THE HEDGING EFFECTIVENESS OF DAX FUTURES

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Abstract

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Recent hedging literature reveals that the performance of dynamic hedging strategies over constant ones tends to differ across various financial markets in terms of the percentage reduction in portfolio variance attainable. This paper analyzes the hedging effectiveness of DAX index futures on the underlying index. This study builds on previous work on futures hedging of stock risk by allowing for time-varying correlations and cointegrativeness, and by assessing hedging effectiveness from a welfare standpoint. It is found that while the dynamic models proposed are statistically superior to the static models, they do not yield greater risk reduction.

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Heather for going beyond administrative assistance.

Dedications

I would like to dedicate this thesis:

To my family for supporting me during my studies.

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1. Introduction

Ever since the launching of the first foreign currency futures contracts on May 16, 1972 at the Chicago Mercantile Exchange, futures contracts have emerged at various exchanges around the world. The number of exchanges trading financial futures has grown from two to sixty. Stock index futures made their debut in 1982 with the introduction of the Value Line and Standard and Poor's 500 index futures contracts, and others soon followed. The focus of this study, the German stock index DAX, started trading in August, 1990, and its futures and other derivative products were introduced in July, 1991. The volume of DAX derivative products in general has increased by 13% in the first quarter of 1995 compared to last year's quarterly results.[1] The DAX makes a relevant subject for study not only because it is an emerging market but because global risk management strategies are becoming increasingly popular amongst financial institutions.

Our attention in this study is directed to the hedging effectiveness of futures on the DAX, that is, the degree of reduction of German market risk that may be obtained by applying various hedging strategies involving futures on the DAX. The hedging strategies compared here differ according to the underlying model or assumptions made regarding the stochastic evolution of the basis. The main distinction made is between models that assume constant joint distributions of spot and futures price changes and those that admit distributional time variability. Constant distribution models include the Naive and OLS.

[1] Die *Frankfurter Allgemeine Zeitung*, June 25, 1995, F-1.

Under the Naive approach the hedge ratio, or the number of units of the cash asset to sell via futures relative to the number of units held spot, is always one. The Naive approach is optimal only when spot and futures price changes are perfectly correlated and variances are equal. A relaxation of the assumption of perfect correlation and equal variance acknowledges the existence of basis risk and results in the OLS approach described below.

Basis risk may be caused by any or all factors discussed by Figlewski (1984). These include market imperfections such as transactions costs and the asymmetric tax treatment of gains and losses from stock and futures transactions, exchange-imposed regulations such as short sale rules, daily price limits and circuit breakers, dividend uncertainty, inefficient cash positions, and the stochastic evolution of interest rates. All of these factors contribute to loosening up the equilibrium links between cash and futures prices and inhibit arbitrage activity aimed at bringing cash and futures prices back into line.

A method of basis-risk minimization is implicit in Markowitz's (1952) portfolio selection theory. In the mean-variance framework, the efficiency of portfolios accounts for the joint return distributions of their constituent assets. Hedging becomes the minimization of the unconditional variance of changes in portfolio value; when the portfolio consists of a cash position and its corresponding futures, the optimal hedge ratio is estimated by simple linear regression of spot price changes on futures price changes. Ederington (1979), Johnson (1960), and Stein (1961) demonstrate that basis-risk is minimized by, what will be referred to herein as, the OLS approach. The effectiveness of the OLS hedge has been demonstrated by Hill and Schneeweiss (1981), Figlewski (1984, 1985), Junkus and Lee (1985), Kamara and Siegel (1987), Lee, Bubnys, and Lin (1987), Myers and Thompson (1989), and Castelino (1989, 1992) to name a few.

When distributional time variability is admitted, one can model the first and second moments conditional on the supposed nature of the stochastic evolution of the basis. We adopt an error correction to model the first moments based on Engle and Grangers' (1987) notion of cointegrated series, and the GARCH model of Engle and Kroner (1994) to model the second moments.

OLS hedges are not optimal if the cash-futures covariance matrix is time-varying. This has been demonstrated for stock return distributions by Baillie and DeGennaro (1990), Bollerslev (1987), French, Schwert, and Stambaugh (1987), Pindyck (1984), and Poterba and Summers (1986). Since the OLS hedge ratio is the *unconditional* covariance of cash and futures price changes divided by the *unconditional* variance of futures price changes, it does not capture time-variability of the joint distribution which may display dynamic variances, covariances or correlation, and implying a *changing* hedge ratio. Early evidence of instability in hedge ratios appears in Grammatikos and Saunders (1983) and Lypny (1988) for foreign currencies; Ceccetti, Cumby and Figlewski (1988) for long-term debt; Figlewski (1984, 1985) and Lee, Bubnys and Lin (1987) for stock index futures; and Castelino (1992) for wheat, corn, Treasury Bills, and Eurodollar time deposit futures. The presence of distributional time variability implies that hedges may be improved by accounting for relevant conditioning information [Myers and Thompson (1989)], with the the autoregressive conditional heteroscedasticity (ARCH) model, a popular modeling approach, introduced by Engle (1982) and generalized by Bollerslev (1986).[2] Baillie and Myers (1991), Myers (1991), Kroner and Sultan (1993), Park and Switzer (1994), Gagnon, Lypny, and McCurdy (1995) and Gagnon and Lypny (1995) report substantial time variation in hedge ratios for various agricultural commodities, currencies, stock indices, and interest rates. Dynamic hedging strategies based on the GARCH framework can improve hedging performance over OLS where risk is measured as unconditional portfolio variance.

