BUCKLING STRENGTH OF SIMPLY SUPPORTED STIFFENED PLATE HAVING A PLAIN CIRCULAR PERFORATION

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ABSTRACT

A thin plate of constant thickness subjected to a uniform compression applied at its opposite edges, is a common structural element. One or more circular perforations may be provided in such plate element for the purpose of allowing access. The loss in buckling strength, as will be caused by these perforations,must be compensated before the plate can be used to its best advantage. An economical design is possible by introducing stiffeners to reinforce the perforated plate.

A theoretical investigation is conducted in this study to report the structural instability caused by a plain circular perforation for simply supported square plates under edge compression, and also for those plates reinforced by two symmetric stiffeners in longitudinal and in transverse manner. Numerical results show that the presence of a plain circular perforation causes a small reduction in buckling compression of a plate whereas the two stiffeners can be used to carry a substantial part of the applied compression, increase the buckling strength or to perform both these functions.

CONTENTS

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Thanks are extended to Mrs. Shubhangi Purohit for the typing of the manuscript.

LIST OF FIGURES

NOTATIONS

a	Plate length
b	Plate width
C	Correction factor for calculating buckling stress in the inelastic range of the plate material
D	Flexural rigidity of the plate
D_x, D_y	Flexural rigidities in two mutually orthogonal directions of a stiffened plate
D_{xy}	Torsional rigidity
E	Young's Modulus
E_s	Secant Modulus
E_t	Tangent Modulus
K.	Buckling coefficient
ℓ	Length and width of a square plate
m, n	Number of half waves in the buckled form of the compressed plate
-P	Part of the applied compression carried

Q_A	Ratio of cross sectional areas of the plate and of the reinforcing stiffeners
Q_s	Shape factor
$\bar{Q}$	Ratio of rigidities of the reinforcing stiffeners and of the plate
r, θ	Polar coordinates
$-S$	Compressive stress applied to the plate edges, far from the perforation
t	Plate thickness
T	Amount of work done by the compressive forces acting on the plate
U	Energy of the plate
U_o	Loss of energy due to a plain circular perforation
w	Lateral deformation of the edgewise compressed plate
x, y, z	Rectangular coordinates
σ_{cr}	Buckling stress
M_x, M_y, M_{xy}	Moments applied to the plate edges
N_x, N_y, N_{xy}	Normal and shearing force per unit length, acting at the plate edges

1

INTRODUCTION

Perforations are very often included in the stressed skin cover of airplane wings. Webs of wide-flange beams and girders must often be opened up at a number of places in order to accomodate utility components. In the present construction practice perforated cover plates are extensively used to substitute lattice bars and batten plates. Structural performance of all these plate elements greatly depends on their buckling strength.

A perforated plate is less strong as compared to one without any perforations. A greater tendency toward buckling may therefore be expected. A design solution must be deviced to increase the structural stability of such perforated plate before it can be used to its best advantage. This always can be accomplished by selecting a thicker plate but the design solution will not be economical in terms of the weight of material used. It is possible to design an adequately strong and rigid structural plate element by keeping its thickness as small as possible and by introducing reinforcing stiffeners. Economy in weight of material and efficient structural performance are both achieved to a high degree through such design.

The determination of stresses around any plain circular perforation in a stiffened plate under edge compression presents itself as a problem of theoretical analysis. Kirsch[1] has obtained a solution for stresses around a small circular perforation in a plate subjected to uniform tension applied in one direction. The theory of bending of curved bars was used by Timoshenko[2] in obtaining an approximate solution of stresses for plates having circular perforation reinforced by beads. Gurney[3] has presented an exact analysis of plane stress distribution for an infinite plate having a circular perforation reinforced by rectangular bead. Mathematical solution is available for determining stresses around a plain circular perforation in an orthotropic plate[4,5].

The intent of this presentation is to study more fully the effect of a plain circular perforation in reducing the buckling strength of stiffened plates.

Energy solution to the problem of buckling of simply supported rectangular plates compressed uniformly at their opposite edges in one direction, is given by Bryan[6]. A variety of problems concerning stability of plates reinforced by ribs, are discussed by Timoshenko[7] and are presented collectively[8]. Several papers are available on the general topic of instability and failure of stiffened plates. Seide[9] and Stein[9] have used Rayleigh-Ritz method for calculating buckling compression of plates reinforced by longitudinal stiffeners. This theory neglects the torsional rigidity of the reinforcing stiffeners but considers their flexural rigidity alone. Budiansky[10] and Stein[10] have presented an analysis to determine the buckling strength of plates reinforced by transverse stiffeners where the torsional rigidity of the stiffeners is taken into account. Seide[11] has corrected the results given in Ref. 9 for the case of asymmetric stiffeners used on one side of the plate. A biblography[12] on stability behaviour of stiffened plates is available. Also the extensive work on buckling of stiffened plates has been summarized by Gerard[13] and Becker[13]. A correlation between stability problems of isotropic and orthotropic plates under uni-axial and bi-axial stress applied at their opposite edges, is given by Wittrick[14]. Instability in shear for simply supported square plate having a plain circular perforation is obtained by Kroll[15] whereas the method of numerical integration was used by Levy[16], Woolley[16] and Kroll[16] in deriving a solution for critical buckling compression for similar plates. Some instability problems have been noted in the study of open web beams and expanded girders. Cato[17] has reported web buckling in built-up girders with rectangular perforations. Lateral buckling of expanded beams has been reported at University of Illinois[18]. Results on over-all stretching of plates having plain circular perforations are presented by Greenspan[19].

The author feels that too little is known about the decrease in buckling strength of a stiffened plate, when a plain circular perforation is included there in. Also a theoretical relation between the size of such perforation and the resulting loss in plate strength must be established. In absence of a rigorous mathematical solution a simplifying recourse has been adapted in this study to present an approximate analysis based on the principle of minimum potential energy of the following items:-

1. Buckling strength of simply supported square plate under edge compression applied in one direction.

2. Structural instability due to a plain circular perforation for uniformly compressed square plate, same as in item 1 above. Different sizes of such perforation with diameters upto one-half of the plate length have been considered.

3. Stabilizing effect of the two symmetric stiffeners, when they are used to reinforce uniformly compressed perforated plates, same in item 2 above. Longitudinal and transverse ways of reinforcement have been considered.

Numerical results are obtained on buckling strength.

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THEORETICAL ANALYSIS

2.1 METHOD OF ANALYSIS

In absence of rigorous solution giving the smallest Eigenvalue of the differential equation representing the buckled form of a perforated stiffened plate, the approximate value of the buckling compression can be obtained by principle of minimum potential energy. The analysis is based on ensuring equilibrium and compatibility, which are formulated in terms of scalar functions of energy and work. All extraneous details arising from complicated geometric or mechanical reasonings are avoided. The following assumptions are made :-

1. The perforated stiffened plate will buckle under the action of stresses acting in a direction parallel to its middle plane. The stresses acting in a direction perpendicular to that of the middle plane may be disregarded.

2. The slope of the deflected surface is considerably small in any direction.

3. Buckling is considered as limited lateral bending w, and in-plane extension is negligible. The energy due to in-plane stretching may be disregarded.

4. Energy of deformation is not transformed into heat, kinetic energy or any other form.

5. Points which lie normal to the middle plane of a perforated stiffened plate prior to its buckling,remain normal during and after its buckling.

6. Two distinct phases may be distinguished before any buckling is noticed; first, the state of stable equilibrium in which the energy of deformation is always greater than the corresponding amount of work done in compressing the perforated stiffened plate and the second, the state of neutral equilibrium where the magnitude of the applied compression is increased to the extent that the amount of work just equals to the corresponding energy of deformation. It is obvious that any further increase in the applied compression, howsoever small, will cause buckling and the state of nonequilibrium.

The smallest value of the buckling compression is obtained by equating the energy of deformation to the corresponding amount of work done.

2.2 DERIVATION OF BASIC EQUATIONS

An approximate theory for bending and stretching of thin plates is derived in order to avoid the difficulties in its application to the problem of buckling of perforated stiffened plates.

Let us consider a thin plate of constant thickness t which is subjected to bending moments M_x, M_y and twisting moments M_{xy} and M_{yx} per unit length, acting at the opposite two edges all as shown in Fig. 1. Let the plate be referred to rectangular coordinates with origin coinciding with the geometric center of the plate. The axes x and y are directed to the edges and the z axis is directed in the direction of plate thickness . The plane midway between the two faces of the plate is denoted as the middle plane and it is used as the reference surface. Let us consider a state where the plate deforms laterally to a limited bent configuration. Let such deformation be designated as w. The following boundary conditions must be satisfied: at $x = y = \pm \frac{\ell}{2}$ $w = 0$, and at $x = y = 0$ $w \neq 0$. The deformed shape can be expressed in the form of a double trigonometric series

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \cos \frac{m\pi x}{\ell} \cos \frac{n\pi y}{\ell} \quad \ldots(1)$$

The slopes of the deformed surface are given as $\frac{\partial w}{\partial x}$

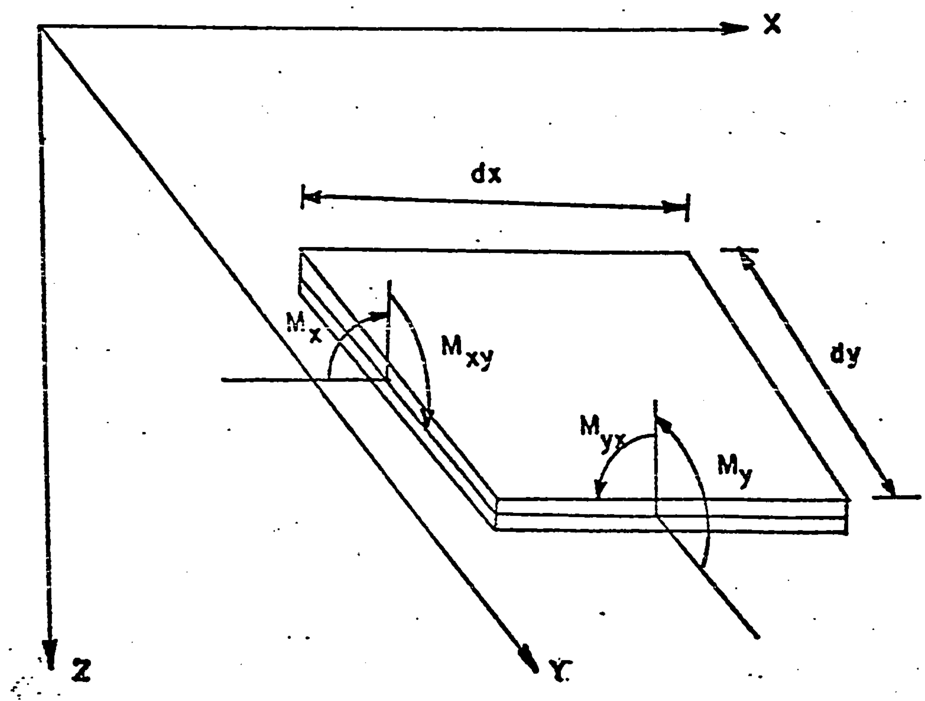


FIG. I MOMENTS APPLIED AT THE PLATE EDGES and $-\frac{\partial w}{\partial y}$, where the minus sign denotes that the slopes decrease with the increase in values of x and y respectively. The curvatures are defined as the rate of change of slope and are given by the following expressions

$$\frac{1}{R_x} = -\frac{\partial^2 w/\partial x^2}{\left\{1 + (\partial w/\partial x)^2\right\}^{3/2}} = -\frac{\partial^2 w}{\partial x^2}$$

$$\frac{1}{R_y} = -\frac{\partial^2 w}{\partial y^2} \qquad \ldots(2)$$

Twist is defined as the rate of change of slope in x direction when measured in y direction or viceversa. The following relation denotes twist

$$\frac{1}{R_{xy}} = \frac{\partial^2 w}{\partial x \, \partial y} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Slopes, curvatures and the deformation geometry of the plate under edge compression are shown in Fig. 2 , whereas the twist of the reference surface of such plate is given in Fig. 3.

