A Realization Technique for Digitally Programmable Analog Active Filters

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ABSTRACT

A Realization Technique for Digitally Programmable Analog Active Filters

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This thesis proposes a realization technique for digitally programmable analog filters. The basic building block in this technique is a Digital to Analog Converter (DAC) in which the weighted output current of the DAC is directly integrated to obtain a programmable integrator. The time constant of this integrator then becomes digitally programmable.

The emphasis is on programmability employing programmable integrator. A second-order digitally programmable filter configuration is designed based on this programmable integrator and its results are analyzed. A 4th order version of the same filter configuration is constructed using cascading method and also using Butterworth (BW) and Chebyshev (CS) approximations. The responses of this 4th order version are analyzed to see how the programmability concept performs in these cases.

The effects of non-ideal characteristics of *real* components, especially Operational Amplifier (OP-AMP) and DAC, on the performance and limitation for the proposed programmable filter structure are investigated. Experimental results on a second order digitally programmable filter are also obtained and compared with the theoretical results obtained earlier to verify the proposed concept.

Applications of this filter structure in realizing general purpose programmable filters, tracking filters, adaptive filters, speech and frequency synthesizers, etc. are briefly mentioned. Realization of some of these configurations are discussed.

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Dedicated to the Memory of

my Parents

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LIST OF SYMBOLS

A_0	: dc gain of OP-AMP
B	: Bandwidth
BP	: Bandpass
BW	: Butterworth
CS	: Chebyshev
C_A	: W_0 control capacitor
C_C	: Q-control capacitor
CMOS	: Complementary metal oxide semiconductor
D/D(s)	: Denominator polynomial
DAC	: Digital to analog converter
DTL	: Diode transistor logic
f_l	: Lower passband limit in bandpass response
f_u	: Upper passband limit in bandpass response
H	: Transfer function
$H_{BP}(s)/(jw)$	: Second order bandpass transfer function/amplitude response
$H_{4BP}(s)/(jw)$	: Fourth order transfer function/amp. response
$H_{CS-BP}(s)/(jw)$	: Chebyshev bandpass transfer function/amp. response
$H_{CS-LP}(s)/(jw)$	: Chebyshev bandpass lowpass transfer function/amp. response
$H_{LP}(s)/(jw)$	: Second order lowpass transfer function/amp. response
$H_{4LP}(s)/(jw)$	: Fourth order lowpass transfer function/amplitude response
$H_{BW-BP}(s)/(jw)$	: Butterworth bandpass transfer function/amplitude response

$H_{BW-LP}(s)/(jw)$	: Butterworth lowpass transfer function/amplitude response
G	: DAC
I_B	: Bias current
I_{os}	: Offset current
L(s)	: Loop gain
LP	: Lowpass
LSB	: Least significant bit
M(s)	: Denominator polynomial
$M_2(s)$	: Denominator polynomial (even)
MSB	: Most significant bit
$N_2(s)$	: Denominator polynomial (odd)
OP-AMP	: Operational amplifier
PLL	: Phase locked loop
$\phi(w)$	: Phase response
Q	: Q-factor
S	: Sensitivity
SC/C_R	: Switched-capacitor
$T(w)$	: Group delay
τ	: Time constant
$T_{BW-BP}(w)$	: Group delay of Butterworth bandpass filter
$T_{BW-LP}(w)$	: Group delay of Butterworth lowpass filter
$T_{CS-BP}(w)$	: Group delay of Chebyshev bandpass filter
$T_{CS-LP}(w)$	: Group delay of Chebyshev lowpass filter

TTL	: Transistor transistor logic
VLSI	: Very large scale integration
V_{os}	: Offset voltage
W_l	: Lower cutoff frequency in bandpass
W[illegible]	: Cutoff frequency at 3dB point
W_0	: Center frequency
W_u	: Upper cutoff frequency in bandpass
X_f	: Output of feedback network
X_s	: Input signal
X_o	: Output signal

CHAPTER 1

INTRODUCTION

1.1 General

In general, a filter is a two-port network that transforms an input signal in some specified way to yield a desired output signal. The signals may be considered in the time domain or in the frequency domain. In the latter case, a filter is often a frequency selective device which passes signals in a certain frequency band and blocks or attenuates signals in other frequencies. Filters are important building blocks of many electronic and communication systems.

Filters may be classified in a number of ways. Analog filters are used to process analog signals, which are functions of a continuous-time variable. Digital filters process digitized waveforms. We may also classify filters as passive or active depending on the type of elements used in their constructions. In addition, some filters employ mechanical, crystal and switching devices [1].

In this thesis, the discussion is focussed on digitally programmable active analog filters. Digital-to-Analog Converter (DAC) and Operational-Amplifier (OP-AMP) are the two main components of this programmable filter. Its transfer functions, group delay characteristics and the effects of non-ideal components of its performance will be considered.

The principle behind the digitally programmable filter is simple and straight forward. The output current of the DAC is directly integrated to obtain a programmable integrator. The time constant (τ) of this integrator then becomes digitally programmable with the binary input of the DAC. In this thesis, the emphasis is on programmability of the filter's characteristics, both digitally and independently. Employing programmable DAC and capacitor, the characteristics of the filter can be independently controlled. The center frequency, the bandwidth sharpness (Q-factor) and the gain of the filter are all independently programmable. There are other ways to obtain digitally programmable filter. Digital filter and Switched-Capacitor filter are such examples. In fact, digital filters are well-known for their programmability; adaptability and flexibility. However, they are also known to require an extensive development system, thus making them expensive for most applications. On the other hand, analog filters, though inexpensive, do not provide many of the features found in digital filters. For these reasons it is necessary to develop (digitally programmable) design techniques so that analog filters become flexible and easy to use. The switched-capacitor filter technique is based on the realization that a capacitor switched between two circuit nodes at a sufficiently high rate is equivalent to a resistor connecting these two nodes. The switched-capacitor programmable filters have some drawbacks. The main one being its frequency limitation. These filters also use large selectable-capacitor arrays to achieve programmability; in most general case where each coefficient of a transfer function must be programmable individually and many such arrays are required, occupying a lot of space in a chip area. Even if the capacitor arrays were to be time-shared, these have to be large in order to obtain sufficient resolution and such a resolution is fixed once the filter has been fabricated. Finally, under VLSI (very-large scale integration) processing capacitor ratio accuracy may deteriorate in certain fabrication process [2].

In our approach, the programmability is obtained by using DAC thus, removing some of the problem faced by conventional filters. Digitally programmable active filter design technique is unique and straightforward in the areas of electronic circuit technology. It is fast gaining popularity because it offers cost-effective design of analog chip in the VLSI environment.

The main components required for the design of this type of programmable filter are OP-AMPs and DAC's. The availability of monolithic OP-AMPs as active elements has made significant impacts on the design of active filters. The amplifier, which was a system composed of many discrete components in the early days, has evolved into a discrete component itself. This has changed the entire picture of linear circuit design. The IC OP-AMP has redefined the ground rules of electronic circuits design by placing the emphasis of circuit design on a systems basis [3].

The Digital-to-Analog Converter (DAC) provides an interface between the digital signals of discrete systems and continuous signals of the analog world. The basic converter consists of an arrangement of weighted resistance values, each controlled by a particular level or significance of digital input data, that develops varying output voltages or currents in accordance with the digital input code. We employ current output DAC in this thesis. Although converters have fixed references, for most applications, a special class of DAC's exists, with a capability of handling variable and even ac reference sources. These devices are termed multiplying DAC's (MDAC's) because their output value is the product of the number represented by the digital input code and the analog reference voltage [4].

The important properties and applications of this type of programmable filters will be discussed and analyzed in detail in the chapters ahead.

1.2 Scope and Organization of the Thesis

The entire thesis is organized in six chapters. Chapter one introduces the subject with a brief summary.

In chapter two, a complete overview of the filter design approach is considered. The concept of programmability and its application in designing digitally programmable filter is developed.

In chapter three, the concept of programmability is applied to propose a configuration of 2nd order digitally programmable filter and its results are analyzed. A study on a 4th order version of the same filter is done using Butterworth (BW) and Chebyshev (CS) approximations.

Chapter four investigates the effects of practical DAC's and OP-AMPs, such as nonlinearity, finite gain, etc. on the performance of the proposed filter structure. New transfer functions were derived using non-ideal elements. Using the theoretical knowledge already discussed, a 2nd order structure is implemented using current-type DAC's and wide bandwidth OP-AMPs. Experimental results of this filter are obtained and analyzed. Limitation on the center frequency due to OP-AMP saturation has been studied and methods to eliminate impedance mismatching are also discussed. Finally, a design guideline is presented for future designers.

Chapter five looks into the applications of this type of filter in the field of communications. Designing various filter structure using the basic element (programmable integrator) are discussed.

Finally, conclusions and some suggestions towards further research are made in chapter six.

1.3 Research Contributions

The major contributions of research work include the proposal of a design technique for digitally programmable analog filter based on programmable integrator. Some specific contributions are summarized below:

* Proposal of programmable integrator based on DAC for digital programmability.

* Analysis of programmable filter using Butterworth and Chebyshev approximations.

* Investigation of non-ideal properties of DAC's and OP-AMPs, derivation of transfer functions using non-ideal components and their effects on digitally programmable filter responses.

* Building and testing of a 2nd order digitally programmable filter structure and compare the experimental results with the theoretical ones. Observation of limitation and variation of the programmable filter parameters. Discussion of design guideline for future designers.

* Discussion of programmability concept in designing various filters useful in the field of electronics and communications.

CHAPTER 2

AN OVERVIEW

2.1 Criteria & Parameters

The technological achievement in the area of integrated circuit combined with the theoretical developments have enormously broadened the scope of filter design techniques which, not too many years ago, was primarily concerned only with methods for interconnecting lumped resistors, capacitors and inductors. Active filters are now available in hybrid integrated circuit forms and are used in many areas. Their applications include telephony, voice channel and some channel bank filters, telemetry and tracking filter for satellite systems, phased locked loop (PLL) filters, etc.... The performance of active filters has many advantages over passive filters. Some of these advantages are mentioned here [5,6]:

1) Active filters are more practical for low frequency applications because they can be designed at higher impedance levels which reduce the capacitor magnitudes. At sub audio frequencies, the passive filter design require bulky inductors, therefore active filter is a good choice to eliminate bulky inductors.

2) Active filters are generally smaller (size) in the low frequency range than their passive counterparts. Further reduction in size is possible with VLSI technology. This could be accomplished by using SC (switched-capacitor) networks and monolithic operational amplifier chips or with hybrid technology.

3) Active filters can realize any circuit while passive filters

can realize only real rational functions which are analytic in the left half planes.

4) Active filters can be cascaded without buffers as long as they are designed with low output impedance or high input impedances.

In modern filter design techniques, the realization of active filters involves many criteria and parameters. The performance of a filter may be represented by its amplitude and/or phase response. The amplitude response is a plot of the amplitude $|H(jw)|$ of its transfer function H (s) versus frequency. The phase response is a plot of the phase $\phi(w)$ of its transfer function versus frequency. An example of a lowpass filter with its amplitude response is shown in Fig. 2.1 [7]. In actual practice it is impossible to achieve the ideal response (Fig. 2.1) because of the sharp corners it requires. A central problem in filter design is, therefore, to approximate the ideal response to some prescribed degree of accuracy with a realizable response. One such realizable amplitude response is represented by the solid line in Fig. 2.1. In practice, the passband, stopband, cutoff frequency and other parameters are essential to obtain a particular filter response. The principal characteristics commonly used to specify the desired performance of an active filter are mentioned here:

Passband is defined as the frequency band ($0 < f < f_c$) occupied by the desired signal. In this band the ideal requirement for the filter is to provide constant gain and constant envelope delay so that the wanted signal will be transmitted with no distortion.

In practice, one cannot achieve exactly constant gain over a finite band of frequencies, so it is customary to specify some acceptable upper and lower limits (fig. 2.1, 2.2) between which the gain can vary. These limits are chosen to accommodate not only the variations of gain that would exist even with an ideal network, but also the effect of component tolerances.

Stopband is referred to the bands ($f > f_c$) occupied by the unwanted signals in a filter response. In these bands the common form of specification merely requires the gain, relative to the lower limit set for the passband, to be equal to or greater than some minimum amount, such as A_2 in Fig. 2.1. This property is important in the sense that it is a measure of the extent to which it rejects unwanted signal. Typical values of this type of signal may range from a modest 20 dB for certain high-power transmitter filters to a more than 100 dB for some tunable filters [8].

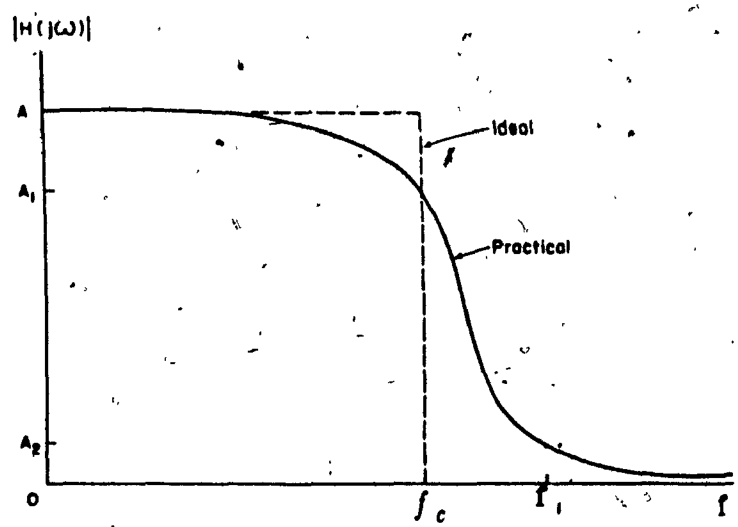

Fig. 2.1. Amplitude characteristic of a lowpass filter.

The interval in which the response continually decreases from the passband to the stopband is called the *transition band.* In Fig. 2.1 the transition band is between $f_c < f < f_1$

Cut-off Frequency is defined to be the frequency between the signal acceptance and rejection bands. Theoretically the cutoff frequency could be infinity and practically it could range from 30, 40,50 Hz... depending on the application.

Bandwidth (B) is the frequency acceptance window. For lowpass filter the bandwidth is between zero frequency and the cut-off frequency or simply the cutoff frequency ($B = f_c$). For bandpass filter, the bandwidth is defined as $B = f_u - f_l$, where f_u is the upper cut-off and f_l is the lower cut-off frequency.

Q-Factor is the ratio of the center frequency to 3 dB bandwidth in describing bandpass filters. The center frequency is the geometric mean frequency between the 3-dB cut-off frequencies and may be approximated by the arithmetic mean when Q factor is higher than about 10.

All these parameters are clearly indicated in Fig. 2.2 [8].

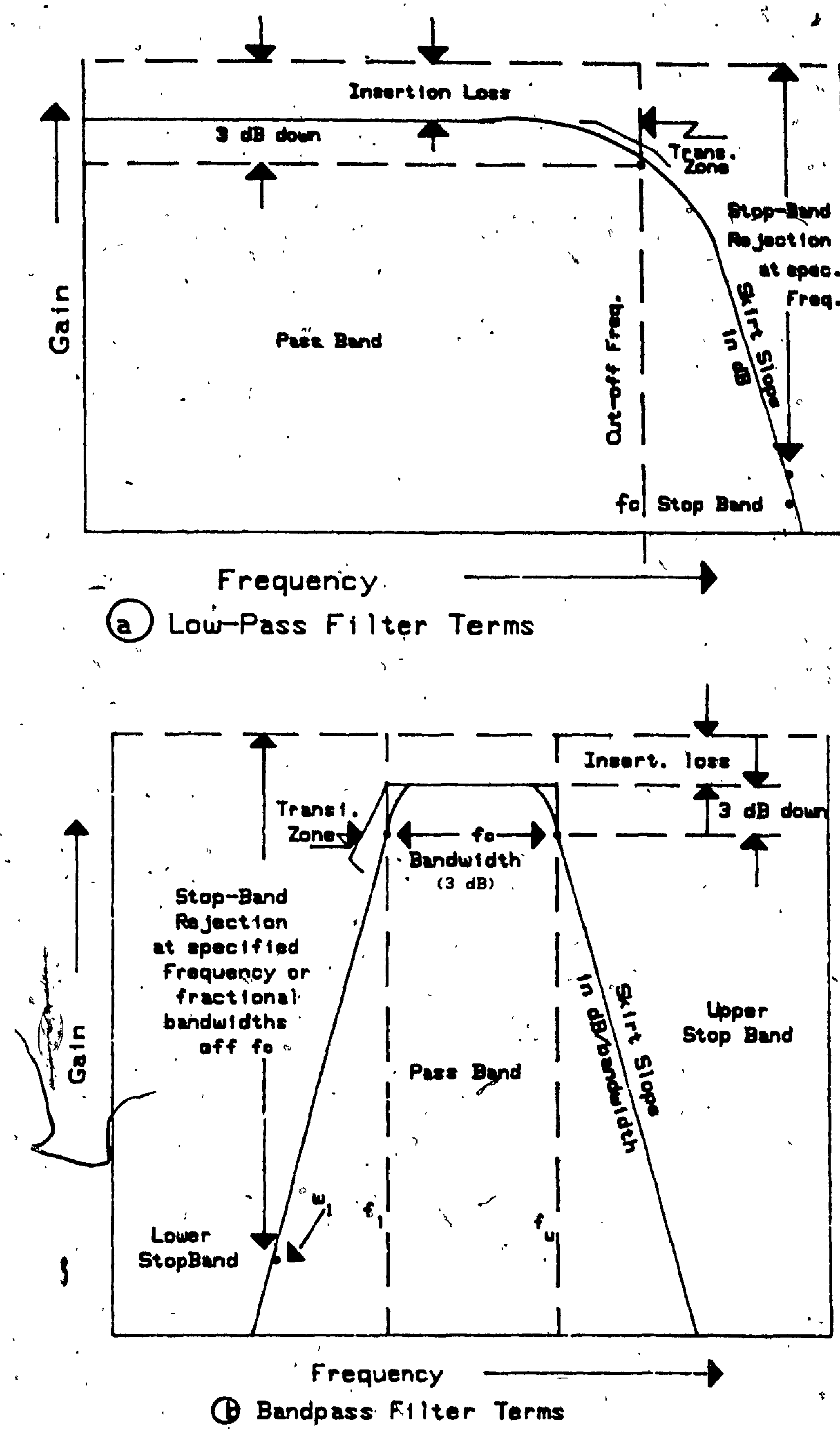

Fig. 2.2. Terminology in filter's characteristics.

2.2 Configuration & Selection

In the previous section the important criteria & parameters required to design any filter structure are considered. In this section the realization procedures of filter network are briefly looked into.

The realization of a desired filter network may be categorized into various steps: selecting the desired frequency response; synthesizing a network which will yield transformation of the desired response; transforming the prototype to the final filter configuration; and the physical realization of the filter network; including component selection and tuning and measuring [8].

Let us focus on the transfer function first. To obtain it, we consider a nth-order linear network whose output signal can be found in terms of the input signal by solving a linear nth order differential equation of the form [9]:

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m x}{dt^m} + b_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \cdots + b_1 \frac{dx}{dt} + b_0 x \quad (2.1)$$

where x(t) is the input signal, y(t) is the output signal and $n \geq m$.

Applying the Laplace transform to the Eq. 2.1 we obtain the transfer function T(s) $= Y(s)/X(s)$ as

$$T(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0} \quad (2.2)$$

where s $= \sigma + jw$ is the complex frequency and N(s) and D(s) are polynomials in s with real coefficients a_i and b_i. Expressing Eq. 2.2 in factored form, we obtain poles and zeros of the transfer function [9]:

$$T(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)} = K\frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=i}^{n}(s-p_j)} \tag{2.3}$$

The poles p_j and zeros z_i may be either real or complex conjugate. Combining conjugate pole pair, we obtain the special case of a second order transfer function which represents a general form of biquad.

$$T(s) = K\frac{s^2 + (w_z/q_z)s + w_z^2}{s^2 + (w_p/q_p)s + w_p^2} \tag{2.4}$$

$$\text{where } z = -\sigma_z \pm j\overline{w}_z$$

$$p = -\sigma_z \pm j\overline{w}_p$$

In Eq. 2.4, the poles (w_p, q_p) and zeros(w_z, q_z) are used to control the filter characteristics and hence these poles and zeros can be digitally and independently programmed to obtain a suitable filter characteristic when the proposed digitally programmable concept is applied. Eq. 2.4 can be modified to obtain any particular transfer function by substituting the appropriate values. Lowpass and bandpass transfer functions are considered for analysis of the programmable filter in this thesis.

Bandpass filters fall into two categories, narrow-band and wide-band. If the ratio of the upper cutoff frequency to the lower cutoff frequency is 2 (an octave), the filter is considered as a wide-band type. Wide-band filter specifications can be separated into individual lowpass and highpass requirements which are accomplished independently.

Narrow-Band Bandpass filters have a ratio of upper cutoff frequency to lower cutoff frequency of approximately 2 or less and can not be designed as separate lowpass and highpass filters. This is, since the ratio of upper cutoff to lower cutoff decreases, the loss at center frequency will increase and consequently it may become prohibitive for ratios near unity [10].

The center frequency for bandpass filter is defined as

$$f_0 = \sqrt{f_l f_u} \tag{2.5}$$

where f_l is the lower passband limit and f_u is the upper passband limit, usually 3-dB attenuation frequencies.

An important parameter of bandpass filters which measures bandwidth sharpness is the Q factor defined as

$$Q = \frac{f_0}{BW} \tag{2.6}$$

where BW is the passband bandwidth $f_u - f_l$.

Our discussion so far revolved around frequency response of filters where input signal was a sine wave. In real-world application of filters input signal consists of a complex wave forms. The response of filters to these non sinusoidal inputs is called transient response.

The group delay is one transient response curve that is useful for estimating filter responses to nonsinusoidal signals. The group delay is defined as the derivative of phase versus frequency as linear phase shift results in

$$T = \frac{d\phi}{dw} \tag{2.7}$$

constant group delay since the linear function is a constant. If the group delay is not constant over the bandwidth of the modulated signal, waveform distortion will occur. Narrow bandwidth signals are more likely encounter constant group delay than signals having a wider spectrum. It is a common practice to use group delay variation as criterion to evaluate phase non linearity and subsequent waveform distortion.

2.3 Concept of Programmability

The application of the programmability concept in realizing active analog filter is the main focus if this thesis. Standard active filter design involves resistor, capacitor and readily available OP-AMPs. In the present age of VLSI technology, the standard RC-active filters are not very attractive because they lack many properties (such as parameters of RC filters are not easily tunable, lack of accurate RC time constant and they occupy larger space) required to program a circuit digitally or to miniaturize a circuit into a VLSI chip. It is also not cost effective to make them into analog IC's. It is well known though that analog IC's could easily be integrated with different digital systems such as microprocessor for wide range of applications. Therefore the problem can be avoided by replacing R in an RC integrator circuit by a DAC. This will enable a particular filter's coefficients to be digitally programmable and be suitable for chip implementation. This will also enable the filter structure to be easily integrated with various systems and be useful in many areas of communication.

The principle involved in realizing programmable filter is straightforward. The output current of DAC is directly integrated to obtain a programmable integrator. The resulting time constant of the DAC-based integrator then becomes digitally programmable with n-bit binary digital input of the DAC.

In the proposed programmable filter, the center frequency and the gain are controlled digitally by employing DAC's. Similarly, to obtain Q programmability, a capacitor is controlled digitally. This is achieved by arranging an array of binary weighted capacitor with switches in series forming the digital Q-control as shown in Fig. 2.3.

