THE USE OF SIGNAL FLOW GRAPHS IN THE SYNTHESIS OF OPERATIONAL AMPLIFIER NETWORKS

Christian Samir Gargour

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ABSTRACT

This report is an attempt to apply the Signal Flow Graph technique to the synthesis of active networks using finite gain operational amplifiers.

A few Signal Flow Graphs for some basic and simple finite gain operational amplifier networks have been deduced and used as basic blocks to synthesize second order transfer functions with real or complex poles and zeros. The Q_p and ω_p sensitivities with respect to the different elements of one of the realization have been computed and it has been shown that they are low.

A method for the computation of the transfer function sensitivity with respect to the gain of the amplifier directly from the Signal Flow Graph and applicable only to some of the basic networks considered has been proposed.

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CHAPTER I

Introduction

1-1 General:-

The Signal Flow Graph technique, first introduced by Mason in 1953 has been mainly used as an analysis tool. It provides a method of solving linear algebraic equations and also is a convenient method of representation and simulation of control systems. However this technique has not found much use in the synthesis of networks.

Since a few years, synthesis using active elements has gained considerable importance. The continual development of the semi-conductor technology has permitted the production of cheap, reliable and small active components making the area of active network synthesis one of the most promising branches of circuit theory.

This report is an attempt to apply the Signal Flow Graph technique to the synthesis of active networks using finite gain operational amplifiers. The report is by no means a complete solution of the problem. The author tries only to present a few solutions to some aspects of the problem and has suggested some ways in which investigations could prove to be fruitful.

1-2 Scope of the Report:-

The text is divided into seven chapters. Chapter two provides a short survey of Signal Flow Graphs and Flow Graphs including sensitivity analysis using Signal Flow Graphs.

Chapter three is a short survey of the operational amplifier, including short discussions on the idealized device, the practical one, stability considerations, and some compensation techniques.

Chapter four deals with the summary of the realization of transfer functions by Signal Flow Graph technique, using infinite gain operational amplifiers.

Chapters five and six contain the author's contribution to the synthesis of transfer functions by Signal Flow Graph technique, using finite gain operational amplifiers. Chapter five contains a few Signal Flow Graphs for some basic and simple finite gain operational amplifier networks and these Signal Flow Graphs are used in chapter six as basic blocks for the synthesis of transfer functions with real or complex poles and zeros. Chapter six contains also a technique which is not general but can be used in several cases to compute the sensitivity of the transfer function with respect to the gain of the amplifier.

Chapter seven discusses some of the further possibilities for investigations.

CHAPTER II

A Short Survey of Signal Flow Graphs and Flow Graphs

2-1 Introduction:-

A Signal Flow Graph [1], as the name implies, is an oriented graph describing the flow of signals from one joint to another of a system and providing cause and effect relationships. Its applications are mainly in the analysis of numerous kinds of linear systems and networks with continuous or discrete signals.

In the literature, there exists a modification of the Signal Flow Graph SFG. called the Flow Graph (FG). The Flow Graph is essentially an alternative to pological representation of a set of linear algebraic equations. It does not depict any flow of signal. It may be simpler than the SFG for some complex systems. Even though the formation and interpretation of these two graphs are basically different, the two graphs are closely related and can be converted to each other easily.

2-2 Construction of a Signal Flow Graph:-

Before proceeding to give the rules for the construction of a S.F.G., it is essential to define the terms: nodes, branches, and transmittances.

Nodes:- They are used to represent the signals or variables of a given system. Every node is associated with a node variable, which represents the strength of the signal.

Branches:- A branch connecting two nodes is used to indicate the function dependence of one signal (variable) upon the other. Thus it forms the cause and effect relationship between the signals. The direction of flow of the signal is indicated by an arrow on the branch. This direction is always from the cause to the effect.

Transmittances:-

The transmittance of the branch represents the algebraic relation between the two variables. Consider the graph shown in figure 2-1-a which represents the two nodes x_1 and x_2 joined by the branch, with the transmittance t_{12}, oriented from x_1 to x_2.

x_1 and x_2 are the system variables. Hence the equation representing this graph is

$$x_2 = t_{12}\, x_1 \tag{2-1}$$

From equation (2-1) it can be seen that

$$x_1 = \left(\frac{1}{t_{12}}\right) x_2 \tag{2-2}$$

This equation is to be represented by the graph shown in figure 2-1-b. It is clear that the two graphs are different. In figure 2-1-a the signal flows from x_1 to x_2 while in figure 2-1-b the signal flows from x_2 to x_1.

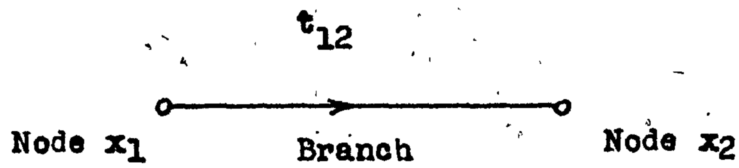
It is obvious that in the construction of a S F G the above should be kept in mind.

As an example consider the set of equations:-

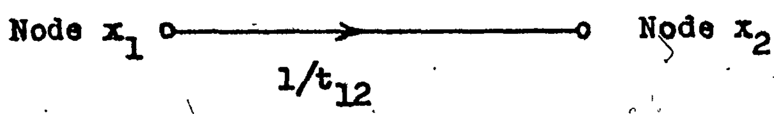
$$\begin{aligned} x_2 &= t_{12}\, x_2 + t_{32}\, x_3 \\ x_3 &= t_{23}\, x_2 + t_{43}\, x_4 \\ x_4 &= t_{24}\, x_2 + t_{34}\, x_3 + t_{44}\, x_4 \\ x_5 &= t_{25}\, x_2 + t_{45}\, x_4 \end{aligned} \tag{2-3}$$

Equations (2-3) are represented by the S.F.G. shown in figure 2-2.

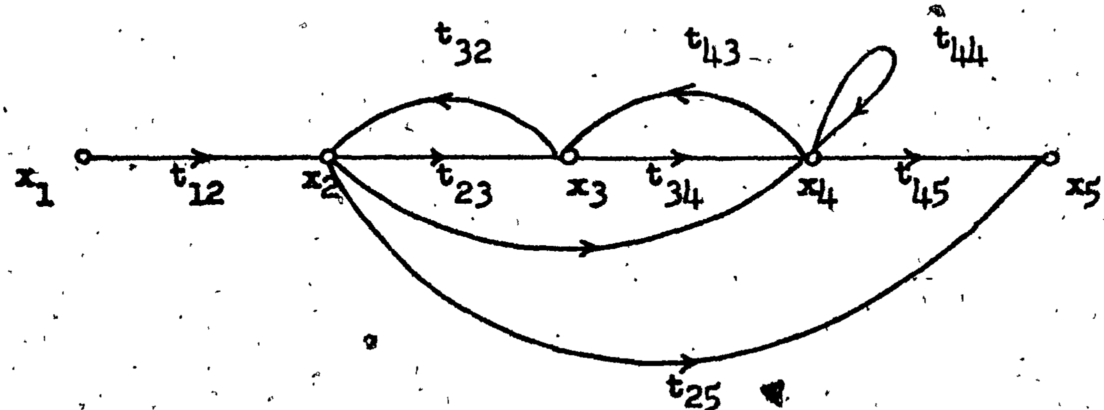
The foregoing discussion results in the following rules to be followed in the construction of S F G s -



<u>Figure 2-1-a</u>. A typical branch of a SFG



<u>Figure 2-1-b</u>.



<u>Figure 2-2</u>. An example of SFG

1) Nodes representing variables are arranged in any desired order, following a set of causes and effects through the system.

2) Branches originating from node x_k and terminating in node x_j represent the dependence of the variable x_j upon x_k but not the opposite.

3) Signals travel along branches only in the direction stipulated by the arrows.

4) A signal x_k travelling along a branch between nodes x_k and x_j is multiplied by the transmittance t_{kj} of the branch so that a signal $t_{kj}\ x_k$ appears at node x_j.

The S.F.G. is completed if necessary by adding either the input node or the output node or both

i) The input node, also known as the source, is that node which has only outgoing branches.

ii) The output node, also known as the sink, is that node which has only incoming branches.

Any non input node x_j can be made an output node simply by introducing a branch with unity gain from x_j to another node marked also x_j. This represents a superfluous equation $x_j = x_j$ and thus does not affect the system in any way.

In figure 2-3-a, it is shown how the node x_3 is converted into an output node.

A similar procedure is used to make an input node from any non output node. In figure 2-3-b it is shown how the node x_1 is converted into an input node.

2-3 Some basic Signal Flow Graph algebra:-

The following definitions are used in S.F.G. algebra

1) Path:- Any continuous, unidirectional succession of branches

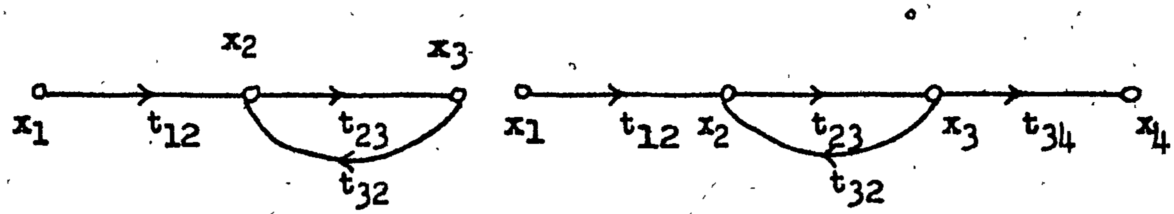


Figure 2-3-a. The transformation of a non-input node into an output node.

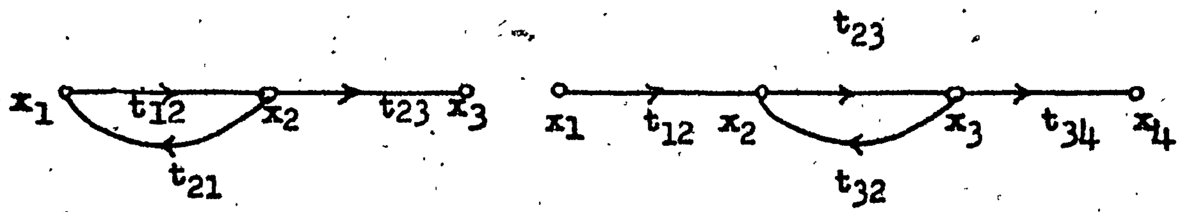


Figure 2-3-b. The transformation of a non-output node into an input node.

traversed in the indicated direction.

2) Forward Path:- A Path from the input node to the output node along which no node is encountered more than once.

3) Loop:- A path which originates and ends on the same node along which no node is encountered more than once.

4) Path Gain:- The product of the transmittances of the branches encountered in traversing the path.

5) Loop Gain:- The product of the transmittances of the branches encountered in traversing the loop.

The following are used in the reduction of S F G s

1) The summing node:-

The value of the variable represented by a node is equal to the sum of the signals entering the node. From figure 2-4-a, it is seen that

$$x_j = \sum_{k=1}^{n} t_{kj}\, x_k$$

2) The transmitting node:-

The value of the variable represented by any node is transmitted on all branches starting from the node. From figure 2-4-b, it is seen that

$$x_1 = t_{k1}\, x_k,\quad x_2 = t_{k2}\, x_k,\quad \ldots\ldots,\quad x_n = t_{kn}\, x_k$$

3) Series connection of branches:-

From figure 2-4-c, it is seen that the branches with gains $t_{12}, t_{23}, \ldots,$ $t_{(n-1)n}$ can be replaced by a single branch with gain equal to

$$t_{12}\, t_{23}\, \ldots\, t_{(n-1)n} = T_{1n}$$

that is

$$x_n = t_{12}\, t_{23}\, \ldots\, t_{(n-1)n}\, x_1 = T_{1n}\, x_1$$

4) Parallel connection of branches:-

From figure 2-4-d, it is seen that the branches with gains $t_1, t_2, t_3, \ldots,$

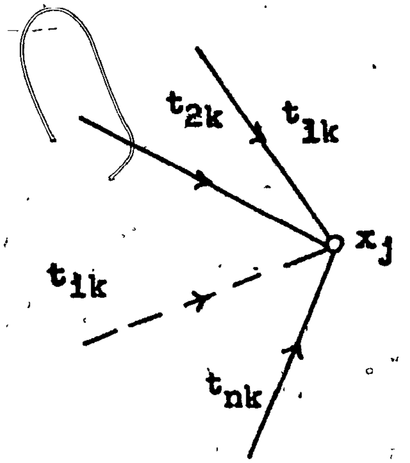


Figure 2-4-a. The summing node.

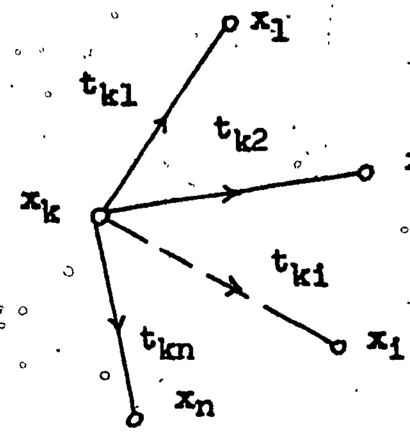


Figure 2-4-b. The transmitting node.

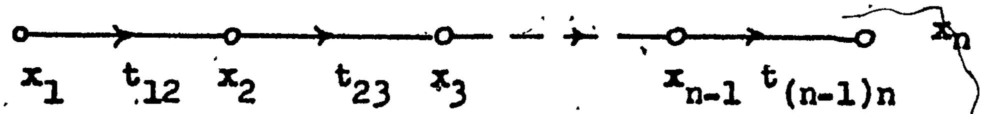


Figure 2-4-c. Series connection of branches.

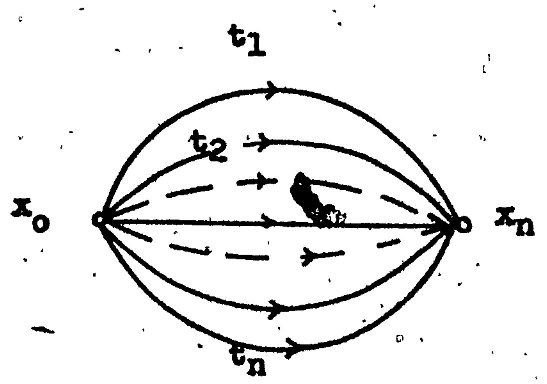


Figure 2-4-d. Parallel connection of branches.

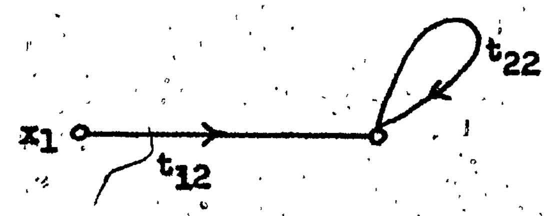


Figure 2-4-e. The self loop.

t_n can be replaced by a single branch with gain equal to

$$T_n = t_1+t_2+t_3+ \ldots +t_n \quad \text{that is}$$

$$x_n = (t_1+t_2+t_3+ \ldots +t_n)x_o$$

5) <u>The Self loop</u>:-

Referring to figure 2-4-e we see that the transmittance between x_2 and x_1 will be

$$x_2 + \frac{t_{12}}{1-t_{22}} x_1$$

<u>2-4 Mason's Gain Formula for the S.F.G.</u> :-

To determine the transmittance between the input node and the output node it is not essential to adopt the different steps of the reduction procedure. Instead we can use Mason's gain formula given by:-

$$M = \Sigma \frac{M_k \Delta_k}{\Delta} \qquad (2-4)$$

where:-

M_k = Gain product of the kth forward path.

Δ = 1 - [Sum of all the individual loop gains]

+ [Sum of the gain products of all possible combinations of two non touching loops]

- [Sum of the gain products of all possible combinations of 3 non touching loops]

+

-

Δ_k = The value of Δ for that part of the graph not touching the kth forward path.

<u>2-5 Some remarks concerning the Signal Flow Graph of a network</u>:-

1) The S F G of a system is not unique. We can draw many different

S F G s for the same network depending on how the network equations are written. All these different S F G are equally valid.

2) Loop and node equations, in general, are not suitable for S F G studies because they do not always follow the cause to effect sequence. State equations are much more suitable for this purpose. The "state transition signal flow graphs" may also be utilized in a very direct way for an analog simulation or for analytical solution of the network.

3) A S F G may usually be drawn directly from a set of network equations which relates branch currents and voltages.

2-6 Sensitivity analysis using Signal Flow Graphs:-

In this section, we shall show how S F G s can be used for sensitivity computation also [2].

Before proceeding with the computation, we shall give the definition and some expressions for the computations of sensitivity functions.

The sensitivity $S_k^{T(s)}$ of a function T(s,k) due to the variation of the parameter k is defined as

$$S_k^{T(s)} \triangleq \frac{d\ \ln(T(s))}{d\ \ln(k)} = \frac{dT(s,k)/T(s,k)}{dk/k} \tag{2-5}$$

Directly from the definition of $S_k^{T(s)}$, the following identities can be easily established.

$$S_{1/k}^{T(s)} = -S_k^{T(s)} \qquad S_{k_1}^{T(s)} = S_{k_2}^{T(s)} \cdot S_k^{k_2} \tag{2-6}$$

Also if

$$T(s,k) = \frac{N_1(s) + kN_2(s)}{D_1(s) + kD_2(s)} = \frac{N(s,k)}{D(s,k)} \tag{2-7}$$

it is easily shown that:-

$$S_k^{T(s)} = k\left[\frac{N_2(s)}{N(s,k)} - \frac{D_2(s)}{D(s,k)}\right] \tag{2-8}$$

If we are interested in $S_k^{T(j\omega)}$, then

$$S_k^{T(j\omega)} = \frac{D_1(j\omega)}{D(j\omega,k)} - \frac{N_1(j\omega)}{N(j\omega,k)} \qquad (2-9)$$

from which the gain sensitivity $S_k^{\alpha(\omega)}$ and the phase sensitivity $S_k^{\beta(\omega)}$ can be expressed as

$$S_k^{\alpha(\omega)} = \text{Re}\left[\frac{D_1(j\omega)}{D(j\omega,k)} - \frac{N_1(j\omega)}{N(j\omega,k)}\right] \qquad (2-10-a)$$

and

$$S_k^{\beta(\omega)} = \text{Im}\left[\frac{D_1(j\omega)}{D(j\omega,k)} - \frac{N_1(j\omega)}{N(j\omega,k)}\right] \qquad (2-10-b)$$

We are going now to consider some useful concept associated with the notion of sensitivity. We shall consider only the variation of a single parameter.

2-6-1 The signal flow graph representation:-

Let $T(s,k)$ be the network transfer function considered where k is the network parameter of interest.

Then equation (2-7) can be rewritten as:-

$$T(s,k) = \frac{T_o(s) + k\,A(s)B(s)}{1 - k\,C(s)} \qquad (2-11)$$

Where $T_o(s) = T(s,0) = \frac{N_1(s)}{D_1(s)}$

$C(s) = D_2(s)$

$C(s) = \frac{}{D_1(s)}$

and $A(s)\,B(s) = \frac{N_2(s)D_1(s) - N_1(s)D_2(s)}{D_1^2(s)}$

The S F G of T(s) is represented in figure 2-5.

2-6-2 The Return Difference $F_k(s)$:-

This is defined as:-

$$F_k(s) \triangleq 1 - [-kC(s)] = 1 + kC(s) \qquad (2-12)$$

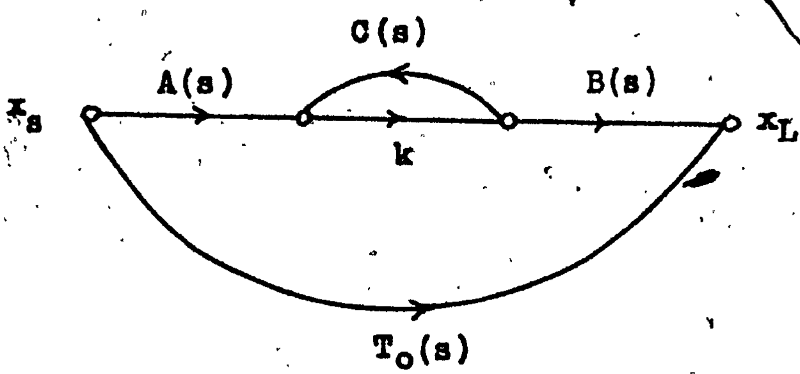


Figure 2-5. SFG of a single loop feedback system.

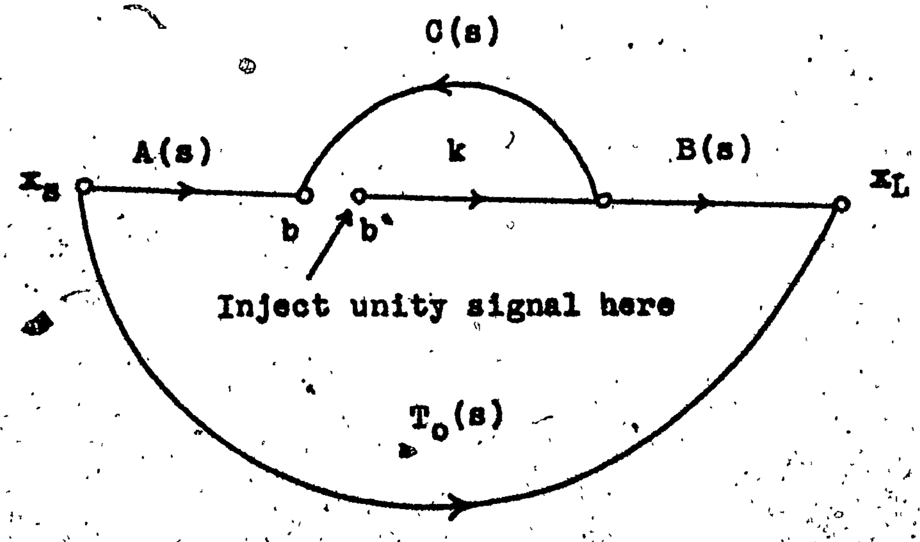


Figure 2-6. Physical interpretation of the concept of return difference.

which becomes:-

$$F_k(s) = 1 + k\frac{D_2(s)}{D_1(s)} = \frac{D_1(s) + kD_2(2)}{D_1(s)} = \frac{D(s,k)}{D_1(s)}$$

Hence $$F_k(s) = \frac{D(s,k)}{D_1(s)} \qquad (2\text{-}13)$$

The physical interpretation of $F_k(s)$ can be given as follows:

i) Let $x_s = 0$

ii) Split the node (b) into (b) and (b´)

iii) Inject a unity signal at b´

Then $F_k(s) = 1$ - (The returned signal) and this is shown in figure 2-6.

2-6-3 The Null Return Difference $F_k^o(s)$:-

This is defined as the Return Difference obtained under the condition of zero output $(x_L = 0)$.

If we inject a unity signal at (b´) we get at (b) a signal $kC(s) + x_sA(s)$

now since $x_L = T_o(s)\ x_s + kB(s)$

and $x_L = 0$ we have $x_s = -\frac{kB(s)}{T_o(s)}$

and the signal at b will be:-

$$k\ C(s) - \frac{kA(s)B(s)}{T_o(s)}$$

Therefore,

$$F_k^o(s) = 1 - k\ C(s) + \frac{kA(s)B(s)}{T_o(s)} \qquad (2\text{-}14)$$

which may also be written as:-

$$F_k^o(s) = 1 + k\frac{N_2(s)}{N_1(s)} = \frac{N(s,k)}{N_1(s)} \qquad (2\text{-}15)$$

2-6-4 Relation between sensitivity and return difference and nul return difference:-

From (2-11) we can get:-

$$\frac{dT}{dk} = \frac{A(s)B(s)}{[Y-kC(s)]^2}$$

and hence

$$S_k^{T(s)} = \frac{k}{T(s,k)} \frac{dT(s,k)}{dk} = \frac{k}{T(s,k)} \frac{A(s)B(s)}{[1-kC(s)]^2}$$

or

$$S_k^{T(s)} = \frac{1}{F_k(s)} [1 - \frac{T_o(s)}{T(s,k)}] \qquad (2\text{-}16)$$

but $kA(s)B(s) = F_k(s)[T(s,k) - T_o(s)]$

which means

$$F_k^o(s) = F_k(s) + \frac{F_k(s)[T(s,k) - T_o(s)]}{T_o(s)}$$

$$F_k^o(s) = F_k(s) \frac{T(s,k)}{T_o(s)}$$

from which we get

$$S_k^{T(s)} = \frac{1}{F_k(s)} - \frac{1}{F_k^o(s)} \qquad (2\text{-}17)$$

2-7 Coates Flow Graphs [Flow Graphs] [1]:-

For the sake of completeness (even though not used in this report anywhere else), Flow graphs are briefly discussed.

This was defined by Coates and is also a collection of branches and nodes, but the interconnections between the nodes do not follow the principle of cause and effect. It is defined with its topological structure, which depends only on the set of algebraic equations, thus avoiding the difficulty inherent to S F G which consists in relating causes and effects. Unless otherwise stated, all the definitions of the S F G s are also applicable to flow graphs.

2-7-1 Construction of Flow Graphs:-

Consider the set of equations

$$\sum_{j=1}^{m} a_{ij} x_j = 0 \qquad (j = 1,2,\ldots\ldots n)$$

If $m \geq n$ these equations show that there are (m-n) independent sources.

The Flow Graph is constructed using the following rules:-

i) Draw nodes $x_1, x_2, \ldots . x_n$ in a convenient manner.

ii) Assign the variable x_p to the ith equation.

iii) Draw the branches with transmittances a_{ij} from nodes x_j $(j=1,2,3, \ldots . n)$ to the node x_p.

iv) Continue the process for all such equations.

The flow graph is obtained by superposing the flow graphs of the equations of the type

$$\Sigma\, a_{ij} x_j = 0 \quad \text{for } i = 1,2,\ldots.,n$$

As an example consider the set of homogeneous equations:-

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0 \end{array}\right\} \qquad (2\text{-}18)$$

We arbitrarily assign x_1 to the first equation x_2 to the second and x_3 to the third. We write the equations and the associated variables in the following way for identification purposes.

$$\begin{array}{c|l} x_1 & a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0 \\ x_2 & a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0 \\ x_3 & a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0 \end{array}$$

and the flow graph is obtained as shown in figure 2-7.

It is obvious that the flow graph representation of a given set is not unique since a different choice of reference variables leads to a completely different flow graph.

<u>2-7-2 The Gain Formula for Flow Graphs:-</u>

The gain or transmittance between an input node and an output node is given by:-

$$M = \sum_k \frac{M_k \Delta_{ck}}{\Delta_c} \qquad (2\text{-}19)$$

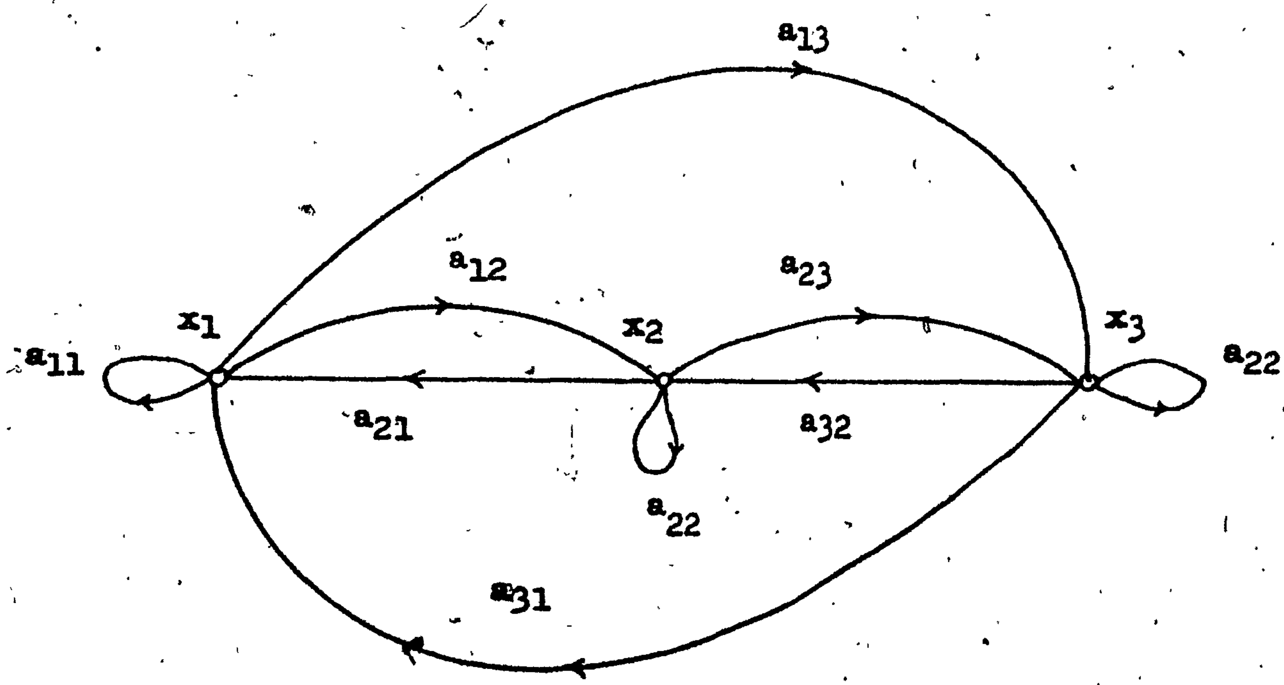


Figure 2-7, An example of Flow Graph.

where:-

M_k = Gain of the kth forward path.

$\Delta_c = \sum_m (-1)^j$ product of the loop gains found in the mth connection graph.

j = number of loops in the mth connection graph.

m = number of all possible connection graphs in the flow graph.

If the flow graph does not have any loop

$$\Delta_c = 1$$

Δ_{ck} = Value of Δ_c for that part of the graph which is not in touch with the kth forward path. If the kth forward path does not have any non touching part $\Delta_{ck} = 1$.

2-8 Conversion of a Signal Flow Graph into a Flow Graph and vice versa:-

Consider the set of equations

$$\sum_{j=1}^{n} a_{ij} x_j = 0 \quad (i = 1, 2, \ldots .n)$$

which are proper for flow graph formulation. Suppose we add a non input variable x_i to both sides of the equations we get:-

$$x_i = x_i + \sum_{j=1}^{n} a_{ij} x_j \quad (i = 1,2,\ldots .n)$$

These are now in proper form for drawing signal flow graphs, if we consider x_i as the effect. Adding x_i to both sides creates a self loop with unity gain at the node x_i on flow graph. That means that a flow graph can be converted into a S F G simply by adding a self loop of unity gain at every non input node of the flow graph. If there is already a self loop at any node of the flow graph, then the gain of the self loop will be increased by unity.

It is readily seen that, by reversing the process, any S F G may be converted into a flow graph by adding self loops of transmittances (-1) to all the non input nodes of the S F G.

CHAPTER III

The Operational Amplifier

(A Short Survey)

3-1 Introduction:-

The operational amplifier [2] is a voltage-controlled voltage-source. Historically, it found use as a basic element in analog computers meant to solve differential equations. However, as time elapsed, it found several other uses in different areas such as, network design, communication systems, controls, etc. It is an extremely versatile commercially manufactured active device.

3-2 The idealized device:-

Theoretically it is a very high gain V.V.T. with two inputs and one output. It has the following properties:

a) Very high voltage gain

b) Infinite (or very high) input impedance

c) Zero (or very low) output impedance

d) Output voltage has the same polarity as one of the inputs and opposite polarity with respect to the other input.

As a result of these properties, it follows that:-

i) The input exitation is not affected by the power drawn by the amplifier.

ii) The amplifier gain is unaffected by load variations.

e) The ideal amplifier is required to have "zero offset" i.e. the output voltage must be zero when the input voltage is zero. The offset of an amplifier is a zero frequency error and should be minimized for numerous reasons, the important ones being the following [3]:-

1) The use of an operational amplifier is limited to signal levels much greater than the offset voltage.

2) Comparator applications require that the output voltage be zero when the two input signals are equal and in phase.

3) In a direct coupled cascade structure using operational amplifiers, the offset limits the total maximum gain.

f) Finally the ideal operational amplifier must be stable. Since we are dealing with a very high gain device, stability is one of the major problems which have to be solved and will be discussed later in this chapter.

The usual representation of the operational amplifier and its corresponding equivalent circuit are shown in figure 3-1.

Analysis shows that

$$V_o = \mu(V_2 - V_1) = -\mu V_i \qquad (3\text{-}1)$$

Zero offset means

$$V_o \to 0 \quad \text{when } (V_2 - V_1) \to 0 \qquad (3\text{-}1a)$$

It is to be noted that the output voltage has the same polarity as the input voltage to the terminal marked (+), while its polarity is opposite to the input voltage to the terminal marked (-). The input terminal marked (+) is called the "non inverting terminal". Often, for many applications the operational amplifier is used "Single-ended" with the non-inverting terminal grounded. The usual representation and the equivalent circuit are shown in figure 3-2.

We have considered, until now, the ideal operational amplifier. However it is usually necessary, to associate it with some passive networks to control its gain and fix it to the desired value as well as to improve its stability and its general performances and this is discussed in the following section.

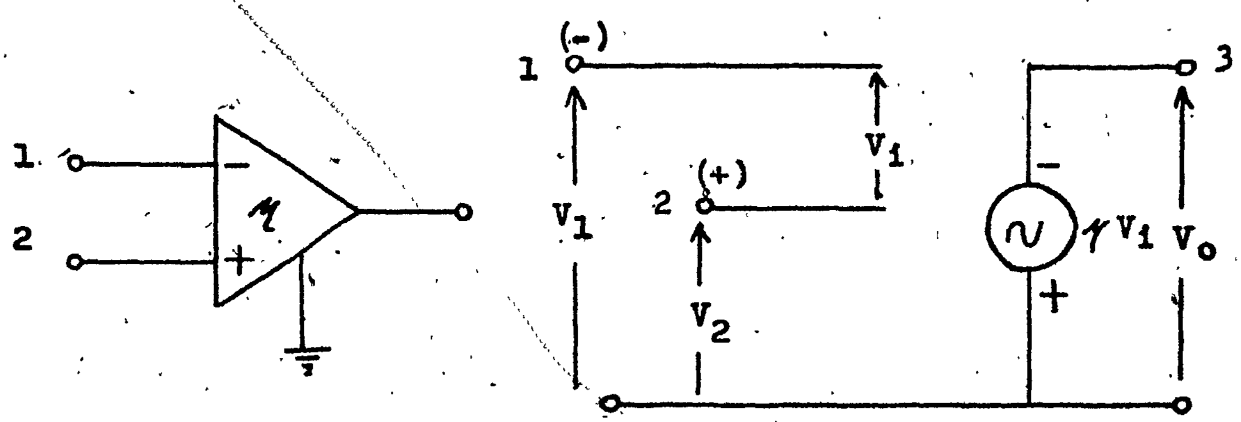


Figure 3-1. Circuit symbol and controlled-source representation of the ideal differential input operational amplifier.

3-3 The effect of negative feedback on the idealized device (Inverting and non inverting modes):-

For linear applications, operational amplifiers are usually used with negative feedback. Two basic arrangements are shown in figure 3-3.

1) For arrangement (a)

$$V_i = \frac{V_o}{\mu}, \quad \frac{1}{R_a} = G_a, \quad \frac{1}{R_b} = G_b$$

and $(V_1 + \frac{V_o}{\mu})\ G_a = -(V_o + \frac{V_o}{\mu})\ G_b$ (3-2)

Hence,

$$\frac{V_o}{V_1} = -\frac{\mu G_a}{(\mu+1)\,G_b + G_a} \quad (3\text{-}2a)$$

Now if we let $\mu \to \infty$,

$$\frac{V_o}{V_1} = -\frac{G_a}{G_b} \quad (3\text{-}3)$$

Because of the negative sign, this arrangement is called the "Inverting mode".

