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Abstract

In this paper, the Asymptotic Homogenization Method (AHM) is applied to anisotropic viscoelastic composites. The local problems are considered and the effective viscoelastic moduli are explicitly determined. A layer viscoelastic composite with periodic structure is studied. Each layer is isotropic and homogeneous. Analytic expressions for the effective coefficients are derived. Numerical results for predicting the viscoelastic properties of layer composite with periodic structure, in particular, two-layer medium is presented. Some comparisons with other theories are done.

1- Introduction

Very often, the behavior of the material is not merely elastic. This is because the material exhibits hereditary properties and, simultaneously, dissipative effects. Apart from the peculiar hysteretic phenomena, hereditary properties are well described by viscoelasticity where, whether or not thermal effects are considered, the mechanical response of the material is taken and can be influenced by the previous behavior of the material itself. It then should not come as a surprise that, along with the progress of continuum mechanics the literature has devoted an increasing attention to viscoelasticity.

The viscoelastic and viscoelastoplastic response is observed in a number of materials widely used in applications: polymers and plastics, metals and alloys at elevated temperatures, concrete, soils, road construction and building materials, biological tissues, and foodstuffs.

In recent years, the estimation of the effective moduli of composites has been in demand in response to the increase in engineering applications. Various mathematically rigorous techniques have been developed to derive the homogenized coefficients of viscoelastic materials. The Asymptotic Homogenization Method (AHM); Bensoussan et al. (1978), Sánchez-Palencia (1980), Pobedria (1984), Bakhvalov and Panasenko (1989), Oleinik et al. (1992) is one of these technique which has been used for calculation of their overall properties of different types of composites, moreover it guarantees convergence, i. e. the solution of the problem with a periodic microstructure converges to the solution of the homogenized problem as the period of the microstructure goes to zero.

Actually, the study of viscoelectroelastic behavior of heterogenous piezoelectric solids has received considerable attention. For example, the procedure to determine the effective complex electroelastic moduli for a fibrous and laminated composite using the existence of a correspondence between quasistatic viscoelectroelasticity and static piezoelectricity when linear constitutive response exists is developed in Li and Dunn (2001). The use of the AHM for predicting the viscoelastic properties of layered materials have been scantily developed in recent works; the problem to estimating the effective moduli is formulated using the asymptotic homogenization method where the computational procedure is divided into two steps: the effective relaxation moduli are computed in Laplace transform domain and are numerically inverse-transformed into time domain in Yeong-Moo et al. (1998); the case of multilayered thermoviscoelastic media is examined in Maghous S. et al. (2003) and recently in Liu S. et al. (2004).

In the present paper the AHM is applied to the determination of the effective coefficients in the case of viscoelastic composites. In Section 2, the three-dimension formulation in displacements for static viscoelastic problems is given. The constitutive relations are expressed by integrals operators. A brief description of the AHM is formulated in Section 3 and the viscoelasic effective coefficients are calculated. In Section 4, an example of layer composite medium is considered, i.e., a composite made of cells that are periodically and perpendicularly along the axis x_3 and each cell may be made of any finite number of layers. Each layer is isotropic, homogeneous elastic material. Some examples for two layer composite periodically distributed in the x_3 direction are analyzed in Section 5, where the analytical expressions for the effective coefficients are obtained and numerical calculations are discussed.

2- Statement of the problem

The mathematical formulation of the viscoelastic problem in terms of displacements deals with the solution of:

$$\left[\overset{\vee}{R}_{ijkl}(\vec{x},t)\, u_{k,l}(\vec{x},t) \right]_{,j} + X_i = 0 \ , \qquad (1)$$

with respect to three unknown components of the displacement vector $u(\vec{x},t)$ under the following boundary conditions:

$$u_i(\vec{x},t)/_{\Sigma_1} = u_i^{\,0}, \quad \overset{\vee}{R}_{ijkl}(\vec{x},t)\, u_{k,l}(\vec{x},t)\, n_j \,/_{\Sigma_2} = S_i^{\,0}, \qquad (2)$$

where $\vec{x} = (x_1, x_2, x_3)$ is the position vector; $u_i^{\,0}, S_i^{\,0}$ are the displacement and forces given on the surface Σ_1 and Σ_2 of the body respectively with surface $\Sigma = \Sigma_1 \cup \Sigma_2$; $\vec{n}$ is the unit normal vector to the surface Σ and X_i denotes the body force components.