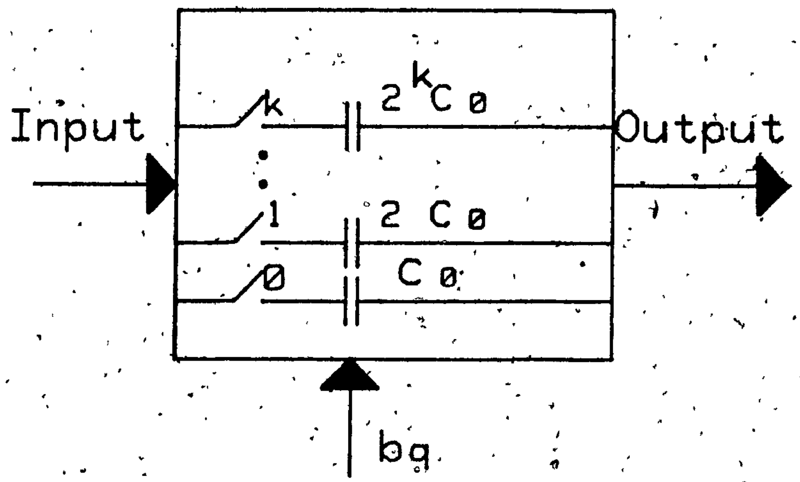

Fig. 2.3. Array of binary weighted capacitor for digital Q-control.

Now the C of the fig. 2.3 can be represented by $\sum_{q=0}^{k} 2^q b_q C_0$ where q is digitally programmable with b_q =0 or 1 and q = 0,1,2, ...,k. Now any combination of Q-factor can be obtained by using various combination of C. The switches are controlled digitally by a logic circuit (DAC). If array of ten binary weighted capacitor is used, 1024 different values of Q could be chosen for Q-programming.

The programmable filter has many advantages. The coefficients are easily programmable (digitally), it occupies less space, it is easily integratable with other systems and most of all it is suitable for analog chip design. Using DAC and C, the characteristics of filter can be independently controlled and digitally programmed as shown in the block diagram of Fig. 2.4.

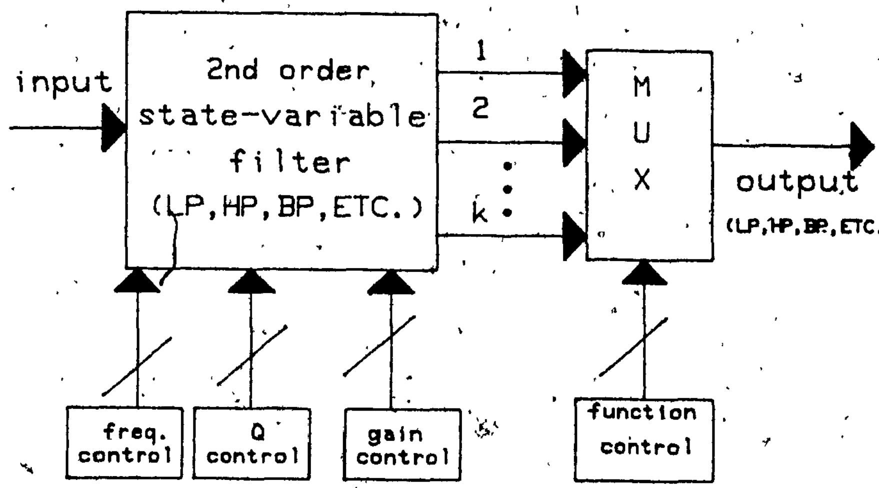

Fig. 2.4. Concept of programmable filter in block diagram.

The center frequency, the bandwidth sharpness (Q-factor) and the gain of filter are independently controlled through the digitally programmable DAC and capacitor. The output of filter responses can also be digitally controlled as shown in the above figure (Fig. 2.4). This is done by connecting all the output of filter responses through an analog multiplexer (readily available as IC) and thereby digitally controlling the filter response at any particular instance.

The DAC's are easily programmable and could easily be integrated with any digital processor for various application. Another important use of this type of filter is in the analog chip design as mentioned earlier. Analog chips are normally costly and are scarcely available compared to digital chips. Since DAC's and OP-AMPs are readily available as standard cell, the proposed analog design is suitable for VLSI implementation in the semi-custom environment; moreover it may be implemented in hybrid technology.

Literature survey on programmable analog filters shows various contributions. P. Anandamahon et all [11] describe a new universal programmable SC filter based on KHN (Kerwin, Huelsman, Newcomb) biquad with independent control of center frequency and Q-factor. The original KHN biquad uses both the inputs of the OP-AMP and thereby causes stray sensitivity in the SC configuration. This is avoided in the proposed structure by simulating the action of OP-AMP one [11] by OP-AMPs with grounded noninverting input. The proposed circuit uses four OP-AMPs for designing a 2nd-order biquad and it requires five capacitor arrays for programming the pole frequency, the Q-factor and the gain. Also, the programming of Q-factor changes the DC gain of the lowpass network. In the proposed circuit, the OP-AMP one [11] output is short circuited to ground during one phase and therefore, the OP-AMP has to slew from ground level to the amplitude of the output sample, which places a restriction on the maximum speed of the operation. D.B. Cox [12] has developed a 2nd order SC filter configuration which has been fabricated on an NMOS chip. This configuration can perform all five basic filter types without requiring any external components. The device has ten TTL compatible programming pins that can be either electrically programmed or hard wired. The primary advantage of this structure is that, Q and center frequency are independently variable. The gain is preset in this design. This particular design has a 2nd order effect of C_C / C_A on the Q and w_0 programming, where C_C controls Q and C_A controls w_0. Though it is possible to compensate for this effect by adjusting C_C and C_A for a specific value of Q and w_0, they can not be matched for all values of Q and w_0. Thus Q and w_0 programming are not totally independent of each other. It also requires large total capacitance area. B.B. Bhattacharyya et all [13] proposed an economical, stray insensitive digitally programmable switched-capacitor biquad based on Cox's biquad [12] but with less total capacitance area. The w_p and Q_p are independently adjustable with a constant gain. However, it requires additional chip area for its seven phase clock which controls the programming coefficients of the configuration. The chip area increases for low value of Q and more for $Q < 1$. References [2,14,15] also investigate various programmability aspects of analog active filters. These have some advantages and disadvantages as with the others already discussed. In light of these, our proposed configuration of programmable filter avoids some of the problem mentioned above. The use of DAC makes it easier for programming the various coefficients of the filter structure. It uses less capacitance arrays compared to others. In SC implementation for analog chip, the digital programmability will be easier to obtain with DAC and therefore would be versatile for various applications. As with the advantages the proposed configuration will have some drawbacks like frequency limitation, non-linearity of DAC's causing problem in programming filter coefficients in higher frequency range, etc. The detail discussions on these will be done later.

2.4 Concept of Digitally Programmable Analog Active Filter

Concept of digitally programmable analog filter is based on digitally programmable integrator as discussed earlier. All basic filter types can be built based on this programmable integrator: lowpass, highpass, bandpass, state-variable etc. A 2nd order state-variable structure is designed and considered here to analyze the concept of programmability as it applies to control and vary the important coefficients of the filter configuration. Center frequency, Q-factor and gain are all digitally programmable and independently controlled in the proposed configuration. Limitation of this programmable concept is also considered in light of the practical applications.

Modern network theory has provided us with many different shapes of amplitude responses. The concept of this digitally programmable is applied in various approximation theory to see how the results vary. The major categories of these responses discussed here are Butterworth (BW) and Chebyshev (CS) approximations.

The BW approximation is applied on the assumption that a flat response at zero frequency is more important than response at other frequencies for a lowpass filter. The BW approximation results in a class of filters which have moderate attenuation steepness and acceptable transient characteristics.

The CS approximation to an ideal filter has a much more rectangular frequency response in the region near cutoff than the BW filter. This is accomplished at the expense of allowing ripples in the passband. Fig. 2.5 compares the response

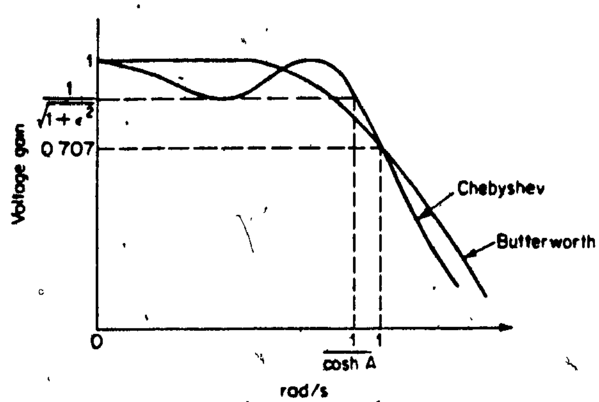

Fig. 2.5. Comparison of BW & CS lowpass response of 3rd order Filter.

of an 3rd order BW & CS lowpass filter [10]. Both filters have the same 3-dB bandwidths normalized to 1 rad/s.

CHAPTER 3

GENERAL STRUCTURE OF A DIGITALLY PROGRAMMABLE ANALOG FILTER

3.1 General

In this chapter a general configuration of a Digitally Programmable Analog Filter is considered. First, the principle behind the digitally programmable concept is developed. Examples are then taken to investigate the principle of digital programmability by programming the various filter coefficients. These programmable parameters (center frequency, Q-factor, gain) are also independently controlled. Limitations of these programmable parameters are also observed. Here digitally programmable means that the filter's characteristics are programmed by some elements which are digitally programmable. The output of the filter response can be digitally controlled as well.

DAC is the important component in this programmable concept. The current-type DAC used here is also called multiplying DAC because its output is the product of the binary digital input and the variable analog input voltage. The DAC uses CMOS current switches and drive circuitry to achieve low power consumption (30 mW max) and low output leakages (200 nA max) [17]. DAC combining with C forms a programmable integrator, the basic element of digitally programmable filters.

3.2 Principle of the Proposed Approach

A novel realization strategy for digitally programmable analog filters is studied here. It employs DAC for programmability instead of regular passive element. The weighted current output of the DAC is directly integrated to obtain a programmable integrator. It is well known that the time constant of an integrator contains the poles and zeros of the filter which, in turn control the filter characteristics. Therefore, an integrator is used as a basic building block in the filter design. In our programmable concept, the time constant of the integrator and hence the poles and zeros of the given filter are digitally programmable.

To illustrate the concept of a digitally programmable filters a simple first order RC active filter (integrator) is considered in Fig. 3.1. In ideal case, the input impedance of OP-AMP is

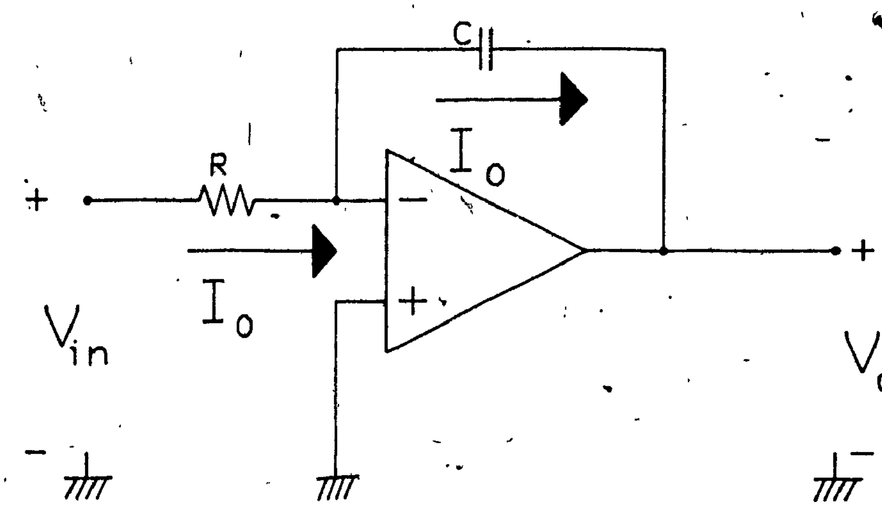

Fig. 3.1. A simple integrator.

infinite, i.e., the current flow into the input terminal is zero. All of this current flows through the feedback capacitor C. Thus

$$V_0 = -\frac{V_{in}}{sCR} \tag{3.1}$$

$$\text{if} \quad I_0(t) = \frac{V_{in}(t)}{R}$$

$$\textit{then}, \quad V_0 = -\frac{1}{C}\int I_0(t)\, dt \tag{3.2}$$

In Eq. 3.2 I_0 contains R but this R is not digitally programmable. To make the digitally programmable concept work, we replace R in Fig. 3.1 with a current-type DAC as shown in Fig. 3.2.

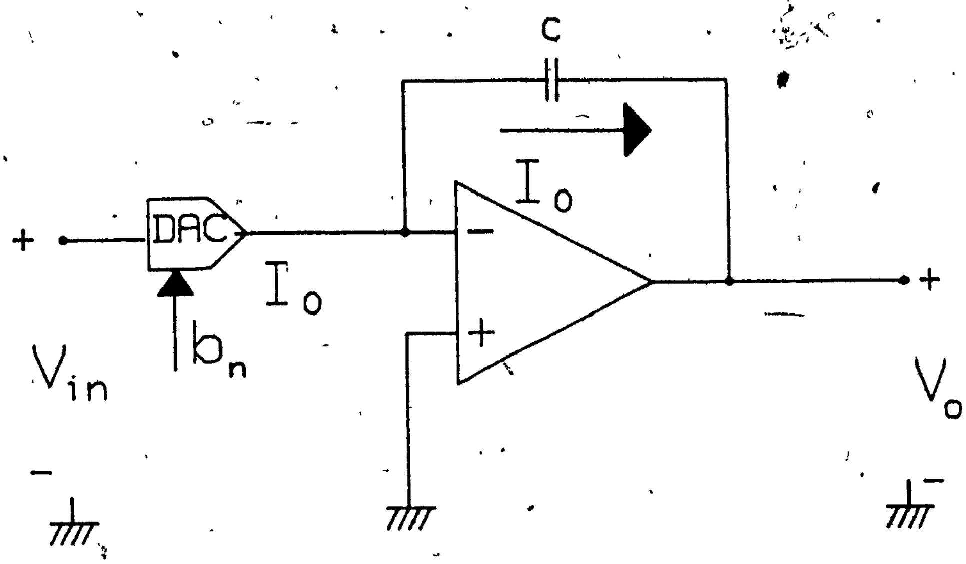

Fig. 3.2. Integrator based on DAC.

The DAC has its weighted output current directly fed into an ideal integrator.

By utilizing the electronic properties, the output I_0 (t) of the DAC is given by

$$I_0(t) = g_m \sum_{n=0}^{k} 2^n b_n V_{in}(t) \tag{3.3}$$

where

g_m is the effective transconductance of the DAC and b_n its binary input:

$b_k, b_{k-1}, \ldots, b_0$

where

b_k is the most significant bit and

b_0 is the least significant bit.

Since these inputs are binary, b_n is either a 1 or 0 and n = 0, 1, 2, ..., k. Substituting Eq. 3.3 in Eq. 3.2 we get

$$V_0(t) = -\frac{g_m}{C}[\int \sum_{n=0}^{k} 2^n b_n] v_{in}(t)\, dt \tag{3.4}$$

In the s-domain Eq. 3.4 can be written as

$$V_0(s) = V_{in}(s) g_m \sum_{n=0}^{k} \frac{2^n b_n}{sC} \quad (3.5)$$

From Eq. 3.5 and comparing it with $V_0(s) = -\frac{V_{in}(s)}{s\tau}$, it can be seen that the time constant τ is

$$\tau = \left[\frac{g_m}{G} \sum_{n=0}^{k} 2^n b_n \right]^{-1} \quad (3.6)$$

To see the relation clearly between passive element R and digitally programmable DAC

we replace τ by RC in Eq. 3.6, we see that

$$R = \left[g_m \sum_{n=0}^{k} 2^n b_n \right]^{-1} \quad (3.7)$$

from Eq. 3.7 it is clear that R is replaced by a component which is digitally programmable using g_m, the transconductance constant and b_n, the binary digit. Hence, τ of Eq. 3.6 is digitally programmable. This forms the basis for the development of the programmable filter. As we mentioned before, programmable means that the important coefficients (center frequency, Q-factor etc.) of filter characteristics can be varied by programmable methods instead of normal (by changing components) methods.

DAC's are available in different bit sizes. It varies from as low as 4-bit to as high as 16-bit. For a 10-bit DAC (DAC 1020) the binary digit varies from 0 to 1023 and g_m varies from 65.01E-9 to 66.60E-6 under ideal conditions. In real situation the ideal conditions such as temperature range, linearity, current offset etc. of DAC are impossible to meet. It is therefore important to consider practical implications of DAC's and study their effects. This will be considered in the next chapter.

3.3 Example of a 2nd Order Filter

In chapter two a general case of a 2nd order biquad transfer function was obtained as:

$$H(s) = K\frac{s^2 + (w_z/q_z)s + w_z^2}{s^2 + (w_p/q_p)s + w_p^2} \tag{3.8}$$

with $z = -\sigma_z \pm j\overline{w}_z$

$p = -\sigma_p \pm j\overline{w}_p$

The poles and zeros of Eq. 3.8 can be programmed to independently vary all filter's characteristics using programmable approach. From Eq. 3.8 any type of filter response can be obtained by substituting appropriate values. Lowpass transfer function is obtained by substituting zero in s^2 and s terms in the numerator. For highpass transfer function, the s and constant terms in the numerator are substituted by zero. Similarly, all other types of filter transfer functions can be derived. Based on Eq. 3.8, we consider a particular example of a 2nd order programmable state-variable filter structure for detail analysis. The structure of a 2nd order state-variable filter based on programmable integrator is proposed here in Fig. 3.3. Both bandpass (BP) and lowpass (LP) filters are incorporated in the design of the circuit of Figure 3.3. Other filter types could be incorporated in this figure if required.

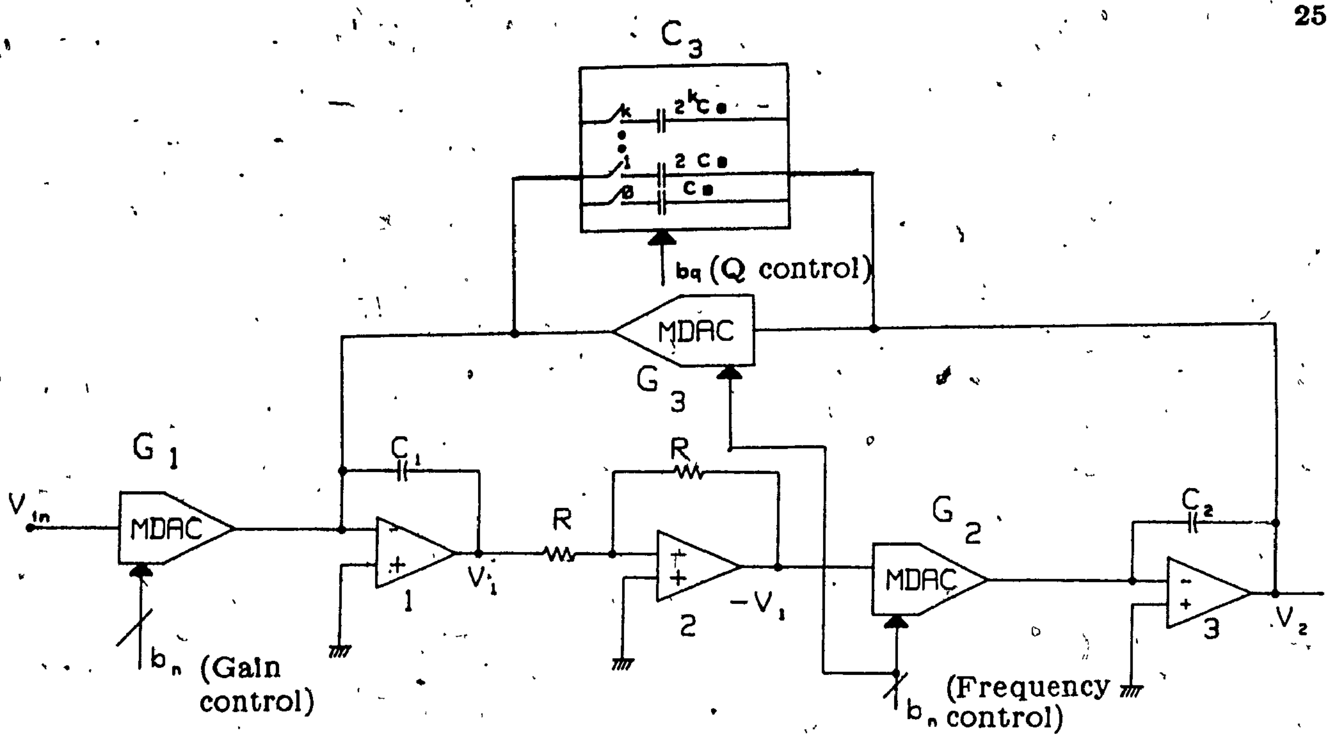

Fig. 3.3. Circuit diagram of 2nd order programmable filter.

If the above figure is represented in block diagram (Fig. 3.4), the important filter characteristics can be clearly and easily explained.

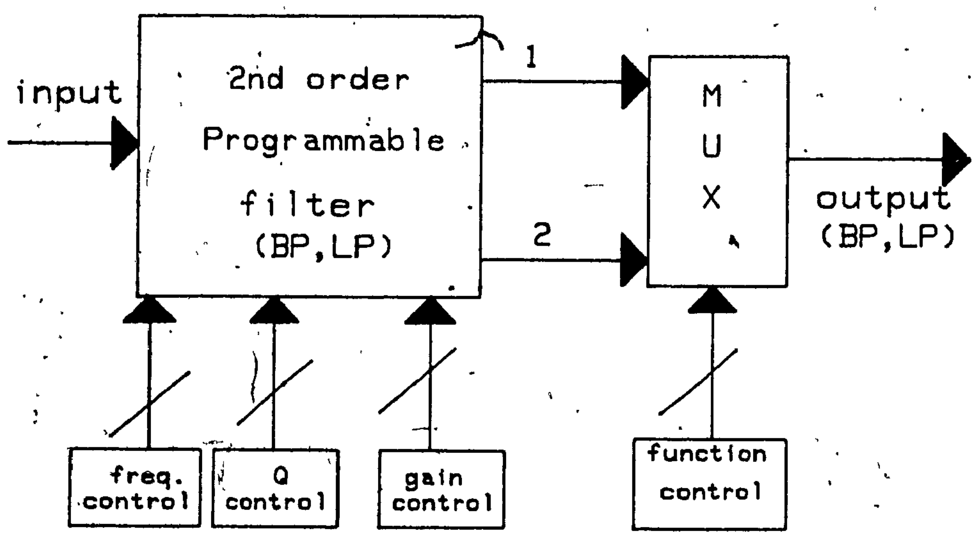

Fig. 3.4. Block diagram of 2nd order programmable filter (Fig. 3.3).

It shows the programmability concept of important filter characteristics using the block diagram approach. In the above block diagram, the second order filter configuration is shown connected to an analog multiplexer (MUX) for digitally controlling the output function responses as an added feature. Any particular function response can be selected at the output employing this technique. Function control switches select the type of response required. The number of function control switches depend upon the number of inputs given at the multiplexer. Suppose, we have k inputs (at the multiplexer) and F = number of control (= log(k)/log(2)). Now if k=4, then F=2 (two control switches required for four inputs) and if k=8, then F=3, etc. Other parameters of filter configuration are also varied digitally as discussed earlier.