With respect to the first moments, it is possible that cash and futures prices follow a long-run stochastic relationship, both being affected similarly by a common "news" variable, and consequently cointegrated in the sense of Engle and Granger. The first two moments or mean equations may be modeled with an error correction term equal to an estimated coefficient multipying the lagged value of the basis.

In this paper we propose hedging models which internalize the time-varying nature of return distributions of a cash stock index and its futures contract by imposing an autoregressive structure on the covariance matrix, specifically, a GARCH (1,1) process, and by correcting means for the possibility that the series are cointegrated. It is found that while the dynamic models possess greater explanatory power, they do not yield better hedges, and, consequently, it must be concluded that the static OLS hedge is preferred for the DAX.

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[2] See Bollerslev, Chou and Kroner (1992) for a review of the ARCH literature.

2. Dynamic Hedging

The random return of a portfolio consisting of a one unit cash position and hedged by b units of corresponding futures is given by:

$$r_t = s_t - b_{t-1} f_t, \tag{1}$$

where

- b_{t-1} is the hedge ratio to be used in period t;
- $s_t = \ln S_t - \ln S_{t-1}$ is the change over the previous period in the natural logarithm of cash price S;
- $f_t = \ln F_t - \ln F_{t-1}$, is the change in futures price F.

The covariance matrix of spot and futures price changes is given by

$$\Sigma \left| \Omega_{t-1} \right. = \begin{bmatrix} \sigma_s^2 & \sigma_{sf} \\ \sigma_{fs} & \sigma_f^2 \end{bmatrix}, \tag{2}$$

where Ω_{t-1} is the information set at t-1.

An investor with quadratic tastes chooses b_{t-1} to maximize end-of-period utility:

$$\max_b E(r_t|\Omega_{t-1}) - \gamma Var(r_t|\Omega_{t-1}), \tag{3}$$

where $\gamma>0$ is the investor's risk aversion parameter. The assumption of time additivity permits a multiperiod objective to be expressed as a sequence of one period choices which is inherent in the conditional hedging strategy described here. The solution to (3) yields the optimal conditional demand for futures contracts as

$$b^{*}_{t-1} = \frac{\mathrm{cov}(s, f|\Omega_{t-1})}{\mathrm{var}(f|\Omega_{t-1})} - \frac{1}{\gamma}\frac{E(f_t|\Omega_{t-1})}{\mathrm{var}(f|\Omega_{t-1})}. \tag{4}$$

The hedge ratio may change over time with changing information pertaining to the covariance structure of returns and mean futures prices. The first term is the risk-minimizing hedge ratio which will obtain if futures prices follow a martingale, $E(f_t|\Omega_{t-1}) = 0$, causing the second term or speculative demand for futures to equal zero. Equation (4) then represents the mean-variance trade-off. Martingale futures prices are sufficient for risk minimization to be consistent with welfare maximization; if not, knowledge of investor preferences is required to make comparisons among alternative hedging strategies. It should also be noted that (4) nests the OLS model if we ignore conditioning information and futures prices follow a martingale, and the Naive model if it is assumed, additionally that $\frac{\mathrm{cov}(s, f)}{\mathrm{var}(f)} = 1$.

3. The Bivariate GARCH Model

The GARCH model employed here specifies the time-varying covariance matrix in (6) based on the mean equations in (5) below.

$$y_t = \mu_t + \varepsilon_t, \tag{5}$$

where $\varepsilon_t|\Omega_{t-1} \sim t_v(0, H_t)$.

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + G'H_{t-1}G \tag{6}$$

$y = (s \quad f)'$ is a vector of observations of cash and futures log-differenced prices, $\mu = (\mu_s \quad \mu_f)'$ is a vector of means to be estimated, and $\varepsilon = (\varepsilon_s \quad \varepsilon_f)'$ is a vector of residuals. It is assumed that the residuals are distributed, conditional on past information, Ω_{t-1}, as bivariate t with ν degrees of freedom and with H the conditional covariance matrix. Equation (5) implies a constant risk premium on cash and futures and includes the martingale model for futures contracts for the special case of a zero risk premium.

The parameterization of the conditional covariance matrix in (6) is adopted from Engle and Kroner (1994) and presented below for the bivariate case. C is a matrix of constants; A is a matrix of coefficients pertaining to lagged, uncentered second moments and cross-moments; and G is a matrix of coefficients pertaining to lagged, centered second moments and cross-moments.

$$H_t = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}' \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$
$$+ \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}' \begin{bmatrix} h_{1,t-1}^2 & h_{12,t-1} \\ h_{12,t-1} & h_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

This parameterization is economical, ensures under mild restrictions that H is positive definite, and is general in that it permits representation of a wide variety of models.

In this study, C, A, and G are restricted to be symmetric. This dynamic model will be referred to as the GARCH model herein.

4. The Error Correction Model

The concept behind the error correction model is that there exists a long-run relationship between the two variables. Although they may deviate from each other in the short run, market forces will bring them back together in the long run. Engle and Granger (1987) show that cointegrated series have an error correction representation stating that a proportion of the disequilibrium in one period should be corrected in the next period.