The constitutive law for a general linear viscoelastic material, which relates the stress to prescribed histories of strain is given by:

$$\overset{\vee}{R}_{ijkl}(\vec{x},t)\, \varepsilon_{kl} \equiv \int_0^t R_{ijkl}(\vec{x},t-\tau)\, d\varepsilon_{kl}(\tau), \qquad (3)$$

where $R_{ijkl}(\bar{x},t)$ are the components of the relaxation tensor. The strain second order tensor ε_{kl} is expressed as:

$$\varepsilon_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}) . \qquad (4)$$

The tensor $R_{ijkl}(\bar{x},t)$ is assumed to be endowed with the same general symmetry and positivity conditions as in elasticity:

$$R_{ijkl}(\bar{x},t) = R_{jikl}(\bar{x},t) = R_{ijlk}(\bar{x},t) = R_{klij}(\bar{x},t) \qquad (5)$$

$\exists \gamma > 0$ such that $R_{ijkl}(\bar{x},t)\,\varepsilon_{ij}\varepsilon_{kl} > \gamma\varepsilon_{ij}\,\varepsilon_{ij}$ for almost every vector $\bar{x} \in \Sigma$.

The conditions $R_{ijkl}(\bar{x},t) = R_{jikl}(\bar{x},t) = R_{ijlk}(\bar{x},t)$ result from the symmetry of the stress and strain tensors (see, Christensen R. M. et al. (1979)). The constitutive assumption of diagonal symmetry $R_{ijkl}(\bar{x},t) = R_{klij}(\bar{x},t)$ is consistent with the reciprocity principle state by Onsager (see Maghous and Creus (2003).

3- Asymptotic Homogenization in Viscoelasticity

3.1- Basics Concepts

We introduce some basic concepts and notations. Let the material function R_{ijkl} be a Y-periodic function. As usual, Y is the typical periodic cell, say:

$Y = (0,Y_1)\times(0,Y_2)\times(0,Y_3)$ where Y_1, Y_2 and Y_3 represent the period of the Y-periodicity.

We set $R_{ijkl} = R_{ijkl}(\vec{\xi})$ where $\vec{\xi} = (\xi_1, \xi_2, \xi_3)$ is the local coordinate (or fast coordinate) and $\bar{x} = (x_1, x_2, x_3)$ is the global (or slow) coordinate; the global and the local coordinates are related to each other by a positive real parameter α (small parameter) as follows:

$\xi = \frac{x}{\alpha}$ and $\alpha = \frac{l}{L}$ which represent the ratio between the characteristic length, l, of the periodic cell Y, and the characteristic length L of the whole domain of the composite.

3.2- Asymptotic Homogenization

The solution of the problem (1)-(2) is found using the following asymptotic expansion analogous to Castillero et al. (1998), Pobedria (1984):

$$\begin{aligned} u_i &= v_i(\bar{x}) + \alpha \overset{\vee}{N}{}^{(1)}_{ilk_1}(\vec{\xi})\, v_{l,k_1}(\bar{x}) + \alpha^2 \overset{\vee}{N}{}^{(2)}_{ilk_1k_2}(\vec{\xi})\, v_{l,k_1k_2}(\bar{x}) + \dots \\ &= \sum_{q=0}^{\infty} \alpha^q \overset{\vee}{N}{}^{(q)}_{ilk_1\cdots k_q}(\vec{\xi})\, v_{l,k_1\cdots k_q}(\bar{x}) \end{aligned} , \qquad (6)$$

where the $(p+2)$ order tensor $\overset{\vee}{N}{}^{(p)}_{ilk_1\cdots k_p}(\vec{\xi})$ is the relaxation local kernel of level p, which depend on the fast variable $\vec{\xi}$. These functions $\overset{\vee}{N}{}^{(p)}_{ilk_1\cdots k_p}(\vec{\xi})$, are local auxiliary Y-periodic functions and satisfy the following conditions:

- The local functions of level zero are unit tensors of second order, and:

$$\overset{\vee}{N}_{mn}^{(0)}(\vec{\xi}) \quad = \delta_{mn} \quad (\delta_{mn} \text{ is the Kronecker symbol}). \tag{7}$$

- The relaxation local kernels of negative level are identically zero, that is:

$$\overset{\vee}{N}_{mn}^{(q)}(\vec{\xi}) \equiv 0\,; \quad for \quad q < 0\,. \tag{8}$$

- Moreover, to make the local auxiliary functions unique, we require:

$$< \overset{\vee}{N}_{mn}^{(q)}(\vec{\xi}) >= 0\,; \quad for\ q > 0\,, \tag{9}$$

where the symbol $< . >$ means the average operator, i.e. $< . > = \frac{1}{V}\int_V (.)\, dV$.

- The local functions $\overset{\vee}{N}_{mnk_1\cdots k_q}^{(q)}(\vec{\xi})$ must satisfy the continuity of the periodic functions:

$$\left[\left[\overset{\vee}{N}_{mnk_1\cdots k_q}^{(q)}(\vec{\xi}) \right]\right] = 0\,, \tag{10}$$

where the symbol $[[.]]$ means the difference of the values of the function (.) on opposite sides of Y .

3.3- Computation of the viscoelastic effective coefficients

Denote the derivative with respect to the fast coordinate ξ_i by the symbol (/) before the subscript and use the symbol (,) to denote the derivative with respect to the slow coordinate x_i .

Equation (1) can be differentiated and the following formula is obtained:

$$\frac{1}{\alpha}\overset{\vee}{R}_{\,i\,j\,k\,l\,/\,j}(\vec{\xi},t)\, u_{\,k\,,\,l} + \overset{\vee}{R}_{\,i\,j\,k\,l}(\vec{\xi},t)\, u_{k\,,\,l\,j} + X_i = 0 \ . \tag{11}$$

We now substitute the expansions (6) into equation (11), and collect the terms of same order of α^q and we have (see Pobedria (1984)):

$$\sum_{q=-1}^{\infty}\alpha^q \left\{ \overset{\vee}{R}_{\,i\,j\,m\,l\,/\,j} \left[\overset{\vee}{N}{}^{(q+2)}_{m\,n\,k_1\,\ldots\,k_{q+2\,/\,l}}\, v_{n\,,\,k_1\,\ldots\,k_{q+2}} + \overset{\vee}{N}{}^{(q+1)}_{m\,n\,k_1\ldots\,k_{q+1}}\, v_{n\,,\,k_1\,\ldots\,k_{q+1}\,l} \right] \right.$$

$$+ \overset{\vee}{R}_{\,i\,j\,m\,l} \left[\overset{\vee}{N}{}^{(q+2)}_{m\,n\,k_1\,\ldots\,k_{q+2}\,/\,l\,j}\, v_{n\,,\,k_1\,\ldots\,k_{q+2}} + \overset{\vee}{N}{}^{(q+1)}_{m\,n\,k_1\,\ldots\,k_{q+1\,/\,l}}\, v_{n\,,\,k_1\,\ldots\,k_{q+1}\,j} \right.$$

$$\left.\left. + \overset{\vee}{N}{}^{(q+1)}_{m\,n\,k_1\,\ldots\,k_{q+1}\,/\,j}\, v_{n\,,\,k_1\,\ldots\,k_{q+1}\,l} + \overset{\vee}{N}{}^{(q)}_{m\,n\,k_1\,\ldots\,k_q}\, v_{n\,,\,k_1\,\ldots\,k_q\,l\,j} \right] \right\} + x_i = 0\,. \tag{12}$$