It is to be noted that:-

$$\frac{V_1 - V_i}{R_a} = \frac{V_i - V_o}{R_b} = \frac{V_i + \mu V_i}{R_b} \quad (3\text{-}4)$$

Hence,

$$V_i = \frac{V_1}{1 + \frac{R_a}{R_b}(1+\mu)} \quad (3\text{-}4a)$$

Thus $V_i \to 0$ as $\mu \to \infty$.

Physically a finite V_i will cause a large output voltage V_o. V_o, being of opposite polarity to V_i, will tend to increase until V_i becomes infinitesimal.

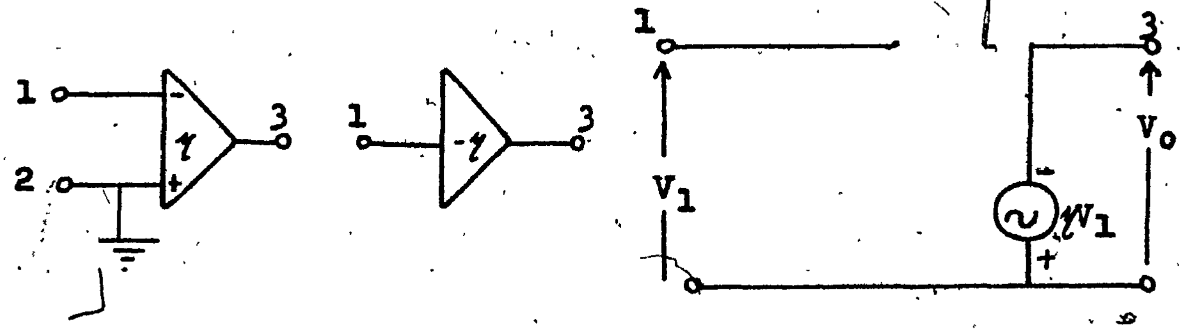


Figure 3-2. The single-ended operational amplifier.

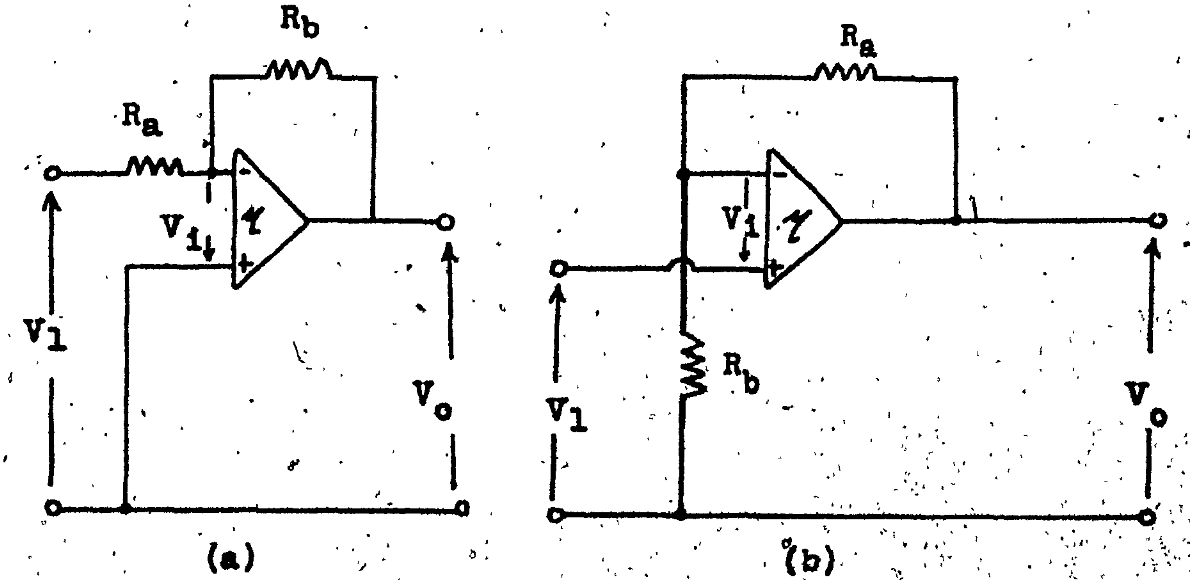


Figure 3-3. Inverting-type and non-inverting-type VVT realizations using the operational amplifier.

2) For arrangement (b)

If $\frac{1}{R_a} = G_a$ and $\frac{1}{R_b} = G_b$

$$G_a(V_1 - \frac{V_o}{\mu}) = (V_o - V_1 + \frac{V_o}{\mu})\ G_b \qquad (3-5)$$

Rearranging,

$$\frac{V_o}{V_1} = \frac{\mu(G_a + G_b)}{(\mu+1)\ G_b + G_a} \qquad (3-5a)$$

If we let $\mu \rightarrow \infty$

$$\frac{V_o}{V_i} = \frac{G_a + G_b}{G_a} \qquad (3-6)$$

Because V_o and V_1 are with the same polarity this arrangement is called the "non-inverting mode".

For both arrangements, when terminal 2 is grounded, terminal 1 behaves as if it too were grounded and is said to be at "virtual ground".

3-4. Characteristics of a practical Operational Amplifier:-

In practice, the operational amplifier is a non-ideal device. It is characterized by a frequency-dependent voltage whose magnitude starts from a very high value at zero frequency (generally in the range of 80 to 120 db) and then monotonically decreases for higher frequencies. Likewise, the phase of the voltage gain is a monotonically decreasing function starting from zero degrees. Demagnitude and phase responses of a typical amplifier are shown in figure 3-4.

A convenient representation of the magnitude response is the straight-line approximation known as the Bode plot. The Bode plot of the magnitude function of a typical amplifier is shown in dotted lines in figure 3-4, and its gain may be expressed as

$$\mu(s) = \frac{\mu_o}{(s+\omega_1)(s+\omega_2)(s+\omega_3)} \qquad (3-7)$$

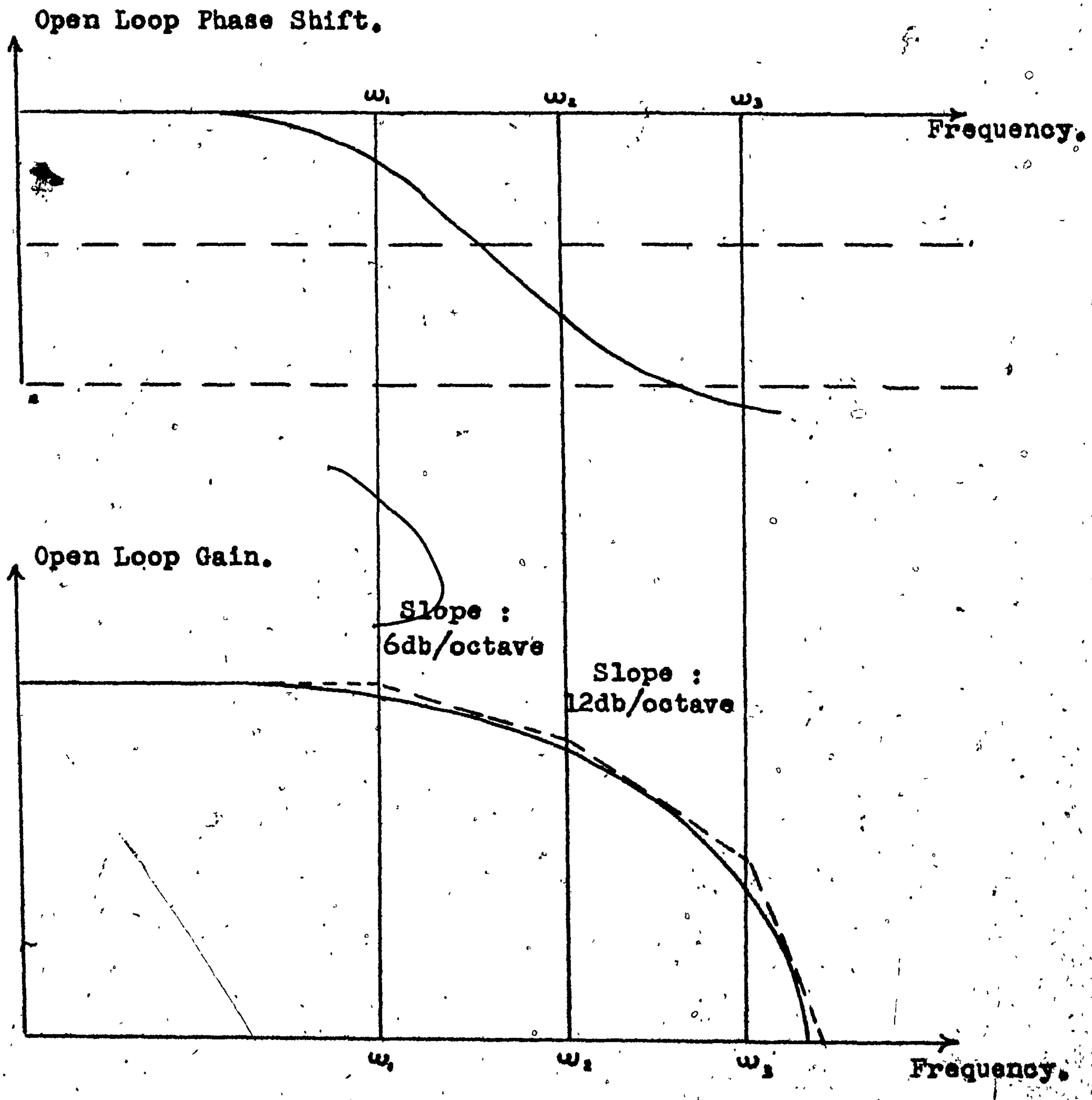


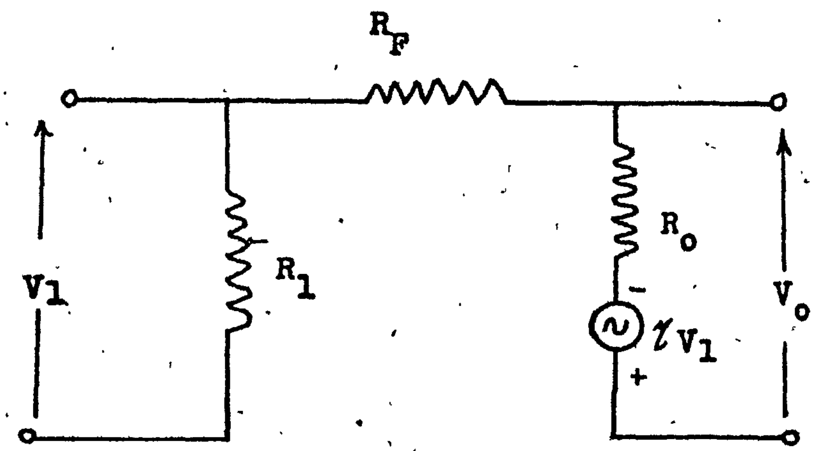
Figure 3-4. The frequency response of a practical operational ampl.

For the frequency range of interest (for this case it is the frequency interval from zero to unity gain crossover frequency), we observe from figure 3-4 that the approximated magnitude of the voltage gain rolls off at the first break frequency ω_1 with a slope of 6dB/octave, then at the second break frequency ω_2 the slope increases to 12dB/octave and after the third break frequency the magnitude rolls at 18dB/octave. A 90° phase occurs between ω_1 and ω_2, a 180° phase shift is somewhere between ω_2 and ω_3, and so on. The Bode plot gives a rough estimate of phase response.

Most of the practical amplifiers have a fairly large bandwidth. Typically the unity gain crossover frequency for an uncompensated integrated operational amplifier is 10MHZ or more.

The input impedance and output impedance are finite and non zero. Typical values are 100KΩ and 100Ω respectively. The controlled-source representation of a non-ideal operational amplifier is shown in figure 3-5. The feedback impedance R_F in most cases is very large and can be neglected. In addition to the above characteristics, a practical operational amplifier has maximum limits on the input and output signals beyond which the input-output relationships become non-linear. One particular limit is the "common mode voltage limit" which is the maximum peak input voltage that can be applied without driving the transistors into saturation.

Even though the figures cited above may appear a little discouraging at first glance, the practical operational amplifiers in general, yield satisfactory results for most purposes, because they are usually used (for linear applications) with negative feedback which improves the actual performance.



<u>Figure 3-5</u>. The controlled source model of a non-ideal operational amplifier.

3-5 Two basic negative feedback arrangements using a practical operational amplifier:-

We shall consider the two basic configurations used in figure 3-3 with the ideal operational amplifier substituted by a non-ideal one.

1) Configuration shown in figure 3-3-a:

Replacing the amplifier by its non ideal model, we get figure 3-6, assuming R_F to be infinite. If R_F is comparable to R_b, it has to be connected as shown with dotted lines.

Analysis gives

$$\frac{V_o}{V_1} = -\frac{G_a}{G_b}\,\frac{1}{1+\frac{(G_a+G_b+G_i)(G_o+G_b)}{G_b(\mu G_o-G_b)}} \qquad (3\text{-}8)$$

$$V_i = -\frac{G_o+G_b}{\mu G_o - G_b}\,V_o \qquad (3\text{-}9)$$

$$Z_o = \frac{G_a+G_b+G_i}{(G_b+G_o)(G_a+G_i)+(1+\mu)G_bG_o} \qquad (3\text{-}10)$$

where $\frac{1}{R_a} = G_a$, $\frac{1}{R_b} = G_b$, $\frac{1}{R_i} = G_i$, $\frac{1}{R_o} = G_o$

If we define

$$A = \frac{\mu_o G_o - G_b}{G_b + G_o} \qquad (3\text{-}11a)$$

$$\beta = \frac{G_b}{G_a + G_b + G_i} \qquad (3\text{-}11b)$$

We observe that A is the voltage V_2 when 1 volt is applied to the input of the operational amplifier with V_1 set equal to zero, while β, called the feedback factor is the fraction of unity output voltage being fed back to the amplifier input.

Thus,

$$\frac{V_o}{V_1} = -\frac{G_a}{G_b}\,\frac{1}{1+\frac{1}{A\beta}} \qquad (3\text{-}12)$$

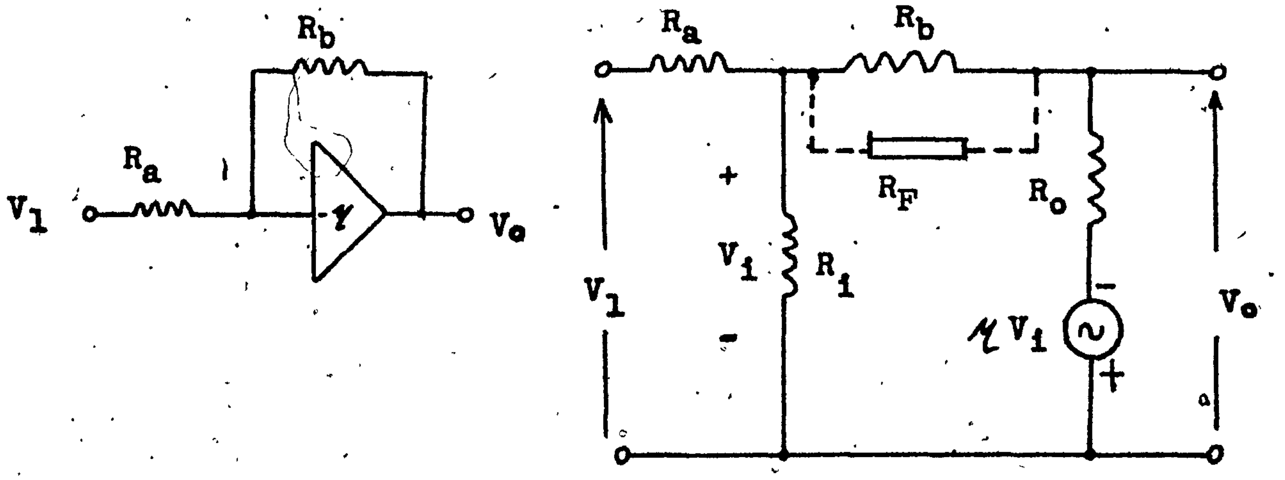


Figure 3-6. An inverting-type VVT realization using a non-ideal operational amplifier and its controlled-source representation.

$$V_i = \frac{1}{A} V_o \tag{3-13}$$

$$Z_o = \frac{1}{\frac{G_o + G_b}{1 + A\beta}} \approx \frac{R_o}{1 + A\beta} \tag{3-14}$$

The quantity $A\beta$ is referred to as the "loop gain". For an ideal operational amplifier:-

$$G_o \to \infty \quad G_i = 0 \quad G_F = 0$$

$$\mu \to \infty$$

$$A = \mu \to \infty \qquad \beta = \frac{G_b}{G_a + G_b} \tag{3-15}$$

Hence, as derived in a previous section

$$\frac{V_o}{V_1} = -\frac{G_a}{G_b} \qquad V_i = 0 \quad Z_o = 0 \tag{3-16}$$

We notice that the arrangement which has been described is in fact an inverting type V.V.T. realization.

2) Configuration used in figure 3-3-b.

Using the approximations (see figure 3-7),

R_o very small,

$R_i(R_a + R_b) > R_a R_b$,

$$\beta = \frac{R_i R_b}{R_i(R_a+R_b) + R_a R_b} \approx \frac{R_a}{R_a + R_b},$$

analysis yields

$$\frac{V_o}{V_1} \approx \frac{R_a + R_b}{R_a} \quad \frac{1}{1 + \frac{1}{\mu\beta}} \tag{3-17}$$

$$Z_{in} \approx R_i(1+\mu\beta) \tag{3-18}$$

$$Z_o \approx \frac{R_o}{1 + \mu\beta} \tag{3-19}$$

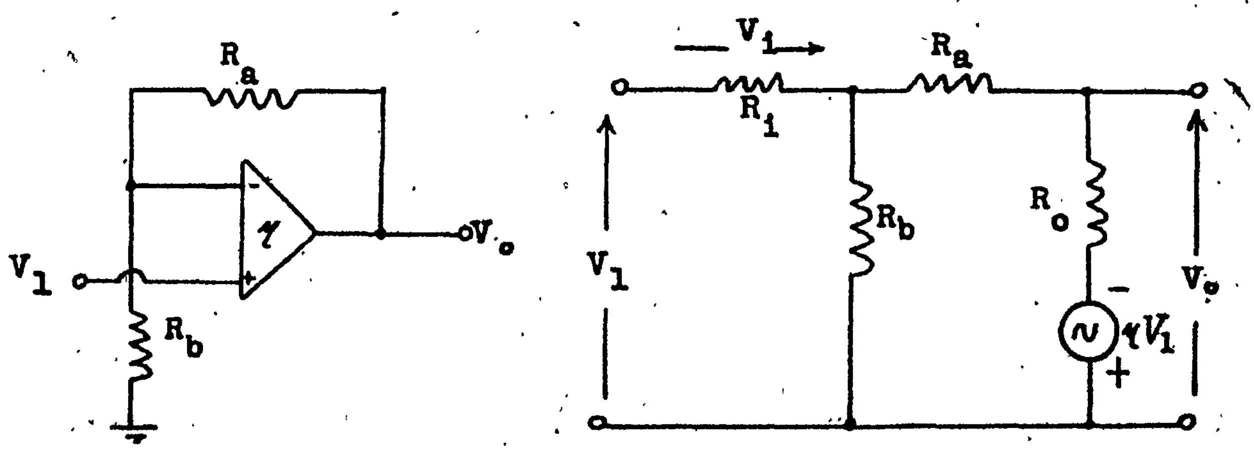


Figure 3-7. A non-inverting-type VVT realization using a non-ideal operational amplifier and its controlled-source representation.

For an ideal amplifier:-

$$\frac{V_o}{V_1} = \frac{R_a + R_b}{R_b} \qquad Z_{in} \rightarrow \infty$$

$$Z_o \rightarrow 0$$

The described circuit is in fact a non inverting V.V.T.

3-6 The Nullator-Norator representation of the Operational Amplifier:-

Consider the network shown in figure 3-8. Because of the nullator at the input, we have $V_1 = I_1 = 0$. On the other hand, at the output, we have V_2, and I_2 arbitrarily, due to the presence of the norator at the output. Thus,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad (3\text{-}20)$$

We shall show that the transmission matrix of the ideal operational amplifier is also a null matrix.

The transmission matrix of the non-ideal operational amplifier with the non-inverting terminal grounded can be shown to be

$$F = \begin{bmatrix} \dfrac{R_o+R_F}{R_o-\mu R_F} & \dfrac{R_oR_F}{R_o-\mu R_F} \\ \dfrac{(R_i+R_F)(R_o+R_F)}{R_iR_F(R_o-\mu R_F)} - \dfrac{1}{R_F} & \dfrac{R_o(R_i+R_F)}{R_i(R_o-\mu R_F)} \end{bmatrix} \qquad (3\text{-}21)$$

In the limiting case of an ideal operational amplifier,

$$R_i \rightarrow \infty \quad R_F \rightarrow \infty \quad R_o \rightarrow 0 \quad \mu \rightarrow \infty$$

and consequently the transmission matrix F becomes a null matrix and the two port represented in the figure represents an operational amplifier. Thus, conversion of nullator-norator circuits into operational amplifier circuits can be achieved by the equivalent circuit shown in figure 3-8.

3-7 Stability considerations in the operational amplifier:-

Introduction:-

One major problem always associated with the operational amplifier is

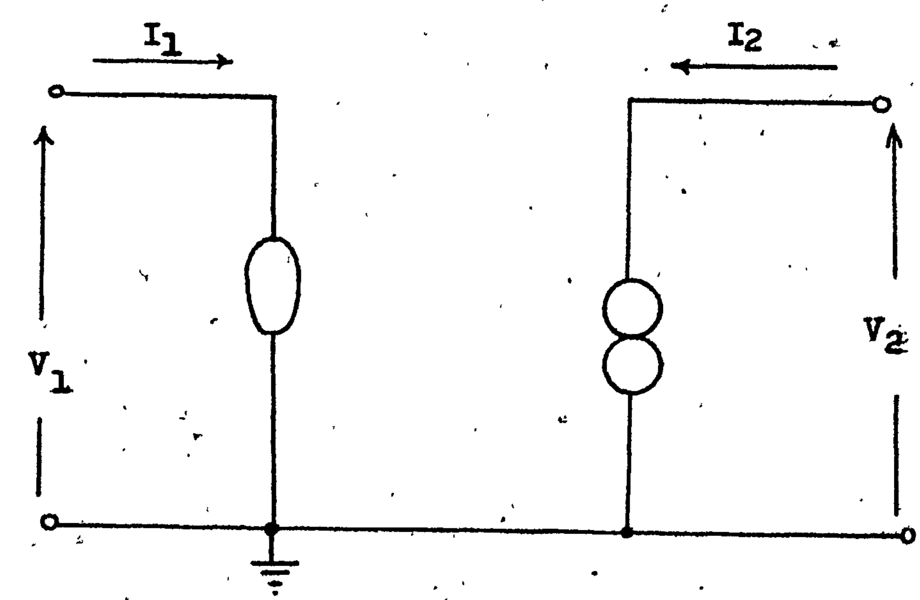


Figure 3-8 . The nullator norator model of an ideal operational amplifier.

its stability. Here, this means that the operational amplifier circuit shall not burst into oscillations. This problem arises due to the frequency dependence of the open loop gain μ of the operational amplifier (as discussed earlier, see figure 3-3-b). For such circuits, we can apply the Bode criterion.

The Bode Criterion:-

A single loop negative feedback operational amplifier can be represented by the S.F.G. of figure 3-9. The closed loop gain is given by

$$A_{cL} = \frac{V_o}{V_1} = -\frac{A_{oL}}{1 + \beta A_{oL}} \qquad (3\text{-}22)$$

where A_{oL} is the open loop gain of the amplifier, β is the feedback factor. Now if the loop gain

$$\beta A_{oL} >> 1 \qquad (3\text{-}23a)$$

which is usually true, we have

$$A_{cL} \approx -\frac{1}{\beta} \qquad (3\text{-}23b)$$

However, practically speaking, A_{oL}, β, are functions of the frequency and the above condition (3-23a) may not hold, thus if, at some frequency ω_o, the quantity $(1+\beta A_{oL})$ becomes equal to zero, A_{cL} will have a jω-axis pole at $\omega = \omega_o$ and the circuit oscillates at a frequency ω_o.

Hence if:

$$|\beta A_{oL}| = 1 \qquad (3\text{-}25)$$

$$\text{Arg}(\beta A_{oL}) = 180° \qquad (3\text{-}26)$$

for some $\omega = \omega_o$, the circuit will oscillate and the loop gain in dB will be zero. If one of the above conditions is not satisfied, the circuit will not oscillate. Specifically, when the loop gain becomes unity the phase shall not become 180°. Since we have observed previously that if the Bode plot rolls off at a slope of 12 dB/octave between the two fre-

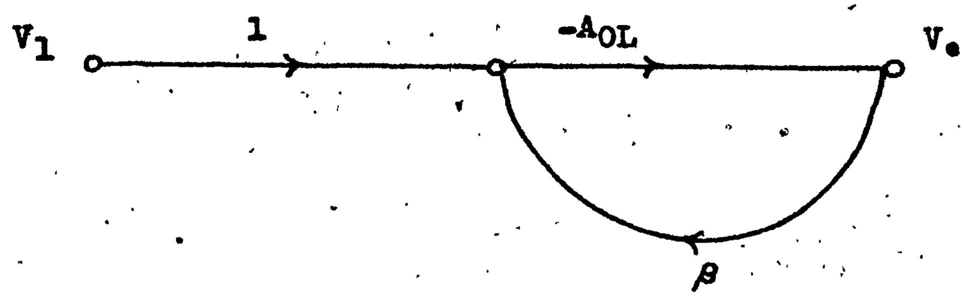


Figure 3-9. SFG representation of a single loop feedback amplifier

quencies ω_a and ω_b, the phase shift of 180° occurs somewhere between ω_a and ω_b. Hence we have:-

"The operational amplifier circuit will be strictly stable if at the zero decibel crossover frequency ω_o of the loop gain, the loop gain has a slope of 6 dB/octave."

3-8 Compensation:-

In an operational amplifier, the phase of the feedback must be controlled to ensure that the above desired gain frequency response is obtained to ensure stability. The design problems (instability and uncontrolled frequency response) can be solved by use of compensation techniques.

If the rate of the roll-off is 12 dB/octave or more, two methods can be used to decrease it. One method, called the internal compensation technique, is achieved by modifying the frequency response of the loop gain. The other method, which is called the external compensation technique, is achieved by modifying the feedback network β.

To illustrate the effect of compensations, let us consider the simple inverting amplifier represented in the figure 3-10. This network has been studied previously.

It has been already shown that

$$\frac{V_o}{V_1} = -\frac{G_a}{G_b}\;\frac{1}{1+\frac{1}{A\beta}}$$

where

$$A = \frac{G_o - G_b}{\mu G_b + G_o} \qquad \beta = \frac{G_b}{G_a + G_b + G_i}$$

We note that the loop gain is given by Aβ. A is approximately equal to μ the loop gain of the amplifier, if G_o is negligibly small and μ very large.

If $G_a \gg (G_b + G_i)$, the closed loop gain is approximately $\frac{1}{\beta}$ in magnitude

($\frac{1}{\beta} = \frac{G_a}{G_b}$). For most practical purposes, the assumptions made are valid and the closed-loop stability can be established from the Bode plots of μ and the closed loop gain $\frac{G_a}{G_b}$. We have represented in the graph shown in figure 3-11, the Bode plot of the frequency response of the open loop gain and also the Bode plot of a typical closed loop gain response (in solid lines). The critical point is ω_{o1} where the open loop gain is equal to the closed loop gain. The rate of roll-off of the two plots is 12 dB/octave in this case, and the result is that the amplifier is marginally stable. To make it strictly stable, we can either modify the closed loop gain i.e. the feedback factor (external compensation) or compensate the open loop gain (internal compensation).

Internal compensation:-

We are going to describe two possible methods for internal compensation.

a) The practical operational amplifier is provided with at least one terminal (such as x) where additional network elements can be connected for internal compensation purposes. [Figure 3-12]. If we connect a capacitor C_1 at point (x), the open loop gain becomes

$$\mu \text{ compensated} = \frac{\omega_c \mu(s)}{s + \omega_c} \tag{3-29}$$

where $$\omega_c = \frac{1}{R_1 C_1} \tag{3-30}$$

C_1 is choosen to make ω_c less than ω_1 the first break frequency of the open loop gain. The closed loop amplifier will be stable, if the zero dB loop gain crossover frequency (ω_{o2}) lies between ω_c and ω_1. This is illustrated in figure 3-11 where the dotted lines represent the response after compensation.

b) The closed loop bandwidth can be increased maintaining stable operation by connecting a series R-C tuned circuit to the compensating ter-

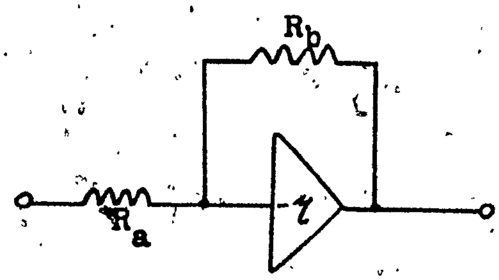


Figure 3-10.

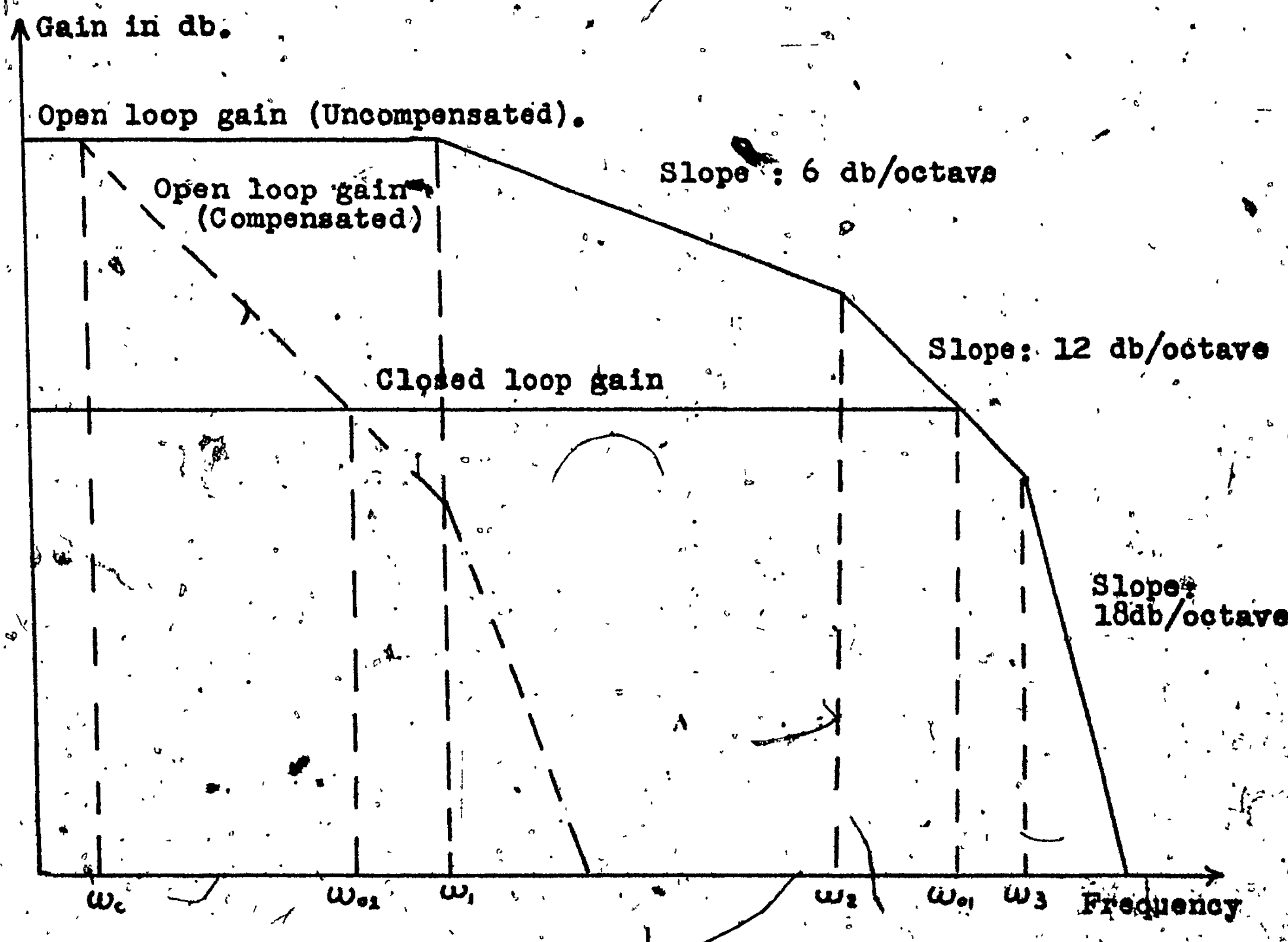


Figure 3-11. Illustration of the effect of the first method of internal compensation of a practical operational amplifier.

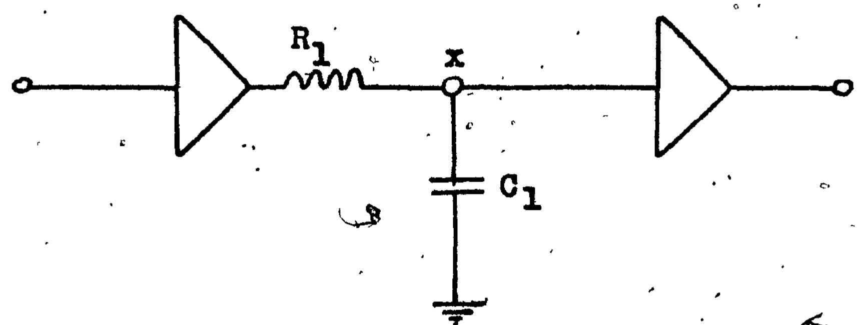


Figure 3-12. A possible arrangement for internal compensation.

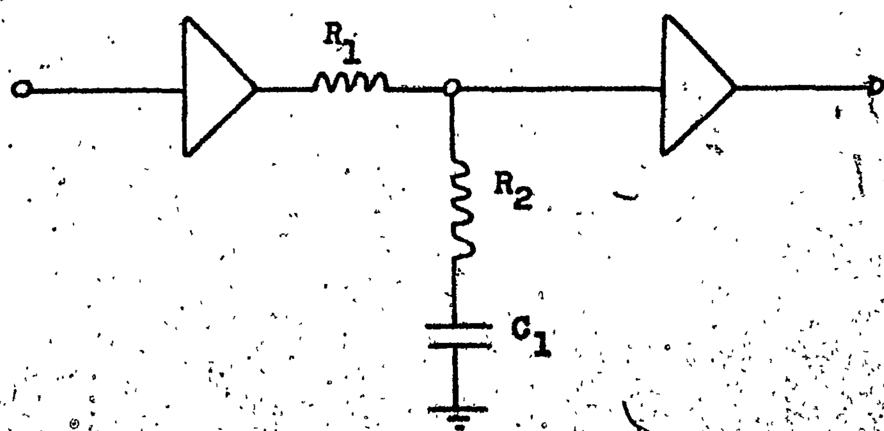


Figure 3-13. Another possible arrangement for internal compensation.

minal x as shown in figure 3-13. The compensated loop gain can be expressed as

$$\mu \text{ compensated} = \frac{R_2}{R_1 + R_2} \mu(s) \frac{s + \omega_{c2}}{s + \omega_{c1}} \qquad (3\text{-}31)$$

where

$$\omega_{c2} = \frac{1}{R_2 C_1} \qquad \omega_{c1} = \frac{1}{(R_1+R_2)C_1} \qquad (3\text{-}32)$$

If ω_{c2} is choosen to be equal to ω_1, the open-loop frequency response will have a slope of 6 dB/octave between ω_{c1} and ω_2. The new zero dB loop gain crossover frequency ω_o now can be made to lie between ω_{c1} and ω_2 by choosing proper ω_{c1} (figure 3-14). This will lead to stable operation, when feedback is applied.