Ever since the introduction of this model, financial researchers have investigated various markets to shed light on the significance of the cointegration model in the financial world. Anderson, Granger, and Hall (1990), for example, analyzed the term structure of US Treasury bills within the framework of cointegration and developed an error correction representation. Szakmary (1991) found that the spot and forward exchange rates are cointegrated and modeled the appropriate error correction. Only marginal support for the error correction hypothesis was given by Bessler and Covey (1991) while investigating the futures commodity market (live cattle). Copeland (1991) looking at cointegration between leading European currencies against the US dollar found that there exists no cointegration between the variables.

Applying an error correction to the mean equations in (5) results in the modified model of

$$y_t = \mu_t + \Psi\Big(\ln(F_{t-1}) - \ln(S_{t-1})\Big) + v_t \qquad (5a)$$

where we make the same distributional assumption for ν_t as ε_t . The model which includes the error correction in the mean equations is referred to as EC. We also estimated a dynamic model with an error correction and a GARCH covariance matrix referred to as GARCH + EC. The GARCH + EC is the most general model and it nests the EC, GARCH, and OLS models.

5. Data

Daily closing spot prices of the German stock index (DAX) and near time delivery of futures were collected for the time period from July 1991 through December 1994. The data series were obtained from the Deutsche Bank, Frankfurt and contain a total of 207 observations. The sample essentially starts with the introduction of the DAX index futures in 1991. The standard contract size is DM100 per index point (current exchange rate ~ 1:1). Dax futures contracts are quoted in index points per DM100 of the contract's value to one decimal place, e.g. 1,505.0. The minimum price movement - referred to as the "tick" - is 0.5. One tick corresponds to a value of DM 50.00 (0.5 x DM100). Wednesday-to-Wednesday percentage changes are collected by computing differences in the natural logarithm of the prices, multiplied by 100. The first 157 observations are used for the estimation period, and the remaining 50 comprise the forecast period Of the three outstanding futures contracts, the price of the nearest futures contract is used. To avoid expiration days and thin markets we will roll over to the next nearest contract one week before expiry.

6. Preliminary Analysis

The time series are first tested for the existence of unit roots by applying several tests. The spot and futures prices are analyzed by using the following augmented Dickey-Fuller test:

$$\Delta y_t = \alpha_0 + \alpha_1 y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \varepsilon_t$$

where enough lagged variables are added to ensure that the error term becomes white noise. However, one of the shortcomings of the ADF test is that if p becomes sufficiently large, it reduces the power of the test. In this case, an alternative test, due to Phillips and Perron (1988) is used, which utilizes a non-parametric correction for serial correlation for the presence of unit roots.

$$y_t = \alpha + \beta\ y_{t-1} + \eta_t$$

where η_t = is the white noise.

Table 1a and 1b report the Phillips and Perron (1988) and Dickey-Fuller tests for a truncation of lag of four. The null hypothesis that unit roots exist in both price series the spot and futures prices cannot be rejected. Figures 1a and 1b exhibit the nonstationary behavior of the spot and futures prices. However, when the spot and futures prices are first-differenced, the null-hypothesis of nonstationarity is rejected. This leads to the conclusion that the differenced series are stationary and integrated in order of 1 which is necessary for testing the existence of cointegration. Table 2 reveals the significance of the cointegration test by using the Phillips and Perron test, the augmented Dickey-Fuller test, and the Durbin-Watson test. The results of Table 2 indicate that both the spot and the futures prices are cointegrated with a cointegration coefficient close to one.

In general, cash and futures series cannot be cointegrated since the basis degenerates to zero at the expiration of the futures contract. However, if both spot and futures prices show a long-run equilibrium relationship, an error correction term should be added to the econometric model to account for the long-run behavior of spot and futures price changes.

While the results of unit-root tests suggest that both return series are stationary, they are characterized by heavy tails and sharp peaks, according to the skewness and kurtosis results in Table 3. The large excess kurtosis is consistent with the time-varying conditional heteroscedasticity model of Bollerslev (1986) and Engle (1982). Ljung-Box (1978) tests for up to 24th order serial correlation in the residuals of each series and is computed as simple deviations from the mean. The Q-Statistics are significant, indicating the presence of serial correlation in the cash and futures return. The ARCH test investigates (Lagrange multiplier tests) for serial correlation in the squared residuals and evidenced serial correlation for the first lag which is consistent with time-varying conditional heteroscedasticity.

7. Results of the Model Estimation

Table 4a and 4b report the maximum likelihood estimates of the conditional means and covariance matrix of cash and futures returns, where **N** denotes the number of observations, **df** denotes the degrees of freedom and **logl** denotes the log-likelihood value. The estimation was conducted by using the algorithm of Broyden, Fletcher, Goldfarb, and Shanno (BFGS). The tables reveal that the unrestricted GARCH + EC, describes well the joint distribution of spot and futures returns. The spot and futures prices series show significant ARCH and GARCH effects, and the error correction coefficients are significant for spot and futures at the 10% and 1% levels, respectively. Noteworthy is the fact of the significant mean futures return (μ_2) which violates the martingale assumption and justifies the introduction of the welfare analysis.

The estimated residuals are assumed distributed as bivariate t with v degrees of freedom. The use of the t distribution is justified given the small estimated degrees of freedom, 6.330 for the GARCH + EC model, 5.795 for the Garch model and 5.163 for the OLS model. The nesting of both Tables, 4a and 4b, is undertaken in order to find the most parsimonious model.

