REALIZATION OF TWO-AMPLIFIER FILTERS HAVING ZERO GAIN POLE-FREQUENCY SENSITIVITY PRODUCTS AND MINIMIZED SUM OF GAIN Q SENSITIVITY PRODUCTS

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A THESIS

in

The Faculty

of

Engineering

Presented in Partial Fulfillment of the Requirements
for the Degree of Doctor of Engineering at
Concordia University
Montreal, Canada
September 77

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ABSTRACT

This thesis discusses the realizations of second order transfer functions by means of two amplifier-active RC networks having the property of zero GωSP (Gain pole frequency sensitivity products) and minimized sum of magnitudes of GQSPs (Gain Q sensitivity products). The realizations of the five commonly occurring types of filters namely Low Pass, High Pass, Band Pass, Null, and All Pass are studied. These filters should be implemented using hybrid IC technology.

The various possible denominator polynomial decompositions suitable to fulfill the condition of zero GωSP are first studied in detail. A two-amplifier configuration (though not unique) is proposed which will realize all the possible decompositions. The entire realization requires the use of generating transfer functions which have to be realized by using single amplifier circuits the output of each of which has to be taken from the output of the amplifier. The generating functions could be of three kinds namely zero order, first order, and second order. The realizations using zero order generating functions are not discussed further as the filters cannot be designed in a relatively simple manner. The other two cases

have been studied in detail. Representative circuits are given where F, the sum of the magnitudes of GQSPs is minimized in each case.

The case where the generating function is first order yields Low Pass, High Pass, and Band Pass only. The case when the generating function is of second order (referred to as Q-multiplication) yields, Band-Pass, Null and All Pass filters. The Q-multiplier circuits are particularly attractive since one can start from an optimized single amplifier filter and then optimize F further. A value of F less than 0.5 Q_P is obtainable while keeping the capacitive spread at a maximum value of three.

Three Null filters and two Band Pass filters have been built and tested. The experimental results conform closely to those from the theoretical studies.

This thesis is dedicated to
my wife Nana P.E. Gargour
and to my daughter Caroline.

TABLE OF CONTENTS

Page

Page

LIST OF FIGURES

N	Null
Null type i	Null with a denominator polynomial decomposition of type i
N(S)	Numerator of T_V
$n_0(S)$	(numerator of t_v)/K_0
OA	Operational Amplifier
Q_P	Designed pole Q of a TAC
Q_{P0}	Designed pole Q of a SAC
QMC	Q-multiplier circuit
RI, RII, RIII	Different types of generating circuits used in QMCs
S	Complex frequency variable
SAC	Single Amplifier Circuit
t_v	Voltage transfer function of a SAC, generating transfer function
T_V	Voltage transfer function of a TAC
TAC	Two-Amplifier Circuit
ω_P	Variable frequency in rad/sec
ω_{P0}	Designed pole frequency in Rad/sec of a SAC
ω_P	Designed pole frequency in Rad/sec or in Hz of a TAC
ω_C, ω_D, ω_m, ω_0	Pole of O.A.
F_{C1}, F_{C2}	Lowest and highest 3 dB frequencies in Hz
$\frac{\Delta Q_P}{Q_P}$	Fractional change in Q_P
$\frac{\Delta\omega_P}{\omega_P}$	Fractional change in ω_P

LIST OF TABLES

LIST OF IMPORTANT ABBREVIATIONS AND SYMBOLS

$A_D(S)$, $A_m(S)$, $A_0(S)$	OAs Open Loop Gains
A_D, A_m, A_0	OAs DC Gains
a_0, a_1, a_2	Coefficients of K_1K_2 in the S^0, S, S^2 terms of $D(S)$ respectively
AP	All Pass
All Pass Type i	All Pass with a denominator polynomial decomposition of Type i
α_0, α_1, α_2	Coefficients of the S^0, S, S^2 terms of $n_0(S)$ respectively
b_0, b_1, b_2	The part of the coefficient of the S^0, S, S^2 term respectively which is independent of K_1, K_2 in $D(S)$
β_1, β_2, β_3	The part of the coefficient of the S^0, S, S^2 term respectively which is independent of K_0 in $D_0(S)$
BP	Band Pass
Band Pass type i	Band Pass with a denominator polynomial decomposition of type i
C	Capacitance of a capacitor
c_0, c_1, c_2	Coefficients of K_1 in the S^0, S, S^2 terms of $D(S)$ respectively
γ_0, γ_1, γ_2	The part of the coefficient of the S^0, S, S^2 term respectively which is associated with K_0 in $D_0(S)$

$D(S)$	Denominator polynomial of T_v
$D_A(S)$	Denominator of T_v expressed as a function of t_v
$D_B(S)$	Denominator of T_v expressed as a function of $D_0(S)$ and $n_0(S)$
$D_{B0}(S)$	$D_B(S)$ when t_v is of zero order
$D_{B1}(S)$	$D_B(S)$ when t_v is of first order
$D_{B2}(S)$	$D_B(S)$ when t_v is of second order
d_0, d_1, d_2	Coefficients of K_2 in the S^0, S, S^2 terms of $D(S)$ respectively
F_{C1}, F_{C2}	Three dB frequencies in HZ
F	Figure of merit
G	Admittance of a resistor
~~GωSP~~	Gain pole frequency sensitivity product
GQSP	Gain Q sensitivity product
Generating Circuit	A SAC used to generate a TAC
Generating Function	Transfer function of a generating circuit
HP	High Pass
High Pass type i	High Pass with a denominator polynomial decomposition of type i
K_0, K_0, K_m, K_1, K_2	Amplifier's gains
LP	Low Pass
Low Pass type i	Low Pass with a denominator polynomial decomposition of type i

N	Null
Null type i	Null with a denominator polynomial decomposition of type i
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Q_P	Designed pole Q of a TAC
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t_V	Voltage transfer function of a SAC, generating transfer function
T_V	Voltage transfer function of a TAC
TAC	Two-Amplifier Circuit
ω_P	Variable frequency in rad/sec
ω_{P0}	Designed pole frequency in Rad/sec of a SAC
ω_P	Designed pole frequency in Rad/sec or in Hz of a TAC
ω_C, ω_D, ω_m, ω_0	Pole of O.A.
F_{C1}, F_{C2}	Lowest and highest 3 dB frequencies in Hz

ACKNOWLEDGEMENTS

The author expresses his appreciation to Professor V. Ramachandran and Professor B.B. Bhattacharyya for their supervision during the course of the investigation.

He expresses his gratitude to Professor V. Ramachandran for having been a source of great inspiration and of confidence during the entire period of the investigation.

Thanks are also due to Professor M.N.S. Swamy for his constant encouragement.

Some suggestions given by Professor T.A. Bickart, the external examiner for the improvement of the thesis are gratefully acknowledged.

The author acknowledges the understanding and encouragement of his wife Mrs. P.E. Gargour and thanks his parents Mr. and Mrs. Charles Gargour for all the efforts they spent towards his education since his childhood.

CHAPTER I

INTRODUCTION

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INTRODUCTION

1.1 General

Passive LC filters have been considerably studied and a large amount of literature [1, 2] exists on their design. These filters are absolutely stable and have no serious sensitivity problems associated with them. However, in spite of these attractive features, they suffer from some serious limitations.

(i) Accurate filter design is complicated by the non-linear frequency dependence of the quality factor Q_p of the inductors and the variation of that Q_p from one inductor to another.

(ii) The inductors may be of large size (and hence of high cost) for low frequency applications such as in analog computers, control systems, etc.

(iii) Problems may arise because of the magnetic coupling between the inductive elements: they may mask weak signals which hence may become not measurable.

Microminiaturization and increasing interest in integrated circuitry (IC) technology has made the use of inductors not practicable, as they cannot be manufactured with reasonable values and quality factors within acceptable tolerance at low frequency.

These difficulties may be overcome by employing active RC filters [3, 4]. Their design requires only resistors and capacitors along with active elements. The main attractions of integrated circuit fabrication are [5, 6] increased system reliability and reduction in size and weight due to microminiaturization. RC active filters can be designed to have several desirable features over the RLC filters and some of these are:

(i) The use of active elements remove the two restrictions from RLC networks namely passivity and reciprocity. Thus active networks not only realize network functions which are realizable by passive RLC networks but also can be used to realize characteristics not achievable with passive networks.

(ii) Input impedance may be made high compared to source impedance and thus these filters draw very little power from the signal source.

(iii) Output impedance can be made low compared to that of the load thereby making the filter response independent of the load impedance. Consequently the filters can be cascaded without additional buffers.

(iv) The RC filters often provide insertion gain which may be desirable in many applications thereby eliminating the need for additional amplifiers.

However, active RC filters if improperly designed have two major drawbacks, namely:

(i) They may become unstable.

(ii) They may be sensitive to network parameter variations.

Thus proper care should be exercised in their design.

1.2 Methods of Realizing RC Active Networks

A large number of RC active network realization procedures have been reported in the literature [7, 8, 9]. These can be classified as:

(i) Direct approach, where the given transfer function is realized as a single section [10, 11, 12].

(ii) Cascade approach where the transfer function is expressed as a product of first and second order transfer functions [13, 14, 15, 16]. Each of these functions is realized independently and the overall network is obtained by cascading them.

The cascade approach requires universal sections and each section can be easily designed and optimized independently [13, 17, 18]. Therefore this approach is used in this thesis.

Several active devices such as operational amplifiers (O.A.), negative impedance converters, and gyrators have been used in the creation of active RC filters [3,4]. The O.A. [19, 20] is the basic active element considered in this thesis.

1.3 The Operational Amplifier

The O.A. is a non-reciprocal two-port device, ideally characterized by an infinite gain, infinite input impedance and zero output impedance. In practice, however, the O.A. has a frequency dependent gain A(S), a finite input impedance and a non-zero output impedance. As an example, the Fairchild μA-741 has a DC gain A_0 of 200,000, an input impedance of 2 MΩ and an output impedance of 75 Ω. Presently the O.A.s are readily available in an integrated form as off-the-shelf components. Commercially available silicon monolithic integrated O.A.s are reliable, versatile, relatively inexpensive (less than $1) and have excellent properties.

The O.A. and its equivalent controlled source representation are shown respectively in Figs. 1.3.1a and 1.3.1b.

The output voltage v_0 is related to the differential input voltage $V_i = (V_2 - V_1)$ by:

$$V_0 = A(S)V_i \qquad (1.3.1)$$

A(S) is the differential open loop gain which for a single pole frequency compensated O.A. is given by:

$$A(S) = \frac{A_0\omega_C}{(S+\omega_C)} \qquad (1.3.2)$$

where A_0, ω_C and $B = A_0\omega_C$ are respectively the D-C gain, the cut-off frequency and the gain bandwidth product of the O.A.

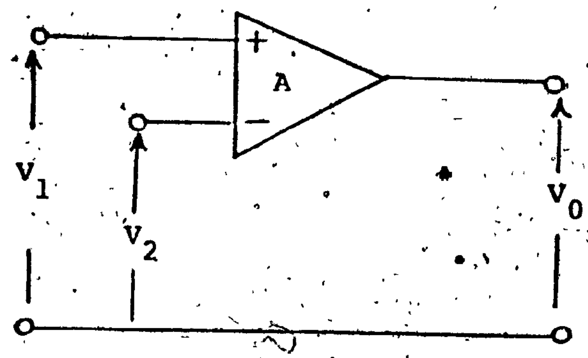

FIGURE 1.3.1a

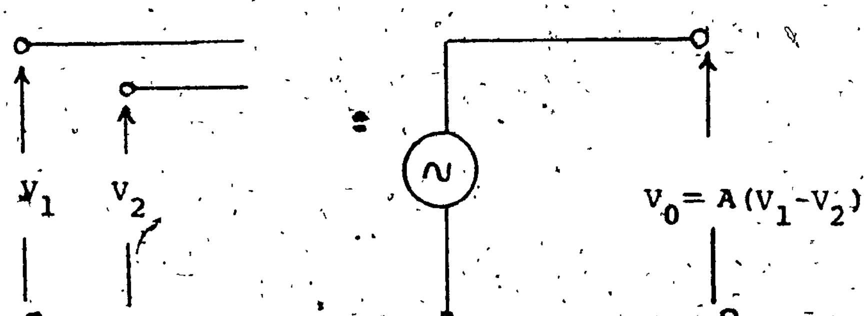

FIGURE 1.3.1b

FIGURE 1.3.1 THE OPERATIONAL AMPLIFIER AND ITS CONTROLLED SOURCE REPRESENTATION.

A_0 and ω_C have large tolerances in addition to their dependence on the temperature and power supply voltages. Hence the characteristics of networks whose response are highly dependent on A_0 and/or ω_C will be subject to variations with changing of temperature and power supply voltage.

1.4 Single and Two-amplifiers Realizations

The number of amplifiers in a biquadratic filter section may be one or more. Various single amplifier and two amplifier networks have been reported in the literature* [17-18, 21-33]. For the realizations where the gains of the amplifiers are finite, the denominator polynomial can be expressed as: [34]

$$D(S) = d_1(S)d_2(S) - K_1n_{11}(S)d_2(S) - K_2n_{22}(S)d_1(S) + K_1K_2[n_{11}(S)n_{22}(S) - n_{12}(S)n_{21}(S)] \quad (1.4.1)$$

where $\frac{n_{ij}(S)}{d_{ij}(S)}$ are passive RC transfer functions and K_1, K_2 are the closed loop gains of the two O.A.s. The decompositions corresponding to a single-amplifier can be obtained from Eqn. (1.4.1) by putting K_1 or K_2 equal to zero. Table 1.4.1 gives the summary of the results contained in [34] as these will be utilized in the present investigation.

*Only some of those networks which have appeared in the recent past have been quoted here and these contain many other references.

TABLE 1.4.1

SUMMARY OF RESULTS CONTAINED IN [34]

TABLE 1.4.1

$$D(S)=d_1(S)d_2(S)-K_1n_{11}(S)d_2(S)-K_2n_{22}(S)d_1(S)+K_1K_2[n_{11}(S)n_{22}(S)-n_{12}(S)n_{21}(S)]$$

$d_1(S)=\delta_{11}S+\delta_{10}$ $d_2(S)=\delta_{21}S+\delta_{20}$	$d_1(S)=\delta_2S^2+\delta_1S+\delta_0$		
$n_{11}(S)=e_1S+e_0$ $n_{12}(S)=g_1S+g_0$ $n_{22}(S)=f_1S+f_0$ $n_{21}(S)=h_1S+h_0$ $n_{11}(S)n_{22}(S)-n_{12}(S)n_{21}(S)=$ $B_2S^2+B_1S+B_0$ $D(S)=[\delta_{21}\delta_{11}-K_1\delta_{21}e_1-$ $K_2\delta_{11}f_1+K_1K_2B_2]S^2$ $+S[\delta_{10}\delta_{21}+\delta_{11}\delta_{20}-$ $K_1(\delta_{21}e_0+\delta_{20}e_1)-$ $K_2(\delta_{11}f_0+\delta_{10}f_1)$ $K_1K_2B_1]+$ $[\delta_{20}\delta_{10}-K_1\delta_{20}e_0-$ $K_2\delta_{10}f_0+K_1K_2B_0]$	$d_2(S)=1$ $n_{22}(S)=\Xi$ (or zero) $n_{11}(S)=m_2S^2-m_1S-m_0$ $n_{11}(S)n_{22}(S)-n_{12}(S)n_{21}(S)=$ $n_2S^2+n_1S-n_0$ $D(S)=[\delta_2-\Xi\delta_2K_1-$ $m_2K_2+n_2K_1K_2]S^2$ $+[\delta_1-\Xi\delta_1K_1+m_1K_2+$ $n_1K_1K_2]S$ $+[\delta_0-\Xi\delta_0K_1-m_0K_2]$ $+n_0K_1K_2]$	$n_{22}(S)=0$ $n_{12}(S)$ or $n_{21}(S)=d_2(S)=$ $n_2S^2+n_1S+n_0$ $n_{11}(S)=m_2S^2-m_1S+m_0$ $D(S)=$ $\{[\delta_2-m_2K_1-n_2K_1K_2]S^2$ $+[\delta_1-m_1K_1-n_1K_1K_2]S$ $+[\delta_0-m_0K_1-n_0K_1K_2]\}d_2(S)$	$d_2(S)=d_1(S)$ $n_{11}(S)n_{22}(S)-n_{12}(S)n_{21}(S)=0$ $n_{11}(S)=m_2S^2-m_1S-m_0$ $n_{22}(S)=n_2S^2-n_1S-n_0$ $D(S)=[[\delta_2-m_2K_1-n_2K_2]S^2$ $+[\delta_1-m_1K_1-n_1K_2]S$ $+[\delta_0-m_0K_1-n_0K_2]]$ $x\,[\delta_2S^2+\delta_1S+\delta_0]$

TABLE 1.4.1
(continued)

$d_1S)=\delta_{11}S+\delta_{10}$ $d_2(S)=\delta_{2t}S+\delta_{20}$	$d_l(S)=\delta_2 S^2+\delta_1 S+\delta_0$
Fialkow Gerst Conditions require:	Fialkow Gerst Conditions require:
$0 < e_0 < \delta_{10}$ $0 < e_1 < \delta_{11}$	$0 < m_0 < \delta_0$, $0 < n_0 < \delta_0$ $0 < m_1 < \delta_1$
$0 < g_0 < \delta_{10}$ $0 < g_1 < \delta_{11}$	$0 < n_1 < \delta_1$ $0 < m_2 < \delta_2$ $0 < m_2 < \delta_2$
$0 < f_0 < \delta_{20}$ $0 < f_1 < \delta_{21}$	$\delta_1^2-4\delta_0\delta_2 > 0$
$0 < h_0 < \delta_{20}$ $0 < h_1 < \delta_{21}$	

Note : Fialkow-Gerst conditions as given above are due to the fact that $t_{ij}(S) = \frac{n_{ij}(S)}{d_{ij}(S)}$ are three-terminal passive RC network transfer functions. [34]

1.5 Pole-Q (Q_P), Pole-Frequency (ω_P) and Their Sensitivities

Consider the second order transfer function $T_v(S)$:

$$T_v(S) \equiv \frac{N(S)}{D(S)} = \frac{N(S)}{p_2S^2+p_1S+p_0} \tag{1.5.1}$$

where N(S) is a polynomial of second order or less which determines the filter's characteristics: Low Pass (LP), High Pass (HP), Band Pass (BP), Null (N) or All Pass (AP).

The quantity Q_P is defined by:

$$Q_P = \frac{\{p_0p_2\}^{\frac{1}{2}}}{p_1} \tag{1.5.2}$$

and the quantity ω_P is defined by:

$$\omega_P = \left\{\frac{p_0}{p_2}\right\}^{\frac{1}{2}} \tag{1.5.3}$$

Hence the transfer function $T_v(S)$ can be rewritten in the form:

$$T_v(S) = \frac{N(S)}{S^2+\frac{\omega_P}{Q_P}S+\omega_P^2} \tag{1.5.4}$$

The sensitivities of Q_P and ω_P with respect to the variation of a network parameter z are respectively defined as:

$$S_z^{Q_P} \triangleq \frac{d(\ln Q_P)}{d(\ln z)} = \frac{z}{Q_P}\frac{dQ_P}{dz} \tag{1.5.6a}$$

$$S_z^{\omega_P} \triangleq \frac{d(\ln\omega_P)}{d(\ln z)} = \frac{z}{\omega_P}\frac{d\omega_P}{dz} \tag{1.5.6b}$$

The fractional changes in Q_P and ω_P due to variations in the network elements are [35, 36] :

$$\frac{\Delta Q_P}{Q_P} = \sum_{\ell} \frac{\Delta K_\ell}{K} S_K^{Q_P} + \sum_i \frac{\Delta G_i}{G_i} S_{G_i}^{Q_P} + \sum_j \frac{\Delta C_j}{C_j} S_{C_j}^{Q_P} \quad (1.5.7a)$$

$$\frac{\Delta \omega_P}{\omega_P} = \sum_{\ell} \frac{\Delta K_\ell}{K_\ell} S_K^{\omega_P} + \sum_i \frac{\Delta G_i}{G_i} S_{G_i}^{\omega_P} + \sum_j \frac{\Delta C_j}{C_j} S_{C_j}^{\omega_P} \quad (1.5.7b)$$

where K is the closed loop gain of the ℓ^{th} amplifier in the network. G_i is the conductance of the i^{th} resistive element in the network. C_j is the capacitance of the j^{th} capacitive element in the network.

Equation (1.5.7) shows that not only the variation in Q_P and ω_P due to K_ℓ should be designed to be low but also the variation due to passive elements should be made low. However, if hybrid IC technology which make use of tantalum thin film RC components and monolithic integrated OAs is used, perfect tracking of passive elements is obtained; that is, the variations in similar passive elements due to changes in temperature can be made equal. It is also possible to control the process such that:

$$\frac{\Delta G_i}{G_i} = \frac{\Delta C_j}{C_j} = \frac{\Delta G}{G} = \text{Constant for all } i \text{ and } j \quad (1.5.8)$$

Hence from Eqn. (1.5,7) and (1.5.8) we get:

$$\frac{\Delta Q_P}{Q_P} = \sum_{\ell} \frac{\Delta K_\ell}{K_\ell} S_K^Q + \frac{\Delta G}{G}\left[\sum_i S_{G_i}^Q + \sum_j S_{C_j}^{Q_P}\right] \quad (1.5.9a)$$

$$\frac{\Delta \omega_P}{\omega_P} = \sum_{\ell} \frac{\Delta K_\ell}{K_\ell} S_{K_\ell}^{\omega_P} + \frac{\Delta G}{G}\left[\sum_i S_{G_i}^{\omega_P} + \sum_j S_{C_j}^{\omega_P}\right] \quad (1.5.9b)$$

As it has been shown in [37]

$$\sum_i S_{G_i}^{Q_P} + \sum_j S_{C_j}^{Q_P} = 0 \quad (1.5.10a)$$

$$\sum_i S_{G_i}^{\omega_P} + \sum_j S_{C_j}^{\omega_P} = 0 \quad (1.5.10b)$$

Equations (1.5.9) reduce to:

$$\frac{\Delta Q_P}{Q_P} = \sum_{\ell} \frac{\Delta K_\ell}{K_\ell} S_{K_\ell}^Q \quad (1.5.11a)$$

$$\frac{\Delta \omega_P}{\omega_P} = \sum_{\ell} \frac{\Delta K_\ell}{K_\ell} S_{K_\ell}^{\omega_P} \quad (1.5.11b)$$

It is clear that the use of thin film technology has eliminated the effect of variations due to passive elements sensitivities. Consequently, not only the networks with low passive sensitivities but also those with high passive sensitivities become attractive.

The relation between the closed loop amplifier gain K and the open loop gain A is: [19, 20]

$$K = \frac{K_0}{1+\frac{K_0}{A}} \simeq \frac{K_0 A}{K_0 + A} \qquad A >> K_0 \quad (1.5.12)$$

where K_0 is determined by a ratio of resistors. Hence

$$S_A^K = \frac{A}{K}\frac{dK}{dA} = \frac{K}{A} \quad \text{for } A >> K \tag{1.5.13}$$

and

$$\frac{\Delta K}{K} = \frac{\Delta A}{A} S_A^K + \frac{\Delta K_0}{K_0} S_{K_0}^K = K\frac{\Delta A}{A^2} \tag{1.5.14}$$

The second term of Eqn. (1.5.14) vanishes because of almost perfect tracking and from [37]. Eqns. (1.5.11a) and (1.5.11b) therefore become:

$$\frac{\Delta Q_P}{Q_P} = \sum_{\ell} K_\ell \frac{\Delta A}{A^2} S_{K_\ell}^{Q_P} \tag{1.5.15a}$$

$$\frac{\Delta \omega_P}{\omega_P} = \sum_{\ell} K_\ell \frac{\Delta A_\ell}{A_\ell^2} S_{K_\ell}^{\omega_P} \tag{1.5.15b}$$

Let, in the case of two amplifiers $(\ell = 1,2)$

$$\frac{\Delta A}{A^2} = \max\left[\frac{\Delta A_1}{A_1^2}, \frac{\Delta A_2}{A_2^2}\right]$$

Hence we have

$$\left|\frac{\Delta Q_P}{Q_P}\right| \leq \sum_{\ell=1}^{2} \left|K_\ell S_{K_\ell}^{Q_P}\right| \frac{\Delta A}{A^2} \tag{1.5.16a}$$

Similarly

$$\frac{\Delta \omega_P}{\omega_P} < \left\{\sum_{\ell=1}^{2} \left|K_\ell S_{K_\ell}^{\omega_P}\right|\right\} \frac{\Delta A}{A^2} \tag{1.5.16b}$$

1.6 The Figure of Merit F

The quantity $\frac{\Delta A}{A^2}$ is completely dependent on the OAs used. Therefore $\frac{\Delta Q_P}{Q_P}$ and $\frac{\Delta \omega_P}{\omega_P}$ are dependent on the factor $\sum_{\ell=1}^{2} K\, S_{K_\ell}^{Q_P}$. It is known that [38] the variation in the transfer function $T_v(S)$ of a second order network whose zeros are located far from the high Q poles is given by:

$$\left.\frac{\Delta T_v(S)}{T_v(S)}\right|_{S=\delta\omega_P} = \frac{\Delta Q_P}{Q_P} + j\; 2Q_P \frac{\Delta\omega_P}{\omega_P} \tag{1.6.1}$$

Therefore it is desirable to make $\frac{\Delta\omega_P}{\omega_P} = 0$. A sum of two positive quantities can be made zero only when the individual quantities are set to zero therefore:

$$S_{K_1}^{\omega_P} = S_{K_2}^{\omega_P} = 0 \tag{1.6.2}$$

This is referred to as zero gain pole frequency sensitivity product (GωSP) condition

It is further desirable to minimize $\frac{\Delta Q_P}{Q_P}$. This requires the minimization of $K_1 S_{K_1}^{Q_P}$ and $K_2 S_{K_2}^{Q_P}$ which is the sum of the individual GQSPs of the amplifiers. Hence we choose in this thesis to minimize F which is defined as:

$$F = |K_1\; S_{K_1}^{Q_P}| + |K_2\; S_{K_2}^{Q_P}| \tag{1.6.3}$$

which is the sum of the magnitudes of $K_1\, S_{K_1}^{Q_P}$ and $K_2\, S_{K_2}^{Q_P}$ and which could be interpreted as the worst case deviation.

1.7 Realizations with Zero GωSP

It is known that [17, 18] it is possible to obtain single amplifier active RC networks(SAC) having the property of zero GωSP and minimized GQSP. However to the best of the author's knowledge, any study of the second order two amplifiers networks on the basis of a general configuration able to generate all the polynomial decompositions suitable for the fulfilment of the condition $\frac{\Delta\omega_P}{\omega_P} = 0$ independently of the gains of the amplifiers together with the minimization of F does not appear to be available in the literature. Such a study is desirable in the case of hybrid IC implementation and an attempt is made in this thesis towards the solution of this problem.

1.8 Scope of the Thesis

This thesis discusses realizations of second order transfer functions by two-amplifier RC active filters (TAC) possessing the property of zero GωSP and minimized F.

In Chapter II, the various possible denominator polynomial decompositions suitable for zero GωSP are discussed. The different properties of such decompositions are studied. A two-amplifier configuration (not unique) able to realize all the above possible polynomial decompositions is given. The use of a generating function is required to realize a given transfer function. This generating function could be of three types, namely zero order, first order, or second order. The zero order is not discussed in detail, as it cannot realize in a simple manner the five commonly occurring filters namely: Low Pass (LP), High Pass (HP), Band Pass (BP), Null (N) and All Pass (AP). Chapter III discusses the case of the first order generating function. This provides a systematic generation of such circuits. For the filters obtained, the GωSPs is zero and the quantity F is minimized and design equations and/or curves are provided.

Chapter IV considers the case when the generating function is of second order. These generating functions are realized by single amplifiers and have the property of zero GωSP and minimized GωSP. The overall two amplifier realization provides Q-multiplication while preserving the zero GωSP property. The resulting Q-multiplied circuits have been optimized to minimize F, keeping the capacitive spread low.

Null and Band-Pass filters were built and tested. The experimental results are given. Chapter IV summarizes the results of the present investigation and proposes possible extensions of this work.

CHAPTER II

POSSIBLE DENOMINATOR POLYNOMIAL DECOMPOSITIONS FOR TWO AMPLIFIER RC ACTIVE NETWORKS HAVING ZERO GAIN POLE FREQUENCY (ω_P) SENSITIVITY PRODUCTS

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POSSIBLE DENOMINATOR POLYNOMIAL DECOMPOSITIONS FOR TWO AMPLIFIER RC ACTIVE NETWORKS HAVING ZERO GAIN POLE FREQUENCY (ω_P) SENSITIVITY PRODUCTS

2.1 Introduction

In this chapter we discuss the various possible denominator polynomial decompositions suitable for zero gain-pole frequency sensitivity product (GωSP). Further , a general two-amplifier network configuration (TAC) is proposed which can realize all the above mentionned polynomial decompositions.

2.2 Possible polynomial decompositions for zero GωSP

It is known that ω_P is a function of K_1, K_2, as well as of resistances (R_i) and capacitances (C_i). If the active network is implemented using hybrid IC technology, $\frac{\Delta\omega_P}{\omega_P} = 0$ is desirable and as given by Eqn. (1.6.2), we should then have

$$S_{K_1}^{\omega_P} = S_{K_2}^{\omega_P} = 0 \qquad (2.2.1)$$

In what follows we shall discuss the conditions and hence the different ways of arranging D(S) so that Eqn. (2.2.1) is satisfied.

As stated in Chapter I, the denominator of any two amplifier network can be written in the general form:

$$D(s) = [b_2+K_1K_2a_2-K_1c_2-K_2d_2]S^2 + [b_1+K_1K_2a_1-K_1c_1-K_1d_1]S + [b_0+K_1K_2a_0-K_1c_0-K_1d_0] \quad (2.2.2)$$

K_1 and K_2 are the closed loop gains of the two amplifiers and are treated here as algebraic quantities, positive for the non-inverting amplifier and negative for the inverting one. Parameters d, c, b are always positive and the a parameters can be either positive or negative. The pole frequency is given by:

$$\omega_p = \left[\frac{b_0+K_1K_2a_0-K_1c_0-K_2d_0}{b_2+K_1K_2a_2-K_1c_2-K_2d_2}\right]^{\frac{1}{2}} \quad (2.2.3)$$

and the ω_p sensitivities with respect to the amplifier gains are given by:

$$S_{K_1}^{\omega_p} = \left[\frac{1}{2}\;\frac{K_1K_2a_0-K_1c_0}{b_0+K_1K_2a_0-K_1c_0-K_2d_2} - \frac{K_1K_2a_2-K_1c_2}{b_2+K_1K_2a_2-K_1c_2-K_2d_2}\right] \quad (2.2.4a)$$

$$S_{K_2}^{\omega_p} = \left[\frac{1}{2}\;\frac{K_1K_2a_0-K_2d_0}{b_0+K_1K_2a_0-K_1c_0-K_2d_0} - \frac{K_1K_2a_2-K_2d_2}{b_2+K_1K_2a_2-K_1c_2-K_2d_2}\right] \quad (2.2.4b)$$

If it is required that

$$S_{K_1}^{\omega_P} = S_{K_2}^{\omega_P} = 0 \qquad (2.2.1)$$

then the relationship to be satisfied are:

$$\frac{b_0+K_1K_2a_0-K_1c_0-K_2d_0}{b_2+K_1K_2a_2-K_1c_2-K_2d_0} = \frac{K_2a_0-c_0}{K_2a_2-c_2} = \frac{K_1a_0-d_0}{K_1a_2-d_2} \qquad (2.2.5)$$

Then, from (2.2.3) we have:

$$\omega_P^2 = \frac{K_2a_0-c_0}{K_2a_2-c_2} = \frac{K_1a_0-d_0}{K_1a_2-d_2} \qquad (2.2.6)$$

It is desirable to fulfill this condition independently of K_1 and K_2 for the following reasons:

i) If the condition (2.2.6) is fulfilled independently of K_1 and K_2, ω_P will be dependent only on the passive elements. This should make the tuning simpler.

ii) This should also permit us to utilize K_1 and K_2 as parameters in minimizing $\frac{\Delta Q_P}{Q_P}$ for any specified value of Q_P.

A little algebraic manipulations will show that the condition (2.2.1) can be satisfied independently of K_1 and K_2, provided the following is satisfied:

$$\frac{a_0}{a_2} = \frac{b_0}{b_2} = \frac{c_0}{c_2} = \frac{d_0}{d_2} = \omega_P^2 \qquad (2.2.7)$$

It is clear then, that the corresponding coefficients of K_1, K_2, K_1K_2 in the S^2, and S^0 terms of D(s) should be present and satisfy Eqn. (2.2.7). All realizable polynomial decompositions that satisfy the conditions expressed by Eqn. 2.2.7 are tabulated in Table 2.2.1. The expressions for the corresponding Q_P and ω_P sensitivities with respect to the parameters K_1 and K_2 as well as the bounds on these sensitivities when all the terms in D(s) are positive are also given in the same table. If all the terms are not positive the bounds may not exist in many cases.

Polynomial decompositions 25 to 27 are respectively realizable [34] only in the forms:

$$D(s) \quad \left[(\delta_2 S^2 + \delta_1 S + \delta_0)\right]\left[(\delta_2 - K_2 m_2) S^2 + (\delta_1 - K_1 n_1) S + (\delta_0 - K_2 m_0)\right] \tag{2.2.8a}$$

$$D(s) \quad \left[(\delta_2 S^2 + \delta_1 S + \delta_0)\right]\left[\delta_2 S^2 + (\delta_1 - K_1 n_1 - K_2 m_1) S + \delta_0\right] \tag{2.2.8b}$$

$$D(s) \quad \left[(\delta_2 S^2 + \delta_1 S + \delta_0)\right]\left[(\delta_2 - K_1 n_2 - K_2 m_2) S^2 + \delta_1 S + (\delta_0 - K_1 n_0 - K_0 m_0)\right] \tag{2.2.8c}$$

These denominators can, provided suitable numerators are chosen, lead to second order transfer functions. However, at least four capacitors will be required and therefore polynomial decompositions 25 to 27 will not be considered further in this thesis.

TABLE 2.2.1

THE POLYNOMIAL DECOMPOSITIONS YIELDING ZERO GωSP AND THE CORRESPONDING Q_{p}-SENSITIVITIES

TABLE 2.2.1

No	Active Parameters Present in s^2	s	s^0	Q_p	$S^{Q_p}_{K_1}$	Bounds on $S^{Q_p}_{K_1}$ *	$S^{Q_p}_{K_2}$	Bounds on $S^{Q_p}_{K_2}$ *
1		K_1K_2		$\frac{[b_2b_0]^{\frac{1}{2}}}{b_1+a_1K_1K_2}$	$\frac{-a_1K_1K_2}{b_1+a_1K_1K_2}$	-1,0	$S^{Q}_{K_1}$	-1,0
2		K_1K_2 K_1		$\frac{[b_2b_0]^{\frac{1}{2}}}{b_1+a_1K_1K_2-c_1K_1}$	$\frac{c_1K_1-a_1K_1K_2}{b_1+a_1K_1K_2-c_1K_1}$	-1,0	$S^{Q_p}_{K_1}[1-\frac{c_1K_1}{b_1-a_1K_1K_2-c_1K_1}]$	-1,0
3	K_1K_2		K_1K_2	$\left[\frac{[b_2+a_2K_1K_2]\times}{[b_0+a_0K_1K_2]}\right]^{\frac{1}{2}}$	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2]}$	0,1	$S^{Q_p}_{K_1}$	0,1
4	K_1K_2	K_1K_2	K_1K_2	$\frac{\left[\frac{[b_2+a_2K_1K_2]\times}{[b_0+a_0K_1K_2]}\right]^{\frac{1}{2}}}{[b_1+a_1K_1K_2]}$	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2]}-\frac{a_1K_1K_2}{b_1+a_1K_1K_2}$	-1,1	$S^{Q_p}_{K_1}$	-1,1
5	K_1K_2	K_1K_2 K_1	K_1K_2	$\frac{\left[\frac{[b_2+a_2K_1K_2]\times}{[b_0+a_0K_1K_2]}\right]^{\frac{1}{2}}}{[b_1+a_1K_1K_2-c_1K_1]}$	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2]}-\frac{a_1K_1K_2-c_1K_1}{b_1+a_1K_1K_2-c_1K_1}$	-1,1	$S^{Q_p}_{K_1}-\frac{c_1K_1}{b_1+a_1K_1K_2-c_1K_1}$	-1,1

TABLE 2.2.1 (Continued)

6	K_1K_2	K_1	K_1K_2	$\frac{\left\{[b_2+a_2K_1K_2]\times[b_0+a_0K_1K_2]\right\}^{1/2}}{[b_1-c_1K_1]}$	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2]}+\frac{K_1c_1}{b_1-K_1c_1}$	-1,1	$S^{Q_P}_{K_1}-\frac{K_1c_1}{b_1-K_1c_1}$	0,1
7	K_1K_2 K_1		K_1K_2 K_1	$\left\{[b_2+a_2K_1K_2-c_2K_1]\times[b_0+a_0K_1K_2-c_0K_1]\right\}^{1/2}$	$\frac{a_2K_1K_2-c_2K_1}{2[b_2+a_2K_1K_2-c_2K_1]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_0K_1]}$	0,1	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2-c_1K_1]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2-c_0K_1]}$	0,1
8	K_1K_2 K_1	K_1K_2	K_1K_2 K_1	$\frac{\left\{[b_2+a_2K_1K_2-c_2K_1]\times[b_2+a_0K_1K_2-c_0K_1]\right\}^{1/2}}{[b_1-a_1K_1K_2]}$	$\frac{a_2K_1K_2-c_2K_1}{2[b_2+a_2K_1K_2-c_2K_1]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_0K_1]}-\frac{a_1K_1K_2}{b_1+a_1K_1K_2}$	-1,1	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2-c_2K_1]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2-c_0K_1]}-\frac{a_1K_1K_2}{b_1+a_1K_1K_2}$	-1,1
9	K_1K_2 K_1	K_1K_2 K_1	K_1K_2 K_1	$\frac{\left\{[b_2+a_2K_1K_2-c_1K_1]\times[b_0+a_0K_1K_2-c_0K_1]\right\}^{1/2}}{[b_1+a_1K_1K_2-c_1K_1]}$	$\frac{a_2K_1K_2-c_2K_1}{2[b_2+a_2K_1K_2-c_2K_1]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_0K_1]}-\frac{a_1K_1K_2+c_1K_1}{b_1+a_1K_1K_2-c_1K_1}$	-1,1	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2-c_2K_1]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2-c_0K_1]}-\frac{a_1K_1K_2}{b_1+a_1K_1K_2-c_1K_1}$	-1,1
10	K_1K_2 K_1	K_1K_2 K_1 K_2	K_1K_2 K_1	$\frac{\left\{[b_2+a_2K_1K_2-c_1K_1]\times[b_0+a_0K_1K_2-c_0K_1]\right\}^{1/2}}{[b_1+a_1K_1K_2-c_1K_1-d_1K_2]}$	$\frac{a_2K_1K_2-c_1K_1}{2[b_2+a_2K_1K_2-c_2K_1]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_0K_1]}-\frac{a_1K_1K_2+c_1K_1}{b_1+a_1K_1K_2-c_1K_1-d_1K_2}$	-1,1	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2-c_2K_1]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2-c_0K_1]}-\frac{a_1K_1K_2+d_1K_2}{b_1+a_1K_1K_2-c_1K_1-d_1K_2}$	-1,1

TABLE 2.2. (Continued)

11	K_1K_2 K_1	K_1 K_2	K_1K_2 K_1	$\frac{\{[b_2+a_2K_1K_2-c_1K_1]\times[b_0+a_0K_1K_2-c_0K_1]\}^{1/2}}{b_1-c_1K_1-d_1K_2}$	$\frac{a_2K_1K_2-c_1K_1}{2[b_2+a_2K_1K_2-c_1K_1]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_0K_1]}+\frac{c_1K_1}{b_1-c_1K_1-d_1K_2}$	-1,1	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2-c_1K_1]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2-c_0K_1]}+\frac{d_1K_2}{b_1-c_1K_1-d_1K_2}$	-1,1
12	K_1K_2 K_1	K_1	K_1K_2 K_1	$\frac{\{[b_2+a_2K_1K_2-c_2K_1]\times[b_0+a_0K_1K_2-c_0K_1]\}^{1/2}}{b_1-c_1K_1}$	$\frac{a_2K_1K_2-c_2K_1}{2[b_2+a_2K_1K_2-c_1K_1]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_0K_1]}+\frac{c_1K_1}{b_1-c_1K_1}$	-1,1	$\frac{a_2K_1K_2}{2[b_2+a_2K_1K_2-c_1K_1]}+\frac{a_0K_1K_2}{2[b_0+a_0K_1K_2-c_0K_1]}$	0,1
13	K_1K_2 K_1 K_2	K_1K_2 K_1	K_1K_2 K_1 K_2	$\frac{\{[b_2+a_2K_1K_2-c_2K_1-d_2K_2]\times[b_0+a_0K_1K_2-c_0K_1-d_0K_2]\}^{1/2}}{[b_1+a_1K_1K_2-c_1K_1]}$	$\frac{a_2K_1K_2-c_2K_1}{2[b_2+a_2K_1K_2-c_2K_1-d_2K_2]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_0K_1-d_0K_2]}-\frac{a_1K_1K_2+c_1K_1}{b_1+a_1K_1K_2-c_1K_1}$	-1,1	$\frac{a_2K_1K_2-d_2K_2}{2[b_2+a_2K_1K_2-c_2K_1-d_2K_2]}+\frac{a_0K_1K_2-d_0K_2}{2[b_0+a_0K_1K_2-c_0K_1-d_0K_2]}-\frac{a_1K_1K_2}{b_1+a_1K_1K_2-c_1K_1}$	-1,1
14	K_1K_2 K_1 K_2	K_1K_2 K_1 K_2	K_1K_2 K_1 K_2	$\frac{\{[b_2+a_2K_1K_2-c_2K_1-d_2K_2]\times[b_0+a_0K_1K_2-c_0K_1-d_0K_2]\}^{1/2}}{[b_1+a_1K_1K_2-c_1K_1-d_1K_1]}$	$\frac{a_2K_1K_2-c_2K_1}{2[b_2+a_2K_1K_2-c_2K_1-d_2K_2]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_0K_1-d_0K_0]}-\frac{a_1K_1K_2+c_1K_1}{b_1+a_1K_1K_2-c_1K_1-d_1K_2}$	-1,1	$\frac{a_2K_1K_2-d_2K_2}{2[b_2+a_2K_1K_2-c_2K_1-d_2K_2]}+\frac{a_1K_1K_2-d_0K_2}{2[b_0+a_0K_1K_2-c_0K_1-d_0K_1]}-\frac{a_1K_1K_2+d_1K_2}{b_1+a_1K_1K_2-c_1K_1-d_1-d_1K_2}$	-1,1

TABLE 2.2.1 (Continued)

15	K_1K_2 K_1 K_2	K_1 K_2	K_1K_2 K_1 K_2	$\frac{\{[b_2+a_2K_1K_2-c_2K_1-d_2K_2]\lambda[b_0+a_0K_1K_2-c_0K_1-d_0K_2]\}^{1/2}}{[b_1-c_1K_1-d_1K_2]}$	$\frac{a_2K_1K_2-c_2K_1}{2[b_2+a_2K_1K_2-c_2K_1-d_2K_2]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_1K_1-d_1K_1]}+\frac{c_1K_1}{b_1-c_1K_1-d_1K_1}$	-1,1	$\frac{a_2K_1K_2-d_2K_2}{2[b_2+a_2K_1K_2-c_2K_1-d_2K_2]}+\frac{a_0K_1K_2-d_0K_2}{2[b_0+a_0K_1K_2-c_0K_1-d_0K_2]}+\frac{d_1K_2}{b_1-c_1K_1-d_1K_2}$	-1,1
16	K_1K_2 K_1 K_2	K_1	K_1K_2 K_1 K_2	$\frac{\{[b_2+a_2K_1K_2-c_2K_1-d_2K_2]\lambda[b_0+a_0K_1K_2-c_0K_1-d_0K_2]\}^{1/2}}{b_1-c_1K_1}$	$\frac{a_2K_1K_2-c_2K_1}{2[b_2+a_2K_1K_2-c_2K_1-d_2K_2]}+\frac{a_0K_1K_2-c_0K_1}{2[b_0+a_0K_1K_2-c_0K_1-d_0K_2]}+\frac{c_1K_1}{b_1-c_1K_1}$	-1,1	$\frac{a_2K_1K_2-d_0K_2}{2[b_2+a_2K_1K_2-c_2K_1-d_2K_1]}+\frac{a_0K_1K_2-d_0K_2}{2[b_0+a_0K_1K_2-c_0K_1-d_0K_1]}$	0,1
17	K_1 K_2	K_1K_2 K_1	K_1 K_2	$\frac{\{[b_2-c_2K_1-d_2K_2]\lambda[b_0-c_0K_1-d_0K_2]\}^{1/2}}{[b_1+a_1K_1K_2-c_1K_1]}$	$\frac{-c_2K_1}{2[b_2-c_2K_1-d_2K_2]}+\frac{-c_0K_1}{2[b_0-c_0K_1-d_0K_2]}-\frac{a_1K_1K_2+c_1K_1}{b_1+a_1K_1K_2-c_1K_1}$	-1,1	$\frac{-d_2K_2}{2[b_2-c_2K_1-d_2K_2]}+\frac{-d_0K_2}{2[b_2-c_2K_1-d_2K_2]}-\frac{a_1K_1K_2}{b_1+a_1K_1K_2-c_1K_1}$	-1,1
18	K_1 K_2	K_1K_2 K_1 K_2	K_1 K_2	$\frac{\{[b_2-c_2K_1-d_2K_2]\lambda[b_0-c_0K_1-d_0K_2]\}^{1/2}}{[b_1+a_1K_1K_2-c_1K_1-d_1K_2]}$	$\frac{-c_2K_1}{2[b_2-c_2K_1-d_2K_2]}+\frac{-c_0K_1}{2[b_0-c_0K_1-d_0K_2]}-\frac{a_1K_1K_2+c_1K_1}{b_1+a_1K_1K_2-c_1K_1-d_1K_2}$	-1,1	$\frac{-d_2K_2}{2[b_2-c_2K_1-d_2K_2]}+\frac{-d_0K_2}{2[b_0-c_0K_1-d_0K_2]}-\frac{a_1K_1K_2+d_1K_2}{b_1+a_1K_1K_2-c_1K_1-d_1K_2}$	-1,1

TABLE 2.2: (Contunued)

19	K_1 K_2	K_1 K_2	K_1 K_2	$\frac{\left[\begin{matrix}[b_2-c_2K_1-d_2K_2]x \\ [b_0-c_0K_1-d_0K_2]\end{matrix}\right]^{\frac{1}{2}}}{b_1c_1K_1-d_1K_2}$	$\frac{-c_2K_1}{2[b_2-c_2K_1-d_2K_2]}+\frac{-c_0K_1}{2[b_0-c_0K_1-d_0K_2]}+\frac{c_1K_1}{b_1-c_1K_1-d_1K_2}$	-1,1	$\frac{-d_2K_1}{2[b_2-c_2K_1-d_2K_2]}+\frac{-d_0K_2}{2[b_0-c_0K_1-d_0K_2]}+\frac{d_1K_2}{b_1-c_1K_1-d_1K_2}$	-1,1
20	K_1 K_2	K_1	K_1 K_2	$\frac{\left[\begin{matrix}[b_2-c_2K_1-d_2K_2]x \\ [b_0-c_0K_1-d_0K_2]\end{matrix}\right]^{\frac{1}{2}}}{b_1-c_1K_1}$	$\frac{-c_2K_1}{2[b_2-c_2K_1-d_2K_2]}+\frac{-c_0K_1}{2[b_0-c_0K_1-d_0K_2]}+\frac{c_1K_1}{b_1+c_1K_1}$	-1,1	$\frac{-d_2K_2}{2[b_2-c_2K_1-d_2K_2]}+\frac{-d_0K_2}{2[b_0-c_0K_1-d_0K_2]}$	0,1
21	K_1	K_1K_2	K_1	$\frac{\left[\begin{matrix}[b_2-c_2K_1]x \\ [b_0-c_0K_1]\end{matrix}\right]^{\frac{1}{2}}}{[b_1+a_1K_1K_2]}$	$\frac{-c_2K_1}{2[b_2-c_2K_1]}+\frac{-c_0K_1}{2[b_0-c_0K_1]}-\frac{a_1K_1K_2}{b_1+a_1K_1K_2}$	-1,1	$\frac{-a_1K_1K_2}{b_1+a_1K_1K_2}$	-1,0
22	K_1	K_1K_2 K_1	K_1	$\frac{\left[\begin{matrix}[b_2-c_2K_1]x \\ [b_0-c_0K_1]\end{matrix}\right]^{\frac{1}{2}}}{[b_1+a_1K_1K_2-c_1K_1]}$	$\frac{-c_2K_1}{2[b_2-K_1c_2]}+\frac{-c_0K_1}{2[b_0-K_1c_0]}-\frac{K_1K_2a_1+K_1c_1}{b_1+K_1K_2a_1-K_1c_1}$	-1,1	$\frac{-a_1K_1K_2}{b_1+a_1K_1K_2-K_1c_1}$	-1,0

TABLE 2.2.1 (Continued)

23	K_1	K_1K_2 K_1 K_2	K_1	$\frac{\left\{[b_2-c_2K_1][b_0-c_0K_1]\right\}^{1/2}}{[b_1+a_1K_1K_2-c_1K_1-d_1K_2]}$	$\frac{-c_2K_1}{2[b_2-c_2K_1]}+\frac{-c_0K_1}{2[b_0-c_0K_1]}-\frac{K_1K_2a_1+K_1c_1}{b_1+K_1K_2a_1-K_1c_1-K_2d_1}$	-1,1	$\frac{-K_1K_2a_1+K_1d_1}{b_1+K_1K_2a_1-K_1c_1-K_2d_1}$	-1,0
24	K_1	K_1 K_2	K_1	$\frac{\left\{[b_2-c_2K_1][b_0-c_0K_1]\right\}^{1/2}}{b_1+c_1K_1-d_1K_2}$	$\frac{-c_2K_1}{2[b_2-c_2K_1]}+\frac{-c_0K_1}{2[b_0-c_0K_1]}+\frac{c_1K_1}{b_1-c_1K_1-d_1K_2}$	-1,1	$\frac{d_1K_2}{b_1-c_1K_1-d_1K_2}$	-1,0
25	K_2	K_1	K_2	$\frac{\left\{[b_2-d_2K_2][b_0-d_0K_2]\right\}^{1/2}}{[b_1-c_1K_1]}$	$\frac{c_1K_1}{b_1-c_1K_1}$	-1,0	$\frac{-d_2K_2}{2[b_2-K_2d_2]}+\frac{-d_0K_0}{2[b_0-K_0d_0]}$	0,1
26		K_1 K_2		$\frac{[b_0b_2]^{1/2}}{b_1-c_1K_1-d_1K_2}$	$\frac{c_1K_1}{b_1-c_1K_1-d_1K_2}$	-1,0	$\frac{d_1K_2}{b_1-c_1K_1-d_1K_2}$	-1,0
27	K_1 K_2		K_1 K_2	$\left\{[b_2-c_2K_1-d_2K_2][b_0-c_0K_1-d_0K_2]\right\}$	$\frac{-c_2K_1}{2[b_2-c_2K_1-d_2K_2]}+\frac{-c_0K_1}{2[b_0-c_0K_1-d_0K_2]}$	1,0	$\frac{-d_2K_2}{2[b_2-c_2K_1-d_2K_2]}+\frac{-d_0K_2}{2[b_0-c_0K_1-d_0K_2]}$	1,0

*These are the bounds when all the terms in D(S) are positive.