3-8-2 External compensation:-

In order to make the amplifier completely stable it is also possible, to connect a small capacitor C_b in parallel with R_b [figure 3-15]. The new loop gain is:-

$$\frac{V_2}{V_1} = - \frac{G_a}{sC_b + G_b} = - \frac{G_b/C_b}{s + \omega_b)} \cdot \frac{G_a}{G_b} \qquad (3\text{-}33)$$

$$\text{where } \omega_b = \frac{1}{R_b C_b} \qquad (3\text{-}34)$$

If C_b is chosen to introduce a break in the closed-loop frequency response at ω_b, which is slightly smaller than ω_o, the rate of closure is reduced to 6 dB/octave leading to a stable closed-loop operation. An approximate value of C_b is $C_b \simeq \frac{R_a C_{in}}{R_b}$ where C_{in} is equal to the total stray capacitances at the input of the operational amplifier.

Another cause for oscillations is the presence of excessive capacitive load at the output. In this case, it is preferable to isolate the capacitance by placing a small resistor R_y in the feedback loop as indicated in figure 3-16. The value of R_y is usually in the order of the loop out-

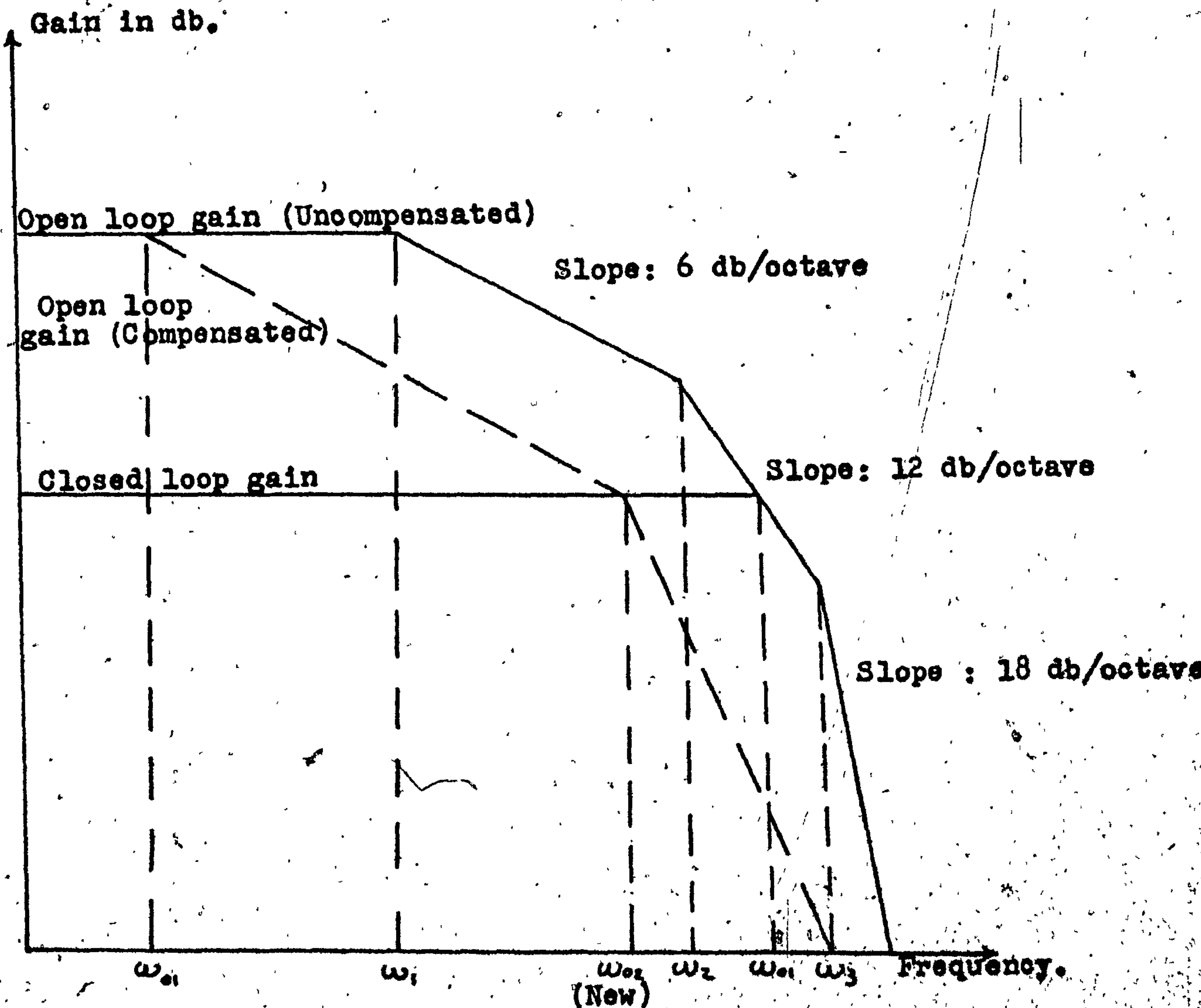


Figure 3-14. Illustration of the effect of the second method of internal compensation for a practical operational amplifier.

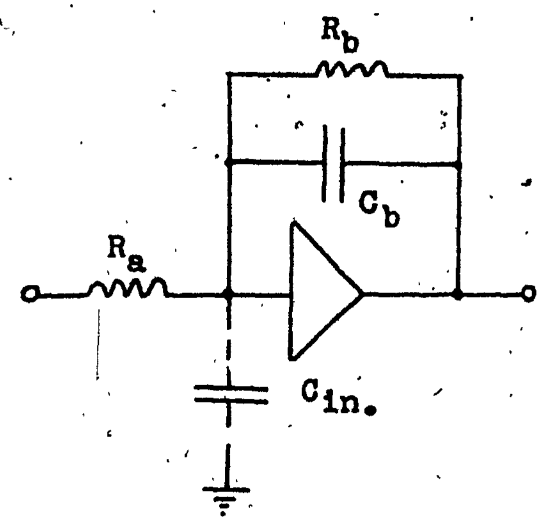


Figure 3-15. Externally compensated inverting type VVT.

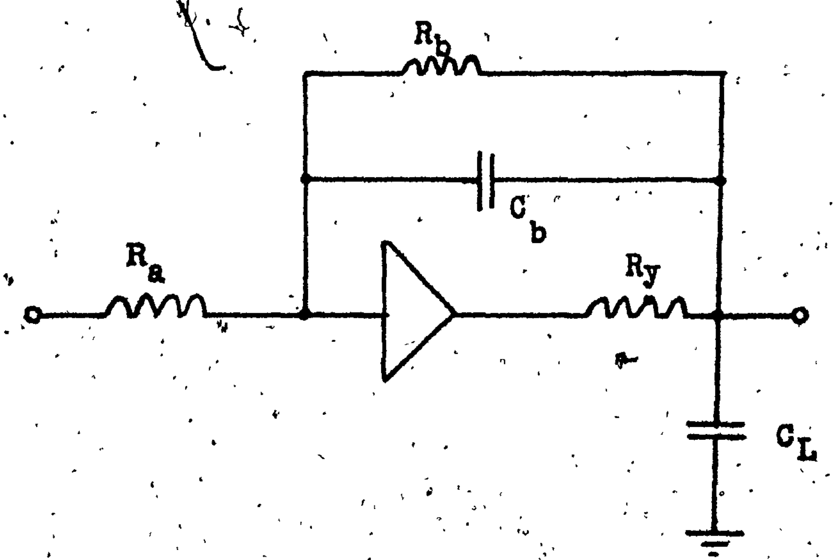


Figure 3-16. Extennally compensated inverting-type VVT driving a capacitive load.

put impedance of the amplifier.

3-9 Sensitivity considerations:-

Another problem encountered in operational amplifier network synthesis (as well as in any kind of network synthesis) is the sensitivity of the network function due to the variation of network parameters. The characteristics of passive and active components may vary because of a change in the external (environmental) and the internal conditions. Such variations cause the pole and zeros to be displaced from their nominal positions. The effects of these displacements may cause the designed network not to exhibit the desired performance; in addition, the active networks may become unstable. The problem of designing a complete insensitive network has not been solved yet. However for specific cases, methods do exist that minimize the sensitivity due to the variation of one or more components. Hence it is highly desirable to design networks using operational amplifiers to have as low a sensitivity as possible and good stability properties.

CHAPTER IV

Realization of transfer functions using infinite gain operational amplifier networks

4-1 Introduction:-

The methods and configurations that can be used in active synthesis using operational amplifiers are too numerous to be mentioned here; however it is possible to subdivide them into the following categories:-

1) Methods based upon polynomial decomposition [2].

2) Methods based upon coefficient matching [2].

3) Methods based upon the association of passive networks (B-A association) [4,5].

It should be noted that in the last category mentioned, the pole sensitivity with respect to the amplifier gain can be prescribed before synthesis, while in the first and second categories it appears that this is not possible. These methods are not discussed in this report. However, in this section, we are going to describe a synthesis procedure developed by F. Anday [6,7,8] for designing low sensitivity active RC circuits using S F G representation. This method is based on drawing the S F G of the bi-quadratic transfer function and obtaining the active circuit from the graph. This may be considered to fall in the category of coefficient matching. It is to be noted that, in this method, the active network contains, apart from resistances and capacitances, only single ended infinite gain operational amplifiers. This method was later extended by Anday to realize second order transfer functions using a minimum number of elements. In addition Anday developed the realization of transfer functions with prescribed sensitivity functions also using S F G s.

4-2 Synthesis of low sensitivity active circuits using S F G for the bi-quadratic transfer function:-

Consider the single-ended infinite gain operational amplifier shown in figure 4-1. According to the defining equations of the operational amplifier, the node voltage equation corresponding to the input can be written as:-

$$V_1Y_1 + V_2Y_2 + \dots + V_iY_i + \dots + V_nY_n + V_oY_F = 0 \qquad (4\text{-}1)$$

$$-V_1\frac{Y_1}{Y_F} - V_2\frac{Y_2}{Y_F} - \dots\dots - V_i\frac{Y_i}{Y_F} - \dots\dots - V_n\frac{Y_n}{Y_F} = V_o \qquad (4\text{-}2)$$

which can be represented by the S F G shown in figure 4-2.

The graph is then transformed to the form shown in figure 4-3 by the self loop rule.

If a given transfer function can be represented by a S F G in such a way that each node except the input node has a self loop with loop transmittance equal to the sum of [1+ any RC admittance function] and each branch has a transmittance equal to any RC admittance function, then the corresponding circuit realization can be easily found by the aid of figure 4-3.

Let the general bi-quadratic transfer function be given by:-

$$T_v(s) = \frac{V_o}{V_i} = -\frac{s^2 + as + b)}{k(s^2 + cs + d)} \qquad (4\text{-}3)$$

Anday [6] has shown that the corresponding flow graph is represented in figure 4-4 and its realization is shown in figure 4-5.

By choosing the forward paths in a suitable form, Anday has shown that, the method can also be applied to the synthesis of transfer functions in which some of the coefficients of the numerator polynomial have negative values. As an example, if $a < 0$ the S F G is shown in figure 4-6

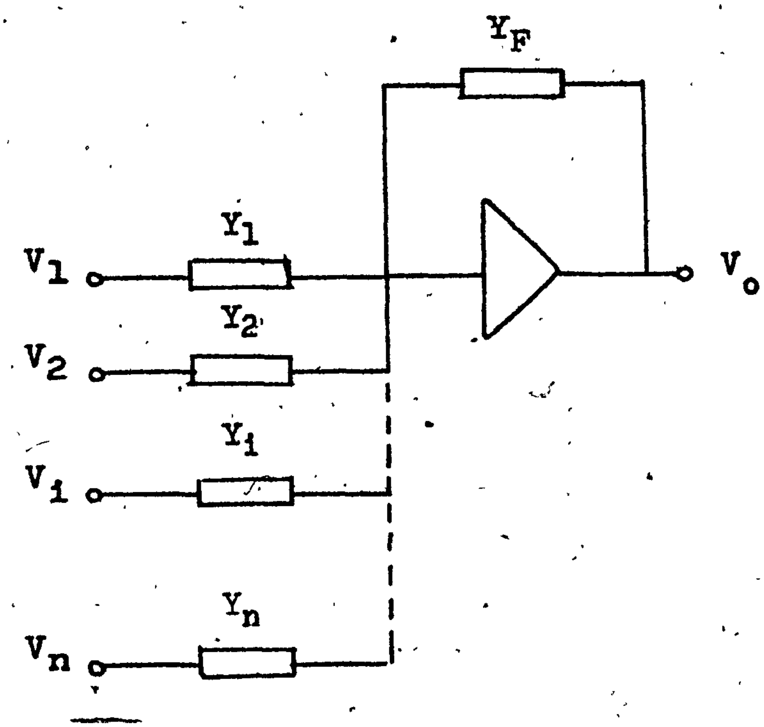


Figure 4-1. Operational amplifier circuit with n inputs

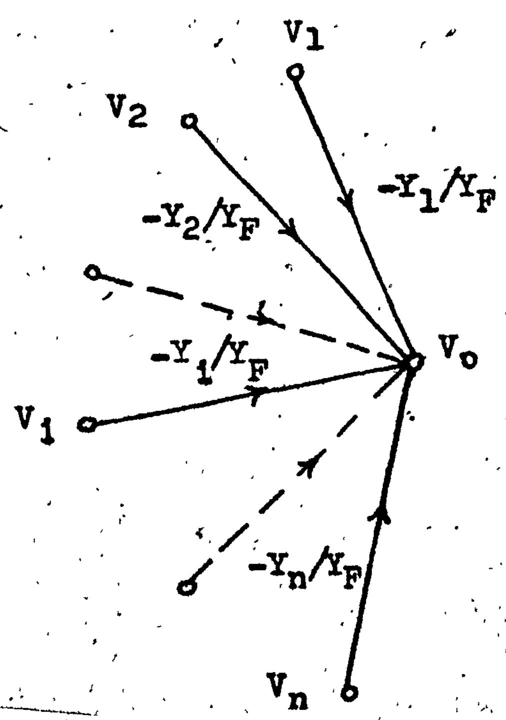


Figure 4-2. SFG of circuit shown in figure 4-1

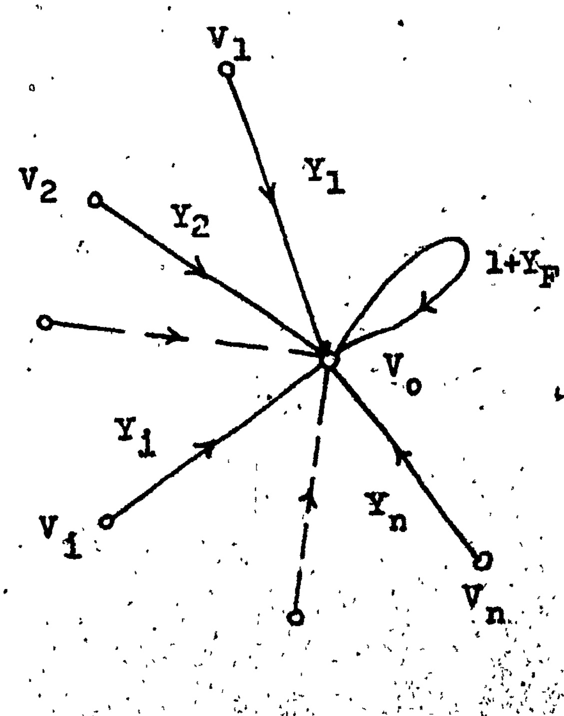


Figure 4-3. Transformation of SFG shown in figure 4-2

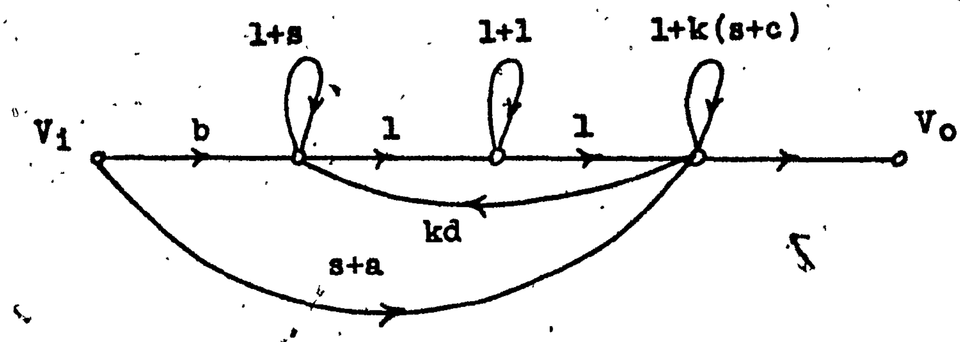


Figure 4-4. SFG of the transfer function: $T_v = \frac{-(s^2+as+b)}{k(s^2+cs+d)}$

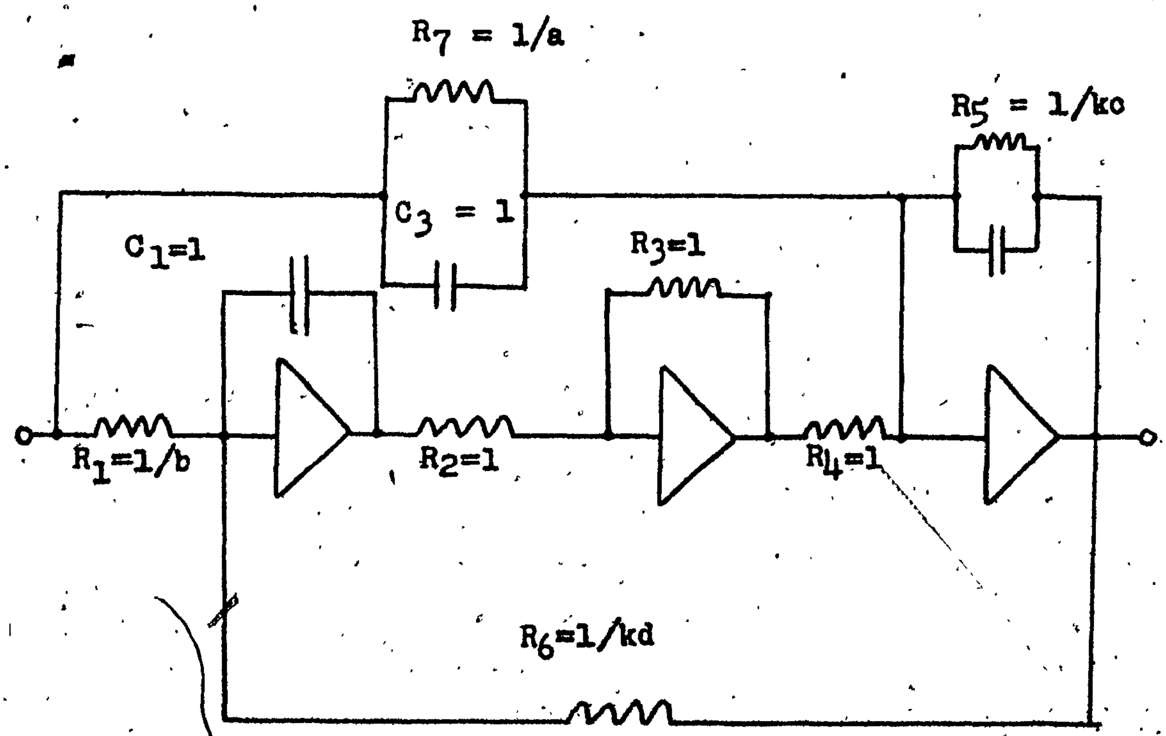


Figure 4-5. Realization of an active RC circuit with transfer function : $T_v = \frac{-(s^2+as+b)}{k(s^2+cs+d)}$

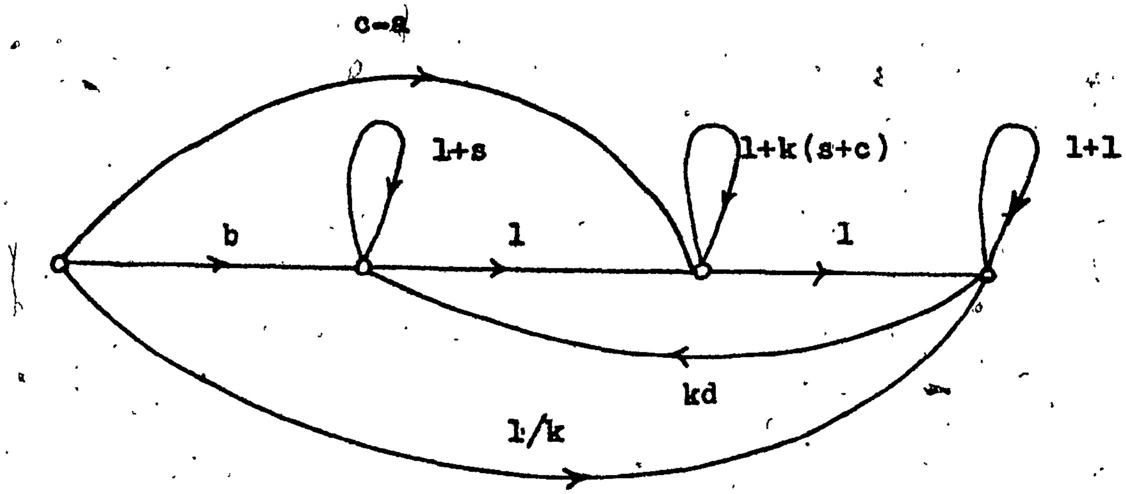


Figure 4-6. SFG of the transfer function; $T_V = \frac{-(s^2-as+b)}{k(s^2+cs+d)}$

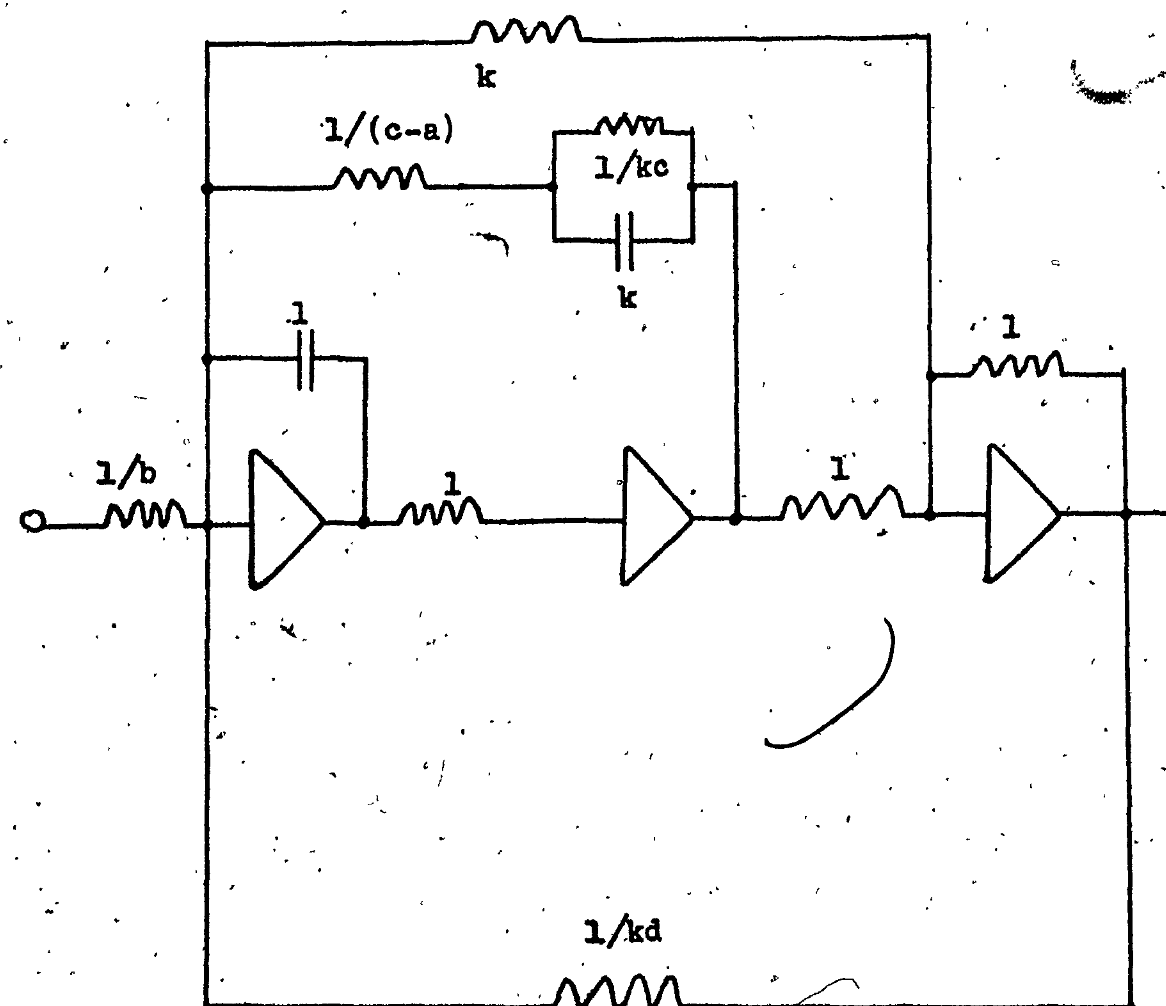


Figure 4-7. Realization of an active RC circuit with a transfer function : $T_v = \dfrac{-(s^2-as+b)}{k(s^2+cs+d)}$

and its realization in figure 4-7. It is clear that we must have $c > |a|$.

4-2-1 Sensitivity considerations:-

The denominator of the transfer function $\frac{V_o}{V_i} = T_v$ evaluted in terms of the circuit elements becomes

$$s^2 + \frac{G_5}{C_2} s + \frac{G_2 G_4 G_6}{C_1 C_2 C_4}$$

Since no difference terms are involved, the pertinent Q and pole sensitivities are very small.

We are now going to compute the sensitivity of the transfer function of the network shown in figure 4-5 with respect to its passive elements. Let us first redraw the S F G of the network shown in figure 4-5 in the form shown in figure 4-4-a. Hence, applying Mason's Formula, we get the transfer function T_v in the form

$$T_v = \frac{C_1 C_3 G_3 S^2 + C_1 G_3 G_7 S + G_1 G_2 G_4}{C_1 C_2 G_3 S^2 + C_1 G_3 G_5 S + G_2 G\ G_6} = \frac{N}{D} \qquad (4\text{-}3\text{-}a)$$

Hence applying the formula

$$S_k^{T(s)} = k \quad \frac{N_2(s)}{N(s,k)} - \frac{D_2(s)}{D(s,k)}$$

where

$$T(s,k) = \frac{N_1(s) + k\ N_1(s)}{D_1(s) + k\ D_1(s)} = \frac{N(s,k)}{D(s,k)}$$

We get the following expressions for the passive elements sensitivities:

$$S_{G_1}^{T(s)} = \frac{G_1\ G_2\ G_4}{N}$$

$$S_{G_2}^{T(s)} = \frac{G_1\ G_2\ G_4}{N} - \frac{G_2\ G_4\ G_6}{D}$$

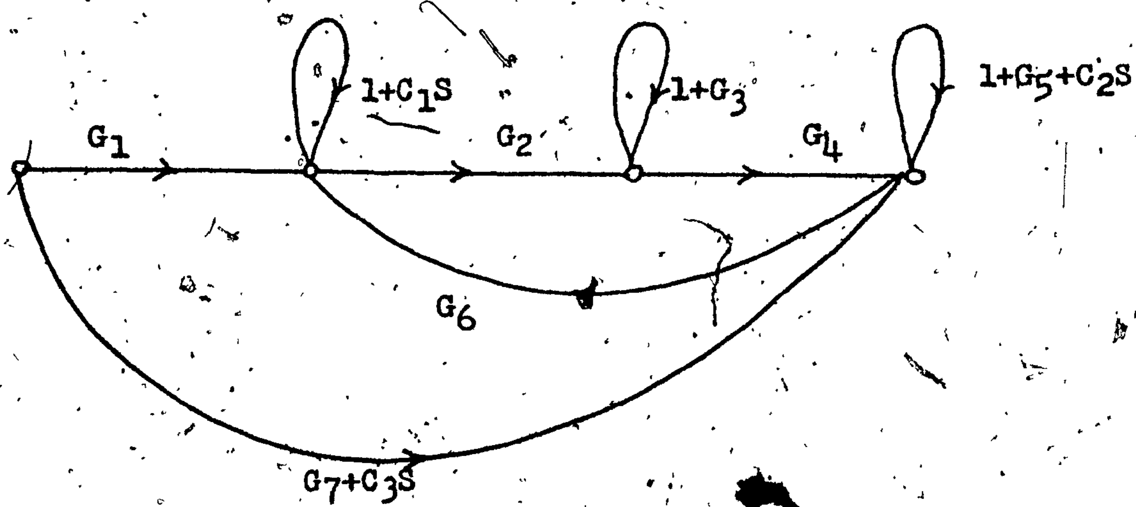


Figure 4-4-a. SFG of the network shown in figure 4-5.

$$S_{G_3}^{T(s)} = \frac{C_1C_3G_3S^2 + C_1G_3G_7S}{N} - \frac{C_1C_2G_3S^2 + C_1G_3G_5S}{D}$$

$$S_{G_4}^{T(s)} = \frac{G_1\ G_2\ G_4}{N} - \frac{G_2\ G_4\ G_6}{D}$$

$$S_{G_5}^{T(s)} = -\frac{C_1\ G_3\ G_7\ S}{D}$$

$$S_{G_6}^{T(s)} = -\frac{G_2\ G_4\ G_6}{D}$$

$$S_{G_7}^{T(s)} = \frac{C_1\ G_3\ G_7\ S}{N}$$

$$S_{C_1}^{T(s)} = \frac{C_1C_3G_3S^2 + C_1G_3G_7S}{N} - \frac{C_1C_3G_3S^2 + C_1G_3G_5S}{D}$$

$$S_{C_2}^{T(s)} = -\frac{C_1C_2G_3S^2}{D}$$

$$S_{C_3}^{T(s)} = \frac{C_1C_3G_3S^2}{N}$$

4-3 Realization of 2nd Order transfer functions using a minimum number of elements:-

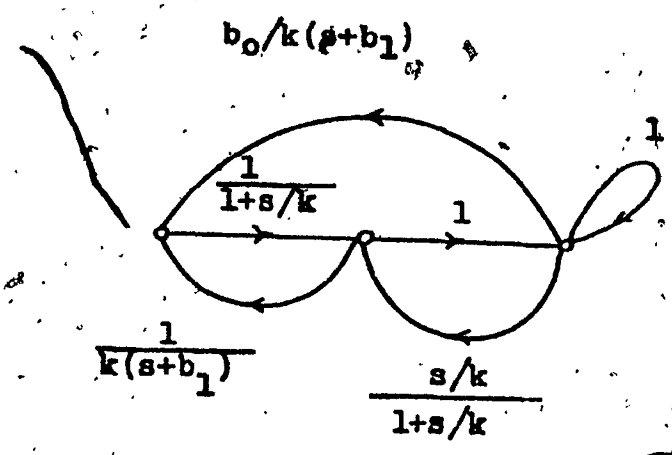
Anday has shown [7] a S F G procedure for the realization of 2nd order transfer functions using a minimum number of passive elements and a single operational amplifier.

Consider the sub-graphs shown in figure 4-8. The determinants of these graphs are respectively:

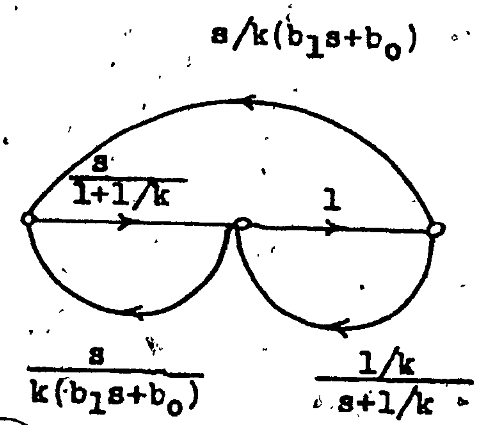
$$\Delta_a = -\frac{s^2}{k(s+b_1)(\frac{s}{k}+1)} \tag{4-4-a}$$

and

$$\Delta_b = -\frac{s^2 + b_1 s + b_o}{k(b_1 s + b_o)(s + 1/k)} \tag{4-4-b}$$

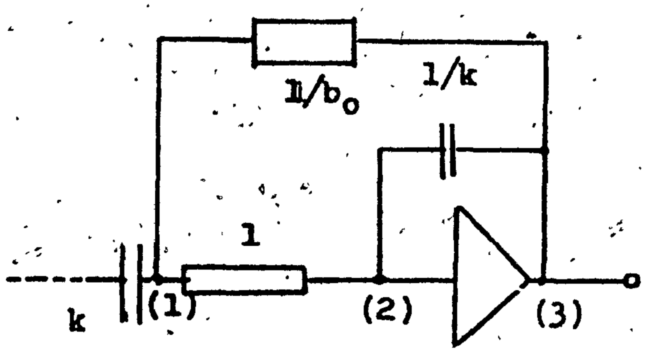


(a)



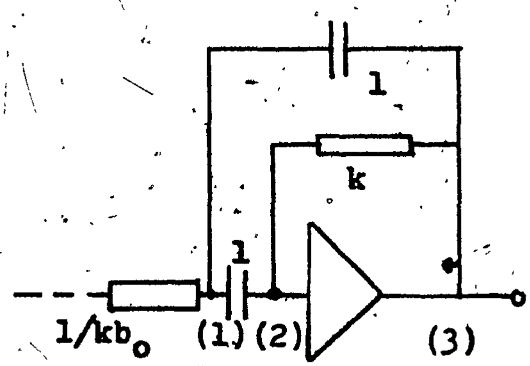
(b)

Figure 4-8. Subgraphs which possess $(s^2+b_1s+b_o)$ as the numerator of their determinants.



$k=(b_o+1)/b_1$

(a)



$k=2/b_1$

(b)

Figure 4-9. Minimal subnetworks which realize $(s^2+b_1s+b_o)$

using the synthesis procedure previously shown two active RC subnetworks, shown in figure 4-9 each containing a single operational amplifier and four passive elements are obtained from these graphs. It can be shown that these active subnetworks can realize the 2^{nd} order polynomial and are minimal in the sense of using minimum number of elements.

Now let the transfer function to be realized be given as:

$$T(s) = \frac{-P(s)}{s^2 + b_1 s + b_o} \tag{4-5}$$

The synthesis procedure which generates minimal 2^{nd} order active networks consist in the following steps:

a) Branches must be inserted into the subgraphs (a) or (b) of figure 4-4 so that equation (4-4-a) or (4-4-b) holds respectively:

$$\Sigma\, T_k \Delta_k = \frac{P(s)}{k(s+b_1)(\frac{s}{k}+1)} \tag{4-6-a}$$

$$\Sigma\, T_k \Delta_k = \frac{P(s)}{k(\hat{b}\ s+b_o)(s+\frac{1}{k})} \tag{4-6-b}$$

where $\Sigma\, T_k \Delta_k$ is the numerator of Mason's Gain Formula.

b) The transmittances of the inserted branches must be suitably selected so that each term of the numerator polynomial P(s) can be realized with a single passive element.

c) Suitable selection of the transmittances and the method of obtaining networks corresponding to the final graph can be carried out to the above procedure.

The realizations for different P(s) are tabulated in Table 4-1.

