AN ELASTICITY THEORY FOR THE ANALYSIS OF PRISMATIC FOLDED SANDWICH PLATE STRUCTURES

by

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ABSTRACT

In this thesis, the theory of prismatic folded plates, developed by J. E. Goldberg and H.L. Leve [1], is extended to sandwich panels. Basically, this means that besides deformation due to bending, the shear deformation of the sandwich core has to be considered. This is best accomplished using the method of partial deflections, as explained in F.J. Plantema's book on "Sandwich Construction" [2]. Using the elasticity method, the load deformation relationships are established. These relationships are used to assemble a stiffness matrix for a sandwich panel element in a folded plate structure.

This stiffness matrix is then used in a direct stiffness analysis for folded plates, as outlined in a paper by A. De Fries-Skene and A.C. Scordelis [3]. This approach is also employed in "The Experimental and Theoretical Study of Aluminum Sandwich Elements", by P.P. Fazio and J.B. Kennedy [4].

An existing computer program written by P.P. Fazio [4], using the ordinary method, was modified to accept the elasticity method as an option. Both methods were run on the computer consecutively, using the same input, and the results were compared with the experiment. The

conclusion is that the elasticity method gives better correlation with the experiment than the ordinary method.

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NOMENCLATURE

A = Cross-section area of edge stiffener.
A_1, A_2, A_3, A_4 = Constants in general solution for displ. u.
a_1, a_2, a_3, a_4 = Constants in fixed edge force analysis.
B_1, B_2, B_3, B_4 = Constants in general solution for displ.v.
b = width of sandwich panel
C_1, C_2, C_3, C_4 = Constants in general solution for displ.w
c = Core thickness.

D = Flexural rigidity of an isotropic sandwich panel.
E = Modulus of elasticity of facing material.

G_c = Modulus of rigidity of core material.

H_y, h_y, h_{ay}, h_{ty} = Column matrices of hyperbolic functions.

k_{ij} = Stiffness coefficients.
k_c, k_t = Functions of $\coth\beta$ and $\tanh\beta$ respectively.
k_s = Sandwich parameter.

L = Length of Panel.

m = Harmonic number of the Fourier Series expansion.
M_x, M_y = Internal bending moments of a sandwich panel in x and y direction.
M_{xy}, M_{yx} = Internal twisting moments.

M_1, M_2 = Edge moments along longitudinal edges

N_x, N_y = Normal forces in x and y direction.

N_{xy}, N_{yx} = Shear forces in plane of panel.

p = Parameter

q = Uniform pressure loading over whole panel.

q_1, q_2 = Pressure loading at edge 1 and 2 uniform in x-direction varying linearly in y-direction.

Q_x, Q_y = Internal shear forces acting in cross-sections parallel to yz and xz plane, respectively.

S = Shear rigidity of an isotropic sandwich panel.

S_1, S_2 = Tangential edge shear forces.

t = Facing thickness.

U = Function of y in expression for u

u = Internal displacement in x-direction.

u_1, u_2 = Tangential edge displacements in x-direction.

V = Function of y in expression for v.

V_1, V_2 = Edge shear forces normal to panels.

v = Internal displacement in y-direction.

v_1, v_2 = Edge displacements in y-direction.

W = Function of y in expression for w_{by}.

W_1, W_2, W_3 = Constants in general solution for the characteristic equation for sandwich panels.

w = Internal displacements in z-direction.

w_1, w_2 = Edge displacements in z-direction.

w_{bx}, w_{by} = Bending deflection, components of w due to M_x and M_y respectively.

w_{sx}, w_{sy} = Shear deflection, components of w due to Q_x and Q_y respectively.

x, y, z = Coordinates in local coordinate system of sandwich panel.

α = Roots of characteristic equation for sandwich panel.

β = Argument of hyperbolic functions at edges.

$\lambda_1, \cdots \lambda_8$ = Denominators in the expressions for stiffness coefficients.

ν = Poisson's ratio.

θ_1, θ_2 = Angles of rotation at edges.

σ_x, σ_y = Stresses in x and y direction, respectively.

τ_{xy}, τ_{yx} = Shear stresses.

FOREWORD

This investigation was carried out as an extension of the study performed by Paul P. Fazio and J. B. Kennedy, on the use of aluminum sandwich elements in a folded plate structure. It was intended to compare the experimental results with the results of the sandwich plate theory, based on The Theory of Elasticity.

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CHAPTER I

GENERAL INTRODUCTION

I INTRODUCTION

A prismatic folded plate structure is formed by a number of panels or slabs rigidly joined at edges running parallel with each other and simply supported at the ends by diaphragms normal to the joints. An illustration of a folded plate structure is shown in Fig. 1.

The theoretical analysis for this type of structure was outlined in a paper by J.E. Goldberg and H.L. Leve [1], which was based on the Theory of Elasticity and the Theory of Plates.

Each joint has four degrees of freedom, two translations and one rotation in a plane normal to the joint, and a contraction or extension of the joint line. Associated with each degree of freedom is a joint force.

Both joint forces and displacements are distributed symmetrically, with respect to the midspan of the joint. The theory assumes that the joint displacements can be expanded into Fourier Series components.

Considering one panel or slab element of the structure, each of the generalized distributed forces is linearly dependent on the four components of displacement at both edges of the panel. An arbitrary harmonic (Fourier term) of one of the displacements is applied to one edge of the panel and the resulting homogeneous bound-

FIGURE 1

TYPICAL FOLDED PLATE STRUCTURE

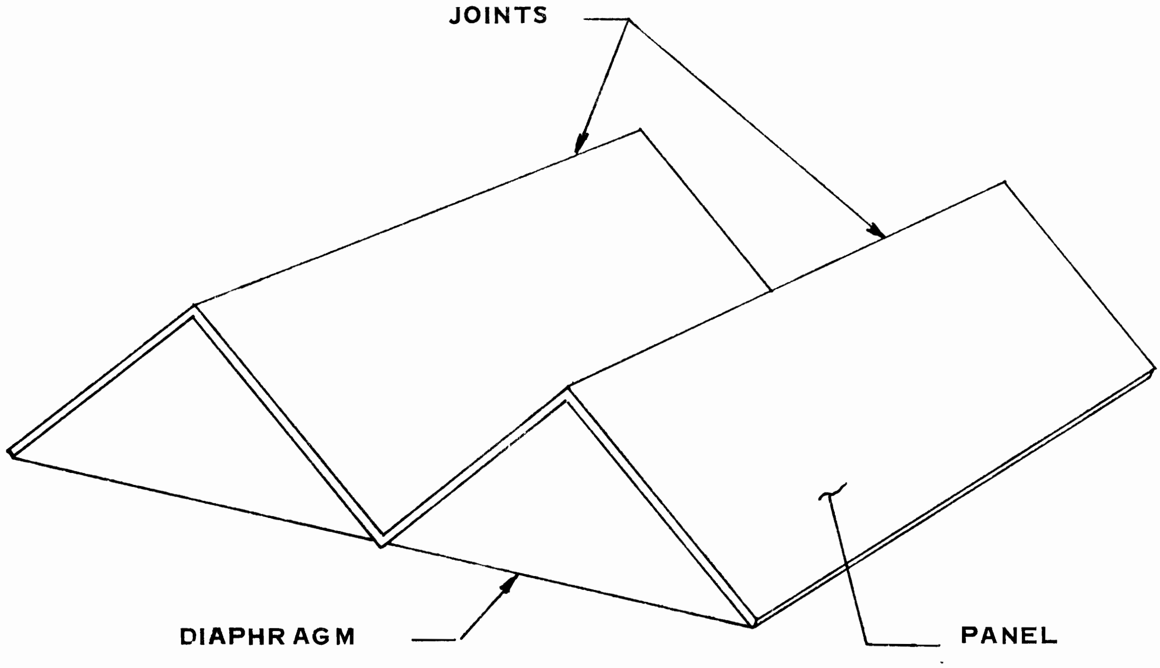


FIG. 1 TYPICAL F OLDED PLATE STR UCTURE

ary value problem is solved for the displacements within the panel. From these displacements, the resulting internal forces and edge forces are derived. The purpose of this paper is to find similar expressions relating edge displacements and edge forces for isotropic sandwich panels.

This problem is more complicated than for uniform plates, as deflections due to shear in the core of the sandwich have to be added to the deflections due to bending. Furthermore, to join sandwich panels rigidly together, some kind of an edge member has to be employed. These edge members add considerably to the in-plane stiffness of the panel and have to be taken into consideration.

The theoretical part of this paper is considered in two parts:

1) <u>Out of plane displacements of the panel.</u>

This problem is solved by introducing partial deflection, as explained by F.J. Plantema in [2], following the procedure of Goldberg and Leve [1].

2) <u>In-plane displacements of the panel.</u>

This is basically similar to the uniform slab and is rewritten from [1]. The effect of the edge stiffener is included.

Part IV of this paper deals with the practical application of the theory developed in the first two parts. Use is made here of a paper by A. DeFries-Skene, and A.C. Scordelis [3], and of the work done by P.P. Fazio and J.B. Kennedy [4], where the method of Direct Stiffness is employed in a computer analysis of Folded Plates.

With the edge force - edge displacement relationship developed in Chapt.II and III an element stiffness matrix can be assembled in the local coordinate system for each harmonic in turn.

After transformation into the global coordinate system, all the element stiffness matrices of the Folded Plate structure, can be combined into the general stiffness matrix for the harmonic being analysed.

The fixed edge forces for pressure loading, and the forces directly applied to the joints, are resolved into their Fourier Series components, and listed in the joint force vector.

The joint displacements,and from these the edge displacements, and forces, and also the internal displacement forces and stresses, can now be found by multiplying the joint force vector by the inverse of the general stiffness matrix. Taking each harmonic in turn, and adding the calculated displacements, forces and stresses for as many harmonics, as is sufficient for the required accuracy, gives the final results for the Folded Plate structure.

To verify the theory, an example from the experimental work by P.P. Fazio and J.B. Kennedy [4], was run on the computer at Sir George Williams University. A program developed by P.P. Fazio and modified to include the Elasticity Method, as derived in this paper, was used and the computed results were compared with the experimental results.

An Appendix was added to derive fixed edge forces for linearly varying and uniform pressure loading.

CHAPTER II

OUT OF PLANE DISPLACEMENTS OF THE PANEL

II OUT OF PLANE DISPLACEMENTS OF THE PANEL

General Solution

The behaviour of an isotropic sandwich panel, Fig. 2, with arbitrary boundary conditions along the longitudinal joints, can best be analyzed by the method of partial deflections[2]. In this approach, the total deflection w at any point of the panel is represented by

$$w = w_{bx} + w_{sx} = w_{by} + w_{sy} \tag{1}$$

w_{bx} = deflection due to bending moment, M_x

w_{sx} = deflection due to shear force, Q_x

w_{by} = deflection due to bending moment, M_y

w_{sy} = deflection due to shear force , Q_y

The generalized stress displacement relationships[2] are (Fig.3)

$$M_x = -D\left(\frac{\partial^2 w_{bx}}{\partial x^2} + \nu \frac{\partial^2 w_{by}}{\partial y^2}\right) \tag{2}$$

$$M_y = -D\left(\frac{\partial^2 w_{by}}{\partial y^2} + \nu \frac{\partial^2 w_{bx}}{\partial x^2}\right) \tag{3}$$

$$M_{xy} = -M_{yx} = \frac{1-\nu}{2} D \frac{\partial^2 (w_{bx} + w_{by})}{\partial x \, \partial y} \tag{4}$$

$$Q_x = S \frac{\partial w_{sx}}{\partial x} \tag{5}$$

$$Q_y = S \frac{\partial w_{sy}}{\partial y} \tag{6}$$

where D and S are respectively, the bending and

FIGURES 2, 3 AND 4

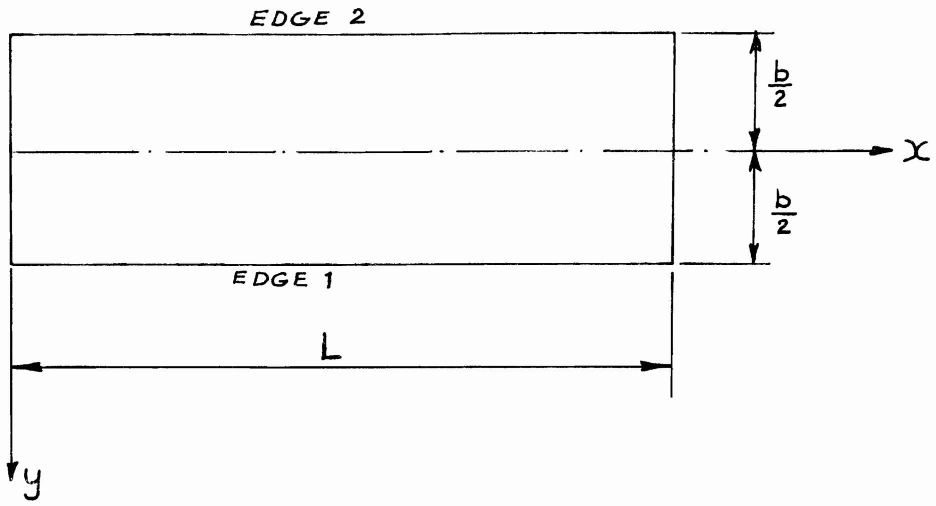


FIG. 2 SANDWICH PANEL GEOMETRY

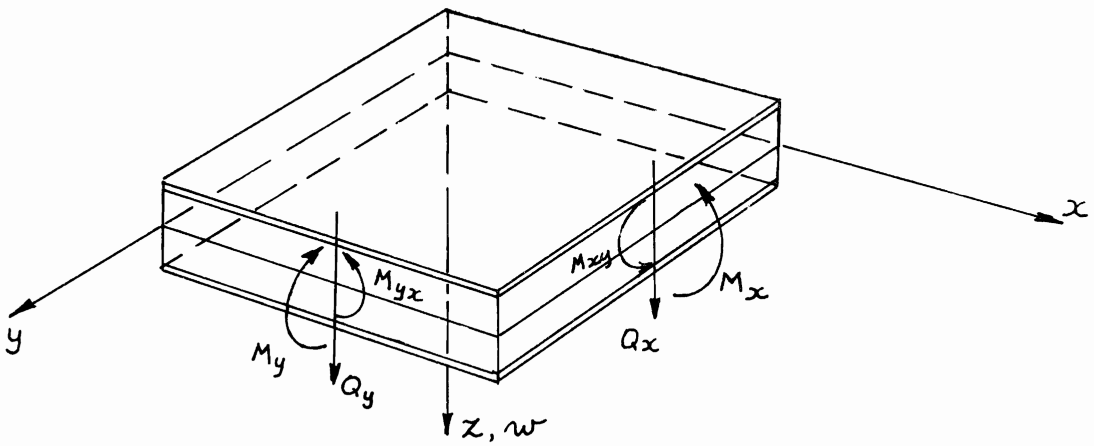


FIG. 3 SIGN CONVENTION OF GENERALIZED STRESSES

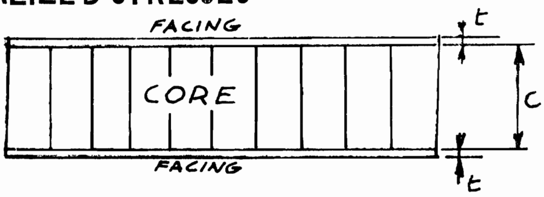


FIG 4 CROSS SECTION OF A SANDWICH

shear stiffness coefficients for a sandwich panel.

$$\left.\begin{aligned} D &= \frac{1}{2}\,\frac{E_f\,t}{1-\nu^2}\,(c+t)^2 \\ S &= \frac{(c+t)^2}{c}\,G_c \end{aligned}\right\} \quad \text{see Fig.4.} \qquad (7)$$

The partial deflection w_{by} can be expressed in the Fourier series form

$$w_{by} = \sum_{m=1,3,5\ldots}^{\infty} W \sin\frac{m\pi x}{L} \qquad (8)$$

admitting only the m th term

$$w_{by}^{m} = W^{m} \sin\frac{m\pi x}{L}$$

where W^m is a function of y only.

Keeping always in mind that the following theory deals with the m th term, we can delete the superscript m . As a result,

$$w_{by} = W \sin\frac{m\pi x}{L} \qquad (9)$$

For an isotropic sandwich plate, the following characteristic equation exists

$$\frac{\partial^2}{\partial x \partial y}\left[\frac{1-\nu}{2}\, D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(w_{bx} - w_{by}\right) - S\left(w_{bx} - w_{by}\right)\right] = 0 \qquad (10)$$

This equation can be solved by substituting

$$\left(w_{bx} - w_{by}\right) = \sin\frac{m\pi x}{L} \sum_{i=1}^{3} W_i\, e^{\alpha_i y} \qquad (11)$$

where W_i and α_i are constants.

The roots of the characteristic equation are

$$\alpha_i = 0 \quad \text{and} \quad \alpha_i = \pm\alpha$$

where $$\alpha^2 = \left(\frac{m\pi}{L}\right)^2 + \frac{2S}{(1-\nu)D} \tag{12}$$

Now,

$$w_{bx} - w_{by} = \left(W_1 + W_2 \cosh\alpha y + W_3 \sinh\alpha y\right) \sin\frac{m\pi x}{L} \tag{12a}$$

then

$$w_{bx} = \left(W + W_1 + W_2 \cosh\alpha y + W_3 \sinh\alpha y\right) \sin\frac{m\pi x}{L} \tag{13}$$

The following relationships between shear deflections and bending deflections exist [2] :

$$-S\frac{\partial w_{sx}}{\partial x} = D\left(\frac{\partial^3 w_{bx}}{\partial x^3} + \nu\frac{\partial^3 w_{by}}{\partial x \partial y^2}\right) + \frac{1-\nu}{2} D \frac{\partial^3 (w_{bx} + w_{by})}{\partial x \partial y^2} \tag{14}$$

$$-S\frac{\partial w_{sy}}{\partial y} = D\left(\frac{\partial^3 w_{by}}{\partial y^3} + \nu\frac{\partial^3 w_{bx}}{\partial x^2 \partial y}\right) + \frac{1-\nu}{2} D \frac{\partial^3 (w_{bx} + w_{by})}{\partial x^2 \partial y} \tag{15}$$

Integration of these equations after substitution of equations (9) and (13) yield

$$w_{sx} = \frac{D}{S}\left[\left(\frac{m\pi}{L}\right)^2 (W + W_1) - \frac{d^2 W}{dy^2} + \left\{\frac{1+\nu}{2}\left(\frac{m\pi}{L}\right)^2 - \frac{S}{D}\right\}\left(W_2 \cosh\alpha y + W_3 \sinh\alpha y\right)\right] \sin\frac{m\pi x}{L} \tag{16}$$

$$w_{sy} = \frac{D}{S}\left[\left(\frac{m\pi}{L}\right)^2 W - \frac{d^2 W}{dy^2} + \left\{\frac{S}{D} + \left(\frac{m\pi}{L}\right)^2\right\} W_1 + \frac{1+\nu}{2}\left(\frac{m\pi}{L}\right)^2 \left(W_2 \cosh\alpha y + W_3 \sinh\alpha y\right)\right] \sin\frac{m\pi x}{L} \tag{17}$$

$$w = \frac{D}{S}\left[\left\{\frac{S}{D} + \left(\frac{m\pi}{L}\right)^2\right\}(W + W_1) - \frac{d^2 W}{dy^2} + \frac{1+\nu}{2}\left(\frac{m\pi}{L}\right)^2 \left(W_2 \cosh\alpha y + W_3 \sinh\alpha y\right)\right] \sin\frac{m\pi x}{L} \tag{18}$$

The arbitrary functions f(y) and g(x) permitted by integration of equations (14) and (15) have been omitted for the reason that on substitution in Eq. (1), they have to be zero, in order to satisfy Eq. (1). The solution for the partial deflections now contain three constants, W_1, W_2, W_3.

The general solution for the function W is found by substitution of equations (9) and (13) in governing plate bending equation (19) for zero load,

$$\frac{\partial^4 w_{bx}}{\partial x^4} + \frac{\partial^4 (w_{bx} + w_{by})}{\partial x^2 \partial y^2} + \frac{\partial^4 w_{by}}{\partial y^4} = 0 \qquad (19)$$

resulting in

$$\frac{d^4 W}{d y^4} - 2\left(\frac{m\pi}{L}\right)^2 \frac{d^2 W}{d y^2} + \left(\frac{m\pi}{L}\right)^4 W = -\left(\frac{m\pi}{L}\right)^4 W_1 + \frac{2}{1-\nu} \frac{S}{D}\left(\frac{m\pi}{L}\right)^2 \left(W_2 \cosh\alpha y + W_3 \sinh\alpha y\right) \qquad (20)$$

The general solution for equation (20) is

$$W = C_1 \sinh \frac{m\pi y}{L} + C_2 \cosh \frac{m\pi y}{L} + \frac{m\pi y}{L}\left(C_3 \sinh \frac{m\pi y}{L} + C_4 \cosh \frac{m\pi y}{L}\right) + \frac{1-\nu}{2}\left(\frac{m\pi}{L}\right)^2 \frac{D}{S}\left(W_2 \cosh\alpha y + W_3 \sinh\alpha y\right) - W_1 \qquad (21)$$

C_1, C_2, C_3 and C_4 are constants.

On substitution of this result into equations (9), (13), (16), (17) and (18), it is found that the constant W_1 disappears from the expressions for w_{bx}, w_{sx} and w. It also disappears on differentiation of w_{sy}, with respect to y and therefore, has no physical significance, and can be set equal to zero.

The equations for the partial deflections then become

$$w_{by} = \left[C_1 \sinh\frac{m\pi y}{L} + C_2 \cosh\frac{m\pi y}{L} + \frac{m\pi y}{L}\left(C_3 \sinh\frac{m\pi y}{L} + C_4 \cosh\frac{m\pi y}{L}\right) + k_s\left(W_2 \cosh\alpha y + W_3 \sinh\alpha y\right)\right] \sin\frac{m\pi x}{L} \qquad (22)$$

$$w_{bx} = \left[C_1 \sinh\frac{m\pi y}{L} + C_2 \cosh\frac{m\pi y}{L} + \frac{m\pi y}{L}\left(C_3 \sinh\frac{m\pi y}{L} + C_4 \cosh\frac{m\pi y}{L}\right) + (1 + k_s)\left(W_2 \cosh\alpha y + W_3 \sinh\alpha y\right)\right] \sin\frac{m\pi x}{L} \qquad (23)$$

where $k_s = \frac{1-\nu}{2} \frac{D}{S}\left(\frac{m\pi}{L}\right)^2$ (24)

Then $\alpha = \left(1 + \frac{1}{k_s}\right)^{1/2} \frac{m\pi}{L}$ (24a)

$$w_{sx} = -\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(C_3 \cosh\frac{m\pi y}{L} + C_4 \sinh\frac{m\pi y}{L}\right) + (1 + k_s)\left(W_2 \cosh\alpha y + W_3 \sinh\alpha y\right) \sin\frac{m\pi x}{L} \qquad (25)$$

$$w_{sy} = -\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(C_3 \cosh\frac{m\pi y}{L} + C_4 \sinh\frac{m\pi y}{L}\right) + k_s\left(W_2 \cosh\alpha y + W_3 \sinh\alpha y\right) \sin\frac{m\pi x}{L} \qquad (26)$$

$$w = C_1 \sinh\frac{m\pi y}{L} + C_2 \cosh\frac{m\pi y}{L} + C_3\left(\frac{m\pi y}{L} \sinh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 \cosh\frac{m\pi y}{L}\right) + C_4\left(\frac{m\pi y}{L} \cosh\frac{m\pi y}{L} - 2\frac{D}{S}\left(\frac{m\pi}{L}\right)^2 \sinh\frac{m\pi y}{L}\right) \sin\frac{m\pi x}{L} \qquad (27)$$

The general boundary condition

is readily satisfied by (27).

We need 3 boundary conditions on each of the longitudinal edges, in order to determine the 6 constants C_1, C_2, C_3, C_4, W_2 and W_3.

Two boundary conditions exist for the slope and displacements of each edge. The third one follows if we assume the presence of edge stiffeners, i.e., no shear deformation along the edges, then

$$\frac{\partial w_{sx}}{\partial x} = 0 \quad , \quad y = \pm \frac{b}{2} \tag{i}$$

making use of Equation (25) and the boundary condition (i), we can obtain the following expressions for W_2 and W_3:

In the symmetric case $C_1 = C_4 = W_3 = 0$

$$W_2 = -\frac{2D}{(1+k_s)S}\left(\frac{m\pi}{L}\right)^2 \frac{\cosh\beta}{\cosh\frac{\alpha b}{2}} C_3 \tag{28}$$

In the antisymmetric case $C_2 = C_3 = W_2 = 0$

$$W_3 = -\frac{2D}{(1+k_s)S}\left(\frac{m\pi}{L}\right)^2 \frac{\sinh\beta}{\sinh\frac{\alpha b}{2}} C_4 \tag{29}$$

where $$\beta = \frac{m\pi b}{2L} \tag{30}$$

Now the partial deflection can be expressed in terms of the unknown constants C_1, C_2, C_3 & C_4 only

$$W_{by} = \Big[C_1 \sinh\frac{m\pi y}{L} + C_2 \cosh\frac{m\pi y}{L} \tag{31}$$
$$+ C_3\left\{\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{k_s}{1+k_s}\,\frac{\cosh\beta}{\cosh\frac{\alpha b}{2}}\cosh\alpha y\right\}$$
$$+ C_4\left\{\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{k_s}{1+k_s}\,\frac{\sinh\beta}{\sinh\frac{\alpha b}{2}}\sinh\alpha y\right\}\Big]\sin\frac{m\pi x}{L}$$

$$W_{bx} = \Big[C_1 \sinh\frac{m\pi y}{L} + C_2 \cosh\frac{m\pi y}{L} \tag{32}$$
$$+ C_3\left\{\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{\cosh\beta}{\cosh\frac{\alpha b}{2}}\cosh\alpha y\right\}$$
$$+ C_4\left\{\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{\sinh\beta}{\sinh\frac{\alpha b}{2}}\sinh\alpha y\right\}\Big]\sin\frac{m\pi x}{L}$$

$$W_{sy} = -\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\Big[C_3\left\{\cosh\frac{m\pi y}{L} - \frac{k_s}{1+k_s}\,\frac{\cosh\beta}{\cosh\frac{\alpha b}{2}}\cosh\alpha y\right\} \tag{33}$$
$$+ C_4\left\{\sinh\frac{m\pi y}{L} - \frac{k_s}{1+k_s}\,\frac{\sinh\beta}{\sinh\frac{\alpha b}{2}}\sinh\alpha y\right\}\Big]\sin\frac{m\pi x}{L}$$

$$W_{sx} = -\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\Big[C_3\left\{\cosh\frac{m\pi y}{L} - \frac{\cosh\beta}{\cosh\frac{\alpha b}{2}}\cosh\alpha y\right\} \tag{34}$$
$$+ C_4\left\{\sinh\frac{m\pi y}{L} - \frac{\sinh\beta}{\sinh\frac{\alpha b}{2}}\sinh\alpha y\right\}\Big]\sin\frac{m\pi x}{L}$$

The sandwich plate problem may now be solved by imposing boundary conditions for the longitudinal edges of the panel, edges 1 and 2. To facilitate the procedure, the problem is separated into two parts.

a) Rotation of edges with no translation.

b) Translation of edges with no rotation.