2.3 A general, two amplifier network configuration

A two amplifier configuration (TAC) is shown in Fig. 2.3.1a. It will be shown that this configuration will permit us to realize all the twenty-four polynomial decompositions discussed in Section 2.2 The sub-network which realizes t_v (called the generating function) is restricted only by the condition that its output is taken from the output of an amplifier. This means that a suitable single amplifier circuit (SAC) is required for realizing t_v. The admittance Y_L should be set to zero whenever possible in order to reduce the number of elements; however it will be shown later that in some cases Y_L cannot be set to zero if the condition expressed by Eqn. 2.2.7 is to be satisfied.*

Analysis of the configuration shown in Fig. 2.3.1a gives:

$$(V_1-V_2)Y_0 = (V_2-V_4)Y_H+V_2Y_L+(V_2-V_3)Y_F \quad (2.3.1)$$

$$V_3 = K_mV_2 \quad (2.3.2)$$

$$V_4 = t_vV_3 \quad (2.3.3)$$

Combining these equations we get:

*It should be noted that in the following derivation K_m and K_0 are not interchangeable and could correspond respectively to K_1 and K_2 or vice-versa. Depending on the realization considered both or only one of these identifications is possible.

$$\frac{V_4}{V_1} = \frac{K_m Y_0 t_v}{D_A(S)} \tag{2.3.4a}$$

$$\frac{V_3}{V_1} = \frac{K_m Y_0}{D_A(S)} \tag{2.3.4b}$$

$$\frac{V_2}{V_1} = \frac{Y_0}{D_A(S)} \tag{2.3.4c}$$

where

$$D_A(S) = Y_0+Y_F+Y_H+Y_L-K_m Y_F-K_m Y_H t_v \tag{2.3.5}$$

Since the sub-network generating t_v is a single amplifier network and since the output is taken from the amplifier output terminal it is possible to express t_v as:

$$t_v = \frac{K_0 n_0(S)}{D_0(S)} \tag{2.3.6}$$

Hence Eqn. 2.3.4 to 2.3.7 could respectively be rewritten as:

$$\frac{V_4}{V_1} = \frac{K_m K_0 Y_0 n_0(S)}{D_B(S)} \tag{2.3.7a}$$

$$\frac{V_3}{V_1} = \frac{K_m Y_0 D_0(S)}{D_B(S)} \tag{2.3.7b}$$

$$\frac{V_2}{V_1} = \frac{Y_0 D_0(S)}{D_B(S)} \tag{2.3.7c}$$

$$D_B(S) = [Y_0+Y_F+Y_H+Y_L-K_m Y_F]\, D_0(S)-K_m K_0 Y_H n_0(S) \tag{2.3.8}$$

Because $D_0(S)$ can always be written [17] as:

$$D_0(S) = f_1(S)+K_0f_2(S) \quad (2.3.9)$$

where $f_2(S)$ is that portion of $D_0(S)$ with which K_0 is associated and $f_1(S)$ is that portion of $D_0(S)$ with which K_0 is not associated. It is clear that the denominator $D_B(S)$ contains K_0, K_m, and K_mK_0 terms in the manner indicated in Eqn. (2.2.3).

Since it is more convenient for several reasons such as the possibility of cascading several sections without buffers, and the independence of the transfer function from the nature of the load, we will restrict our attention on the transfer functions taken from the output of an amplifier. Also $\frac{V_3}{V_1}$ contains zeros of $D_0(S)$ in the numerator which will be of the form $(A_2S^2+A_1S+A_0)$. It is obvious that since $D_0(S)$ should be a strictly Hurwitz polynomial, the basic five filter types namely Low Pass, High Pass, Band Pass, Null and All Pass cannot be obtained. Therefore our attention in this thesis will only be focused on $T_v = \frac{V_4}{V_1}$ given by Eqn. (2.3.7a).

A particular case will be considered now. If we let $Y_F = 0$ as shown in Fig. 2.3.1b the denominator of Eqn. (2.3.7) is given by:

$$D_B(S) = (Y_0+Y_L+Y_H)D_0(S)-K_mK_0Y_Hn_0(S) \quad (2.3.10)$$

This form of D(S) is not quite general as the one described by Eqn. (2.3.8) because it could contain only K_0 and K_mK_0 terms; however it will be shown at a later stage that this configuration is useful to generate several of the polynomial decompositions.

The network shown in Fig. 2.3.1a is by no means unique. It can be modified in several ways and a number of other two-amplifier configurations can be generated. As an example the circuit shown in Fig. 2.3.1c is one possible modification for which analysis using Eqn. 2.3.6 gives:

$$\frac{V_5}{V_1} = \frac{Y_0\,[K_mK_0Y_Hn_0(S) + (K_mY_{F2}+Y_{F1})D_0(S)]}{D_C(S)} \tag{2.3.11a}$$

$$\frac{V_4}{V_1} = \frac{K_mK_0Y_0\,[Y_{F1}+Y_{F2}+Y_H+Y_L]\,n_0(S)}{D_C(S)} \tag{2.3.11b}$$

$$\frac{V_3}{V_1} = \frac{K_mY_0\,[Y_{F1}+Y_{F2}+Y_H+Y_L]\,D_0(S)}{D_C(S)} \tag{2.3.11c}$$

$$\frac{V_2}{V_1} = \frac{Y_0\,[Y_{F1}+Y_{F2}+Y_H+Y_L]\,D_0(S)}{D_C(S)} \tag{2.3.11d}$$

where

$$D_C(S) = [Y_0(Y_{F1}+Y_{F2}+Y_H)+Y_{F1}(Y_{F2}+Y_H+Y_L)-K_mY_{F1}Y_{F2}]\,D_0(S) - K_mK_0Y_HY_{F1}n_0(S) \tag{2.3.12}$$

It is clear that a second order transfer function T_V can be generated by using any of the following possibilities.

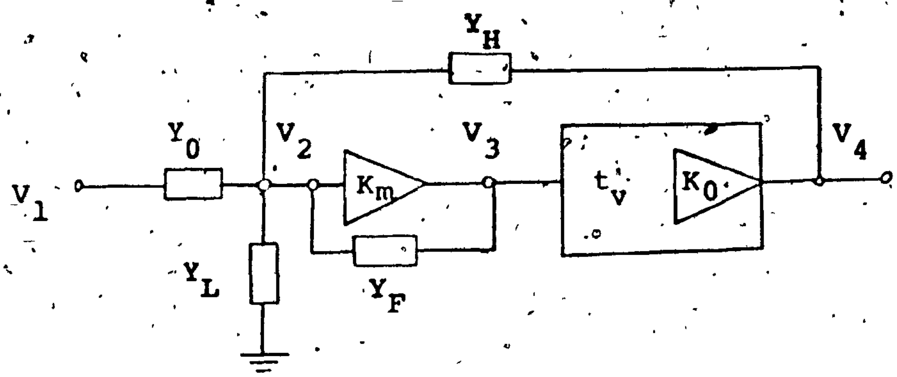

Fig. 2.3.1.a

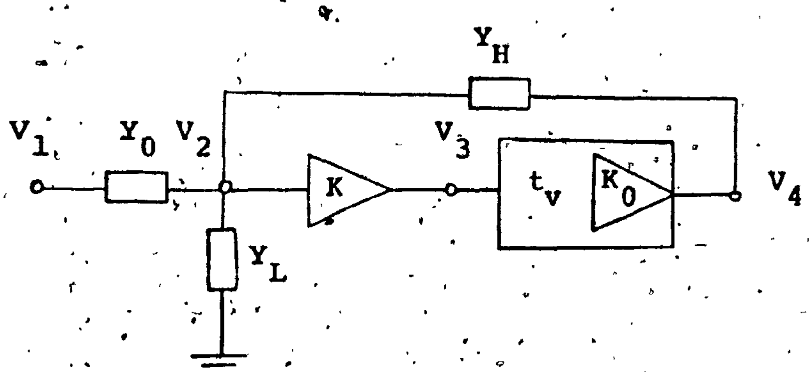

Fig. 2.3.1.b

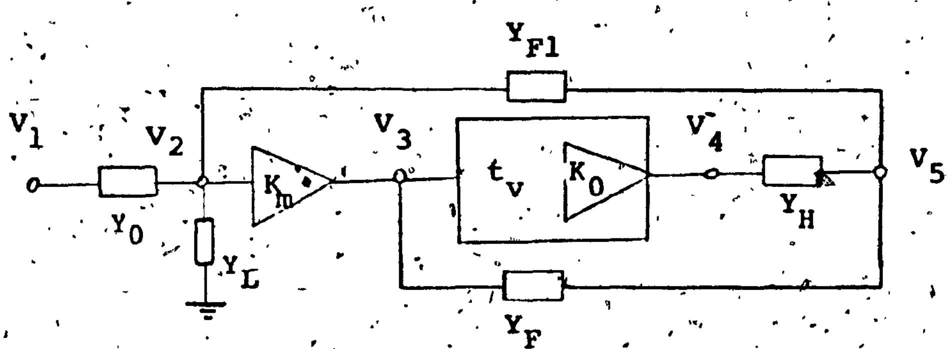

Fig. 2.3.1.c

Proposed configurations capable of realizing the different polynomial decompositions.

i) $D_0(S)$ being a zero degree polynomial

ii) $D_0(S)$ being a First degree polynomial

iii) $D_0(S)$ being a Second degree polynomial

We will now show that the proposed configuration shown in Fig. 2.3.1a can realize all the twenty-four possible polynomial decompositions.

2.4 The case of zero degree $D_0(S)$

Without any loss of generality a zero order transfer function realized by a SAC could be expressed, provided that the output is located at the output terminal of the amplifier as:

$$t_v = \frac{K_0 n_0(S)}{D_0(S)} = \frac{K_0 \alpha_0}{\beta_0 - K_0 \gamma_0} \qquad (2.4.1a)$$

Fialkow-Gerst conditions require that

$$0 < \gamma_0 < \beta_0 \qquad (2.4.1b)$$

Obviously γ_0 is a non-negative quantity while K_0, α_0 could be either positive or negative. Using Eqn. 2.4.1 and Eqn. 2.3.8, we get $D_B(S)$ as

$$D_{B0}(S) = [\beta_0 - K_0\gamma_0][Y_0 + Y_F + Y_H + Y_L - K_m Y_F] - K_m K_0 Y_H \alpha_0 \qquad (2.4.2)$$

---

*This is designated as $D_{B0}(S)$ because this polynomial is obtained when $D_0(S)$ is a zero degree polynomial.

It is obvious that a second degree polynomial cannot be obtained using simple firstorder admittances of the form :

$$Y = CS + G$$

For this purpose admittances of the general form:

$$\prod_i \frac{k_i (S + z_i)}{(S + p_i)}$$

where

$$0 \leqslant z_i \leqslant p_i$$

alone or togoether with admittances of the form shown in Eqn. (2.4.3) should be used.However ,their use necessarily produces terms of the form $(S + p_i)$ in the numerator of the transfer function and hence the five basic types of transfer functions namely LP,HP,BP,N,AP cannot be obtained without one or more pole-zero cancellations. Hence the case of the zero order degree $D_0(S)$ is not further pursued in this thesis.

2.5 The case of first degree $D_0(S)$

Without any loss of generality , a first order transfer function realized by a SAC could be expressed , provided that the output is taken through the output of the amplifier , as:

$$t_v = K_0 \frac{n_0(S)}{D_0(S)} = \frac{K_0(\alpha_1 S+\alpha_0)}{(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)} \quad (2.5.1a)$$

Fialkow-Gerst conditions require that

$$0 < \gamma_1 < \beta_1 \qquad 0 < \gamma_0 < \beta_0 \quad (2.5.1b)$$

where γ_1, γ_0 are non-negative quantities while K_0, α_1, α_0 could be either positive or negative. Using Eqn. 2.5.1 and Eqn. 2.3.8 we get $D_B(S)$ as:

$$D_{B1}{}^{*} = [Y_0+Y_F+Y_H+Y_L-K_m Y_F]\,[(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)] - K_0K_m(\alpha_1 S+\alpha_0)Y_H \quad (2.5.2)$$

To obtain a second degree denominator, the admittances Y_0, Y_F, Y_H, Y_L, should be chosen properly. One of the possible forms is $Y = mS+n$ and therefore:

$$Y_H = G_H+C_HS \qquad Y_F = G_F+C_FS$$
$$Y_L = G_L+C_LS \qquad Y_0 = G_0+C_0S \quad (2.5.4.)$$

Using these expressions in $D_{B1}(S)$ given by Eqn. (2.5.2) we get:

---

*This is designated as $D_{B1}(S)$ because this polynomial is obtained when $D_0(S)$ is a first degree polynomial.

$$D_{B1}(S) = S^2\{\beta_1(C_H+C_F+C_0+C_L)-K_m\beta_1C_F$$
$$-K_0\gamma_1(C_H+C_F+C_0+C_L)+K_mK_0(\gamma_1C_F-\alpha_1C_H)\}$$
$$+ S\{\beta_1(G_H+G_F+G_0+G_L)+\beta_0(C_H+C_F+C_0+C_L)$$
$$-K_m(\beta_1G_F+\beta_0C_F)-K_0[\gamma_1(G_F+G_H+G_0+G_L)$$
$$+\gamma_0(C_H+C_F+C_0+C_L)]+K_mK_0(\gamma_1G_F+\gamma_0C_F-\alpha_1G_H-\alpha_0C_H)\}$$
$$+ \{\beta_0(G_H+G_F+G_0+G_L)-K_m\beta_0G_F-K_0\gamma_0(G_H+G_F+G_L+G_0)$$
$$+K_mK_0(\gamma_0G_F-\alpha_0G_H)\}$$

(2.5.5)

For this denominator, the condition expressed in Eqn. 2.2.7 is given by:

$$\frac{\beta_1(C_H+C_F+C_0+C_L)}{\beta_0(G_H+G_F+G_0+G_L)} = \frac{\beta_1C_F}{\beta_0G_F} = \frac{\gamma_1(C_H+C_F+C_0+C_L)}{\gamma_0(G_H+G_F+G_0+G_L)}$$
$$= \frac{\gamma_1C_F-\alpha_1C_H}{\gamma_0G_F-\alpha_0G_H} = \omega_P^2 \qquad (2.5.6)$$

It is to be expected that depending on the realization considered it may become possible or necessary to set some of the capacitances and/or conductances to zero. However in the realization, the conditions:

$$\frac{\beta_1}{\beta_0} = \frac{\gamma_1}{\gamma_0} \quad \text{and} \quad \frac{\beta_1}{\beta_0} = \frac{\alpha_1}{\alpha_0} \qquad (2.5.7)$$

if satisfied simultaneously give rise to a constant t_v and

TABLE 2.5.1

POSSIBLE POLYNOMIAL DECOMPOSITIONS WHEN $D_0(S)$ IS A FIRST DEGREE POLYNOMIAL

TABLE 2.5.1

POSSIBLE POLYNOMIAL DECOMPOSITIONS WHEN $D_0(S)$ IS A FIRST DEGREE POLYNOMIAL

Decomposition No.	Active terms present in			Quantities to be set to zero in $D_{B1}(S)$
	S^2	S	S^0	
1		K_mK_0		$C_F, G_F, \gamma_1, \gamma_0$ (C_H, α_0) or (G_H, α_1)
4	K_mK_0	K_mK_0	K_mK_0	$C_F, G_F, \gamma_1, \gamma_0$
9a	K_mK_0, K_0	K_mK_0, K_0	K_mK_0, K_0	C_F, G_F
9b	K_mK_0, K_m	K_mK_0, K_m	K_mK_0, K_m	γ_1, γ_0
10	K_mK_0, K_m, K_0	K_mK_0, K_m, K_0	K_mK_0, K_m, K_0	-
15	K_mK_0, K_m, K_0	K_m, K_0	K_mK_0, K_m, K_0	B_1
18	K_m, K_0	K_mK_0, K_m, K_0	K_m, K_0	B_2, B_0
19	K_m, K_0	K_m, K_0	K_m, K_0	B_2, B_1, B_0

TABLE 2.5.1
(continued)

22a	K_0	$K_m K_0$, K_0	K_0	C_F, G_E, (C_H, α_0) or (G_H, α_1)
22b	K_m	$K_m K_0$, K_m	K_m	γ_1, γ_0, (C_H, α_0) or (G_H, α_1)

$B_2 \triangleq \gamma_1 C_{F2} - \alpha_1 C_H$

$B_1 \triangleq \gamma_1 G_{F2} + \gamma_0 C_{F2} - (\alpha_1 G_H + \alpha_0 C_H)$

$B_0 \triangleq \gamma_0 G_{F2} - \alpha_0 G_H$

hence the resulting T_v is not a second order transfer function. This possibility should be avoided.

From Eqn. (1.4.1) and Fialkow Gerst conditions, it follows that for this case only the polynomials decompositions 1, 4, 9, 10, 15, 18, 19, 22 are possible. These are tabulated in Table 2.5.1 where the quantities to be set to zero in $D_{B1}(S)$ are also given for each case. It should be noted that in the cases of decompositions 9, and 22 there exist two possibilities which are also shown. This case is further discussed in Chapter III.

2.6 The case of second degree $D_0(S)$

Without any loss of generality, a second order transfer function realized using a single finite gain amplifier could be expressed, provided that the output is taken through the output of the amplifier, as:

$$t_v = \frac{K_0 n_0(S)}{D_0(S)} = \frac{K_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2+(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)} \tag{2.6.1a}$$

Fialkow-Gerst conditions require that:

$$0 \leq \gamma_2 \leq \beta_2, \quad 0 \leq \gamma_1 \leq \beta_1, \quad 0 \leq \gamma_0 \leq \beta_0 \tag{2.6.1b}$$

where γ_2, γ_1, γ_0 are non-negative quantities while K_0, α_2, α_1, α_0 could be either positive or negative. Substituting

Eqn. 2.6.1 into 2.3.8 with

$$Y_0 = G_0 \quad Y_F = G_F \quad Y_H = G_H \quad Y_L = G_L \tag{2.6.2}$$

we get $D_B(S)$ as:

$$\begin{aligned} D_{B2}(S)^* = {} & S^2[\beta_2(G_0+G_F+G_H+G_L)-K_m\beta_2G_F-K_0\gamma_2(G_0+G_H+G_H+G_L) \\ & +K_mK_0(\gamma_2G_F-\alpha_2G_H)] \\ & + S[\beta_1(G_0+G_F+G_H+G_L)-K_m\beta_1G_F-K_0\gamma_1(G_0+G_H+G_F+G_L) \\ & +K_mK_0(\gamma_1G_F-\alpha_1G_H)] \\ & + [\beta_0(G_0+G_F+G_H+G_L)-K_m\beta_0G_F-K_0\gamma_0(G_0+G_H+G_F+G_L) \\ & +K_mK_0(\gamma_0G_F-\alpha_0G_H)] \end{aligned} \tag{2.6.3}$$

From Eqn. 2.2.7 we get

$$\frac{\beta_0}{\beta_2} = \frac{\gamma_0}{\beta_2} = \frac{\alpha_0}{\beta_2} = \omega_P^2 \tag{2.6.4}$$

It is known [17,18] that t_v can be realized by various decompositions of $D_0(S)$ involving K_0. By proper choice of $D_0(S)$ and the conductances G_0, G_F, G_H, G_L, the decompositions 1 to 24 can be achieved. These are tabulated in Table 2.6.1. The quantities to be set to zero in D_{B2} are also given for each case. It should be noted that in the cases of

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*This is designated as $D_{B2}(S)$ because this polynomial is obtained when $D_0(S)$ is a second degree polynomial.

TABLE 2.6.1

POSSIBLE POLYNOMIAL DECOMPOSITIONS WHEN $D_0(S)$ IS A SECOND DEGREE POLYNOMIAL

TABLE 2.6.1

POSSIBLE POLYNOMIAL DECOMPOSITIONS WHEN $D_0(S)$ IS A SECOND DEGREE POLYNOMIAL

Decomposition No.	Active terms present in			Quantities to be set to zero in D_{B2}
	s^2	s	s^0	
1		K_mK_0		$G_F, \alpha_2, \alpha_0, \gamma_2, \gamma_1, \gamma_0$
2		K_mK_0, K_0		$G_F, \alpha_2, \alpha_0, \gamma_2, \gamma_0$
3	K_mK_0		K_mK_0	$G_F, \alpha_1, \gamma_2, \gamma_1, \gamma_0$
4	K_mK_0	K_mK_0	K_mK_0	$G_F, \gamma_2, \gamma_1, \gamma_0$
5	K_mK_0	K_mK_0, K_0	K_mK_0	G_F, γ_2, γ_0
6	K_mK_0	K_0	K_mK_0	$G_F, \alpha_1, \gamma_2, \gamma_0$
7	K_mK_0, K_0		K_mK_0, K_0	G_F, α_1, γ_1
8	K_mK_0, K_0	K_mK_0	K_mK_0, K_0	G_F, γ_1
9a	K_mK_0, K_0	K_mK_0, K_0	K_mK_0, K_0	G_F
9b	K_mK_0, K_m	K_mK_0, K_m	K_mK_0, K_m	$\gamma_2, \gamma_1, \gamma_0$

TABLE 2.6.1
(continued)

10	K_mK_0, K_m	K_mK_0, K_m, K_0	K_mK_0, K_m	γ_2, γ_0
11	K_mK_0, K_m	K_m, K_0	K_mK_0, K_m	B_1, γ_2, γ_0
12a	K_mK_0, K_0	K_0	K_mK_0, K_0	G_F, α_1
12b	K_mK_0, K_m	K_m	K_mK_0, K_m	$\alpha_1, \gamma_2, \gamma_1, \gamma_0$
13	K_mK_0, K_m, K_0	K_mK_0, K_m	K_mK_0, K_m, K_0	γ_1
14	K_mK_0, K_m, K_0	K_mK_0, K_m, K_0	K_mK_0, K_m, K_0	-
15	K_mK_0, K_m, K_0	K_m, K_0	K_mK_0, K_m, K_0	B_1
16	K_mK_0, K_m, K_0	K_m	K_mK_0, K_m, K_0	α_1, α_1
17	K_m, K_0	K_mK_0, K_m	K_m, K_0	B_2, B_0, γ_1
18	K_m, K_0	K_mK_0, K_m, K_0	K_m, K_0	B_2, B_0

TABLE 2.6.1
(continued)

19	K_m, K_0	K_m, K_0	K_m, K_0	B_2, B_1, B_0
20	K_m, K_0	K_m	K_m, K_0	$B_2, B_0, \gamma_1, \alpha_1$
21	K_0	$K_m K_0$	K_0	$G_F, \alpha_2, \alpha_0, \gamma_1$
22a	K_0	$K_m K_0, K_0$	K_0	G_F, α_2, α_0
22b	K_m	$K_m K_0, K_m$	K_m	$\alpha_2, \alpha_0, \gamma_2, \gamma_1, \gamma_0$
23	K_m	$K_m K_0, K_m, K_0$	K_m	$\alpha_2, \alpha_0, \gamma_2, \gamma_0$
24	K_m	K_m, K_0	K_m	$\alpha_2, \alpha_0, \gamma_2, \gamma_0, B_1$

$B_2 \triangleq \gamma_2 G_F - \alpha_2 G_H$

$B_1 \triangleq \gamma_1 G_F - \alpha_1 G_H$

$B_0 \triangleq \gamma_0 G_F - \alpha_0 G_H$

decompositions 9, 12, 22 there exist two possibilities which are also shown. The pole frequency of the denominator of the transfer function given by Eqn. 2.6.1 is:

$$\omega_{P0} = \left[\frac{\beta_0 - K_0\gamma_0}{\beta_2 - K_0\gamma_2}\right]^{\frac{1}{2}} \tag{2.6.5}$$

while the pole frequency of $D_{B2}(S)$ given by Eqn. (2.4.3) is:

$$\omega_P = \left[\frac{\beta_0(G_0+G_F+G_H+G_L) - K_m\beta_0 G_F - K_0\gamma_0(G_0+G_F+G_H+G_L) + K_mK_0(\gamma_0 G_F - \alpha_0 G_H)}{\beta_2(G_0+G_F+G_H+G_L) - K_m\beta_2 G_F - K_0\gamma_2(G_0+G_H+G_H+G_L) + K_mK_0(\gamma_2 G_F - \alpha_2 G_H)}\right]^{\frac{1}{2}} \tag{2.6.6}$$

If the condition expressed by Eqn. (2.2.7) is fulfilled we have $\omega_P = \omega_{P0}$. The Q_P of $D_{B2}(S)$ is given by:

$$Q_P = \frac{\left\{\left[\begin{array}{l}\beta_2(G_0+G_F+G_H+G_L) - K_m\beta_2 G_F \\ -K_0\gamma_2(G_0+G_F+G_H+G_L) \\ +K_mK_0(\gamma_2 G_F - \alpha_2 G_H)\end{array}\right]\left[\begin{array}{l}\beta_0(G_0+G_F+G_H+G_L) - K_m\beta_0 G_F \\ -K_0\gamma_0(G_0+G_F+G_H+G_L) \\ +K_mK_0(\gamma_2 G_F - \alpha_2 G_H)\end{array}\right]\right\}^{\frac{1}{2}}}{\beta_1(G_0+G_F+G_H+G_L) - K_m\beta_1 G_F - K_0\gamma_1(G_0+G_H+G_F+G_L) + K_mK_0(\beta_1 G_F - \gamma_1 G_H)} \tag{2.6.7}$$

This could be expressed as:

$$Q_P = Q_{P0}\frac{\left\{\left[(G_0+G_F+G_H+G_L)+\frac{K_mK_0(\gamma_2G_F-\alpha_2G_H)-K_m\beta_2G_F}{\beta_2-K_0\gamma_2}\right]\left[(G_0+G_F+G_H+G_L)+\left(\frac{K_mK_0(\gamma_0G_F-\alpha_0G_H)-K_m\beta_0G_F}{\beta_0-K_0\gamma_0}\right)\right]\right\}^{\frac{1}{2}}}{(G_0+G_F+G_H+G_L)\left(\frac{K_0K_m(\gamma_1G_F-\alpha_1G_H)-K_m\beta_1G_F}{\beta_1-K_0\gamma_1}\right)} \tag{2.6.8}$$

where

$$Q_{P0} = \frac{\{(\beta_2-K_0\gamma_2)(\beta_0-K_0\gamma_Q)\}^{\frac{1}{2}}}{\beta_1-K_1\gamma_1} \tag{2.6.9}$$

is the Q of the SAC transfer function given in Eqn. 2.6.1. Hence it is clear that if the passive and active parameters are properly chosen and if the filters are fabricated in hybrid IC technology, the given configuration achieves a Q multiplication (multiplication of Q of t_V) while keeping the pole frequency ω_P invariant.

Q multiplication has already been mentioned in the literature and a signal flow graph for achieving it is given [31]. This paper does not discuss any realization or further properties. Among the important advantages of Q multiplication the following could be mentioned:

i) Low-Q circuits having good circuit properties can be used in suitable Q multiplier circuits in order to obtain high values of Q and still preserve their desirable properties. This should enable us to design high-Q circuits making use of low-Q ones having their properties optimized independently.

ii) High-Q positive feedback circuits have a tendency to become unstable although they may have other good properties. This situation could be improved by a combination of low-Q positive feedback circuits and a Q multiplier possessing a negative feedback loop.

iii) In many instances tuning is simplified.

The case of second order $D_0(S)$ is further discussed in Chapter IV where the resulting filter networks are referred to as the "Q-multiplier cicuits" (QMC)

2.7 Summary and Discussions

In this chapter, we have considered the various possible denominator polynomial decompositions for two amplifier filter networks having zero GωSP. For hybrid IC implementation, this will lead to design of filters having zero functional deviation of ω_p which is a highly desirable feature for high Q applications.

A configuration is proposed (though not unique) which is shown to be capable of realizing all the above polynomial decompositions. This is possible by the use of three types of generating functions. These generating functions have to be realized such that the output is taken from the output of an amplifier. Out of these three possibilities, the cases of the first and second order $D_0(S)$ will be discussed further. The case of zero order $D_0(S)$ will not be discussed any more because it does not appear to lead to any filter design in a simple manner. The case of the second order $D_0(S)$ appears to be particularly attractive as in this case starting from an optimized SAC filter. A TAC filter can be designed such that "Q-multiplication" is achieved while keeping the pole frequency ω_p invariant.

CHAPTER III

THE CASE OF THE FIRST ORDER GENERATING FUNCTION

CHAPTER III

THE CASE OF THE FIRST ORDER GENERATING FUNCTION

3.1 Introduction

In the previous chapter, we considered the different polynomial decompositions having the property of zero GωSP and proposed a general configuration for realizing them. Further it was shown that the generating function could be either of first or second order. In this chapter, we shall discuss realizations in which the generating circuit produces a first order transfer function. The quantity F of the resulting two amplifier network will be minimized. This results in filters having zero GωSP and minimized $\frac{\Delta Q_p}{Q_p}$. The technique is illustrated by selecting appropriate generating circuits.

3.2 The Basic Configuration with First Order Generating Function

The proposed configuration is shown in Fig. 3.2.1. Using Eqns. 2.3.4a, 2.3.5 and setting $V_4 = V_0$ we get

$$\frac{V_0}{V_1} = \frac{K_m Y_0 t_v}{Y_0 + Y_F + Y_H + Y_L - K_m Y_F - K_m Y_H t_v} \qquad (3.2.1)$$

From Eqns. 3.2.1, 2.5.2, and 2.5.1 we obtain the transfer function T_v of the configuration shown in Fig. 3.2.1 as

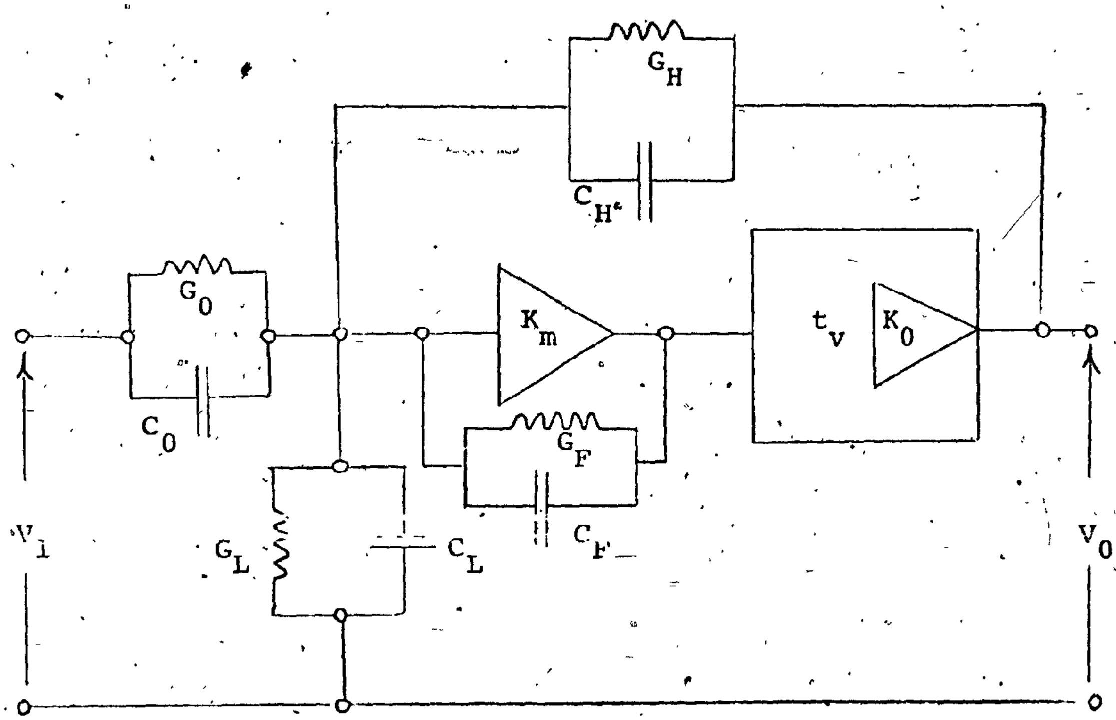


FIGURE 3.2.1 THE PROPOSED CONFIGURATION WITH FIRST ORDER t_v

$$T_V = \frac{K_m K_0 (G_0 + C_0 S)(\alpha_1 S + \alpha_0)}{\begin{array}{l} S^2\{(\beta_1 - K_0\gamma_1)(C_H + C_F + C_0 + C_L) - K_m\beta_1 + K_m K_0(\gamma_1 C_F - \alpha_1 C_H)\} \\ + S\{(G_H + G_F + G_0 + G_L)(\beta_1 - K_0\gamma_1) + (C_H + C_F + C_0 + C_L)(\beta_0 - K_0\gamma_0) \\ - K_m(\gamma_1 G_{F2} + \gamma_0 C_{F2}) + K_m K_0(\alpha_1 G_F + \alpha_0 C_F - \alpha_1 G_H - \alpha_0 C_H) \\ + \{(\beta_0 - K_0\gamma_0)(G_H + G_F + G_0 + G_L) - K_m\beta_0 G_F + K_m K_0(\gamma_0 G_F - \alpha_0 G_H)\} \end{array}} \qquad (3.2.2)$$

Table 3.2.1 gives all the possible polynomial decompositions and for each one the corresponding generating function t_V, the transfer function of the entire circuit T_V, its pole Q (Q_P) and active parameters sensitivities ($S_{K_m}^{Q_P}$) and ($S_{K_0}^{Q_P}$) as well as their bounds when all the denominator D(S) terms are positive.

It is clear from the numerator of T_V that we should expect to be able to obtain Low Pass, Band Pass, and High Pass transfer functions but no Null or All Pass ones. The number of elements (particularly the capacitors) may seem to be high in Fig. 3.2.1. However it should be noted that several of these elements could or have to be set to zero in order to achieve the different polynomial decompositions and the different kinds of numerators of T_V.

Table 3.2.2 gives the quantities which can or have to be set to zero in order to obtain a T_V. It should be noted that in some cases the elements are kept for the only reason that their removal would make the condition expressed in Eqn. 2.5.6 (which ensures zero GωSP independently of K_0 and K_m)

TABLE 3.2.1

POLYNOMIAL DECOMPOSITIONS SUITABLE FOR ZERO GωSP OBTAINABLE FROM FIRST ORDER t_V AND THE CORRESPONDING T_V, Q_P, AND SENSITIVITIES

TABLE 3.2.1

POLYNOMIAL DECOMPOSITION TYPE 1

$$t_V = \frac{K_0\alpha_1 S}{\beta_1 S-\beta_0}$$

$$T_V = \frac{K_m K_0 (G_0+C_0 S)\alpha_1 S}{S^2\beta_1(C_0+C_L)+S[\beta_0(C_0-C_L)+\beta_1(G_H+G_0+G_L)-K_m K_0 G_H\alpha_1] + \beta_0(G_H+G_0+G_L)}$$

$$Q_P = \frac{-\{\beta_1(C_0+C_L)\beta_0(G_H-G_0+G_L)\}^{\frac{1}{2}}}{\beta_0(C_0-C_L)-\beta_1(G_H-G_0-G_L)-K_m K_0 G_H\alpha_1}$$

$$S_{K_0}^{Q_P} = \frac{K_m K_0 G_H\alpha_1}{\beta_0(C_0+C_L)-\beta_1(G_H-G_0-G_L)-K_m K_0 G_H\alpha_1}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,0

$$S_{K_m}^{Q_P} = S_{K_0}^{Q_P}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,0

TABLE 3.2.[illegible] (Continued)

t_v	$= \dfrac{K_0\alpha_0}{\beta_1 S - \beta_0}$
T_v	$= \dfrac{K_m K_0 (G_0 + C_0 S)\alpha_0}{S^2\beta_1 (C_H + C_0 + C_L) + S[\beta_0 (C_H + C_0 + C_L) + \beta_1 (G_H + G_0 + G_L) - C_H\alpha_0 K_m K_0] + \beta_0 (G_H + G_0 + G_L)}$
Q_P	$= \dfrac{\{\beta_1 (C_H + C_0 + C_L)\beta_0 (G_H + G_0 + G_L)\}^{\frac{1}{2}}}{\beta_0 (C_H + C_0 - C_L) - \beta_1 (G_H - G_0 - G_L) - K_m K_0 C_H \alpha_0}$
$S^{Q_P}_{K_0}$	$= \dfrac{K_m K_0 G_H \alpha_0}{\beta_0 (C_H - C_0 + C_L) - \beta_1 (G_H - G_0 - G_L) - C_H \alpha_0 K_m K_0}$ Bounds of $S^{Q_P}_{K_0}$ when all terms in D(S) are positive: -1,0
$S^{Q_P}_{K_m}$	$= S^{Q_P}_{K_0}$ Bounds of $S^{Q_P}_{K_m}$ when all terms in D(S) are positive: -1,0
POLYNOMIAL DECOMPOSITION TYPE 4	
t_v	$= \dfrac{K_0(\alpha_1 S + \alpha_0)}{\beta_1 S + \beta_0}$
T_v	$= \dfrac{K_m K_0 (G_0 + C_0 S)(\alpha_1 S + \alpha_0)}{S^2 [\beta_1 (C_H + C_0 - C_L) - K_m K_0 C_H \alpha_1] + S[\beta_1 (G_H + G_0 + G_L) + \beta_0 (C_H + C_0 + C_L) - K_m K_0 (\alpha_1 G_H + \alpha_0 C_H)] + [\beta_0 (G_H + G_0 + G_L) - K_m K_0 G_H \alpha_0]}$

TABLE 3.2.1 (Continued)

Q_P	$= \dfrac{\{[\beta_1(C_H+C_0+C_L)-K_mK_0C_H\alpha_1][\beta_0(G_H+G_0+G_L)-K_mK_0G_H\alpha_0]\}^{\frac{1}{2}}}{\beta_1(G_H+G_0+G_L)+\beta_0(C_H+C_0+C_L)-K_mK_0(G_H\alpha_1+C_H\alpha_0)}$
$S_{K_0}^{Q_P}$	$= \dfrac{1}{2}\left[\dfrac{-K_mK_0G_H\alpha_0}{\beta_0(G_H+G_0+G_L)-K_mK_0G_H\alpha_0} + \dfrac{-K_mK_0C_H\alpha_1}{\beta_1(C_H+C_0+C_L)-K_mK_0C_H\alpha_1}\right] - \dfrac{-K_mK_0(G_H\alpha_1+C_H\alpha_0)}{\beta_1(G_H+G_0+G_L)+\beta_0(C_H+C_0+C_L)-K_mK_0(G_H\alpha_1+C_H\alpha_0)}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in $D(S)$ are positive: $-1,1$
$S_{K_m}^{Q_P}$	$= S_{K_0}^{Q_P}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in $D(S)$ are positive: $-1,1$
POLYNOMIAL DECOMPOSITION TYPE 9A	
t_v	$= \dfrac{K_0(\alpha_1 S+\alpha_0)}{(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)}$
T_v	$= \dfrac{K_mK_0(G_0+C_0S)(\alpha_1 S+\alpha_0)}{S^2[(C_H+C_0+C_L)(\beta_1-K_0\gamma_1)+K_mK_0C_H\alpha_1] + S[(G_H+G_0+G_L)(\beta_1-K_0\gamma_1)+(C_H+C_0+C_L)(\beta_0-K_0\gamma_0)-K_mK_0(G_H\alpha_1+C_H\alpha_0)] + [(G_H+G_0+G_L)(\beta_0-K_0\gamma_0)-K_mK_0G_H\alpha_0]}$
Q_P	$= \dfrac{\{[(G_H+G_0+G_L)(\beta_0-K_0\gamma_0)-K_mK_0G_H\alpha_0][(C_H+C_0+C_L)(\beta_1-K_0\gamma_1)-K_mK_0C_H\alpha_1]\}^{\frac{1}{2}}}{(G_0+G_H+G_L)(\beta_1-K_0\gamma_1)+(C_0+C_H+C_L)(\beta_0-K_0\gamma_0)-K_mK_0(G_H\alpha_1+C_H\alpha_0)}$

TABLE 3.2.1 (Continued)

$S_{K_0}^{Q_P}$	$= \frac{1}{2}\left[\frac{-K_0\gamma_0(G_H+G_0+G_L)-K_mK_0G_H\alpha_0}{(G_0+G_H+G_L)(\beta_0-K_0\gamma_0)-K_mK_0G_H\alpha_0} + \frac{K_0\gamma_1(C_H+C_0+C_L)-K_mK_0C_H\alpha_1}{(C_0-C_H-C_L)(\beta_1-\beta_0\gamma_1)-K_mK_0C_H\alpha_1}\right]$ $- \frac{-K_0[\gamma_1(G_0+G_H+G_L)+\gamma_0(C_0+C_H+C_L)]-K_mK_0(G_H\alpha_1+C_H\alpha_0)]}{(G_0-G_H+G_L)(\beta_1-K_0\gamma_1)+(C_0+C_H+C_L)(\beta_0-K_0\gamma_0)-K_mK_0(G_H\alpha_1+C_H\alpha_0)}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1
$S_{K_m}^{Q_P}$	$= \frac{1}{2}\left[\frac{-K_mK_0G_H\alpha_0}{(G_H+G_0-G_L)(\beta_0-K_0\gamma_0)-K_mK_0G_H\alpha_0} + \frac{-K_mK_0C_H\alpha_1}{(C_H+C_0+C_L)(\beta_1-\beta_0\gamma_1)-K_mK_0C_H\alpha_1}\right]$ $- \frac{-K_mK_0(G_H\alpha_1+C_H\alpha_0)}{(G_H+G_0+G_L)(\beta_1-K_0\gamma_1)+(C_H-C_0+C_L)(\beta_0-K_0\gamma_0)-K_mK_0(G_H\alpha_1+C_H\alpha_0)}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1
POLYNOMIAL DECOMPOSITION TYPE 9B	
t_v	$= \frac{K_0(\alpha_1S+\alpha_0)}{\beta_1S+\beta_0}$
T_v	$= \frac{K_mK_0(G_0-C_0S)(\alpha_1S+\alpha_0)}{S^2[\beta_1(C_H+C_F+C_0+C_E)-K_m\beta_1C_F-K_mK_0C_H\alpha_1] + S[\beta_1(G_H+G_F+G_0+G_L)+\beta_0(C_H+C_F+C_0+C_L)-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0(\alpha_1G_H+\alpha_0C_H)] + [\beta_0(G_H+G_F+G_0+G_L)-K_m\beta_0G_F-K_mK_0G_H\alpha_0]}$

TABLE 3.2.1 (Continued)

Q_p	$= \dfrac{\{[\beta_1(C_H+C_F+C_0+C_L)-K_m\beta_1C_F-K_mK_0C_H\alpha_1][\beta_0(G_H+G_F+G_0+G_L)-K_m\beta_0G_F-K_mK_0G_H\alpha_0]\}^{\frac{1}{2}}}{\beta_1(G_H-G_F-G_0-G_L)-\beta_0(C_H-C_F-C_0-C_L)-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0(G_H\alpha_1-C_H\alpha_0)}$
$S_{K_0}^{Q_p}$	$= \dfrac{1}{2}\left[\dfrac{-K_mK_0G_H\alpha_0}{\beta_0(G_H+G_F+G_0-G_L)-K_m\beta_0G_F-K_mK_0G_H\alpha_0} + \dfrac{-K_mK_0C_H\alpha_1}{\beta_1(C_H-C_F-C_0-C_L)-K_m\beta_1C_F-K_mK_0C_H\alpha_1}\right] - \dfrac{-K_mK_0(G_H\alpha_1+C_H\alpha_0)}{\beta_1(G_H-G_F-G_0+G_L)+\beta_0(C_H+C_F+C_0-C_L)-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0(G_H\alpha_1+C_H\alpha_0)}$ Bounds of $S_{K_0}^{Q_p}$ when all terms in D(S) are positive: -1,1
$S_{K_m}^{Q_p}$	$= \dfrac{1}{2}\left[\dfrac{-K_m\beta_0G_F-K_mK_0G_H\alpha_0}{\beta_0(G_H-G_F-G_0-G_L)-K_m\beta_0G_F-K_mK_0G_H\alpha_0} + \dfrac{-K_m\beta_1C_F-K_mK_0C_H\alpha_1}{\beta_1(C_H+C_F-C_0-C_L)-K_m\beta_1C_F-K_mK_0C_H\alpha_1}\right] - \dfrac{-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0(G_H\alpha_1+C_H\alpha_0)}{\beta_1(G_H-G_F-G_0+G_L)+\beta_0(C_H+C_F+C_0+C_L)-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0(G_H\alpha_1+C_H\alpha_0)}$ Bounds of $S_{K_m}^{Q_p}$ when all terms in D(S) are positive: -1,1
POLYNOMIAL DECOMPOSITION TYPE 10	
t_v	$= \dfrac{K_0(\alpha_1S+\alpha_0)}{(\beta_1-K_0\alpha_1)S-(\beta_0-K_0\alpha_0)}$