The likelihood ratio tests in Table 5 reports that the dynamic models have a significantly greater explanatory power than OLS (tests III and IV). However, the explanatory power seems to reside in the error correction adjustment more so than the

GARCH adjustment since removing the GARCH adjustment from GARCH + EC does not significantly reduce explanatory power (test II), whereas removal of the error correction does (test I). Therefore, the most parsimonious model is the EC model.

The next section reports the results of an analysis of hedging effectiveness, employing both risk-minimization and welfare maximization criteria.

8. Hedging Effectiveness

In Table 6a and 6b the different hedging strategies are compared to determine if the anticipated superior result of the dynamic hedge, due to the dynamic specification of the covariance matrix and mean equations, does increase the efficiency of the hedge ratio estimates. The parameters are estimated for each model and then applied to calculate the individual hedge ratios. Portfolio return over a 156-week estimation period and a 50-week forecast period are computed according to equation (1). The tables show portfolio mean return, variance and the percentage change in variance relative to the OLS model. Figure 2 shows the considerable volatility of the hedge ratios for the dynamic models. The hedge ratios are computed as follows:

Risk-Minimization

$$b = \frac{COV(s,f)}{VAR(f)}$$ for the OLS and EC model

$$b = \frac{COV(s,f \mid \Omega_{t-1})}{VAR(f \mid \Omega_{t-1})}$$ for the GARCH Models

Welfare-Maximization

$$b = \frac{COV(s,f)}{VAR(f)} - \frac{1}{\gamma}\frac{E(f)}{Var(f)} \text{ for the OLS}$$

$$b = \frac{COV(s,f)}{VAR(f)} - \frac{1}{\gamma}\frac{E(f_t \mid \Omega_{t-1})}{Var(f_t)} \text{ for the ECM}$$

$$b = \frac{COV(s,f \mid \Omega_{t-1})}{VAR(f \mid \Omega_{t-1})} - \frac{1}{\gamma}\frac{E(f_t \mid \Omega_{t-1})}{Var(f_t \mid \Omega_{t-1})} \text{ for the GARCH}$$

It is assumed that $\gamma = 3$.

Table 6a and 6b report the percentage variance reduction of the dynamic models over OLS for the estimation and forecast periods. For the within-sample period we can not detect risk reduction of the dynamic models over the OLS. This is inconsistent with the statistical analysis where we demonstrated that the dynamic model has a significantly greater explanatory power than the OLS model, and may be attributed to overfitting of the model. In the out-of-sample comparisons, the dynamic models outperform OLS with the exception of the GARCH model under a risk-minimization criteria. The efficiency analysis is inconsistent with the statistical analysis for the in-sample period, and the more favourable performance of the dynamic models out-of-sample leads to greater ambiguity.

It is notable that the GARCH + EC model yielded a higher mean return than OLS in-sample, under risk-minimization and welfare maximization.

To address the risk-return trade-off problem, we analyze hedging performance from an utility standpoint.[3]

9. Utility Comparison

In order to determine whether the statistically superior results of the GARCH + EC model over the OLS model are also economically significant, one has to take the investor's preferences into account. According to Kroner and Sultan (1993), the superiority of the dynamic hedge model over the constant model is only valid if the outcomes of the dynamic hedge result in higher expected utility, net of transaction costs, than the static models.

Investors' preferences are assumed to be quadratic and the optimal hedge ratio is chosen to maximize the investors' end-of-period utility:

$$MAX\left[E\left(r_t \mid \Omega_{t-1}\right)-\gamma\ VAR\left(r_t \mid \Omega_{t-1}\right)\right]$$

where γ is the risk aversion parameter.

Therefore the optimal conditional demand for futures contracts can be written as:

$$b=\frac{COV(s,f \mid \Omega_{t-1})}{VAR(f \mid \Omega_{t-1})}-\frac{1}{\gamma}\frac{E\ (\ f_t\ \mid \Omega_{t-1})}{Var\ (\ f_t\ \mid \Omega_{t-1})}$$

[3] As proposed by Ceccetti et al (1988), Kroner and Sultan (1993), and Sephton (1993)

Given the assumption of a martingale, the second term of the right hand side of the equation, the speculative demand for futures, is equal to zero.

It is assumed that investors engaged in dynamic hedging are rebalancing their portfolios in each period and incur transaction costs, **c**, each time when the hedge ratio is altered. Therefore, the difference in average utility over any hedging period for an investor undertaking dynamic hedging is the difference in portfolio variance times the degree of risk-aversion minus the round-trip transaction cost, **c**, for each period of rebalancing. If **o** stands for the OLS hedge and **d** for the dynamic (GARCH + ECM) hedging, we can state that an investor is better off performing the dynamic hedge as opposed to the constant hedge if:

$$-c-\gamma\,\sigma^2(r_d) > -\gamma\,\sigma^2(r_o)$$

under risk minimization.

Note, that the only difference in the above stated utility equation is that transaction costs are incurred by periodically rebalancing the investor's portfolio.

The equation can be changed to:

$$\left|\frac{\sigma^2(r_d)}{\sigma^2(r_o)}-1\right| > \frac{c}{\gamma\,\sigma^2(r_o)}$$

This equation shows that an investor would prefer the dynamic hedging to the OLS hedge if the percentage reduction in variance, demonstrated by the left-hand side of the equation, is greater than the ratio of the transaction costs to expected utility under the constant hedge, exhibited by the right-hand side of the equation. To clarify equation (6.4), one could state that, with the investor's risk preference parameter $\gamma = 3$ and $\sigma^2(r_0) = 0.5$ and $c = .01\%$ ($20 for a contract with an underlying value of $100*DAX-Index ~ $200000), a reduction in variance of only .666% is necessary to justify pursuing the dynamic hedging strategy as opposed to the constant hedging strategy.

