A Study of Canonic Sinusoidal RC-Oscillators Using Operational Amplifiers

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ABSTRACT

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In this thesis, systematic generation and design of operational amplifier based RC oscillators are studied. Both single output operational amplifier based RC oscillators (Type-I) and differential output operational amplifier based RC oscillators (Type-II) are investigated. The study includes both single and variable frequency operation for both types of RC oscillators. RC oscillators with special features such as grounded capacitors and/or frequency controlling grounded variable resistor are also examined.

Towards this end, a unified and systematic approach is developed first for generation of Type-I oscillators. Using this approach, two complete sets of canonic RC oscillators are identified for single and variable frequency operations respectively. In the set of single frequency oscillators, it is established that there are only 12 circuits each consisting of one operational amplifier, 2 capacitors and 4 resistors. Three of these circuits are new. In the set of variable frequency oscillators, it is shown that there are only 16 circuits each containing one resistor more than that of single frequency oscillator circuits. Eight of the

circuits in this set are also new. A complete set of 8 canonic grounded capacitor variable frequency oscillators is then derived. A start up problem in this set is identified and the proper solution is developed. The method of solution leads to another complete set of 12 easily tunable variable frequency canonic grounded capacitor RC oscillators. All the grounded capacitor oscillators are new.

Extentson of the approach to Type-II circuits leads to four complete sets of oscillators of this type. The first set contains single frequency oscillators. The second set consists of variable frequency oscillators. Each of the circuits in the above two sets requires 2 capacitors, 3 resistors and one differential output operational amplifier. The other two sets are grounded capacitor circuits. One set contains single frequency oscillators while the other consists of variable frequency oscillators. In the single frequency oscillator set, each circuit consists of 2 capacitors, 4 resistors as well as one differential output operational amplifier. In the variable frequency oscillator set, each circuit requires one additional resistor. All the Type-II circuits are new.

The theoretical analyses are verified by extensive experimental tests. The test results agree closely with theoretical predictions.

DEDICATED TO :

My Parents

My Daughter, Negar

My Son, Erfan

ACKNOWLEDGEMENTS

I wish to record my deep sense of gratitude to my wife Shahla for her immense underestanding and patience during the long and often testing period of the research and the time of writing this thesis.

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LIST OF ABBREVIATIONS AND SYMBOLS

SU_s	: Stable unstable	16
Y_i	: Admittance of branch i	17
G_i	: Conductance of branch i	17
C_i	: Capacitance of branch i	17
Y_{nz}	: Non-zero admittance	22
l	: Number of loops in a graph	33
b	: Number of branches in a graph	33
n	: Number of nodes in a graph	33
p	: Non-negative variable	44
f( )	: Function of	44
g( )	: Function of	44
U	: A function of circuit elements	44
W	: A function of circuit elements	44
k	: Ratio of two elements	49
R	: Resistor	61
R_i	: Resistance of branch i	61
R_v	: Variable resitor	61
$S^{\omega_s}_{R_v}$	: Sensitivity of ω_s with respect to R_v	61
FET	: Field effect transistor	73
BSFO	: Base circuit	74
m	: Ratio of two elements	85
EVE	: Equal valued elements	87
ε	: A small positive quantity	90
r	: A function of circuit elements	92
N_{ij}	: Numerator polynomial associated with points i and j	106

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CHAPTER I

INTRODUCTION

CHAPTER I

INTRODUCTION

1.1 General

Sinusoidal oscillators have always been an important part of electronic systems [1]. Today, these oscillators play an essential role in most of the existing electronic systems. They are widely used in communications [2], instrumentation and measurements[3], etc. One can generate a long list of applications for such a widely used device. However, we mention in the following only a few of the more important ones.

Low frequency oscillators are used in sonar engineering and biomedical engineering (rehabilitation devices for handicapped). Audio frequency oscillators are used for measurments of impedances, harmonic distortions, power output, damping factor, etc. They are used in bridge measurements and signal regeneration [4]. Radio frequency oscillators can be found in modulators and coherent demodulators. Most digital communication systems use oscillators for modulating and demodulating the digital signal for transmision [5]. Sinusoidal oscillators are also used for analysing the responses of active and passive filters. One of the important parts of a phase-locked loop is its oscillator [6]. Sinusoidal oscillators can also be used in low power inverters. These inverters are used in small strobe lights and photoflash units [7]. A considerable reduction in switching transient is possible due to the fact that, the harmonic output from these inverters is much less than that of a conventional square-wave inverter.

1.2 RC-Oscillators

Sinusoidal oscillators are generally divided into three categories (RC, LC, RLC), out of which RC oscillators (RCOs) are most suitable for low and very low frequency operations. This is because at these frequencies, the inductors become physically large and of poor quality, while on the other hand RCOs remain compact and economical.

Further, in recent years tremendous advances in integrated circuit (IC) technology have provided the opportunity for producing miniaturized and light weight systems. Limitations in this technology for realization of inductors make the RCOs even more attractive and sometimes essential.

1.3 RCOs With Special Features

RCOs can be divided into two categories, single frequency oscillators (SFOs) and variable frequency oscillators (VFOs). In both of these categories, some of the circuits have some advantages over the rest in the same family. For example, in both cases grounded capacitor RC-oscillators (GCRCOs) are preferred over the other types of RCOs. In VFOs, it is highly desirable to have the frequency varied by varying only one element of the circuit. For many practical reasons, the variable element is preferred to be a resistor. Further, it is important to have the variable resistor also grounded, in which case, by making the variable resistor voltage dependent, a voltage controlled oscillator (VCO) can be easily obtained.

1.4 Canonic RCOs

Numerous studies have been reported in the literature for the design and implementation of RC - active oscillators [8-20]. With the exception of a very few, the approach to most of these studies has been on an ad hoc basis, that is, a circuit is selected at random and its properties are examined, as opposed to a systematic and unified treatment that yields a complete set of results. Recently one such treatment was reported in [8]. However, the investigations were restricted to the class of SFOs which use only three terminal controlled sources as active elements.

The workhorse of analog circuits is the operational amplifier (OA). Any other type of active element can, in fact, be derived from the OA with a suitable passive feedback around it. Thus, it is desirable to have a study of systematic generation of RCOs, using OAs directly as active elements. Further, it is of interest to study canonic circuits for RCOs. The term canonic is used in this thesis to mean the RCOs with the minimum number of components, active as well as passive.

Canonic realizations are important for many reasons. They consume the least amount of power. This reduces the problem of heat sinking. Consequently, the volume and weight of the system is reduced. Further, for low and very low frequency applications, passive components tend to be normally large in value and more costly, hence canonic circuits will be desirable from these considerations. Finally, in IC technology, it can be shown that the minimum obtainable frequency is directly proportional to the number of passive components in the network [21].

1.5 Scope

In this thesis, the aim is to present the results of a general, unified and systematic approach to the realization of OA based canonic RC-active oscillators. Both single frequency and variable frequency applications are investigated. The study undertakes both single ended OA, (an OA with a single output with respect to the ground, simply called an OA in this thesis) based RCOs and differential output OA (DOOA) based RCOs. The results for

OA based RCOs are given in chapters 2, 3 and 4. The results for DOOA based RCOs are given in chapter 5. The results of experimental varifications are also given in the corresponding chapters.

Chapter 2 presents the realization of RCOs for single frequency applications. The general theory of oscillators is discussed. This theory is also used in the latter chapters. Systematically, a complete set of 12 canonic circuits are derived. Each circuit requires only one OA, 2 capacitors and 4 resistors. One of these circuits has both of its capacitors grounded.

Chapter 3 examines the generation of VFOs. In this chapter, first, the theory of VFOs is discussed. Based on this theory and that of chapter 2, a set of 16 canonic VFOs are found. It is shown that the set is complete. Each circuit employs one OA, 2 capacitors and 5 resistors. Frequency of oscillation in each circuit can be controlled by a single variable resistor. Three of the circuits have the variable resistor grounded.

Chapter 4 extends the theories developed in previous chapters to the realization and design of GCRCOs. First, a complete set of 8 canonic GCRCOs is found, each containing one OA, 2 capacitors and 6 resistors. The circuits in this set, however, have either a limited frequency range of variation or a poor start up property. Based on this set, another set of 12 easily tunable and wide range GCRCOs is found. This second set is also complete, and each circuit in this set employs one resistor more than the number of resistors used in the first set.

Chapter 5 presents the results for systematic generation of DOOA based RCOs. Using the DOOA as the active part of the circuits allows further reduction in the number of resistors in the circuits. In this chapter a total of 10 circuits are found, out of which, two are 5 element SFOs. Three are 6 element grounded capacitor single frequency oscillators (GCSFOs). Two are 5 element VFOs with the variable resistor grounded. Three are 7 element grounded capacitor variable frequency oscillators (GCVFOs). One of the GCVFOs has its variable resistor also grounded.

Chapter 6 contains the conclusion of the thesis. Some possible directions for future research in this field are also given in this chapter.

CHAPTER II

CANONIC SINGLE FREQUENCY RC-OSCILLATORS USING OAs

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CANONIC SINGLE FREQUENCY RC-OSCILLATORS USING OAs

2.1 Introduction

RCOs can be broadly divided into two categories: SFOs and VFOs. In SFOs, the oscillation condition (OC) is dependent on the oscillation frequency (OF). To vary the frequency of oscillation, either the design should be changed or at least more than one element should be varied simultaneously. Consequently, the most convenient applications for this type of oscillators are in the area of fixed frequency operation.

2.2 Theory

It is convenient to assume that an RCO consists of two parts, active and passive. Since the aim is to find canonic RCOs, we should minimize both passive and active parts. Using OAs directly as the active element, it is clear that, the minimum number of OAs that can be used in the circuit is one. Fig.2.1(a) shows the OA as the active part of an oscillator. This active part has two input and one output ports. Now, we should find a passive network that contains all of the passive elements of the circuit. Keeping in mind that oscillators do not have inputs and they should be derived through feedbacks, the passive network must have two

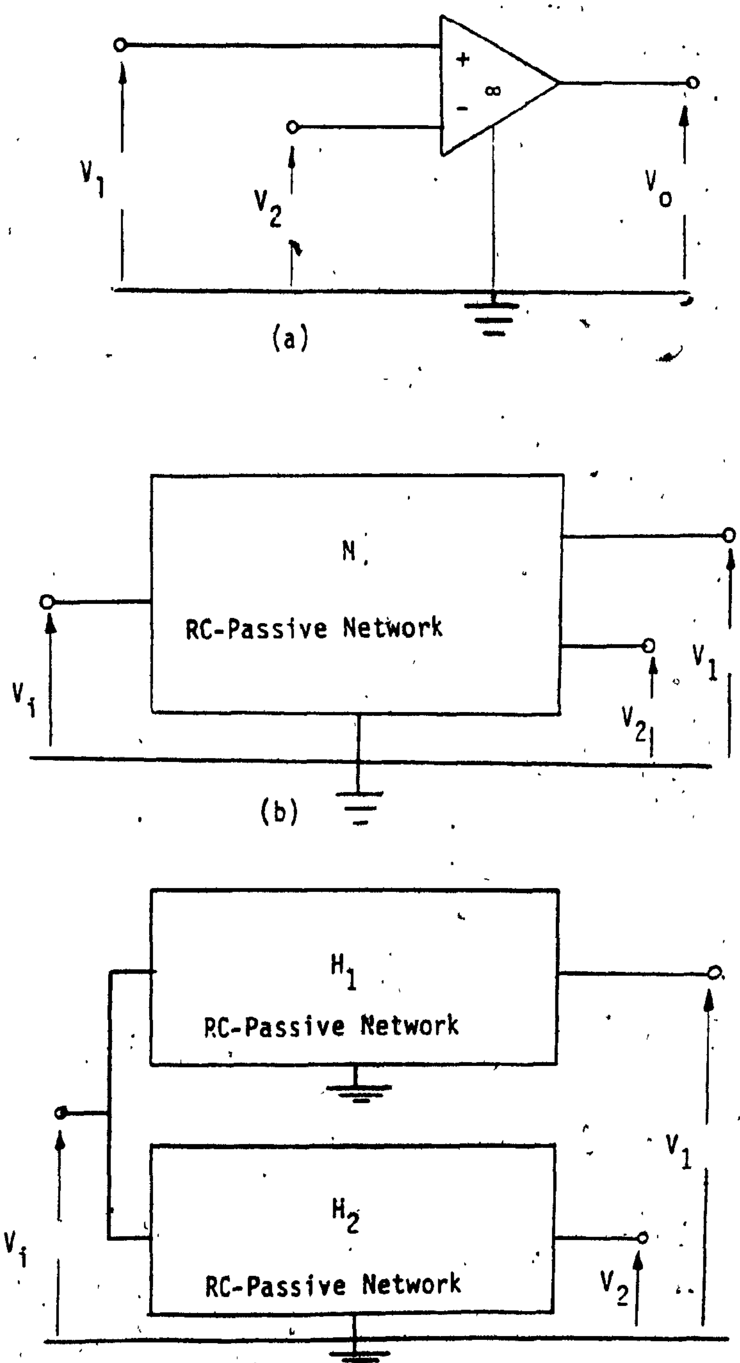

Fig. 2.1(a): Single output operational amplifier.
(b): Four terminal 3 port RC network with connected graph.
(c): Four terminal 3 port RC network with non-connected graph.

outputs and one input to match the inputs and the output of the active part rspectively. There exist only two of such networks; they are shown in Figs.2.1(b) and 2.1(c). Connecting the outputs of passive part to the inputs of active part and assuming unity feedback from output of active part to input of passive part, circuits of Figs.2.2(a) and 2.2(b) are obtained. In Fig.2.2(a), the passive network has a connected graph while the passive part of Fig.2.2(b) has two separate graphs. In Appendix A, it is shown that Fig.2.2(b) is in fact, a special case of Fig.2.2(a). Therefore, analysis of Fig.2.2(a) will automatically include the analysis of Fig.2.2(b).

Consider the circuit in Fig.2.2(a). It consists of one OA of differential gain A_d. Ideally A_d is infinity. The network N is a four-terminal, three port network. The transfer functions of interest are:

$$T_{11}(S) = \frac{V_1}{V_i} = \frac{N_1(S)}{D(S)} \qquad (2\text{-}1)$$

$$T_{12}(S) = \frac{V_2}{V_i} = \frac{N_2(S)}{D(S)} \qquad (2\text{-}2)$$

The characteristic equation(CE) of the passive network is D(S)=0 and its roots are on the negative real axis [22]. The unity feedback loop, causes the following equation to

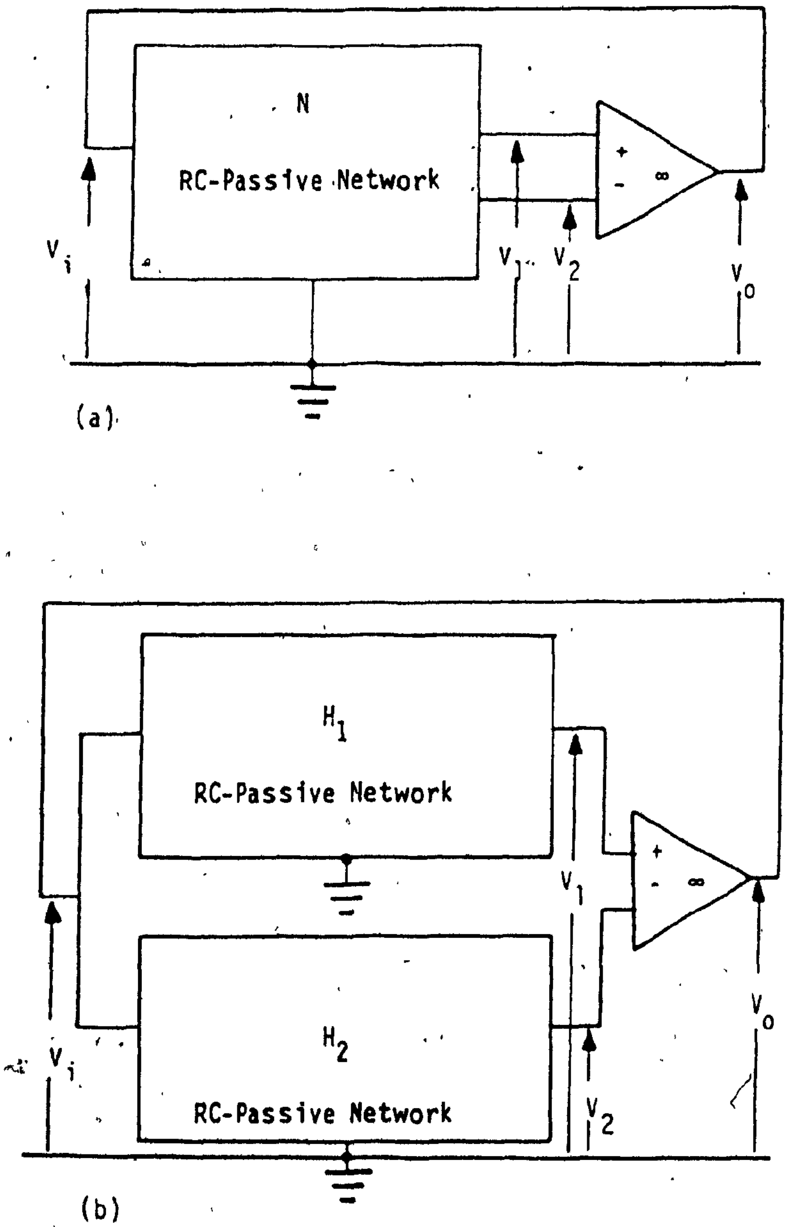

Fig. 2.2(a): General configuration of RCO with a single OA and connected graph RC network.
(b): General configuration of RCO with a single OA and non-connected graph RC network.

hold

$$V_o = V_i \qquad (2\text{-}3)$$

From the gain characteristics of the OA, we have

$$V_o = A_d(V_1 - V_2) \qquad (2\text{-}4)$$

substituting for V_o from (2-3) and dividing both sides by $V_i A_d$ we have,

$$\frac{1}{A_d} = \frac{V_1}{V_i} - \frac{V_2}{V_i} \qquad (2\text{-}5)$$

Substituting for the right side of (2-5) from (2-1) and (2-2) and letting A_d go to infinity, we can write

$$\frac{N_1(S)}{D(S)} - \frac{N_2(S)}{D(S)} = 0 \qquad (2\text{-}6)$$

Since, D(S) is neither zero nor infinity on the $j\omega$ axis, the over all CE of the system is :

$$N_1(S) - N_2(S) = 0 \qquad (2\text{-}7)$$

To ensure a pure sinusoidal oscillation, Eqn.(2-7) should be a second order polynomial. Furthere, its roots must be on the $j\omega$ axis. This requires that at least two capacitors be present in the network N. The general form of (2-7) is:

$$(\alpha_1 S^2+\beta_1 S+\gamma_1)-(\alpha_2 S^2+\beta_2 S+\gamma_2)= 0 \qquad (2\text{-}8)$$

which can be put in the form

$$\alpha S^2 + \beta S + \gamma = 0 \qquad (2\text{-}9)$$

where

$$\alpha=\alpha_1-\alpha_2 \qquad (2\text{-}10a)$$

$$\beta=\beta_1-\beta_2 \qquad (2\text{-}10b)$$

$$\gamma=\gamma_1-\gamma_2 \qquad (2\text{-}10c)$$

Equation (2-9) will have pure imaginary roots (excluding origin and infinity) if:

$$\alpha \neq 0 \qquad (2\text{-}11a)$$

$$\beta = 0 \qquad (2\text{-}11b)$$

$$\gamma/\alpha > 0 \qquad (2\text{-}11c)$$

then CE is:

$$\alpha S^2 + \gamma = 0 \qquad (2\text{-}12)$$

The roots of (2-12) are now on the $j\omega$ axis and the OF is

$$\omega_s = (\gamma/\alpha)^{1/2} \qquad (2\text{-}13)$$

A close examination of (2-7)-(2-12) provides some rules that are found useful in developing a general approach for identifing specific circuits from the configuration of Fig.2.2(a). They are as follows.

I- For CE to be a second order polynomial, N must contain at least two capacitors. Since it is desirable to have the minimum number of capacitors in the network, it will be assumed hereafter that N contains only two capacitors and all the other elements are resistors.

II- To satisfy (2-11b), either $\beta_1 = \beta_2 = 0$ or $\beta_1 = \beta_2 \neq 0$. It can be shown that an RC unbalanced network that contains only two capacitors can not have a null at a nonzero and finite frequency. Also, if the pair T_{11} and T_{12} is any combination of pure low pass or high pass transfer functions, then (2-11c) would be violated. Therefore, the "S" term should be present in both $N_1(S)$ and $N_2(S)$, and $\beta_1 = \beta_2 = 0$ is not possible.

III- For (2-11c) to be satisfied, the same polynomial ($N_1(S)$ or $N_2(S)$) which contains second order "S" term must also contain a constant term.

Combining rules I to III, it is concluded that at least one of $N_1(S)$ or $N_2(S)$ should be a complete second degree polynomial in "S".

IV- At least two branches should be incident on every node in the network N, (see Appendix B).

2.3 Stable-Unstable Pair Circuits

If the input terminals of the OA in Fig.2.2(a) are interchanged, the CE of the system will still be as given by (2-7). Therefore, if N has been designed such that Fig.2.2(a) produces an RCO, then, theoretically, interchanging the inputs to the OA should still yield an RCO circuit. However, it is shown in Appendix C that only one of the two resulting circuits will work properly while the other will latch up in an unstable mode of operation. Consequently, the above pair of circuits will be termed stable-unstable (SU) pair circuits.

2.4 Realization of SFO Circuits

The configuration of Fig.2.2(a) contains the minimum number of OAs, namely one. Therefore, it is only necessary to realise the network N using the minimum number of passive components. It has been established so far that N has only two capacitors. Then, it is only required to find the minimum number of resistors to generate a canonic SFO. For a systematic method to obtain complete results the approach is as follows. First, assume that there is no independent node inside N. The network N then has four nodes. Next assume that there is one independent node inside N (5 node network) and so on. Then, for every case all the possible configurations with the minimum number of components are found, while disregarding all non canonic circuits. As we are seeking canonic realizations, it is obvious that the network N should contain the fewest number of nodes as well as branches. Also, the two nodes of a branch must be clearly identifiable with the two terminals of an element. Hence, it is assumed that a branch admittance Y_i between any two nodes corresponds only to a conductance G_i or to the admittance of a capacitor SC_i or to the admittance of a parallel connection of a resistor and a capacitor G_i+SC_i.

2.4.1 Four-Node Network

As it is shown in Fig.2.3, a four node network can have a maximum of 6 branches. Analysis shows that the CE contains four branch admittances, that is,

$$Y_1Y_2 - Y_3Y_4 = 0 \qquad (2\text{-}14)$$

where Y_i is the admittance of the branch i. Note that, as is expected, branches Y_6 and Y_5 do not contribute to the CE. Therefore, they can be removed from the network without affecting the operation of the oscillator. Using the most general form of the branch admittance SC_i+G_i for all the branches, Eqn.(2-14) can be written as

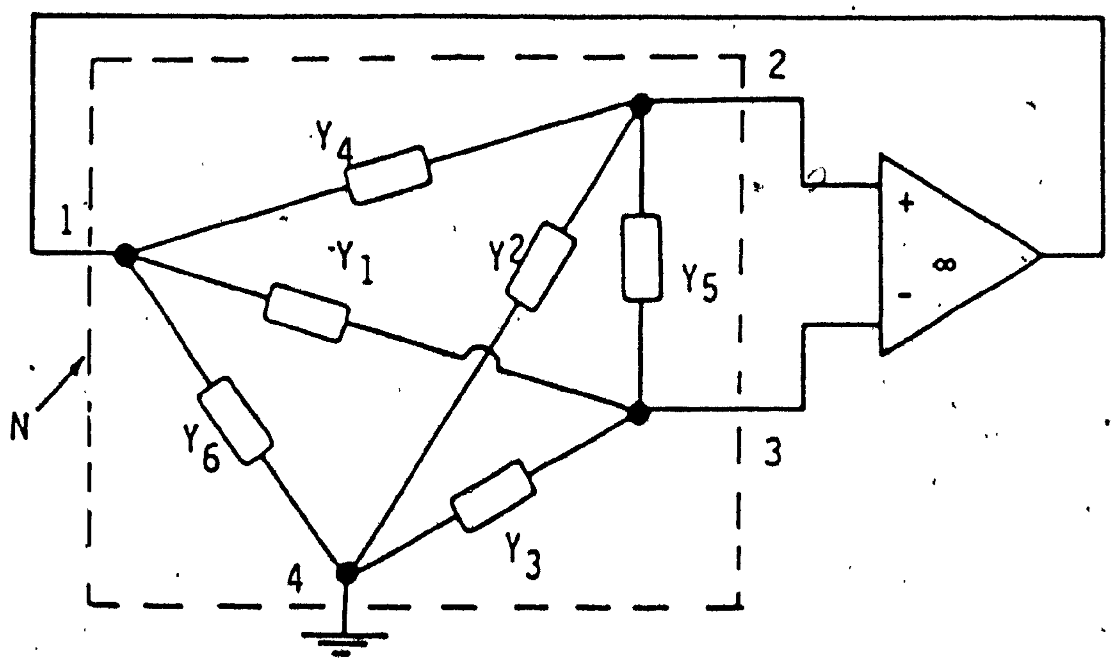

Fig. 2.3: Oscillator with the general four node passive RC network.

$$S^2(C_1C_2-C_3C_4)+S(C_1G_2+C_2G_1-C_3G_4-G_3C_4)+(G_1G_2-G_3G_4)=0 \quad (2\text{-}15)$$

and (2-11a) becomes

$$C_1C_2-C_3C_4\neq 0 \quad (2\text{-}16)$$

As mentioned before, we are interested in realizations that contain only two capacitors. Hence, the only choices are

(a) $$C_1=C_2=0 \quad (2\text{-}17)$$

(b) $$C_3=C_4=0. \quad (2\text{-}18)$$

Let us choose the condition (a). Then, Eqn.(2-11b) becomes:

$$-C_3G_4-G_3C_4=0 \quad (2\text{-}19)$$

Since the value of any element can not be negative and C_3 and C_4 are non zero, Eqn.(2-19) can be satisfied only if

$$G_4=G_3=0 \quad (2\text{-}20)$$

then the CE is:

$$S^2(-C_3C_4)+G_1G_2=0. \quad (2\text{-}21)$$

Equation (2-21) shows that (2-11c) can not be satisfied, therefore, the system will not oscillate. Similarly, assuming condition (b), it can be shown that no oscillation is possible.

Thus, we conclude that a four node topology for N will not produce any oscillator circuit. Clearly then, the oscillator circuit should have at least five nodes.

2.4.2 Five-Node Network

In this case the most general form of N is as shown in Fig.2.4. The CE is

$$Y_8Y_5(Y_1+Y_2+Y_3+Y_4)+Y_5Y_3Y_2+Y_1Y_4Y_8$$

$$-Y_7Y_6(Y_1+Y_2+Y_3+Y_4)-Y_6Y_4Y_2-Y_1Y_3Y_7= 0 \qquad (2\text{-}22)$$

Again, note that Y_9 and Y_{10} do not contribute to the CE and hence they need not be considered. There are eight branch admitances in the CE given by (2-22).
It is observed that the choice $Y_1= Y_2= Y_3= Y_4= 0$ or $Y_5= Y_6= Y_7= Y_8= 0$ will reduce the CE identically to zero. Therefore, we proceed in the following way.

(i) Let any three out of the four admittances Y_1 to Y_4 be set to zero. Then (2-22) can be written as

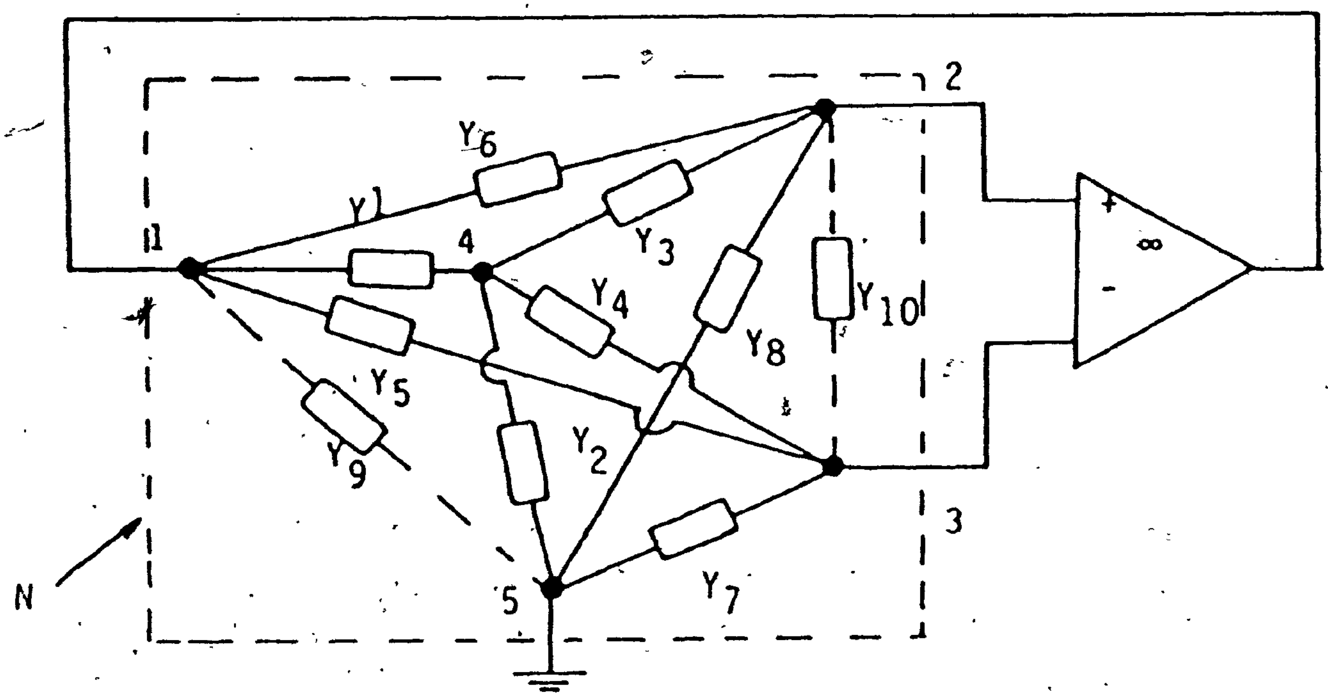

Fig. 2.4: Oscillator with the general five node passive RC network.

$$Y_8Y_5(Y_{nz}) - Y_7Y_6(Y_{nz}) = 0 \qquad (2\text{-}23)$$

where Y_{nz} is the remaining nonzero branch in the set Y_1 to Y_4. Hence, the CE reduces to

$$Y_8Y_5 - Y_7Y_6 = 0. \qquad (2\text{-}24)$$

Equation (2-24) is mathematically the same as (2-14), which was shown to produce no oscillation. Therefore, more than two branches in the set can not be set to zero.

(ii) Choose any three out of Y_5 to Y_8 equal to zero. The CE then reduces to one term, incapable of generating oscillation. Choosing any two out of Y_5 to Y_8 equal to zero, we obtain

$$Y_8 = Y_7 = 0,\ CE = Y_5Y_3 - Y_6Y_4 = 0 \qquad (2\text{-}25)$$

$$Y_8 = Y_6 = 0,\ CE = Y_5Y_2 - Y_7Y_1 = 0 \qquad (2\text{-}26)$$

$$Y_7 = Y_5 = 0,\ CE = Y_8Y_1 - Y_6Y_2 = 0 \qquad (2\text{-}27)$$

$$Y_6 = Y_5 = 0,\ CE = Y_8Y_4 - Y_7Y_3 = 0 \qquad (2\text{-}28)$$

$$Y_8 = Y_5 = 0,\ CE = -Y_6Y_7(Y_1+Y_2+Y_3+Y_4) - Y_1Y_3Y_7 - Y_6Y_4Y_2 = 0 \qquad (2\text{-}29)$$

$$Y_7 = Y_6 = 0,\ CE = Y_5Y_8(Y_1+Y_2+Y_3+Y_4) + Y_1Y_4Y_8 + Y_5Y_3Y_2 = 0 \qquad (2\text{-}30)$$

Equations (2-25)-(2-28) are similar to (2-14) and can be discarded. Equations (2-29) and (2-30) also can not produce imaginary roots since all their coefficients are of the same sign. Clearly then, only one branch admittance in the set Y_5 to Y_8 can be set to zero. Therefore, from parts (i) and (ii), the conclusion is that a total of three branch admittances out of eight can be set to zero. Thus at least five branches should be present in the passive network. The only possible choices for three branch admittances as zeros are:

$$Y_5=Y_3=Y_2=0,\quad CE=Y_1Y_4Y_8-Y_6Y_7(Y_1+Y_4)=0 \qquad (2\text{-}31)$$

$$Y_8=Y_1=Y_4=0,\quad CE=Y_5Y_3Y_2-Y_6Y_7(Y_3+Y_2)=0 \qquad (2\text{-}32)$$

$$Y_6=Y_4=Y_2=0,\quad CE=Y_1Y_3Y_7-Y_8Y_5(Y_1+Y_3)=0 \qquad (2\text{-}33)$$

$$Y_7=Y_1=Y_3=0,\quad CE=Y_6Y_4Y_2-Y_8Y_5(Y_2+Y_4)=0 \qquad (2\text{-}34)$$

Mathematically (2-31)-(2-34) are of the form

$$Y_aY_b(Y_c+Y_d)-Y_eY_dY_c=0 \qquad (2\text{-}35)$$

Therefore, we shall analyse (2-35) and then put the results back into (2-31)-(2-34).

Comparing (2-35) with (2-7), we get

$$N_1(S)= Y_aY_b(Y_c+ Y_d), \qquad N_2(S)= Y_eY_dY_c.$$

It is necessary that either Y_c or Y_d contain a capacitor(Rule II). To have $N_1(S)$ or $N_2(S)$ as a complete second-order polynomial(Rule IV), at least one branch should be of the form SC_i+G_i, hence at least six elements are required to produce the proper CE. Therefore, the derived circuit must have five branches and six elements. Clearly, the number of resistors is four since the number of capacitors is two. To illustrate how a specific oscillator circuit is generated, let $Y_c=SC_c$, $Y_b=SC_b+G_b$, $Y_a=G_a$, $Y_d=G_d$ and $Y_e=G_e$. Then the CE is

$$G_aC_bC_cS^2+S(G_aG_dC_b+G_aG_bC_c-G_eG_dC_c)+G_aG_bG_d=0 \qquad (2\text{-}36)$$

The OC and OF are, respectively,

$$\text{OC:} \quad G_aG_dC_b+G_aG_bC_c-G_eG_dC_c= 0 \qquad (2\text{-}37)$$

$$\text{OF:} \quad \omega_s=\left(\frac{G_bG_d}{C_bC_c}\right)^{1/2} \qquad (2\text{-}38)$$

Substituting these results into (2-31)-(2-34), different oscillator circuits can now be found. For example putting the results into (2-32) we have,

$$Y_2=Y_c=SC_2,\ Y_b=Y_8=sC_8+G_8,\ Y_a=G_5,\ Y_d=Y_4=G_4,\ Y_e=Y_6=G_6 \quad (2-39)$$

and

$$\omega_s=\left(\frac{G_8G_4}{C_8C_2}\right)^{1/2} \quad (2-40)$$

The resulting circuit is shown in Fig.2.5(a). Using the same procedure, four oscillator circuits are derived from each equation, leading to a total of eight SU pair oscillator circuits. The eight circuits each have a resistor and a capacitor in series. An interchange of the capacitor and the resistor will not affect the OC or OF. Thus, these eight circuits are further reduced to four topologically different circuits. These four circuits are shown in Figs.2.5(a)-(d).

Now that it is known that at least six elements are required to build an oscillator, let us investigate circuits with six branch admittances with one element per branch.

2.4.3 Five-Node Six-Branch Six-Element Network

Let any two out of eight branches in (2-22) be set to zero.

$$Y_7=Y_3=0,\ CE=Y_8Y_5(Y_1+Y_2+Y_4)+Y_1Y_4Y_8-Y_6Y_4Y_2=0 \quad (2-41)$$

$$Y_7=Y_1=0,\ CE=Y_8Y_5(Y_2+Y_3+Y_4)+Y_5Y_3Y_2-Y_6Y_4Y_2=0 \quad (2-42)$$

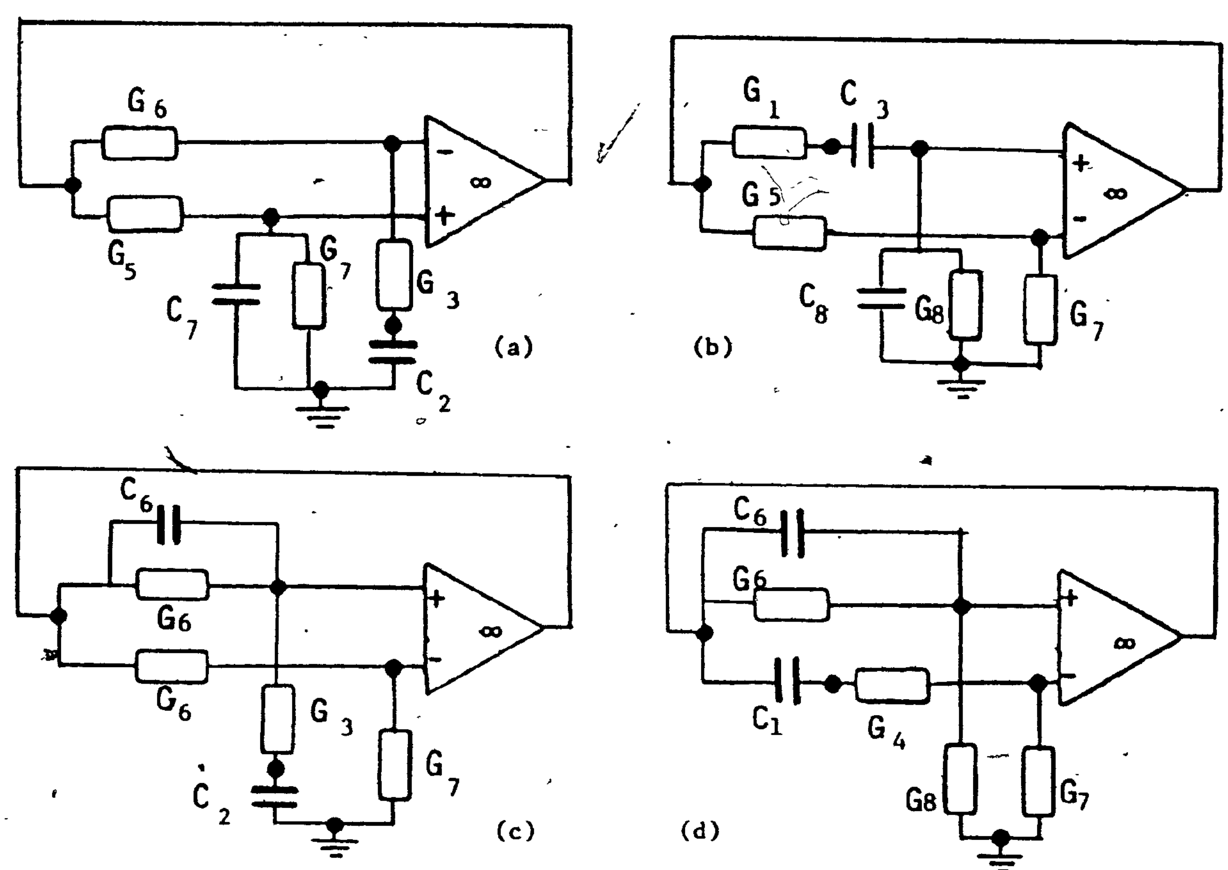

Fig. 2.5: The set of 12 canonic single frequency oscillators (Continued).

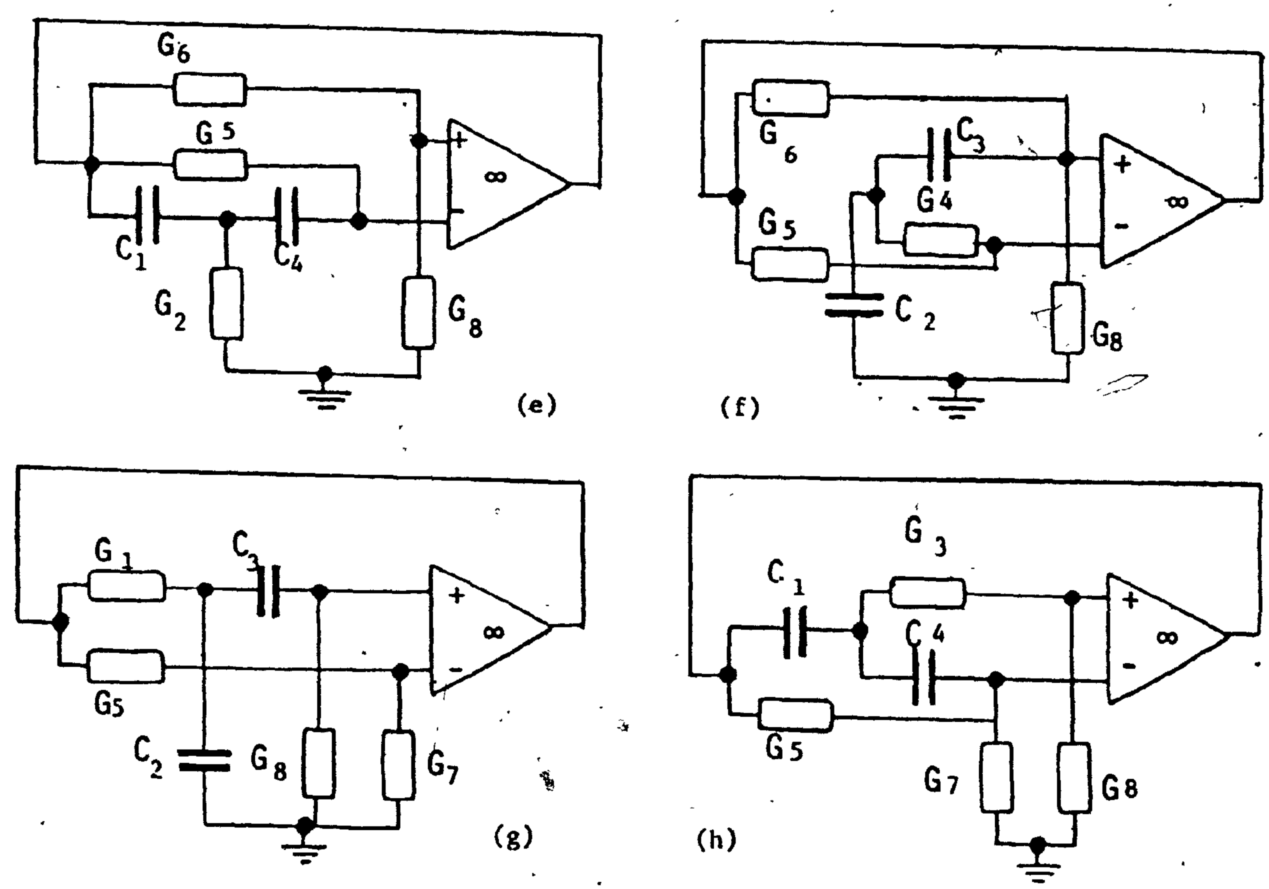

Fig. 2.5: The set of 12 canonic single frequency oscillators (Continued).