TABLE 3.2.1 (Continued)

$$T_Y = \frac{K_m K_0 (G_0 + C_0 S)(\alpha_1 S + \alpha_0)}{\begin{aligned} &S^2[(C_H - C_F + C_0 - C_L)(\beta_1 - K_0\gamma_1) - K_m\beta_1 C_F + K_m K_0(\gamma_1 C_F - \alpha_1 C_H)] \\ &+ S[(G_H - G_F + G_0 - G_L)(\beta_1 - K_0\gamma_1) + (C_H - C_F + C_0 - C_L)(\beta_0 - K_0\gamma_0) - K_m(\beta_1 G_F + \beta_0 C_F) + K_m K_0(\gamma_1 G_F + \gamma_0 C_F - \alpha_1 G_H - \alpha_0 C_H)] \\ &+ [(G_H - G_F + G_0 - G_L)(\beta_0 - K_0\gamma_0) - K_m\beta_0 G_F + K_m K_0(\gamma_0 G_F - \alpha_0 G_H)] \end{aligned}}$$

$$Q_P = \frac{\{[(C_H - C_F - C_0 - C_L)(\beta_1 - K_0\gamma_1) - K_m\beta_1 C_F - K_m K_0(\gamma_1 C_F - \alpha_1 C_H)][(G_H + G_F + G_0 + G_L)(\beta_0 - K_0\gamma_0) - K_m\beta_1 G_F + K_m K_0(\gamma_0 G_F - \alpha_0 G_H)]\}^{\frac{1}{2}}}{(G_H + G_F - G_0 - G_L)(\beta_1 - K_0\gamma_1) - (C_H - C_F - C_0 - C_L)(\beta_0 - K_0\gamma_0) - K_m(\beta_1 G_F + \beta_0 C_F) - K_m K_0(\gamma_1 G_F + \gamma_0 C_F - \alpha_1 G_H - \alpha_0 C_H)}$$

$$S_{K_0}^{Q_P} = \frac{1}{2}\left[\frac{-K_0\gamma_1(C_H - C_F + C_0 - C_L) + K_m K_0(\gamma_1 C_F - \alpha_1 C_H)}{(C_H - C_F - C_0 - C_L)(\beta_1 - K_0\gamma_1) - K_m\beta_1 C_F - K_m K_0(\gamma_1 C_F - \alpha_1 C_H)} + \frac{-K_0\gamma_0(G_H + G_0 + G_F + G_L) + K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}{(G_H + G_F + G_0 + G_L)(\beta_0 - K_0\gamma_0) - K_m\beta_0 G_F - K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}\right]$$

$$- \left[\frac{-K_0[\gamma_1(G_F + G_H + G_0 - G_L) + \gamma_0(C_F + C_H + C_0 + C_L)] + K_m K_0(\gamma_1 G_F + \gamma_0 C_F - \alpha_1 G_H - \alpha_0 C_H)}{(G_H - G_F - G_0 + G_L)(\beta_1 - K_0\gamma_1) + (C_H - C_F - C_0 + C_L)(\beta_0 - K_0\gamma_0) - K_m(\beta_1 G_F + \beta_0 C_F) + K_m K_0(\gamma_1 G_F + \gamma_0 C_F - \alpha_1 G_H - \alpha_0 C_H)}\right]$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = \frac{1}{2}\left[\frac{-K_m\beta_1 C_F - K_m K_0(\gamma_1 C_F - \alpha_1 C_H)}{(C_H - C_F - C_0 - C_L)(\beta_1 - K_0\gamma_1) - K_m\beta_1 C_F - K_m K_0(\gamma_1 C_F - \alpha_1 C_H)} + \frac{-K_m\beta_0 G_F + K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}{(G_H + G_F + G_0 + G_L)(\beta_0 - K_0\gamma_0) - K_m\beta_0 G_F + K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}\right]$$

$$- \left[\frac{-K_m(\beta_1 G_F + \beta_0 C_F) + K_m K_0(\gamma_1 G_F + \gamma_0 C_F - \alpha_1 G_H - \alpha_0 C_H)}{(G_H + G_0 - G_F - G_L)(\beta_1 - K_0\gamma_1) + (C_H - C_0 + C_F + C_L)(\beta_0 - K_0\gamma_0) + K_m(\beta_1 G_F + \beta_0 C_F) - K_m K_0(\gamma_1 G_F + \gamma_0 C_F - \alpha_1 G_H - \alpha_0 C_H)}\right]$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 3.2.1. (Continued)

POLYNOMIAL DECOMPOSITION TYPE 15

$$t_v = \frac{K_0(\alpha_1 S+\alpha_0)}{(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)}$$

$$T_v = \frac{K_0(G_0+C_0S)(\alpha_1S+\alpha_0)}{S^2[(C_H+C_F+C_0+C_L)(\beta_1-K_0\gamma_1)-K_m\beta_1C_F+K_mK_0(\gamma_1C_F-\alpha_1C_H)] + S[(G_H+G_F+G_0+G_L)(\beta_1-K_0\gamma_1)+(C_H+C_F+C_0+C_L)(\beta_0-K_0\gamma_0)] + [(C_H+C_F+C_0+C_L)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F+K_mK_0(\gamma_0G_F-\alpha_0G_H)]}$$

$$Q_P = \frac{\{[(G_H+G_F+G_0-G_L)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F+K_mK_0(\gamma_0G_F-\alpha_0G_H)][(C_H+C_F+C_0+C_L)(\beta_1-K_0\gamma_1)-K_m\beta_1C_F+K_mK_0(\gamma_1C_F-\alpha_1C_H)]\}^{\frac{1}{2}}}{(G_H+G_F+G_0+G_L)(\beta_1-K_0\gamma_1)-(C_H+C_F-C_0+C_L)(\beta_0-K_0\gamma_0)-K_m(\beta_1G_F+\beta_0C_F)}$$

$$S_{K_0}^{Q_P} = \frac{1}{2}\left[\frac{-K_0\gamma_0(G_H-G_0+G_F+G_L)+K_mK_0(\gamma_0G_F-\alpha_0G_H)}{(G_H+G_0-G_F-G_L)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F-K_mK_0(\gamma_0G_F-\alpha_0G_H)} + \frac{-K_0\gamma_1(G_F+G_H+G_L)-K_0\alpha_0(C_F+C_H+C_0+C_L)}{(C_H+C_0+C_F+C_L)(\beta_1-K_0\gamma_1)-K_m\beta_1C_F+K_mK_0(\gamma_1C_F-\alpha_1C_H)}\right]$$

$$-\left[\frac{-K_0\gamma_1(C_H+C_0+C_F+C_L)-K_0\gamma_0(G_H+G_0+G_F-G_L)}{(G_H+G_0+G_F+G_L)(\beta_1-K_0\gamma_1)+(C_H+C_0+C_F+C_L)(\beta_0-K_0\gamma_0)-K_m(\beta_1G_F+\beta_0C_F)}\right]$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = \frac{1}{2}\left[\frac{K_m\beta_0G_F+K_mK_0(\gamma_0G_F-\alpha_0G_H)}{(G_H+G_0+G_F+G_L)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F+K_mK_0(\gamma_0G_F-\alpha_0G_H)} + \frac{-K_m\beta_1C_F+K_mK_0(\gamma_1C_F-\alpha_1C_H)}{(C_H+C_0+C_F+C_L)(\beta_1-K_0\gamma_1)-K_m\beta_1C_F+K_mK_0(\gamma_1C_F-\alpha_1C_H)}\right]$$

$$-\left[\frac{-K_m(\beta_1G_F+\beta_0C_F)}{(G_H+G_0+G_F-G_L)(\beta_1-K_0\gamma_1)+(C_H-C_0+C_F+C_L)(\beta_0-K_0\gamma_0)-K_m(\beta_1G_F+\beta_0C_F)}\right]$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 3.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 18	
t_V	$= \dfrac{K_0(\alpha_1 S+\gamma_0)}{(\delta_1-K_0\gamma_1)S-(\delta_0-K_0\gamma_0)}$
T_V	$= \dfrac{K_mK_0(G_0+C_0S)(\alpha_1S+\alpha_0)}{S^2[(C_H+C_F+C_0+C_L)(\delta_1-K_0\gamma_1)-K_m\delta_1C_F] + S[(G_H+G_F+C_0+G_L)(\delta_1-K_0\gamma_1)+(C_H+C_F+G_0+C_L)(\delta_0-K_0\gamma_0)-K_m(\delta_1G_F+\delta_0C_F) + K_mK_0(\gamma_1G_F+\gamma_0C_F-\alpha_1G_H-\alpha_0C_H)] + [(G_H+G_F+G_0+G_L)(\delta_0-K_0\gamma_0)-K_m\delta_0G_F]}$
Q_P	$= \dfrac{\{[(C_H-C_F+C_0-C_L)(\delta_1-K_0\gamma_1)-K_m\delta_1C_F][(G_H-G_F-G_0)(\delta_0-K_0\gamma_0)-K_m\delta_0G_F]\}^{\frac{1}{2}}}{(G_H-G_F-G_0-G_L)(\delta_1-K_0\gamma_1)+(C_H-C_F-C_0-C_L)(\delta_0-K_0\gamma_0)-K_m(\delta_1G_F+\delta_0C_F)+K_mK_0(\gamma_1G_F+\gamma_0C_F-\alpha_1G_H-\alpha_0C_H)}$
$S^{Q_P}_{K_0}$	$= \dfrac{1}{2}\left[\dfrac{-K_0\gamma_1(C_H+C_F-C_0-C_L)}{(C_H-C_F+C_0-C_L)(\delta_1-K_0\gamma_1)-K_m\delta_1C_F} + \dfrac{-K_0\gamma_0(G_H+G_F-G_0-G_L)}{(G_H-G_F-G_0-G_L)(\delta_0-K_0\gamma_0)-K_m\delta_0G_F}\right]$ $- \left[\dfrac{-K_0\gamma_1(G_H-G_F+G_0+G_L)-K_0\gamma_0(G_H+G_F+G_0+G_L)+K_mK_0(\gamma_1G_F\gamma_0C_F-\alpha_1G_H-\alpha_0C_H)}{(G_H+G_0-G_F-G_L)(\delta_1-K_0\gamma_1)+(C_H-C_F-C_0-C_L)(\delta_0-K_0\gamma_0)-K_m(\delta_1G_F+\delta_0C_F)+K_mK_0(\gamma_1G_F+\gamma_0C_F-\alpha_1G_H-\alpha_0C_H)}\right]$ Bounds of $S^{Q_P}_{K_0}$ when all terms in D(S) are positive: -1,1
$S^{Q_P}_{K_m}$	$= \dfrac{1}{2}\left[\dfrac{K_m\delta_1C_F}{(C_H\ C_F\ C_0\ C_L)(\delta_1-K_0\gamma_1)-K_m\delta_1C_F}\ \dfrac{K_m\delta_0G_F}{(G_H\ G_F\ G_0\ G_L)(\delta_0-K_0\gamma_0)-K_m\delta_0G_F}\right]$ $- \left[\dfrac{-K_m(\delta_1G_F\ \delta_0C_F)\ K_mK_0(\gamma_1G_F\ \gamma_0C_F-\alpha_1G_H-\alpha_0C_H)}{(G_H\ G_0\ G_F\ G_L)(\delta_1-K_0\gamma_1)\ (C_H\ C_F\ C_0\ C_L)(\delta_0-K_0\gamma_0)-K_m(\delta_1G_F\ \delta_0C_F)\ K_mK_0(\gamma_1G_F\ \gamma_0C_F-\alpha_1G_H-\alpha_0C_H)}\right]$ Bounds of $S^{Q_P}_{K_m}$ when all terms in D(S) are positive: -1,1

TABLE 3.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 19

$$t_V = \frac{K_0(\alpha_1 S+\alpha_0)}{(\beta_1-K_0\gamma_1)S-(\beta_0-K_0\gamma_0)}$$

$$T_V = \frac{K_mK_0(G_0-C_0S)(\alpha_1S+\alpha_0)}{S^2[(C_H+C_F+C_0-C_L)(\beta_1-K_0\gamma_1)-K_m\beta_1C_F] + S[(G_H+G_F+G_0-G_L)(\beta_1-K_0\gamma_1)+(C_H+C_F+C_0+C_L)(\beta_0-K_0\gamma_0)-K_m(\beta_1G_F+\beta_0C_F)] + [(G_H+G_F+G_0+G_L)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F]}$$

$$Q_P = \frac{\{[(C_H+C_F+C_0-C_L)(\beta_1-K_0\gamma_1)-K_m\beta_1C_F][(G_H+G_F+G_0+G_L)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F]\}^{\frac{1}{2}}}{(G_H-G_F-G_0-G_L)(\beta_1-K_0\gamma_1) - (C_H-C_F+C_0-C_L)(\beta_0-K_0\gamma_0) - K_m(\beta_1G_F-\beta_0C_F)}$$

$$S_{K_0}^{Q_P} = \frac{1}{2}\left[\frac{-K_0\gamma_1(C_H-C_F+C_0+C_L)}{(C_H+C_F+C_0+C_L)(\beta_1-K_0\gamma_1)-K_m\beta_1C_F} + \frac{-K_0\gamma_0(G_H+G_F+G_0+G_L)}{(G_H+G_F+G_0+G_L)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F}\right]$$

$$-\left[\frac{-K_0\gamma_1(G_F+G_H+G_0-G_L)-K_0\gamma_0(C_F+C_H+C_0-C_L)}{(G_H-G_0+G_F-G_L)(\beta_1-K_0\gamma_1)+(C_H-C_0-C_F+C_L)(\beta_0-K_0\gamma_0)-K_m(\beta_1G_F+\beta_0C_F)}\right]$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = \frac{1}{2}\left[\frac{-K_m\beta_1C_F}{(C_H-C_F+C_0+C_L)(\beta_1-K_0\gamma_1)-K_m\beta_1C_F} + \frac{-K_m\beta_0G_F}{(G_H+G_F+G_0+G_L)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F}\right]$$

$$-\left[\frac{-K_m(\beta_1G_F+\beta_0C_F)}{(G_H-G_0+G_F-G_L)(\beta_1-K_0\gamma_1)+(C_H-C_0+C_F-C_L)(\beta_0-K_0\gamma_0)-K_m(\beta_1G_F+\beta_0C_F)}\right]$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 3.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 22A
$t_V = \dfrac{K_0\alpha_1 S}{(\beta_1 - K_0\gamma_1)S + (\beta_0 - K_0\gamma_0)}$
$T_V = \dfrac{K_m K_0 (G_0 + C_0 S)\alpha_1 S}{S^2[(C_0+C_L)(\beta_1 - K_0\gamma_1)] + S[(G_H+G_0+G_L)(\beta_1 - K_0\gamma_1) + (C_0+C_L)(\beta_0 - K_0\gamma_0) - K_m K_0 \alpha_1 G_H] + [(G_0+G_H+G_L)(\beta_0 - K_0\gamma_0)]}$
$Q_P = \dfrac{\{(C_0 - C_L)(\beta_1 - K_0\gamma_1)(G_0+G_H+G_L)(\beta_0 - K_0\gamma_0)\}^{\frac{1}{2}}}{(G_H+G_0-G_L)(\beta_1 - K_0\gamma_1) - (C_0+C_L)(\beta_0 - K_0\gamma_0) - K_m K_0 G_H \alpha_1}$
$S_{K_0}^{Q_P} = \dfrac{1}{2}\left[\dfrac{-K_0\gamma_1}{\beta_1 - K_0\gamma_1} - \dfrac{K_0\gamma_0}{\beta_0 - K_0\gamma_0}\right] - \dfrac{-K_0\gamma_0(C_0+C_L) - K_0\gamma_1(G_H+G_0+G_L)}{(G_H+G_0+G_L)(\beta_1 - K_0\gamma_1) + (C_0 - C_L)(\beta_0 - K_0\gamma_0) - K_m K_0 G_H \alpha_1}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1
$S_{K_m}^{Q_P} = \dfrac{K_m K_0 G_H \alpha_1}{(G_H+G_0-G_L)(\beta_1 - K_0\gamma_1) - (C_0 - C_L)(\beta_0 - K_0\gamma_0) - K_m K_0 G_H \alpha_1}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,0
$t_v = \dfrac{K_0\alpha_0}{(\beta_1 - K_0\gamma_1)S + (\beta_0 - K_0\gamma_0)}$

TABLE 3.2.1 (Continued)

T_v	$= \dfrac{K_m K_0 (G_0 - C_0 S)\alpha_0}{S^2 (C_H - C_0 + C_L)(\beta_1 - K_0\gamma_1) + S[(G_0 - G_L)(\beta_1 - K_0\gamma_1) - (C_H - C_0 + C_L)(\beta_0 - K_0\gamma_0) - K_m K_0 C_H \alpha_0] - (G_0 + G_L)(\beta_0 - K_0\gamma_0)}$
Q_P	$= \dfrac{\{(C_H + C_0 - C_L)(\beta_1 - K_0\gamma_1)(G_0 - G_L)(\beta_0 - K_0\gamma_0)\}^{\frac{1}{2}}}{(G_0 + G_L)(\beta_1 - K_0\gamma_1) - (C_H - C_0 - C_L)(\beta_0 - K_0\gamma_0) - K_m K_0 C_H \alpha_0}$
$S_{K_0}^{Q_P}$	$= \dfrac{1}{2}\left[\dfrac{K_0\gamma_1}{\beta_1 - K_0\gamma_1} - \dfrac{K_0\gamma_0}{\beta_0 - K_0\gamma_0}\right] - \dfrac{-K_0\gamma_1(G_0 + G_L) - K_0\gamma_0(C_H + C_0 + C_L) - K_m K_0 C_H \alpha_0}{(G_0 + G_L)(\beta_1 - K_0\gamma_1) - (C_H - C_0 - C_L)(\beta_0 - K_0\gamma_0) - K_m K_0 C_H \alpha_0}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1
$S_{K_m}^{Q_P}$	$= \dfrac{K_m K_0 C_H \alpha_0}{(G_0 + G_L)(\beta_1 - K_0\gamma_1) - (C_H - C_0 - C_L)(\beta_0 - K_0\gamma_0) - K_m K_0 C_H \alpha_0}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,0
POLYNOMIAL DECOMPOSITION TYPE 22B	
t_v	$= \dfrac{K_0\alpha_0}{\beta_1 S - \beta_0}$
T_v	$= \dfrac{K_m K_0 (G_0 + C_0 S)\alpha_0}{S^2 \beta_1 [C_H - C_F + C_0 + C_L - K_m C_F] - S[(G_F - G_0 + G_L)\beta_1 + (C_H + C_F + C_0 + C_L)\beta_0 - K_m(\beta_1 G_F + \beta_0 C_F) - K_m K_0 \alpha_0 C_H] + \beta_0 [G_F + G_0 + G_L - K_m G_F]}$

TABLE 3.2.1 (Continued)

$$Q_P = \frac{\{[\beta_1(C_H+C_F+C_0+C_L-K_mC_F)\beta_0(G_F+G_0-K_mG_F)]\}^2}{(G_F+G_0-G_L)\beta_1-\beta_0(C_H-C_F-C_0-C_L)+K_m(\beta_1G_F+\beta_0C_F)-K_mK_0\alpha_0C_H}$$

$$S_{K_0}^{Q_P} = \frac{K_mK_0C_H\alpha_0}{\beta_1(G_F-G_0-G_L)-\beta_0(C_H+C_F+C_0+C_L)-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0\alpha_0C_H}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,0

$$S_{K_m}^{Q_P} = \frac{1}{2}\left[\frac{-K_mG_F}{G_F-G_0-G_L-K_mG_F} + \frac{-K_mC_F}{C_H-C_0-C_F+C_L+K_mC_F}\right] - \frac{-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0C_H\alpha_0}{\beta_1(G_F+G_0-G_L)-\beta_0(C_H+C_F+C_0+C_L)-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0C_H\alpha_0}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$t_v = \frac{K_0\alpha_1S}{\beta_1S+\beta_0}$$

$$T_v = \frac{K_mK_0(G_0+C_0S)\alpha_1S}{S^2[\beta_1(C_F+C_0-C_L)-K_m\beta_1C_F]-S[\beta_1(G_H-G_F+G_0+G_L)-\beta_0(C_F+C_0+C_L)-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0G_H\alpha_1]+[\beta_0(G_H+G_F+G_0+G_L)-K_m\beta_0G_F]}$$

$$Q_P = \frac{\{\beta_1(C_F-C_0-C_L-K_mC_F)\beta_0(G_H-G_F+G_0+G_2-K_mG_F)\}^{\frac{1}{2}}}{\beta_1(G_H+G_F+G_0-G_L)-\beta_0(C_F-C_0+C_L)-K_m(\beta_1G_F-\beta_0C_F)-K_mK_0G_H\alpha_1}$$

$$S_{K_0}^{Q_P} = \frac{K_mK_0G_H\alpha_1}{\beta_1(G_H-G_F-G_0-G_L)-\beta_0(C_F-C_0-C_L)-K_m(\beta_1G_F+\beta_0C_F)-K_mK_0G_H\alpha_1}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,0

TABLE 3.2.1 (Continued)

$$S_{K_m}^{Q_P} = \frac{1}{2}\left[\frac{-K_m G_F}{G_H - G_F + G_0 - G_L - K_m G_F} + \frac{-K_m C_F}{C_F + C_0 - C_L - K_m C_F}\right] - \frac{-K_m(\beta_1 G_F + \beta_0 C_F) - K_m K_0 \alpha_1 G_H}{\beta_1(G_H - G_F + G_0 - G_L) + \beta_0(C_F + C_0 - C_L) - K_m(\beta_1 G_F + \beta_0 C_F) - K_m K_0 \alpha_1 G_H}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 3.2.2

QUANTITIES WHICH CAN OR HAVE TO BE SET TO ZERO IN ORDER TO OBTAIN A T_v WHEN t_v IS OF FIRST ORDER

TABLE 3.2.2

No.	Active terms present in s^2	s	s^0	Quantities to be set to zero G_F	C_F	G_H	C_H	G_L	C_L	G_0	C_0	α_1	γ_2	γ_1	γ_0	B_2	B_1	B_0	$\left(\frac{\text{Numerator}}{K_m K_0}\right)$	
1		$K_m K_0$		O	O		O	X	X	Δ			O	O	O				$\alpha_1 C_0 S^2$	
				O	O		O	X			Δ		O	O	O				$\alpha_1 G_0 S$	(i)
				O	O		O						O	O	O				$\alpha_1 S(G_0+C_0 S)$	
				O	O	O			X	Δ		O		O	O				$\alpha_0 C_0 S$	(ii)
				O	O	O		X	X		Δ	O		O	O				$\alpha_0 G_0$	
				O	O	O						O		O	O				$\alpha_0(G_0+C_0 S)$	
4	$K_m K_0$	$K_m K_0$	$K_m K_0$	O	O			X	X	Δ				O	O				$C_0 S(\alpha_1 S+\alpha_0)$	
				O	O			X	X		Δ			O	O				$G_0(\alpha_1 S+\alpha_0)$	
				O	O			X	X					O	O				$(G_0+C_0 S)(\alpha_1 S+\alpha_0)$	
9a	$K_m K_0$ K_0	$K_m K_0$ K_0	$K_m K_0$ K_0	O	O			X	X	Δ									$C_0 S(\alpha_1 S+\alpha_0)$	
				O	O			X	X		Δ								$G_0(\alpha_1 S+\alpha_0)$	
				O	O			X	X										$(G_0+C_0 S)(\alpha_1 S+\alpha_0)$	
9b	$K_m K_0$ K_m	$K_m K_0$ K_m	$K_m K_0$ K_m					X	X	Δ				O	O				$C_0 S(\alpha_1 S+\alpha_0)$	
								X	X		Δ			O	O				$G_0(\alpha_1 S+\alpha_0)$	
								X	X					O	O				$(G_0+C_0 S)(\alpha_1 S+\alpha_0)$	

TABLE 3.2.[illegible] (CONTINUED)

No.	Active terms present in S^2	S	S^0	Quantities to be set to zero G_F	C_F	G_H	C_H	G_L	C_L	G_0	C_0	α_1	D	γ_1	γ_0	β_2	β_1	β_0	$\left(\frac{\text{Numerator}}{K_m K_0}\right)$	
10	$K_m K_0$	$K_m K_0$	$K_m K_0$				□	X	X	Δ									$C_0\alpha_1 S^2$	
	K_m	K_m	K_m			□		X	X	Δ		Δ							$\alpha_0 C_0 S$	(i)
	K_0	K_0	K_0				□	X			Δ								$G_0\alpha_1 S$	(ii)
						□		X	X		Δ	Δ							$G_0\alpha_0$	
								X	X	Δ									$C_0 S(\alpha_1 S+\alpha_0)$	
							□	X	X										$\alpha_1 S(C_0 S+G_0)$	
								X	X		Δ								$G_0(\alpha_1 S+\alpha_0)$	
							□	X	X			Δ							$\alpha_0(G_0+C_0 S)$	
								X	X										$(G_0+C_0 S)(\alpha_1 S+\alpha_0)$	
15	$K_m K_0$	K_m	$K_m K_0$				□	X	X	Δ							O		$\alpha_1 C_0 S^2$	
	K_m	K_0	K_m			□		X	X	Δ		Δ					O		$\alpha_0 C_0 S$	(i)
	K_0		K_0				□	X			Δ						O		$G_0\alpha_1 S$	(ii)
						□		X	X		Δ	Δ					O		$G_0\alpha_0$	
								X	X	Δ							O		$C_0 S(\alpha_1 S+\alpha_0)$	
						□		X	X			Δ					O		$G_0(\alpha_1 S+\alpha_0)$	
								X	X		Δ						O		$C_0 S(\alpha_1 S+\alpha_0)$	
						□		X	X			Δ					O		$\alpha_0(G_0+C_0 S)$	
								X	X								O		$(G_0+C_0 S)(\alpha_0+\alpha_1 S)$	

TABLE 3.2.3 (CONTINUED)

No.	Active terms present in S^2	S	S^0	Quantities to be set to zero G_F	C_F	G_H	C_H	G_L	G[illegible]	G_0	C_0	α_1	α_0	γ_1	γ_0	B_2	B_1	B_0	$\left(\frac{\text{Numerator}}{K_m K_0}\right)$	
18	K_m	K_m	K_m					X	X	Δ						O		O	$C_0S(\alpha_1S+\alpha_0)$	
	K_0	K_0	K_0					X	X		Δ					O		O	$G_0(\alpha_1S+\alpha_0)$	
		K_mK_0						X	X							O		O	$(G_0+C_0S)(\alpha_1S+\alpha_0)$	
19	K_m	K_m	K_m					X	X	Δ						O	O	O	$C_0S(\alpha_1S+\alpha_0)$	
	K_0	K_0	K_0					X	X		Δ					O	O	O	$G_0(\alpha_1S+\alpha_0)$	
								X	X							O	O	O	$(G_0+C_0S)(\alpha_1S+\alpha_0)$	
22a	K_0	K_0	K_0	O	O		O	X	X	Δ			O						$C_0\alpha_1S^2$	
		K_mK_0		O	O		O	X			Δ		O						$G_0\alpha_1S$	(i)
				O	O	O		X	X	Δ		O							α_0C_0S	(ii)
				O	O	O		X	X		Δ	O							α_0G_0	
				O	O			X	X										$\alpha_0(G_0+C_0S)$	
22b	K_m	K_m	K_m				O	X	X	Δ			O	O	O				$C_0\alpha_1S^2$	
		K_mK_0					O	X			Δ		O	O	O				$G_0\alpha_1S$	(i)
						O		X	X	Δ		O		O	O				$C_0\alpha_0S$	(ii)
						O		X	X		Δ	O		O	O				$G_0\alpha_0$	
								X	X					O	O				$(G_0+C_0S)(\alpha_0+\alpha_1S)$	

TABLE 3.2.2 (CONTINUED)

○	Elements which have to be set to zero to realize the polynomial decomposition under consideration. (These are already given in Table 2.5.1.)
△	Elements which have to be set to zero to realize the numerator given in the right-hand side column.
X	Elements which could be set to zero if desired in order to reduce the total number of elements.
□	Elements which should be set to zero to achieve the condition necessary for $\Delta\omega_p/\omega_p = 0$.

impossible to fulfill. It is clearly seen from Table 3.2.2 that L.P., B.P., and H.P. transfer functions are obtainable using polynomial decompositions 1, 10, 15, and 22.

From the foregoing discussion it is clear that the eight polynomial decompositions can be obtained by using only six generating functions. These are tabulated in Table 3.2.3 together with the polynomial decompositions which are realizable by each one. In fact the generating functions, LO and BO can be considered as two particular cases of LBO; also L1 as well as B1 can be considered as two particular cases of LB1, thus reducing the number of generating functions to two only.

The first order generating circuits can also be classified as:

(1) Passive circuits cascaded with an amplifier (called RI).

(2) Active circuits (called RII).

A detailed study of four representative TAC's will be given at the end of this chapter.

Table 3.2.4 gives as illustrative examples TAC's classified according to the kind of generating circuits from which they are derived. These TAC's have been chosen among those which realize LP , HP , or BP transfer functions with the use of no more than three capacitors. In each case the identification with the general parameters used in Table 3.2.1

TABLE 3.2.3

POLYNOMIAL DECOMPOSITIONS OBTAINABLE FROM GIVEN FIRST ORDER GENERATING FUNCTIONS t_v

TABLE 3.2.3

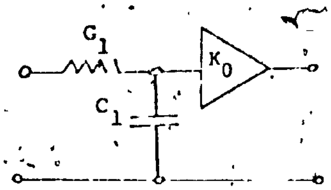
	Generating Transfer Function	Obtainable Polynomial Decompositions							
		1	4	9	10	15	18	19	22
L0	$\frac{K_0\alpha_0}{\beta_1 S+\beta_0}$	O							O (b)
B0	$\frac{K_0\alpha_1 S}{\beta_1 S+\beta_0}$	O							O (b)
LB0	$\frac{K_0(\alpha_1 S+\alpha_0)}{\beta_1 S+\beta_0}$		O	O (b)					
L1	$\frac{K_0\alpha_0}{(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)}$				O	O			O (a)
B1	$\frac{K_0\alpha_1 S}{(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)}$				O	O			O (a)
LB1	$\frac{K_0(\alpha_1 S\ \alpha_0)}{(\beta_1-K_0\gamma_1)+(\beta_0-K_0\gamma_0)}$		O	O (a)	O	O	O	O	O (a)

TABLE 3.2.4

DIFFERENT TACs OBTAINABLE FROM FIRST ORDER GENERATING FUNCTIONS REALIZED BY SACs

TABLE 3.2.4

GENERATING FUNCTION TYPE L_0

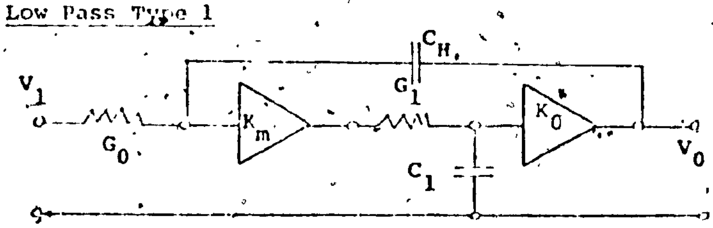


$$t_v = \frac{K_0 G_1}{C_1 S + G_1}$$

Identification with the general parameters.

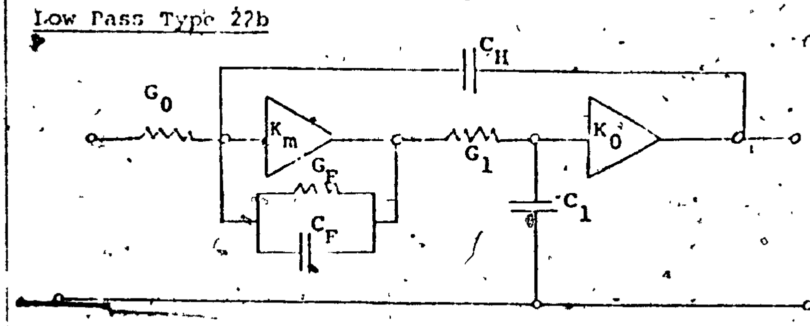
$\alpha_0 = \beta_0 = G_1$ $\qquad$ $\beta_1 = C_1$

Low Pass Type 1



$$T_{LP1} = \frac{K_m K_0 G_0 G_1}{S^2 C_1 C_H + S\,[G_1 C_H + C_1 G_0 - K_m K_0 G_1 C_H] + G_1 G_0}$$

Low Pass Type 22b

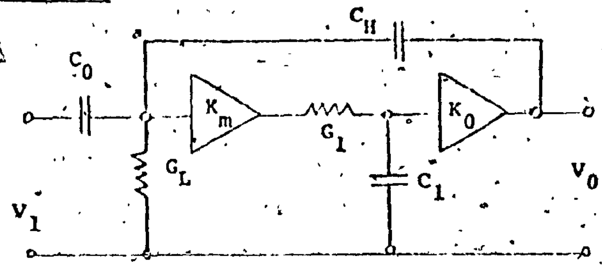


$$T_{LP22} = \frac{K_m K_0 G_0 G_1}{\begin{array}{l} S^2\,[(C_H + C_F) - K_m C_F]\,C_1 + \\ S\,[C_1 G_F + G_1 (C_H + C_F) - K_m (C_1 G_F + G_1 C_F)] \\ \quad - K_m K_0 C_H G_1 \\ + G_1\,[G_F + G_0 - K_m G_F] \end{array}}$$

with $\dfrac{C_F}{G_F} = \dfrac{C_H}{G_0}$

TABLE 3.2.4 (Continued)

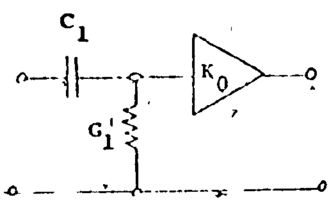
Band Pass Type 1(ii)



$$T_{BP1} = \frac{K_m K_0 G_1 C_0 S}{S^2 C_1 (C_0 + C_H) + S [G_1 (C_H + C_0) + C_1 G_L - K_m K_0 C_H G_1] + G_1 G_L}$$

TABLE 3.2.4 (Continued)

GENERATING FUNCTION TYPE B_0

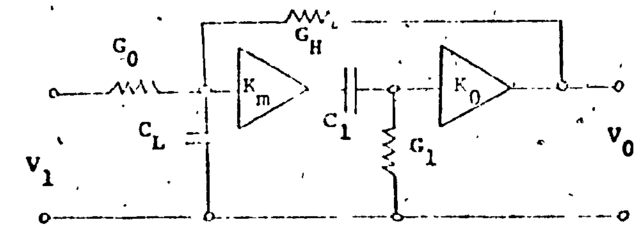


$$t_v = \frac{C_1 S}{C_1 S + G_1}$$

Identification with the general parameters.

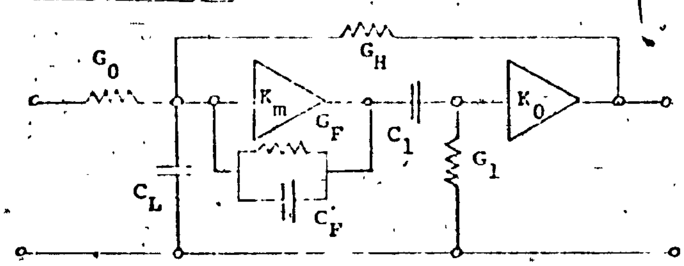
$$\alpha_1 = \beta_1 = C_1 \qquad \beta_0 = G_1$$

Band Pass Type 1(i)



$$T_{BP1} = \frac{K_m K_0 G_0 C_1 S}{C_1 C_L S^2 + S[G_1 C_L + C_1 (G_H + G_0) - K_m K_0 G_H C_1] + G_1 (G_H + G_0}$$

Band Pass Type 22b(i)

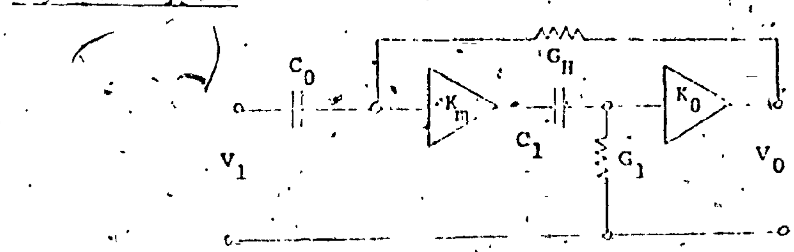


$$T_{BP22} = \frac{K_m K_0 G_0 C_1 S}{\begin{array}{l} S^2 [(C_F + C_L) - K_m C_F] C_1 + \\ S[C_1 (G_F + G_0) + G_1 (C_F + C_L) - K_m (C_1 G_F + G_1 C_F) \\ \quad - K_m K_0 G_H C_1] \\ + G_1 [G_H + G_F + G_L + G_0) - K_m G_F] \end{array}}$$

with $\dfrac{C_L}{G_H + G_0 + G_L} = \dfrac{C_F}{G_F}$

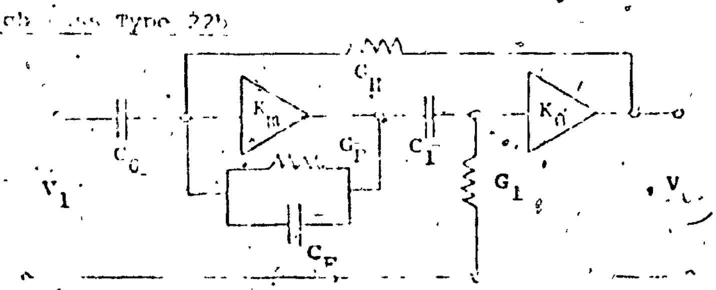
TABLE 3.2.4 (Continued)

High Pass Type 1



$$T_{HP1} = \frac{K_m K_0 C_1 C_0 S^2}{C_1 C_0 S^2 + S(G_1 C_0 + C_1 G_H - K_m K_0 C_H C_1) + G_1 G_H}$$

High Pass Type 2b



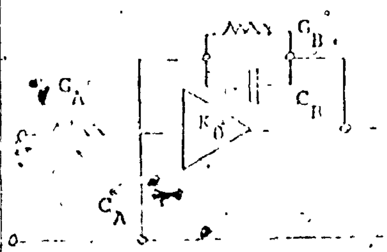
$$T_{HP2b} = \frac{K_m K_0 C_0 C_1 S^2}{\begin{aligned}&S^2[(C_F + C_0) - K_m C_F] + \\ &S[C_1(G_H + G_F) + G_1(C_F + C_0) \\ &\quad - K_m(C_1 G_F + G_1 C_F) - K_m K_0 G_H C_1] + \\ &G_1[(G_H + G_F + G_0) - K_m G_F]\end{aligned}}$$

with

$$\frac{C_0}{G_0 + G_H} = \frac{C_F}{G_F}$$

TABLE 3.2.4 (Continued)

GENERATING FUNCTION TYPE L_1

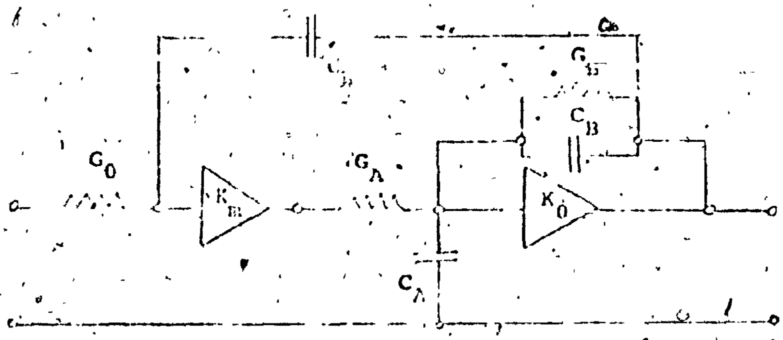


$$t_V = \frac{K_0 G_A}{[C_A (C_B - K_0 C_B)] \ldots [illegible] \ldots (G_B - K_0 G_B)}$$

Identification with the general parameters

$\beta_1 = C_A C_B$	$\gamma_1 = C_B$	$\alpha_1 = 0$
$\beta_0 = G_A G_B$	$\gamma_0 = G_B$	$\alpha_0 = G_A$

Low Pass [illegible] 22a



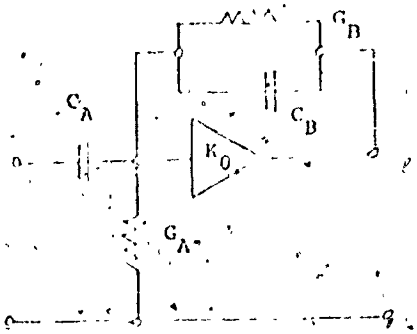
$$T_{LP22a} = \frac{K_m K_0 G_0 G_A}{S^2 (C_A C_B - K_0 C_B) C_H + S[(C_A + C_B - K_0 C_B) G_0 + (G_A + G_B - K_0 G_B) C_H - K_m K_0 G_A C_H] + (G_A + G_B - K_0 G_B) G_0}$$

with

$$\frac{C_A}{C_B} = \frac{G_A}{G_B}$$

TABLE 3.2.4 (Continued)

GENERATING FUNCTION TYPE B_1



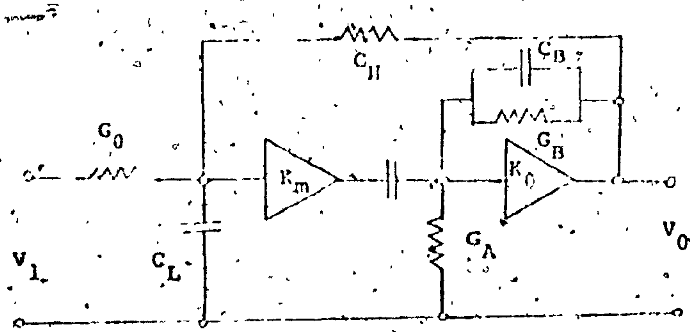
$$t_V = \frac{K_0 C_A s}{s[C_A + C_B - K_0 C_B] + [G_A + G_B - K_0 G_B]}$$

Identification with the general parameters

$\beta_1 = C_A + C_B$ $\quad \gamma_1 = C_B$ $\quad \alpha_1 = C_A$

$\beta_0 = G_A + G_B$ $\quad \gamma_0 = G_B$ $\quad \alpha_0 = 0$

Band Pass Type 22a(i)



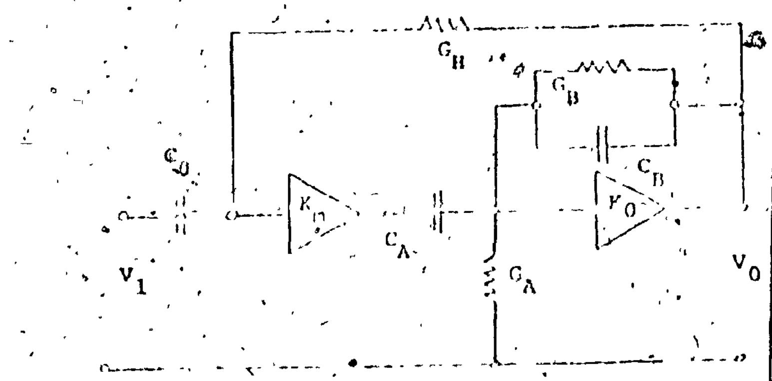
$$T_{BP22a} = \frac{K_m K_0 G_0 C_A s}{s^2 C_L (C_A + C_B - K_0 C_B) + s[(G_H + G_0)(C_A + C_B - K_0 C_B) + (G_A + G_B - K_0 G_B) C_L - K_m K_0 G_H C_A] + (G_A + G_B - K_0 G_B)(G_0 + G_H)}$$

with

$$\frac{C_A}{C_B} = \frac{G_A}{G_B}$$

TABLE 3.2.4 (Continued)

High Pass Type 22a



$$T_{HP22a} = \frac{K_m K_0 C_0 C_A S^2}{S^2 (C_A + C_B - K_0 C_B) C_0 + S[(C_A + C_B - K_0 C_B) G_H + (G_A + G_B - K_0 G_B) C_0 - K_m K_0 C_A G_A] + (G_A + G_B - K_0 G_B) G_H}$$

$$\frac{C_A}{C_B} = \frac{G_A}{G_B}$$

is given and hence expressions for the pole Q (Q_p) and the active sensitivities $S_{K_m}^{\omega_p}$, $S_{K_0}^{\omega_p}$ are easily obtainable.

Some of these circuits might have appeared in the literature, however it should be noted that they are derived here systematically as special cases so as to satisfy the zero GωSP condition.