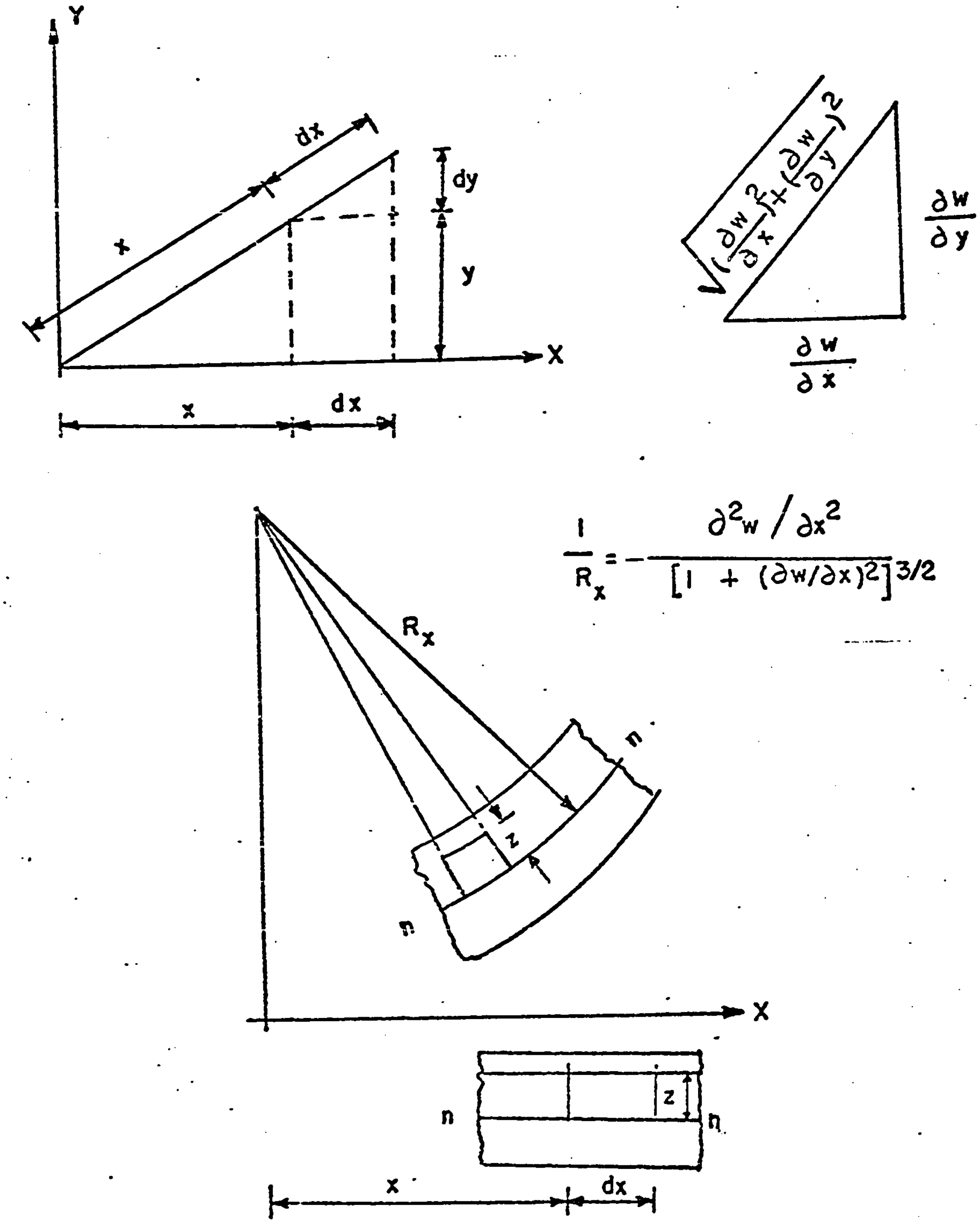


FIG 2- SLOPE, CURVATURE & DEFORMATION GEOMETRY FOR THIN PLATE

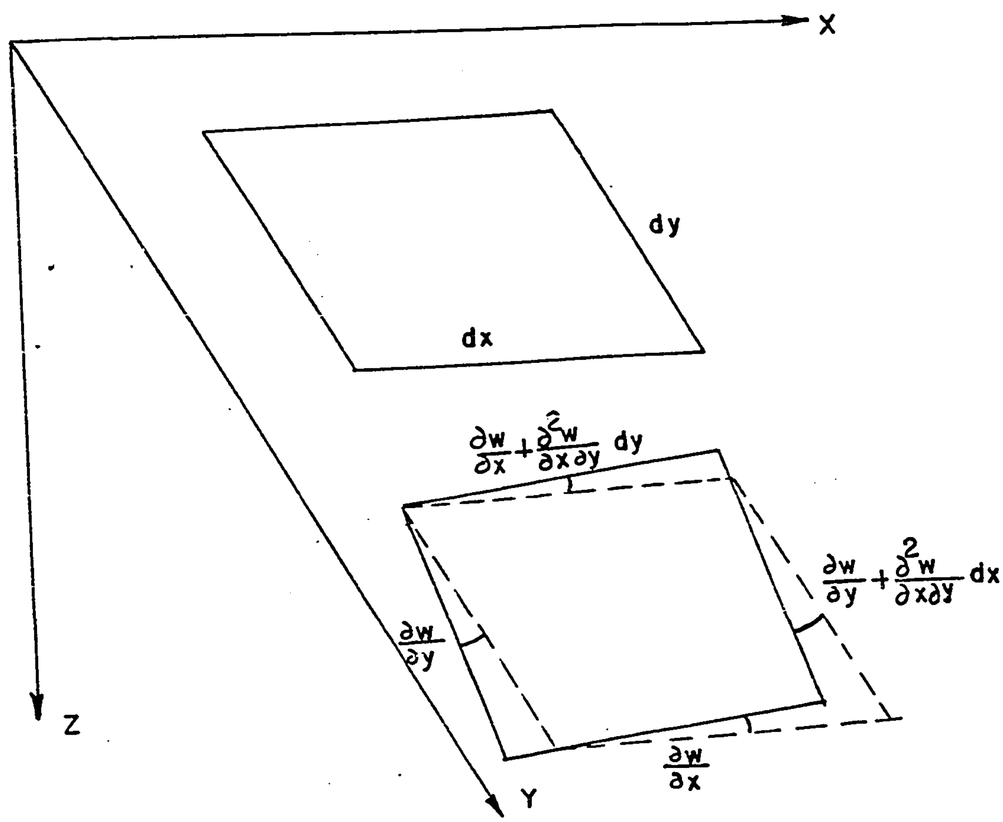


FIG. 3- TWIST OF THE REFERENCE SURFACE

The relation between normal stresses and the curvatures is

$$\sigma_x = \frac{E\,z}{(1-\mu^2)}\;\frac{1}{R_x}\;+\mu\frac{1}{R_y}$$

$$\sigma_y = \frac{E\,z}{(1-\mu^2)}\;\frac{1}{R_y}\;+\mu\frac{1}{R_x} \qquad \ldots\ldots(4)$$

Similarly the shearing stress is related to the twist by the following expression .

$$\tau_{xy} = 2\,G\,z\,\frac{1}{R_{xy}} \qquad \ldots\ldots(5)$$

The moments causing bending of the plate are obtained from the integrals

$$M_x = \sigma_x\,z\,dz \quad \text{and} \quad M_y = \sigma_y\,z\,dz \qquad \ldots.(6)$$

And the twisting moment M_{xy} is obtained from

$$M_{xy} = \tau_{xy}\,z\,dz \qquad \ldots\ldots(7)$$

After substituting values of stresses from equations (4) and (5) into equations(6) and (7), the moments are expressed in terms of curvatures and twist

$$M_x = D\left[\frac{1}{R_x} + \mu\frac{1}{R_y}\right] \text{ and } M_y = D\left[\frac{1}{R_y} + \mu\frac{1}{R_x}\right] \quad \ldots\ldots (8)$$

$$M_{xy} = D\,(1-\mu)\,\frac{\partial^2 w}{\partial x\,\partial y} \quad \ldots\ldots\ldots\ldots (9)$$

The energy due to bending is given as

$$dU_1 = \tfrac{1}{2}\left[M_x\frac{1}{R_x} + M_y\frac{1}{R_y}\right] dx\,dy \quad \ldots\ldots\ldots\ldots (10)$$

And the energy due to twisting of the plate is

$$dU_2 = \tfrac{1}{2}\,(1-\mu)\,D\left[\frac{\partial^2 w}{\partial x\,\partial y}\right]^2 dx\,dy \quad \ldots\ldots\ldots\ldots (11)$$

After substituting for moments their expressions (8) and (9) into (10) we get the energy of deformation of an infinitesimal differential element(dx dy)by adding dU_1 and dU_2 . The total energy of deformation of the plate will be obtained by integrating the energy of the differential element over the entire area of the plate.

$$U = (dU_1 + dU_2)$$

$$U = \frac{1}{2}D \iint \left[\left\{ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right\}^2 - \right.$$

$$\left. - 2(1-\mu) \left\{ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right\} - \left\{ \frac{\partial^2 w}{\partial x \, \partial y} \right\}^2 \right] dx\, dy \quad \ldots .(12)$$

It must be noted that the notation D represents the bending rigidity of the plate and

$$D = \frac{E t^3}{12(1-\mu^2)}$$

The work done by the forces applied at the boundary of the plate and acting in its middle plane is represented as

$$T = -\frac{1}{2} \iint \left[N_x \left\{ \frac{\partial w}{\partial x} \right\}^2 + N_y \left\{ \frac{\partial w}{\partial y} \right\}^2 + 2\, N_{xy} \times \right.$$

$$\left. \times \left\{ \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right\} \right] dx\, dy \quad \ldots\ldots\ldots\ldots(13)$$

3

BUCKLING OF SIMPLY SUPPORTED SQUARE PLATE UNDER UNIFORM EDGE COMPRESSION APPLIED IN ONE DIRECTION

Let us consider a thin plate of length = width = ℓ and thickness = t which is compressed uniformly at the opposite two edges $y = \pm \frac{\ell}{2}$ while being held as simply supported at the remaining two edges $x = \pm \frac{\ell}{2}$, all as shown in Fig. 4. Let the magnitude of the average compression per unit length be denoted as N_y. The plate boundaries are free of any other normal and shearing forces N_x , N_{xy} . It is assumed that the compressed plate would undergo a limited lateral bending w, which can be represented by a double trigonometric series as given in equation (1). In considering buckling of this plate the stretching in its plane is neglected. The value of critical stress is obtained by equating the energy of deformation to the corresponding amount of work done.