Since four passive elements are required for the denominator polynomial and one for each term of the numerator polynomial, it can be shown that the minimum number of passive components for the realization of the 2^{nd} order transfer function with a single operational amplifier is (n+4) where

TABLE 4-1

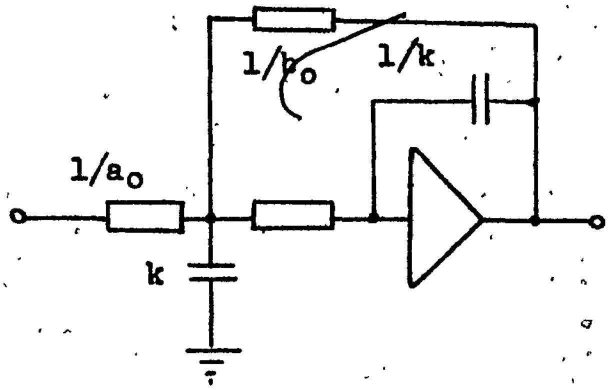
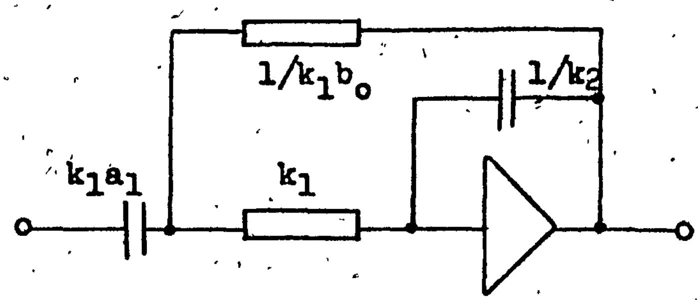
Transfer Function	Minimal second order active networks
$\frac{-a_o}{s^2+b_1s+b_o}$	 $k = \frac{a_o+b_o+1}{b_1}$
$\frac{-a_1s}{s^2+b_1s+b_o}$	 $k_1 = 1/\sqrt{a_1b_1-b_o}$ $k_2 = a_1/\sqrt{a_1b_1-b_o}$ $a_1b_1 > b_o$

TABLE 4-1 (Continued)

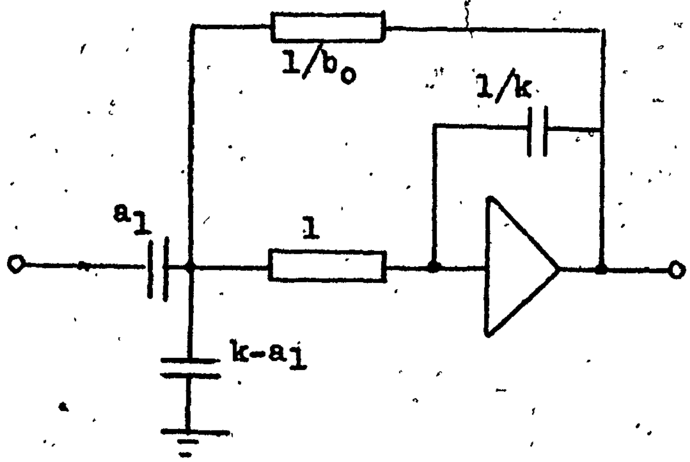
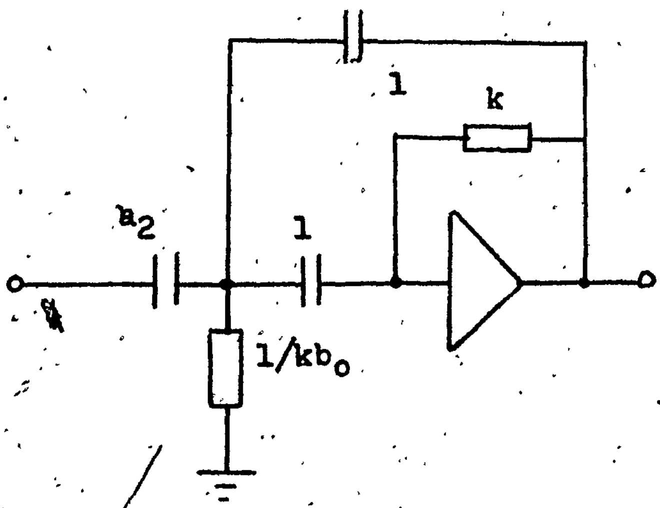
$\dfrac{-a_1 s}{s^2+b_1 s+b_o}$	 $k = (b_o+1)/b_1$ $a_1 b_1 \leqslant b_o$
$\dfrac{a_2 s^2}{s^2+b_1 s+b_o}$	 $k = \dfrac{a_2+2}{b_1}$

TABLE 4-1 (Continued)

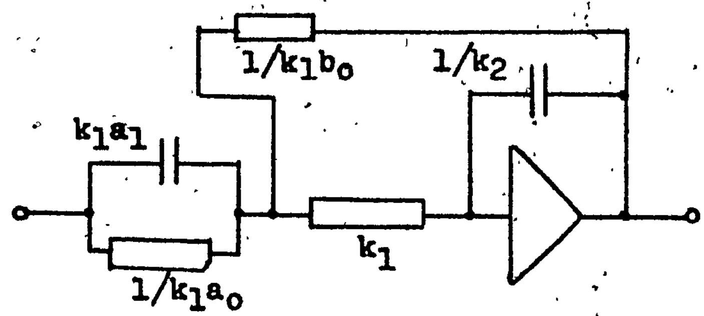
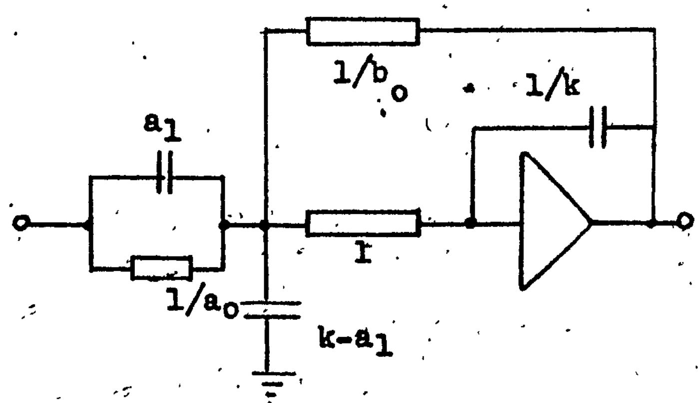
	 $k_1 = b_1/a_1$ $k_1 = a_1/\sqrt{a_1b_1-a_0-b_0}$ $a_1b_1 > (a_0+b_0)$
$\frac{-(a_1s+a_0)}{s^2+b_1s+b_0}$	 $k = (a_0+b_0+1)$ $a_1b_1 \leqslant (a_0+b_0)$

TABLE 4-1 (Continued)

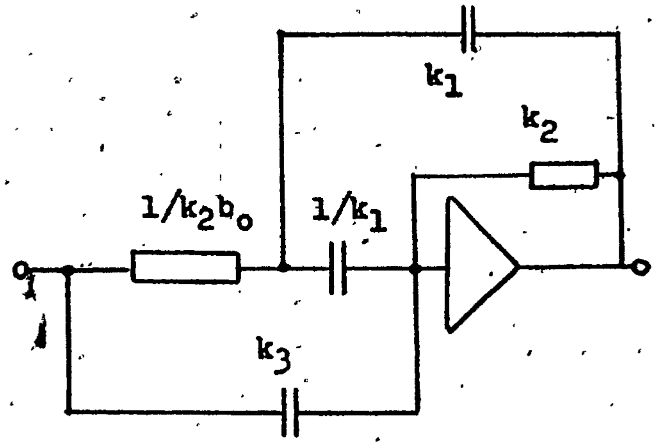
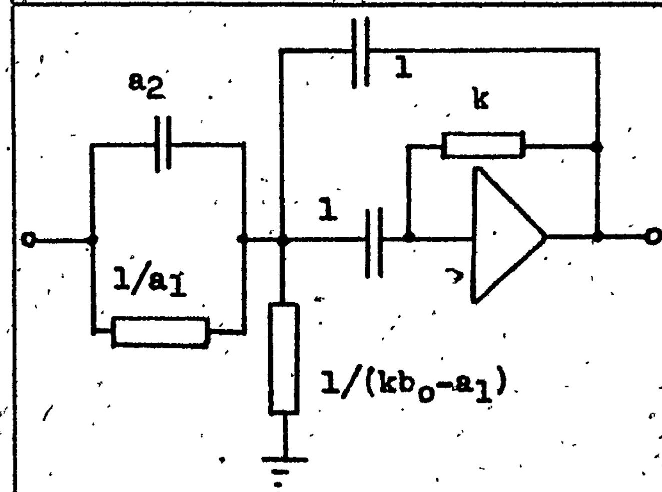
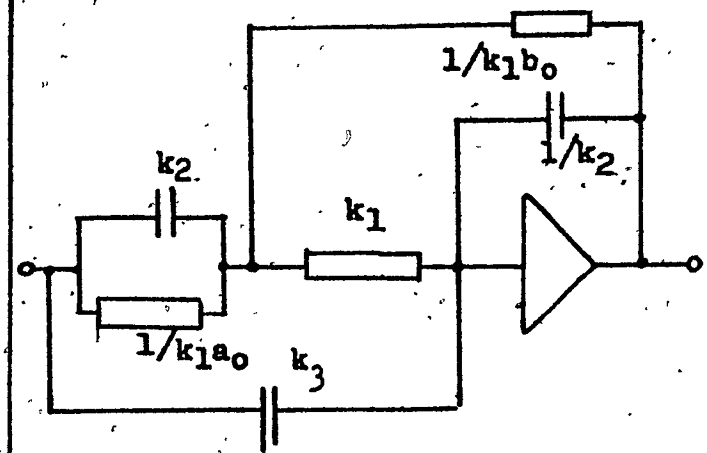
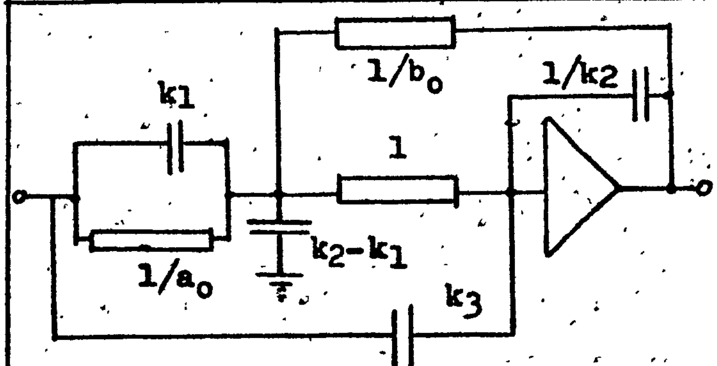
$\frac{-s(a_2 s+a_1)}{s^2+b_1 s+b_o}$	 $k_1 = \sqrt{b_o/\{a_1 b_1 - b_o(a_2+1)\}}$ $k_2 = \frac{a_1 b_1 - b_o a_2}{b_1\sqrt{[b_o\{a_1 b_1 - b_o(a\ +1)\}]}}$ $k_3 = a_2\sqrt{[b_o\{a_1 b_1 - b_o(a_2+1)\}]}$
	 $k = (a_2+2)/b_1$ $a_1 b_1 \leq (a_2+1)b_o$

TABLE 4-1 (Continued)

$\dfrac{-a_2s^2+a_1s+a_o}{s^2+b_1s+b_o}$	 $k_1 = 1/\sqrt{b_1(a_1-a_2b_1)-a_o-b_o}$ $k_2 = \dfrac{a_1-a_2b_1}{\sqrt{b_1(a_1-a_2b_1)-a_o-b_o}}$ $a_1-a_2b_1 \quad 0$ $a_1-a_2b_1 \quad (a_o+b_o)/b_1$
	 $k_1 = a_1-a_2b_1$ $k_2 = (a_o+b_o+1)/b_1$ $k_3 = a_2b_1/(a_o+b_o+1)$ $a_1-a_2b_1 > 0$ $a_1-a_2b_1 \leq (a_o+b_o)/b_1$

n is the number of terms of the numerator polynomial P(s). It should be noted that this number can be reduced by one if the numerator and denominator coefficients satisfy the conditions given in Table 4-1.

4-4 Realization of transfer functions with a prescribed sensitivity function:-

In this section we shall discuss a S F G approach due to Anday [8] for the simultaneous realization of a 2^{nd} order transfer function and the sensitivity function of this transfer function with respect to the variation of some passive network parameter.

Consider a S F G with node variable X_i $(i = 0,1,2, \ldots n)$ and let $T_{on} = \frac{X_n}{X_o}$ be the transfer function between the output variable X_n and the input variable X_o and let

$$t_{ij} = a_{ij}s + b_{ij} \qquad a_{ij} \geq 0 \quad b_{ij} \geq 0 \tag{4-7}$$

be the form of the transmittance branch (i,j). The sensitivity function of the transfer function T_{on} with respect to the variation of the transmittance can be defined as

$$S_{t_{ij}}^{T_{on}} = \frac{d\,T_{on}}{d\,t_{ij}} \cdot \frac{t_{ij}}{T_{on}} \tag{4-8}$$

It is shown that $\frac{d\,T_{on}}{d\,t_{ij}}$ can be evaluated from a S F G as follows:

$$\frac{d\,t_{on}}{d\,t_{ij}} = \frac{\Sigma\, P_{oi}\Delta_{oi}}{\Delta} \cdot \frac{\Sigma\, P_{jn}\Delta_{jn}}{\Delta} \tag{4-9}$$

where Δ is the determinant of the S F G, P_{oi} and P_{jn} are the gains of the forward paths from the input to the node i, and from the node j to the output node n respectively. Δ_{oi} and Δ_{jn} are the determinants of the S F G that do not touch the forward paths o to i and j to n respectively. By using equation (4-9) and Mason's gain formula in equation (4-8) we get

$$S^{T_{on}}_{t_{ij}} = \frac{\Sigma P_{oi}\Delta_{oi} \; \Sigma P_{jn}\Delta_{jn}}{\Sigma P_{on}\Delta_{on}} \cdot \frac{t_{ij}}{\Delta} \tag{4-10}$$

Now consider the S F G model shown in figure 4-10 with loop and branch transmittances in the form:

$$t_{ij} = a_{ij}\, s + b_{ij} \qquad (i = 0,1,2,3 \ldots) \quad (j = 1,2,3 \ldots) \tag{4-11}$$

$$a_{ij} \geq 0$$

$$b_{ij} \geq 0$$

A 2^{nd} order transfer function can be represented by a S F G of this type and the active network which corresponds to this graph can be easily synthesized. Using Mason's gain formula and equation (4-11) one obtains equations (4-12) and (4-13) respectively for the graph of figure 4-10:

$$T_{o3} = \frac{\Sigma P_{o3}\Delta_{o3}}{\Delta} = \frac{t_{o1}t_{12}t_{26}}{\Delta} \tag{4-12}$$

$$S^{T_{o3}}_{t_{11}} = -S^{T_{o3}}_{t_{12}} = \frac{t_{11}(t_{22}t_{23} - t_{23}t_{32})}{\Delta} \tag{4-13-a}$$

$$S^{T_{o3}}_{t_{22}} = \frac{t_{11}t_{22}t_{33}}{\Delta} \tag{4-13-b}$$

$$S^{T_{o3}}_{t_{33}} = -S^{T_{o3}}_{t_{23}} = \frac{t_{33}(t_{11}t_{12} - t_{12}t_{21})}{\Delta} \tag{4-13-c}$$

$$S^{T_{o3}}_{t_{21}} = -\frac{t_{12}t_{21}t_{33}}{\Delta} \tag{4-13-d}$$

$$S^{T_{o3}}_{t_{31}} = \frac{t_{12}t_{23}t_{31}}{\Delta} \tag{4-13-e}$$

$$S^{T_{o3}}_{t_{32}} = -\frac{t_{11}t_{23}t_{32}}{\Delta} \tag{4-13-f}$$

In the above equations

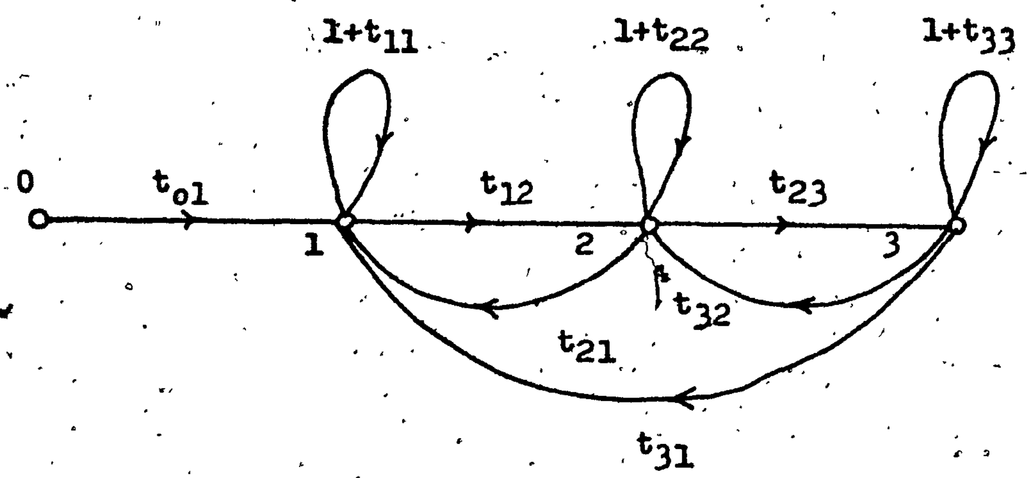


Figure 4-10. Model of a second order lowpass bandpass, or highpass transfer function.

$$\Delta = -(t_{11}t_{22}t_{33} + t_{12}t_{23}t_{31} - t_{11}t_{23}t_{32} - t_{12}t_{21}t_{33}) \quad (4\text{-}14)$$

Let $$T_{o3}(s) = \frac{P(s)}{Q(s)} \quad (4\text{-}15)$$

and $$S_k^{T_{o3}}(s) = \frac{R(s)}{Q(s)} \quad (4\text{-}16)$$

be the forms of the transfer and sensitivity functions. Comparing equation (4-15) with equation (4-12) and equations (4-13) with equation (4-16) one can see that the sensitivity functions of equations (4-13) are in the form described by equation (4-16). Thus by using the graph model of figure 4-10, a transfer function and a sensitivity function with the transfer function poles only, can be simultaneously realized. The synthesis procedure consists in the following steps:

a) Select one of t_{11}, t_{22}, t_{33}, t_{12}, t_{23}, t_{21}, t_{31}, or t as a varying parameter.

b) Using equations (4-13), select and determine the values of the transmittances in the numerator polynomial of the sensitivity function corresponding to the preselected varying transmittance in such a way that this polynomial is equal to R(s).

c) Find the values of the unknown transmittances in equation (4-14) so that the determinant of the S F G is equal to Q(s).

d) Determine t_{o1} in such a way that

$$t_{o1}t_{12}t_{13} = P(s)$$

e) Synthetize the S F G thus obtained using the above mentioned procedure.

An example is worked out:

$$T(s) = \frac{-5}{s^2+3s+3} \quad (4\text{-}17\text{-}a)$$

$$S_k^T = \frac{-s(s+4)}{s^2+3s+4} \quad (4\text{-}17\text{-}b)$$

According to the synthesis procedure t_{22} is taken as the varying transmittance. Then by using the numerator polynomials of equations (4-13-b) and (4-17-b) the value of t_{22} is selected and t_{11} and t_{33} are determined as follows:

$$t_{11} = s+4 \qquad t_{22} = s \qquad t_{33} = 1$$

In the following step the values of t_{12}, t_{23}, t_{31}, t_{32} and t_{21} are determined by using equation (4-14) and the polynomial (s^2+3s+3) (see figure 4-11-b).

Finally t_{o1} is taken equal to 5 to realize the numerator polynomial of T(s) (see figure 4-11-c). The network which realizes simultaneously equations (4-17-a) and 4-17-b) is then obtained by using the above procedure (see figure 4-11-d).

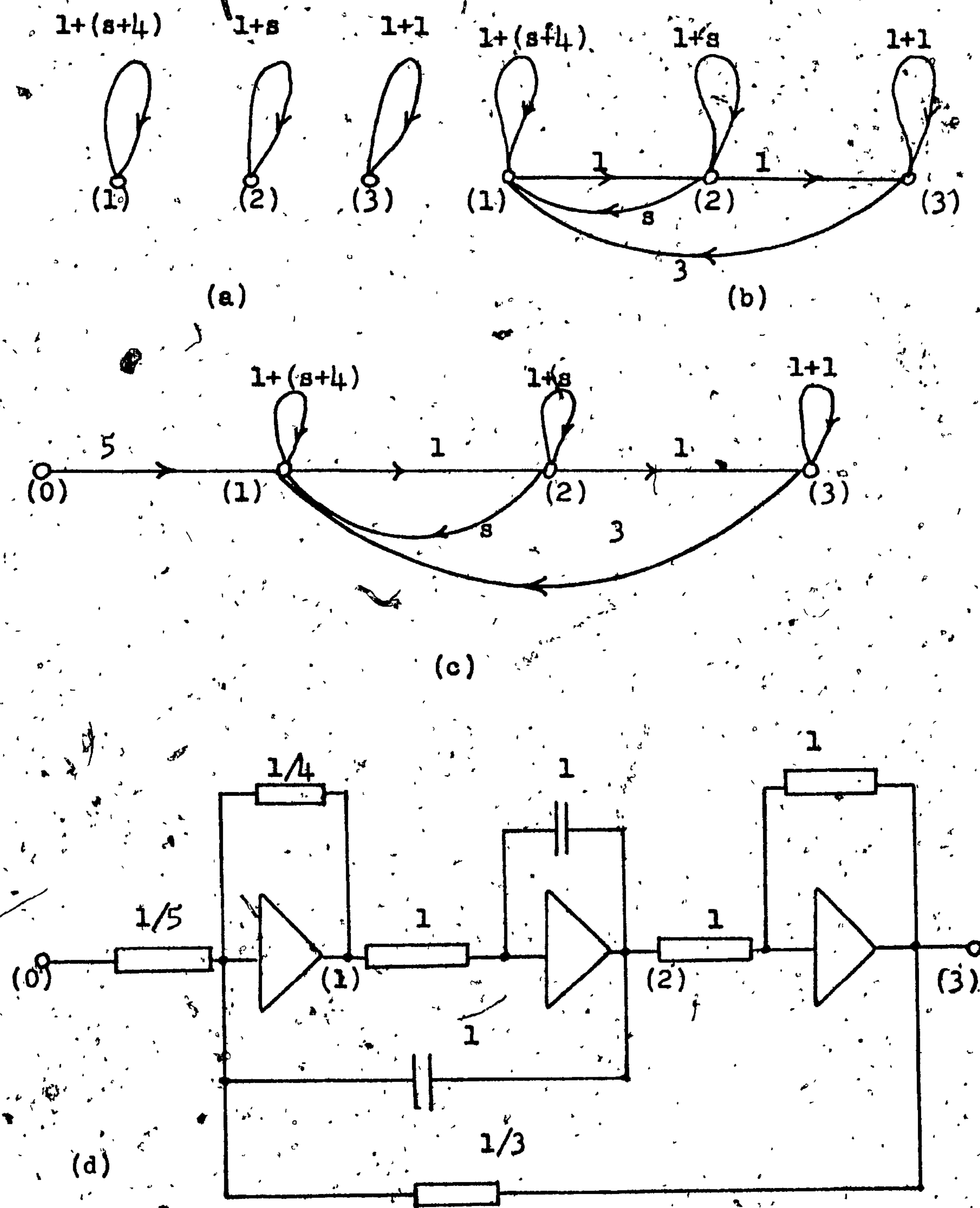


Figure 4-11. Simultaneous realization of : $T(s) = \dfrac{-5}{s^2+3s+3}$

and, $S_k^T = \dfrac{-s(s+4)}{s^2+3s+3}$

CHAPTER V

Different Signal Flow Graphs for some basic finite gain operational amplifier networks

5-1 Introduction:-

As mentioned earlier, signal flow graphs have been mainly used as an analysis tool for feedback networks, sensitivity calculations, and analog simulation using state variable approach, etc... As discussed in the previous chapter, S F G's are used also for the synthesis of active networks using infinite gain operational amplifiers.

Before attempting to use S F G s in the synthesis of transfer functions using finite gain operational amplifier as active elements, we will deduce a few S F G s for some basic and simple finite gain operational amplifier networks. Since for each operational amplifier network there is a great number of possible S F G s it would be a tremendous work, (if possible) to deduce all the possible S F G s of a single network configuration; hence only a few of them will be given here.

We consider here finite gain operational amplifiers and we will assume that these amplifiers are ideal with respect to all other characteristics. (Infinite input impedance, zero output impedance, etc....).

We will show later (in the next chapter) how these simple S F G s can be used as basic blocks to synthetize transfer functions. We will focus our interest and our work on second order transfer functions only, because of sensitivity considerations.

5-2 Configuration 1 (Figure 5-1):-

The different S F G s considered for configuration 1 are tabulated in Table 5-1. We note that for the cases 1-b and 1-c if $K \to \infty$, the self loop transmittance reduces to unity.

5-3 Configuration 2 (Figure 5-2):-

The different S F G s considered for configuration 2 are tabulated in Table 5-2. We note that for each of cases 2-a and 2-b if $K \to \infty$ the self loop transmittance becomes unity.

5-4 Configuration 3 (Figure 5-3):-

Analysis yields

$$[V_1 - (\frac{V_o}{-K})]\ Y_1 = [(\frac{V_o}{-K}) - V_o]\ Y_F \qquad (5\text{-}1)$$

Rearranging we get

$$\frac{V_o}{V_1} = - \frac{1}{Z_1Y_F + \frac{1}{K}(1 + Z_1Y_F)} \qquad (5\text{-}2)$$

The above can be generalized to n entries (Figure 5-4).

Analysis yields

$$(V_1Y_1 + V_2Y_2 + .. + V_nY_n) + \frac{V_o}{K}(Y_1 + Y_2 + .. + Y_n) + V_oY_F(1+K) = 0 \qquad (5\text{-}3)$$

The different S F G s considered for configuration 3 are tabulated in Table 5-3. We note that if $K \to \infty$ S F G s, 3-a and 3-e reduce to the S F G s used by Anday while the self loop transmittance of each of S F G s 3-b and 3-c reduces to unity. Also the self loop of configuration 3-d vanishes.

5-5 Configuration 4 (Figure 5-5):-

Analysis yields

$$(V_{in} - V_2)\ Y_1 = (V_2 - V_o)\ Y_F \qquad (5\text{-}4)$$

$$(V_1 - V_2)\ Y_1 = (V_2 - V_o)\ Y_F \qquad (5\text{-}5)$$

$$\frac{V_o}{K}\ Y_1 = V_2Y_F - V_oY_F \qquad (5\text{-}6)$$

$$V_o[\frac{Y_1}{K} + Y_F] = V_2Y_F = [V_1 - \frac{V_o}{K}]\ Y_F \qquad (5\text{-}7)$$

$$V_o[\frac{Y_1}{K} + Y_F + \frac{Y_F}{K}] = V_1Y_F \qquad (5\text{-}8)$$

$$\frac{V_o}{V_1} = \frac{Y_F}{\frac{Y_1}{K} + Y_F + \frac{Y_F}{K}} \tag{5-9}$$

$$\frac{V_o}{V_1} = \frac{1}{1 + \frac{1}{K} + \frac{1}{K}\frac{Y_1}{Y_F}} \tag{5-10}$$

It is clear that the value of Y_2 does not affect the transfer function, hence we may set it to any value and one possibility is $Y = \infty$ and the network of figure 5-5 becomes the one shown in figure 5-6.

The different S F G s presented for configuration 4 are tabulated in Table 5-4. We notice that if $K \to \infty$ each of the self loop transmittance of S F G 4-a and 4-b reduces to unity while the value of the transmittance of the self loop of S F G 4-c reduces to two; also the feedback loops of S F G 4-d vanish.

5-6 Configuration 5 (Figure 5-7):-

Analysis yields

$$Y_3(V_2 - V_1) + Y_1(V_2 - V_3) + Y_4(V_2 - V_o) = 0 \tag{5-11}$$

$$Y_1(V_3 - V_2) + Y_2(V_3 - V_o = 0 \tag{5-12}$$

$$V_3 = -\frac{1}{K}V_o \tag{5-13}$$

Solving and rearranging we get

$$\frac{V_o}{V_1} = \frac{-1}{[Y_2(Z_1+Z_3)+Z_3Y_4+Z_1Z_3Y_2Y_4][1+\frac{1}{K}]+\frac{1}{K}} \tag{5-14}$$

which, if $K \to \infty$ reduces to

$$\frac{V_o}{V_1} = \frac{-1}{Y_2(Z_1+Z_3)+Z_3Y_4+Z_1Z_3Y_2Y_4} \tag{5-15}$$

The considered S F G s for configuration 5 are tabulated in Table 5-5. We notice that if $K \to \infty$ each of the S F G 5-b and 5-c reduces to S F G 5-a. We notice also that the 3 S F G 5-a, 5-b, 5-c represent the transfer function of configuration 5 within a constant factor equal to two. These S F G s can be made to represent exactly the transfer function by adding to each of the S F G s a forward branch of transmittance value $(\frac{1}{2})$ in front of node V_1.

In this report, only operational amplifiers are considered, but it is felt that these methods can be extended to other active elements also. However this is not attempted here.

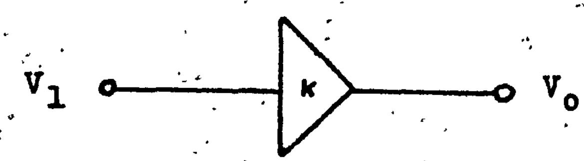


Figure 5-1. Configuration 1 (K may be positive or negative)
$V_o = KV_1$

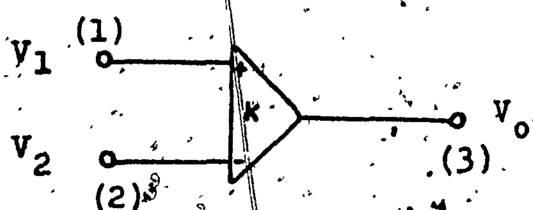


Figure 5-2. Configuration 2 $V_o = K(V_1 - V_2)$ $K \gtrless 0$

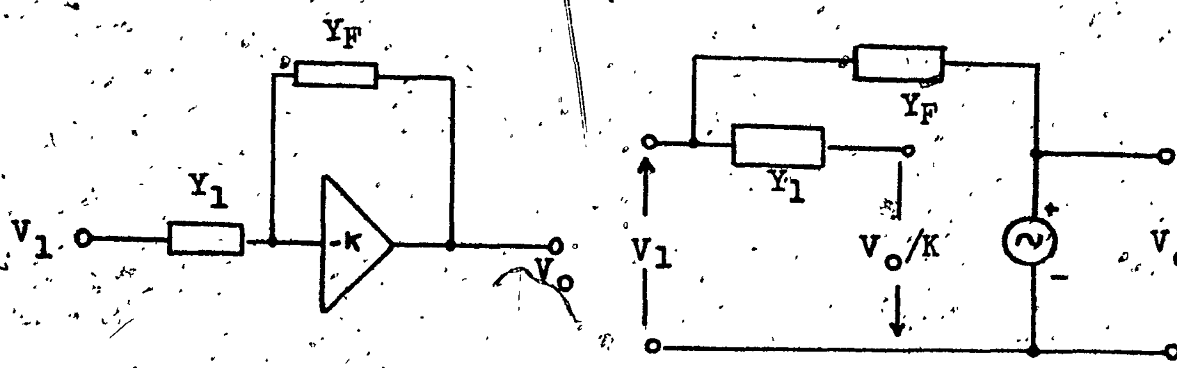


Figure 5-3. Configuration 3 , and its controlled voltage model.

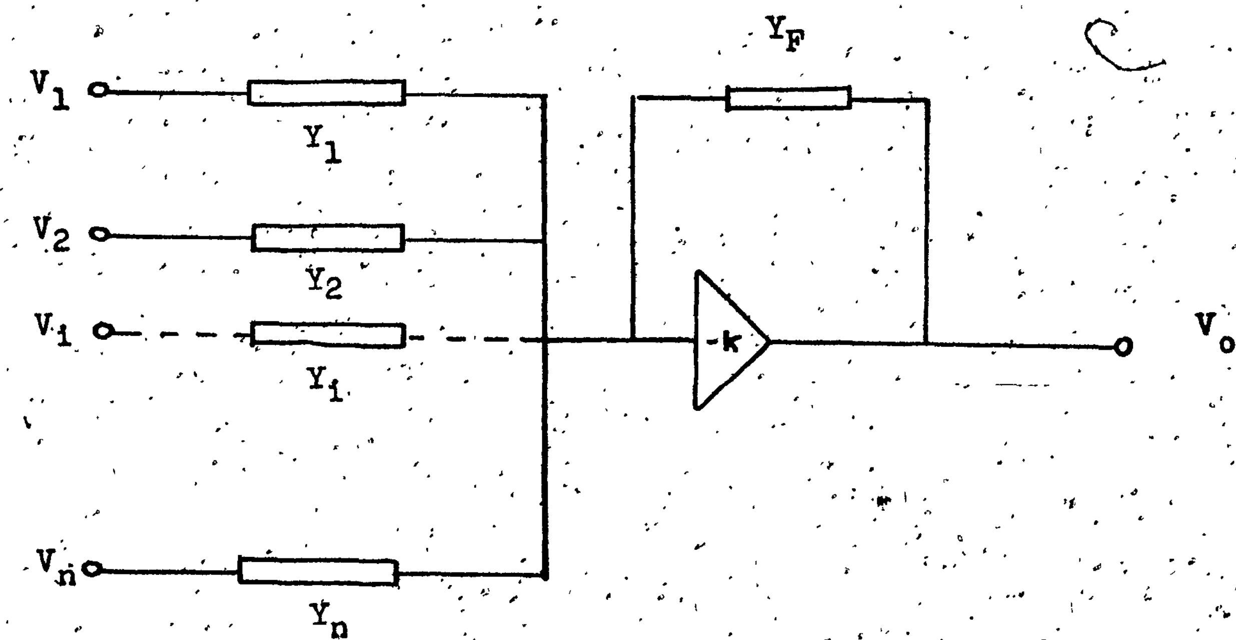


Figure 5-4. Configuration 3 generalized to n inputs.

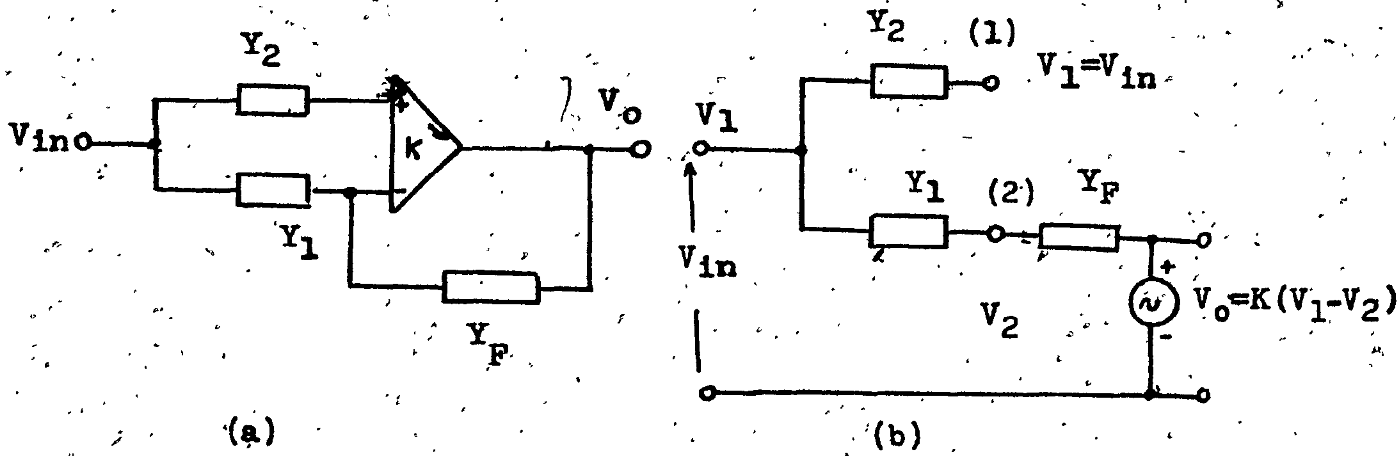


Figure 5-5. Configuration 4 ; and its controlled voltage model.