To start the analysis of Fig. 3.3, we obtain the transfer functions first. Amplitude and group delay responses were then derived from these transfer functions and are used to investigate the programmability concept of the filter parameters. The derivation of transfer functions is shown in Appendix A. The transfer functions for BP and LP filters are

$$H_{BP}(s) = \frac{N_1(s)}{D(s)} = \frac{-SC_2G_1}{s^2C_1C_2 + sC_3G_2 + G_2G_3} \tag{3.9}$$

$$H_{LP}(s) = \frac{N_2(s)}{D(s)} = \frac{-G_2G_1}{s^2C_1C_2 + sC_3G_2 + G_2G_3} \tag{3.10}$$

The above equations are rewritten in standard format as

$$H_{BP}(s) = -s\left(\frac{C_2G_1}{C_1C_2}\right)\frac{1}{s^2 + s\left(\frac{C_3G_2}{C_1C_2}\right) + \frac{G_2G_3}{C_1C_2}} \tag{3.11}$$

$$H_{LP}(s) = -\left(\frac{G_2G_1}{C_1C_2}\right)\frac{1}{s^2 + s\left(\frac{C_3G_2}{C_1C_2}\right) + \frac{G_2G_3}{C_1C_2}} \tag{3.12}$$

where the output of DAC is represented by

$$g_m \sum_{n=0}^{k} 2^n b_n$$

DAC is represented by G in all equations

and C_3 is represented (based on Fig. 2.3 of chap. 2)by

$$C_3 = \sum_{q=0}^{k} 2^q b_q C_0$$

with b_q is either a 0 or 1 and q=0,1,2,...,k. Comparing the denominators of Eq. 3.11 & 3.12 with standard second order equation, $s^2+s(\frac{w_0}{Q})+w_0^2$, give the following identities

$$w_0 = \sqrt{\frac{G_2G_3}{C_1C_2}} = \frac{G}{\sqrt{C_1C_2}}, \quad center\,frequency \tag{3.13}$$

with $G_2=G_3=G$

$$Q = \frac{w_0C_1C_2}{C_3G_2} = \frac{\sqrt{C_1C_2}}{C_3}, \quad Q-factor \tag{3.14}$$

$$G_1 = g_{m_1} \sum_{n=0}^{k} 2^n b_n, \quad gain control \tag{3.14a}$$

Ideally, for given values of capacitors and identical DAC's ($G_2=G_3=G$) Q-factor will remain unchanged, w_0 will vary linearly with G and 3 dB-bandwidth will also increase linearly with G. If we want to keep 3 dB-bandwidth unchanged (common in most applications) then we have to vary w_0 and Q in the same direction as G so that $\frac{w_0}{Q}$ = constant. The DAC G_1 appears in the numerators of the transfer functions indicating G_1 controls gain. Eq. 3.14 indicates that Q is independent of the DAC if the DAC's are identical. For the DAC's to be identical, the transconductance constant g_m of all DAC's should be exactly same and the linearity of the binary word ($\sum_{n=0}^{k} 2^n b_n$) should hold. In practice, these conditions are difficult to achieve. These practical (non-ideal) cases will be considered in the next chapter.

Sensitivity is an important parameter, particularly for higher frequency active filters. The components of active filters are subject to change due to temperature, humidity, aging etc. It is defined as the change in filter performance due to drift or change in component values. Mathematically, the measure of the change ΔF in some performance characteristic F, resulting from a change Δx_i in a network parameter x_i to be the sensitivity (S) with respect to x_i, given by

$$S_{x_i}^{F} = \frac{\Delta F / F}{\Delta x_i / x_i}$$

where F is function and x_i is element in that function. In the proposed programmable filter, we have

$$w_0 = \sqrt{\frac{G_2 G_3}{C_1 C_2}} = \frac{G}{\sqrt{C_1 C_2}}, \quad center\,frequency$$

with $G_2 = G_3 = G$

and

$$Q = \frac{w_0 C_1 C_2}{C_3 G_2} = \frac{\sqrt{C_1 C_2}}{C_3}, \quad Q-factor$$

Thus W_0 and Q sensitivities are as follows:

$$S_G^{W_0} = \frac{G}{W_0} \frac{\partial W_0}{\partial G} = 1$$

$$S_{C_1}^{W_0} = \frac{C_1}{W_0} \frac{\partial W_0}{\partial C_1} = -1/2$$

$$S_{C_2}^{W_0} = \frac{C_2}{W_0} \frac{\partial W_0}{\partial C_2} = -1/2$$

$$S_{C_1}^{Q} = \frac{C_1}{Q} \frac{\partial Q}{\partial C_1} = 1/2$$

$$S_{C_2}^{Q} = \frac{C_2}{Q} \frac{\partial Q}{\partial C_2} = 1/2$$

$$S_{C_3}^{Q} = \frac{C_3}{Q} \frac{\partial Q}{\partial C_3} = -1$$

Sensitivities are identical for both lowpass and bandpass filter configurations. In the above analysis, all sensitivity values are found to be ≤ 1.

3.4 Discussion & Analysis

In a conventional active analog filter, the capacitors and resistors are generally fixed, although variable capacitors may be used in tunable filters. The aim of this new concept is to realize a programmable filter by using digitally programmable elements; this would then permit the filter to be programmed by a digital methods, such as, logic circuit or microprocessor. To accomplish this, the conventional resistors are replaced by DAC's.

Considering Eq. 3.13, 3.14 & 3.14a it is clear that the ability to alter the value of G and C_3 can be employed in the filter function to program the center frequency, quality factor and the gain of the response. Hence, Q is dictated by the value of C_3, while the center frequency is determined by G and C_1 & C_2 which are common in both parameters. Gain varies as w_0 is varied. Gain can be programmed independently if G_1 is assumed independent of G.

Like any other filters, programmable filters has its limitation in respect to its components and parameters. Most OP-AMPs above 50 kHz have insufficient open-loop gain for the active filter requirement. However, OP-AMPs with extended bandwidth are available at frequencies upto 500 kHz. The OP-AMP used in the experimental realization is wide bandwidth (5 MHz) type and has high input impedance. The circuit works well at frequencies upto about 150 kHz. Above that frequency the OP-AMP starts saturating due to the variation in the input impedances and hence limiting the center frequency. Bandwidth of the filter response is controlled by capacitors value. Q-factor upto 10 give good accuracy. For Q's above 10, the Q capacitor C_3 must be made smaller to obtain desirable response.

3.4.1 Analysis of Amplitude Response

The amplitude function is used to classify the various types of filters according to the locations of their poles and zeros. The amplitude response is the most important characteristic since its value at a certain frequency determines whether the frequency passes or blocked.

The amplitude responses of Fig. 3.3 (in dB) are:

$$|H_{BP}(jw)| = 20\log_{10} wC_2G_1 - 20\log_{10} D \tag{3.15}$$

similarly,

$$|H_{LP}(jw)| = 20\log_{10} G_1G_2 - 20\log_{10} D \tag{3.16}$$

where

$$D = \sqrt{(G_2^2G_3^2 - 2w^2G_2G_3C_1C_2 + w^4C_1^2C_2^2 + w^2G^2C_3^2)}$$

The amplitude response of Eqs. 3.15, 3.16 are plotted in Fig. 3.5, 3.6 for various values of Q and G. The amplitude response shifts as the center frequency is moved by programmable G. The plots indicate that w_0 varies linearly with G and 3 dB-bandwidth also increases linearly for fixed Q. The important filter parameters are digitally programmed as evident in these graphs. The center frequency is shifted by changing the bits of DAC's. The Q-factor is varied by digitally programming the C parameter. The bandwidth of response curve becomes sharper as Q is increased.

3.4.2 Analysis of Group delay Response

Another quantity of interest in our study of filters is the group delay. The group delay response is used to see if constant delay occurs when the desired sig-

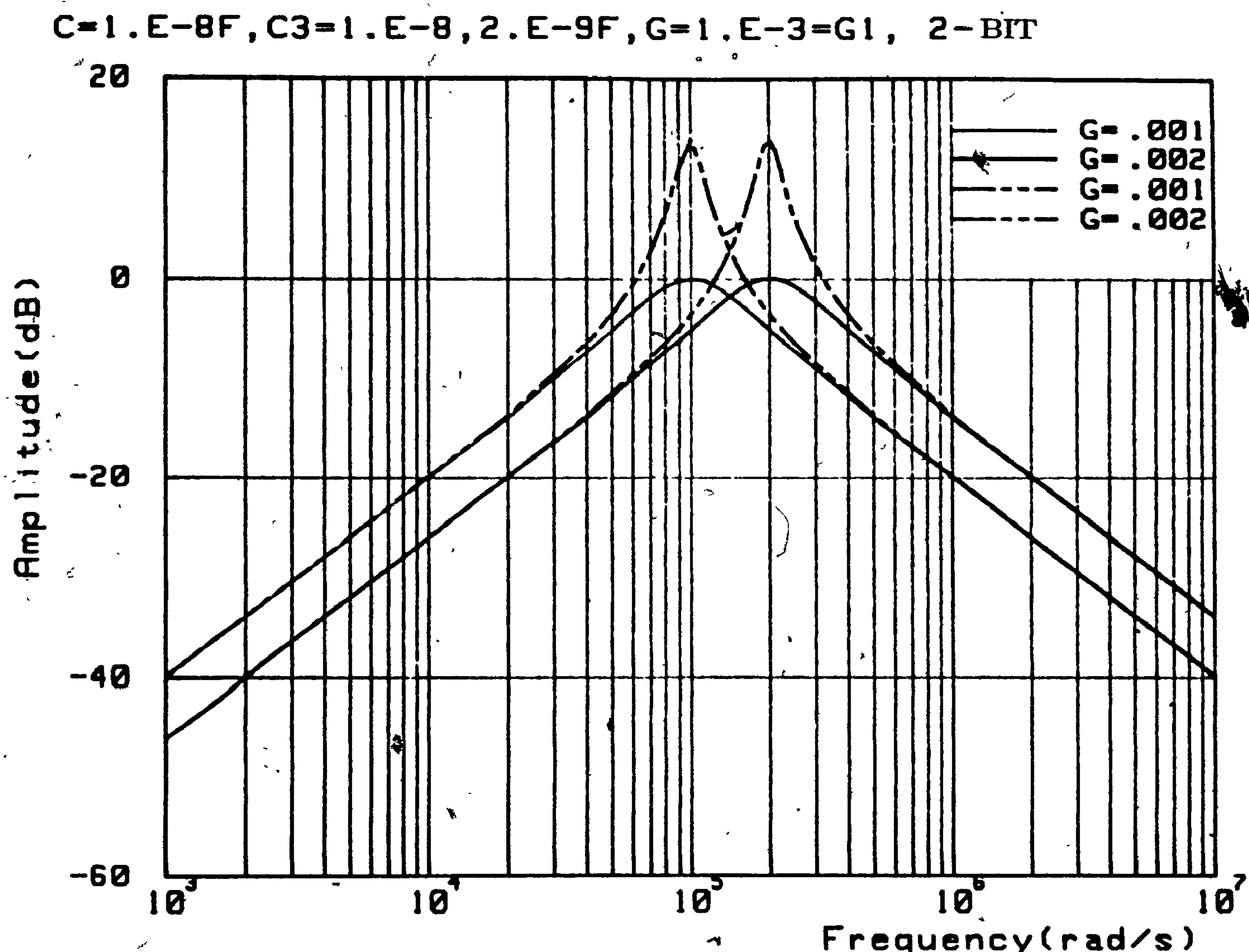

FIG. 3.5 AMPLITUDE RESPONSES OF 2ND ORDER BPF, Q=1,5

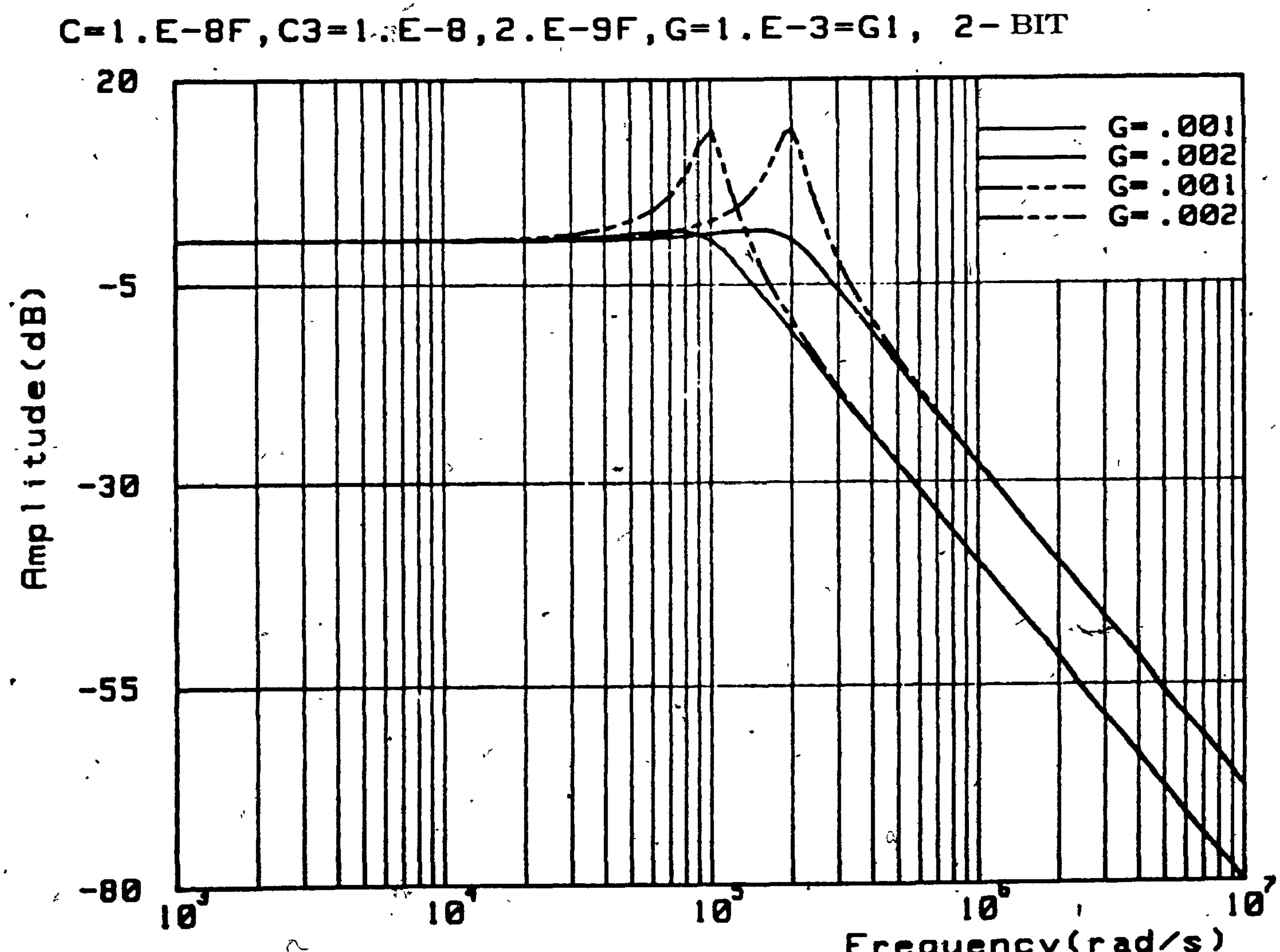

FIG. 3.6 AMPLITUDE RESPONSES OF 2ND ORDER LPF, Q=1,5

nal pass through the filter [3]. This is the second parameter that will be used to verify the programmability concept of digitally programmable analog filter.

The group delay is defined as

$$T(w) = \frac{d\phi}{dw}$$

where ϕ is the phase response. The group delay is derived in Appendix B. The group delay response of both BP and LP filter of Fig. 3.3 is

$$T(w) = \frac{C_3 G_2^2 G_3 + w^2 G_2 C_1 C_2 C_3}{(G_2 G_3 - w^2 C_1 C_2)^2 + (w G_2 C_3)^2} \tag{3.17}$$

The group delay response of Eq. 3.17 is plotted in Fig. 3.7 for various values of Q and G. The response shows that the center frequency is varied by programming the G. The concept of w_0 and Q programming in a filter response are verified through plots of group delay response as well.

3.5 Higher Order Filter Realization

This section deals with the construction of higher order filter. There are different ways of constructing a filter with higher order (n>2) transfer function. One popular method is cascading method. In this method, the transfer function is expressed as a product of factors H_1, H_2, ...,H_m and construct sections corresponding to each factor. Finally, the sections are cascaded (the output of the first is the input of the 2nd and so forth) as shown in the example of Fig. 3.8. Fig. 3.8 is constructed using cascading method where each block represents an independent second order filter structure. In this fashion any filter structure of desired order can be constructed and employing the programmability concept, the parameters of this type of filter structure can be digitally programmed and

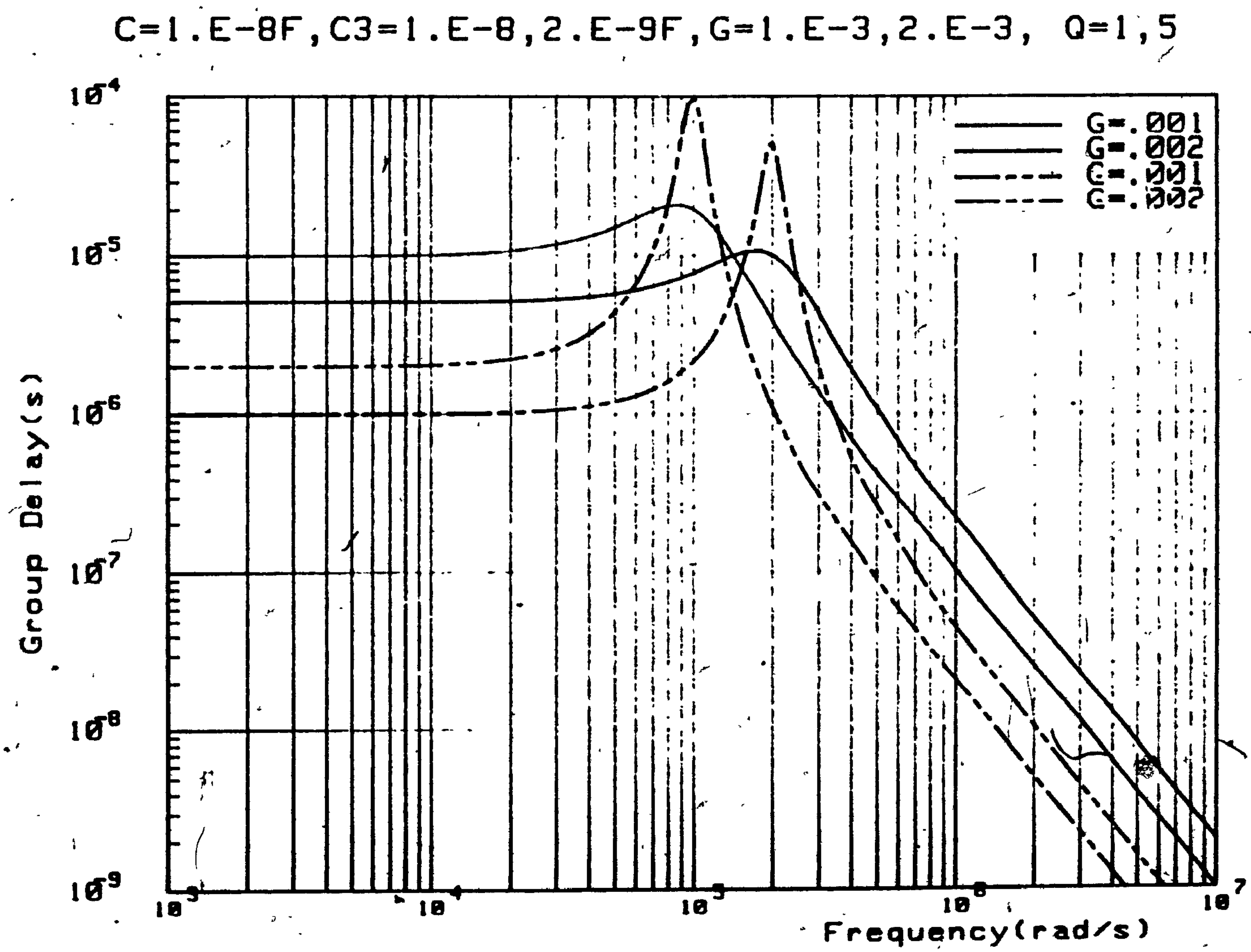

FIG. 3.7 GROUP DELAY RESPONSES OF 2ND ORDER FILTER

independently controlled as shown in the block diagram (Fig. 3.8).

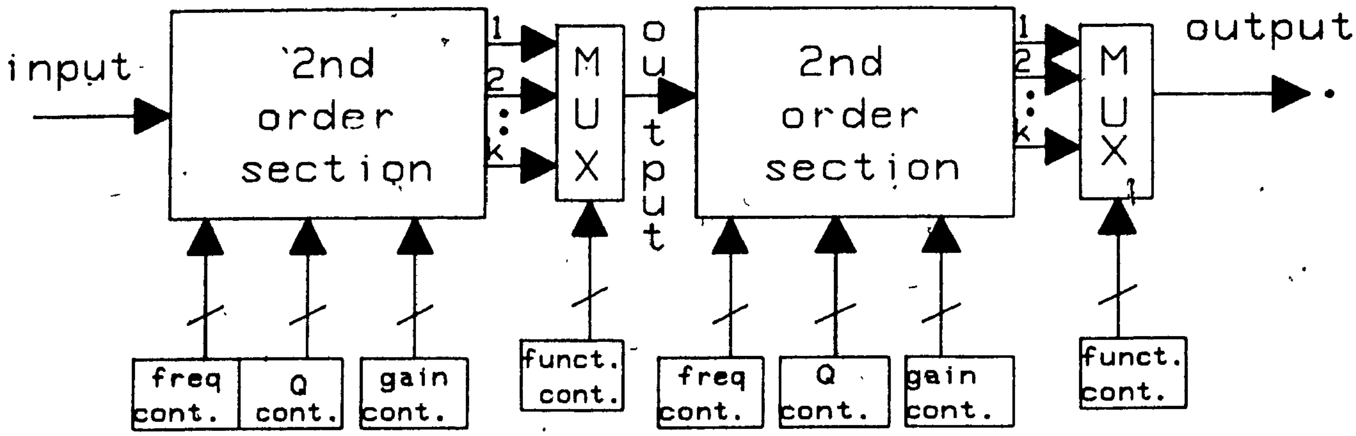

Fig. 3.8. Block diagram of a higher order filter.

If the sections do not interact with each other to change their individual transfer functions, the overall configuration has the transfer function of given order n [16]. Cascading structure of Fig. 3.8 can accommodate any filter response (LP, HP, BP, etc.) if so desired. To illustrate the concept of realizing higher order filter using cascading method and the straightforward application of digital programmability, we discuss a design of bandpass filter with variable bandwidth using a LP and HP filter section (assuming both filter types are available in Fig. 3.8). Suppose, a lowpass filter response is chosen from the first section with a cutoff frequency f_2 and a highpass filter response is chosen from the second section with a cutoff frequency of f_1. The two response will combine to give a bandpass response at the output as shown in Fig. 3.9. Using the programmability concept the bandwidth can be controlled by controlling f_1 and f_2 digitally. Using the Cascading and the programmability concept various higher order filter response can be easily obtained and their parameters can easily be digitally programmed and independently controlled. For even order $n>2$, the usual cascaded circuit has $n/2$ second-order sections. If the order $n>2$ is odd, there will be $(n-1)/2$ second-order sections and one first order section [16].

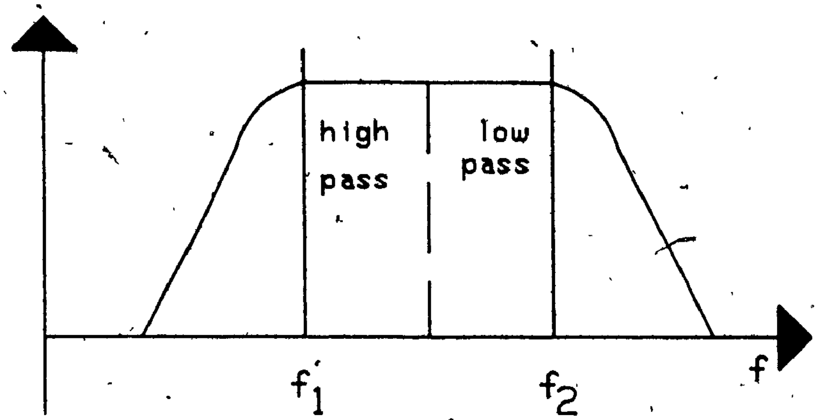

Fig. 3.9. Amplitude response of a cascading 4th order bandpass filter.