3.3 A Band Pass Realization of Type 1

The overall realization consisting of two amplifiers is shown in Fig. 3.3.1a (Figs. 3.3.1b and 3.3.1c show the actual realizations of the two amplifiers contained in Fig. 3.3.1a). The generating circuit is of type RI. The generating transfer function t_v is of type B0. The TAC is a Band Pass filter of type 1. Its transfer function is given by

$$T_{BP1} = \frac{K_m K_0 G_0 C_1 S}{C_1 C_L S^2 + [G_1 C_L + C_1(G_0+G_H) - K_m K_0 C_1 G_H] S + G_1(G_0+G_H)} \tag{3.3.1}$$

$$K_m = 1 + \frac{G_A}{G_B} \tag{3.3.1a}$$

$$K_0 = 1 + \frac{G_{A'}}{G_{B'}} \tag{3.3.1b}$$

For the transfer function given in Eqn. 3.3.1 we have

$$\omega_p = \left[\frac{G_1 G_H}{C_1 C_L}\left(1+\frac{G_0}{G_H}\right)\right]^{\frac{1}{2}} \tag{3.3.2}$$

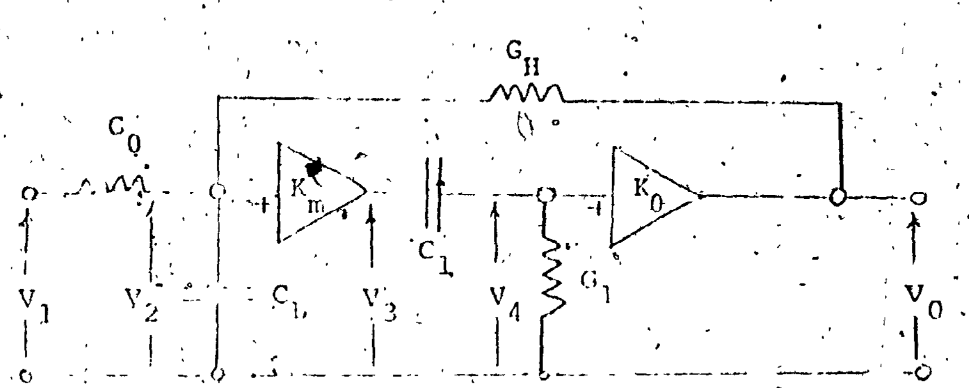


FIGURE 3.3.1a

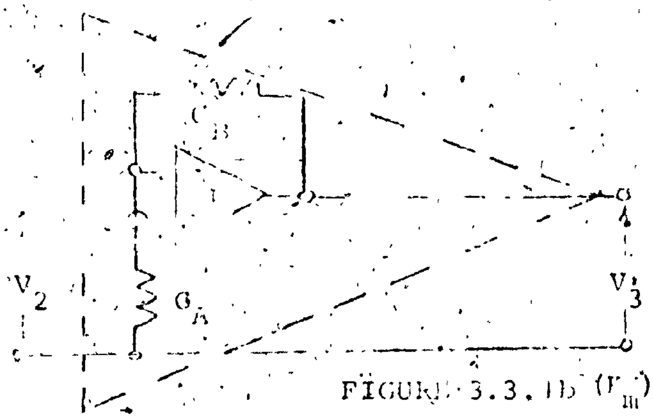


FIGURE 3.3.1b (K_m)

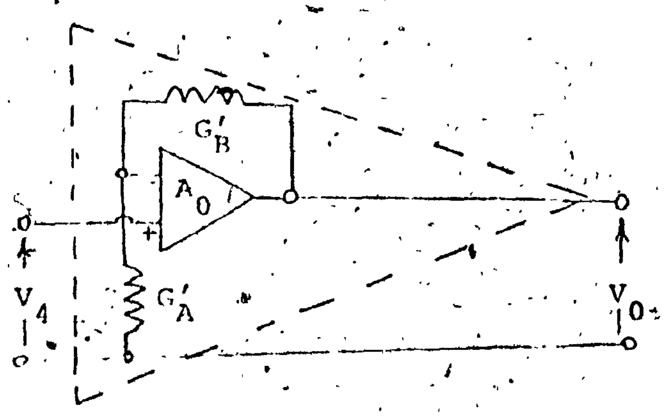


FIGURE 3.3.1c (K_0)

FIGURE 3.3.1 A BAND PASS TYPE 1 TAC REALIZATION

$$S_{K_Q}^{\omega_P} = S_{K_m}^{\omega_P} = 0 \qquad (3.3.3)$$

$$Q_P = \frac{\left[\frac{C_L G_1}{C_1 G_H}\left(1+\frac{G_0}{G_H}\right)\right]^{\frac{1}{2}}}{1+\frac{G_0}{G_H}+\frac{C_L G_1}{C_1 G_H}-K_m K_0} \qquad (3.3.4)$$

$$S_{K_0}^{Q_P} = S_{K_m}^{Q_P} = \frac{K_m K_0}{1+\frac{G_0}{G_H}+\frac{C_L G_1}{C_1 G_H}-K_m K_0} \qquad (3.3.5)$$

$$|K_m S_{K_m}^{Q_P}| = \frac{K_m^2 K_0}{1+\frac{G_0}{G_H}+\frac{C_L G_1}{C_1 G_H}-K_m K_0} \qquad (3.3.6)$$

$$|K_0 S_{K_0}^{Q_P}| = \frac{K_m K_0^2}{1+\frac{G_0}{G_H}+\frac{C_L G_1}{C_1 G_H}-K_m K_0} \qquad (3.3.7)$$

$$F = \frac{(K_m+K_0) K_m K_0}{1+\frac{G_0}{G_H}+\frac{C_L G_1}{C_1 G_H}-K_m K_0} \qquad (3.3.8)$$

From Eqn. 3.3.4 it is possible to express the quantity $K_m K_0$ as:

$$K_m K_0 = 1+\frac{G_0}{G_H}+\frac{G_1 C_L}{G_H C_1} - \frac{1}{Q_P}\left[\frac{G_1 C_L}{G_H C_1}\left(1+\frac{G_0}{G_H}\right)\right]^{\frac{1}{2}} \qquad (3.3.9)$$

Hence using Eqn. 3.3.8 and 3.3.9

$$F = \frac{(K_m + K_0)\left[1+\frac{G_0}{G_H}+\frac{G_1}{G_H}\cdot\frac{C_L}{C_1} - \frac{1}{Q_P}\left[\frac{G_1}{G_H}\cdot\frac{C_L}{C_1}\left(1+\frac{G_0}{G_H}\right)\right]^{\frac{1}{2}}\right]}{\frac{1}{Q_P}\left[\frac{G_1}{G_H}\cdot\frac{C_L}{C_1}\left(1+\frac{G_0}{G_H}\right)\right]^{\frac{1}{2}}} \qquad (3.3.10)$$

From Eqn. 3.3.9 and Eqn. 3.3.10 it is ~~clear~~ that for any prescribed value of Q_p and for given values of the passive elements the smallest value of F is obtained if we let

$$K_m = K_0 = K \qquad (3.3.11)$$

Under this condition

$$K\left[1+\frac{G_0}{G_H}+\frac{G_1}{G_H}\cdot\frac{C_L}{C_1} - \frac{1}{Q_P}\left[\frac{C_L}{C_1}\cdot\frac{G_1}{G_H}\left(1+\frac{G_0}{G_H}\right)\right]^{\frac{1}{2}}\right] \qquad (3.3.12)$$

Assuming now

$$1+\frac{G_0}{G_H}+\frac{G_1}{G_H}\cdot\frac{C_L}{C_1} \gg \frac{1}{Q_P}\left[\frac{C_L}{C_1}\cdot\frac{G_1}{G_H}\left(1+\frac{G_0}{G_H}\right)\right]^{\frac{1}{2}} \qquad (3.3.13)$$

which is a valid assumption as long as the spreads are moderate and Q_p high and defining:

$$\frac{C_L}{C_1} = \gamma_c \qquad (3.3.14a)$$

$$\frac{G_1}{G_H} = \gamma_1 \qquad (3.3.14b)$$

$$\frac{G_0}{G_H} = \gamma_0 \qquad (3.3.14c)$$

We get

$$F_m \doteq 2Q_p \frac{[1+\gamma_0+\gamma_1\gamma_c]^{3/2}}{[\gamma_1\gamma_c(1+\gamma_0)]^{1/2}} \quad (3.3.15)$$

Further if we define

$$1 + \gamma_0 = x \quad (3.3.16a)$$

$$\gamma_1\gamma_c = \gamma_{1c} \quad (3.3.16b)$$

F_m becomes

$$F_m = 2Q_p \frac{[x+\gamma_{1c}]^{3/2}}{(x\gamma_{1c})^{1/2}} \quad (3.3.17)$$

Fig. 3.3.2 shows the variation of $\frac{F_m}{Q_p}$, versus γ_{1c} for several values of x. The lowest value of F_m is $5.2Q_p$ and is obtained for $\gamma_{1c} = 0.5$ and x = 1 which implies an unrestricted resistive spread. The following set of design equations appears to be a good compromise able to achieve low values of resistive spread γ_R and of F_m for specified values of Q and of the capacitive spread γ_c. Letting

$$\gamma_1 = \gamma_0 = \gamma \quad (3.3.19)$$

in the expression of $\frac{F_m}{Q_p}$ given in Eqn. (3.3.15). Minimization of $(\frac{F_m}{Q_p})$ using $\frac{\partial}{\partial\gamma}(\frac{F_m}{Q_p}) = 0$ yields

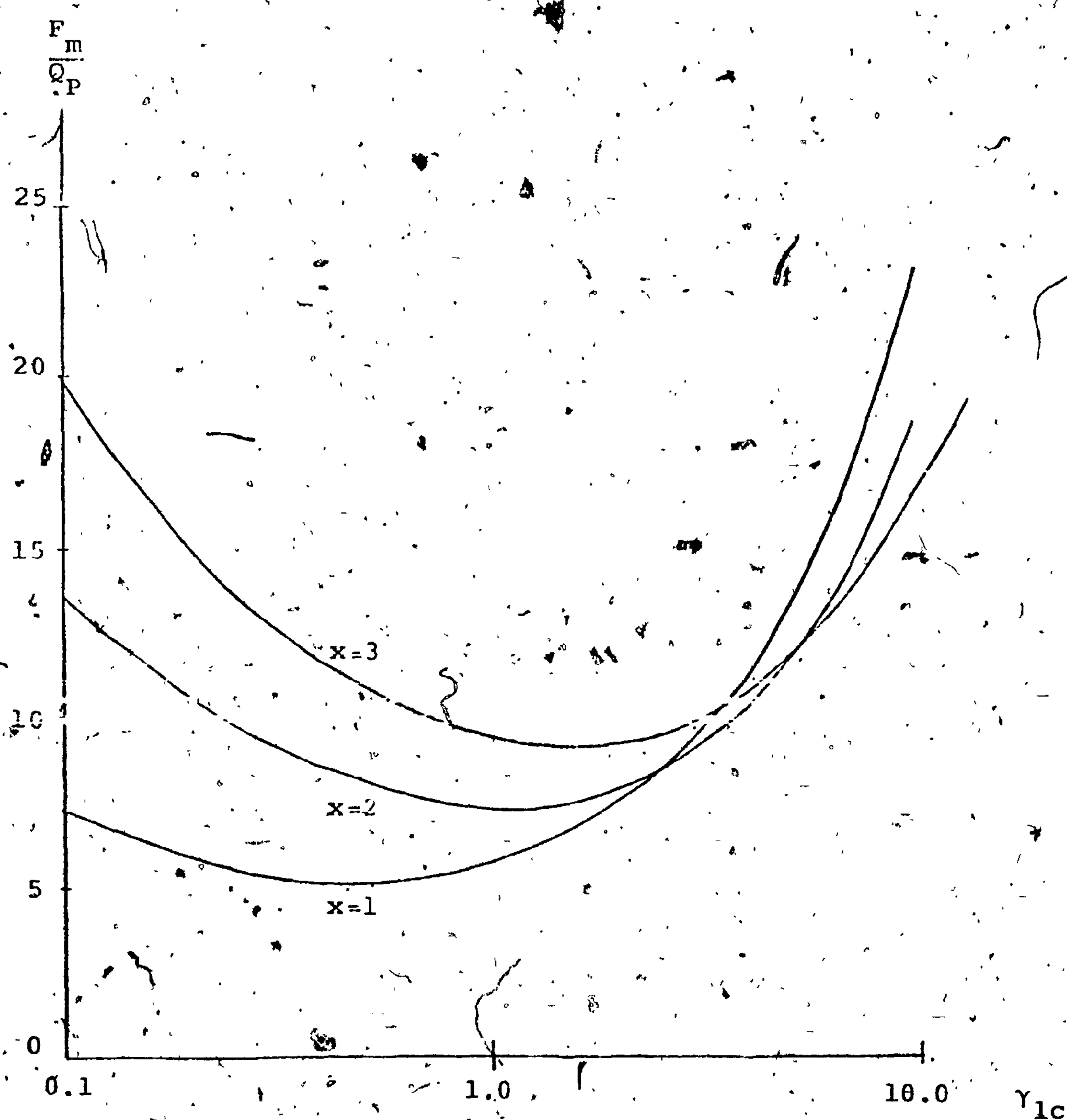


FIGURE 3.3.2 F_m/Q_P VERSUS γ_{1c} OF THE BAND PASS TYPE 1 TAC FOR SEVERAL VALUES OF x

$$\gamma = \frac{-\gamma_c+\{\gamma_c^2+\gamma_c+1\}^{1/2}}{1+\gamma_c} \tag{3.3.20}$$

The corresponding values of $K_m = K_0 \stackrel{\Delta}{=} K$ and $\frac{F_m}{Q_R}$ can be obtained respectively from Eqn. 3.3.12 and 3.3.15 as

$$K = [1+\gamma(1+\gamma_c) - \frac{1}{Q_p}\{\gamma\gamma_c(1+\gamma)\}^{1/2}]^{1/2} \tag{3.3.21}$$

and

$$\frac{F_m}{Q_p} = \frac{2[1+\gamma(1+\gamma_c)]^{3/2}}{[\gamma\gamma_c(1+\gamma)]^{1/2}} \tag{3.3.22}$$

Fig. 3.3.3 shows the variation of γ, $\frac{F_m}{Q_p}$, γ_R versus γ_c. ω_p and Q_p can be expressed as

$$\omega_p = \frac{G_H}{C_L}\gamma_c\gamma(1+\gamma)^{1/2} \tag{3.3.23a}$$

$$Q_p = \frac{\{\gamma\gamma_c(1+\gamma)\}^{1/2}}{1+\gamma(1+\gamma_c)-K^2} \tag{3.3.23b}$$

The elemental values are determined when ω_p, Q_p, γ_c (and hence γ) are prescribed using Eqns. 3.3.1a, b, 3.3.14, 3.3.19, 3.3.20, 3.3.21, 3.3.23.

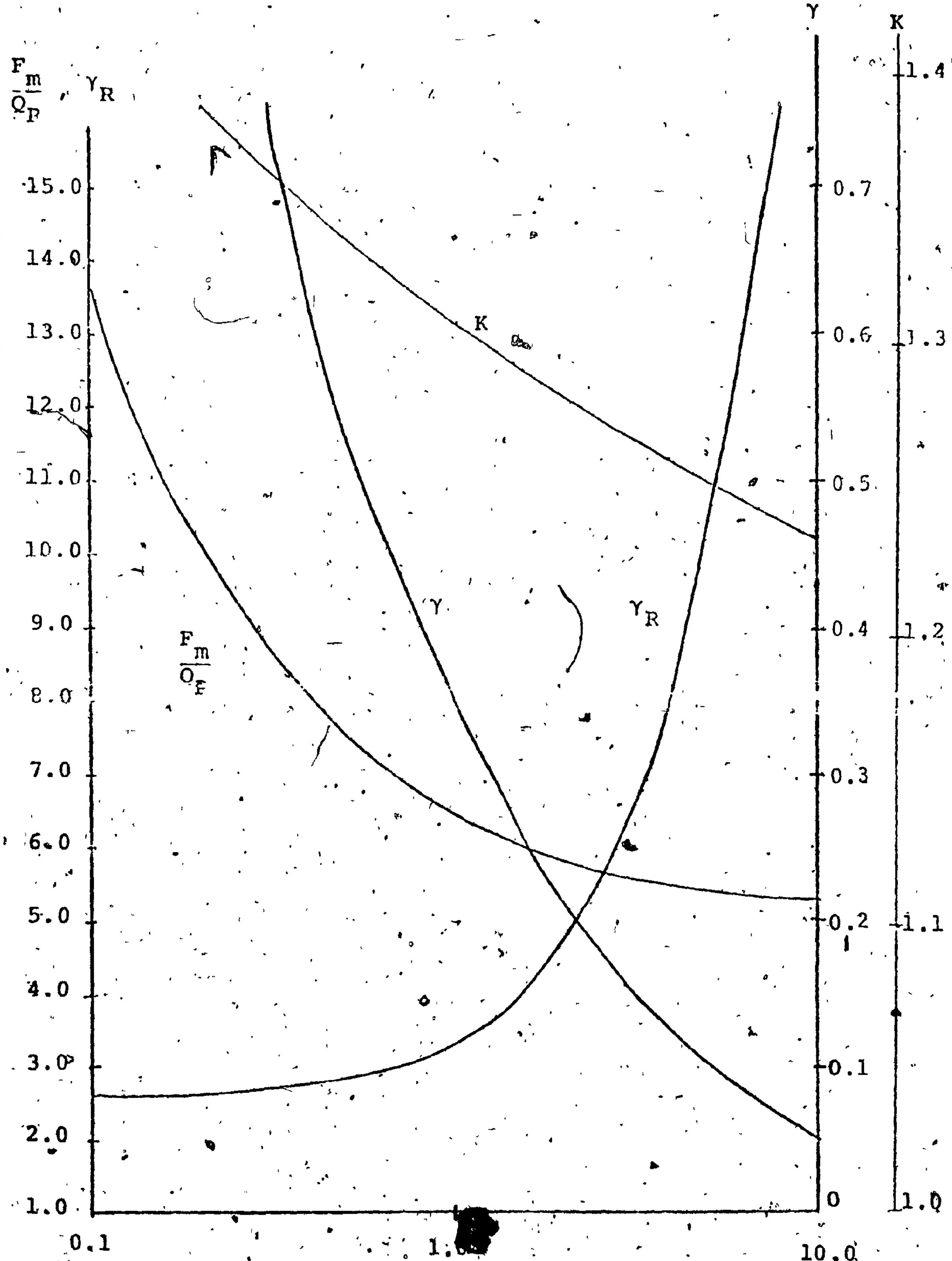


FIGURE 3.3.3 F_m/Q_P, K, γ, γ_R, OF THE BAND PASS TYPE 1 TAC, VERSUS γ_c

3.3.1 Experimental Results

The network shown in Fig. 4.3.1 was built up using discrete elements and tested. The results are summarized below:

Designed values:

$\omega_P = 6283.185$ Rad/sec. = 1000 Hz.

$Q_P = 50$

$\gamma_c = 1$

$\gamma_0 = \gamma_1 = 0.366$

$K_0 = K_m = 1.3107$

$C_1 = C_L = 15$ KPF.

$R_H = R_B = R'_B = 7.502$ KΩ

$R_1 = R_0 = 20.497$ KΩ

$R_A = R'_A = 24.145$ KΩ

GQSP Values:

$$\frac{\left|K_0 S_{K_0}^{Q_P}\right|}{Q_P} = \frac{\left|K_m S_{K_m}^{Q_P}\right|}{Q_P} = 3.224$$

$$\frac{F}{Q_P} = 6.45$$

Element spread:

Capacitive 1:1

Resistive 3.257:1

The 3dB frequencies are:

$F_{C1} = 990.05$ Hz. $F_{C2} = 1010.05$ Hz.

Any of G_1, G_R or G_0 can be used to achieve the tuning of ω_P. Then any or both of K_m, K_0 can be used to achieve the tuning of Q_P without affecting the value of ω_P because ω_P is independent of both K_0 and K_m.

The circuit was implemented using 1% tolerance resistors and 2% tolerance capacitors with values chosen to be, within the range of currently available elements, as close as possible to the designed ones. (Trim pots were used wherever necessary.) The OA's which have been used were LM741. Actual values obtained are:

ω_P = 1000.6 Hz.

F_{C1} = 990.2 Hz. F_{C2} = 1010.4 Hz.

Q_P = 49.53

Q_P and ω_P variations from their designed values are 0.93% and 0.06% respectively; this is due to the small differences between the designed values and the real ones as well as to the tolerance of the elements. Power supply voltages used were ±10V and ±15V. Also the OA's were heated and their temperature controlled. The response was experimentally measured and plotted at 22°C (room temperature) and 70°C for both power supply voltages used. Only the OA's were heated; the passive elements were not heated in order to simulate Hybrid Integrated Circuit technology.

As shown in Fig. 3.3.4 no appreciable change in the response was observed; thus the experimental results confirm

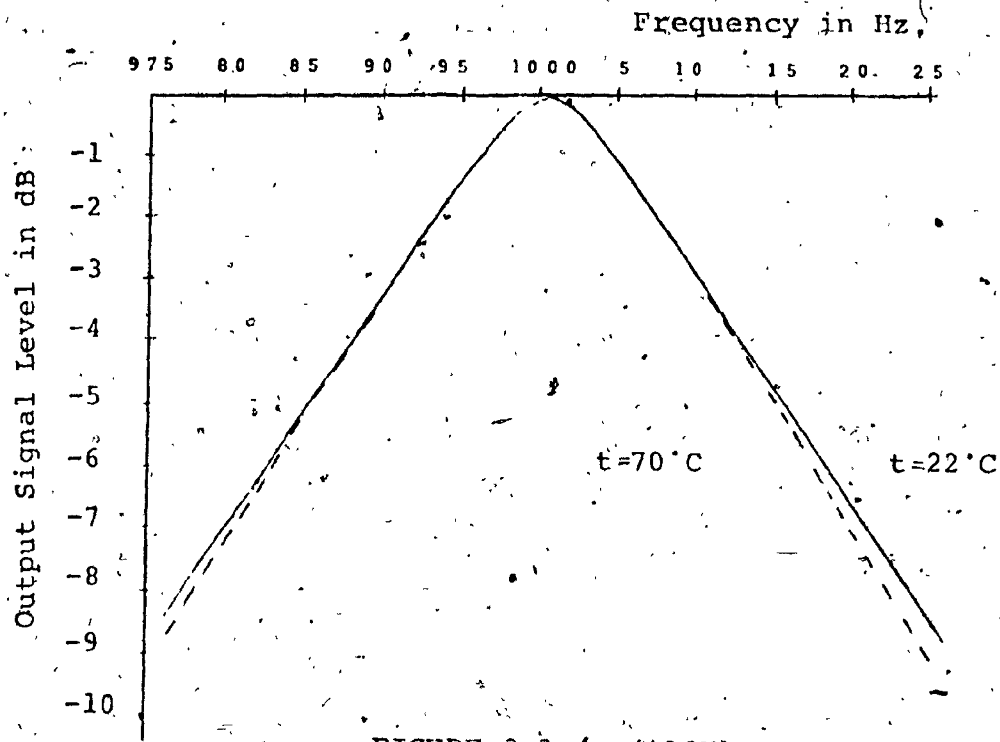


FIGURE 3.3.4a (±10V)

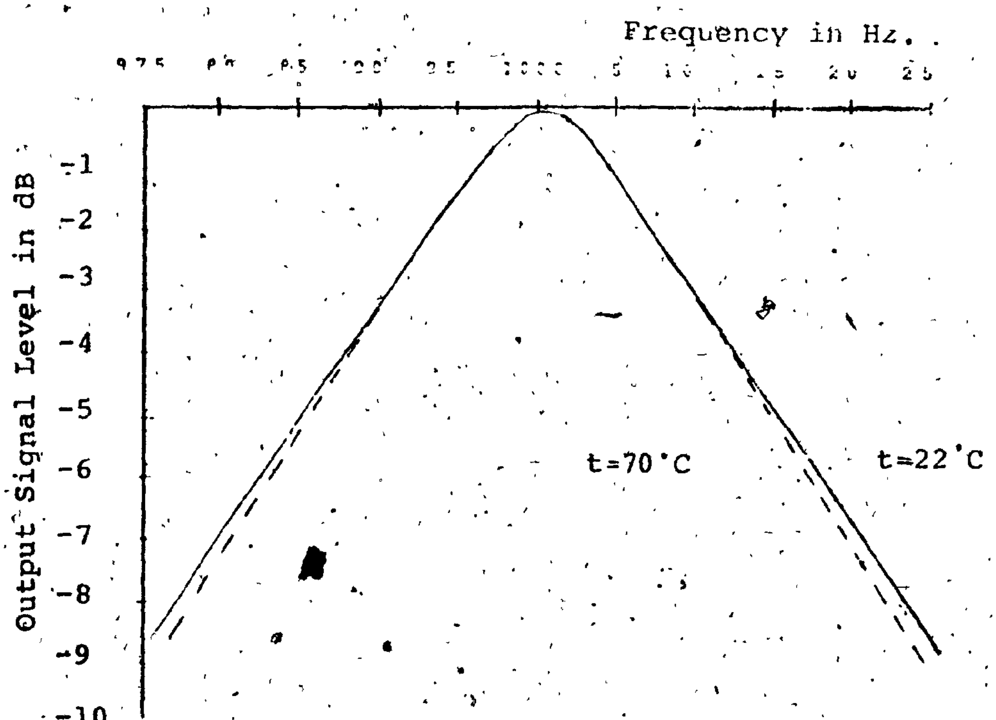


FIGURE 3.3.4b (±15V)

FIGURE 3.3.4 FREQUENCY RESPONSE OF THE BAND PASS TYPE 1 TAC FOR $Q_P = 50$

the theoretical predictions.

3.3.2 Effect of the pole of the OA on Q_P and ω_P [17, 33]

Replacing the amplifiers of Fig. 3.3.1a by the networks of Fig. 3.3.1b and 3.3.1c the analysis gives

$$D(S) = S^2 + \frac{S}{x+\gamma_{1c}-K_mK_0} \cdot \frac{\omega_P}{Q_P}\left[x + \gamma_{1c} - \frac{1}{(\frac{1}{K_0}+\frac{1}{A_0(S)})(\frac{1}{K_m}+\frac{1}{A_m(S)})}\right] + \omega_P^2 \tag{3.3.24}$$

Using

$$A_0(S) = \frac{B_0}{S+\omega_0} \tag{3.3.25a}$$

$$A_m(S) = \frac{B_m}{S+\omega_m} \tag{3.3.25b}$$

where

B_0, B_m are the unity gain bandwidth of the amplifiers. ω_0, ω_m are the poles of the amplifiers. $A_0 = B_0/\omega_0$, $A_m = B_m/\omega_m$ are the DC gains of the amplifiers. We get

$$D(S) = a_4S^4 + a_3S^3 + a_2S^2 + a_1S = (S+P_1)(S+P_2)(S^2 + \frac{\hat{\omega}_{Pa}}{\hat{Q}_{Pa}} + \hat{\omega}_P^2)\ ^* =$$

$$S^4\left[\frac{1}{B_0B_m}\right] + S^3\left[\frac{1}{B_0}\left(\frac{1}{K_m}+\frac{\omega_m}{B_m}\right) + \frac{1}{B_m}\left(\frac{1}{K_0}+\frac{\omega_0}{B_0}\right) + \left(\frac{\omega_P}{Q_P}\right)\frac{x+\gamma_{1c}}{x+\gamma_{1c}-K_0K_m}\frac{1}{B_0B_m}\right] +$$

---

* The dominant roots of D(S) are contained in the factor $(S^2 + \frac{\hat{\omega}_{Pa}}{\hat{Q}_{Pa}}S + \hat{\omega}_{Pa}^2)$

$$+ S^2\left[\left(\frac{1}{K_0}+\frac{\omega_0}{B_0}\right)\left(\frac{1}{K_m}+\frac{\omega_m}{B_m}\right)+\left(\frac{\omega_P}{Q_P}\right) \frac{(x+\gamma_{1c})}{x+\gamma_{1c}-K^2}\left[\frac{1}{B_0}\left(\frac{1}{K_m}+\frac{\omega_m}{B_m}\right)+\frac{1}{B_m}\left(\frac{1}{K_0}+\frac{\omega_0}{B_0}\right) + \right.\right.$$

$$\left.\frac{\omega_P^2}{B_0 B_m}\right]$$

$$+ S\left[\frac{\left(\frac{\omega_P}{Q_P}\right)}{x+\gamma_{1c}-K_m K_0}\left[(x+\gamma_{1c})\left(\frac{1}{K_0}+\frac{\omega_0}{B_0}\right)\left(\frac{1}{K_m}+\frac{\omega_m}{B_m}\right) - 1\right] + \right.$$

$$\left.\omega_P^2\left[\frac{1}{B_0}\left(\frac{1}{K_m}+\frac{\omega_m}{B_m}\right)+\frac{1}{B_m}\left(\frac{1}{K_0}+\frac{\omega_0}{B_0}\right)\right]\right]$$

$$+ \omega_P^2\left(\frac{1}{K_0}+\frac{\omega_0}{B_0}\right)\left(\frac{1}{K_m}+\frac{\omega_m}{B_m}\right) \qquad (3.3.26)$$

If we let

$$K_m = K_0 = K$$

$$B_m = B_0 = B \qquad A_m = A_0 = A$$

$$\omega_m = \omega_0 = \omega_c .$$

and use

$$B = \omega_c A$$

we get

$$D(S) =$$

$$= \frac{S^4}{B^2}$$

$$+ S^3\left[\frac{2}{B}\left(\frac{1}{K}+\frac{1}{A}\right) + \frac{\omega_P}{Q_P}\frac{x+\gamma_{1c}}{x+\gamma_{1c}-K^2}\cdot\frac{1}{B^2}\right] +$$

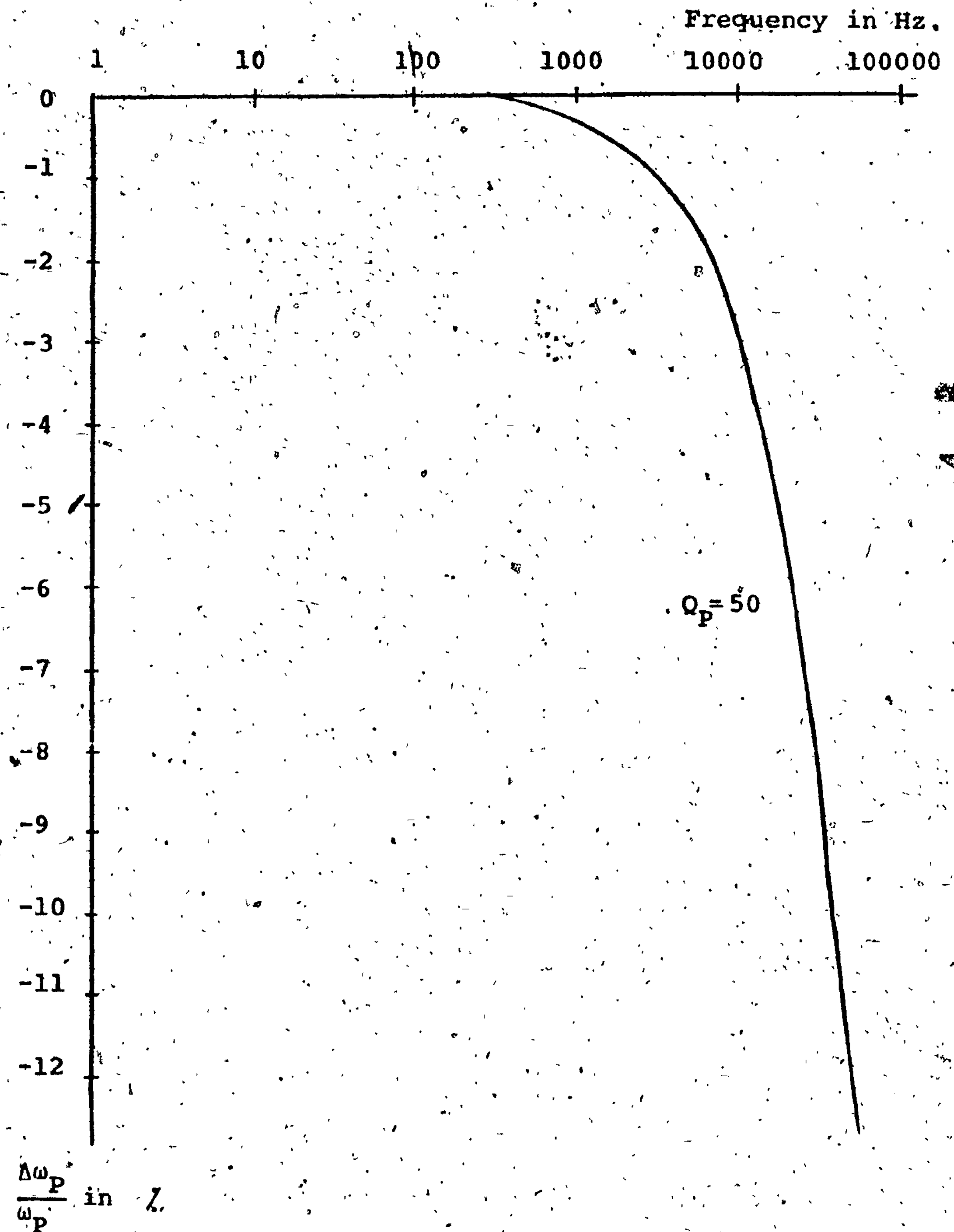


FIGURE 3.3.5 THE EFFECT OF THE OAs POLES ON ω_P FOR THE BAND PASS TYPE 1 TAC

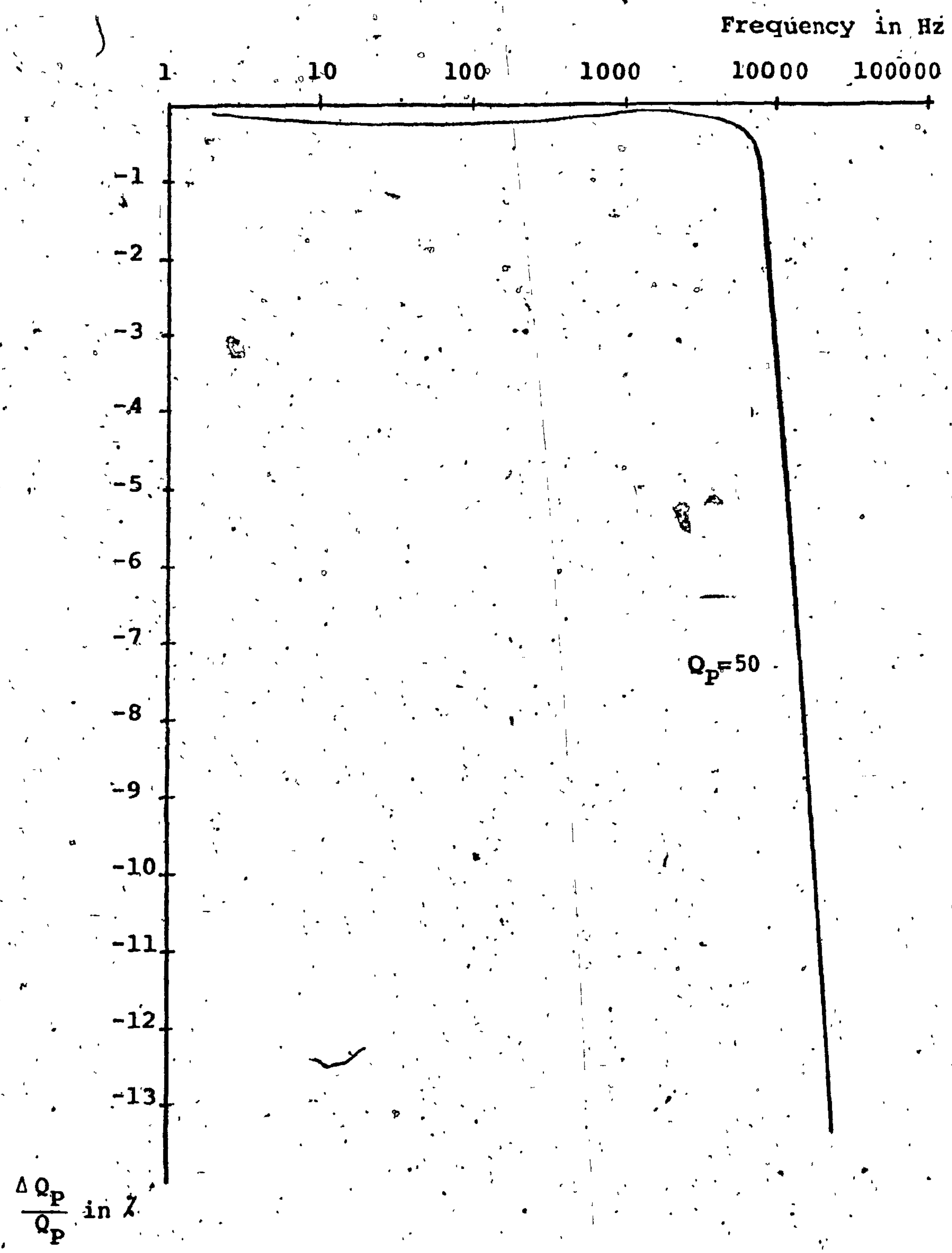


FIGURE 3.3.6 THE EFFECT OF THE OAs POLES ON Q_P FOR THE BAND PASS TYPE 1 TAC

$$s^2\left[\left(\frac{1}{K}+\frac{1}{A}\right)^2+\frac{\omega_P}{Q_P}\left(\frac{x+Y_{1c}}{x+Y_{1c}-K^2}\right)\left(\frac{1}{K}+\frac{1}{A}\right)\frac{2}{B}+\frac{\omega_P^2}{B^2}\right]+$$

$$+\,s\left[\frac{\omega_P}{Q_P}\left(\frac{(Y_{1c}+x)\left(\frac{1}{K}+\frac{1}{A}\right)^2-1}{Y_{1c}+x-K^2}\right)+\omega_P^2\left(\frac{1}{K}+\frac{1}{A}\right)\frac{2}{B}\right]+\left(\frac{1}{K}+\frac{1}{A}\right)^2\omega_P^2$$

(3.3.27)

Using Eqn. 3.3.24 the values of $\frac{\Delta\omega_P}{\omega_P}$ and $\frac{\Delta Q_P}{Q_P}$ due to the effect of the pole of the amplifier have been obtained for various values of ω_P and are given in Fig. 3.3.5 and 3.3.6 respectively. The use of OA LM741 has been assumed in these computations.

3.4 A Low Pass Realization of Type 1

The overall realization consisting of two amplifiers is shown in Fig. 3.3.1a. (Figs. 3.3.1b and 3.3.1c show the actual realizations of the two amplifiers contained in Fig. 3.3.1a). This is a Low Pass filter containing an RI generating circuit having a t_v of type L0 and the denominator of T_v is of type 1. The transfer function is given by:

$$T_{LP1}=\frac{K_mK_0G_0G_1}{s^2C_1C_H+s\,[G_1C_H+G_0C_1-K_mK_0G_1C_H]+G_1G_0} \qquad (3.4.1a)$$

$$K_m = 1+\frac{G_A}{G_B} \qquad (3.4.1b)$$

$$K_0 = 1+\frac{G_A'}{G_B'} \qquad (3.4.1c)$$

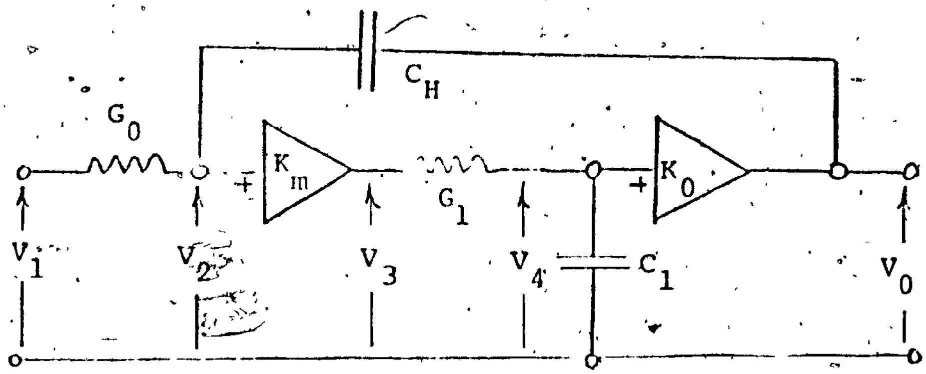


FIGURE 3.4.1a

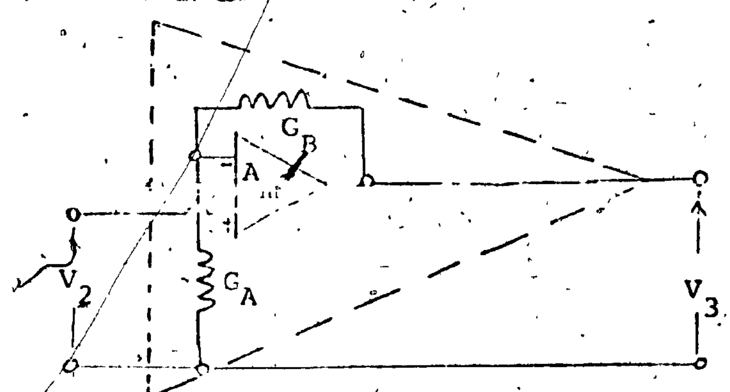


FIGURE 3.4.1b

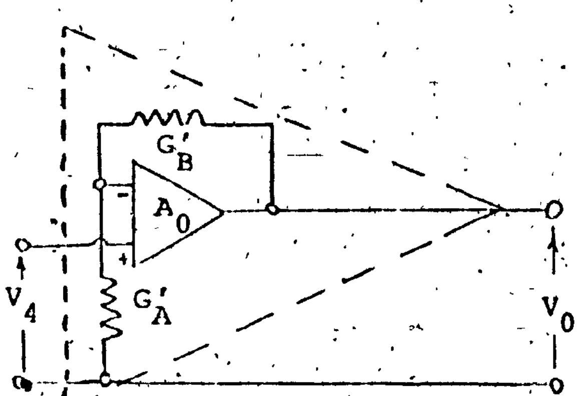


FIGURE 3.4.1c

FIGURE 3.4.1 A LOW PASS TYPE 1 TAC REALIZATION

For the transfer function given in Eqn. 3.4.1 we have:

$$\omega_P = \left\{\frac{G_1 G_0}{C_1 C_H}\right\}^{1/2} \quad (3.4.2)$$

$$S_{K_0}^{\omega_P} = S_{K_m}^{\omega_P} = 0 \quad (3.4.3)$$

$$Q_P = \frac{\left\{\frac{G_0}{G_1}\frac{C_1}{C_H}\right\}^{1/2}}{1+\frac{G_0}{G_1}\frac{C_1}{C_H}-K_m K_0} \quad (3.4.4)$$

$$S_{K_0}^{Q_P} = S_{K_m}^{Q_P} = \frac{K_m K_0}{1+\frac{G_0}{G_1}\frac{C_1}{C_H}-K_m K_0} \quad (3.4.5)$$

$$|K_m S_{K_m}^{Q_P}| = \frac{K_m^2 K_0}{1+\frac{G_0}{G_1}\frac{C_1}{C_H}-K_m K_0} \quad (3.4.6)$$

$$|K_0 S_{K_0}^{Q_P}| = \frac{K_m K_0^2}{1+\frac{G_0}{G_1}\frac{C_1}{C_H}-K_m K_0} \quad (3.4.7)$$

$$F = \frac{(K_m+K_0)K_m K_0}{1+\frac{G_0}{G_1}\frac{C_1}{C_H}-K_m K_0} \quad (3.4.8)$$

The technique of minimization of F is the same as before;

however since the equations are different, it is discussed below. From Eqn. 3.4.4, it is possible to express the quantity $K_m K_0$ as

$$K_m K_0 = 1 + \frac{G_0}{G_1}\cdot\frac{C_1}{C_H} - \frac{1}{Q_P}\{\frac{G_0}{G_1}\cdot\frac{C_1}{C_H}\}^{1/2} \qquad (3.4.9)$$

Hence using (3.4.8) and (3.4.9)

$$F = \frac{(K_m + K_0)\,[1+\frac{G_0}{G_1}\cdot\frac{C_1}{C_H} - \frac{1}{Q_P}\{\frac{G_0}{G_1}\cdot\frac{C_1}{C_H}\}^{1/2}\,]}{\frac{1}{Q_P}\{\frac{G_0}{G_1}\cdot\frac{C_1}{C_H}\}^{1/2}} \qquad (3.4.10)$$

From Eqn. 3.4.9 and 3.4.10 it is clear that for any prescribed value of Q_P and for given values of the passive elements the smallest value of F is obtained if we let

$$K_m = K_0 = K \qquad (3.4.11)$$

Under this condition we have

$$K = [1 + \frac{G_0}{G_1}\cdot\frac{C_1}{C_H} - \frac{1}{Q_P}\{\frac{G_0}{G_1}\cdot\frac{C_1}{C_H}\}^{1/2}\,]^{1/2} \qquad (3.4.12)$$

Assuming now that

$$1 + \frac{G_0}{G_1}\cdot\frac{C_1}{C_H} \gg \frac{1}{Q_P}\{\frac{G_0}{G_1}\cdot\frac{C_1}{C_H}\}^{1/2} \qquad (3.3.13)$$

which is a valid assumption as long as the spreads are moderate and Q_P high; and defining

$$\frac{C_1}{C_H} = \gamma_c \tag{3.3.14a}$$

$$\frac{G_0}{G_1} = \gamma_0 \tag{3.3.14b}$$

We get

$$F_m = \frac{2Q_P\,[1+\gamma_0\gamma_c]^{3/2}}{[\gamma_0\gamma_c]^{1/2}} \tag{3.3.15}$$

If we define again

$$\gamma_0\gamma_c = \gamma_{0c} \tag{3.3.16}$$

then

$$\frac{F_m}{Q_P} = \frac{2(1+\gamma_{0c})^{3/2}}{\gamma_{0c}^{1/2}} \tag{3.4.17}$$

Fig. 3.4.2 shows the variation of F_m/Q_P versus γ_{0c}. The lowest value of F_m is 5.2 Q_P and is obtained for γ_{0c} = 0.5 which means that we should have

$$\frac{C_1}{C_H} \cdot \frac{G_0}{G_1} = 0.5 \tag{3.4.18}$$

The value of K is obtained using Eqn. 3.4.12 and 3.5.18 as

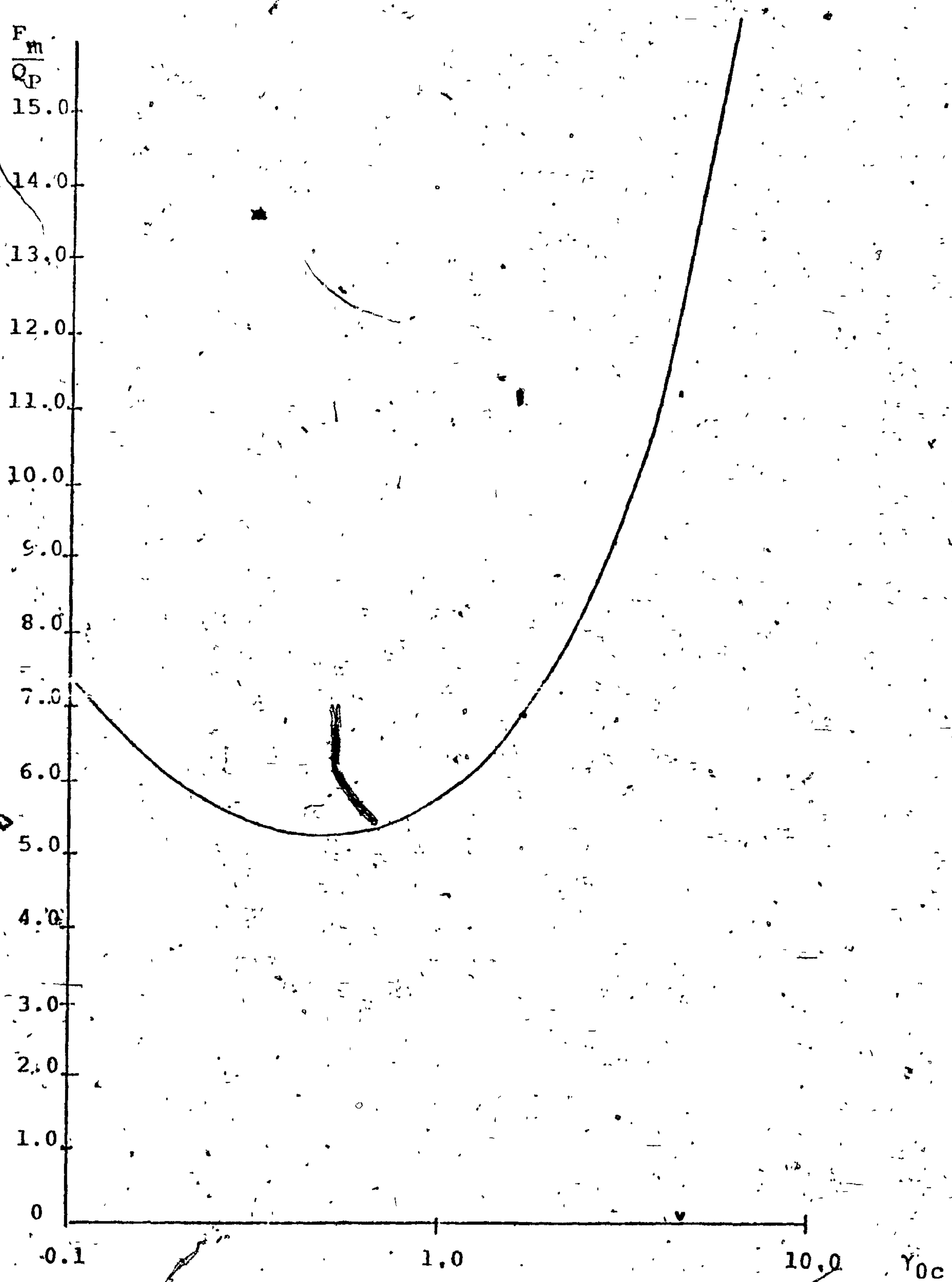


FIGURE 3.4.2 F_m/Q_p VERSUS γ_{0c} FOR THE LOW PASS TYPE 1 TAC

$$K = [1.5 - \frac{1}{\sqrt{2}\, Q_P}] \tag{3.4.19}$$

ω_P can be expressed using Eqns. 3.4.2 and 3.4.18 as

$$\omega_P = \frac{1}{\sqrt{2}} \frac{G_1}{C_1} \tag{3.4.20}$$

The elemental values are obtained when ω_P and Q_P are prescribed using Eqns. 3.4.1a, b, 3.4.18, 3.4.19, 3.4.20.

Although this circuit has been already reported in the literature [23, 24] it has been studied here and design equations for a minimized value of F given to show that it is only a particular case of the general configuration presented in this thesis.

3.5 A High Pass Realization of Type 1

The overall realization consisting of two amplifiers is shown in Fig. 3.5.1a (Figs. 3.5.1b abd 3.5.1c show the actual realizations of the two amplifiers contained in Fig. 3.5.1a). It consists of an RI generating circuit of Type B0. The entire circuit is a High Pass filter having a denominator of Type 1. Its transfer functions is given by

$$T_{HP1} = \frac{K_m K_0 C_0 C_1 S^2}{C_0 C_1 S^2 + (G_1 C_0 + G_H C_1 - K_m K_0 G_H C_1) S + G_1 G_H} \tag{3.5.1a}$$

$$K_m = 1 + \frac{G_A}{G_B} \tag{3.5.1b}$$

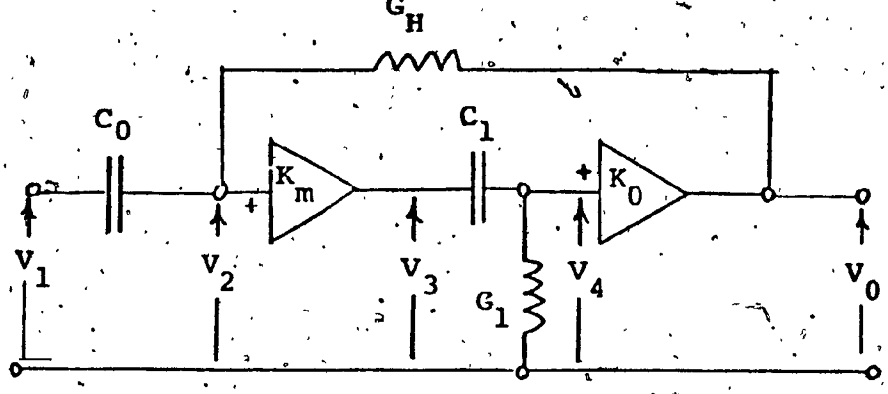


FIGURE 3.5.1a

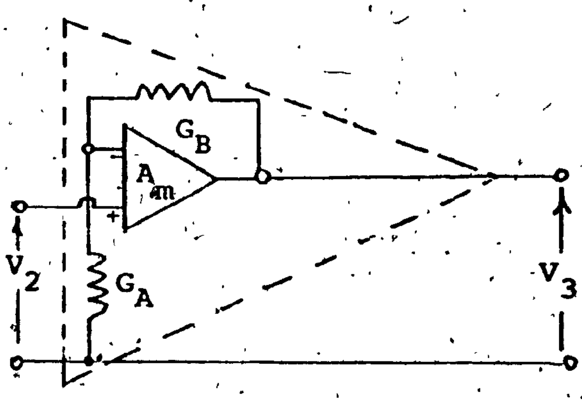


FIGURE 3.5.1b

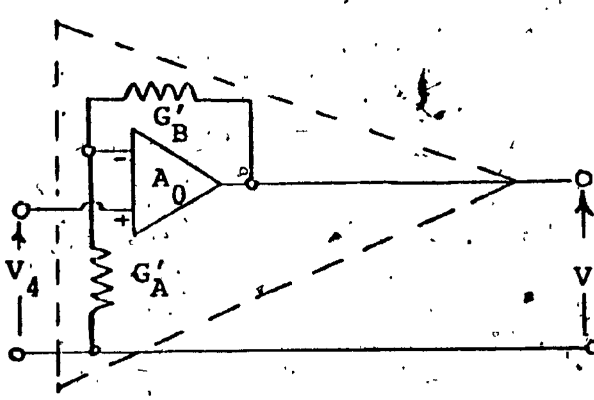


FIGURE 3.5.1c

FIGURE 3.5.1 A HIGH PASS TYPE 1 TAC REALIZATION

$$K_0 = 1 + \frac{G'_A}{G'_B} \qquad (3.5.1c)$$

The design equations of the High Pass filter are obtained from those of the Low Pass Filter by RC-CR transformation. Therefore the curve given in Fig. 3.4.2 and the expressions for $\frac{F_m}{Q_p}$ and K obtained in Eqns. 3.4.17 and 3.4.19 are valid in this case also.