After some convenient manipulations in equation (15), the coefficients, for each degree q of the parameter α and for each $v_{n\,,\,k_1\,\ldots\,k_{q+2}}$ can be equated to a tensorial constant $\overset{\vee}{c}{}^{(q)}_{ink_1\cdots k_{q+2}}$ and the following expression is obtained as Pobedria and Ilyushin (1970);

$$\left(\check{R}_{ijml}\check{N}^{(q+2)}_{mnk_1\ldots k_{q+2}/l}\right)_{/j}+\left(\check{R}_{ijmk_{q+2}}\check{N}^{(q+1)}_{mnk_1\ldots k_{q+1}}\right)_{/j}$$

$$+\check{R}_{ik_{q+2}ml}\check{N}^{(q+1)}_{mnk_1\ldots k_{q+1}/l}+\check{R}_{ik_{q+2}mk_{q+1}}\check{N}^{(q)}_{mnk_1\ldots k_q}=\check{c}^{(q)}_{ink_1\ldots k_{q+2}},\quad where\quad q=-1,0,1,\ldots \tag{13}$$

Now, problem (1)-(2) is reduced to the homogeneous problem of the viscoelasticity theory and the $(4+q)$ order tensors $\check{c}^{(q)}_{ink_1\cdots k_{q+2}}$ are called viscoelastic effective tensors of q level, moreover:

$$\check{c}^{(q)}_{ink_1\ldots k_{q+2}}\equiv 0,\quad \text{for}\, q<0. \tag{14}$$

The solution of the heterogeneous problem (1)-(2) is reduced to the result of two sequences of recurrent problems in the periodic cell (Pobedria (1984**,** chapter 8))**.**

The effective coefficients $\check{c}^{(q)}_{ink_1\cdots k_{q+2}}$ can be calculated by the following operator formulae:

$$\check{c}^{(q)}_{ink_1\ldots k_{q+2}}=<\check{R}_{ik_{q+2}ml}\check{N}^{(q+1)}_{mnk_1\ldots k_{q+1}/l}+\check{R}_{ik_{q+2}mk_{q+1}}\check{N}^{(q)}_{mnk_1\ldots k_q}>. \tag{15}$$

In order to find the local function $\check{N}^{(1)}_{mnk}$, we need first to solve the following local problem analogous to Castillero et al. (1998),

$$\left[\check{R}_{ijml}\check{N}^{(1)}_{mnk/l}+\check{R}_{ijnk}\right]_{/j}=0, \tag{16}$$

with conditions (8) and (10); the relaxation local kernel of first order $\check{N}^{(1)}_{mnk}$ is calculated and the relaxation kernel tensor of the null approximation is obtained as follows:

$$\check{R}^{(0)}_{ijnk}=\check{R}_{ijml}\check{N}^{(1)}_{mnk/l}+\check{R}_{ijnk}, \tag{17}$$

thus the viscoelasic effective coefficients are calculated as follows:

$$\check{c}_{ijnk}=<\check{R}^{(0)}_{ijnk}>. \tag{18}$$

4-Two phase viscoelastic composite

We consider a layered medium, i.e., a composite formed by cells that are periodically distributed along the axis x_3 and each cell is made of a finite number of layers. The axes of symmetry of each layer are parallel to each other and the x_3-axis is perpendicular to the layers. The relaxation modulus tensor R is a periodic function of the coordinate x_3 and it does not depend on x_1 and x_2.

Under the above considerations, the fast coordinate has the following form $\vec{\xi}=(0,0,\xi)$ where

$$\xi\equiv\xi_3=\frac{x_3}{\alpha} \tag{19}$$

From the local problem (19), the following expression for the effective coefficients is obtained:

$$\check{c}_{ijkl} = <\check{R}_{ijkl}> + <\check{R}_{ijm3}\check{R}^{-1}_{m3n3}><\check{R}^{-1}_{n3p3}>^{-1}<\check{R}^{-1}_{p3q3}\check{R}_{q3kl}> - <\check{R}_{ijm3}\check{R}^{-1}_{m3n3}\check{R}_{n3kl}> \qquad (20)$$

In particular, for a binary layered, the average $< f >$ is understood as:

$$< f > = p_1 \; f^{(1)} + p_2 \; f^{(2)} \qquad (21)$$

where $p_1 + p_2 = 1$ and $p_\beta \;\; (\beta = 1,2)$ means the volume fraction of every layer.