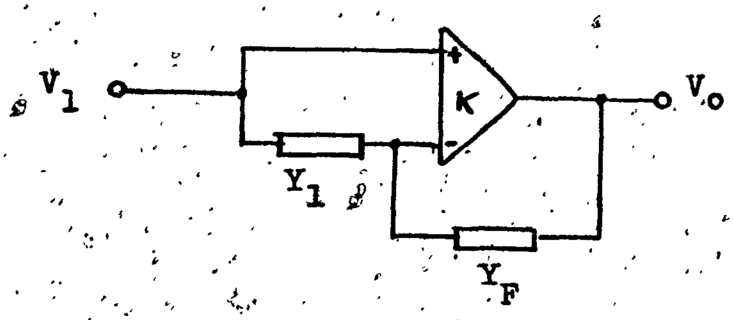


Figure 5-6. Configuration 4 , with $Z_2=0$

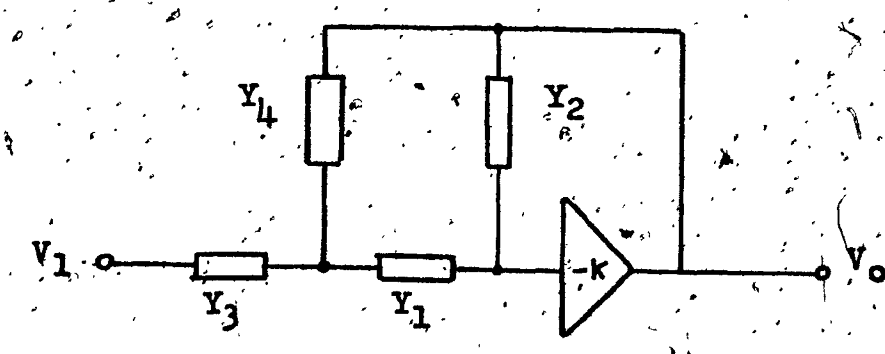


Figure 5-7. Configuration 5.

TABLE 5-1

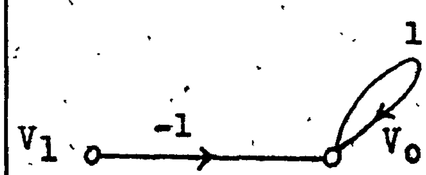
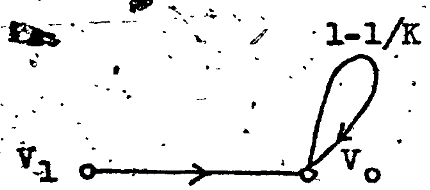
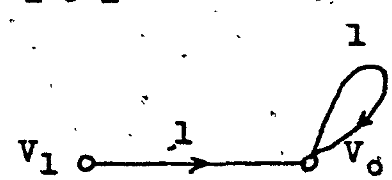
Possible SFG	Proof using Mason's Formula	SFG when $K \longrightarrow \infty$
1-a 	$V_o = KV_1$	1-a-1
1-b 	$V_O = -V_1 + V_O(1+1/K)$ $V_o - V_o - V_o/K = -V_1$ $V_o = KV_1$	1-b-1 
1-c 	$V_o = V_1 + V_o(1 - 1/K)$ $V_o - V_o + V_o/K = V_1$ $V_o = KV_1$	1-c-1 

TABLE 5-2

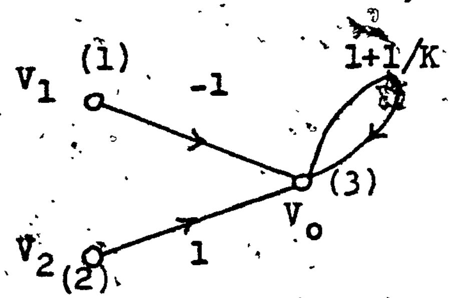
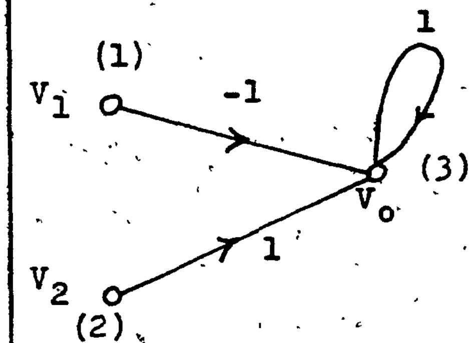
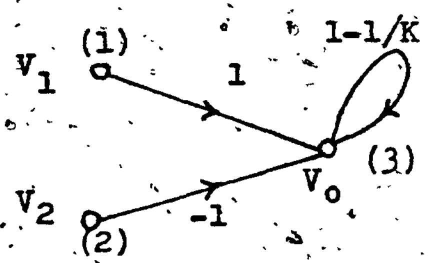
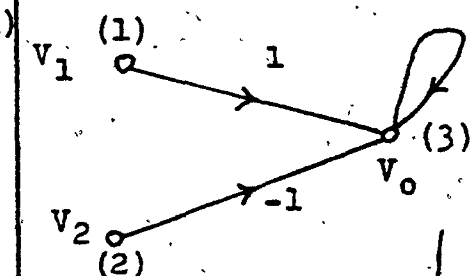
Possible SFG	Proof using Mason's Formula	SFG when K ⟶ ∞
2-a 	$V_o = V_2 - V_1 + V_o(1+1/K)$ $V_o - V_o - V_o/K = V_2 - V_1$ $V_o = K(V_1 - V_2)$	2-a-1 
2-b 	$V_o = V_1 - V_2 + V_o(1-1/K)$ $V_o - V_o + V_o/K = V_1 - V_2$ $V_o = K(V_1 - V_2)$	2-b-1 

TABLE 5-3

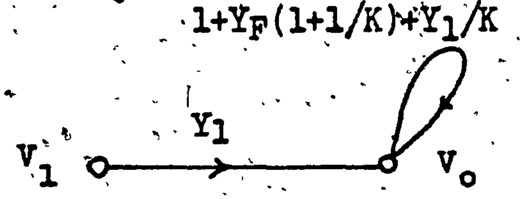
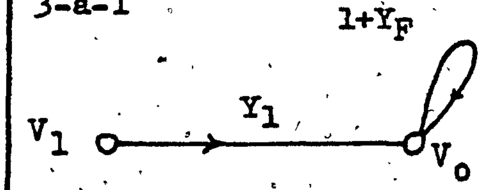
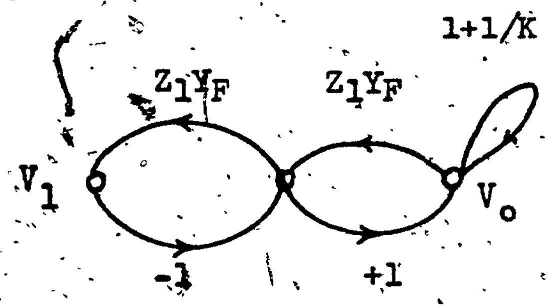
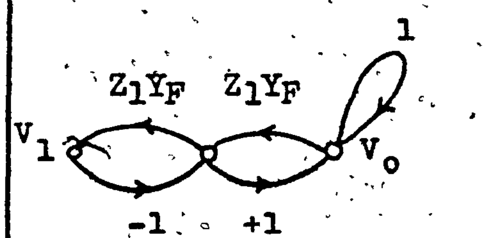
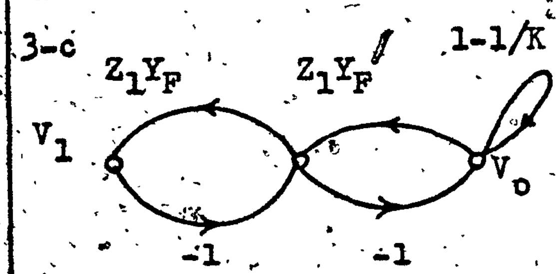
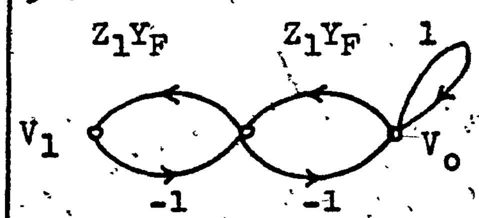
Possible SFG	Proof using Mason's Formula	SFG when K $\longrightarrow \infty$
3-a 	$\frac{V_o}{V_1} = \frac{Y_1}{1-[1+Y_F(1+1/K)+Y_1/K]}$ $= \frac{-1}{Z_1Y_F+(1+Z_1Y_F)/K}$	3-a-1 
3-b 	$\frac{V_o}{V_1} = \frac{-1 \times 1}{1-[-Z_1Y_F+Z_1Y_F+1+1/K] + [-Z_1Y_F(1+1/K)]}$ $= \frac{1}{Z_1Y_F+(1+Z_1Y_F)/K}$ Which is correct within a constant multiplier (-1)	3-b-1 
3-c 	$\frac{V_o}{V_1} = \frac{-1 \times -1}{1-[-Z_1Y_F-Z_1Y_F+1-1/K] + [-Z\ Y\ (1-1/K)]}$ $\frac{1}{Z_1Y_F+(1+Z_1Y_F)/K}$ Which is correct within a constant multiplier (-1)	3-c-1 

TABLE 5-3 (Continued)

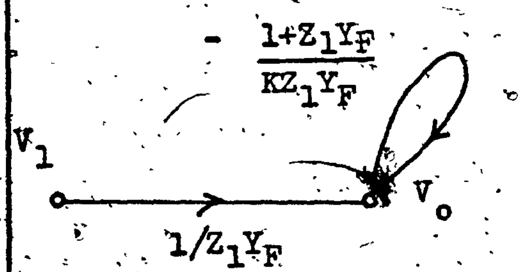
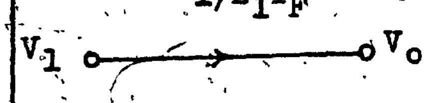
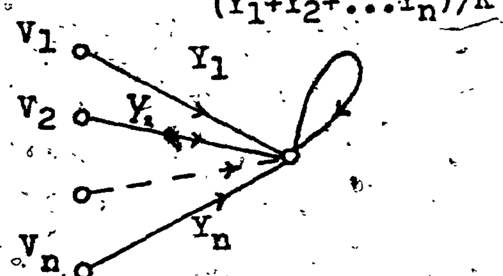
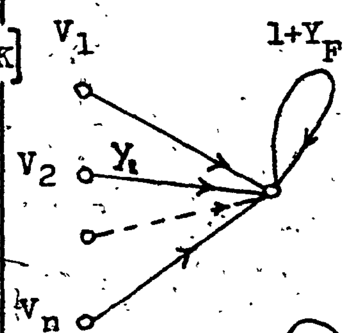
Possible SFG	Proof using Mason's Formula	SFG when $K \longrightarrow \infty$
3-d 	$\frac{V_o}{V_1} = \frac{(1/Z_1Y_F)}{1-\left[-\frac{1+Z_1Y_F}{KZ_1Y_F}\right]}$ $\frac{1}{Z_1Y_F+(1+Z_1Y_F)/K}$ Which is correct within a constant multiplier (-1)	3-d-1 
3-e 	$V_o = V_1Y_1+V_2Y_2+\ldots\ldots+V_nY_n$ $+V_o[1+Y_F(1+1/K)+(Y_1+Y_2+\ldots+Y_n)/K]$ $(V_1Y_1+V_2Y_2+\ldots\ldots+V_nY_n)$ $+(Y_1+Y_2+\ldots\ldots+Y_n)/K$ $+Y_F(1+1/K) = 0$ Which is correct for the generalized case with n inputs.	3-e-1 

TABLE 5-4

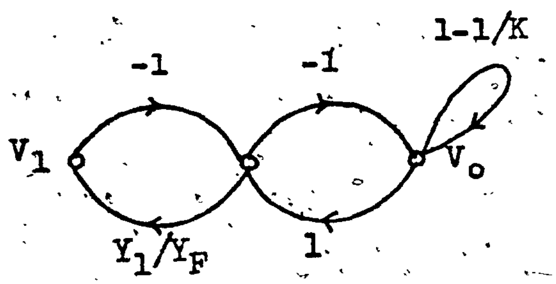
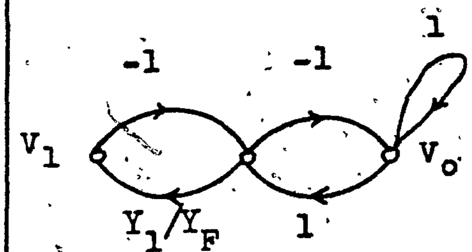
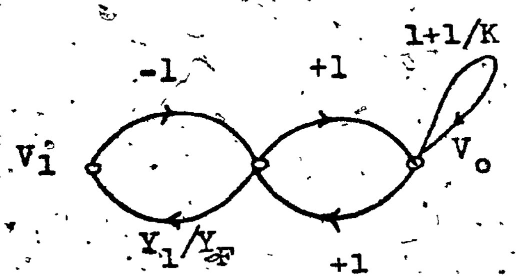
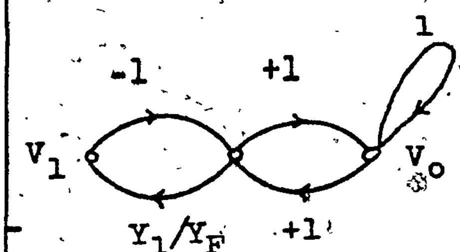
Possible SFG	Proof using Mason's Formula	SFG when $K \longrightarrow \infty$
4-a 	$$\frac{V_o}{V_1} = \frac{-1X-1}{1-\left[-(Y_1/Y_F)-1+1-1/K\right]+\left[-(Y_1/Y_F)(1-1/K)\right]}$$ $$\frac{1}{1+(1/K)(1+Y_1/Y_F)}$$	4-a-1 
4-b 	$$\frac{V_o}{V_1} = \frac{-1X1}{1-\left[-(Y_1/Y_F)+1+1+1/K\right]+\left[-(Y_1/Y_F)(1+1/K)\right]}$$ $$\frac{1}{1+(1/K)(1+Y_1/Y_F)}$$	4-b-1 

TABLE 5-4 (Continued)

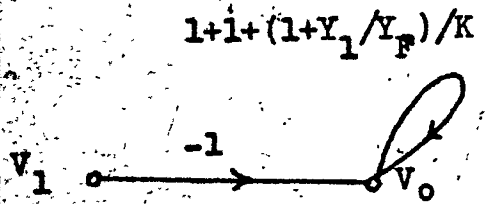
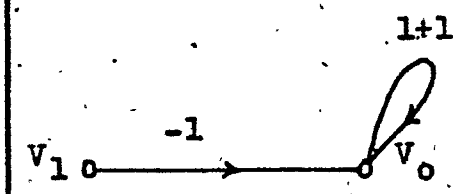
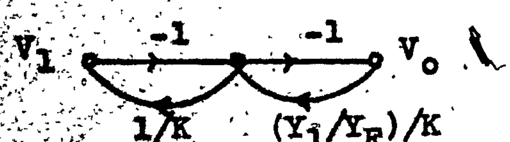
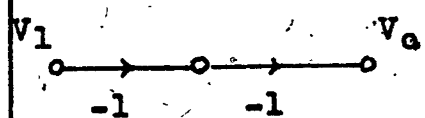
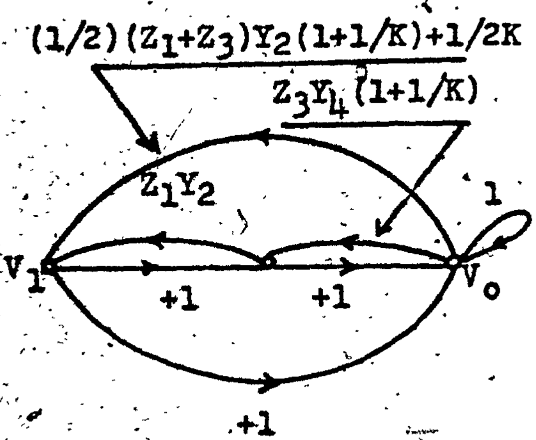
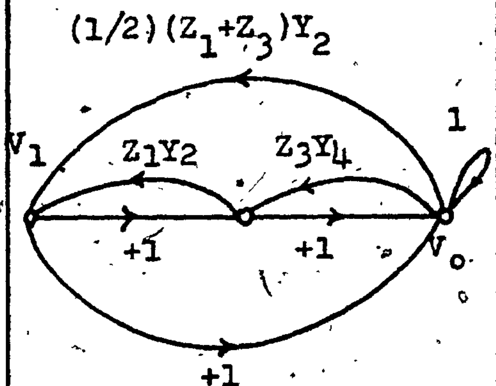
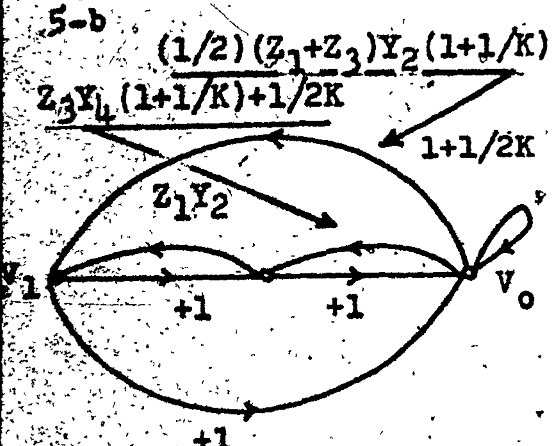
Possible SFG	Proof using Mason's Formula	SFG when K → ∞
4-c 	$\frac{V_o}{V_1} = \frac{-1}{1-[1+1+(1/K)(1+Y_1/Y_F)]}$ $\frac{1}{1+(1/K)(1+Y_1/Y_F)}$	4-c-1 
4-d 	$\frac{V_o}{V_1} \triangleq \frac{-1X-1}{1-[-(1/K)-(1/K)(Y_1/Y_F)]}$ $= \frac{1}{1+(1/K)(1+Y_1/Y_F)}$	4-d-1 

TABLE 5-5

Possible SFG	Proof using Mason's Formula	SFG when $K \longrightarrow \infty$
5-a 	$\frac{V_o}{V_1} = \frac{1X1+1}{1-\left[Z_1Y_2+Z_3Y_4(1+1/K) + Z_1Y_2Z_3Y_4(1+1/K) + (Z_1+Z_3)Y_2(1+1/K) + (1/K)+1+Z_1Y_2\right]}$ $= \frac{-2}{(1+1/K)\left[Z_1Z_3Y_2Y_4+(Z_1+Z_3)Y_2 + Z_3Y_4\right] + 1/K}$ Which is correct within a constant multiplier (2)	5-a-1 
5-b 	$\frac{V_o}{V_1} = \frac{1X1+1}{1-\left[Z_1Y_2+Z_3Y_4(1+1/K)+1/2K \; Z_1Z_3Y_2Y_4(1+1/K)+Z_1Y_2/2K + (Z_1+Z_3)Y_2(1+1/K) +1 + Z_1Y_2(1+1/2K)\right]}$ $= \frac{-2}{(1+1/K)\left[Z_1Z_3Y_2Y_4+Z_3Y_4+(Z_1+Z_3)Y_2\right] +1/K}$ Which is correct within a constant multiplier (2)	5-b-1 SAME AS 5-a-1

CHAPTER VI

Transfer function synthesis of finite gain operational amplifier networks using Signal Flow Graph

6-0 Introduction:-

We show in this chapter how the SFGs, which have been deduced in Chapter V can be used to realize transfer functions. We will consider three cases:-

Case (s-a):- Synthesis of transfer functions having negative real axis poles and zeros.

Case (s-b):- Synthesis of transfer functions of lowpass filters with real or complex poles and zeros.

Case (s-c):- Synthesis of any second order transfer functions with real or complex poles and zeros.

Case (s-a) is considered because the treatment (and hence the realizations) in case (s-b) and (s-c) at times may depend on case (s-a).

6-1 Case (s-a) synthesis of transfer functions having poles and zeros on the negative real axis:-

Case (s-a-1):-

Consider the transfer function

$$T_v = \frac{1}{(s+\sigma_1)(s+\sigma_2)} \tag{6-1}$$

where σ_1 and σ_2 are positive real constants.

If we choose to realize this transfer function using configurations similar to configuration 3, we may represent the quantity

$$T_{1v} = \frac{-1}{s+\sigma_1} \tag{6-2}$$

by the SFG shown in figure 6-1 which is similar to SFG(3-c).

We may also represent the quantity

$$T_{2v} = \frac{-1}{s+\sigma_2} \tag{6-3}$$

by the SFG shown in figure 6-2.

which has also the general form of SFG (3-a). Since we are using configuration 3 for the realization of T_{1v} and T_{2v}, we can write

$$T_v = T_{1v} T_{2v} \tag{6-4}$$

which may be considered to represent the cascading of the two networks. Identification of the network parameters can be achieved by several way. Some of them are shown here.

i) Equal resistances prescribed to a certain value α.

Let

$$Z_1^{(1)} = Z_1^{(2)} = \alpha \tag{6-5-a}$$

and

$$Y_F^{(1)} = \frac{s}{(1+\sigma_1)\alpha} \qquad Y_F^{(2)} = \frac{s}{(1+\sigma_2)\alpha} \tag{6-5-b}$$

This identification leads to the network shown in figure 6-3.

ii) Equal capacitances prescribed to a certain value α.

Let

$$Y_F^{(1)} = Y_F^{(2)} = \alpha s \tag{6-6-a}$$

and

$$Z_1^{(1)} = \frac{1}{(1+\sigma_1)\alpha} \qquad Z_1^{(2)} = \frac{1}{(1+\sigma_2)\alpha} \tag{6-6-b}$$

$$K_1 = \frac{1}{\sigma_1} \qquad K_2 = \frac{1}{\sigma_2} \tag{6-6-c}$$

This identification leads to the network shown in figure 6-4.

iii) Arbitrary choice of resistances and capacitances.

Let

$$Z_1^{(1)} = \frac{1}{\alpha_1} \qquad Z_1^{(2)} = \frac{1}{\alpha_2} \tag{6-7-a}$$

and

$$Y_F^{(1)} = \frac{\alpha_1 s}{1+\sigma_1} \qquad Y_F^{(2)} = \frac{\alpha_2 s}{1+\sigma_2} \tag{6-7-b}$$

$$K_1 = \frac{1}{\sigma_1} \qquad K_2 = \frac{1}{\sigma_2} \tag{6-7-c}$$

α_1 and α_2 are positive constant parameters which can be chosen in order to fulfill some special conditions or requirements.

This identification leads to the network shown in figure 6-5.

iv) Minimization of C_{Tot} with R_{Tot} prescribed to a desired value K.

Using the identification which has been already used in (iii) (equation (6-7)) and the network shown in figure 6-5, we have:-

$$R_{Tot} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} = K \tag{6-8}$$

$$C_{Tot} = \frac{\alpha_1}{1+\sigma_1} + \frac{\alpha_2}{1+\sigma_2} \tag{6-9}$$

$$\alpha_1 = \frac{\alpha_2}{\alpha_2 - K} \tag{6-10}$$

$$C_{Tot} = \frac{\alpha_2}{(\alpha_2-K)(1+\sigma_1)} + \frac{\alpha_2}{1+\sigma_2} \tag{6-11}$$

Putting

$$\frac{\partial C_{Tot}}{\partial \alpha_2} = \frac{1}{1+\sigma_1} \cdot \frac{(\alpha_2-K)-\alpha_2}{(\alpha_2-1)^2} + \frac{1}{1+\sigma_2} = 0 \tag{6-12}$$

we get:-

$$\frac{1}{1+\sigma_2} - \frac{K}{(1+\sigma_1)(\alpha_2-K)^2} = 0 \tag{6-13}$$

$$(\alpha_2-K)^2 = \frac{1+\sigma_2}{1+\sigma_1} \tag{6-14}$$

$$\alpha_2 = K + \sqrt{\frac{1+\sigma_2}{1+\sigma_1}} \tag{6-15}$$

$$\alpha_1 = \frac{K+\sqrt{(1+\sigma_2)/(1+\sigma_1)}}{\sqrt{(1+\sigma_2)/(1+\sigma_1)}} \tag{6-16}$$

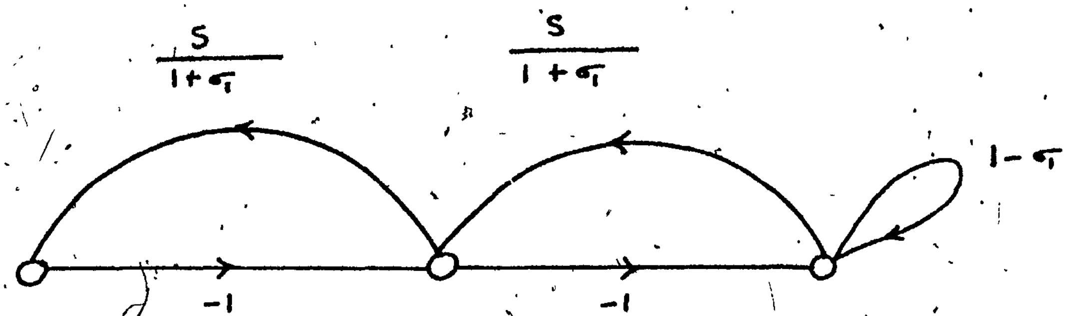


Figure 6-1. A possible SFG for $T_{1v} = \dfrac{-1}{S + \sigma_1}$

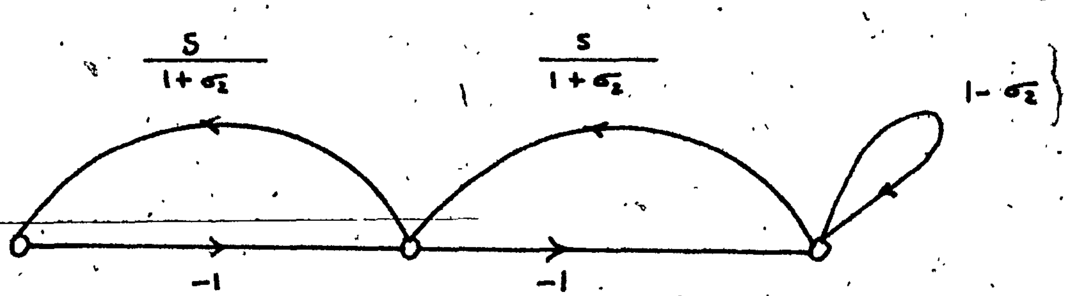


Figure 6-2. A possible SFG for $T_{2v} = \dfrac{-1}{S + \sigma_2}$

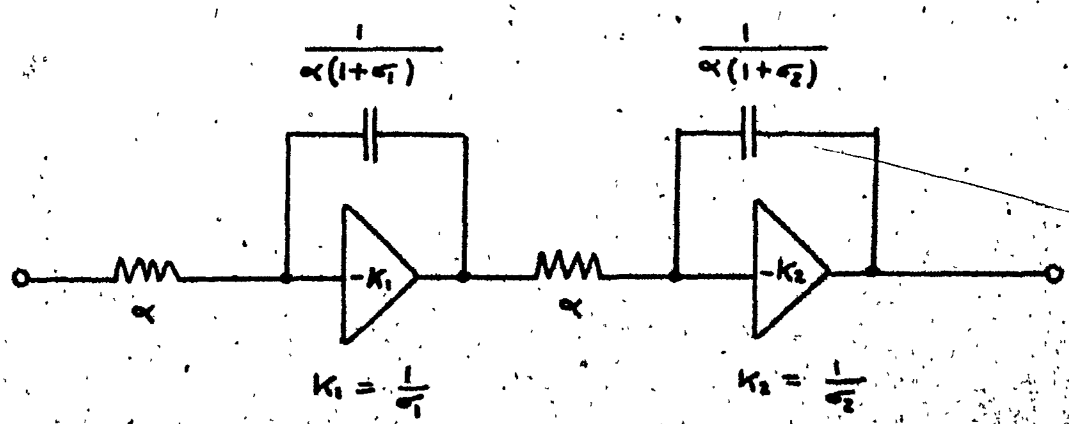


Figure 6-3. A network realizing the transfer function $T_v = \dfrac{1}{(S+\sigma_1)(S+\sigma_2)}$

with equal resistances prescribed to a desired value α.

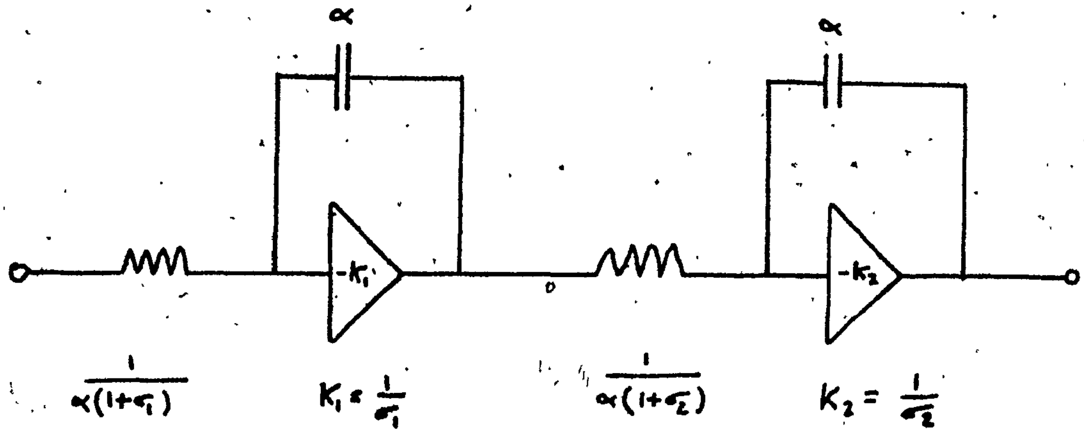


Figure 6-4. A network realizing the transfer function $T_v = \frac{1}{(S+\sigma_1)(S+\sigma_2)}$ with equal capacitances prescribed to a desired value α

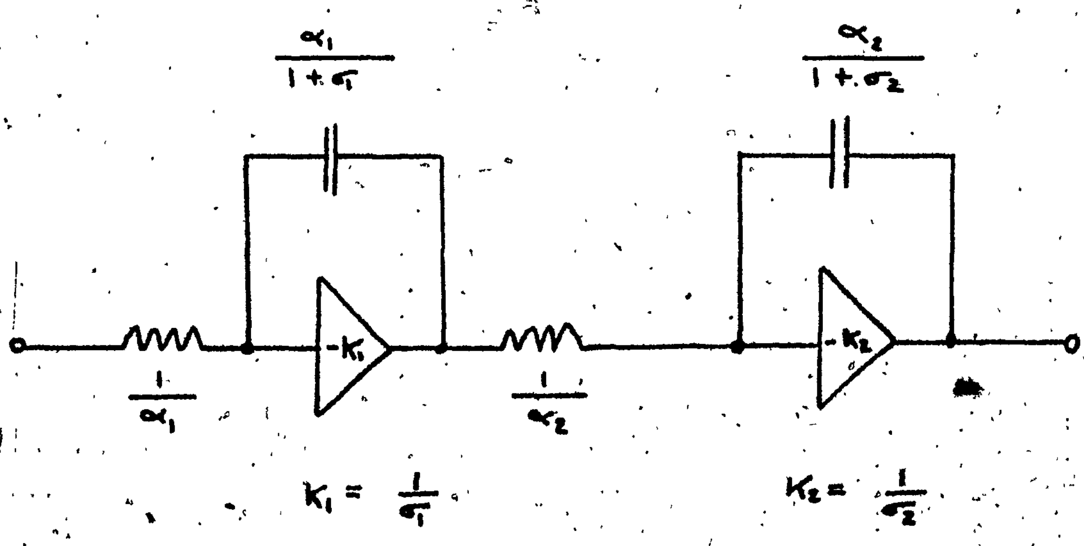


Figure 6-5. A network realizing the transfer function $T_v = \frac{1}{(S+\sigma_1)(S+\sigma_2)}$ and fulfilling different possible prescribed conditions according to the values assigned to the parameters α_1 and α_2

It is obvious that it is mandatory for the physical realization of the network to be possible that α_1 and α_2 be positive quantities. Notice that we chose the positive value of the square root in equation (6-14) because the negative value would lead to a negative value of either α_1 or α_2. It could be easily verified that this leads to a minimum value of C_{Tot}.

v) Minimization of R_{Tot} with C_{Tot} prescribed to a desired value (k):-

Using the identification which has been already used in (iii) and (iv) (equation (6-7) and the network shown in figure 6-5; we have:-

$$C_{Tot} = \frac{\alpha_1}{1+\sigma_1} + \frac{\alpha_2}{1+\sigma_2} = K \tag{6-17}$$

$$R_{Tot} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \tag{6-18}$$

Let $(1+\sigma_1) = x_1$ and $(1+\sigma_2) = x_2$ (6-19)

$$\frac{\alpha_1}{x_1} = K - \frac{\alpha_2}{x_2} \tag{6-20}$$

$$\alpha_1 = Kx_1 - \left(\frac{x_1}{x_2}\right)\alpha_2 \tag{6-21}$$

$$R_{Tot} = \frac{1}{Kx_1 - \left(\frac{x_1}{x_2}\right)\alpha_2} + \frac{1}{\alpha_2} \tag{6-22}$$

$$R_{Tot} = \frac{x_1 x_2}{Kx_1x_2 - x_1x_2} + \frac{1}{\alpha_2} \tag{6-23}$$

To minimize R_{Tot} we should have

$$\frac{\partial R_{Tot}}{\partial \alpha_2} = \frac{x_1 x_2}{(Kx_1x_2 - x_1\alpha_2)^2} - \frac{1}{\alpha_2^2} \tag{6-24}$$

$$\text{or } x_1 x_2 \alpha_2^2 - \alpha_2^2 (Kx_1x_2 - x_1\alpha_2)^2 = 0 \tag{6-25}$$

$\alpha_2 = 0$ is a trivial solution.

The other value of α_2 would be

$$\alpha_2 = Kx_2 \pm \frac{\sqrt{x_1 x_2}}{x_1} \tag{6-26}$$

To obtain the minimization of R_{Tot} we choose

$$\alpha_2 = Kx_2 - \sqrt{\frac{x_2}{x_1}} \tag{6-27}$$

Hence

$$\alpha_1 = \sqrt{\frac{x_2}{x_1}} \tag{6-28}$$

It is clear that K is bounded by the relation

$$K \geq \sqrt{\frac{1}{x_1 x_2}} \tag{6-29}$$

because α_1 and α_2 have to be positive quantities for physical realizability.

Case(s-a-2):-

Consider the transfer function

$$T_v = \frac{(s+\sigma_1)(s+\sigma_2)}{(s+\sigma_3)(s+\sigma_4)} \tag{6-30}$$

where σ_1, σ_2, σ_3, σ_4 are positive real constants. If we choose to realize this transfer function by configurations similar to configuration 3, we may represent the quantity

$$T_{1v} = -\frac{(s+\sigma_1)}{(s+\sigma_3)} \tag{6-31}$$

by the S F G shown in figure 6-6 which is similar to S F G (3-c) and the quantity

$$T_{2v} = -\frac{(s+\sigma_2)}{(s+\sigma_4)} \tag{6-32}$$

by the S F G shown in figure 6-7.

For the networks which we are using we have

$$T_v = T_{1v}\ T_{2v} \tag{6-33}$$

and this can be realized by the cascading of two networks similar to configuration 3. Identification of the network elements can be achieved in several ways and some of them are shown here.