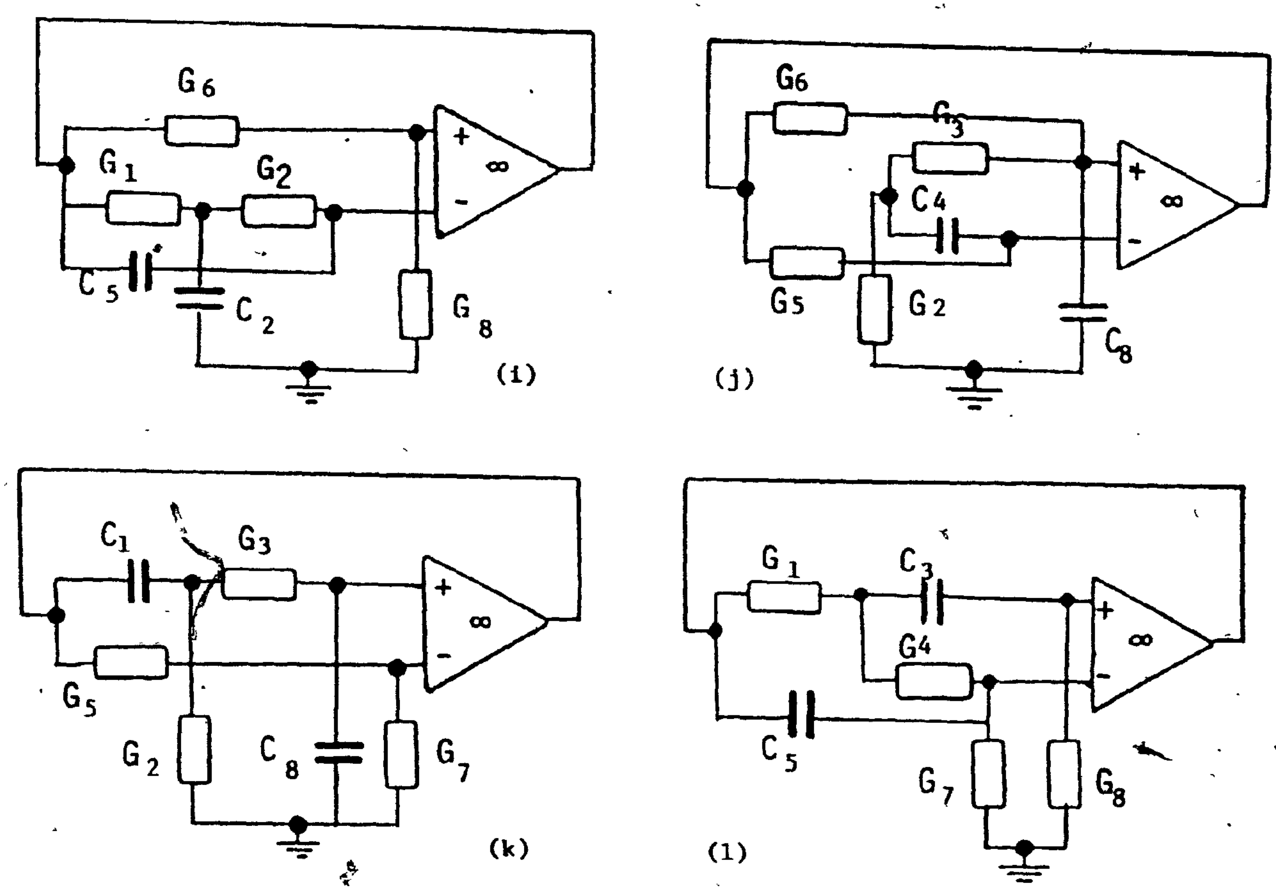

Fig. 2.5: The set of 12 canonic single frequency oscillators (Continued).

$$Y_6=Y_4=0,\ CE=Y_8Y_5(Y_1+Y_2+Y_3)+Y_5Y_3Y_2-Y_1Y_3Y_7=0 \qquad (2\text{-}43)$$

$$Y_6=Y_2=0,\ CE=Y_8Y_5(Y_1+Y_3+Y_4)+Y_1Y_4Y_8-Y_1Y_3Y_7=0 \qquad (2\text{-}44)$$

$$Y_5=Y_2=0,\ CE=Y_1Y_4Y_8-Y_6Y_7(Y_1+Y_3+Y_4)-Y_1Y_3Y_7=0 \qquad (2\text{-}45)$$

$$Y_5=Y_3=0,\ CE=Y_1Y_4Y_8-Y_6Y_7(Y_1+Y_4+Y_2)-Y_6Y_4Y_2=0 \qquad (2\text{-}46)$$

$$Y_8=Y_4=0,\ CE=Y_5Y_3Y_2-Y_6Y_7(Y_1+Y_2+Y_3)-Y_1Y_3Y_7=0 \qquad (2\text{-}47)$$

$$Y_8=Y_1=0,\ CE=Y_5Y_3Y_2-Y_6Y_7(Y_2+Y_3+Y_4)-Y_6Y_4Y_2=0 \qquad (2\text{-}48)$$

Mathematically (2-41)-(2-48) are of the form

$$Y_aY_b(Y_c+Y_d+Y_e)+Y_cY_eY_b-Y_fY_eY_d=0 \qquad (2\text{-}49)$$

In (2-49), branches Y_e or Y_d should be capacitors (Rule II). Let $Y_d=SC_d$. Hence, the only choice for the other capacitive branch that can lead to an oscillator is $Y_a=SC_a$. The CE is

$$S^2(C_aC_dG_b)+S(C_aG_b(G_c+G_e)-G_fG_eC_d)+G_cG_eG_b=0 \qquad (2\text{-}50)$$

$$\text{OC:}\quad C_aG_b(G_c+G_e)-G_fG_eC_d=0 \qquad (2\text{-}51a)$$

$$\text{OF:}\quad \omega_s=\left(\frac{G_cG_e}{C_aC_d}\right)^{1/2} \qquad (2\text{-}51b)$$

If we choose $Y_e = SC_e$, then the only choice for the other capacitive branch would be $Y_c = SC_c$, then CE is

$$S^2 C_c C_e G_b + S[G_a G_b (C_c + C_e) - G_d G_f C_e] + G_a G_b G_d = 0 \qquad (2\text{-}52)$$

$$\text{OC:} \quad G_a G_b (C_c + C_e) - G_d G_f C_e = 0 \qquad (2\text{-}53)$$

$$\text{OF:} \qquad \omega_s = \left(\frac{G_a G_b}{C_c C_e}\right)^{1/2} \qquad (2\text{-}54)$$

Substituting the results from (2-50) and (2-52) into (2-41)-(2-48), 16 circuits are found, one from each substitution. However, these 16 circuits are eight SU pairs. Therefore, there are only eight stable circuits which are shown in Figs.2.5(e)-(l).

Since we are concerned with canonic realizations only and have found circuits with six elements, the analysis of seven-branch networks will not be necessary. This is because they require at least seven elements and hence will no longer be canonic. The next type of network that should be considered, is one with two independent nodes inside it.

2.4.4 Six-Node Network

In this case N contains two internal nodes. The general form of the corresponding network is shown in Fig.2.6. The CE is

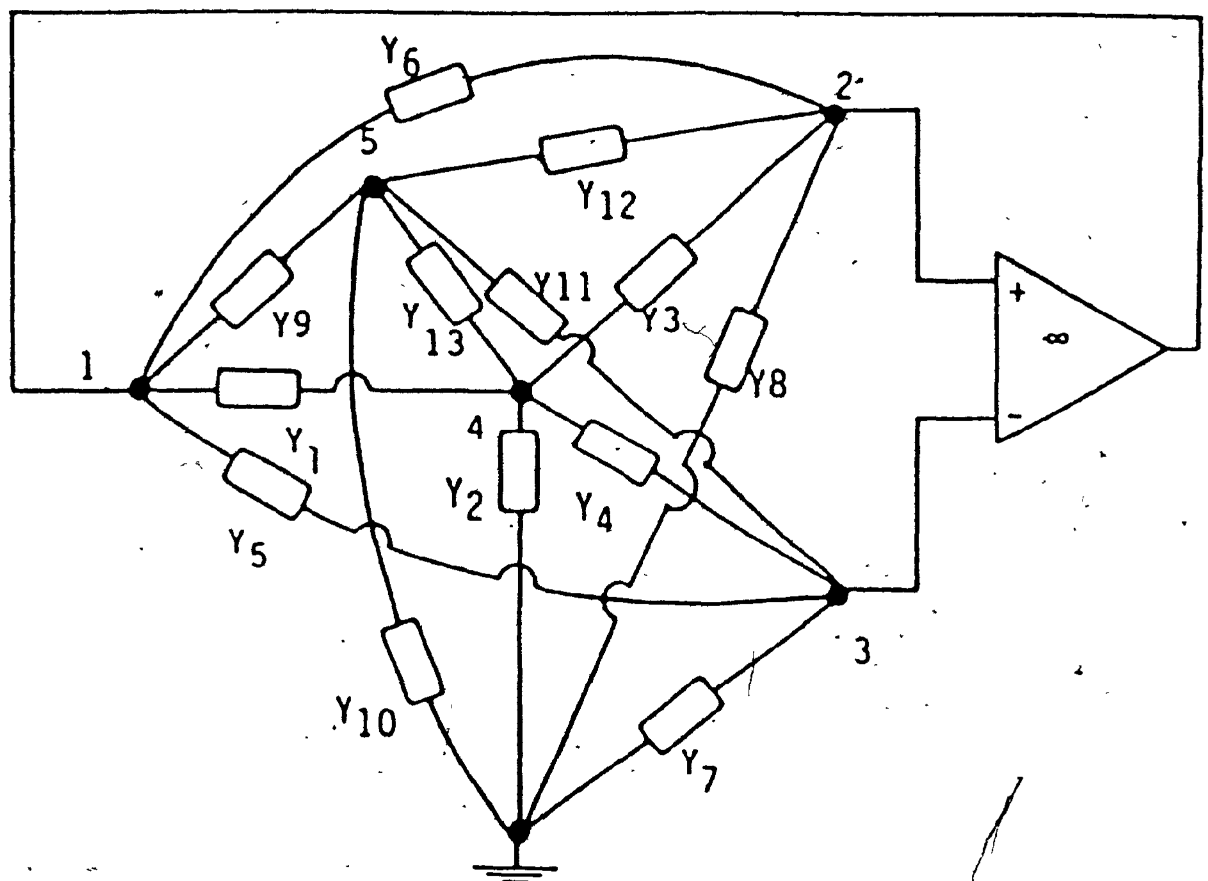

Fig. 2.6: Oscillator with the general six node passive network.

$$Y_6Y_{10}Y_7Y_2+Y_6Y_{10}Y_7Y_4+Y_6Y_{10}Y_7Y_{13}+Y_6Y_{10}Y_{11}Y_2+Y_6Y_{10}Y_{11}Y_4$$

$$+Y_6Y_{10}Y_{11}Y_{13}+Y_6Y_7Y_{11}Y_2+Y_6Y_7Y_{11}Y_4+Y_6Y_7Y_{11}Y_{13}+Y_6Y_{10}Y_2Y_4$$

$$+Y_6Y_{11}Y_2Y_4+Y_6Y_7Y_2Y_{13}+Y_6Y_{11}Y_2Y_{13}+Y_6Y_7Y_{13}Y_4+Y_6Y_{10}Y_{13}Y_4$$

$$+Y_6Y_2Y_4Y_{13}+Y_6Y_{12}Y_2Y_4+Y_6Y_{12}Y_7Y_4+Y_6Y_{12}Y_7Y_2+Y_6Y_9Y_2Y_4$$

$$+Y_6Y_9Y_7Y_4+Y_6Y_9Y_2Y_7+Y_6Y_{11}Y_{10}Y_1+Y_6Y_{10}Y_{11}Y_3+Y_6Y_{11}Y_7Y_1$$

$$+Y_6Y_{11}Y_7Y_3+Y_6Y_7Y_{10}Y_1+Y_6Y_7Y_{10}Y_3+Y_7Y_{11}Y_1Y_3+Y_7Y_{10}Y_1Y_3,$$

$$+Y_7Y_9Y_1Y_3+Y_7Y_6Y_9Y_1+Y_7Y_6Y_9Y_3+Y_7Y_{12}Y_1Y_3+Y_7Y_6Y_{12}Y_1$$

$$+Y_7Y_6Y_{12}Y_3+Y_7Y_3Y_{13}Y_1+Y_7Y_6Y_{13}Y_1+Y_7Y_6Y_{13}Y_3+Y_7Y_{12}Y_{13}Y_1$$

$$+Y_7Y_{12}Y_{13}Y_6+Y_7Y_9Y_{13}Y_3+Y_7Y_9Y_{13}Y_6+Y_7Y_9Y_{12}Y_3+Y_7Y_9Y_{12}Y_1$$

$$+Y_7Y_9Y_{12}Y_{13}+Y_2Y_9Y_{12}Y_4+Y_9Y_{12}Y_7Y_4+Y_9Y_{12}Y_7Y_2+Y_{10}Y_{11}Y_1Y_3$$

$$-Y_5Y_{10}Y_8Y_2-Y_5Y_{10}Y_8Y_3-Y_5Y_{10}Y_8Y_{13}-Y_5Y_{10}Y_{12}Y_2-Y_5Y_{10}Y_{12}Y_3$$

$$-Y_5Y_{10}Y_{12}Y_{13}-Y_5Y_8Y_{12}Y_2-Y_5Y_8Y_{12}Y_3-Y_5Y_8Y_{12}Y_{13}-Y_5Y_{10}Y_2Y_3$$

$$-Y_5Y_{12}Y_2Y_3-Y_5Y_8Y_2Y_{13}-Y_5Y_{12}Y_2Y_{13}-Y_5Y_8Y_{13}Y_3-Y_5Y_{10}Y_{13}Y_3$$

$$-Y_5Y_2Y_3Y_{13}-Y_5Y_{11}Y_2Y_3-Y_5Y_{11}Y_8Y_3-Y_5Y_{11}Y_8Y_2-Y_5Y_9Y_2Y_3$$

$$-Y_5Y_9Y_8Y_3-Y_5Y_9Y_2Y_8-Y_5Y_{12}Y_{10}Y_1-Y_5Y_{10}Y_{12}Y_4-Y_5Y_{12}Y_8Y_1$$

$$-Y_5Y_{12}Y_8Y_4-Y_5Y_8Y_{10}Y_1-Y_5Y_8Y_{10}Y_4-Y_8Y_{12}Y_1Y_4-Y_8Y_{10}Y_1Y_4$$

$$-Y_8Y_9Y_1Y_4-Y_8Y_5Y_9Y_1-Y_8Y_5Y_9Y_4-Y_8Y_{11}Y_1Y_4-Y_8Y_5Y_{11}Y_1$$

$$-Y_8Y_5Y_{11}Y_4-Y_8Y_4Y_{13}Y_1-Y_8Y_5Y_{13}Y_1-Y_8Y_5Y_{13}Y_4-Y_8Y_{11}Y_{13}Y_1$$

$$-Y_8Y_{11}Y_{13}Y_5-Y_8Y_9Y_{13}Y_4-Y_8Y_9Y_{13}Y_5-Y_8Y_9Y_{11}Y_4-Y_8Y_9Y_{11}Y_1$$

$$-Y_8Y_9Y_{11}Y_{13}-Y_2Y_9Y_{11}Y_3-Y_9Y_{11}Y_8Y_3$$

$$-Y_9Y_{11}Y_8Y_2-Y_{10}Y_{12}Y_1Y_4=0 \quad (2\text{-}55)$$

There are 13 branch admittances in N. The problem is one of assigning six elements (four resistors and two capacitors) properly to these admittances. Even assuming one element per branch, choosing six out of 13 and then choosing two out of six as capacitors, requires $\binom{13}{6}\cdot\binom{6}{2}=25{,}740$ combinations to be investigated. This would be an extremely time consuming and a tedious job. Fortunately an elegant and simple approach is possible.

Let n, b and l be the number of nodes, branches and loops in the passive network respectively. Then l is given by

$$l = b - n + 1 \quad (2\text{-}56)$$

In order to have at least one loop in the network b=6. This means at least six elements are required to construct the network. Figure 2.7(a) shows one loop with six nodes distributed on it. Choose any node as input node(for example,node 1). Then any other node (not next to input node) can be chosen as ground node (for example, 3,4 or 5). There can not be any branch connected directly between the input terminals of OA. Therefore, the OA's input nodes

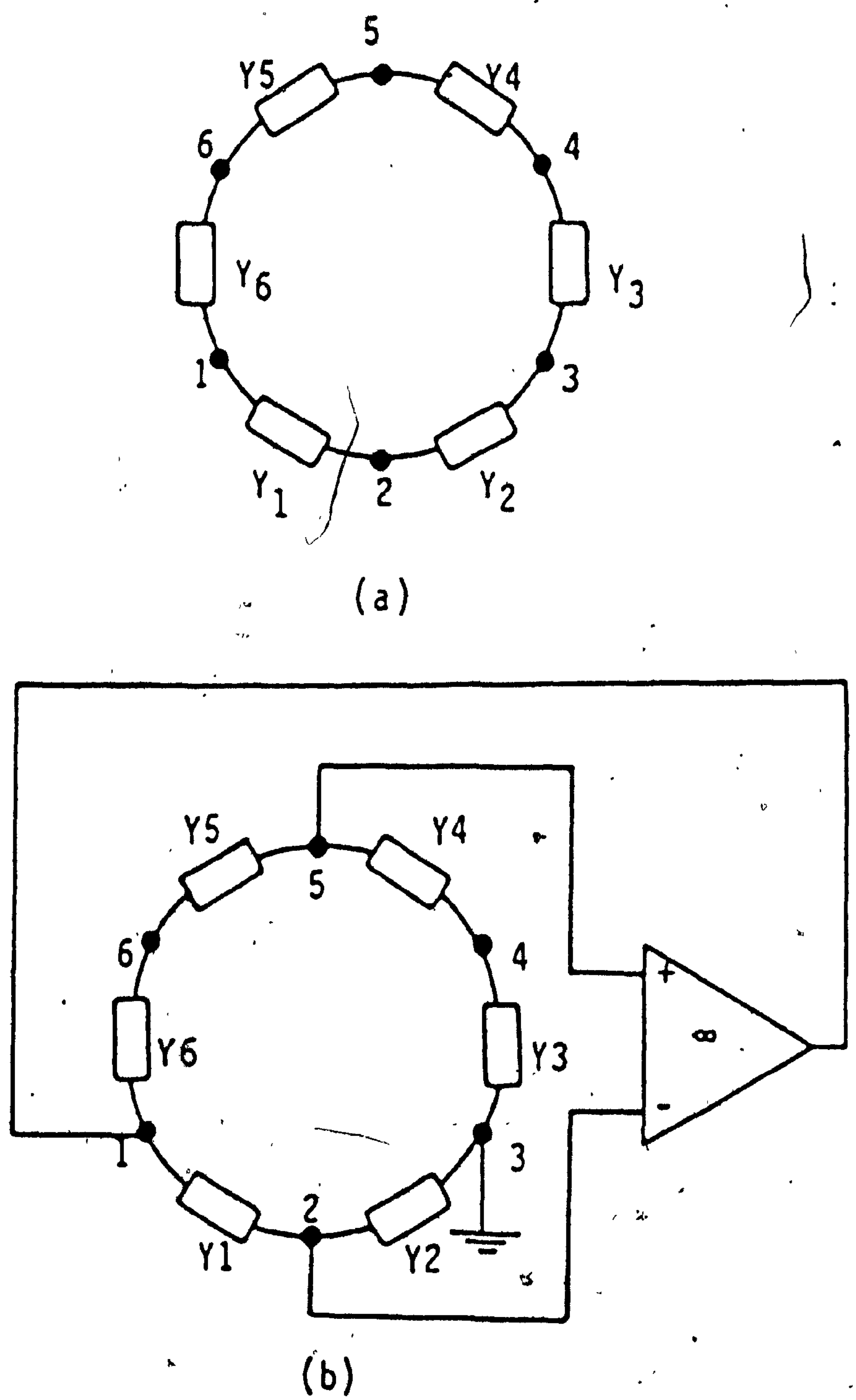

Fig. 2.7(a): Passive six node, one loop, six branch network.
(b): Configuration with one loop passive network.

should be between input node and the ground. Figure 2.7(b) shows an example, the CE of Fig.2.7(b) is

$$Y_a Y_b - Y_c Y_d = 0 \qquad (2\text{-}57)$$

where

$$Y_a = Y_1, \qquad Y_b = \frac{Y_3 Y_4}{Y_3 + Y_4}, \qquad Y_c = \frac{Y_6 Y_5}{Y_6 + Y_5}, \qquad Y_d = Y_2.$$

Analysis of (2-57) does not lead to any oscillator circuit. In fact, it is shown in Appendix D that a single loop RC network with more than five nodes, two Cs, one OA and any number of Rs can neither produce a canonic SFO, nor can it produce a VFO.

The study of a six-node network with seven branches is unnecessary, since it is not canonic.

2.4.5 Seven-Node Network

In this case, N has three internal nodes. From the previous discussion, it is clear that N must have at least two loops to be able to produce oscillation. From (2-56) it can be shown that at least eight branches are then required in N. This requires at least seven elements. Hence, the circuits will be non-canonic. Clearly then study of seven and higher node networks is not warranted. Thus, the complete set of canonic SFOs consists of 12 circuits as given in Fig.2.5. Circuits of Figs.2.5(a), (f) and (j) are new SFOs.

2.5 Design Procedures

The design procedures have been derived for all the 12 SFOs. However, the procedure is illustrated here only for the circuit of Fig.2.5(j). The same procedure can be used for all remaining 11 circuits. Design equations for all the circuits are given in Table 2-I. The CE of the circuit of Fig.2.5(j) is

$$Y_5Y_8(Y_2+Y_3+Y_4)+Y_5Y_3Y_2-Y_6Y_4Y_2=0 \tag{2-58}$$

choosing $Y_4=SC_4$, $Y_8=SC_8$, $Y_2=G_2$, $Y_3=G_3$, $Y_6=G_6$, $Y_5=G_5$, (2-58) can be written as

$$S^2G_5C_8C_4+S[G_5C_8(G_2+G_3)-G_6G_2C_4]+G_5G_3G_2=0 \tag{2-59}$$

$$\text{OC:}\quad G_5C_8(G_2+G_3)-G_6G_2C_4=0 \tag{2-60}$$

$$\text{OF:}\quad \omega_s=\left(\frac{G_3G_2}{C_8C_4}\right)^{1/2} \tag{2-61}$$

Choosing $G_2=G_3=G_6=G=\frac{1}{R}$ and $C_4=C_8=C$ from (2-60) and (2-61)

$$G_5=\frac{1}{2R}\ \text{and}\ \omega_s=\frac{1}{RC} \tag{2-62}$$

Note that G_6 and G_5 can be varied without affecting ω_s. Thus one of them can be adjusted to set the circuit to

Table 2-I : Design equations for circuits of Fig.2.5.

(a)	$C_2 = C_7 = C$, $G_6 = G_7 = G_3 = G = 1/R$ OC: $G_5 = 2G$, $\omega_s = \frac{1}{RC}$
(b)	$C_3 = C_8 = C$, $G_8 = G_5 = G_1 = G = 1/R$ OC: $G_7 = 2G$, $\omega_s = \frac{1}{RC}$
(c)	$C_6 = C_2 = C$, $G_6 = G_7 = G_3 = G = 1/R$ OC: $G_5 = 2G$, $\omega_s = \frac{1}{RC}$
(d)	$C_1 = C_6 = C$, $G_6 = G_7 = G_4 = G = 1/R$ OC: $G_8 = 2G$, $\omega_s = \frac{1}{RC}$
(e)	$C_1 = C_4 = C$, $G_8 = G_5 = G_2 = G = 1/R$ OC: $G_6 = 2G$, $\omega_s = \frac{1}{RC}$
(f)	$C_2 = C_3 = C$, $G_8 = G_5 = G_4 = G = 1/R$ OC: $G_6 = 2G$, $\omega_s = \frac{1}{RC}$
(g)	$C_2 = C_3 = C$, $G_8 = G_5 = G_1 = G = 1/R$ OC: $G_7 = 2G$, $\omega_s = \frac{1}{RC}$
(h)	$C_1 = C_4 = C$, $G_8 = G_5 = G_3 = G = 1/R$ OC: $G_7 = 2G$, $\omega_s = \frac{1}{RC}$
(i)	$C_2 = C_5 = C$, $G_8 = G_4 = G_1 = G = 1/R$ OC: $G_6 = 2G$, $\omega_s = \frac{1}{RC}$
(j)	$C_4 = C_8 = C$, $G_5 = G_2 = G_3 = G = 1/R$ OC: $G_6 = 2G$, $\omega_s = \frac{1}{RC}$
(k)	$C_1 = C_8 = C$, $G_5 = G_2 = G_3 = G = 1/R$ OC: $G_7 = 2G$, $\omega_s = \frac{1}{RC}$
(l)	$C_3 = C_5 = C$, $G_8 = G_4 = G_1 = G = 1/R$ OC: $G_7 = 2G$, $\omega_s = \frac{1}{RC}$

oscillation.

2.6 About the 12 circuits

It has been shown that there are only 12 canonic SFOs. We have mentioned that 3 of these circuits are new, the other 9 circuits have been reported in [8]. However, [8] gives also 12 canonic SFOs. In the following we clarify the apparent discrepency.

In [8], 16 SFOs are found using controlled sources as active elements. The configurations are shown in Figs.6-9 of the same reference. In realization of controlled sources however, four of the circuits need more than one OA which are not canonic. The remaining 12 realizations are shown in Table I as cases (a)-(l). These 12 circuits are realizations of the configurations (a)-(f) of Figs.6 and 8. Three circuits, namely, (j)-(l) of Table I are physically the same as the three circuits (i), (g) and (h), respectively, of Table I [8].

2.7 Experimental Results

All the 12 SFO circuits have been tested experimentally. Experimental results agree with the theoretical analysis. In all the experiments the amplitude of the output has been constant (25 volts peak to peak). The maximum distortion measured for any circuit has been less than 1.5% . The

Signal has been measured between output of OA (Fairchild 741) and the ground node. The power supply of ±15 volts has been used. Only the experimental results of one circuit are given here as representitive sample. Figure 2.8 shows the output signal of oscillator circuit corresponding to Fig.2.5(a) at two frequencies: (a) 0.104 kHz and (b) 1.326 kHz. The component values for two circuits are:

(a) R_5=67.2 kΩ, R_6=36.6 kΩ, R_3=R_7=1.2 kΩ, C_7=C_2=0.1 µF,

(b) R_5=225 kΩ, R_6=129.2 kΩ, R_3=R_7=15.2 kΩ, C_7=C_2=0.1 µF,

All the resistors are carbon film of tolerance 5%. All the capacitors are polystyrene of tolerance 5%.

2.8 Summary

Using a new and unified approach for generating canonic RCOs, it is established that the minimum number of passive components required to generate an OA based SFO is 6. A set of 12 single frequency oscillators is found. The set is shown to be complete, that is, no additional single frequency RC oscillator circuits can be generated using one OA, two capacitors and four resistors. All the 12 circuits have been tested in the laboratory. The results of the tests are in close agreement with the theory. While nine of these oscillators have already been reported in the literature, three are new oscillator circuits. One of the

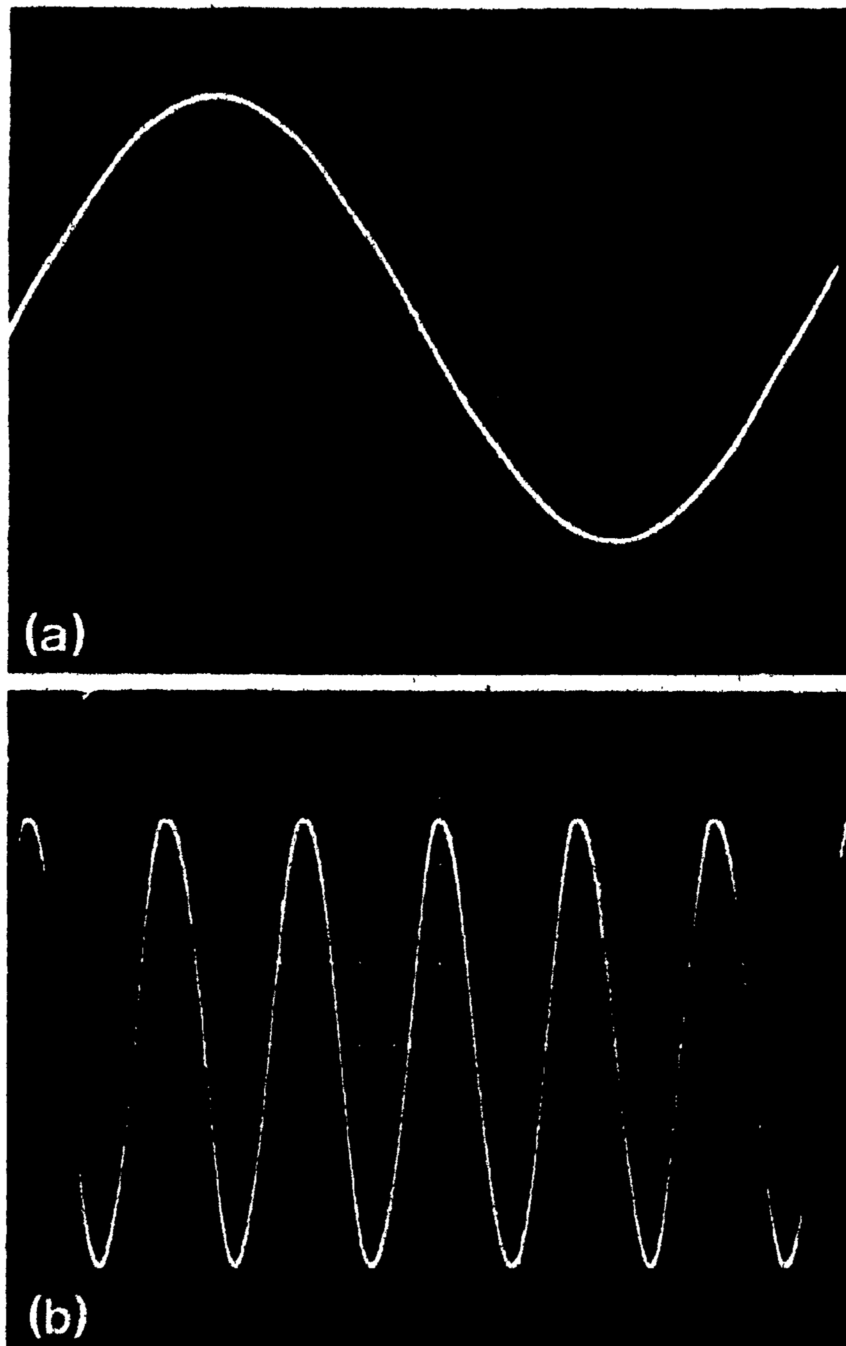

Fig. 2.8: Output signal of circuit of Fig.2.5(a), at two frequencies. (a) 104 Hz, (b) 1326 Hz.

new circuits has both the capacitors grounded, which is an attractive feature for IC technology.

CHAPTER III

CANONIC VARIABLE FREQUENCY RC-OSCILLATORS USING OAs

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CANONIC VARIABLE FREQUENCY RC-OSCILLATORS USING OAs

3.1 Introduction

In variable frequency oscillators the condition of oscillation is independent of the frequency of oscillation. Thus, the frequency can be varied over a range without affecting the condition of oscillation. It is important that the frequency can be varied by varying the value of only one element. For many practical reasons the variable element should be a resistor. For example, by simply making the variable resistor to be voltage dependent, a voltage controlled oscillator (VCO) can be obtained.

3.2 Theory

It is necessary in a VFO that the frequency can be varied while the circuit is in operation. In mathematical terms, this is equivalent to having OF varied without affecting the OC. This means the expressions for OF and OC should be such that OF contains an element (variable element) of the circuit which OC is independent of.

Since Fig.2.2(a) is the most general configuration for RCOs using OAs, it is, necessary to take Fig.2.2(a) as the general configuration for the study of VFOs.

Consider Fig.2.2(a), The CE is given by (2-7). From the previous chapter, the experessions for OC and OF are

$$\text{OC:} \quad \beta = 0 \tag{3-1}$$

$$\text{OF:} \quad \omega_s = (\gamma/\alpha)^{1/2} \tag{3-2}$$

To vary the frequency from (3-2) α and/or γ should be varied. Assuming that p is the variable element of the network, then OF and OC can be written as:

$$\omega_s = (\gamma/\alpha)^{1/2} = f(p) \tag{3-3}$$

$$\beta = g(p) \tag{3-4}$$

Any network coefficient can be expresed as a linear function of any of the network elements [23]. Hence, the most general form of (3-4) is

$$\beta = 0 = (U_1 - U_2)p + (U_3 - U_4) = 0 \tag{3-5}$$

where U_1, U_2, U_3, and U_4 are functions of the elements of the circuit other than p. To satisfy (3-5), it is necessary that

$$U_1 - U_2 = 0 \tag{3-6a}$$

$$U_3 - U_4 = 0 \qquad (3\text{-}6b)$$

If (3-6) is satisfied, it is obvious that variations of p cannot alter (3-5). Therefore, f(p) can be varied while g(p) is always zero.

3.3 Realization of VFOs

It was shown in chapter 2 that four node networks cannot generate any oscillator circuits. Therefore, it is clear that for VFOs, the network N must have at least five nodes. It is also shown in Appendix D that a single loop network can not generate a VFO. Therefore, N must have a minimum of 2 loops. Since at least two capacitors and four resistors are required for an SFO, a VFO has at least two capacitors, and five resistors. Finally, from (2-56) a five-node network with two loops must have at least six branches. Therefore, the first network to be considered is a five node six branch network having seven elements.

3.3.1 Five-Node Six-Branch Seven-Element Network

Consider Fig.2.4. It is the most general configuration of a five node network. As it was mentioned before, branches Y_9 and Y_{10} do not contribute to the CE and are removed. The CE of the network is given by (2-22). A six-branch network can have seven elements only if one of the branches is of the form SC_1+G_1 and others are single element branches. To obtain a six-branch network any two out of eight branches present in N can be removed. In this way only eight CEs that are potential candidate for generating oscillators are found. These CEs are the same as Eqns. (2-41) to (2-48). These equations are similar to (2-49). A detailed analysis of (2-49) shows that no VFO circuit can be found from this type of network.

3.3.2 Five-Node Seven-Branch Network

Again, Fig.2.4 repersents the general circuit and CE is given by Eqn.(2-22). There are eight branches in N. To generate a seven branch network it is only necessary to set one branch admittance at a time to zero. In this way eight CE can be found as:

$$Y_1=0, \quad Y_6Y_7(Y_2+Y_3+Y_4)+Y_6Y_2Y_4 -Y_5Y_8(Y_2+Y_3+Y_4)-Y_5Y_2Y_3=0 \tag{3-7a}$$

$$Y_2=0, \quad Y_7Y_6(Y_1+Y_4+Y_3)+Y_7Y_1Y_3 -Y_8Y_5(Y_1+Y_4+Y_3)-Y_8Y_1Y_4=0 \tag{3-7b}$$

$$Y_3=0, \quad Y_7Y_6(Y_4+Y_1+Y_2)+Y_6Y_4Y_2 -Y_8Y_5(Y_4+Y_1+Y_2)-Y_8Y_4Y_1=0 \tag{3-7c}$$

$$Y_4=0, \quad Y_7Y_6(Y_3+Y_2+Y_1)+Y_7Y_3Y_1$$

$$-Y_8Y_5(Y_3+Y_2+Y_1)-Y_5Y_3Y_2=0 \qquad (3\text{-}7d)$$

$Y_5=0,$
$$Y_7Y_6(Y_3+Y_4+Y_1+Y_2)+Y_3Y_1Y_7$$
$$+Y_6Y_4Y_2-Y_8Y_4Y_1=0 \qquad (3\text{-}8a)$$

$Y_6=0,$
$$Y_5Y_8(Y_3+Y_4+Y_1+Y_2)+Y_4Y_1Y_8$$
$$+Y_5Y_3Y_2-Y_7Y_3Y_1=0 \qquad (3\text{-}8b)$$

$Y_7=0,$
$$Y_5Y_8(Y_1+Y_2+Y_4+Y_3)+Y_1Y_4Y_8$$
$$+Y_5Y_2Y_3-Y_6Y_2Y_4=0 \qquad (3\text{-}8c)$$

$Y_8=0,$
$$Y_6Y_7(Y_1+Y_2+Y_3+Y_4)+Y_1Y_3Y_7$$
$$+Y_6Y_2Y_4-Y_5Y_2Y_3=0 \qquad (3\text{-}8d)$$

Equations (3-7) and (3-8) are mathematically the same as (3-9) and (3-10) respectively, as given below

$$Y_aY_f(Y_b+Y_c+Y_d)+Y_gY_bY_d-Y_hY_e(Y_b+Y_c+Y_d)$$
$$-Y_hY_bY_c=0 \qquad (3\text{-}9)$$

$$Y_gY_f(Y_a+Y_b+Y_c+Y_d)+Y_aY_cY_f+Y_gY_bY_d-Y_hY_bY_c=0 \qquad (3\text{-}10)$$

Analysing (3-9), it is found that the only choices for capacitive and resistive branches leading to VFOs are,

(i) $Y_c=SC_c$, $Y_f=SC_f$, $Y_b=G_b$, $Y_d=G_d$, $Y_g=G_g$, $Y_h=G_h$, $Y_e=G_e$ and

(ii) $Y_d=SC_d$, $Y_e=SC_e$, $Y_h=G_h$, $Y_b=G_b$, $Y_c=G_c$, $Y_f=G_f$, $Y_g=G_g$.

For choice(i), the CE is:

$$S^2G_gC_fC_c+S[\ G_b(G_gC_f-G_hC_c)+(G_gG_dC_f$$

$$-G_hG_eC_c)]\ +G_gG_bG_d-G_hG_e(G_b+G_d)\ =\ 0 \qquad (3\text{-}11a)$$

$$OC:\ \ G_gC_f(G_b+G_d)-G_hG_eC_c-\ G_hG_bC_c=\ 0 \qquad (3\text{-}11b)$$

$$OF:\ \ \omega_s=\{\ \frac{G_gG_bG_d-G_hG_e(G_b+G_d)}{G_gC_fC_c}\ \}^{1/2} \qquad (3\text{-}11c)$$

Writing (3-11) in the form of (3-3), we have

$$G_b(G_gC_f-G_hC_c)+(G_gG_dC_f-G_hG_gC_c)=0 \qquad (3\text{-}12)$$

Comparing (3-12) with (3-6), OC is:

$$G_gC_f-G_hC_c=\ 0 \qquad (3\text{-}13a)$$

$$G_gG_dC_f-G_hG_eC_c=\ 0 \qquad (3\text{-}13b)$$

and OF is: $$\omega_s=\{\ \frac{G_gG_bG_d-G_hG_e(G_b+G_d)}{G_gC_fC_c}\ \}^{1/2}\ >0 \qquad (3\text{-}13c)$$

or

$$G_d = G_e, \quad G_g/G_h = C_c/C_f = k, \; G_b > \frac{G_d}{k-1}$$

where G_b is the variable element. For choice (ii), the CE is

$$-S^2 G_h C_e C_d + S[G_g G_f C_d + G_g G_b C_d - G_h C_e (G_b + G_c)]$$
$$+ G_g G_f (G_b + G_c) - G_h G_b G_c = 0 \qquad (3\text{-}14)$$

yielding:

$$\text{OC:} \quad G_f = G_c \text{ and } G_g/G_h = C_e/C_d = k \text{ and } G_b > \frac{G_c}{k-1} \qquad (3\text{-}15a)$$

$$\text{OF:} \quad \omega_s = \left\{ \frac{G_g G_f (G_b + G_c) - G_h G_b G_c}{G_h C_e C_d} \right\}^{1/2} \qquad (3\text{-}15b)$$

and G_b is the variable element.

When the results from part (i) are put into (3-7a)-(3-7d), four VFOs are found. For example, from (3-7a) it is found that

$$\text{OC:} \quad G_8 = G_4, \; G_6/G_5 = C_3/C_7 = k, \; G_2 > \frac{G_4}{k-1} \qquad (3\text{-}16a)$$

where G_2 is the variable element and,

$$\text{OF:} \quad \omega_s = \left\{ \frac{G_6G_4G_2 - G_5G_8(G_2+G_4)}{G_6C_3C_7} \right\}^{1/2} \tag{3-16b}$$

Using the results of part (ii) for (3-8a)-(3-8d) another four VFOs are found that are SU pairs of the four VFOs from part(i). Figures 3.1(a)-(d) show the final four stable VFOs obtained from (3-7a)-(3-7d).

Analysing (3-10), the possible choices for capacitive and resistive branches leading to VFOs are :

(i) $Y_a=SC_a$, $Y_c=SC_c$, $Y_b=G_b$, $Y_d=G_d$, $Y_g=G_g$, $Y_f=G_f$, $Y_h=G_h$

(ii) $Y_b=SC_b$, $Y_d=SC_d$, $Y_a=G_a$, $Y_c=G_c$, $Y_g=G_g$, $Y_f=G_f$, $Y_h=G_h$.

For choice (i), the CE from (3-10) is:

$$S^2G_fC_aC_c+S[G_gG_f(C_a+C_c)-G_hG_bG_c] +G_gG_f(G_b+G_d)+G_gG_bG_d=0 \tag{3-17}$$

and:

$$\text{OC:} \quad G_gG_f(C_a+C_c)-G_hG_bC_c=0 \tag{3-18a}$$

$$\text{OF:} \quad \omega_s = \left\{ \frac{G_gG_f(G_b+G_d)+G_gG_bG_d}{G_fC_aC_c} \right\}^{1/2} \tag{3-18b}$$

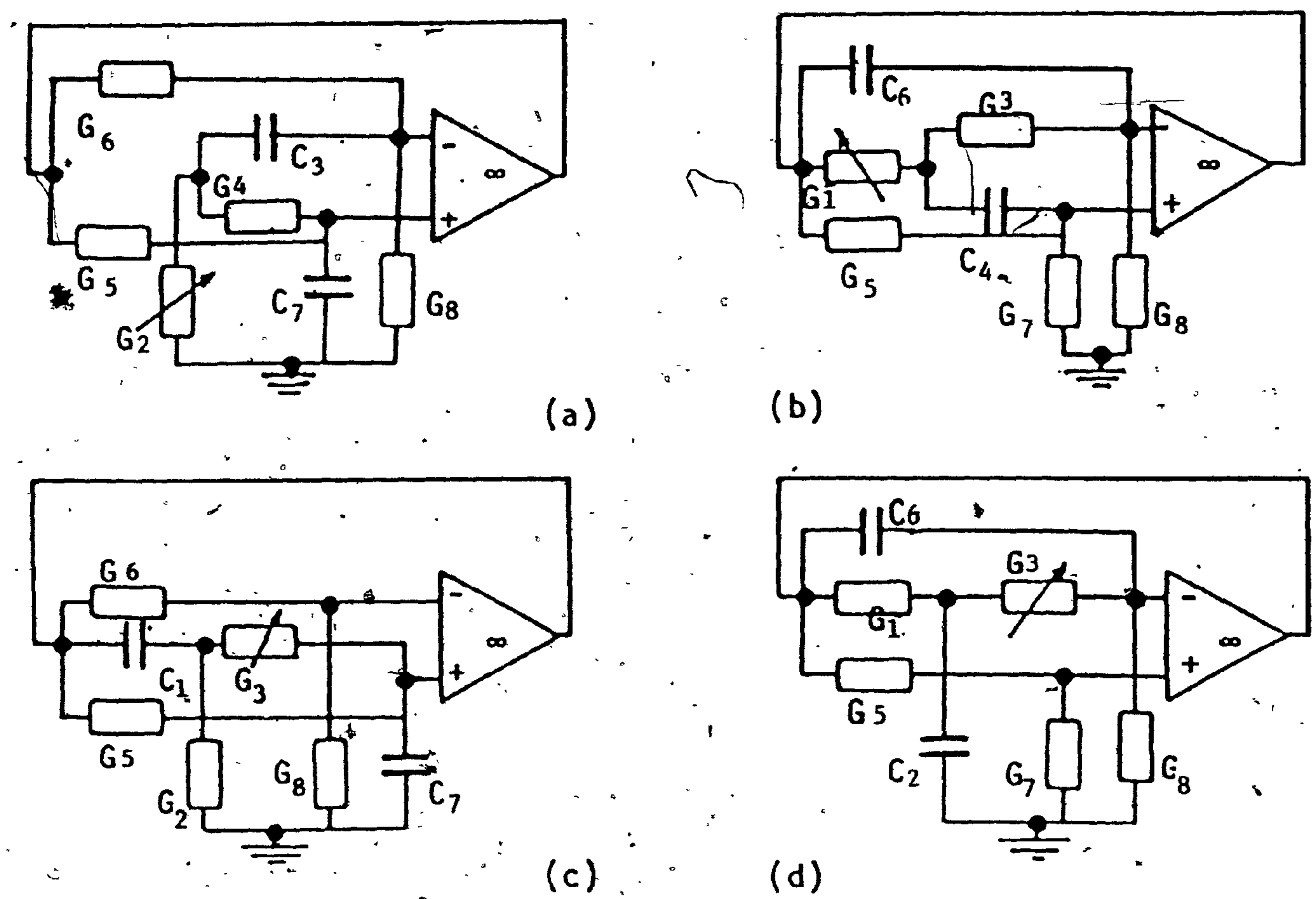


Fig. 3.1: The set of 16 canonic variable frequency oscillators (Continued).