Using the following mapping of adjacent subscripts $(11)\to 1$, $(22)\to 2$, $(33)\to 3$, $(23)=(32)\to 4$, $(31)=(13)\to 5$, $(12)=(21)\to 6$, the analytical expressions for the viscoelastic effective coefficients can be written as follows,

$$\check{c}_{11} = \check{c}_{22} = <\check{R}_{11}> - <\frac{\check{R}_{13}\check{R}_{31}}{\check{R}_{33}}> + \frac{<\frac{\check{R}_{13}}{\check{R}_{33}}><\frac{\check{R}_{31}}{\check{R}_{33}}>}{<\frac{1}{\check{R}_{33}}>} \qquad , \qquad (22a)$$

$$\check{c}_{12} = <\check{R}_{12}> - <\frac{\check{R}_{13}\check{R}_{32}}{\check{R}_{33}}> + \frac{<\frac{\check{R}_{13}}{\check{R}_{33}}><\frac{\check{R}_{32}}{\check{R}_{33}}>}{<\frac{1}{\check{R}_{33}}>}, \qquad (22b)$$

$$\check{c}_{13} = \frac{<\frac{\check{R}_{13}}{\check{R}_{33}}>}{<\frac{1}{\check{R}_{33}}>} \qquad , \qquad (22c)$$

$$\check{c}_{33} = <\frac{1}{\check{R}_{33}}>^{-1} \qquad , \qquad (22d)$$

$$\check{c}_{55} = <\frac{1}{\check{R}_{55}}>^{-1} \qquad , \qquad (22e)$$

$$\check{c}_{66} = <\check{R}_{66}> - <\frac{\check{R}_{65}\check{R}_{56}}{\check{R}_{55}}> + \frac{<\frac{\check{R}_{65}}{\check{R}_{55}}><\frac{\check{R}_{56}}{\check{R}_{55}}>}{<\frac{1}{\check{R}_{55}}>} \qquad . \qquad (22f)$$

In the case where every individual layer is assumed to be isotropic, we have:

$$\check{R}_{33} = \check{R}_{11} = \lambda + 2\mu\,, \quad \check{R}_{55} = \check{R}_{66} = \mu\,, \quad \check{R}_{65} = \check{R}_{56} = 0\,, \quad \check{R}_{12} = \check{R}_{32} = \check{R}_{31} = \check{R}_{13} = \lambda\,.$$

where μ is the shear relaxation modulus and λ is such that $3\lambda + 2\mu$ characterizes the relaxation modulus under isotropic compression. (λ and μ are Lame's constants).

Then, equations (22) can be written in the following form:

$$\check{c}_{11} = <\lambda + 2\mu> - <\lambda^2(\lambda+2\mu)^{-1}> + <\lambda(\lambda+2\mu)^{-1}>^2 <\lambda(\lambda+2\mu)^{-1}>^{-1} \;, \qquad (23a)$$

$$\check{c}_{12} = <\lambda> - <\lambda^2(\lambda+2\mu)^{-1}> + <\lambda(\lambda+2\mu)^{-1}>^2 <(\lambda+2\mu)^{-1}>^{-1}, \qquad (23b)$$

$$\check{c}_{13} = <\lambda(\lambda+2\mu)^{-1}><(\lambda+2\mu)^{-1}>^{-1}, \qquad (23c)$$

$$\check{c}_{33} = <\lambda(\lambda+2\mu)^{-1}>^{-1} \;, \qquad (23d)$$

$$\check{c}_{55} = <\mu^{-1}>^{-1}, \qquad (23e)$$

$$\check{c}_{66} = <\mu> \;. \qquad (23f)$$

The above effective relaxation moduli are the same that the coefficients reported in Maghous et al. (2003).

5- Numerical examples

Example 1

We consider a two-layer composite, one layer is made of an elastic material and the other is made of a viscoelastic material. The moduli of the elastic layer1 is $E = 20,\ \upsilon = 0.21$, and the moduli of the viscoelastic layer2, which is given, using the standard linear solid model Christensen (1982) by:

$$E(t) = 3 + 17\,e^{-t},\ \upsilon = 0.38 \qquad (24)$$

See Yeong-Moo et al. (1998).