3.5.1 Amplitude Response of 4th Order LP & BP Filter

In the previous section a BP filter is designed using a LP and BP section, merely to demonstrate the cascading method of building filter. In this section a 4th order programmable filter configuration is constructed using cascading method using two second order BP filter section. The transfer functions for this 4th order programmable configuration are obtained using Eq. 3.9, 3.10

$$H_{4BP}(s) = H_{2BP}(s) * H_{2BP}(s)$$

$$H_{4BP}(s) = \left\{ \frac{-SC_2G_1}{s^2C_1C_2 + sC_3G_2 + G_2G_3} \right\}^2$$

$$H_{4BP}(s) = \frac{s^2C_2^2G_1^2}{M(s)} \tag{3.18}$$

similarly, for 4th order LP filter

$$H_{4LP}(s) = H_{2LP}(s) * H_{2LP}(s)$$

$$= \left\{ \frac{-G_2G_1}{s^2C_1C_2 + sC_3G_2 + G_2G_3} \right\}^2$$

$$H_{4LP}(s) = \frac{G_1^2G_2^2}{M(s)} \tag{3.19}$$

where M for both Eq. 3.18, 3.19 is

$$M(s) = s^4C_1^2C_2^2 + 2s^3C_1C_2C_3G + 2s^2C_1C_2G^2 + s^2C_3^2G^2 + 2sC_3G^3 + G^4$$

The corresponding magnitude responses in dB are given by:

$$|H_{4BP}(jw)| = 20\log_{10}w^2C_2^2G_1^2 - 20\log_{10}D \qquad (3.20)$$

and

$$|H_{4LP}(jw)| = 20Log_{10}G_1^2G^2 - 20\log_{10}D \qquad (3.21)$$

where

$$D = w^8C_1^4C_2^4 - 4w^6C_1^3C_2^3G^2 + 2w^6C_1^2C_2^2C_3^2G_2 + 6w^4C_1^2C_2^2G^4 - 4w^4C_1C_2C_3^2G^4 + w^4C_3G^4 - 4w^2C_1C_2G^6 + 2w^2C_3^2G^6 + G^8 \qquad (3.22)$$

The amplitude responses of Eq. 3.20 & 3.21 are plotted in Fig. 3.10,3.11 with $Q = 1.55Q_1$ where $Q_1 = 1,5$. When 4th order filter is obtained by cascading method, the new Q-factor is calculated using $Q = 1.55Q_1$ [7], where Q_1 is of 2nd order filter. The response is flat for Q < 1. The response shows sharp peak for Q > 1. G_1 is used to program gain and is independent of Q variation.

The procedure for finding practical element values for cascading filter is identical to that of 2nd-order filter, except, of course there are more elements. For 4th-order cascading filter configuration, each of the sections has to be designed to have the square root of the gain. Thus the 4th-order cascading configuration has a gain of the product of square root of the individual second order gain. If the designer wishes, the section gains may be assigned differently as long as their product is the gain.

3.5.2 Group Delay Response of 4th Order Filter

We use previously derived Eq. 2B from appendix B to obtain group delay response of 4th order filter as

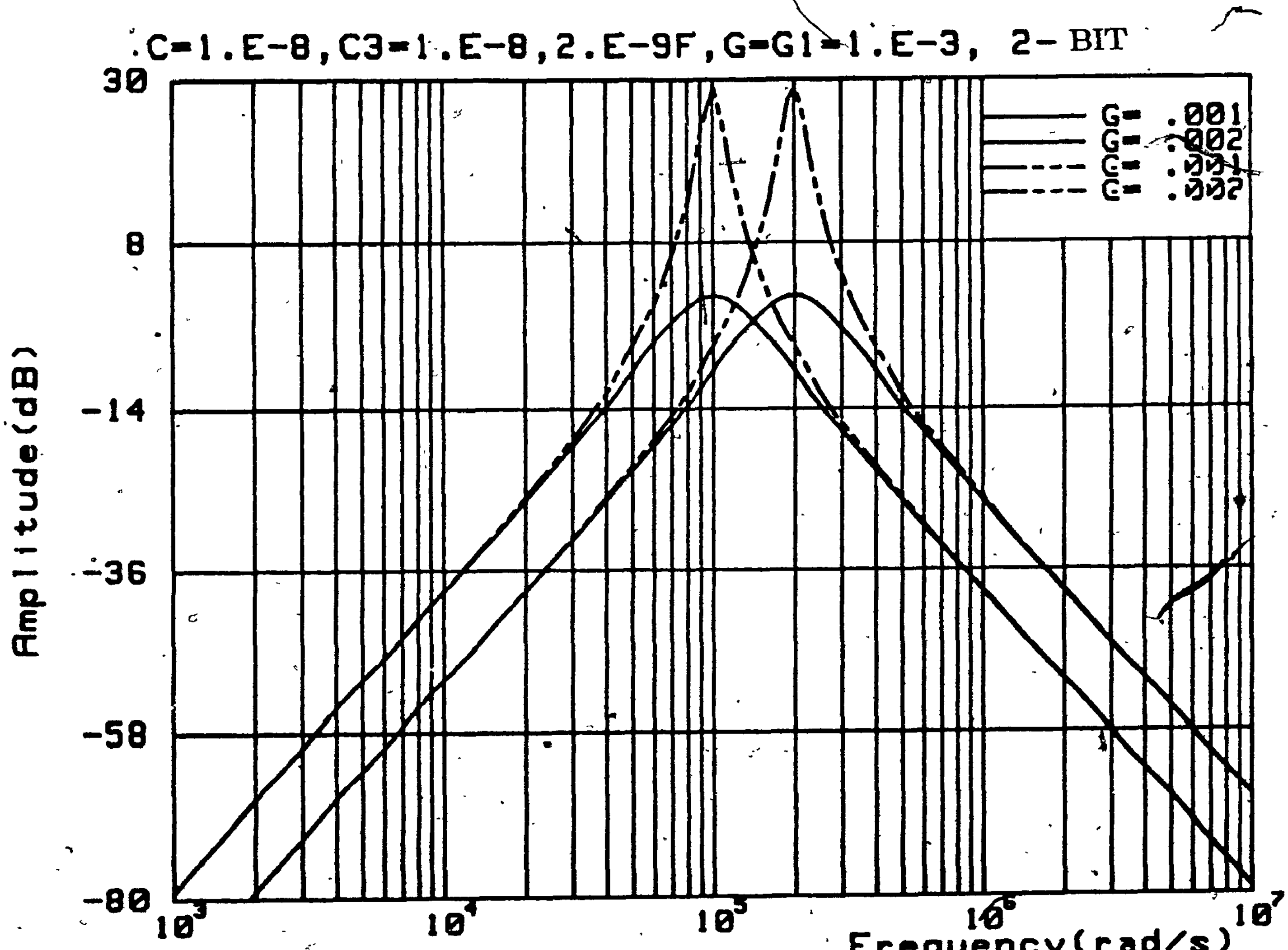

FIG. 3.10 AMPLITUDE RESPONSES OF 4TH ORDER BPF, Q=1,5

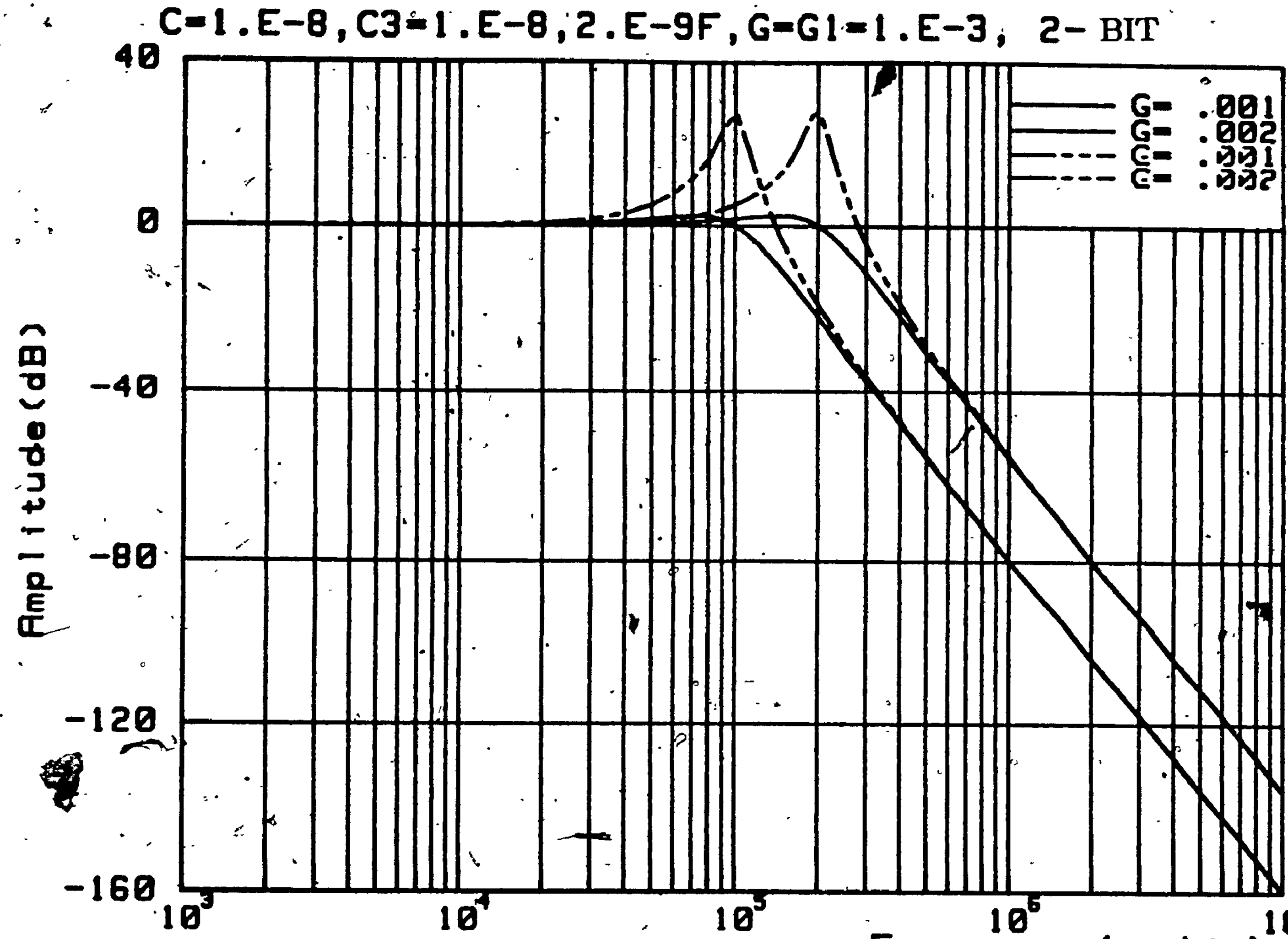

FIG. 3.11 AMPLITUDE RESPONSES OF 4TH ORDER LPF, Q=1,5

$$T(w) = \frac{2w^6C_1^3C_2^3C_3G - 2w^4C_1^2C_2^2C_3G^3 + 2w^4C_1C_2C_3^3G^3}{w^8C_1^4C_2^4 - 4w^6C_1^3C_2^3G^2 + 2w^6C_1^2C_2^2C_3^2G_2 + 6w^4C_1^2C_2^2G^4}$$
$$\frac{-2w^2C_1C_2C_3G^5 + 2w^2C_3^3G^5 + 2G^7C_3}{-4w^4C_1C_2C_3^2G^4 + w^4C_3G^4 - 4w^2C_1C_2G^6 + 2w^2C_3^2G^6 + G^8} \quad (3.23)$$

Fig. 3.12 shows group delay responses of 4th order filter for various Q's. The responses are similar to LP amplitude responses with peak amplifying distortions. The center frequency and Q-factor are programmed to obtained desired responses.

3.6 Analysis of Higher Order Filter using Approximations

In modern filter theory the design problem is divided into two distinct parts- the approximation problem and the realization problem. In the following sections we shall consider ways to approximate the given response by a realizable transfer function. Many such approximations have been derived in the field of filter theory. We employ commonly used Butterworth (BW) and Chebyshev (CS) approximation to analyze the proposed programmable filter configuration. Both approximations are useful in the field of filter design because of their many desirable characteristics.

3.7 Butterworth (BW) Approximation

The ideal response of filter is unrealizable in physical sense for two reasons: They are (1) transfer functions are ratios of polynomials and (2) the magnitude of the transfer function cannot possess the discontinuities necessary for the clearly defined boundaries between pass and stopbands. Therefore, various approximations are used to analyze the filter responses. We start with Butterworth approximation. The Butterworth (BW) filter is characterized by its monotonic response. It has maximally flat response in the passband. The BW response is

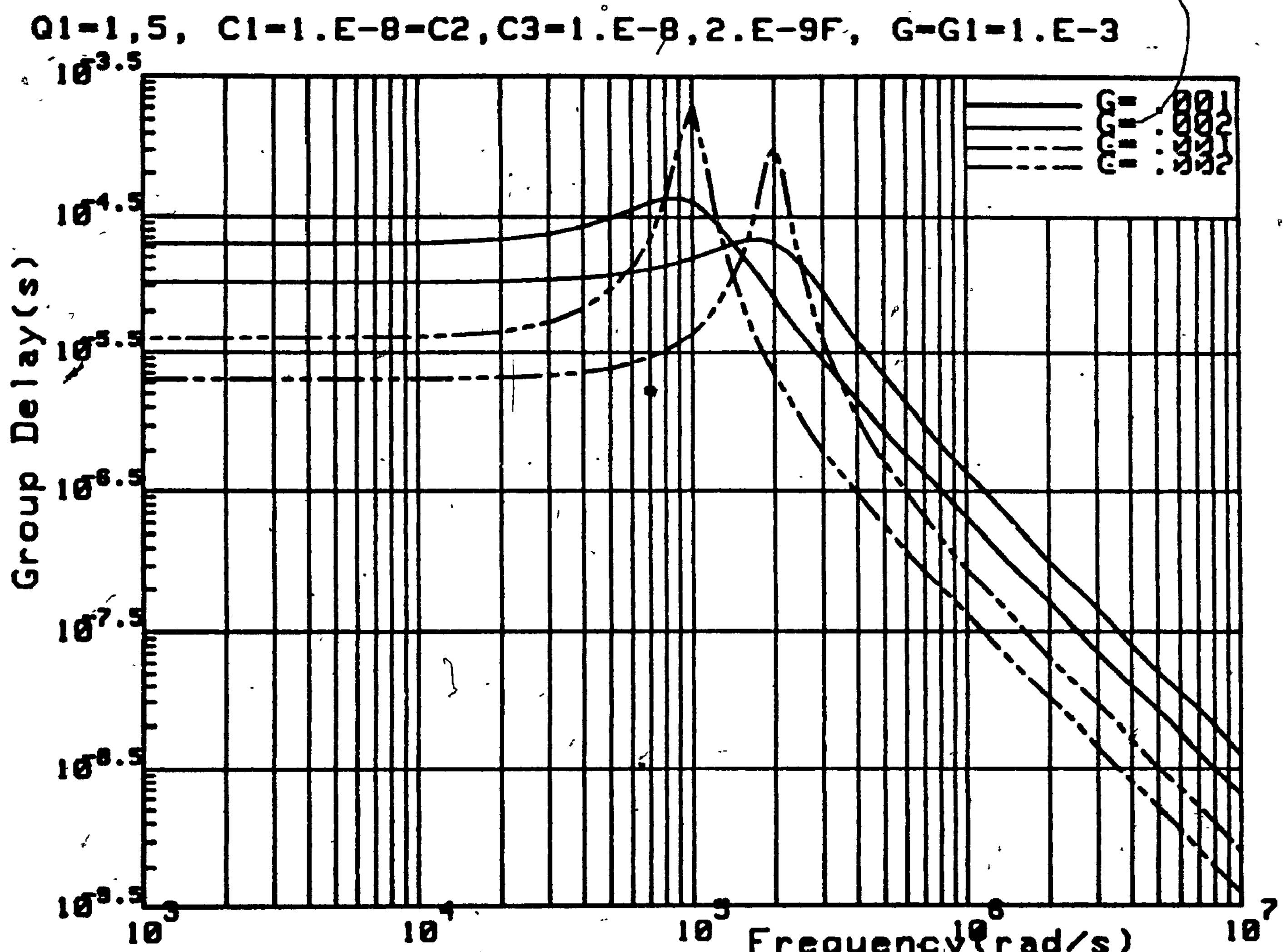

FIG. 3.12 GROUP DELAY RESPONSES OF 4TH ORDER FILTER

monotonically decreasing in the stopband and thus it attains its maximum value, $|H(jw)|_{max}=1$, at w=0 [1]. The overall response improves as n (order of the filter) increases, since for $n_1 >> n_2$, we have $w^{2n_1} << w^{2n_2}$ on $0 < w < 1$, and $w^{2n_1} >> w^{2n_2}$ for $w > 1$ [1]. The amplitude function is defined in the n-th order case by

$$|H(jw)| = \frac{1}{\sqrt{1 + (w/w_c)^{2n}}}; \quad n = 1,2,3,... \qquad (3.24)$$

The response is flattest near w=0 of any nth order all-pole filter and is called maximally flat for this reason. Mathematically, this can be seen by expanding Eq. 3.24 using the binomial theorem; this results in [1]

$$|H(jw)| = 1 - \frac{1}{2}w^{2n} + \frac{3}{8}w^{4n} - \frac{5}{10}w^{6n} + \frac{35}{128}w^{8n} .. \qquad (3.25)$$

which is valid for w near 0. The first 2^n-1 derivatives of $|H(jw)|$ in Eq. 3.25 will contain a factor w, and thus will be zero at w = 0, therefore for n large the function $|H(jw)|$ near w = 0 is exceedingly flat or maximally flat as it was mentioned earlier.

3.7.1 Amplitude Response of BW-LP & BW-BP Filter

Butterworth approximation method is used to design a circuit model of a lowpass second-order filter (Fig. 3.13). Using gain one, the transfer function is found to be

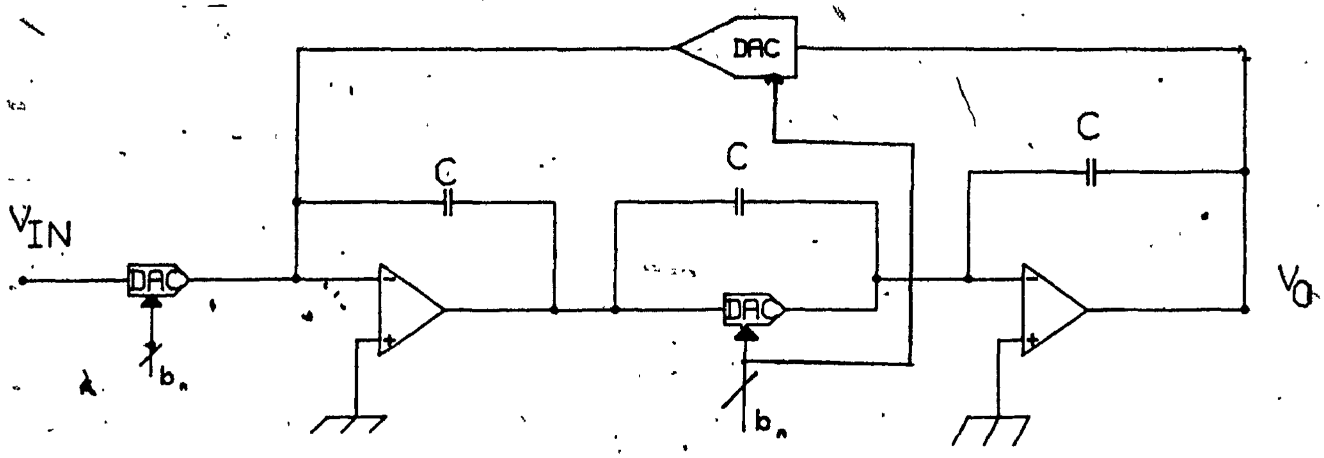

Fig. 3.13. Butterworth n=2 lowpass programmable filter.

$$H_{BW-LP}(s) = \frac{\frac{G^2}{C_1C_2}}{s^2 + s(\frac{2G}{C_1}) + \frac{G^2}{C_1C_2}}$$

where

$$w_c = \sqrt{\frac{G^2}{C_1C_2}}$$

The logarithmic amplitude response of BW-LP filter is

$$|H_{BW}(jw)| = -20\log_{10}\sqrt{1 + (\frac{w}{w_c})^{2n}}; \; n=2,4 \qquad (3.26)$$

The amplitude response is plotted in Fig. 3.14 for 2nd and 4th filter. The plots verify that the digitally programmable concept based on DAC's works. The w_c is shifted by programmable DAC's. The plots also indicate that indeed the general characteristics of amplitude response improves as order of the filter increases. The plots show that Q has no effect on the response as flat passband is the characteristic of BW-LP filter. The response is flat in the passband as expected. In the BW-LP filter response, for frequencies near the cutoff point and in the stopband the response is distinctly inferior to CS response as we shall see

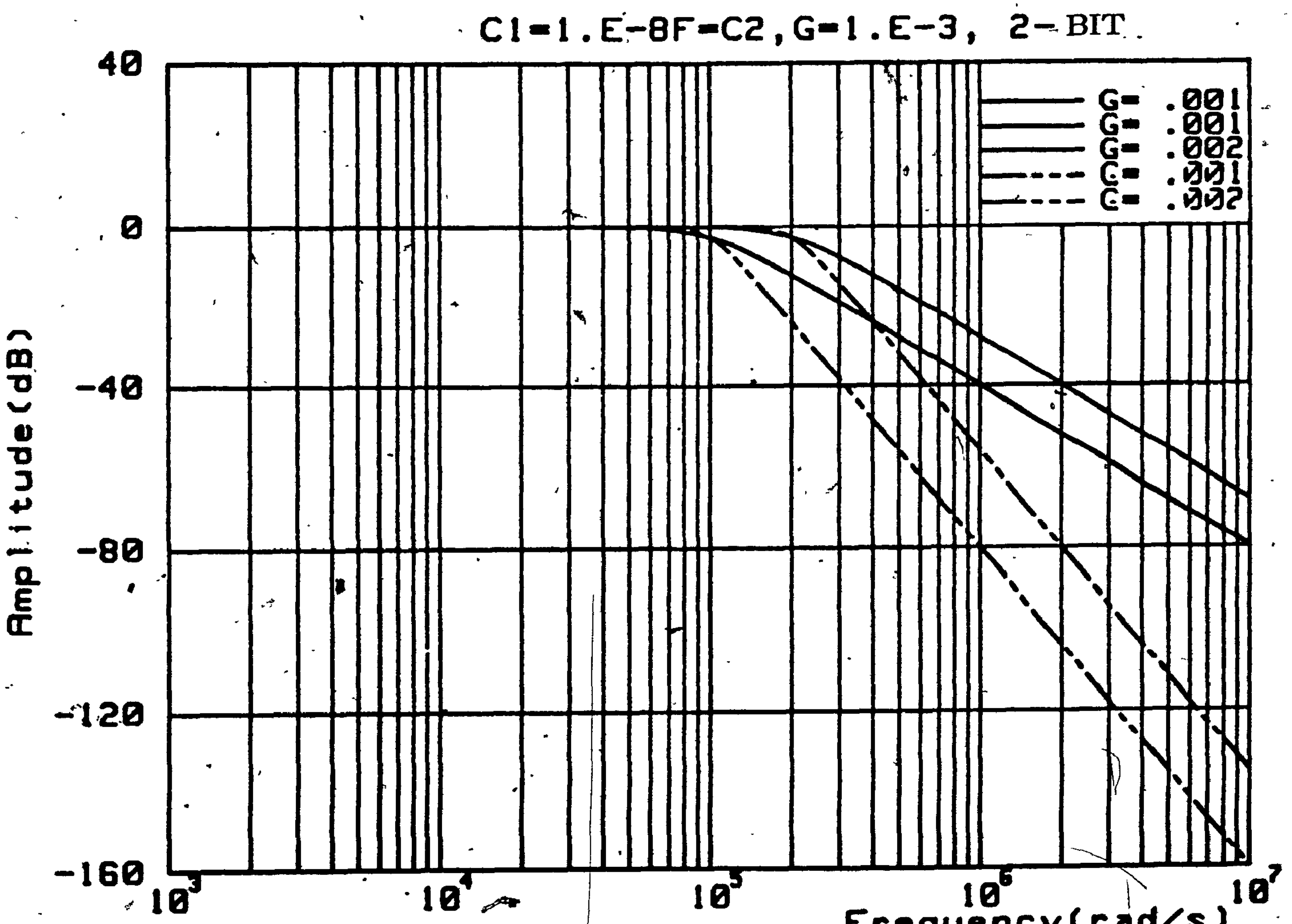

FIG. 3.14 AMPLITUDE RESPONSES(BW) OF 2ND,4TH ORDER LPF

later.