Furthermore, each part of the problem will be subdivided into a symmetric and an antisymmetric case.

FIGURES 5 AND 6

5) SYMMETRIC EDGE ROTATIONS

6) ANTISYMMETRIC EDGE ROTATIONS

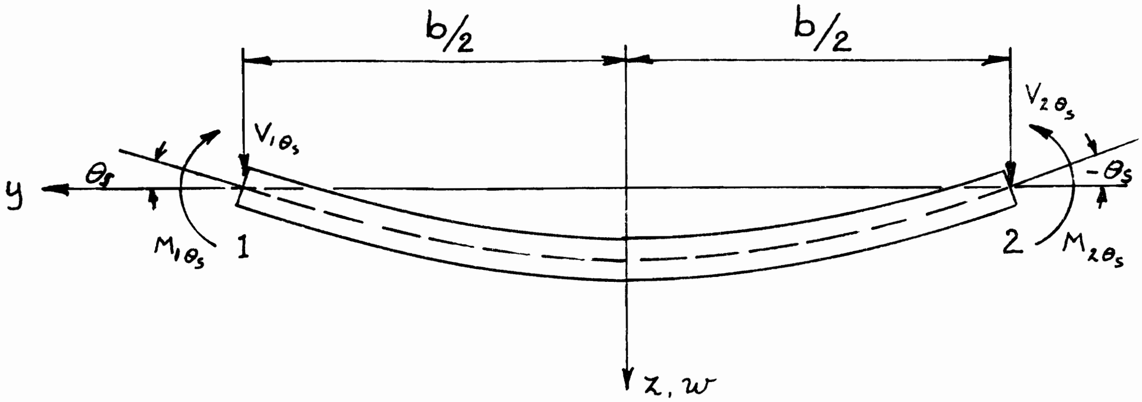


FIG. 5 SYMMETRIC EDGE ROTATIONS

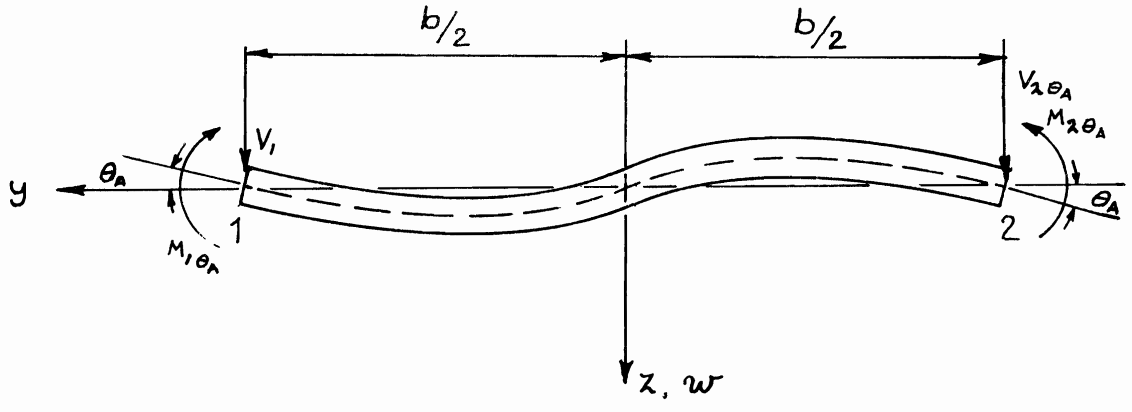


FIG.6 ANTISYMMETRIC EDGE ROTATIONS

(A) Rotation of edges 1 and 2

Boundary Conditions:

$$w = 0 \quad , \quad y = \pm \frac{b}{2} \tag{ii}$$

Assume

$$\left.\begin{aligned} \left(\frac{\partial w_{by}}{\partial y}\right)_{y=\frac{b}{2}} &= \theta_1 \sin\frac{m\pi x}{L} \\ \left(\frac{\partial w_{by}}{\partial y}\right)_{y=-\frac{b}{2}} &= \theta_2 \sin\frac{m\pi x}{L} \end{aligned}\right\} \tag{iii}$$

Symmetric Case (Fig.5)

$$\left(\frac{\partial w_{by}}{\partial y}\right)_{y=\frac{b}{2}} = \left(\frac{\partial w_{by}}{\partial y}\right)_{y=-\frac{b}{2}}$$

For symmetry w must be an even function of y ; $C_1 = C_4 = 0$

$$w = \left[C_2 \cosh\frac{m\pi y}{L} + C_3\left(\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\cosh\frac{m\pi y}{L}\right)\right]\sin h\frac{m\pi x}{L}$$

From B.C. (ii) , it follows that

$$C_2 = -C_3\left(\beta\tanh\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)$$

and from equation (31)

$$w_{by} = C_3\left[\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{k_s}{1+k_s}\frac{\cosh\beta}{\cosh\frac{\alpha b}{2}}\cosh\alpha y - \left(\beta\tanh\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)\cosh\frac{m\pi y}{L}\right]\sin\frac{m\pi x}{L}$$

$$\frac{\partial w_{by}}{\partial y} = C_3\left(\frac{m\pi}{L}\right)\left[\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} + \sinh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(\frac{k_s}{1+k_s}\right)^{1/2}\frac{\cosh\beta}{\cosh\frac{\alpha b}{2}}\sinh\alpha y - \left(\beta\tanh\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)\sinh\frac{m\pi y}{L}\right]\sin\frac{m\pi x}{L}$$

For the symmetrical case, the rotations along the edges $y = \pm\frac{b}{2}$ are taken from B.C. (iii)

$$\theta_s = \bar{\theta}_s \sin\frac{m\pi x}{L}$$

Then

$$C_3 = \left(\frac{L}{m\pi}\right)\left[\beta \operatorname{sech}\beta + \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\tanh\beta p}{p\tanh\beta}\right)\right\}\sinh\beta\right]^{-1}\bar{\theta}_s$$

where sub.s indicates symmetry and a bar denotes maximum amplitude of harmonic m. The expression for C_3 can be written as follows

$$C_3 = \left(\frac{L}{m\pi}\right)\lambda_1\,\bar{\theta}_s$$

where

$$\lambda_1 = \left[\beta \operatorname{sech}\beta + \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\tanh\beta p}{p\tanh\beta}\right)\right\}\sinh\beta\right]^{-1} \qquad (35)$$

and $\quad p = \left(1 + \frac{1}{k_s}\right)^{1/2}, \quad \beta p = \frac{\alpha b}{2}$

<u>Antisymmetric Case (Fig.6)</u>

$$\left(\frac{\partial w_{by}}{\partial y}\right)_{y=\frac{b}{2}} = \left(\frac{\partial w_{by}}{\partial y}\right)_{y=-\frac{b}{2}}$$

For antisymmetry w must be an odd function of y : $C_2 = C_3 = 0$

$$w = \left[C_1 \sinh\frac{m\pi y}{L} + C_4\left(\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\sinh\frac{m\pi y}{L}\right)\right]\sin\frac{m\pi x}{L}$$

From B.C. (ii) : $\quad C_1 = -C_4\left(\beta\coth\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)$

$$w_{by} = C_4\left[\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{k_s}{1+k_s}\frac{\sinh\beta}{\sinh\frac{\alpha b}{2}}\sinh\alpha y - \left(\beta\coth\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)\sinh\frac{m\pi y}{L}\right]\sin\frac{m\pi x}{L}$$

$$\frac{\partial w_{by}}{\partial y} = C_4\left(\frac{m\pi}{L}\right)\left[\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} + \cosh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(\frac{k_s}{1+k_s}\right)^{1/2}\frac{\sinh\beta}{\sinh\frac{\alpha b}{2}}\cosh\alpha y - \left(\beta\coth\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)\cosh\frac{m\pi y}{L}\right]\sin\frac{m\pi x}{L}$$

For the antisymmetrical case, the rotations along the edges $y = \pm \frac{b}{2}$ are taken as

B.C. (iii): $\theta_A = \bar{\theta}_A \sin \frac{m\pi x}{L}$

$$C_4 = -\frac{L}{m\pi}\left[\beta \operatorname{csch}\beta - \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\coth\beta p}{p\coth\beta}\right)\right\}\cosh\beta\right]^{-1}\bar{\theta}_A$$

where subscript A indicates antisymmetry, the expression for C_4 can be written as follows

$$C_4 = -\left(\frac{L}{m\pi}\right)\lambda_2 \bar{\theta}_A$$

where

$$\lambda_2 = \left[\beta \operatorname{csch}\beta - \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\coth\beta p}{p\coth\beta}\right)\right\}\cosh\beta\right]^{-1} \quad (36)$$

Substituting the expressions for the constants C_1, C_2, C_3 & C_4 into equation (27) and combining symmetric and antisymmetric cases by substitution of

$\bar{\theta}_S = \frac{1}{2}\left(\bar{\theta}_1 - \bar{\theta}_2\right)$ for symmetric case

and $\bar{\theta}_A = \frac{1}{2}\left(\bar{\theta}_1 + \bar{\theta}_2\right)$ for antisymmetric case,

the following expression for the total deflection due to edge rotation is obtained, and is expressed in matrix notation.

$$w = \frac{1}{2}\left(\frac{L}{m\pi}\right)\sin\frac{m\pi x}{L}\lfloor \bar{\theta}_1 \ \bar{\theta}_2 \rfloor \left[W_\theta\right]\left\{H_y\right\} \quad (37a)$$

where $$\left[W_\theta\right] = \begin{bmatrix} \lambda_1 & -\lambda_1 k_t & -\lambda_2 & \lambda_2 k_c \\ -\lambda_1 & \lambda_1 k_t & -\lambda_2 & \lambda_2 k_c \end{bmatrix}$$

$$K_t = \beta \tanh \beta$$

$$K_c = \beta \coth \beta$$

$$\{H_y\} = \begin{Bmatrix} \frac{m\pi y}{L} \sinh \frac{m\pi y}{L} \\ \cosh \frac{m\pi y}{L} \\ \frac{m\pi y}{L} \sinh \frac{m\pi y}{L} \\ \sinh \frac{m\pi y}{L} \end{Bmatrix}$$

Similarly from equations (33) and (34)

$$W_{sx} = \frac{D}{S} \left(\frac{m\pi}{L}\right) \sin \frac{m\pi x}{L} \lfloor \bar{\theta}_1 \ \bar{\theta}_2 \rfloor [S_\theta] \left(\{h_y\} - \{h_{\alpha y}\}\right) \quad (37b)$$

$$W_{sy} = \frac{D}{S} \left(\frac{m\pi}{L}\right) \sin \frac{m\pi x}{L} \lfloor \bar{\theta}_1 \ \bar{\theta}_2 \rfloor [S_\theta] \left(\{h_y\} - \frac{1}{p^2}\{h_{\alpha y}\}\right) \quad (37c)$$

where

$$[S_\theta] = \begin{bmatrix} -\lambda_1 & \lambda_2 \\ \lambda_1 & \lambda_2 \end{bmatrix}$$

$$\{h_y\} = \begin{Bmatrix} \cosh \frac{m\pi y}{L} \\ \sinh \frac{m\pi y}{L} \end{Bmatrix}$$

$$\{h_{\alpha y}\} = \begin{Bmatrix} \frac{\cosh \beta}{\cosh \beta p} \cosh \alpha y \\ \frac{\sinh \beta}{\sinh \beta p} \sinh \alpha y \end{Bmatrix}$$

Furthermore,

$$W_{bx} = W - W_{sx} = \frac{1}{2}\left(\frac{L}{m\pi}\right) \sin \frac{m\pi x}{L} \lfloor \bar{\theta}_1 \ \bar{\theta}_2 \rfloor \times \left([W_\theta]\{H_y\} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_\theta]\left(\{h_y\} - \{h_{\alpha y}\}\right)\right) \quad (37d)$$

$$w_{by} = w - w_{sy} = \frac{1}{2}\left(\frac{L}{m\pi}\right)\sin\frac{m\pi x}{L}\lfloor\bar{\theta}_1 \quad \bar{\theta}_2\rfloor \times \tag{37e}$$

$$\left([W_\theta]\{H_y\} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2[S_\theta]\left(\{h_y\} - \frac{1}{p^2}\{h_{\alpha y}\}\right)\right)$$

To obtain expressions for the generalized stresses, the equations for the partial displacements are substituted into eequations (2) to (6), giving the following results:

$$M_x = \frac{1-\nu}{2}D\left(\frac{m\pi}{L}\right)\sin\frac{m\pi x}{L}\lfloor\bar{\theta}_1 \quad \bar{\theta}_2\rfloor\left([W_\theta]\{H_y\} + \left(\frac{2\nu}{1-\nu} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)[S_\theta]\{h_y\}\right.$$
$$\left. + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2[S_\theta]\{h_{\alpha y}\}\right) \tag{38a}$$

$$M_y = -\frac{1-\nu}{2}D\left(\frac{m\pi}{L}\right)\sin\frac{m\pi x}{L}\lfloor\bar{\theta}_1 \quad \bar{\theta}_2\rfloor\left([W_\theta]\{H_y\} - \left(\frac{2}{1-\nu} + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)[S_\theta]\{h_y\}\right.$$
$$\left. + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2[S_\theta]\{h_{\alpha y}\}\right) \tag{38b}$$

$$M_{xy} = -M_{yx} = \frac{1-\nu}{2}D\left(\frac{m\pi}{L}\right)\cos\frac{m\pi x}{L}\lfloor\bar{\theta}_1 \quad \bar{\theta}_2\rfloor\left([W_\theta]\{H'_y\}\right.$$
$$\left. - \left(1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)[S_\theta]\{h'_y\} + \frac{D}{S}\left(\frac{m\pi}{L}\right)^2\left(p + \frac{1}{p}\right)[S_\theta]\{h'_{\alpha y}\}\right) \tag{38c}$$

$$Q_x = D\left(\frac{m\pi}{L}\right)^2\cos\frac{m\pi x}{L}\lfloor\bar{\theta}_1 \quad \bar{\theta}_2\rfloor[S_\theta]\left(\{h_y\} - \{h_{\alpha y}\}\right) \tag{38d}$$

$$Q_y = D\left(\frac{m\pi}{L}\right)^2\sin\frac{m\pi x}{L}\lfloor\bar{\theta}_1 \quad \bar{\theta}_2\rfloor[S_\theta]\left(\{h'_y\} - \frac{1}{p}\{h'_{\alpha y}\}\right) \tag{38e}$$

where

$$\{H'_y\} = \left\{\begin{matrix} \frac{m\pi y}{L}\cosh\frac{m\pi y}{L} \\ \sinh\frac{m\pi y}{L} \\ \frac{m\pi y}{L}\sinh\frac{m\pi y}{L} \\ \cosh\frac{m\pi y}{L} \end{matrix}\right\}$$

$$\{h'_y\} = \begin{Bmatrix} \sinh \frac{m\pi y}{L} \\ \cosh \frac{m\pi y}{L} \end{Bmatrix}$$

$$\{h'_{\alpha y}\} = \begin{Bmatrix} \frac{\cosh\beta}{\cosh\beta p} \cosh \alpha y \\ \frac{\sinh\beta}{\sinh\beta p} \sinh \alpha y \end{Bmatrix}$$

Forces along edges 1 and 2 due to edge rotations

$$M_{1\theta} = -(M_y)_{y=\frac{b}{2}} = \frac{1-\nu}{2} D\left(\frac{m\pi}{L}\right) \sin\frac{m\pi x}{L} \lfloor \bar{\theta}_1\ \bar{\theta}_2 \rfloor \times \tag{39a}$$

$$\left([W_\theta] \begin{Bmatrix} \beta \sinh\beta \\ \cosh\beta \\ \beta\cosh\beta \\ \sinh\beta \end{Bmatrix} - \frac{2}{1-\nu} [S_\theta] \begin{Bmatrix} \cosh\beta \\ \sinh\beta \end{Bmatrix} \right)$$

$$M_{2\theta} = (M_y)_{y=-\frac{b}{2}} = -\frac{1-\nu}{2} D\left(\frac{m\pi}{L}\right) \sin\frac{m\pi x}{L} \lfloor \bar{\theta}_1\ \bar{\theta}_2 \rfloor \times \tag{39b}$$

$$\left([W_\theta] \begin{Bmatrix} \beta \sinh\beta \\ \cosh\beta \\ -\beta\cosh\beta \\ -\sinh\beta \end{Bmatrix} - \frac{2}{1-\nu} [S_\theta] \begin{Bmatrix} \cosh\beta \\ -\sinh\beta \end{Bmatrix} \right)$$

$$V_{1\theta} = \left(Q_y + \frac{M_{yx}}{\partial x}\right)_{y=\frac{b}{2}} = \frac{1-\nu}{2} D\left(\frac{m\pi}{L}\right)^2 \sin\frac{m\pi x}{L} \lfloor \bar{\theta}_1\ \bar{\theta}_2 \rfloor \times$$

$$\left([W_\theta] \begin{Bmatrix} \beta \cosh\beta \\ \sinh\beta \\ \beta\sinh\beta \\ \cosh\beta \end{Bmatrix} + \left(\frac{1+\nu}{1-\nu} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)[S_\theta] \begin{Bmatrix} \sinh\beta \\ \cosh\beta \end{Bmatrix} + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_\theta] \begin{Bmatrix} \frac{\tanh\beta p}{p\tanh\beta} \sinh\beta \\ \frac{\coth\beta p}{p\coth\beta} \cosh\beta \end{Bmatrix} \right) \tag{39c}$$

$$V_{2\theta} = -\left(Q_y + \frac{M_{yx}}{\partial x}\right)_{y=-\frac{b}{2}} = -\frac{1-\nu}{2} D\left(\frac{m\pi}{L}\right)^2 \sin\frac{m\pi x}{L} \lfloor \bar{\theta}_1\ \bar{\theta}_2 \rfloor \times$$

$$\left([W_\theta] \begin{Bmatrix} -\beta \cosh\beta \\ -\sinh\beta \\ \beta\sinh\beta \\ \cosh\beta \end{Bmatrix} + \left(\frac{1+\nu}{1-\nu} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)[S_\theta] \begin{Bmatrix} -\sinh\beta \\ \cosh\beta \end{Bmatrix} + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_\theta] \begin{Bmatrix} -\frac{\tanh\beta p}{p\tanh\beta} \sinh\beta \\ \frac{\coth\beta p}{p\coth\beta} \cosh\beta \end{Bmatrix} \right) \tag{39d}$$

FIGURES 7 AND 8

7) SYMMETRIC NORMAL EDGE TRANSLATIONS

8) ANTISYMMETRIC NORMAL EDGE TRANSLATIONS

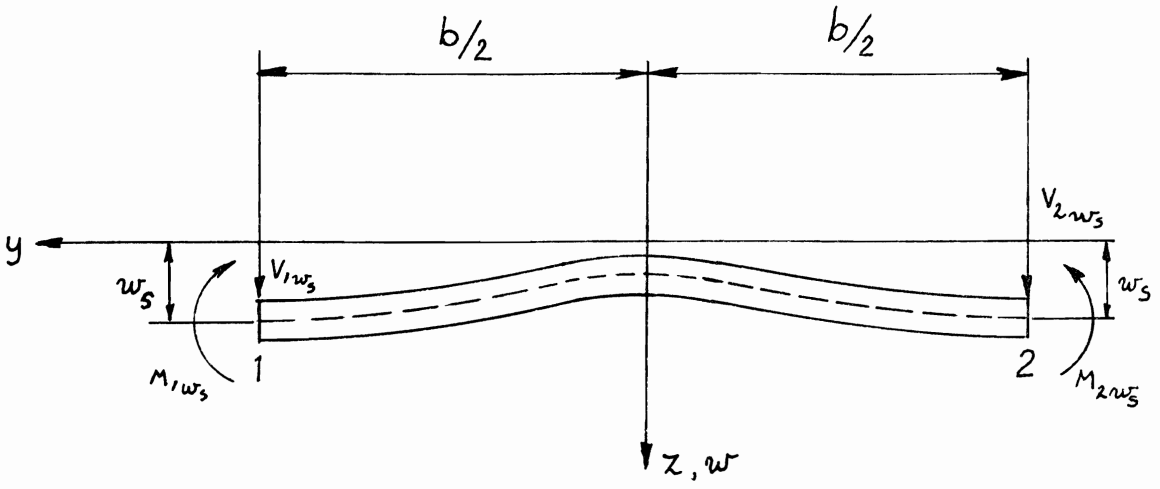


FIG. 7 SYMMETRIC NORMAL EDGE TRANSLATIONS

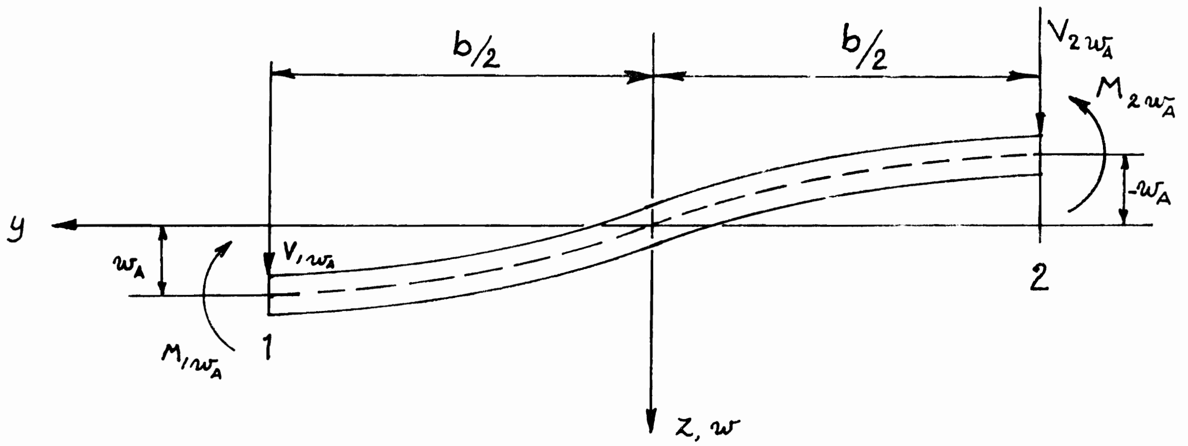


FIG. 8 ANTISYMMETRIC NORMAL EDGE TRANSLATIONS

(B) Translation of edges 1 and 2 (Normal to Panel)

Boundary Conditions

$$\frac{\partial w_{by}}{\partial y} = 0 \qquad \text{for} \quad y = \pm \frac{b}{2} \qquad \text{(iv)}$$

$$(w)_{y=\frac{b}{2}} = \bar{w}_1 \sin \frac{m\pi x}{L}$$

$$(w)_{y=-\frac{b}{2}} = \bar{w}_2 \sin \frac{m\pi x}{L} \qquad \text{(v)}$$

Symmetrical Case (Fig. 7)

$$(w)_{y=\frac{b}{2}} = (w)_{y=-\frac{b}{2}}$$

For symmetry w must be an even function of y

$$C_1 = C_4 = 0$$

$$\frac{\partial w_{by}}{\partial y} = \left(\frac{m\pi}{L}\right)\left[C_2 \sinh \frac{m\pi y}{L} + C_3 \left\{\frac{m\pi y}{L} \cosh \frac{m\pi y}{L} + \sinh \frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 \frac{\cosh\beta}{p\cosh\beta p} \sinh \alpha y\right\}\right] \sin \frac{m\pi x}{L}$$

From B.C. (iv)

$$C_2 = -C_3 \left\{\beta \coth\beta + 1 - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 \frac{\tanh\beta p}{p\tanh\beta}\right\}$$

Substituting for the constants C_1, C_2, C_3 in equation (27),

$$w = C_3 \left[\frac{m\pi y}{L} \sinh \frac{m\pi y}{L} - \left\{\beta\coth\beta + 1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\tanh\beta p}{p\tanh\beta}\right)\right\} \cosh \frac{m\pi y}{L}\right] \sin \frac{m\pi x}{L}$$

For the symmetric case, the normal displacements along the edges $y=\pm\frac{b}{2}$ are taken as

$$w_s = \bar{w}_s \sin\frac{m\pi x}{L}$$

From B.C. (v) , it follows that

$$C_3 = -\left[\beta\,\mathrm{csch}\,\beta + \left\{1+\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1-\frac{\tanh\beta p}{p\tanh\beta}\right)\right\}\cosh\beta\right]^{-1}\bar{w}_s$$

or $C_3 = -\lambda_3\,\bar{w}_s$

where $$\lambda_3 = \left[\beta\,\mathrm{csch}\,\beta + \left\{1+\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1-\frac{\tanh\beta p}{p\tanh\beta}\right)\right\}\cosh\beta\right]^{-1} \quad (40)$$

Antisymmetric Case (Fig. 8)

$$(w)_{y=\frac{b}{2}} = -(w)_{y=-\frac{b}{2}}$$

For antisymmetry w must be an odd function of y

$$C_2 = C_3 = 0$$

$$\frac{\partial w_{by}}{\partial y} = \frac{m\pi}{L}\left[C_1\cosh\frac{m\pi y}{L} + C_4\left\{\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} + \cosh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{\sinh\beta}{p\sinh\beta p}\cosh\alpha y\right\}\right]\sin\frac{m\pi x}{L}$$

From B.C. (iv)

$$C_1 = -C_4\left\{\beta\tanh\beta + 1 - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{\coth\beta p}{p\coth\beta}\right\}$$

$$w = C_4\left[\left(\frac{m\pi y}{L}\right)\cosh\frac{m\pi y}{L} - \left\{\beta\tanh\beta + 1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1-\frac{\coth\beta p}{p\coth\beta}\right)\right\}\sinh\frac{m\pi y}{L}\right]\sin\frac{m\pi x}{L}$$

For the antisymmetric case, the normal displacements along the edges $y=\pm\frac{b}{2}$ are taken as

$$w_A = \bar{w}_A \sin\frac{m\pi x}{L}$$

From B.C. (v) , it follows that

$$C_4 = \left[\beta \operatorname{sech}\beta - \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\coth\beta p}{p\coth\beta}\right)\right\}\sinh\beta\right]^{-1}\bar{w}_A$$

$$\text{or} \quad C_4 = \lambda_4 \bar{w}_A$$

where $$\lambda_4 = \left[\beta \operatorname{sech}\beta - \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\coth\beta p}{p\coth\beta}\right)\right\}\sinh\beta\right]^{-1} \quad (41)$$