3.6 A Band Pass Realization of Type 22b

For the sake of completeness, we show that the method of minimization of F can be applied to any polynomial decomposition by giving here a Band Pass realization of Type 22b. The overall realization consisting of two amplifiers is shown in Fig. 4.6.1a (Figs. 4.6.1b abd 4.6.1c show the actual realizations of the two amplifiers contained in Fig. 3.6.1a). It consists of an RI generating circuit having a t_v of Type B0. The entire circuit corresponds to a Band Pass filter having a denominator of Type 22b. Its transfer function is given by:

$$T_{BP22} = \frac{K_m K_0 G_0 C_1 S}{\begin{aligned} &S^2 [C_1 (C_F + C_L) + K_m C_1 C_F] + \\ &S [(G'_0 + G_H + G_F) C_1 + G'_1 (C_L + C_F) - K_m K_0 G_H C_1 + K_m (G_F C_1 + C_F G'_1)] \\ &+ [(G'_0 + G_H + G_F) G'_1 + K_m G'_1 G_F] \end{aligned}} \qquad (3.6.1a)$$

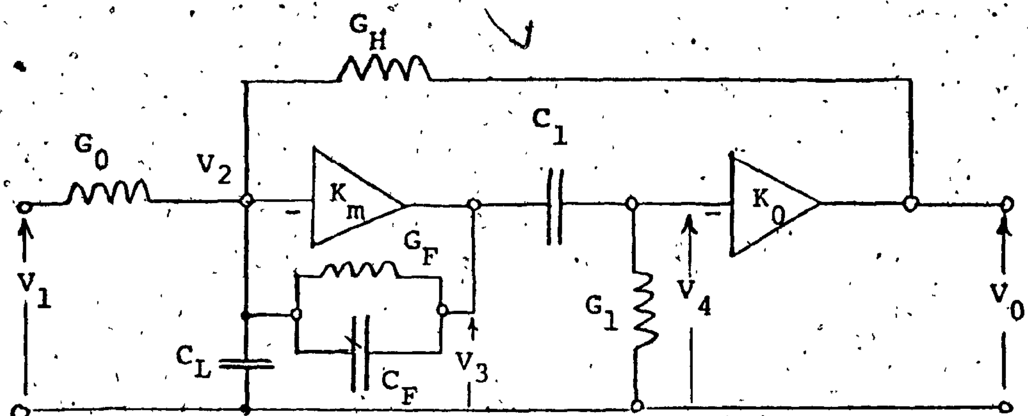


FIGURE 3.6.1a

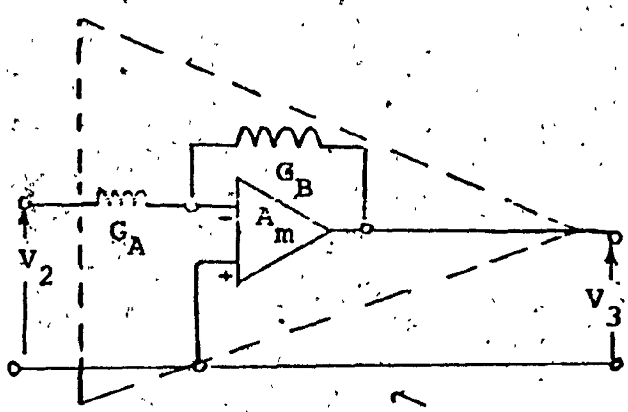


FIGURE 3.6.1b

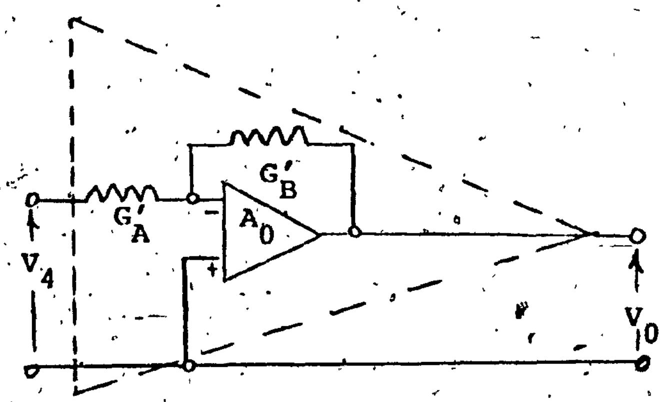


FIGURE 3.6.1c

FIGURE 3.6.1 A BAND PASS TYPE 22b TAC REALIZATION

With

$$K_m = \frac{G_A}{G_B} \quad (3.6.1b)$$

$$K_0 = \frac{G'_A}{G'_B} \quad (3.6.1c)$$

$$G'_0 = G_0 + G_A \quad (3.6.1d)$$

$$G'_1 = G_1 + G_A \quad (3.6.1c)$$

These equations take into account the loading effect of the amplifiers. The condition which assures a zero pole frequency deviation is given by

$$\frac{G'_1(G'_0+G_H+G_F)}{C_1(C_F+C_L)} = \frac{G'_1 G_F}{C_1 G_F} = \omega_P^2 \quad (3.6.2)$$

Under this condition

$$S_{K_m}^{\omega_P} = S_{K_0}^{\omega_P} = 0 \quad (3.6.3)$$

$$Q_P = \frac{\{[(G'_0+G_H+G_F)G'_1+K_m G'_1 G_F][(C_F+C_L)C_1+K_m C_1 C_F]\}^{1/2}}{C_1(G'_0+G_H+G_F)+G'_1(C_L+C_F)+K_m[G_F C_1+C_F G'_1-K_0 G_H C_1]} \quad (3.6.4)$$

$$S_{K_m}^{Q_P} = \frac{1}{2}\left[\frac{K_m C_F}{C_F+C_L+K_m C_F} + \frac{K_m G_F}{G'_0+G_H+G_F+K_m G_F}\right] - \frac{K_m[G_F C_1+C_F G'_1-K_0 G_H C_1]}{C_1(G'_0+G_H+G_F)+G'_1(C_L+C_F)+K_m[G_F C_1+C_F G'_1-K_0 G_H C_1]} \quad (3.6.5)$$

$$S_{K_0}^{Q_P} = \frac{K_m K_0 G_H C_1}{C_1(G'_0+G_H+G_F)+G'_1(C_L+C_F)+K_m[G_F C_1+C_F G'_1-K_0 G_H C_1]} \qquad (3.6.6)$$

Using Eqn. 3.6.2 let us define

$$\frac{C_L}{C_F} = \frac{G'_0+G_H}{G_F} = \lambda \qquad (3.6.7a)$$

and

$$\left\{\frac{G'_1}{G_F}\cdot\frac{C_1}{C_F}\right\}^{1/2} = a \qquad (3.6.7b)$$

$$\frac{G'_1}{G_F} + \frac{C_1}{C_F} = b \qquad (3.6.7c)$$

$$\frac{G_H}{G_F}\cdot\frac{C_1}{C_F} = c \qquad (3.6.7d)$$

We have

$$Q_P = \frac{a(1+K_m+\lambda)}{b(1+K_m+\lambda)-K_m K_0 c} \qquad (3.6.8)$$

$$S_{K_m}^{Q_P} = \frac{K_m}{1+\lambda+K_m} - \frac{K_m(b-K_0 c)}{(1+\lambda)b+K_m(b-K_0 c)} \qquad (3.6.9)$$

$$S_{K_0}^{Q_P} = \frac{K_m K_0 c}{(1+\lambda)b+K_m(b-K_0 c)} = \frac{b}{a}Q_P - 1 \qquad (3.6.10)$$

$$K_0 = \frac{\frac{a}{c}\left(\frac{b}{a}Q_P-1\right)(1+K_m+\lambda)}{K_m Q_P} \qquad (3.6.11)$$

Where it is necessary that

$$K_0 < \frac{b}{c} \cdot \frac{1+\lambda+K_m}{K_m} \tag{3.6.12}$$

Using Eqn. 3.6.8, 3.6.9, and 3.6.10 we get

$$|K_m S_{K_m}^{Q_P}| = \frac{K_m}{1+\left(\frac{K_m}{1+\lambda}\right)} \left[\frac{b}{a} Q_P - 1\right] \tag{3.6.13a}$$

$$\left(\frac{b}{a} Q_P - 1\right) > 0 \tag{3.6.13b}$$

$$|K_0 S_{K_0}^{Q_P}| = \frac{1}{Q_P}\left(\frac{b}{a} Q_P - 1\right)^2 \left(\frac{1+\lambda}{K_m} + 1\right) \frac{a}{c} \tag{3.6.14}$$

Hence if $\frac{b}{a} Q_P >> 1$

$$F = \frac{b}{a} Q_P \left[\frac{b}{c}\left(\frac{1+\lambda}{K_m} + 1\right) + \frac{K_m}{1+\frac{K_m}{1+\lambda}}\right] \tag{3.6.15}$$

A set of design equations can be obtained as follows. It is clear from Eqn. 3.6.7 that the quantity $\frac{b}{a}$ has a minimum value which is two and which is obtained if we let

$$\frac{G_1'}{G_F} = \frac{C_1'}{C_F} \tag{3.6.16}$$

Also if we let

$$\frac{C_L}{C_F} = \frac{C_1}{C_F} = \frac{G_H}{G_F} + \varepsilon_0 = \lambda \tag{3.6.17}$$

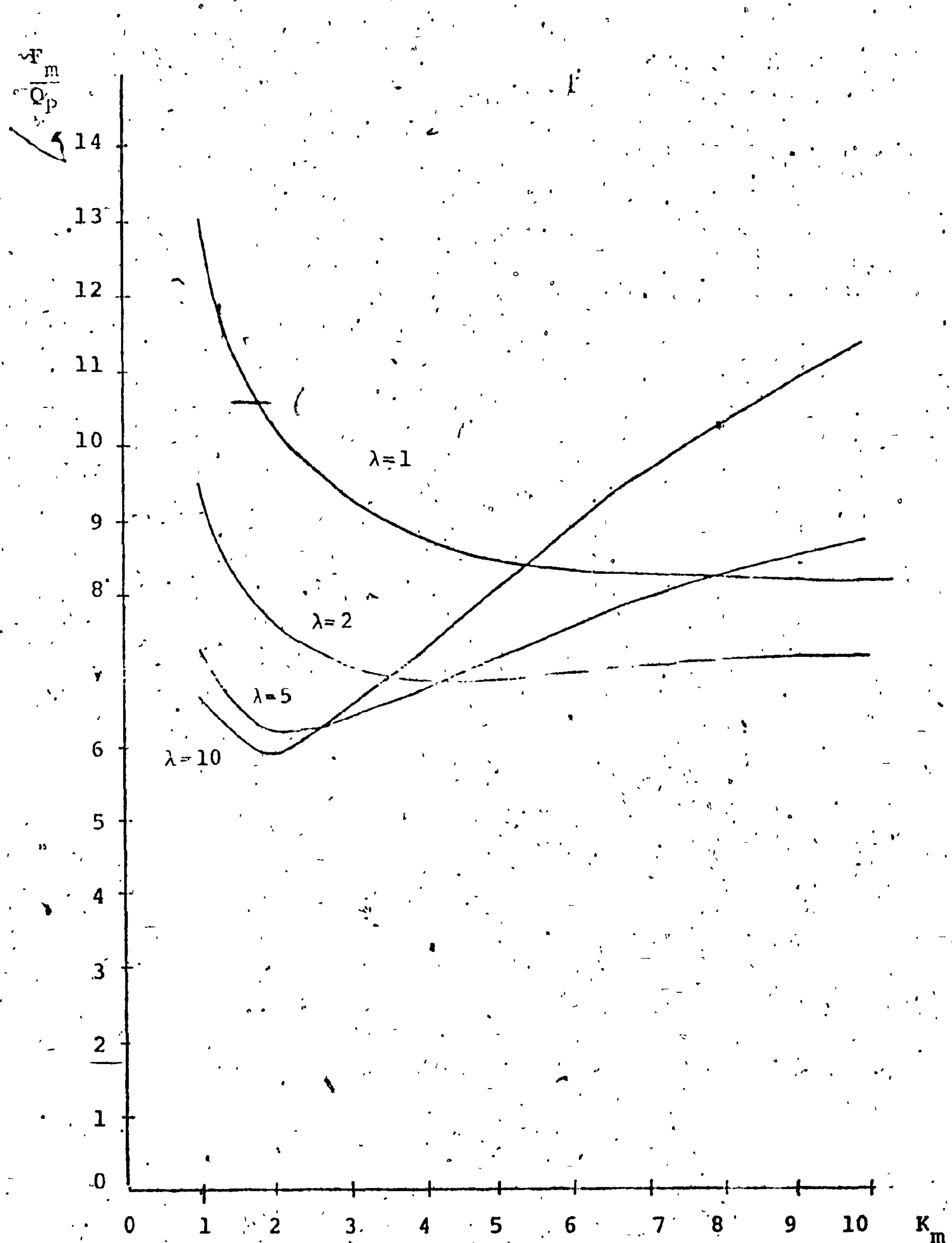


FIGURE 3.6.2 F_m/Q_p VERSUS K_m OF THE BAND PASS TYPE 22b TAC FOR SEVERAL VALUES OF λ

where

$$\varepsilon_0 = \frac{G'_0}{G_F} \tag{3.6.18}$$

$$\frac{F_m}{Q_p} = 2\left[\frac{2}{(\lambda-\varepsilon_0)}\left(\frac{1+\lambda}{K_m} + 1\right) + \frac{K_m}{1+\frac{K_m}{1+\lambda}}\right] \tag{3.3.19}$$

And assume that

$$\varepsilon_0 << \lambda \tag{3.6.20}$$

Then

$$\frac{F_m}{Q_p} \doteq 2\left[\frac{2}{\lambda}\left[\frac{1+\lambda}{K_m} + 1\right] + \frac{K_m}{1+\frac{K_m}{\lambda+1}}\right] \tag{3.6.21}$$

The variation of F_m versus K'_m for several values of the capacitive spread λ is shown in Fig. 3.6.2 according to Eqn. 3.6.21. The corresponding value of K_0 for any prescribed value of K_m can be obtained using Eqn. 3.6.11.

The elemental values are obtained when ω_p, Q_p, K_m, λ, ε_0, are prescribed using Eqns. 3.6.1b to 3.6.1e, 3.6.2, 3.6.11, 3.6.15, 3.6.16, 3.6.17, 3.6.18.

3.6.1 Experimental results

The network shown in Fig. 3.6.1 was built up using discrete elements and tested. The results are summarized below.

Designed values:

ω_p = 6283.185 Rad/sec. = 1000 Hz

Q_p = 50

λ = 5

K_m = 2

ϵ_0 = 0.4

a = 5

b = 10

c = 23

K_0 = 1.72174

$C_I = C_1$ = 22 KPF C_F = 4.4 KPF

R_F = 36.172 KΩ

$R_1 = R_A$ = 14.468 KΩ

R_H = 7.863 KΩ

$R_0 = R_A$ = 180.857 KΩ

R_B = 361.715 KΩ

R'_B = 24.910

GQSP values:

$$\frac{|K_m S_{K_m}^{Q_p}|}{Q_p} = 3$$

$$\frac{|K_0 S_{K_0}^{Q_p}|}{Q_p} = 3.48$$

$$\frac{F_m}{Q} = 6.48$$

Element spread:

Capacitive 1:1

Resistive 46:1

The 3dB frequencies are:

$F_{C1} = 990.05$ Hz. $F_{C2} = 1010.05$ Hz.

Any of G_1, G_H or G_0 can be used to achieve the tuning of ω_P. Then K_0 can be used to achieve the tuning of Q_P without affecting the value of ω_P because ω_P is independent of both K_0 and K_m.

The circuit has been implemented using 1% tolerance resistors and 1% tolerance capacitors with values chosen to be, within the range of currently available elements to be as close as possible to the designed ones. (Trim pots have been used wherever necessary.) The OAs which have been used were LM741. Actual values obtained

$\omega_P = 999$ Hz.

$F_{C1} = 989.3$ Hz. $F_{C2} = 1009.6$ Hz.

$Q_P = 49.211$

Q_P and ω_P variations from their designed values are 1.58% and 0.1% respectively. This is due to the small differences between the designed values and the real ones as well as to the tolerances of the elements. Power supply voltage used were ±10V and ±15V. Also the OAs were heated and their

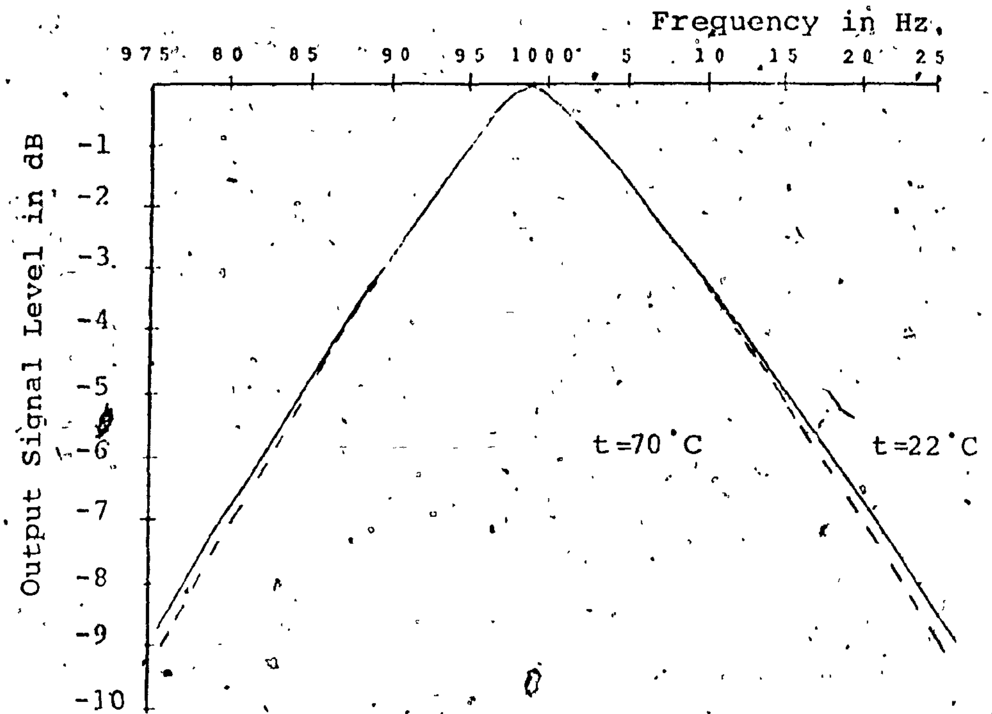


FIGURE 3.6.3a (±10V)

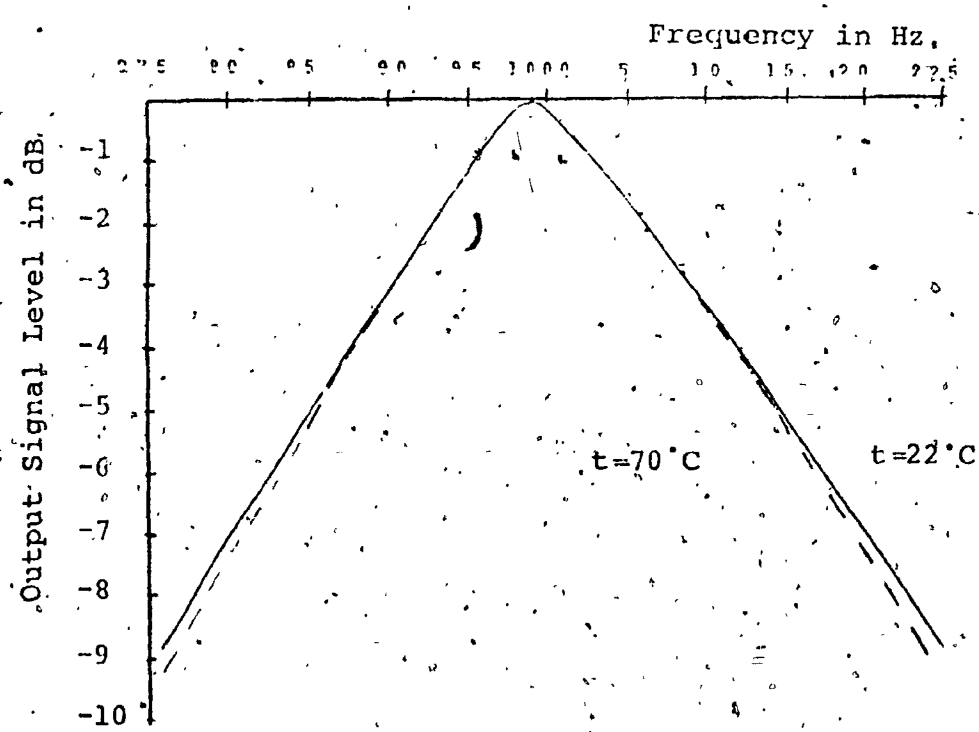


FIGURE 3.6.3b (±15V)

FIGURE 3.6.3 FREQUENCY RESPONSE OF THE BAND PASS TYPE 22b TAC FOR $Q_P = 50$

temperature controlled. The response was experimentally measured and plotted at 22° C (room temperature) and 70° C for both power supply voltages. Only the OAs were heated. The passive elements were not heated in order to simulate Hybrid Integrated Circuit technology.

As shown in Fig. 3.6.3 no appreciable change in the response was observed; thus the experimental results confirm the theoretical predictions. It was found that the circuit was stable during activation.

3.6.2 Effect of the poles of the OAs on ω_P and Q_P [17,33]

Replacing the amplifiers of Fig. 3.4.1a by the networks of Fig. 3.4.1b and 3.4.1c analysis yields

$$D(S) =$$

$$S^2 \left[e+\frac{g k_m}{A(S)}+\frac{ek_0}{A_0(S)}+g\,\frac{k_m k_0}{A(S)A_0(S)}\right] +$$

$$S\,\frac{\omega_P}{Q_P}\left\{e+Yb\left[\frac{g k_m}{A(S)}+\frac{ek_0}{A_0(S)}+\frac{g\,k_m k_0}{A(S)A_0(S)}\right]\right\} +$$

$$\omega_P^2\left[e+\frac{g k_m}{A(S)}+\frac{ek_0}{A_0(S)}+\frac{g k_m k_0}{A(S)A_0(S)}\right] \quad (3.6.22a)$$

where

$$e = 1 + \lambda + K_m \quad (3.6.22b)$$

$$k_m = 1 + K_m \quad (3.6.22c)$$

$$k_0 = 1 + K_0 \quad (3.6.22d)$$

$$g = 1 + \lambda \quad (3.6.22e)$$

$$Y = \frac{e}{be - K_m K_0 c} \tag{3.6.22f}$$

$$\lambda = \frac{C_L}{C_F} = \frac{G_0' + G_H}{G_F} = \frac{C_1}{C_F} \tag{3.6.22g}$$

$$\frac{G_1'}{G_F} + \frac{C_1}{C_F} = b \tag{3.3.22h}$$

$$\frac{G_H}{G_F} - \frac{C_1}{C_F} = c \tag{3.3.22i}$$

Using

$$A_0(S) = \frac{B_0}{S+\omega_0} \tag{3.3.23a}$$

$$A_m(S) = \frac{B_m}{S+\omega_m} \tag{3.3.23b}$$

where

B_m, and B_0 are the unity gain bandwidth of the amplifiers. ω_0, ω_m are the poles of the amplifiers. A_0, A_m are the DC gains of the amplifiers. We get

$$D(S) =$$

$$S^4 \left[\frac{g\ kk_0}{B_m B_0}\right] +$$

$$S^3 \left[\frac{g\ k_m}{B_m} + \frac{ek_0}{B_0} + \frac{g\ k_m k_0}{B_m B_0}(\omega_m + \omega_0) + Yb\ \frac{\omega_P}{Q_P}\ \frac{g\ k_m k_0}{B_m B_0}\right]$$

$$+\ S^2 \left[e + \frac{g\ k_m \omega_m}{B_m} + \frac{ek_0 \omega_0}{B_0} + g\ k_m k_0 \cdot \frac{\omega_m \omega_0}{B_m B_0} + \frac{\omega_P}{Q_P}\ Y\, b \left[\frac{g\ k_m}{B_m} + \frac{ek_0}{B_0} + \frac{g\ k_m k_0 (\omega_m + \omega_0)}{B_m B_0}\right.\right.$$

$$\left.+\omega_P^2\ \frac{g\ k_m k_0}{B_m B_0}\right].$$

$$+ S\left[\frac{\omega_P}{Q_P}\{e+\gamma b\,[\frac{g\ k_m\omega_m}{B_m}+\frac{e\ k_0\omega_0}{B_0}+\frac{g\ k_m k_0\omega_m\omega_0}{B_m B_0}]\}\right.$$

$$\left.+\ \omega_P^2(\frac{g\ k_m}{B_m}+\frac{ek_0}{B_0}+\frac{g\ k_m k_0(\omega_m+\omega_0)}{B_m B_0})\right]$$

$$+\ \omega_P^2[e+\frac{g\ k_m\omega_m}{B_m}+\frac{e\ k_0\omega_0}{B_0}+\frac{g\ k_m k_0\omega_m\omega_0}{B_m B_0}] \qquad (3.3.24)$$

If we let

$$B_m = B_0 = B \quad A_m = A_0 = A \quad \omega_m = \omega_0 = \omega_c$$

$$D(S) =$$

$$S^4[\frac{g\ k_m k_0}{B^2}] +$$

$$S^3[\frac{1}{B}(g\ k_m+e\ k_0)+\frac{2}{AB}\ g\ k_m k_0+\frac{\omega_P}{Q_P}\gamma b\ \frac{g\ k_m k_0}{B^2}]$$

$$+\ S^2[e+\frac{1}{A}(g\ k_m+e\ k_0)+\frac{g\ k_m k_0}{A^2}+\omega_P^2\ \frac{g\ k_m k_0}{B^2}$$

$$+\ \frac{\omega_P}{Q_P}\gamma b\,[\frac{1}{B}(g\ k_m+e\ k_0)+\frac{2}{AB}\ g\ k_m k_0]]$$

$$+\ S\left[\frac{\omega_P}{Q_P}\{e+\gamma b[\frac{1}{A}(g\ k_m+e\ k_0)+\frac{g\ k_m k_0}{A^2}]\}\right. +$$

$$\left.\omega_P^2[\frac{1}{B}(g\ k_m+e\ k_0)+\frac{2g\ k_m k_0}{AB}\right]+[e+\frac{1}{A}(g\ k_m+e\ k_0)+\frac{g\ k_m k_0}{A^2}]$$

$$(3.3.25)$$

Using Eqn.3.3.25 the values of $\frac{\Delta\omega_P}{\omega_P}$ and $\frac{\Delta Q_P}{Q_P}$ due to the effect of the pole of the amplifiers have been obtained for various

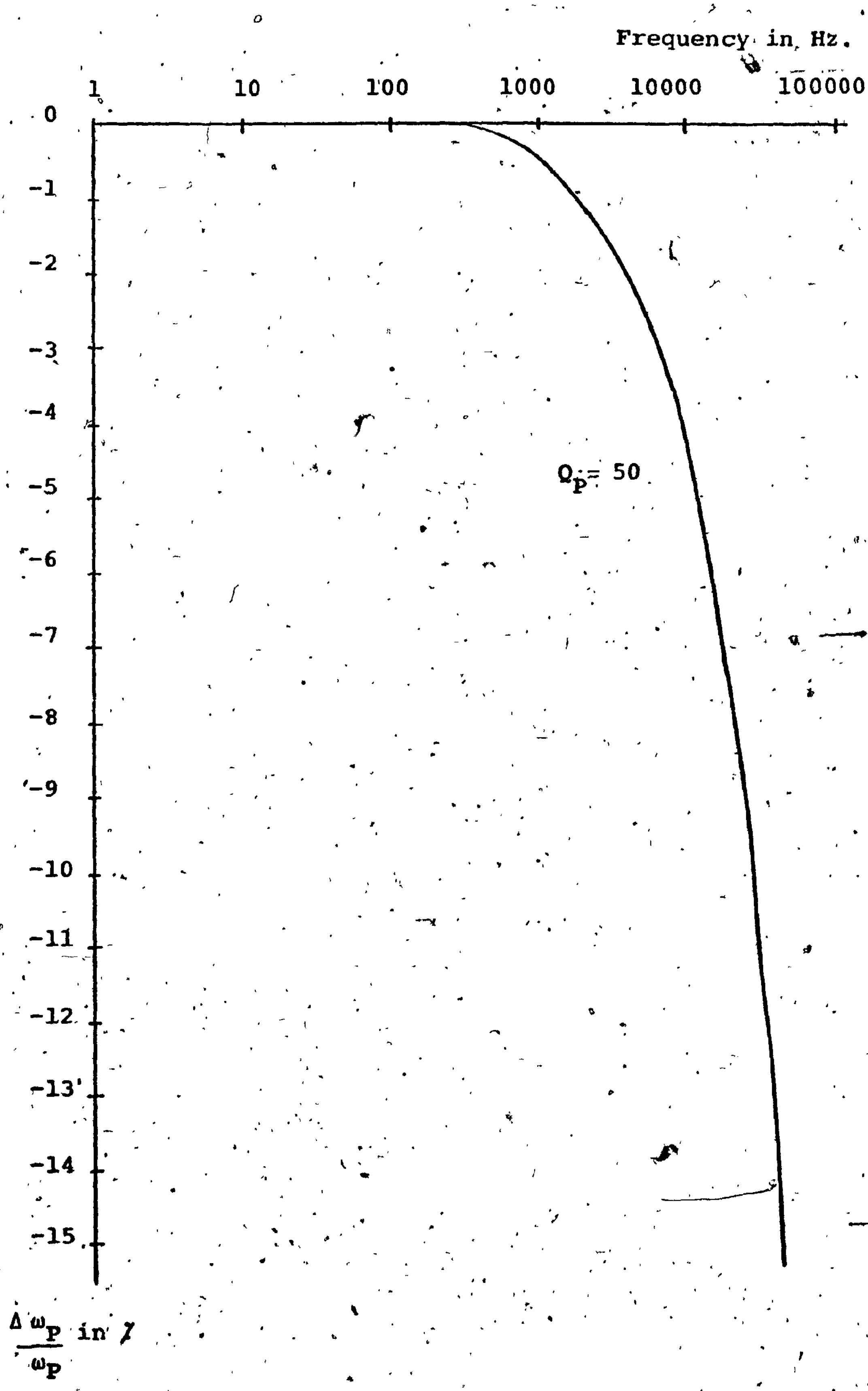


FIGURE 3.6.4 THE EFFECT OF THE OAs POLES ON ω_P FOR THE BAND PASS TYPE 22b TAC

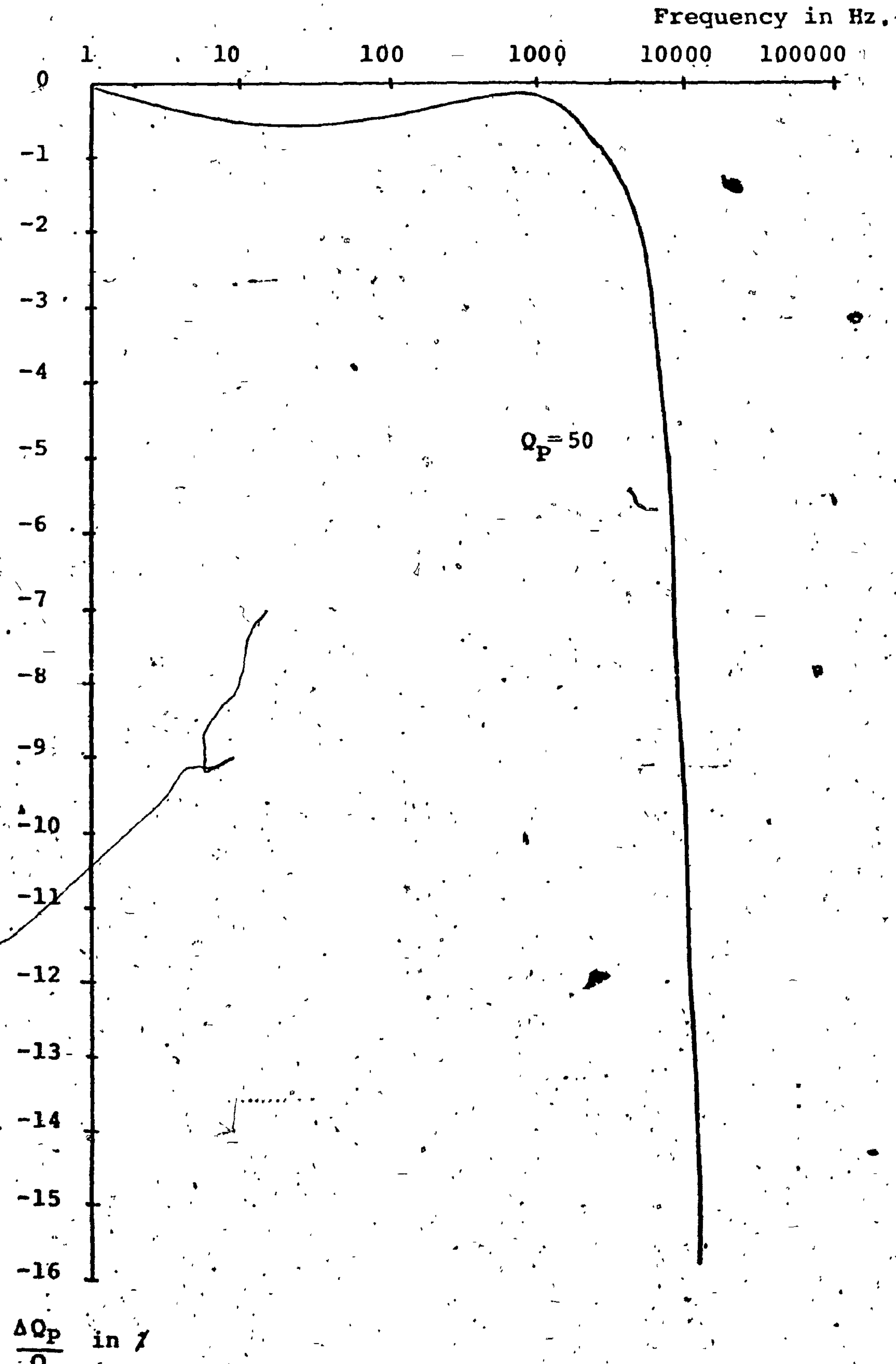


FIGURE 3.6.5 THE EFFECT OF THE OAs POLES ON Q_P FOR THE BAND PASS TYPE 22b TAC

values of ω_p and are given in Fig. 3.6.4 and 3.6.5 respectively. The use of OA LM741 have been assumed in these computations.

3.7 Summary and Discussion

In this chapter we have considered TACs using a first order generating function t_v. It is shown that eight polynomial decompositions can be obtained when the admittance are of the form y = CS+G and require only two types of t_v. Only Low Pass, High Pass and Band Pass filters can be realized. These filters are designed to yield zero GωSP and minimized F. Hence the quantity $\left|\frac{\Delta T_v(S)}{T_v}\right|_{S=j\omega p}$ is also minimized.

Two Band Pass filters were built and tested. Experimental results confirm the theoretical studies.

CHAPTER IV

ZERO $G\omega SP$ Q-MULTIPLIED CIRCUITS

CHAPTER IV

ZERO GωSP Q-MULTIPLIED CIRCUITS

4.1 Introduction

In the previous chapter we considered realizations using two finite gain amplifiers when the generating circuit consisted of first order transfer functions. In this chapter we will discuss realizations when the generating circuit consists of second order transfer functions. This results in Q-multiplication. The quantity F of the two amplifier network will be minimized to obtain Q multiplier circuits having zero GωSP and minimized $\frac{\Delta Q_P}{Q_P}$. The technique is illustrated by selecting appropriate single-amplifier generating circuits.

4.2 The Basic Q-Multiplier Configuration

The proposed configuration is shown in Fig. 4.2.1 which is obtained from Fig. 2.3.1a by letting $Y_L = 0$.

Since

$$t_v = K_0 \frac{n_0(S)}{D_0(S)} = \frac{K_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2+(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)} \tag{4.2.1a}$$

$$0 < \gamma_2 < \beta_2 \qquad 0 < \gamma_1 < \beta_1 \tag{4.2.1b}$$

The transfer function T_v of the configuration shown in Fig. 4.2.1 will be

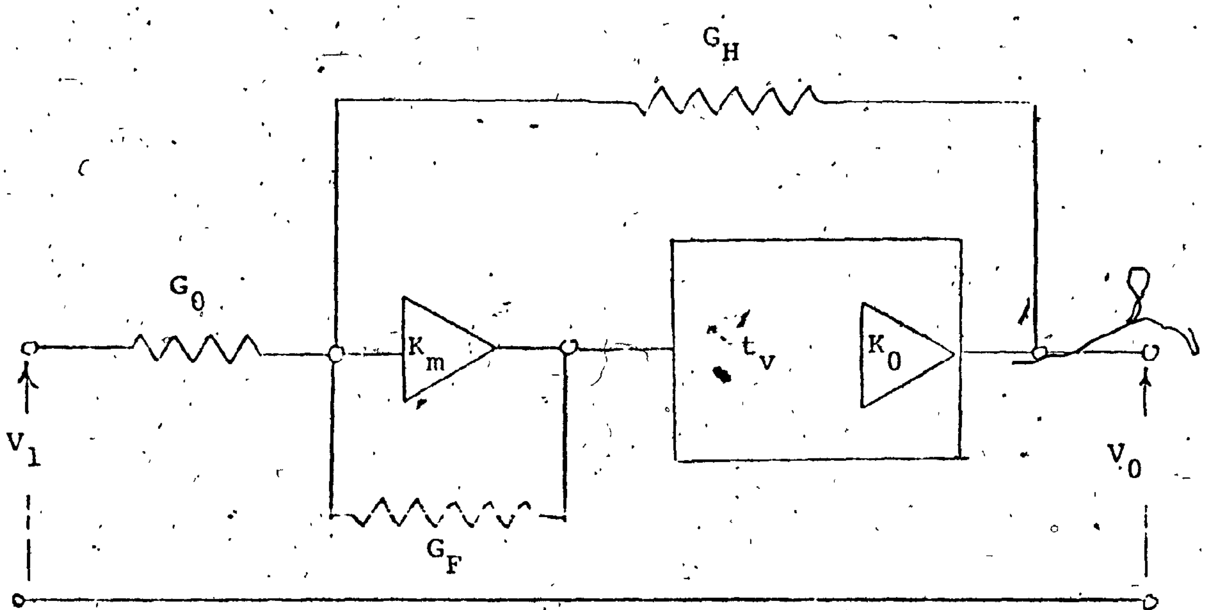


FIGURE 4.2.1 THE PROPOSED QMC WITH $Y_L = 0$ AND ALL THE IMPEDANCES PURELY RESISTIVE.

$$T_v = \frac{K_m K_0 G_0 (\alpha_2 S^2 + \alpha_1 S + \alpha_0)}{\begin{aligned} & S^2 [\beta_2 (G_0 + G_F + G_H) - K_m \beta_2 G_F - K_0 \gamma_2 (G_0 + G_F + G_H) \\ & \quad + K_m K_0 (\gamma_2 G_F - \alpha_2 G_H)] \\ & + S [\beta_1 (G_0 + G_F + G_H) - K_m \beta_1 G_F - K_0 \gamma_1 (G_0 + G_F + G_H) \\ & \quad + K_m K_0 (\gamma_1 G_F - \alpha_1 G_H)] \\ & + [\beta_0 (G_0 + G_F + G_H) - K_m \beta_0 G_F - K_0 \gamma_0 (G_0 + G_F + G_H) \\ & \quad + K_m K_0 (\gamma_0 G_F - \alpha_0 G_H)] \end{aligned}} \qquad (4.2.1)$$

It is known from Chapter II that for the condition $\frac{\Delta\omega_P}{\omega_P} = 0$ to be satisfied independently of K_m and K_0, it is required that

$$\frac{\beta_0}{\beta_2} = \frac{\gamma_0}{\gamma_2} = \frac{\alpha_0}{\alpha_2} = \omega_P^2 \qquad (2.6.4)$$

This restricts the choice of suitable t_v to the following types: Band Pass, Null, and All Pass. The Low Pass and High Pass types have to be excluded since any one of the quantities α_0 or α_2 cannot be set to zero without making the other zero.

The numerator of T_v being identical (within a constant multiplier) to the numerator of t_v, we should then expect to be able to obtain Band Pass, Null and All Pass transfer functions but no Low Pass or High Pass ones through the use of the second order t_v and the configuration shown in Fig. 4.2.1.

Table 4.2.1 gives all the possible polynomial decompositions and some of the properties of each decomposition based on Table 2.2.1. It has been shown in Section 2.6 that provided that the conditions of Eqn. 2.6.4 are fulfilled, the pole frequency ω_p of T_V is the same as the pole frequency ω_{p0} of t_V and its sensitivities with respect to the active parameters K_m, K_0 are equal to zero.

An examination of Table 4.2.1 shows that the twenty-four polynomial decompositions can be obtained by using only twelve generating functions $t_V = K_0 n_0(S)/D_0(S)$. These are tabulated in Table 4.2.2 together with the polynomial decompositions which are respectively realizable by each generating function.

It is clearly noted that these generating functions have to be realized by SACs satisfying the condition given in Eqn. 2.4.4. When this is done all the possible polynomial decompositions shown in Table 4.2.1 can be obtained. This enables us to realize two amplifier networks with the property of zero GωSP starting from optimized SACs [17, 18]. These realizations are by no means unique because each generating function can be realized in several ways, thus leading to several corresponding TACs.

In our study we are restricting our attention to Band Pass, Null or All Pass transfer functions only. Therefore the generating functions can only be Band Pass, Null or All Pass. In addition these generating functions will have to

TABLE 4.2.1

THE POLYNOMIAL DECOMPOSITIONS SUITABLE FOR Q-MULTIPLICATION WITH ZERO GωSP AND THE CORRESPONDING T_V, Ω_P AND SENSITIVITIES

TABLE 4.2.1

POLYNOMIAL DECOMPOSITION TYPE 1	
$t_v(S) = \dfrac{K_0\alpha_1 S}{\beta_2 S^2 - \beta_1 S + \beta_0}$	
$Q_{P0} = \dfrac{[\beta_0\beta_2]^{\frac{1}{2}}}{\beta_1}$	
$S_{K_0}^{Q_{P0}} = 0$	Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: 0,0
$T_v(S) = \dfrac{K_m K_0 G_0 \alpha_1 S}{S^2\beta_2(G_H+G_0) + S[\beta_1(G_H+G_0) - K_m K_0 \alpha_1 G_H] + \beta_0(G_H+G_0)}$	
$Q_P = Q_{P0}\left[1 - K_m K_0 G_H \alpha_1/(G_0 + G_H)\beta_1\right]^{-1}$	
$S_{K_0}^{Q_P} = \dfrac{Q_P}{Q_{P0}} - 1$	Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,0
$S_{K_m}^{Q_P} = S_{K_0}^{Q_P}$	Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,0

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 2	
$t_v(S) = \dfrac{K_0\alpha_1 S}{\beta_2 S^2-(\beta_1-K_0\gamma_1)S-\beta_0}$	
$Q_{P0} = \dfrac{\left[\beta_0\beta_2\right]^{\frac{1}{2}}}{\beta_1-K_0\gamma_1}$	
$S_{K_0}^{Q_{P0}} = \dfrac{K_0\gamma_1}{\beta_1-K_0\gamma_1}$	Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: -1,0
$T_v(S) = \dfrac{K_m K_0 G\alpha_1 S}{S^2\beta_2(G_H+G_0)+S[(G_H+G_0)(\beta_1-K_0\gamma_1)-K_m K_0\alpha_1 G_H]+\beta_0(G_H+G_0)}$	
$Q_P = \dfrac{Q_{P0}}{1-\dfrac{K_m K_0\alpha_1 G_H}{(\beta_1-K_0\gamma_1)(G_H+G_0)}}$	
$S_{K_0}^{Q_P} = \dfrac{K_m K_0\alpha_1 G_H+K_0\gamma_1(G_H-G_0)}{(G_H-G_0)(\beta_1-K_0\gamma_1)-K_m K_0\alpha_1 G_H}$	Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,0
$S_{K_m}^{Q_P} = \dfrac{Q_P}{Q_{P0}} -1$	Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,0

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 3
$t_v(S) = \dfrac{K_0(\alpha_2 S^2+\alpha_0)}{\beta_2 S^2+\beta_1 S+\beta_0}$
$Q_{P0} = \dfrac{\{\beta_0\beta_2\}^{\frac{1}{2}}}{\beta_1}$
$S_{K_0}^{Q_{P0}} = 0$ Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: 0,0
$T_v(S) = \dfrac{K_m K_0 G_0(\alpha_2 S^2+\alpha_0)}{S^2[\beta_2(G_H+G_0)-K_m K_0\alpha_2 G_H]+S\beta_1(G_H+G_0)+[\beta_2(G_H-G_0)-K_m K_0\alpha_0 G_H]}$
$Q_P = Q_{P0}\left\{\left[1-\dfrac{K_m K_0\alpha_0 G_H}{\beta_0(G_H-G_0)}\right]\left[1-\dfrac{K_m K_0\alpha_2 G_H}{\beta_2(G_H+G_0)}\right]\right\}^{\frac{1}{2}}$
$S_{K_0}^{Q_P} = -\dfrac{1}{2}\left\{\dfrac{K_m K_0\alpha_0 G_H}{\beta_0(G_H-G_0)-K_m K_0\alpha_0 G_H}+\dfrac{K_m K_0\alpha_2 G_H}{\beta_2(G_H+G_0)-K_m K_0\alpha_2 G_H}\right\}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: 0,1
$S_{K_m}^{Q_P} = S_{K_0}^{Q_P}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: 0,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 4

$$t_v(S) = \frac{K_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{\beta_2 S^2+\beta_1 S+\beta_0}$$

$$Q_{P0} = \frac{\left\{\beta_0\beta_2\right\}^{\frac{1}{2}}}{\beta_1}$$

$$S_{K_0}^{Q_{P0}} = 0$$

Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: 0,0

$$T_v(S) = \frac{K_m K_0 G_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{S^2[\beta_2(G_H+G_0)-K_m K_0\alpha_2 G_H]+S[\beta_1(G_H+G_0)-K_m K_0\alpha_1 G_H]+[\beta_0(G_H+G_0)-K_m K_0\alpha_0 G_H]}$$

$$Q_P = \frac{\Omega_{P0}\left\{\left[1-\frac{K_m K_0\alpha_0 G_H}{\beta_0(G_H+G_0)}\right]\left[1-\frac{K_m K_0\alpha_2 G_H}{\beta_2(G_H+G_0)}\right]\right\}^{\frac{1}{2}}}{1-\frac{K_m K_0\alpha_1 G_H}{\beta_1(G_H+G_0)}}$$

$$S_{K_0}^{Q_P} = -\frac{1}{2}\left[\frac{K_m K_0\alpha_0}{\beta_0(G_H+G_0)-K_m K_0\alpha_0 G_H}+\frac{K_m K_0\alpha_2 G_H}{\beta_2(G_H+G_0)-K_m K_0\alpha_2 G_H}\right]+\frac{K_m K_0\alpha_1}{\beta_1(G_H+G_0)-K_m K_0\alpha_1 G_H}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = S_{K_0}^{Q_P}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 5
$t_v(S) = \dfrac{K_0(\alpha_2 S^2 + \alpha_1 S - \alpha_0)}{\beta_2 S^2 + (\beta_1 - K_0\gamma_1)S + \beta_0}$
$Q_{P0} = \dfrac{\{\beta_0\beta_2\}^{\frac{1}{2}}}{\beta_1 - K_0\gamma_1}$
$S^{Q_{P0}}_{K_0} = \dfrac{K_0\gamma_1}{\beta_1 - K_0\gamma_1}$ Bounds of $S^{Q_{P0}}_{K_0}$ when all terms in D(S) are positive: -1,0
$T_v(S) = \dfrac{K_m K_0(\alpha_2 S^2 - \alpha_1 S + \alpha_0)}{S^2[\beta_2(G_H+G_0) - K_m K_0\alpha_2 G_H] + S[\beta_1(G_H+G_0) - K_0\gamma_1(G_H+G_0) - K_m K_0 \alpha_1 G_H] + [\beta_0(G_H+G_0) - K_m K_0 \alpha_0 G_H]}$
$Q_P = \dfrac{Q_{P0}\left\{\left[1 - \dfrac{K_m K_0 \alpha_0 G_H}{\beta_0(G_H+G_0)}\right]\left[1 - \dfrac{K_m K_0 \alpha_2 G_H}{\beta_2(G_0+G_H)}\right]\right\}^{\frac{1}{2}}}{1 - \dfrac{K_m K_0 \alpha_1 G_H}{(\beta_1 - K_0\gamma_1)(G_0+G_H)}}$
$S^{Q_P}_{K_0} = -\dfrac{1}{2}\left[\dfrac{K_m K_0\alpha_0 G_H}{\beta_0(G_H+G_0) - K_m K_0 \alpha_0 G_H} + \dfrac{K_m K_0 \alpha_2 G_H}{\beta_2(G_H+G_0) - K_m K_0 \alpha_2 G_H}\right] + \dfrac{K_m K_0 \alpha_1 G_H + K_0\gamma_0(G_H+G_0)}{\beta_1(G_H+G_0) - K_0\gamma_0(G_H+G_0) - K_m K_0 \alpha_1 G_H}$ Bounds of $S^{Q_P}_{K_0}$ when all terms in D(S) are positive: -1,1
$S^{Q_P}_{K_m} = S^{Q_P}_{K_0} - \dfrac{K_0\gamma_0(G_H+G_0)}{\beta_1(G_H+G_0) - K_0\gamma_0(G_H+G_0) - K_m K_0 \alpha_1 G_H}$ Bounds of $S^{Q_P}_{K_m}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 6
$t_v = \dfrac{K_0(\alpha_2 S^2+\alpha_0)}{\beta_2 S^2+(\beta_1-K_0\gamma_1)S+\beta_0}$
$C_P = \dfrac{\{\beta_0\beta_2\}^{\frac{1}{2}}}{\beta_1}$
$S_{K_0}^{Q_P} = 0$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: 0,0
$T_v(S) = \dfrac{K_m K_0 G_0(\alpha_2 S^2+\alpha_0)}{S^2[\beta_2(G_H+G_0)-K_m K_0\alpha_2 G_H]+S[\beta_1(G_H+G_0)-K_0\gamma_1(G_H+G_0)]+[\beta_0(G_H+G_0)-K_m K_0\alpha_0 G_H]}$
$Q_P = Q_{P0}\left\{\left[1-\dfrac{K_m K_0\alpha_2 G_H}{(G_H+G_0)\beta_2}\right]\left[1-\dfrac{K_m K_0\alpha_2 G_H}{(G_H+G_0)\beta_2}\right]\right\}^{\frac{1}{2}}$
$S_{K_0}^{Q_P} = -\dfrac{1}{2}\left[\dfrac{K_m K_0\alpha_0 G_H}{\beta_0(G_0+G_H)-K_m K_0\alpha_0 G_H}+\dfrac{K_m K_0\alpha_2 G_H}{\beta_2(G_0+G_H)-K_m K_0\alpha_2 G_H}\right]+\dfrac{K_0\gamma_1}{1-K_0\gamma_1}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1
$S_{K_m}^{Q_P} = S_{K_0}^{Q_P}-\dfrac{K_0\gamma_1}{1-K_0\gamma_1}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: 0,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 7
$t_v = \dfrac{K_0(\alpha_2 S^2+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2+\beta_1 S+(\beta_0-K_0\gamma_0)}$
$Q_P = \dfrac{\left[(\beta_2-K_0\gamma_2)(\beta_0-K_0\gamma_0)\right]^{\frac{1}{2}}}{\beta_1}$
$S_{K_0}^{Q_P} = -\dfrac{1}{2}\left[\dfrac{K_0\gamma_0}{\beta_0-K_0\gamma_0}+\dfrac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right]$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,0
$T_v(S) = \dfrac{K_m K_0 G_0(\alpha_2 S^2-\alpha_0)}{S^2[\beta_2(G_H+G_0)-K_0\gamma_2(G_H+G_0)-K_m K_0\alpha_2 G_H]+S\beta_2(G_H-G_0)+\beta_0(G_H+G_0)-K_0\gamma_0(G_H+G_0)-K_m K_0\alpha_0 G_H}$
$Q_P = Q_{P0}\left\{\left[1-\dfrac{K_m K_0\alpha_0 G_H}{(\beta_0-K_0\gamma_0)(G_H+G_0)}\right]\left[1-\dfrac{K_m K_0\alpha_2 G_H}{(\beta_2-K_0\gamma_2)(G_H+G_0)}\right]\right\}^{\frac{1}{2}}$
$S_{K_0}^{Q_P} = -\dfrac{1}{2}\left[\dfrac{K_0\gamma_0(G_H+G_0)+K_m K_0\alpha_0 G_H}{(\beta_0-K_0\gamma_0)(G_H+G_0)-K_m K_0\alpha_0 G_H}+\dfrac{K_0\gamma_2(G_H+G_0)+K_m K_0\alpha_2 G_H}{(\beta_2-K_0\gamma_2)(G_H+G_0)-K_m K_0\alpha_2 G_H}\right]$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: 0,1
$S_{K_m}^{Q_P} = -\dfrac{1}{2}\left[\dfrac{K_m K_0\alpha_0 G_H}{(\beta_0-K_0\gamma_0)(G_H+G_0)-K_m K_0\alpha_0 G_H}+\dfrac{K_m K_0\alpha_2 G_H}{(\beta_2-K_0\gamma_2)(G_H+G_0)-K_m K_0\alpha_2 G_H}\right]$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: 0,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 8
$t_v = \dfrac{K_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2+\beta_1 S+(\beta_0-K_0\gamma_0)}$
$Q_P = \{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)\}^{\frac{1}{2}}$
$S_{K_0}^{Q_P} = -\frac{1}{2}\left[\dfrac{K_0\gamma_2}{\beta_2-K_0\gamma_2} + \dfrac{K_0\gamma_0}{\beta_0-K_0\gamma_0}\right]$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: $-\frac{1}{2}, \frac{1}{2}$
$T_v(S) = \dfrac{K_m K_0 G_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{S^2[(\beta_2-K_0\gamma_2)(G_H+G_0)-K_m K_0\gamma_2 G_H]+S[\beta_1(G_H+G_0)-K_m K_0\alpha_1 G_H]+[(\beta_0-K_0\gamma_0)(G_0+G_H)-K_m K_0\alpha_0 G_H]}$
$Q_P = \dfrac{Q_{P0}\left[\left(1-\dfrac{K_m K_0\alpha_0 G_H}{(\beta_0-K_0\gamma_0)(G_0+G_H)}\right)\left(1-\dfrac{K_m K_0\alpha_2 G_H}{(\beta_2-K_0\gamma_2)(G_H+G_0)}\right)\right]^{\frac{1}{2}}}{1-\dfrac{K_m K_0\alpha_1 G_H}{\beta_1(G_0+G_H)}}$
$S_{K_0}^{Q_P} = -\left[\frac{1}{2}\left(\dfrac{K_m K_0\alpha_1 G_H+K_0\gamma_0(G_H+G_0)}{(\beta_0-K_0\gamma_0)(G_H+G_0)-K_m K_0\alpha_0 G_H} + \dfrac{K_m K_0\alpha_2 G_H+K_0\gamma_0(G_H+G_0)}{(\beta_2-K_0\gamma_0)(G_H+G_0)-K_m K_0\alpha_2 G_H}\right)\right] + \dfrac{K_m K_0\alpha_2 G_H}{\beta_1(G_H+G_0)-K_m K_0\alpha_1 G_H}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: $-1, 1$