However, in the real world, market participants face much more attractive round-trip transaction costs. At the German Exchange, round-trip transaction costs for one DAX futures contract is $3 for the floor traders, $10 for institutional investors, but around $150 for retail customers.[4]

Table 7 shows average utility under the relevant hedging period for the OLS and GARCH + EC models as a function of the degree of risk aversion. The table shows that the dynamic model cannot yield higher utility: the higher mean return does not compensate for the higher variance in the absence of transactions costs, and, therefore, cannot yield higher utility in the presence of transactions costs.

[4] According to figures handed out by the Dresdner Bank, Frankfurt, Germany.

10. Conclusion

This study demonstrates that two dynamic models of the joint distribution of spot and futures prices for the DAX index, an error correction model of the means and a GARCH model of the covariance matrix, possess significantly greater explanatory power than the OLS model. However, in-sample efficiency and welfare analyses indicate that both dynamic models perform worse than simple OLS in reducing portfolio variance or increasing utility when applying hedging strategies. This contradicts the statistical superiority of the former and may be attributed to overfitting of the data.

While more favorable out-of sample results are obtained, these cannot override the in-sample results, and it is concluded that static hedging remains the preferred strategy for the DAX.

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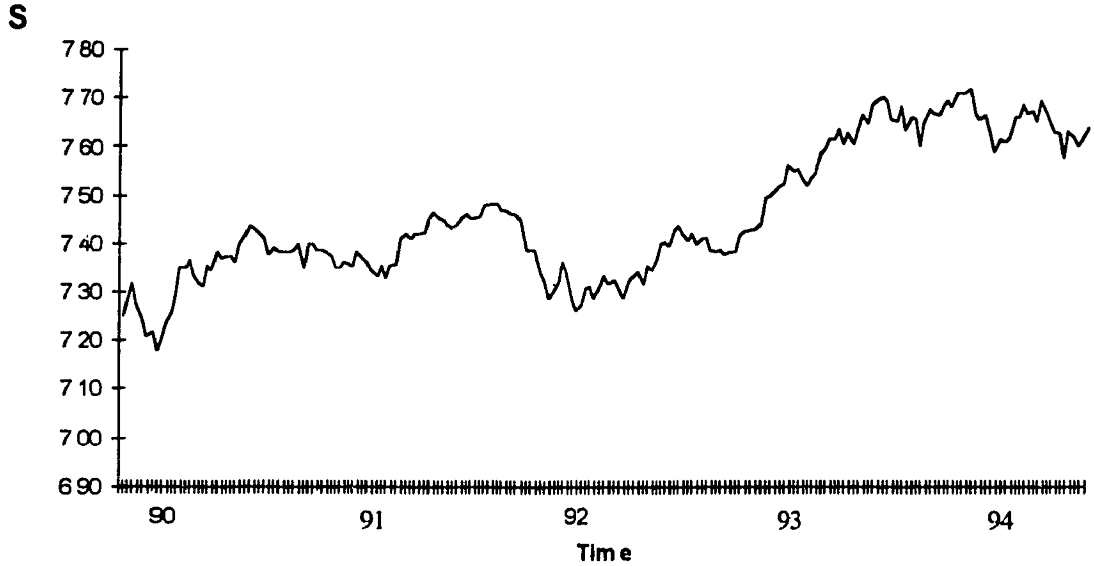
Figure 1a