The program for the calculation of the effective coefficients and their graphical time dependence representation was made in *Mathematica.*

Fig.1-a and Fig.1-b illustrate the variation in time of the effective relaxation modulus homogenized $\check{c}_{33}$ and $\check{c}_{55}$ respectively for different volume fractions, see equations (21), (23d), (23e).

When the volume fractions for layer1 increase, the behavior of these magnitudes do not vary qualitatively since in the initial moment they fall abruptly and after some few seconds they remain constant. The increase fraction volume produces an increase of the relaxation modules.

Different volume fractions of layer1 are used:
VF I: the volume fraction of layer1 is 20%.
VF II: the volume fraction of layer1 is 40%.
VF III: the volume fraction of layer1 is 50%.

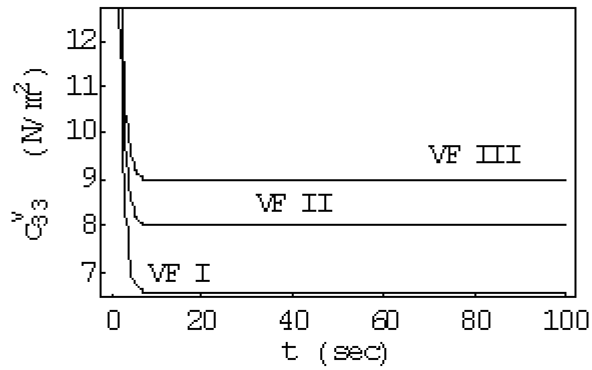

Fig.1-a. Effective relaxation modulus $\overset{\vee}{c}_{33}$ in time, example 1.

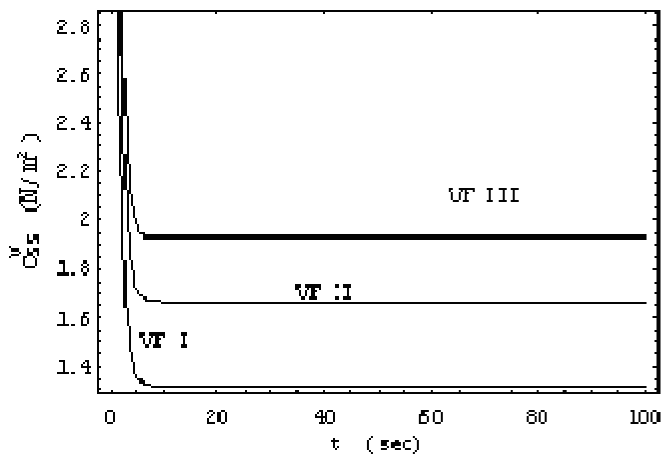

Fig.1-b. Effective relaxation modulus $\overset{\vee}{c}_{55}$ in time, example 1.

Example 2

We consider a two-layer composite where each layer is made up of an isotropic viscoelastic material with different relaxation times. The relaxation moduli are given as follows:

$E(t) = 3 + 17e^{-t/10}, \ \upsilon = 0.38$ for layer1 (25)

$E(t) = 3 + 17e^{-t}, \ \upsilon = 0.38$ for layer2. (26)

The following functional dependence for the Lame's constants, λ and μ is considered (see Yeong-Moo et al. (1998)),

$$\lambda = \frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)}, \tag{27}$$

$$\mu = \frac{E}{2(1+\upsilon)} \tag{28}$$

Analogous to the previous example, Fig.2-a and 2-b show the behavior of the effective relaxation moduli $\overset{\vee}{c}_{33}$ and $\overset{\vee}{c}_{55}$ for different levels of the volume fraction related to layer1. In the case of the effective relaxation modulus $\overset{\vee}{c}_{33}$, the behavior of this quantity is similar in time for different volume fractions of layer 1.