In general we can let:-

$$Z_1^{(1)} = \frac{1}{\alpha_1[s(1+\frac{1}{K_1})+\sigma_1(1+\frac{1}{K_1})]} \qquad Y_F^{(1)} = \alpha_1[s(1-\frac{1}{K_1})+(\sigma_3-\frac{\sigma_1}{K_1})] \tag{6-34-a}$$

$$Z_1^{(2)} = \frac{1}{\alpha_2[s(1+\frac{1}{K_2})+\alpha_2(1+\frac{1}{K_2})]} \qquad Y_F^{(2)} = \alpha_2[s(1-\frac{1}{K_2})+(\sigma_4-\frac{\sigma_2}{K_2})] \tag{6-34-b}$$

with the conditions

$$K_1 \geq 1 \quad \sigma_3 \geq \frac{\sigma_1}{K_1} \quad \sigma_4 \geq \frac{\sigma_2}{K_2} \quad K_2 \geq 1 \tag{6-34-c}$$

Hence the general form of the network will be the one shown in figure 6-8. It may be mentioned here that there exist other identifications for $Z_1^{(1)}$ $Z_1^{(2)}$ $Y_F^{(1)}$ and $Y_F^{(2)}$ and hence different solutions are possible.

i) Equal resistances prescribed to a certain value $\frac{1}{K}$.

The conditions are:-

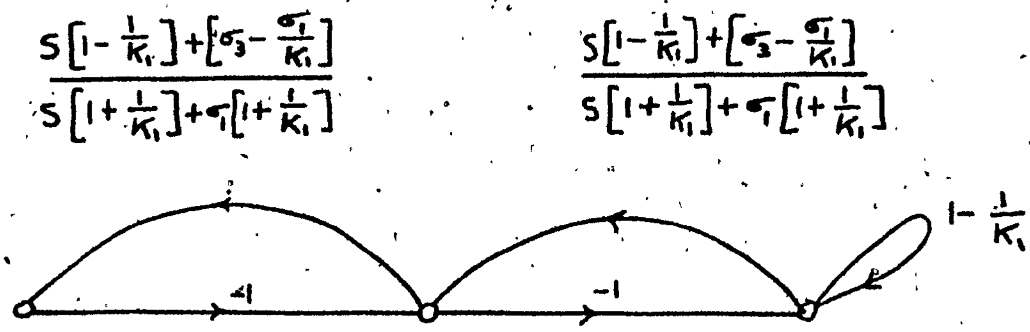
$$\alpha_1\sigma_1(1+\frac{1}{K_1}) = K \tag{6-35-a}$$

$$\alpha_1(\sigma_3-\frac{\sigma_1}{K_1}) = K \tag{6-35-b}$$

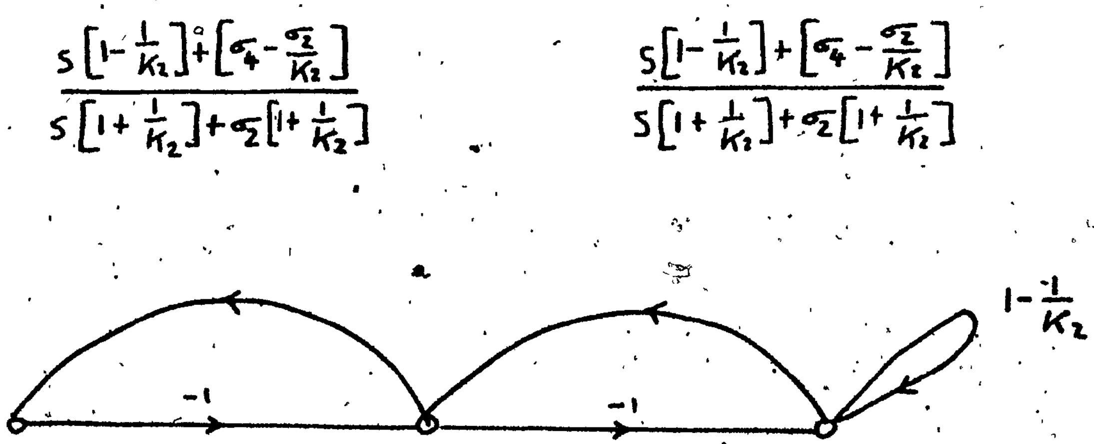
$$\alpha_2\sigma_2(1+\frac{1}{K_2}) = K \tag{6-35-c}$$

$$\alpha_2(\sigma_4-\frac{\sigma_2}{K_2}) = K \tag{6-35-d}$$

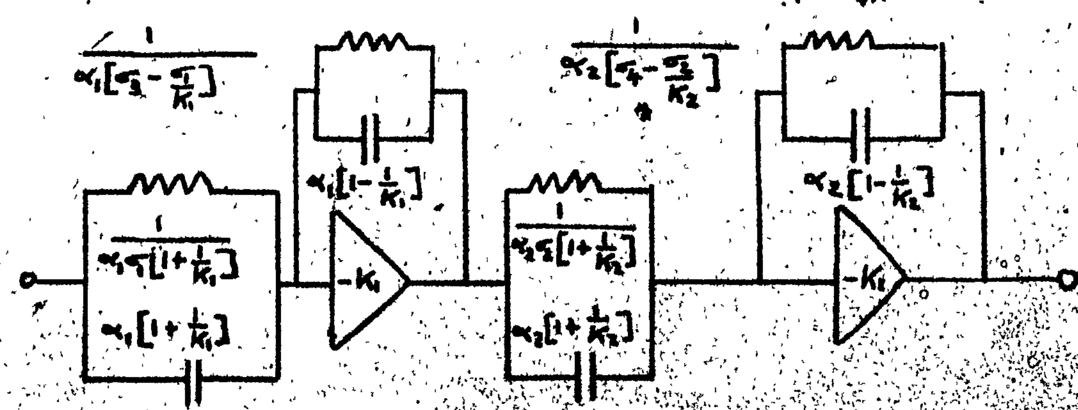
Hence



<u>Figure 6-6.</u> A possible SFG for $T_{1v} = - \dfrac{(S+\sigma_1)}{(S+\sigma_3)}$



<u>Figure 6-7.</u> A possible SFG for $T_{2v} = - \dfrac{(S+\sigma_2)}{(S+\sigma_4)}$



<u>Figure 6-8.</u> A network realizing the transfer function $T_v = \dfrac{(S+\sigma_1)(S+\sigma_2)}{(S+\sigma_3)(S+\sigma_4)}$ and fulfilling different possible prescribed conditions according to the values assigned to the parameters [illegible] and [illegible]

$$\sigma_1(1+\frac{1}{K_1}) = (\sigma_3 - \frac{\sigma_1}{K_1}) \qquad (6\text{-}36)$$

$$K_1 = \frac{2\sigma_1}{\sigma_3-\sigma_1} \qquad (6\text{-}37)$$

$$\alpha_1 = \frac{K}{\sigma_1(1+\frac{1}{K_1})} \qquad (6\text{-}38)$$

$$\alpha_1 = \frac{K}{\sigma_1(1+\frac{\sigma_3-\sigma_1}{2\sigma_1})} = \frac{K}{\sigma_1+\frac{\sigma_3-\sigma_1}{2}} = \frac{2K}{2\sigma_1-\sigma_3} \qquad (6\text{-}39)$$

Also

$$K_2 = \frac{2\sigma_2}{\sigma_4-\sigma_2} \qquad (6\text{-}40)$$

$$\alpha_2 = \frac{2K}{2\sigma_2-\sigma_4}$$

Since α_1 and α_2 must be positive quantities the network is physically realizable if

$$\sigma_3 > \sigma_1 \qquad 2\sigma_1 > \sigma_3 \qquad (6\text{-}40\text{-a})$$

and

$$\sigma_4 > \sigma_2 \qquad 2\sigma_2 > \sigma_4 \qquad (6\text{-}40\text{-b})$$

We notice that the admissible values of σ_1, σ_2, σ_3, σ_4 are highly restrictive and K_1 and K_2 cannot be arbitrarily chosen but must have definite values.

ii) Equal capacitances prescribed to a certain value K.

The conditions are:-

$$\alpha_1[1+\frac{1}{K_1}] = K \qquad (6\text{-}41\text{-a})$$

$$\alpha_1[1-\frac{1}{K_1}] = K \qquad (6\text{-}41\text{-b})$$

$$\alpha_2[1+\frac{1}{K_2}] = K \qquad (6\text{-}41\text{-c})$$

$$\alpha_2[1-\frac{1}{K_2}] = K \qquad (6\text{-}41\text{-d})$$

It is clear that these requirements can be fulfilled only if:-

$$K_1 = K_2 \rightarrow \infty \tag{6-42}$$

$$\alpha_1 = \alpha_2 = K \tag{6-43}$$

iii) Prescribed values of two elements of the network.

It is clear that by adjusting the values of the two parameters α_1 and α_2, it is possible to prescribe the values of two elements of the network to any desired value with arbitrary choice of K_1 and K_2. Aslo α_1 and α_2 can be chosen in order to fulfill any given condition or requirement.

iv) Minimization of C_{Tot} with a prescribed value for $R_{Tot} = K$.

$$R_{Tot} = \frac{1}{\alpha_1}\left[\frac{1}{\sigma_1(1+\frac{1}{K_1})} + \frac{1}{\sigma_3 - \frac{\sigma_1}{K_1}}\right] + \frac{1}{\alpha_2}\left[\frac{1}{\sigma_2(1+\frac{1}{K_2})} + \frac{1}{\sigma_4 - \frac{\sigma_2}{K_2}}\right] \tag{6-44}$$

Let

$$\left[\frac{1}{\sigma_1(1+\frac{1}{K_2})} + \frac{1}{\sigma_3 - \frac{\sigma_1}{K_1}}\right] = x_1 \tag{6-45-a}$$

and

$$\left[\frac{1}{\sigma_2(1+\frac{1}{K_2})} + \frac{1}{\sigma_4 - \frac{\sigma_2}{K_2}}\right] = x_2 \tag{6-45-b}$$

Hence

$$K = \frac{x_1}{\alpha_1} + \frac{x_2}{\alpha_2} \tag{6-46}$$

$$C_{Tot} = \alpha_1\left[1+\frac{1}{K_1} + 1 - \frac{1}{K_1}\right] + \alpha_2\left[1+\frac{1}{K_2} + 1 - \frac{1}{K_2}\right] \tag{6-47}$$

$$C_{Tot} = 2\alpha_1 + 2\alpha_2 \tag{6-48}$$

$$\alpha_1 = \frac{x_1\alpha_2}{\alpha_2 K - x_2} \tag{6-49}$$

$$C_{Tot} = \frac{2x_1\alpha_2}{\alpha_2 K - x_2} + 2\alpha_2 \tag{6-50}$$

To minimize C_{Tot}, we should have:-

$$\frac{\partial C_{Tot}}{\partial \alpha_2} = \frac{2x_1[\alpha_2 K - x_2 - K\alpha_2]}{\alpha_2 K - x_2)^2} + 2 = 0 \tag{6-51}$$

$$x_1 x_2 = (\alpha_2 K - x_2)^2 = 0 \tag{6-52}$$

$$\alpha_2 = \frac{x_2 \pm \sqrt{x_1 x_2}}{K} \tag{6-53}$$

To obtain minimization of C_{Tot} and to avoid negative values of α_1 and α_2, we must take

$$\alpha_2 = \frac{x_2 + \sqrt{x_1 x_2}}{K} \tag{6-54}$$

Hence

$$\alpha_1 = \frac{x_1 + \sqrt{x_1 x_2}}{K} \tag{6-55}$$

v) Minimization of R_{Tot} with a prescribed value for $C_{Tot} = K$.

$$R_{Tot} = \frac{1}{\alpha_1}\left[\frac{1}{\sigma_1(1+\frac{1}{K_1})} + \frac{1}{\sigma_3 - \frac{\sigma_1}{K_1}}\right] + \frac{1}{\alpha_2}\left[\frac{1}{\sigma_2(1+\frac{1}{K_2})} + \frac{1}{\sigma_4 - \frac{\sigma_2}{K_2}}\right] \tag{6-56}$$

Let

$$\frac{1}{\sigma_1(1+\frac{1}{K_1})} + \frac{1}{\sigma_3 - \frac{\sigma}{K_1}} = x_1 \tag{6-56-a}$$

and

$$\frac{1}{\sigma_2(1+\frac{1}{K_2})} + \frac{1}{\sigma_4 - \frac{\sigma_2}{K_2}} = x_2 \tag{6-56-b}$$

$$R_{Tot} = \frac{x_1}{\alpha_1} + \frac{x_2}{\alpha_2} \tag{6-57}$$

$$C_{Tot} = 2\alpha_1 + 2\alpha_2 \tag{6-58}$$

$$\alpha_1 = \frac{K-2\alpha_2}{2} = \frac{K}{2} - \alpha_2 \tag{6-59}$$

$$R_{Tot} = \frac{x_1}{\frac{K}{2} - \alpha_2} + \frac{x_2}{\alpha_2} \tag{6-60}$$

In order to minimize R_{Tot} we must have

$$\frac{\partial R_{Tot}}{\partial \alpha_2} = \frac{x_1}{\left(\frac{K}{2} - \alpha_2\right)^2} - \frac{x_2}{\alpha_2^2} = 0 \tag{6-61}$$

$\alpha_2 \to \infty$ is a trivial solution, the other solution is

$$x_1\alpha_2^2 - x_2\left(\frac{K}{2} - \alpha_2\right)^2 = 0 \tag{6-62}$$

$$[\sqrt{x_1}\alpha_2 - \sqrt{x_2}(\frac{K}{2} - \alpha_2)][\sqrt{x_1}\,\alpha_2 + \sqrt{x_2}(\frac{K}{2} - \alpha_2)] = 0 \tag{6-63}$$

Hence

$$\alpha_2 = \frac{K\sqrt{x_2}}{2(\sqrt{x_1} + \sqrt{x_2})} \tag{6-64-a}$$

or

$$\alpha_2 = \frac{K\sqrt{x_2}}{2[\sqrt{x_2} - \sqrt{x_1}]} \tag{6-64-b}$$

Hence corresponding to (6-64-a) we have

$$\alpha_1 = \frac{K}{2}[1 - \frac{\sqrt{x_2}}{\sqrt{x_1} + \sqrt{x_2}}] \tag{6-65-a}$$

and corresponding to (6-64-b) we have

$$\alpha_1 = \frac{K}{2}[1 - \frac{\sqrt{x_2}}{\sqrt{x_2} - \sqrt{x_1}}] \tag{6-65-b}$$

The two different solutions and the corresponding restrictions are shown below:-

Solution No. 1

$$\alpha_1 = \frac{K}{2}\left[1 - \frac{\sqrt{x_2}}{\sqrt{x_1} + \sqrt{x_2}}\right]$$

$$\alpha_2 = \frac{K}{2}\,\frac{\sqrt{x_2}}{\sqrt{x_1} + \sqrt{x_2}}$$

Solution No. 2

$$\alpha_1 = \frac{K}{2}\left[1 - \frac{\sqrt{x_2}}{\sqrt{x_2} + \sqrt{x_1}}\right]$$

$$\alpha_2 = \frac{K\sqrt{x_2}}{2[\sqrt{x_2} - \sqrt{x_1}]}$$

with $\sqrt{x_2} > \sqrt{x_1}$

Case (s-a-3):-

Let us now consider again the transfer function

$$T_v = \frac{1}{(s+\sigma_1)(s+\sigma_2)} \tag{6-66}$$

We realize it now, using the network number 4 as a basic configuration. The S F G of the quantity

$$T_{1v} = \frac{1}{s+\sigma_1} \tag{6-67}$$

can be drawn in a form similar to S F G (4-a) and is represented in figure 6-9.

Similarly, the S F G of the quantity

$$T_{2v} = \frac{1}{s+\sigma_2} \tag{6-68}$$

may be represented by figure 6-10 which has also the general form of S F G (4-a).

Since we are using configuration 4 for the realization of T_{1v} and T_{2v}, we can write

$$T_v = T_{1v}\, T_{2v} \tag{6-69}$$

Using network number 4 as a basic block, several possible networks can realize the transfer function T_v. These networks depend upon the identification of the elements.

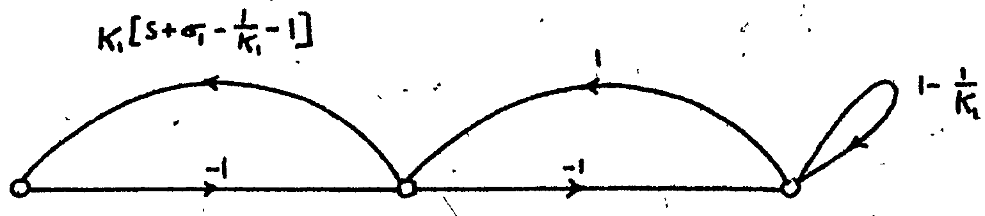


Figure 6-9. A possible SFG for $T_{1v} = \frac{1}{(S+\sigma_1)}$

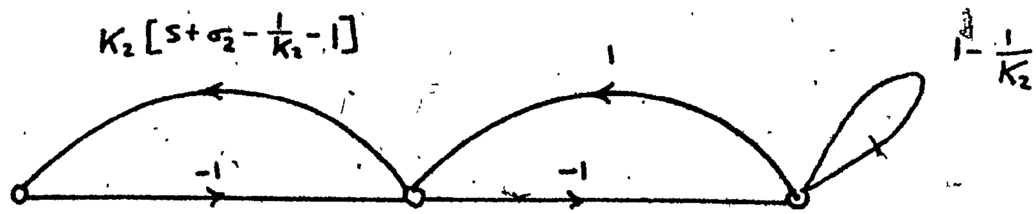


Figure 6-10. A possible SFG for $T_{2v} = \frac{1}{(S+\sigma_2)}$

It is evident that for physical realizability we should satisfy the conditions:-

$$\sigma_1 - 1 - \frac{1}{K_1} \geq 0 \qquad \text{(6-70-a)}$$

$$\sigma_2 - 1 - \frac{1}{K_2} \geq 0 \qquad \text{(6-70-b)}$$

making the realizable T_v very restrictive. However, if one is permitted to realize a given T_v within a constant multiplier, the above restrictive nature can be avoided. If $1/\beta_1$ is the constant mulitplier associated with T_{1v}, then we have the S F G and the realized network as shown in figures 6-11 and 6-12 respectively and β_1 is chosen such that

$$\beta_1\sigma_1 - 1 - \frac{1}{K_1} \geq 0 \qquad \text{(6-71-a)}$$

Similarly for T_{2v} the S F G and the realized network are shown in figures 6-13 and 6-14 respectively, with the constant multiplier $1/\beta_2$ and β_2 is chosen such that

$$\beta_2\sigma_2 - 1 - \frac{1}{K_2} \geq 0 \qquad \text{(6-71-b)}$$

Hence the total network realizing T_v within a factor $\frac{1}{\beta_1\beta_2}$ is shown in figure 6-15.

It is to be noted that the identification which has been used is

$$Z_F^{(1)} = K_1 \quad Y_1^{(1)} = \beta_1 s + (\beta_1\sigma_1 - 1 - \frac{1}{K_1}) \qquad \text{(6-72-a)}$$

$$Z_F^{(2)} = K_2 \quad Y_1^{(2)} = \beta_2 s + (\beta_2\sigma_2 - 1 - \frac{1}{K_2} \qquad \text{(6-72-b)}$$

i) Equal capacitances form prescribed to a desired value α.

Using the following identification:-

$$Z_F^{(1)} = \frac{K_1}{\alpha_1} \quad Y_1^{(1)} = \alpha_1\beta_1 s + \alpha_1(\beta_1\sigma_1 - 1 - \frac{1}{K_1}) \qquad \text{(6-73-a)}$$

$$Z_F^{(2)} = \frac{K_2}{\alpha_2} \quad Y_1^{(2)} = \alpha_2\beta_2 s + \alpha_2(\beta_2\sigma_2 - 1 - \frac{1}{K_2}) \qquad \text{(6-73-b)}$$

With the conditions

$$\alpha_2\beta_2\sigma_2 - 1 - \frac{1}{K_2} \geq 0 \quad \alpha_1\beta_1\sigma - 1 - \frac{1}{K_1} \geq 0 \qquad \text{(6-73-c)}$$

The network representing T_v with this identification is shown in figure 6-16. To determine the values of α_1 and α_2 for equal capacitances prescribed to a certain value α we must set

$$\alpha_1\beta_1 = \alpha \qquad \text{(6-74-a)}$$

$$\alpha_2\beta_2 = \alpha \qquad \text{(6-74-b)}$$

Since the values of β_1 and β_2 are determined from equation (6-73-c) we must have

$$\alpha_1 = \frac{\alpha}{\beta_1} \qquad 2 = \frac{\alpha}{\beta_2} \qquad \text{(6-75)}$$

ii) Equal resistances form prescribed to a certain value α.

Using the same identification and general network as in (i) it is clear that the conditions are

$$\frac{K_1}{\alpha_1} = \alpha \qquad \text{(6-76-a)}$$

$$\frac{K_2}{\alpha_2} = \alpha \qquad \text{(6-76-b)}$$

$$\frac{1}{\alpha_1(\beta_1\sigma_1 - 1 - \frac{1}{K_1})} = \alpha \qquad \text{(6-76-c)}$$

$$\frac{1}{\alpha_2(\beta_2\sigma_2 - 1 - \frac{1}{K_2})} = \alpha \qquad \text{(6-76-d)}$$

Hence

$$\alpha_1 = \frac{K_1}{\alpha} \qquad \alpha_2 = \frac{K_2}{\alpha} \qquad \text{(6-77)}$$

Replacing in (6-76-c) and (6-76-d) we get

$$\frac{\alpha}{K_1(\beta_1\sigma_1 - 1 - \frac{1}{K_1})} = \alpha \qquad \text{(6-78-a)}$$

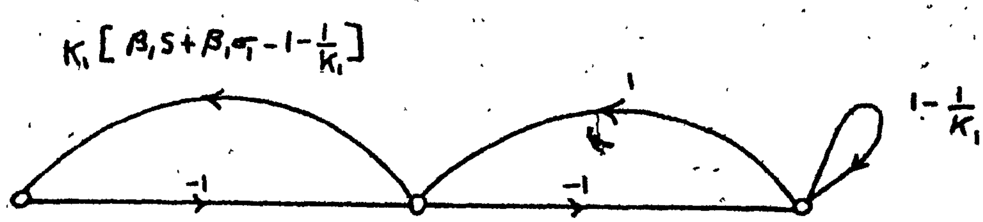


Figure 6-11. A possible SFG for $T_{1v} = \dfrac{1}{\beta_1(S+\sigma_1)}$

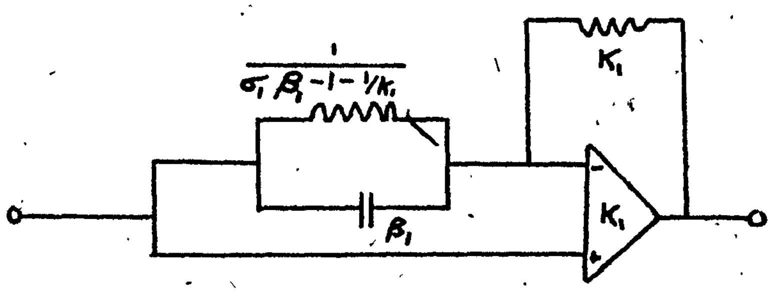


Figure 6-12. A network realizing the transfer function $T_{1v} = \dfrac{1}{\beta_1(S+\sigma_1)}$

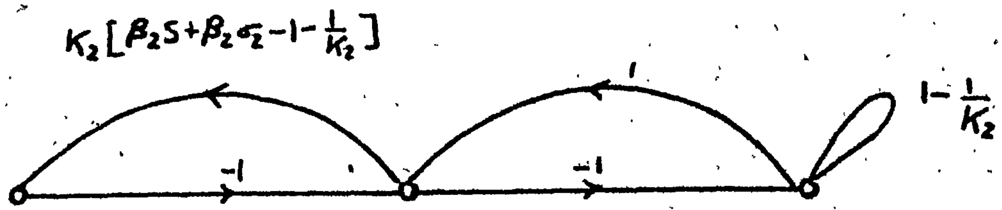


Figure 6-13. A possible SFG for $T_{2v} = \dfrac{1}{\beta_2(S+\sigma_2)}$

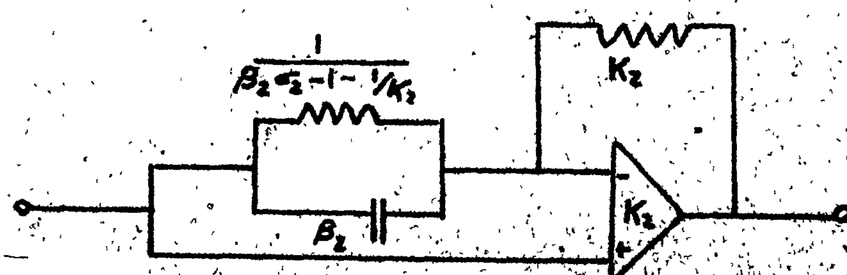


Figure 6-14. A network realizing the transfer function $T_{2v} = \dfrac{1}{\beta_2(S+\sigma_2)}$

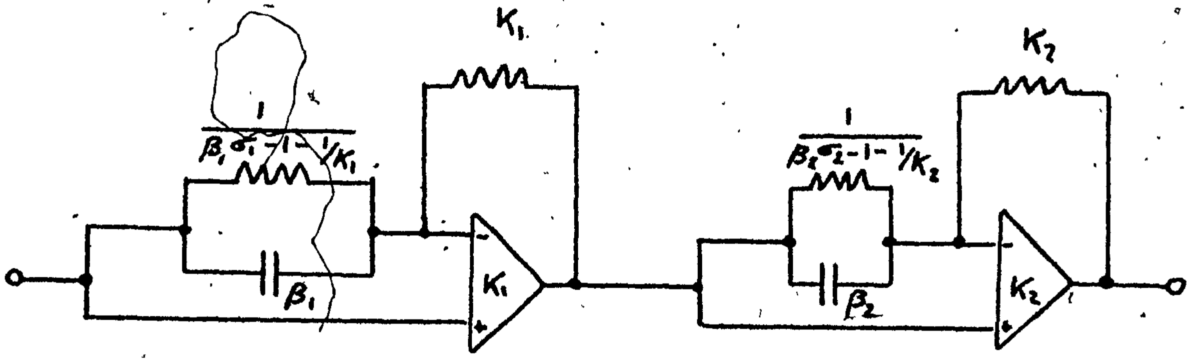


Figure 6-15. A network realizing the transfer function $T_V = \frac{1}{(S+\sigma_1)(S+\sigma_2)}$ within a constant multiplier.

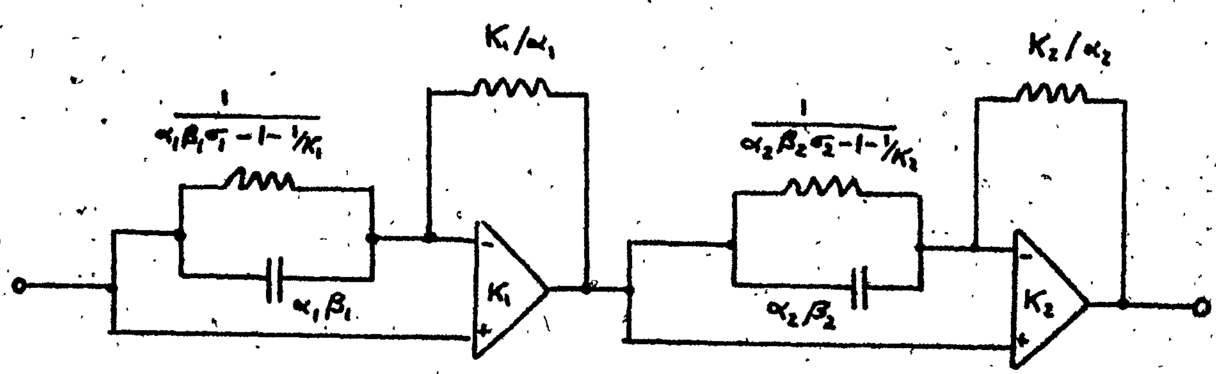


Figure 6-16. A network realizing the transfer function $T_V = \frac{1}{(S+\sigma_1)(S+\sigma_2)}$ within a constant multiplier and fulfilling different prescribed conditions according to the values assigned to the parameters β_1 and β_2

$$\frac{\alpha}{K_2(\beta_2\sigma_2 - 1 - \frac{1}{K_2})} = \alpha \tag{6-78-b}$$

Hence

$$K_1(\beta_1\sigma_1 - 1 - \frac{1}{K_1}) = 1 \tag{6-79-a}$$

and

$$K_2(\beta_2\sigma_2 - 1 - \frac{1}{K_2}) = 1 \tag{6-79-b}$$

Hence

$$K_1 = \frac{2}{\beta_1\sigma_1 - 1} \qquad K_2 = \frac{2}{\beta_2\sigma_2 - 1} \tag{6-80}$$

Now since from condition (6-73-c)

$$K_1 \geq \frac{1}{\beta_1\sigma_1 - 1} \qquad K_2 \geq \frac{1}{\beta_2\sigma_2 - 1} \tag{6-81}$$

It is clear that the four conditions can be fulfilled by a proper choice of K_1, K_2, β_1, β_2; but it is to be noted that the quantities K_1, K_2 are no more arbitrary but have to be fixed to a certain value.

iii) Arbitrary choice of two network elements.

Using again the same identification and the same network as in (i), it is possible by a proper choice of the parameters α_1 and α_2 to assign specified values to two elements of the network or to fulfill some special condition or requirement.

iv) Minimization of C_{Tot} with a prescribed value (K) of R_{Tot}.

Using again the same identification and the same general network as in (i), we have:-

$$C_{Tot} = \alpha_1\beta_1 + \alpha_2\beta_2 \tag{6-82}$$

$$R_{Tot} = \frac{1}{\alpha_1}[K_1 + \frac{1}{\beta_1\sigma_1 - 1 - \frac{1}{K_1}}] + \frac{1}{\alpha_2}[K_2 + \frac{1}{\beta_2\sigma_2 - 1 - \frac{1}{K_2}}] \tag{6-83}$$

Let

$$\beta_1 K_1 + \frac{\beta_1}{\beta_1\sigma_1 - 1 - \frac{1}{K_1}} = x_1 \tag{6-84-a}$$

and

$$\beta_2 K_2 + \frac{\beta_2}{\beta_2\sigma_2 - 1 - \frac{1}{K_2}} = x_2 \tag{6-84-b}$$

Hence

$$K = \frac{x_1}{\beta_1\alpha_1} + \frac{x_2}{\beta_2\alpha_2} \tag{6-85}$$

and if we rewrite equation (6-76) in the form

$$C'_{Tot} = 2\,C_{Tot} = 2\,\alpha_1\beta_1 + 2\,\alpha_2\beta_2 \tag{6-86}$$

we notice that equations (6-85) and (6-86) are respectively similar to equations (6-46) and (6-48) if the following transformation is used:-

Replace α_1 by $\alpha_1\beta_1$ (6-87-a)

Replace α_2 by $\alpha_2\beta_2$ (6-87-b)

The problem being the same as in case(s-a-2)(iv) the solution is given by equations (6-54) and (6-55). Using the mentioned above transformation we have

$$\alpha_2 = \frac{x_2 + \sqrt{x_1 x_2}}{\beta_2 K} \tag{6-88-a}$$

$$\alpha_1 = \frac{x_1 + \sqrt{x_1 x_2}}{\beta_1 K} \tag{6-88-b}$$

with the values of x_1 and x_2 as defined in equations (6-84-a) and (6-84-b).

v) Minimization of R_{Tot} with a prescribed value for $C_{Tot} = \frac{K}{2}$.

Using the same identification and the same general network as in (i) we have

$$C_{Tot} = \frac{K}{2} = \alpha_1\beta_1 + \alpha_2\beta_2 \qquad (6\text{-}89)$$

$$K = 2\beta_1\alpha_1 + 2\beta_2\alpha_2 \qquad (6\text{-}90)$$

$$R_{Tot} = \frac{1}{\alpha_1}[K_1 + \frac{1}{\beta_1\sigma_1 - 1 - \frac{1}{K_1}}] + \frac{1}{\alpha_2}[K_2 + \frac{1}{\beta_2\sigma_2 - 1 - \frac{1}{K_2}}] \qquad (6\text{-}91)$$

Using the same definition for x_1 and x_2 as in equations (6-84), we may write

$$R_{Tot} = \frac{x_1}{\alpha_1\beta_1} + \frac{x_2}{\alpha_2\beta_2} \qquad (6\text{-}92)$$

Using the transformations (6-87), equations (6-90) and (6-92) are similar respectively to equations (6-57) and (6-58) and since the problem is similar to case(s-a-2)(iv) the solution is given by equations (6-64) and (6-65).

Hence using transformations (6-87) and with the values of x_1 and x_2 as defined in equations (6-84) we have

<u>1st Solution</u>

$$\alpha_1 = \frac{K}{2\beta_1}[1 - \frac{\sqrt{x_2}}{\sqrt{x_2} + \sqrt{x_1}}]$$

$$\alpha_2 = \frac{K}{2\beta_2}[\frac{\sqrt{x_2}}{\sqrt{x_2} + \sqrt{x_1}}]$$

<u>2nd Solution</u>

$$\alpha_1 = \frac{K}{2\beta_1}[1 - \frac{\sqrt{x_2}}{\sqrt{x_2} + \sqrt{x_1}}]$$

$$\alpha_2 = \frac{K\sqrt{x_2}}{2\beta_2[\sqrt{x_2} - \sqrt{x_1}]}$$

with $\sqrt{x_2} > \sqrt{x_1}$

(6-93)

6-2 Part(s-b): Synthesis of lowpass second order transfer functions with real or complex poles:-

We shall discuss two methods and illustrate them by some examples.

6-2-1 Method 1

Any second order lowpass transfer function with real or complex poles can be expressed in the form

$$T_v = \frac{1}{D(s)} = \frac{1}{as^2 + bs + c} \tag{6-94}$$

The denominator D(s) can always be split into two parts

$$D(s) = D_1(s) + D_2(s) \tag{6-95}$$

such as

$$D_1(s) = k_1(s+\sigma_1)(s+\sigma_2) \tag{6-96-a}$$

$$D_2(s) = k_2(s+\sigma_3)(s+\sigma_4) \tag{6-96-b}$$

where σ_1, σ_2, σ_3, σ_4 are on the negative real axis of the s-plane.

It is easily verified using Mason's Formula that the S F G of T_v can be represented as shown in figure 6-17.

Using S F G(2-b)for identification purposes it is clear that the block diagram of the S F G of figure 6-17 can be represented as shown in figure 6-18.

Now since $\frac{1}{D_1(s)}$ and $\frac{D_1(s)}{KD_2(s)}$ are second order transfer functions with poles and zeros lying on the negative real axis of the s-plane we are able, using the techniques described in part(s-a),to realize them by finite gain operational amplifier networks.