TABLE 4.2.1 (Continued)

$S_{K_m}^{Q_P}$	$= -\frac{1}{2}\left[\frac{K_m K_0 \alpha_0 G_H}{(\beta_0 - K_0\gamma_0)(G_0+G_H) - K_m K_0 \alpha_0 K_H} + \frac{K_m K_0 \alpha_2 G_H}{(\beta_2 - K_0\gamma_2)(G_0+G_H) - K_m K_0 \alpha_2 G_H}\right] + \frac{K_m K_0 \alpha_1 G_H}{\beta_1 (G_H - G_0) - K_m K_0 \alpha_1 G_H}$. Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1
POLYNOMIAL DECOMPOSITION TYPE 9a	
t_v	$= \frac{K_0(\alpha_2 S^2 + \alpha_1 S + \alpha_0)}{(\beta_2 - K_0\gamma_2)S^2 + (\beta_1 - K_0\gamma_1)S + (\beta_0 - K_0\gamma_0)}$
Q_{P0}	$= \frac{\{(\beta_0 - K_0\gamma_0)(\beta_2 - K_0\gamma_2)\}^{\frac{1}{2}}}{(\beta_1 - K_0\gamma_1)}$
$S_{K_0}^{Q_P}$	$= -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0 - K_0\gamma_0} + \frac{K_0\gamma_2}{\beta_2 - K_0\gamma_2}\right] + \frac{K_0\gamma_1}{\beta_1 - K_0\gamma_1}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1
T_v	$= \frac{K_m K_0 G_0(\alpha_2 S^2 + \alpha_1 S + \alpha_0)}{S^2[(\beta_2 - K_0\gamma_2)(G_H+G_0) - K_m K_0 \alpha_2 G_H] + S[(\beta_1 - K_0\gamma_1)(G_H+G_0) - K_m K_0 \alpha_1 G_H] + [(\beta_0 - K_0\gamma_0)(G_H+G_0) - K_m K_0 \alpha_0 G_H]}$
Q_P	$= \frac{Q_{P0}\left\{\left[1 - \frac{K_m K_0 \alpha_0 G_H}{(G_0+G_H)(\beta_0 - K_0\gamma_0)}\right]\left[1 - \frac{K_m K_0 \alpha_2 G_H}{(G_0+G_H)(\beta_2 - K_0\gamma_2)}\right]\right\}^{\frac{1}{2}}}{1 - \frac{K_m K_0 \alpha_1 G_H}{(G_0+G_H)(\beta_1 - K_0\gamma_1)}}$

TABLE 4.2.1 (Continued)

$S_{K_0}^{Q_P}$	$= -\frac{1}{2}\left\{\frac{K_m K_0 \alpha_0 G_H + K_0 \gamma_0 (G_H - G_0)}{(G_H+G_0)(\beta_0 - K_0\gamma_0) - K_m K_0 \alpha_0 G_H} + \frac{K_m K_0 \alpha_2 G_H + K_0 \gamma_2 (G_H+G_0)}{(G_H+G_0)(\beta_2 - K_0\gamma_2) - K_m K_0 \gamma_2 G_H}\right\} + \frac{K_m K_0 \alpha_1 G_H + K_0 \gamma_1 (G_0+G_H)}{(G_0-G_H)(\beta_1 - K_0\gamma_1) - K_m K_0 \alpha_1 G_H}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1
$S_{K_m}^{Q_P}$	$= -\frac{1}{2}\left\{\frac{K_m K_0 \alpha_0 G_H}{(G_H-G_0)(\beta_0 - K_0\gamma_0) - K_m K_0 \alpha_0 G_H} + \frac{K_m K_0 \alpha_2 G_H}{(G_H+G_0)(\beta_0 - K_0\gamma_0) - K_m K_0 \alpha_2 G_H}\right\} + \frac{K_m K_0 \alpha_1 G_H}{(G_0-G_H)(\beta_1 - K_0\gamma_1) - K_m K_0 \alpha_1 G_H}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1
POLYNOMIAL DECOMPOSITION TYPE 9b	
t_v	$= \frac{K_0(\alpha_2 S^2 + \alpha_1 S + \alpha_0)}{\beta_2 S^2 - \beta_1 S + \beta_0}$
Q_{P0}	$= \frac{\beta_0 \beta_2}{\beta_1}$
$S_{K_0}^{Q_P}$	$= 0$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: 0,0
T_v	$= \frac{K_m K_0 G_0 (\alpha_2 S^2 + \alpha_1 S + \alpha_0)}{S^2[\beta_0(G_H+G_0+G_F) - K_m\beta_2\gamma_F - K_m K_0 \alpha_2 G_H] + S[\beta_1(G_H+G_0+G_F) - K_m\beta_1\gamma_F - K_m K_0 \alpha_1 G_H] + [\beta_0(G_H+G_0+G_H) - K_m\beta_0\gamma_F - K_m K_0 \alpha_0 G_H]}$

TABLE 4,2,1 (Continued)

$$Q_P = \frac{Q_{P0}\left[\left(1-\frac{K_m\beta_0 G_F - K_m K_0 \alpha_0 G_H}{\beta_0(G_H-G_0-G_F)}\right)\left(1-\frac{K_m\beta_2 G_F + K_m K_0 \alpha_2 G_H}{\beta_2(G_H+G_0+G_F)}\right)\right]^{\frac{1}{2}}}{1-\frac{K_m\beta_1 G_F - K_0 K_m \alpha_1 G_H}{\beta_1(G_H-G_0-G_F)}}$$

$$S_{K_0}^{Q_P} = -\frac{1}{2}\left[\frac{K_m K_0 \alpha_0 G_H}{\beta_0(G_H+G_0+G_F)-K_m\beta_0 G_F - K_m K_0 \alpha_0 G_H} + \frac{K_m K_0 \alpha_2 G_H}{\beta_2(G_H+G_0+G_F)-K_m\beta_0 G_F - K_m K_0 \alpha_0 G_H}\right] + \frac{K_m K_0 \alpha_1 G_H}{\beta_1(G_H+G_0+G_F)-K_m\beta_1 G_F - K_m K_0 \alpha_1 G_H}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = -\frac{1}{2}\left[\frac{K_m\beta_0 G_F\ K_m K_0 \alpha_0 G_H}{\beta_0(G_H+G_0-G_F)-K_m\beta_0 G_F - K_m K_0 \alpha_0 G_H} + \frac{K_m\beta_2 G_F\ K_m K_0 \alpha_2 G_H}{\beta_2(G_H-G_0-G_F-K_m\beta_2 G_F - K_m K_0 \alpha_2 G_H}\right] + \frac{K_m\beta_1 G_F\ K_m K_0 \alpha_1 G_H}{\beta_1(G_H+G_0+G_F)-K_m\beta_1 G_F - K_m K_0 \alpha_1 G_H}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

POLYNOMIAL DECOMPOSITION TYPE 10

$$t_v = \frac{K_0(\alpha_2 S^2 + \alpha_1^{(1)} S + \alpha_0)}{\beta_2 S^2 - (\beta_1 - K_0\gamma_1)S - \beta_0}$$

$$Q_{P0} = \frac{\beta_0\beta_2^{\frac{1}{2}}}{\beta_1 - K_0\gamma_1}$$

$$S_{K_0}^{Q_P} = \frac{K_0\gamma_1}{\beta_1 - K_0\gamma_1}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,0

TABLE 4.2.1 (Continued)

$$T_v(S) = \frac{K_m K_0 G_0 [\alpha_2 S^2 + \alpha_1 S - \alpha_0]}{S^2 [\beta_2 (G_H + H_0 + G_F) - K_m \beta_2 - K_m K_0 \alpha_2 G_H] + S[(\beta_1 - K_0 \gamma_1)(G_H - G_0 + G_F) - K_m \beta_1 G_F + K_m K_0 (\gamma_1 G_F - \alpha_1^{(1)} G_H)]}$$

$$+ [\beta_0 (G_H + G_0 + G_F) - K_m \beta_0 G_F - K_m K_0 \alpha_0 G_H]$$

$$Q_P \doteq \frac{Q_{P0} \left[1 - \dfrac{K_m \beta_0 G_F + K_m K_0 \alpha_0 G_H}{\beta_0 (G_H - G_0 + G_F)} \quad 1 - \dfrac{K_m \beta_2 G_F + K_m K_0 \alpha_2 G_H}{\beta_2 (G_H + G_0 + G_F)} \right]^{\frac{1}{2}}}{1 - \dfrac{K_m \beta_1 G_F - K_m K_0 (\gamma_1 G_F - \alpha_1^{(1)} G_H)}{(\beta_1 - K_0 \gamma_1)(G_H + G_0 + G_F)}}$$

$$S_{K_0}^{Q_{P0}} = -\frac{1}{2} \left[\frac{K_m K_0 \alpha_0 G_H}{\beta_0 [G_H - G_0 - G_F] - K_m \beta_0 G_F - K_m K_0 \alpha_0 G_H} + \frac{K_m K_0 \alpha_2 G_H}{\beta_2 [G_H - G_0 + G_F] - K_m \beta_2 G_F - K_m K_0 \alpha_2 G_H} \right]$$

$$- \frac{K_m K_0 (\gamma_0 G_F - \alpha_1^{(1)} G_H) - K_0 \gamma_1 (G_H + G_0 + G_F) - K_0 \gamma_1 (G_H + G_0 + G_H)}{(\beta_1 - K_0 \gamma_1)(G_H - G_0 \; G_F) - K_m \beta_1 G_F - K_m K_0 (\gamma_1 G_F - \alpha_1^{(1)} G_H)}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = -\frac{1}{2} \left[\frac{K_m K_0 \alpha_0 G_H + K_m \beta_0 G_F}{\beta_0 [G_H + G_0 - G_F] - K_m \beta_0 G_F - K_m K_0 \alpha_0 G_H} + \frac{K_m K_0 \alpha_2 G_H + K_m \beta_2 G_F}{\beta_2 [G_H - G_0 - G_F] - K_m \beta_2 G_F - K_m K_0 \alpha_2 G_H} \right] + \frac{K_m K_0 (\gamma_1 G_F - \alpha_1 G_H) - K_m \beta_1 G_F}{(\beta_1 - K_0 \gamma_1)(G_H + G_0 + G_F) - K_m \beta_1 G_F - K_m K_0 (\gamma_1 G_F - \alpha_1^{(1)} G_H)}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 11	
t_v	$= \dfrac{K_0(\alpha_2 S^2+\alpha_1 S-\alpha_0)}{\beta_2 S^2+(\beta_1-K_0\gamma_1)S+\beta_0}$
Q_{P0}	$= \dfrac{[\beta_0\beta_2]^{\frac{1}{2}}}{\beta_1-K_0\gamma_1}$
$S_{K_0}^{Q_{P0}}$	$= \dfrac{K_0\gamma_1}{\beta_1-K_0\gamma_1}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: 0,1
$T_v(S)$	$= \dfrac{K_mK_0G_0(\alpha_2S^2+\alpha_1S+\alpha_0)}{S^2[\beta_2(G_H+G_0+G_F)-K_m\beta_2G_F-K_mK_0G_H\alpha_2] + S[(\beta_1-K_0\gamma_1)(G_H+G_0+G_F)-K_m\beta_1G_F] + [\beta_0(G_H+G_0+G_F)-K_m\beta_0G_F-K_mK_0G_H\alpha_0]}$
Q_P	$= \dfrac{Q_{P0}\left[\left(1-\dfrac{K_m\beta_0G_F+K_mK_0\alpha_0G_H}{\beta_0[G_H+G_0+G_F]}\right)\left(1-\dfrac{K_m\beta_2G_F+K_mK_0\alpha_2G_H}{\beta_2[G_H+G_0+G_F]}\right)\right]^{\frac{1}{2}}}{1-\dfrac{K_m\beta_1G_F}{(\beta_1-K_0\gamma_1)(G_H+G_0+G_F)}}$
$S_{K_0}^{Q_P}$	$= -\dfrac{1}{2}\left[\dfrac{K_mK_0\alpha_0G_H}{\beta_0[G_H+G_0+G_F]-K_m\beta_0G_F-K_mK_0\alpha_0G_H} + \dfrac{K_mK_0\alpha_0G_H}{\beta_2[G_H+G_0+G_F]-K_m\beta_2G_F-K_mK_0\alpha_2G_H}\right] + \dfrac{K_0\gamma_1[G_H+G_0+G_P]}{(\beta_1-K_0\gamma_1)(G_H+G_0+G_F)-K_m\beta_1G_F}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

$S_{K_m}^{Q_P} = -\frac{1}{2}\left[\frac{K_m\beta_0 G_F + K_m K_0\alpha_0 G_H}{\beta_0(G_H-G_0-G_F) - K_m\beta_0 G_F - K_m K_0\alpha_0 G_H} + \frac{K_m\beta_2 G_F + K_m K_0\alpha_2 G_H}{\beta_2(G_H-G_0-G_F) - K_m\beta_2 G_F - K_m K_0\alpha_2 G_H}\right] + \frac{K_m\beta_1 G_F}{(\beta_1-K_0\gamma_1)(G_H+G_0-G_F) - K_m\beta_1 G_F}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1
POLYNOMIAL DECOMPOSITION TYPE 12a
$t_v = \frac{K_0(-\alpha_2 S^2 - \alpha_0)}{(\beta_2-K_0\gamma_2)S^2 + (\beta_1-K_0\gamma_1)S - (\beta_0-K_0\gamma_0)}$
$Q_{P0} = \frac{\{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)\}^{\frac{1}{2}}}{\beta_1-K_0\gamma_1}$
$S_{K_0}^{Q_{P0}} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0} + \frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right] + \frac{K_0\gamma_1}{\beta_1-K_0\gamma_1}$ Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: -1,1
$T_v(S) = \frac{K_m K_0 G_0(\alpha_2 S^2 + \alpha_0)}{S^2[(\beta_2-K_0\gamma_2)(G_H-G_0-G_F) - K_m K_0\alpha_2 G_H] + S[(\beta_1-K_0\gamma_1)(G_0+G_H-G_F)] + [(\beta_0-K_0\gamma_0)(G_H+G_0+G_F) - K_m K_0\alpha_0 G_H]}$
$Q_P = Q_{P0}\left[1 - \frac{K_m K_0\alpha_2 G_H}{(\beta_0-K_0\gamma_0)(G_H-G_0-G_F)}\right]^{\frac{1}{2}}\left[1 - \frac{K_m K_0\alpha_0 G_H}{(\beta_2-K_0\gamma_2)(G_H-G_0-G_F)}\right]^{\frac{1}{2}}$
$S_{K_0}^{Q_P} = -\frac{1}{2}\left[\frac{K_0\gamma_0(G_H+G_0-G_F) - K_m K_0\alpha_0 G_H}{(\beta_0-K_0\gamma_0)(G_H-G_0-G_F) - K_m K_0\alpha_0 G_H} + \frac{K_0\gamma_2(G_H+G_0+G_F) + K_m K_0\alpha_2 G_H}{(\beta_2-K_0\gamma_2)(G_H-G_0-G_F) - K_m K_0\alpha_2 G_H}\right] + \frac{K_0\gamma_1}{\beta_1-K_0\gamma_1}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

$$S^{Q_P}_{K_m} = -\frac{1}{2}\left[\frac{K_m K_0 \alpha_0 G_H}{(\beta_0 - K_0\gamma_0)(G_0 - G_H - G_F) - K_m K_0 \alpha_0 G_H} + \frac{K_m K_0 \alpha_2 G_H}{(\beta_2 - K_0\gamma_2)(G_0 - G_F + G_H) - K_m K_0 \alpha_2 G_H}\right]$$

Bounds of $S^{Q_P}_{K_m}$ when all terms in D(S) are positive: 0,1

POLYNOMIAL DECOMPOSITION TYPE 12b

$$t_v = \frac{K_0(\alpha_2 S^2 + \alpha_0)}{\beta_2 S^2 - \beta_1 S - \beta_0}$$

$$Q_{P0} = \frac{\{\beta_0 \beta_2\}^{\frac{1}{2}}}{\beta_1}$$

$$S^{Q_{P0}}_{K_0} = 0$$ Bounds of $S^{Q_{P0}}_{K_0}$ when all terms in D(S) are positive: 0,0

$$T_v(S) = \frac{K_m K_0 G_I(\alpha_2 S^2 + \alpha_0)}{S^2[\beta_2(G_0 + G_H + G_F) - K_m\beta_2\gamma_F + K_m K_0(\gamma_2 G_F - \alpha_2 G_H)] + S[\beta_1(G_H - G_0 + G_F) - K_m\beta_1 G_F] + [\beta_0(G_H + G_0 + G_F) - K_m\beta_0\gamma_F + K_m K_0(\gamma_0 G_F - \alpha_0 G_H)]}$$

$$Q_P = \frac{Q_{P0}\left\{\left[\begin{matrix}(G_H - G_0 + G_F) - K_m G_F \\ -K_m K_0\left(\frac{\gamma_0}{\beta_0} G_F - \frac{\alpha_0}{\beta_0} G_H\right)\end{matrix}\right]\left[\begin{matrix}G_H + G_0 + G_F) - K_m G_F \\ +K_m K_0\left(\frac{\gamma_2}{\beta_2} G_F - \frac{\alpha_2}{\beta_2} G_H\right)\end{matrix}\right]\right\}^{\frac{1}{2}}}{(G_H - G_0 + G_F) - K_m G_F}$$

$$S^{Q_P}_{K_0} = \frac{1}{2}\left[\frac{K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}{\beta_0(G_H - G_0 - G_F) - K_m\beta_0 G_F - K_m K_0(\gamma_0 G_F - \alpha_0 G_H)} + \frac{K_m K_0(\gamma_2 G_F - \alpha_2 G_H)}{\beta_2(G_H + G_0 + G_F) - K_m\beta_2 G_F + K_m K_0(\gamma_2 G_F - \alpha_2 G_H)}\right]$$

Bounds of $S^{Q_P}_{K_0}$ when all terms in D(S) are positive: 0,1

TABLE 4.2.1 (Continued)

$$S^{Q_P}_{K_m} = -\frac{1}{2}\left[\frac{-K_m\beta_0 G_F - K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}{\beta_0(G_H - G_0 - G_F) - K_m\beta_0 G_F - K_m K_0(\gamma_0 G_F - \alpha_0 G_H)} + \frac{-K_m\beta_2 G_F + K_m K_0(\gamma_2 G_F - \alpha_2 G_H)}{\beta_2(G_H + G_0 - G_F) - K_m\beta_2 G_F - K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}\right] + \frac{K_m G_F}{G_H + G_0 + G_F - K_m G_F}$$

Bounds of $S^{Q_P}_{K_m}$ when all terms in D(S) are positive: -1,1

POLYNOMIAL DECOMPOSITION TYPE 13

$$t_v = \frac{K_0(\alpha_2^{(1)} S^2 - \alpha_1 S + \alpha_0^{(1)})}{(\beta_2 - K_0\gamma_2)S^2 - (\beta_1 - K_0\gamma_1)S - (\beta_0 - K_0\gamma_0)}$$

$$Q_{P0} = \frac{\{(\beta_0 - K_0\gamma_0)(\beta_2 - K_0\gamma_2)\}^{\frac{1}{2}}}{\beta_1 - K_0\gamma_1}$$

$$S^{Q_{P0}}_{K_0} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0 - K_0\gamma_0} + \frac{K_0\gamma_2}{\beta_2 - K_0\gamma_2}\right] + \frac{K_0\gamma_1}{\beta_1 - K_0\gamma_1}$$

Bounds of $S^{Q_{P0}}_{K_0}$ when all terms in D(S) are positive: -1,1

$$T_v(S) = \frac{K_m K_0 G_0(\alpha_2^{(1)} S^2 + \alpha_1 S + \alpha_0^{(1)})}{S^2\{(\beta_2 - K_0\gamma_2)(G_H - G_0 + G_F) - K_m\beta_2 G_F + K_m K_0(\gamma_2 G_F - \alpha_2^{(1)} G_H)\} + S[(G_H + G_0 - G_F)\beta_1 - K_m\beta_1 G_F - K_m K_0\alpha_1 G_H] + [(\beta_0 - K_0\gamma_0)(G_H + G_0 + G_F) - K_m\beta_0 G_F + K_m K_0(\gamma_0 G_F - \alpha_0^{(1)} G_H)]}$$

$$Q_P = \frac{Q_{P0}\left[1 - \frac{K_m\beta_0 G_F - K_m K_0(\gamma_0 G_F - \alpha_0^{(1)} G_H)}{(\beta_0 - K_0\gamma_0)(G_H - G_0 - G_F)} \times \frac{K_m\beta_0 G_F - K_m K_0(\gamma_0 G_F - \alpha_0^{(1)} G_H)}{(\beta_2 - K_0\gamma_2)(G_H - G_0 + G_F)}\right]^{\frac{1}{2}}}{1 - \frac{K_m\beta_1 G_F - K_m K_0\alpha_1}{G_H - G_0 + G_F + \beta_1}}$$

TABLE 4.2.1 (Continued)

$$S_{K_0}^{Q_P} = \frac{1}{2}\left[\frac{K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)-K_0\gamma_0(G_H+G_0+G_F)}{(\beta_0-K_0\gamma_0)(G_H-G_0+G_F)-K_m\beta_0G_F-K_mK_0(\gamma_0G_F-\alpha_1^{(1)}G_H)} - \frac{K_0K_m(\gamma_0G_F-\alpha_2^{(1)}G_H)-K_0\gamma_2(G_H+G_F+G_0)}{(\beta_2-K_0\gamma_2)(G_H+G_0-G_F)-K_m\beta_2G_F-K_mK_0(\gamma_2G_F-\alpha_2^{(1)}G_H)}\right] + \frac{K_mK_0\alpha_1G_H}{\beta_1(G_H-G_0+G_F)-K_m\beta_1G_F-K_mK_0\alpha_1G_H}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = \frac{1}{2}\left[\frac{K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)-K_m\beta_0G_F}{(\beta_0-K_0\gamma_0)(G_H+G_0+G_F)-K_m\beta_0G_F-K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)} + \frac{K_mK_0(\gamma_2G_F-\alpha_2^{(1)}G_H)-K_m\beta_2G_F}{(\beta_2-K_0\gamma_2)(G_H+G_0+G_F)-K_m\beta_2G_F-K_mK_0(\gamma_2G_F-\alpha_2^{(1)}G_H)}\right] - \frac{K_mK_0\alpha_1G_H-K_m\beta_1G_F}{\beta_1(G_H-G_0-G_F)-K_m\beta_1G_F-K_mK_0\alpha_1G_H}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

POLYNOMIAL DECOMPOSITION TYPE 14

$$t_v = \frac{K_0(\alpha_2^{(1)}S^2+\alpha_1^{(2)}S+\alpha_0^{(1)})}{(\beta_2-K_0\gamma_2)S^2+(\beta_1-K_1\gamma_1)S+(\beta_0-K_0\gamma_0)}$$

$$Q_{P0} = \frac{\left\{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)\right\}^{\frac{1}{2}}}{\beta_1-K_0\gamma_1}$$

$$S_{K_0}^{Q_{P0}} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0} + \frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right] + \frac{K_0\gamma_1}{\beta_1-K_0\gamma_1}$$

Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

$$T_v(S) = \frac{K_m K_0 G_0(\alpha_2^{(1)} S^2 + \alpha_1^{(2)} S + \alpha_0^{(1)})}{S^2[(\beta_2-K_0\gamma_2)(G_0-G_H+G_F)-K_m\beta_2 G_F+K_m K_0(\gamma_2 G_F-\alpha_2^{(1)} G_H)] + S[(\beta_1-K_0\gamma_1)(G_0+G_H+G_F)-K_m\beta_1 G_F+K_m K_0(\gamma_1 G_F-\alpha_1^{(2)} G_H)] + [(\beta_0-K_0\gamma_0)(G_0-G_H+G_F)-K_m\beta_0 G_F+K_m K_0(\gamma_0 G_F-\alpha_0^{(1)} G_H)]}$$

$$Q_P = \frac{Q_{P0}\left[1-\frac{K_m\beta_0 G_F-K_m K_0(\gamma_0 G_F-\alpha_0^{(1)} G_H)}{(\beta_0-K_0\gamma_0)(G_H-G_0-G_F)} \times 1-\frac{K_m\beta_2 G_F-K_m K_0(\alpha_2 G_F-\alpha_2^{(1)} G_H)}{(\beta_0-K_0\gamma_0)(G_H-G_0+G_F)}\right]^{\frac{1}{2}}}{1-\frac{K_m\beta_1 G_F-K_m K_0(\gamma_1 G_F-\alpha_1^{(2)} G_H)}{(\beta_1-K_0\gamma_1)(G_H+G_0-G_F)}}$$

$$S_{K_0}^{Q_P} = \frac{1}{2}\left[\frac{-K_0\gamma_0(G_H+G_0+G_F)+K_m K_0(\gamma_0 G_F-\alpha_0^{(1)} G_H)}{(G_H+G_0+G_F)(\beta_0-K_0\gamma_0)-K_m\beta_0 G_F-K_m K_0(\gamma_0 G_F-\alpha_0^{(1)} G_H)} - \frac{-K_0\gamma_2(G_H+G_0+G_F)+K_m K_0(\alpha_2 G_F-\alpha_2^{(1)} G_H)}{(G_H+G_0+G_F)(\beta_2-K_0\gamma_2)-K_m\beta_2 G_F+K_m K_0(\gamma_2 G_F-\alpha_2^{(1)} G_H)}\right]$$

$$- \frac{-K_0\gamma_0(G_0+G_H-G_F)+K_m K_0(\gamma_1 G_F-\alpha_1^{(2)} G_H)}{(\beta_1-K_0\gamma_1)(G_H+G_0+G_F)-K_m\beta_1 G_F+K_m K_0(\gamma_1 G_F-\alpha_1^{(2)} G_H)}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = \frac{1}{2}\left[\frac{-K_m\beta_0 G_F+K_m K_0(\gamma_0 G_F-\alpha_0^{(1)} G_H)}{(\beta_0-K_0\gamma_0)(G_H-G_0-G_F)-K_m\beta_0 G_F+K_m K_0(\beta_0 G_F-\alpha_0^{(1)} G_H)} + \frac{-K_m\beta_2 G_F+K_m K_0(\gamma_0 G_F-\alpha_0^{(1)} G_H)}{(\beta_2-K_0\gamma_2)(G_H+G_0+G_F)-K_m\beta_2 G_F+K_m K_0(\gamma_0 G_F-\alpha_2^{(1)} G_H)}\right]$$

$$- \frac{-K_m\beta_1 G_F-K_m K_0(\gamma_1 G_F-\alpha_1^{(1)} G_H)}{(\beta_1-K_0\gamma_1)(G_H+G_0-G_F)-K_m\beta_1 G_F+K_m K_0(\gamma_1 G_F-\alpha_1^{(2)} G_H)}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 15

$$t_v = \frac{K_0(\alpha_2^{(1)}S^2+\alpha_1 S+\alpha_1^{(1)})}{(\beta_2-K_0\gamma_2)S^2+(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)}$$

$$Q_{P0} = \frac{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)}{\beta_1-K_0\gamma_1}$$

$$S_{K_0}^{Q_{P0}} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0}+\frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right]+\frac{K_0\gamma_1}{\beta_1-K_0\gamma_1}$$

Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: -1,1

$$T_v(S) = \frac{K_mK_0G_0(\alpha_2^{(1)}S^2+\alpha_1 S\ \alpha_0^{(1)})}{S^2[(\beta_2-K_0\gamma_2)(G_H+G_0-G_F)-K_m\beta_2G_F+K_mK_0(\gamma_2G_F-\alpha_2^{(1)}G_H)]+S[(\beta_1-K_0\gamma_1)(G_H+G_0+G_F)-K_m\beta_1G_F]+[(\beta_0-K_0\gamma_0)(G_H+G_0+G_F)-K_m\beta_0G_F+K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)]}$$

$$Q_P = \frac{Q_{P0}\left\{\left[1-\frac{K_m\beta_0G_F+K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)}{(\beta_0-K_0\gamma_0)(G_H+G_0+G_F)-K_m\beta_0G_F+K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)}\right]\left[1-\frac{K_m\beta_2G_F+K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)}{(\beta_2-K_0\gamma_2)(G_H+G_0+G_F)-K_m\beta_2G_F+K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)}\right]\right\}^{\frac{1}{2}}}{1-\frac{K_m\beta_1G_F}{(\beta_1-K_0\gamma_1)(G_H+G_0+G_F)}}$$

TABLE 4.2.1 (Continued)

$$S_{K_0}^{Q_P} = \frac{1}{2}\left[\frac{-K_0\gamma_0(G_H+G_0+G_F)+K_0(\gamma_0G_F-\alpha_0^{(1)}G_H)}{(\beta_0-K_0\gamma_0)(G_H-G_0-G_F)-K_m\beta_0G_F+K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)} + \frac{-K_0\gamma_2(G_H+G_0+G_F)+K_mK_0(\gamma_2G_F-\alpha_2^{(1)}G_H)}{(\beta_2-K_0\gamma_2)(G_H-G_0+G_F)-K_m\beta_2G_F+K_mK_0(\gamma_2G_F-\alpha_2G_H)}\right]$$

$$\frac{K_0\gamma_1(G_H+G_0-G_F)}{(\beta_1-K_0\gamma_1)(G_H-G_0-G_F)-K_m\beta_1G_F}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = \frac{1}{2}\left[\frac{-K_m\beta_0G_F-K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)}{(\beta_0-K_0\gamma_0)(G_H+G_0-G_F)-K_m\beta_0G_F-K_mK_0(\gamma_0G_F-\alpha_0^{(1)}G_H)} + \frac{-K_m\beta_2G_F+K_mK_0(\gamma_2G_F-\alpha_2^{(1)}G_H)}{(\beta_2-K_0\gamma_2)(G_H+G_0+G_F)-K_m\beta_2G_F+K_mK_0(\gamma_2G_F-\alpha_2G_H)}\right]$$

$$+ \frac{K_m\beta_1G_F}{(\beta_1-K_0\gamma_1)(G_H+G_0-G_F)-K_m\beta_1G_F}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

POLYNOMIAL DECOMPOSITION TYPE 16

$$t_Y = \frac{K_0(\alpha_2S^2+\alpha_0)}{S^2(\beta_2-K_0\gamma_2)+\beta_1S+(\beta_0-K_0\gamma_0)}$$

$$Q_{P0} = \frac{\left\{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_0)\right\}^{\frac{1}{2}}}{\beta_1-K_0\gamma_1}$$

$$S_{K_0}^{Q_{P0}} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0}+\frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right]$$

Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: 0,1

TABLE 4.2.1 (Continued)

$$T_v(S) = \frac{K_m K_0 G_0 (\alpha_2 S^2 + \alpha_0)}{S^2[(\beta_2 - K_0\gamma_2)(G_H+G_0+G_F) - K_m\beta_2 G_F + K_m K_0(\gamma_2 G_F - \alpha_2 G_H)] - S[\beta_1(G_H+G_0-G_F) - K_m\beta_1 G_F] + [(\beta_0 - K_0\gamma_0)(G_H+G_0+G_F) - K_m\beta_0 G_F + K_m K_0(\gamma_0 G_F - \alpha_0 G_H)]}$$

$$Q_P = \frac{Q_{P0}\left[1 - \frac{K_m\beta_0 G_F - K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}{(\beta_0 - K_0\gamma_0)(G_H - G_F - G_0)} \times 1 - \frac{K_m\beta_2 G_F - K_m K_0(\gamma_2 G_F - \alpha_2 G_H)}{(\beta_2 - K_0\gamma_2)(G_H - G_F - G_0)}\right]^{\frac{1}{2}}}{1 - \frac{K_m G_F}{G_H + G_0 + G_F}}$$

$$S^{Q_P}_{K_0} = \frac{1}{2}\left[\frac{K_0\gamma_0(G_0+G_H-G_F)\ K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}{(-K_0\gamma_0)(G_H-G_0+G_F) - K_m\beta_0 G_F + K_0(\gamma_0 G_F - \alpha_0 G_H)} + \frac{-K_0\gamma_2(G_0+G_H+G_F)\ K_m K_0(\gamma_2 G_F - \alpha_2 G_H)}{(\beta_2 - K_0\gamma_2)(G_H - G_0 + G_F) - K_m\beta_2 G_F + K_m K_0(\gamma_2 G_F - \alpha_2 G_H)}\right]$$

Bounds of $S^{Q_P}_{K_0}$ when all terms in D(S) are positive: 0,1

$$S^{Q_P}_{K_m} = \frac{1}{2}\left[\frac{-K_m\beta_0 G_F + K_m K_0(\gamma_0 G_F - \alpha_0 G_H)}{(\beta_0 - K_0\gamma_0)(G_H - G_0 - G_F) - K_m\beta_0 G_F - K_m K_0(\gamma_0 G_F - \alpha_0 G_H)} + \frac{-K_m\beta_2 G_F + K_m K_0(\gamma_2 G_F - \alpha_2 G_H)}{(\beta_2 - K_0\gamma_2)(G_H + G_0 + G_F) - K_m\beta_2 G_F - K_m K_0(\gamma_2 G_F - \alpha_2 G_H)}\right]$$

$$+ \frac{K_m G_F}{(G_H + G_0 + G_F) - K_m G_F}$$

Bounds of $S^{Q_P}_{K_m}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 17
$t_v = \dfrac{K_0(\alpha_2 S^2+\alpha_1 S-\alpha_0)}{S^2(\beta_2-K_0\gamma_2)-\beta_1 S-(\beta_0-K_0\gamma_0)}$
$Q_{P0} = \dfrac{[(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)]^{\frac{1}{2}}}{\beta_1}$
$S_{K_0}^{Q_{P0}} = -\dfrac{1}{2}\left[\dfrac{K_0\gamma_0}{\beta_0-K_0\gamma_0}+\dfrac{K_0\gamma_2}{\beta_0-K_0\gamma_2}\right]$ Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: 0,1
$T_v = \dfrac{K_m K_0 G_0(\alpha_2 S^2+\alpha_1 S-\alpha_0)}{S^2[(G_0+G_H+G_F)(\beta_2-K_0\gamma_2)-K_m\beta_2 G_F]+S[(G_0-G_H-G_F)\beta_1-K_m\beta_1 G_F-K_m K_0\alpha_1 G_H]+[(G_0+G_H+G_F)(\beta_0-K_0\gamma_0)-K_m\beta_0 G_F]}$
$Q_P = \dfrac{Q_{P0}\left[\left(1-\dfrac{K_m\beta_0 G_F}{(\beta_0-K_0\gamma_0)(G_0-G_H-G_F)}\right)\left(1-\dfrac{K_m\beta_2 G_F}{(\beta_2-K_0\gamma_2)(G_0-G_H-G_F)}\right)\right]^{\frac{1}{2}}}{1-\dfrac{K_m\beta_1 G_F+K_m K_0\alpha_1 G_H}{\beta_1(G_H+G_0-G_F)}}$
$S_{K_0}^{Q_P} = -\dfrac{1}{2}\left[\dfrac{K_0\gamma_0[G_H+G_0-G_F]}{[G_H-G_0+G_F](\beta_0-K_0\gamma_0)-K_m\beta_0 G_F}+\dfrac{K_0\gamma_2[G_H+G_0+G_F]}{[G_H-G_0-G_F](\beta_2-K_0\gamma_2)-K_m\beta_2 G_F}\right]+\dfrac{K_m K_0\alpha_1 G_H}{[G_H+G_0+G_F]\beta_1-K_m\beta_1 G_F-K_m K_0\alpha_1 G_H}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

$$S_{K_m}^{Q_P} = -\frac{1}{2}\left[\frac{K_m\beta_0 G_F}{[(G_H-G_0-G_F)(\beta_0-K_0\gamma_0)-K_m\beta_0 G_F]} + \frac{K_m\beta_2 G_F}{[(G_H-G_0-G_F)(\beta_2-K_0\gamma_2)-K_m\beta_2 G_F]}\right] + \frac{K_m\beta_1 G_F - K_m K_0\alpha_1 G_H}{[(G_0-G_H+G_F)(\beta_1-K_0\gamma_1)-K_m\beta_1 G_F-K_m K_0\alpha_1 G_H]}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

POLYNOMIAL DECOMPOSITION TYPE 18

$$t_v = \frac{K_0(\alpha_2 S^2-\alpha_1^{(1)}S-\alpha_0)}{(\beta_2-K_0\gamma_2)S^2+(\beta_1-K_0\gamma_1)S-(\beta_0-K_0\gamma_0)}$$

$$Q_{P0} = \frac{\{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)\}^{\frac{1}{2}}}{\beta_1-K_0\gamma_1}$$

$$S_{K_0}^{Q_{P0}} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0} - \frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right] + \frac{K_0\gamma_1}{\beta_1-K_0\gamma_1}$$

Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: -1,1

$$T_v = \frac{K_m K_0 G_0(\alpha_2 S^2+\alpha_1^{(1)}S-\alpha_0)}{S^2[(\beta_2-K_0\gamma_2)(G_0-G_H+G_F)-K_m\beta_2 G_F] + S[(\beta_1-K_0\gamma_1)(G_0-G_H+G_F)-K_m\beta_1 G_F+K_m K_0(\gamma_1 G_F-\alpha_1^{(1)}G_H)] + [(\beta_0-K_0\gamma_0)(G_0+G_F-G_F)-K_m\beta_0 G_F]}$$

$$Q_P = \frac{Q_{P0}\left[1-\frac{K_m\beta_0 G_F}{(\beta_0-K_0\gamma_0)(G_H-G_0-G_F)}\cdot 1-\frac{K_m\beta_2 G_F}{(\beta_2-K_0\gamma_2)(G_H-G_0+G_F)}\right]^{\frac{1}{2}}}{1-\frac{K_m\beta_1 G_F-K_m K_0(\gamma_1 G_F-\alpha_1^{(1)}G_H)}{(G_H-G_0+G_F)(\beta_1-K_0\gamma_1)}}$$

TABLE 4.2.1 (Continued)

$$S_{K_0}^{Q_P} = -\frac{1}{2}\left[\frac{K_0\gamma_0(G_H+G_0-G_F)}{(G_0-G_H+G_F)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F} + \frac{K_0\gamma_2(G_H+G_0+G_F)}{(G_0-G_H-G_F)(\beta_2-K_0\gamma_2)-K_m\beta_2G_F}\right] - \frac{K_mK_0(\gamma_1G_F-\alpha_1^{(1)}G_H)}{(G_0+G_H-G_F)(\beta_1-K_0\gamma_1)-K_m\beta_1G_F+K_mK_0(\gamma_1G_F-\alpha_1G_H)}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = -\frac{1}{2}\left[\frac{K_m\beta_0G_F}{(G_0-G_H+G_F)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F} + \frac{K_m\beta_2G_F}{(G_0-G_H-G_F)(\beta_2-K_0\gamma_2)-K_m\beta_2G_F}\right] - \frac{-K_m\beta_1G_F-K_mK_0(\gamma_1G_F-\alpha_1^{(1)}G_H)}{(G_0-G_H-G_F)(\beta_1-K_0\gamma_1)-K_m\beta_1G_F-K_mK_0(\gamma_1G_F-\alpha_1G_H)}$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

POLYNOMIAL DECOMPOSITION TYPE 19

$$t_v = \frac{K_0(\alpha_2S^2+\alpha_1S+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2-(\beta_1-K_0\gamma_1)S-(\beta_0-K_0\gamma_0)}$$

$$Q_{P0} = \frac{\{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)\}^{1/2}}{\beta_1-K_0\gamma_1}$$

$$S_{K_0}^{Q_{P0}} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0} + \frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right] + \frac{K_0\gamma_1}{\beta_1-K_0\gamma_1}$$

Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

T_V	$= \dfrac{K_m K_0 G_D(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{S^2[(\beta_2-K_0\gamma_2)(G_0-G_H-G_F)-K_m\beta_2 G_F] + S[(\beta_1-K_0\gamma_1)(G_0-G_H-G_F)-K_m\beta_1 G_F] + [(\beta_0-K_0\gamma_0)(G_0+G_H+G_F)-K_m\beta_0 G_F]}$
Q_P	$= \dfrac{Q_{P0}\left[\left(1-\dfrac{K_m\beta_0 G_F}{(\beta_0-K_0\gamma_0)(G_0-G_H-G_F)}\right)\left(1-\dfrac{K_m\beta_2 G_F}{(\beta_2-K_0\gamma_2)(G_0-G_H-G_F)}\right)\right]^{\frac{1}{2}}}{1-\dfrac{K_0\beta_1 G_F}{(\beta_1-K_0\gamma_1)(G_0+G_H+G_F)}}$
$S_{K_0}^{Q_P}$	$= -\dfrac{1}{2}\left[\dfrac{K_0\gamma_0(G_H-G_0-G_F)}{(G_H-G_0-G_F)(\beta_0-K_0\gamma_0)-K_m\beta_0 G_F} + \dfrac{K_0\gamma_2(G_H-G_0-G_F)}{(G_H-G_0-G_F)(\beta_2-K_0\gamma_2)-K_m\beta_2 G_F}\right] + \dfrac{K_0\gamma_1(G_H+G_0+G_F)}{(\beta_1-K_0\gamma_1)(G_H+G_0+G_F)-K_m\beta_1 G_F}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1
$S_{K_m}^{Q_P}$	$= -\dfrac{1}{2}\left[\dfrac{K_m\beta_0 G_F}{(G_H-G_0-G_F)(\beta_0-K_0\gamma_0)-K_m\beta_0 G_F} + \dfrac{K_m\beta_2 G_F}{(G_H-G_0-G_F)(\beta_2-K_0\gamma_2)-K_m\beta_2 G_F}\right] + \dfrac{K_m\beta_1 G_F}{(\beta_1-K_0\gamma_1)(G_H+G_0+G_F)-K_m\beta_1 G_F}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1
POLYNOMIAL DECOMPOSITION TYPE 20	
t_V	$= \dfrac{K_0(\alpha_2 S^2-\alpha_0)}{(\beta_2-K_0\gamma_2)S^2-\beta_1 S+(\beta_0-K_0\gamma_0)}$
Q_{P0}	$= \dfrac{\{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)\}^{\frac{1}{2}}}{\beta_1}$

TABLE 4.2.1 (Continued)

$S_{K_0}^{Q_{P0}}$	$= -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0} + \frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right]$ Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: 0,1
T_v	$= \frac{K_m K_0 G_0 (\beta_2 S^2 - \gamma_0)}{S^2[(G_H+G_0-G_F)(\beta_2-K_0\gamma_2)-K_m\beta_2G_F] + S[(G_H+G_0-G_F)\beta_1-K_m\beta_1G_F] + [(G_H-G_0+G_F)(\beta_0-K_0\gamma_0)-K_m\beta_0G_F]}$
Q_P	$= \frac{Q_{P0}\left[\left(1-\frac{K_m\beta_0G_F}{(\beta_0-K_0\gamma_0)(G_H-G_0-G_F)}\right)\left(1-\frac{K_m\beta_2G_F}{(\beta_2-K_0\gamma_2)(G_H-G_0-G_F)}\right)\right]^{\frac{1}{2}}}{1-\frac{K_mG_F}{G_0+G_H+G_F}}$
$S_{K_0}^{Q_P}$	$= -\frac{1}{2}\left[\frac{K_0\gamma_0(G_H+G_0+G_F)}{(\beta_0-K_0\gamma_0)(G_H-G_0-G_F)-K_m\beta_0G_F} + \frac{K_0\gamma_2(G_H+G_0+G_F)}{(\beta_2-K_0\gamma_2)(G_H-G_0-G_F)-K_m\beta_2G_F}\right]$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: 0,1
$S_{K_m}^{Q_P}$	$= -\frac{1}{2}\left[\frac{K_m\beta_0G_F}{(\beta_0-K_0\gamma_0)(G_H+G_0-G_F)-K_m\beta_0G_F} + \frac{K_m\beta_2G_F}{(\beta_2-K_0\gamma_2)(G_H+G_0-G_F)-K_m\beta_2G_F}\right] + \frac{K_mG_F}{G_0+G_H+G_F}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 21

$$t_v = \frac{K_0\alpha_1 S}{(\beta_2-K_0\gamma_2)S^2 + \beta_1 S + (\beta_0-K_0\gamma_0)}$$

$$Q_{P0} = \frac{\left\{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)\right\}^{\frac{1}{2}}}{\beta_1}$$

$$S_{K_0}^{Q_{P0}} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0} + \frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right]$$

Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: 0,1

$$T_v = \frac{K_m K_0 G_0 \alpha_1 S}{S^2[(\beta_2-K_0\gamma_2)(G_H+G_0+G_F)] + S[\beta_1(G_H+G_0+G_F)-K_m K_0\alpha_1] + [(\beta_0-K_0\gamma_0)(G_H+G_0+G_F)]}$$

$$Q_P = \frac{Q_{P0}}{1-\dfrac{K_m K_0\alpha_1}{\beta_1(G_H-G_0-G_F)}}$$

$$S_{K_0}^{Q_P} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0} + \frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right] + \frac{K_m K_0\alpha_1 G_H}{\beta_1(G_H-G_0-G_F)-K_m K_0\alpha_1 G_H}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = S_{K_0}^{Q_P} + \frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0} + \frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right]$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,0

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 22a

$$t_v = \frac{K_0\alpha_1 S}{(\beta_2-K_0\gamma_2)S^2+(\beta_1-K_0\gamma_1)S-(\beta_0-K_0\gamma_0)}$$

$$Q_{P0} = \frac{\{(\beta_0-K_0\gamma_0)(\beta_2-K_0\gamma_2)\}^{\frac{1}{2}}}{\beta_1-K_0\gamma_1}$$

$$S_{K_0}^{Q_{P0}} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0}+\frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right]+\frac{K_0\gamma_1}{\beta_1-K_0\gamma_1}$$

Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: -1,1

$$T_v = \frac{K_m K_0 G_0\alpha_1 S}{S^2[(\beta_2-K_0\gamma_2)(G_H+G_0+G_F)] + S[(\beta_1-K_0\gamma_1)(G_H+G_0-G_F)+K_m K_0(\gamma_1 G_F-\alpha_1 G_H)] + [(\beta_0-K_0\gamma_0)(G_H+G_0+G_F)]}$$

$$Q_P = \frac{Q_{P0}}{1+\dfrac{K_m K_0(\gamma_1 G_F-\alpha_1 G_H)}{(\beta_1-K_0\gamma_1)(G_H-G_0+G_F)}}$$

$$S_{K_0}^{Q_P} = -\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0}+\frac{K_0\gamma_2}{\beta_2-K_0\gamma_2}\right]-\frac{K_m K_0(\gamma_1 G_F-\alpha_1 G_H)}{(\beta_1-K_0\gamma_1)(G_H+G_0-G_F)+K_m K_0(\gamma_1 G_F-\alpha_1 G_H)}$$

Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,1

$$S_{K_m}^{Q_P} = S_{K_0}^{Q_P}+\frac{1}{2}\left[\frac{K_0\gamma_0}{\beta_0-K_0\gamma_0}+\frac{K_0\gamma_2}{\beta_0-K_0\gamma_2}\right]$$

Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,0

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 22b	
t_v	$= \dfrac{K_0 \alpha_1 S}{\beta_2 S^2 + \beta_1 S + \beta_0}$
Q_{P0}	$= \dfrac{\{\beta_0 \beta_2\}^{\frac{1}{2}}}{\beta_1}$
$S_{K_0}^{Q_{P0}}$	$= 0$ Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: 0,0
T_v	$= \dfrac{K_m K_0 G_0 \alpha_1 S}{S^2 [\beta_2 (G_H + G_0 + G_F) - K_m \beta_2 G_F] + S[\beta_1 (G_H + G_0 + G_F) - K_m \beta_1 G_F - K_m K_0 \alpha_1 G_H] + [\beta_0 (G_H + G_0 + G_F) - K_m \beta_0 G_F]}$
Q_P	$= \dfrac{Q_{P0}\left[1 - \dfrac{K_m G_F}{G_H + G_0 + G_F}\right]}{1 - \dfrac{K_m \beta_1 G_F - K_m K_0 \alpha_1 G_H}{\beta_1 (G_H + G_0 + G_F)}}$
$S_{K_0}^{Q_P}$	$= \dfrac{K_m K_0 \alpha_1 G_H}{\beta_1 (G_H + G_0 + G_F) - K_m \beta_1 G_F - K_m K_0 \alpha_1 G_H}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,0
$S_{K_m}^{Q_P}$	$= \dfrac{-K_m G_F}{G_H + G_0 + G_F - K_m G_F} + \dfrac{K_m \beta_1 G_F + K_m K_0 \alpha_1 G_H}{\beta_1 [G_H + G_0 + G_F] - K_m \beta_1 G_F - K_m K_0 \alpha_1 G_H}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 23
$t_{v_H} = \dfrac{K_0\alpha_1 S}{\beta_2 S^2 + (\beta_1 - K_0\gamma_1)S - \beta_0}$
$Q_{P0} = \dfrac{\{\beta_0\beta_2\}^{\frac{1}{2}}}{(\beta_1 - K_0\gamma_1)}$
$S_{K_0}^{Q_{P0}} = \dfrac{-K_0\gamma_1}{\beta_1 - K_0\gamma_1}$ Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: -1,0
$T_v(S) = \dfrac{K_m K_0 G_0 \alpha_1 S}{S^2[\beta_2(G_H+G_0+G_F) - K_m\beta_2 G_F] + S[(\beta_1 - K_0\gamma_1)(G_H+G_0-G_F) - K_m\beta_1 G_F + K_m K_0(\gamma_1 G_F - \alpha_1 G_H)] + [\beta_0(G_H - G_0 - G_F) - K_m\beta_0 G_F]}$
$Q_P = \dfrac{Q_{P0}\left[1 - \dfrac{K_m G_{F2}}{G_H - G_0 - G_F}\right]}{1 - \dfrac{K_m\beta_1 G_F + K_m K_0(\gamma_1 G_F - \alpha_1 G_H)}{\beta_1 - K_0\gamma_1}}$
$S_{K_0}^{Q_P} = \dfrac{-K_m K_0(\gamma_1 G_F - \alpha_1 G_H)}{(\beta_1 - K_0\gamma_1)(G_H - G_0 - G_F) - K_m\beta_1 G_F - K_m K_0(\gamma_1 G_F - \alpha_1 G_H)}$ Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: -1,0
$S_{K_m}^{Q_P} = \dfrac{-K_m G_F}{(G_H + G_0 + G_F) - K_m G_F} + \dfrac{K_m\beta_1 G_F - K_m K_0(\gamma_1 G_F - \alpha_1 G_H)}{(\beta_1 - K_0\gamma_1)(G_H - G_0 - G_F) - K_m\beta_1 G_F + K_m K_0(\gamma_1 G_F - \alpha_1 G_H)}$ Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: -1,1

TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 24	
$t_v = \dfrac{K_0 \alpha_1 S}{\beta_2 S^2 + (\beta_1 - K_0 \gamma_1) S + \beta_0}$	
$Q_{P0} = \dfrac{\{\beta_0 \beta_2\}^{1/2}}{\beta_1 - K_0 \gamma_1}$	
$S_{K_0}^{Q_{P0}} = \dfrac{K_0 \gamma_1}{\beta_1 - K_0 \gamma_1}$	Bounds of $S_{K_0}^{Q_{P0}}$ when all terms in D(S) are positive: $-1, 0$
$T_v(S) = \dfrac{K_m K_0 G_0 \alpha_1 S}{S^2 [\beta_2 (G_H + G_0 + G_F) - K_m \beta_2 G_F] + S[(\beta_1 - K_0 \gamma_1)(G_H + G_0 + G_F) - K_m \beta_1 G_F] + [\beta_0 (G_H + G_0 + G_F) - K_m \beta_0 G_F]}$	
$Q_P = \dfrac{Q_{P0} \left[1 - \dfrac{K_m G_F}{G_H + G_0 + G_F}\right]}{1 - \dfrac{K_m \beta_1 G_F}{(G_H + G_0 + G_F)(\beta_1 - K_0 \gamma_1)}}$	
$S_{K_0}^{Q_P} = \dfrac{K_0 \gamma_1 (G_H + G_0 + G_F)}{(\beta_1 - K_0 \gamma_1)(G_H + G_0 + G_F) - K_m \beta_1 G_F}$	Bounds of $S_{K_0}^{Q_P}$ when all terms in D(S) are positive: $-1, 0$
$S_{K_m}^{Q_P} = \dfrac{-K_m G_F}{(G_H + G_0 + G_F) - K_m G_F} + \dfrac{K_m \beta_1 G_F}{(\beta_1 - K_0 \gamma_1)(G_H + G_0 + G_F) - K_m \beta_1 G_F}$	Bounds of $S_{K_m}^{Q_P}$ when all terms in D(S) are positive: $-1, 1$

Note: In some cases quantities are marked (1) or (2). Either of these but not both can be set to zero.

TABLE 4.2.2

POLYNOMIAL DECOMPOSITIONS OBTAINABLE FROM GIVEN SECOND ORDER GENERATING FUNCTION t_v

TABLE 4.2.2

	Generating Transfer Function t_v	Obtainable polynomial decompositions																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
N0	$\frac{K_0(\alpha_2S^2+\alpha_0)}{\beta_2S^2+\beta_1S+\beta_0}$												x b												
N1	$\frac{K_0(\alpha_2S^2+\alpha_0)}{\beta_2S^2+(\beta_1-K_0\gamma_1)S+\beta_0}$						x				x														
N2	$\frac{K_0(\alpha_2S^2+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2+\beta_1(S)+(\beta_0-K_0\gamma_0)}$							x									x				x				
N3	$\frac{K_0(\alpha_2S^2+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2+(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)}$												x a		x				x						
B0	$\frac{K_0\alpha_1S}{\beta_2S^2+\beta_1S+\beta_0}$	x																					x b		
B1	$\frac{K_0\alpha_1S}{\beta_2S^2+(\beta_1-K_0\gamma_1)S+\beta_0}$		x																					x	x

TABLE 4.2.2 (CONTINUED)

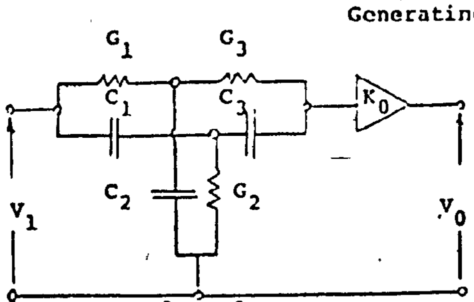
	Generating Transfer Function t_v	Obtainable polynomial decompositions 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
B2	$\frac{K_0\alpha_1 S}{(\beta_2-K_0\gamma_2)S^2+\beta_1 S+(\beta_0-K_0\gamma_0)}$													x								x			
B3	$\frac{K_0\alpha_1 S}{(\beta_2-K_0\gamma_2)S^2+(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)}$													x	x	x							x a		
C0	$\frac{K_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{\beta_2 S^2+\beta_1 S+\beta_0}$				x					x b															
C1	$\frac{K_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{\beta_2 S^2+(\beta_1-K_0\gamma_1)S+\beta_0}$					x					x	x													
C2	$\frac{K_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2+\beta_1 S+(\beta_0-K_0\gamma_0)}$								x					x				x							
C3	$\frac{K_0(\alpha_2 S^2+\alpha_1 S+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2+(\beta_1-K_0\gamma_1)S+(\beta_0-K_0\gamma_0)}$									x a				x	x	x			x	x					

TABLE 4.2.3

DIFFERENT QMCs OBTAINABLE FROM SECOND ORDER GENERATING FUNCTIONS REALIZED BY SACs.

TABLE 4.2.3

Generating function type: NO



Design Equations: [40]

$\omega_{P0}G_1 = C_1 = (\alpha)^{\frac{1}{2}}(\alpha-1)^{-1}$

$\omega_{P0}G_2 = C_2 = (\alpha^2+1)[2\alpha(\alpha-1)]^{-1}$

$\omega_{P0}G_3 = C_3 = (\alpha-1)[2(\alpha)^{\frac{1}{2}}]^{-1}$

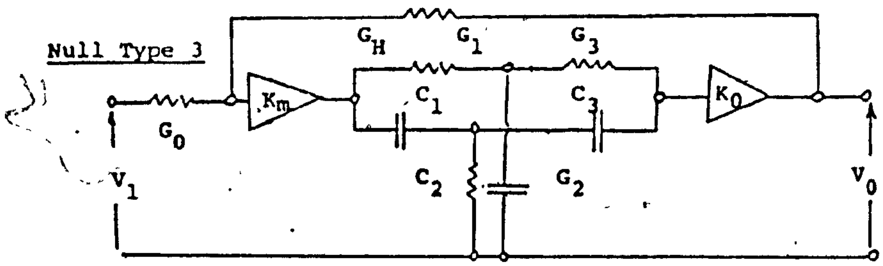
$Q_{P0} = [1+\frac{1}{\alpha}]^{-1}$

$\alpha > 1$

$$t_{N0} = \frac{s^2 + \omega_{P0}^2}{s^2 + \frac{\omega_{P0}}{Q_{P0}} s + \omega_{P0}^2}$$

Identifications with the general parameters, $\alpha_2=\beta_2=1$, $\alpha_0=\beta_0=\omega_{P0}^2$, $\beta_1=\omega_{P0}(\alpha+\frac{1}{\alpha})$

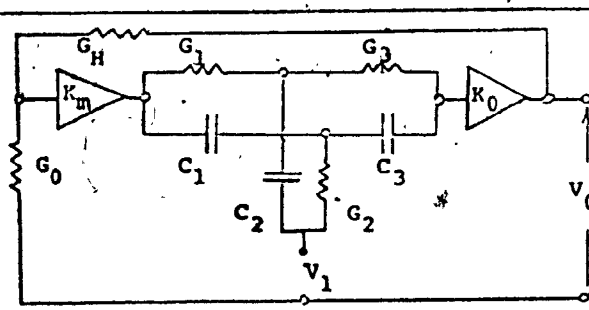
Null Type 3



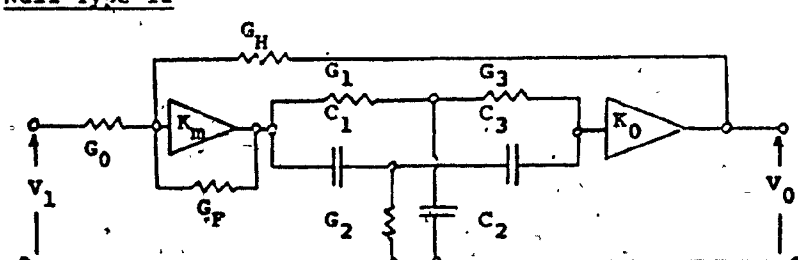
$$T_{N3} = \frac{n_N(s)}{D_3(s)} = \frac{K_m K_0 G_0 (S^2+\omega_{P0}^2)}{S^2[G_0+G_H-G_H K_m K_0] + \frac{\omega_{P0}}{Q_{P0}} S + [G_0+G_H-G_H K_m K_0]}$$

Band Pass Type 3

$$T_{BP3} = \frac{K_m K_0 (G_0+G_H)(\frac{\omega_{P0}}{Q_{P0}}) S}{D_3(s)}$$



Null Type 12



$$T_{N12} = \frac{n_N(s)}{D_{N12}(s)} = \frac{K_m K_0 G_0 (S^2+\omega_{P0}^2)}{[G_0+G_F(1-K_m)+G_H(1-K_mK_0)]S^2 + (\frac{\omega_{P0}}{Q_{P0}})[G_0+G_F(1-K_m)]S + \omega_{P0}^2[G_0+G_F(1-K_m)+G_H(1-K_mK_0)]}$$

Band Pass Type 12

$$T_{BP12} = \frac{K_m(\frac{\omega_{P0}}{Q_{P0}})[G_0+G_H+G_F(1-K_m)]S}{D_{12}(s)}$$

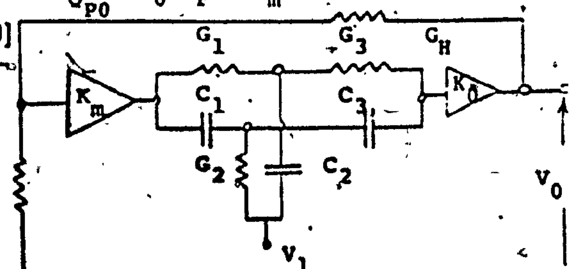
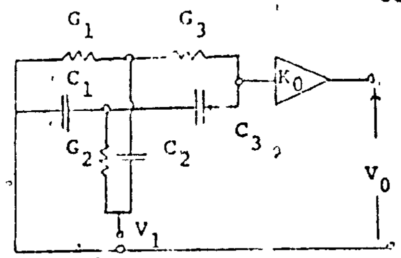


TABLE 4.2.3 (Continued)

Generating function type: BO

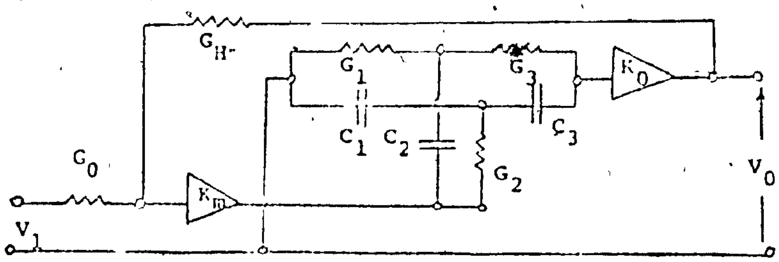


Design Equations: Identical to equations given for the NO type

$$t_{B0} = \frac{K_0\left(\frac{\omega_{P0}}{Q_{P0}}\right)S}{S^2+\frac{\omega_{P0}}{Q_{P0}}S+\omega_{P0}^2}$$

Identification with the general parameters $\beta_2=1$, $\beta_0=\omega_{P0}^2$, $\beta_1=\omega_{P0}(\alpha+\frac{1}{\alpha})^{-1}$, $\alpha_1=\frac{\omega_{P0}}{Q_{P0}}$

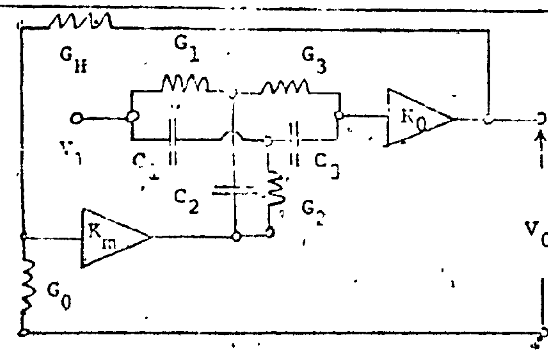
Band Pass Type 1



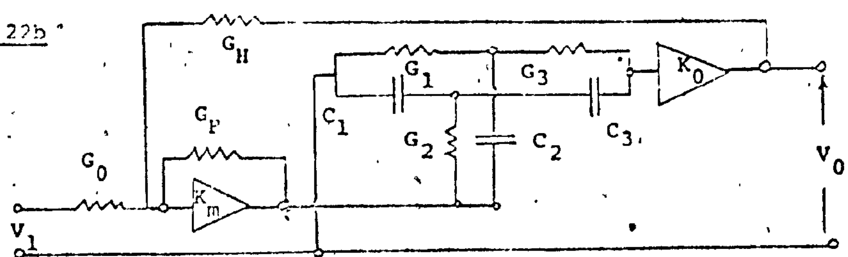
$$T_{BP1} = \frac{n_{BP}(s)}{D_1(s)} = \frac{K_mK_0(\omega_{P0}/Q_{P0})S}{S^2[G_0+G_H]+\frac{\omega_{P0}}{Q_{P0}}[(G_0+G_H)-K_mK_0G_H)-K_mK_0G_H]S+(G_0+G_H)\omega_{P0}^2}$$

Null Type 1

$$T_{N1} = \frac{K_0(G_0+G_H)(S^2+\omega_{P0}^2)}{D_1(s)}$$



Band Pass Type 22b



$$T_{BP22} = \frac{n_{BP}(s)}{D_{22}(s)} = \frac{K_mK_0\left(\frac{\omega_{P0}}{Q_{P0}}\right)S}{S^2[G_0+G_H+G_F(1-K_m)]+S\left(\frac{\omega_{P0}}{Q_{P0}}\right)[G_0+G_F(1-K_m)+G_H(1-\frac{\omega_{P0}}{Q_{P0}}K_mK_0)] + [G_0+G_H+G_F(1-K_m)]\omega_{P0}^2}$$

Null Type 22b

$$T_{N22} = \frac{K_0\left(\frac{\omega_{P0}}{Q_{P0}}\right)[G_0+G_H+G_F(1-K_m)]S}{D_{22}(s)}$$

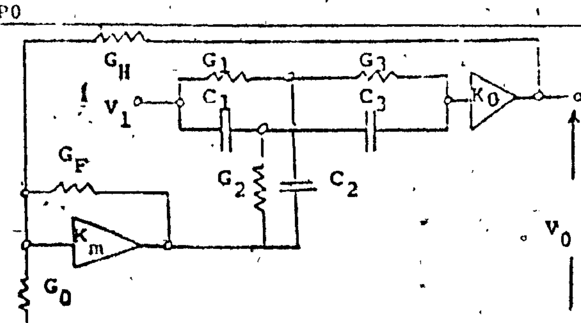
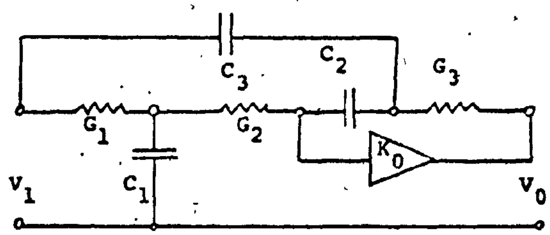


TABLE 4.2.3. (Continued)

Generating function type N1



$$t_{N1} = \frac{K_0[C_1C_2C_3S^2 + G_1G_2(C_2+C_3)]}{C_1C_2C_3S^2+S[C_1C_2G_3(1-K_0)+C_1G_2(C_2+C_3)]+G_1G_2(C_2+C_3)}$$

$$\frac{C_1}{G_1+G_2} = \frac{C_2+C_3}{G_3}$$

$$\omega_{P0} = \left[\frac{G_1G_2(C_2+C_3)}{C_1C_2C_3}\right]^{\frac{1}{2}}$$

$$Q_{P0} = \left[\frac{G_1G_2(C_2+C_3)}{C_1C_2C_3}\right]^{\frac{1}{2}}\left[\frac{G_3}{C_3}(1-K_0) + \frac{G_2(C_2+C_3)}{C_2C_3}\right]^{-1}$$

Design Equations: [17]

$$\omega_{P0} = 1$$

$$G_2 = c \qquad G_1 = c(c\varepsilon_1 - 1)$$

$$G_3 = [b-c+\frac{1}{Q_{P0}}]^2\,[b-c-\varepsilon_1+\frac{1}{Q_{P0}}]^{-1}$$

$$c_1 = \frac{c^2}{c\varepsilon_1 - 1}$$

$$c_2 = [b-c+\frac{1}{Q_{P0}}]/\varepsilon_1$$

$$c_3 = [b-c+\frac{1}{Q_{P0}}]\,[b-c-\varepsilon_1+\frac{1}{Q_{P0}}]^{-1}$$

$$K_0 = b[b-c+\frac{1}{Q_{P0}}]^{-1}$$

$$\frac{1}{c} < \varepsilon_1 < b-c+\frac{1}{Q_{P0}}$$

Identification with the general parameters:

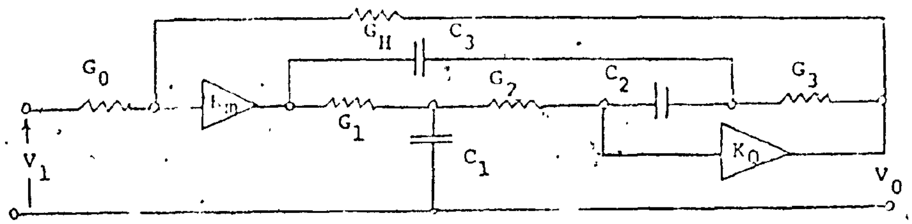
$$\beta_2 = \alpha_2 = C_1C_2C_3$$

$$\beta_0 = \alpha_0 = G_1G_2(C_2+C_3)$$

$$\beta_1 = C_1C_2G_3+C_1G_2(C_2+C_3)$$

$$\gamma_1 = C_1C_2G_3$$

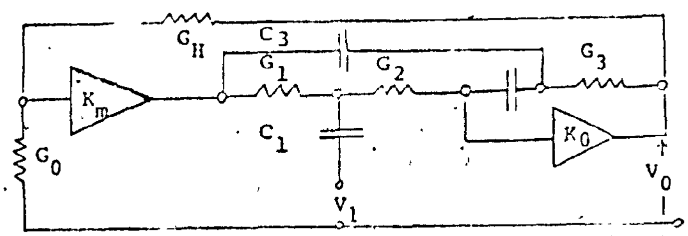
Null type 6



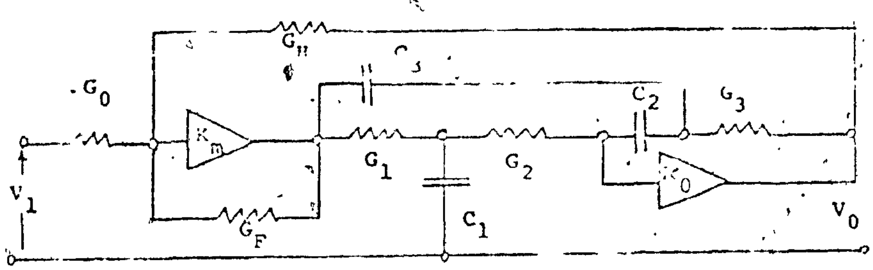
$$T_{N6} = \frac{n_N(s)}{D_6(s)} = \frac{-K_m K_0 G_0 [C_1 C_2 C_3 S^2 + G_1 G_2 (C_2+C_3)]}{C_1 C_2 C_3 [G_0+G_H-K_m K_0 G_H] S^2 + [C_1 C_2 C_3 (1-K_0) + G_2 C_1 (C_1+C_3)] (G_0+G_H) S + G_1 G_2 (C_2+C_3) [G_0+G_H-K_m K_0 G_H]}$$

Band Pass Type 6

$$T_{BP6} = \frac{K_0 (G_0+G_H) C_1 G_2 S}{D_6(s)}$$



Null Type 10



$$T_{N10} = \frac{n_N(s)}{D_{10}(s)} = \frac{-K_m K_0 G_0 [C_1 C_2 C_3 S^2 + G_1 G_2 (C_2+C_3)]}{S^2 [G_0+G_H+G_F(1-K_m)-K_m K_0 G_H] C_1 C_2 C_3 + S \begin{bmatrix} [(G_0+G_F+G_H) [C_1 C_2 G_3 + C_1 G_2 (C_2+C_3) - K_0 C_1 C_2 G_3] \\ -K_m G_F [C_1 C_2 G_3 + C_1 G_2 (C_2+C_3)] + K_m K_0 G_F C_1 C_2 G_3 \end{bmatrix} + G_1 G_2 (C_2+C_3) [G_0+G_F+G_H+K_m G_F - K_m K_0 G_H]}$$

Band Pass Type 10

$$T_{BP10} = \frac{K_0 [G_0+G_H+G_F(1-K_m)] C_1 G_2 S}{D_{10}(s)}$$

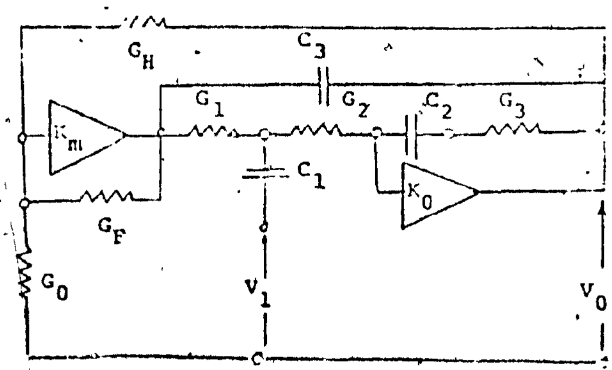
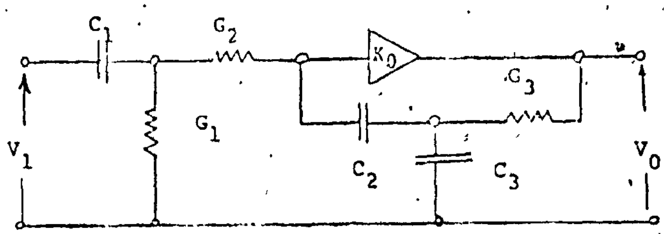


TABLE 4.2.3 (Continued)

Generating function type B1



$$t_{B1} = \frac{K_0(C_2+C_3)C_1G_2S}{C_1C_2C_3S^2+[C_1C_2G_3(1-K_0)+C_1G_2(C_2+C_3)]S+G_1G_2(C_2+C_3)}$$

$$\frac{C_1}{G_1+G_2} = \frac{C_2+C_3}{G_3}$$

$$\omega_{P0} = \left[\frac{G_1G_2(C_2+C_3)}{C_1C_2C_3}\right]^{\frac{1}{2}}$$

$$Q_{P0} = [C_1C_2C_3G_1G_2(C_2+C_3)]^{\frac{1}{2}}[C_1C_2G_3(1-K_0)+C_1G_2(C_2+C_3)]^{-1}$$

Design Equations: [17]

$$\omega_{P0} = 1$$

$$G_1 = [b(1-\frac{1}{K_0})+\frac{1}{Q_{P0}}][b(1-\frac{1}{K_0})+\frac{1}{Q_{P0}}-\frac{1}{\varepsilon_1}]^{-1}$$

$$G_2 = [b(1-\frac{1}{K_0})+\frac{1}{Q_{P0}}]\ \varepsilon_1$$

$$G_3 = b^2[K_0(b/\varepsilon_1-K_0)]^{-1}$$

$$C_1 = [b(1-\frac{1}{K_0})+\frac{1}{Q_{P0}}]^2[b(1-\frac{1}{K_0})+\frac{1}{Q_{P0}}-\frac{1}{\varepsilon_1}]^{-1}$$

$$C_2 = \frac{b}{K_0} \qquad C_3 = b[\frac{b}{\varepsilon_1}-K_0]^{-1}$$

$$\frac{b}{\varepsilon_1} > K_0 > 0 \qquad b(1-\frac{1}{K_0})+\frac{1}{Q_{P0}} > \frac{1}{\varepsilon_1} > 0$$

Identification with the general parameters:

$\beta_2 = C_1C_2C_3$

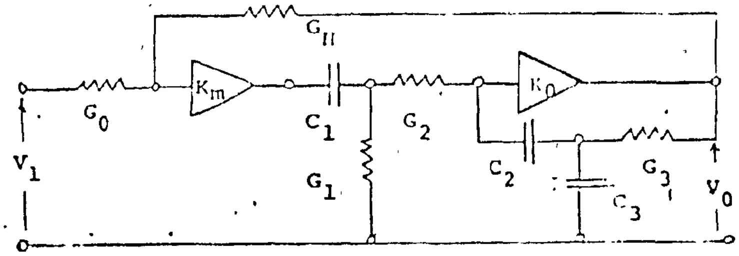
$\beta_1 = C_1C_2G_3 + C_1G_2(C_2+C_3)$

$\beta_0 = C_1G_2(C_1+C_3)$

$\gamma_1 = C_1C_2G_3$

$\alpha_1 = C_1G_2(C_2+C_3)$

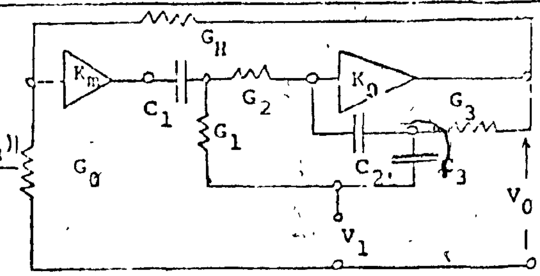
Band Pass Type 2



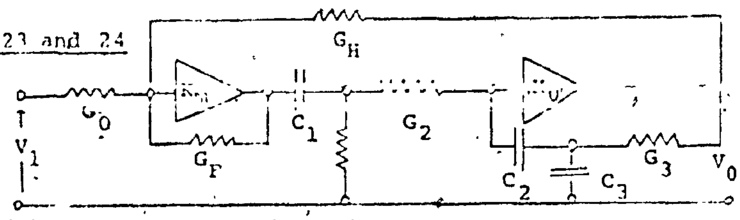
$$T_{BP2} = \frac{n_{BP}(s)}{D_2(s)} = \frac{K_m K_0 G_0 (C_2+C_3) G_2 C_1 S}{C_1C_2C_3(G_0+G_H)S^2 + S\left[\begin{array}{l}((G_0+G_H)\ [C_1C_2G_3+C_1G_2(C_2+C_3)-K_0C_1C_2G_3] \\ -K_mK_0G_HC_1G_2(C_2+C_3)\end{array}\right] + G_1G_2(C_2+C_3)(G_0+G_H)}$$

Null Type 2

$$T_{N2} = \frac{K_0(G_0+G_H)\ [C_1C_2C_3S^2+G_1G_2(C_2+C_3)]}{D_2(s)}$$



Band Pass Type 23 and 24



$$T_{BP23} = \frac{n_{BP}(s)}{D_{23}(s)} = \frac{K_mK_0G_0C_1G_2(C_2+C_3)S}{\begin{array}{l}C_1C_2C_3\ [G_0+G_F+G_H-K_mG_F]\ S^2 + \\ S\left\{\begin{array}{l}(G_0+G_F+G_H)\ [(1-K_m)(C_1C_2G_3+C_1G_2(C_2+C_3))-K_0C_1C_2G_3] \\ + K_mK_0\ [C_1C_2C_3G_F-C_1G_2(C_2+C_3)G_H]\end{array}\right. \\ + G_1G_2(C_2+C_3)\ [G_0+G_F+G_H-K_mG_F]\end{array}}$$

If $C_2C_3G_F-G_2(C_2+C_3)G_H = 0$, T_{BP23} is transformed into $T_{BP24} = n_{BP}/D_{24}$

Null Type 23 and 24

$$T_{N23} = \frac{n_N(S)}{D_{23}(s)} = \frac{K_0\ [G_0+G_H+G_F(1-K_m)]\ \times\ [C_1C_2C_3S^2+G_1G_2(C_2+C_3)]}{D_{24}(s)}$$

If $C_2C_3G_F-G_2(C_2+C_3)G_H = 0$

T_{N23} is transformed into $T_{N24} = \frac{n_N(S)}{D_{24}(s)}$

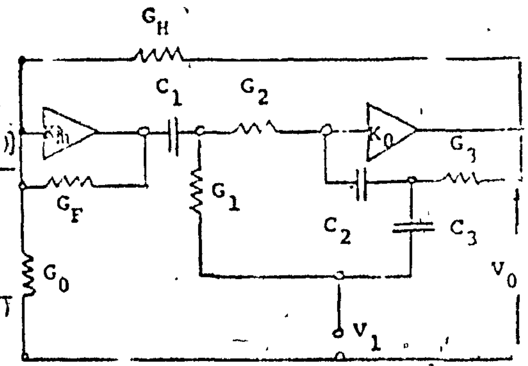
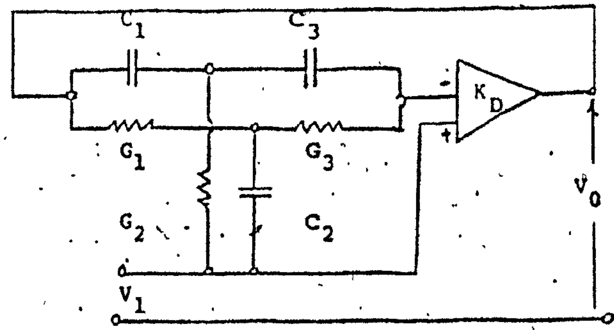


TABLE 4.2.3 (Continued)

Generating function type N2



$$t_{N2} = \frac{K_D\,[C_1C_2C_3s^2 + G_1G_3(C_1+C_3)]}{C_1C_2C_3(1+K_D)s^2 + [C_2C_3G_2 + G_3C_2(C_1+C_3)]\,s + G_1G_3(C_1+C_3)(1+K_D)}$$

$$\frac{C_1+C_3}{G_2} = \frac{C_2}{G_1+G_3}$$

$$\omega_{PO} = \left[\frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}\right]^{\frac{1}{2}}$$

$$Q_{PO} = \left[C_1C_2C_3G_1G_3(C_1+C_3)\right]^{\frac{1}{2}}(1+K_0)\,[C_2C_3G_2+G_3(C_1+C_3)]^{-1}$$

Design Equations: [17]

$\omega_{PO} = 1$ $\qquad K_D = (bQ_{PO}-1)$

$G_2 = \frac{G_1}{4}[b^2\eta - 4(\eta+1)]$

$G_3 = G_1[b^2\eta - 4(\eta+1)]$

$C_1 = G_1[b^2\eta - 4(\eta+1)]/2b$

$C_2 = \frac{b}{2}G_1$

$C_3 = G_1[b^2\eta - 4(\eta+1)]/2b$

Identification with the general parameters:

$\beta_2 = \gamma_2 = -\alpha_2 = C_1C_2C_3$

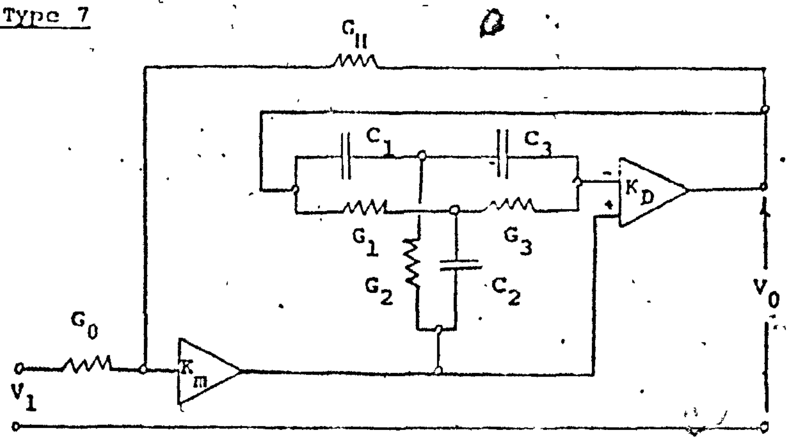
$\beta_0 = \gamma_0 = -\alpha_0 = G_1G_3(C_1+C_3)$

$\beta_1 = C_2C_3G_2 + G_3(C_1+C_3)$

$K_0 = -K_D$

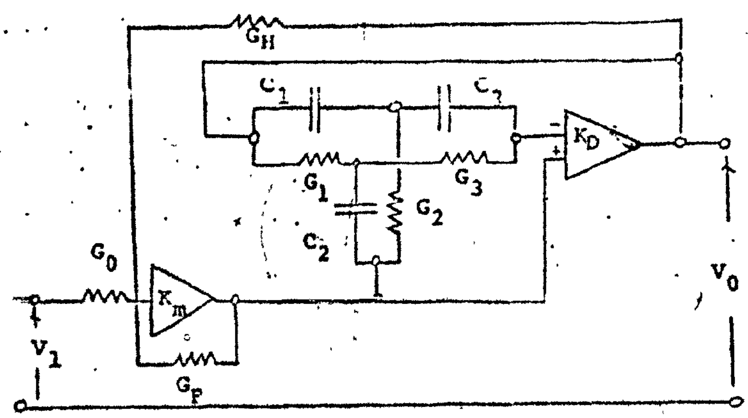
TABLE 4.2.3 (Continued)

Null Type 7



$$T_{N7} = \frac{K_m K_D G_0 [C_1 C_2 C_3 S^2 + G_1 G_3 (C_1 + C_3)]}{\begin{array}{l} C_1 C_2 C_3 S^2 [(G_0 + G_H)(1 + K_D) - K_m K_D G_H] \\ + S(G_0 + G_H)[C_2 C_3 G_2 + G_3 (C_1 + C_3)] \\ + G_1 G_3 (C_1 + C_3)[(G_0 + G_H)(1 + K_D) - K_m K_D G_H] \end{array}}$$

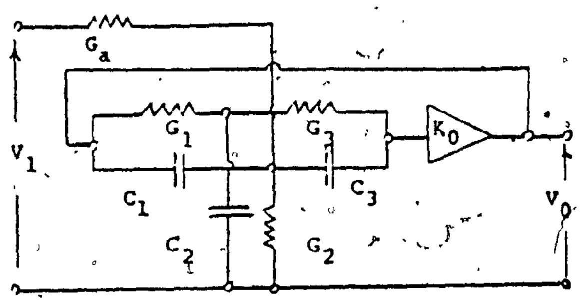
Null Type 16



$$T_{N16} = \frac{K_m K_D G_0 [C_1 C_2 C_3 S^2 + G_1 G_3 (C_1 + C_3)]}{\begin{array}{l} S^2 C_1 C_2 C_3 [(G_0 + G_F + G_H)(1 + K_D) - K_m G_F - K_m K_D (G_H + G_F)] \\ + S[C_2 C_3 G_2 + G_3 (C_1 + C_3)][G_0 + G_H + G_F (1 - K_m)] \\ + G_1 G_3 (C_1 + C_3)[(G_0 + G_F + G_H)(1 + K_D) - K_m G_F - K_m K_D (G_H + G_F)] \end{array}}$$

TABLE 4.2.3 (Continued)

Generating function type B2



$$t_{B2} = \frac{-K_0 C_2 C_3 G_a S}{C_1C_2C_3(1-K_0)S^2 + S[C_2C_3(G_\alpha+G_2)+C_2(C_1+C_3)G_2] + G_1(G_2+G_\alpha)G_3(1-K_0)}$$

$$\frac{C_2}{C_1+C_3} = \frac{G_1 G_3}{G_\alpha+G_2}$$

$$\omega_{PO} = \left\{\frac{G_1(G_a+G_2)G_3}{C_1C_2C_3}\right\}^{\frac{1}{2}}$$

$$Q_{PO} = (C_1C_2C_3G_1G_2G_3)^{\frac{1}{2}}(1+K_p)[C_3C_2(G_\alpha+G_2)+C_2(C_1+C_3)G_3]^{-1}$$

Design Equations: [17]

$$\omega_{PO} = 1$$

$$G_a = G_2 = G_1[b^2\eta - 4(\eta+1)]/8$$

$$G_3 = G_1[b^2\eta-4(\eta+1)]/4(\eta+1)$$

$$C_1 = G_1[b^2\eta-4(\eta+1)]/2b$$

$$C_2 = G_1\frac{b}{2}$$

$$C_3 = [b^2\eta-4(\eta+1)]/2b$$

$$K_0 = -(bQ_{PO}-1)$$

$$b^2 > 4(1+\frac{4}{\eta})$$

Identification with the general parameters:

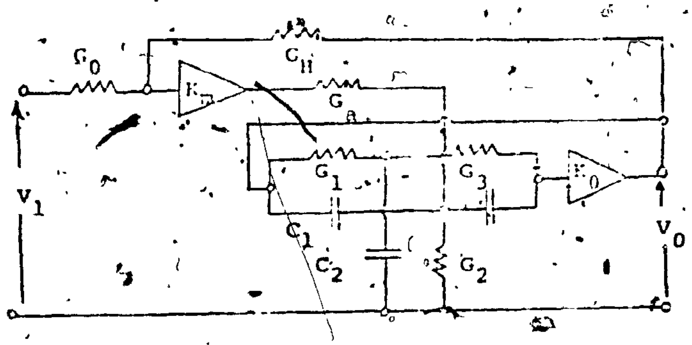
$$\beta_2 = \gamma_2 = C_1C_2C_3$$

$$\beta_0 = \gamma_0 = G_1(G_2+G_\alpha)G_3$$

$$\beta_1 = C_2C_3(G_\alpha+G_2)+C_2G_3(C_1+C_3)$$

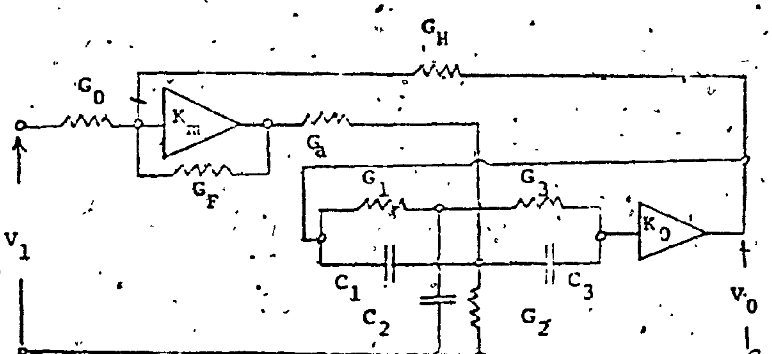
$$\alpha_1 = C_2C_3G_0$$

Band Pass Type 21



$$T_{BP21} = \frac{-K_m K_0 G_0 C_2 C_3 G_a S}{C_1 C_2 C_3 (G_0+G_H)(1-K_0)S^2 + S\{(G_0+G_H)[C_2 C_3 (G_a+G_2)+C_2 G_3 (C_1+C_3)] - K_m K_0 G_H C_2 C_3 G_a\} + G_1 G_3 (G_0+G_H)(G_2+G_a)(1-K_0)}$$

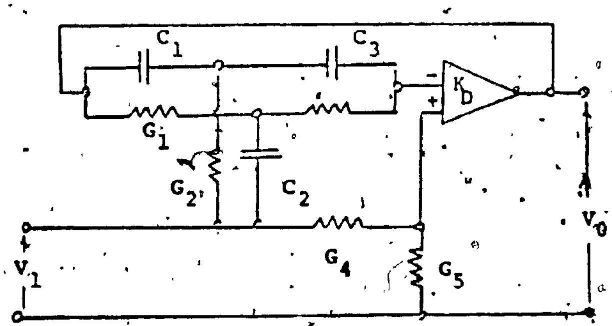
Band Pass Type 13



$$T_{BP13} = \frac{-K_m K G_0 C_2 C_3 G S}{S^2 C_1 C_2 C_3 [(G_0+G_F+G_H)(1-K_0)-K_m G_F - K_m K_0 G_F] + S\{[C_2 C_3 (G_a+G_2)+C_2 G_3 (C_1+C_3)][G_0+G_H+G_F(1-K_m)] + K_m K_0 G_a G_H\} + [G_1 G_3 (G_2+G_a)][(G_0+G_F+G_H)(1-K_0)-K_m G_F - K_m K_0 G_F]}$$

TABLE 4.2.3 (Continued)

Generating function type C2



$$t_{C2} = \frac{K_D \frac{G_4}{G_4+G_5}\left[C_1C_2C_3S^2 - S\frac{G_5}{G_4}(C_2G_3(C_1+C_3)\ \ C_2G_3G_2) + G_1G_3(C_1+C_3)\right]}{S^2C_1C_2C_3(1+K_D) + S[C_2C_3G_2+G_3(C_1+C_3)C_2] + G_1G_3(C_1+C_3)(1+K_D)}$$

$$\frac{C_1+C_3}{G_2} = \frac{C_2}{G_1+G_3}$$

$$\omega_{PO} = \left[\frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}\right]^{\frac{1}{2}}$$

$$Q_{PO} = \{C_1C_2C_3G_1G_3(C_1+C_3)\}^{\frac{1}{2}}(1+K_D)[C_2C_3G_2+G_3(C_1+C_3)]^{-1}$$

Design Equations: [17]

$$\omega_{PO} = 1$$

$$G_2 = \frac{G_1}{4}[b^2\eta - 4(\eta+1)]$$

$$G_3 = G_1[b^2\eta - 4(\eta+1)]$$

$$C_1 = G_1[b^2\eta - 4(\eta+1)]/2b$$

$$C_2 = G_1 b/2$$

$$C_3 = G_1[b^2\eta - 4(\eta+1)]/2b\eta$$

$$K_P = bQ_{PO} - 1$$

Identification with the general parameters:

$$\beta_2 = \gamma_2 = C_1C_2C_3$$

$$\beta_0 = \gamma_0 = G_1G_3(C_1+C_3)$$

$$\beta_1 = C_2C_3G_2 + G_3(C_1+C_3)G_2$$

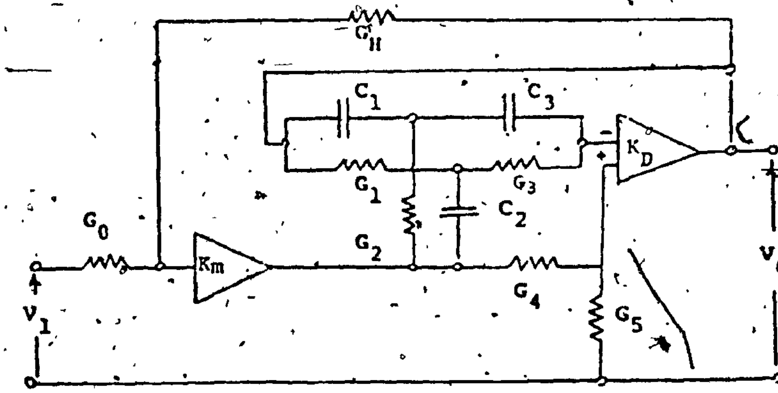
$$\alpha_2 = \frac{-G_4}{G_4+G_5}\beta_2$$

$$\alpha_0 = \frac{-G_4}{G_4+G_5}\beta_0$$

$$\alpha_1 = \frac{G_5}{G_4+G_5}\beta_1 \qquad K_0 = -K_D$$

TABLE 4.2.3 (Continued)

All Pass Type 8



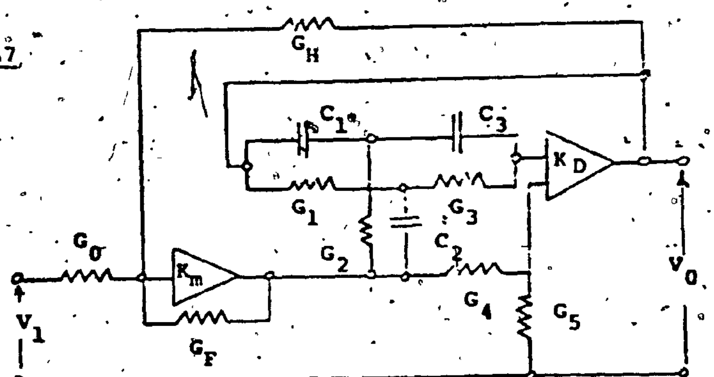
$$T_{AP8} = \frac{\frac{K_m K_D G_0 G_4}{(G_4+G_5)(G_0+G_H)}\left(C_1C_2C_3 s^2 - \frac{G_5}{G_4}\left[C_2G_3(C_1+C_3)+C_2C_3G_2\right]s + G_1G_3(C_1+C_3)\right)}{s^2 C_1C_2C_3\left[1+K_D - \frac{K_mK_DG_HG_4}{(G_0+G_H)(G_4+G_5)}\right]}$$

$$+ s\left[C_2C_3G_2 + C_2(C_1+C_3)G_3\right]\left(1+\frac{K_mK_DG_5G_H}{(G_0+G_H)(G_4+G_5)}\right)$$

$$+ C_1G_3(C_1+C_3)\left[1+K_D - \frac{K_mK_DG_HG_4}{(G_0+G_H)(G_4+G_5)}\right]$$

with $\frac{G_5}{G_4}\left[1+\frac{K_mK_DG_HG_5/G_4}{(G_0+G_H)(1+G_5/G_4)}\right] = \left[1+K_D-\frac{K_mK_DG_H}{(G_0+G_H)(1+G_5/G_4)}\right]$

All Pass Type 12 and 17



$$T_{AP12} = \frac{\frac{K_mK_DG_4}{(G_4+G_5)(G_0+G_F+G_H)}\left[C_1C_2C_3s^2 - \frac{G_5}{G_4}\left[C_2G_3(C_1+C_3)+C_2C_3G_2\right]s+G_1G_3(C_1+C_3)\right]}{s^2C_1C_2C_3\left[1+K_D\frac{K_mG_F}{G_0+G_F+G_H} - \frac{K_mK_D}{G_0+G_F+G_H}\left[\frac{G_HG_4}{G_4+G_5}-G_F\right]\right]}$$

$$+ s\left[\left(C_2C_3G_2+C_2G_3(C_1+C_3)\right)\left(1 - \frac{K_mG_F}{G_0+G_F+G_H} + \frac{K_mK_DG_HG_4}{(G_0+G_F+G_H)(G_4+G_5)}\right)\right]$$

$$+ G_1G_3(C_1+C_3)\left[1+K_D - \frac{K_mG_F}{G_0+G_F+G_H} - \frac{K_mK_D}{G_0+G_F+G_H}\left[\frac{G_HG_4}{G_4+G_5}-G_F\right]\right]$$

with $\frac{G_5}{G_4}\left[1+K_D-\frac{K_mG_F+K_mK_D(G_HG_4/(G_4+G_5)-G_F)}{(G_0+G_F+G_H)}\right] = \left[1-\frac{K_mG_F}{(G_0+G_F+G_H)}+\frac{K_mK_DG_HG_4}{(G_0+G_F+G_H)(G_4+G_5)}\right]$

If we let also $\frac{G_F}{G_H} = \frac{G_4}{G_4+G_5}$ we transform T_{AP12} into an All Pass of type 17 be realized using optimized circuits containing not more than three capacitors [17, 18]. Hence our choice of generating functions will be restricted to the N_0, N_1, N_2, B_0, B_1, B_2 and C_2 types only.