∴ Energy of deformation of the buckled plate, from equations (1) and (12) is

$$U = \frac{\pi^4 D}{8 \ell^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} a_{mn}^2 (n^2 + m^2)^2 \quad \ldots\ldots\ldots (14)$$

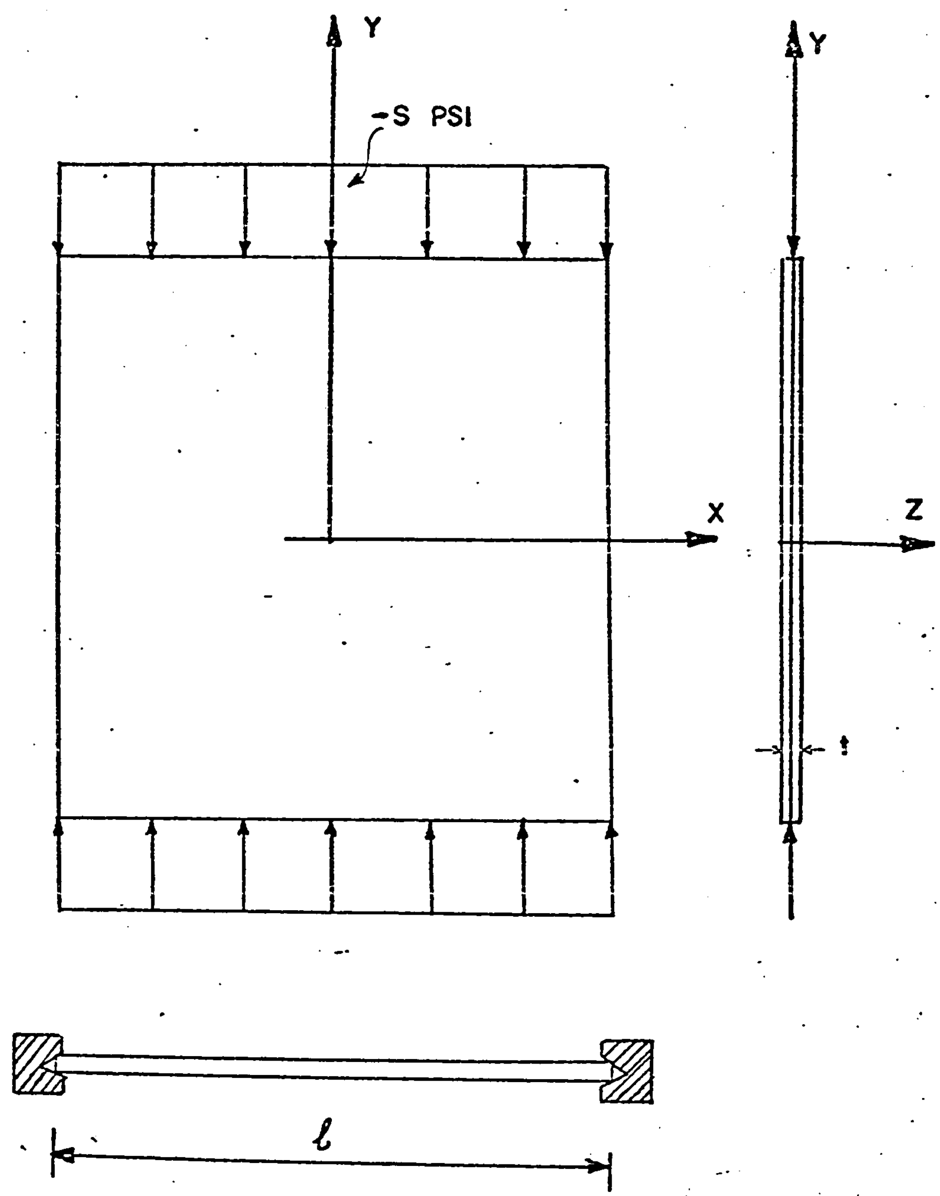


FIG.4 - DIMENSIONS & DETAILS OF SIMPLY SUPPORTED SQUARE PLATE UNDER EDGE COMPRESSION

The work done by the compressive forces causing buckling of the plate, from equation (13), will be

$$T = \frac{\pi^2}{8} N_Y \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} n^2 a_{mn}^2 \qquad \text{........(15)}$$

Critical value of the buckling compression is obtained by equating the energy of plate deformation U, to the corresponding amount of work T. The following characteristic equation must be satisfied and all values of the coefficients a_{mn} be considered as zero except one.

$$(n^2+m^2)^2 - N_Y n^2 \ell^2 (\pi^2 D)^{-1} = 0 \qquad \text{........ (16)}$$

The plate will begin to buckle at the smallest value of the applied compression that satisfies equation (16). n and m represent the length of the buckle and are expressed as positive integers. The minimum value of the buckling coefficient K is obtained by considering that the plate would buckle into one single wave in each direction parallel to x and y axes. The following relation holds good for determining k

$$K = \left(n + \frac{m^2}{n} \right)^2 \qquad \text{..................(17)}$$

We observe

$$D = \frac{E\,t^3}{12(1-\mu^2)}, \quad N_y = \sigma_{cr} t \quad \text{and} \quad \mu = 0.30$$

The critical stress will be given as

$$\sigma_{cr} = 3.62\,E\left[\frac{t}{l}\right]^2 \quad \ldots\ldots\ldots\ldots\ldots(18)$$

3.1 ELASTIC BUCKLING OF THE COMPRESSED PLATE

The critical stress σ_{cr} as calculated from equation (18) will give a true value only when the proportional limit of the plate material is not exceeded. Elastic buckling of the compressed plate is distinguished from its inelastic buckling by obtaining the value of critical stress well below the proportional limit and within the elastic range.

3.2 INELASTIC BUCKLING OF THE COMPRESSED PLATE

In case of a thin plate where the ratio $\frac{t}{\ell}$ is considerably small, the critical stress may exceed proportional limit of the plate material. Use of equation (18) will give exaggerated results. Necessarily a plasticity correction factor C must be introduced.

In the inelastic region of the stress-strain curve as shown in Fig. 5, the stress is no longer related to strain by linear relationship. The value Poisson's ratio μ tends to increase to a numerical value equal to one-half, especially when the plastic strains are large. Therefore Young's modulus E, as used in relation (18), must be replaced either by the ratio of inelastic stress to corresponding plastic strain, that is defined as secant modulus E_s or by the variable slope of the stress-strain curve and defined as tangent modulus E_t. The correction factor C must include all these inelastic effects. By using the results of the unified theory of plastic buckling of columns and plates given by Stowell[20] the following relation holds good for theoretical calculation of C

$$C = \frac{1-\mu_e^2}{1-\mu_p^2}\,\frac{E_s}{E}\left[\tfrac{1}{2}+\tfrac{1}{2}\left(\sqrt{\frac{1+3E_t/E_s}{4}}\right)\right] \qquad (19)$$

and the critical stress, beyond the proportional limit of the plate material is given as

$$\sigma_{cr_i} = 3.62\ CE\left(\frac{t}{\ell}\right)^2 \qquad \dots\dots\dots\dots \quad (20)$$

where we observe μ_e and μ_p as the values of Poisson's

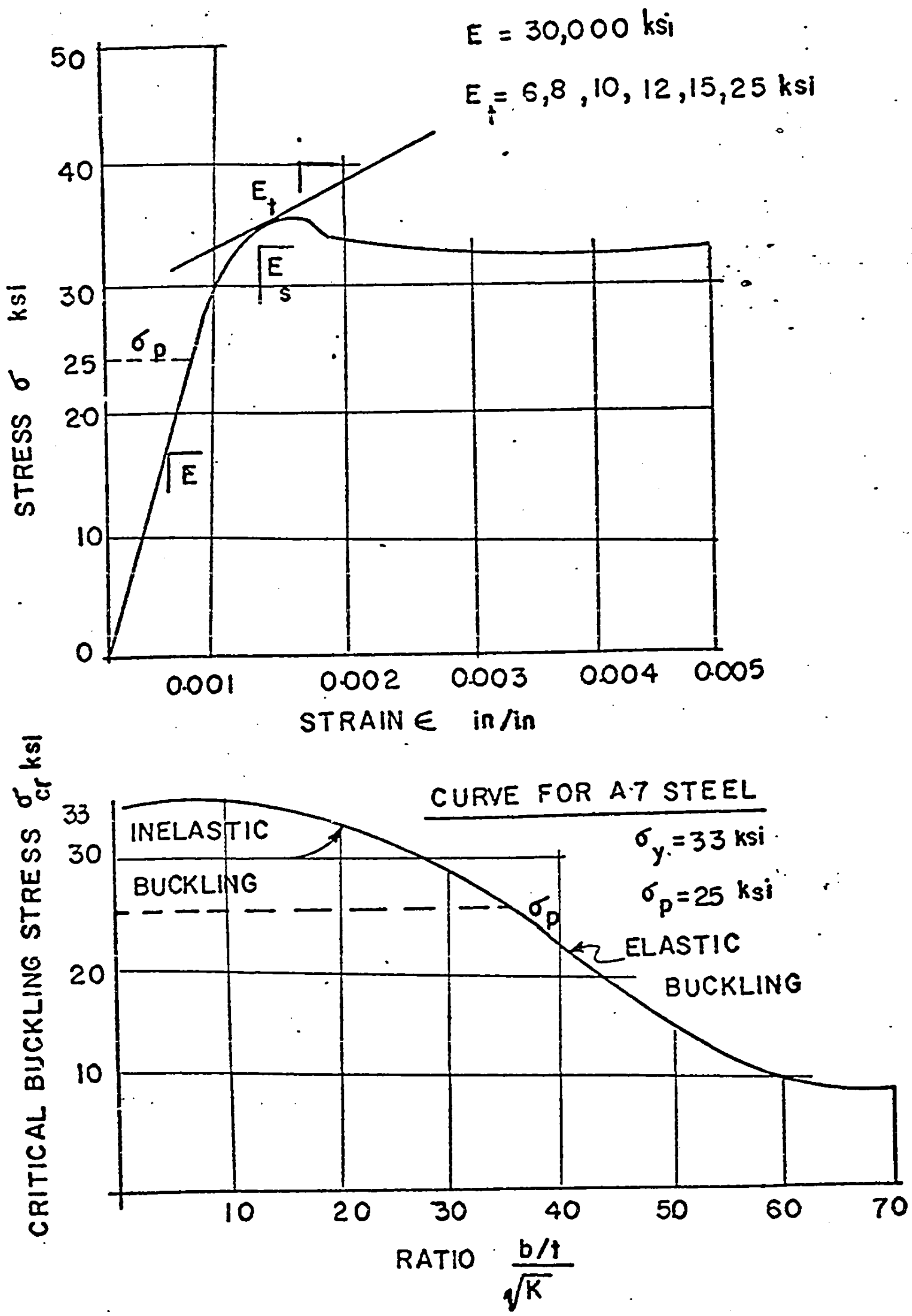


FIG 5- (A) STRESS STRAIN CURVE

(B) ELASTIC & INELASTIC BUCKLING

ratios corresponding to elastic and platic strains. The critical stress beyond the proportional limit of the plate material is denoted as σ_{cr_i}. In order to obtain equation (20), the correction factor C must be multiplied to both sides of equation (18). By comparing these equations (18) and (20) we also find that $C = \dfrac{\sigma_{cr_i}}{\sigma_{cr}}$

3.3 FAILURE OF THE COMPRESSED PLATE

A compressed plate must be deformed plastically before it would begin to fail. Necessarily the nonlinear inelastic stress-strain behaviour must be taken into consideration. The ultimate compression, at which failure may be noticed, can be expressed as

$$\sigma_{ultimate} \geq \sigma_{cr_i} \qquad \text{..........(21)}$$

4. AN ANALYSIS OF STRUCTURAL INSTABILITY OF SIMPLY SUPPORTED SQUARE PLATE HAVING A PLAIN CIRCULAR PERFORATION

A theoretical analysis is presented for calculating an approximate value of buckling compression of simply supported square plate having a plain circular perforation. We consider a case where the compression is applied along opposite two edges of such perforated plate. No attempt is made to reinforce the plate at the periphery of the perforation. Results, as obtained in this analysis, indicate that a plain circular perforation will decrease the critical compression of the plate. Structural instability, as would be caused by any such perforation, will depend on its size.

4.1 EFFECT OF PLAIN CIRCULAR PERFORATION ON THE STATE OF STRESS

A thin plate under edge compression or tension, is in state of plane stress. If a small circular perforation is made in the middle of this plate, the stress distribution in the immediate neighbourhood of such perforation, will change. In considering buckling of this perforated plate the state of redistributed stresses must be known.