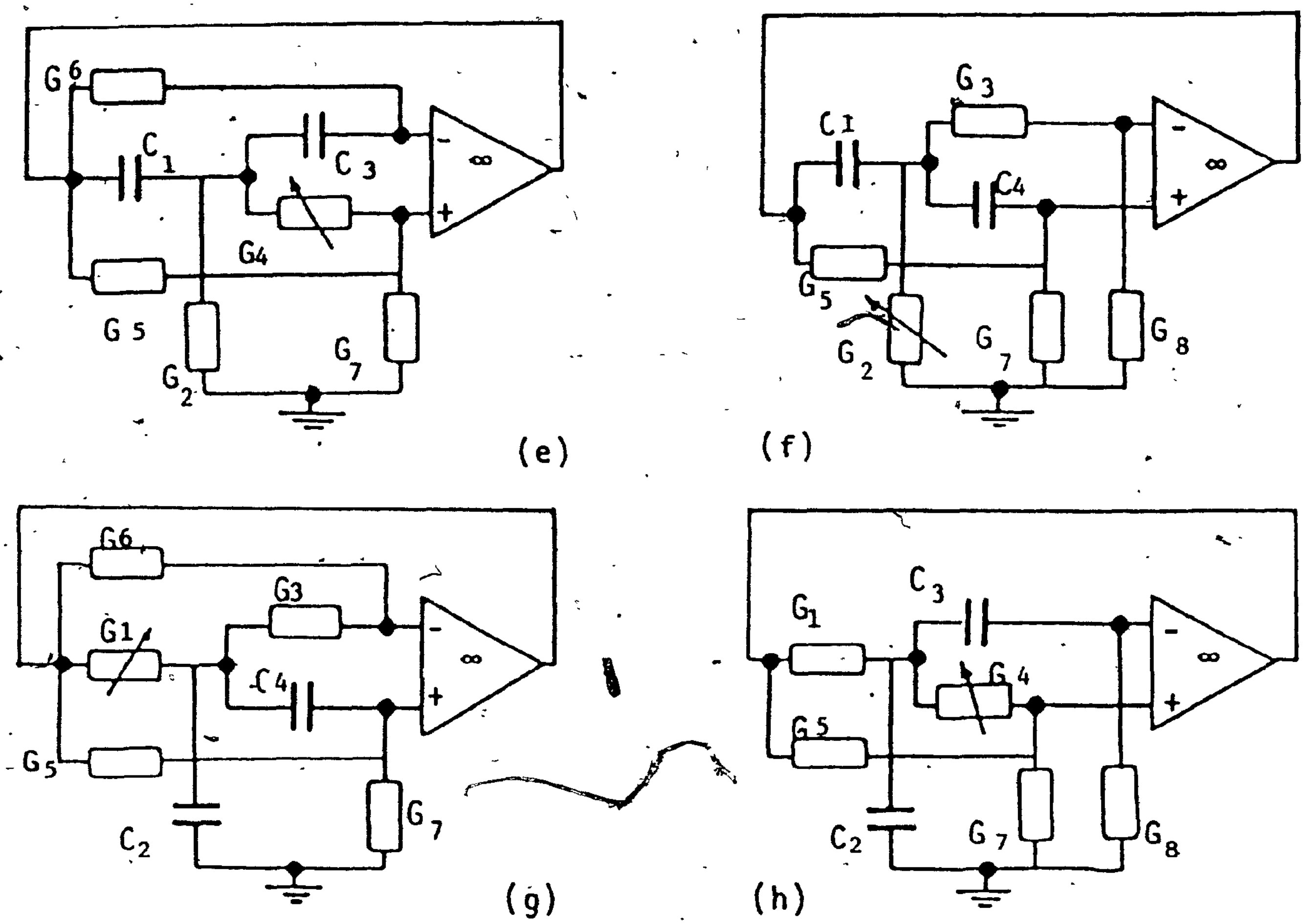


Fig. 3.1: The set of 16 canonic variable frequency oscillators (Continued).

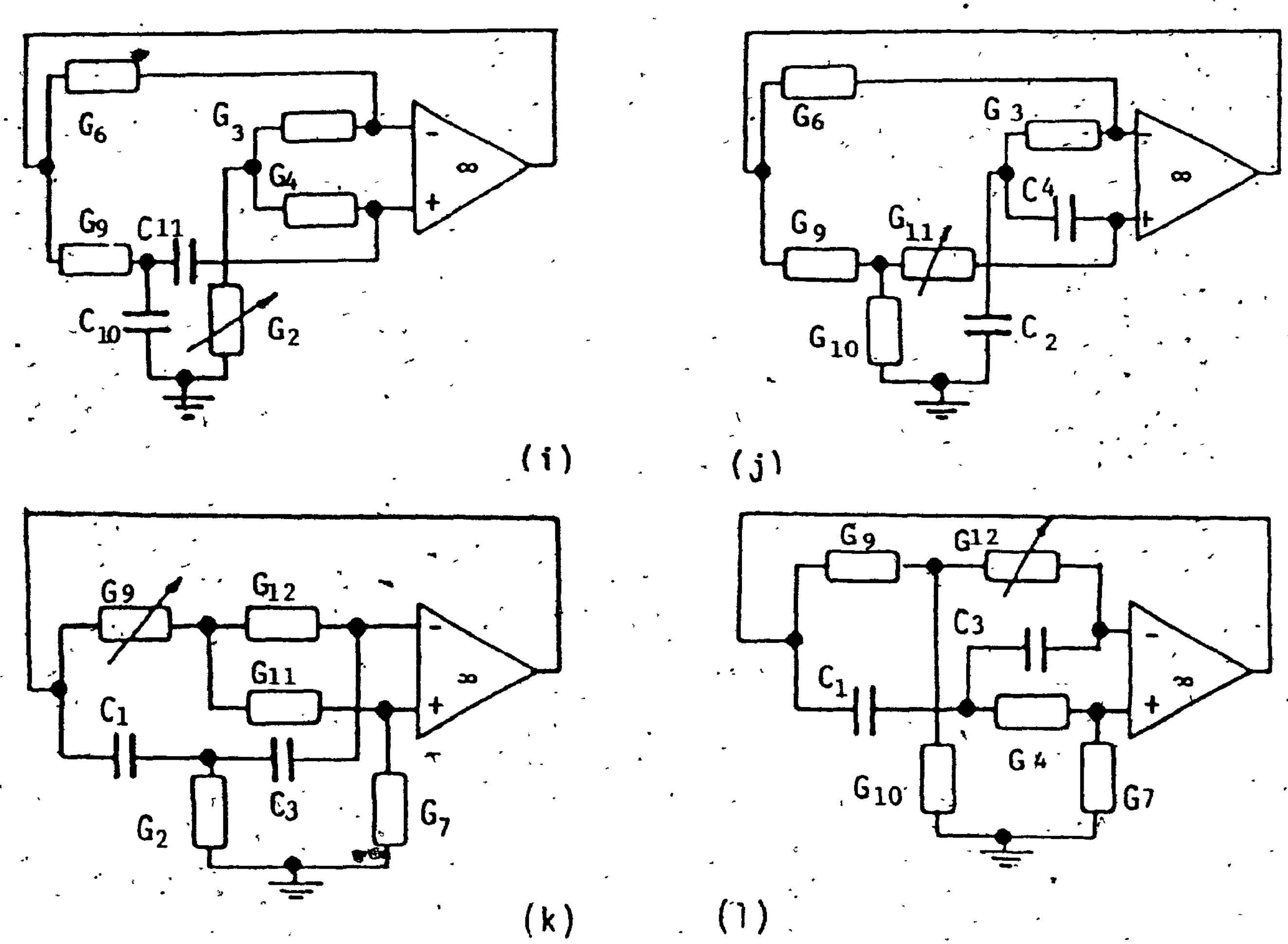


Fig. 3.1: The set of 16 canonic variable frequency oscillators (Continued).

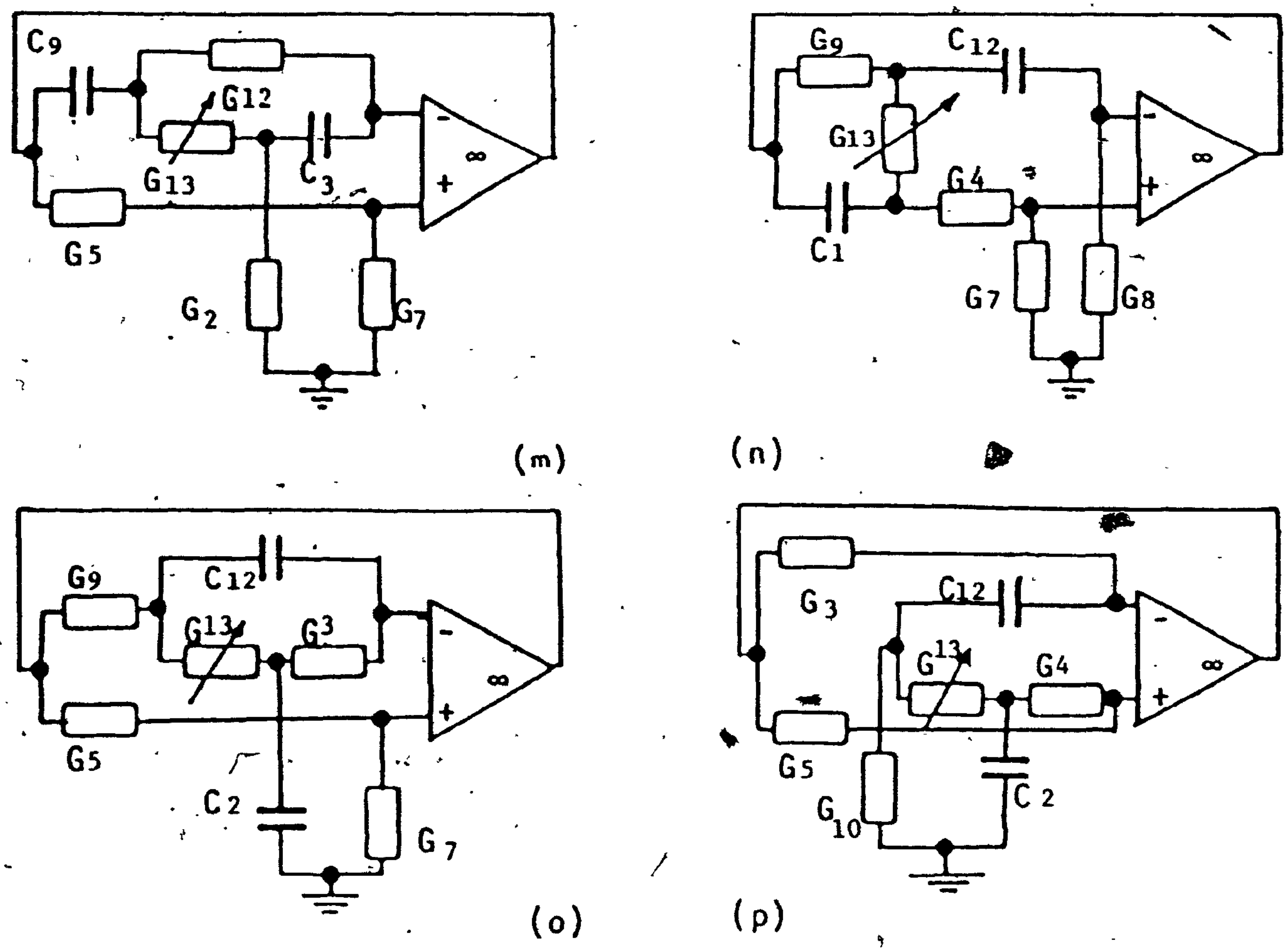


Fig. 3.1: The set of 16 canonic variable frequency oscillators (Continued).

Since (3-18a) is independent of G_d, satisfying (3-18a) allows (3-5) to be always satisfied when G_d is the variable element. Putting these results into (3-8a)-(3-8d) four VFOs are found. These VFOs are found to be two SU pairs yielding only two stable VFOs.

For choice (ii), the CE is

$$S^2G_gC_bC_d+S[G_gG_f(C_b+C_d)-G_hG_cC_b]$$

$$+G_gG_f(G_a+G_c)+G_aG_cG_f=0 \qquad (3\text{-}19)$$

$$\text{OC:}\quad G_gG_f(C_b+C_d)-G_hG_cC_b=0 \qquad (3\text{-}20a)$$

$$\text{OF:}\quad \omega_s=\left\{\frac{(G_gG_f(G_a+G_c)+G_aG_cG_f)}{G_gC_bC_d}\right\}^{1/2} \qquad (3\text{-}20b)$$

Here (3-20a) is independent of G_a. Therefore, G_a can be taken as the variable element. From these results, and (3-8a)-(3-8d), four VFOs are found, which are again two SU pairs, yielding only two stable VFOs. Figures 3.1(e)-(h) show the final four stable VFOs. Note that $\omega_s>0$ is always satisfied in both parts (i) and (ii).

Clearly, a VFO for which N has five nodes and eight branches or more is not canonic. Thus, a study of such networks is unwarranted.

3.3.3 Six-Node Network

Figure 2.6 shows the network with all its branches. The CE is given by (2-55). It is necessary to have at least two loops in the passive network (see Appendix D). From (2-56), in order to have at least two loops, N must have at least seven branches. There are a total of 13 branches in N. The number of ways the branches may be chosen, as well as the two capacitive and the variable resistive branches may be selected, are

$$\binom{13}{7}.\binom{7}{2}.\binom{5}{1}=180,180$$

Clearly, it is an enormous task for straight forward verification. Thus, we follow the approach adopted in the case of SFOs.

Figure 3.2 shows all the possible graphs with six nodes, seven branches and two loops. Consequently, a total of 61 cases should be considered for all the four graphs of Fig.3.2. However, no VFOs are found from graphs of Figs.3.2(b) and (c). To illustrate this, let us follow the steps for the case of Fig.3.2(a).

For the graph of Fig.3.2(a), any of the nodes can be chosen as the ground node (for example, node 1). Then only nodes 3,5, or 6 can be chosen as input node (for example, node 3). The possible input terminals of the OA are then 5 and 6 or 2 and 4 or 6 and 2 (for example, 5 and 6).

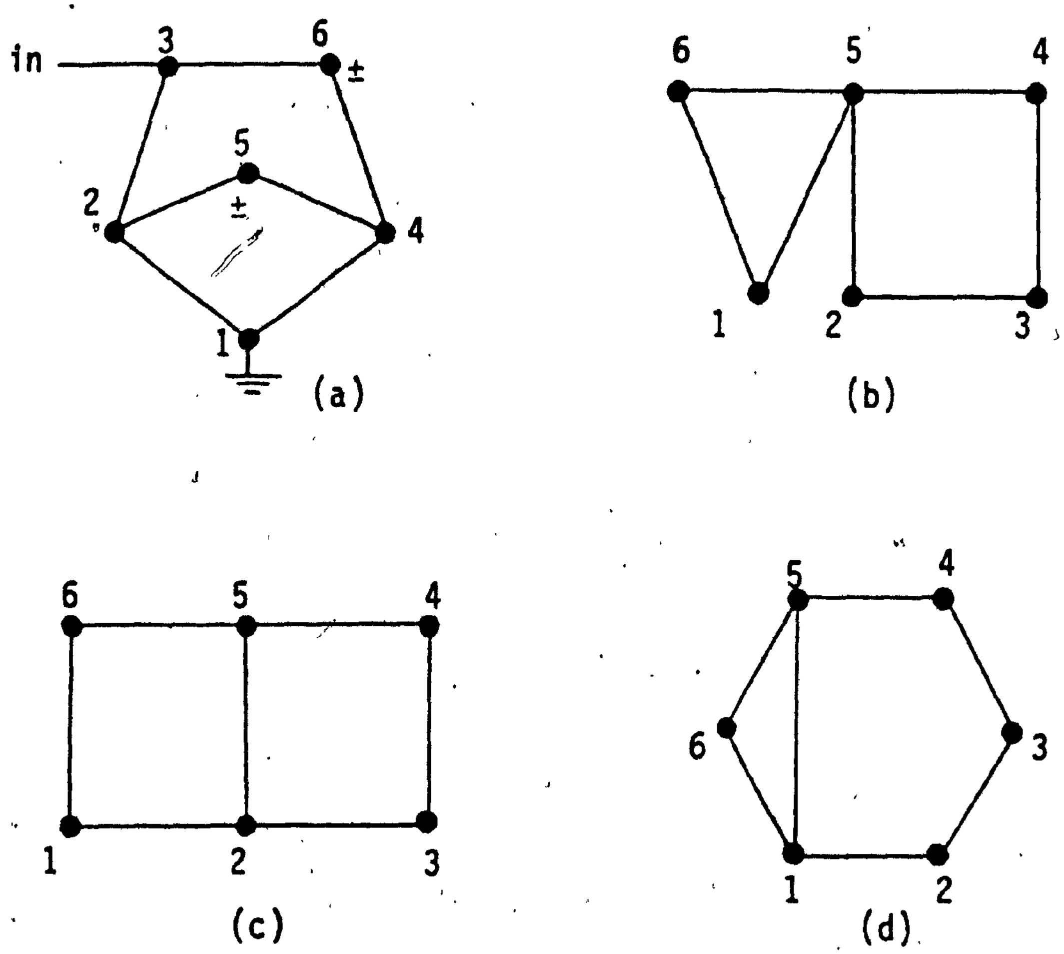


Fig. 3.2: All the possible graphs of a six node, seven branch, two loop connected network.

Comparing the resulting circuit with Fig.2.6, the nodes and branches can be renamed. Figure 3.3 shows the resulting circuit and CE can be found from (2-55) as:

$$Y_6Y_2Y_4Y_{11}+Y_6Y_2Y_4Y_{10}+Y_6Y_2Y_{10}Y_{11}+Y_6Y_4Y_{10}Y_{11}$$
$$+Y_6Y_3Y_{10}Y_{11}+Y_6Y_9Y_2Y_4-Y_2Y_3Y_9Y_{11}=0 \quad (3\text{-}21)$$

Using rules I-IV, one possible choice for capacitive branches is $Y_{11}=SC_{11}$, $Y_{10}=SC_{10}$. All other branches are then resistive. Then (3-21) can be written as

$$S^2[(G_2+G_3+G_4)G_6C_{10}C_{11}]+S[G_6G_4G_2(C_{10}+C_{11})$$
$$-G_2G_3G_9C_{11}]+G_6G_4G_2G_9 = 0 \quad (3\text{-}22)$$

$$\text{OF:} \quad \omega_s=\left\{\frac{G_4G_2G_9}{(G_2+G_3+G_4)C_{10}C_{11}}\right\}^{1/2} \quad (3\text{-}23a)$$

$$\text{OC:} \quad G_2(G_6G_4(C_{10}+C_{11})-G_3G_9C_{11})=0 \quad (3\text{-}23b)$$

Comparing (3-23) with (3-5) shows that $U_3 = U_4 = 0$

$$U_1=G_6G_4(C_{10}+C_{11}), \qquad U_2=G_3G_9C_{11}$$

and G_2 is the variable element. Figure 3.1(i) shows the final circuit.

Note that ω_s is independent of G_6. Thus, G_6 can be used to set the OC of the circuit. Using the same procedure, a total of 16 VFOs are found. However, eight of them can be derived from the other eight simply by interchanging the two

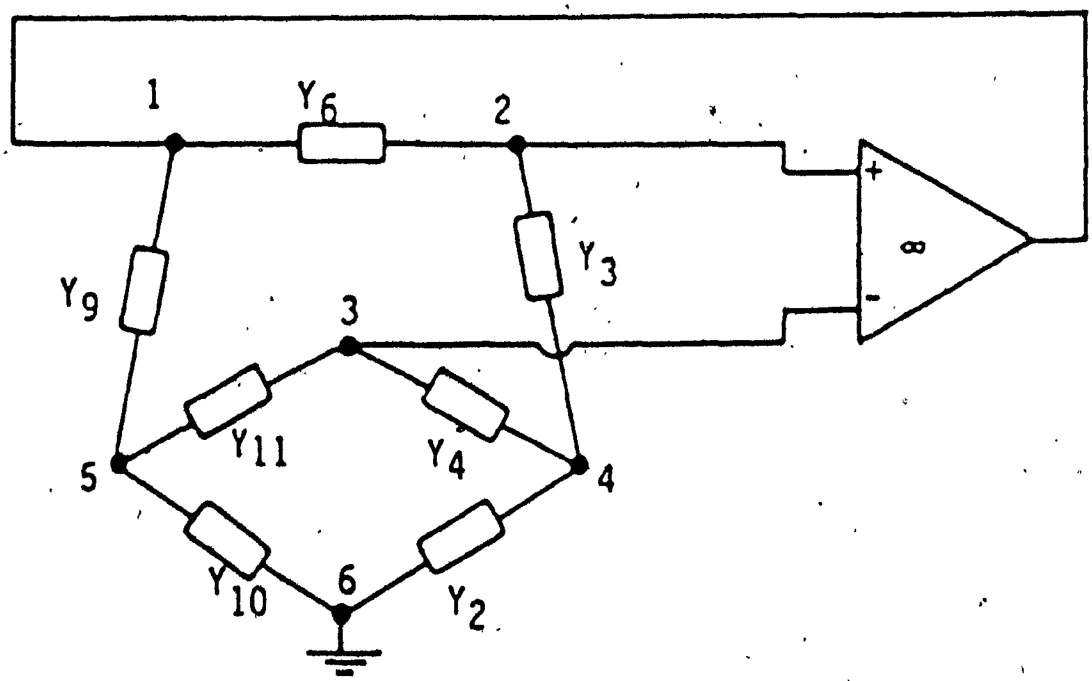


Fig. 3.3: Active network with a six node, seven branch, two loop passive network.

internal nodes. Finally, the eight circuits are four SU pairs. Figures 3.1(i)-(l) show the final four stable VFOs.

Using the same procedure for the graph of Fig.3.2(d), a total of 16 VFOs are found. Again, these 16 circuits are reduced to distinct eight for internal nodes. Further, the eight circuits are found to be four SU pairs. Figures 3.1(m)-(p) show the four circuits.

Analysis of an eight branch network is not required because it needs eight elements which is not canonic.

3.3.4 Seven-Node Network

In a seven-node network, at least eight branches should be present to generate two loops. Eight branches require at least eight elements. Since VFOs with seven elements have been found, eight element circuits are not canonic. Therefore, the study of networks with seven or more nodes is not required.

3.4 Design procedures

Design procedures are given here for Fig.3.1(i) only. Similar procedures can be used for all other VFO circuits. The OC and OF are given by (3-23a) and (3-23b). Choosing $C_{10}=C_{11}=C$ and $G_3=G_9=G_4=G$, (3-23a) and (3-23b) can be written as:

$$\text{OF:} \quad \omega_s = \frac{G}{C}\cdot\left(\frac{G_2}{G_2+2G}\right)^{1/2} \qquad (3\text{-}24a)$$

$$OC: \quad 2G_6GC-G^2C=0 \text{ or } G_6=\frac{G}{2} \qquad (3\text{-}24b)$$

Let $G=\frac{1}{R}$. Then the design equations are

$C_{10} = C_{11} = C$, $R_3= R_9= R_4= R$, $R_6= 2R$

$$\omega_s= \frac{1}{RC}\cdot\left(\frac{R}{R+2R_2}\right)^{1/2} \qquad (3\text{-}25)$$

Where R_2 is the variable resistor. The designs for all the VFOs are given in Table 3-I.

3.5 Classification of VFOs

The 16 VFOs can be classified into four classes according to the variation of their frequency as a function of the variable resistor.

3.5.1 Class A

The circuits of this class are shown in Figs.3.1(a)-(d). The OF is

$$\omega_s = \frac{1}{kRC}\cdot\left(\frac{kR-R-R_v}{R_v}\right)^{1/2} \qquad (3\text{-}26)$$

where $k>1$ is a constant and R_v is the variable resistor. Sensitivity analysis shows that this class has high sensitivity to R_v at low frequencies. Figure 3.4(a) shows $S_{R_v}^{\omega_s}$ (sensitivity of ω_s with respect to R_v). A plot of ω_s vs R_v is also included in Fig.3.4(a).

Table 3-I : Design equations for circuits of Fig.3.1

(a)	$C_7=C,\ G_5=G_4=G_8=G,\ \frac{G_6}{G_5}=\frac{C_3}{C_7}=K$ $\omega_s=\frac{G}{kC}\left(\frac{G_2(k-1)-G}{G}\right)^{1/2}$ OC: $G_2>\frac{G}{k-1},\ K>1$
(b)	$C_6=C,\ G_5=G_3=G_8=G,\ \frac{G_7}{G_8}=\frac{C_4}{C_6}=K$ $\omega_s=\frac{G}{kC}\left(\frac{G_1(k-1)-G}{G}\right)^{1/2}$ OC: $G_1>\frac{G}{k-1},\ K>1$
(c)	$C_7=C,\ G_5=G_2=G_8=G,\ \frac{G_6}{G_8}=\frac{C_1}{C_7}=K$ $\omega_s=\frac{G}{kC}\left(\frac{G_4(k-1)-G}{G}\right)^{1/2}$ OC: $G_4>\frac{G}{k-1},\ K>1$
(d)	$C_6=C,\ G_5=G_1=G_8=G,\ \frac{G_7}{G_5}=\frac{C_2}{C_6}=K$ $\omega_s=\frac{G}{kC}\left(\frac{G_3(k-1)-G}{G}\right)^{1/2}$ OC: $G_3>\frac{G}{k-1},\ K>1$

(i)	$C_{10}=C_{11}=C,\ G_9=G_3=G_4=G,\ G_6=G/2$ $\omega_s=\frac{G}{C}\left(\frac{G_2}{2G+G_2}\right)^{1/2}$
(j)	$C_2=C_4=C,\ G_9=G_3=G_{10}=G,\ G_6=G/2$ $\omega_s=\frac{G}{C}\left(\frac{G_{11}}{2G+G_{11}}\right)^{1/2}$
(k)	$C_1=C_3=C,\ G_2=G_{11}=G_{12}=G,\ G_7=G/2$ $\omega_s=\frac{G}{C}\left(\frac{G_9}{2G+G_9}\right)^{1/2}$
(l)	$C_1=C_3=C,\ G_9=G_{10}=G_4=G,\ G_7=G/2$ $\omega_s=\frac{G}{C}\left(\frac{G_{12}}{2G+G_{12}}\right)^{1/2}$

(e)	$C_1=C_3=C,\ G_2=G_7=G_6=G,$ OC: $G_5=2G$ $\omega_s=\frac{G}{C}\left(\frac{G+2G_4}{G}\right)^{1/2}$
(f)	$C_1=C_4=C,\ G_3=G_8=G_5=G,$ OC: $G_7=2G$ $\omega_s=\frac{G}{C}\left(\frac{G+2G_2}{G}\right)^{1/2}$
(g)	$C_2=C_4=C,\ G_3=G_6=G_7=G,$ OC: $G_5=2G$ $\omega_s=\frac{G}{C}\left(\frac{G+2G_1}{G}\right)^{1/2}$
(h)	$C_2=C_3=C,\ G_1=G_5=G_8=G,$ OC: $G_7=2G$ $\omega_s=\frac{G}{C}\left(\frac{G+2G_4}{G}\right)^{1/2}$

(m)	$C_3=C,\ \frac{C_9}{C_3}=\frac{G_5}{G_7}=k,\ G_{12}=G_2=G_7=G,\ G_{13}<G(k-1)$ $\omega_s=\frac{G}{C}\left(\frac{G_{13}}{G(k-1)-G_{13}}\right)^{1/2}$ $k>1$
(n)	$C_1=C,\ \frac{C_{12}}{C_1}=\frac{G_8}{G_7}=k,\ G_9=G_4=G_7=G,\ G_{13}<G(k-1)$ $\omega_s=\frac{G}{C}\left(\frac{G_{13}}{G(k-1)-G_{13}}\right)^{1/2}$ $k>1$
(o)	$C_{12}=C,\ \frac{C_2}{C_{12}}=\frac{G_7}{G_5}=k,\ G_3=G_9=G_5=G,\ G_{13}<G(k-1)$ $\omega_s=\frac{G}{C}\left(\frac{G_{13}}{G(k-1)-G_{13}}\right)^{1/2}$ $k>1$
(p)	$C_2=C,\ \frac{C_{12}}{C_2}=\frac{G_6}{G_5}=k,\ G_{10}=G_4=G_5=G,\ G_{13}<G(k-1)$ $\omega_s=\frac{G}{C}\left(\frac{G_{13}}{G(k-1)-G_{13}}\right)^{1/2}$ $k>1$

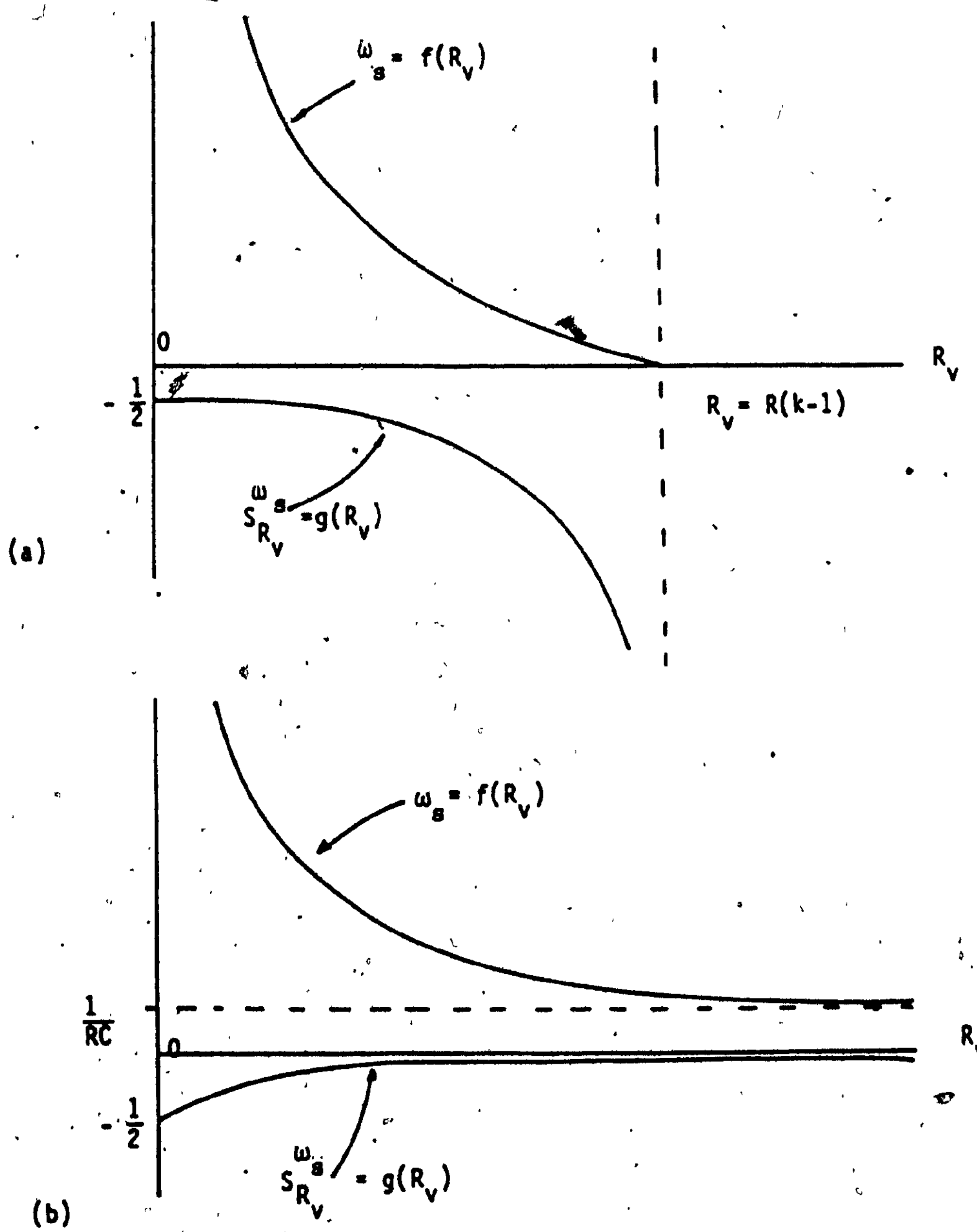
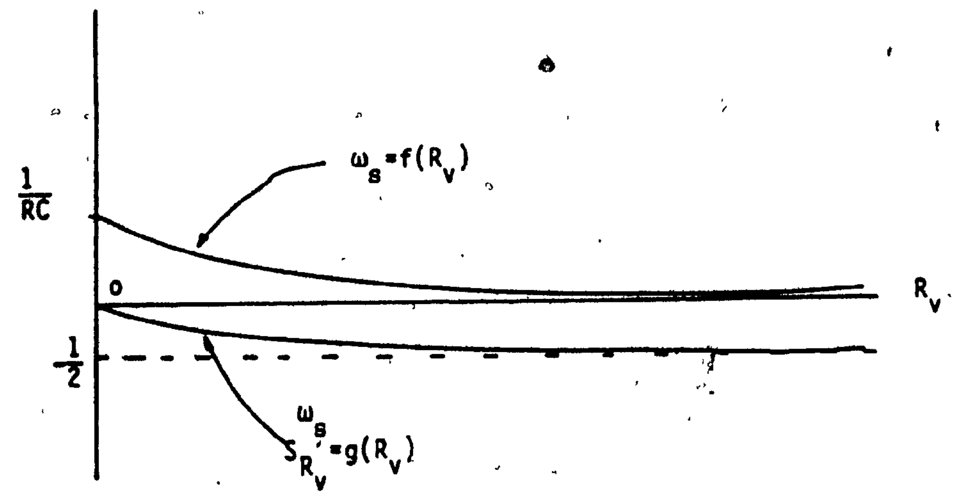


Fig. 3.4: Variation of sensitivity and frequency versus variable resistor, for classes A, B, C, and D, shown in (a), (b), (c), and (d) respectively (Continued).

(c)



(d)

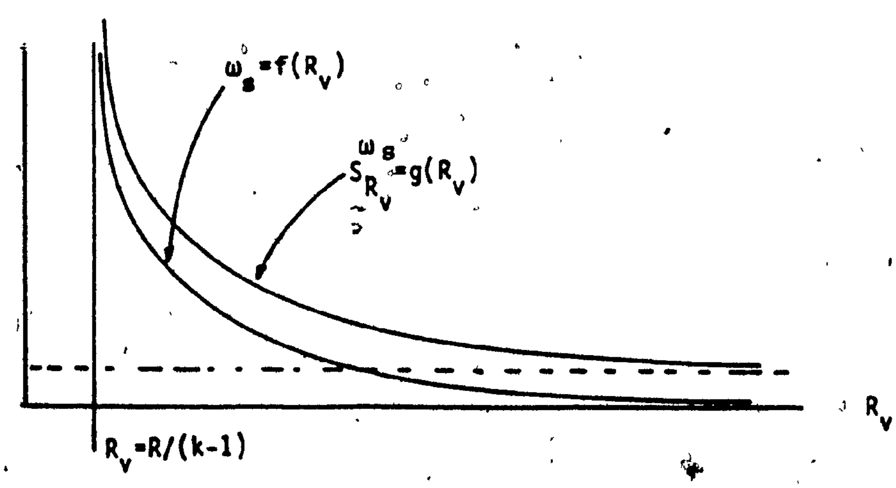


Fig. 3.4: Variation of sensitivity and frequency versus variable resistor, for classes A, B, C, and D, shown in (a), (b), (c), and (d) respectively (Continued).

3.5.2 Class B

Circuits of this class are shown in Figs.3.1(e)-(h).

OF is:
$$\omega_s = \frac{1}{RC}\cdot\left(\frac{2R+R_v}{R_v}\right)^{1/2} \qquad (3\text{-}27)$$

These circuits have low sensitivity to R_v at low frequencies and even at high frequencies the sensitivity never exceeds -1/2. Figure 3.4(b) shows sensitivity as well as ω_s vs R_v curves for this class.

3.5.3 Class C

The circuits are shown in Figs.3.1(i)-(l). The OF is:

$$\omega_s = \frac{1}{RC}\cdot\left(\frac{R}{R+2R_v}\right)^{1/2} \qquad (3\text{-}28)$$

The sensitivity of this group approaches zero at high frequencies and never exceeds -1/2 at low frequencies. Figure 3.4(c) shows sensitivity and ω_s as a function of R_v.

3.5.4 Class D

The circuits are shown in Figs.3.1(m)-(p). And OF is:

$$\omega_s = \frac{1}{RC}\cdot\left(\frac{R}{R_v(k-1)-R}\right)^{1/2} \qquad (3\text{-}29)$$

These circuits have high sensitivity at high frequencies and at low frequencies the sensitivity approaches 1/2. Figure

3.4(d) shows sensitivity and ω_s as functions of R_v.

3.6 Experimental Results

All the 16 VFO circuits have been built and tested in the laboratory. They all performed as expected according to the theoretical analysis. The power supply of ±15 volts has been used in all the cases. The output signal is taken from the output of the OA (Fairchild 741). The amplitude of oscillation has been relatively constant. However, it varies from circuit to circuit. The experimental results of one circuit from each class are presented here.

Class A: Figure 3.1(a) is taken as the represent tive circuit for this class. Design values for the circuit elements are:

C_7=0.1 μF, C_3=1 μF, R_5=100 kΩ, R_4=R_8=1 kΩ, R_6=10 kΩ, R_2 is variable element.

The theoretical and experimental variation of frequency as a function of variable resistor as well as distortion measurements are given in Fig.3.5(a).

Class B: For this class, the experimental results of circuit of Fig.3.1(n) are given as sample. Design values for components are:

C_1=C_4=0.1μF, R_5=R_3=R_8=1 kΩ, R_7=0.5 kΩ, R_2 is variable.

Figure 3.5(b) shows the theoretical and experimental variation of frequency vs variable resistor. Distortion measurements are also included in the same Figure.

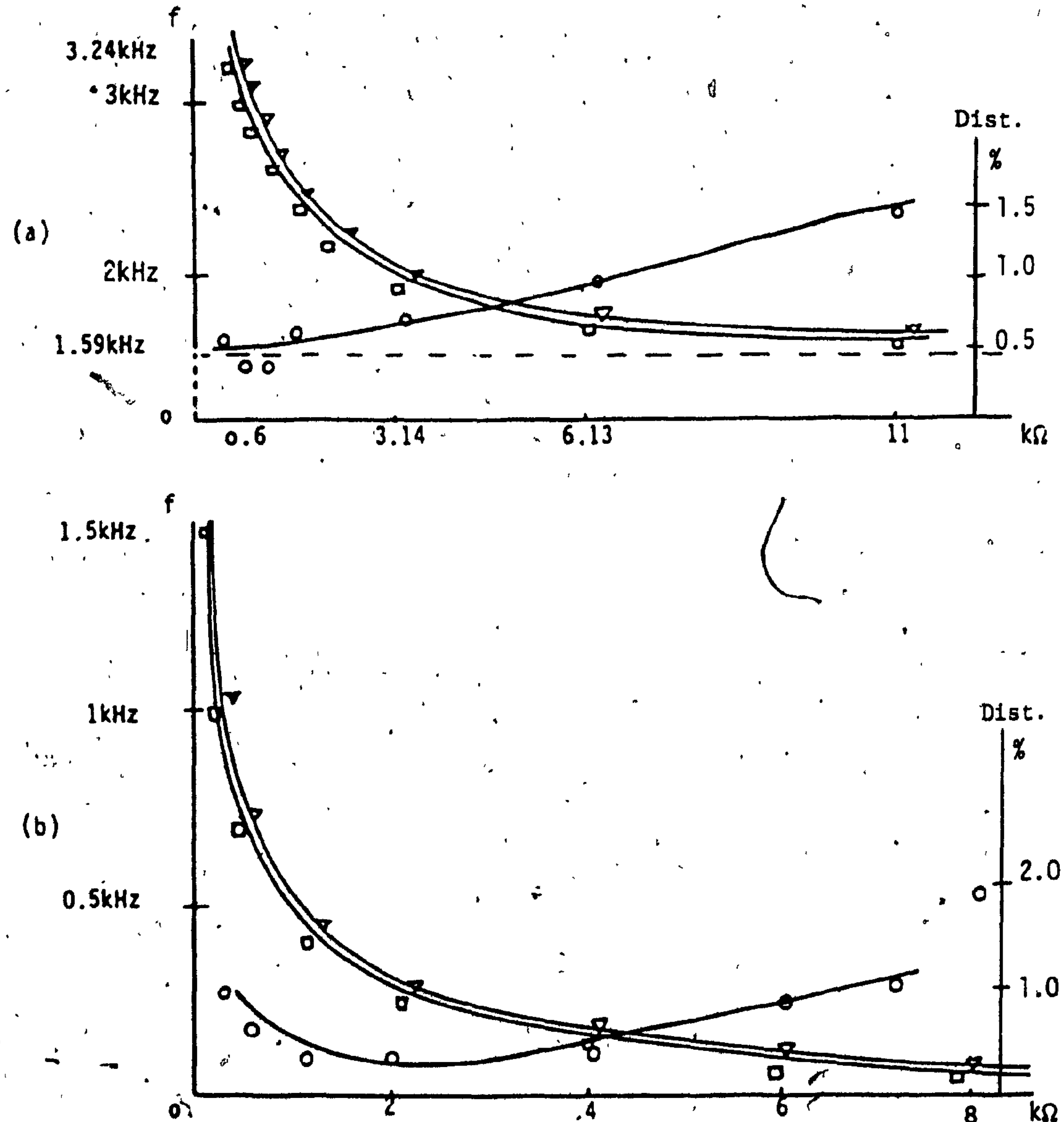
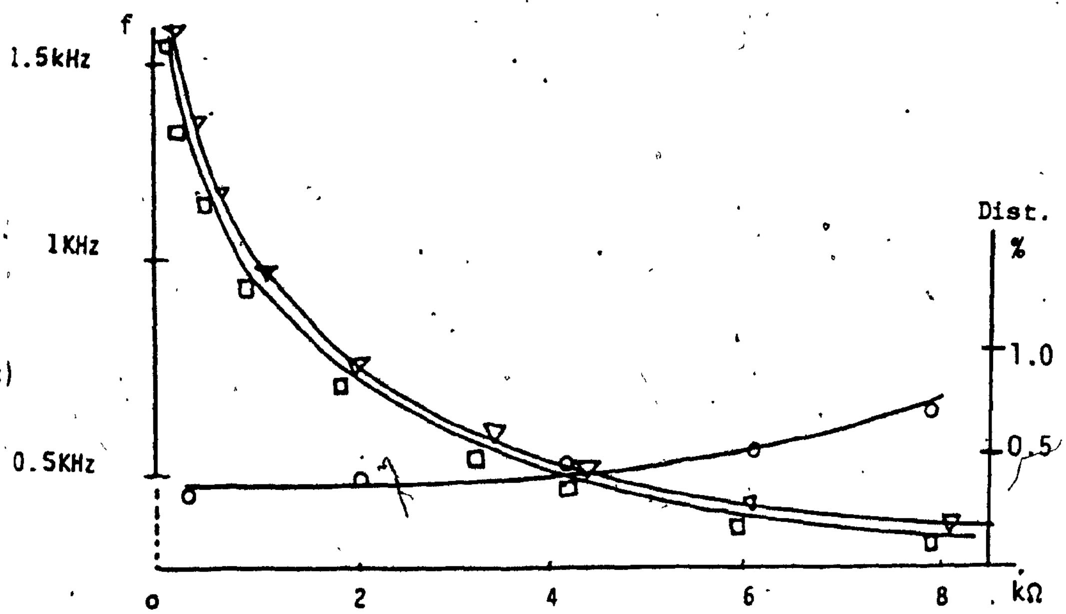


Fig. 3.5: Theoretical and experimental variation of frequency versus variable resistor for classes A,B,C and D shown in (a),(b),(c) and (d) respectively (Continued).

(c)



(d)

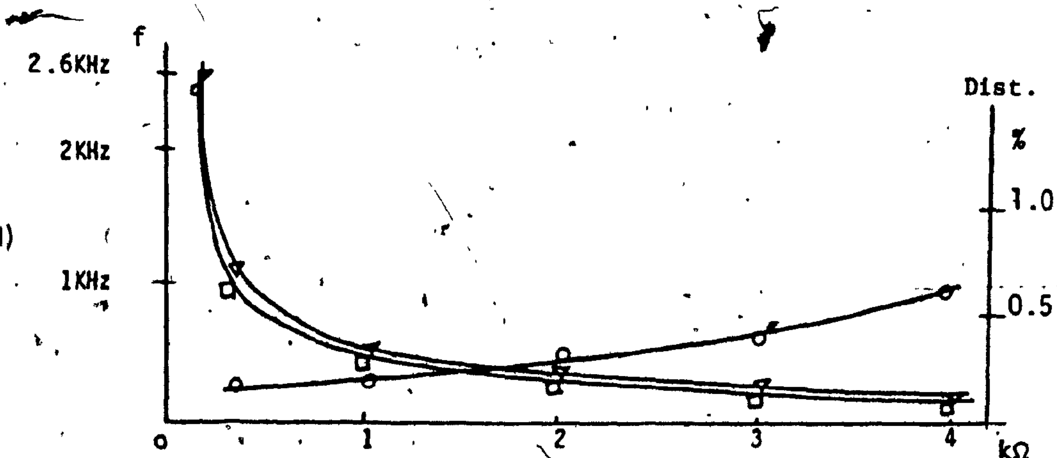


Fig. 3.5: Theoretical and experimental variation of frequency versus variable resistor for classes A,B,C and D shown in (a),(b),(c) and (d) respectively (Continued).