Superposition of symmetric and antisymmetric cases, after the following substitution

Symmetric case: $\bar{w}_S = \frac{1}{2}\left(\bar{w}_1 + \bar{w}_2\right)$

Antisymmetric Case: $\bar{w}_A = \frac{1}{2}\left(\bar{w}_1 - \bar{w}_2\right)$

yields in matrix notation

$$w = \frac{1}{2}\sin\frac{m\pi x}{L}\lfloor \bar{w}_1 \quad \bar{w}_2 \rfloor\left[\left[W_w\right]\left\{H_y\right\} + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left[S_w\right]\left(\left\{h_y\right\} - \left\{h_{ty}\right\}\right)\right] \quad (42a)$$

where $$\left[W_w\right] = \begin{bmatrix} -\lambda_3 & \lambda_3(k_c+1) & \lambda_4 & -\lambda_4(k_t+1) \\ -\lambda_3 & \lambda_3(k_c+1) & -\lambda_4 & \lambda_4(k_t+1) \end{bmatrix}$$

$$\left[S_w\right] = \begin{bmatrix} \lambda_3 & -\lambda_4 \\ \lambda_3 & \lambda_4 \end{bmatrix}, \quad \left\{h_{ty}\right\} = \begin{Bmatrix} \frac{\tanh\beta p}{p\tanh\beta}\cosh\frac{m\pi y}{L} \\ \frac{\coth\beta p}{p\coth\beta}\sinh\frac{m\pi y}{L} \end{Bmatrix}$$

The partial deflections are

$$w_{sx} = \frac{D}{S}\left(\frac{m\pi}{L}\right)^2\sin\frac{m\pi x}{L}\lfloor \bar{w}_1 \quad \bar{w}_2 \rfloor\left[S_w\right]\left(\left\{h_y\right\} - \left\{h_{\alpha y}\right\}\right) \quad (42b)$$

$$w_{sy} = \frac{D}{S}\left(\frac{m\pi}{L}\right)^2\sin\frac{m\pi x}{L}\lfloor \bar{w}_1 \quad \bar{w}_2 \rfloor\left[S_w\right]\left(\left\{h_y\right\} - \frac{1}{p^2}\left\{h_{\alpha y}\right\}\right) \quad (42c)$$

$$w_{bx} = \frac{1}{2} \sin \frac{m\pi x}{L} \lfloor \bar{w}_1 \ \ \bar{w}_2 \rfloor \left[[W_w]\{H_y\} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_w]\left(\{h_{ty}\} - \{h_{\alpha y}\}\right)\right] \tag{42d}$$

$$w_{by} = \frac{1}{2} \sin \frac{m\pi x}{L} \lfloor \bar{w}_1 \ \ \bar{w}_2 \rfloor \left[[W_w]\{H_y\} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_w]\left(\{h_{ty}\} - \frac{1}{p^2}\{h_{\alpha y}\}\right)\right] \tag{42e}$$

Substitution of these into equations (2) to (6) give the generalized stress equations.

$$M_x = \frac{1-\nu}{2} D \left(\frac{m\pi}{L}\right)^2 \sin \frac{m\pi x}{L} \lfloor \bar{w}_1 \ \ \bar{w}_2 \rfloor \times$$
$$\left[[W_w]\{H_y\} + \frac{2\nu}{1-\nu}[S_w]\{h_y\} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_w]\left(\{h_{ty}\} - \{h_{\alpha y}\}\right)\right] \tag{43a}$$

$$M_y = -\frac{1-\nu}{2} D \left(\frac{m\pi}{L}\right)^2 \sin \frac{m\pi x}{L} \lfloor \bar{w}_1 \ \ \bar{w}_2 \rfloor \times$$
$$\left[[W_w]\{H_y\} - \frac{2}{1-\nu}[S_w]\{h_y\} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_w]\left(\{h_{ty}\} - \{h_{\alpha y}\}\right)\right] \tag{43b}$$

$$M_{xy} = -M_{yx} = \frac{1-\nu}{2} D \left(\frac{m\pi}{L}\right)^2 \cos \frac{m\pi x}{L} \lfloor \bar{w}_1 \ \ \bar{w}_2 \rfloor \times$$
$$\left[[W_w]\{H_y'\} - [S_w]\{h_y\} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_w]\left(\{h_{ty}'\} - \frac{1}{2}\left(p + \frac{1}{p}\right)\{h_{\alpha y}'\}\right)\right] \tag{43c}$$

$$Q_x = D\left(\frac{m\pi}{L}\right)^3 \cos \frac{m\pi x}{L} \lfloor \bar{w}_1 \ \bar{w}_2 \rfloor [S_w]\left(\{h_y\} - \{h_{\alpha y}\}\right) \tag{43d}$$

$$Q_y = D\left(\frac{m\pi}{L}\right)^3 \sin \frac{m\pi x}{L} \lfloor \bar{w}_1 \ \ \bar{w}_2 \rfloor [S_w]\left(\{h_y'\} - \frac{1}{p}\{h_{\alpha y}'\}\right) \tag{43e}$$

where

$$\{h_{ty}'\} = \begin{Bmatrix} \dfrac{\tanh \beta p}{p \tanh \beta} \sinh \dfrac{m\pi y}{L} \\[2ex] \dfrac{\coth \beta p}{p \coth \beta} \cosh \dfrac{m\pi y}{L} \end{Bmatrix}$$

Reactions along edges 1 and 2, due to translation normal to the panel

$$M_{1w} = -(M_y)_{y=\frac{b}{2}} = \frac{1-\nu}{2} D\left(\frac{m\pi}{L}\right)^2 \sin\frac{m\pi x}{L} \lfloor \bar{w}_1 \quad \bar{w}_2 \rfloor \times$$
$$\left([W_w] \begin{Bmatrix} \beta\sinh\beta \\ \cosh\beta \\ \beta\cosh\beta \\ \sinh\beta \end{Bmatrix} - \left(\frac{2}{1-\nu} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)[S_w]\begin{Bmatrix} \cosh\beta \\ \sinh\beta \end{Bmatrix} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_w] \begin{Bmatrix} \frac{\tanh\beta p}{p\tanh\beta}\cosh\beta \\ \frac{\coth\beta p}{p\coth\beta}\sinh\beta \end{Bmatrix} \right) \tag{44a}$$

$$M_{2w} = (M_y)_{y=-\frac{b}{2}} = -\frac{1-\nu}{2} D\left(\frac{m\pi}{L}\right)^2 \sin\frac{m\pi x}{L} \lfloor \bar{w}_1 \quad \bar{w}_2 \rfloor \times$$
$$\left([W_w] \begin{Bmatrix} \beta\sinh\beta \\ \cosh\beta \\ -\beta\cosh\beta \\ -\sinh\beta \end{Bmatrix} - \left(\frac{2}{1-\nu} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)[S_w]\begin{Bmatrix} \cosh\beta \\ -\sinh\beta \end{Bmatrix} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 [S_w] \begin{Bmatrix} \frac{\tanh\beta p}{p\tanh\beta}\cosh\beta \\ -\frac{\coth\beta p}{p\coth\beta}\sinh\beta \end{Bmatrix} \right) \tag{44b}$$

$$V_{1w} = \left(Q_x + \frac{\partial M_{yx}}{\partial x}\right)_{y=\frac{b}{2}} = \frac{1-\nu}{2} D\left(\frac{m\pi}{L}\right)^3 \sin\frac{m\pi x}{L} \lfloor \bar{w}_1 \quad \bar{w}_2 \rfloor \times$$
$$\left([W_w] \begin{Bmatrix} \beta\cosh\beta \\ \sinh\beta \\ \beta\sinh\beta \\ \cosh\beta \end{Bmatrix} + \frac{1+\nu}{1-\nu}[S_w]\begin{Bmatrix} \sinh\beta \\ \cosh\beta \end{Bmatrix} \right) \tag{44c}$$

$$V_{2w} = -\left(Q_x + \frac{M_{yx}}{\partial x}\right)_{y=-\frac{b}{2}} = -\frac{1-\nu}{2} D\left(\frac{m\pi}{L}\right)^3 \sin\frac{m\pi x}{L} \lfloor \bar{w}_1 \quad \bar{w}_2 \rfloor \times$$
$$\left([W_w] \begin{Bmatrix} -\beta\cosh\beta \\ -\sinh\beta \\ \beta\sinh\beta \\ \cosh\beta \end{Bmatrix} + \frac{1+\nu}{1-\nu}[S_w]\begin{Bmatrix} -\sinh\beta \\ \cosh\beta \end{Bmatrix} \right) \tag{44d}$$

Combined forces along edges 1 and 2 are obtained by adding equations (39) and (44).

$$M_1 = M_{1\theta} + M_{1w}$$

$$M_2 = M_{2\theta} + M_{2w}$$

$$V_1 = V_{1\theta} + V_{1w}$$

$$V_2 = V_{2\theta} + V_{2w}$$

$$\begin{aligned} M_1 = D \sin\frac{m\pi x}{L}\Big[& \frac{m\pi}{L}\left(\lambda_1 \cosh\beta - \lambda_2 \sinh\beta\right) \bar{\theta}_1 \\ & - \frac{m\pi}{L}\left(\lambda_1 \cosh\beta + \lambda_2 \sinh\beta\right) \bar{\theta}_2 \\ & - \left(\frac{m\pi}{L}\right)^2\left\{\left(\lambda_3 \cosh\beta - \lambda_4 \sinh\beta\right) - (1-\nu)\right\} \bar{w}_1 \\ & - \left(\frac{m\pi}{L}\right)^2\left\{\lambda_3 \cosh\beta + \lambda_4 \sinh\beta\right) \bar{w}_2 \Big] \end{aligned} \quad (45a)$$

$$\begin{aligned} M_2 = D \sin\frac{m\pi x}{L}\Big[& - \frac{m\pi}{L}\left(\lambda_1 \cosh\beta + \lambda_2 \sinh\beta\right) \bar{\theta}_1 \\ & + \frac{m\pi}{L}\left(\lambda_1 \cosh\beta - \lambda_2 \sinh\beta\right) \bar{\theta}_2 \\ & + \left(\frac{m\pi}{L}\right)^2\left(\lambda_3 \cosh\beta + \lambda_4 \sinh\beta\right) \bar{w}_1 \\ & + \left(\frac{m\pi}{L}\right)^2\left\{\left(\lambda_3 \cosh\beta - \lambda_4 \sinh\beta\right) - (1-\nu)\right\} \bar{w}_2 \Big] \end{aligned} \quad (45b)$$

$$\begin{aligned} V_1 = -D \sin\frac{m\pi x}{L}\Big[& \left(\frac{m\pi}{L}\right)^2\left\{\left(\lambda_1 \sinh\beta - \lambda_2 \cosh\beta\right) - (1-\nu)\right\} \bar{\theta}_1 \\ & - \left(\frac{m\pi}{L}\right)^2\left(\lambda_1 \sinh\beta + \lambda_2 \cosh\beta\right) \bar{\theta}_2 \\ & - \left(\frac{m\pi}{L}\right)^3\left(\lambda_3 \sinh\beta - \lambda_4 \cosh\beta\right) \bar{w}_1 \\ & - \left(\frac{m\pi}{L}\right)^3\left(\lambda_3 \sinh\beta + \lambda_4 \cosh\beta\right) \bar{w}_2 \Big] \end{aligned} \quad (46a)$$

$$\begin{aligned} V_2 = -D \sin\frac{m\pi x}{L}\Big[& \left(\frac{m\pi}{L}\right)^2\left(\lambda_1 \sinh\beta + \lambda_2 \cosh\beta\right) \bar{\theta}_1 \\ & - \left(\frac{m\pi}{L}\right)^2\left\{\left(\lambda_1 \sinh\beta - \lambda_2 \cosh\beta\right) - (1-\nu)\right\} \bar{\theta}_2 \\ & - \left(\frac{m\pi}{L}\right)^3\left(\lambda_3 \sinh\beta + \lambda_4 \cosh\beta\right) \bar{w}_1 \\ & - \left(\frac{m\pi}{L}\right)^3\left(\lambda_4 \sinh\beta - \lambda_4 \cosh\beta\right) \bar{w}_2 \Big] \end{aligned} \quad (46b)$$

Note that

$$\lambda_3 \cosh\beta = \lambda_1 \sinh\beta$$

$$\lambda_4 \sinh\beta = \lambda_2 \cosh\beta$$

CHAPTER III

IN-PLANE DISPLACEMENTS OF THE PANEL

III IN-PLANE DISPLACEMENTS OF THE PANEL

Let us take
$$u^m = U^m_{(y)} \cos \frac{m\pi x}{L}$$
$$v^m = V^m_{(y)} \sin \frac{m\pi x}{L} \tag{47}$$

Remembering that the following analysis is for an m^{th} harmonic, the superscripts are dropped.

The stresses in panel are given by [1]

$$\sigma_x = \frac{E}{1-\nu^2}\left(\frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y}\right)$$
$$= \frac{E}{1-\nu^2}\left(-\frac{m\pi}{L}U + \nu \dot{V}\right) \sin \frac{m\pi x}{L} \tag{48a}$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\frac{\partial v}{\partial y} + \nu \frac{\partial u}{\partial x}\right)$$
$$= \frac{E}{1-\nu^2}\left(\dot{V} - \nu \frac{m\pi}{L}U\right) \sin \frac{m\pi x}{L} \tag{48b}$$

$$\tau_{xy} = G\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{48c}$$
$$= \frac{E}{2(1+\nu)}\left(\dot{U} + \frac{m\pi}{L}V\right) \cos \frac{m\pi x}{L}$$

where $V = V(y)$, $\dot{V} = \frac{dV(y)}{dy}$, $U = U(y)$, $\dot{U} = \frac{dU(y)}{dy}$

Substitution of these stresses into the equilibrium equations,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{49a}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \tag{49b}$$

yields two simultaneous differential equations [1] :

$$\frac{E}{2(1+\nu)}\left[\ddot{U}-\frac{2}{1-\nu}\left(\frac{m\pi}{L}\right)^2 U+\frac{1+\nu}{1-\nu}\,\frac{m\pi}{L}\,\dot{V}\right]\cos\frac{m\pi x}{L} \qquad (50a)$$

$$\frac{E}{2(1+\nu)}\left[-\frac{1+\nu}{1-\nu}\,\frac{m\pi}{L}\,\dot{U}+\frac{2}{1-\nu}\,\ddot{V}-\left(\frac{m\pi}{L}\right)^2 V\right]\sin\frac{m\pi x}{L} \qquad (50b)$$

where $\ddot{U}=\dfrac{d^2U(y)}{dy^2}$, $\ddot{V}=\dfrac{d^2V(y)}{dy^2}$

It can be verified [1] that a solution to these differential equations is given by

$$U = A_1\cosh\frac{m\pi y}{L}+A_2\sinh\frac{m\pi y}{L} + A_3\frac{m\pi y}{L}\cosh\frac{m\pi y}{L}+A_4\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} \qquad (51a)$$

$$V = B_1\cosh\frac{m\pi y}{L}+B_2\sinh\frac{m\pi y}{L} + B_3\frac{m\pi y}{L}\cosh\frac{m\pi y}{L}+B_4\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} \qquad (51b)$$

where

$$B_1 = A_2-\frac{3-\nu}{1+\nu}A_3$$
$$B_2 = A_1-\frac{3-\nu}{1+\nu}A_4$$
$$B_3 = A_4 \qquad (51c)$$
$$B_4 = A_3$$

A_i , B_i $(i=1,2,3,4)$ are constants which can be obtained by making the displacements satisfy the boundary conditions.

FIGURES 9 AND 10

9) IN-PLANE SYMMETRIC EDGE TRANSLATION

10) IN-PLANE ANTISYMMETRIC EDGE TRANSLATION

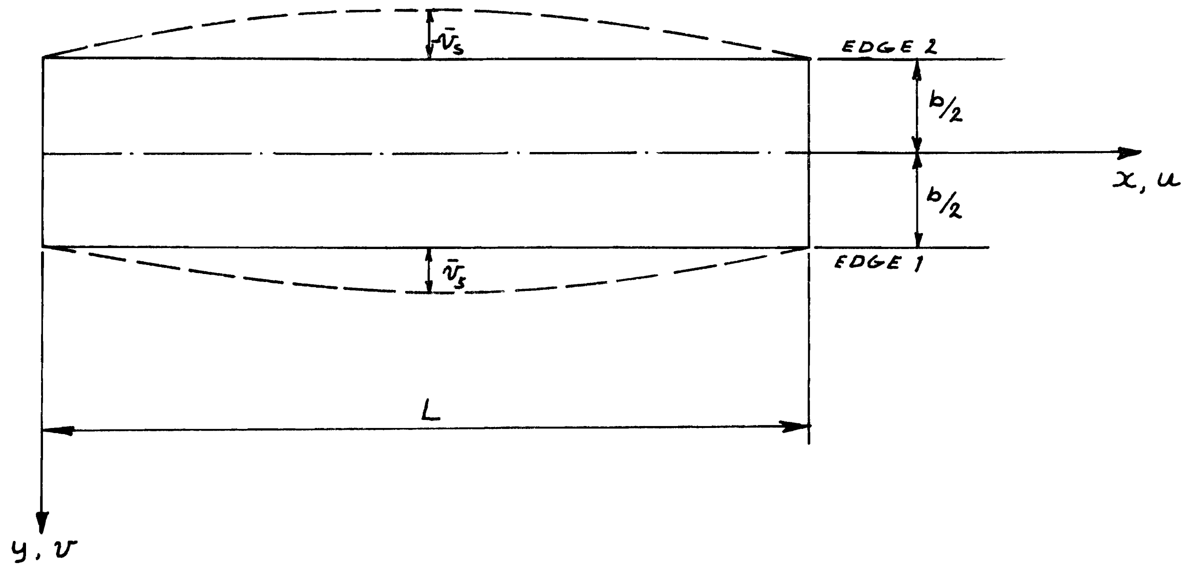


FIG. 9 IN-PLANE SYMMETRIC EDGE TRANSLATION

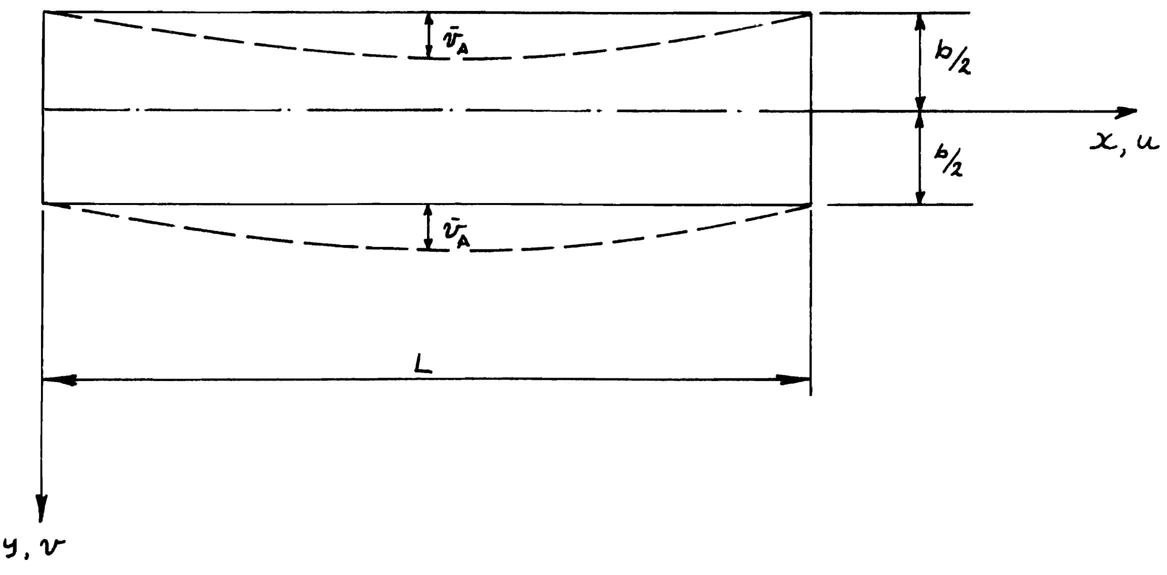


FIG. 10 IN-PLANE ANTISYMMETRIC EDGE TRANSLATION

As in Chapter II of this thesis, the deflections will be imposed one at a time, keeping the other one zero; furthermore, symmetric and antisymmetric parts of each deflection component will be considered separately.

(A) <u>Translation Normal to Edges 1 and 2.</u>

Boundary Conditions

$$u = 0 \quad , \quad y = \pm \frac{b}{2} \qquad \text{(vi)}$$

$$\left.\begin{aligned} (v)_{y=\frac{b}{2}} &= \bar{v}_1 \sin \frac{m\pi x}{L} \\ (v)_{y=-\frac{b}{2}} &= \bar{v}_2 \sin \frac{m\pi x}{L} \end{aligned}\right\} \qquad \text{(vii)}$$

<u>Symmetric Case (Fig. 9)</u>

$$(v)_{y=\frac{b}{2}} = -(v)_{y=-\frac{b}{2}}$$

In view of symmetry and definition (51c),

$$B_1 = B_4 = A_2 = A_3 = 0$$

From B.C. (vi) : $A_1 = -A_4 \beta \tanh\beta$

$$B_2 = -A_4 \left(\beta \tanh\beta + \frac{3-\nu}{1+\nu}\right)$$

And therefore, we have from (47) and (51),

$$u_s = A_4 \left(\frac{m\pi y}{L} \sinh \frac{m\pi y}{L} - \beta \tanh\beta \cosh \frac{m\pi y}{L}\right) \cos \frac{m\pi x}{L}$$

$$v_s = A_4 \left[\frac{m\pi y}{L} \cos \frac{m\pi y}{L} - \left(\beta \tanh\beta + \frac{3-\nu}{1+\nu}\right) \sin \frac{m\pi y}{L}\right] \sin \frac{m\pi x}{L}$$

From B.C. (vii): $A_4 = \left(\beta \operatorname{sech}\beta - \frac{3-\nu}{1+\nu} \sinh\beta\right)^{-1} \bar{v}_s = \lambda_5 \bar{v}_s$

where $\lambda_5 = \left(\beta \operatorname{sech}\beta - \frac{3-\nu}{1+\nu} \sinh\beta\right)^{-1}$ (52)

$$u_s = \lambda_5 \bar{v}_s \cos\frac{m\pi x}{L}\left(\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - k_t \cosh\frac{m\pi y}{L}\right)$$

$$v_s = \lambda_5 \bar{v}_s \sin\frac{m\pi x}{L}\left(\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \left(k_t + \frac{3-\nu}{1+\nu}\right)\sinh\frac{m\pi y}{L}\right)$$

where $k_t = \beta \tanh\beta$ as in Chapter II, and subscripts s indicates as before, a symmetric case.

Antisymmetric Case (Fig.10)

$$(v)_{y=\frac{b}{2}} = (v)_{y=-\frac{b}{2}}$$

From antisymmetry and definition (51c),

$$B_2 = B_3 = A_1 = A_4 = 0 .$$

From B.C. (vi) : $A_2 = -A_3 \beta \coth\beta$

$$B_1 = -A_3\left(\beta \coth\beta + \frac{3-\nu}{1+\nu}\right)$$

And therefore, we have from (47) and (51)

$$u_A = A_3\left(\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \beta\coth\beta \sinh\frac{m\pi y}{L}\right)\cos\frac{m\pi x}{L}$$

$$v_A = A_3\left(\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \left(\beta\coth\beta + \frac{3-\nu}{1+\nu}\right)\cosh\frac{m\pi y}{L}\right)\sin\frac{m\pi x}{L}$$

From B.C. (vii) : $A_3 = \left(\beta \operatorname{csch}\beta + \frac{3-\nu}{1+\nu}\cosh\beta\right)^{-1} \bar{v}_A = \lambda_6 \bar{v}_A$

where $\lambda_6 = \left(\beta \operatorname{csch}\beta + \frac{3-\nu}{1+\nu}\cosh\beta\right)^{-1}$ (53)

$$u_A = \lambda_6 \bar{v}_A \cos\frac{m\pi x}{L}\left(\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - k_c \sinh\frac{m\pi y}{L}\right)$$

$$v_A = \lambda_6 \bar{v}_A \sin\frac{m\pi x}{L}\left(\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \left(k_c + \frac{3-\nu}{1+\nu}\right)\cosh\frac{m\pi y}{L}\right)$$

where $k_c = \beta\coth\beta$ as before and subscript A denotes antisymmetric case.