An analysis for calculating stresses in a plate having a plain circular perforation when subjected to a uniform tension applied in one direction, is given by Kirsch.[1] A review of this solution shows that the same may be used to determine the redistributed stresses in a uniformly compressed plate, just prior to its buckling. Since we consider buckling under the action of uniform compression applied in one direction, a minus sign must be introduced to preceede S in kirsch's solution. The stresses will be given by the following relations

$$\sigma_r = \left\{ -\frac{S}{2}\left[1 - \frac{R^2}{r^2}\right] - \frac{S}{2}\left[1 + \frac{3R^4}{r^4} - \frac{4R^2}{r^2}\right]\cos 2\theta \right\},$$

$$\sigma_\theta = \left\{ -\frac{S}{2}\left[1 + \frac{R^2}{r^2}\right] + \frac{S}{2}\left[1 + \frac{3R^4}{r^4}\right]\cos 2\theta \right\} \text{ and}$$

$$\tau_{r\theta} = -\frac{S}{2}\left[\frac{3R^4}{r^4} - \frac{2R^2}{r^2} - 1\right]\sin 2\theta \qquad \ldots\ldots\ldots(22)$$

Where

r, θ = polar coordinates with origin coinciding with the geometric center of the plate

R = radius of perforation

$-S$ = edge compression acting far from the perforation

σ_r = stress in the radial direction

σ_θ = stress in the tangential direction

$\tau_{r\theta}$ = shear stress

Two distinct stress zones may be distinguished as shown in Fig. 6; the first, far from the perforation and the second, in its immediate neighbourhood. Stresses in the first zone may be considered as if unaffected by the presence of the perforation. In the second zone, a high concentration of stresses may be noticed. When r is very large, as will be at any point in the first stress zone of the compressed perforated plate, the stresses are given by the following relations

$$\sigma_\theta = - S, \ \sigma_r = -\tfrac{1}{2} S \ (1 + \cos 2\theta), \ \tau_{r\theta} = \tfrac{1}{2} S \sin 2\theta \ldots(23)$$

Whereas the stresses at any point where r = R will be given as

$$\sigma_r = \tau_{I\theta} = 0, \ \sigma_\theta = - (S - 2S \cos 2\theta) \ \ldots\ldots\ldots\ldots(24)$$

4.2 EFFECT OF LATERAL TENSION

In obtaining solution for redistributed stresses at any point of the perforated plate, as given in equations(22),(23) and (24) we consider the action of compression applied in one direction. In fact the perforated plate will undergo some

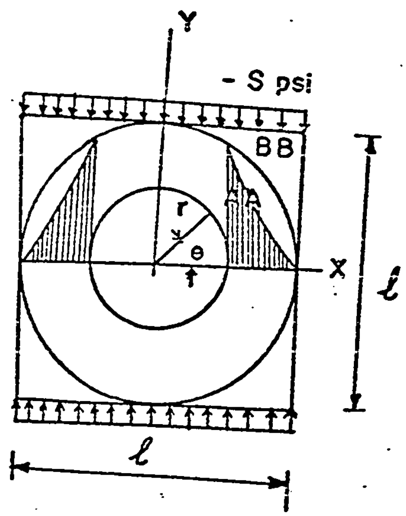


FIG 6 - EFFECT OF PLAIN CIRCULAR PERFORATION ON THE STATE OF STRESS contraction in a direction parallel to that of the applied compression. An extension will be noticed in a direction perpendicular to that of the applied compression. IF the stresses corresponding to such extension are included, equation (24) will be modified as

$$\sigma_\theta = - \left[S - 2S \cos(2\theta - \Pi)\right] + \left[S - 2S \cos 2\theta\right] \quad \ldots\ldots\ldots (25)$$

The stresses as referred to polar coordinates may be converted to those in rectangular coordinates by using the following relations

$$\left.\begin{aligned}\sigma_x &= \sigma_r \cos^2\theta + \sigma_\theta \sin^2\theta - \tau_{r\theta} \sin 2\theta,\\ \sigma_y &= \sigma_r \sin^2\theta + \sigma_\theta \cos^2\theta + \tau_{r\theta} \sin 2\theta\\ \tau_{xy} &= \tfrac{1}{2}(\sigma_r - \sigma_\theta)\sin 2\theta + \tau_{r\theta} \cos 2\theta\end{aligned}\right\} \ldots\ldots (26)$$

4.3 BUCKLING OF SIMPLY SUPPORTED SQUARE PLATE HAVING A PLAIN CIRCULAR PERFORATION UNDER EDGE COMPRESSION

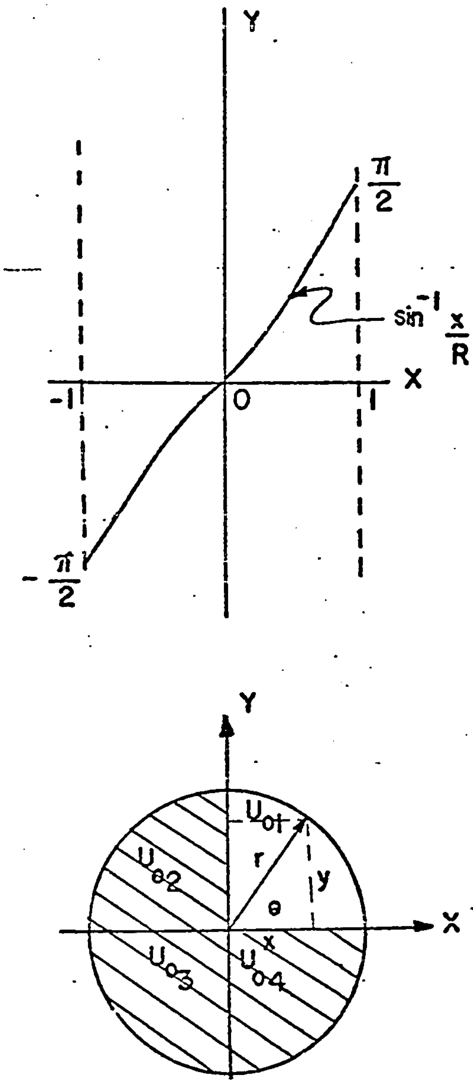
The critical stress at which the perforated plate would begin to buckle, is obtained by equating the energy of deformation to the corresponding amount of work done by the compressive forces. By using equations (22) through (26) in the calculation of energy and work, we obtain

$$U_p = \iint_{Area} \tfrac{1}{2} \left\{ -\left[\int_t \sigma_x \, z \, dz \, \frac{\partial^2 w}{\partial x^2}\right] - \left[\int_t \sigma_y \, z \, dz \, \frac{\partial^2 w}{\partial y^2}\right] + \left[2\int_t \tau_{xy} \, z \, dz \, \frac{\partial^2 w}{\partial x \, \partial y}\right] \right\} dx \, dy \quad \text{........(27)}$$

Again it may be tentatively assumed that the energy of the perforated plate can be obtained by the relation $U_p = U - U_o$, where the notation U represents the energy of the plate prior to making any perforation therein. U_o denotes the energy as will be obtained by integrating equation (12) over the entire portion of the plate that has been removed. The energy of the perforated plate U_p will be given as

$$U_p = \frac{\Pi^4 D}{8 \ell^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 (n^2+m^2)^2 - \frac{4 \, \Pi^4 D}{8} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 (n^2+m^2)^2 \int_0^R \sqrt{(R^2-x^2)} \, dx \quad \text{..(28)}$$

Similarly the work done in compressing the perforated plate will be given as $T_p = T - T_o$ where T_o is obtained by integrating equation (13) over the portion of the plate that has been removed.



$$U_o = U_{o1} + U_{o2} + U_{o3} + U_{o4} = 4\,U_{o1}$$

$$U_{o1} = \left[\frac{D}{2}\iint \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^2 (m^2\pi^2 + n^2\pi^2)^2 \cos^2 m\pi x \cos^2 n\pi y \; dx\, dy\right] \times$$

$$\times \left[\frac{x\sqrt{R^2 - x^2}}{2} + \frac{R^2}{2}\sin^{-1}\frac{x}{R}\right]_{x=0}^{x=R}$$

FIG. 7- SUBDIVISION OF THE REMOVED PORTION OF THE PLATE INTO FOUR QUADRANTS IN CALCULATING THE LOSS OF ENERGY U_o

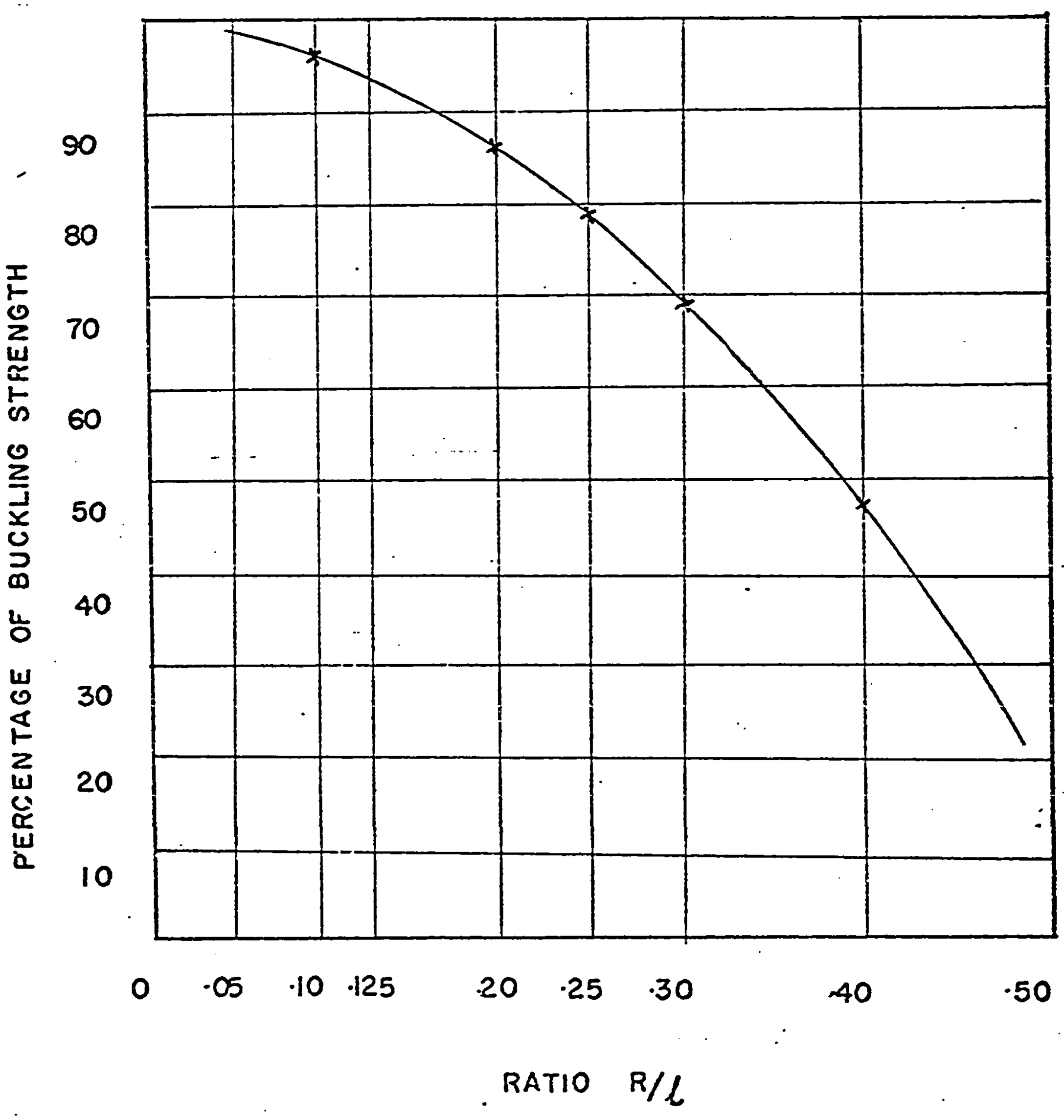


FIG.8 - INSTABILITY DUE TO PLAIN CIRCULAR PERFORATION

The smallest value of the buckling stress will be obtained by making the expression $\frac{U_p}{T_p}$ a minimum. The necessary and sufficient condition to obtain this minima will be satisfied if the determinant of the coefficients $a_{11}, \ldots a_{mn}$ reduces to zero in the following set of simultaneous equations

$$\left.\begin{array}{l} \dfrac{U_p}{a_{11}} + S\,\dfrac{T_p}{a_{11}} = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \dfrac{U_p}{a_{mn}} + S\,\dfrac{T_p}{a_{mn}} = 0 \end{array}\right\} \ldots\ldots\ldots\ldots (29)$$

The loss in buckling strength of the plate as caused by the presence of a circular perforation, will be given as

$$\sigma_{cr}\, t \iint dx\, dy = \frac{\pi^2 D \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^2 (n^2+m^2)^2}{\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^2 n^2}\, 4\int_0^R \sqrt{(R^2-x^2)}\, dx =$$

$$= \frac{3.62\, E \lambda^2 \pi t^3}{\ell^2} \quad \ldots\ldots\ldots\ldots (30)$$

It must be noted that in deriving equation (28), we replace the bending rigidity of the plate D by the term $\frac{Et^3}{12(1-\mu^2)}$ and the radius of the perforation R which is expressed as a fraction of the plate length ℓ, is replaced by a parameter $\frac{\lambda}{\ell}$. The critical stress of the perforated plate will be given as

$$\sigma_{cr_p} = 3.62 \, Et^2 \left[\frac{1 - \lambda^2 \Pi}{\ell^2} \right] \qquad \text{..........(31)}$$

5

BUCKLING STRENGTH OF SIMPLY SUPPORTED STIFFENED PLATE HAVING A PLAIN CIRCULAR PERFORATION

In case of buckling of simply supported square plates with or without any perforations, the value of critical stress is directly proportional to the square of the ratio $\frac{t}{l}$. One or more stiffeners always can be provided to increase buckling strength of these plates, to carry a part of the applied compression or to perform both these functions. In some cases the plate is reinforced locally at the periphery of the perforation by doubler plates whereas in other cases no such local reinforcement is provided and the perforation is left plain. The intent of this study is confined to the later cases where we consider plain perforations alone.