The 4th order BW-BP transfer function is obtained from normalized 2nd order BW-LP transfer function of S by the following transformation [5]:

$$S = \frac{Q(s^2 + w_0^2)}{w_0 \cdot s} \tag{3.27}$$

When transformation is used to derive a BP filter transfer function the order of BP filter is twice that of its corresponding LP filter and is always even. The resulting BP amplitude response resembles its LP counterpart shifted upward in frequency from 0 to w_0. A structure for second order Butterworth-BP filter is illustrated in Fig. 3.13. The 2nd order BW-LP transfer function is [6]

$$H_{2LP}(S) = \frac{1}{S^2 + \sqrt{2}S + 1} \tag{3.28}$$

substituting Eq. 3.27 into Eq. 3.28 we obtain new 4th order Butterworth-BP transfer function:

$$H_{BW\text{-}BP}(s) = \frac{s^2 w_0^2}{D(s)} \tag{3.29}$$

where

$$D(s) = s^4Q^2 + 2s^2w_0^2Q^2 + w_0^4Q^2 + \sqrt{2}s^3w_0Q + \sqrt{2}sw_0^3Q + s^2w_0^2$$

The amplitude response in logarithmic form:

$$|H_{BW\text{-}BP}(jw)| = 20\log_{10}(w^2w_0^2) - 20\log_{10}\sqrt{(M_2^2 - N_2^2)}\,\Big|_{s=jw} \tag{3.30}$$

where

$$M_2(s) = s^4Q^2 + 2s^2w_0^2Q^2 + w_0^4Q^2 + s^2w_0^2$$
$$N_2(s) = \sqrt{2}(s^3w_0Q + sw_0^3Q)$$
$$w_0 = \frac{G}{C}$$

$$Q = \frac{C}{C_3}$$

The BW-BP amplitude response is plotted in Fig. 3.15 for various Q. The digitally programmable concept is employed to shift center frequency and Q-factor. The BW-BP amplitude responses vary monotonically on both sides of its peak with a maximally flat passband. It is clear from the graph that higher Q's correspond to sharper passbands.

3.7.2 Group Delay Response of BW-LP & BW-BP Filter

The group delay response for 4th order BW-LP filter is derived using standard polynomial [9]. The 4th order polynomial is given as

$$\begin{aligned}&(s^2 + .765s + 1)(s^2 + 1.848s + 1)\\&= s^4 + 2.613s^3 + 3.4142s^2 + 2.613s + 1\end{aligned}$$

using Eq. 2B from Appendix B, we obtain

$$T_{BW-LP}(w) = \frac{2.6131w^6 + 1.082847w^4 + 1.0820847w^2 + 2.6131}{w^8 + 1} \quad (3.31)$$

where

$$w \rightarrow \left[\frac{w}{w_0}\right]$$

$$w_0 = \frac{G}{C}$$

The 4th order BW-BP filter group delay response is derived from Eq. 3.29 as follows:

$$T_{BW-BP}(w) = \sqrt{2}\frac{(w^6 w_0 Q^3 - w^4 w_0^3 Q^3 + w^4 w_0^3 Q - w^2 w_0^5 Q^3}{w^8 Q^4 - 4w^6 w_0^2 Q^4 + 6w^4 w_0^4 Q^4 - 4w^2 w_0^6 Q^4} \frac{+ w^2 w_0^5 Q + w_0^7 Q^3)}{+ w^4 w_0^4 + w_0^8 Q^4} \quad (3.32)$$

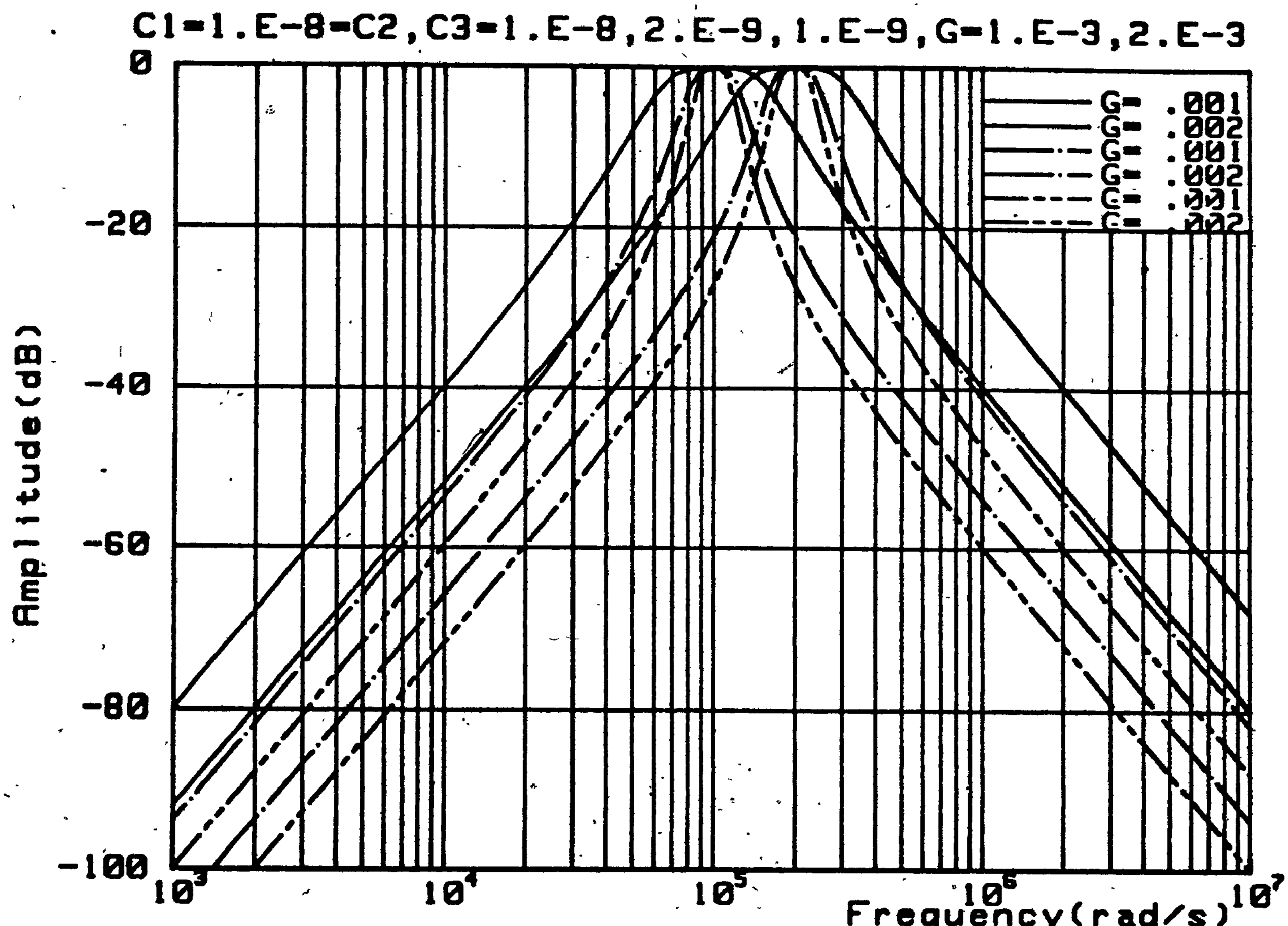

FIG. 3.15 AMPLITUDE(BW) RESP. OF 4TH ORDER BPF, Q=1,5,10

Eqs. 3.31 & 3.32 are plotted in Fig. 3.16 & 3.17 for BW-LP and BW-BP filter respectively. The center frequency is changed by programming the DAC's. The BW-LP response does not change as Q-factor does not play any role in this case but for BW-BP, the response becomes sharper as Q gets higher.

3.8 Chebyshev (CS) Approximation for 4th Order Filter

CS is the 2nd approximation technique used here to analyze the proposed filter configuration. If steepness of attenuation slope, especially in the region of cutoff, is more important than passband flatness or phase linearity, then CS response is often applicable. The CS filter response is superior (in the sense that the Chebyshev transition width is smaller) to the BW at cutoff and in the stop-band, it is, in fact the optimum all-pole filter in this respect. The BW amplitude response is better than the Chebyshev amplitude response in the vicinity of w=0. because it is flattest in the vicinity of w=0. However, in almost all other respects, the CS response is better. To illustrate all these points, we proceed to analyze filter characteristics employing CS approximation.

3.8.1 CS-LP & CS-BP Approximations of Amplitude Responses

We have seen in section 3.6.1 that the BW-LP amplitude response is the flattest in the vicinity of w=0 and for large values of w than the Chebyshev response. The Chebyshev amplitude response has better roll off (>6 dB/octave) than the Butterworth (6 dB/octave roll off) response.

The CS-LP filter is an optimum all-pole filter. Its amplitude response is given by [1]

$$|H(jw)| = \frac{k}{\sqrt{1 + \epsilon^2 C_n^2(\frac{w}{w_c})}}, \quad n = 1,2,3,\ldots \qquad (3.33)$$

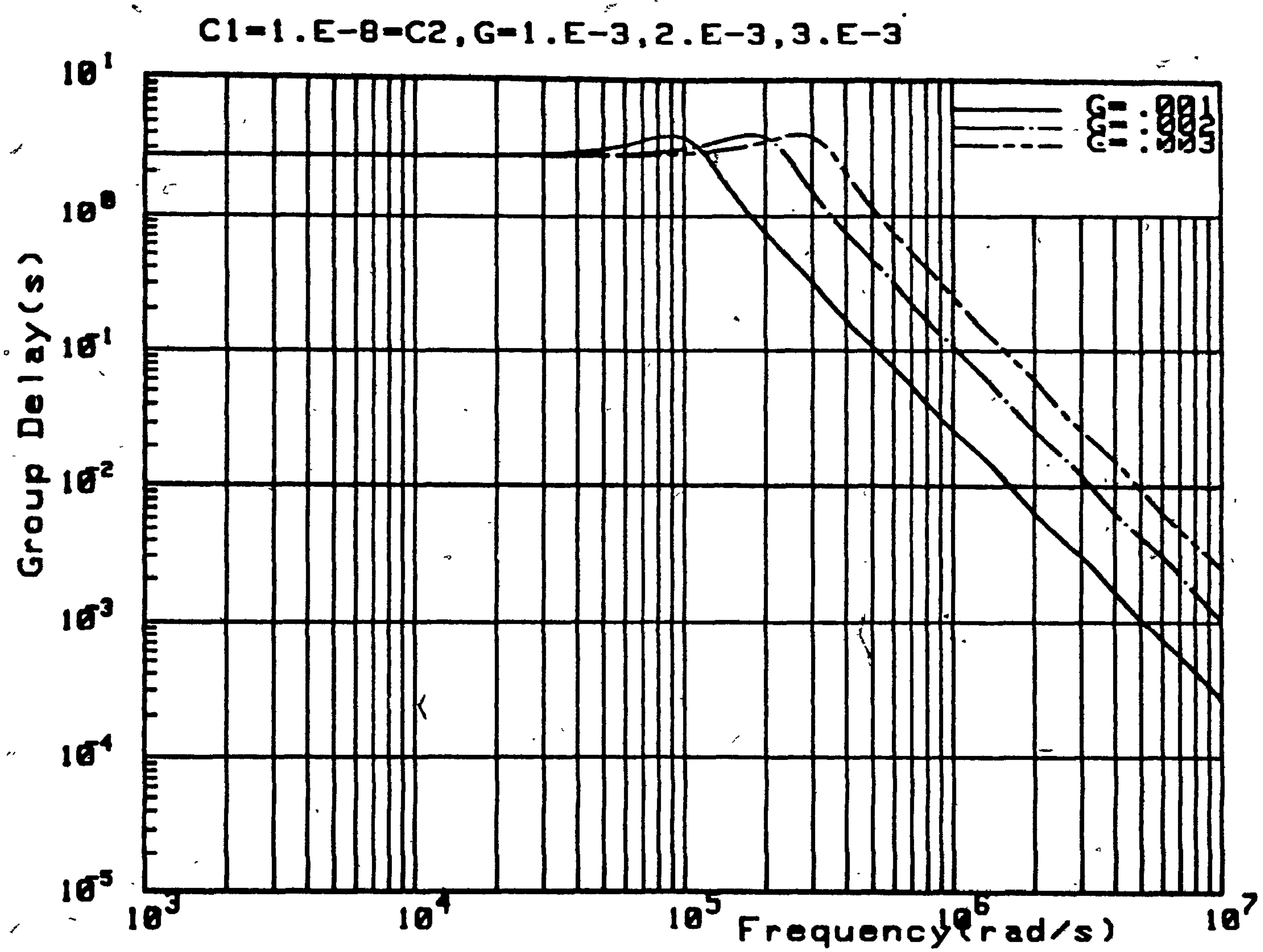

FIG. 3.16 GROUP DELAY RESP(BW) OF 4TH ORD. LP FILTER

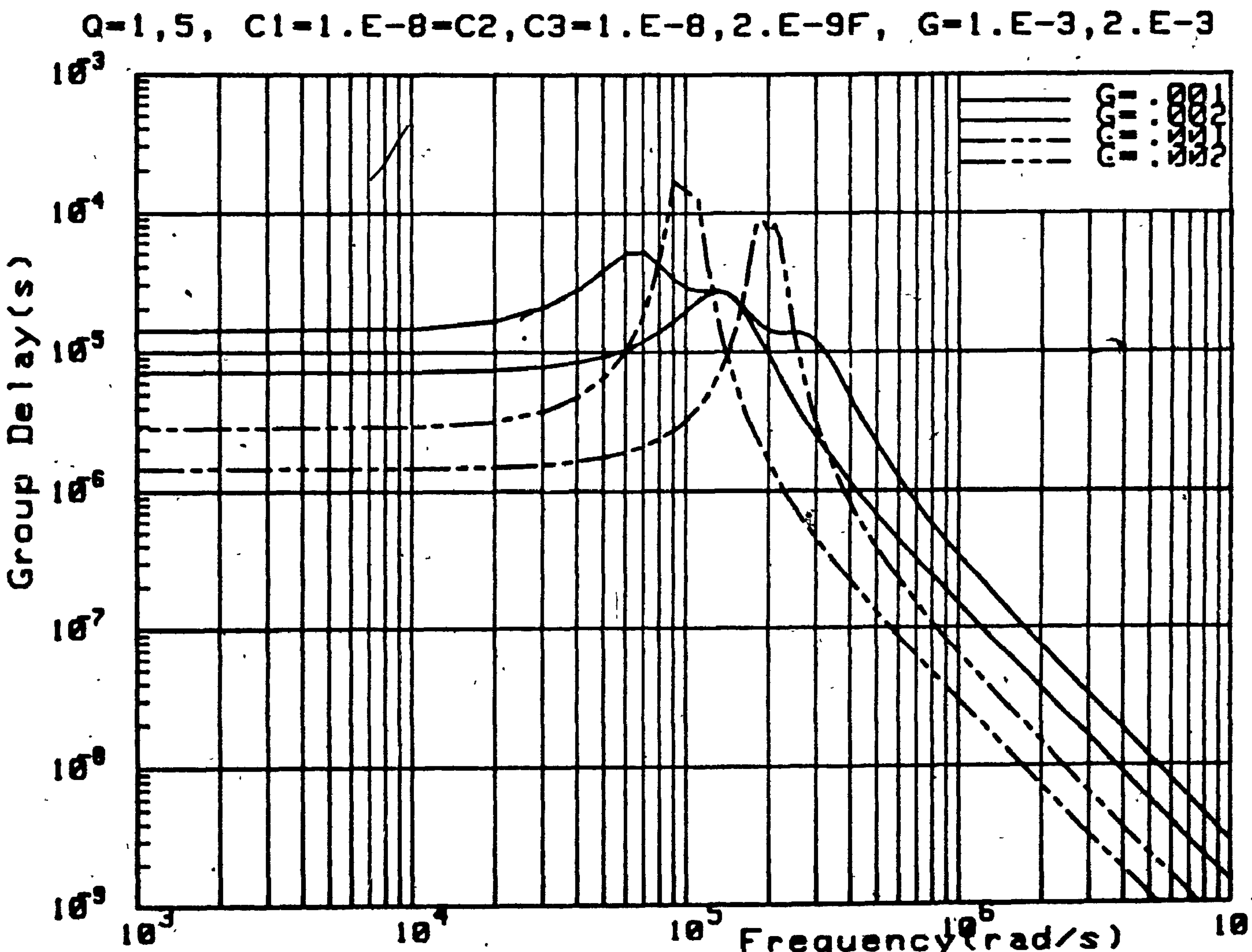

FIG. 3.17 GROUP DELAY RESP(BW) OF 4TH ORD. BP FILTER

where

ϵ is a constant and

$$C_n(w) = \cos(n \cos^{-1} w) \tag{3.34}$$

is the CS polynomial of the first kind of degree n.

and $w = \cos\theta$

By Eq: 3.34 we see that

$$C_0(w) = \cos 0 = 1$$
$$C_1(w) = \cos(\cos^{-1} w) = w \tag{3.35}$$

The amplitude reaches its peak value of k at the points where C_n is zero. Since these points are distributed across the passband, the CS response has ripples in the passband and is monotonic decreasing elsewhere. The width of the ripples is determined by the value of ϵ and the number of ripples is determined by n. The value k determines the gain of the filter.

The CS-LP response for 4th order filter is plotted in Fig. 3.18 with 3 dB ripple width. To find ϵ we use

$$-3 = 20\log_{10}\frac{1}{\sqrt{1+\epsilon^2}},\ \epsilon = .99763$$

The CS polynomials are taken from table [1].

For

2nd order, $C_n(w) = (2w^2 - 1)$

4th order, $C_n(w) = (8w^4 - 8w^2 + 1)$

The amplitude response for 4th order CS-LP (in dB) is

$$|H_{CS-LP}(jw)| = -20\log_{10}\sqrt{1 + \epsilon\, 2(8w^4 - 8w^2 + 1)^2} \tag{3.36}$$

The passband shows ripples as its characteristics. In return for the lack of smoothness in the passband there are advantages in a very much higher roll off

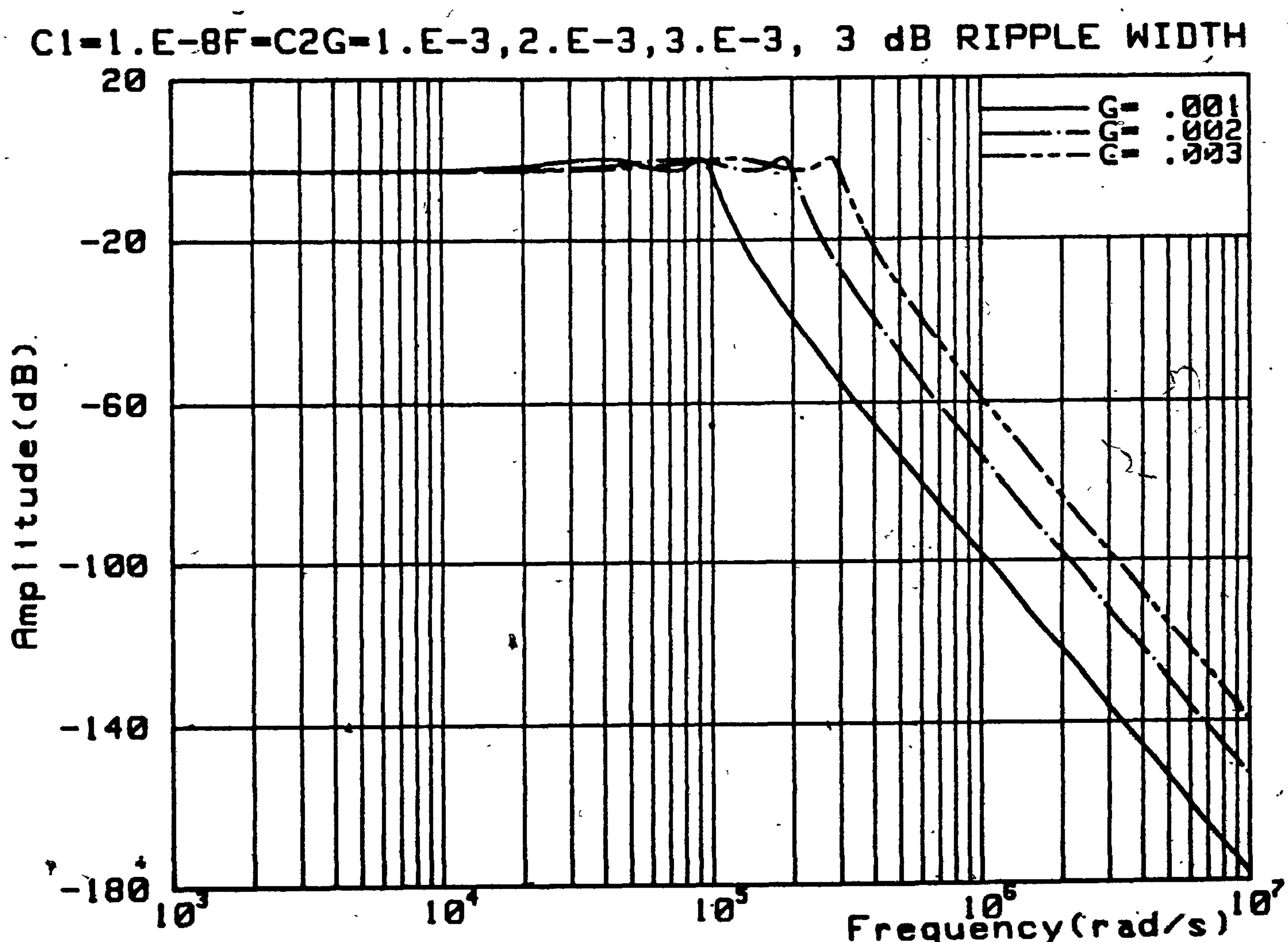

FIG. 3.18 AMPLITUDE(CS) RESP. OF 4TH ORDER LP FILTER

(>6 dB/octave) of cutoff point around the edge of the passband. The center frequency is varied by using programmable components in the amplitude response.

To understand why the CS polynomial yields a good approximation in Eq. 3.33, let us consider some of its properties. From Eq. 3.35 we have

$$C_n(1) = \cos(n \cos^{-1} 1) = \cos 0 = 1 \tag{3.37}$$

Using the expression $\cos^{-1}(-w) = \pi + \cos^{-1} w$, we may write

$$C_n(-w) = \cos\left[n \cos^{-1}(-w)\right] = \cos\left[n(\pi + \cos^{-1} w)\right] \tag{3.38}$$
$$= \cos n\pi \, \cos(n \cos^{-1} w) - \sin n\pi \, \sin(n \cos^{-1} w)$$

or

$$C_n(-w) = (-1)^n C_n(w) \tag{3.39}$$

Thus $C_n(w)$ is even or odd according to whether n is even or odd, and therefore $C_n^2(w)$ is an even function [1].

The zeros of $C_n(w)$ are all real, distinct and lie in the interval $-1 < w < 1$. Since $C_n(w) = 0$ requires $\theta = (2k-1)\frac{\pi}{2n}$ and thus the zeros w_k are given by

$$w_k = \cos(2k-1)\frac{\pi}{2n} \; ; \; k = 1,2,..n$$

also on $-1 \le w \le 1$ we have $|C_n(w)| \le 1$ since $C_n(w)$ is the cosine of a real number in that case. Outside this range, on $|w| > 1$, $C_n(w)$ has no more zeros and thus is a monotonically increasing (or decreasing in the case of negative w) function [1].