Substituting, $\bar{v}_S = \frac{1}{2}\left(\bar{v}_1 - \bar{v}_2\right)$ symmetric part

$\bar{v}_A = \frac{1}{2}\left(\bar{v}_1 + \bar{v}_2\right)$ antisymmetric part

Superimposing, $u = u_S + u_A$

$v = v_S + v_A$

The internal deformations are

$$u = \frac{1}{2}\cos\frac{m\pi x}{L}\lfloor \bar{v}_1 \quad \bar{v}_2 \rfloor [U_v]\{H_y\} \qquad (54a)$$

where $[U_v] = \begin{bmatrix} \lambda_5 & -\lambda_5 k_t & -\lambda_6 & \lambda_6 k_c \\ -\lambda_5 & \lambda_5 k_t & -\lambda_6 & \lambda_6 k_c \end{bmatrix}$

$\{H_y\}$ as in Eq. 37a

$$v = \frac{1}{2}\sin\frac{m\pi x}{L}\lfloor \bar{v}_1 \quad \bar{v}_2 \rfloor [V_v]\{H_y\} \qquad (54b)$$

where $[V_v] = \begin{bmatrix} -\lambda_6 & \lambda_6\left(k_c + \frac{3-\nu}{1+\nu}\right) & \lambda_5 & -\lambda_5\left(k_t + \frac{3-\nu}{1+\nu}\right) \\ -\lambda_6 & \lambda_6\left(k_c + \frac{3-\nu}{1+\nu}\right) & -\lambda_5 & \lambda_5\left(k_t + \frac{3-\nu}{1+\nu}\right) \end{bmatrix}$

Making use of equations (48) and (54) the in-plane generalized stresses are defined as follows:

$$N_x = 2\sigma_x t = \frac{2Et}{1-\nu^2}\left(\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y}\right)$$

(t = facing thickness)

$$= -\frac{Et}{1+\nu}\left(\frac{m\pi}{L}\right)\sin\frac{m\pi x}{L}\lfloor \bar{v}_1 \quad \bar{v}_2 \rfloor [N_{vx}]\{H_y\} \qquad (55a)$$

where $[N_{vx}] = \begin{bmatrix} \lambda_5 & -\lambda_5\left(k_t - \frac{2\nu}{1+\nu}\right) & -\lambda_6 & \lambda_6\left(k_c - \frac{2\nu}{1+\nu}\right) \\ -\lambda_5 & \lambda_5\left(k_t - \frac{2\nu}{1+\nu}\right) & -\lambda_6 & \lambda_6\left(k_c - \frac{2\nu}{1+\nu}\right) \end{bmatrix}$

$$N_y = 2\sigma_y t = \frac{2Et}{1-\nu^2}\left(\frac{\partial v}{\partial y} + \nu \frac{\partial u}{\partial x}\right)$$

$$= \frac{Et}{1+\nu}\left(\frac{m\pi}{L}\right) \sin\frac{m\pi x}{L} \lfloor \bar{v}_1 \quad \bar{v}_2 \rfloor [N_{vy}] \{H_y\} \qquad (55b)$$

where $[N_{vy}] = \begin{bmatrix} \lambda_5 & -\lambda_5\left(k_t + \frac{2}{1+\nu}\right) & -\lambda_6 & \lambda_6\left(k_c + \frac{2}{1+\nu}\right) \\ -\lambda_5 & \lambda_5\left(k_t + \frac{2}{1+\nu}\right) & -\lambda_6 & \lambda_6\left(k_c + \frac{2}{1+\nu}\right) \end{bmatrix}$

$$N_{yx} = 2\tau_{yx} t = \frac{Et}{(1+\nu)}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$= \frac{Et}{1+\nu} \frac{m\pi}{L} \cos\frac{m\pi x}{L} \lfloor \bar{v}_1 \quad \bar{v}_2 \rfloor [N_{vxy}] \{H_y\} \qquad (55c)$$

where $[N_{vyx}] = \begin{bmatrix} -\lambda_6 & \lambda_6\left(\frac{1-\nu}{1+\nu} + k_c\right) & \lambda_5 & -\lambda_5\left(\frac{1-\nu}{1+\nu} + k_t\right) \\ -\lambda_6 & \lambda_6\left(\frac{1-\nu}{1+\nu} + k_c\right) & -\lambda_5 & \lambda_5\left(\frac{1-\nu}{1+\nu} + k_t\right) \end{bmatrix}$

(B) <u>Tangential Translations along Edges 1 and 2</u>

Boundary Conditions

$$v = 0 \quad , \quad y = \pm\frac{b}{2} \qquad \text{(viii)}$$

$$\left.\begin{aligned} (u)_{y=\frac{b}{2}} &= \bar{u}_1 \cos\frac{m\pi x}{L} \\ (u)_{y=-\frac{b}{2}} &= \bar{u}_2 \cos\frac{m\pi x}{L} \end{aligned}\right\} \qquad \text{(ix)}$$

FIGURES 11 AND 12

11) SYMMETRIC TANGENTIAL EDGE TRANSLATION

12) ANTISYMMETRIC TANGENTIAL EDGE TRANSLATION

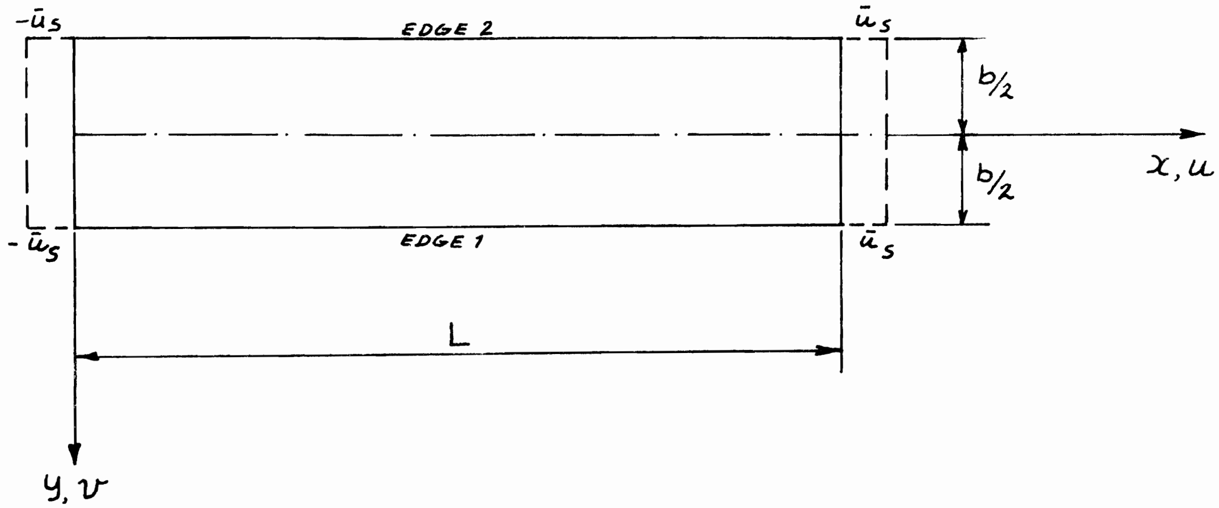


FIG. 11 SYMMETRIC TANGENTIAL EDGE TRANSLATION

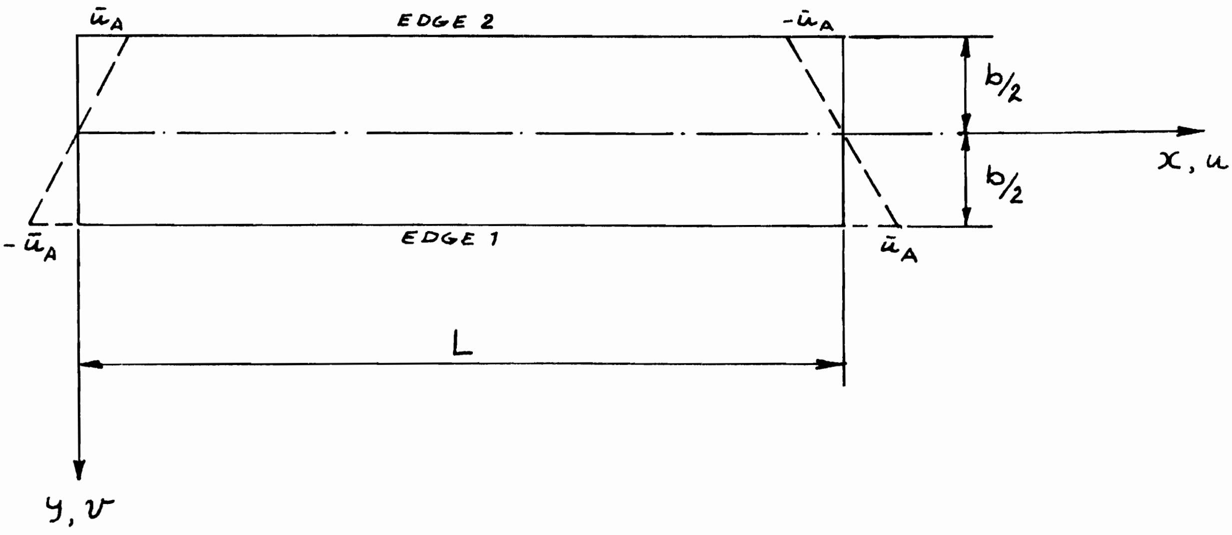


FIG. 12 ANTISYMMETRIC TANGENTIAL EDGE TRANSLATION

Symmetrical Case (Fig.11)

$$(u)_{y=\frac{b}{2}} = (u)_{y=-\frac{b}{2}}$$

since U is an even function of y : $A_2 = A_3 = 0$;

and from equation (51c), $B_1 = B_4 = 0$.

From B.C.(viii) : $B_2 = -B_3 \beta \coth\beta$

$$A_1 = -A_4 \left(\beta\coth\beta - \frac{3-\nu}{1+\nu}\right)$$

$$u_s = A_4\left[\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \left(\beta\coth\beta - \frac{3-\nu}{1+\nu}\right)\cosh\frac{m\pi y}{L}\right]\cos\frac{m\pi x}{L}$$

$$v_s = A_4\left[\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \beta\coth\beta\sinh\frac{m\pi y}{L}\right]\sin\frac{m\pi x}{L}$$

From B.C. (ix) : $A_4 = -\left(\beta\,\mathrm{csch}\beta - \frac{3-\nu}{1+\nu}\cosh\beta\right)^{-1}\bar{u}_s = -\lambda_7\bar{u}_s$

where $\lambda_7 = \left(\beta\,\mathrm{csch}\beta - \frac{3-\nu}{1+\nu}\cosh\beta\right)^{-1}$ (56)

$$u_s = -\lambda_7\bar{u}_s\cos\frac{m\pi x}{L}\left[\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \left(k_c - \frac{3-\nu}{1+\nu}\right)\cosh\frac{m\pi y}{L}\right]$$

$$v_s = -\lambda_7\bar{u}_s\sin\frac{m\pi x}{L}\left[\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - k_c\sinh\frac{m\pi y}{L}\right]$$

Antisymmetrical Case (Fig.12)

$$(u)_{y=\frac{b}{2}} = -(u)_{y=-\frac{b}{2}}$$

since U is an odd function of y : $A_1 = A_4 = 0$;

it then follows from equation (51c), $B_2 = B_3 = 0$.

From B.C.(viii) : $B_1 = -B_4 \beta\tanh\beta$

$$A_2 = -A_3\left(\beta\tanh\beta - \frac{3-\nu}{1+\nu}\right)$$

$$u_A = A_3\left[\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \left(\beta\tanh\beta - \frac{3-\nu}{1+\nu}\right)\sinh\frac{m\pi y}{L}\right]\cos\frac{m\pi x}{L}$$

$$v_A = A_3\left[\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \beta\tanh\beta\cosh\frac{m\pi y}{L}\right]\sin\frac{m\pi x}{L}$$

From B.C. (ix) : $A_3 = \left(\beta\,\mathrm{sech}\beta + \frac{3-\nu}{1+\nu}\sinh\beta\right)^{-1}\bar{u}_A = \lambda_8\bar{u}_A$

where $\lambda_8 = \left(\beta \operatorname{sech}\beta + \frac{3-\nu}{1+\nu} \sinh\beta\right)^{-1}$

$$u_A = \lambda_8 \bar{u}_A \cos\frac{m\pi x}{L}\left[\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \left(k_t - \frac{3-\nu}{1+\nu}\right)\sinh\frac{m\pi y}{L}\right] \quad (57)$$

$$v_A = \lambda_8 \bar{u}_A \sin\frac{m\pi x}{L}\left[\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - k_t \cosh\frac{m\pi y}{L}\right]$$

Substituting, $\bar{u}_S = \frac{1}{2}(\bar{u}_1 + \bar{u}_2)$ for the symmetric part,

$\bar{u}_A = \frac{1}{2}(\bar{u}_1 - \bar{u}_2)$ for the antisymmetric part

and superimposing, $u = u_S + u_A$

$v = v_S + v_A$

results in internal deformations

$$u = \frac{1}{2}\cos\frac{m\pi x}{L}\lfloor \bar{u}_1 \quad \bar{u}_2 \rfloor [U_u]\{H_y\} \quad (58a)$$

where $[U_u] = \begin{bmatrix} -\lambda_7 & \lambda_7\left(k_c - \frac{3-\nu}{1+\nu}\right) & \lambda_8 & -\lambda_8\left(k_t - \frac{3-\nu}{1+\nu}\right) \\ -\lambda_7 & \lambda_7\left(k_c - \frac{3-\nu}{1+\nu}\right) & -\lambda_8 & \lambda_8\left(k_t - \frac{3-\nu}{1+\nu}\right) \end{bmatrix}$

$$v = \frac{1}{2}\sin\frac{m\pi x}{L}\lfloor \bar{u}_1 \quad \bar{u}_2 \rfloor [V_u]\{H_y\} \quad (58b)$$

where $[V_u] = \begin{bmatrix} \lambda_8 & -\lambda_8 k_t & -\lambda_7 & \lambda_7 k_c \\ -\lambda_8 & \lambda_8 k_t & -\lambda_7 & \lambda_7 k_c \end{bmatrix}$

Substitution of equations (58) into equations (48) yield the generalized stresses

$$N_x = \frac{Et}{1+\nu}\left(\frac{m\pi}{L}\right)\sin\frac{m\pi x}{L}\lfloor \bar{u}_1 \quad \bar{u}_2 \rfloor [N_{ux}]\{H_y\} \quad (59a)$$

where $[N_{ux}] = \begin{bmatrix} \lambda_7 & -\lambda_7\left(k_c - \frac{3+\nu}{1+\nu}\right) & -\lambda_8 & \lambda_8\left(k_t - \frac{3+\nu}{1+\nu}\right) \\ \lambda_7 & -\lambda_7\left(k_c - \frac{3+\nu}{1+\nu}\right) & \lambda_8 & -\lambda_8\left(k_t - \frac{3+\nu}{1+\nu}\right) \end{bmatrix}$

$$N_y = \frac{Et}{1+\nu}\left(\frac{m\pi}{L}\right)\sin\frac{m\pi x}{L}\lfloor \bar{u}_1 \quad \bar{u}_2 \rfloor [N_{uy}]\{H_y\} \quad (59b)$$

where $[N_{uy}] = \begin{bmatrix} -\lambda_7 & \lambda_7\left(k_c - \frac{1-\nu}{1+\nu}\right) & \lambda_8 & -\lambda_8\left(k_t - \frac{1-\nu}{1+\nu}\right) \\ -\lambda_7 & \lambda_7\left(k_c - \frac{1-\nu}{1+\nu}\right) & -\lambda_8 & \lambda_8\left(k_t - \frac{1-\nu}{1+\nu}\right) \end{bmatrix}$

$$N_{yx} = \frac{Et}{1+\nu}\left(\frac{m\pi}{L}\right)\cos\frac{m\pi x}{L}\lfloor \bar{u}_1 \quad u_2 \rfloor \left[N_{uyx}\right]\{H_y\} \tag{59c}$$

where
$$\left[N_{uyx}\right] = \begin{bmatrix} \lambda_8 & -\lambda_8\left(k_t - \frac{2}{1+\nu}\right) & -\lambda_7 & \lambda_7\left(k_c - \frac{2}{1+\nu}\right) \\ -\lambda_8 & \lambda_8\left(k_t - \frac{2}{1+\nu}\right) & -\lambda_7 & \lambda_7\left(k_c - \frac{2}{1+\nu}\right) \end{bmatrix}$$

The edge forces are obtained by adding N_y, N_{yx} from equations (54) and (59) for $y = \pm\frac{b}{2}$

$$\left.\begin{aligned} N_1 &= \left(N_y\right)_{y=\frac{b}{2}} \\ N_2 &= \left(N_y\right)_{y=-\frac{b}{2}} \\ S_1 &= -\left(N_{yx}\right)_{y=\frac{b}{2}} \\ S_2 &= \left(N_{yx}\right)_{y=\frac{b}{2}} \end{aligned}\right\} \tag{60}$$

The sign convention for forces and displacements is shown in Fig. 14.

Edge forces for edge deformations in plane of panel

$$\begin{aligned} N_1 = -\frac{2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\sin\frac{m\pi x}{L}\Big[&\left(\lambda_5\cosh\beta - \lambda_6\sinh\beta\right)\bar{v}_1 \\ &+ \left(\lambda_5\cosh\beta + \lambda_6\sinh\beta\right)\bar{v}_2 \\ &+ \left\{\left(\lambda_7\cosh\beta - \lambda_8\sinh\beta\right) + (1+\nu)\right\}\bar{u}_1 \\ &+ \left(\lambda_7\cosh\beta + \lambda_8\sinh\beta\right)\bar{u}_2\Big] \end{aligned} \tag{61a}$$

$$N_2 = -\frac{2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\sin\frac{m\pi x}{L}\Big[(\lambda_5\cosh\beta + \lambda_6\sinh\beta)\bar{v}_1 + (\lambda_5\cosh\beta + \lambda_6\sinh\beta)\bar{v}_2 + \{(\lambda_7\cosh\beta - \lambda_8\sinh\beta) + (1+\nu)\}\bar{u}_1 + (\lambda_7\cosh\beta + \lambda_8\sinh\beta)\bar{u}_2\Big] \quad (61b)$$

$$S_1 = \frac{-2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\cos\frac{m\pi x}{L}\Big[\{(\lambda_5\sinh\beta - \lambda_6\cosh\beta) + (1+\nu)\}\bar{v}_1 + (\lambda_5\sinh\beta + \lambda_6\cosh\beta)\bar{v}_2 + (\lambda_7\sinh\beta - \lambda_8\cosh\beta)\bar{u}_1 + (\lambda_7\sinh\beta + \lambda_8\cosh\beta)\bar{u}_2\Big] \quad (62c)$$

$$S_2 = -\frac{2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\cos\frac{m\pi x}{L}\Big[(\lambda_5\sinh\beta + \lambda_6\cosh\beta)\bar{v}_1 + \{(\lambda_5\sinh\beta - \lambda_6\cosh\beta) + (1+\nu)\}\bar{v}_2 + (\lambda_7\sinh\beta + \lambda_8\cosh\beta)\bar{u}_1 + (\lambda_7\sinh\beta - \lambda_8\cosh\beta)\bar{u}_2\Big] \quad (62d)$$

Note: $\lambda_5\sinh\beta = \lambda_7\cosh\beta$

$\lambda_6\cosh\beta = \lambda_8\sinh\beta$

(C) Edge Stiffener Deformation (Fig. 13)

Let us consider the free body of the stiffener shown in Figure 13, and let us assume that the axial dis-

FIGURES 13 AND 14

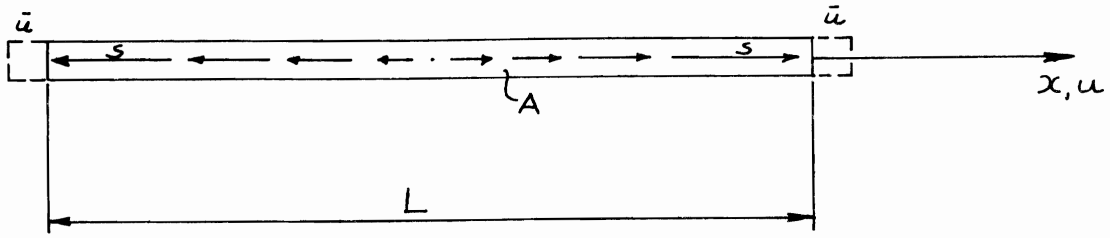


FIG. 13 **EDGE STIFFENER AXIAL DEFORMATION**

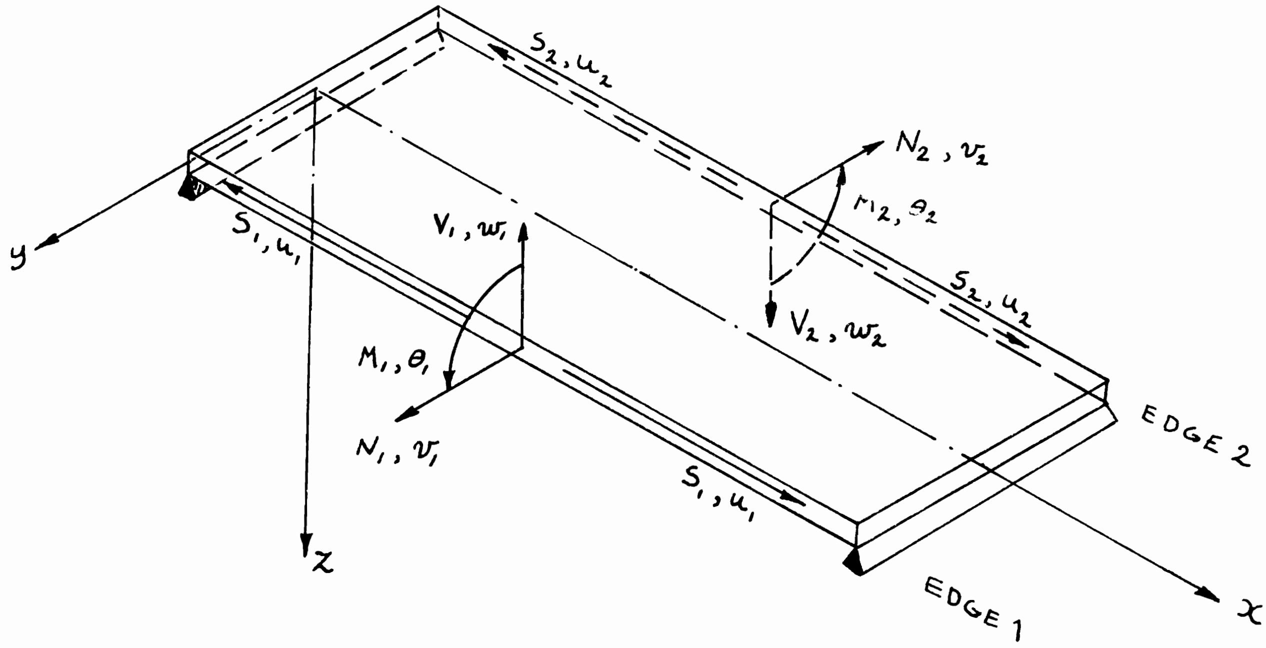


FIG. 14 **SIGN CONVENTION OF EDGE FORCES & DISPLACEMENTS**

$$u = \bar{u} \cos \frac{m\pi x}{L}$$

Stress in edge stiffener

$$\sigma_x = E \frac{du}{dx} = -E\bar{u}\left(\frac{m\pi}{L}\right) \sin \frac{m\pi x}{L}$$

The force transmitted across any arbitrary cross-section of area A, is $P_x = \sigma_x A$. Then, the shear stress resultant S per unit length is given by

$$S = \frac{dP_x}{dx}$$
$$= -EA\bar{u}\left(\frac{m\pi}{L}\right)^2 \cos \frac{m\pi x}{L}$$

Substituting the appropriate values for $\bar{u}$ we obtain the following expressions for the shear stress resultants at the edges 1 and 2 :

$$S_1 = -EA\,\bar{u}_1 \left(\frac{m\pi}{L}\right)^2 \cos \frac{m\pi x}{L}$$

$$S_2 = -EA\,\bar{u}_2 \left(\frac{m\pi}{L}\right)^2 \cos \frac{m\pi x}{L} \qquad (63)$$

These 2 expressions can be added to the edge force-edge displacement relationships for the whole panel.

Considering the bending stiffness is negligible in this analysis, we only consider the axial deformation of the stiffener.

CHAPTER IV

DIRECT STIFFNESS ANALYSIS

IV DIRECT STIFFNESS ANALYSIS

From the previous theory the stiffness matrix for an isotropic sandwich element of a folded plate structure can now be assembled, using the sign convention for edge forces, and displacements shown in Fig. 14.

The element stiffness matrix can be written [3] as

$$\begin{Bmatrix} M_1 \\ M_2 \\ V_1 \\ V_2 \\ S_1 \\ S_2 \\ N_1 \\ N_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & & & & \\ k_{21} & k_{22} & k_{23} & k_{24} & & \text{ZERO'S} & & \\ k_{31} & k_{32} & k_{33} & k_{34} & & & & \\ k_{41} & k_{42} & k_{43} & k_{44} & & & & \\ & & & & k_{55} & k_{56} & k_{57} & k_{58} \\ & \text{ZERO'S} & & & k_{65} & k_{66} & k_{67} & k_{68} \\ & & & & k_{75} & k_{76} & k_{77} & k_{78} \\ & & & & k_{85} & k_{86} & k_{87} & k_{88} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ w_1 \\ w_2 \\ u_1 \\ u_2 \\ v_1 \\ v_2 \end{Bmatrix} \quad (64)$$

It should be noted that the sign convention for the displacements u_1 , v_2 , w_1 , θ_1 and forces V_1 and S_1, M_1 are opposite to the one used in the theory.