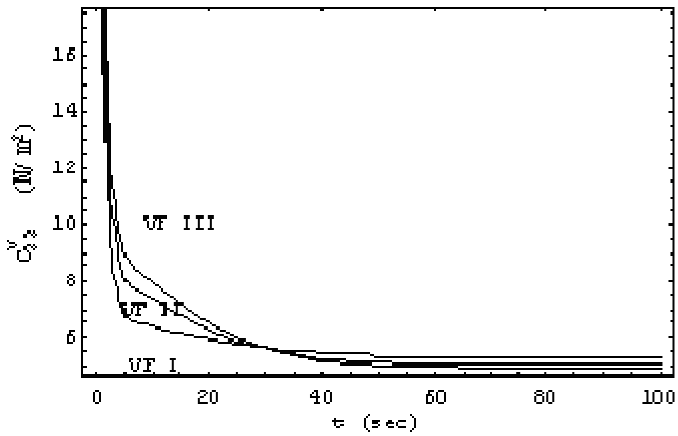

Fig.2-a. Effective relaxation modulus $\check{c}_{33}$ in time, example 2

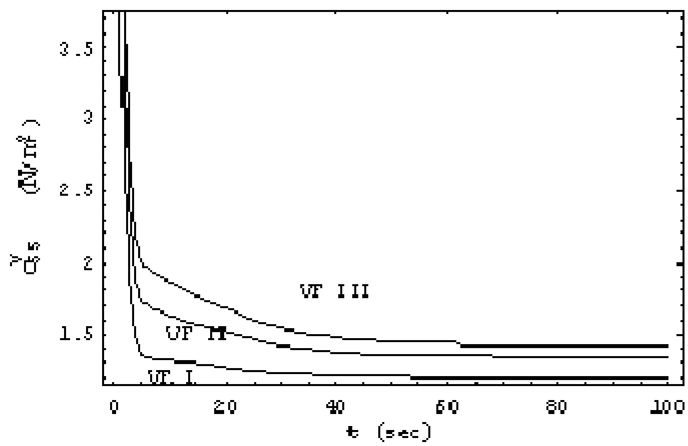

Fig.2-b. Effective relaxation modulus $\check{c}_{55}$ in time, example 2.

Example 3

In order to validate our theoretical approach, we consider the numerical data given by Maghous S. et al. (2003). A linear elastic behavior is adopted for the layer1 (delayed effects are neglected). A Dischinger model is used for the layer2 in the shear mode and linear elasticity in the dilatation mode.

The relaxation functions are defined for $\tau \leq t$ by:

$$\varphi(\tau,t) = \exp(-\alpha t) - \exp(-\alpha\tau)$$
$$\mu_2(\tau,t) = \mu_{2,0}\exp(\frac{\mu_{2,0}}{\alpha\beta}\varphi(\tau,t))$$
$$\lambda_2(\tau,t) = K - \frac{2}{3}\mu_2(\tau,t)$$

where *K is* the (elastic) bulk modulus and $\mu_{2,0} = \mu_2(\tau)$ is considered as constant, here α and β are model's parameters**.**

The following data have been selected for the viscoelastic layer2

$\beta_2 = 0.5$, $K = 10000 MPa$ $\mu_{2,0} = 8571\, MPa$

$\alpha = 0.026\ days^{-1}$, $\alpha\beta = 35667 MPa$.

The stiffness of the elastic layer1 is taken comparable to that of the viscoelastic layer2:

$\beta_2 = 0.5$, $\mu_1 = \mu_{2,0}$ and $\lambda_1 = \lambda_2(\tau,\tau) = K - \frac{2}{3}\mu_{2,0}$.

Fig. 3-a and 3-b display for $\tau = 0$ the variation in time of the homogenized relaxation moduli $\check{c}_{11}$ and $\check{c}_{33}$ (normalized by the oedometric modulus $\lambda_1 + 2\mu_1$) for AHM and Maghous et al. (2003). Similar behavior is observed, in fact, both coefficients decrease in time.