□ Theoretical ▽ Experimental o Distortion

Class C: The representative circuit for this class is that of Fig.3.1(i). In this circuit, the design values for components are:

$C_{10}=C_{11}=0.1\ \mu F$, $R_6=R_3=R_4=1\ k\Omega$, $R_9=2\ k\Omega$, R_2 is variable.

Theoratical and experimental variation of frequency vs variable resistor is given in Fig.3.5(c). Distortion mesurments are also included.

Class D: Figure 3.1(p) is taken as representative circuit of this class. Figure 3.5(d) shows the theoretical and experimental variation of frequency as a function of variable element. The Figure 3.5(d) also shows the distortion variations. Design values for circuit elements are:

$C_2=0.1\mu F$, $C_{11}=1\ \mu F$, $R_6=R_{10}=R_3=1\ k\Omega$, $R_5=10k\Omega$, R_{13} is variable.

All the resistors are carbon film of tolerance 5%. All the capacitors are polystyrene of tolerance 5%.

3.7 Summary

A unified approach for design of VFOs has been described. The approach is general and complete. Further, it yields canonic circuits, that is, circuits comprising of minimum number of resistors, capacitors and OAs. It is shown that a minimum of two capacitors and five resistors and one OA is required for variable frequency operation. A set of 16 circuits for variable frequency operation is derived. Eight of the circuits have been published before [34] and the remaining eight of the VFOs are completely new. It has been systematically shown that the set is complete, that is, no additional canonic oscillator circuits can be generated. In all of the circuits, the frequency of oscillation is controlled by a single variable resistor. In three of them the variable resistor is grounded which is desirable for VCO operation. The VFOs are classified in four groups, each a group of four circuits, according to the nature of dependence of the VFO frequency on the variable resistor. This should facilitate selection of a VFO for a particular application.

CHAPTER IV

CANONIC GROUNDED CAPACITOR VARIABLE FREQUENCY RC-OSCILLATORS USING OAs

CHAPTER IV

CANONIC GROUNDED CAPACITOR VARIABLE FREQUENCY RC-OSCILLATORS USING OAs

4.1 Introduction

It is highly desirable to have all the capacitors grounded in RC circuits [24-30]. Grounded capacitor circuits are attractive for several reasons. For a ganged variation of a pair of capacitors used in most tunable oscillators, it is desirable to have the capacitors grounded. This grounded capacitor pair then allows the rotor and the case of tuning unit to be kept at the same ground potential, making it possible to have a reduction in mechanical and electrical complexity of the system [24]. Grounded capacitor circuits are also suitable for monolithic IC technology [31-33]. Also for thin film fabrication, it is desirable to have all the capacitors grounded [25], in which case the elimination of etching process and the reduction of number of contacts is achieved. Furthermore, the parasitic capacitances surrounding the capacitors can be easily accounted for or tuned out as they are now in parallel with the grounded capacitors [26]. It is also desirable to have equal valued capacitors in a circuit from fabrication point of view [27]. Consequently, there has been considerable interest in design and implementation of

RC networks with grounded capacitors[24-30]. However, in almost all of the studies the approach has been on an ad hoc basis, rather than a systematic and unified approach that yields a complete set of results.

This chapter presents the results of a systematic approach for the generation of canonic OA based GCRCOs for variable frequency operations. A set of 8 canonic circuits is first derived for GCVFOs. This set is proven to be complete. It is shown that GCVFOs need at least 8 elements (2 capacitors and 6 resistors) in addition to the OA. All of these circuits are new. However, either they have a poor start up property or they have a narrow range of frequency variation. Consequently, another set of 12 easily tunable GCVFOs are found from these 8 circuits. In the new set, each circuit uses nine elements(2 capacitors and 7 resistors).This set is also complete and all the 12 circuits are new. In all of the GCVFOs, the frequency can be varied by a single variable resistor. They all allow equal valued capacitor design. Two of these oscillators have their variable resistors grounded. Hence, they can be made VCOs by a single grounded FET. Three of GCVFOs have an attractive property in that they all have equal valued elements (all the resistors are equal and all the capacitors are equal).

4.2 Grounded Capacitor SFOs

In chapter 2 a complete set of canonic SFOs has been derived. Out of the 12 circuits in that set, only one of them namely the circuit of Fig.2.5(a) has both of its capacitors grounded. Since the set is complete, that circuit is, in fact, the only canonic GCSFO possible. For convenience, the circuit of Fig.2.5(a) is also given in Fig.4.1. This circuit is called the Base Circuit and it is referred to as BSFO circuit throughout this chapter.

4.3 Grounded Capacitor VFOs

4.3.1 Theory

Since it is already known that, there exists only one canonic GCSFO, it is only logical to argue the following. If there exists any canonic GCVFO circuit, that circuit must always contain all the elements of this BSFO circuit. In other words, if one GCVFO is found, it is always possible to reduce it to the BSFO circuit by removing one or more elements from that circuit. Thus, it should be possible to find all the canonic GCVFOs by suitably introducing new resistor(s) to the canonic BSFO circuit. Therefore, one can adopt the following approach.

Determine the number of all possible ways that one can add a single resistor to the BSFO circuit. If no GCVFO is found for any of these ways, then proceed to determine the

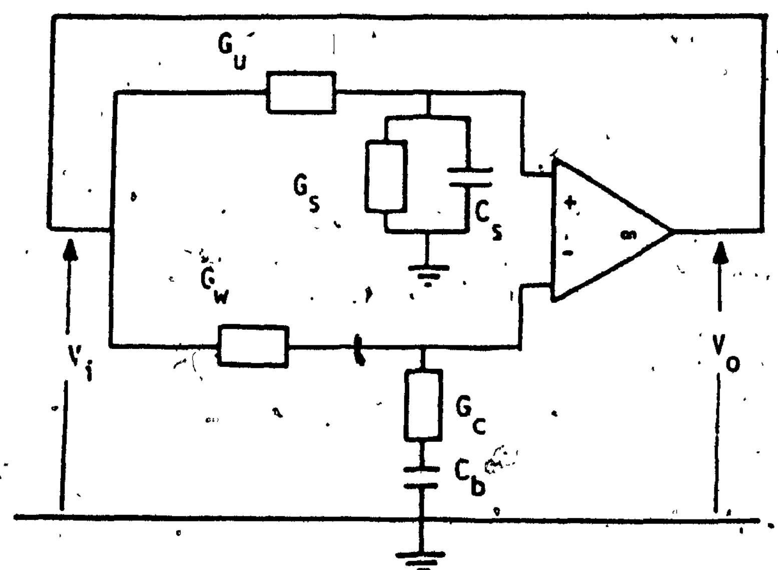


Fig. 4.1: Grounded capacitor single frequency oscillator.

number of all possible ways that one can add two resistors to BSFO and so on. Therefore, to find a complete set of GCVFOs, it is only necessary to find all the possible resistive modifications of BSFO circuit. If it is required to introduce more than one resistor to the BSFO circuit, one can enter all the resistors at once or add one resistor at a time. Since in practice even if one decides to introduce all the resistors at once, ultimately, the resistors are added one by one, it is much easier and systematic to enter a resistor into the circuit and study the resulting circuits to find whether or not they are GCVFOs. If no GCVFO is found, then these circuits will be the basis for introducing the second resistor to them and so on.

4.3.2 Resistive Modification of GCSFO

There are only three ways that a single resistor can be added to the BSFO circuit. They are as following.

(a) Introducing a resistor between any two nodes of the circuit.

(b) Entering a resistor in series with a resistor or a capacitor of the circuit.

(c) Breaking a node into two nodes and connecting them with the new resistor.

There are 5 nodes in the BSFO circuit. Using procedure (a), there are $\binom{5}{2}=10$ ways that a resistor can be added to the circuit. Only four of which make the new circuits to differ from its original configuration. These four circuits are shown in Fig.4.2(a)-(d). Further investigation of these circuits show that none of them can be a GCVFO. Application of procedure (b) to the BSFO circuit generates five circuits. The circuits are shown in Fig.4.3(a)-(e). In four of these circuits, there are two resistors in series and they may be regarded as one resistor in which case the circuits reduce to their original configuration. However, these circuits should be kept for introduction of the second resistor to them, if no GCVFO is found from the seven element networks. It is only the circuit of Fig.4.3(c) that differs from the BSFO circuit. However, further study of this circuit does not lead to any GCVFO circuit either. Finally, using procedure (c) and the BSFO circuit, five circuits are found, they are shown in Fig.4.4(a)-(e). None of these circuits can be a GCVFO either. Since no GCVFO was found from the seven element networks, the next step is to introduce another resistor to all the above 14 circuits.

Using the 14 circuits obtained above and the procedures (a), (b) and (c) the following are found.

Application of procedure (a) to Fig.4.2(b) leads only to one GCVFO. It is shown in Fig.4.5(a).

Application of procedure (a) to Fig.4.2(d) leads also to the GCVFO circuit of Fig.4.5(a).

Application of procedure (a) to Fig.4.2(c) generates four GCVFOs. They are shown in Fig.4.5(b)-(e).

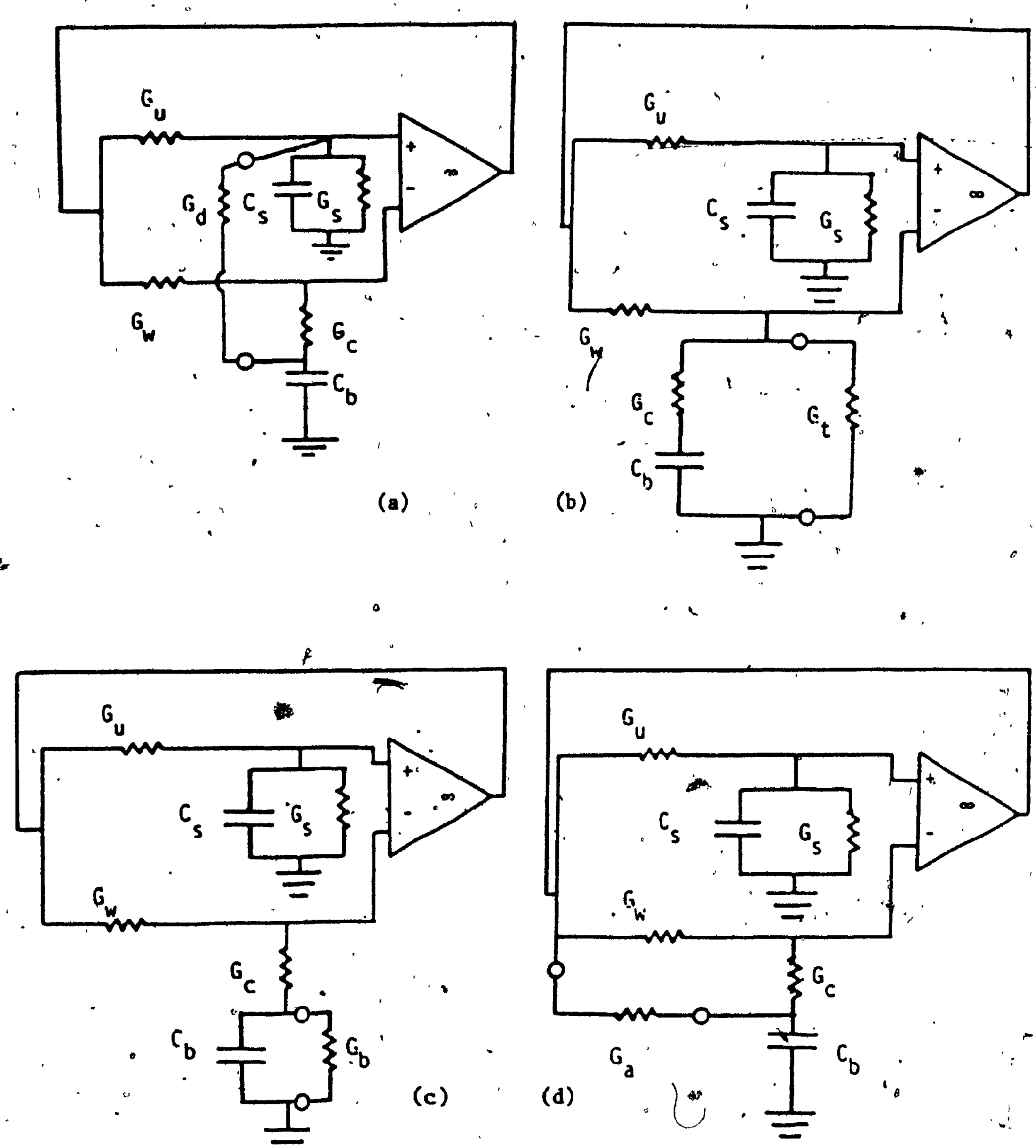


Fig. 4.2: Modified grounded capacitor single frequency oscillators, using procedure (a).

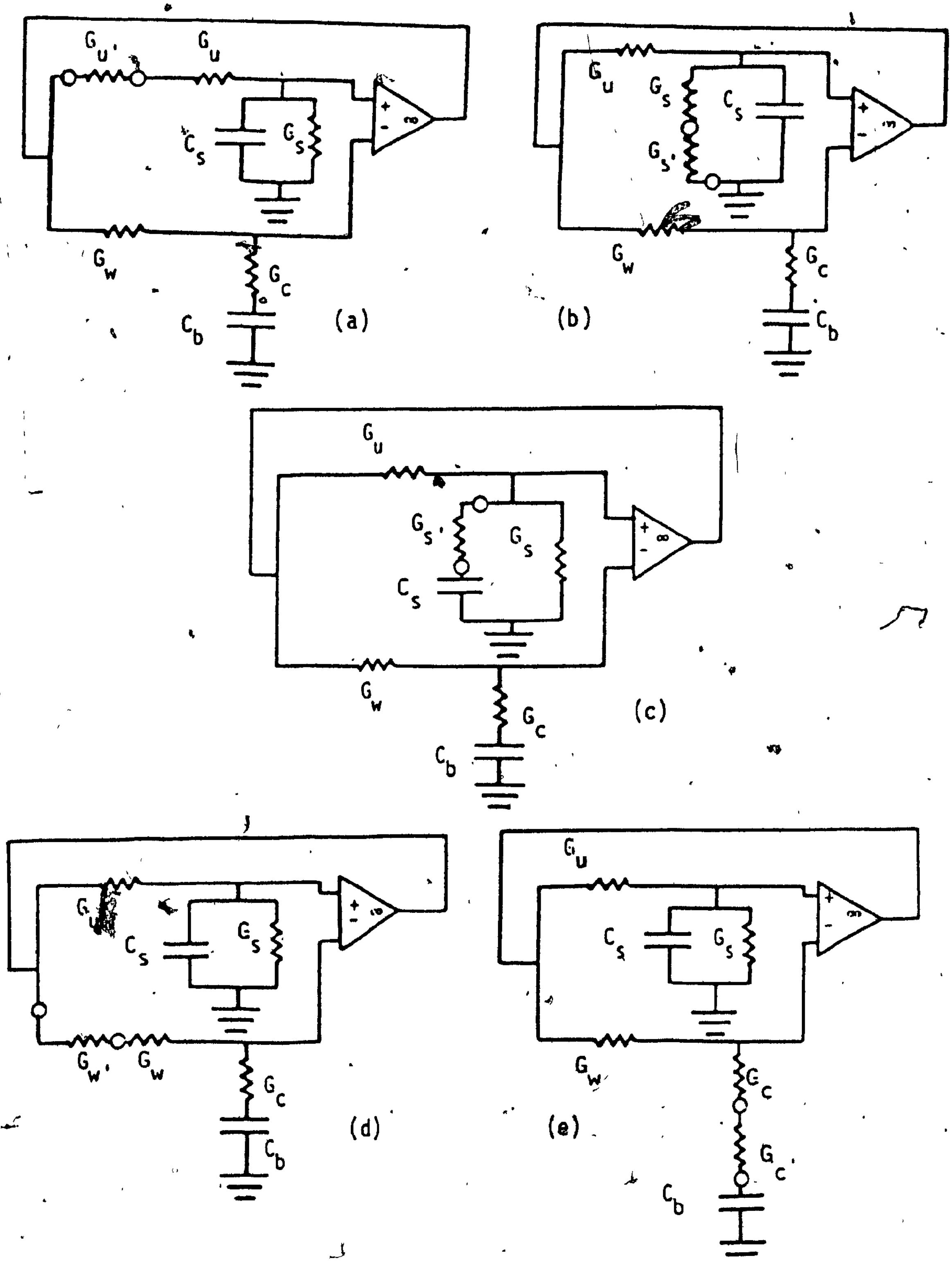


Fig. 4.3: Modified grounded capacitor single frequency oscillators, using procedure (b).

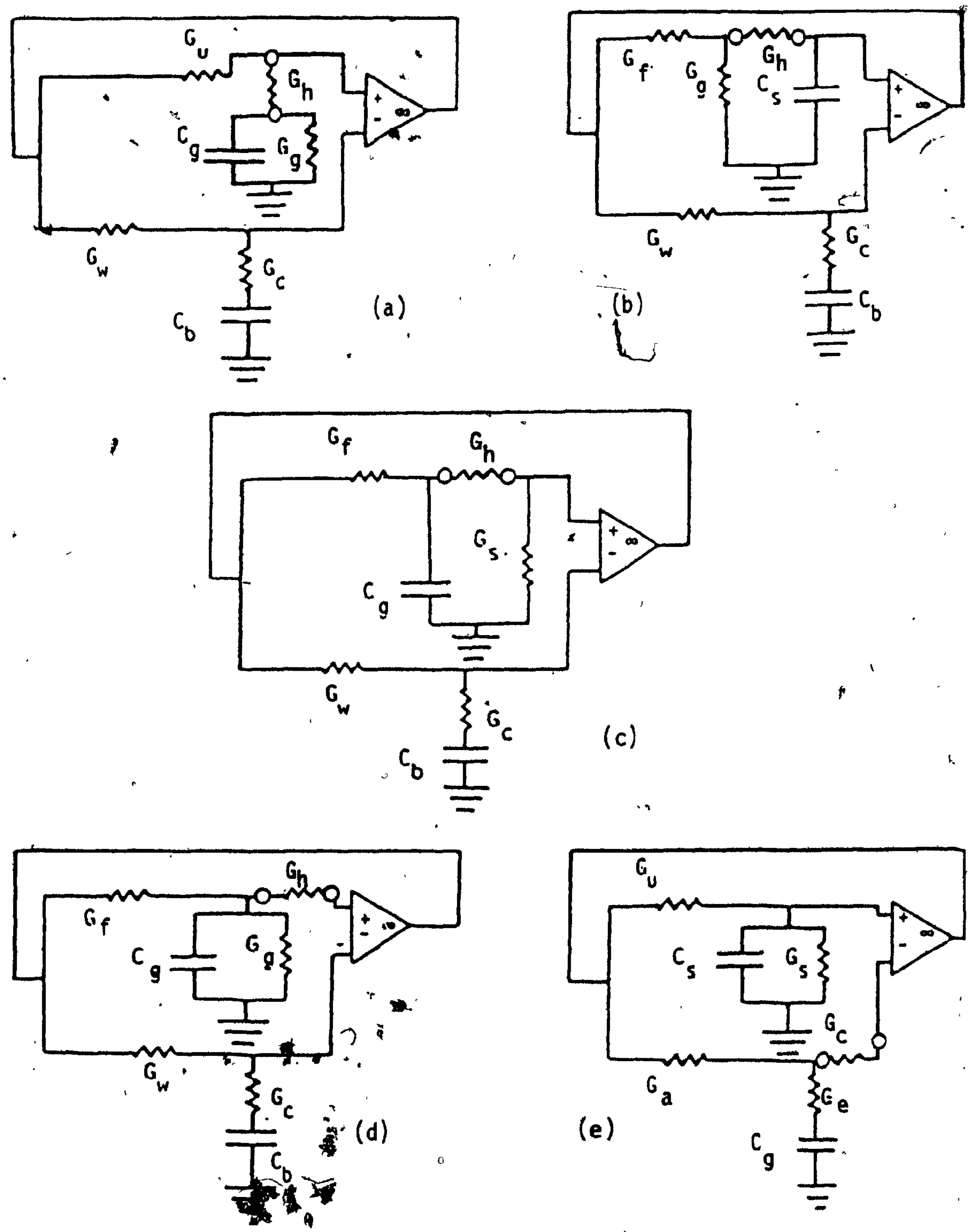


Fig. 4.4: Modified grounded capacitor single frequency oscillators, using procedure (c).

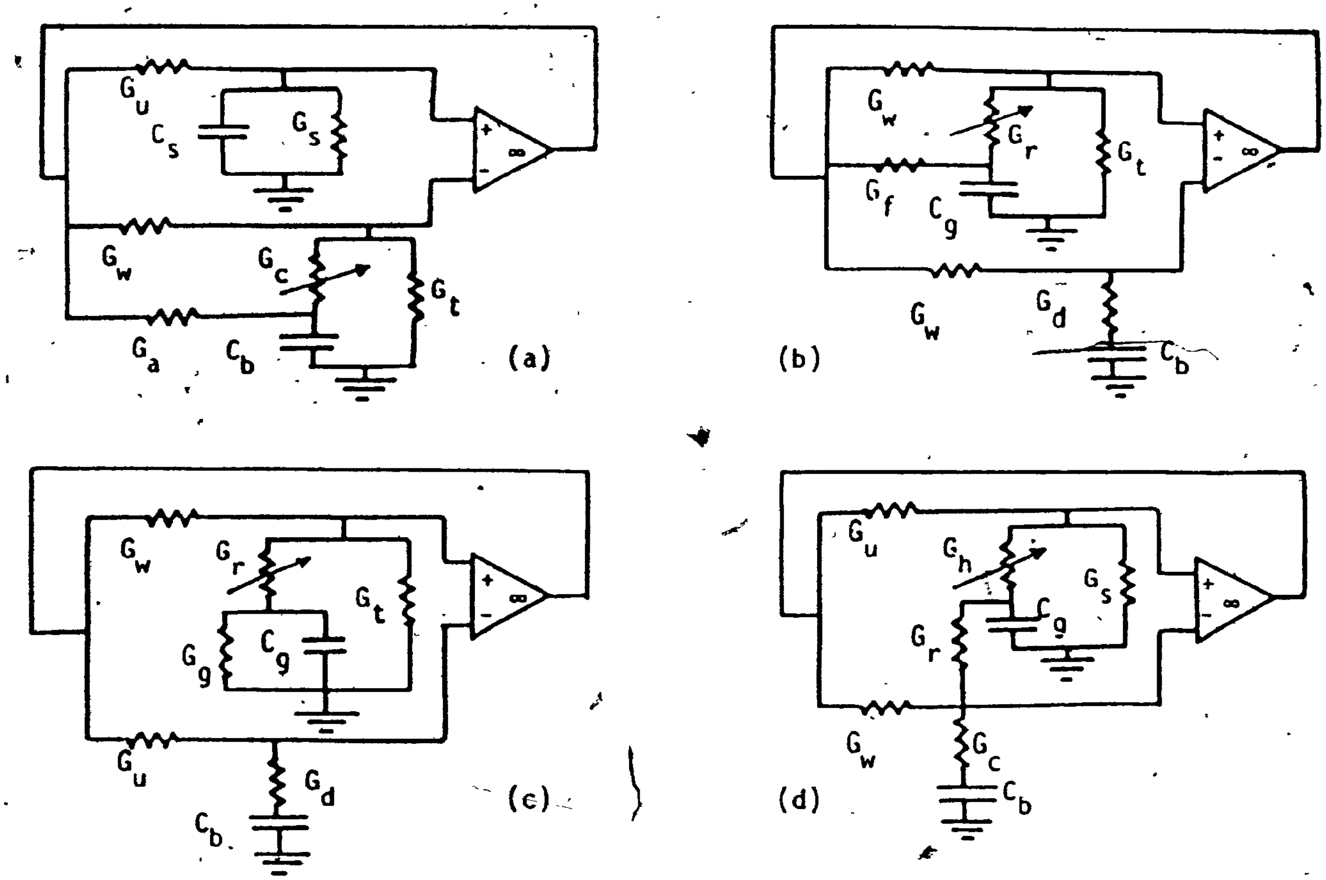


Fig. 4.5: The set of 8 canonic GCVFOs (Continued).

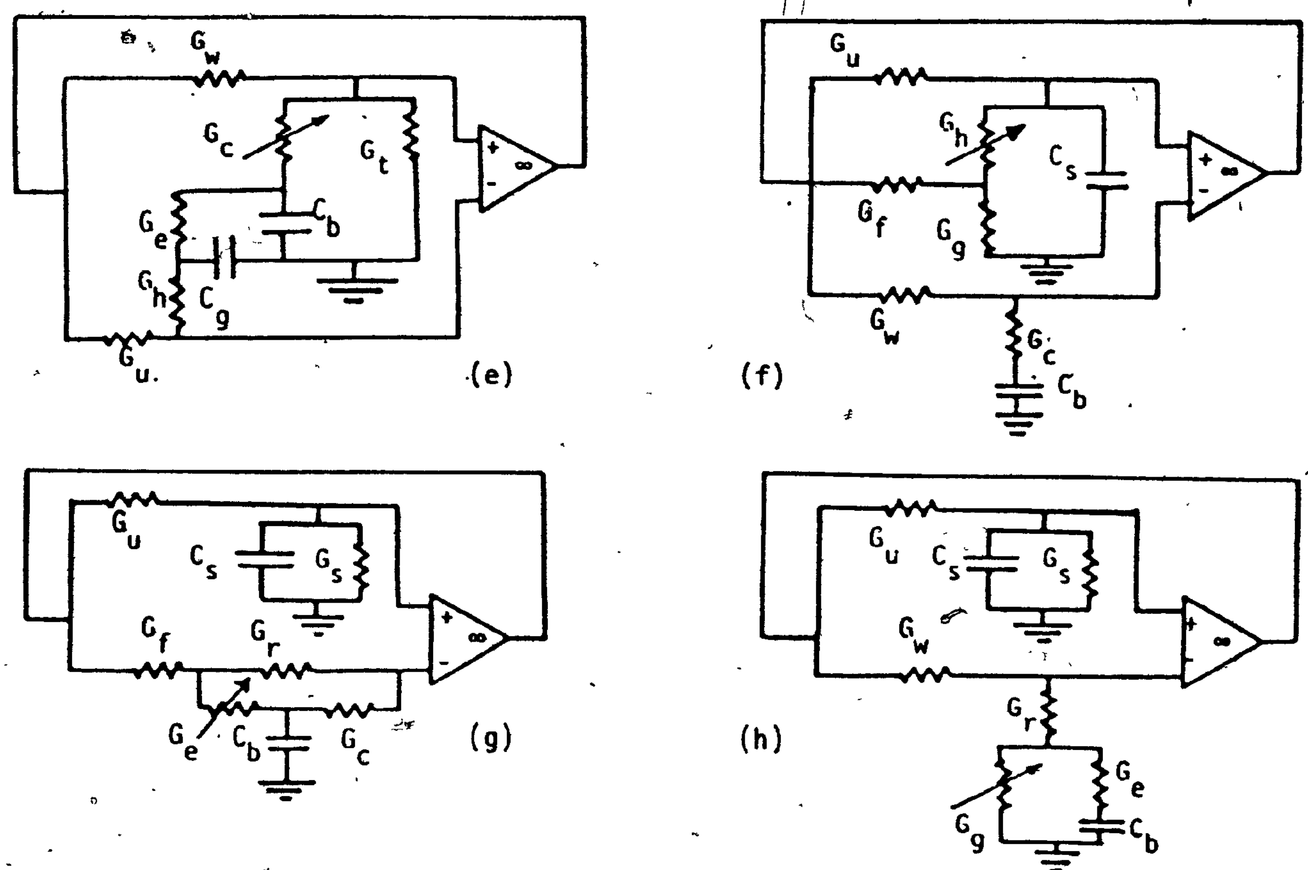


Fig. 4.5: The set of 8 canonic GCVFOs (Continued).

Application of procedure (a) to Fig.4.3(b), (d) and (e) produces three GCVFOs that are shown in Figs.4.5(f), (g), and (h) respectively. Application of procedure ~~(a)~~ to Fig.4.4(a), (b) and (c) also leads to three GCVFOs which are the same as the ones shown in Fig.4.5(c), (f) and (b) respectively.

Overall a set of 8 canonic GCVFOs with different configurations are derived. Each circuit consists of one OA, 2 capacitors and 6 resitors.

Application of procedures (b) and (c) to some of the seven element networks also leads to canonic GCVFO circuits, however, it is found that the resulting circuit(s) is similar to one of the above 8 GCVFOs. Since it is found that a GCVFO requires at least 8 elements, it is not necessary to search for GCVFOs with higher number of elements because the aim is to find canonic GCVFOs.

4.3.3 Design Procedures

Design procedure is given for only one of the circuits, but similar procedures can be used to derive the design equations for other circuits as well.

Let us take the circuit of Fig.4.5(a) as an example. The CE of the circuit is given by (4-1),

$$\alpha S^2 + \beta S + \gamma = 0 \tag{4-1}$$

where:

$$\alpha = G_w C_s C_b \tag{4-2a}$$

$$\beta = G_w C_s (G_a + G_c) + G_w G_s C_b + G_a G_c C_s - G_u (G_t + G_c) C_b \tag{4-2b}$$

$$\gamma = G_w G_s (G_a + G_c) + G_a G_c G_s - G_u G_t (G_a + G_c) \tag{4-2c}$$

Equation (4-2b) can be written as equation (3-5), where:

$$p = G_c \tag{4-3a}$$

$$U_1 = C_s (G_w + G_a) \tag{4-3b}$$

$$U_2 = G_u C_b \tag{4-3c}$$

$$U_3 = G_w G_a C_s + G_w G_s C_b \tag{4-3d}$$

$$U_4 = G_t G_u C_b \tag{4-3e}$$

To satisfy (3-6), let us assume $C_s/C_b = k$, then (3-6a) and (3-6b) can be written as (4-4a) and (4-4b), respectively.

$$U_1 - U_2 = k(G_w + G_a) - G_u = 0 \tag{4-4a}$$

$$U_3 - U_4 = kG_w G_a + G_w G_s - G_u G_t = 0 \tag{4-4b}$$

Another condition imposed on the above two equations comes from the fact that the OF should always be positive or:

$$\omega_s = \left\{ \frac{G_wG_s(G_a+G_c)+G_aG_cG_s-G_uG_t(G_a+G_c)}{G_wC_sC_b} \right\}^{1/2} > 0 \quad (4-5)$$

Substituting for G_uG_t from (4-4b), then:

$$G_aG_cG_s-k(G_wG_aG_a+G_wG_aG_c) > 0 \quad (4-6a)$$

or $$G_c(G_s-kG_w)>kG_wG_a \quad (4-6b)$$

or $$G_s>kG_w \quad (4-6c)$$

Equation (4-4) and (4-6c) guarantee that the circuit will always oscillate and the frequency can be changed during the operation. To build the circuit let:

$G_c=1/R_v$, $C_s=C_b=C$, $k=1$, $G_w=G_a=G=1/R$

for $G_s>kG_w$, let $G_s=mG$ where $m>1$, from (4-4) and (4-6), $G_u=2G$, $G_t=(G+mG)/2$. Therefore,

$$\omega_s = \frac{1}{RC} \cdot \left(\frac{R(m-1)-R_v}{R_v} \right)^{1/2} \quad (4-7)$$

Designs for all the circuits are given in Table 4-I.

4.4 Experimental Results.

All of 8 circuits have been built and tested in the laboratory. Only the results of the circuits of

Table 4-I Design equations for circuits of Fig.4.5.

(a)	$C_s = C_b = C, \quad G_s = G_u = \frac{2}{R}, \quad G_t = \frac{3}{2R}, \quad G_w = G_a = \frac{1}{R}, \quad G_c = \frac{1}{R_v}$ $\omega = \frac{1}{RC}\sqrt{\frac{R-R_v}{R_v}} \qquad R_v < R$
(b)	$C_b = C_g = C, \quad G_f = G_d = \frac{1}{R}, \quad G_t = \frac{1}{2R}, \quad G_u = G_w = \frac{2}{R}, \quad G_r = \frac{1}{R_v}$ $\omega = \frac{1}{RC}\sqrt{\frac{R+R_v}{2R-R_v}} \qquad R_v < 2R$
(c)	$C_b = C_g = C, \quad G_d = G_u = \frac{1}{R}, \quad G_t = \frac{2}{R}, \quad G_w = \frac{4}{R}, \quad G_r = \frac{1}{R_v}$ $\omega = \frac{1}{RC}\sqrt{\frac{3R+2R_v}{R-2R_v}} \qquad R_v < \frac{R}{2}$
(d)	$C_b = C_g = C, \quad G_s = G_u = G_c = \frac{1}{R}, \quad G_w = \frac{1}{2R}, \quad G_r = \frac{1}{3R}, \quad G_h = \frac{1}{R_v}$ $\omega = \frac{1}{RC}\sqrt{\frac{3R+R_v}{3R-5R_v}} \qquad R_v < \frac{3R}{5}$
(e)	$C_b = C_g = C, \quad G_e = G_u = \frac{1}{R}, \quad G_t = G_w = \frac{4}{R}, \quad G_h = \frac{2}{R}, \quad G_c = \frac{1}{R_v}$ $\omega = \frac{2}{RC}\sqrt{\frac{3R+2R_v}{R-4R_v}} \qquad R_v < \frac{R}{4}$
(f)	$C_s = C_b, \quad G_f = G_g = G_w = G_c = G_u = \frac{1}{R}, \quad G_h = \frac{1}{R_v}$ $\omega = \frac{1}{RC}\sqrt{\frac{R}{R+2R_v}}$
(g)	$C_s = C_b = C, \quad G_f = G_u = G_r = G_c = G_s = \frac{1}{R}, \quad G_e = \frac{1}{R_v}$ $\omega = \frac{1}{RC}\sqrt{\frac{2R+R_v}{R_v}}$
(h)	$C_s = C_b = C, \quad G_r = \frac{1}{R}, \quad G_w = G_e = G_s = \frac{2}{R}, \quad G_u = \frac{1}{2R}, \quad G_g = \frac{1}{R_v}$ $\omega = \frac{2}{RC}\sqrt{\frac{R_v-R}{R+3R_v}} \qquad R_v > R$

Fig.4.5(f)and (h) are given here as an example. The chosen values for the elements are:

Fig.4.5(f): $R_u = R_f = R_g = R_w = R_c = 1\ k\Omega$, $C_b = C_s = 0.1\mu F$, R_h=variable

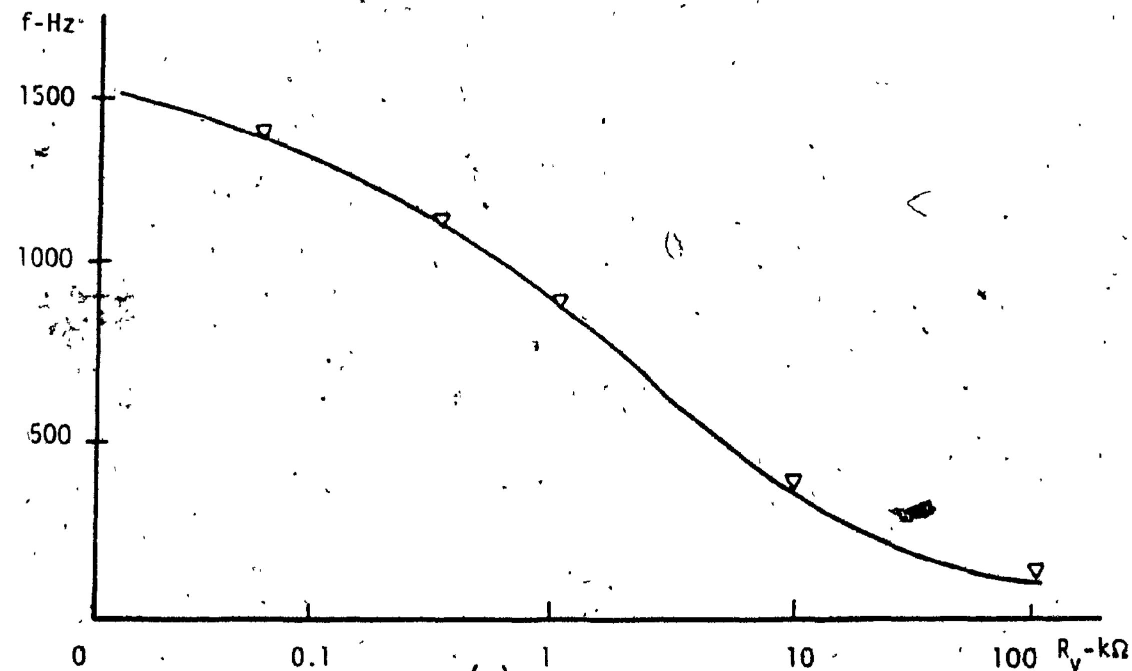
Fig.4.5(h): $R_e = R_w = R_s = 1k\Omega, R_r = 2k\Omega$, $R_u = 4k\Omega, C_b = C_s = 0.1\mu F$

R_g=variable

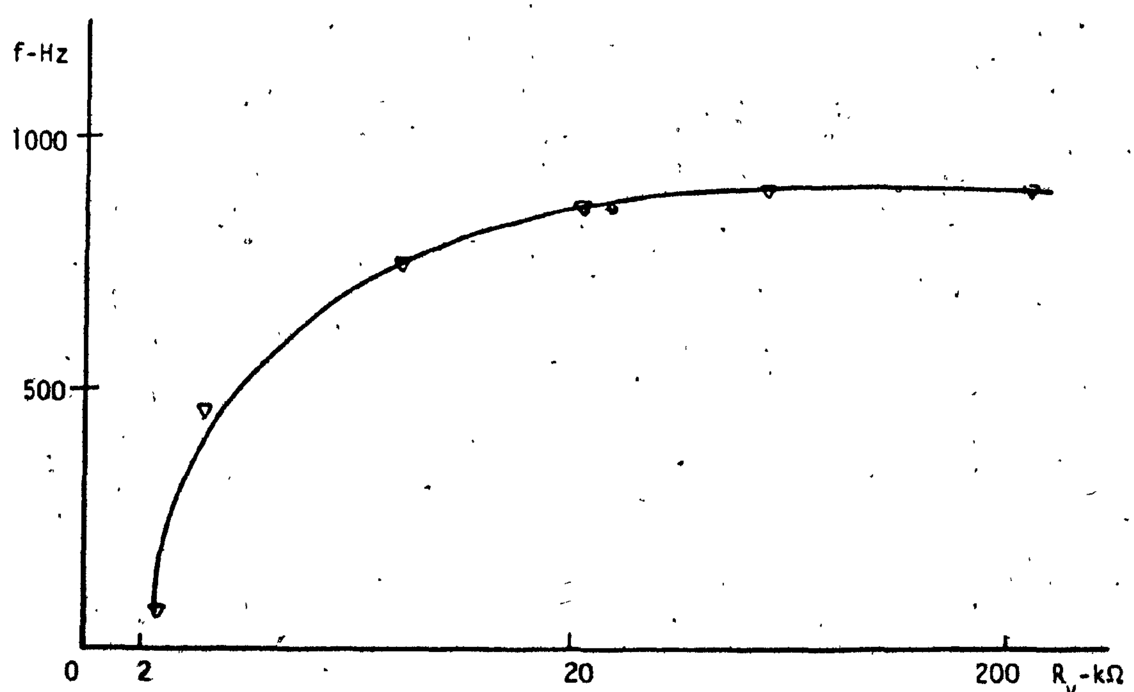
All the capacitors are polystyrene of tolerance 1%(measured). All the resistors are carbon film of tolerance 1%. Figs.4.6(a) and (b) show the variation of frequency versus the variable resistor for the circuits of Figs.4.5(f) and 4.5(h) respectively. In all the experiments, a ±15volts power supply has been used. The output is taken from the output of the OA (Fairchild 741).

4.5 Summary

Systematically, a set of 8 citcuits for canonic GCVFOs have been derived. The set is shown to be complete. The frequency of oscillation of all the GCVFOs is single resistor controlled. One of the circuits (Fig.4.5(h)) has also its variable resistor grounded (desirable for VCOs). Three of the circuits (Fig.4.5(a),(f),(h)) have a range of frequency variation of more than one decade. The circuits of Figs.4.5(f)and (g) have equal valued elements (EVE), that is, all the resistors are equal and all the capacitors are equal. All of the canonic GCVFOs are new circuits.



(a)



(b)

Fig. 4.6: Variation of frequency versus variable resistor, (a) for Fig.4.5(f), (b) for Fig.4.5(h), (Experimental).

Unfortunately, it was also noted that in practice all but one (Fig.4.5(d)) of the GCVFOs have difficulty in starting the oscillations. The oscillator of Fig.4.5(d), however, has a very limited range of frequency variation.

4.6 Easily Tunable GCVFOs

4.6.1 General Considerations

From experemental tests, it was noted that all but one of the 8 canonic GCVFOs found (Fig.4.5(d)) have a rather difficult problem in starting to oscillate. However, the circuit of Fig.4.5(d) can be shown to have a limited frequency range of variation, namely, the normalized range of 1 to 4. In what follows the problem is investigated in detail.

Let us assume the condition of oscillation is:

$$\beta = (W_1-W_2)G_v+(W_3-W_4)=0 \qquad (4\text{-}8)$$

For a continuous variation of frequency we must have

$$W_1-W_2=0 \qquad (4\text{-}9a)$$

$$W_3-W_4=0 \qquad (4\text{-}9b)$$

In general,

$$W_i=f_i(R,C) \qquad (4\text{-}10)$$

Thus when the values of the circuit elements are slightly different from their designed values (which is always the case due to component tolerances), we have,

$$W_1-W_2\neq 0 \qquad (4\text{-}11a)$$

$$W_3-W_4\neq 0 \qquad (4\text{-}11b)$$

To correct the problem, one should find a variable in(4-11a) such that it has small or no effect on (4-11b). Also, there should be a variable in (4-11b) such that it has small or no effect on (4-11a). By varying these variables back and forth, one could eventually satisfy (4-9). However, it is not always possible to find such variables to begin with. Even if there are variables such as the ones described, there is no way of knowing when both of (3-6a) and (3-6b) are completely satisfied. For example if :
$W_1-W_2=+\varepsilon_1$ and $W_3-W_4=-\varepsilon_2$, then (4-8) can be written as:

$$G_v(\varepsilon_1)-(\varepsilon_2)=0 \qquad (4\text{-}12)$$

This equation can be satisfied without $\varepsilon_1=\varepsilon_2=0$ and the circuit will oscillate, but G_v can not be varied as desired. From the above considerations, it is necessary to find a method that allows us to set $W_1-W_2=0$ and $W_3-W_4=0$ independently of each other. There are two ways in which the problem can be rectified. In these methods there are three steps following which the circuit is guaranteed to start oscillating and will remain oscillating as the variable element is continuously adjusted.

4.6.2 Method I

In (4-8), G_v is first set to zero and then by varying a resistive element of (4-11b) equation(4-8) is satisfied. At this point, the circuit will start oscillating, indicating that $W_3-W_4=0$. Now making G_v a nonzero quantity causes the oscillaton to stop. By varying another resistive element which is present in (4-11a) but not in (4-11b) the circuit can restart oscillations, indicating that $W_1-W_2=0$. Therefore, both (4-11a) and (4-11b) are satisfied independently. From the above procedure, it is concluded that first, it should be possible to set $G_v=0$, second there should exist an element "e" in (4-11a) which is not contained in (4-11b). Then, the general form of OC for an easily tunable circuit should be :

$$g_1(e)G_v+g_2=0 \qquad (4-13)$$

where

$$g_1(e)=W_1-W_2 \qquad (4-14)$$

$$g_2=W_3-W_4 \qquad (4-15)$$

4.6.3 Method II

Writing (4-8) in terms of resistances rather than conductances and multiplying by $R_v=1/G_v$ (4-8) becomes :

$$r_1+(r_2)R_v=0 \quad (4-16)$$

If there is an element in r_2 that r_1 is independent of, then (4-16) can be written as:

$$r_1+R_v(r_2(e))=0 \quad (4-17)$$

Equation (4-17) is similar to (4-13), and similar three steps now can be performed to start the oscillation.

4.6.4 Identification of Easily Tunable GCVFOs

From the above two methods it is concluded that there are two conditions to be satisfied if the circuit is to be an easily tunable GCVFO. Condition one: the variable element should be able to take on a value of either zero or infinity for finite frequencies. Condition two: the coefficient of the variable element(g_1 in(4-13) and r_2 in (4-17)) should contain an element which does not exist in the rest of the equation, for example, g_2 in (4-13) and r_1 in (4-17) are independent of "e" while g_1 in (4-13) and r_2 in (4-17) are functions of "e".