Making use of the following definitions:

$$D = \frac{1}{2}\,\frac{E\,t}{1-\nu^2}\,(c+t)^2$$

$$S = \frac{(c+t)^2}{c}\,G_c \quad , \quad k_s = \frac{1-\nu}{2}\,\frac{D}{S}\left(\frac{m\pi}{L}\right)^2$$

$$\beta = \frac{m\pi b}{2L} \quad , \quad p = \left(1 + \frac{1}{k_s}\right)^{1/2}$$

$$\lambda_1 = \left[\beta\,\mathrm{sech}\,\beta + \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\tanh\beta p}{p\tanh\beta}\right)\right\}\sinh\beta\right]^{-1}$$

$$\lambda_2 = \left[\beta\,\mathrm{csch}\,\beta - \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\coth\beta p}{p\coth\beta}\right)\right\}\cosh\beta\right]^{-1}$$

$$\lambda_3 = \left[\beta\,\mathrm{csch}\,\beta + \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\tanh\beta p}{p\tanh\beta}\right)\right\}\cosh\beta\right]^{-1}$$

$$\lambda_4 = \left[\beta\,\mathrm{sech}\,\beta - \left\{1 + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\coth\beta p}{p\coth\beta}\right)\right\}\sinh\beta\right]^{-1}$$

The m^{th} harmonic element stiffness coefficients consisting of bending and plate stiffness coefficients are given as follows:

Bending stiffness coefficients for a sandwich panel:

$$\left.\begin{aligned}
K_{11} &= K_{22} = D\left(\frac{m\pi}{L}\right)\left(\lambda_1\cosh\beta - \lambda_2\sinh\beta\right)\\
K_{12} &= K_{21} = -D\left(\frac{m\pi}{L}\right)\left(\lambda_1\cosh\beta + \lambda_2\sinh\beta\right)\\
K_{13} &= K_{31} = D\left(\frac{m\pi}{L}\right)^2\left\{\left(\lambda_3\cosh\beta - \lambda_4\sinh\beta\right) - (1-\nu)\right\}\\
K_{14} &= K_{41} = -D\left(\frac{m\pi}{L}\right)^2\left(\lambda_3\cosh\beta + \lambda_4\sinh\beta\right)\\
K_{23} &= K_{32} = K_{14}\\
K_{24} &= K_{42} = K_{13}\\
K_{33} &= K_{44} = D\left(\frac{m\pi}{L}\right)^3\left(\lambda_3\sinh\beta - \lambda_4\cosh\beta\right)\\
K_{34} &= K_{43} = -D\left(\frac{m\pi}{L}\right)^3\left(\lambda_3\sinh\beta + \lambda_4\cosh\beta\right)\\
&\text{Plate stiffness coefficients:}\\
K_{55} &= K_{66} = -\frac{2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\left(\lambda_7\sinh\beta - \lambda_8\cosh\beta\right) + AE\left(\frac{m\pi}{L}\right)^2\\
K_{56} &= K_{65} = -\frac{2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\left(\lambda_7\sinh\beta + \lambda_8\cosh\beta\right)\\
K_{57} &= K_{75} = -\frac{2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\left\{\left(\lambda_7\cosh\beta - \lambda_8\sinh\beta\right) + (1+\nu)\right\}\\
K_{58} &= K_{85} = -\frac{2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\left(\lambda_7\cosh\beta + \lambda_8\sinh\beta\right)\\
K_{67} &= K_{76} = K_{58}\\
K_{68} &= K_{86} = K_{57}\\
K_{77} &= K_{88} = -\frac{2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\left(\lambda_5\cosh\beta - \lambda_6\sinh\beta\right)\\
K_{78} &= K_{87} = -\frac{2Et}{(1+\nu)^2}\left(\frac{m\pi}{L}\right)\left(\lambda_5\cosh\beta + \lambda_6\sinh\beta\right)
\end{aligned}\right\} \quad (65)$$

where

$$\lambda_5 = \left[\beta \operatorname{sech}\beta - \frac{3-\nu}{1+\nu} \sinh\beta\right]^{-1}$$

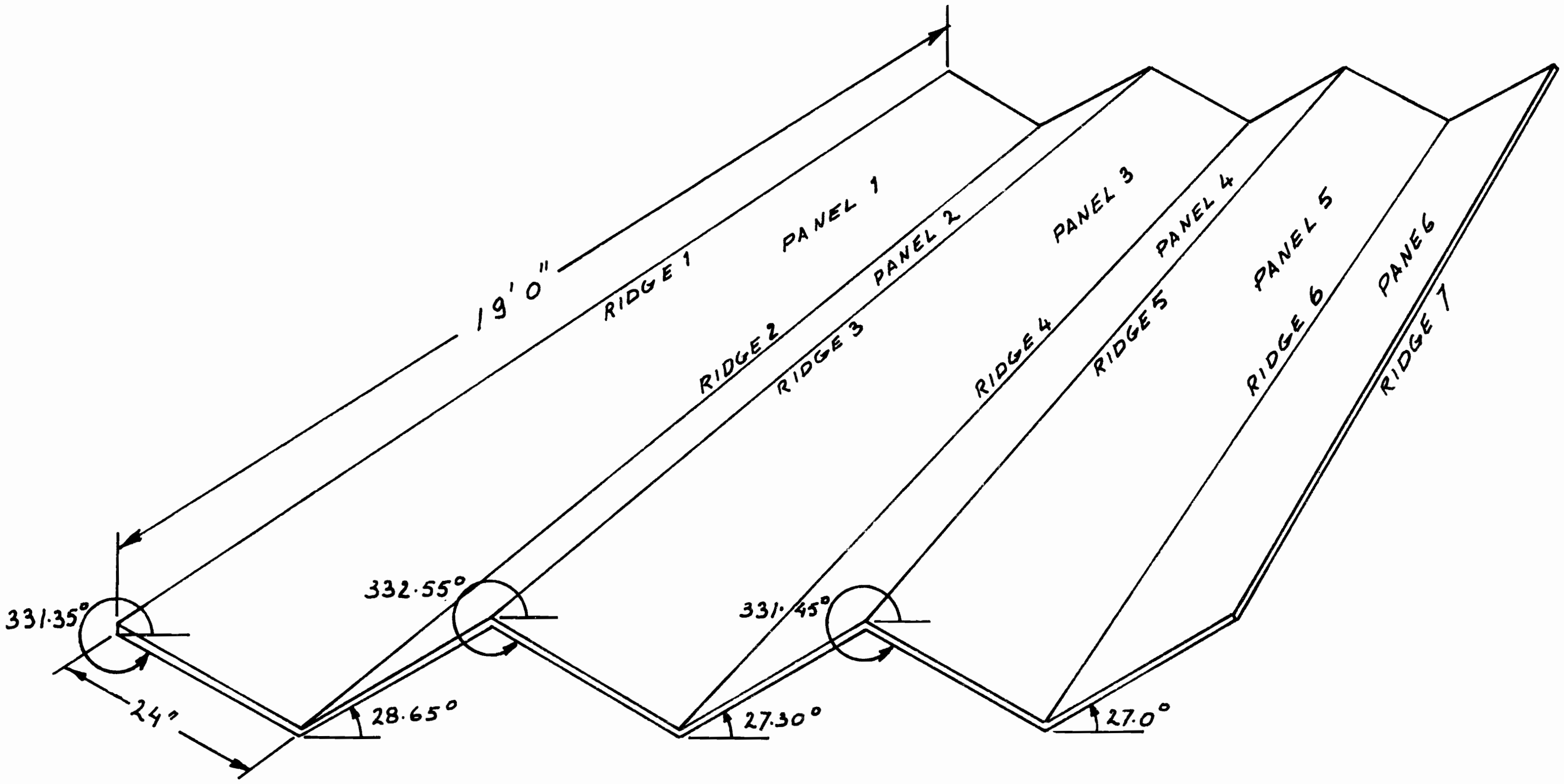
$$\lambda_6 = \left[\beta \operatorname{csch}\beta + \frac{3-\nu}{1+\nu} \cosh\beta\right]^{-1}$$

$$\lambda_7 = \left[\beta \operatorname{csch}\beta - \frac{3-\nu}{1+\nu} \cosh\beta\right]^{-1}$$

$$\lambda_8 = \left[\beta \operatorname{sech}\beta + \frac{3-\nu}{1+\nu} \sinh\beta\right]^{-1}$$

FIGURE 15

FOLDED PLATE MODEL



ALL PANELS ARE IDENTICAL
ALUMINUM FACINGS .025" THICK
STYROLITE CORE 1.0" THICK

FIG. 15 19FT. FOLDED PLATE MODEL

COMPUTER PROGRAMME

The programme as written by Dr. P.P. Fazio and described in [4] was rewritten to include the analysis by the Elasticity Method, as an additional option.

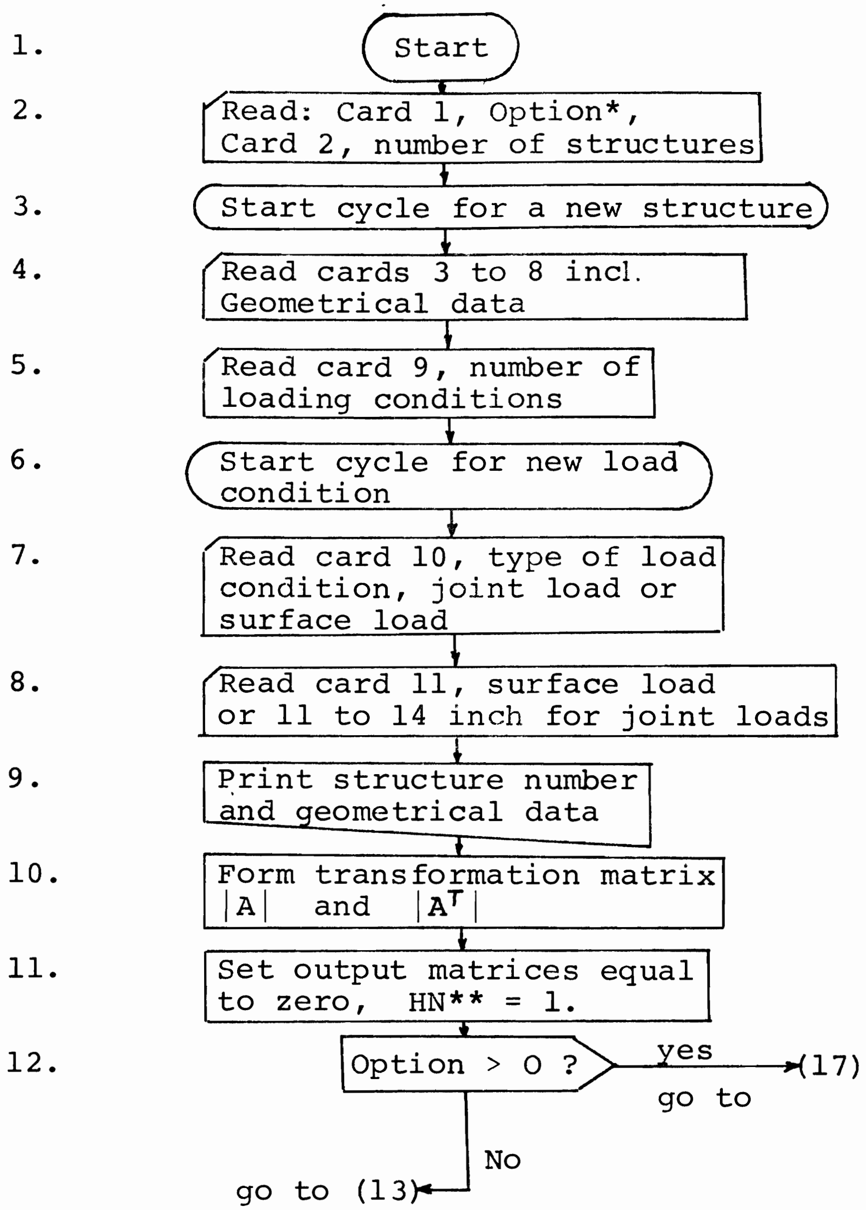
The original programme used the analysis by the ordinary method to find the theoretical stresses and deflections of folded sandwich plate structures.

Experimental results are available for the 19 ft. folded plate model (Fig. 15) in [4] . In order to compare the theory with the experiment,numerical results were obtained for both methods.

A flow chart of the programme is shown in Figure 16.

The computed vertical displacements and longitudinal stresses at midspan of the model for all the ridges, are plotted in Figures 17 to 30. These plots were compared with the corresponding plots from the experiment.

The plotted experimental stresses are average values of measured stresses in upper and lower facings at the ridges.



* Option = 0, ordinary method
= 1, elastic method
= 2, both methods

** HN = Harmonic Number

FIGURE 16 FLOWCHART

FIGURE 16 FLOWCHART (CONTINUED)

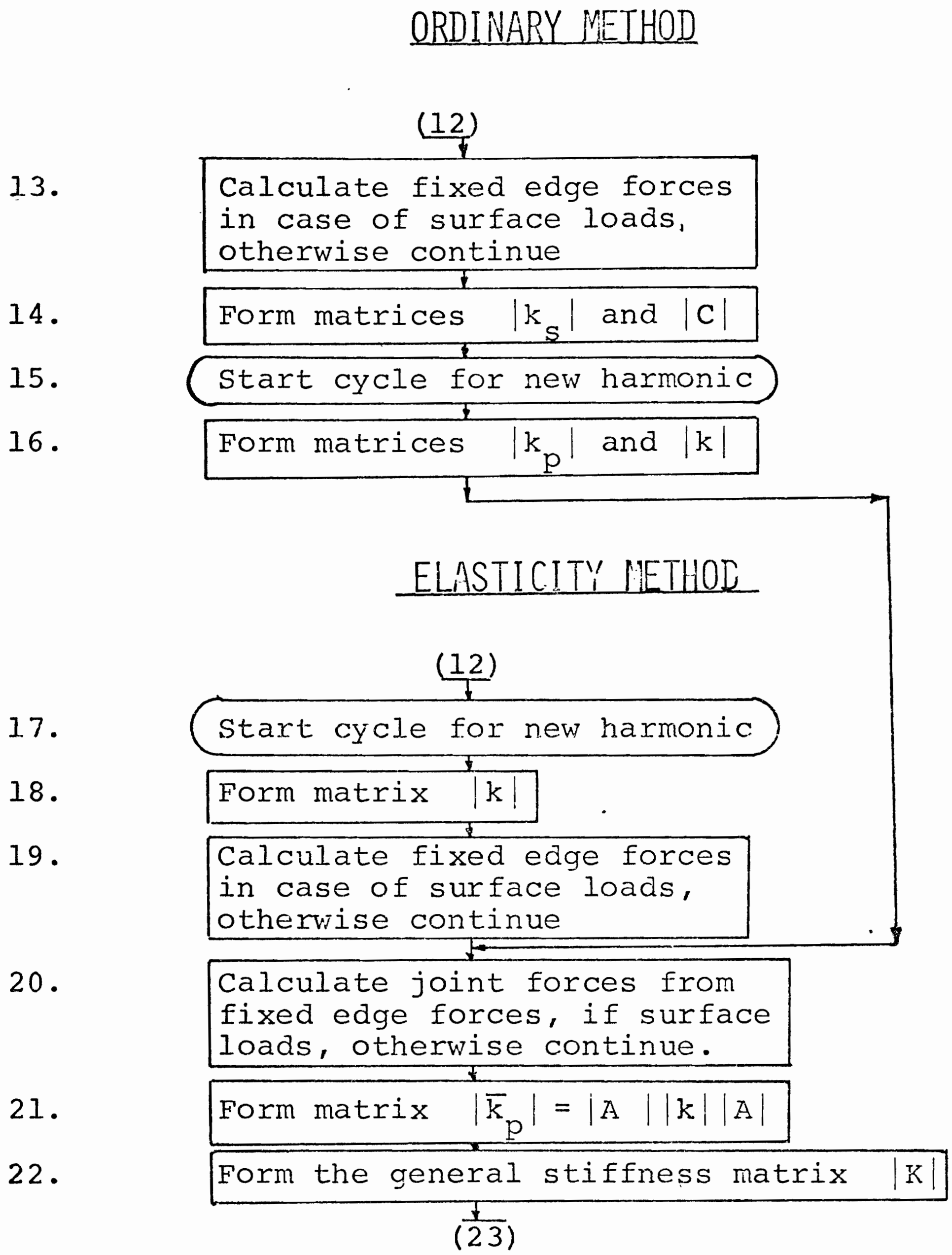


FIGURE 16 FLOWCHART (CONTINUED)