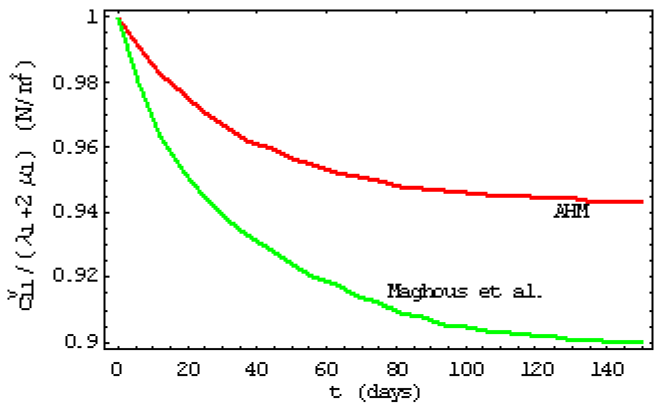

Fig.3-a. Effective relaxation modulus $\check{c}_{11}$ in time, example 3.

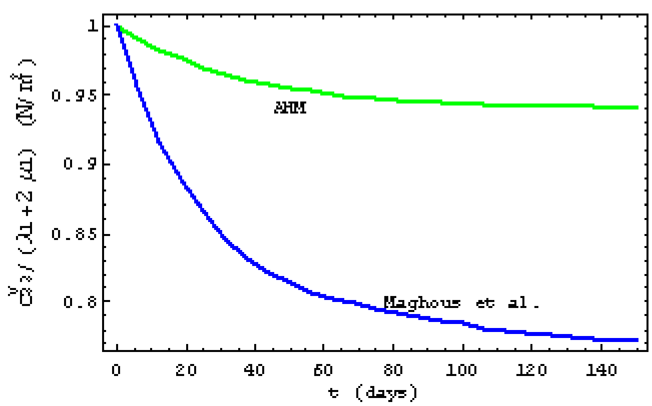

Fig.3-b. Effective relaxation modulus $\check{c}_{33}$ in time, example 3.

Example 4

Every individual layer is assumed to be isotropic with different relaxation time**:**
The relaxation moduli are given as follows (similar to example 2):

$$E(t) = q_{oi} + q_{1i}\ e^{-p_i t},\ \upsilon_i, \quad i = 1,2\ ,\ \text{for layer } i$$

The material constants of both layers are shown in the following table: (see Liu et al. (2004)).

	q_{0i} (MPa)	q_{1i} (MPa)	p_i (1/day)	υ
Layer-I	9.67×10^8	3.22×10^8	0.00658	0.24
Layer-II	6.1×10^7	1.84×10^8	0.00125	0.2

Figure 4 shows the effective relaxation modulus homogenized $\check{c}$ in time, when the volume fraction of Layer-II is 10 %.

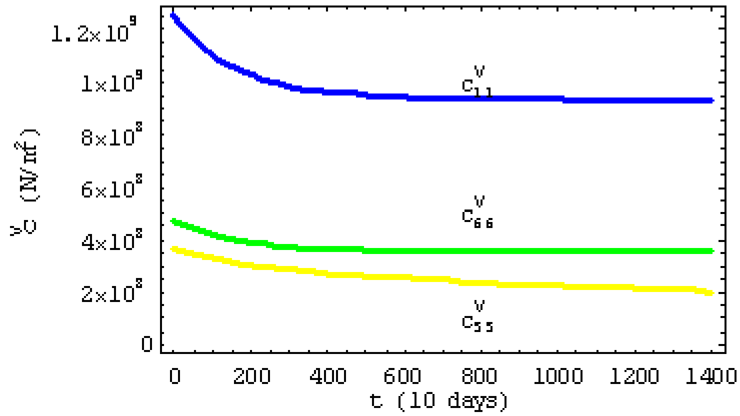

Fig.4. Effective relaxation modulus $\check{c}$ in time, example 4.

6- Conclusions

The Asymptotic Homogenization Method is applied to viscoelastic composite media for computing their effective coefficients. The local problem has ben considered and the effective viscoelastic moduli are explicitly determined.

A two-layer composite where the laminates are perpendicular to axis x_3, is studied. The effective viscoelastic coefficients are derived. Numerical results for different configurations of the layers are reported. From the explicit solution (22)-(23), the effective coefficients for the pure elastic case are the same as those reported in Rodríguez et al. (1997) and Castillero et al. (1998). The effective relaxation moduli (22)-(23) calculated using AHM are the same that the coefficients reported in Maghous et al. (2003).

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