The 3 dB frequency is

$$w_{3dB} = \cosh\left(\frac{1}{n}\cosh^{-1}\frac{1}{\epsilon}\right) \tag{3.40}$$

we note that for exactly $w_c = w_{3dB}$ if ϵ=.99763 in which case we have the Chebyshev filter with 3 dB ripple width.

Chebyshev bandpass transfer function is obtained from normalized CS-LP 2nd order function of s by using the transformation of Eq. 3.27. From Table [9] 2nd order CS-LP transfer function with 3 dB ripple is given as:

$$H_{CS-LP}(s) = \frac{1}{s^2 + .645s + .708} \tag{3.41}$$

using transformations of Eq. 3.27

$$= \frac{1}{\left(\frac{s^2+w_0^2}{\frac{w_0}{Q}}.s\right)^2 + .645\frac{s^2+w_0^2}{\frac{w_0}{Q}}.s + .708}$$

$$= \frac{1}{\frac{s^4Q^2 + 2s^2w_0^2Q^2 + w_0^4Q^2}{w_0^2s^2} + \frac{.645(s_2 + w_0^2)Q}{w_0.s} + .708}$$

$$= \frac{w_0^2s^2}{s^4Q^2 + 2s^2w_0^2Q^2 + w_0^4Q^2 + .645s^3w_0Q + .645sw_0^3Q + .708s^2w_0^2} \tag{3.42}$$

where

$$w_0 = \frac{G}{C}$$

$$Q = \frac{C}{C_3}$$

multiplying the denominator of Eq. 3.42 by its complex conjugate we obtain the amplitude response (in dB) of 4th order CS-BP filter.

$$|H_{CS-BP}(jw)| = 20\log_{10}(w^2w_0^2) - 20\log_{10}\sqrt{D} \tag{3.43}$$

where

$$D = \sqrt{(M_2^2 - N_2^2)}\,|_{s=jw}$$

$$M_2 = s^4 + 2s^2 w_0^2 Q^2 + w_0^4 Q^2 + .708 s^2 w_0^2$$
$$N_2 = .645 s^3 w_0 Q + .645 s w_0^3 Q$$

The CS-BP amplitude response is plotted in Fig. 3.19 for various Q's. The responses show passband ripples (varies smooth to sharp as Q increases). In Chebyshev response the Q programmability method can be applied to obtain programmable ripple in the passband. For example, to obtain a 4th order (2 ripples) CS response, two second order responses with appropriate Q should be cascaded using programmable method as described earlier which will give a 4th order response with two ripples. Programmable DAC's are used to shift the center frequency as desired. In BP filter the center and cutoff frequencies are related by

$$w_0 = \sqrt{w_u w_l}$$

where

$$w_l = w_0(-\frac{1}{2}Q + \sqrt{1 + \frac{1}{4}Q^2})$$
$$w_u = w_0(\frac{1}{2}Q + \sqrt{1 + \frac{1}{4}Q^2})$$

The CS-BP amplitude responses are plotted for 3 dB ripple.

3.8.2 CS-LP & CS-BP Approximations of Group Delay Responses

Derivation of 4th order group delay response is done as follows: From Table [9], the denominator with 3 dB ripple is given as

$$(s^2 + .411s + .196)(s^2 + .170s + .903)$$
$$= s^4 + .581s^3 + 1.16887s^2 + .404453s + .176988$$

so the group delay response of CS-LP filter is

$$T_{CS\text{-}LP}(jw) = \frac{.58w^6 - .53424553w^4 + .16426289w^2 + .071583328}{w^8 - 2w^6 + 1.25025869w^4 - .256169636w^2 + .031324752} \quad (3.44)$$

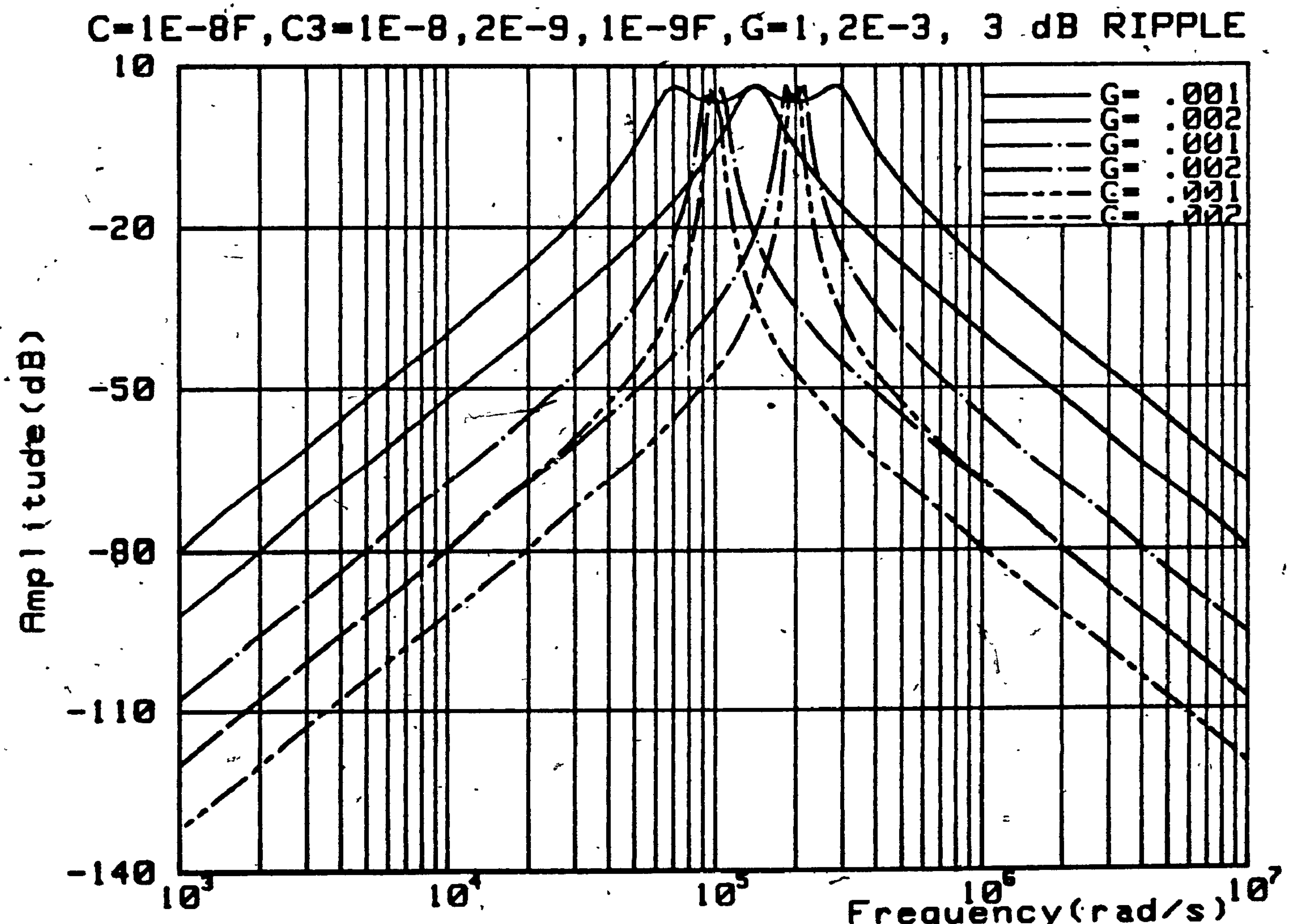

FIG. 3.19 AMPLITUDE(CS) RESP. OF 4TH ORDER BPF,Q=1,5,10

similarly, the group delay response of CS-BP filter is derived from Eq. 3.42 and found to be

$$T_{CS\text{-}BP}(jw) = \frac{N}{D} \tag{3.45}$$

where

$$N = .645w^6w_0Q^3 - .645w^4w^0Q^3 + .456655w^4w_0^3Q$$
$$- .645w^2w_0^5Q^3 + .456665w^2w_0^5Q + .645w_0^7Q^3$$

and

$$D = w^8Q^4 - 4w^6w_0^2Q^4 - w^6w_0^2Q^2 + 6w^4w_0^4Q^4$$
$$+ 2w^4w_0^4Q^2 - 4w^2w_0^6Q^4 + .501264w^4w_0^4 - w^2w_0^6Q^2 + w_0^8Q^4$$

The responses for both Eqs. 3.44 & 3.45 are plotted in Fig. 3.20, 21 for various Q's. The response shows 2 ripples as expected. As the order goes higher, the distortion of the signal is peaked more.

In this chapter a structure of a digitally programmable analog filter is presented and throughly analyzed. At first, a 2nd order circuit diagram is developed based on the proposed principle. Amplitude and the group delay responses of this circuit were analyzed with emphasis on the programmability of different parameters. The various filter parameters are programmed digitally and independently as was the objective of the proposed approach.

Secondly, higher order filter realization was considered. The cascading method was employed to realize higher order filter structure and again the programmability concept was applied to verify the proposed approach. The results were as expected. Finally, the approximation methods (Butterworth & Chebyshev) were applied to verify the programmability approach. In approximation analyses, a 4th order filter structure was considered. Responses for 4th order filter were better than 2nd order responses in many aspects. The 4th-order filter

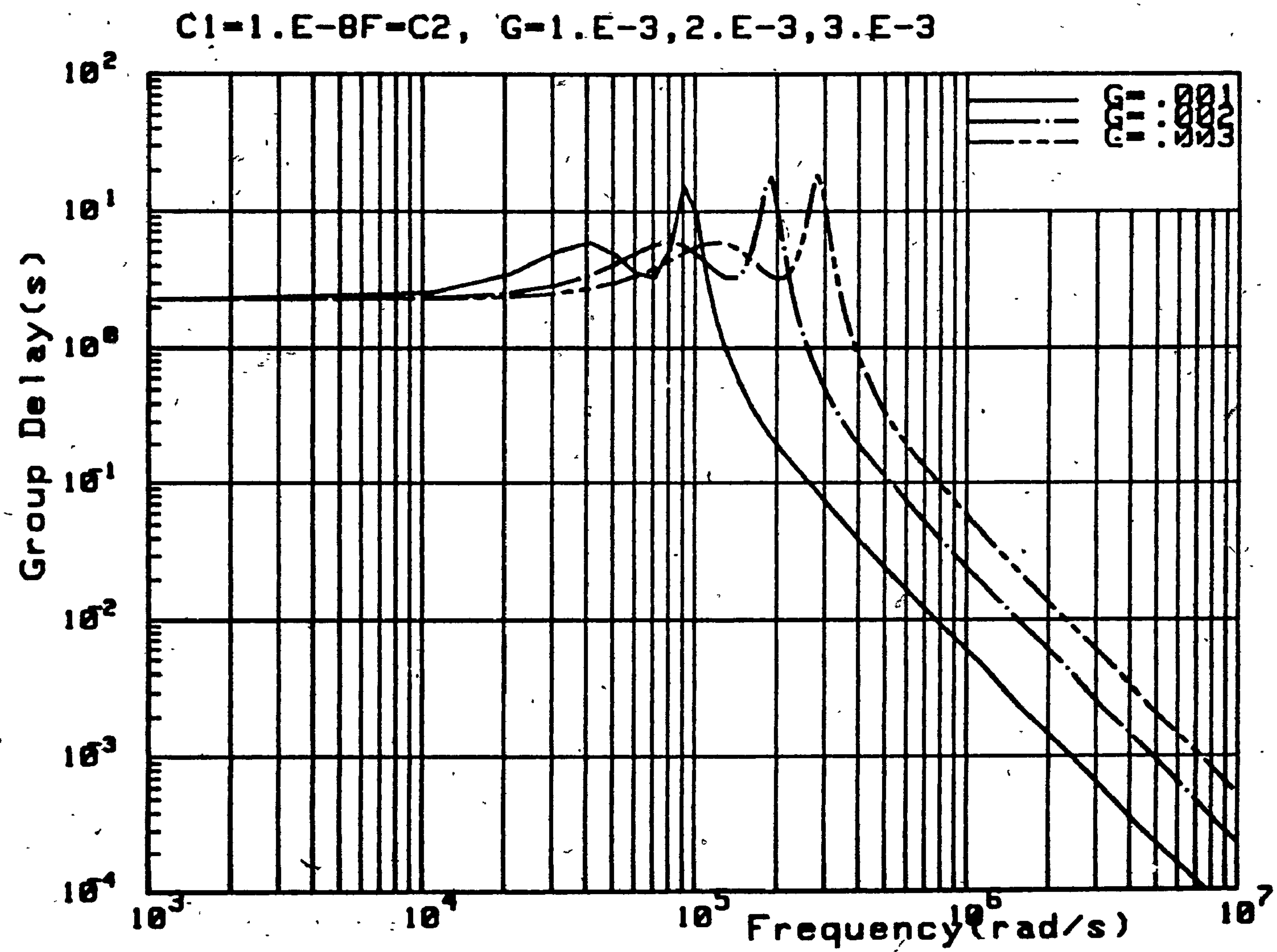

FIG. 3.20 GROUP DELAY RESP(CS) OF 4TH ORD. LP FILTER

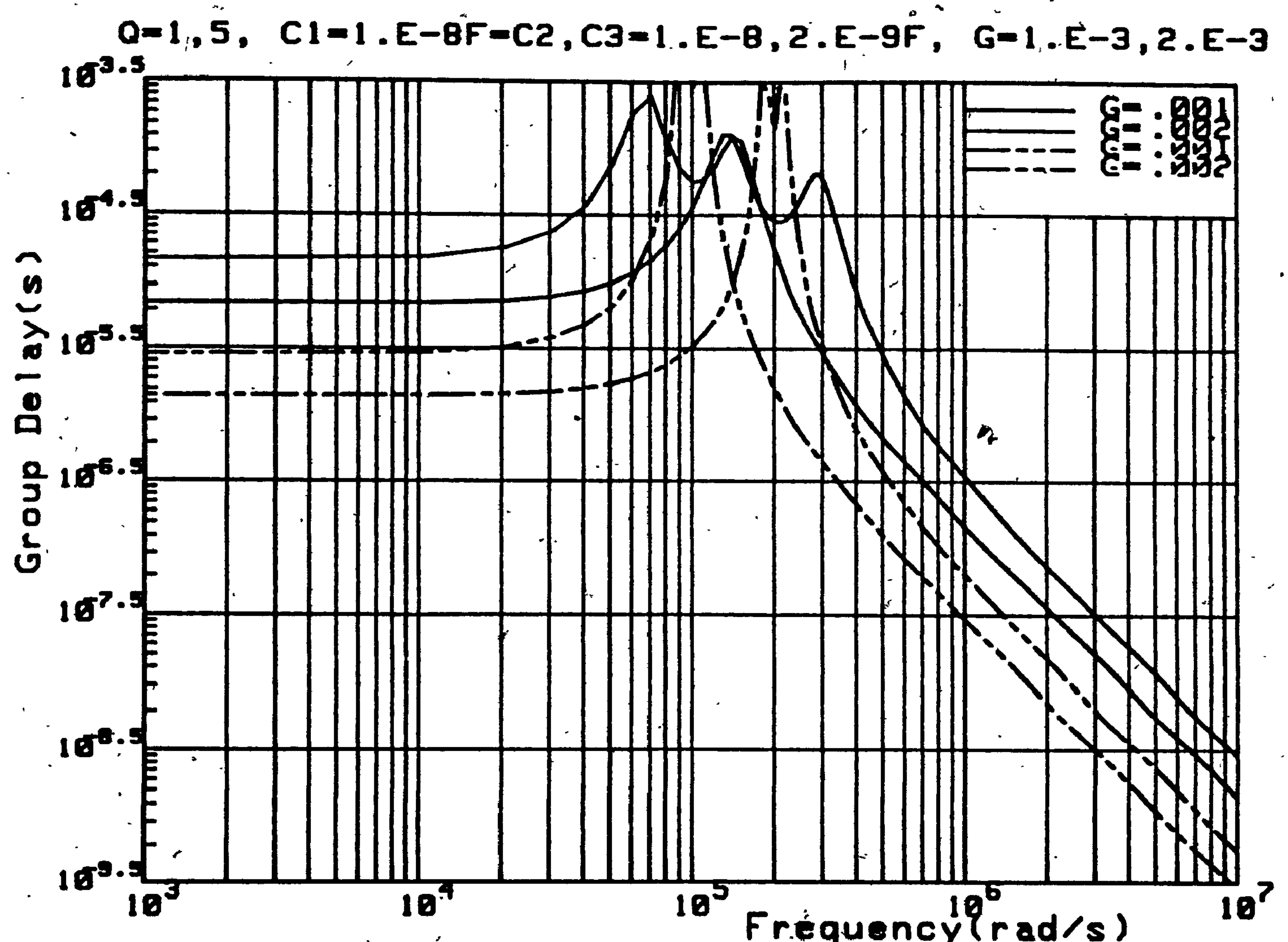

FIG. 3.21 GROUP DELAY RESP(CS) OF 4TH ORD. BP FILTER

have a sharper response. The concept of digitally programming the center frequency and other parameters were verified for both BW and CS responses.

In this chapter the digitally programmable filter principle is examined assuming ideal conditions and ideal components. In the next chapter, the non-ideality of these components is examined and a second order structure of digitally programmable filter is implemented and its results are obtained and compared with the theoretical results. The practical limitations of the programmable parameters are investigated as well.

CHAPTER 4

NON-IDEAL EFFECTS OF OP-AMPS AND DAC'S IN PROGRAMMABLE FILTER CHARACTERISTICS

In the preceding chapter we have introduced a concept of digitally programmable analog filter realized by current-type DAC's and OP-AMPs. It was assumed that the OP-AMPs and DAC's were ideal. In practical realizations, such a network will deviate from the ideal responses mainly due to finite and frequency dependent gain of the OP-AMP, due to input impedance, offset voltage and currents. Similarly, DAC's introduce various undesirable parameters such as non-linearity, finite resolution and speed. Still other effects are introduced by the feedthrough capacitors of the switches in the DAC. In this chapter, first, the aim is to study some of the effects of these non-ideal characteristics of these *real* components, especially on the performance, limitation, design guides for the proposed programmable filter structure. Secondly, a hardware implementation of the proposed filter configuration on a second order programmable filter is realized to verify the proposed concept and the analytical results obtained in the last chapter.

4.1 The Non-ideal OP-AMP

In analyzing the filter characteristics in the previous chapter we have assumed ideal condition for OP-AMP. In practice, the influence of finite and frequency dependent gain of the OP-AMP is well known and has to be taken into consideration. The typical one pole model of a finite gain OP-AMP is represented by [3]

$$A(s) = \frac{A_0 w_p}{s + w_p} \qquad (4.1)$$

where A_0 denotes the dc gain, i.e., the gain at s=0 and w_p is the 3 dB frequency of the finite gain OP-AMP. The above model will be substituted for finite gain OP-AMP to derive new transfer function in the next section.

4.2 The Transfer Functions with Finite Gain OP-AMPs

In chapter three we obtained transfer function of the proposed filter configuration with ideal parameters. In this section, we proceed to derive new transfer function of Fig. 4.1 using a model for finite OP-AMP and see how the *real* OP-AMP affects the programmability of the various filter coefficients.

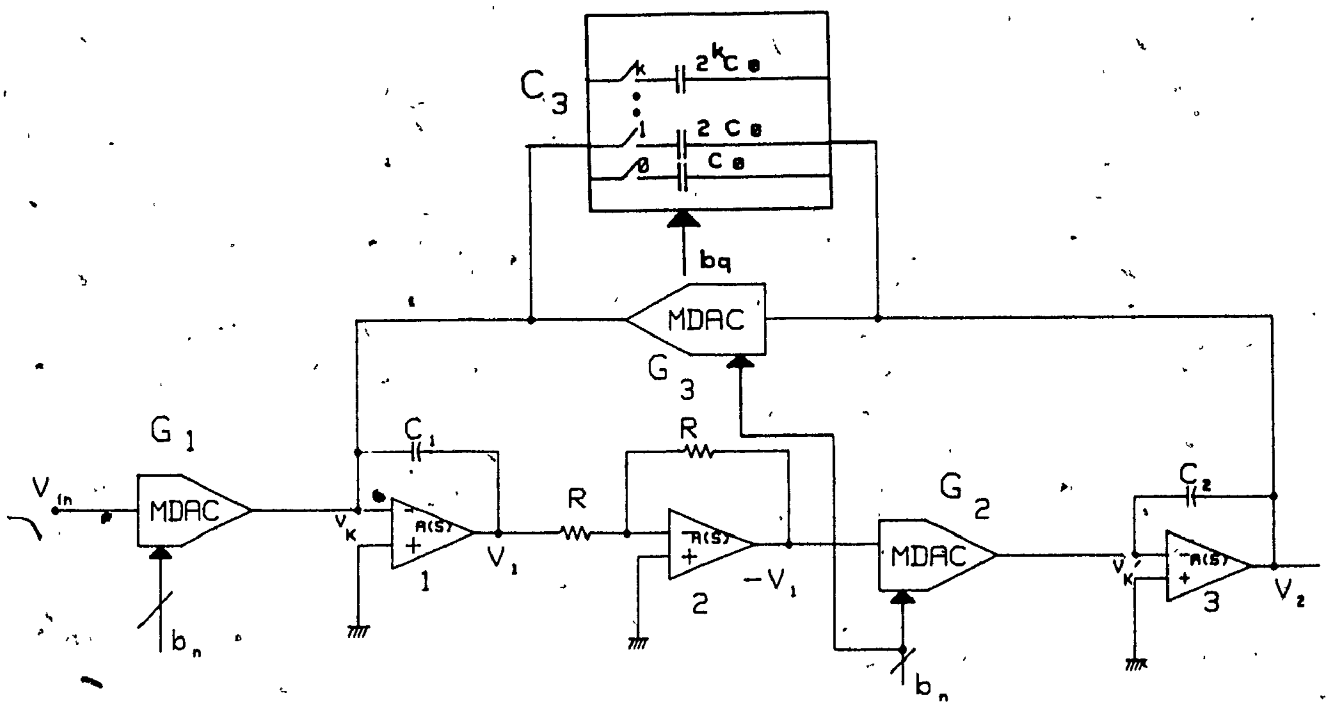

Fig. 4.1. Second order programmable filter with non-ideal OP-AMPs.

Using Eq. 4.1 for the finite gain OP-AMP model and the Fig. 4.2,

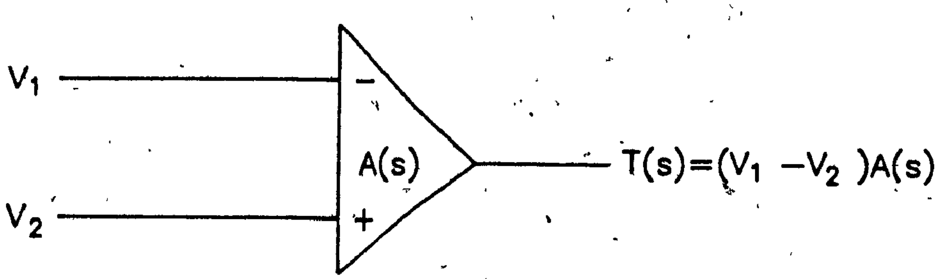

Fig. 4.2. Finite Gain OP-AMP.

we synthesize new transfer function for the second order programmable filter of Fig. 4.1.

$$V_k (G + G_1 + sC_1 + sC_3) = V_{in} G_1 a + V_1 sC_1 + V_2(G + sC_3) \quad (4.2)$$

$$V'_k (G + sC_2) = V_1 G + V_2 sC_2 \quad (4.3)$$

where $V_k = -\frac{1}{A(s)}(V_1 + V_2)$

$$V'_k = -\frac{1}{A(s)} V_2$$

$$A(s) = A_0 \frac{w_p}{s + w_p}$$

$$G = g_m \sum_{n=0}^{k} 2^n b_n$$

and $G_1 = g_{m1} \sum_{n=0}^{k} 2^n b_n$

from Eq. 4.3

$$V_1 = \frac{V_2\left(G + sC_2 + A(s)sC_2\right)}{A(s)G} \quad (4.4)$$

$$V_2 = \frac{V_1 A(s)G}{G + sC_2 + A(s)sC_2} \quad (4.5)$$

substituting Eqs. 4.4, 4.5 in Eq. 4.2 we obtain expressions for BP and LP transfer functions.