S
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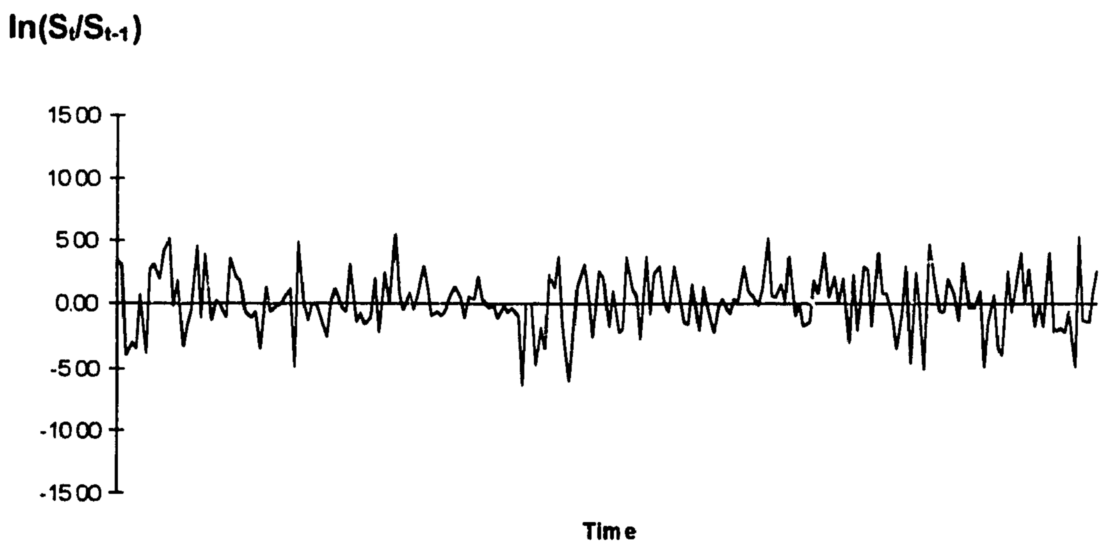
Tim e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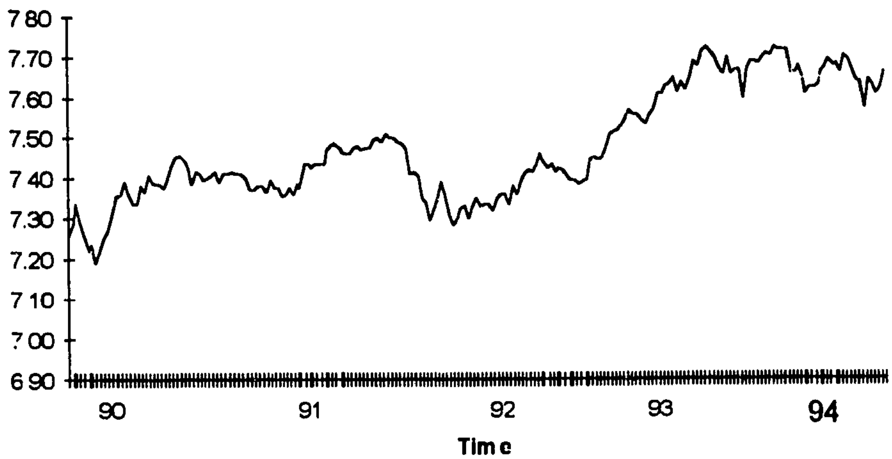
-15 00
Tim e

Figure 1b

F



ln(F_t/F_{t-1})

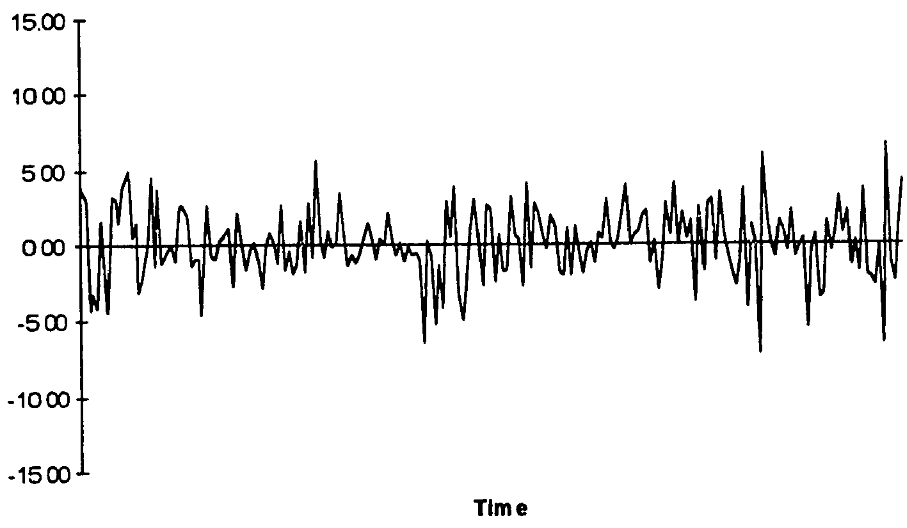
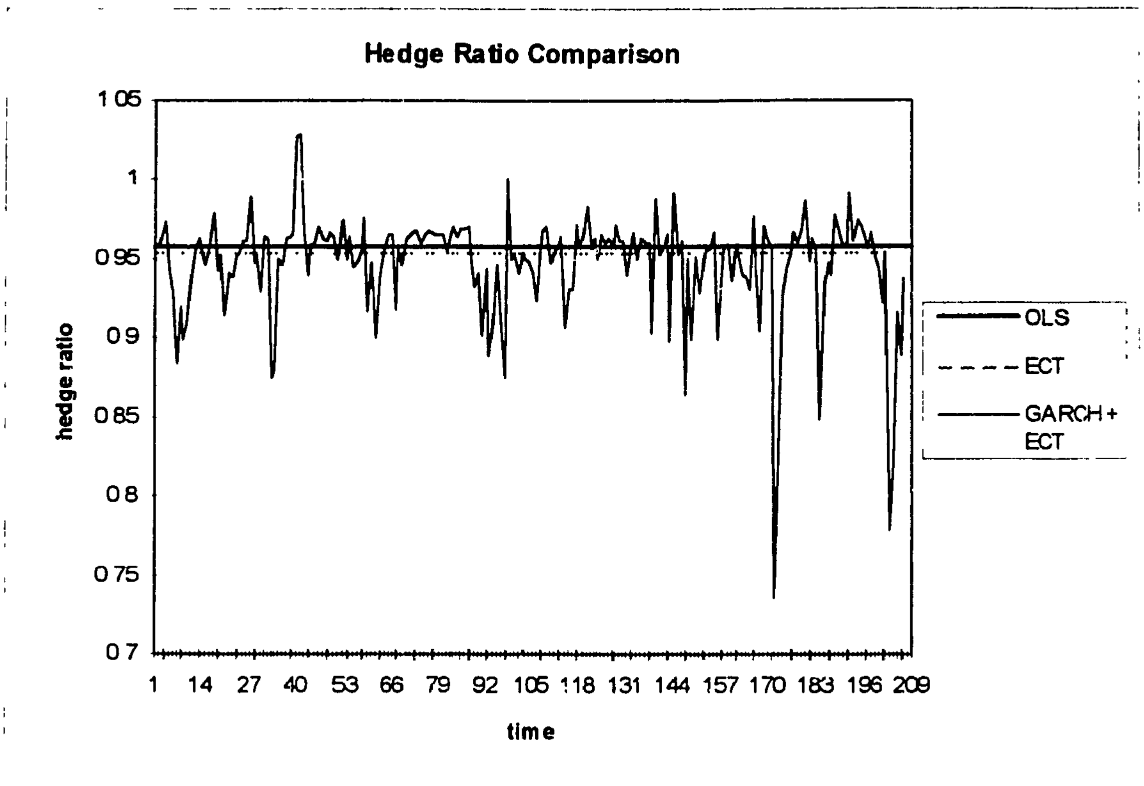