Out of the eight circuits of previous sections, only one (Fig.4.5(d)) satisfies the above two conditions. However, this circuit has a rather limited frequency range of variation (normalized range of 1 to 4).

To convert the other 7 circuits to easily tunable GCVFOs and to improve the range of Fig.4.5(d), it is necessary to introduce another resistor into the circuits such that, OC is allowed to be written in the form of either (4-13) or (4-17). The following interpretation of the mathematics involved in (4-13) and (4-17) allows us to find exactly where the new resistor is to be entered.

When the variable resistor is set to zero or infinity, the OC becomes independent of the new resistor "e". Physically, this means that when R_v is set to zero or infinity, "e" should be positioned such that it has no contribution to the CE. This means setting R_v to zero or infinity should position "e" between the input and ground or between the inputs of OA. For example in Fig.4.5(d) when $R_v=(1/G_h)=0$ the resistor $R_r=1/G_r$ will be positioned between the two inputs of OA. Hence, it has no contribution to CE.

Taking the above consideration into account and using the methods previously developed for introducing a new resistor to the circuit, the following is found.

The three circuits of Figs.4.5(a),(g)and (h) are not convertible. One circuit is derived from Fig.4.5(f). It is shown in Fig.4.7(h). One circuit (Fig.4.7(i)) is derived from the non-oscillating 8 element circuit previously established. From the remaining four circuits of Fig.4.5, a total of 10 circuits are derived as shown in Fig.4.7. These circuits construct a set of 12 easily tunable GCVFOs. All of the 12 circuits are new. One of which (Fig.4.7(i))has

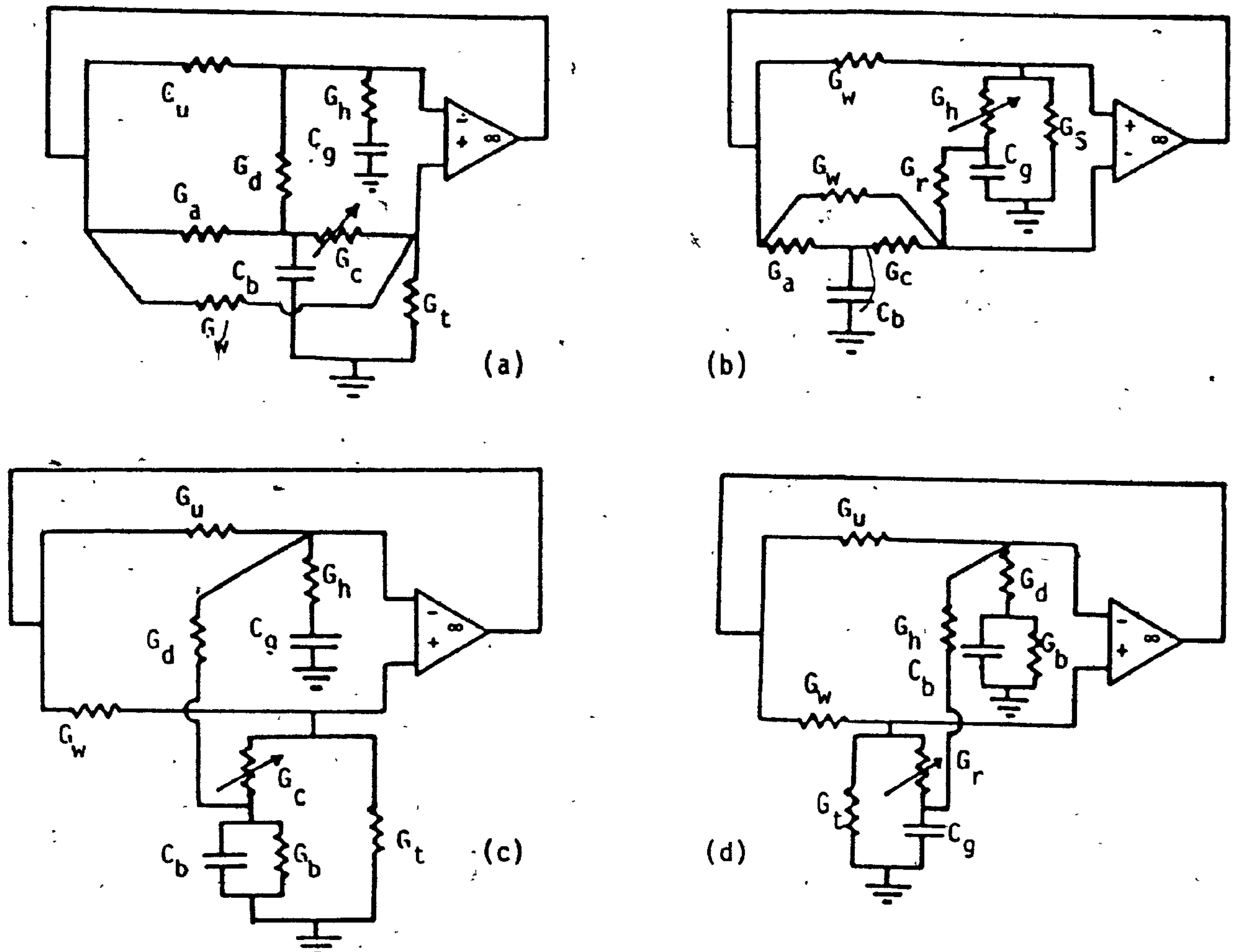


Fig. 4.7: The set of 12 canonic GCVFOs (Continued).

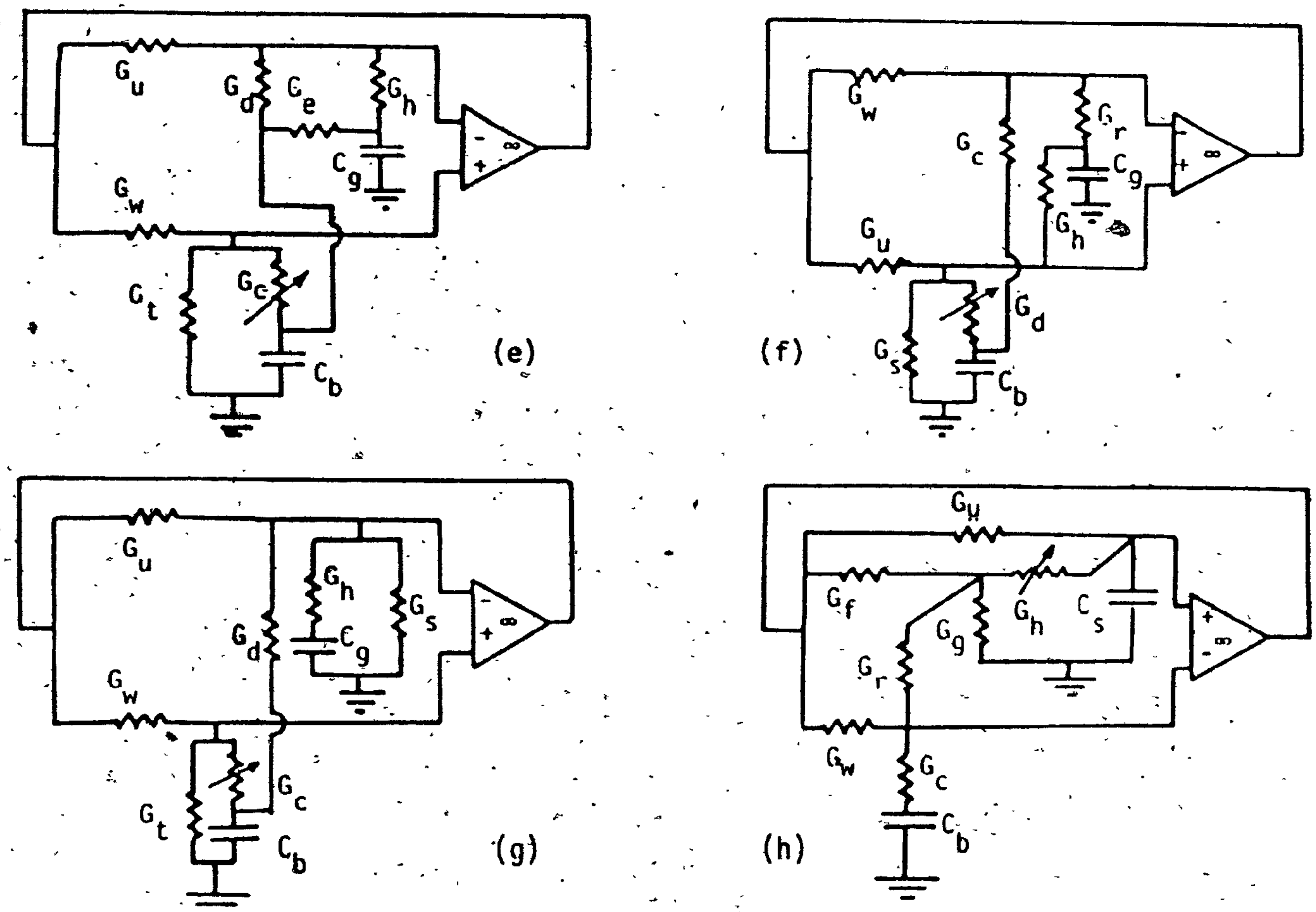


Fig. 4.7: The set of 12 canonic GCVFOs (Continued).

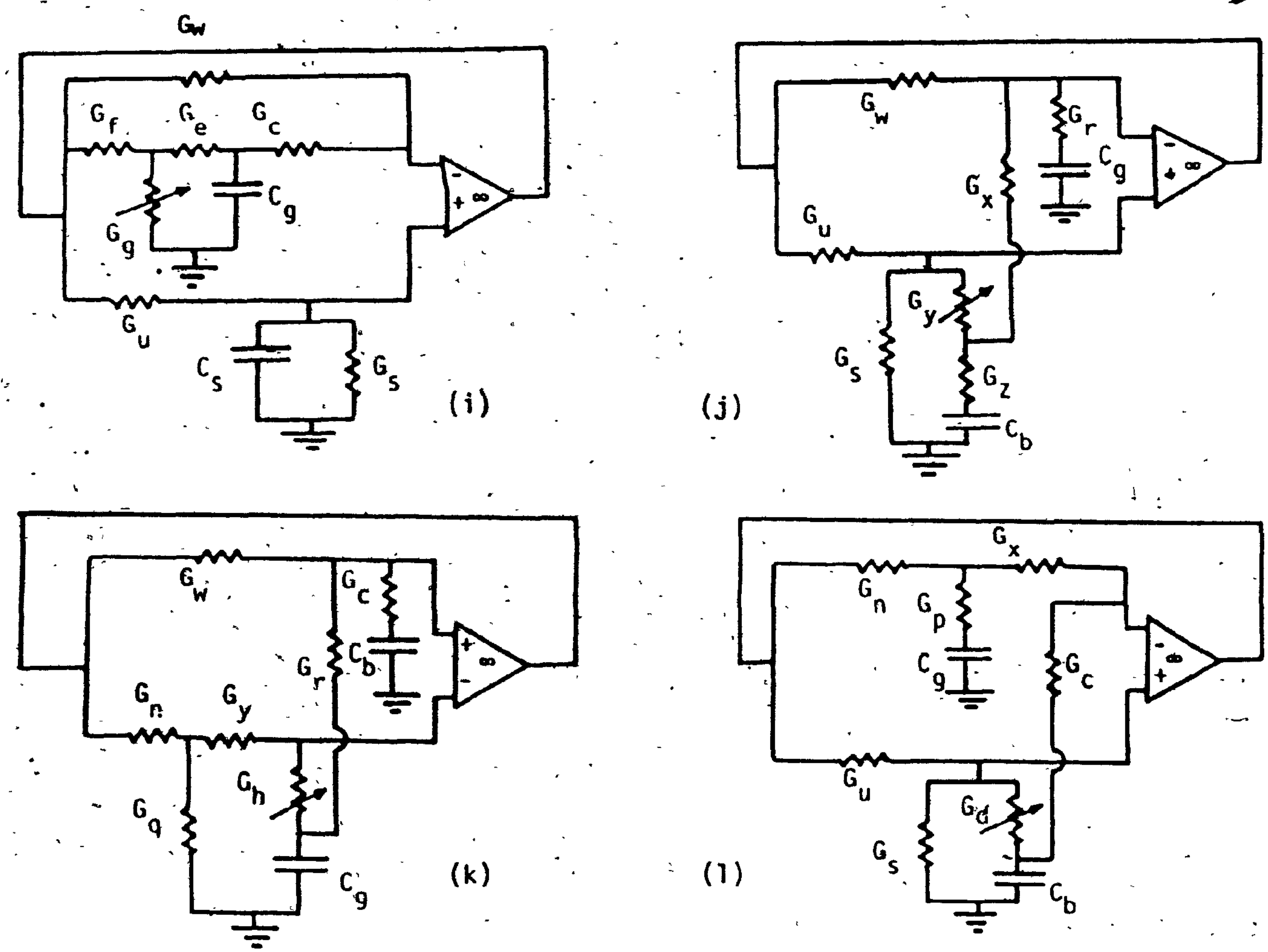


Fig. 4.7: The set of 12 canonic GCVFOs (Continued).

also its variable resistor grounded. Figure 4.7(h) is an EVE circuit.

4.7 Design Procedures

The same design procedure as in section 4.3 can be used. The design equations are shown in Table 4-II. It should be noted that design equations are not unique.

4.8 Experimental Results

All the circuits have been tested and they perform according to the theory. As representitive, only the results of circuits of Figs.4.7(h) and (i) are given. The quality and tolerance of all the resistors and all the capacitors are the same as in the previous sections. The values chosen for the elements are:

Fig.4.7(h): $R_f = R_r = R_g = R_w = R_u = R_c = 1\ k\Omega$, $C_b = C_s = 0.1\mu F$
R_h=variable.

Fig.4.7(i): $R_f = 2.76k\Omega$, $R_e = 3k\Omega$, $R_s = 14.77k\Omega$, $R_u = 0.6k\Omega$,
$R_w = 2.4k\Omega$, $C_g = C_s = 0.1\ \mu F$ R_g= variable.

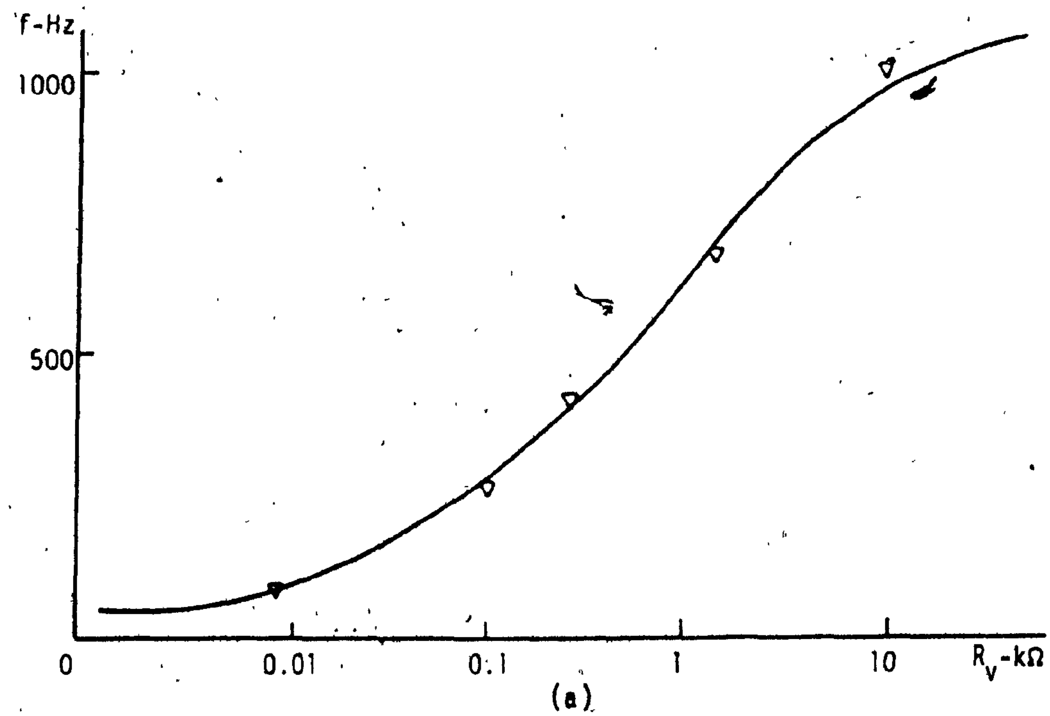
Figures 4.8(a) and (b) show the experimental variation of frequency versus the variable resistor for the circuits of Figs.4.7(i) and 4.7(h), respectively. For all the experiments, a ±15volts power supply has been used. The output is taken from the output of OA (Fairchild 741).

Table 4-II Design equations for circuits of Fig.4.7. (Continued)

(a)	$C_b = C_g = C,\quad G_t = G_w = G_a = G_d = \frac{1}{R},\quad G_u = \frac{\sqrt{2}}{R},$ $G_h = (1+\sqrt{2})/R,\quad G_c = \frac{1}{R_v}\qquad K_1 = 1+\sqrt{2},\quad K_2 = 3 + \frac{5}{\sqrt{2}}$ $\omega = \frac{1}{RC}\sqrt{\frac{K_1R+K_2R_v}{R-\sqrt{2}\,R_v}}\qquad R_v < \frac{R}{\sqrt{2}}$
(b)	$C_b = C_g = C,\quad G_c = G_w = G_a = G_R = \frac{1}{R},\quad G_u = \frac{12}{R},\quad G_s = \frac{9}{R},$ $G_h = \frac{1}{R_v}\qquad \omega = \frac{3}{RC}\sqrt{\frac{3(R_v+R)}{R-15R_v}}$
(c)	$C_b = C_g = C,\quad G_b = \frac{4}{R},\quad G_h = \frac{8}{R},\quad G_t = \frac{12}{R},\quad G_w = 3G_u,\quad G_d = \frac{1}{R},$ $G_c = \frac{1}{R_v}\qquad R_v < \frac{R}{15}\qquad \omega = \frac{8}{RC}\sqrt{\frac{2(R+3R_v)}{R-15R_v}}$
(d)	$C_b = C_g = C,\quad G_b = G_d = \frac{1}{R},\quad G_w = \frac{2}{R},\quad G_t = \frac{4}{R},\quad G_h = \frac{1}{7R},$ $G_u = \frac{1}{3R}\qquad R_v < \frac{7R}{20}\qquad \omega = \frac{1}{RC}\sqrt{\frac{2(7R+R_v)}{7R-20R_v}}$
(e)	$C_b = C_g = C,\quad G_u = G_e = \frac{1}{R},\quad G_w = \frac{2}{R},\quad G_h = \frac{5}{R},\quad G_t = \frac{3}{R},$ $G_d = \frac{1}{21R},\quad G_c = \frac{1}{R_v}\quad R_v < \frac{21R}{149}\qquad \omega = \frac{3}{RC}\sqrt{\frac{126R+111R_v}{21R-149R_v}}$
(f)	$C_b = C_g = C,\quad G_w = G_h = \frac{1}{R},\quad G_u = \frac{3}{R},\quad G_r = \frac{2}{R},\quad G_s = \frac{2}{R},$ $G_c = \frac{1}{2R},\quad G_d = \frac{1}{R_v}\quad R_v < \frac{2R}{9}\qquad \omega = \frac{3}{RC}\sqrt{\frac{4R+2R_v}{2R-9R_v}}$

Table 4-II Design equations for circuits of Fig.4.7. (Continued)

(g)	$C_b = C_g = C$, $G_s = G_u = \frac{1}{R}$, $G_w = G_h = \frac{2}{R}$, $G_t = \frac{4}{R}$, $G_d = \frac{2}{3R}$, $G_c = \frac{1}{R_v}$ $R_v < \frac{3R}{10}$ $\omega_s = \frac{1}{RC}\sqrt{\frac{12R+8R_v}{3R-10R_v}}$
(h)	$C_s = C_g = C$, $G_u = G_w = G_f = G_r = G_g = G_c = \frac{1}{R}$, $G_h = \frac{1}{R_v}$ $R_v < R$ $\omega_s = \frac{1}{RC}\sqrt{\frac{R-R_v}{R+4R_v}}$
(i)	$C_b = C_s = C$, $G_w = G_f = G_e = G_c = \frac{1}{R}$, $G_s = \frac{3}{R}$, $G_u = \frac{5}{R}$, $G_g = \frac{1}{R_v}$ $\omega_s = \frac{1}{RC}\sqrt{\frac{R+12R_v}{R+2R_v}}$
(j)	$C_b = C_g = C$, $G_s = G_R = G_u = \frac{1}{R}$, $G_z = \frac{2}{R}$, $G_w = \frac{2}{5R}$, $G_x = \frac{2}{7R}$, $G_y = \frac{1}{R_v}$ $R_v < \frac{R}{68}$ $\omega_s = \frac{2}{RC}\sqrt{\frac{7R+2R_v}{R-68R_v}}$
(k)	$C_g = C_B = C$, $G_y = G_n = G_c = G_q = \frac{1}{R}$, $G_r = \frac{1}{7R}$, $G_w = \frac{1}{4R}$, $G_h = \frac{1}{R_v}$ $R_v < \frac{21R}{25}$ $\omega_s = \frac{1}{RC}\sqrt{\frac{7R+R_v}{21R-25R_v}}$
(ℓ)	$C_g = C_b = C$, $G_u = G_x = G_p = G_s = \frac{1}{R}$, $G_c = \frac{1}{4R}$, $G_n = \frac{1}{2R}$, $G_d = \frac{1}{R_v}$ $R_v < \frac{4R}{9}$ $\omega = \frac{1}{RC}\sqrt{\frac{4R+R_v}{4R-9R_v}}$



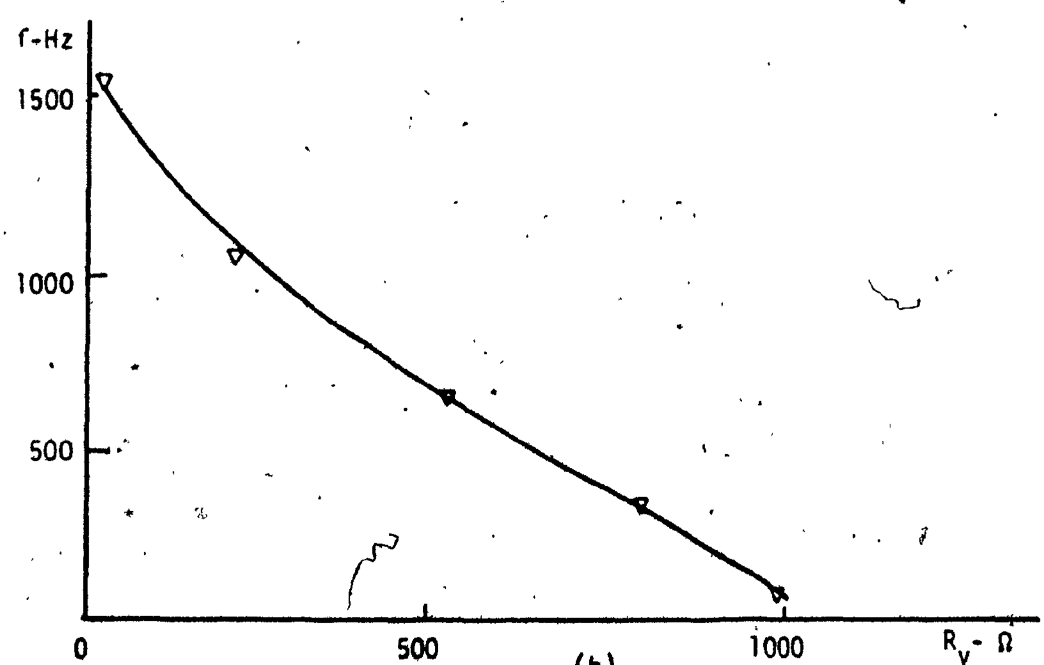


Fig. 4.8: Variation of frequency versus variable resistor, (a) for Fig.4.7(i), (b) for Fig.4.7(h),(Experimental).

4.9 Summary

A systematic approach for the realization of canonic GCVFOs has been described. The approach is general and complete. Using this approach, a set of 8 canonic GCVFOs is found. All of the circuits in this set are completely new. The set is proven to be complete. The circuits in this set require six resistors, two capacitors and one OA. Two of these circuits have equal valued elements (all the resistors are equal and all the capacitors are equal). However, all but one of these circuits have difficulty in starting up oscillations. Further, the one that has no difficulty in starting the oscillations has a rather limited frequency range of variation. Based on this set, another set of 12 easily tunable GCVFOs are found. This set is shown to be complete. Each requires 7 resistors, 2 capacitors and one OA. All the circuits of this set are also completely new. One of the circuits in this set has equal valued elements. In all of the GCVFOs, the frequency of operation is varied by a single variable resistor. Each set has one circuit in which the variable resistor is also grounded (a desirable feature for voltage controlled oscillators). All the circuits can be designed with equal capacitor values. All of the oscillator circuits have been built and tested. The results show that the circuits perform closely to their theoretically predicted behaviour.

CHAPTER V

CANONIC RC-OSCILLATORS USING DIFFERENTIAL OUTPUT OAs

CHAPTER V

CANONIC RC-OSCILLATORS USING DIFFERENTIAL OUTPUT OAs

5.1 Introduction

In the previous chapters, attention has been restricted to canonic RCOs using single ended OAs.

When a DOOA is used in place of an OA, a reduction in the number of passive components in the oscillator circuits may be possible. In the recent past this device has attracted significant attention. DOOA has also been put out as an IC chip [35]. Even though there has been many studies based on DOOAs for RC circuits,[36-38] none has been made for RCOs.

This chapter presents a systematic and general approach to the realization of RCOs based on DOOAs. Using this approach, a total of 10 canonic RCO circuits are found. All the circuits obtained use a single DOOA. Two of the RCOs are single frequency oscillators consisting of 2 capacitors and 3 resistors. Two VFOs are found, using the same number of components. In both of these VFOs the variable resistor is grounded. Three GCSFOs are derived, each consisting of 2 capacitors and 4 resistors. Finally, three GCVFOs are found, each using 2 capacitors and 5 resistors. In one of the GCVFOs the variable resistor is also grounded.

5.2 Theory

5.2.1 General Considerations

As shown in previous sections, it is convenient to devide the RCO circuit into two parts, active and passive. The minimum number of DOOAs that can be used in the circuit is one. Therefore, it is assumed that only one DOOA is in the circuit. Figure 5.1(a) shows the DOOA as the active part of the system. It is assumed that the DOOA is ideal. Since the active part has two inputs and two outputs, the passive RC network N must have two outputs and two inputs to match the terminals of the active part. Figures 5.1(b) and 5.1(c) show the passive RC network with the required terminals. The ports of the RC network then can be connected to the ports of active part as shown in Figs.5.2(a), and 5.2(b). There is unity feed-back in the final circuits. Figure 5.2(a) is the most general configuration of a DOOA based RCO. In Appendix E, it is shown that Fig.5.2(b) is a stable system and hence, it cannot generate oscillator circuits. Therefore, the investigation of Fig.5.2(a) is sufficiant for completeness.

5.2.2 Characteristic Equation

Consider the Fig.5.2(a), the transfer functions of interest are:

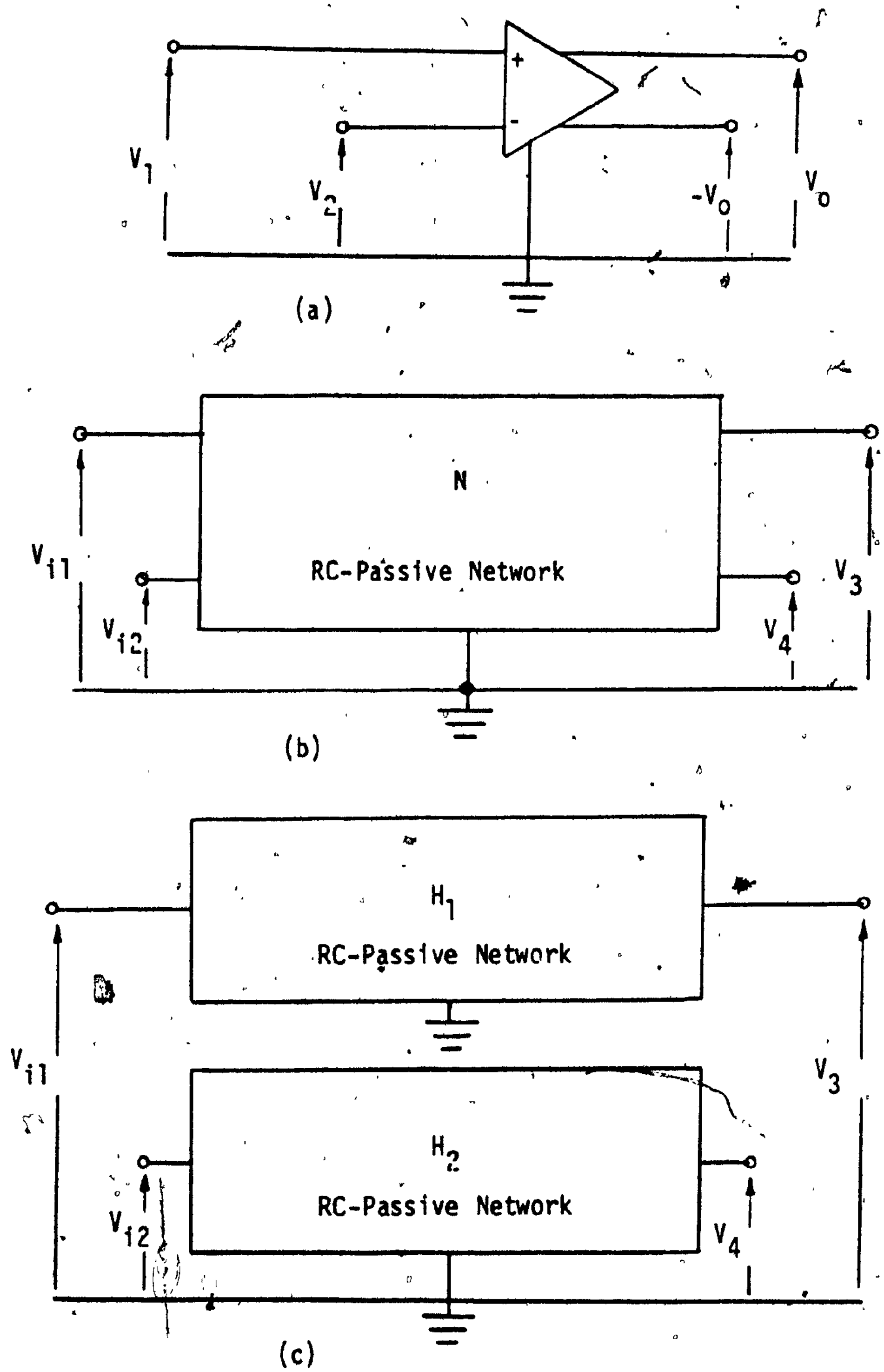


Fig. 5.1(a): A differential output operational amplifier.
(b): A four port passive network with connected graph.
(c): A four port passive network with non-connected graph.

$$T_{13} = \frac{V_3}{V_{11}} \Bigg|_{V_{12}=0} = \frac{N_{13}(S)}{D(S)} \tag{5-1a}$$

$$T_{23} = \frac{V_3}{V_{12}} \Bigg|_{V_{11}=0} = \frac{N_{23}(S)}{D(S)} \tag{5-1b}$$

By superposition,

$$V_3 = V_{11}\left(\frac{N_{13}(S)}{D(S)}\right) + V_{12}\left(\frac{N_{23}(S)}{D(S)}\right) \tag{5-2}$$

similarly,

$$T_{14} = \frac{V_4}{V_{11}} \Bigg|_{V_{12}=0} = \frac{N_{14}(S)}{D(S)} \tag{5-3a}$$

$$T_{24} = \frac{V_4}{V_{12}} \Bigg|_{V_{11}=0} = \frac{N_{24}(S)}{D(S)} \tag{5-3b}$$

or,

$$V_4 = V_{11}\left(\frac{N_{14}(S)}{D(S)}\right) + V_{12}\left(\frac{N_{24}(S)}{D(S)}\right) \tag{5-4}$$

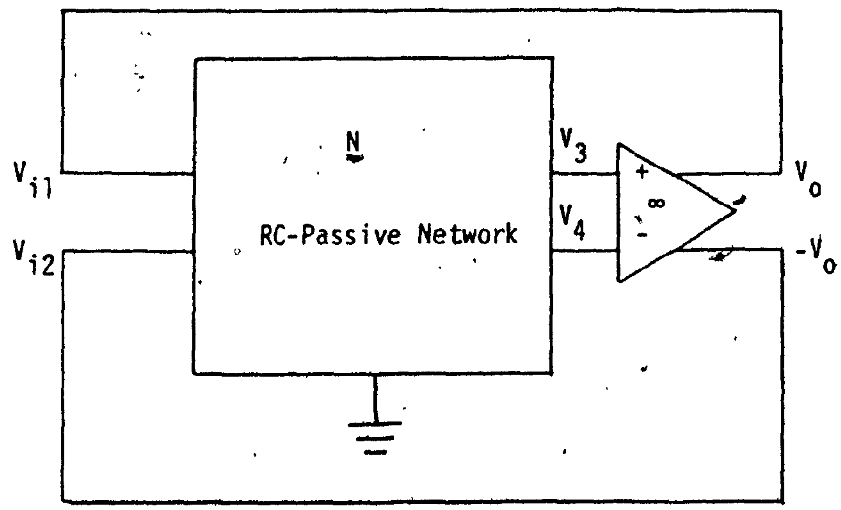
D(S) is the CE of the passive network and its roots are on the negative real axis. The DOOA allows us to write,

$$V_o = A_d(V_3 - V_4) \tag{5-5}$$

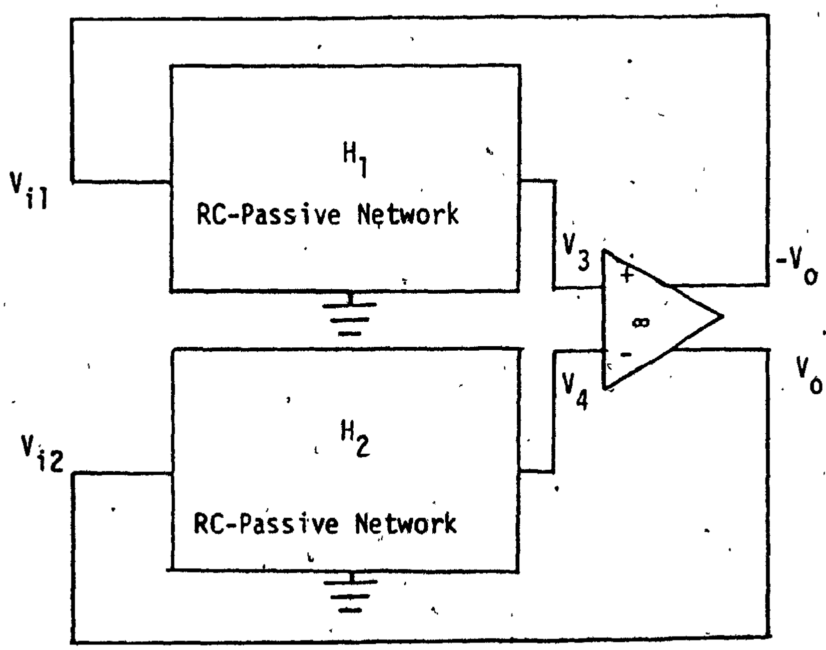
From unity feed-back, we can write,

$$V_{11} = V_o \tag{5-6a}$$

$$V_{12} = -V_o \tag{5-6b}$$



(a)



(b)

Fig. 5.2(a): General configuration of RCO using differential output OA.
(b): An RC stable system, using a differential output OA.

Substituting from (5-2), (5-4) and (5-6) into (5-5) and allowing A_d to go to infinity, the over all CE of the system is

$$[N_{13}(S)+N_{24}(S)]-[N_{14}(S)+N_{23}(S)] = 0 \quad (5\text{-}7)$$

Equation (5-7) is similar to (2-7) in nature, therefore, we can write the CE as

$$N_1(S)-N_2(S) = 0 \quad (5\text{-}8)$$

where,

$$N_1(S)=N_{13}(S)+N_{24}(S) \quad (5\text{-}9a)$$

$$N_2(S)=N_{14}(S)+N_{23}(S) \quad (5\text{-}9b)$$

Clearly, (5-8) should have all the properties of (2-7), namely all the equations (2-8) to (2-13) are also valid here. Also rules I to III apply to (5-7).

5.3 Realization of DOOA Based RCOs

As in section 2.6, for a systematic approach to identify the canonic RCOs we proceed as follows.

First we assume there are no independent nodes inside the passive network, in which case a five node network is obtained. Analysis of this network just obtained may or may not lead to any RCO. Then we assume there are two nodes inside the passive RC network and so on. This procedure is continued until non-canonicity is reached.

5.3.1 Five-Node Network

Figure 5.3 shows the general configuration of an RCO with a five node network. The CE of the circuit can be derived as:

$$Y_uY_t+Y_cY_s+2(Y_cY_u)-Y_bY_t-Y_wY_s-2(Y_bY_w) = 0 \qquad (5\text{-}10)$$

From rule II, one capacitive branch should be common to $N_1(S)$ and $N_2(S)$. Then, the possible choices for capacitive branches are:

(a) $Y_s=SC_s+G_s$

(b) $Y_t=SC_t+G_t$

For option (a), the choices for the second capacitive branch are:

(a-1) $Y_w=SC_w+G_w$

(a-2) $Y_c=SC_c+G_c$

Taking option (b), the choices for the second capacitive branch are:

(b-1) $Y_u=SC_u+G_u$

(b-2) $Y_b=SC_b+G_b$

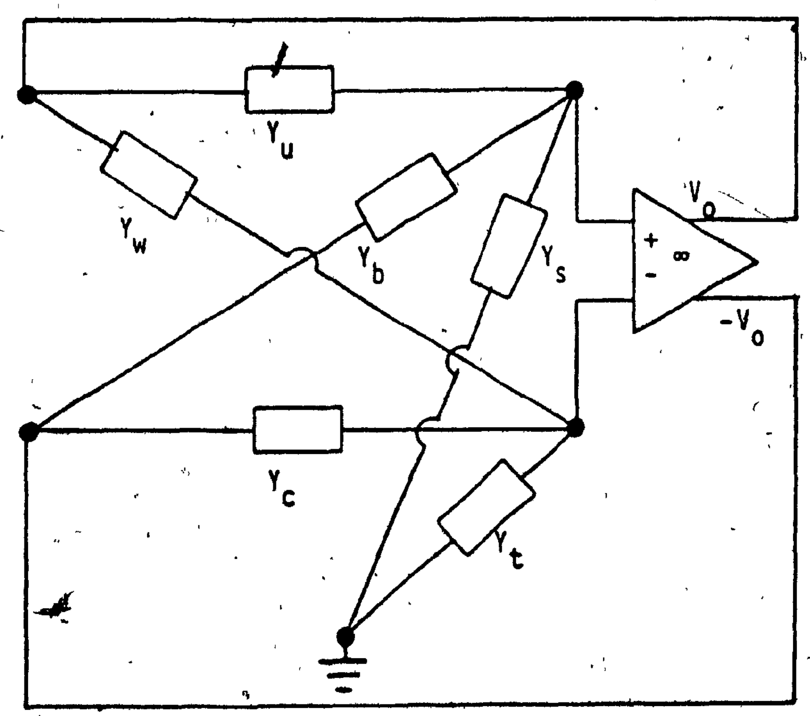


Fig. 5.3: General configuration of an RCO with a five node network and DOOA.

Over all the following four cases should be considered.

Case I: Y_s and Y_w are capacitive.

Case II: Y_s and Y_c are capacitive.

Case III: Y_t and Y_u are capacitive.

Case IV: Y_t and Y_b are capacitive.

It can be shown that cases II and IV are equal, that is, they always generate SU pair circuits. Similarly cases I and III are equal and generate SU pair circuits. Furthermore, cases II and III will lead to the same circuits. Therefore, only case II is investigated.

Let $Y_s=SC_s+G_s$ and $Y_c=SC_c+G_c$ then the CE is:

$$S^2C_cC_s+S[G_sC_c+G_cC_s+2G_uC_c-G_wC_s]+G_uG_t+G_cG_s$$
$$+2G_uG_c-G_bG_t-G_wG_s-2G_wG_b=0 \qquad (5\text{-}11)$$

For a canonic circuit, we should set to zero as many branch admittances as possible. In (5-11) there are two sets of G_is that can be set to zero simultaneously:

(i) $G_s=G_b=G_c=0$

(ii) $G_s=G_t=G_b=0$

Setting the elements in (i) equal to zero we can write:

$$\text{OC:} \qquad 2G_uC_c-G_wC_s=0 \qquad (5\text{-}12a)$$

$$\text{OF} \qquad \omega_s=\left(\frac{G_uG_t}{C_cC_s}\right)^{1/2} \qquad (5\text{-}12b)$$

Since OC is independent of G_t and G_t is present in OF the circuit is a VFO and G_t is the variable element. Figure 5.4(a) shows the final circuit.

Setting the elements in (ii) equal to zero we have:

$$OC: \quad G_c C_s + 2G_u C_o - G_w C_s = 0 \qquad (5\text{-}13a)$$

$$OF: \quad \omega_s = \left\{ \frac{2G_c G_u}{C_s C_o} \right\}^{1/2} \qquad (5\text{-}13b)$$

All the elements in OF are also present in OC. Therefore, the circuit is an SFO. The final circuit is shown in Fig.5.5(a).

5.3.2 Six-Node Network

There is one internal node in a six node network, as it is shown in Fig.5.6. This Figure presents a general configuration of a DOOA based RCO with 6 nodes. All the possible branch admittances are present in the passive network. It should be noted that, there are some branches in the network whose presence does not contribute to the CE of the circuit. Hence, they are removed. These branches originally were:

Between the two inputs of the RC network.

Between the two inputs of DOOA.

Between the inputs of RC network and ground node.

There are 11 branch admittances in Fig.5.6, and the CE is:

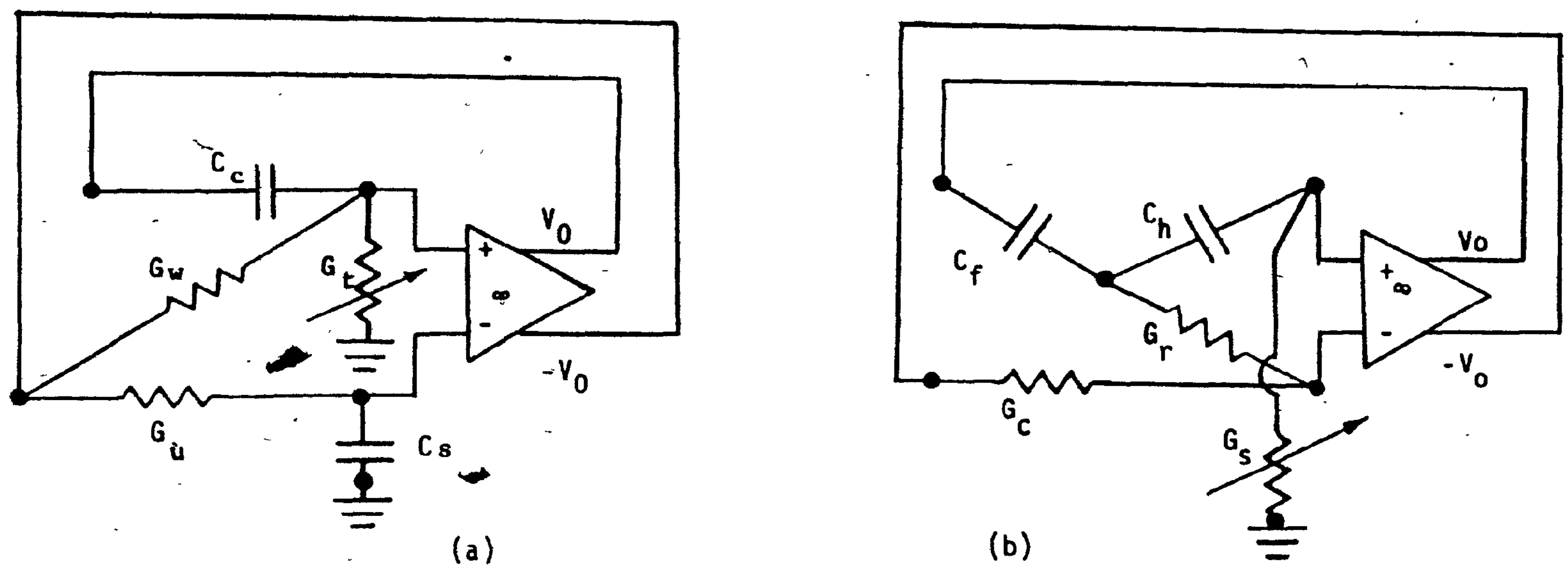


Fig. 5.4: The set of two canonic variable frequency oscillators using differential output amplifier.