Case (s-b-1):-

Consider the transfer function with complex poles

$$T_v = \frac{1}{s^2 + bs + c} = \frac{1}{D(s)} \tag{6-97}$$

Let us split the denominator D(s) in the following way:-

$$D(s) = D_1(s) + D_2(s) \tag{6-98}$$

where $D_1(s) = (c - b^2/4)$ (6-99-a)

$$D_2(s) = (s + b/2)^2 \tag{6-99-b}$$

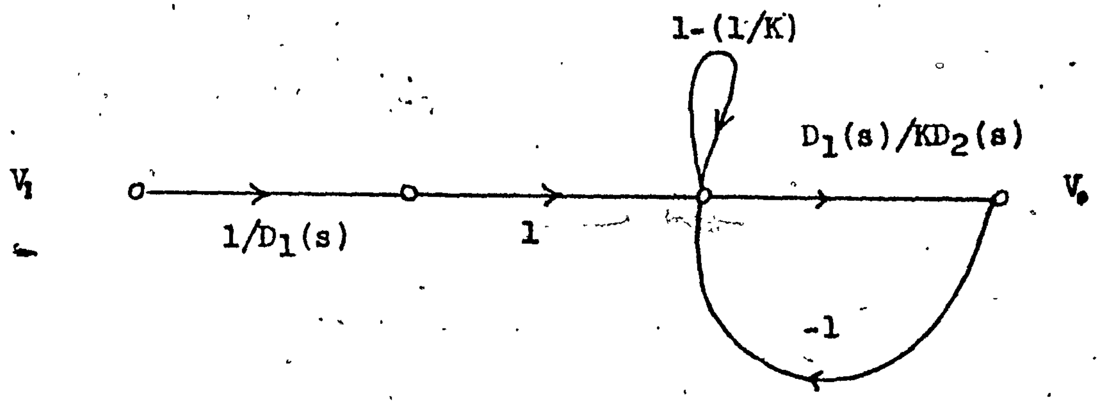


Figure 6-17. A possible SFG representing the transfer function:

$$T_v = \frac{1}{D_1(s) + D_2(s)}$$

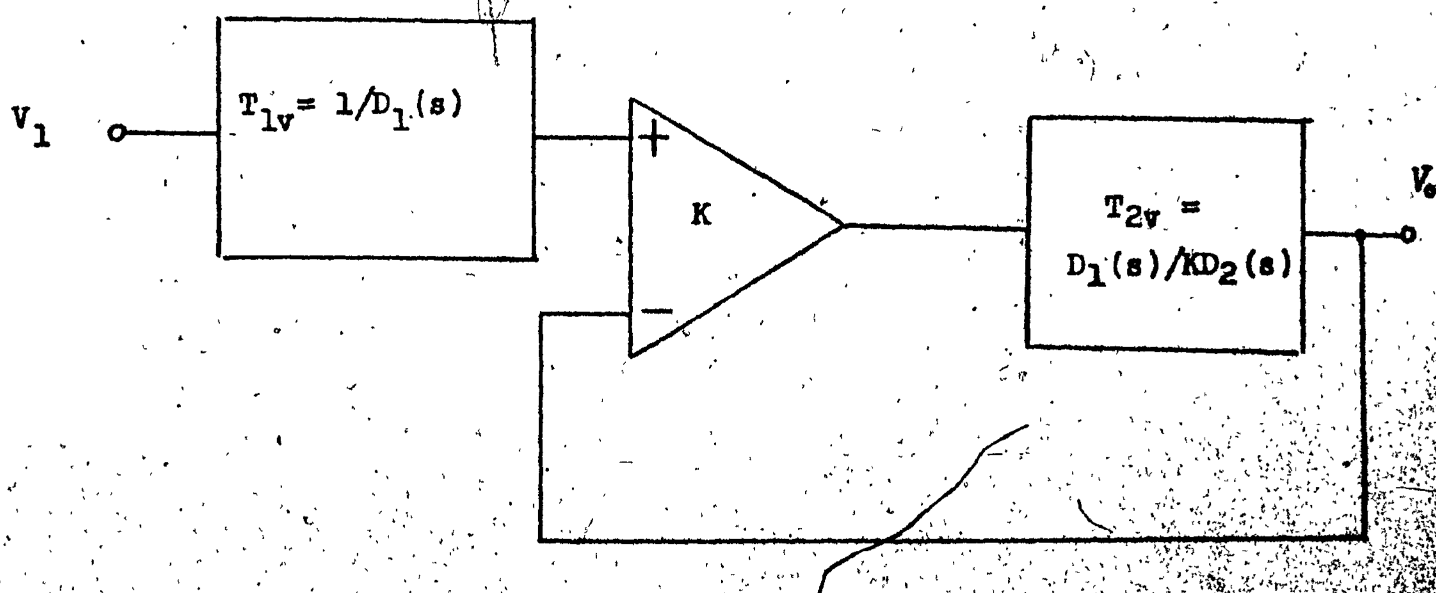


Figure 6-18. A block diagram representing a network realizing the transfer function $T_v = \frac{1}{D_1(s) + D_2(s)}$

Let us consider first the transfer function

$$T_{2v} = \frac{D_1(s)}{KD_2(s)} = \frac{c - b^2/4)}{K(s + b/2)^2} \qquad (6\text{-}100)$$

This transfer function may be written in the form

$$T_{2v} = \frac{(c - b^2/4)}{K} \times \frac{1}{(s + b/2)^2} \qquad (6\text{-}101)$$

Now since the poles of T_v are complex $c - b^2/4 > 0$ it is possible to let

$$K = c - b^2/4 \qquad (6\text{-}102)$$

and T_{2v} reduces to

$$T_{2v} = \frac{1}{(s + b/2)^2} \qquad (6\text{-}103)$$

A network representing T_{2v} has been deduced in case(s-a-1) and is shown in figure 6-5 provided that we set $\sigma_1 = \sigma_2 = \frac{b}{2}$.

Also

$$T_{1v} = \frac{1}{D_1(s)} = \frac{1}{(c - b^2/4)} \qquad (6\text{-}104)$$

If one is permitted to realize T_v within a constant multiplier $\frac{1}{(c - b^2/4)}$ the subnetwork T_{1v} may be removed from the total network representing T_v (see figure 6-20) and the network realizing $\hat{T}_v$ within a constant multiplier $\frac{1}{c - b^2/4}$ is shown in figure 6-19.

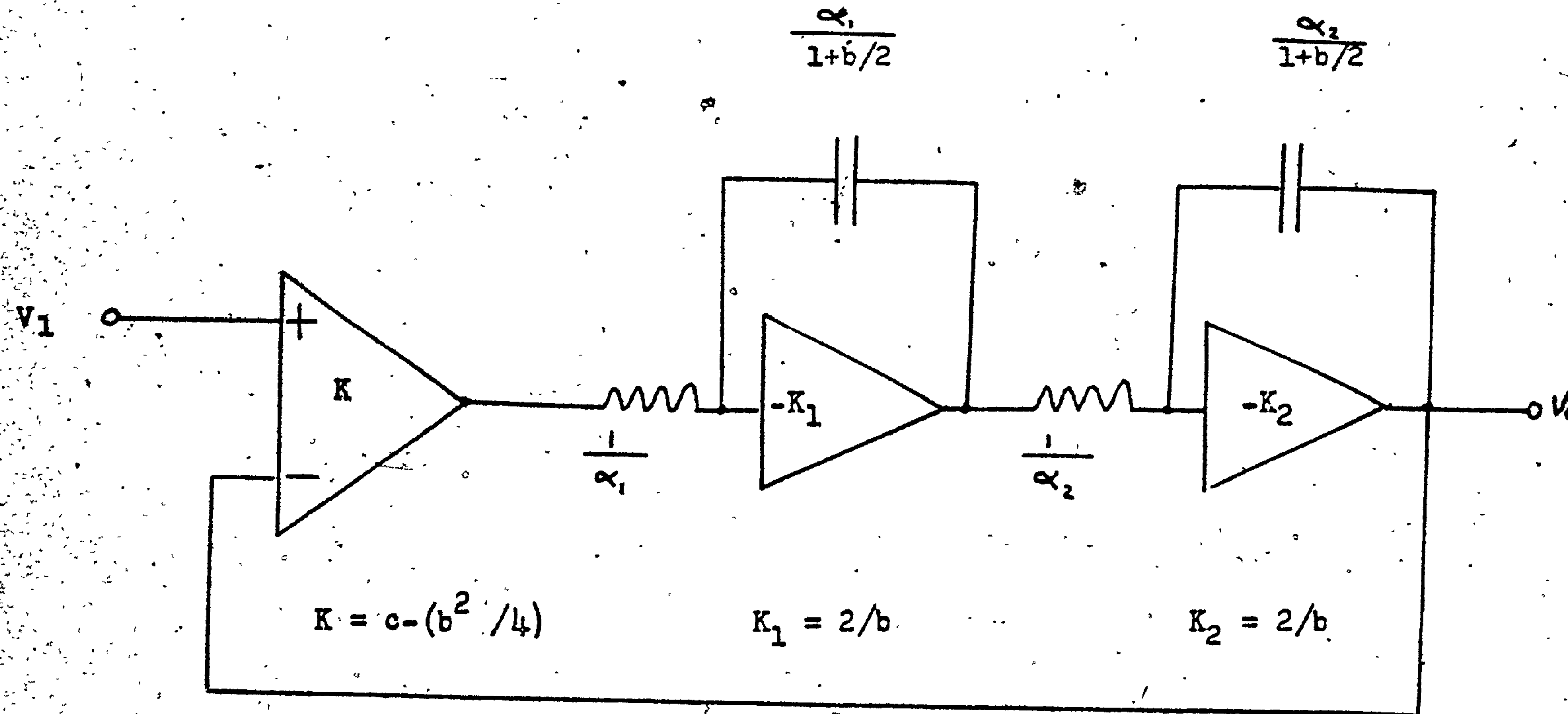
Case (s-b-2):-

Consider again the transfer function with complex poles

$$T_v = \frac{1}{s^2 + bs + c} \qquad (6\text{-}105)$$

and let us split the denominator as in case (s-b-1) paragraph 6-3-2.

Consider first the transfer function



<u>Figure 6-19.</u> A network realizing the transfer function $T_v = \frac{1}{S^2 + aS + b}$ within a constant multiplier.

$$T_{2v} = \frac{D_1(s)}{KD_2(s)} = \frac{c - b^2/4}{K(s + b/2)^2} \quad (6\text{-}106)$$

T_{2v} can be written in the form

$$T_{2v} = \frac{(c - b^2/4)\beta_1\beta_2}{K} \times \frac{1}{(s + b/2)^2 \beta_1\beta_2} \quad (6\text{-}107)$$

Now since the poles of T_v are complex it is possible to set

$$\beta_1\beta_2(c - b^2/4) = K \quad (6\text{-}108)$$

and T_{2v} reduces to

$$T_{2v} = \frac{1}{\beta_1\beta_2(s + b/2)^2} \quad (6\text{-}109)$$

A network representing exactly T_{2v} has been deduced in case (s-a-3) and is shown in figure 6-16, provided that we set $\sigma_1 = \sigma_2 = b/2$.

Now since $T_{1v} = \frac{1}{D_1(s)} = \frac{1}{c - b^2/4}$ (6-110)

if one is permitted to realize T_v within a constant multiplier $\frac{1}{c - b^2/4}$ the subnetwork T_{1v} may be removed from the total network representing T_v (see figure 6-20) and the network realizing T_v within a constant multiplier $\frac{1}{c - b^2/4}$ is shown in figure 6-20.

6-2-2 Method 2

Method (2) is based on drawing the S F G from the given transfer function and on obtaining the active circuit directly from the S F G by indentifying properly each part of it.

Case (s-b-3):-

Consider the transfer function with real or complex poles given by

$$T_v = \frac{+1}{k[s^2 + cs + d]} \quad (6\text{-}111)$$

where k, c, and d are positive real constants.

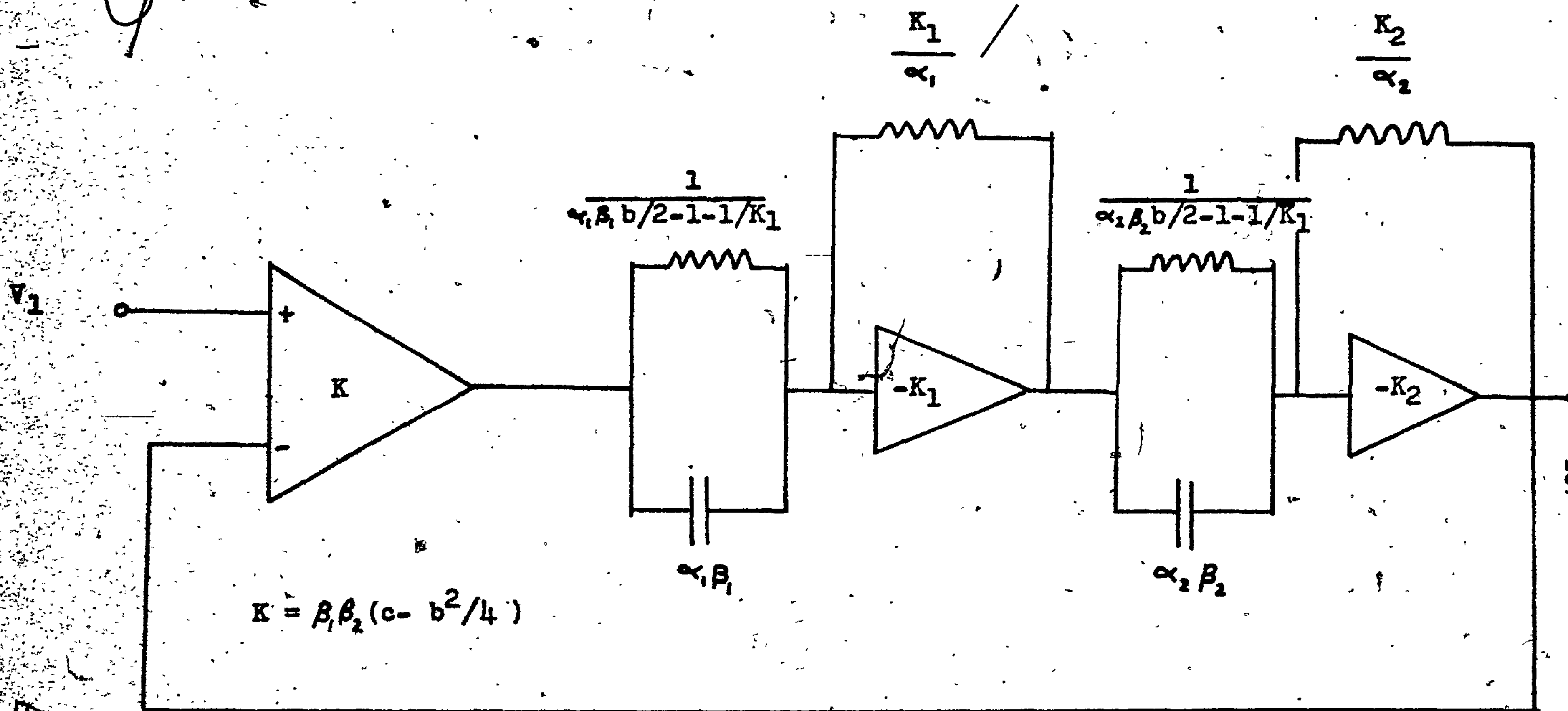


Figure 6-20. A network realizing the transfer function $T_v = \frac{1}{S^2 + aS + b}$ within a constant multiplier.

It is easily verified, using Mason's Gain Formula, that the S F G shown in figure 6-21 represents T_v where β is a positive real constant which will be determined later.

Using SFG (3-e) for identification purpose the network representing T_v is directly deduced and is shown in figure 6-22.

It is to be noted that the framed part of the SFG represented in figure 6-21 is realized by the framed part of the network represented in figure 6-22 while the remaining part of the SFG is realized by the remaining part of the network.

It is obvious that for physical realizability purposes the following conditions are mandatory:-

$$\beta \geq \frac{d}{c} \qquad (6\text{-}112\text{-}a)$$

$$K_k \geq k\beta \qquad (6\text{-}112\text{-}b)$$

$$K_1 \geq \frac{1+k(\beta c-d)}{\beta} \qquad (6\text{-}112\text{-}c)$$

$$K_2 \geq \frac{1}{kc} \qquad (6\text{-}112\text{-}d)$$

Several other suitable SFGs of T_v are possible and would lead to other representing networks but we shall not discuss them here.

<u>Case (s-b-4)</u>:-

Consider the transfer function

$$T_v = \frac{1}{as^2+bs+c} \qquad (6\text{-}113)$$

where $a \geq 0 \quad b \geq 0 \quad c \geq 0$

we will realize it now using a multiple feedback operational amplifier network similar to configuration 5.

The SFG representing T_v within a constant multiplier 1/2 is shown in figure 6-23 and is similar to SFG 5-1, A is defined by the equation

$$A = a(1+\frac{1}{C}) \qquad (6\text{-}114)$$

Using the following identifications of the network elements:-

$$K = \frac{1}{C} \qquad (6\text{-}115\text{-}a)$$

$$Z_1 = Z_3 = \frac{1}{1+\frac{1}{K}} \qquad (6\text{-}115\text{-}b)$$

$$Y_2 = \left(\frac{b \pm \sqrt{b^2-8A}}{4}\right)s \qquad (6\text{-}115\text{-}c)$$

$$Y_4 = \left(\frac{b \mp \sqrt{b^2-8A}}{4}\right)s \qquad (6\text{-}115\text{-}d)$$

The corresponding network is shown in figure 6-24.

It is obvious that it is mandatory for the physical realizability of the network that the condition

$$b^2 - 8A > 0 \qquad (6\text{-}116)$$

is fulfilled.

If this condition is not fulfilled the entire transfer function has to be multiplied by a constant factor $\frac{1}{k}$ where k is chosen so as to fulfill condition (6-116) in the form

$$b^2k^2 - ak\left(1 + \frac{1}{ck}\right) \geq 0 \qquad (6\text{-}117)$$

and in that case the transfer function will be realized by the network shown in figure 6-24 within a constant $\frac{1}{2k}$.

It is to be noted that the realization shown in figure 6-18 is an equal resistances realization. Also if $b^2 = 8A$ it becomes also an equal capacitances realization.

It also should be noted that this realization is valid whether the poles of T_v are real or complex as long as conditions (6-116) or (6-117) are fulfilled.

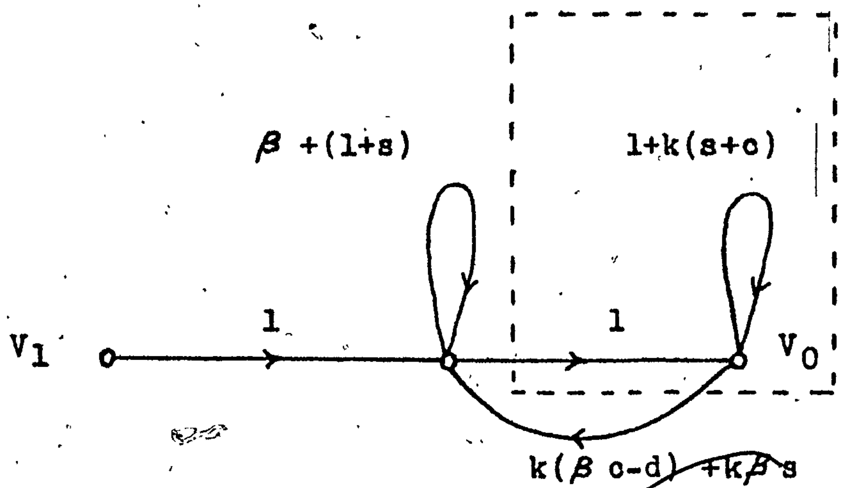


Figure 6-21. Apossible SFG for the transfer function $T_v = \frac{1}{k(s^2+cs+d)}$

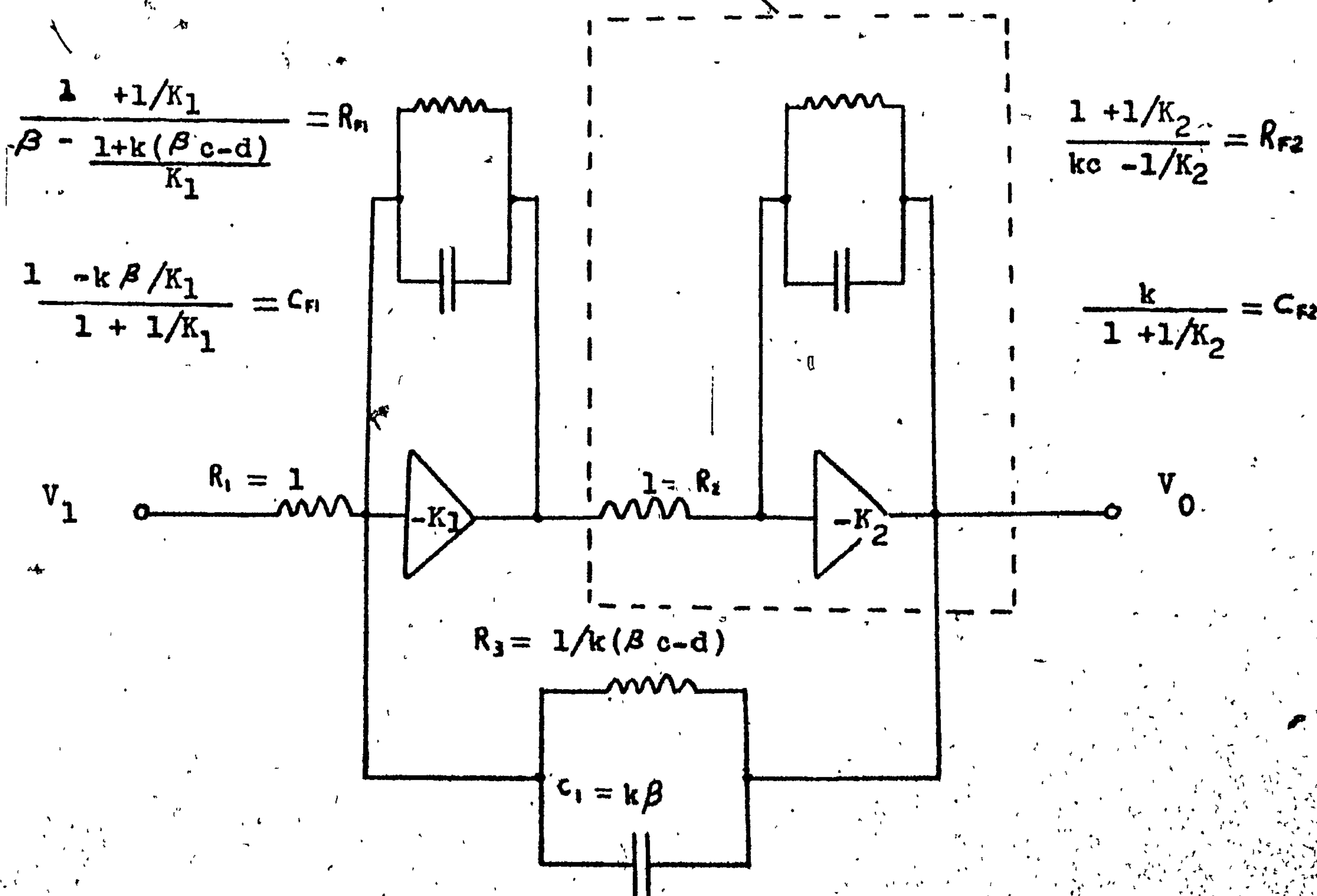


Figure 6-22. A network realizing the transfer function $T_v = \frac{1}{k(s^2+cs+d)}$

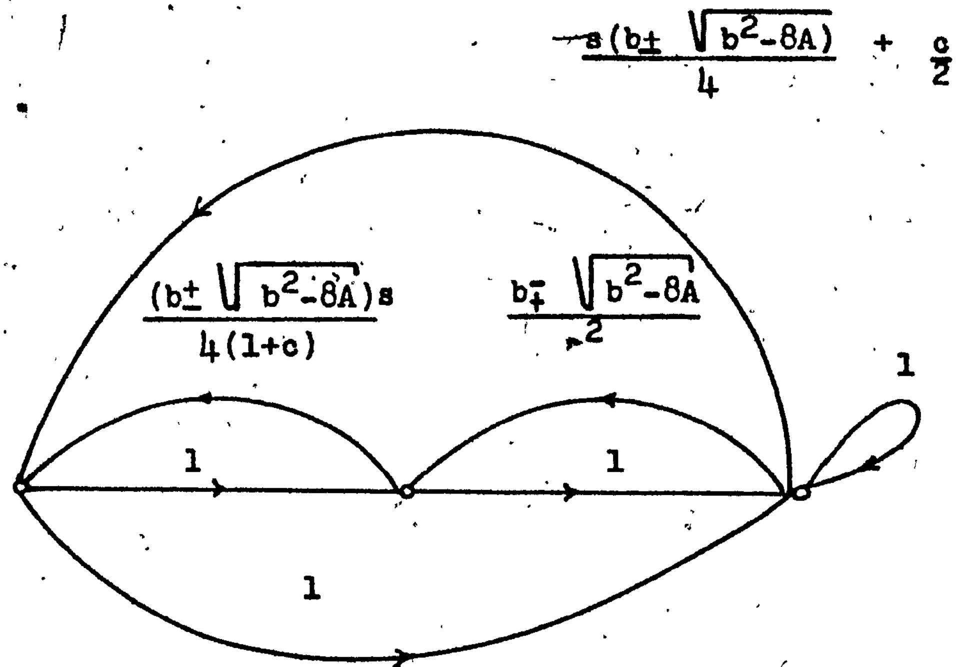


Figure 6-23. A possible SFG for the transfer function $T_v = \dfrac{2}{s^2+as+b}$

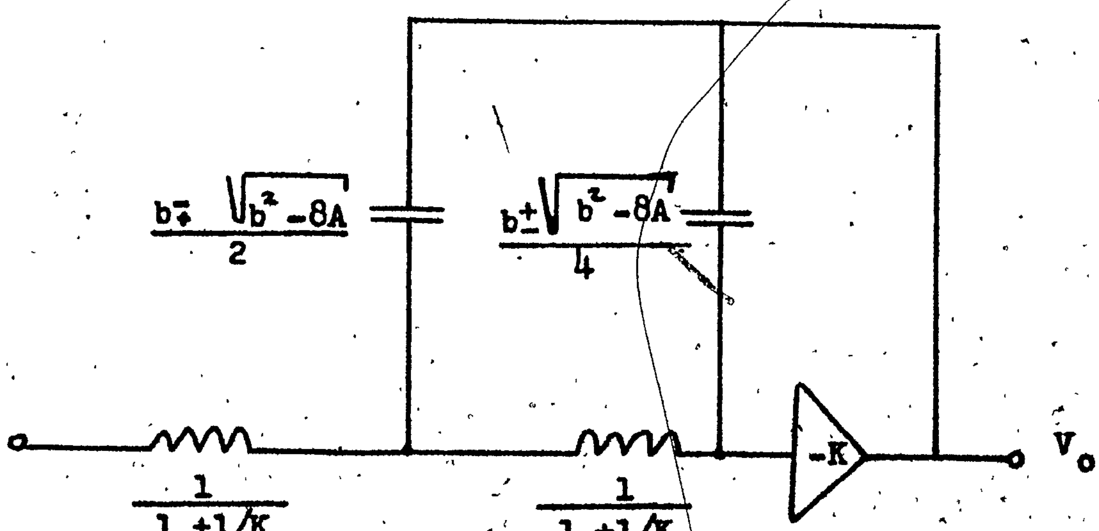


Figure 6-24. A network realizing the transfer function $T_v = \dfrac{1}{s^2+as+b}$ within a constant multiplier.

6-3 Synthesis of any second order transfer function with real or complex poles and zeros:-

We shall discuss here only one method and illustrate it by a typical example.

6-3-1 The Method

The method here is similar to Method 2 in case (s-b). It is based on drawing the SFG from the given transfer function and on obtaining the circuit directly from the SFG by identifying properly each part of it.

Case (s-c-1:-

Let the general biquadratic transfer function be given by

$$T_v(s) = \frac{-(s^2+as+b)}{k(s^2+cs+d)} \tag{6-118}$$

where k is the multiplying constant of T_v.

It is easily verified, using Mason's gain formula that the SFG shown in figure 6-25 is an exact representation of T_v.

β is a positive constant parameter which will be determined later.

Using SFG (3-e) for identification purposes the network representing T_v is directly derived and is shown in figure 6-26.

It should be noted that the part of the SFG shown in dotted lines in figure 6-25 is realized by the part of the network shown in dotted lines in figure 6-26.

The remaining part of the SFG is realized by the remaining part of the network.

It is obvious that for physical realization purposes, the following conditions are mandatory

$$\beta \geq \frac{d}{c} \tag{6-119-a}$$

$$\beta + \frac{b}{\beta} - a \geq 0 \tag{6-119-b}$$

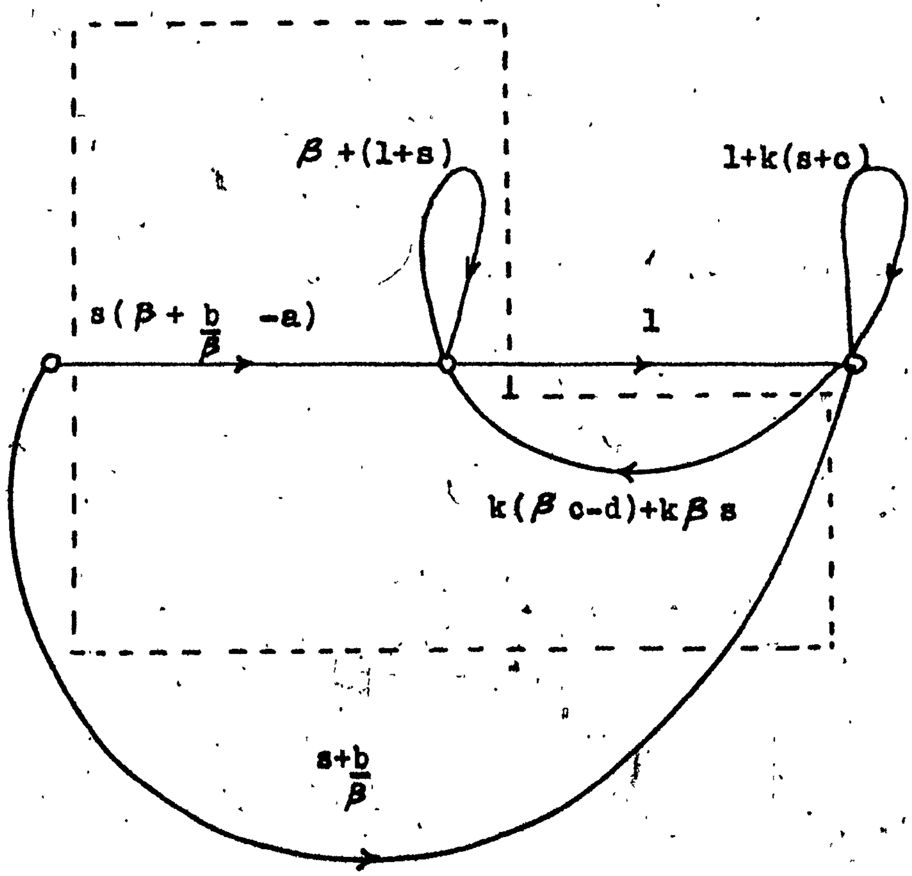


Figure 6-25. A possible SFG representation for the transfer function:

$$T_v = \frac{s^2+as+b}{k(s^2+cs+d)}$$

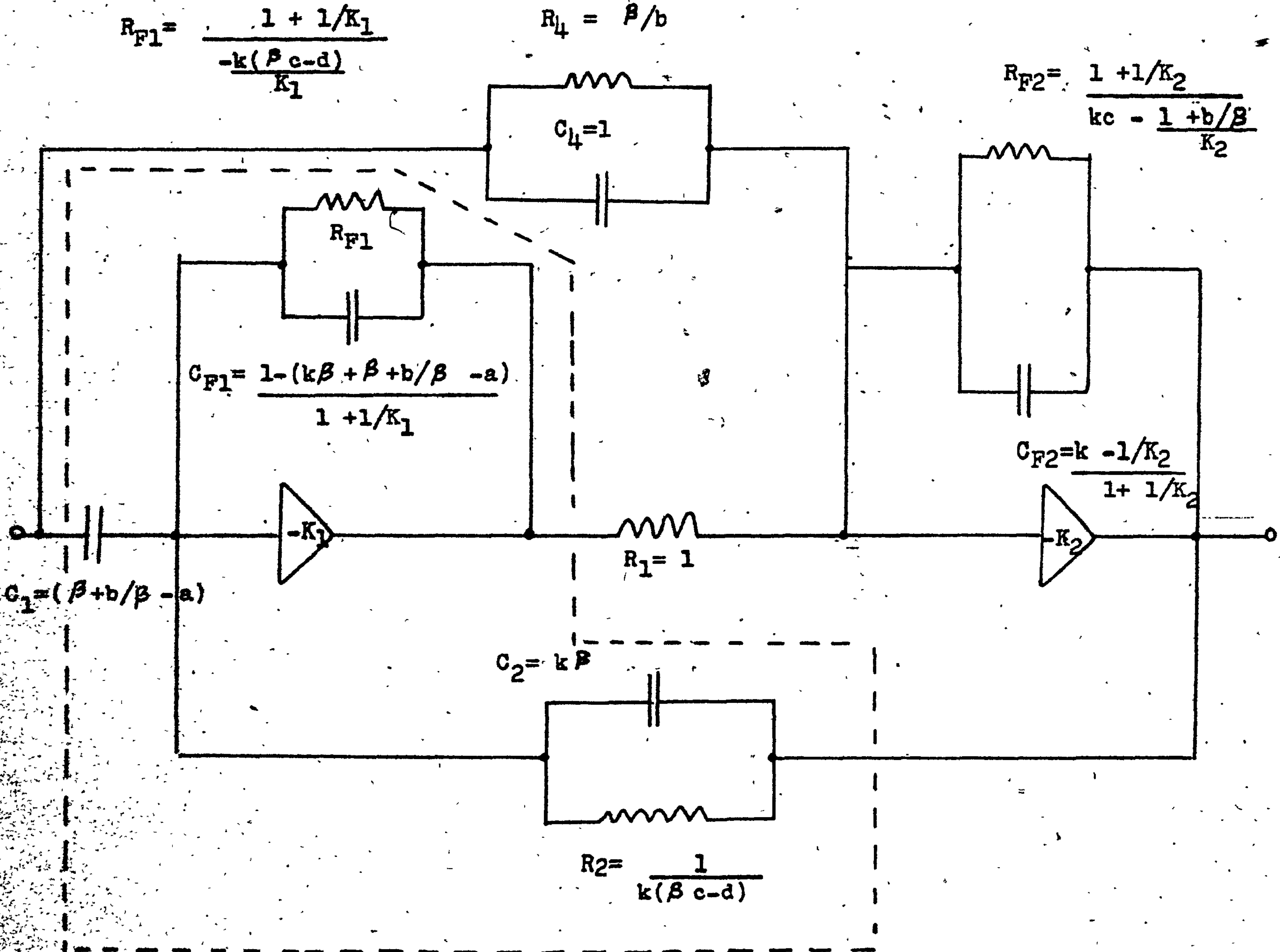


Figure 6-26. A network realizing the transfer function $T_V = \frac{s^2+as+b}{(s^2+cs+d)k}$

It is clear that the two previous conditions are compatible, since it would be sufficient to let $\beta \geq a$ to fulfill condition (6-119-b) and this would be compatible with condition (6-119-a) . Also, if the numerator of T(s) has complex zeros of transmission condition (6-119-b) is always satisfied.

$$K_1 \geq \beta(k+1) + \frac{b}{\beta} - a \qquad (6\text{-}119\text{-}c)$$

$$K_1' \geq \frac{k(\beta c-d)}{\beta} \qquad (6\text{-}119\text{-}d)$$

$$K_2 \geq \frac{1}{k} \qquad (6\text{-}119\text{-}e)$$

$$K_2 \geq \frac{1+b/\beta}{kc} \qquad (6\text{-}119\text{-}f)$$

For the sake of completeness we shall compute the sensitivities ω_p and Q_p with respect to the elements of the circuit shown in figure 6-26. The denominator $D(s) = ks^2+kcs+kd$ of the transfer function, computed from the realized network will be in the form:-

$$D(s) = \left[C_{F1}C_{F2}(1+\frac{1}{K_1})(1+\frac{1}{K_2}) + C_{F1}C_4(1+\frac{1}{K_1})\frac{1}{K_2} + C_{F2}(C_1+C_2)(1+\frac{1}{K_2})\frac{1}{K_1} + C_4(C_1+C_2)\frac{1}{K_1K_2}\right]s^2$$

$$+ \left[(C_{F1}C_{F2}+C_{F1}G_{F2})(1+\frac{1}{K_1})(1+\frac{1}{K_2}) + (G_{F1}C_4+C_{F1}G_3+C_{F1}G_4)(1+\frac{1}{K_1})\frac{1}{K_2} + (G_{F2}C_1+G_{F2}C_2+C_FG_2)(1+\frac{1}{K_2})\frac{1}{K_1} + (C_1G_3+C_1G_4+C_2G_3+C_2G_4+G_2C_4)\frac{1}{K_1K_2} - G_3C_2\right]s$$

$$+\left[\begin{array}{l} G_{F1}G_{F2}(1+\frac{1}{K_1})(1+\frac{1}{K_2}) \\ +(G_{F1}G_3+G_{F1}G_4)(1+\frac{1}{K_1})\frac{1}{K_2} \\ +(G_{F2}G_2)(1+\frac{1}{K_2})\frac{1}{K_1} \\ +(G_2G_3+G_2G_4)\frac{1}{K_1K_2} \end{array}\right] \quad (6\text{-}120)$$

It is seen that this realization contains negative parts in the s and s° terms. The negative part in the s° term can be eliminated by making $G_2 = 0$, which means that β shall be chosen as $\beta = \frac{d}{c}$ under this condition C_1 is always positive if we consider only complex zeros of transmission. However if we try to put the condition $G_{F2} = K_1G_3$, we see that the negative parts in both the s and s° terms are eliminated. Under this condition we show below that conditions (6-119) are always compatible.