5.1 STABILIZING EFFECT OF LONGITUDINAL STIFFENERS

Buckling strength of a plate with or without any perforation can be increased by providing longitudinal stiffeners which are parallel to the direction of the applied compression. These stiffeners subdivide the plate into panels of smaller widths. Each panel may be considered as simply supported in between the stiffeners and to act as a long plate with a greater value of the ratio $\frac{t}{b}$, where b denotes the width of the panel. It is easily verified that the buckling strength of the plate will increase because $\left[\frac{t}{b}\right]^2 > \left[\frac{t}{\ell}\right]^2$ also because the stiffeners will carry a part of the applied compression. The relation between the dimensions of stiffeners and the corresponding increase in buckling strength of the perforated plate can be obtained by using energy method.

5.2 BUCKLING STRENGTH OF A SIMPLY SUPPORTED SQUARE PLATE REINFORCED BY TWO LONGITUDINAL STIFFENERS AND HAVING A PLAIN CIRCULAR PERFORATION

Let us consider a square plate of length = width = ℓ that is reinforced by two equidistant stiffeners subdividing the plate into three panels of equal width b. Let a small circular perforation be made in the middle of this stiffened plate and let a uniform compression -S be applied at the opposite two edges $y = \pm \frac{1}{2}\ell$, in a direction parallel to that of the two stiffeners, all as shown in Fig. 9. The minimum value of the applied compression at which the perforated stiffened plate will begin to buckle, is obtained by equating the total energy of deformation of such plate to the corresponding amount of work done during compressing. By designating the energy and work done in compressing of the two stiffeners

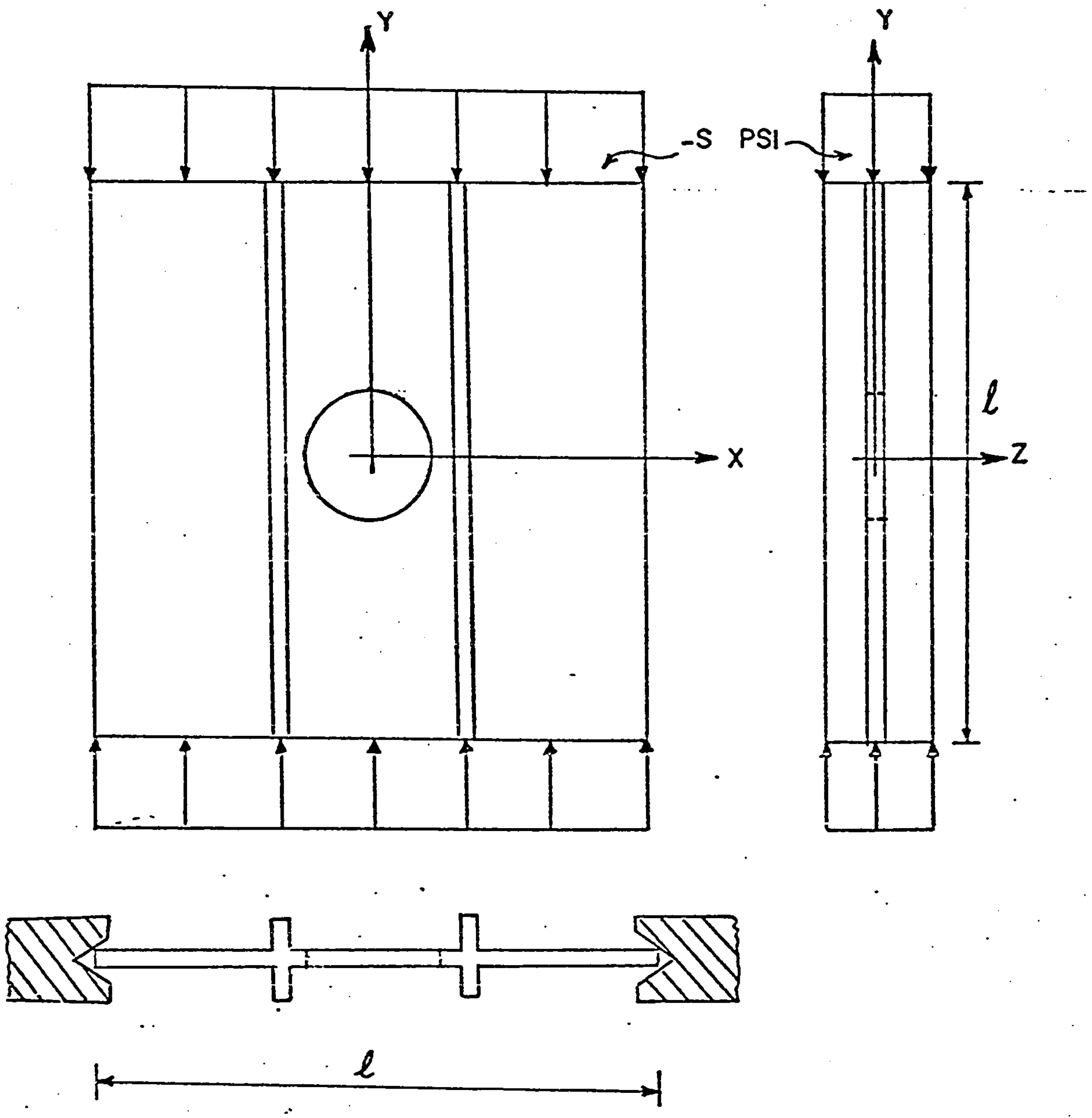


FIG. 9- LOADING OF SIMPLY SUPPORTED SQUARE PLATE REINFORCED BY TWO SYMMETRIC STIFFENERS AND HAVING PLAIN CIRCULAR PERFORATION

by the notations U_s and T_s respectively, we obtain

$$U - U_o + U_s = T - T_o + T_s \qquad \text{.............(32)}$$

We denote the bending rigidity of a stiffener as EI, also we introduce the following other notations

$-S$ = uniform compression applied along the opposite two edges, far from the perforation

$-P$ = part of the applied compression carried by the stiffeners

Q_s = shape factor $= \dfrac{\text{plate length}}{\text{plate width}} = 1$

Q_A = ratio of the crosssectional areas of the stiffeners and the plate

$\bar{Q}$ = ratio of rigidities $= \dfrac{EI}{D}$.

The energy of deformation of a single stiffener at a distance p from the origin is given as

$$U_s = \tfrac{1}{2}EI \int \left[\frac{\partial^2 w}{\partial y^2}\right]^2_{x=p} dy = \tfrac{1}{4}EI \; \frac{\pi^4}{\ell^3} \sum_{n=1}^{\infty} n^4 \left(a_{n1} \cos \frac{\pi p}{\ell} + \right.$$

$$\left. + a_{n2} \cos \frac{2\pi p}{\ell} + \ldots\right)^2 \qquad \text{.......(33)}$$

And the work done by the part of the applied compression -P acting on the stiffeners is

$$T_s = \sum_{1}^{2} Q_A \; -\tfrac{1}{2}P \int \left[\frac{\partial w}{\partial y}\right]^2_{x=p} dy = \sum_{1}^{2} Q_A \; -\tfrac{1}{4}P \frac{\Pi^2}{\ell} \sum_{n=1}^{\infty} n^2 (a_{n1} \times$$

$$\times \cos\frac{\Pi p}{\ell} + a_{n2} \cos\frac{2\Pi p}{\ell} + \ldots\ldots)^2 \qquad \ldots\ldots\ldots\ldots(34)$$

The general equation for calculating the critical stress is obtained from equations (32),(33) and (34)

$$\sigma_{cr} = 0.905\ (1 - \lambda^2 \Pi)\ E \frac{t^2}{\ell^2} \times$$

$$\times \frac{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{mn}^2 (n^2 + m^2 Q_s^2)^2 + 2 \sum_{1}^{2} \bar{Q} \sum_{n=1}^{\infty} n^4 \left(\sum_{m=1}^{\infty} a_{mn} \cos\frac{m\Pi p}{\ell}\right)^2}{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} n^2 a_{mn}^2 + 2 \sum_{1}^{2} Q_A \sum_{n=1}^{\infty} n^2 \left(\sum_{m=1}^{\infty} a_{mn} \cos\frac{m\Pi p}{\ell}\right)^2} \ldots(35)$$

In order to obtain the smallest value of the critical stress the derivative of equation (35) is equated to zero. The approximate expression for the critical stress is given as

$$\sigma_{cr} = \frac{3\bar{Q} + 4}{3 Q_A + 1}\ 0.905\ (1 - \lambda^2 \Pi)\ E \frac{t^2}{\ell^2} \qquad \ldots\ldots\ldots\ldots(36)$$

After rearranging the terms in equation (36) we obtain

$$\sigma_{cr} = \frac{3.62\ Et^2}{\ell^2}\left[1 - \lambda^2\Pi + \left(\frac{0.75\ \bar{Q} + 1}{3\ Q_A + 1} - 1\right)\right] \quad \ldots(37)$$

where the last term in the parentheses of equation (37) represents the stabilizing effect of the two symmetric stiffeners reinforcing the perforated plate longitudinally in a direction parallel to that of the applied compression.

Let us consider a particular case where the two stiffeners are of equal rigidity and their size is selected such that the ratio of cross sectional areas Q_A becomes 0.10 and the ratio of rigidities $\bar{Q}$ is 5.00. The critical stress in this particular case, from equation (37), will be

$$\sigma_{cr} = \frac{3.62\ E\ t^2}{\ell^2}\ (1 - \lambda^2\Pi + 2.66) \quad \ldots\ldots\ldots\ldots(38)$$

5.3 STABILIZING EFFECT OF TRANSVERSE STIFFENERS

Let us consider a square plate of length = width = ℓ having a small circular perforation. Let this perforated plate be reinforced by two equidistant stiffeners transversely in a direction perpendicular to that of the applied compression, as shown in Fig. 10. The two transverse stiffeners