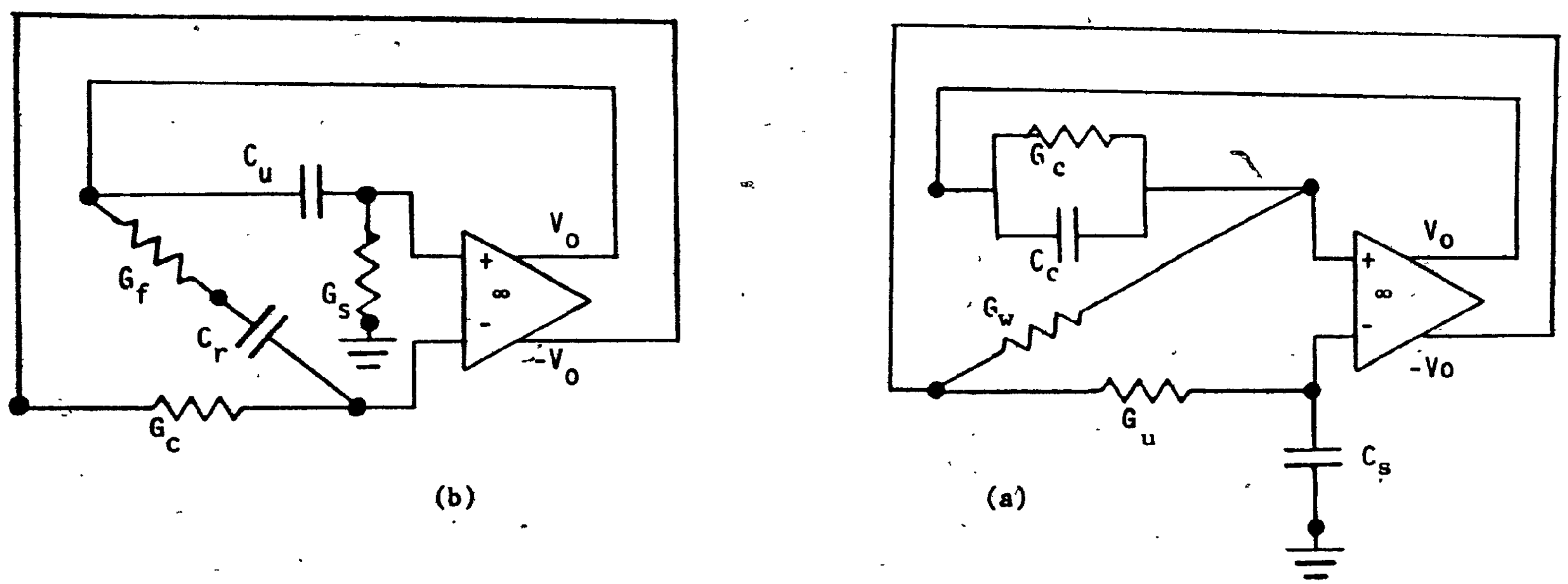


Fig. 5.5: The set of two canonic single frequency oscillators using differential output amplifier.

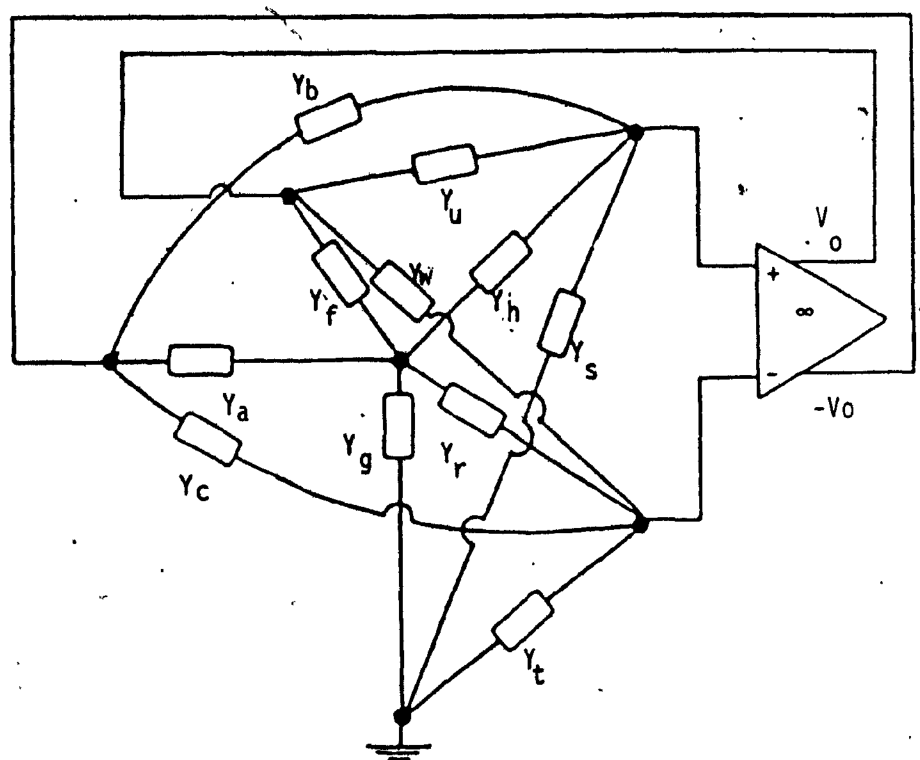


Fig. 5.6: General configuration of an RC-oscillator with a six node network and DOOA.

$$Y_wY_sY_f+Y_wY_sY_g+Y_wY_sY_h+Y_wY_sY_r+Y_wY_sY_a+Y_wY_hY_g+Y_fY_rY_s$$
$$+Y_tY_bY_f+Y_tY_bY_g+Y_tY_bY_h+Y_tY_bY_r+Y_tY_bY_a+Y_bY_rY_g+Y_aY_hY_t$$
$$+2(Y_wY_bY_f+Y_wY_bY_g+Y_wY_bY_h+Y_wY_bY_r+Y_wY_bY_a+Y_wY_hY_a+Y_fY_rY_b)$$
$$-Y_uY_tY_f-Y_uY_tY_g-Y_uY_tY_h-Y_uY_tY_r-Y_uY_tY_a-Y_uY_rY_g-Y_fY_hY_t$$
$$-Y_sY_cY_f-Y_sY_cY_g-Y_sY_cY_h-Y_sY_cY_r-Y_sY_cY_a-Y_cY_hY_g-Y_aY_rY_s$$
$$-2(Y_uY_cY_f+Y_uY_cY_g+Y_uY_cY_h+Y_uY_cY_r+Y_uY_cY_a$$
$$+Y_uY_rY_a+Y_cY_hY_f)=0 \qquad (5-14)$$

Since the DOOA is assumed to be ideal, the input impedances of the active part are infinity and no currents are flowing into the DOOA. This allows us to represent the circuit as shown in Fig.5.7 . In this Figure the controlled sources (CSs) are shown between the inputs of passive network and ground node.

It is clear that at least two loops are in the circuit, each containing one of the controlled sources. Therefore, the number of loops in this network are:

$$l = b - n + 1 \geq 2 \qquad (5-15a)$$

or

$$b \geq n + 1 \qquad (5-15b)$$

Let l=2, since there are six nodes in N, therefore, there must be seven branches in the RCO circuit. It is known that 2 of the branches are controlled sources, therefore, only 5 admittance branches must be in the circuit.

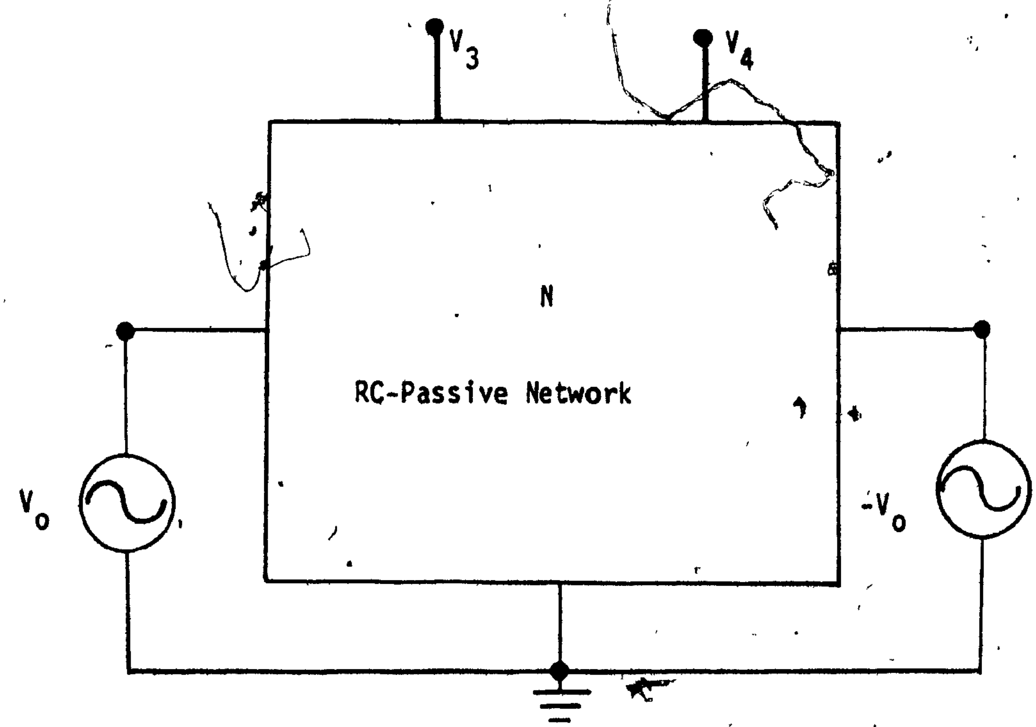


Fig. 5.7: General configuration of an RCO with a model for DOOA.

Given 7 branches and 6 nodes only 4 different graphs with two loops can be generated. It should be pointed out that no two branches must be in parallel. Figures 5.8(a) to 5.8(d) show the resulting graphs. To identify the RCO circuits each graph is examined in detail as follows.

(i) Graph (a)

There are three grounded branches in Fig.5.7. Therefore, only node 6 of grahp (a) can be chosen as ground node. In this case, the graph (a) becomes a non-connected graph and according to Appendix E no osillator circuit can be generated from it.

(ii) Graph (b)

Only node 2 and 5 qualify for becoming the ground node. Since graph (b) is symmetrical with respect to the axis passing through nodes 1 and 6, it is sufficieht to study one of the cases.

Let node 2 be the ground node. There are three grounded branches in the graph. Both CSs correspondingto the $\pm$ outputs of the DOOA should be grounded. There are $\binom{3}{2}=3$ ways that controlled sources can be assigned. Because of symmetry only two of them are examined. They are shown in Figs.5.9(a) and 5.9(b). None of the node pairs of Fig.5.9(a) can be chosen as inputs of DOOA. Therefore, this graph is disregarded. In graph of Fig.5.9(b), only nodes 4

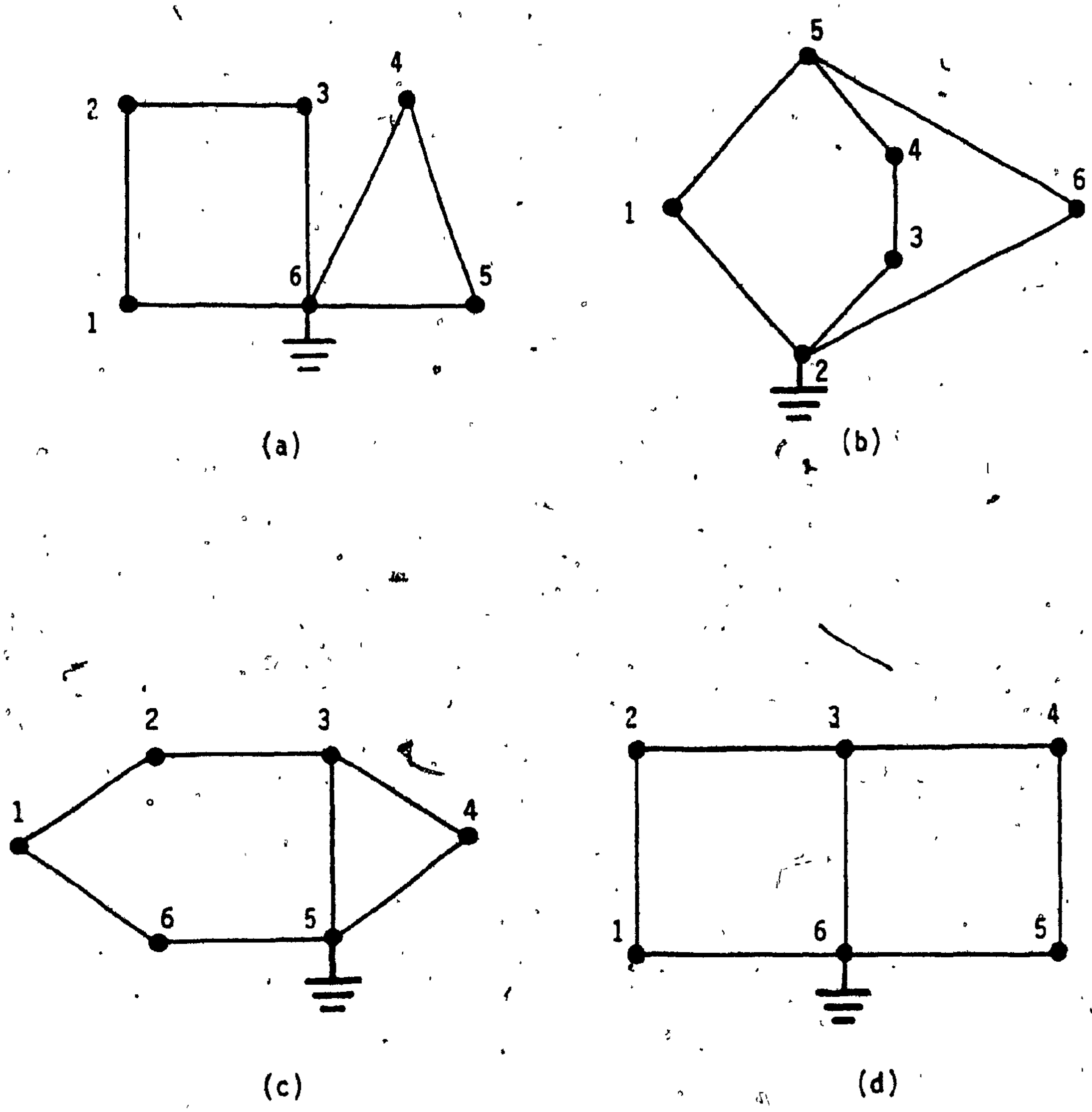


Fig. 5.8: All the distinct graphs of a 6 node 7 branch 2 loop network.

and 6 can be taken as inputs to the DOOA. There are 5 branches in the graph that are to be assigned passive components. It is already known that canonic RCOs using DOOAs require 5 passive elements. Therefore, we can start with assigning one element per branch. Two of these 5 branches should be capacitors, therefore total of $\binom{5}{2}=10$ possible ways are to be examined. Comparing the circuits obtained with the general configuration of Fig.5.6, the branch admittances are named and the CEs are found from (5-14). Out of the 10 cases only one canonic RCO is found, it is shown in Fig.5.9(c). The CE of the circuit is:

$$-2S^2G_cC_fC_h+S[G_rG_sC_f-G_sG_cC_f-G_sG_cC_h]$$
$$-G_sG_cG_r = 0 \qquad (5\text{-}16)$$

and:

$$\text{OC:} \qquad G_rC_f-G_cC_f-G_cC_h = 0 \qquad (5\text{-}17a)$$

$$\text{OF:} \qquad \omega_s=\{\frac{G_sG_r}{2C_fC_h}\}^{1/2} \qquad (5\text{-}17b)$$

Since OC is independent of G_s and OF is a function of G_s, the circuit is a VFO. The final circuit is shown in Fig.5.4(b).

(iii) Graph (c)

Only nodes 3 and 5 qualify to be considered as ground nodes. Because of symmetry node 5 is taken as ground node

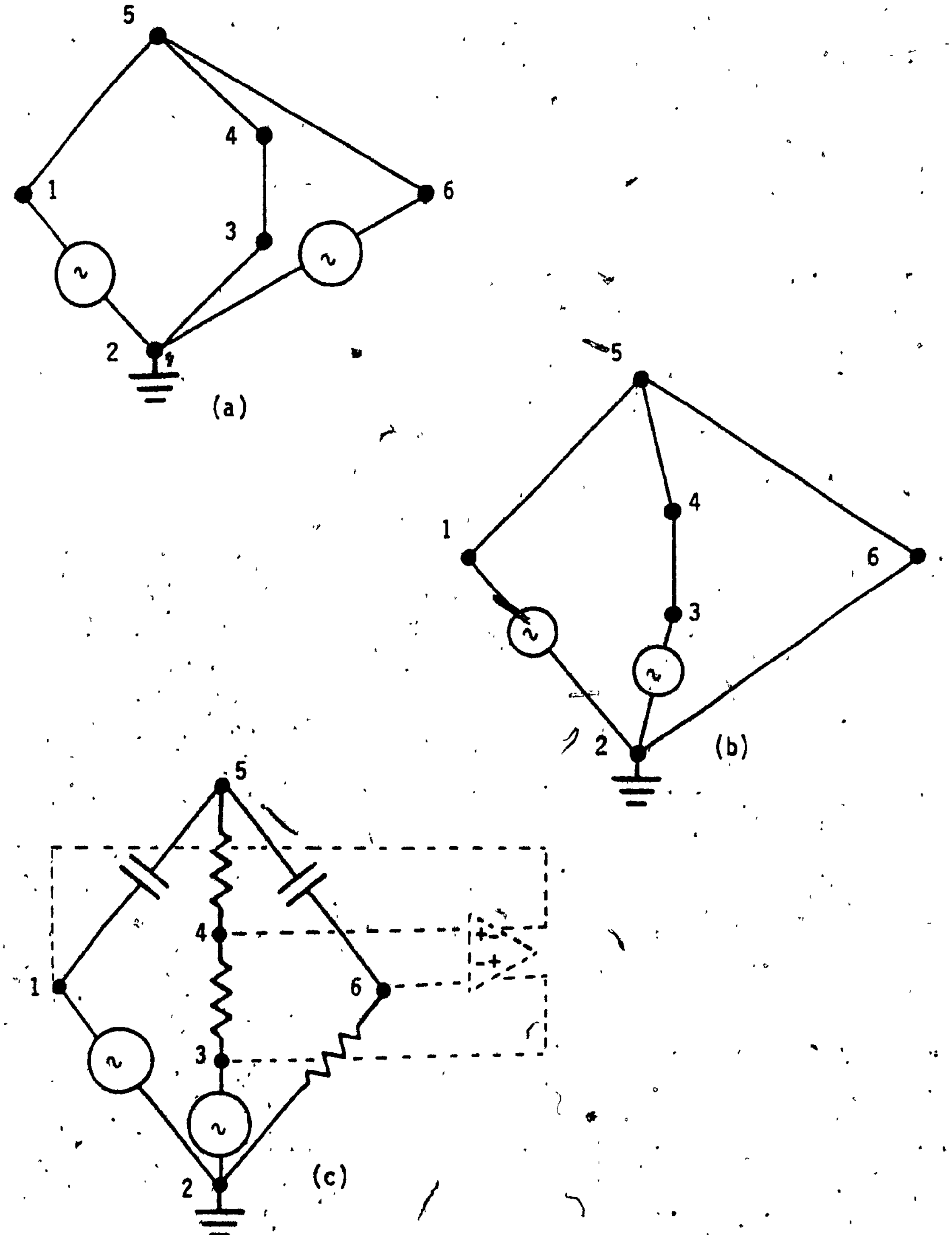


Fig. 5.9: Distribution of active branches for Fig.5.8(b).

only. Three branches are grounded and two should be CSs. Hence, a total of three cases are to be examined. They are shown in Figs.5.10(a) to 5.10(c).

In graph of Fig.5.10(a), there is one branch between the two outputs of the CSs and therefore, it can be disregarded.

In graph of Fig.5.10(b), no pair nodes can be found as the inputs to the DOOA and it is disregarded.

In graph of Fig.5.10(c), either node pairs (2,4) or (1,4) could be taken as inputs to the DOOA. In each case, 5 branches are to be filled with 2 capacitors and 3 resistors. A total of $2\cdot\binom{5}{2}=20$ cases are to be examined. Out of these 20 cases only two are found to be oscillator circuits. They are shown in Figs.5.11(a) and 5.11(b). These two circuits both have a resistor in series with a capacitor and each can be derived from the other by interchanging the resistor and capacitor that are in series. Therefore, only one of these circuits is taken as a distinct RCO. The final circuit is shown in Fig.5.5(b) and the CE of the circuit is:

$$-2S^2G_cC_uC_r+S[G_fG_sC_r-G_cG_fC_u-G_sG_cC_r-G_cG_fC_u]-G_sG_cG_f = 0 \qquad (5\text{-}18)$$

and:

$$\text{OC:} \quad G_fG_sC_r-G_cG_fC_u-G_sG_cC_r-G_cG_fC_u= 0 \qquad (5\text{-}19a)$$

$$\text{OF:} \quad \omega_s=\{\frac{G_sG_f}{2C_uC_r}\}^{1/2} \qquad (5\text{-}19b)$$

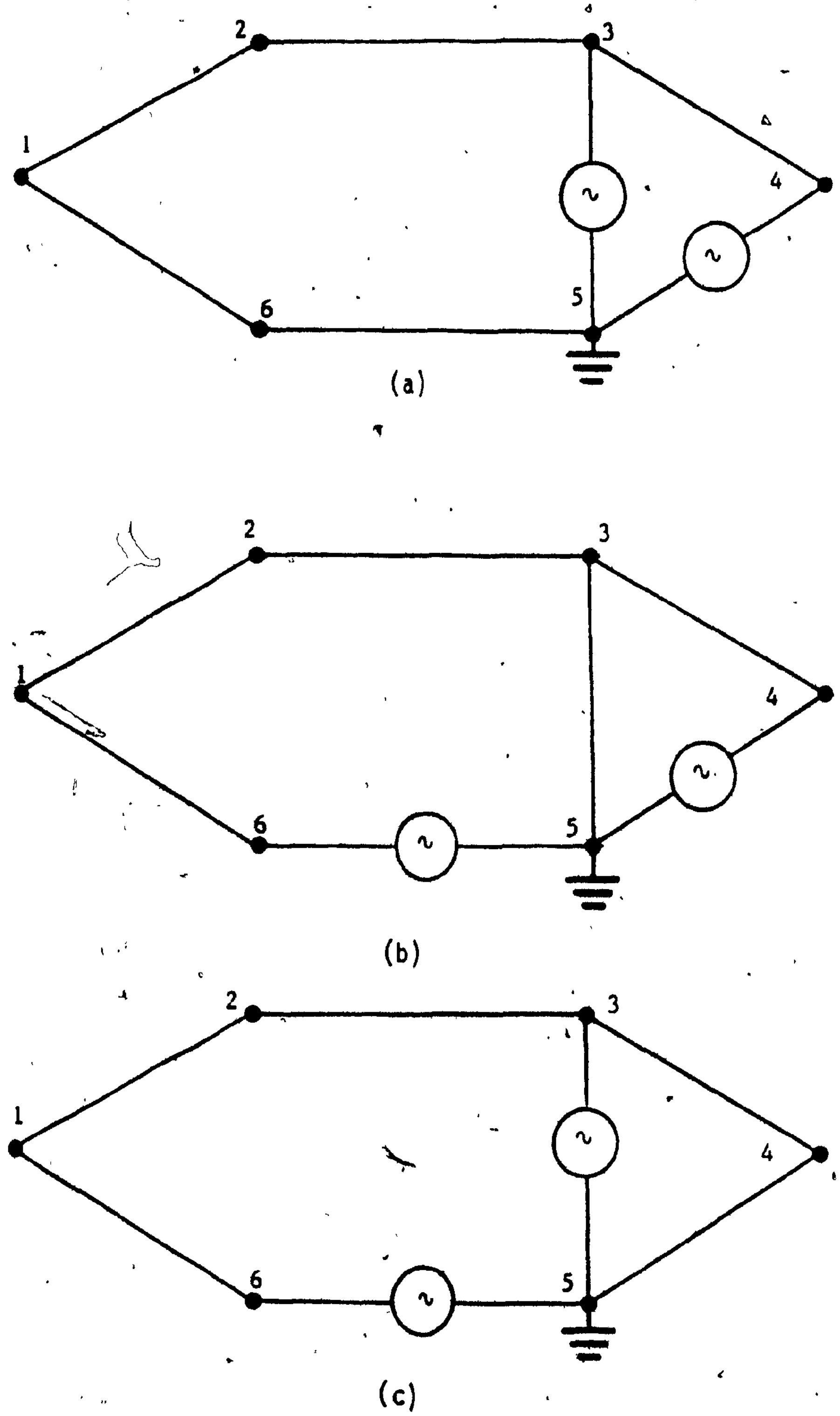


Fig. 5.10: Distribution of active branches for Fig.5.8(c).

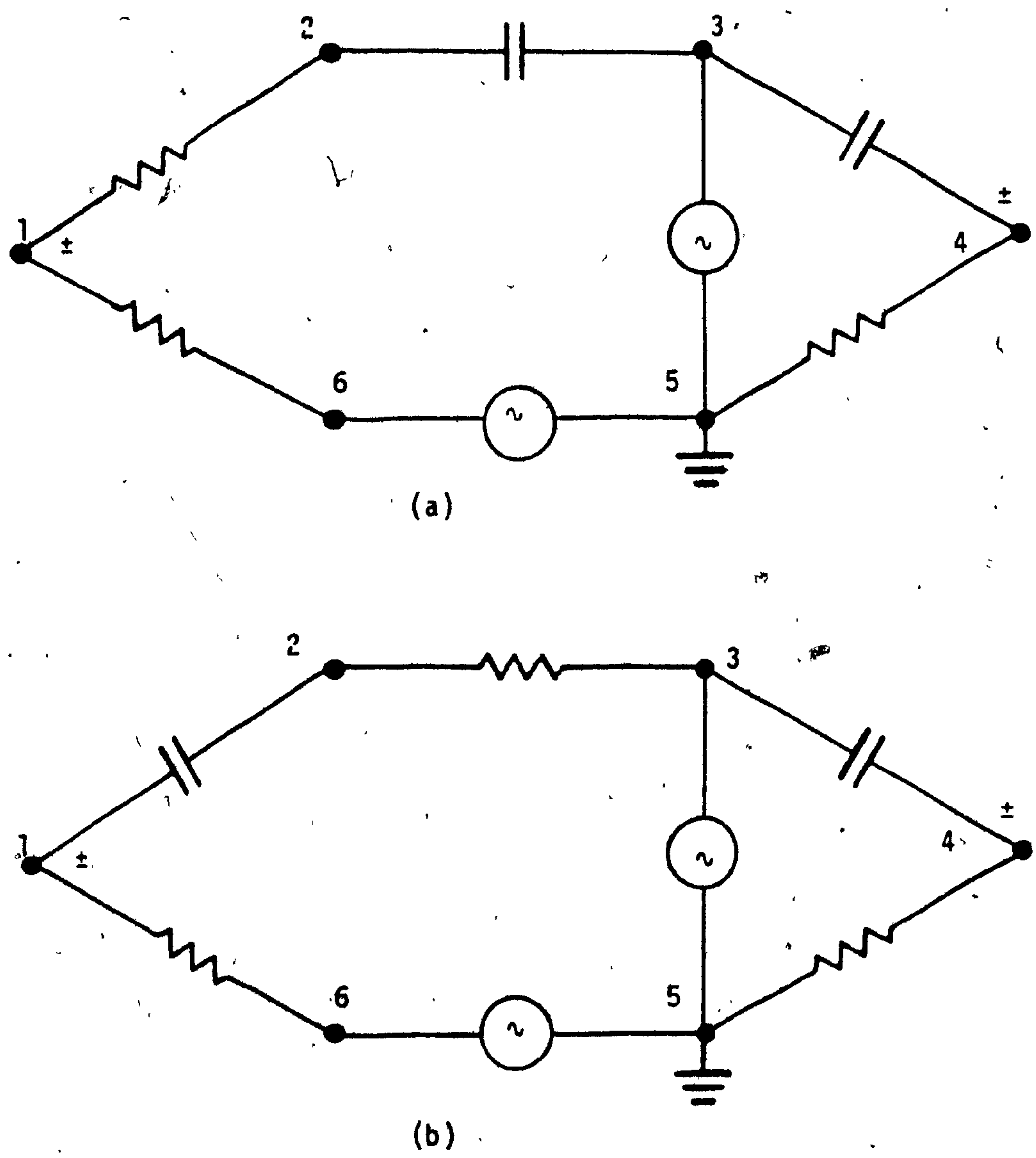


Fig. 5.11: Distribution of elements for Fig.5.10(b).

As G_s and G_f are common between OC and OF the circuit is an SFO circuit.

(iv) Graph (d)

In this graph, only nodes 3 and 6 can be taken as ground nodes. Because of symmetry, node 6 is chosen as ground node only. The CS branches and input node pairs can be identified in three ways as it is shown in Fig.5.12. There are a total of 30 combinations for the passive elements to be distributed. Further examination of these combinations shows that no oscillator circuit can be generated from these graphs.

5.3.3 Seven-Node Network

In a seven node network, the minimum number of branches required to construct a two loop graph is 8. Two of the branches being CSs, minimum of 6 passive branches remain to be filled with passive components. At least 6 components are needed for distribution over the 6 passive branches. It has been established that a canonic DOOA based RCO needs only 5 passive components. Therefore, 7 node networks do not generate canonic RCOs and hence do not need to be examined.

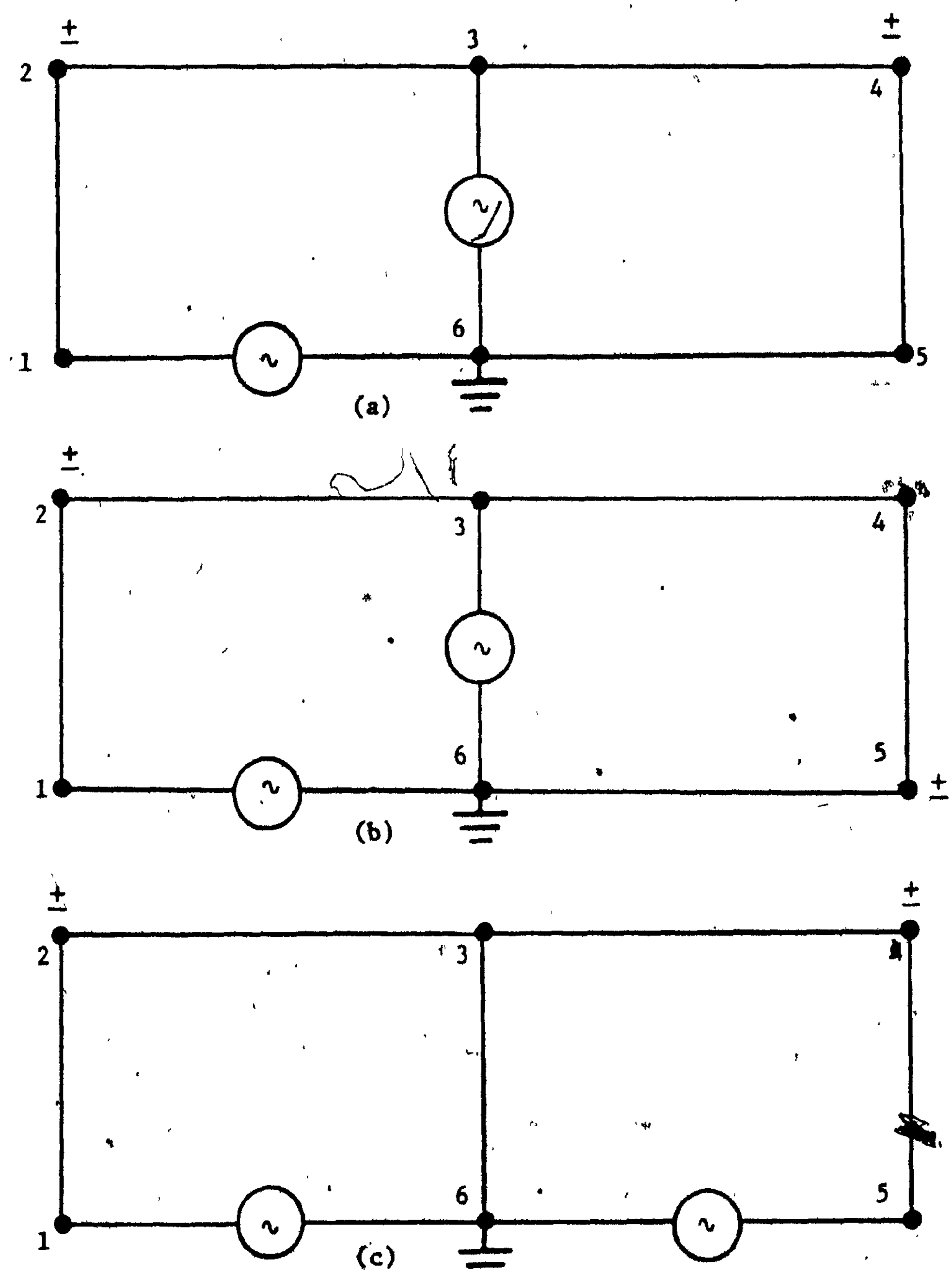


Fig. 5.12: Distribution of active branches for Fig.5.8(d).

5.4 Realization of GCRCOs Using DOOAs

None of the circuits derived so far in this chapter has been a GCRCO circuit. It is, therefore, logical to conclude that a GCRCO using DOOA must have more than 2 loops that have been examined so far. This fact can be easily seen from Fig.5.13, since there are 2 capacitors in the circuit, they are not parallel and they must both be grounded. Figure 5.13 represents the most general GCRCO configuration that uses a DOOA. It is not necessary to know the number of internal nodes. The only way that the circuit of Fig.5.13 can have 2 loops, is that there be no path from the non-grounded terminal of one capacitor to the non-grounded terminal of the other. In such a case, Fig.5.13 assumes the configuration of Fig.5.2(b) which according to Appendix E can not generate an oscillator. From above discussions, the number of loops in a GCRCO is:

$$l = b - n + 1 \geq 3 \qquad (5\text{-}20)$$

or:

$$b \geq n + 2 \qquad (5\text{-}21)$$

5.4.1 Five-Node Network

The most general configuration of a five node network (no internal node) is given by Fig.5.3 and the CE is given by (5-10) The only grounded branches in that network are Y_s and Y_t. Choosing these two branches as capacitive branches the CE from (5-10) is only a first degree polynomial and therefore, no oscillator circuit can be generated.

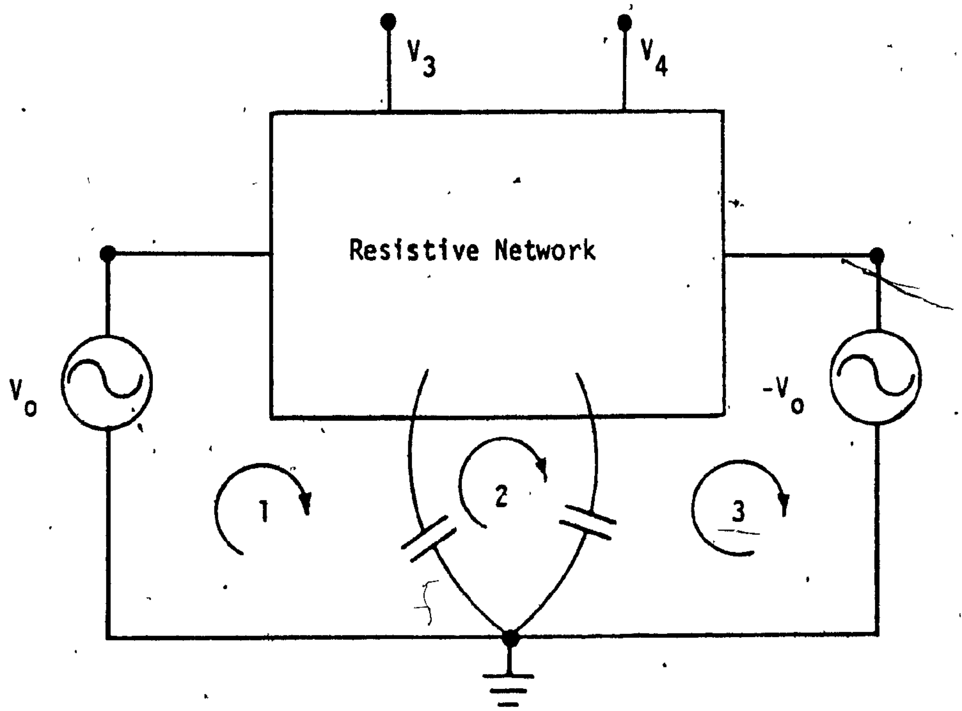


Fig. 5.13: General configuration of a GCRCO with a DOOA model.

5.4.2 Six-Node Network

According to (5-20), the number of loops in the system should be $l \geq 3$. Let us start with $l=3$. Hence, from (5-21), the number of branches in the network is $b=6+2=8$. Having two CSs in the circuit, only 6 passive branches are left. Assuming minimum of one element per branch, 6 passive components are required (2 capacitors and 4 resistors) to construct a circuit with the above requirments.

Given 6 nodes and 8 branches, a total of 9 distinct graphs can be found with 3 loops. These graphs are shown in Figs.5.14(a) to 5.14(i). Since two branches as CSs and two branches as capacitive branches should be grounded, a total of four grounded branches must be in the graph. Therefore only those nodes that are the intersection of more than 3 branches can be chosen as the ground node. In which case, graphs (a) and (b) of Fig.5.14 are disregarded because of lack of qualified ground node. Graphs (c) and (d) are non-connected graphs and can also be disregarded (see Appendix E). Graphs (e) to (i) are examined in detail in the following.

(i) Graph (e)

Only node 6 can be taken as ground node. There are four grounded branches, two should be capacitive and two should be CSs. In a total of six ways can the branches be distributed. They are shown in Figs.5.15(a) to 5.15(f).

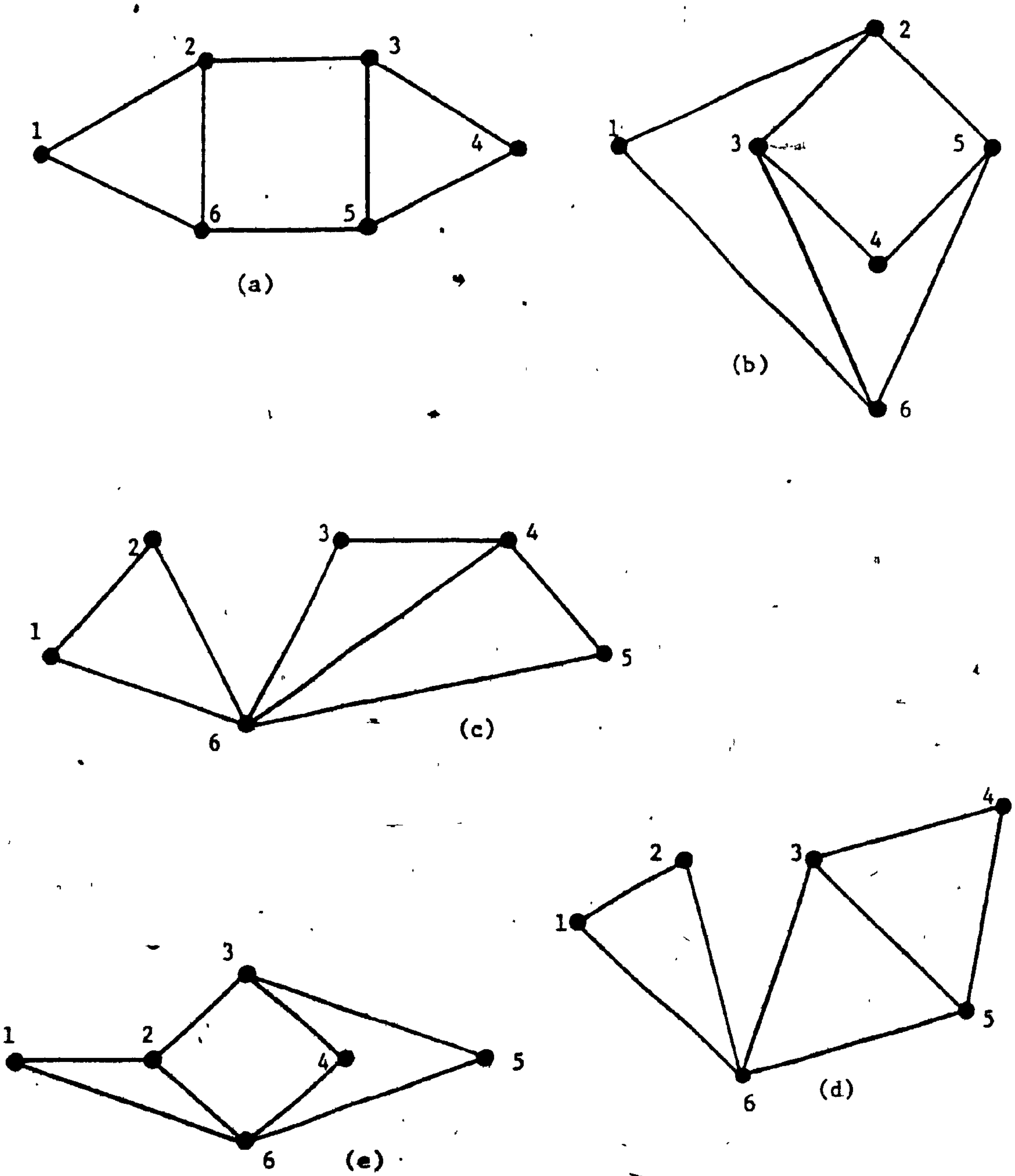


Fig. 5.14: All the possible 3 loop, 6 node, and 8 branch graphs (Continued).

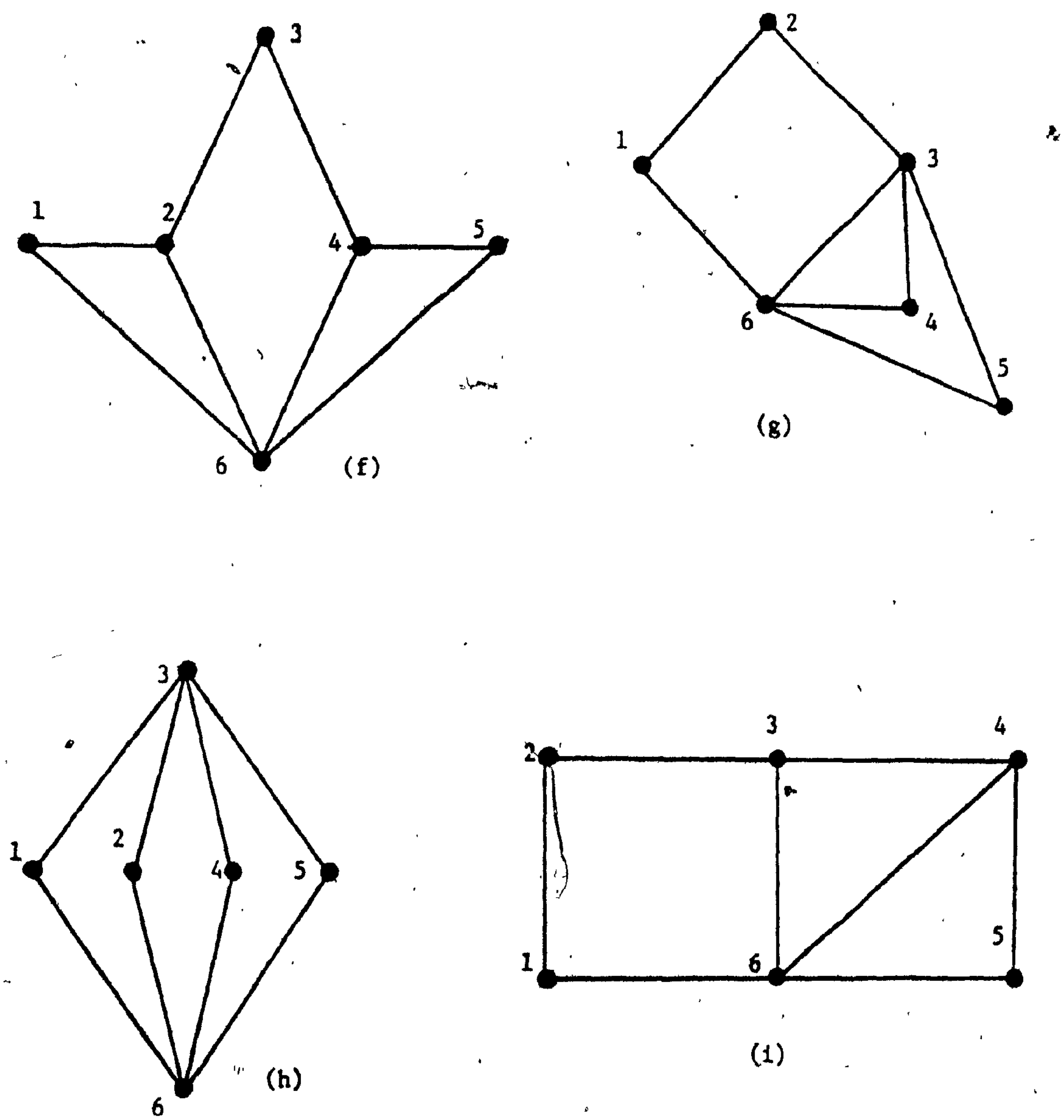


Fig. 5.14: All the possible 3 loop, 6 node, and 8 branch graphs (Continued).