The condition $G_{F2} = K_1G_3$ can be written in the form

$$k = \frac{K_1}{C} + \frac{K_1+1+b/\beta}{K_2C} \quad (6\text{-}121)$$

Condition (6-119-f) can be written in the form

$$k \geq \frac{1+b/\beta}{K_2C} \quad (6\text{-}119\text{-}f1)$$

and is always fulfilled, if condition (6-121) is fulfilled.

Condition (6-119-e) can be written in the form

$$k \geq \frac{1}{k_2} \quad (6\text{-}119\text{-}e1)$$

and is surely compatible with condition (6-121) if

$$K_1 \geq C \quad (6\text{-}122)$$

Condition (6-119-c) is compatible with condition (6-121) if

$$K_1 \geq \frac{\frac{\beta+b}{K_2C} + \beta + \frac{b}{\beta} - a}{1 - \frac{\beta}{C} - \frac{\beta}{K_2C}} \tag{6-123}$$

Condition (6-119-a) and (6-119-b) are always compatible with condition (6-121).

Hence β will be chosen from conditions (6-119-a) and (6-119-b), K_2 will be chosen arbitrarily, K_1 will be chosen from conditions (6-119-d), (6-122) and (6-123) and k will be calculated using condition (6-121).

Under the conditions $\beta = \frac{d}{c}$ and $G_{F2} = K_1G_3$ we have computed the sensitivities.

It can be seen that all the ω_p sensitivities are smaller than 1/2 while all the Q_p sensitivities are smaller than 1.

Since $\omega_p = \sqrt{\frac{kd}{k}}$ (6-124)

we may express ω_p as a function of C_{F1}, C_{F2}, C_1, C_2, C_4, G_{F1}, G_{F2}, G_3, G_4, K_1 and K_2 and hence compute the sensitivity of ω_p with respect to each of these parameters computation leads to:-

$$S^{\omega}_{G_{F1}} = \frac{G_{F1}G_{F2}(1+\frac{1}{K_1})(1+\frac{1}{K_2})+G_{F1}(G_3+G_4)(1+\frac{1}{K_1})\frac{1}{K_2}}{2kd} \tag{6-125-a}$$

$$S^{\omega}_{G_{F2}} = \frac{G_{F2}G_{F1}(1+\frac{1}{K_1})(1+\frac{1}{K_2})}{2kd} \tag{6-125-b}$$

$$S^{\omega}_{G_3} = \frac{G_3G_{F1}(1+\frac{1}{K_1})\frac{1}{K_2}}{2kd} \tag{6-125-c}$$

$$S^{\omega}_{G_4} = \frac{G_4G_{F1}(1+\frac{1}{K_1})\frac{1}{K_2}}{2kd} \tag{6-125-d}$$

$$S^{\omega}_{C_{F1}} = -\frac{C_{F1}C_{F2}(1+\frac{1}{K_1})(1+\frac{1}{K_2})+C_{F1}C_4(1+\frac{1}{K_1})\frac{1}{K_2}}{2k} \quad (6\text{-}125\text{-}e)$$

$$S^{\omega}_{C_{F2}} = -\frac{C_{F2}C_{F1}(1+\frac{1}{K_1})(1+\frac{1}{K_2})+C_{F2}(C_1+C_2)(1+\frac{1}{K_2})\frac{1}{K_1}}{2k} \quad (6\text{-}125\text{-}f)$$

$$S^{\omega}_{C_1} = -\frac{C_1C_{F2}(1+\frac{1}{K_2})\frac{1}{K_1} + C_1C_4(\frac{1}{K_1K_2}}{2k} \quad (6\text{-}125\text{-}g)$$

$$S^{\omega}_{C_2} = -\frac{C_2CF_2(1+\frac{1}{K_2})\frac{1}{K_1} + C_2C_4(\frac{1}{K_1K_2})}{2k} \quad (6\text{-}125\text{-}h)$$

$$S^{\omega}_{C_4} = -\frac{C_4(C_1+C_2)(\frac{1}{K_1K_2})+C_4C_{F1}(1+\frac{1}{K_1})\frac{1}{K_2}}{2k} \quad (6\text{-}125\text{-}i)$$

$$S^{\omega}_{K_1} = \frac{C_{F1}C_{F2}(1+\frac{1}{K_2})\frac{1}{K_1}+C_{F1}C_4(\frac{1}{K_1K_2})+C_{F2}(C_1+C_2)(1+\frac{1}{K_2})\frac{1}{K_2}+(C_1+C_2)\frac{C_4}{K_1K_2}}{2k}$$

$$-\frac{G_{F1}G_{F2}(1+\frac{1}{K_2})\frac{1}{K_1}+G_{F1}(G_3+G_4)\frac{1}{K_1K_2}}{2kd} \quad (6\text{-}125\text{-}j)$$

$$S^{\omega}_{K_2} = \frac{C_{F1}C_{F2}(1+\frac{1}{K_1}+C_{F1}C_4(1+\frac{1}{K_1})\frac{1}{K_2}+C_{F2}(C_1+C_2)(1+\frac{1}{K_1})\frac{1}{K_2}+C_4(C_2+C_1)\frac{1}{K_1K_2}}{2k}$$

$$-\frac{G_{F1}G_{F2}(1+\frac{1}{K_1})\frac{1}{K_2}+G_{F1}(G_3+G_4)(1+\frac{1}{K_1})\frac{1}{K_2}}{2kd} \quad (6\text{-}125\text{-}k)$$

Since

$$Q_p = \sqrt{\frac{k^2d}{kc}} \quad (6\text{-}126)$$

it is possible to express Q_p as a function of C_{F1}, C_{F2}, C_1, C_2, C_4, G_{F1}, G_{F2}, G_3, G_4, K_1 and K_2 and hence to compute the sensitivity of Q_p with respect to each of these parameters. Computation leads to:-

$$S^{Q}_{G_{F1}} = \frac{G_{F1}G_{F2}(1+\frac{1}{K_1})(1+\frac{1}{K_2})+G_{F1}(G_3+G_4)(1+\frac{1}{K_1})\frac{1}{K_2}}{2kd} - \frac{G_{F1}C_{F2}(1+\frac{1}{K_1})(1+\frac{1}{K_2})+G_{F1}C_4(1+\frac{1}{K_1})\frac{1}{K_2}}{kc} \quad (6\text{-}127\text{-}a)$$

$$S^{Q}_{G_{F2}} = \frac{G_{F2}G_{F1}(1+\frac{1}{K_1})(1+\frac{1}{K_2}}{2kd} - \frac{G_{F2}C_{F1}(1+\frac{1}{K_1})(1+\frac{1}{K_2})+G_{F2}C_1(1+\frac{1}{K_2})\frac{1}{K_1}+G_{F2}C_2(\frac{1}{K_1K_2})}{kc} \quad (6\text{-}127\text{-}b)$$

$$S^{Q}_{G_3} = \frac{G_3G_{F1}(1+\frac{1}{K_1})\frac{1}{K_2}}{2kd} - \frac{G_3(C_1+C_2)\frac{1}{K_1K_2}}{kc} \quad (6\text{-}127\text{-}c)$$

$$S^{Q}_{G_4} = \frac{G_4G_{F1}(1+\frac{1}{K_1})\frac{1}{K_2}}{2kd} - \frac{G_4G_{F1}(1+\frac{1}{K_1})\frac{1}{K_2}+G_4(C_1+C_2)\frac{1}{K_1K_2}}{kc} \quad (6\text{-}127\text{-}d)$$

$$S^{Q}_{C_{F1}} = \frac{C_{F1}C_{F2}(1+\frac{1}{K_1})(1+\frac{1}{K_2})+C_{F1}C_4(1+\frac{1}{K_1})\frac{1}{K_2}}{2k} - \frac{C_{F1}G_{F2}(1+\frac{1}{K_1})(1+\frac{1}{K_2})+C_{F1}(G_3+G_4)(1+\frac{1}{K_1})\frac{1}{K_2}}{kc} \quad (6\text{-}127\text{-}e)$$

$$S^{Q}_{C_{F2}} = \frac{C_{F2}C_{F1}(1+\frac{1}{K_1})(1+\frac{1}{K_2})+C_{F2}(C_1+C_2)(1+\frac{1}{K_2})\frac{1}{K_1}}{2k} - \frac{C_{F2}G_{F1}(1+\frac{1}{K_1})(1+\frac{1}{K_2})}{kc} \quad (6\text{-}127\text{-}f)$$

$$S^{Q}_{C_1} = \frac{C_1C_{F2}(1+\frac{1}{K_2})\frac{1}{K_1}+C_1C_4(\frac{1}{K_1K_2})}{2k} - \frac{C_1C_{F2}(1+\frac{1}{K_2})\frac{1}{K_1}+(G_3+G_4)C_1\frac{1}{K_1K_2}}{kc} \quad (6\text{-}127\text{-}g)$$

$$S^{Q}_{C_2} = \frac{C_2C_{F2}\left(1+\frac{1}{K_2}\right)\frac{1}{K_1} + C_2C_4\left(\frac{1}{K_1K_2}\right)}{2k} - \frac{C_2G_{F2}\left(\frac{1}{K_1K_2}\right)+C_2G_4\left(\frac{1}{K_1K_2}\right)}{kc} \tag{6-127-h}$$

$$S^{Q}_{C_4} = \frac{C_4C_{F1}\left(1+\frac{1}{K_1}\right)\frac{1}{K_2} + C_4(C_1+C_2)\frac{1}{K_1K_2}}{2k} - \frac{C_4G_{F1}\left(1+\frac{1}{K_1}\right)\frac{1}{K_2}}{kc} \tag{6-127-i}$$

$$S^{Q}_{K_1} = \frac{\left[\begin{array}{l}(G_{F1}C_{F2}+C_{F1}G_{F2})\left(1+\frac{1}{K_2}\right)\frac{1}{K_1} + (G_{F1}C_4+C_{F1}G_3+C_{F1}G_4)\frac{1}{K_1K_2} \\ +(G_{F2}C_1)\left(1+\frac{1}{K_2}\right)\frac{1}{K_1} + (C_1G_3+C_1G_4+C_2G_3+C_2G_4)\frac{1}{K_1K_2}\end{array}\right]}{kc} - \frac{G_{F1}G_{F2}\left(1+\frac{1}{K_2}\right)\frac{1}{K_1} + G_{F1}(G_3+G_4)\frac{1}{K_1K_2}}{2kd} - \frac{\left[\begin{array}{l}C_{F1}C_{F2}\left(1+\frac{1}{K_2}\right)\frac{1}{K_1} + C_{F1}C_4\left(\frac{1}{K_1K_2}\right) + C_{F2}(C_1+C_2)\left(1+\frac{1}{K_2}\right)\frac{1}{K_1} \\ +C_4(C_1+C_2)\frac{1}{K_1K_2}\end{array}\right]}{2k} \tag{6-127-j}$$

$$S^{Q}_{K_2} = \frac{\left[\begin{array}{l}(G_{F1}C_{F2}+C_{F1}G_{F2})\left(1+\frac{1}{K_1}\right)\frac{1}{K_2} + (G_{F1}C_4+C_{F1}G_3+C_{F1}G_4)\left(1+\frac{1}{K_1}\right)\frac{1}{K_2} \\ G_{F2}C_1\left(\frac{1}{K_1K_2}\right) + (C_1G_3+C_1G_4+C_2G_3+C_2G_4)\frac{1}{K_1K_2}\end{array}\right]}{kc} - \frac{G_{F1}G_{F2}\left(1+\frac{1}{K_1}\right)\frac{1}{K_2} + G_{F1}(G_3+G_4)\left(1+\frac{1}{K_1}\right)\frac{1}{K_2}}{2kd} - \frac{\left[\begin{array}{l}C_{F1}C_{F2}\left(1+\frac{1}{K_1}\right)\frac{1}{K_2} + C_{F1}C_4\left(1+\frac{1}{K_1}\right)\frac{1}{K_2} + C_{F2}(C_1+C_2)\frac{11}{K_1K_2} \\ + C_4(C_1+C_2)\frac{1}{K_1K_2}\end{array}\right]}{2kd} \tag{6-127-k}$$

6-4 A Method valid for the transfer function sensitivity computation, with respect to the gain of the operational amplifier directly from the SFG:-

We shall discuss here a method for the transfer function sensitivity computation with respect to the gain of the operational amplifier directly from the SFG applicable to some of the basic configurations which have been discussed in chapter 4.

6-4-1 Configuration No. 3:-

Consider basic network No. 3, for this network it is easily deduced from the definition of sensitivity that

$$S_K^{T(s)} = \frac{Z_1Y_F + 1}{-[Z_1Y_F + \frac{1}{K}(1+Z_1Y_F)]\,[-K]} \qquad (6\text{-}128)$$

Now the term $\frac{+1}{-[Z_1Y_F + \frac{1}{K}(1+Z_1Y_F)]}$ can be identified as the transfer function $T_v = \frac{V_o}{V_1}$ of the network 3 while the term $\frac{1}{Z_1Y_F + 1}$ can be identified as the transfer function $T_p = \frac{V_o'}{V_1}$ of the passive network obtained from network 3 by removing the operational amplifier and grounding the output terminal as shown in figure 6-27.

Hence $S_K^T(s)$ can be written in the form

$$S_K^{T(s)} = \frac{T_v}{-(K)\,T_p} \qquad (6\text{-}129)$$

Now T_v can be obtained directly from the SFG of the network using Mason's Gain Formula while T_p can also be obtained from the SFG using the following procedure:

1) Isolate and remove from the SFG the branches representing the operational amplifier gain.

2) Identify the node x which represents the net output V_o' of the passive circuit.

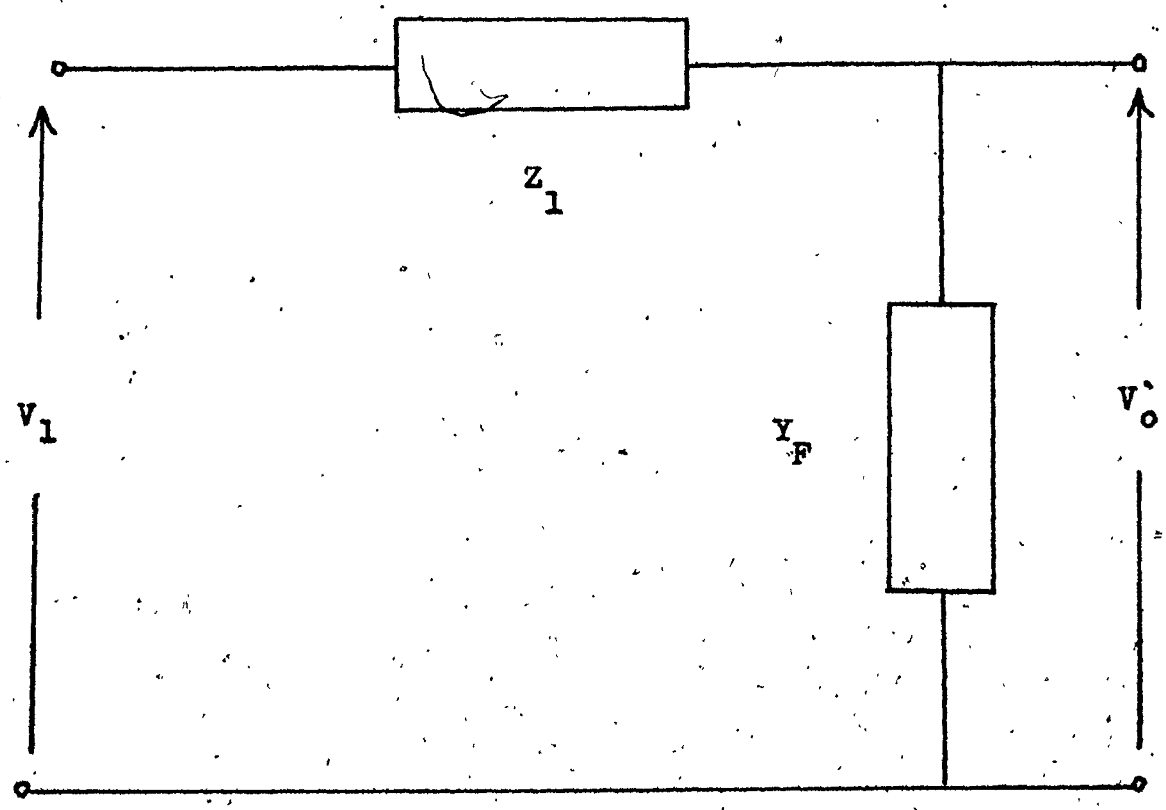


Figure 6-27. The passive circuit associated with basic network N• 3 .

3) Ground the active network output node (V_o).

4) Compute the transfer function $T_p = \frac{V_o'}{V_1}$ network using again Mason's Gain Formula.

Using this procedure and starting from two different SFGs representing network No. 3, two possible SFGs representing the passive circuit associated with network No. 3 are deduced in figure 6-28 and figure 6-29 and it is easily deduced from any one of them that

$$T_p = \frac{1}{1 + Z_1 Y_F}$$

6-4-2 Configuration No. 5:-

Consider now basic network No. 5. It can be deduced from the definition of the sensitivity that

$$S_K^{T(s)} = \frac{1+Y_2(Z_1+Z_3)+Z_3Y_4+Z_1Z_3Y_2Y_4}{K[Y_2(Z_1+Z_3)+Z_3Y_4+Z_1Z_3Y_2Y_4]+[1+Y_2(Z_1+Z_3)+Z_3Y_4+Z_1Z_3Y_2Y_4]} \qquad (6\text{-}130)$$

Now the term

$$\frac{-K}{K[Y_2(Z_1+Z_3)+Z_3Y_4+Z_1Z_3Y_2Y_4]+[1+Y_2(Z_1+Z_3)+Z_3Y_4+Z_1Z_3Y_2Y_4]}$$

can be identified as the transfer function $T_v = \frac{V_o}{V_1}$ of network 5 while the term

$$\left(\frac{1}{[1+Y_2(Z_1+Z_3)+Z_3\ 1/4+Z_1Z_3Y_2Y_4]}\right)$$

can be identified as the transfer function $T_p = \frac{V_o'}{V_1}$ of the passive network obtained from network 5 by removing the operational amplifier and grounding the output terminal as shown in figure 6-30.

Hence $S_K^{T(s)}$ can be written in the form

$$S_K^{T(s)} = \frac{T_v}{(-K)T_p} \qquad (6\text{-}131)$$

which is exactly the equation deduced previously for network 3. Again

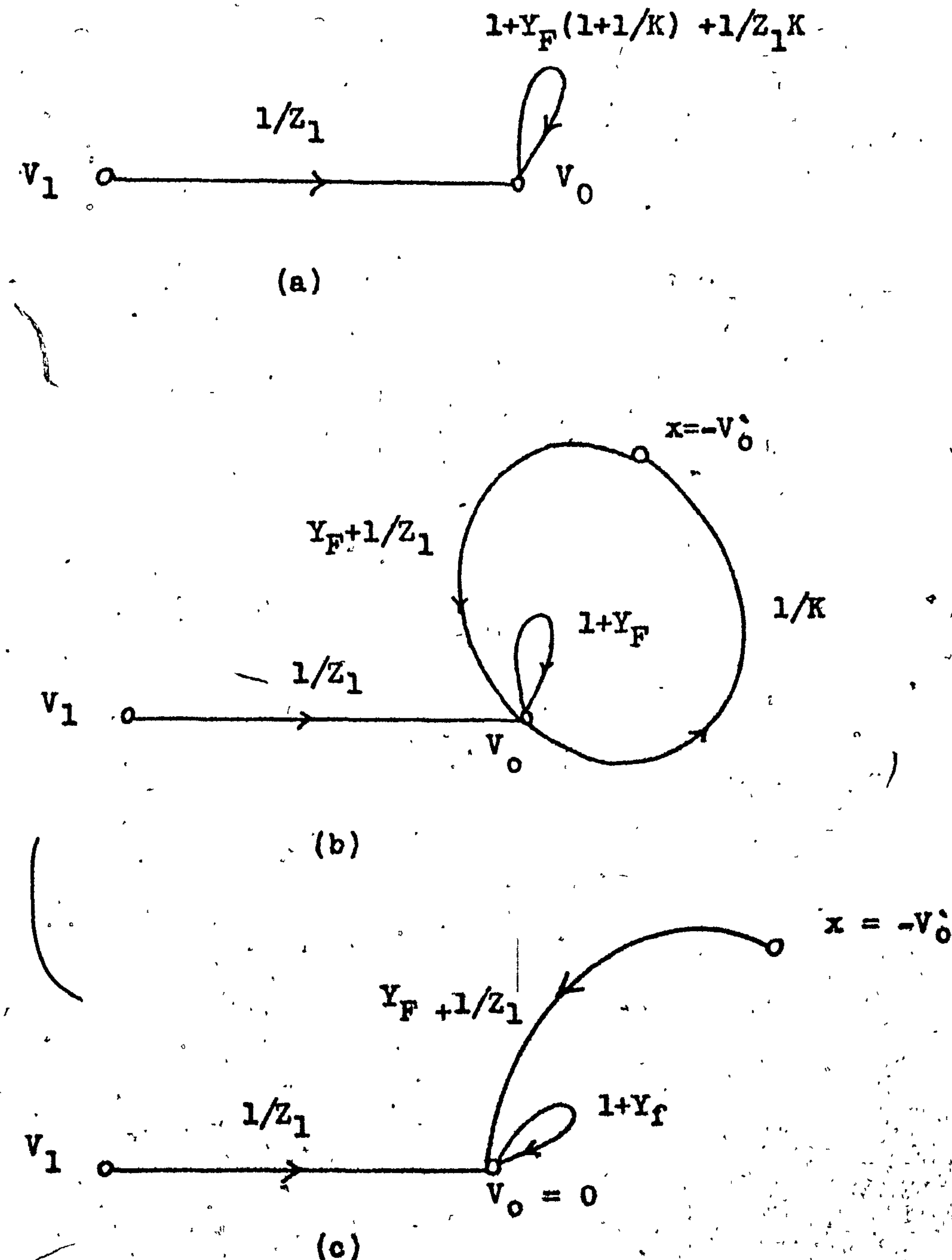
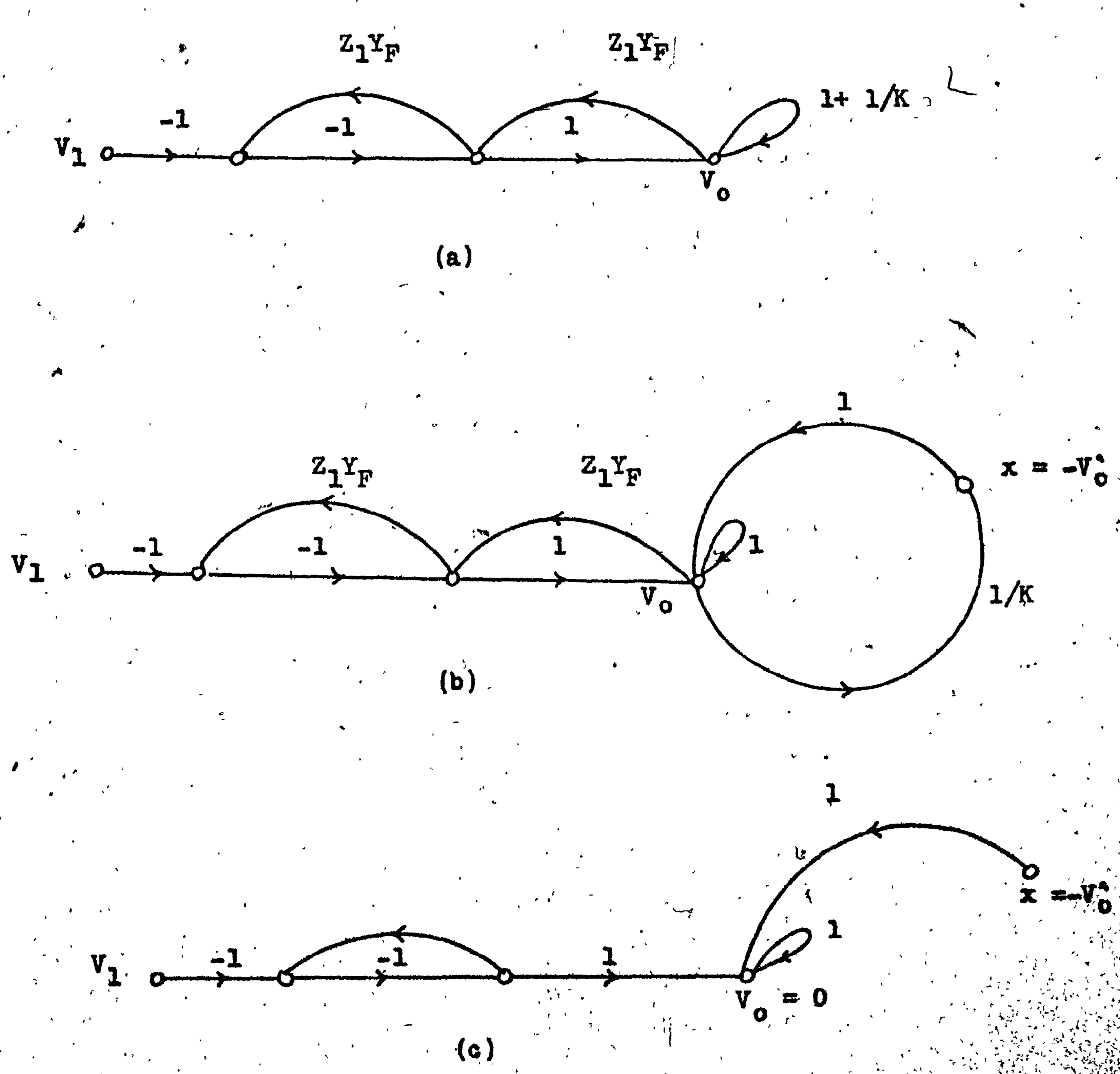


Figure 6-28. The deduction of the SFG of the passive network associated with basic network N^a 3 starting from SFG 3-a.



<u>Figure 6-29</u>. The deduction of the SFG of the passive network associated with basic network N°3 starting from SFG 3-b.

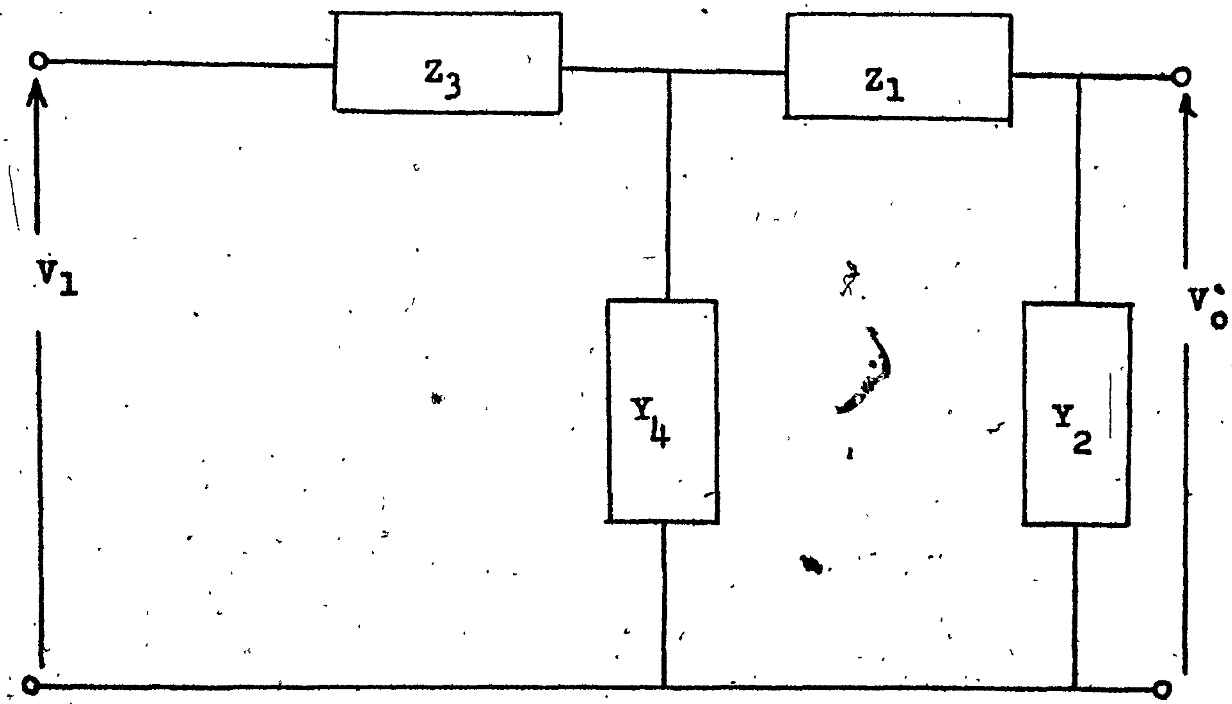


Figure 6-30. The passive circuit associated with basic network N° 5.

this shows that $S_K^{T(s)}$ can be deduced directly from the SFG by computing T_v directly using Mason's Gain Formula and T_p using the procedure described formerly.

The deduction of the SFG of the passive network associated with network 5 is shown in figure 6-31 and it is easily verified using Mason's Gain Formula that

$$T_p = \frac{1}{1+Y_2(Z_3+Z_1)+Z_3Y_4+Z_1Z_3Y_2Y_4}$$

6-5 Discussion:-

A method of realizing finite gain operational amplifier networks using SFGs is given. The method consists of:

1) Constructing suitable SFGs to represent the given transfer function.
2) Identifying proper portions of the SFGs with operational amplifiers networks and connecting them properly,

It is to be noted that the solution is not unique and that a wide variety of SFGs and corresponding networks are available.

Also, a method of computing the sensitivity of the transfer function with respect to the gain of the operational amplifier for some basic networks is given. However, it is to be noted that this is not general and is applicable for some specified configurations only.

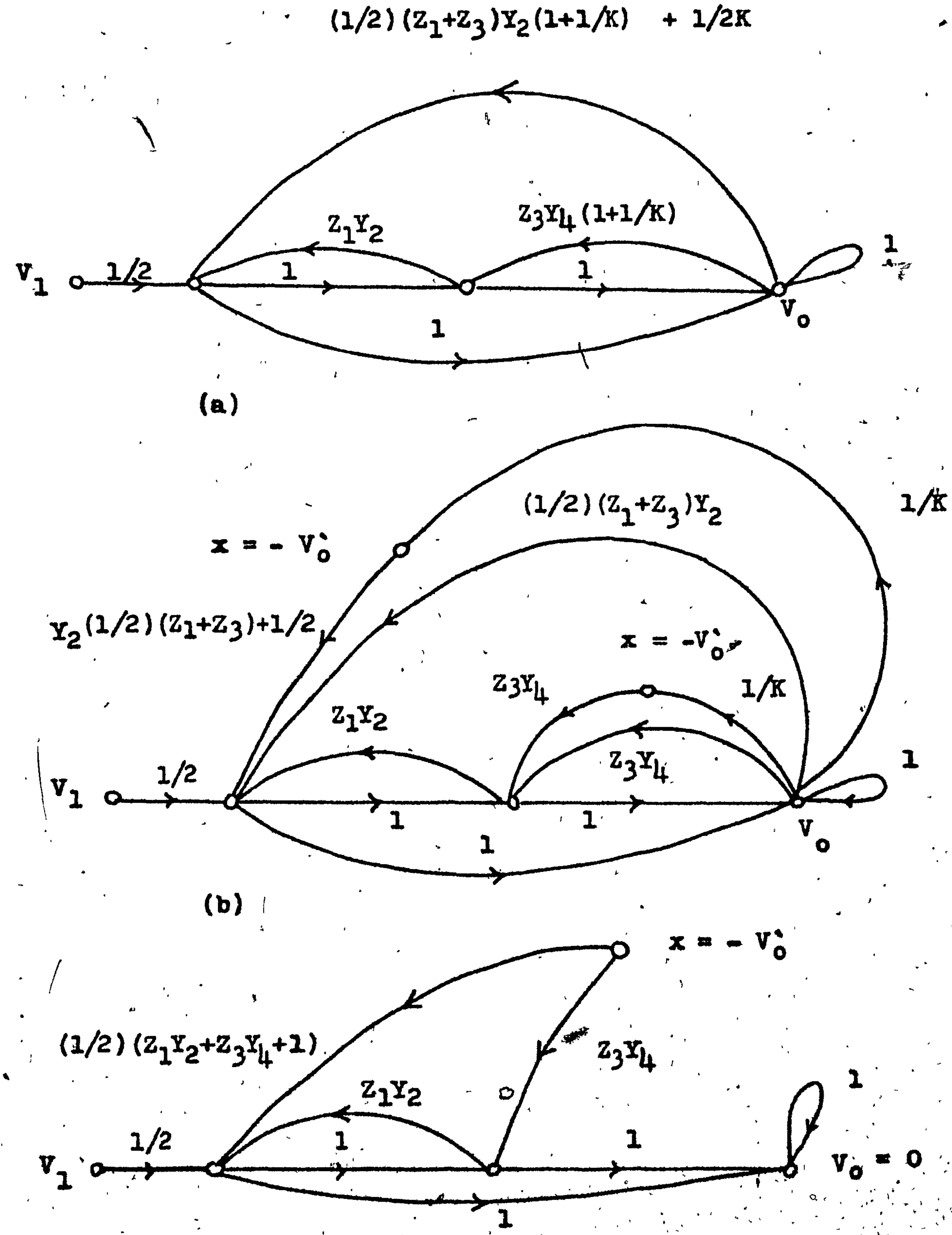


Figure 6-37. The deduction of the SFG of the passive network associated with basic network N° 5 starting from SFG 5-a.

CHAPTER VII

Conclusions

The earlier portions of the report (specifically chapters II, III and IV) contain a short survey of SFGs, operational amplifiers and realizations of transfer functions using infinite gain operational amplifiers by SFGs.

In chapter V we have developed a few SFGs for some basic and simple finite gain operational amplifier networks.

Using these simple SFGs as basic blocks, we have shown how second order transfer functions can be synthetized starting from suitable SFGs representing the transfer function.

Three cases are considered:- (i) The synthesis of transfer functions having negative real axis poles and zeros, (ii) The synthesis of low-pass filters transfer functions with real or complex poles, (iii) The synthesis of any second order transfer functions with real or complex poles and zeros. The first case is mainly considered, because the treatment (and hence the realizations) in the other cases, at times may depend upon it. It is evident that apart from the SFGs and the networks proposed in this report, several others are possible and suitable for our purpose.

For the active network realized in the third case (the most general) the ω_p and Q_p sensitivities with respect to different elements of the circuit have been computed and it is shown that the magnetudes of these sensitivities are low.

We have also proposed a method which in spite of the fact that it is not general could be used for the computation of the transfer function sensitivity with respect to the gain of the operational amplifier for some of the basic configurations which have been used throughout this report.

It is felt that further investigations could be fruitful in the following domains:

1) Synthesis of active networks using the SFGs method applied to active elements.

2) Using the SFG technique for synthesis of transfer functions with prescribed sensitivities.

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