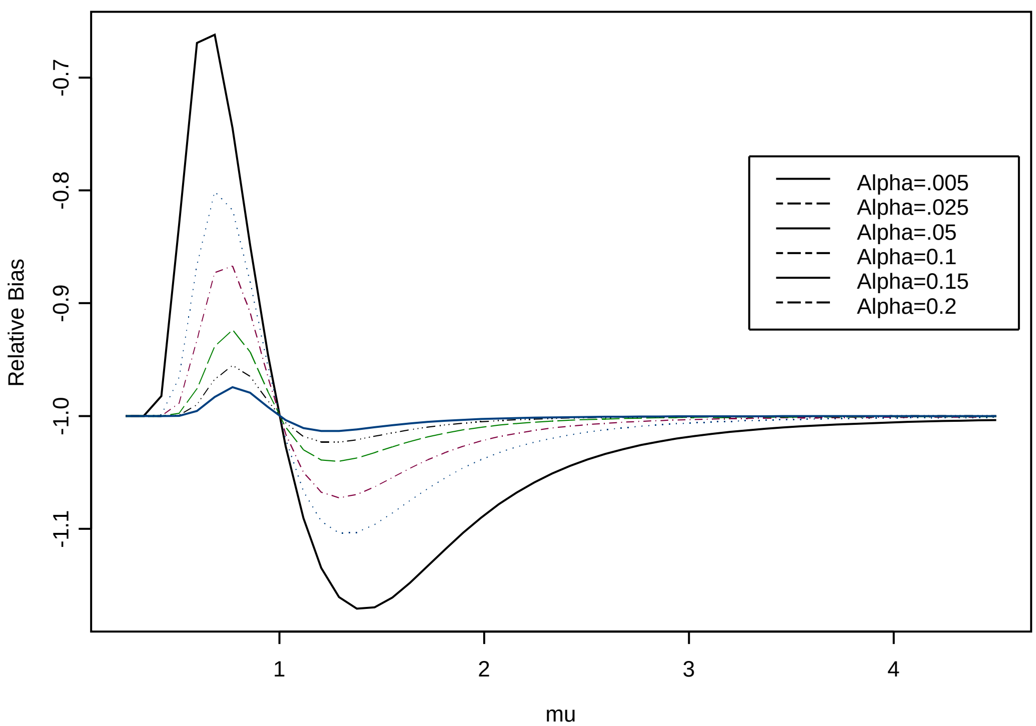

Figure 4.2: Relative Bias of $\hat{\mu}_t$ for $n = 16$

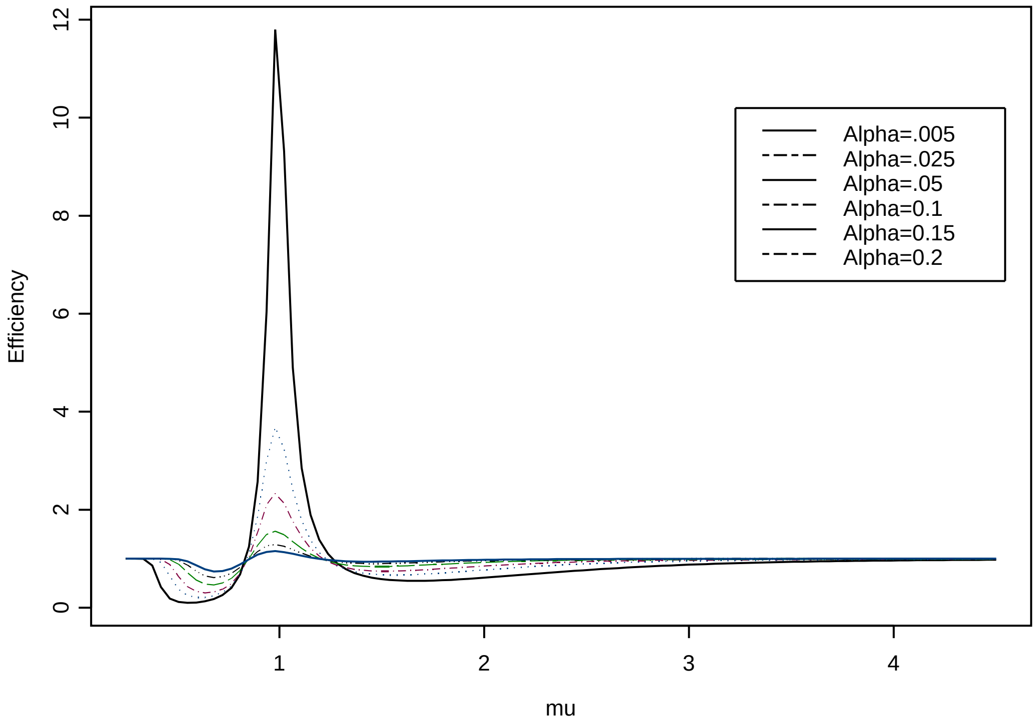

Figure 4.3: Relative Efficiency of $\hat{\mu}_z$ for $n = 16$

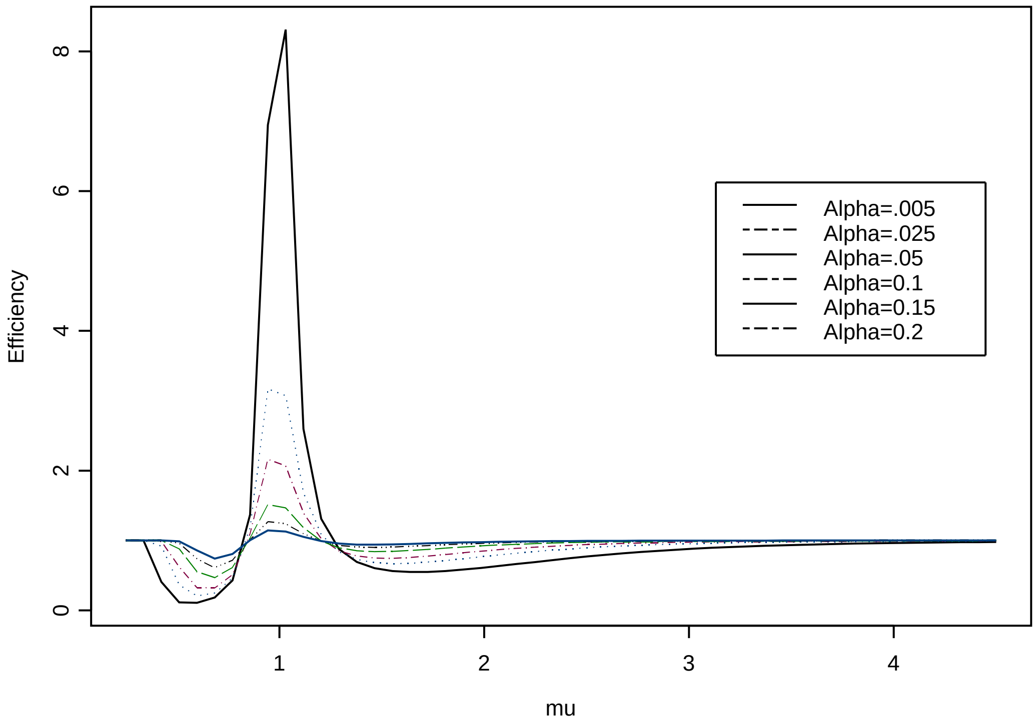

Figure 4.4: Relative Efficiency of $\hat{\mu}_t$ for $n = 16$

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Prof. Xiaowen Zhou
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7141, Sherbrooke Street West
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