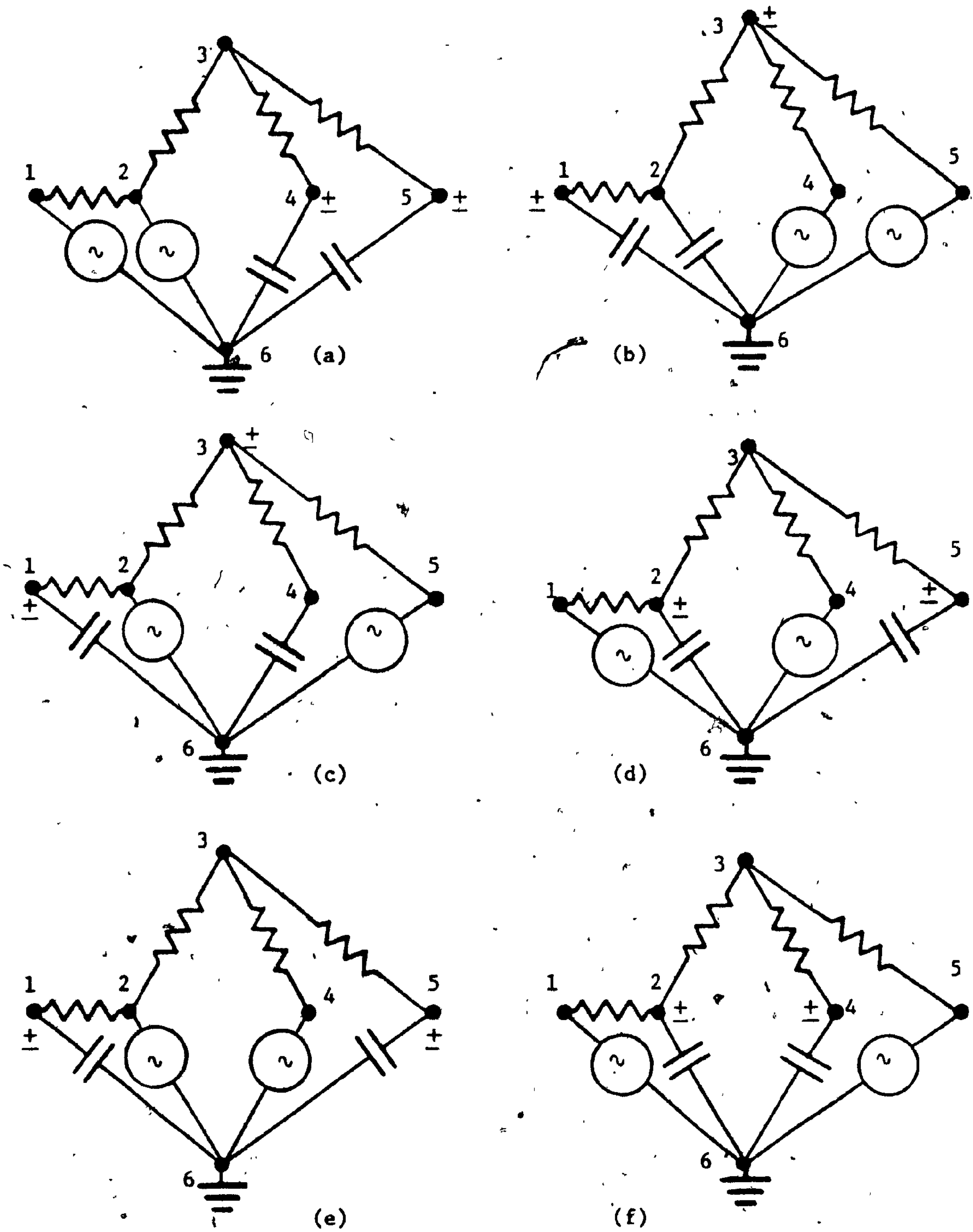


Fig. 5.15: Distribution of elements for Fig.5.14(e), with grounded capacitors and controlled sources.

The graph of Fig.5.15(a) in which one branch is between the outputs of CSs is disregarded.

Choosing proper nodes for the inputs of DOOA, a total of 7 cases are to be examined for the remaining five graphs of Fig.5.15. However, none of these graphs lead to a GCRCO.

(ii) Graph (f)

Only node 6 can be the ground node. There are 6 ways that CSs and capacitive branches can be distributed. However, five of the resulting graphs can be disregarded for different reasons. Only one (Fig.5.16) needs further study. In the final analysis, this graph is also disregarded and no oscillator circuit is found.

(iii) Graph (g)

Nodes 3 and 6 both qualify for being the grounded node. Because of symmetry only node 6 is taken as grounded node. In six ways capacitive branches and CSs are distributed over the graph. Resulting graphs are shown in Figs.5.17(a) to 5.17(f). In Fig.5.17 the graphs (c), and (d) are the same as graphs (d), and (f) respectively. Graphs (a), (b) and (c) do not generate RCO circuits. Further analysis shows that only graph (f) generates an oscillator circuit. The final circuit is shown in Fig.5.18(a). The CE of the circuit is:

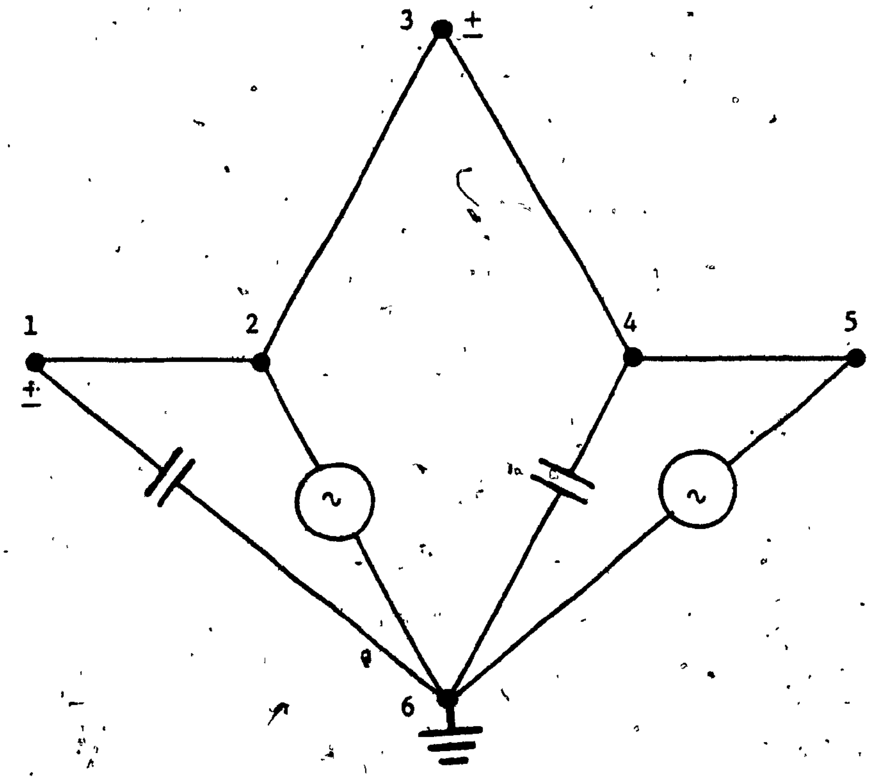


Fig. 5.16: Distribution of elements for Fig.5.14(f), with grounded capacitors and controlled sources.

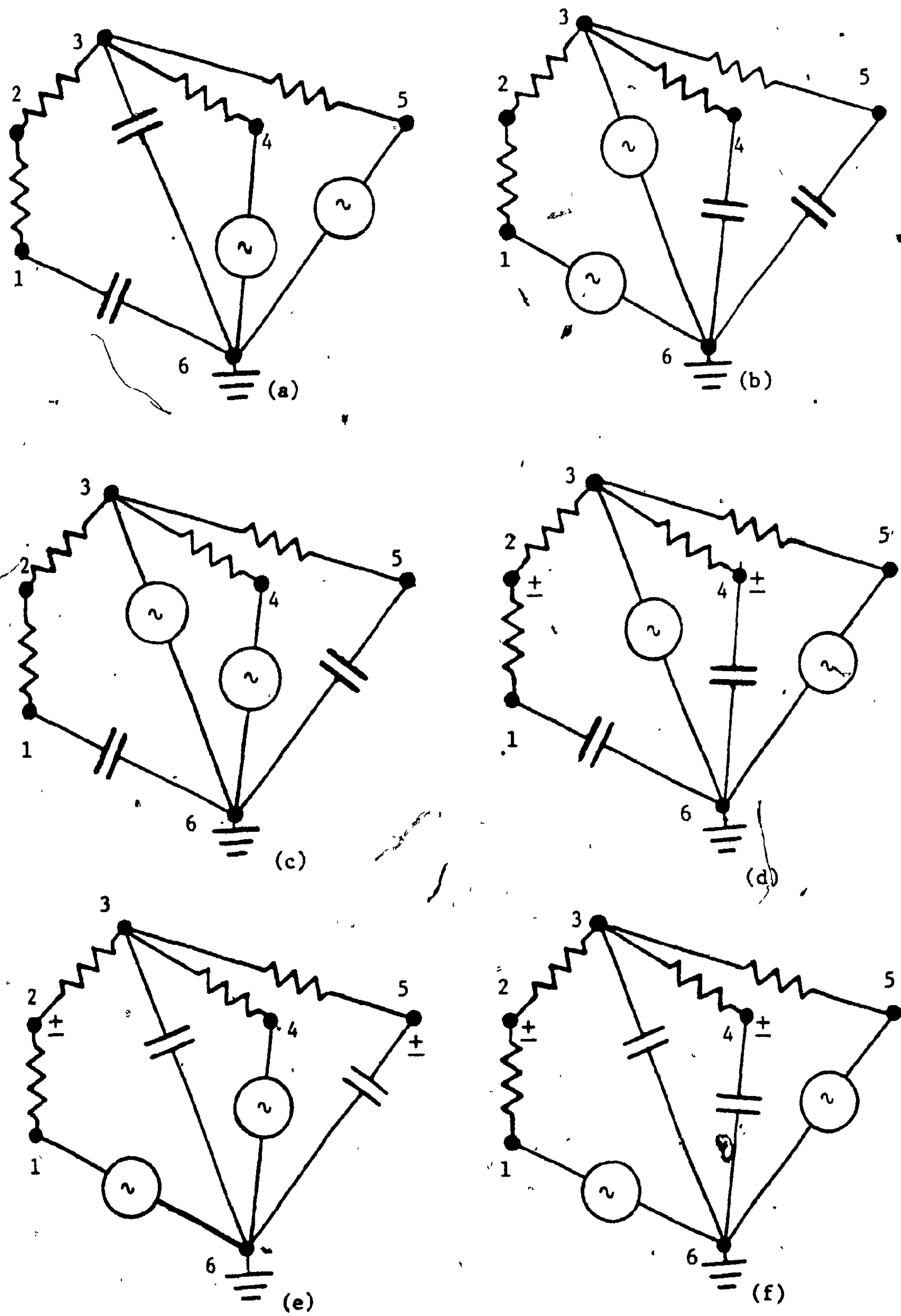


Fig. 5.17: Distribution of elements for Fig.5.14(g), with grounded capacitors and controlled sources.

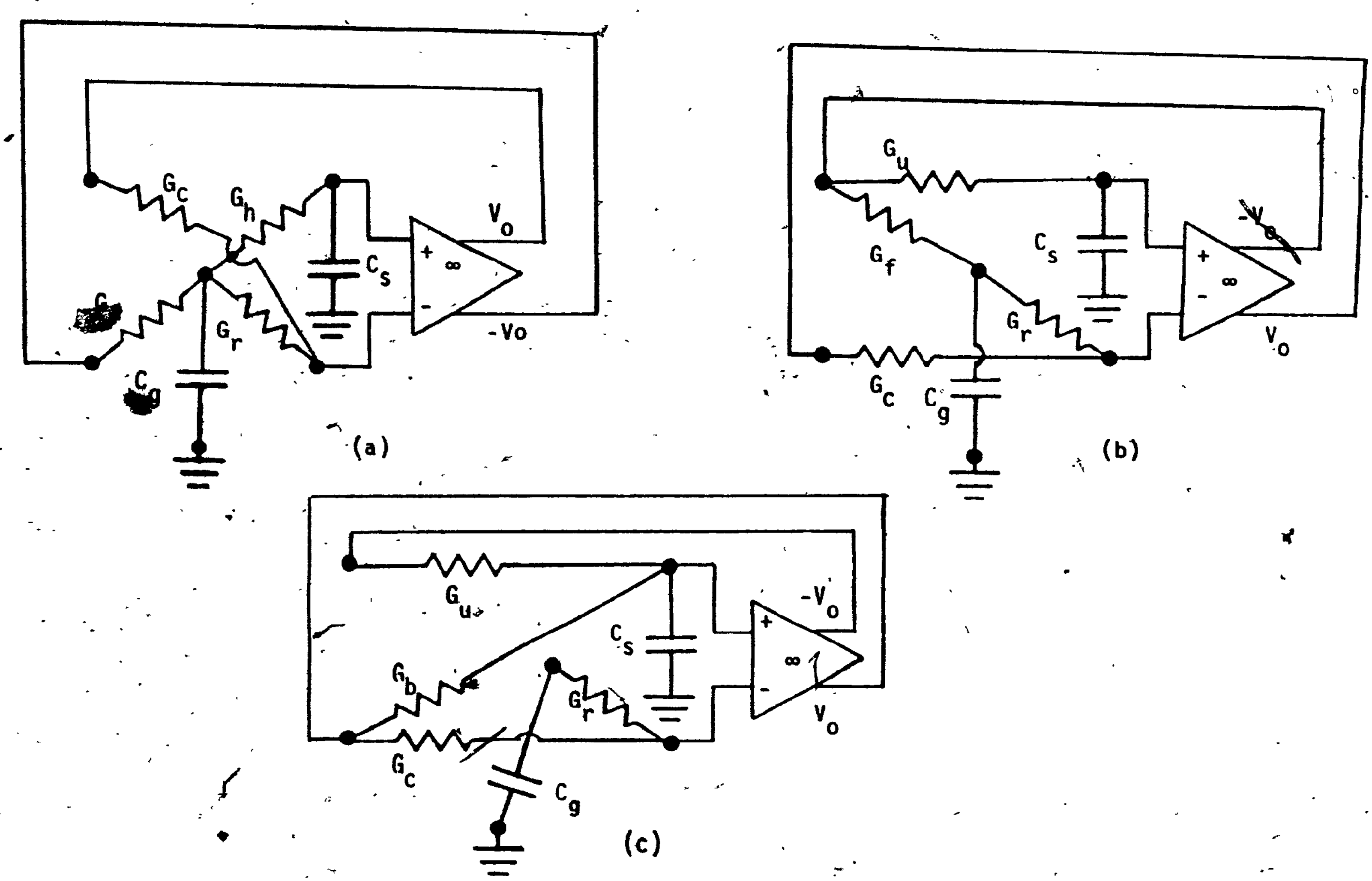


Fig. 5.18: The set of three canonic GCSFO circuits, using DOOA.

$$-S^2G_cC_gC_s+S[G_fG_rC_s-G_cG_fC_s-G_cG_hC_s$$

$$-G_cG_rC_s-G_cG_hC_g]-2G_fG_oG_h = 0 \qquad (5\text{-}22)$$

and:

OC: $$G_fG_rC_s-G_cG_fC_s-G_cG_hC_s-G_cG_rC_s$$

$$-G_cG_hC_g = 0 \qquad (5\text{-}23a)$$

OF: $$\omega_s = \{ \frac{2G_fG_h}{C_gC_s} \}^{1/2} \qquad (5\text{-}23b)$$

Since G_f and G_h are common in both OC and OF, the circuit is a GCSFO.

(iv) Graph (h)

Either of nodes 3 or 6 can be taken as the ground node. CSs and capacitive branches are distributed over the graph in six ways. However, because of symmetry they are all the same as Fig.5.19. Detailed analysis of Fig.5.19 shows that no oscillator circuit can be generated from the graph.

(v) Graph (i)

Only node 6 is qualified to be the ground node. Distributing CSs and capacitive branches generates the 6 graphs of Figs.5.20(a) to 5.20(f). Analysis of these 6 graphs show that only Figs.5.20(b) and 5.20(c) lead to oscillator circuits. The final circuits corresponding to these two graphs are shown in Figs.5.18(b) and 5.18(c), respectively. The CE of the circuit of Fig.5.18(b) is:

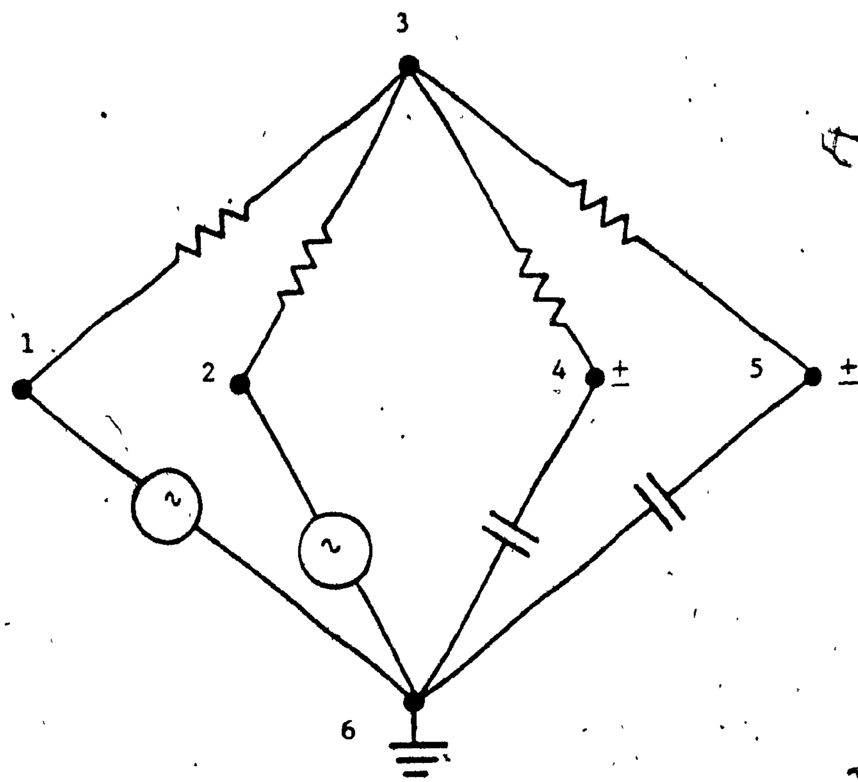


Fig. 5.19: Distribution of elements for Fig.5.14(h), with grounded capacitors and conlrolled sources.

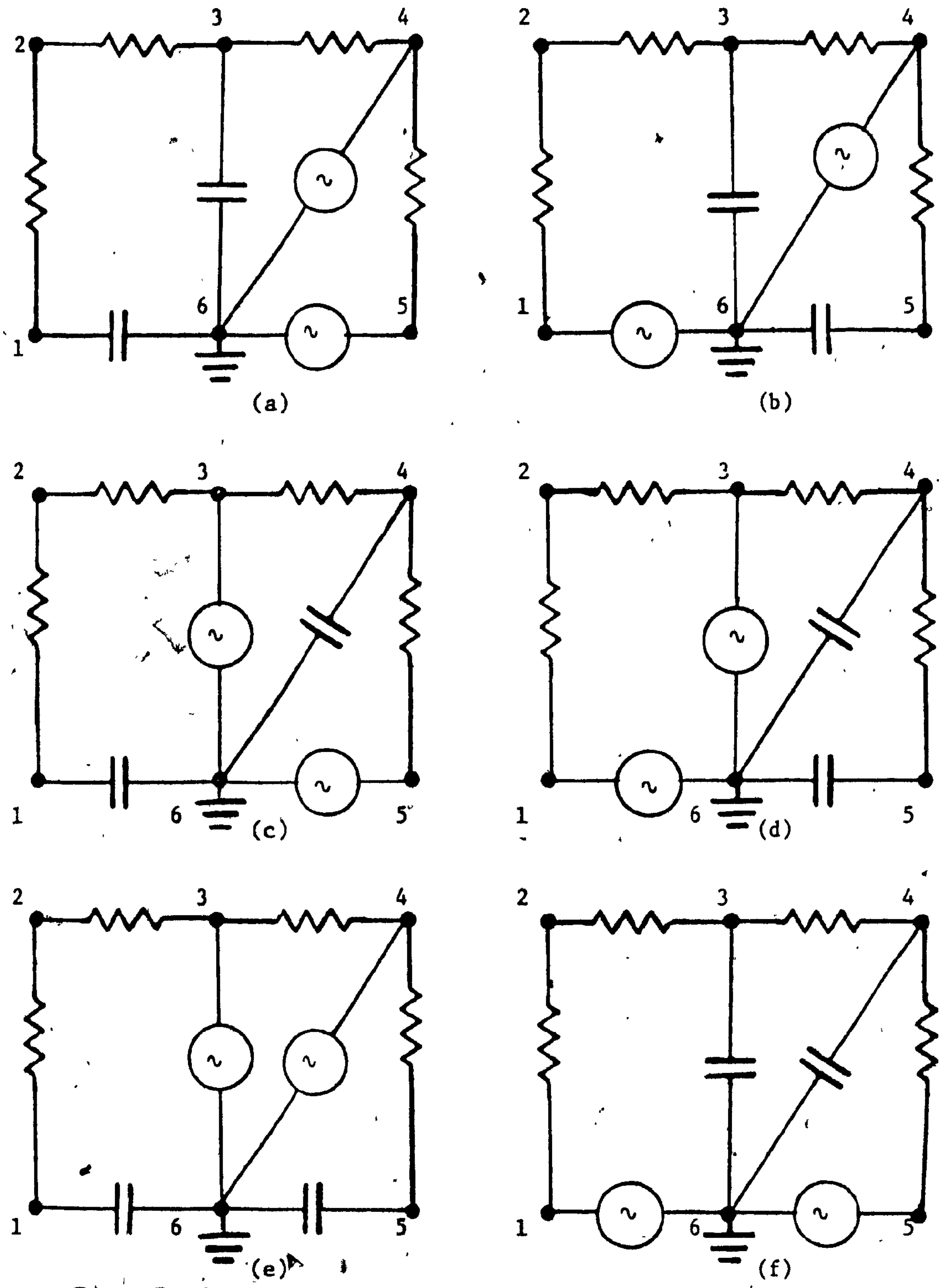


Fig. 5.20: Distribution of elements for Fig.5.14(i), with grounded capacitors and conlrolled sources.

$$-S^2G_cC_gC_s+S[G_fG_rC_s-G_uG_cC_g -G_uG_rC_g-G_cG_fC_s$$

$$-G_cG_rC_s-G_uG_cC_g]-2G_uG_cG_f -2G_uG_cG_r=0 \quad (5\text{-}24)$$

and:

$$\text{OC:} \quad G_fG_rC_s-G_uG_cC_g-G_uG_rC_g$$

$$-G_cG_fC_s-G_cG_rC_s-G_uG_cC_g = 0 \quad (5\text{-}25a)$$

$$\text{OF} \quad \omega_s=\{ \frac{2G_u(G_f+G_r)}{C_gC_s} \}^{1/2} \quad (5\text{-}25b)$$

Since G_u, G_f and G_r are common between OC and OF, the circuit is a GCSFO.

The CE of the circuit of Fig.5.18(c) is:

$$-S^2G_cC_gC_s+S[G_bG_rC_g -G_uG_cC_g-G_uG_rC_g$$

$$-G_rG_cC_s-G_uG_cC_g]-G_uG_cG_r = 0 \quad (5\text{-}26)$$

$$\text{OC:} \quad G_bG_rC_g-G_rG_cC_s-G_uG_cC_g-G_uG_rC_g-G_uG_cC_g = 0 \quad (5\text{-}27a)$$

$$\text{OF:} \quad \omega_s=\{ \frac{2G_uG_r}{C_gC_s} \}^{1/2} \quad (5\text{-}27b)$$

This circuit is also a GCSFO.

5.4.3 Seven-Node Network

If the network contains 7 nodes, then $l=b-7+1\geq3$ or $b\geq9$. Since two branches are CSs, minimum of seven passive branches remain. Even assuming a minimum of one element per branch, seven passive components are required to construct the circuit. It was already shown that a canonic GCSFO contains six passive elements, consequently circuits with seven elements are non-canonic. Therefore, the study of a seven node network is not warranted.

5.5 Realization of GCVFOs Using DOOAs

In previous sections, all the canonic GCSFOs have been identified. Since the canonic GCSFOs require 6 passive elements, clearly a GCVFO must consist of at least 7 passive elements. Starting with 7 passive elements and one DOOA, all the 5,6,7.... node networks must be examined.

5.5.1 Five-Node Network

It was shown in section 5.5.1 that a five node network can not generate a GCSFO. Consequently, it can not generate a GCVFO either. Therefore, this case is disregarded.

5.5.2 Six-Node Network

It was established in section 5.4.2 that the graph of a GCRCO with 6 nodes contains at least 6 passive branches. Therefore, the seven elements can be distributed over 6 branches in two ways.

(i) The seven passive elements can be distributed over 6 branches, in which case one branch is of the form G_1+SC_1 and others have one element each. There are total of 8 branches in the circuit, of which two are CSs. From section 5.4.2, with 8 branches and 6 nodes, a total of 9 three loop graphs can be generated. Out of these 9 graphs 4 can be disregarded. The remaining 5 graphs generate a total of 14 potential GCSFOs.

Taking these 14 configurations and adding one resistor in parallel with one of the capacitors at a time, 28 cases are to be examined. Analysis of these 28 cases leads to one circuit. This circuit is the result of connecting one resistor in parallel with the capacitor between nodes 1 and 6 of Fig.5.16. The final circuit is shown in Fig.5.21(b). The CE of the circuit is:

$$-S^2G_cC_gC_s+S[G_f(G_rC_s-G_cC_s)+G_bG_rG_g-G_sG_cC_g$$
$$-G_rG_cC_s]+G_fG_rG_s+2G_fG_rG_b-G_sG_cG_f-G_rG_cG_s=0 \quad (5\text{-}28)$$

and, OC: $G_f(G_rC_s-G_cC_s)+G_bG_rG_g-G_sG_cC_g$

$$-G_rG_cC_s=0 \qquad (5\text{-}29a)$$

$$\text{OF:}\quad \omega_s=\left\{\frac{G_fG_rG_s+2G_fG_rG_b-G_sG_cG_f-G_rG_cG_s}{-G_cC_gC_s}\right\}^{1/2} \qquad (5\text{-}29b)$$

Since the coefficient of G_f in OC can be set to zero and OF is function of G_f, the circuit is a GCVFO and G_f is the variable element.

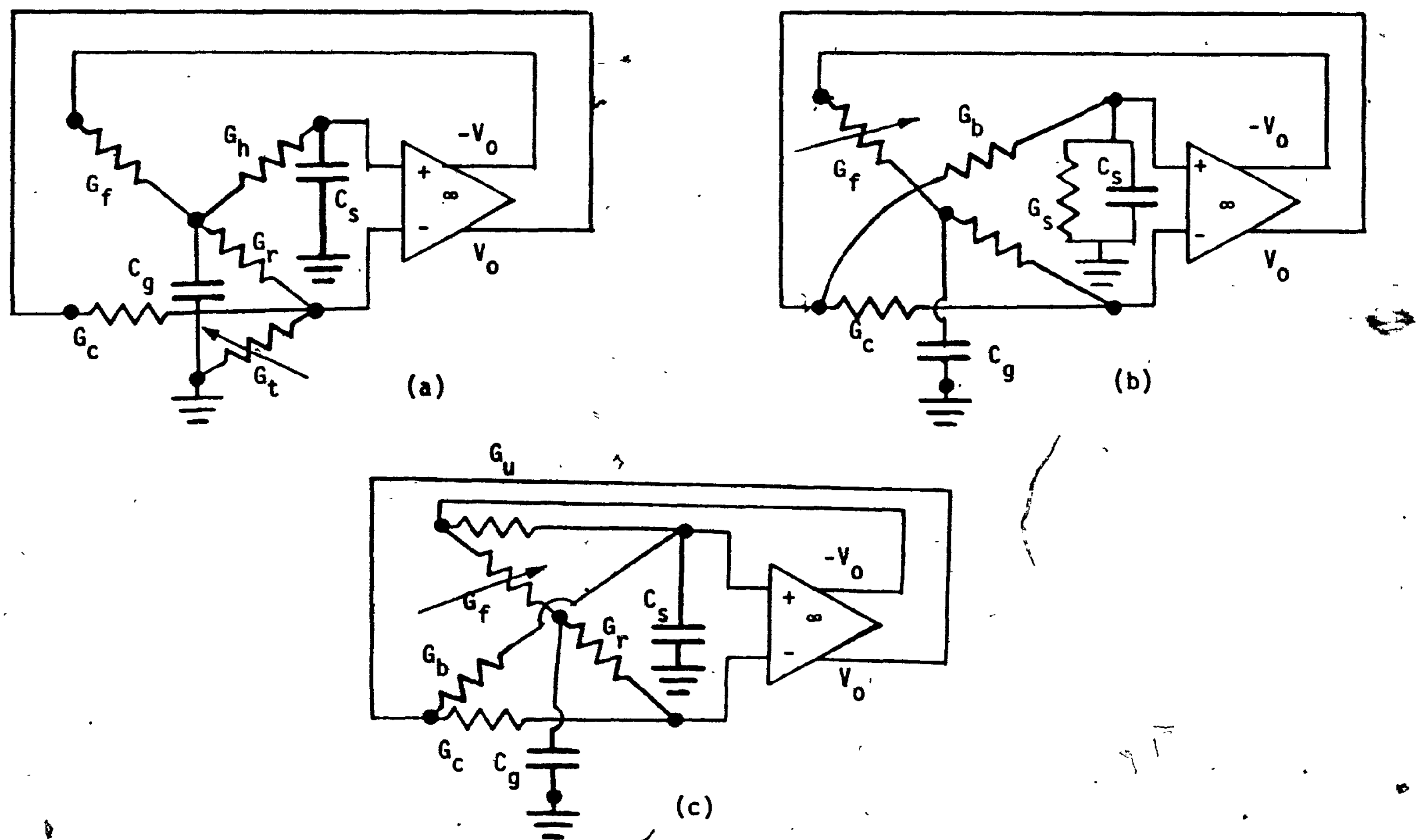


Fig. 5.21: The set of 3 canonic GCVFOs, using DOOA.

(ii) The seven elements can be distributed over seven branches, that is, one element per branch. Taking the CSs into account there are total of 9 branches in the network.

Given 6 nodes and 9 branches the number of loops in the graph is:

$$l=9-6+1=4$$

There are total of 14 different graphs that can be generated with 4 loops, 6 nodes and 9 branches. These graphs are shown in Figs.5.22(a) to 5.22(n). The graph of Fig.5.22(a) is a non-connected graph and hence it is disregarded. Graphs of Figs.5.22(b) and 5.22(c) do not have four branches connected to one node, and therefore, they are ignored.

The remaining 11 graphs generate 60 potential cases that need to be investigated. Out of these 60 cases, however, two lead to GCVFO circuits. The first circuit is obtained from graph of Fig.5.22(d) and the second circuit is obtained from that of 5.22(e). They are shown in Figs.5.23(a) and 5.23(b) rspectively. The final circuit for graph of Fig.5.23(a) is shown in Fig.5.21(a). The CE of the circuit is:

$$-S^2 G_c C_g C_s + S[G_f G_r C_s - G_c G_f C_s - G_c G_h C_s - G_c G_r C_s - G_c G_h C_g] - G_f G_h G_t - 2G_f G_h G_c = 0 \qquad (5\text{-}30)$$

and:

$$OC: \quad G_f G_r C_s - G_c G_f C_s - G_c G_h C_s - G_c G_r C_s - G_c G_h C_g = 0 \qquad (5\text{-}31a)$$

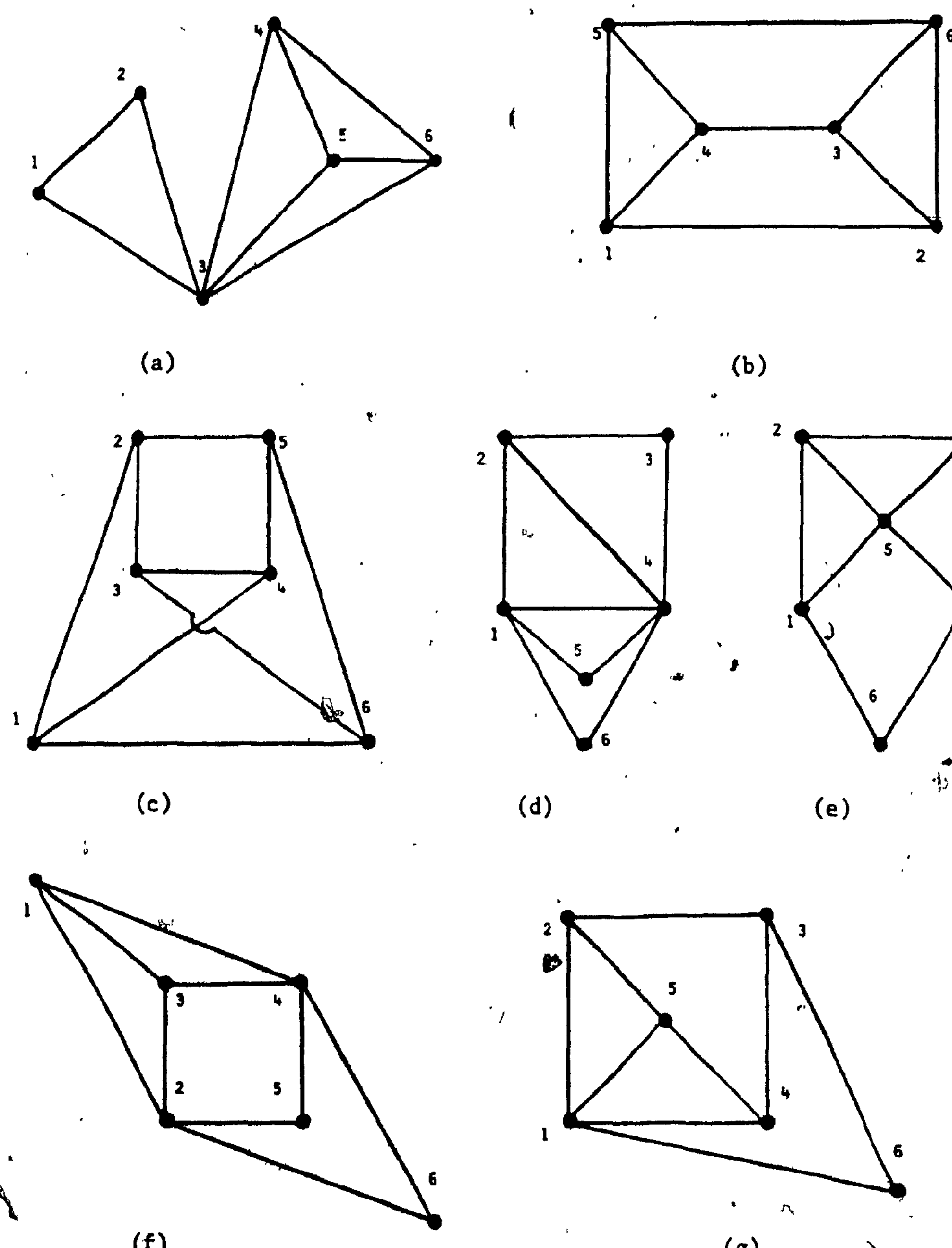


Fig. 5.22: All the possible graphs of 6 node, 4 loop, and 9 branch networks (Continued).

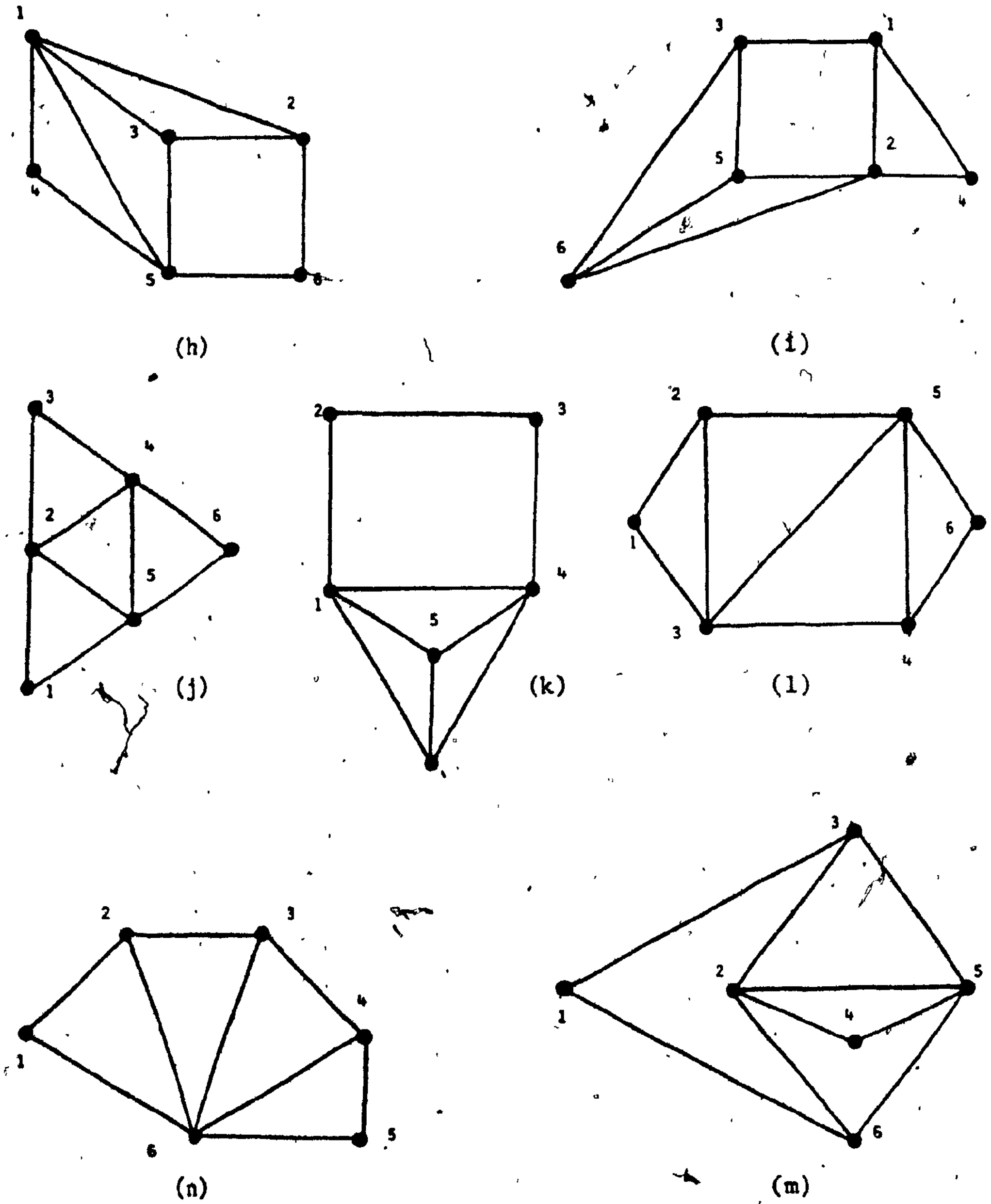
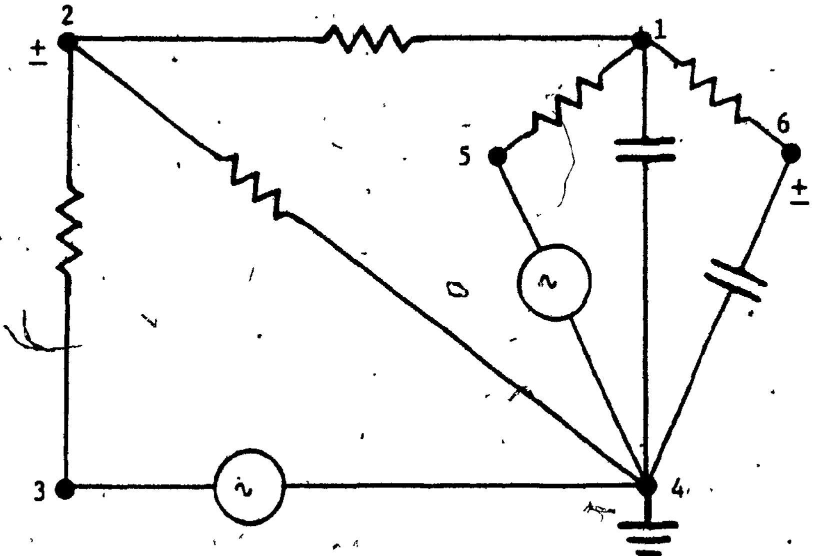
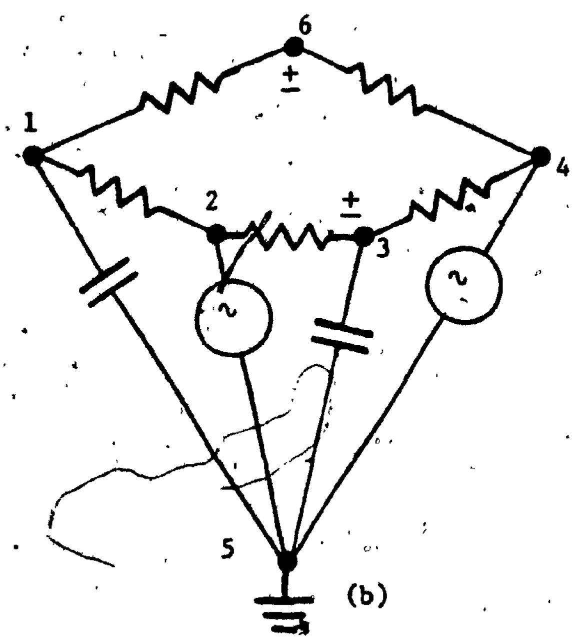


Fig. 5.22: All the possible graphs of 6 node, 4 loop, and 9 branch networks (Continued).



(a)



(b)

Fig. 5.23: Distribution of elements over a 4 loop, 6 node, 9 branch graph, leading to an oscillator.

OF: $$\omega_s = \left\{ \frac{G_f G_h G_t + 2G_f G_h G_c}{G_c C_g C_s} \right\}^{1/2} \quad (5\text{-}31b)$$

Since OC is independent of G_t and OF is dependent on G_t, therefore, G_t is the variable element and the circuit is a GCVFO.

The final circuit for graph of Fig.5.23(b) is shown in Fig.5.21(c). The CE of the circuit is:

$$-S^2 G_c C_g C_s + S[G_f(G_r C_s - G_c C_s) + G_b G_r C_g - G_u G_c C_g$$
$$-G_u G_r C_g - G_c G_r C_s - G_u G_c C_g] + 2G_f G_r G_b - 2G_u G_c G_f$$
$$-2G_u G_r G_c = 0 \quad (5\text{-}32)$$

OC: $$G_f(G_r C_s - G_c C_s) + G_b G_r C_g - G_u G_c C_g$$
$$-G_u G_r C_g - G_c G_r C_s - G_u G_c C_g = 0 \quad (5\text{-}33a)$$

OF: $$\omega_s = \left\{ \frac{2G_u G_c G_f + 2G_u G_c G_r - 2G_f G_r G_b}{G_c C_g C_s} \right\}^{1/2} \quad (5\text{-}33b)$$

The coefficient of G_f in OC can be set to zero and OF is function of G_f, therefore, the circuit is a GCVFO and G_f is the variable element.

5.5.3 Seven-Node Network

In a seven node network, in order to have 9 branches, there must be 3 loops. There are a total of 16 three loop graphs with 7 nodes and 9 branches as they are shown in Figs.5.24(a) to 5.24(p). Graphs of Figs.5.24(a) to 5.24(e) do not have a qualified ground node and are ignored. From the remaining 11 graphs a total of 36 potential GCRCOs are generated. However, further analysis of these 36 cases show that no GCVFO circuit can be generated.