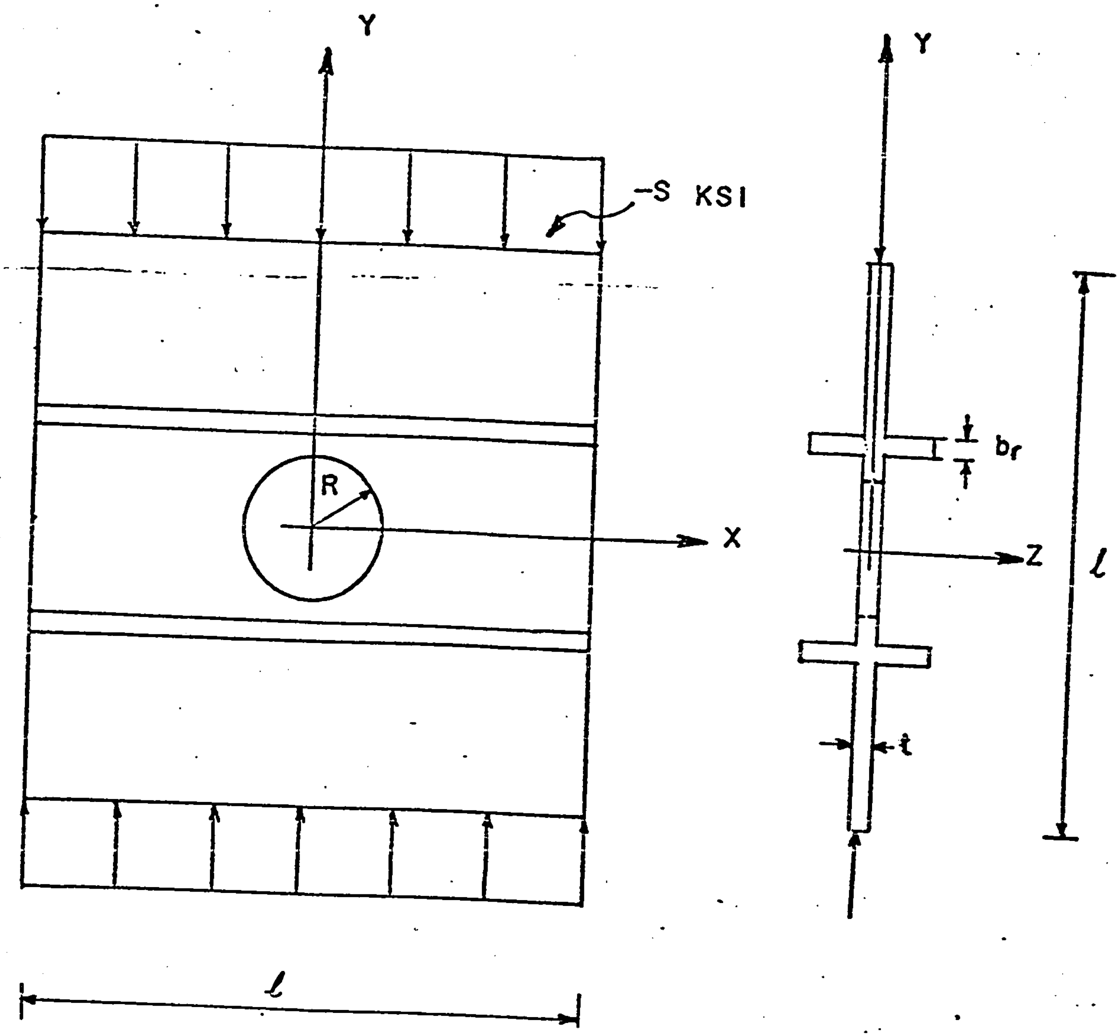


FIG. 10 DIMENSIONS & DETAILS OF TRANSVERSELY STIFFENED PERFORATED PLATE USED FOR CALCULATING EQUATION (39).

will not participate in carrying the applied compression yet the buckling strength of the perforated plate will be increased to some extent. This increase in buckling strength will be noticed because the two stiffeners subdivide the perforated plate into three panels each of equal width b and each panel will act as a long plate.The critical stress will be obtained by equating the total energy of deformation to the corresponding amount of work done. The following equation holds good

$$\sigma_{cr} = \frac{3.62\ Et^2}{l^2}\left\{1- \lambda^2\Pi+\left[\frac{(n^2+1)+3\bar{Q}}{4n^2} - 1\right]\right\} \qquad \text{.......(39)}$$

Where n denotes the number of half-waves into which the plate would buckle. It is noticed that the equation (39) will acquire its minimum value when

$$n^2 + n - 9 = 0 \qquad \text{................(40)}$$

After substituting $n = 2.54 \quad 3$ and $\bar{Q} = 5.00$ into equation (39),we obtain

$$\sigma_{cr} = \frac{3.62\ Et^2}{l^2}\,(1- \lambda^2\Pi + 1.70) \qquad \text{............(41)}$$

5.4 ORTHOTROPIC IMPLICATIONS IN BUCKLING OF PERFORATED STIFFENED PLATES

Buckling stress of a perforated plate with three or more stiffeners will differ very little from that of a plate reinforced closely by infinite number of stiffeners. As a result, the buckling stress in such case may be calculated with orthotropic plate theory[21].

The bending and twisting moments in a stiffened plate will be related to curvatures and twist by the following relations

$$M_x = D_x \frac{\partial^2 w}{\partial x^2} + D_1 \frac{\partial^2 w}{\partial y^2}$$

$$M_y = D_y \frac{\partial^2 w}{\partial y^2} + D_1 \frac{\partial^2 w}{\partial x^2} \qquad \ldots\ldots(42)$$

$$M_{xy} = 2 D_{xy} \frac{\partial^2 w}{\partial x \, \partial y}$$

We consider the stiffened plate as a technically orthotropic plate[22] and the relations in equation (42) are based on different elastic properties in two orthogonal directions of such plate. Again it must be noted that the relations in equations (8) and (9) which are based on isotropic plate theory, will be substituted by those in equation (42).

For the particular case when the size of the perforation is considerably small, the buckled form of such perforated stiffened plate may be represented by the differential equation

$$D_x \frac{\partial^4 w}{\partial x^4} + 2(D_1 + 2D_{xy}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} =$$

$$= - N_y \frac{\partial^2 w}{\partial y^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(43)$$

The critical stress at which the compressed plate will begin to buckle, is given by the relation

$$\sigma_{cr} = \frac{\pi^2 D_x D_y}{b^2 t} \quad K - 2\left\{1 - \frac{D_1 + 2 D_{xy}}{\sqrt{D_x D_y}}\right\} \ldots.(44)$$

The theoretical determination of the plate rigidities in the two mutually orthogonal directions of a stiffened plate is not simple especially when the stiffeners are asymmetric. The following relations hold good provided that the stiffeners are symmetric and are provided on both sides of the perforated plate.

$$D_x = D + \frac{EI_x}{p_y} , \quad D_y = D + \frac{EI_y}{p_x} \qquad \ldots..(45)$$

and

$$D_1 = \left[D - 2\, D_{xy} \right] \qquad \text{.................} (46)$$

The notations p_x and p_y are used to represent the stiffener spacing in the directions of x and y axes.

5.3 BUCKLING OF STIFFENED PLATES WITH TWO OR MORE CIRCULAR PERFORATIONS.

A stiffened plate with two or more closely spaced perforations is more likely to be buckled in between such perforations, when compressed beyond the critical capacity. Exact analysis of the effect of adjecent perforations entails considerable complications because of the complex nature of the locally concentrated stress.

When the perforations are spaced at a distance greater than that of the stiffeners, it will be simplifying to neglect the effect of adjecent perforations and to consider the stiffened plate to be subdivided into smaller segments each containing a single perforation and acting as simply supported in between the stiffeners. Buckling behaviour of each perforated segment will necessarily depend on the degree of elastic restraint available at the supporting stiffeners therefore the stiffeners must be sturdy in size.

6

NUMERICAL RESULTS

We obtain several values of critical buckling compression for simply supported square plates of different cross sections, also when a small circular perforation is included there in, and when these perforated plates are reinforced by two symmetric stiffeners in longitudinal and in transverse manners. These numerical results formulate basis for predicting relative buckling strength of plates with plain circular perforations with or without any stiffeners.

The values given in Table No. 1 are calculated from equation (18) for simply supported square plates of different cross sections. Several values of the ratio t/ℓ are considered and corresponding buckling compressions are calculated. The results show that the buckling strength of a plate will increase when a higher ratio t/ℓ is selected.

The values given in Table No. 2 are calculated from equation (31) where we consider the presence of a plain circular perforation in simply supported square plates of identical dimensions and cross sections as were considered in calculating the values given in Table No. 1, above. Perforations with diameters upto one-half of the plate length. The results given in Table No. 2 show that the buckling strength of simply supported square plates is hardly affected by the presence of a perforation when the size of the perforation is considerably small as compared to plate dimensions. There is a rapid decrease in buckling strength with the increase in size of the perforation.

We also consider longitudinal and transverse stiffening arrangements in reinforcing of simply supported square plates with plain circular perforations. The values of critical buckling compression for these cases are calculated from equations (37) and (38) respectively and are given in Table No. 3 and Table No. 4.

TABLE NO. 1 - VALUES OF CRITICAL STRESS σ_{cr} IN KSI AS CALCULATED FROM EQUATION (18) FOR SIMPLY SUPPORTED SQUARE PLATES UNDER UNIFORM EDGE COMPRESSION APPLIED IN ONE DIRECTION

$$E = 30 \times 10^{3} \text{ KSI}$$

Ratio t/ℓ	0.001	0.002	0.004	0.006	0.008	0.01	0.02
* Critical Stress σ_{cr} in KSI	0.00362E	0.01448E	0.05492E	0.13032E	0.23168E	0.36200E	1.44800E

* ALL VALUES TO BE MULTIPLIEDBY 10^{-3}

TABLE NO.2 - VALUES OF CRITICAL STRESS σ_{cr_p} IN KSI CALCULATED FROM EQUATION (31) FOR SIMPLY SUPPORTED SQUARE PLATE HAVING A PLAIN CIRCULAR PERFORATION OF RADIUS R, WHEN COMPRESSED AT OPPOSITE EDGES IN ONE DIRECTION

$E = 30 \times 10^3$ KSI

R/ℓ	0.00000	0.05000	0.06250	0.10000	0.12500	0.20000	0.25000	0.50000
t/ℓ	* Values of critical stress σ_{cr_p}							
0.001	0.00362E	0.00359E	0.00358E	0.00351E	0.00345E	0.00317E	0.00219E	0.00078E
0.002	0.01448E	0.01436E	0.01432E	0.01404E	0.01380E	0.01268E	0.01164E	0.00312E
0.004	0.05792E	0.05744E	0.05728E	0.05626E	0.05520E	0.05072E	0.04656E	0.01249E
0.006	0.13032E	0.12924E	0.12888E	0.12636E	0.12420E	0.12412E	0.10476E	0.02798E
0.010	0.36200E	0.35900E	0.35800E	0.35100E	0.34500E	0.31700E	0.21900E	0.07800E

* ALL VALUES TO BE MULTIPLIED BY 10^{-3}

TABLE NO. 3 - VALUES OF CRITICAL STRESS CALCULATED FROM EQUATION (37), $E = 30 \times 10^3$ KSI

$Q_A = 0.10$, $\bar{Q} = 5.0$, $\mu = 0.30$

R/ℓ	0.00000	0.05000	0.06250	0.10000	0.12500	0.20000	0.25000	0.50000
t/ℓ	* Values of critical stress as calculated from equation(37)							
0.001	0.01327E	0.01324E	0.01323E	0.01316E	0.01310E	0.01282E	0.01184E	0.01043E
0.002	0.05310E	0.05298E	0.05294E	0.05266E	0.05242E	0.05130E	0.05026E	0.04174E
0.004	0.21237E	0.21189E	0.21173E	0.21071E	0.20965E	0.20517E	0.20101E	0.16694E
0.006	0.47784E	0.47676E	0.47640E	0.47388E	0.47172E	0.47164E	0.45228E	0.37550E
0.010	1.32734E	1.31450E	1.31290E	1.30634E	1.30320E	1.27184E	1.24634E	1.03334E

* ALL VALUES TO BE MULTIPLIED BY 10^{-3}

TABLE NO. 4 - VALUES OF CRITICAL STRESS CALCULATED FROM EQUATION (39), $E = 30 \times 10^{3}$ KSI

$Q_A = 0.10$, $\bar{Q} = 5.0$, $\mu = 0.30$

R/ℓ	0.00000	0.05000	0.06250	0.10000	0.12500	0.20000	0.25000	0.50000
t/ℓ	* Values of critical stress as calculated fromequation (39)							
0.001	0.00977E	0.00974E	0.00973E	0.00966E	0.00960E	0.00932E	0.00843E	0.00693E
0.002	0.04765E	0.04753E	0.04749E	0.04721E	0.04697E	0.04585E	0.04481E	0.03629E
0.004	0.15139E	0.15091E	0.15075E	0.14973E	0.14867E	0.14419E	0.14003E	0.08596E
0.006	0.35186E	0.35078E	0.35048E	0.34790E	0.34574E	0.34566E	0.32630E	0.24952E
0.010	0.97440E	0.97240E	0.89990E	0.89860E	0.89204E	0.88890E	0.85754E	0.61904E

* ALL VALUES TO BE MULTIPLIED BY 10^{-3}

7
CONCLUSIONS

Based on the results obtained in this study, the following conclusions are formulated with regard to buckling of simply supported square plates reinforced by two symmetric stiffeners and having small circular perforations.

1. Perforated plates are subject to failure by buckling at relatively low stresses, frequently below the proportional limit of the plate material and seldom above the yield stress.