Figure 2



Hedge Ratio Comparison
1 05
1
0 95
0 9
0 85
0 8
0 75
0 7
hedge ratio
1 14 27 40 53 66 79 92 105 118 131 144 157 170 183 196 209
time
OLS
ECT
GARCH + ECT

Table 1a

Unit Root Tests

July 1991 - December 1994

	Prices		Differences	
	PPT and 4 lags	**PP with 4 lags**	**PPT and 4 lags**	**PP with 4 lags**
S	-2.163	-1.327	-14.997	-15.103
F	-2.250	-1.385	-15.346	-15.346
Critical values	-3.410	-2.860	-3.410	-2.860

S = Spot, F = Futures, and PPT stands for Phillips and Perron Test with a time trend. PP is the corresponding statistic without a time trend. The critical values are exhibited in Engle and Granger (1987) and can also be found in Phillips and Ouliaris (1990).

Table 1b

Unit Root Tests

July 1991 - December 1994

	Prices		Differences	
	DFT and 4 lags	**DF with 4 lags**	**DFT and 4 lags**	**DF with 4 lags**
S	-2.072	-1.356	-6.952	-6.955
F	-2.088	-1.345	-6.967	-6.975
Critical values	-3.410	-2.860	-3.410	-2.860

S = Spot, F = Futures, and DFT stands for Dickey Fuller Test with a time trend. DF is the corresponding statistic without a time trend. The critical values are exhibited in Engle and Granger (1987) and can also be found in Phillips and Ouliaris (1990).

Table 2

Cointegration Tests

Log Spot = λ + δ log Futures + ε

July 1991 - December 1994

	DW	**PP(4)**	**ADF(4)**	δ	λ
	1.118995	-9.14454	-6.23610	1.007358424	0.064883341
95 % c.v	0.86	-3.37	-3.37	0.003740793	0.023740793

The DW statistic is the Durbin Watson statistic of the above stated cointegration equation between the logcash and logfutures prices. PP stands for the Phillips and Perron t-statistic and ADF is the forth order augmented Dickey-Fuller test. The critical values can be found in Engle and Granger (1987).

Table 3

Descriptive Statistics

Log-differenced Spot and Futures prices

July 1991 - December 1994

	Skewness	Kurtosis	Q(24)	ARCH(1)
S	-0.114	0.366	20.481	37.968
F	-0.140	-0.019	17.853	37.968
Critical value 95 %			36.42	p = 0.00006

where, S = Spot, F = Futures.

Table 4a

Maximum Likelihood Estimation

Hedging Effectiveness Comparisons between OLS, EC, GARCH + EC

N=155, df=149, Logl = -282.02 | **N=155, df=147, Logl = -267.49** | **N=155, df=141, Logl = -263.51**

		OLS				EC				GARCH + EC		
	Coefficient	Std.Error	T-Stat	P-Val	Coefficient	Std.Error	T-Stat	P-Val	Coefficient	Std.Error	T-Stat	P-Val
$\mu 1$	0.17770	0.16428	1.0817	0.2793	0.35886	0.28889	1.2422	0.2141	0.46728	0.13364	3.4964	0.0004
$\mu 2$	0.04360	0.16989	0.2566	0.7974	0.56024	0.29529	1.8972	0.0577	0.62372	0.13809	4.5167	0.0000
$\psi 1$					-0.143110	0.23845	-0.6000	0.5483	-0.181900	0.09815	1.8530	0.0638
$\psi 2$					-0.449130	0.24188	-1.8560	0.0633	-0.446540	0.09506	4.6970	0.0000
C11	1.77722	0.14540	12.222	0.0000	1.75817	0.14872	11.822	0.0000	1.31384	0.29739	4.4178	0.0000
C12	1.43839	0.13362	10.764	0.0000	1.43234	0.13383	10.702	0.0000	1.06843	0.35303	3.0264	0.0024
C22	1.87159	0.15602	11.995	0.0000	1.82159	0.15573	11.697	0.0000	0.95614	0.44065	2.1698	0.0300
A11									0.30509	0.10894	2.8005	0.0051
A12									-0.125840	0.11697	1.0750	0.2820
A22									0.33956	0.10904	3.1139	0.0018
B11									0.36436	0.13429	2.7131	0.0066
B12									0.24975	0.24811	1.0066	0.3141
B22									0.50890	0.12791	3.9784	0.0000
NU	5.16316	1.44968	3.5615	0.0003	5.05096	1.47587	3.4223	0.0006	6.33026	2.21620	2.8563	0.0042