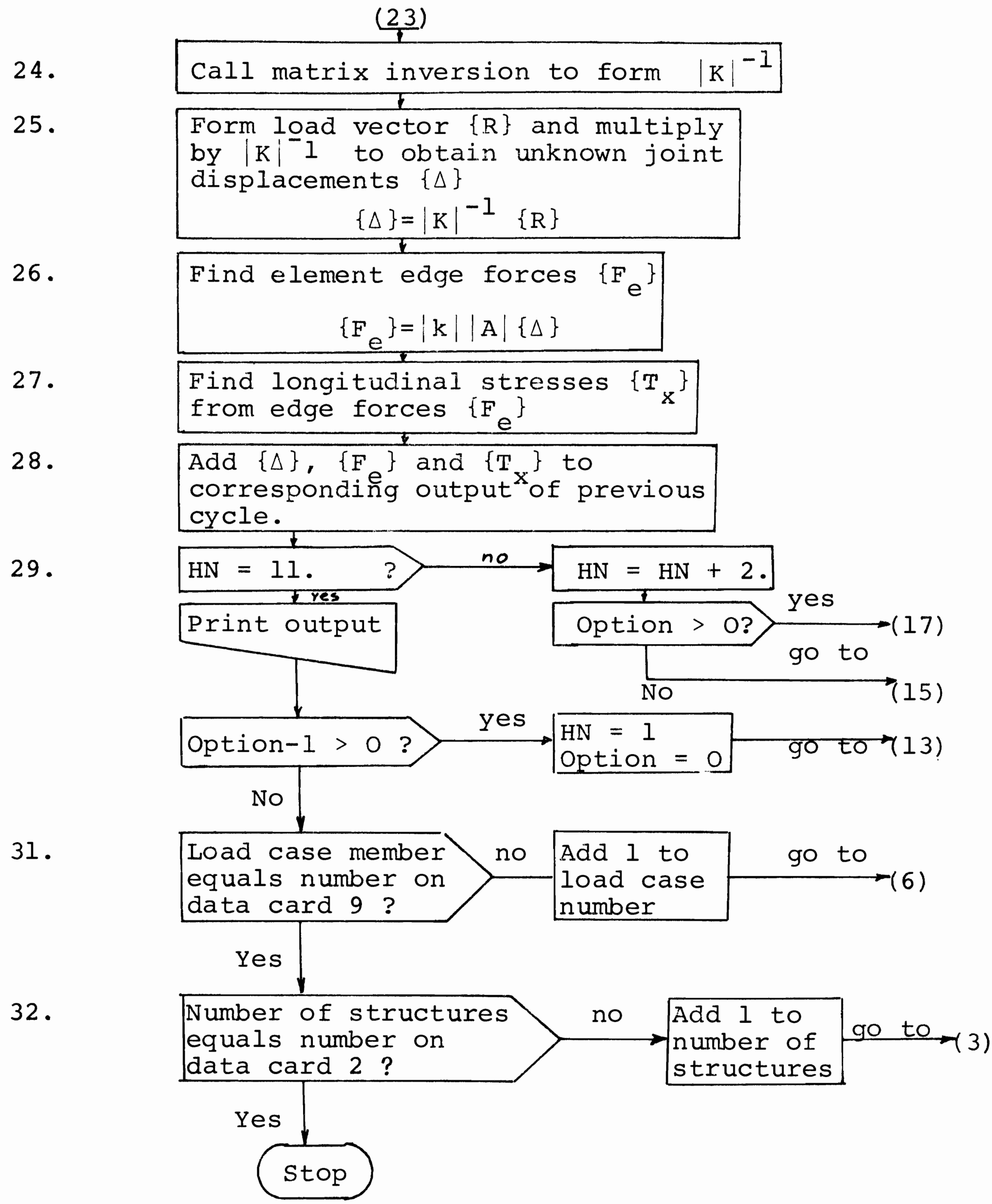


FIG. 17 THEORETICAL AND EXPERIMENTAL VERTICAL DISPLACEMENTS AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE

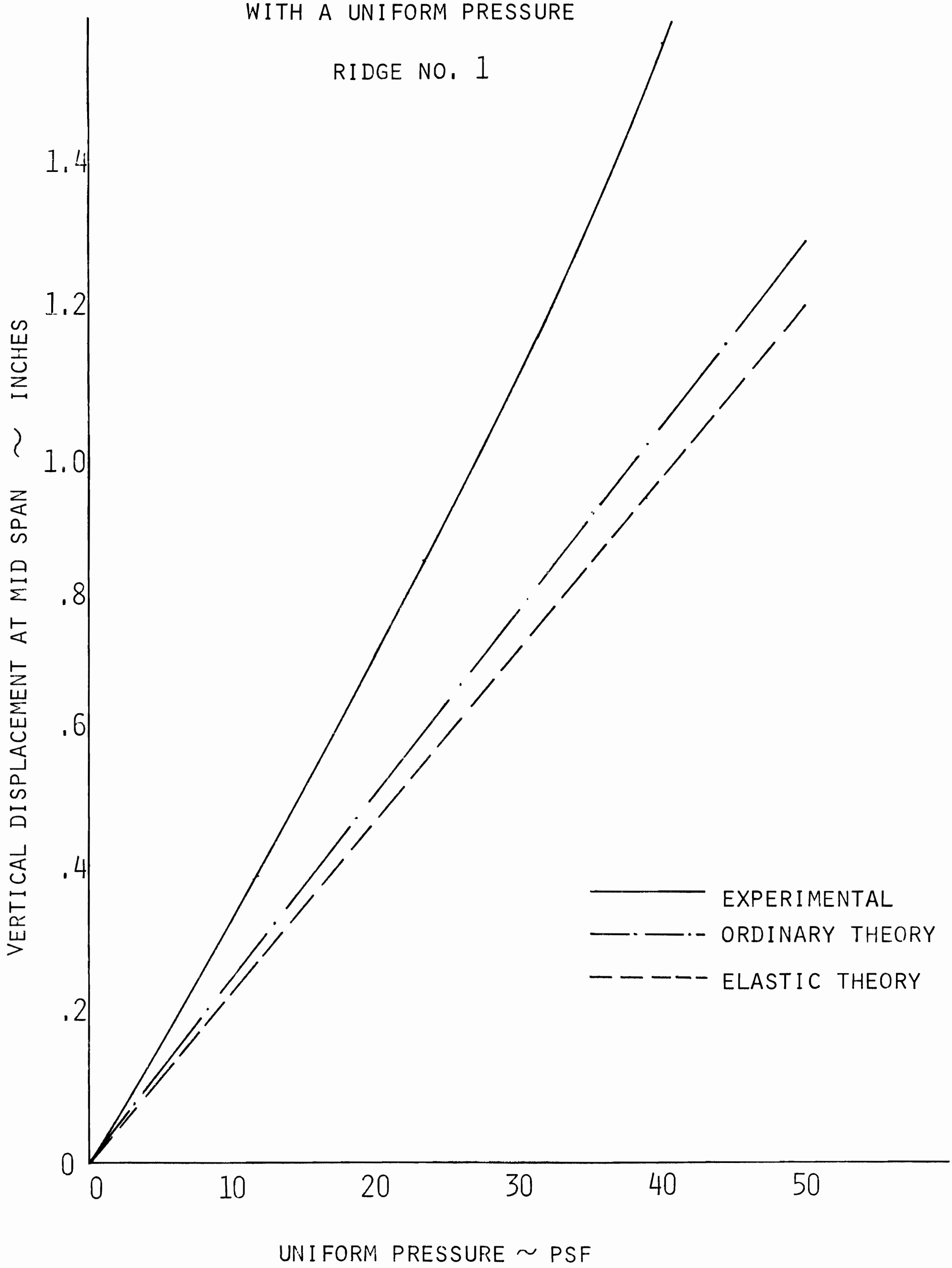


FIG. 18 THEORETICAL AND EXPERIMENTAL VERTICAL DISPLACEMENTS AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE

RIDGE NO. 2

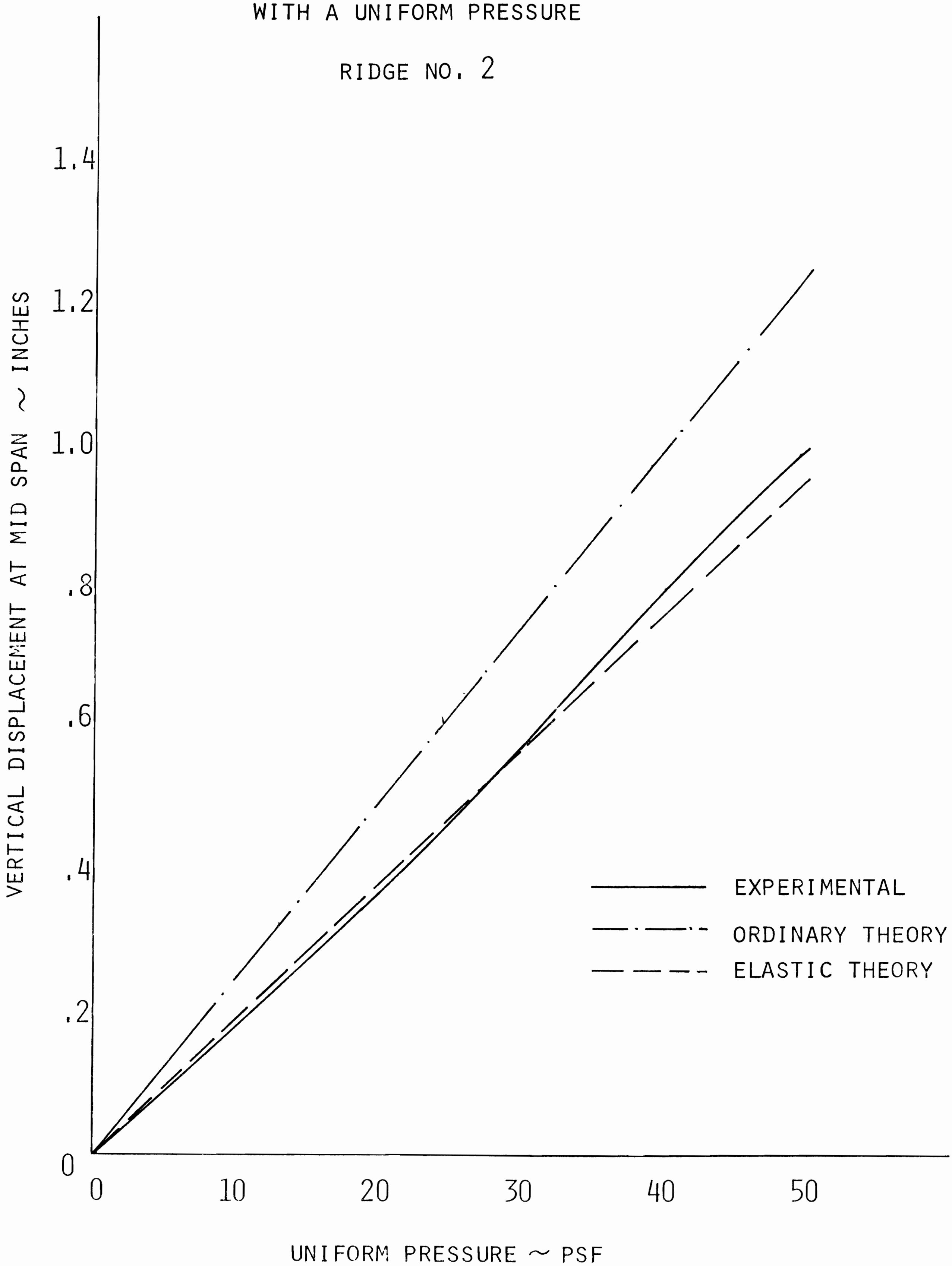


FIG. 19 THEORETICAL AND EXPERIMENTAL VERTICAL DISPLACEMENTS AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE

RIDGE NO. 3

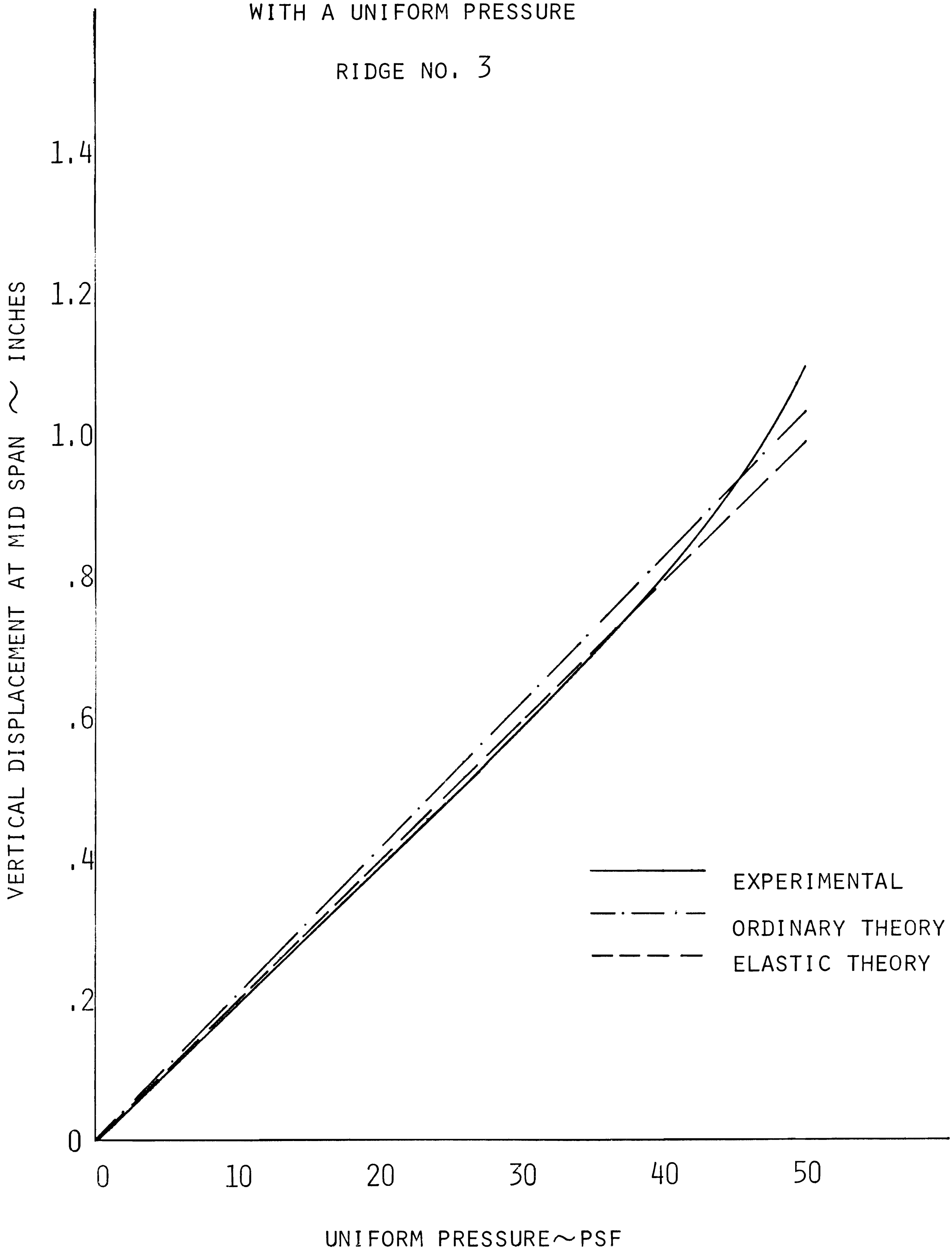


FIG. 20 THEORETICAL AND EXPERIMENTAL VERTICAL DISPLACEMENTS AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE

RIDGE NO. 4

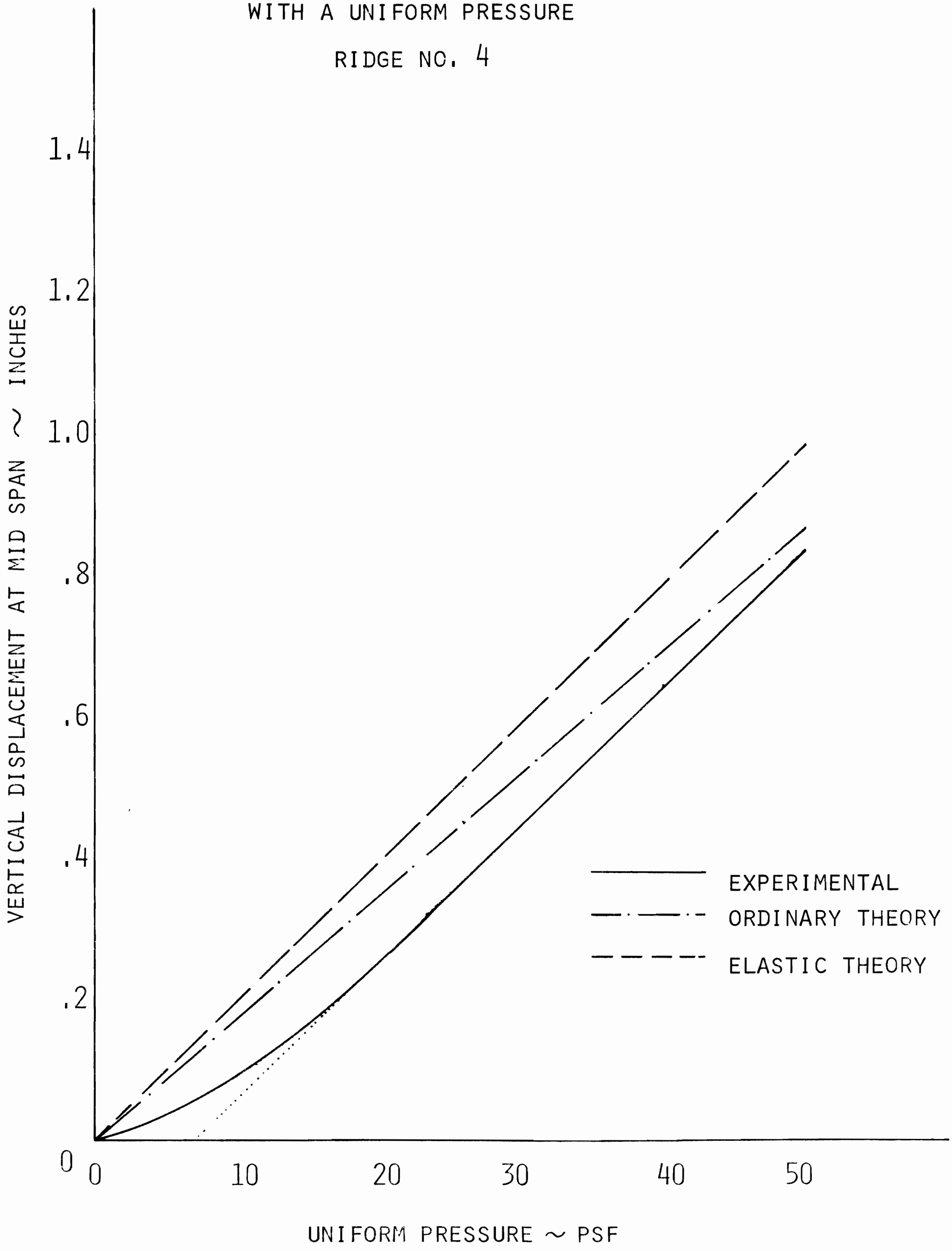


FIG. 21 THEORETICAL AND EXPERIMENTAL VERTICAL DISPLACEMENTS AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE

RIDGE NO. 5

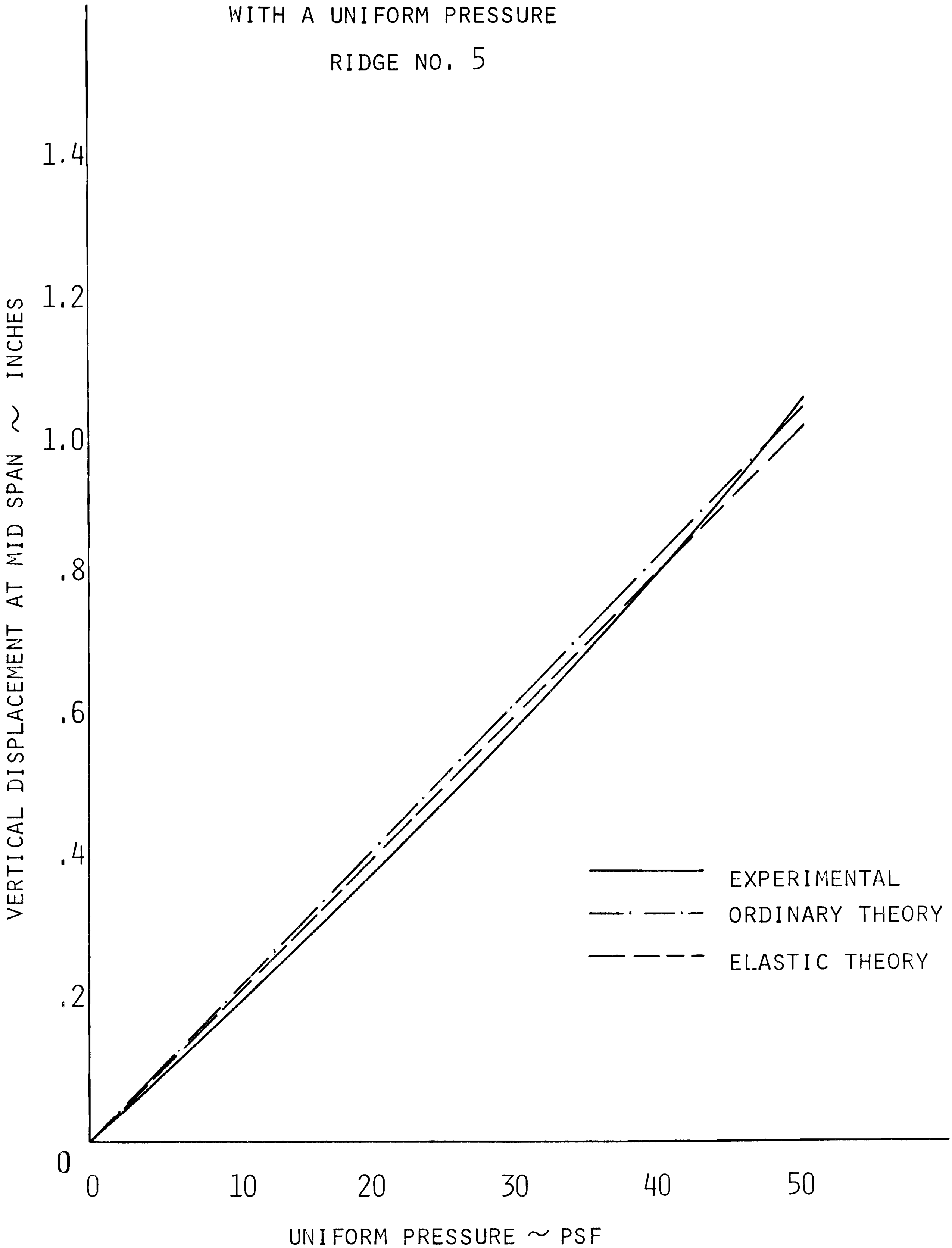


FIG. 22 THEORETICAL AND EXPERIMENTAL VERTICAL DISPLACEMENTS AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE

RIDGE NO. 6

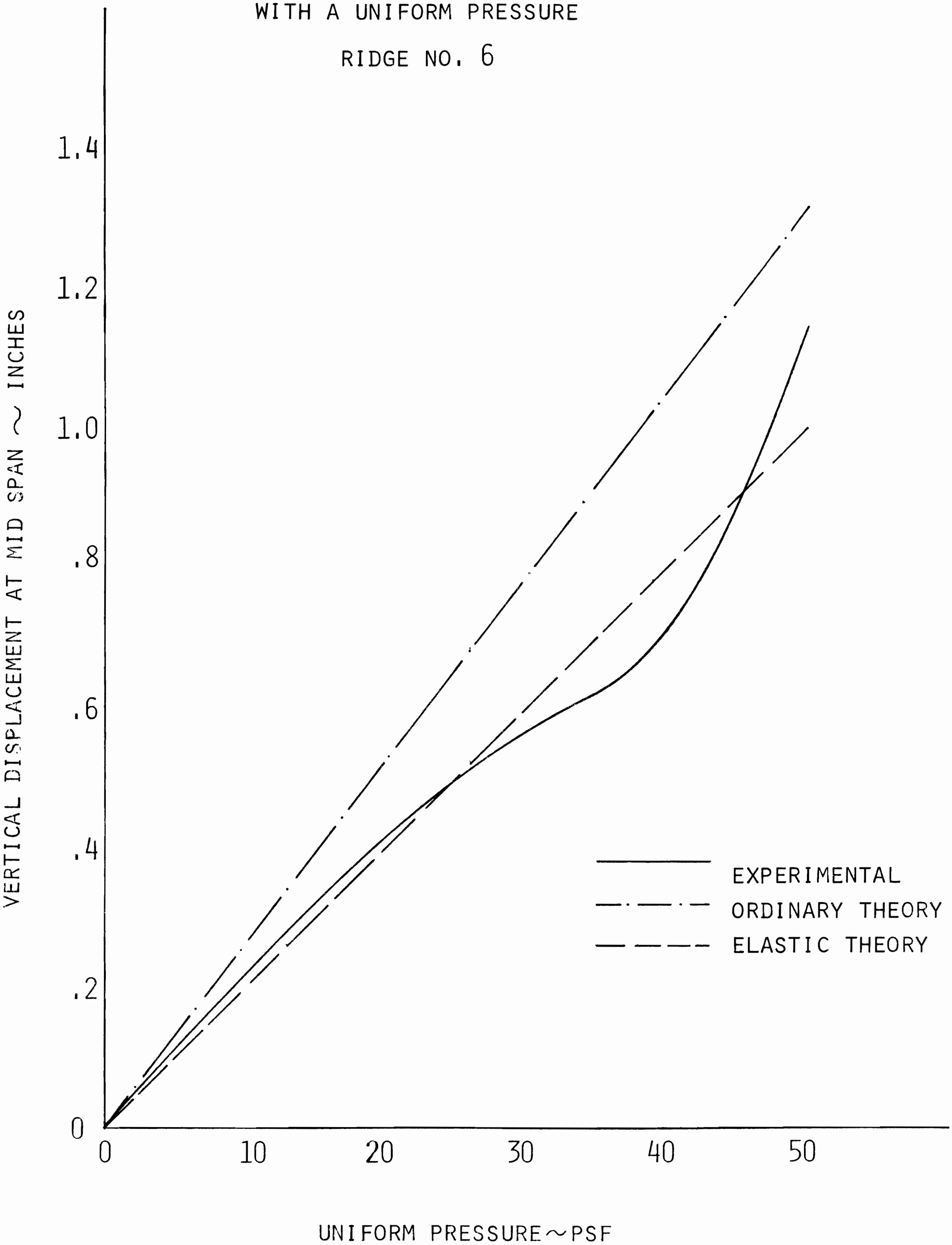


FIG. 23 THEORETICAL AND EXPERIMENTAL VERTICAL DISPLACEMENTS AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE

RIDGE NO. 7

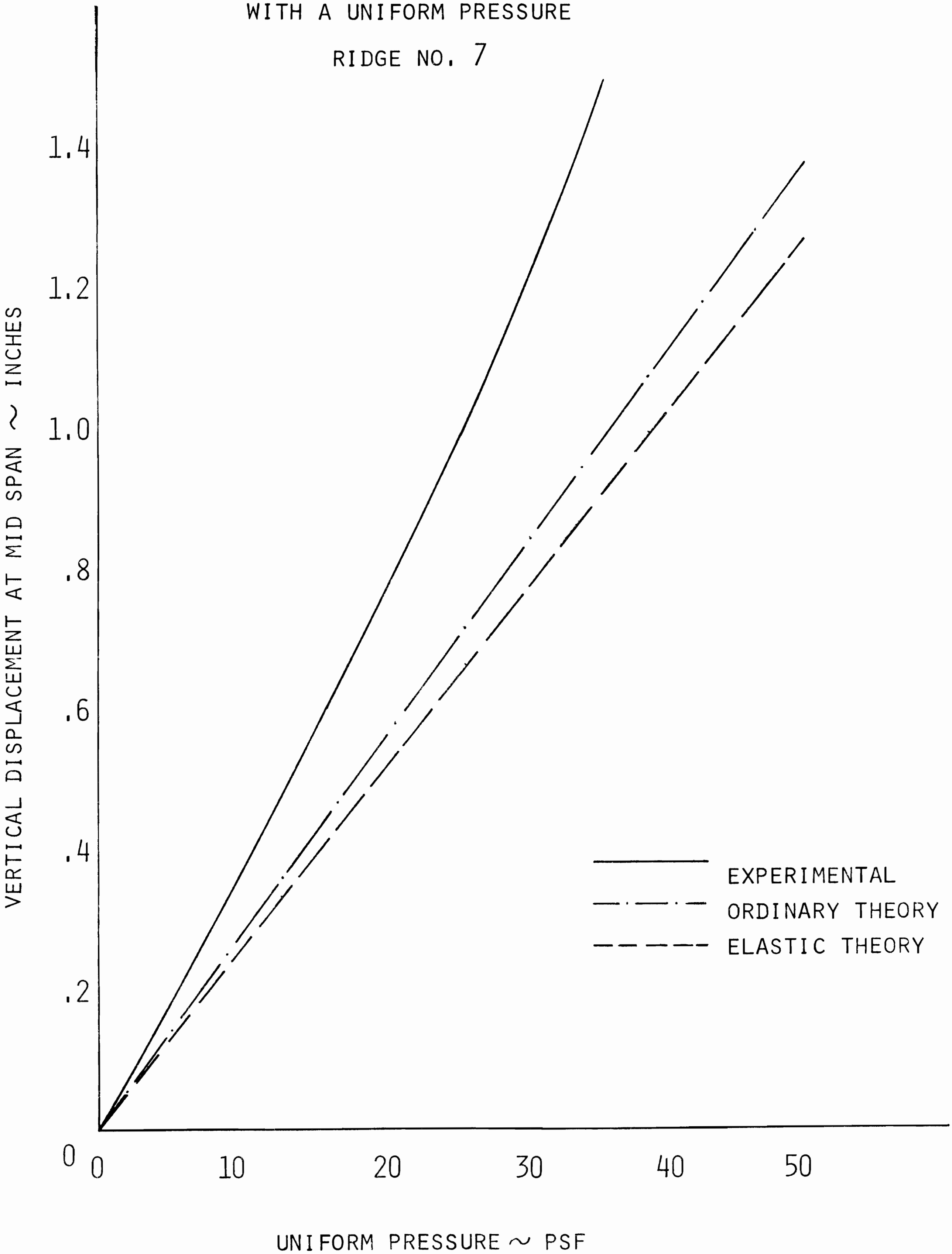


FIG. 24 THEORETICAL AND EXPERIMENTAL LONGITUDINAL STRESSES AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE

RIDGE NO. 1

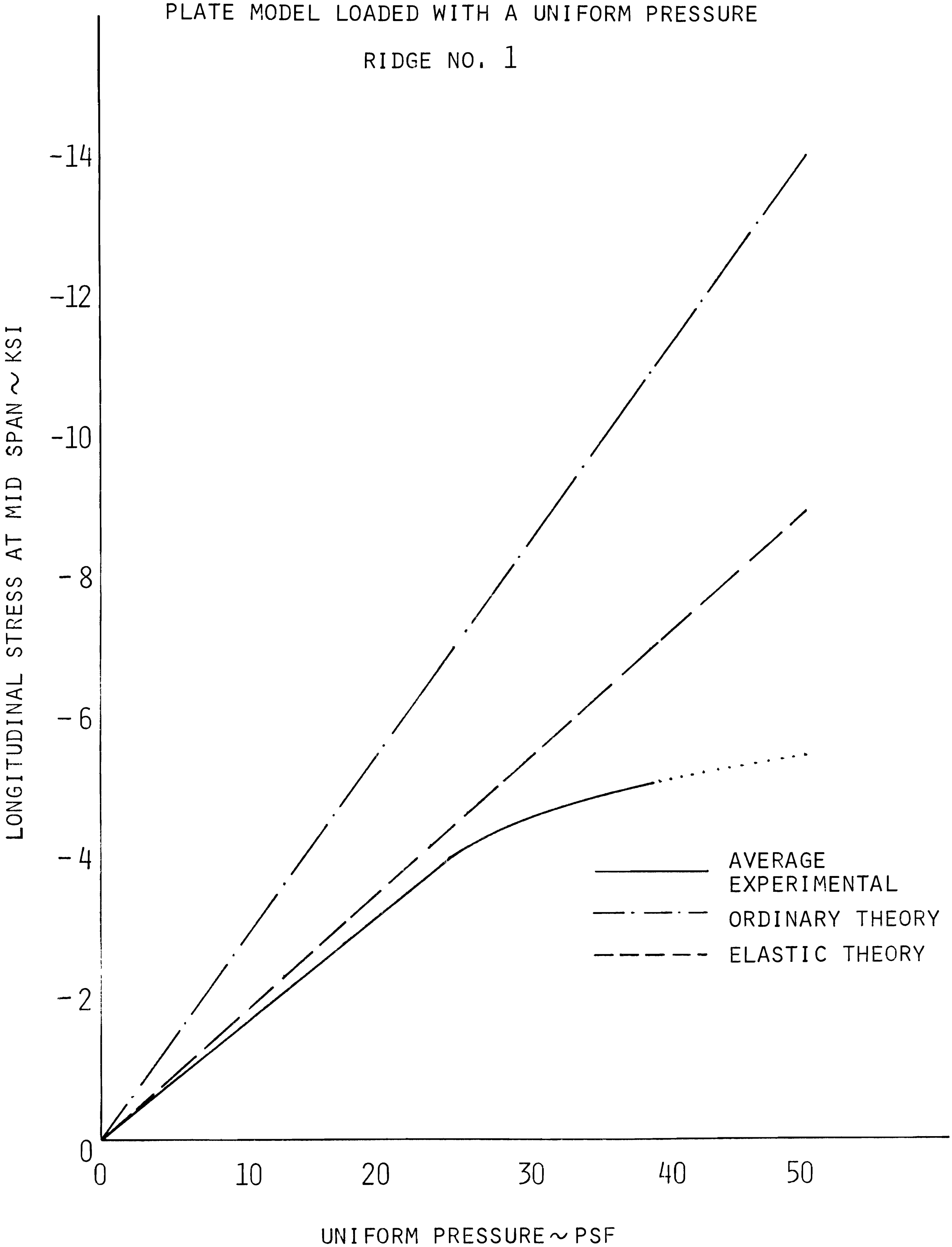


FIG. 25 THEORETICAL AND EXPERIMENTAL LONGITUDINAL STRESSES AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE RIDGE NO. 2

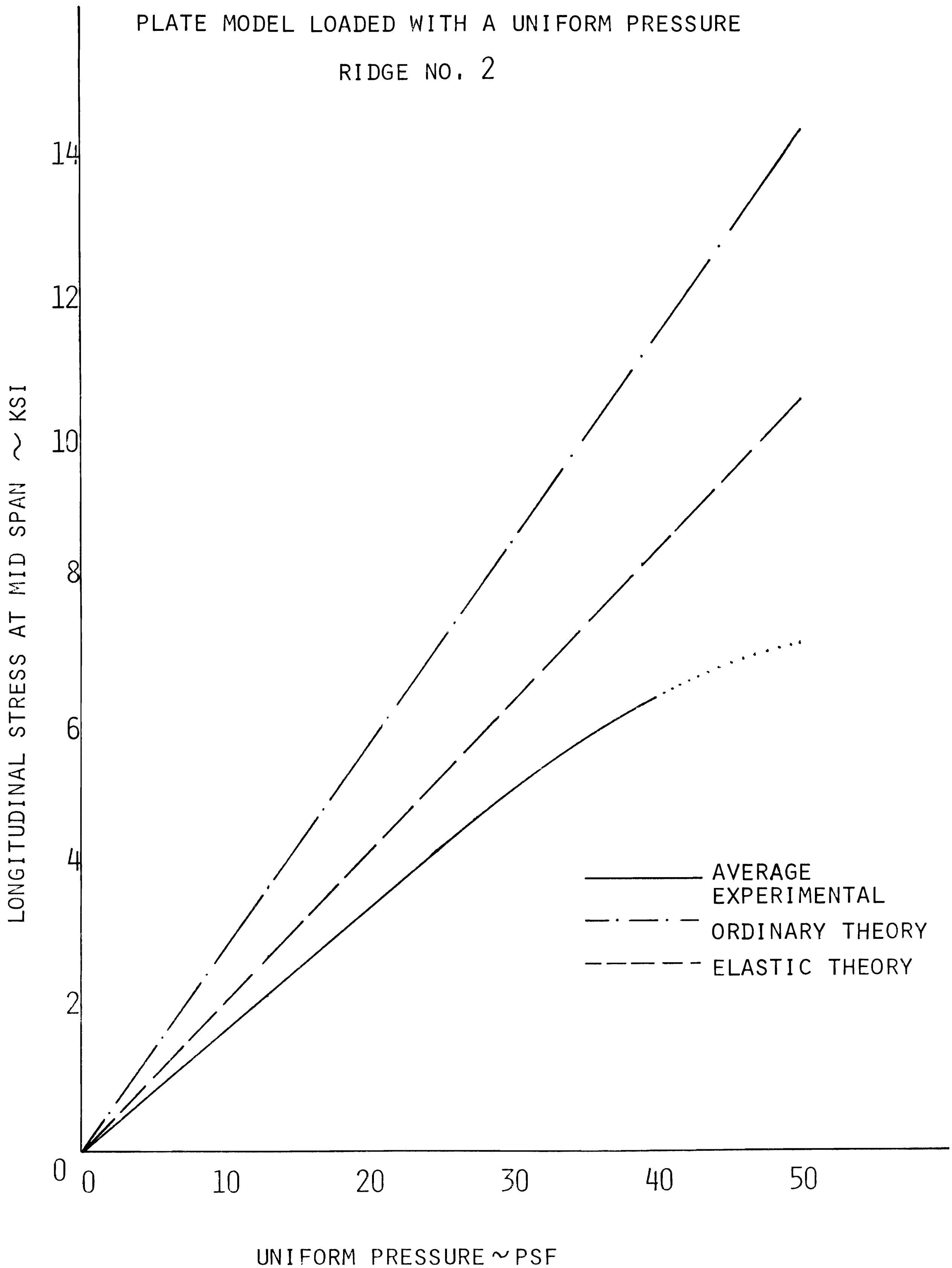


FIG. 26 THEORETICAL AND EXPERIMENTAL LONGITUDINAL STRESSES AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE

RIDGE NO. 3

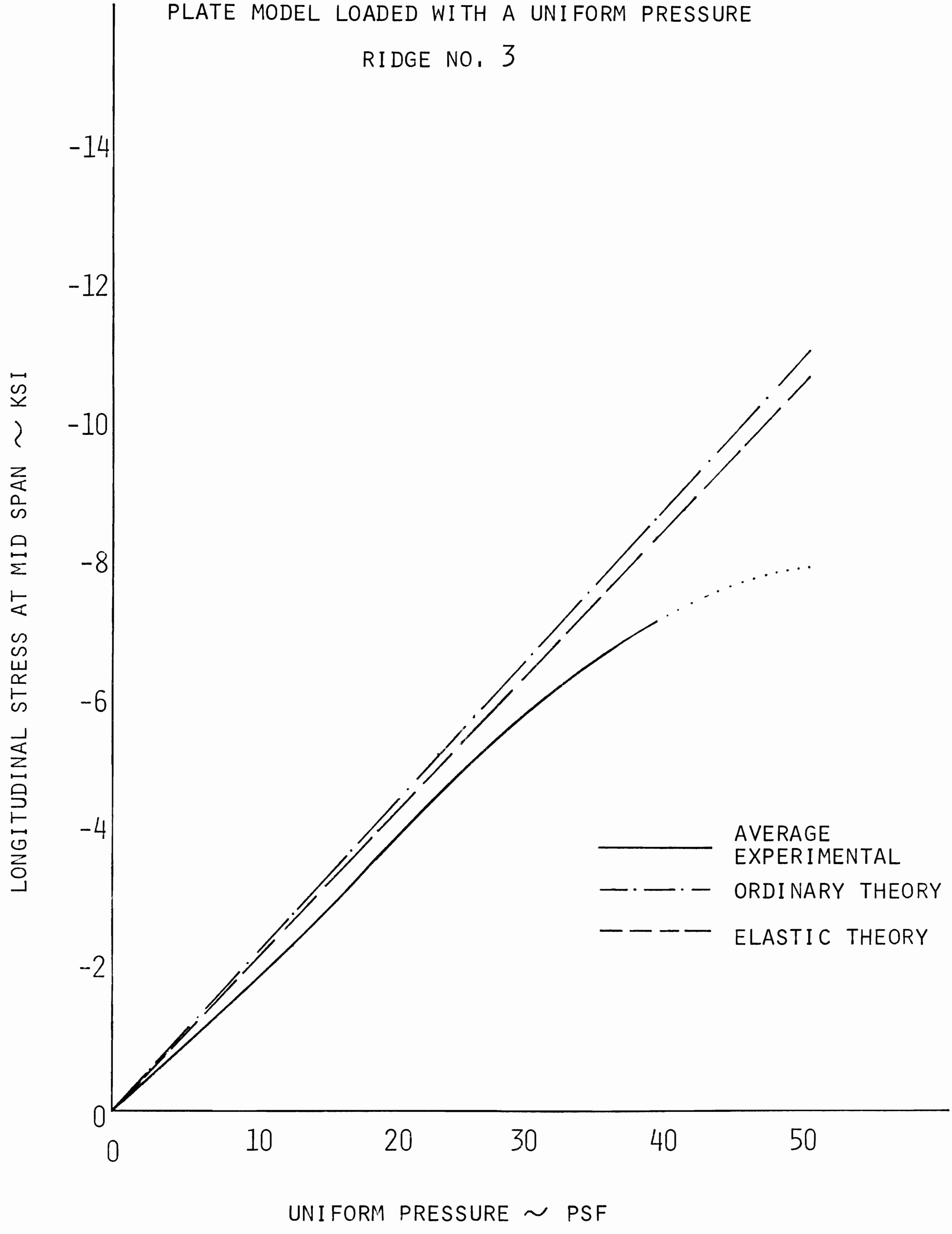


FIG. 27 THEORETICAL AND EXPERIMENTAL LONGITUDINAL STRESSES AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE RIDGE NO. 4