2. Buckling is the critical mode of failure for the major portion of the compressed plate especially when its thickness is considerably small.

3. Buckling may occour due to lateral bending of the compressed plate or under action of both bending and twisting. The wave length of the buckle depends on the cross sectional dimensions of the plate and on the shape factor Q_s. Values of critical buckling stress for simply supported plates may be predicted from a direct reading chart given in Fig.11.

4. Perforated plates when reinforced by two symmetric stiffeners will buckle at relatively high stress frequently above the proportional limit of the plate material. The

chart given in Fig. 12 indicates the effect of flexural rigidity of the stiffeners on the buckling strength of plates having plain circular perforations. A marked increase in the the buckling compression may be accomplished by selecting more rigid and sturdy stiffeners.

Critical buckling stress depends on elastic properties of the plate material. The values of Poisson's ratio increase in the inelastic range of the plate material. At large values of plastic strains such increase is almost one hundred per cent. The chart in Fig. 13 is obtained by plotting the values of plate buckling coefficient K against the corresponding values of Poisson's ratio in the elastic as well as in the inelastic range of the plate material.

The chart shown in Fig. 14 is developed by Gerard[13] which aids in calculating the plate buckling coefficient while taking into consideration the torsional rigidity of the reinforcing stiffeners. The upper curve will be applicable if the stiffeners are considered as torsionally stiff, and are attached to both sides of the perforated plate by means of double row of fasteners. The lower curve will be applicable if the stiffeners behave as torsionally flexible.

5. In presenting the buckling formulae in this study the energy of twist is disregarded based on the assumption that the stiffeners would behave as absolutely stiff against

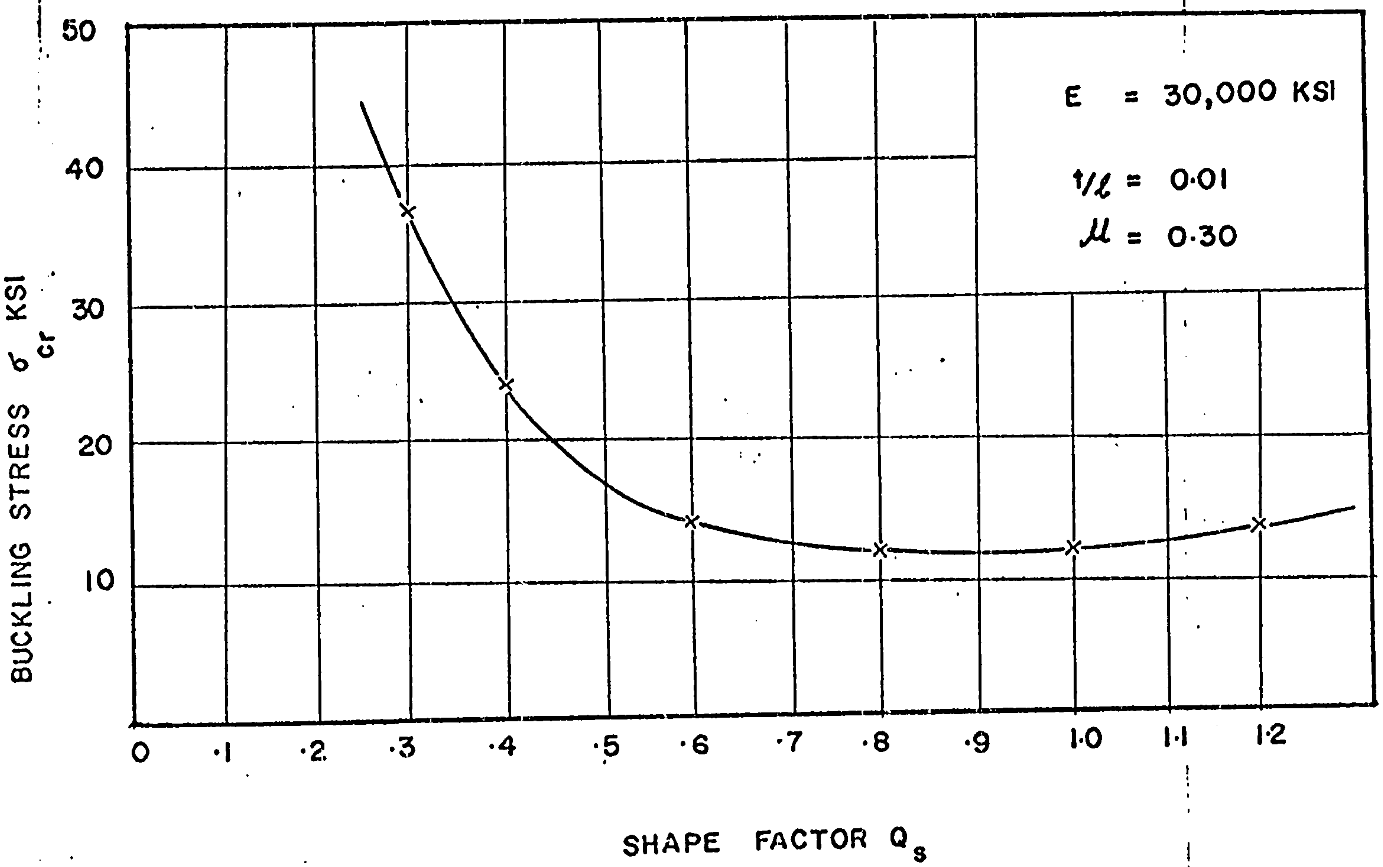


FIG.11 - BUCKLING STRESS VS SHAPE FACTOR FOR SIMPLY SUPPORTED PLATES UNDER EDGE COMPRESSION

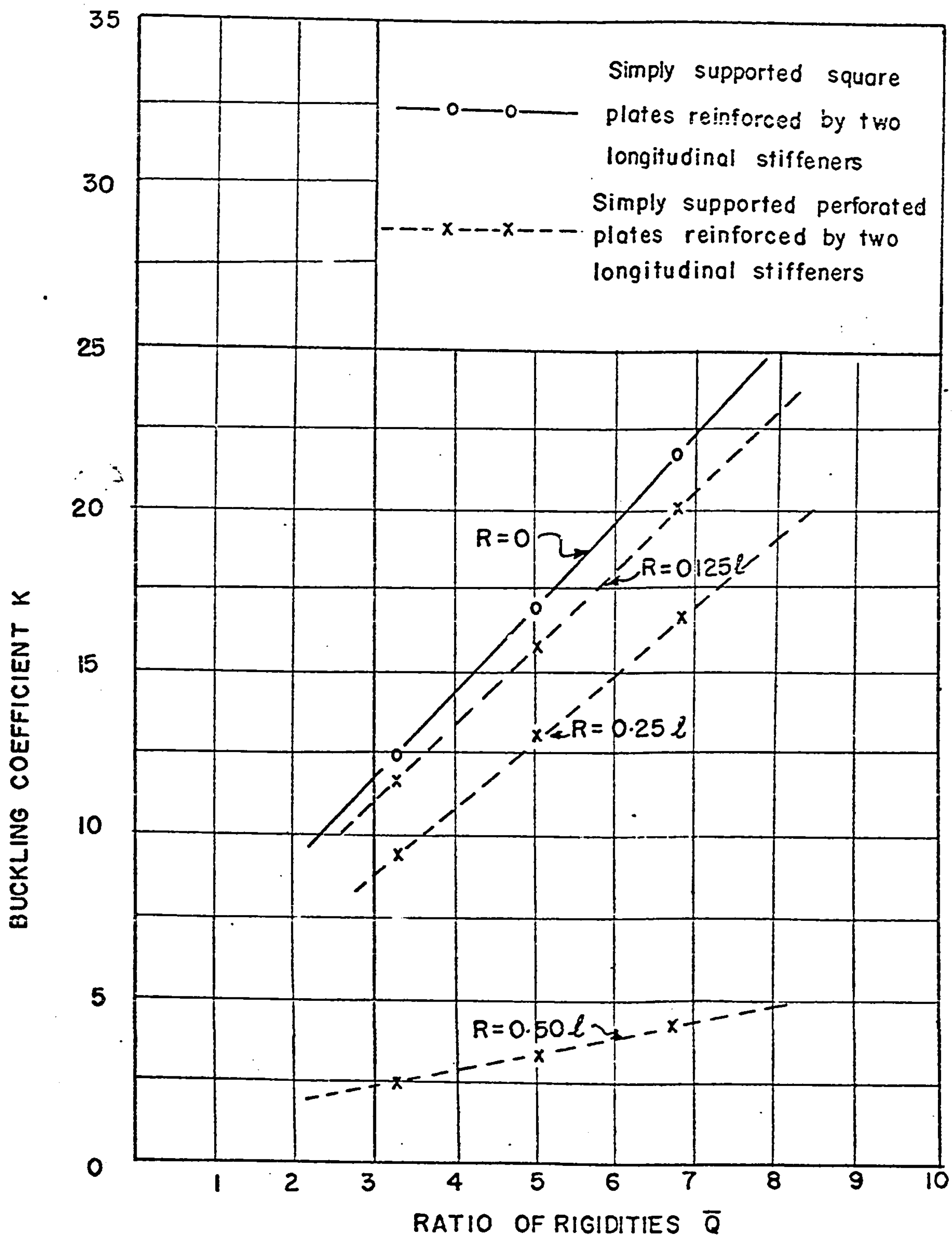


FIG. 12 - EFFECT OF STIFFENER RIGIDITY ON PLATE BUCKLING COEFFICIENT

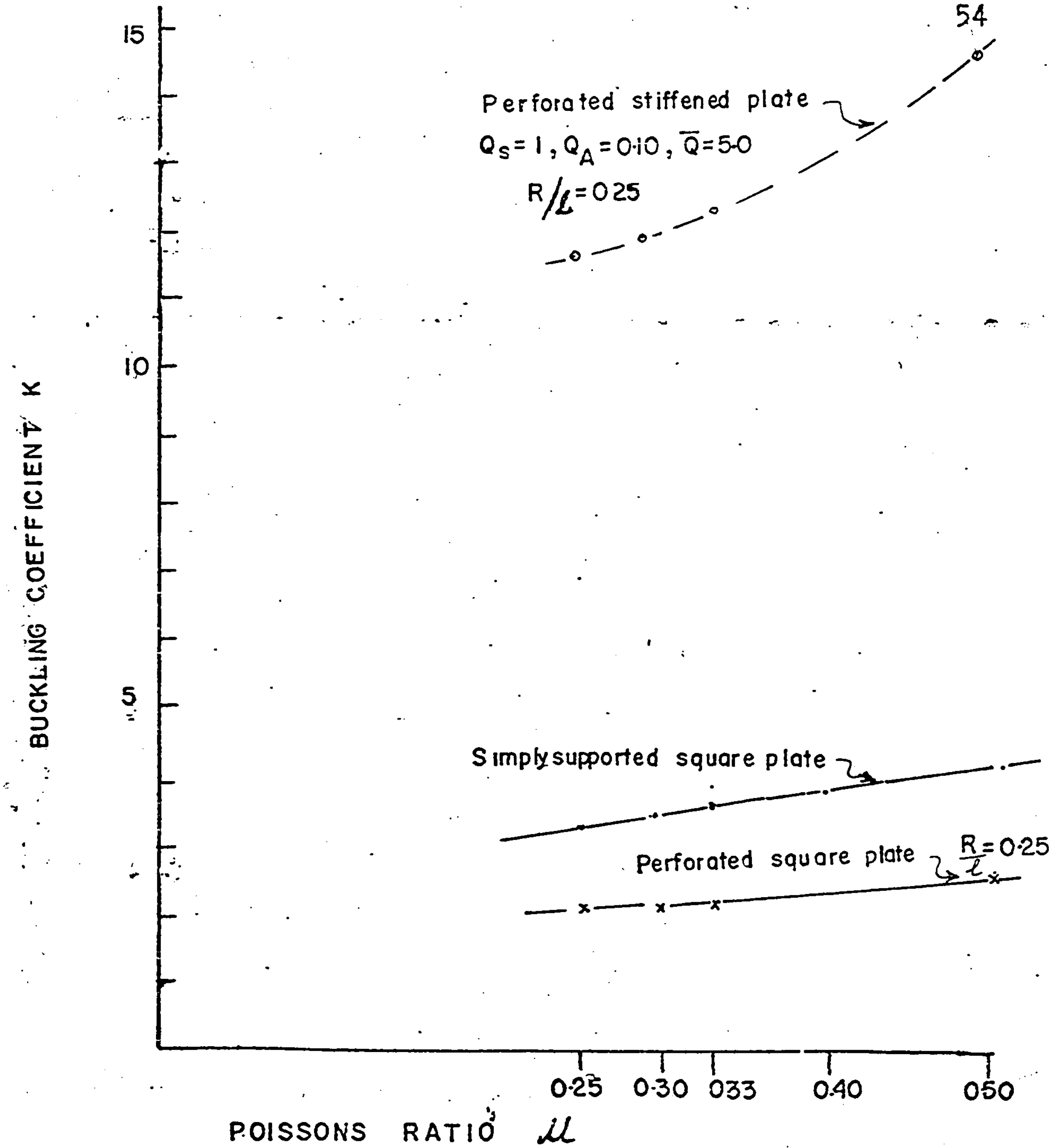


FIG 13 - EFFECT OF POISSONS RATIO ON BUCKLING COEFFICIENT K twist.