5.5.4 Eight-Node Network

In an eight node, three loop network b=3+8-1=10 and two of the branches are CSs. Therefore, at least 8 passive branches are required. For 8 passive branches at least 8 elements are necessary, making it a non-canonic GCRCO. Therefore, the study of 8 and higher node networks is not required.

5.6 Design Procedures

Design procedures are given here for one of the SFO and one of the VFO circuits only. Similar steps can be taken for deriving the design equations for all the other circuits.

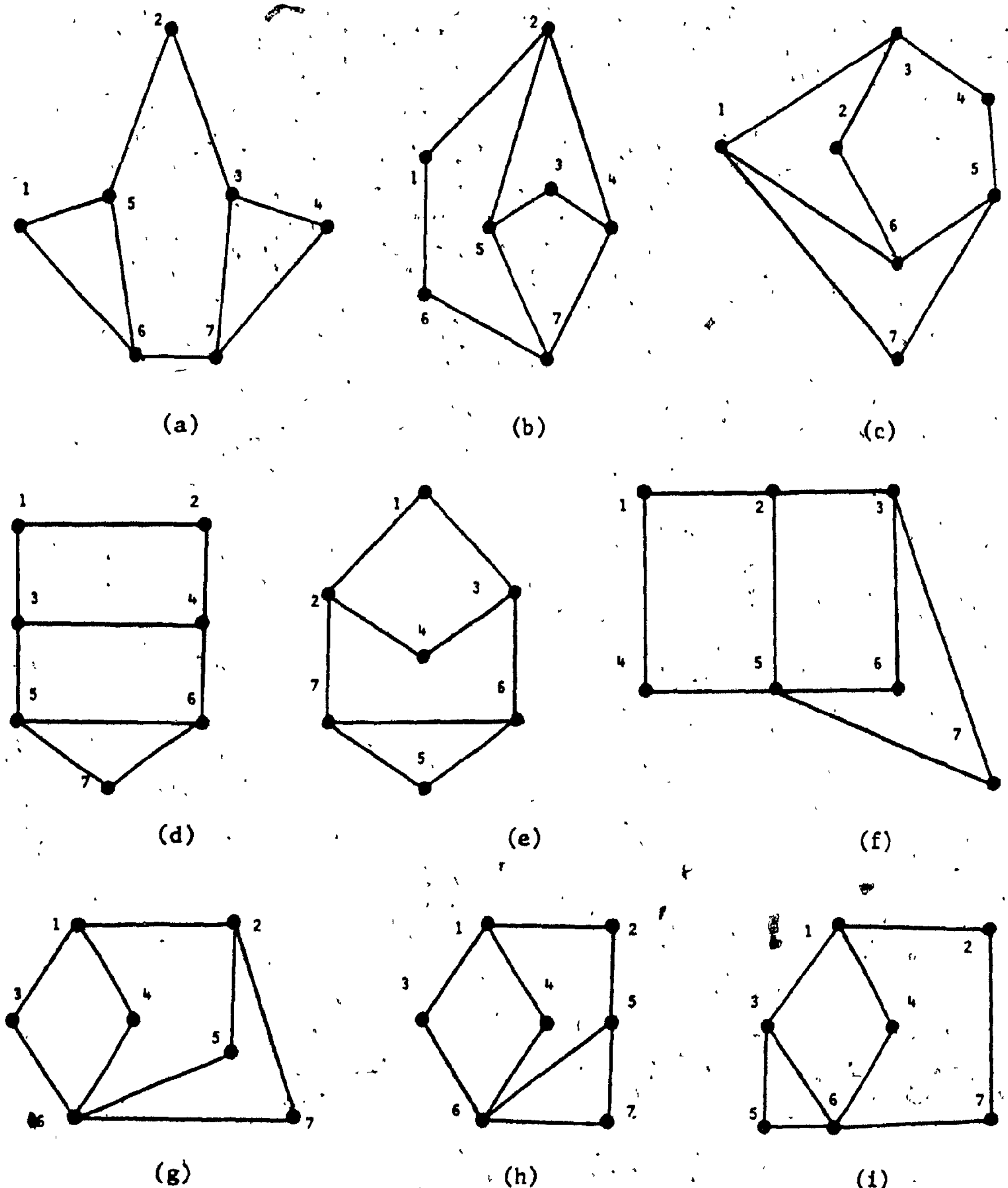


Fig. 5.24: All the possible graphs of a 6 node, 9 branch, 3 loop network (Continued).

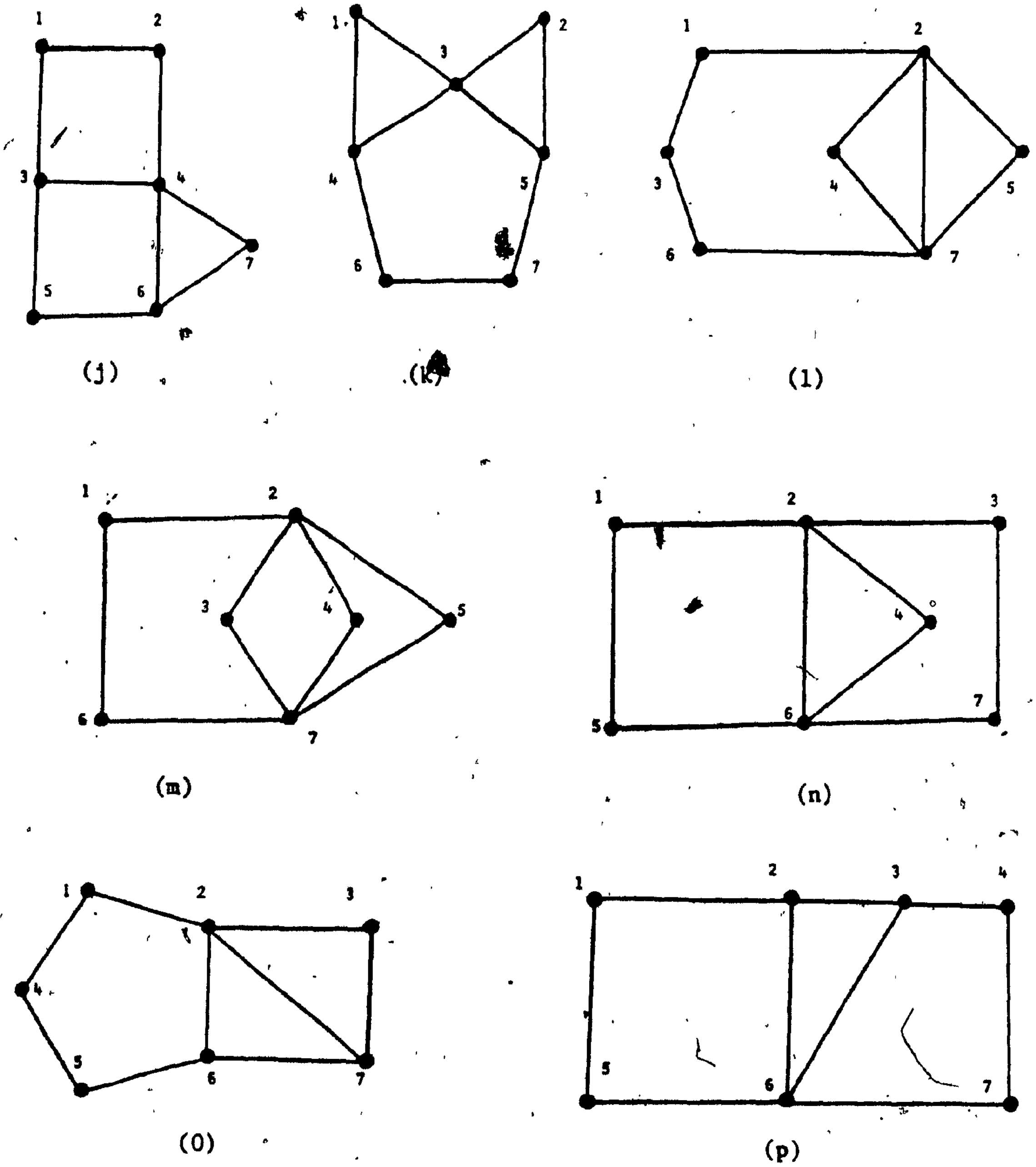


Fig. 5.24: All the possible graphs of a 6 node, 9 branch, 3 loop network (Continued).

Take the circuit of Fig.5.4(b) for example. The CE of this circuit is given by (5-16). Let $C_f=C_h$, then from (5-17a) the OC is:

$$G_r=2G_c \qquad (5\text{-}34)$$

From (5-17b), then the OF is:

$$\omega_s=\frac{(G_sG_c)^{1/2}}{C} \qquad (5\text{-}35)$$

Similarly, take the circuit of Fig.5.5(b). The CE of this circuit is given by (5-18). Let $C_r=C_u=C$ then the OC from (5-19a) is:

$$G_fG_s-2G_cG_f-G_cG_s = 0 \qquad (5\text{-}36)$$

One solution of (5-36) is $G_s=G_f=3G_c=G$. Then substitution of these results in (5-19b) gives OF as:

$$\omega_s=\frac{G}{C}.2^{-1/2} \qquad (5\text{-}37)$$

The design equations for all the circuits are given in Table 5-I.

5.7 Experimental Results

All the 10 circuits found in this chapter have been built and tested. The experimental results show that the circuits behave according to their theoretical expectations. As examples, only the results from circuits of Figs.5.4(b) and 5.5(b) are presented here. The design values for

Table 5-I : Design equations for all the circuits of chapter V.

Fig.5.4(a)	$C_c=C_s=C$, $G_w=2G_u$, $\omega_s=\frac{\sqrt{G_uG_t}}{C}$
Fig.5.4(b)	$C_f=C_h=C$, $G_r=2G_c$, $\omega_s=\frac{\sqrt{G_sG_c}}{C}$
Fig.5.5(a)	$C_s=C_c=C$, $G_w=G_u+2G_c$, $\omega_s=\frac{\sqrt{2G_cG_u}}{C}$
Fig.5.5(b)	$C_u=C_r=C$, $G_f=G_s=3G_c=G$, $\omega_s=2^{-1/2}\ \frac{G}{C}$
Fig.5.18(a)	$C_g=C_s=C$, $G_r=2G_c$, $G_f=2G_h+G_r$, $\omega_s=\frac{\sqrt{2G_fG_h}}{C}$
Fig.5.18(b)	$C_g=C_s=C$, $G_u=G_c=G$, $G_f=3G$, $G_r=5G$, $\omega_s=4\cdot\frac{G}{C}$
Fig.5.18(c)	$C_g=C_s=C$, $G_r=G_u=G_c=G$, $G_b=4G$, $\omega_s=\sqrt{2}\cdot\frac{G}{C}$
Fig.5.21(a)	$C_g=C_s=C$, $G=G_f=2G_c=2G_h=G_r/2$, $\omega_s=\frac{\sqrt{G(G_t+G)}}{C}$
Fig.5.21(b)	$C_g=C_s=C$, $G=G_r=G_c=G_s=G_b/2$, $\omega_s=\frac{\sqrt{G(G-4G_f)}}{C}$ $G_f<\frac{G}{4}$
Fig.5.21(c)	$C_g=C_s=C$, $G=G_r=G_c=G_u=G_b/4$, $G_f<\frac{G}{3}$ $\omega_s=\frac{\sqrt{G(2G-6G_f)}}{C}$

Components of the two circuits are:

Fig.5.4(b): $C_f=C_h=1\ \mu F$, $R_r=1\ k\Omega$, $R_c=2\ k\Omega$, R_s=variable.

Fig.5.5(b): $C_f=C_u=0.1\mu F$, $R_r=1\ k\Omega$, $R_c=3\ k\Omega$, $R_s=1\ k\Omega$.

All the capacitors are polystyrene of tolerance 1% (measured). All the resistors are carbon film of tolerance 1%. Figure 5.25 shows the variation of frequency and distortion versus the variable resistor for the circuit of Fig.5.4(b). Figure 5.26 shows the output signal of the circuit of Fig.5.5(b) at f=796Hz. The DOOA was not available during the time of experimental verifications and it was simulated as it is shown in Fig.5.27. In all the experiments a $\pm$15 volts power supply has been used. The output is taken from between the output of the OA (Fairchiled 741) and the ground node.

5.8 Summary

A systematic approach for the realization of canonic RCOs using DOOAs has been presented. Using this appraoch all the canonic DOOA based RCOs have been identified. For single frequency operations, a set of RCO circuits has been found. This set consists of two canonic circuits. Each using 2 capacitors and 3 resistors. Also for variable frequency operations, a set of two canonic VFOs has been derived. Each circuit in this set contains 2 capacitors and

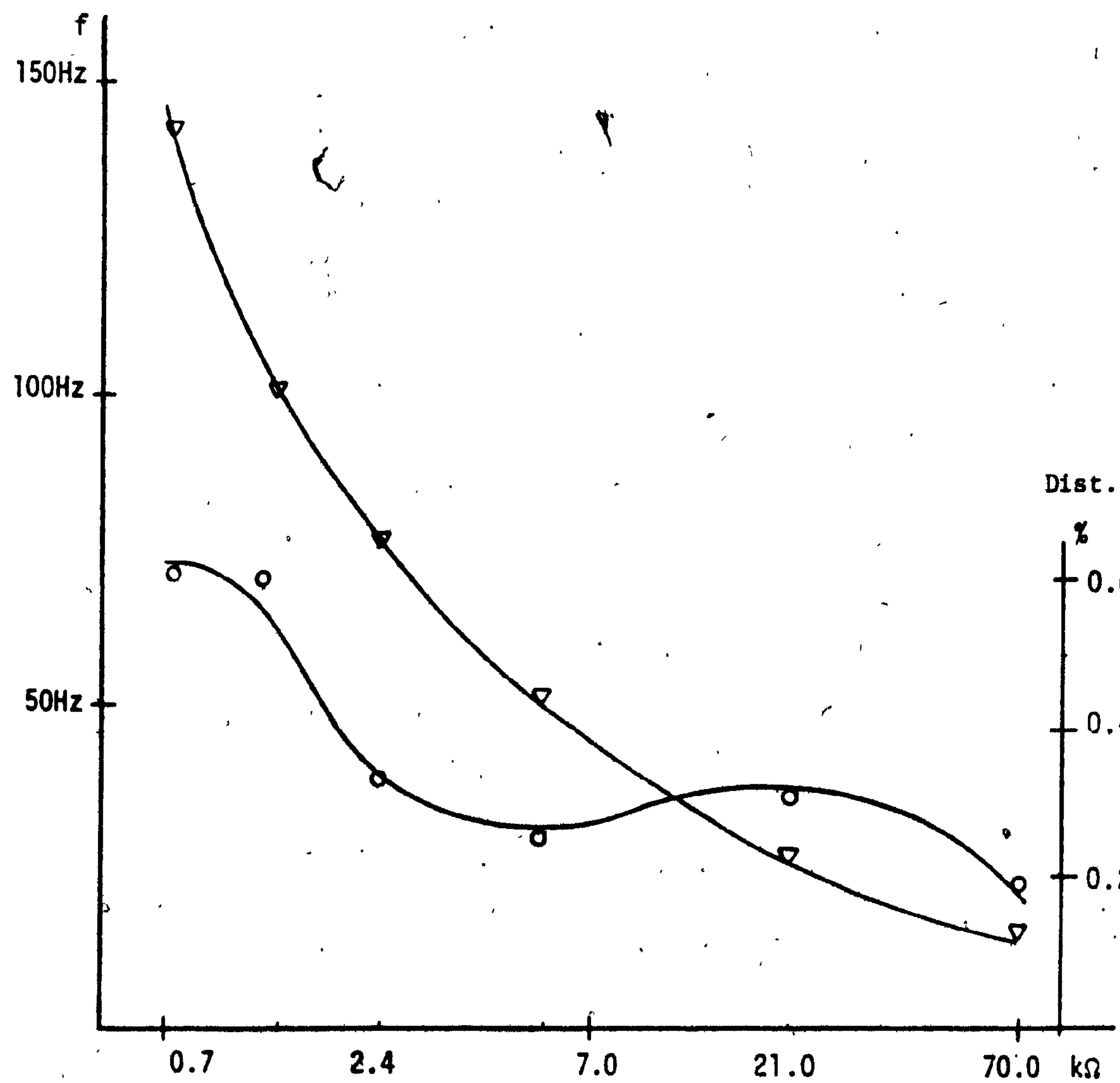


Fig. 5.25: Variation of frequency and distortion versus variable resistor for the circuit of Fig.5.4(b). ▽:Frequency, O:Distortion.

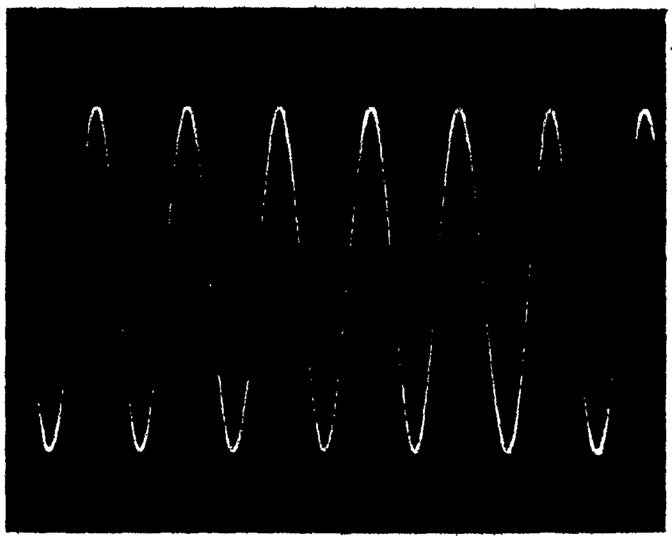


Fig. 5.26: Output signal of the circuit of Fig.5.4(b) at f=796Hz.

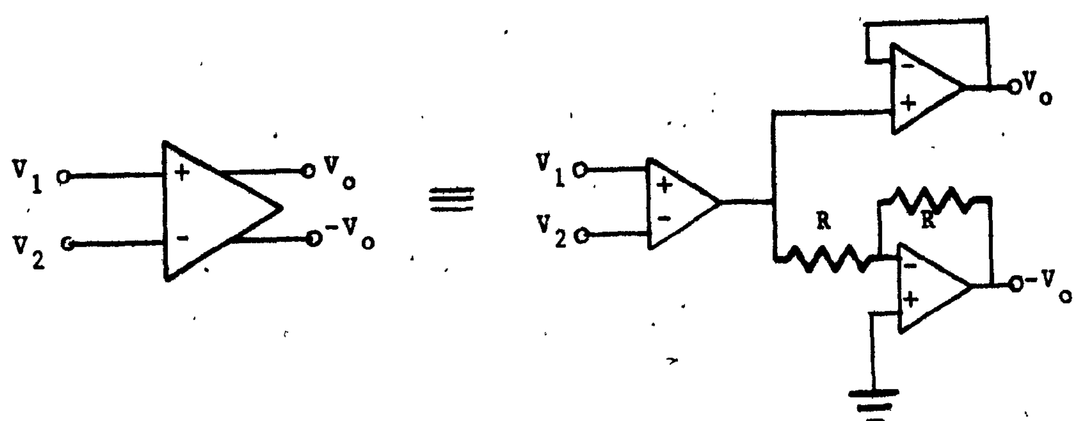


Fig.5.27 : Equivalet circuit for DOOA using OAs.

3 resistors. It is shown that both the sets are complete.

For grounded capacitor circuits, two sets of RCOs are obtained. The first set belongs to the SFO class and the second set is of the VFO type. There are three circuits in each set. The GCSFO circuits each use 2 capacitors and 4 resistors while the GCVFOs use 2 capacitors and 5 resistors each. All the VFOs and GCVFOs are single resistor frequency controlled. Both the sets are canonic and complete. Both the VFOs and one of the GCVFOs have their variable resistors grounded. All the circuits allow equal capacitor designs. All the circuits are reported for the first time.

CHAPTER VI

CONCLUSION

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CONCLUSION

In this thesis, a systematic study has been undertaken for generation, classification and design of OA based sinusoidal RC oscillators. Both single frequency and variable frequency operations using single ended as well as differential output OAs have been considered.

In chapter 2 the theory of generating canonic RCOs based on OAs is developed. The theory is new, in that, the OA is not used for implementation of controlled sources, it is rather used directly as the active part of the system. In this way it is possible to generate circuits that can not be recognized otherwise. Using this theory a set of 12 canonic SFOs are generated. Each canonic circuit requires 2 capacitors, 4 resistors and a single OA. Three of this SFOs are new circuits, one of them has all of its capacitors grounded (an attractive feature for IC fabriction). Eventhough only 3 of these circuits are new, but two important facts are established in the chapter. First, it is established for the first time that the minimum number of passive components for a canonic OA based SFO is 6 (2 capacitors and 4 resistors). Second, it is shown for the first time that, there exist only 12 canonic SFOs of this type. Comparing this set with the set of SFO circuits reported in [8], it is clear that, even though the set in [8] is a complete set, the 3 new circuits are not identified in [8]. This is because of the fact that in [8], controlled sources are used as active elements; as a result, the above three circuits are not recognized. Finally, in this chapter the experimental results show that all the circuits perform according to their expected behaviour.

Chapter 3 introduces the thoery of the VFOs. A systematic approach for the identification of VFOs is developed. The approach is unified, general and complete. It is established for the first time that the minimum number of passive components required for generating a VFO is 7, five resistors, two cpacitors. It is also shown for the first time that the maximum number of canonic VFOs that can be generated using the above components is 16. Out of the 16 VFOs, eight are completely new. In all of the VFOs found, the frequency is varied by a single variable resistor. Three of them have their variable resistor grounded (attractive feature for VCO operation). The VFOs are classified into four groups according to their nature of dependence of the VFO frequency on the variable resistor. Each group consists of four circuits.

In chapter 4 a unique new approach is developed for realization of GCVFOs. The approach is systematic, unified and complete. It yields a set of eight canonic GCVFO circuits, each using one OA, 2 capacitors and 6 resistors.

It is shown that the set is complete. All the circuits in this set are completely new. Two of these circuits have equal valued elements (all the capacitors are equal and all the resistors are equal). In this chapter a new problem is identified for VFOs that have difficulty in starting the operation. A proper solution to this problem is developed using which another set of 12 canonic easily tunable GCVFOs is found. This set is also shown to be complete. Each consists of one OA, 2 capacitors and 7 resitors. All the circuits in this set are completely new. One of the circuits in this set has equal valued elements. In all of the 20 GCVFOs the frequency of operation is varied by a single variable resistor. In each set, one circuit has its variable resistor grounded. All the circuits in this chapter can be designed with equal valued cpacitors. All the circuits are built and tested. The results show that the circuits perform closely to their theoretically expected behaviour.

In chapter 5 a new device, namely, differential output OA is used as the active part of the network. This is the first time that such a device is used for generating RCOs. In this chapter it is shown that by using the DOOAs as active part of the RCOs, in all but one type of RCOs a reduction of one or two resistors is possible. It is only in the case of GCSFOs that no reduction in element count is achieved. Systematically, it is established that a canonic

DOOA based RCO requires at least 2 capacitors and 3 resistors.

A set of two SFO circuits is derived, each requiring 2 capacitors , 3 resistors and one DOOA. Comparing this set with the SFOs in chapter 2, a reduction of one element is obtained.

A set of two VFO circuits is found. It is shown that the set is complete. Each VFO requires 2 capacitors, 3 resistors and one DOOA. The circuits of this set have two resistors less than those found in chapter 3.

A set of 3 GCSFO circuits is developed. This set is also complete. The minimum number of passive components in this set is 6 (2 capacitors and 4 resistors).

Finally, a set of 3 GCVFO circuits is obtained. This set is also proven to be complete, each consisting of 2 capacitors , 5 resistors and one DOOA. Comparison of this set with their counterparts in chapter 4 shows that a reduction of one resistor is acheived.

In all of the VFOs and GCVFOs derived in this chapter the frequency is varied by a single variable resistor. Both the VFOs and one of the GCVFOs have their variable resistors grounded. All the circuits of this chapter allow equal capacitor designs. All the circuits have been built and tested, the experimental results show that they perform as expected from the theory.

6.1 Possible Direction of Future Research

A possible direction for future research is to study in detail all the different properties of all the circuits reported in this thesis. Such a study will lead to identification of best circuit for each property. In the thesis, the operational amplifiers were assumed to be ideal. It would be worth while to look into the effects of the internal pole/s of the amplifiers on the performance of these oscillators and possibly develop circuits with better capabilities.

If one is interested in designing oscillators with less number of passive components (of course, at the price of active elements), one could undertake the investigation of RC-oscillators with two operational amplifiers.

It is hoped that the methods reported in this thesis will be useful in the development of other RC-circuits.

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APPENDICES

APPENDIX A

Consider Fig.2.2(a) and assume that N contains, apart from the input and ground nodes, the set of nodes A ={a_1, a_2, a_3, ... a_t}. The total number of nodes in N is then n=t+2. If all the possible branch admittances are present, (n-1) branches will be incident with each node. The CE of the network is given by Eqn.(2-7). Clearly, any number of branch admittances can be set to zero and the CE of the resulting circuit can be derived from (2-7), provided that no node has been eliminated in the process.

Let the set of nodes A be divided into two subsets, B and C. Such that B={b_1, b_2, b_3, ... b_i} and C={c_1, c_2, c_3, ... c_j}. Also, let each group contain one and only one of the input nodes of the OA. Then t=i+j. In general, we have interconnecting branches between the two sets of nodes. Let all the branch admittances connecting the two sets be set to zero. Then each node in set B is incident with n-1-j=i+1 branch admittances and each node in set C is incident with n-1-i=j+1 branches. Clearly, the resulting network is of the form shown in Fig.2.2(b). The CE of this circuit can be found from (2-7), since no node has been eliminated in the process.

APPENDIX B

To show that every node in the network of Fig.2.2(a) should be incident with at least two branch admittances, we proceed in the following way.

Any internal node to which at least two branches are not connected will not contribute to the CE of the network. Consequently, such a node can be ignored. As far as the nodes which correspond to the OA input terminals are concerned, if only one branch is connected to each of them, then that branch can be short-circuited in the network without affecting the operation in any way. Hence, only the input and ground nodes need to be examined. To examine them, consider Fig.B.1, where it is assumed that only one branch is connected to input as well as the ground node. We then have

$$\frac{V_1}{V_i} = \frac{N_1(S)}{D(S)}, \qquad \frac{V_2}{V_i} = \frac{N_2(S)}{D(S)} \tag{B-1}$$

and

$$V_i\left(\frac{N_1(S)-N_2(S)}{D(S)}\right) = \frac{V_o}{A_d} \tag{B-2}$$

Letting Z_{11} denote the input impedance at 1-1' with terminal 2-2' open circuited, we have,

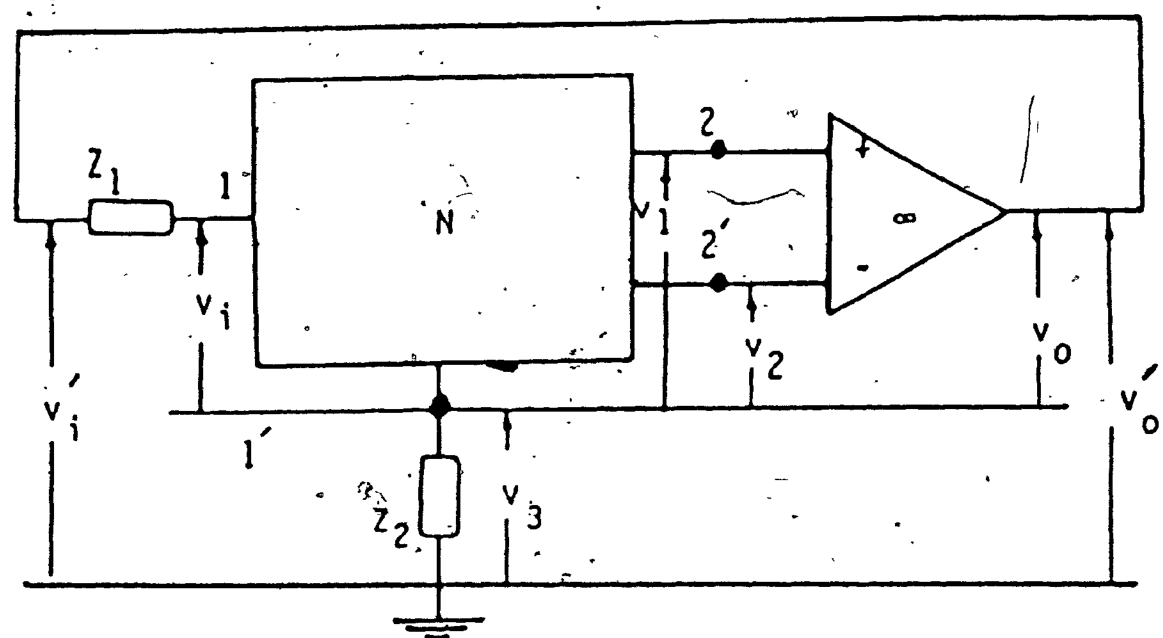

Fig. B.1: General configuration of an oscillator with one OA, assuming only one branch is connected to input node and only one branch is connected to ground node.

$$\frac{V_i' Z_{11}}{Z_1+Z_2+Z_{11}} = V_i \qquad \text{(B-3)}$$

Note that Z_{11} is an RC impedance. Also

$$V_o+V_3 = V_i' = V_o'. \qquad \text{(B-4)}$$

Substituting from (B-3) and (B-4) into (B-2)

$$\left(\frac{Z_{11}}{Z_1+Z_2+Z_{11}}\right)\left(\frac{N_1(S)-N_2(S)}{D(S)}\right) = \frac{1}{A_d}\cdot\left(\frac{V_i'-V_3}{V_i'}\right) \qquad \text{(B-5)}$$

Thus, as far as our interest in the roots on the $j\omega$ axis is concerned, the CE becomes

$$N_1(S) - N_2(S) = 0 \qquad \text{(B-6)}$$

This equation is the same as (2-7). Hence, Z_1 and Z_2 do not contribute to the operation of the circuit and can be easily short-circuited.

APPENDIX C

The circuits shown in Fig.C.1(a) and (b) are the same except that the input terminals of the OA in one are interchanged with respect to the other. Let us assume that N is designed such that, it produces oscillation if A_d goes to infinity. Analysis of Fig.C.1(a) yields

$$A_d[N_1(S)-N_2(S)]-D(S)= 0 \qquad (C-1)$$

where D(S) is a second order RC polynomial, for example,

$$D(S)=d_o[S^2+(\sigma_1+\sigma_2)S+\sigma_1\sigma_2],\ d_o>0,\ \sigma_1>0,\ \sigma_2>0 \qquad (C-2)$$

Since N has been designed to produce oscillation,

$$N_1- N_2= a_o(S^2+\omega_s^2) \qquad (C-3)$$

Substituting (C-2) and (C-3) in (C-1), we get the CE as

$$(-A_d a_o+d_o)S^2 + d_o(\sigma_1+\sigma_2)S + (-A_d a_o\omega_s^2+d_o\sigma_1\sigma_2)=0 \qquad (C-4)$$

Similarly, for Fig.C.1(b), we get the CE as

$$(A_d a_o+d_o)S^2 + d_o(\sigma_1+\sigma_2)S + (A_d a_o\omega_s^2+d_o\sigma_1\sigma_2)=0 \qquad (C-5)$$

In practice, when the power supply is turned on, the gain A_d

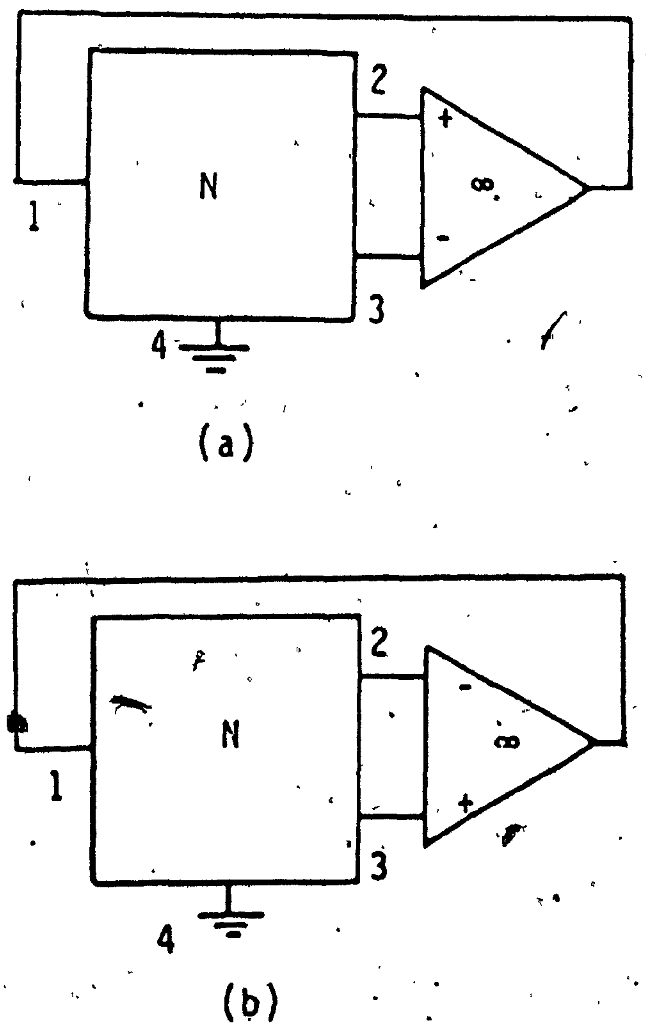

Fig. C.1: A pair of stable-unstable RCOs.

of the OA starts from zero and rises rapidly to a very large value, $A_m > 0$, ideally infinity. As A_d is changing from zero to A_m, the roots of (C-4) and (C-5) change from the negative real axis to close to their equilibrium positions for example, $j\omega_s$. But for the circuit of Fig.C.1(b) for which the CE is (C-5) the roots remain all the while in the left half of the s-plane as A_d changes from zero to A_m. Hence, during the transition to equilibrium position, the circuit remains stable.

On the other hand, for the circuit of Fig.C.1(a) for which the CE is given by (C-4), the roots will migrate to the right-half of the s-plane as A_d changes from zero to A_m. Hence, during the period of the transition the circuit becomes unstable and latch-up will occur. Such behaviour has been reported in other OA circuits, also in [8].

The above two circuits are termed stable-unstable (SU) pair circuits. Obviously then, if one of them which is designed to be an oscillator does not oscillate in practice, interchanging the connection to the OA input terminals will rectify the problem.

APPENDIX D

Figure D.1(a) shows a one loop RC network with more than five nodes. Only two of the branches are capacitive and the rest are resistive. It is shown in this Appendix that such a circuit with one OA can not generate a canonic SFO, nore it can generate a VFO.

Choose any node as ground node and connect the OA to the circuit such that there is no branch/es connected between the inputs of the OA. The resulting circuit can always be represented as Fig.D.1(b). The CE of the system is:

$$Y_1Y_2 - Y_3Y_4 = 0 \qquad \text{(D-1)}$$

From (D-1), the following cases are to be examined.

(a)-Only one of Y_1 to Y_4 is capacitive.

(b)-Capacitive branches are multiplied together.

(c)-Capacitive branches are not multyplied together.

<u>Case (a):</u> Let Y_1 contains the two capacitors of the circuit. Fig.D.2(a) shows the most general form of Y_1. The CE is:

$$\alpha S^2+\beta S+\gamma=0 \qquad \text{(D-2)}$$

where

$$\alpha = C_aC_b(G_cG_2-G_3G_4) \qquad \text{(D-3)}$$

$$\beta = (G_cG_2-G_3G_4)(C_aG_b+G_aC_b)- G_4G_3G_c(C_a+C_b) \qquad \text{(D-4)}$$

$$\gamma = G_aG_b(G_cG_2-G_3G_4)-G_3G_4G_c(G_a+G_b) \qquad \text{(D-5)}$$

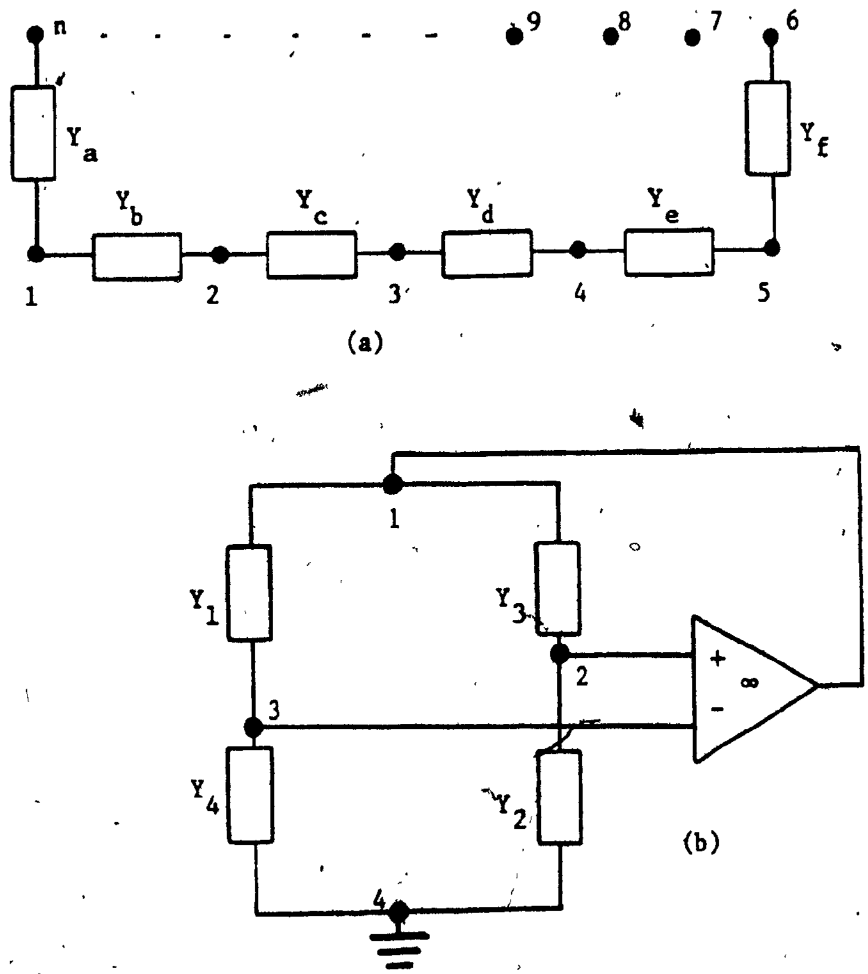

Fig. D.1(a): one loop RC network with more than five nodes.
(b): RC circuit with a one loop RC network

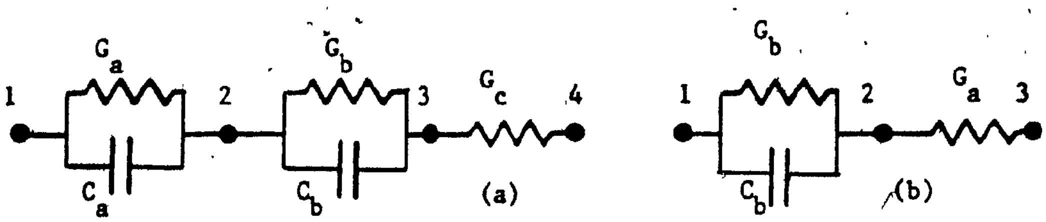

Fig. D.2(a): General form of more than 2 RC branches in series containing 2 capacitors.
(b): General form of more than one branch in series containing one caopacitor.

For none of the resistors (D-4) can be written in the form of

$$G_v(U_1-U_2)+(U_3-U_4)=0 \qquad (D-6)$$

Hence, the circuit can not be a VFO. To generate a canonic SFO two of the resistors should be set to zero to reduce the number of elements to 6. Only elemination of G_a and G_d do not brake the circuit. Putting G_a and G_b to zero makes $\beta \neq 0$ and no oscillation is possible. Therefore a canonic SFO can not be generated.

<u>Case (b):</u> Let Y_1, Y_2 be capacitive and Y_3, Y_4 be resistive. The most general form of Y_1 or Y_2 is as shown in Fig.D.3(a). Let the branch admittances be:

$$Y_1= \frac{G_aG_b+SG_aC_b}{SC_b+G_a+G_b}$$

$$Y_2= \frac{G_cG_d+SG_cC_d}{SC_d+G_c+G_d}$$

$$Y_3= G_3$$

$$Y_4= G_4$$

then the CE is:

$$S^2[G_aG_cC_bC_d-G_3G_4C_bC_d] +S[G_aG_cG_dC_b+G_aG_bG_cC_d$$
$$-G_3G_4C_b(G_c+G_d) -G_3G_4C_d(G_a+G_b)] +G_aG_bG_cG_d$$
$$-G_3G_4(G_a+G_b)(G_c+G_d)=0 \qquad (D-7)$$

and condition of oscillation is:

$$G_aG_cG_dC_b+G_aG_bG_cC_d-G_3G_4C_b(G_c+G_d)$$
$$-G_3G_4C_d(G_a+G_b)=0 \qquad \text{(D-8)}$$

To generate an SFO, (D-8) must be satisfied. There are minimum of 8 elements in the circuit, only G_b and G_d can be set to zero, in which case (D-8) is not satisfied. Bringing back any of the resistors makes the circuit a non canonic SFO configuration. Equation (D-8) can not be written as (D-6). Hence, this circuit is not a VFO.

<u>Case (c)</u>: Let Y_1, Y_3 be capacitive and Y_2, Y_4 resistive. Therefore the branch admittances are:

$$Y_1= \frac{G_aG_b+SG_aC_b}{SC_b+G_a+G_b}$$

$$Y_3= \frac{G_cG_d+SG_cC_d}{SC_d+G_c+G_d}$$

$$Y_2= G_2$$

$$Y_4= G_4$$

then the CE is:

$$s^2G_2G_aC_bC_d+s[G_2G_aC_b(G_c+G_d)+G_2G_aG_bC_d]$$

$$+G_2G_aG_b(G_c+G_d)$$

$$-s^2G_4G_cC_bC_d-s[G_4G_cC_d(G_a+G_b)-G_4G_cG_dC_b]$$

$$-G_4G_cG_d(G_a+G_b) = 0 \qquad \text{(D-9)}$$

and condition of oscillation is:

$$G_2G_aG_cC_b+G_2G_aG_dC_b+G_2G_aG_bC_d$$

$$-G_4G_cG_aC_d-G_4G_cG_bC_d-G_4G_cG_dC_b = 0 \qquad \text{(D-10)}$$

Only G_b and G_d can be set to zero, in which case from (D-10), the frequency of oscillation is zero. Introduction of one element to resulting circuit generates a non-canonic SFO configuration.

Equation (D-10) can not be of the form of (D-6). Hence the circuit can not be a VFO either. Therefore, a single loop RC network with more than five nodes, one OA, 2 capacitors and any number of resistors can not generate a canonic SFO, nor it can generate a canonic VFO.

APPENDIX E

The Fig.5.2(b) shows two passive networks connected to a DOOA with unity feed-back. There is no path from one input of the DOOA to the other. The passive networks have only one node in common, namely, the ground node. The graphs of this type of networks are called non-connected graphs. To show that this type of circuits are stable circuits and hence do not generate any oscillator, we proceed as follows.

Consider the Fig.5.2(b), the transfer functions of interest are:

$$T_{13} = \frac{V_3}{V_{11}} = \frac{N_{13}(S)}{D_1(S)} \qquad (E-1)$$

or:

$$V_3 = V_{11}\left(\frac{N_{13}(S)}{D_1(S)} \right) \qquad (E-2)$$

similarly:

$$T_{24} = \frac{V_4}{V_{12}} = \frac{N_{24}(S)}{D_2(S)} \qquad (E-3)$$

or:

$$V_4 = V_{12}\left(\frac{N_{24}(S)}{D_2(S)} \right) \qquad (E-4)$$

$D_1(S)$ and $D_2(S)$ are the CEs of the passive networks and their roots are on the negative real axis.

The DOOA allows us to write:

$$V_o = A_d(V_3 - V_4) \qquad (E\text{-}5)$$

From the unity feed-back we can write:

$$V_{i1} = -V_o \qquad (E\text{-}6a)$$

$$V_{i2} = V_o \qquad (E\text{-}6b)$$

Substituting from (E-2), (E-4) and (E-6) into (E-5) and allowing A_d to go to infinity, the over all CE of the system is

$$(D_2(S)N_{13}(S) + D_1(S)N_{24}(S)) = 0 \qquad (E\text{-}7)$$

Equation (E-7) is a complete second degree polynomial with positive coefficients. Therefore, its roots are in the negative half of s-plane and it is stable.