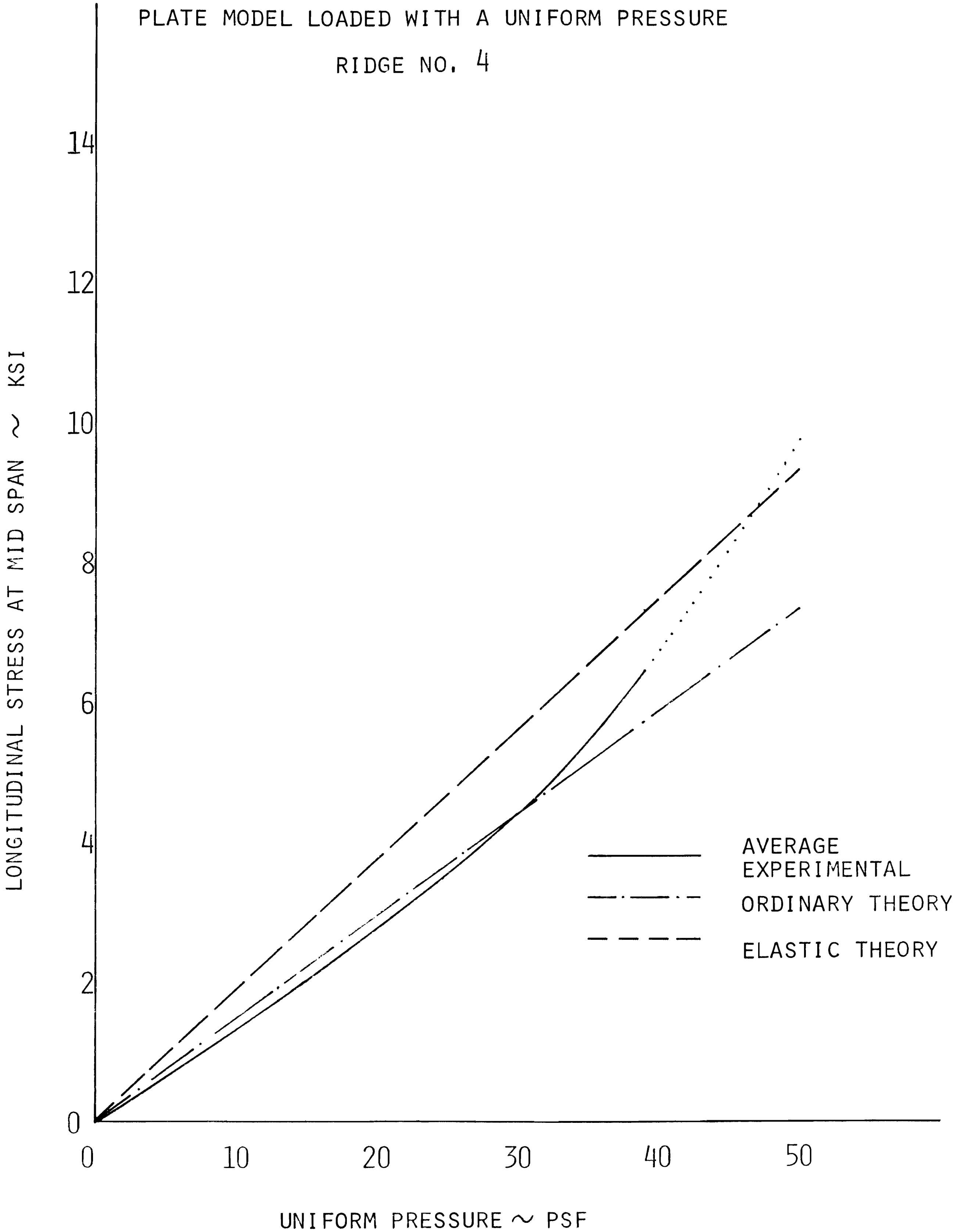


FIG. 28 THEORETICAL AND EXPERIMENTAL LONGITUDINAL STRESSES AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE RIDGE NO.5

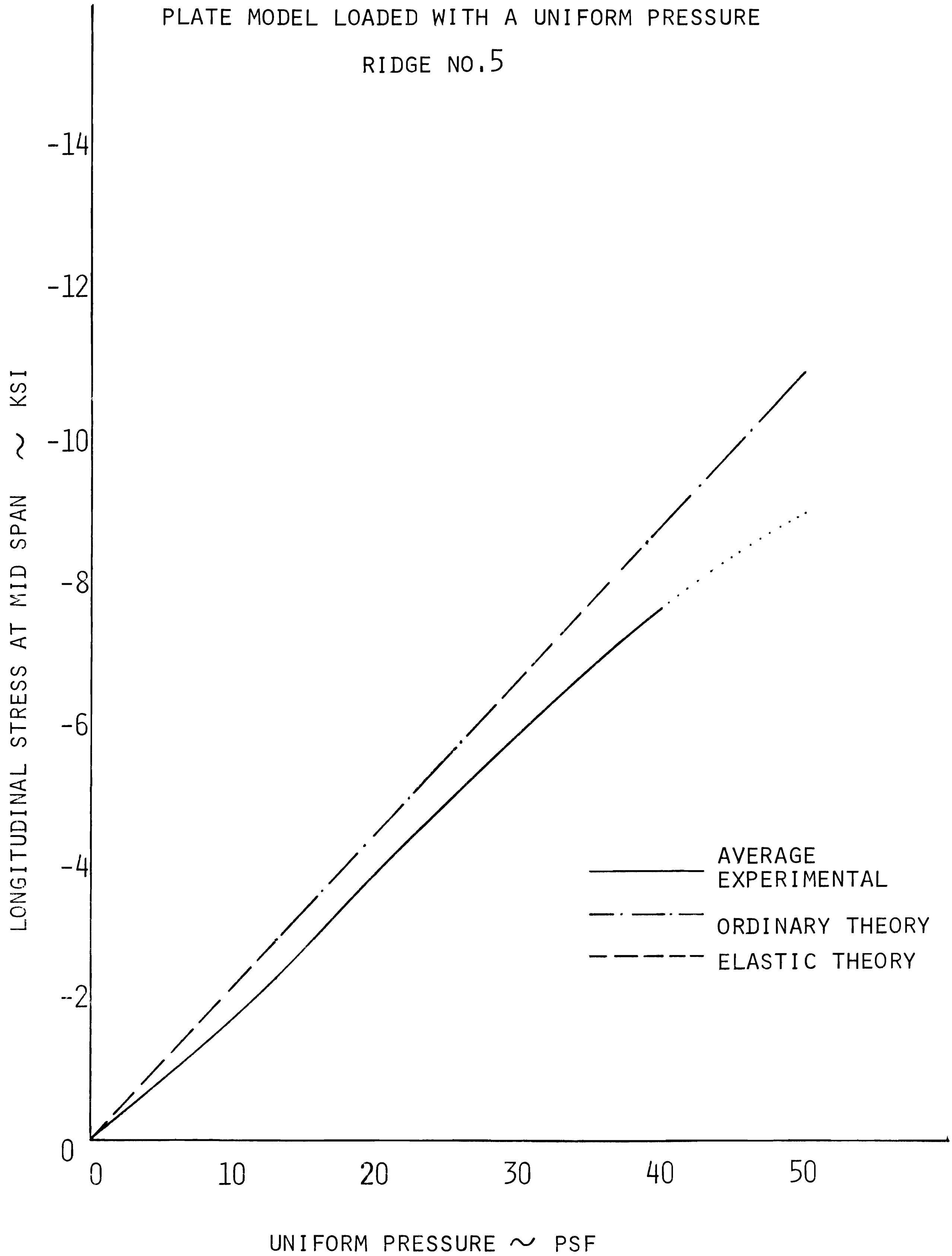


FIG. 29 THEORETICAL AND EXPERIMENTAL LONGITUDINAL STRESSES AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE RIDGE NO. 6

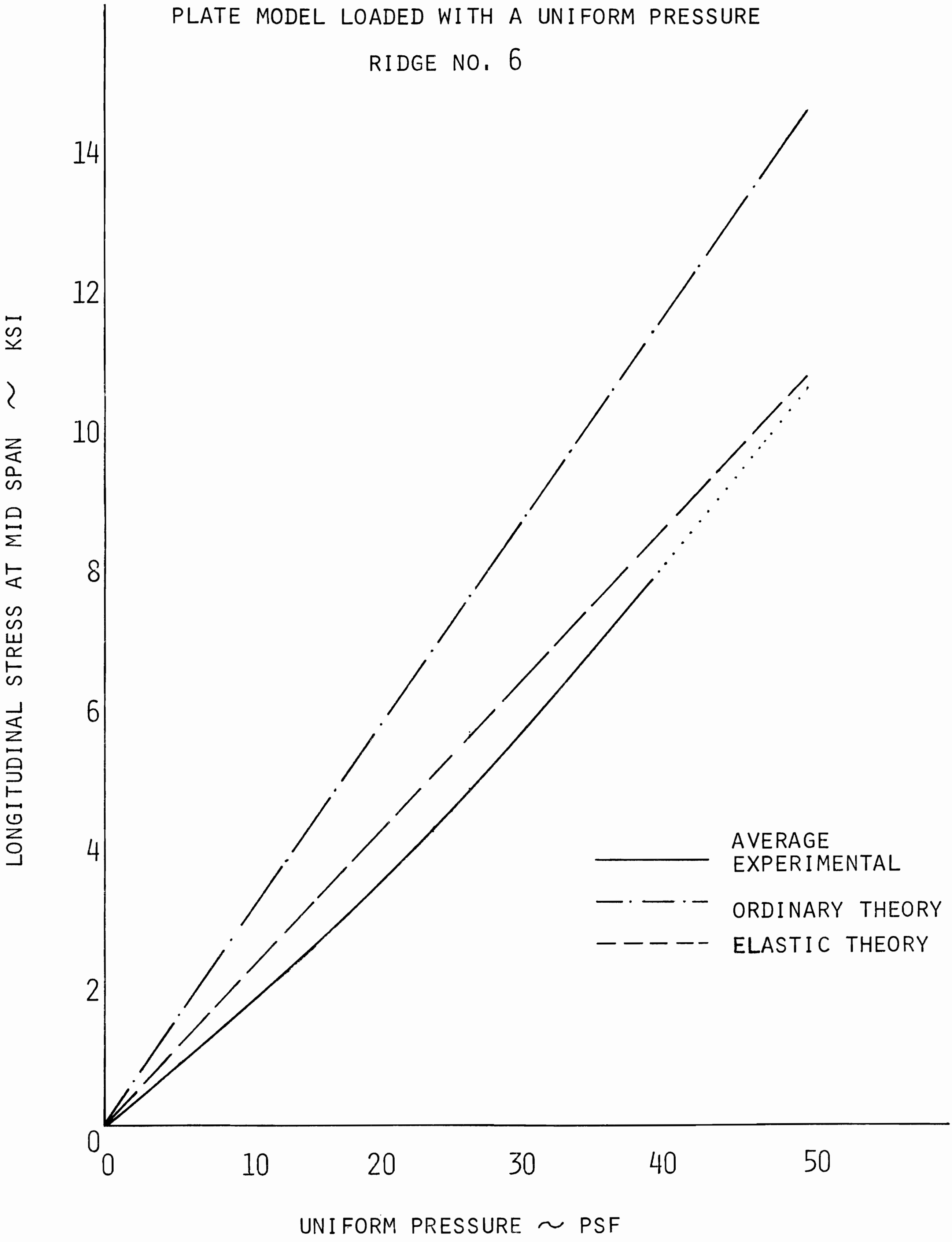
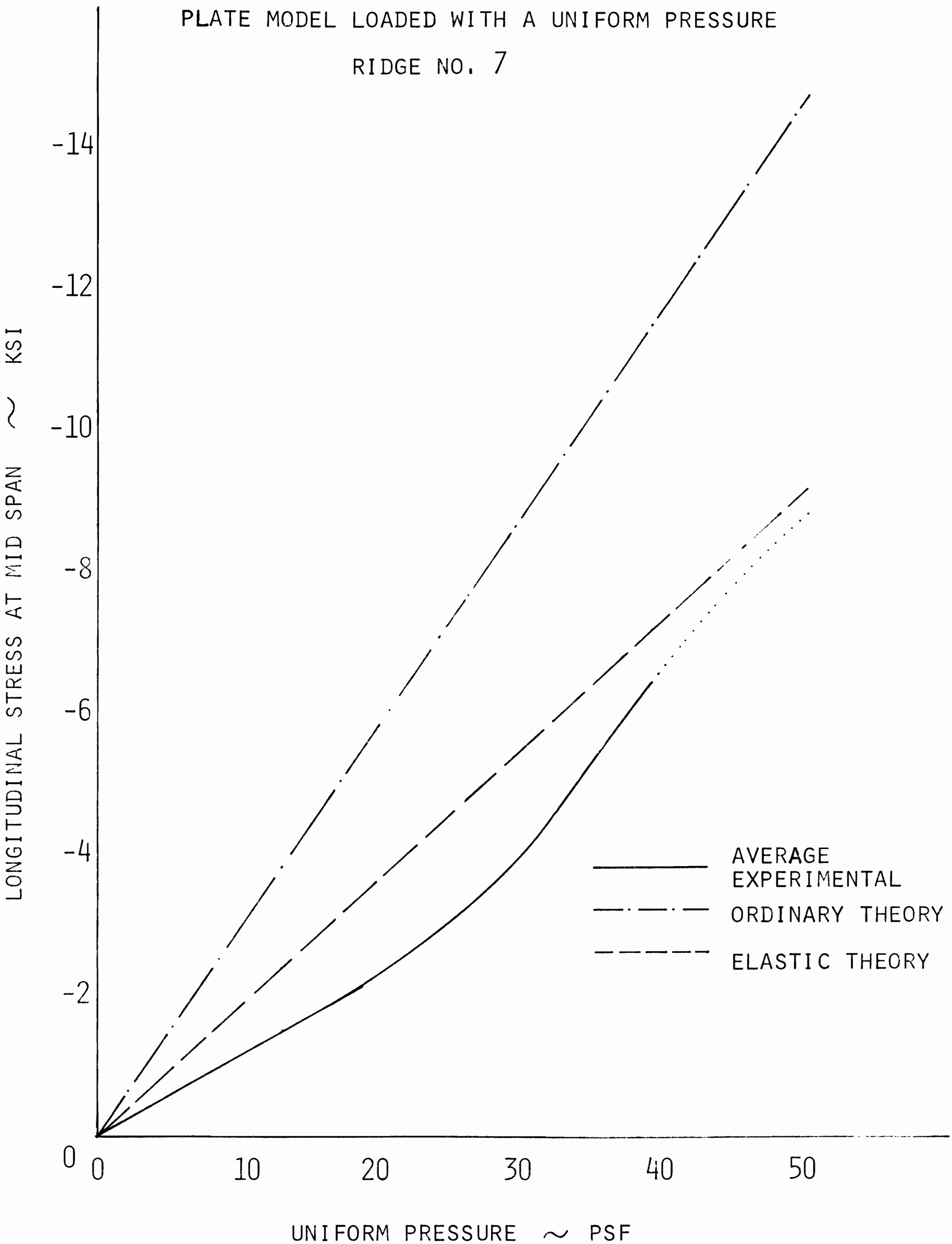


FIG. 30 THEORETICAL AND EXPERIMENTAL LONGITUDINAL STRESSES AT MIDSPAN OF THE 19 FT. FOLDED PLATE MODEL LOADED WITH A UNIFORM PRESSURE RIDGE NO. 7



CHAPTER V

CONCLUSION

V CONCLUSION

The results of the ordinary theory and elasticity theory are compared with the experiment in Figures 17 to 30. The comparison of vertical displacement at the midspan of the screw ridges is shown in Figures 17 to 23; on the other hand, in Figures 24 to 30, the comparison of longitudinal stresses at midspan is shown for ridges 1 to 7.

It is seen from the plotted results that the vertical displacements and longitudinal stresses vary linearly with the uniform pressure loading, which conforms to the basic assumptions.

Discrepancies arise due to the following imperfections in the model.

1) Initial waviness of the surface.
2) Compressibility of the core.
3) Imperfect bonding.
4) Joints between panels not rigid.

Unstable behaviour at the compression ridges of the experimental model can also be observed. See Figures 13, 5 and 7.

Instability is manifested in the displacement plots by an increasing slope of the experimental curve, together with a decreasing slope in the stress plot.

From the figures, it can be observed that symmetrically located ridges or joints show similar behaviour in both theoretical and experimental results.

Except for ridges 1, 7 and 4, the results of the elasticity theory gives better correlation with the experiment.

The apparent discrepancies in ridge 1, 7 and 4, can be explained as follows. For ridge 1 and 7 this was due to the fact that the free edge displacements before relaxation were not calculated in the computer program, however, the stresses show reasonable agreement with the experiment. In the case of ridge 4, comparison of Figures 20 and 27 shows that the experimental results may have been influenced by some local phenomenon.

REFERENCES

REFERENCES

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2. Plantema, F.J., "Sandwich Construction",John Wiley & Sons, Inc., New York.

3. De Fries-Skene, A. and Scordelis, A.C., "Direct Stiffness Solution for Folded Plates" Journal of the Structural Division, Proceedings of the A.S.C.E., August, 1964.

4. Fazio, P.P. and Kennedy, J.B., "Experimental and Theoretical Study of Aluminum Sandwich Elements", June, 1968. SGWU Library, No. TA660-P6F3.

APPENDIX

APPENDIX

FIXED EDGE FORCES DUE TO DISTRIBUTED LOAD

General Theory

Governing equation

$$\frac{\partial^4 \bar{w}_{bx}}{\partial x^4} + \frac{\partial^4 (\bar{w}_{bx} + \bar{w}_{by})}{\partial x^2 \partial y^2} + \frac{\partial^4 \bar{w}_{by}}{\partial y^4} = \frac{q}{D} \qquad \text{(A1)}$$

The loading q is uniform in x direction and can vary in y direction. We will concern ourselves only with a linear variation.

The loading can be represented by

$$q = \frac{4}{\pi} q(y) \sum_{m=1,3,5,\cdots} \frac{1}{m} \sin \frac{m\pi x}{L} \qquad \text{(A2)}$$

Taking as before the m^{th} term

$$q^m = \frac{4q(y)}{m\pi} \sin \frac{m\pi x}{L} \qquad \text{(A3)}$$

In the following, the superscripts will be deleted. Splitting the partial deflections in x and y direction, as follows

$$\bar{w}_{bx} = \tilde{w}_{bx} + w_{bx} \quad , \quad \bar{w}_{sx} = \tilde{w}_{sx} + w_{sx}$$
$$\bar{w}_{by} = \tilde{w}_{by} + w_{by} \quad , \quad \bar{w}_{sy} = \tilde{w}_{sy} + w_{sy} \qquad \text{(A4)}$$

where w_{bx}, w_{by} are solutions of the governing differential equation for zero load (equation (19)). $\tilde{w}_{bx}$ is the solution of 4th order D.E.

$$\frac{\partial^4 \tilde{w}_{bx}}{\partial x} = \frac{4\, q(y)}{m\pi D} \sin \frac{m\pi x}{L} \qquad \text{(A5)}$$

and $\tilde{w}_{sx}$ follows from

$$\frac{\partial \tilde{w}_{sx}}{\partial x} = -\frac{D}{S} \frac{\partial^3 \tilde{w}_{bx}}{\partial x_3} \qquad \text{(A6)}$$

Then $$\tilde{w}_{bx} = \frac{4}{LD} q(y) \left(\frac{L}{m\pi}\right)^5 \sin \frac{m\pi x}{L} \qquad \text{(A7)}$$

and $$\tilde{w}_{sx} = \frac{4}{LS} q(y) \left(\frac{L}{m\pi}\right)^3 \sin \frac{m\pi x}{L} \qquad \text{(A8)}$$

The second part of the solution is similar as that shown in Chapt.II of this paper. See equations (22), (23), (25), (26) for $\bar{w}_{bx}$, $\bar{w}_{by}$, $\bar{w}_{sx}$ and $\bar{w}_{sy}$, respectively. From equation (12a) and from

$$\bar{w} = \bar{w}_{bx} + \bar{w}_{sx} = \bar{w}_{by} + \bar{w}_{sy}$$

it follows that $$\tilde{w}_{by} = \tilde{w}_{bx}$$

$$\tilde{w}_{sy} = \tilde{w}_{sx}$$

Both Edges Fixed

Now all the components of the partial deflections can be added and the unknown constants derived for the following boundary conditions for fixed edges at $y = \pm \frac{b}{2}$

$$\frac{\partial \bar{w}_{sx}}{\partial x} = 0 \qquad \text{(Aa)}$$

$$\bar{w} = 0 \qquad \text{(Ab)}$$

$$\frac{\partial \bar{w}_{by}}{\partial y} = 0 \qquad \text{(Ac)}$$

We will again find a solution by separation of the loading into a symmetric and an antisymmetric case.