The chart given in Fig. 15 is developed by Gerard[13] which aids in calculating the effect of elastic restraint at the uncompressed edges on buckling strength of simply supported plates. In this study we assume that the uncompressed edges remain straight during and after buckling. The elastic restraint against in-plane expansion is due to Poisson's ratio.

In considering buckling of perforated stiffened plate we assume that the reinforcing stiffeners subdivide the plate into segments, each acting as a long plate while being held as simply supported in between the stiffeners. Obviously the elastic restraint against in-plane motion in any such segment must be provided by the stiffeners. The actual amount of restraint depends on cross sectional dimensions and the parameter Q_A contains the effect of elastic restraint on plate buckling coefficient. In Fig. 15 the values of buckling coefficient K are plotted against the corresponding values of the parameter Q_A. Two limiting cases are distinguished: in the first case the ratio of cross sectional areas Q_A is considered as zero. Which means the stiffeners are absolutely flexible or in other words the plate will behave as if unreinforced. In the second limiting case the ratio of cross sectional areas Q_A is considered as

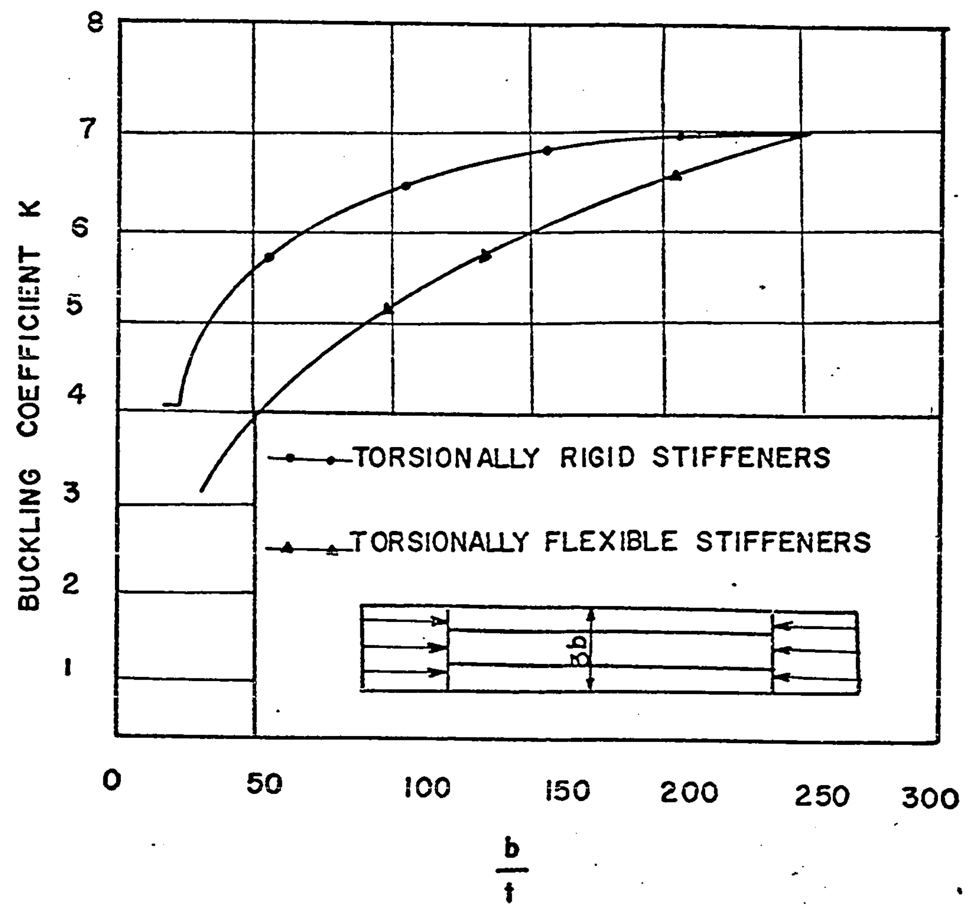


FIG. 14 - EFFECT OF STIFFENER TORSIONAL RIGIDITY ON BUCKLING COEFFICIENT K

Which means that the stiffeners will behave as absolutely rigid. For any practical dimensions of stiffeners, both the limiting cases will not apply and the value of the plate buckling coefficient may be predicted from the chart given in Fig. 15.

6. Presence of a plain circular perforation decrease the buckling strength of a plate however such loss in buckling strength may be economically compensated by providing suitable reinforcement either at the periphery of the perforation or in its immediate vicinity in longitudinal or in transverse manners. Relative buckling strength of a perforated plate reinforced by two symmetric stiffeners may be predicted from Fig. 16.

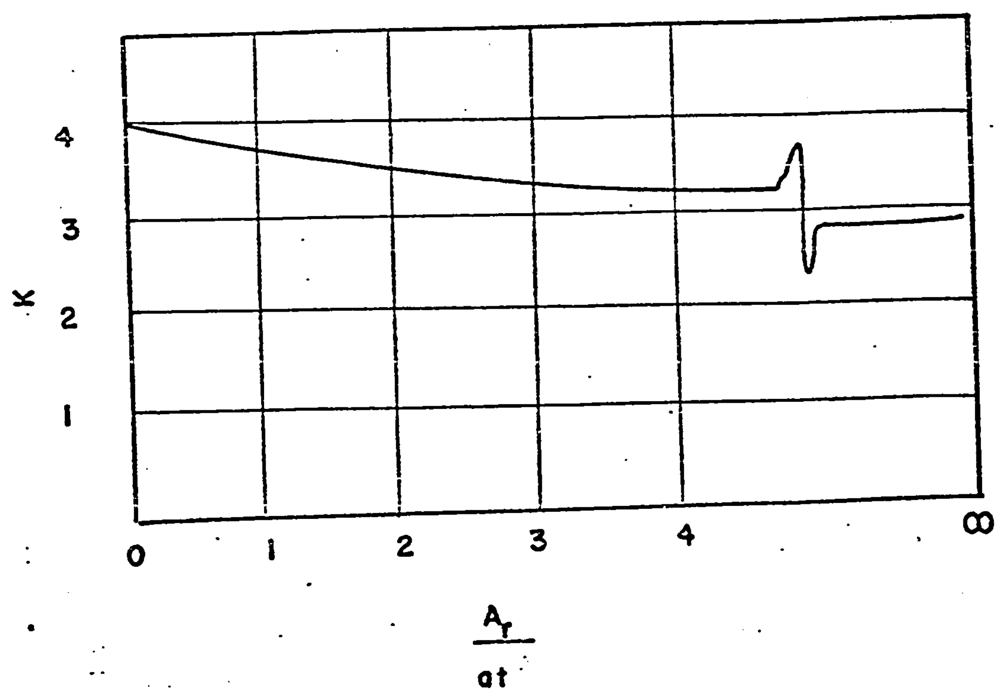


FIG. 15 - EFFECT OF ELASTIC RESTRAINT AT THE UNLOADED PLATE EDGES ON BUCKLING COEFFICIENT K

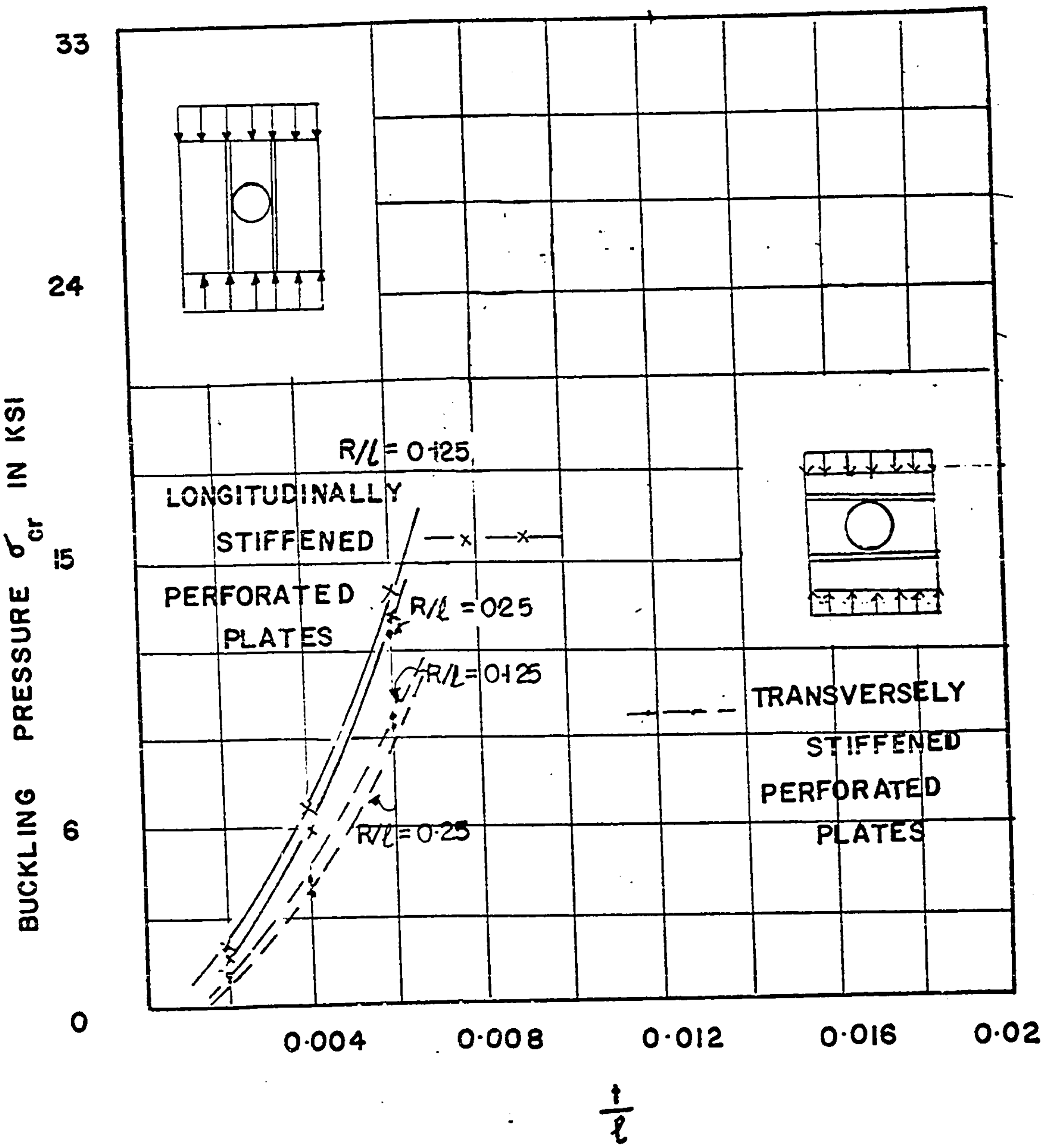


FIG. 16 - BUCKLING OF SIMPLY SUPPORTED SQUARE PLATE REINFORCED BY TWO SYMMETRIC STIFFENERS AND HAVING A PLAIN CIRCULAR PERFORATION

8

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