The new BP transfer function,

$$H_{BP}(s) = \frac{G_1 A_0 w_p (s^2 C_2 + sC_2 w_p + sG + sC_2 A_0 w_p + G w_p)}{M_2 + N_2} \tag{4.6}$$

The LP transfer function,

$$H_{LP}(s) = \frac{A_0^2 w_p^2 G G_1}{M_2 + N_2} \tag{4.7}$$

where

$$M_2 = s^4(C_1 C_2 + C_2 C_3 + s^2(G + GG_1 + 2C_1 G w_p + 2C_3 G w_p + 2C_2 G w_p$$
$$+ 2C_2 G_1 w_p + C_1 C_2 w_p^2 + C_2 C_3 w_p^2 + C_2 G A_0 w_p + C_2 G_1 A_0 w_p + 2C_1 C_2 A_0 w_p^2$$
$$+ C_2 C_3 A_0 w_p^2 + C_1 G a_0 w_p + C_3 G A_0 w_p + C_1 G A_0 w_p + C_1 C_2 A_0^2 w_p^2)$$
$$+ G^2 w_p^2 + G G_1 w_p^2 + G^2 A_0 w_p^2 + G G_1 A_0 w_p^2 + G_2 A_0^2 w_p^2$$

and

$$N_2 = s^3(C_1 G + C_3 G + C_2 G + C_2 G_1 + 2C_1 C_2 w_p + 2C_2 C_3 w_p$$
$$+ 2C_1 C_2 A_0 w_p + C_2 C_3 A_0 w_p) + s(2G^2 w_p + 2GG_1 w_p + C_1 G w_p^2 + C_3 G w_p^2$$
$$+ C_2 G w_p^2 + C_2 G_1 w_p^2 + C_2 G A_0 w_p^2 + C_2 G_1 A_0 w_p^2 + G^2 A_0 w_p$$
$$+ C_1 G A_0 w_p^2 + C_3 G A_0 w_p^2 + C_1 G A_0 w_p^2 + C_3 G A_0^2 w_p^2 + G G_1 A_0 w_p)$$

The amplitude responses for the above transfer functions were plotted using logarithmic function in Fig. 4.3a, 3b, 4a, 4b for center frequency of $w_0 = 10$, 100 K rad/s. The numerical values for A_0, w_p were taken from data sheet of LF 356B, the wide bandwidth OP-AMP used in our experiment. This particular OP-AMP is a monolithic JFET input operational amplifier which incorporates well matched, high voltage JFETs on the same chip with standard bipolar transistor[11]. This OP-AMP is designed for high slew rate (12 $\frac{v}{\mu s}$) and low voltage and current noise. $A_0 = 106$ dB and $w_p = 2\pi * 40$ rad/s [17].

We notice in the plots that no significant changes took place between ideal and non-ideal responses for lower frequency ($w_0 = 10\ Krad/s$). For higher frequency the response vary as observed in the plots. For low frequency the finite

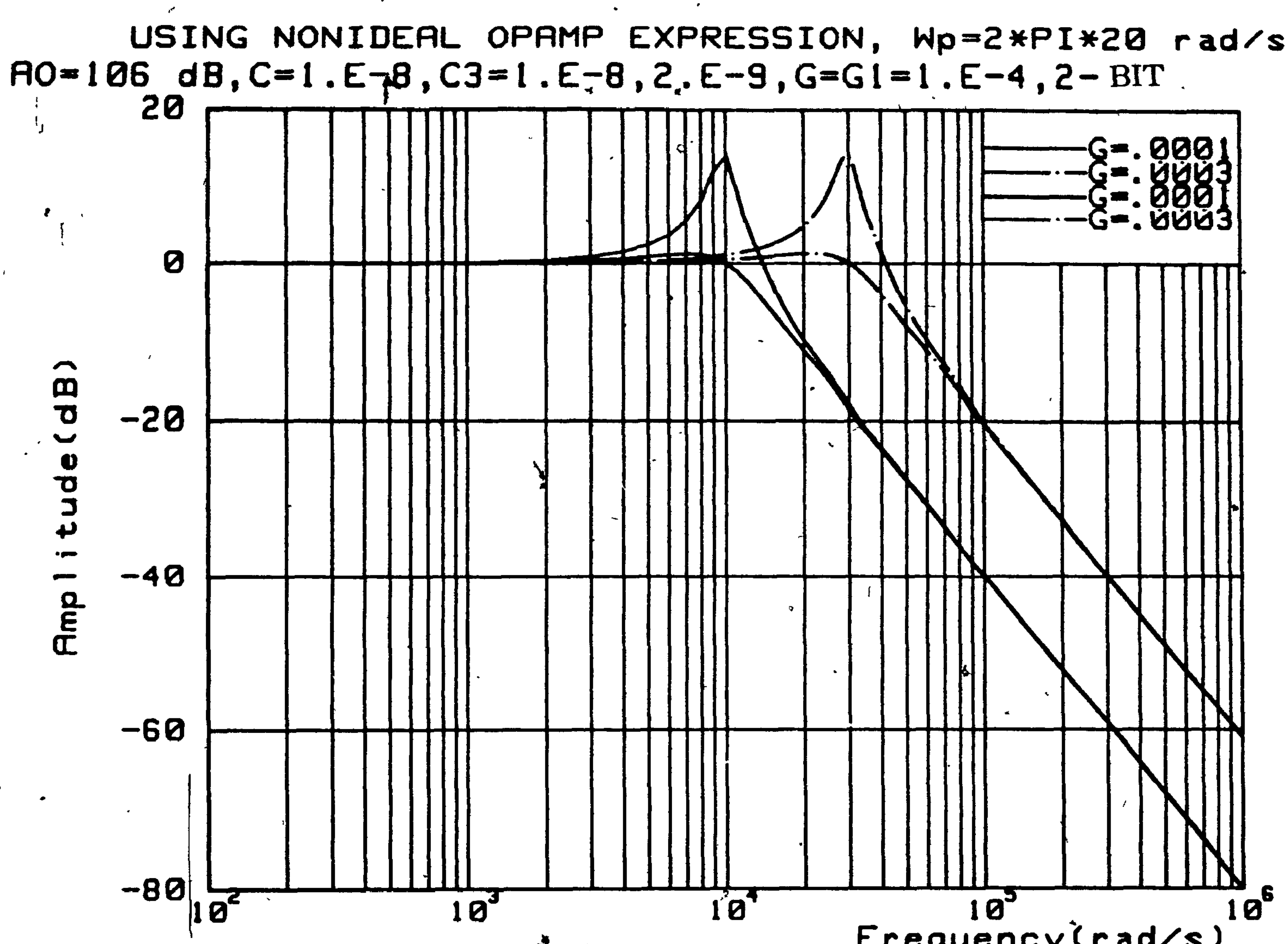

FIG. 4.3a AMPLITUDE RESPONSES OF 2ND ORDER LPF, Q=1,5

USING NONIDEAL OPAMP EXPRESSION, Wp=2*PI*20 rad/s
AO=106 dB,C=1.E-8,C3=1.E-8,2.E-9,G=G1=1.E-4,2-BIT

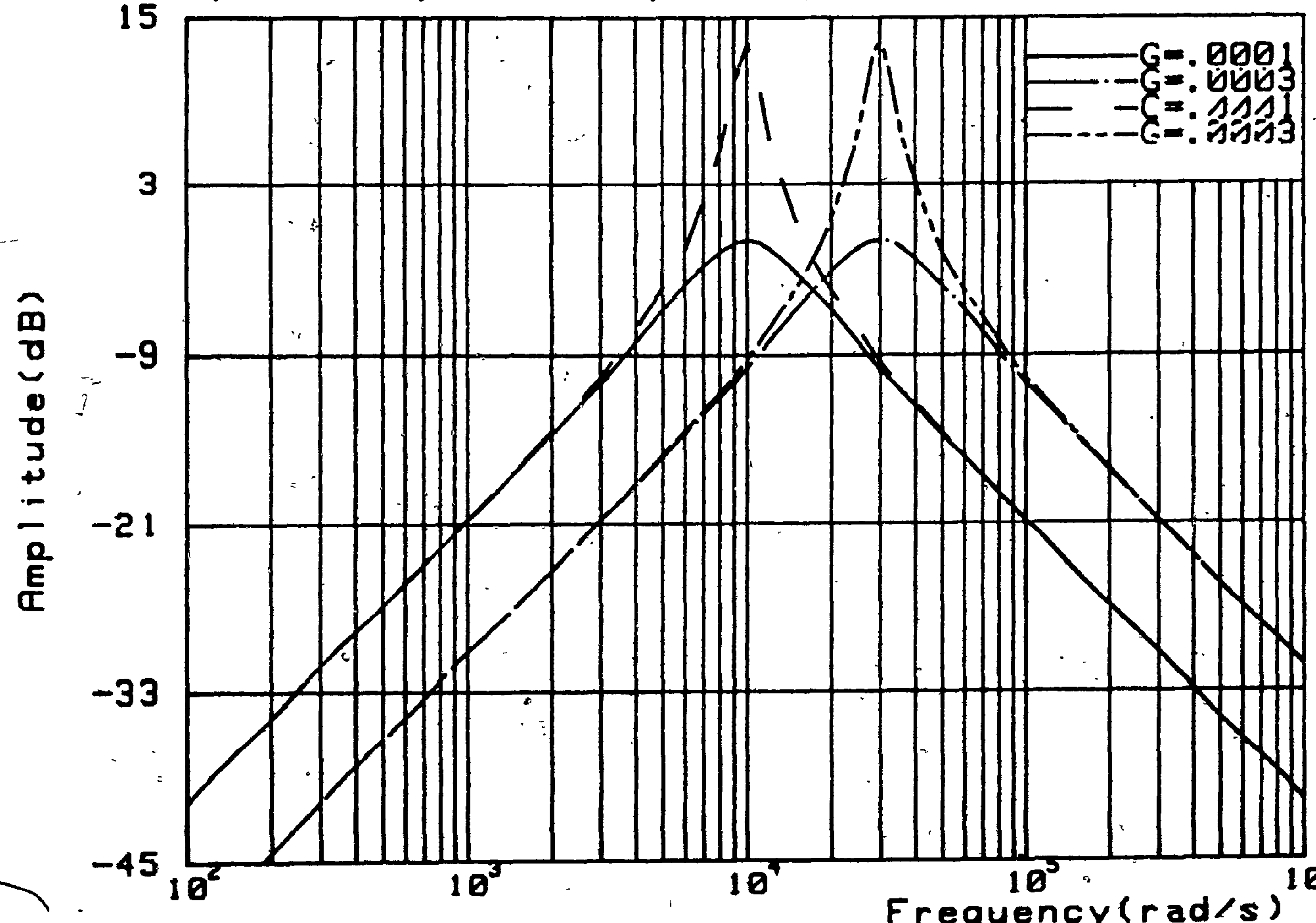

FIG. 4.3b AMPLITUDE RESPONSES OF 2ND ORDER BPF, Q=1,5

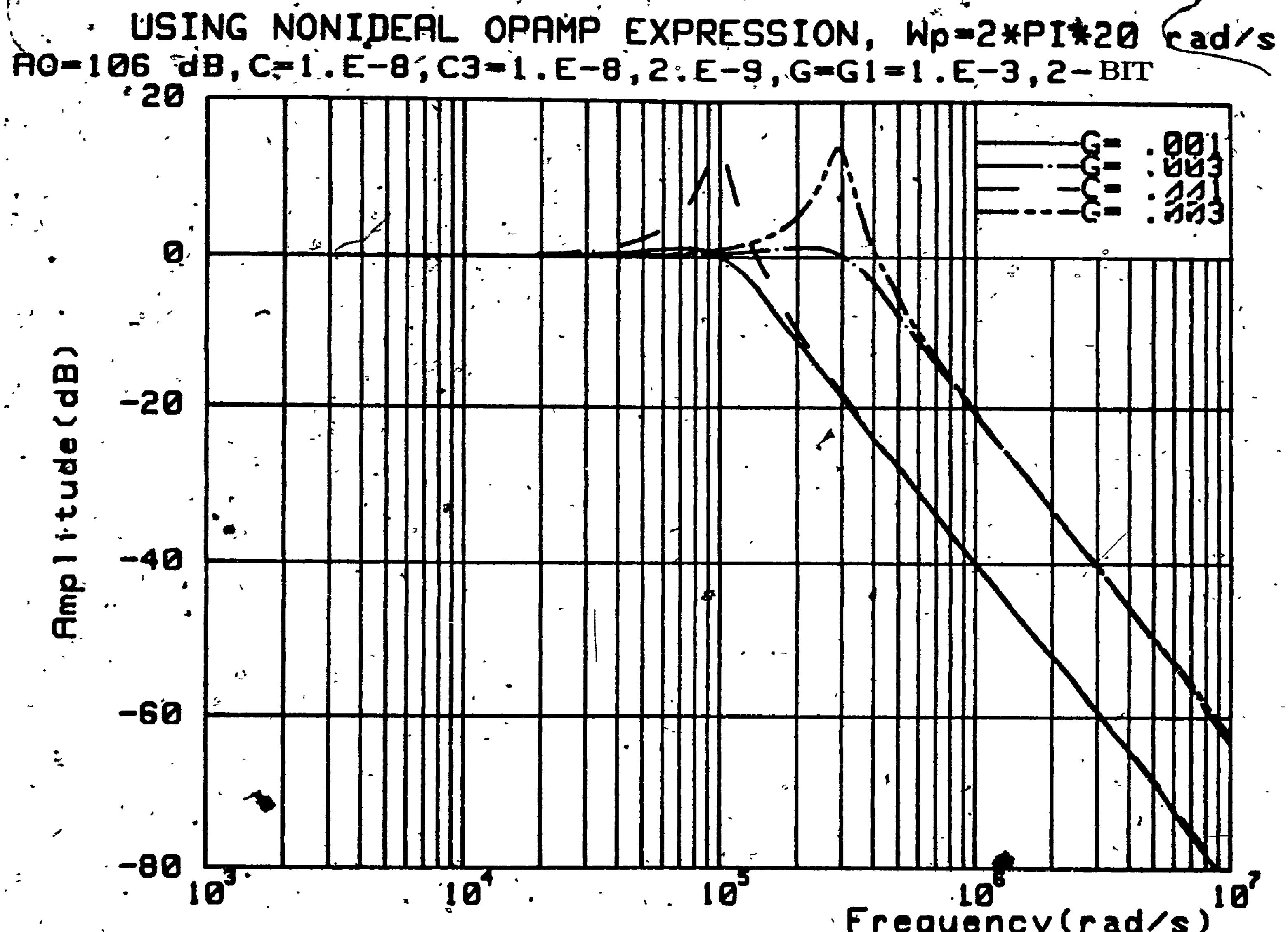

FIG. 4.4a AMPLITUDE RESPONSES OF 2ND ORDER LPF, Q=1,5

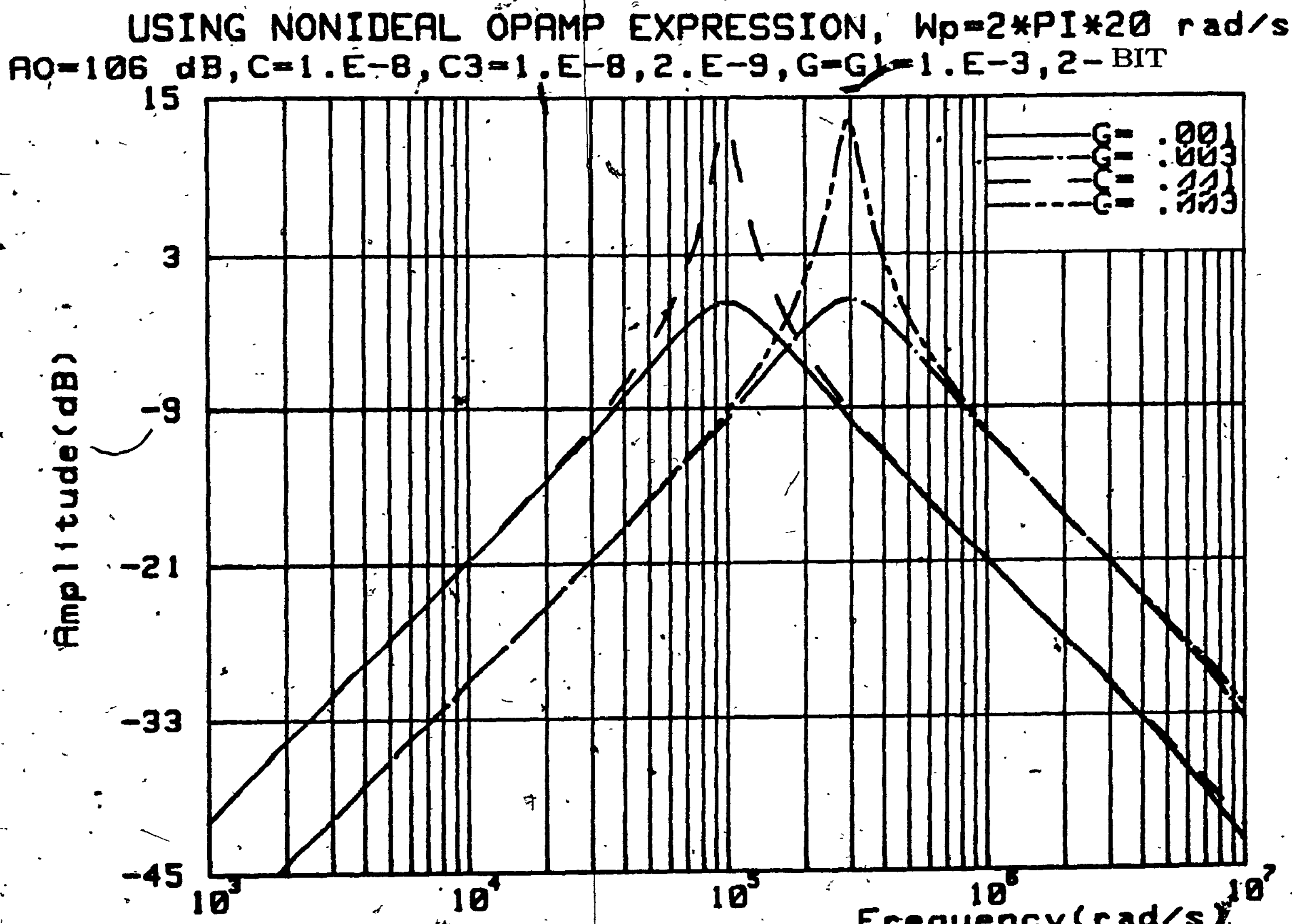

FIG. 4.4b AMPLITUDE RESPONSES OF 2ND ORDER BPF, Q=1,5

gain (106 dB) OP-AMP behaves almost like an ideal OP-AMP. For higher frequencies (in the 100 k range) OP-AMP starts deviating from its ideal characteristics and provide undesirable responses. Evidently, this deviation is due to finite gain bandwidth product of the OP-AMP. The deviation also occurs due to dc errors. There are two sources of such error, namely offset current and offset voltage. The offset current is defined as the difference between two bias currents modeled as I_{B1} and I_{B2} and shown in Fig. 4.4C.

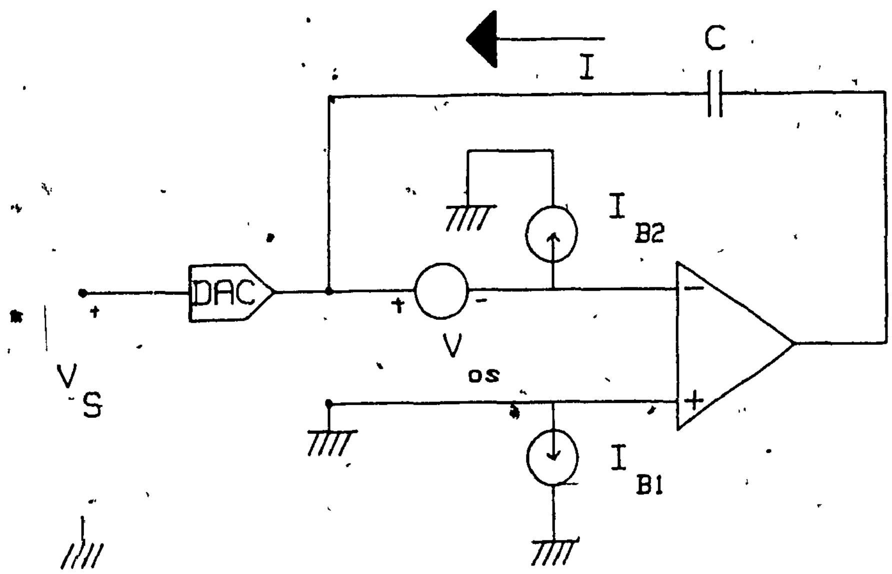

Fig. 4.4C Programmable integrator model with dc errors

Mathematically,

$$I_{os} = | I_{B1} - I_{B2} |$$

where I_{os} = offset current

I_{B1} = bias current in non-inverting terminal

I_{B2} = bias current in inverting terminal

the offset current is typically 5 to 10 percent of the bias current. Bias currents are small, typically in the 100 nA range. Bias currents are even smaller in FET-input OP-AMP (30 pA in LF 356).

The second source of dc error in the OP-AMP characteristics is the offset voltage. When an OP-AMP is operated with no differential input, a voltage will appear at the output even though the input voltage is zero. This effect is modeled by the offset voltage (V_{os}) as shown in Fig. 4.4C. The offset voltage accounts for any transconductance variation of the OP-AMP inputs. The combined effects of these two dc errors on the programmable integrator (Fig. 4.4C) is derived below to see their effects on the programmable filter [25,26].

Irrespective of the input signal there will be a dc current through the feedback capacitor given by

$$I = (V_s - V_{os})G - I_B$$

after a time of integration t has elapsed, the following voltage appears

$$V_0 = V_{os} - \frac{1}{C}\int_0^t I dt$$

$$= V_{os} - \frac{1}{C}\int_0^t ((V_s - V_{os})G - I_B)dt$$

$$= \underbrace{V_{os} + \frac{tG}{C}(V_{os} + \frac{I_B}{G})}_{output\ offset} - \underbrace{\frac{G}{C}\int_0^t V_s\, dt}_{output\ signal} \quad (4.8)$$

The additive error of integration is chiefly given by the second component of the output offset term which increases linearly with time from zero to a maximum value. However, the error contributed by the dc components in the proposed programmable integrator is negligible (.1%). This can be easily seen by doing a sample calculation using Eq. 4.8 (values are taken from LF 356 data sheet where $V_{os} = 1mV$, $I_B = 30pA$). Furthermore, to reduce the output dc voltage due to the bias currents in the proposed configuration, a resistor is connected with the noninverting terminal of the OP-AMP. In this section, we examined the main sources of error in the OP-AMP and their effects on the programmable filter. These effects are negligible in the low (audio) frequency range. The deviation becomes significant if the frequency increases to higher levels.

4.3 The Non-Ideal DAC's

This section deals with the non-ideal parameters of DAC's. The DAC used to construct the programmable integrator is a current-output type DAC. The current output of this DAC is integrated directly using a feedback capacitor across the OP-AMP to obtain the programmable integrator model. The DAC of this type usually consist of weighted resistance values, each controlled by a particular level of digital input data with varying output currents or voltages in accordance with the digital input code. DAC's, made of FET or conventional transistor are also available for this type of application. However, they are not very useful in this particular application since they require a bias resistor. This bias resistor eventually appears in parallel with the integrating capacitor of the integrator. The net result is a lossy integrator which is undesirable in this application.