Table 4b

Maximum Likelihood Estimation

Hedging Effectiveness Comparisons between OLS, GARCH, GARCH + EC

N=155, df=149, Logl = -282.02 **N=155, df=143, Logl = -274.68** **N=155, df=147, Logl = -263.51**

		OLS				GARCH				GARCH + EC		
	Coefficient	Std.Error	T-Stat	P-Val	Coefficient	Std.Error	T-Stat	P-Val	Coefficient	Std.Error	T-Stat	P-Val
μ1	0.17770	0.16428	1.0817	0.2793	0.24076	0.12599	1.9108	0.0560	0.46728	0.13364	3.4964	0.0004
μ2	0.04360	0.16989	0.2566	0.7974	0.11828	0.13317	0.8881	0.3744	0.62372	0.13809	4.5167	0.0000
ψ1									-0.18190	0.09815	1.8530	0.0638
ψ2									-0.44654	0.09506	4.6970	0.0000
C11	1.77722	0.14540	12.222	0.0000	0.05277	0.23122	0.2282	0.8194	1.31384	0.29739	4.4178	0.0000
C12	1.43839	0.13362	10.764	0.0000	0.41334	0.27802	1.4867	0.1370	1.06843	0.35303	3.0264	0.0024
C22	1.87159	0.15602	11.995	0.0000	0.50980	0.33374	1.5275	0.1266	0.95614	0.44065	2.1698	0.0300
A11					0.07402	0.08527	0.8680	0.3853	0.30509	0.10894	2.8005	0.0051
A12					-.30940	0.09176	-3.371	0.0007	-0.12584	0.11697	1.0750	0.2820
A22					-.07121	0.08641	-0.824	0.4098	0.33956	0.10904	3.1139	0.0018
B11					0.71374	0.15528	4.5963	0.0000	0.36436	0.13429	2.7131	0.0066
B12					0.23268	0.15764	1.4759	0.1399	0.24975	0.24811	1.0066	0.3141
B22					0.67052	0.13309	5.0378	0.0000	0.50890	0.12791	3.9784	0.0000
NU	5.16316	1.44968	3.5615	0.0003	5.79544	1.50502	3.8507	0.0001	6.33026	2.21620	2.8563	0.0042

Table 5

Log-Likelihood Estimation

and Tests of Parameter Restrictions

Test	Likelihood ratio	df	95% c.v.
I	22.34	2	5.99
II	7.96	6	12.59
III	29.06	2	5.99
IV	14.68	6	12.59

Test I compares the unrestricted dynamic model (GARCH + EC) with the GARCH model (removing ECM). Test II compares the GARCH + EC with the EC (removing GARCH). Test III compares EC with the OLS model. Test IV compares the GARCH model with the OLS model.

Table 6a

Hedging Effectiveness

In-Sample Comparison

	a) Risk-Minimization			b) Welfare-Maximization		
	Mean	**Variance**	**% ΔVariance**	**Mean**	**Variance**	**% ΔVariance**
OLS	0.130	0.319		0.130	0.319	
EC	0.130	0.319	0.136	0.132	0.327	2.541
GARCH	0.126	0.330	3.488	0.126	0.332	4.077
GARCH + EC	0.133	0.320	0.475	0.135	0.339	4.203

where, % ΔVariance = % change in variance of OLS.

Table 6b

Hedging Effectiveness

Out-of-Sample Comparison

	a) Risk-Minimization			b) Welfare-Maximization		
	Mean	**Variance**	**% ΔVariance**	**Mean**	**Variance**	**% ΔVariance**
OLS	0.066	0.521		0.066	0.517	
EC	0.066	0.518	-.598	0.066	0.492	-4.793
GARCH	0.068	0.539	0.070	0.068	0.532	-0.066
GARCH + EC	0.110	0.475	-8.823	0.106	0.473	-8.515

where, % ΔVariance = % change in variance of OLS.

Table 7

Utility Comparison

The table exhibits the in-sample utility comparisons

between the GARCH + EC and the OLS hedge strategies.

$$-c-\gamma\,\sigma^2(r_d)) > -\gamma\,\sigma^2(r_o)$$

1.) Risk-Minimization

γ	$U(r_o) - U(r_d)$	OLS $U(r_o)$	Dynamic $U(r_d)$
1	0.001519	-0.319	-0.321
2	0.003039	-0.638	-0.641
3	0.004558	-0.958	-0.962
4	0.006078	1.277	-1.283
5	0.007597	-1.597	-1.604
6	0.009116	-1.916	-1.925
7	0.010636	-2.236	-2.246
8	0.012155	-2.555	-2.567
9	0.013675	-2.875	-2.888
10	0.015194	-3.194	-3.209

2.) Welfare-Maximization

γ	$U(r_o) - U(r_d)$	OLS $U(r_o)$	Dynamic $U(r_d)$
1	0.0008	-0.1891	-0.1971
2	0.0215	-0.5086	-0.5301
3	0.0034	-0.0828	-0.8631
4	0.0483	-1.1478	-1.1961
5	0.0618	-1.4673	-1.5291
6	0.0752	-1.7869	-1.8621
7	0.0886	-2.1065	-2.1951
8	0.1021	-2.4260	-2.5281
9	0.1155	-2.7456	-2.8611
10	0.1289	-3.0652	-3.1941