Symmetric Load

$$q(y) = q_s$$

for symmetry $\bar{C}_1 = \bar{C}_4 = \bar{W}_3 = 0$

$$\bar{w}_{by} = \frac{4q_s}{L}\left[\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \bar{C}_2 \cosh\frac{m\pi y}{L} + \bar{C}_3 \frac{m\pi y}{L}\sinh\frac{m\pi y}{L} + k_s \bar{W}_2 \cosh\alpha y\right]\sin\frac{m\pi x}{L}$$

$$\bar{w}_{bx} = \frac{4q_s}{L}\left[\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \bar{C}_2 \cosh\frac{m\pi y}{L} + \bar{C}_3 \frac{m\pi y}{L}\sinh\frac{m\pi y}{L} + (1+k_s)\bar{W}_2 \cosh\alpha y\right]\sin\frac{m\pi x}{L}$$

$$\bar{w}_{sy} = \frac{4q_s}{L}\left[\frac{1}{S}\left(\frac{L}{m\pi}\right)^3 - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\bar{C}_3 \cosh\frac{m\pi y}{L} - k_s \bar{W}_2 \cosh\alpha y\right]\sin\frac{m\pi x}{L}$$

$$\bar{w}_{sx} = \frac{4q_s}{L}\left[\frac{1}{S}\left(\frac{L}{m\pi}\right)^3 - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\bar{C}_3 \cosh\frac{m\pi y}{L} - (1+k_s)\bar{W}_2 \cosh\alpha y\right]\sin\frac{m\pi x}{L}$$

$$\bar{w} = \frac{4q_s}{L}\left[\left\{\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \frac{1}{S}\left(\frac{L}{m\pi}\right)^3\right\} + \bar{C}_2 \cosh\frac{m\pi y}{L} + \right.$$
$$\left. + \bar{C}_3\left(\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\cosh\frac{m\pi y}{L}\right)\right]\sin\frac{m\pi x}{L}$$

The first B.C. (Aa) gives

$$\frac{1}{S}\left(\frac{L}{m\pi}\right)^3 - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\bar{C}_3 \cosh\beta - (1+k_s)\bar{W}_2 \cosh\frac{\alpha b}{2} = 0$$

Then
$$\bar{W}_2 = \frac{\left(\frac{L}{m\pi}\right)^3}{S(1+k_s)\cos\frac{\alpha b}{2}} - \frac{\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\cosh\beta}{(1+k_s)\cosh\frac{\alpha b}{2}}\bar{C}_3$$

The second B.C. (Ab) gives

$$\bar{C}_2 = -\frac{\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \frac{1}{S}\left(\frac{L}{m\pi}\right)^3}{\cosh\beta} - \left(\beta\tanh\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)\bar{C}_3$$

Now

$$\bar{w}_{by} = \frac{4q_s}{L}\Bigg[\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 - \left\{\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \frac{1}{S}\left(\frac{L}{m\pi}\right)^3\right\}\frac{\cosh\frac{m\pi y}{L}}{\cosh\beta} + \frac{1}{S}\left(\frac{k_s}{1+k_s}\right)\left(\frac{L}{m\pi}\right)^3\frac{\cosh\alpha y}{\cosh\frac{\alpha b}{2}}$$
$$+ \bar{C}_3\left\{\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} - \left(\beta\tanh\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)\cosh\frac{m\pi y}{L}\right.$$
$$\left. - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\left(\frac{k_s}{1+k_s}\right)\frac{\cosh\beta}{\cosh\frac{\alpha b}{2}}\cosh\alpha y\right\}\Bigg]\sin\frac{m\pi x}{L}$$

From the last B.C. (Ac), we obtain

$$\bar{C}_3 = \lambda_1\left[\left\{\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \frac{1}{S}\left(\frac{L}{m\pi}\right)^3\right\}\tanh\beta - \frac{1}{S}\left(\frac{L}{m\pi}\right)^3\frac{1}{p}\tanh\beta p\right]$$

where λ_1, is as previously defined (Ref. Equation (35)),

or $\bar{C}_3 = \frac{\lambda_1}{D}\left(\frac{L}{m\pi}\right)^5\left\{1 + \frac{D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\tanh\beta p}{p\tanh\beta}\right)\right\}\tanh\beta$

defining $\bar{\lambda}_1 = \lambda_1\left\{1 + \frac{D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\tanh\beta p}{p\tanh\beta}\right)\right\}\tanh\beta$ (A9)

then $\bar{C}_3 = \frac{1}{D}\left(\frac{L}{m\pi}\right)^5\bar{\lambda}_1$

Antisymmetric Part

$$q(y) = q_A\frac{2y}{b}$$

for antisymmetry $\bar{C}_2 = \bar{C}_3 = \bar{W}_2 = 0$

$$\bar{w}_{by} = \frac{8q_A}{bL}\left[\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 y + \bar{C}_1\sinh\frac{m\pi y}{L} + \bar{C}_4\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} + k_s\bar{W}_3\sinh\alpha y\right]\sin\frac{m\pi x}{L}$$

$$\bar{w}_{bx} = \frac{8q_A}{bL}\left[\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 y + \bar{C}_1\sinh\frac{m\pi y}{L} + \bar{C}_4\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} + (1+k_s)\bar{W}_3\sinh\alpha y\right]\sin\frac{m\pi x}{L}$$

$$\bar{w}_{sy} = \frac{8q_A}{bL}\left[\frac{1}{S}\left(\frac{L}{m\pi}\right)^3 y - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\bar{C}_4\sinh\frac{m\pi y}{L} - k_s\bar{W}_3\sinh\alpha y\right]\sin\frac{m\pi x}{L}$$

$$\bar{w}_{sx} = \frac{8q_A}{bL}\left[\frac{1}{S}\left(\frac{L}{m\pi}\right)^3 y - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 \bar{C}_4 \sinh\frac{m\pi y}{L} - (1+K_s)\bar{W}_3 \sinh\alpha y\right]\sin\frac{m\pi x}{L}$$

$$\bar{w} = \frac{8q_A}{bL}\left[\left\{\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \frac{1}{S}\left(\frac{L}{m\pi}\right)^3\right\} y + \bar{C}_1 \sinh\frac{m\pi y}{L} + \bar{C}_4\left(\frac{m\pi y}{L}\cosh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2 \sinh\frac{m\pi y}{L}\right)\right]\sin\frac{m\pi x}{L}$$

The first B.C. (Aa) gives

$$\bar{W}_3 = \frac{b\left(\frac{L}{m\pi}\right)^3}{2S(1+k_s)\sin\frac{\alpha b}{2}} - \frac{\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2}{1+k_s}\frac{\sinh\beta}{\sinh\frac{\alpha b}{2}}\bar{C}_4$$

The second B.C. (Ab) gives

$$\bar{C}_1 = -\frac{b\left\{\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \frac{1}{S}\left(\frac{L}{m\pi}\right)^3\right\}}{2\sinh\beta} - \left(\beta\coth\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)\bar{C}_4$$

Now

$$\bar{w}_{by} = \frac{8q_A}{bL}\left[\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 y - \frac{b}{2}\left\{\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \frac{1}{S}\left(\frac{L}{m\pi}\right)^3\right\}\frac{\sinh\frac{m\pi y}{L}}{\sinh\beta} + \frac{b}{2S}\left(\frac{L}{m\pi}\right)^3\frac{k_s}{1+k_s}\frac{\sinh\alpha y}{\sinh\frac{\alpha b}{2}} + \bar{C}_4\left\{\frac{m\pi y}{L}\cos\frac{m\pi y}{L} - \left(\beta\coth\beta - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)\times \sinh\frac{m\pi y}{L} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{k_s}{1+k_s}\frac{\sinh\beta}{\sinh\frac{\alpha b}{2}}\sinh\alpha y\right\}\right]\sin\frac{m\pi x}{L}$$

From the last B.C. (Ac) we obtain

$$\bar{C}_4 = \lambda_2\left[\frac{1}{D}\left(\frac{L}{m\pi}\right)^6 - \frac{b}{2}\left\{\frac{1}{D}\left(\frac{L}{m\pi}\right)^5 + \frac{1}{S}\left(\frac{L}{m\pi}\right)^3\right\}\coth\beta + \frac{b}{2S}\left(\frac{L}{m\pi}\right)^3\frac{1}{p}\coth\beta p\right]$$

where λ_2 is as previously defined equation (36)

$$\text{or } \bar{C}_4 = \frac{\lambda_2 b}{2D}\left(\frac{L}{m\pi}\right)^5\left[\frac{1}{\beta} - \left\{1 + \frac{D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\coth\beta p}{p\coth\beta}\right)\right\}\coth\beta\right]$$

defining $\bar{\lambda}_2 = \lambda_2\left[\frac{1}{\beta} - \left\{1 + \frac{D}{S}\left(\frac{m\pi}{L}\right)^2\left(1 - \frac{\coth\beta p}{p\coth\beta}\right)\right\}\coth\beta\right]$ (A10)

then $\bar{C}_4 = \frac{b}{2D}\left(\frac{L}{m\pi}\right)^5\bar{\lambda}_2$

Combining now symmetric and antisymmetric loads by substitution of

symmetric load $q_s = \frac{q_1 + q_2}{2}$

antisymmetric load $q_A = \frac{q_1 + q_2}{2}$

$$\bar{w}_{by} = \frac{2}{DL}\left(\frac{L}{m\pi}\right)^5 \sin\frac{m\pi x}{L}\lfloor q_1\ \ q_2\rfloor \times \tag{A11a}$$

$$\left(\begin{Bmatrix} 1+\frac{2y}{b} \\ 1-\frac{2y}{b}\end{Bmatrix} + \begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2\end{bmatrix}\begin{Bmatrix}\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} \\ \frac{m\pi y}{L}\cosh\frac{m\pi y}{L}\end{Bmatrix} - \begin{bmatrix} a_1 & a_2 \\ a_1 & -a_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L} \\ \sinh\frac{m\pi y}{L}\end{Bmatrix}\right.$$

$$\left. + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L} \\ \sinh\frac{m\pi y}{L}\end{Bmatrix} + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\frac{1}{p^2}\begin{bmatrix} a_3 & a_4 \\ a_3 & -a_4\end{bmatrix}\begin{Bmatrix}\cosh\alpha y \\ \sinh\alpha y\end{Bmatrix}\right)$$

$$\bar{w}_{bx} = \frac{2}{DL}\left(\frac{L}{m\pi}\right)^5 \sin\frac{m\pi x}{L}\lfloor q_1\ \ q_2\rfloor \times \tag{A11b}$$

$$\left(\begin{Bmatrix} 1+\frac{2y}{b} \\ 1-\frac{2y}{b}\end{Bmatrix} + \begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2\end{bmatrix}\begin{Bmatrix}\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} \\ \frac{m\pi y}{L}\cosh\frac{m\pi y}{L}\end{Bmatrix} - \begin{bmatrix} a_1 & a_2 \\ a_1 & -a_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L} \\ \sinh\frac{m\pi y}{L}\end{Bmatrix}\right.$$

$$\left. + \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\lambda_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L} \\ \sinh\frac{m\pi y}{L}\end{Bmatrix} + \frac{D}{S}\left(\frac{m\pi}{L}\right)^2\begin{bmatrix} a_3 & a_4 \\ a_3 & -a_4\end{bmatrix}\begin{Bmatrix}\cosh\alpha y \\ \sinh\alpha y\end{Bmatrix}\right)$$

$$\bar{w}_{sy} = \frac{2}{SL}\left(\frac{L}{m\pi}\right)^3 \sin\frac{m\pi x}{L}\lfloor q_1\ \ q_2\rfloor \times$$

$$\left(\begin{Bmatrix} 1+\frac{2y}{b} \\ 1-\frac{2y}{b}\end{Bmatrix} - 2\begin{bmatrix}\bar{\lambda}_1 & \lambda_2 \\ \bar{\lambda}_1 & \lambda_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L} \\ \sinh\frac{m\pi y}{L}\end{Bmatrix} - \frac{1}{p^2}\begin{bmatrix} a_3 & a_4 \\ a_3 & -a_4\end{bmatrix}\begin{Bmatrix}\cosh\alpha y \\ \sinh\alpha y\end{Bmatrix}\right) \tag{A 11c}$$

$$\bar{w}_{sx} = \frac{2}{SL}\left(\frac{L}{m\pi}\right)^3 \sin\frac{m\pi x}{L}\lfloor q_1\ \ q_2\rfloor \times$$

$$\left(\begin{Bmatrix} 1+\frac{2y}{b} \\ 1-\frac{2y}{b}\end{Bmatrix} - 2\begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L} \\ \sinh\frac{m\pi y}{L}\end{Bmatrix} - \begin{bmatrix} a_3 & a_4 \\ a_3 & -a_4\end{bmatrix}\begin{Bmatrix}\cosh\alpha y \\ \sinh\alpha y\end{Bmatrix}\right) \tag{A 11d}$$

$$\bar{w} = \frac{2}{DL}\left(\frac{L}{m\pi}\right)^5 \sin\frac{m\pi x}{L}\lfloor q_1\ \ q_2\rfloor \times \tag{A 11e}$$

$$\left(\left\{1+\frac{D}{S}\left(\frac{m\pi}{L}\right)^2\right\}\begin{Bmatrix} 1+\frac{2y}{b} \\ 1-\frac{2y}{b}\end{Bmatrix} + \begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2\end{bmatrix}\begin{Bmatrix}\frac{m\pi y}{L}\sinh\frac{m\pi y}{L} \\ \frac{m\pi y}{L}\cosh\frac{m\pi y}{L}\end{Bmatrix}\right.$$

$$\left. - \begin{bmatrix} a_1 & a_2 \\ a_1 & -a_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L} \\ \sinh\frac{m\pi y}{L}\end{Bmatrix}\right)$$

where
$$\left.\begin{aligned}
a_1 &= \frac{1+\bar{\lambda}_1\beta\sinh\beta+\frac{D}{S}\left(\frac{m\pi}{L}\right)^2}{\cosh\beta}\\
a_2 &= \frac{1+\bar{\lambda}_2\beta\cosh\beta+\frac{D}{S}\left(\frac{m\pi}{L}\right)^2}{\sinh\beta}\\
a_3 &= \frac{1-2\bar{\lambda}_1\cosh\beta}{\cosh\beta p}\\
a_4 &= \frac{1-2\bar{\lambda}_2\sinh\beta}{\sinh\beta p}
\end{aligned}\right\} \quad \text{(A12)}$$

Substitution of equations (A11) into equations (2) to (6) yield the internal forces

$$\bar{M}_x = -D\left(\frac{\partial^2\bar{w}_{bx}}{\partial x^2}+\nu\frac{\partial^2\bar{w}_{by}}{\partial y^2}\right)$$
$$= \frac{2}{L}(1-\nu)\left(\frac{L}{m\pi}\right)^3\sin\frac{m\pi x}{L}\lfloor q_1 \quad q_2\rfloor \times$$
$$\left(\frac{1}{1-\nu}\begin{Bmatrix}1+\frac{2y}{b}\\ 1-\frac{2y}{b}\end{Bmatrix}+\begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2\\ \bar{\lambda}_1 & -\bar{\lambda}_2\end{bmatrix}\begin{Bmatrix}\frac{m\pi y}{L}\sinh\frac{m\pi y}{L}\\ \frac{m\pi y}{L}\cosh\frac{m\pi y}{L}\end{Bmatrix}-\begin{bmatrix}a_1 & a_2\\ a_1 & -a_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L}\\ \sinh\frac{m\pi y}{L}\end{Bmatrix}\right.$$
$$\left.-\left\{\frac{2\nu}{1-\nu}-\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right\}\begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2\\ \bar{\lambda}_1 & -\bar{\lambda}_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L}\\ \sinh\frac{m\pi y}{L}\end{Bmatrix}+\frac{D}{S}\left(\frac{m\pi}{L}\right)^2\begin{bmatrix}a_3 & a_4\\ a_3 & -a_4\end{bmatrix}\begin{Bmatrix}\cosh\alpha y\\ \sinh\alpha y\end{Bmatrix}\right)$$

(A13a)

$$\bar{M}_y = -D\left(\frac{\partial^2\bar{w}_{by}}{\partial y^2}+\nu\frac{\partial^2\bar{w}_{bx}}{\partial x^2}\right)$$
$$= -\frac{2}{L}(1-\nu)\left(\frac{L}{m\pi}\right)^3\sin\frac{m\pi x}{L}\lfloor q_1 \quad q_2\rfloor \times$$
$$\left(\frac{-\nu}{1-\nu}\begin{Bmatrix}1+\frac{2y}{b}\\ 1-\frac{2y}{b}\end{Bmatrix}+\begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2\\ \bar{\lambda}_1 & -\bar{\lambda}_2\end{bmatrix}\begin{Bmatrix}\frac{m\pi y}{L}\sinh\frac{m\pi y}{L}\\ \frac{m\pi y}{L}\cosh\frac{m\pi y}{L}\end{Bmatrix}-\begin{bmatrix}a_1 & a_2\\ a_1 & -a_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L}\\ \sinh\frac{m\pi y}{L}\end{Bmatrix}\right.$$
$$\left.+\left\{\frac{2}{1-\nu}+\frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right\}\begin{bmatrix}\bar{\lambda}_1 & \bar{\lambda}_2\\ \bar{\lambda}_1 & -\bar{\lambda}_2\end{bmatrix}\begin{Bmatrix}\cosh\frac{m\pi y}{L}\\ \sinh\frac{m\pi y}{L}\end{Bmatrix}+\frac{D}{S}\left(\frac{m\pi}{L}\right)^2\begin{bmatrix}a_3 & a_4\\ a_3 & -a_4\end{bmatrix}\begin{Bmatrix}\cosh\alpha y\\ \sinh\alpha y\end{Bmatrix}\right)$$

(A13b)

$$\bar{M}_{xy} = -\bar{M}_{yx} = \frac{1-\nu}{2}D\frac{\partial^2(\bar{w}_{bx}+\bar{w}_{by})}{\partial x\,\partial y}$$
$$= \frac{2}{L}(1-\nu)\left(\frac{L}{m\pi}\right)^3\cos\frac{m\pi x}{L}\lfloor q_1 \quad q_2\rfloor \times \quad \text{(A13c)}$$

$$\left(\frac{L}{m\pi} \begin{Bmatrix} \frac{2}{b} \\ -\frac{2}{b} \end{Bmatrix} + \begin{bmatrix} \bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2 \end{bmatrix} \begin{Bmatrix} \frac{m\pi y}{L} \cosh \frac{m\pi y}{L} \\ \frac{m\pi y}{L} \sinh \frac{m\pi y}{L} \end{Bmatrix} - \begin{bmatrix} a_1 & a_2 \\ a_1 & -a_2 \end{bmatrix} \begin{Bmatrix} \sinh \frac{m\pi y}{L} \\ \cosh \frac{m\pi y}{L} \end{Bmatrix} \right.$$

$$\left. + \left\{ 1 + \frac{2D}{S} \left(\frac{m\pi}{L} \right)^2 \right\} \begin{bmatrix} \bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2 \end{bmatrix} \begin{Bmatrix} \sinh \frac{m\pi y}{L} \\ \cosh \frac{m\pi y}{L} \end{Bmatrix} + \frac{D}{2S} \left(\frac{m\pi}{L} \right)^2 \left(\frac{1}{p} + p \right) \begin{bmatrix} a_3 & a_4 \\ a_3 & -a_4 \end{bmatrix} \begin{Bmatrix} \sinh \alpha y \\ \cosh \alpha y \end{Bmatrix} \right)$$

$$\bar{Q}_x = S \frac{\partial \bar{w}_{sx}}{\partial x} = \frac{2}{L} \left(\frac{L}{m\pi} \right)^2 \cos \frac{m\pi x}{L} \lfloor q_1 \quad q_2 \rfloor \times \tag{A13d}$$

$$\left(\begin{Bmatrix} 1 + \frac{2y}{b} \\ 1 - \frac{2y}{b} \end{Bmatrix} - 2 \begin{bmatrix} \bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2 \end{bmatrix} \begin{Bmatrix} \cosh \frac{m\pi y}{L} \\ \sinh \frac{m\pi y}{L} \end{Bmatrix} - \begin{bmatrix} a_3 & a_4 \\ a_3 & -a_4 \end{bmatrix} \begin{Bmatrix} \cosh \alpha y \\ \sinh \alpha y \end{Bmatrix} \right)$$

$$\bar{Q}_y = S \frac{\partial \bar{w}_{sy}}{\partial y} = \frac{2}{L} \left(\frac{L}{m\pi} \right)^2 \sin \frac{m\pi x}{L} \lfloor q_1 \quad q_2 \rfloor \times \tag{A13e}$$

$$\left(\frac{L}{m\pi} \begin{Bmatrix} \frac{2}{b} \\ -\frac{2}{b} \end{Bmatrix} - 2 \begin{bmatrix} \bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2 \end{bmatrix} \begin{Bmatrix} \sinh \frac{m\pi y}{L} \\ \cosh \frac{m\pi y}{L} \end{Bmatrix} - \frac{1}{p} \begin{bmatrix} a_3 & a_4 \\ a_3 & -a_4 \end{bmatrix} \begin{Bmatrix} \sinh \alpha y \\ \cosh \alpha y \end{Bmatrix} \right)$$

The fixed edge forces can now be calculated

$$\bar{M}_1 = -(\bar{M}_y)_{y=\frac{b}{2}}$$

$$= \frac{2}{L}(1-\nu)\left(\frac{L}{m\pi} \right)^3 \sin \frac{m\pi x}{L} \lfloor q_1 \quad q_2 \rfloor \times$$

$$\left(\frac{-\nu}{1-\nu} \begin{Bmatrix} 2 \\ 0 \end{Bmatrix} + \begin{bmatrix} \bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2 \end{bmatrix} \begin{Bmatrix} \beta \sinh \beta \\ \beta \cosh \beta \end{Bmatrix} - \begin{bmatrix} a_1 & a_2 \\ a_1 & -a_2 \end{bmatrix} \begin{Bmatrix} \cosh \beta \\ \sinh \beta \end{Bmatrix} \right.$$

$$\left. + \left\{ \frac{2}{1-\nu} + \frac{2D}{S} \left(\frac{m\pi}{L} \right)^2 \right\} \begin{bmatrix} \bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2 \end{bmatrix} \begin{Bmatrix} \cosh \beta \\ \sinh \beta \end{Bmatrix} + \frac{D}{S} \left(\frac{m\pi}{L} \right)^2 \begin{bmatrix} a_3 & a_4 \\ a_3 & -a_4 \end{bmatrix} \begin{Bmatrix} \cosh \beta p \\ \sinh \beta p \end{Bmatrix} \right)$$

$$= \frac{4}{L} \left(\frac{L}{m\pi} \right)^3 \sin \frac{m\pi x}{L} \left[\left(\bar{\lambda}_1 \cosh \beta + \bar{\lambda}_2 \sinh \beta - 1 \right) q_1 + \left(\bar{\lambda}_1 \cosh \beta - \bar{\lambda}_2 \sinh \beta \right) q_2 \right] \tag{A14a}$$

$$\bar{M}_2 = (\bar{M}_y)_{y=-\frac{b}{2}}$$

$$= -\frac{4}{L} \left(\frac{L}{m\pi} \right)^3 \sin \frac{m\pi x}{L} \left[\left(\bar{\lambda}_1 \cosh \beta - \bar{\lambda}_2 \sinh \beta \right) q_1 + \left(\bar{\lambda}_1 \cosh \beta + \bar{\lambda}_2 \sinh \beta - 1 \right) q_2 \right] \tag{A14b}$$

For uniform load $q_1 = q_2 = q$

$$\bar{M}_1 = -\bar{M}_2 = \frac{4qL^2}{m^3\pi^3} \sin\frac{m\pi x}{L}\left(2\bar{\lambda}_1 \cosh\beta - 1\right) \tag{A15}$$

$$\bar{V}_1 = -\left(\bar{Q}_y + \frac{\partial M_{yx}}{\partial x}\right)_{y=\frac{b}{2}} = \frac{2}{L}\left(\frac{L}{m\pi}\right)^2 \sin\frac{m\pi x}{L} \lfloor q_1 \ q_2 \rfloor \times$$

$$\left((2-\nu)\begin{Bmatrix} \frac{1}{\beta} \\ -\frac{1}{\beta} \end{Bmatrix} + (1-\nu)\begin{bmatrix} \bar{\lambda}_1 & \bar{\lambda}_2 \\ \bar{\lambda}_1 & -\bar{\lambda}_2 \end{bmatrix}\left(\begin{Bmatrix} \beta\cosh\beta \\ \beta\sinh\beta \end{Bmatrix} - \left(\frac{1+\nu}{1-\nu} - \frac{2D}{S}\left(\frac{m\pi}{L}\right)^2\right)\begin{Bmatrix} \sinh\beta \\ \cosh\beta \end{Bmatrix}\right)\right.$$

$$\left. - (1-\nu)\begin{bmatrix} a_1 & a_2 \\ a_1 & -a_2 \end{bmatrix}\begin{Bmatrix} \sinh\beta \\ \cosh\beta \end{Bmatrix} + (1-\nu)\frac{D}{S}\left(\frac{m\pi}{L}\right)^2 \frac{1}{p}\begin{bmatrix} a_3 & a_4 \\ a_3 & -a_4 \end{bmatrix}\begin{Bmatrix} \sinh\beta p \\ \cosh\beta p \end{Bmatrix}\right)$$

$$\bar{V}_1 = -\frac{2}{L}\left(\frac{L}{m\pi}\right)^2 \sin\frac{m\pi x}{L}\left[\left\{\frac{1}{\beta} - 2\left(\bar{\lambda}_1 \sinh\beta - \bar{\lambda}_2 \cosh\beta\right)\right\} q_1 - \left\{\frac{1}{\beta} + 2\left(\bar{\lambda}_1 \sinh\beta + \bar{\lambda}_2 \cosh\beta\right)\right\} q_2\right] \tag{A16a}$$

$$\bar{V}_2 = -\frac{2}{L}\left(\frac{L}{m\pi}\right)^2 \sin\frac{m\pi x}{L}\left[\left\{\frac{1}{\beta} + 2\left(\bar{\lambda}_1 \sinh\beta + \bar{\lambda}_2 \cosh\beta\right)\right\} q_1 - \left\{\frac{1}{\beta} - 2\left(\bar{\lambda}_1 \sinh\beta - \bar{\lambda}_2 \cosh\beta\right)\right\} q_2\right] \tag{A16b}$$

For uniform load $q_1 = q_2 = q$

$$\bar{V}_1 = -\bar{V}_2 = \frac{8qL}{m^2\pi^2} \sin\frac{m\pi x}{L} \bar{\lambda}_1 \sinh\beta \tag{A17}$$

<u>One Edge Fixed the Other Edge Free, Uniform Load</u>

This problem can be solved from the results of the previous section for uniform load equations (A15) and (A17).

$$\bar{M}_1 = -\bar{M}_2 = \frac{4qL}{m^3\pi^3} \sin\frac{m\pi x}{L}\left(2\bar{\lambda}_1 \cosh\beta - 1\right)$$

$$\bar{V}_1 = -\bar{V}_2 = \frac{8qL}{m^2\pi^2} \sin\frac{m\pi x}{L} \bar{\lambda}_1 \sinh\beta$$

The element stiffness matrix for deformations normal to the panel is, (See equations (64) and (65)).

$$\begin{Bmatrix} M_1 \\ M_2 \\ V_1 \\ V_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ w_1 \\ w_2 \end{Bmatrix}$$

Now for edge 2 fixed and edge 1 free

$$\theta_2 = w_2 = 0$$

$$\left.\begin{aligned} \bar{M}_1 + M_1 &= \left\{ \frac{4qL^2}{m^3\pi^3}\left(2\bar{\lambda}_1 \cosh\beta - 1\right) + k_{11}\theta_1 + k_{13} w_1 \right\} \sin\frac{m\pi x}{L} = 0 \\ \bar{V}_1 + V_1 &= \left\{ \frac{8qL}{m^2\pi^2}\left(\bar{\lambda}_1 \sinh\beta\right) + k_{31}\theta_1 + k_{33} w_1 \right\} \sin\frac{m\pi x}{L} = 0 \end{aligned}\right\} \quad (A18)$$

The fixed edge forces at edge 2 are

$$\begin{aligned} M_2' &= \bar{M}_2 + k_{21}\theta_1 + k_{23} w_1 \\ V_2' &= \bar{V}_2 + k_{41}\theta_1 + k_{43} w_1 \end{aligned} \quad (A19)$$

Solving the simultaneous equations A18 for θ_1 and w_1, yield after substitution in equations (A19)

$$M_2' = \frac{-4qL}{m^2\pi^2}\left\{ \left(\frac{L}{m\pi}\right)\left(2\bar{\lambda}_1 \cosh\beta - 1\right)\left(1 + \frac{k_{21}k_{33} - k_{23}k_{13}}{k_{11}k_{33} - k_{13}k_{31}}\right) + 2\bar{\lambda}_1 \sinh\beta \left(\frac{k_{23}k_{11} - k_{21}k_{13}}{k_{11}k_{33} - k_{13}k_{31}}\right) \right\} \sin\frac{m\pi x}{L}$$

$$V_2' = \frac{4qL}{m^2\pi^2}\left\{ \left(\frac{L}{m\pi}\right)\left(2\bar{\lambda}_1 \cosh\beta - 1\right)\left(\frac{k_{43}k_{13} - k_{41}k_{33}}{k_{11}k_{33} - k_{13}k_{31}}\right) - 2\bar{\lambda}_1 \sinh\beta \left(1 + \frac{k_{43}k_{11} - k_{41}k_{13}}{k_{11}k_{33} - k_{13}k_{31}}\right) \right\} \sin\frac{m\pi x}{L} \quad (A20)$$