To overcome these problems, a R-2R resistive ladder DAC is chosen for this design. The DAC used in our hardware experiment (DAC 1020) is a 10-bit current-type multiplying DAC. A deposited thin film R-2R resistor ladder divides the reference current and provides the circuit with excellent characteristics. One of the advantages of the R-2R ladder is that the impedance as seen from the input to the OP-AMP is constant. Hence bandwidth and resolution do not change with digital setting. Another important fact is that all the resistors are either R or 2R and the accuracy is not dependent upon the absolute value of all R's, but rather only their differences. Similarly, temperature effects are only significant with respect to how well all the R and 2R resistors track each other, respectively. The circuit uses CMOS current switches and drive circuitry to achieve low power consumption (30 mW max) and low output leakage currents (200 nA max). The digital inputs are compatible with DTL/TTL logic levels as well as CMOS logic level swings [20]. This part, combined with an external amplifier and voltage reference is very attractive for multiplying applications since its linearity error is essentially independent of the voltage reference. Fig. 4.5 shows the schematic of the DAC where the binary weighted current is given by

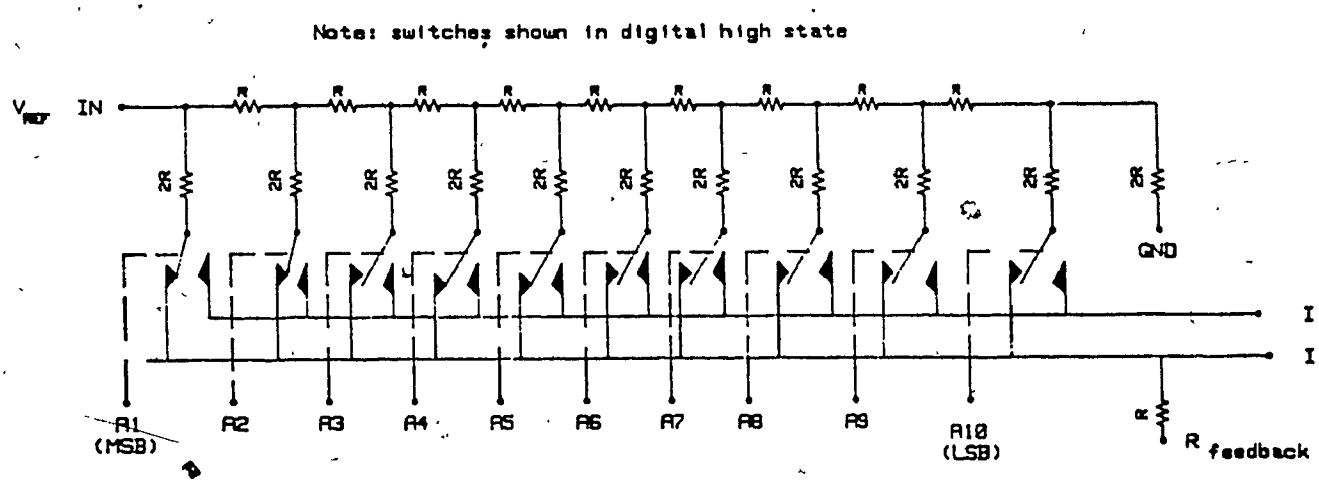

Fig. 4.5. Equivalent circuit of 10 bit DAC 1020 [11].

$$I_0 = KI_{FS}\left[b_1 2^{-1} + b_2 2^{-2} + \cdots + b_n 2^{-n}\right]$$

where

I_0 = binary weighted output current

K = gain

I_{FS} = Full-Scale output current

$b_1, b_2, \ldots, b_n$ = n-bit input word

The key points discussed in the next few sections are the non-ideal parameters of the DAC and their influence to the proposed programmable filter configuration.

4.3.1 Resolution of DAC's

The term resolution as applied to a DAC refers exclusively to the number of bits or the corresponding number of discrete output levels. The resolution of a DAC affects the step size of the programming range in this digitally programmable filter structure. The resolution of a DAC is determined by the width of the input word and by the full-scale output current or voltage of the DAC [4]:

$$DAC\ resolution\ in\ volts = \frac{V_{FS}}{2^n} \tag{4.9}$$

A 10-bit converter with a full-scale voltage of 5-volts has a resolution of 4.88 mv, for example. However, resolution can be stated in different ways. A 10-bit DAC may be said to have 10-bit resolution, a resolution of .097 percent of full-scale or a resolution of 1 part in 1024. DAC's are available with resolutions ranging from as few as 6 bits to 18 or 20 bits. Resolutions of 8, 10, 12 and 14 bits are quite common and economical. Above 14 bits, DAC's become more expensive and greater care must be taken to obtain their full precision [4,22].

4.3.2 The Effects of Non-linear DAC's

Fig. 4.6 shows the graph for an ideal 3-bit DAC where analog output is a linear function of digital input.

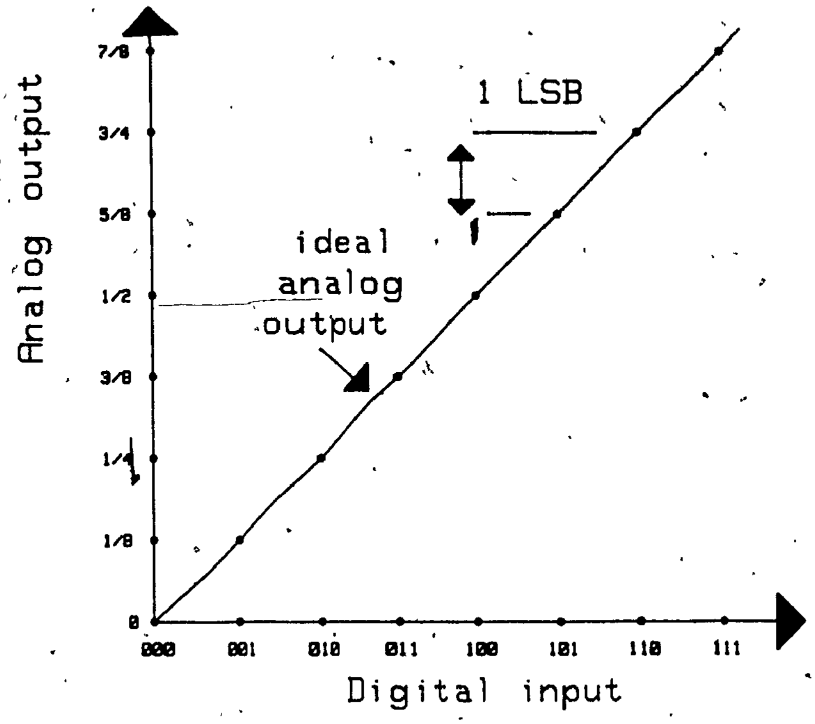

Fig. 4.6. Ideal 3-bit DAC relationship [22].

However, in reality, the performance of the DAC will change due to temperature, age and variations in supply voltage. The gain and offset voltage of a converter drift with time and temperature, as do its integral and differential linearity errors.

Linearity errors, and- more important- their variation with temperature are affected by variations of resistance in both the resistors and switches of the DAC [4]. The errors produced by the analog components used to build a DAC can be mainly attributed to errors caused by bit switches and resistance tolerances. The purpose of this section is to illustrate the important converter limitation (non-linearity) and its effects on the frequency responses of the programmable filter configuration.

The linearity error in a DAC is usually specified as a fraction of an LSB or as a percentage of full-scale voltage or current. Typically, this value is $\pm 1/2$ LSB (DAC 1020). Consequently, an n-bit DAC will have a resolution of the order of

$\frac{1}{2^n}$. Since the proposed technique provides a binary weighted conductance as

$$G = g_m \sum_{n=0}^{k} 2^n b_n$$

We can conclude that the effective transconductance due to a non-ideal DAC will be

$$g'_m = g_m (1+\Delta g_m) \qquad (4.10)$$

where $\Delta g_m = \pm \frac{g_m}{2*2^n}$

We replace g_m by Eq. 4.10 in the appropriate BP and LP transfer function and plot them to see the variation in the amplitude response. Using different n (bit size), we observe how non-linearity varies.

For n = 4

$$w_0 = \frac{G}{C} = g'_m \sum_{n=0}^{k} \frac{2^n b_n}{C} = g_m (1 \pm \frac{1}{2*2^n}) \sum_{n=0}^{k} \frac{2^n b_n}{C}$$

using C = 10^{-8} F, $g_m = 10^{-4}$, highest bit and the positive value only, we get

$$w_0 = (1 \pm \frac{1}{2*2^4}) \frac{15*10^4}{10^{-8}} = 154687.5 rad/s$$

$$\% \ linearity \ error = \frac{w_0 \ linear - w_0 \ non-linear}{w_0 \ linear} * 100$$

$$\% linearityerror = \frac{15*10^4 - 154687.5}{15*10^4} * 100 = 3.125\%$$

similarly, n=6

$$w_0 = (1 \pm \frac{1}{2*2^6}) \frac{63*10^4}{10^{-8}} = 634921.875 rad/s$$

$$\% \ linearity \ error = \frac{63*10^4 - 634921.875}{63*10^4} * 100 = .781\%$$

for n=8

$$w_0 = (1 \pm \frac{1}{2*2^8})\frac{255*10^4}{10^{-8}} = 2554980.469 rad/s$$

$$\%\ linearity\ \ error = \frac{255*10^4 - 2554980.469}{255*10^4}*100 = .19\% = 0.2\%$$

for n = 10 (DAC 1020)

$$w_0 = (1 \pm \frac{1}{2*2^{10}})\frac{1023*10^4}{10^{-8}} = 10234995.11 rad/s$$

$$\%\ linearity\ \ error = \frac{1023*10^4 - 10234995.11}{1023*10^4}*100 = .05\%$$

for n = 12

$$w_0 = (1 \pm \frac{1}{2*2^{12}})\frac{4095*10^4}{10^{-8}} = 40954998.77 rad/s$$

$$\%\ linearity\ \ error = \frac{4095*10^4 - 40954998.77}{4095*10^4}*100 = .012\%$$

To see the numerical value in graphical form, we plot them in Fig. 4.7,8,9 for n=4,6,10 using linear and non-linear expressions of the DAC in the the transfer functions of Eq. 4.6,.4.7. The graphs acknowledge that center frequency does not shift in a linear fashion. This was due to the linearity error in DAC. The linearity error calculated above is same as specified in the data sheet. The linearity error goes lower as resolution of the DAC increases as evident in the above calculations and in the graphs. The error is very low for 10-bit DAC and upwards. It should be mentioned that the DAC above 12 bits are difficult to use because it is more expensive and greater care is needed to obtain full precision as mentioned earlier.

The shift in center frequency due to the non-linearity of the DAC can be corrected in the programmable filter. For a DAC with n (n=0,1,..,k) bits there will be 2^n various bit pattern generating 2^n different center frequency. Any

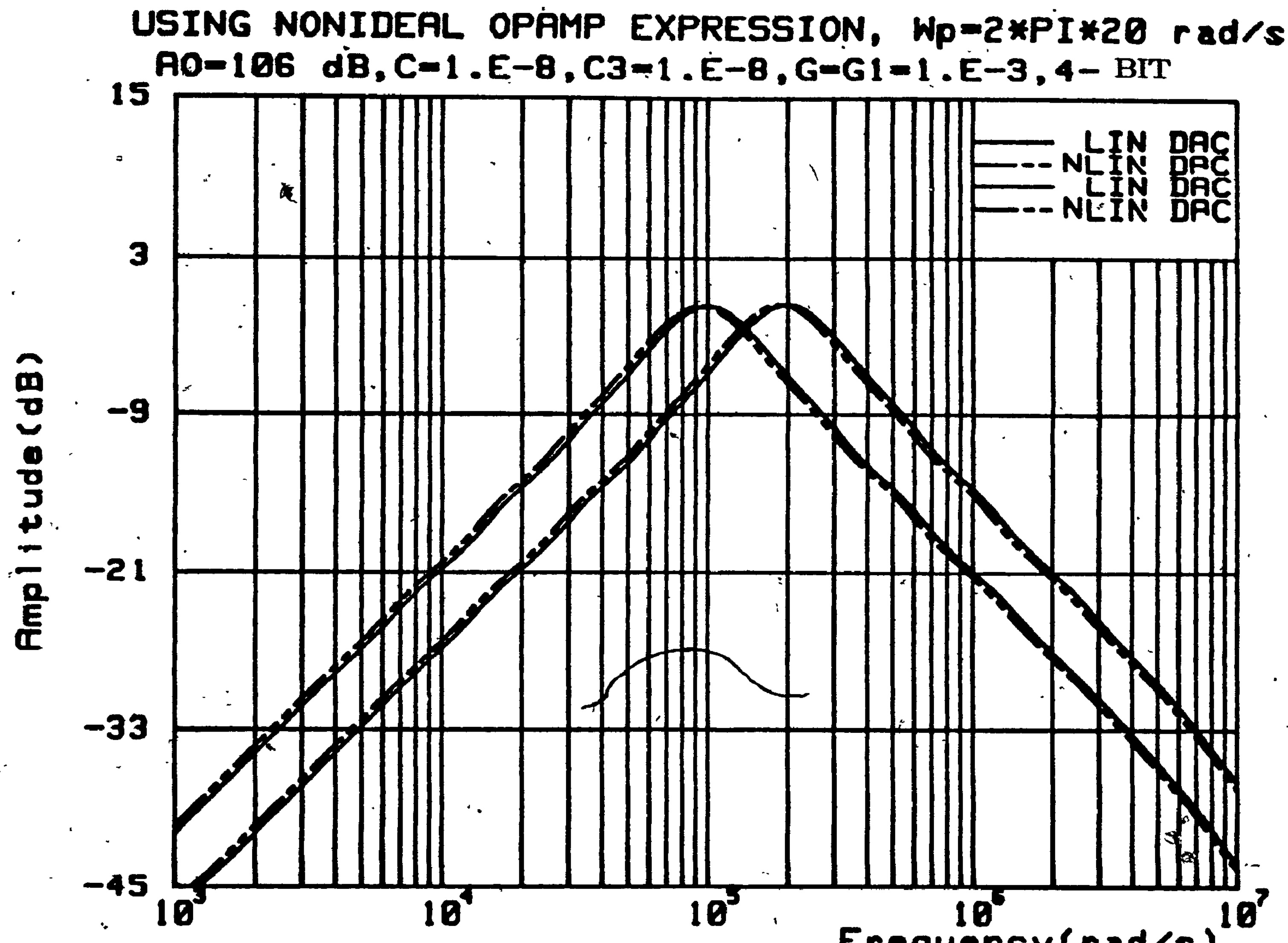

FIG. 4.7. AMPLITUDE RESPONSES OF 2ND ORDER BPF USING LINEAR & NONLINEAR DAC (n=4 bit), G'=G1'=9.375E-4*1,2

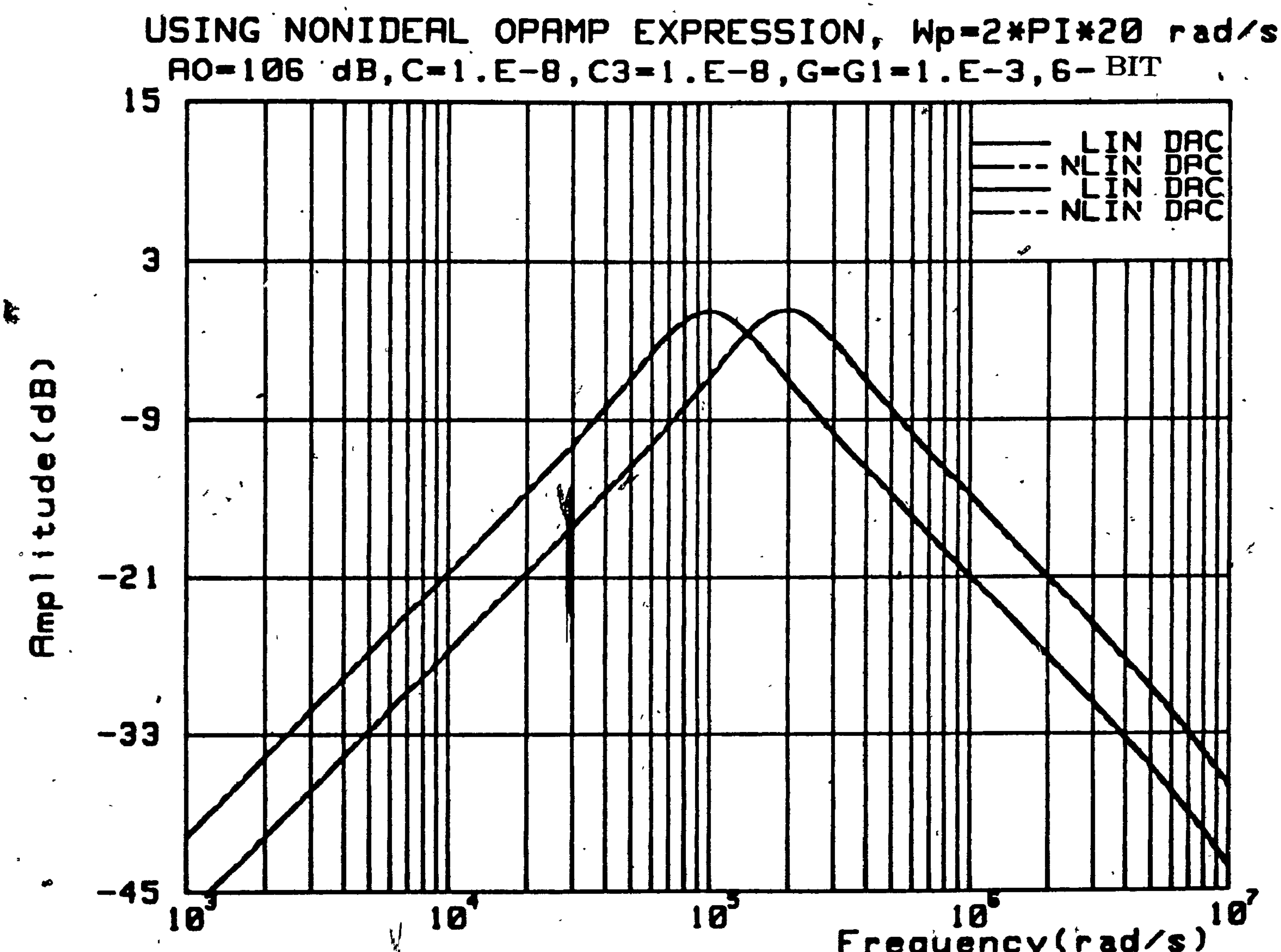

FIG. 4.8 AMPLITUDE RESPONSES OF 2ND ORDER LPF USING LINEAR & NONLINEAR DAC (n=6 bit),G′=G1′=9.8437E-4*1,2

USING NONIDEAL OPAMP EXPRESSION, Wp=2*PI*20 rad/s
AO=106 dB, C=1.E-8, C3=1.E-8, G=G1=1.E-3, 10-BIT

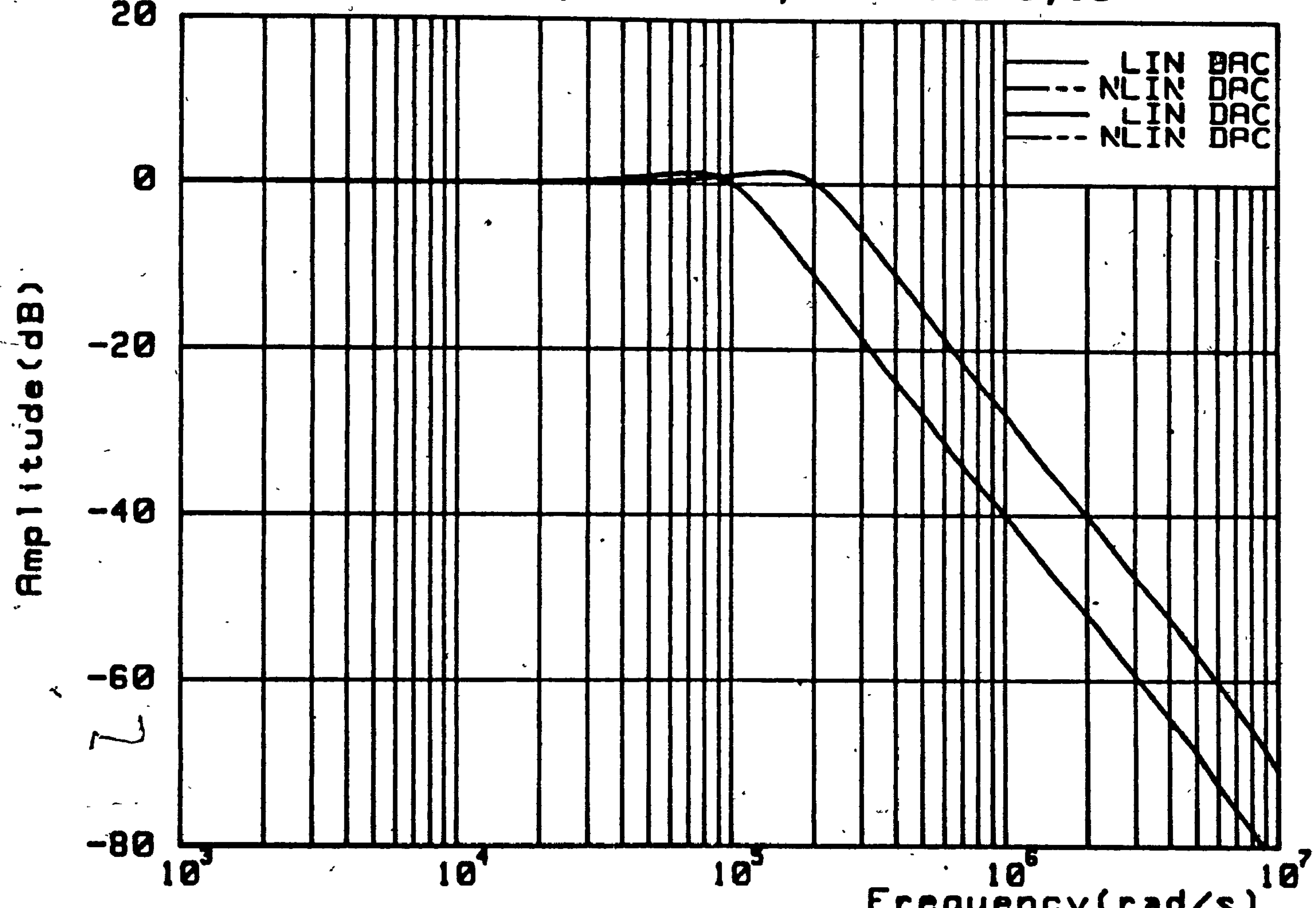

FIG. 4.9 AMPLITUDE RESPONSES OF 2ND ORDER LPF USING LINEAR & NONLINEAR DAC (n=10 bit), G'=G1'=9.9902E-4*1,2

non-linearity in the center frequency can be corrected by appropriately reprogramming the bit pattern of the DAC to take care of the error. There is an exception to this case. When highest bit pattern (all one's) is used to obtain a center frequency and if this cause a shift (error) in the center frequency, this can not be corrected using above mentioned method. This is because the bit pattern can only be changed in the -ve (smaller value) direction to correct the error, not in both direction as may be required.

The leakage currents associated with the OFF state resistors of the switches can be a problem if the circuit is operated at very low current levels (LSB current of $< 10\ \mu A$). This is due loss of charges from the integrating capacitor. In most designs (as in ours) where fairly high (mA) current levels is maintained, the leakage currents are negligible except at very high temperature. Yet another parameter, the OP-AMP bias current (I_B) may introduce error when it flows through the internal feedback resistor (15 k) of the DAC. However, it is found to be negligible ($I_B * R_f = 30pA * 15k = .45\mu$ V) for the given configuration.

4.4 Summary

An overview of DAC characteristics has been discussed with emphasis on its non-linear effects, specifically on the amplitude responses of the programmable lowpass and bandpass filter. We observed that the non-linearity of DAC decreases as resolution of DAC increases which was clear in the plots of the of the amplitude responses. We also observed that non-linearity of DAC in programming the center frequency or gain can be corrected by reprogramming the binary digits of the DAC. Both DAC and OP-AMP have non-ideal parameters that affect the performance of the digitally programmable filter. These non-ideal parameters prevent the filter from performing well at higher frequency range.