Table 4.2.3 gives different TACs obtainable from the above mentioned generating functions and the corresponding transfer functions. Table 4.2.3 can be combined with Table 4.2.1 to obtain the different properties of these circuits.

The following classification of the generating circuits can be made:

(1) Passive circuits cascaded with an amplifier (called RI)

(2) Single ended (inverting or non-inverting) amplifier circuits (called RII)

(3) Differential amplifier circuits (called RIII)

A detailed study of representative QMCs obtained from one of each of the above mentioned types of generating circuits will be given in this chapter.

4.3 A QMC Realization Obtained Using an RI Generating Circuit

The overall realization consisting of two amplifiers is as shown in Fig. 4.3.1a (Figs. 4.3.1b and 4.3.1c show the actual realizations of the two ampliers contained in Fig. 4.3.1a). It consists of a passive optimized twin-T network

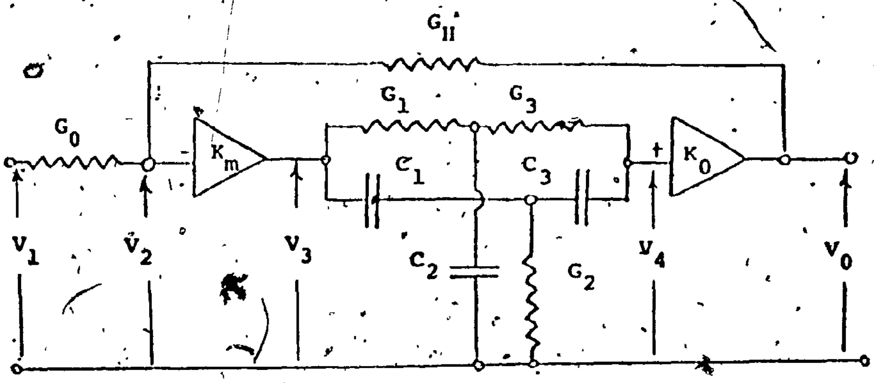


FIGURE 4.3.1a

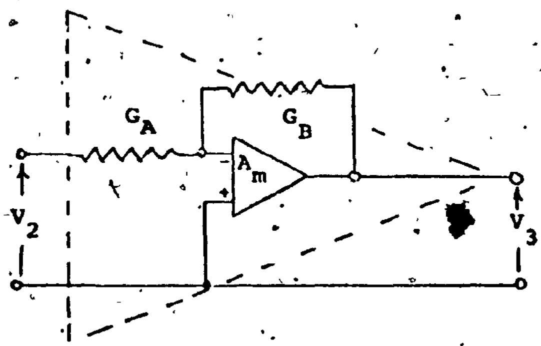


FIGURE 4.3.1b (K_m)

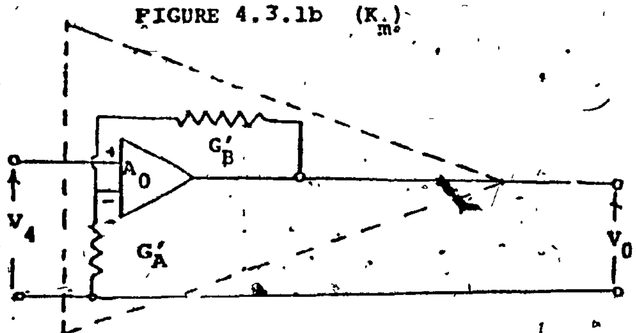


FIGURE 4.3.1c (K_0)

FIGURE 4.3.1 A Null QMC REALIZATION OBTAINED FROM AN RI GENERATING CIRCUIT.

A SET OF DESIGN EQUATIONS

$G_1, G_2, G_3, C_1, C_2, C_3$ are given in Table 4.2.3

$G_H/G_A = 1$

$K_m = K_0 = G_A/G_B = 1 + G'_A/G'_B \quad G_H/G_0$

$K_m \; K_0$ is obtained from Fig. 4.3.6 or Fig. 4.3.7 for specified Q_P

[35,36] cascaded with a finite gain amplifier constituting an RI generating circuit having a t_v of type N_0, used in the configuration of Fig. 4.2.1 with $G_F = 0$. The entire network corresponds to a null filter having a denominator decomposition of type 3. Its transfer function is given by

$$T_3 = \frac{-\frac{K_m K_0}{x}\frac{G_0}{G_F}(S^2+\omega_{P0}^2)}{S^2[1+K_m K_0/x]+S\frac{\omega_{P0}}{Q_{P0}}+[1+K_m K_0/x]\omega_{P0}^2} \qquad (4.3.1a)$$

$$x = 1 + \frac{G_0+G_A}{G_H} \qquad (4.3.1b)$$

$$K_m = \frac{G_A}{G_B} \qquad (4.3.1c)$$

$$K_0 = 1 + \frac{G_{A'}}{G_{B'}} \qquad (4.3.1d)$$

This takes care of the loading effect of the amplifier.

The design equations of the twin T have been given in Table 4.2.3. Fig. 4.3.2 shows the variation of the resistive and capacitive spreads as well as the variation of Q_{P0} (Q of the passive circuit) as functions of the parameter α. From the graph it is clear that the minimum value of spread is 2 and occurs when $Q_{P0} = 0.25$ which corresponds to $\alpha = 3.732$. From the transfer function given in Eqn. (3.3.1) we get

$$\omega_P = \omega_{P0} \qquad (4.3.2)$$

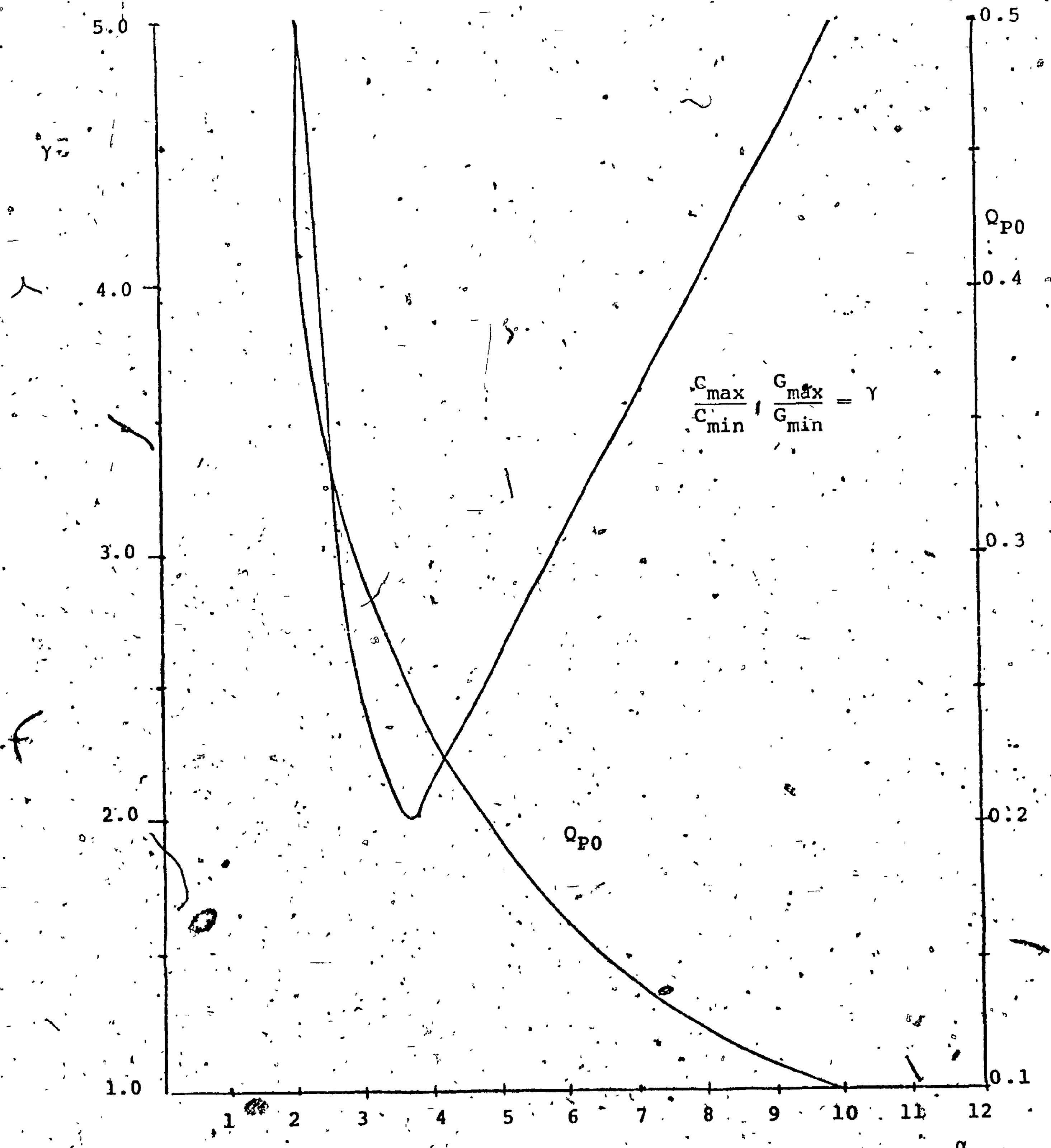


FIGURE 4.3.2 THE VARIATION OF THE CAPACITIVE SPREAD, THE RESISTIVE SPREAD, AND Q_{P0} OF THE PASSIVE TWIN-T VERSUS THE PARAMETER α

$$S_{K_m}^{\omega_P} = S_{K_0}^{\omega_P} = 0 \tag{4.3.3}$$

$$Q_P = Q_{P0}\left[1 + \frac{K_m K_0}{x}\right] \tag{4.3.4}$$

$$S_{K_m}^{Q_P} = S_{K_0}^{Q_P} = \frac{\frac{K_m K_0}{x}}{1 + \frac{K_m K_0}{x}} \tag{4.3.5}$$

$$|K_0 S_{K_0}^{Q_P}| = \frac{\frac{K_0^2 K_m}{x}}{1 + \frac{K_m K_0}{x}} \tag{4.3.6a}$$

$$|K_m S_{K_m}^{Q_P}| = \frac{\frac{K_m^2 K_0}{x}}{1 + \frac{K_m K_0}{x}} \tag{4.3.6b}$$

Defining the Q-multiplication factor ρ as

$$\rho = \frac{Q_P}{Q_{P0}} \tag{4.3.7}$$

we get

$$F = |K_m S_{K_m}^{Q_P}| + |K_0 S_{K_0}^{Q_P}| = (K_m+K_0)(1-\frac{1}{\rho}) \tag{4.3.8}$$

From Eqn. 4.3.4 we have

$$K_m K_0 = x(\rho-1) \tag{4.3.9}$$

Hence it is clear that for a given value of ρ and of the parameter x which is independent of K_m and K_0, the minimum value of F is obtained if we let

$$K_m = K_0 \triangleq K \tag{4.3.10}$$

Hence

$$K = \{x(\rho-1)\}^{\frac{1}{2}} \tag{4.3.11}$$

and F becomes

$$F_m = 2x^{\frac{1}{2}} \frac{(\rho-1)^{3/2}}{\rho} \tag{4.3.12}$$

It is obvious that for a given value of ρ, it is desirable to reduce the parameter x as much as possible. However inspection of Eqn. (4.3.1b) shows that the minimum possible value of x is 1 and this implies unrestricted resistive spread. In which case

$$F_m = \frac{2(\rho-1)^{3/2}}{\rho} \tag{4.3.13}$$

which can be approximated for $\rho >> 1$ as

$$F_m = 2\rho^{\frac{1}{2}} \tag{4.3.14}$$

From the previous equations it is clear that F can be reduced for a given value of Q_P by decreasing ρ which implies increasing Q_{P0}. However, inspection of the graphs of Fig. (4.3.2) shows that this could be done only at the expense of increasing the resistive and capacitive spread. It is known that the maximum theoretical value for Q_{P0} is 0.5 for an infinite capacitive spread. Hence the theoretical lower limit of ρ is $2Q_P$. Figs. (4.3.3) and (4.3.4) show the variation of $\frac{F_m}{Q_P}$ versus Q_P for several values of x when Q_{P0} = 0.25 (corresponding to a capacitive spread of 2) and Q_P = 0.4 (corresponding to a capacitive spread of 5). It is clearly noted from the graphs that $\frac{F_m}{Q_P}$ decreases as Q_P increases. The variation of K for several values of x versus ρ is shown in Fig. 4.3.5.

It is clear from Eqn. 4.3.12 as well from the graphs shown in Fig. 4.3.3 and Fig. 4.3.4 that it is desirable to reduce the value of x in order to reduce the value of $\frac{F}{Q_P}$ corresponding to any prescribed value of Q_P. However decreasing the value of x is associated with an increase in the resistive spread γ. Let

$$x = 1 + \frac{1}{\gamma_A} + \frac{1}{\gamma_0} \qquad (4.3.15a)$$

$$\frac{1}{\gamma_A} = \frac{G_A}{G_H} \qquad (4.3.15b)$$

$$\frac{1}{\gamma_0} = \frac{G_0}{G_H} \qquad (4.3.15c)$$

Where for the purpose of minimizing x it is assumed that $\gamma_A \gg 1$, $\gamma_0 \gg 1$ with these assumptions

$$\frac{G_H}{G_B} = \gamma_A K \qquad (4.3.16)$$

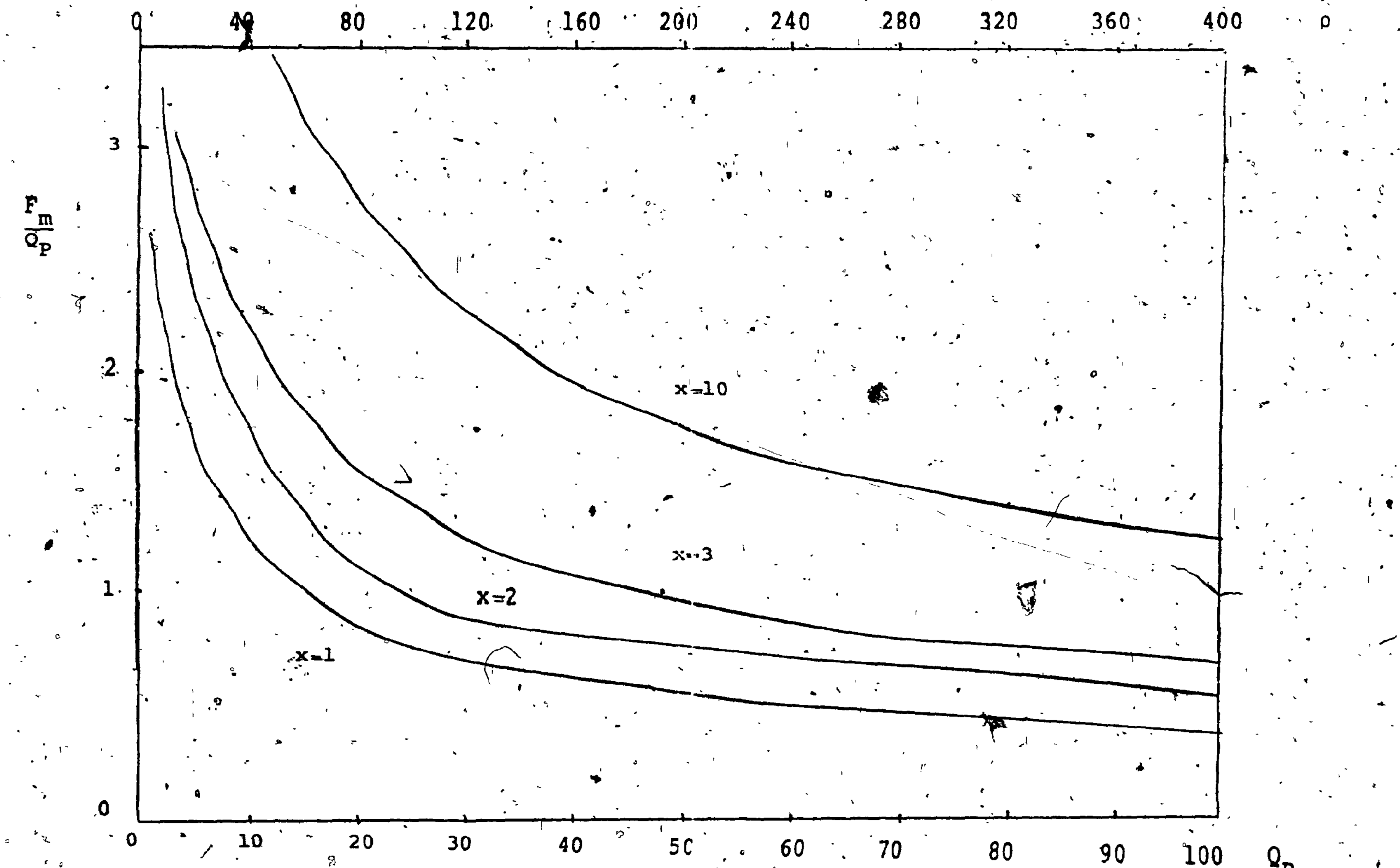


FIGURE 4.3.3

F_m/Q_P OF THE NULL TYPE 3 QMC FOR SEVERAL VALUES OF x VERSUS Q_P WHEN Q_{P0} = 0.25

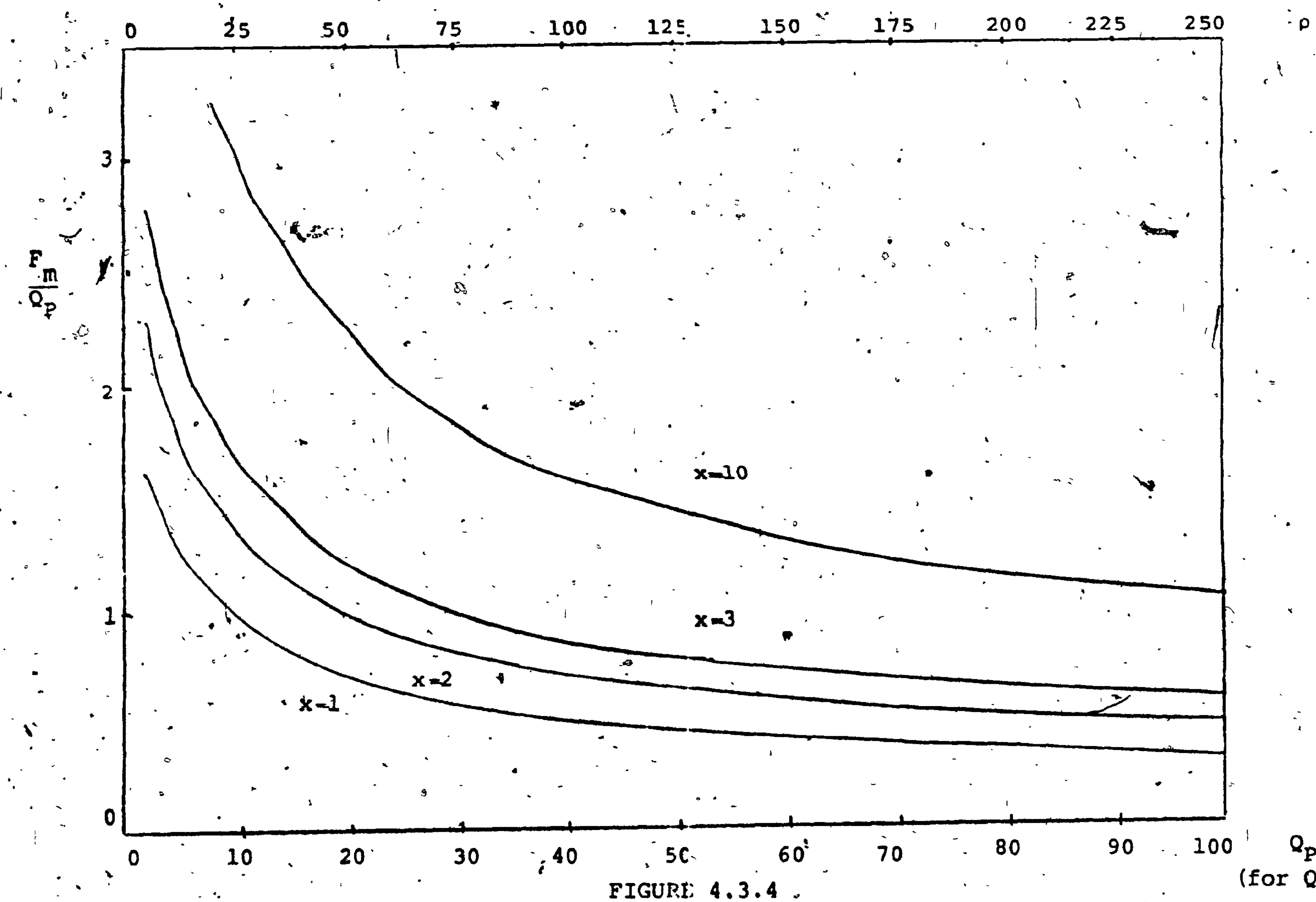


FIGURE 4.3.4

F_m/Q_p OF THE NULL TYPE 3 QMC FOR SEVERAL VALUES OF x VERSUS Q_p WHEN $Q_{p0} = 0.4$

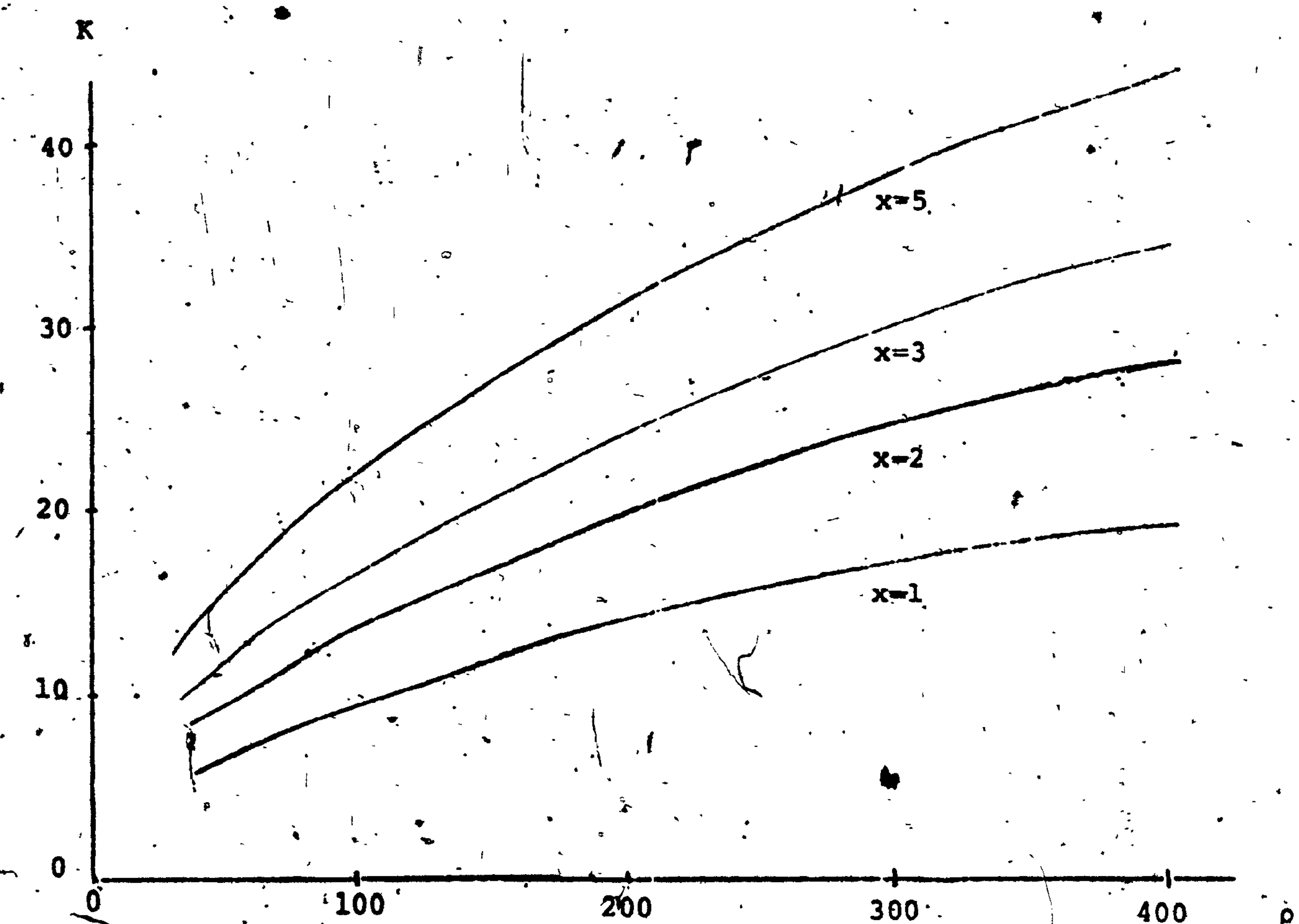


FIGURE 4.3.5

K OF THE NULL TYPE 3 QMC FOR SEVERAL VALUES OF x AND MINIMIZED F VERSUS ρ is the quantity which for low values of Q_{P0} and high values of Q_P (which is true in this thesis) determines the resistive spread γ. Therefore

$$\gamma_A = 1 \qquad (4.3.17a)$$

and

$$\gamma_0 = K \qquad (4.3.17b)$$

appears to be a good compromise to achieve low values of F_m and γ. Hence

$$x = 2 + \frac{1}{K} \qquad (4.3.17c)$$

$$\rho = 1 + \frac{K^2}{2K+1} \qquad (4.3.18a)$$

$$F_m = 2(2+\frac{1}{K})^{\frac{1}{2}} \frac{(\rho-1)^{3/2}}{\rho} \qquad (4.3.18b)$$

A set of curves giving F_m/Q_P, K_m (obtained from Eqn. 4.3.18) and γ versus Q_P for Q_{P0} = 0.25 and Q_{P0} = 0.4 are given respectively in Figs. 4.3.6, 4.3.7.

For the Null filter network shown in Fig. 4.3.1 grounding the input end of G_0 and lifting from ground the common end of G_2 and C_2 to transform it into an input terminal as shown in Table 4.2.3, we get a type 3 Band Pass filter network having the same denominator as that of the Null network. For the same element values this network shall have all the properties, namely sensitivities, gain sensitivity

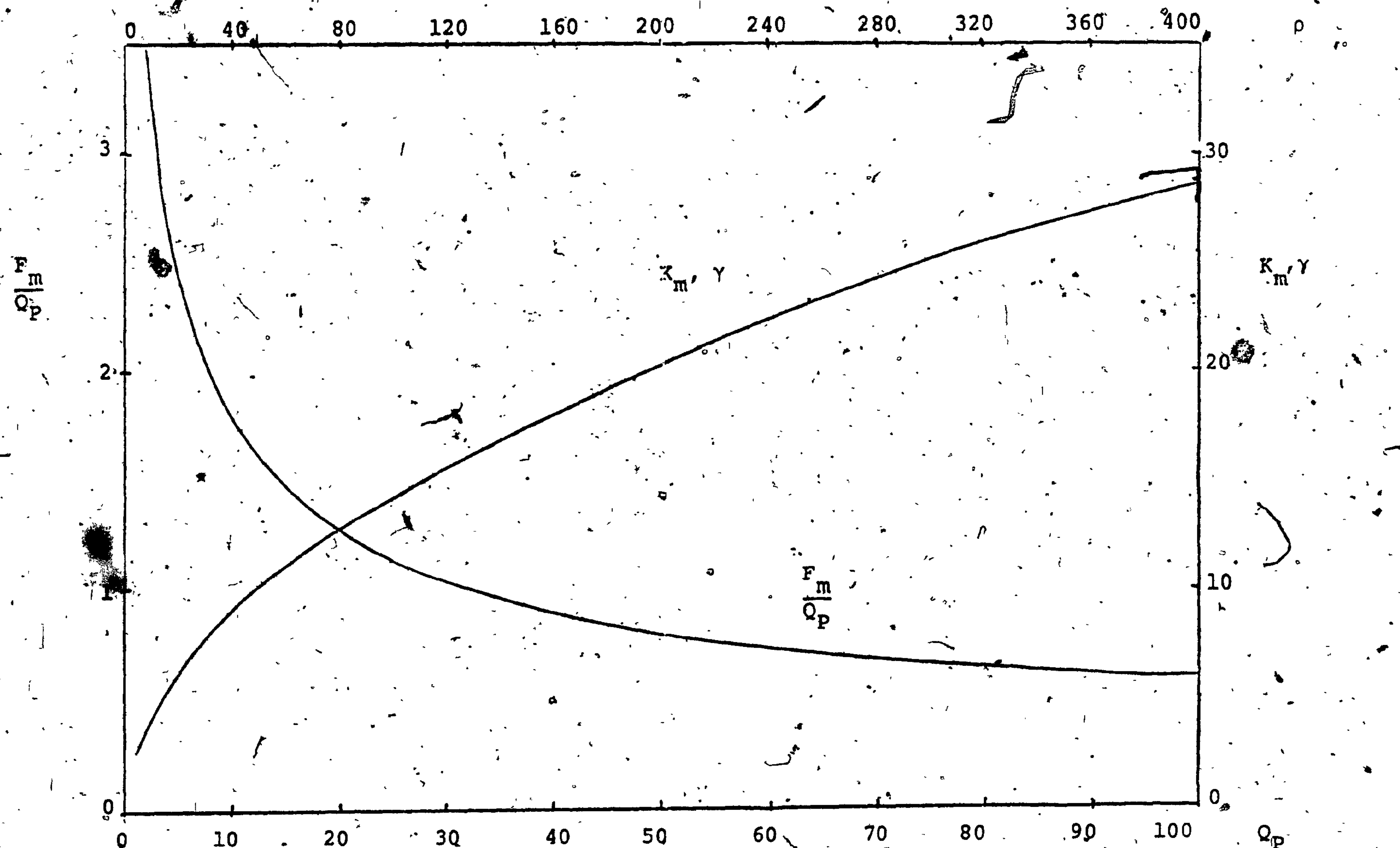


FIGURE 4.3.6

F_m/Q_P, K_m, AND γ OF THE NULL TYPE 3 QMC FOR $x=2+1/K_m$ VERSUS Q_P WHEN $Q_{P0} = 0.25$

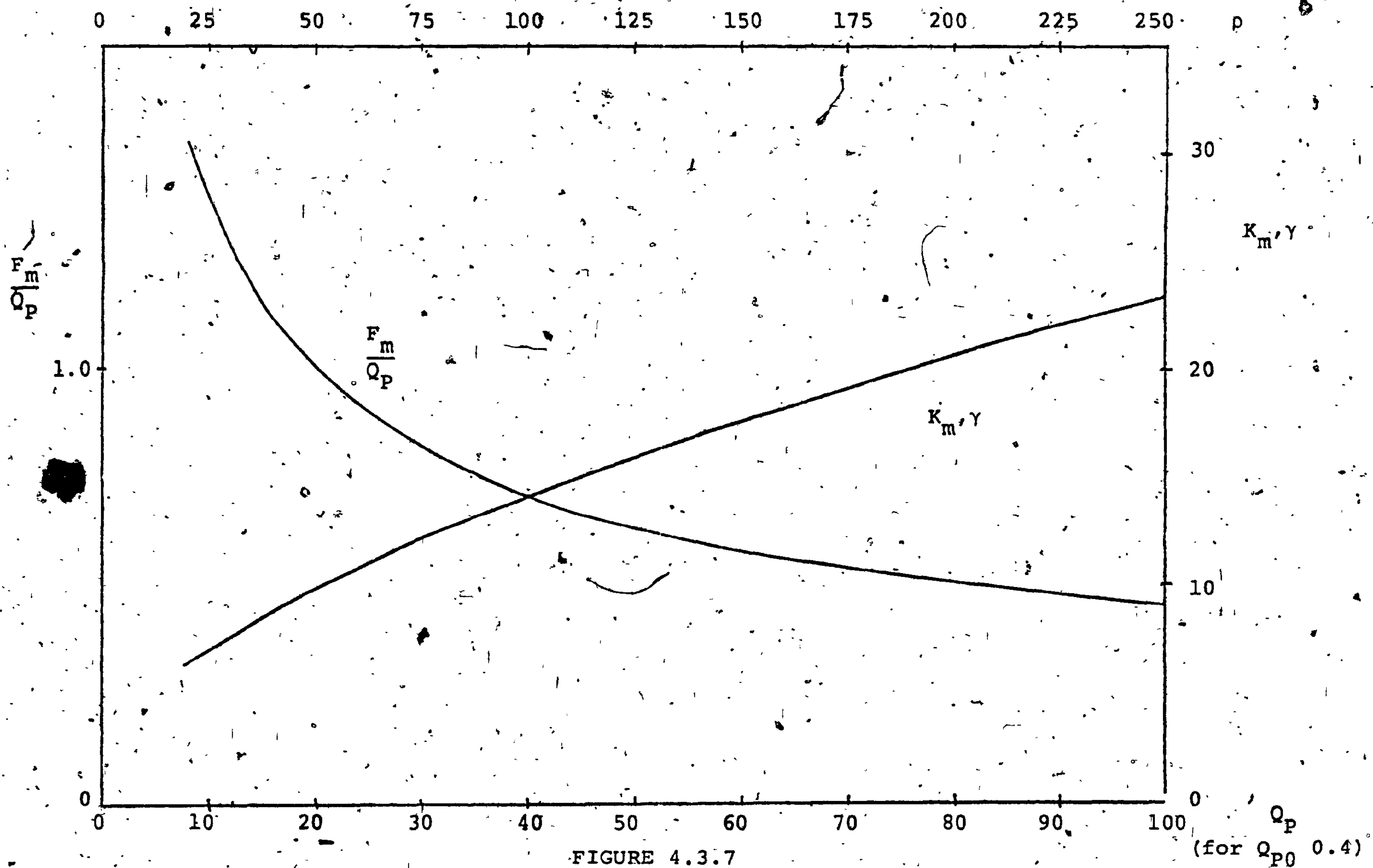


FIGURE 4.3.7

F_m/Q_P, K_m and γ OF THE NULL TYPE 3 QMC FOR $x=2+1/K_m$ VERSUS Q_P WHEN $Q_{P0}= 0.4$

products, F, Q-multiplication, etc. of the Null network and as such no further detailed discussion of this network is necessary.

4.3.1 Experimental Results

The network shown in Fig. 4.3.1 was built up using discrete elements and tested. The results are summarized below.

Designed values:

$\alpha = 3.732$

$\omega_P = 6283.185$ rad/sec. = 1000 Hz.

$Q_{P0} = 0.25$

$Q_P = 50$

$\rho = 200$

$C_1 = C_3 = 11.254$ KPF.

$C_2 = 22.508$ KPF

$R_1 = R_3 = 14.142$ KΩ

$R_2 = R_A = R_{A'} = R_H = 7.071$ KΩ

$K_m = K_0 = 20.194$

$x = 2.049 \qquad \gamma_A = 1$

$R_0 = R_B = 142.792$ KΩ

$R_{B'} = 135.720$ KΩ

GSP Values:

$$K_0 S_{K_0}^{\omega_P} = K_m S_{K_m}^{\omega_P} = 0$$

$$|K_0 S_{K_0}^{Q_p}|/Q_p = |K_m S_{K_m}^{Q_p}|/Q_p = 0.402$$

$$F/Q_p = 0.804$$

Element spread:

Capacitive 2:1

Resistive 20.194:1

The 3 dB frequencies are:

F_{C1} = 990.05 HZ. F_{C2} = 1010.05 Hz.

Any or both of K_m and K_0 can be adjusted in order to achieve the tuning of Q_p without affecting ω_p.

The circuit has been implemented using 1% tolerance resistors and 2% tolerance condensers with values chosen to be, within the range of currently available elements, as close as possible to the designed ones. (Trim pots have been used wherever necessary.) The OAs which have been used were SN72301 properly compensated so as to avoid high frequency oscillations due to excessive phase shift [20].

Actual values obtained.

ω_p = 999.6 Hz.

F_{C1} = 991.7 Hz. F_{C2} = 1011.4 Hz.

Q_p = 50.74

The depth of the notch has been found to be -70 dB. It is found that the circuit was stable during activation. Q_p and

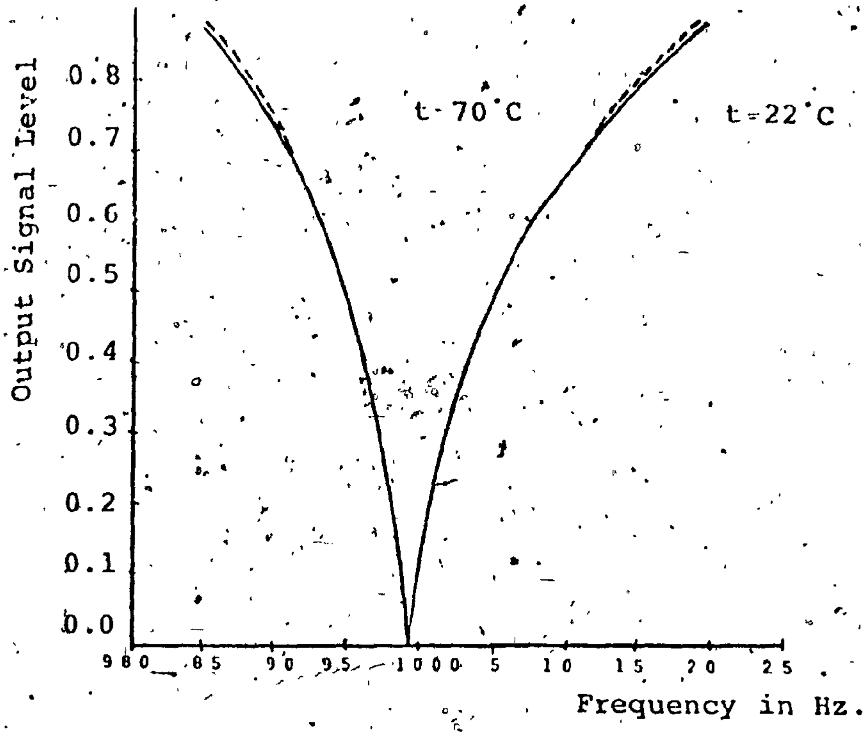


FIGURE 4.3.8a (±10V)

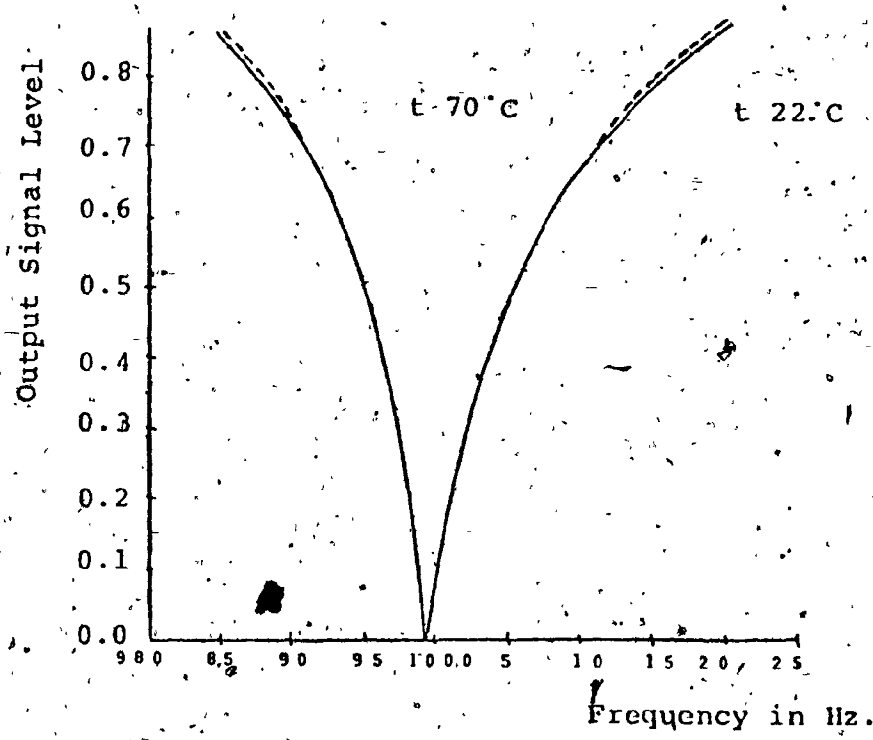


FIGURE 4.3.8b (±15V)

FIGURE 4.3.8 FREQUENCY RESPONSE OF THE NULL TYPE 3 QMC FOR $Q_p=50$

ω_p variations from their designed values are 1.48% and 0.04% respectively. This is due to the small differences between the designed values and the real ones as well as to the tolerance of the elements. The power supply voltages used were ∓10V and ∓15V. Also the OAs were heated and their temperature controlled. The response was experimentally measured and plotted at 22°C (room temperature) and 70°C for both power supplies voltages. Only the OAs were heated; the passive elements were not heated in order to simulate Hybrid Integrated Circuit technology. As shown in Fig. 4.3.8 no appreciable change in the response was observed; thus the experimental results confirm the theoretical predictions.

4.3.2 Effect of the Pole of the O.A. on Q and ω [17,33]

In order to study the effect of the pole of the O.A. on Q_p and ω_p, it is assumed that the desired elements are available and therefore one pole of the passive circuit always cancels with one of its zeros. Replacing the amplifiers of Fig. 3.3.1a by the networks of Fig. 3.3.1b and 3.3.1c the analysis gives:

$$D(S) = S^2\Big[1+\frac{K_m K_0}{x}+\frac{1}{A_m(S)}\Big(1+K_m+\frac{Y}{x}\Big)+\frac{K_0}{A_0(S)}$$

$$+\frac{K_0}{A_m(S)A_0(S)}\Big(1+\frac{Y}{x}\Big)\Big] + \cdots$$

$$[1 + \frac{1}{A_m(S)}(1+K_m\frac{y}{x}) + \frac{K_0}{A_0(S)} + \frac{K_0}{A_m(S)A_0(S)}(1+\frac{y}{x})]\frac{\omega_P}{Q_{P0}}S +$$

$$[1 + \frac{K_mK_0}{x} + \frac{1}{A_m(S)}(1 + K_m\frac{y}{x}) + \frac{K_0}{A_0(S)} +$$

$$\frac{K_0}{A_m(S)A_0(S)}(1+\frac{y}{x})]$$

(4.3.19a)

where

$$y = 1 + \frac{G_0}{G_F} \qquad (4.3.19b)$$

Using

$$A_0(S) = \frac{B_0}{S+\omega_0} \qquad (4.3.20a)$$

$$A_m(S) = \frac{B_m}{S+\omega_m} \qquad (4.3.20b)$$

Where B_0, B_m are the unity gain band width of the OAs, ω_0, ω_m are the poles of the OAs. A_0, A_m are the DC gains of the OAs, we get.

$$D(S) =$$

$$S^4[\frac{K_0(1+\frac{y}{x})}{B_mB_0}] +$$

$$S^3[\frac{(1+K_m\frac{y}{x})}{B_m} + \frac{K_0}{B_0} + \frac{K_0}{B_mB_0}(1+\frac{y}{x})(\omega_m+\omega_0\frac{\omega_P}{Q_{P0}})]$$

$$+ \; s^2 [1 + \frac{K_m K_0}{x} + \frac{(1+K_m \frac{y}{x})}{A_m} + \frac{K_0}{A_0} + \frac{K_0(1+\frac{y}{x})}{A_m A_0} +$$

$$\frac{\omega_{P0}}{Q_{P0}} [\frac{1+K_m \frac{y}{x}}{B_m} + \frac{K_0}{B_0} + \frac{(\omega_m+\omega_0)}{B_m B_0} K_0(1+\frac{y}{x}) + \frac{K_0(1+\frac{y}{x})\omega_P^2}{B_m B_0}$$

$$S [\frac{\omega_P}{Q_{P0}} [1 + \frac{(1+\frac{K_m y}{x})}{A_m} + \frac{K_0}{A_0} + \frac{K_0(1+\frac{y}{x})}{A_m A_0}] +$$

$$\omega_P^2 [\frac{1+\frac{K_m y}{x}}{B_m} + \frac{K_0}{B_0} + \frac{(\omega_m+\omega_0)}{B_m B_0} K_0(1+\frac{y}{x})]] +$$

$$[1 + \frac{K_m K_0}{x} + \frac{(1+K_m \frac{y}{x})}{A_m} + \frac{K_0}{A_0} + \frac{K_0(1+\frac{y}{x})}{A_m A_0}] \omega_P^2 \tag{4.3.21}$$

If we let

$$K_m = K_0 \triangleq K$$

$$B_m = B_0 \triangleq B \qquad A_m = A_0 \triangleq A$$

$$\omega_m = \omega_0 \triangleq \omega_c$$

and use

$$B = \omega_c A$$

we get:

$$D(S) = S^4 [\frac{K_0(1+\frac{y}{x})}{B^2}] +$$

$$S^3 [\frac{1+K(1+\frac{y}{x})}{B} + \frac{2K}{AB}(1+\frac{y}{x}) + \frac{K(1+\frac{y}{x})\omega_P^2}{B^2}]$$

$$S^2 [1 + \frac{K^2}{x} + \frac{1+K(1+\frac{y}{x})}{A} + \frac{K(1+\frac{y}{x})}{A^2} +$$

FIGURE 4.3.9 THE EFFECT OF THE OAs POLES ON ω_p FOR THE TYPE 3 QMC

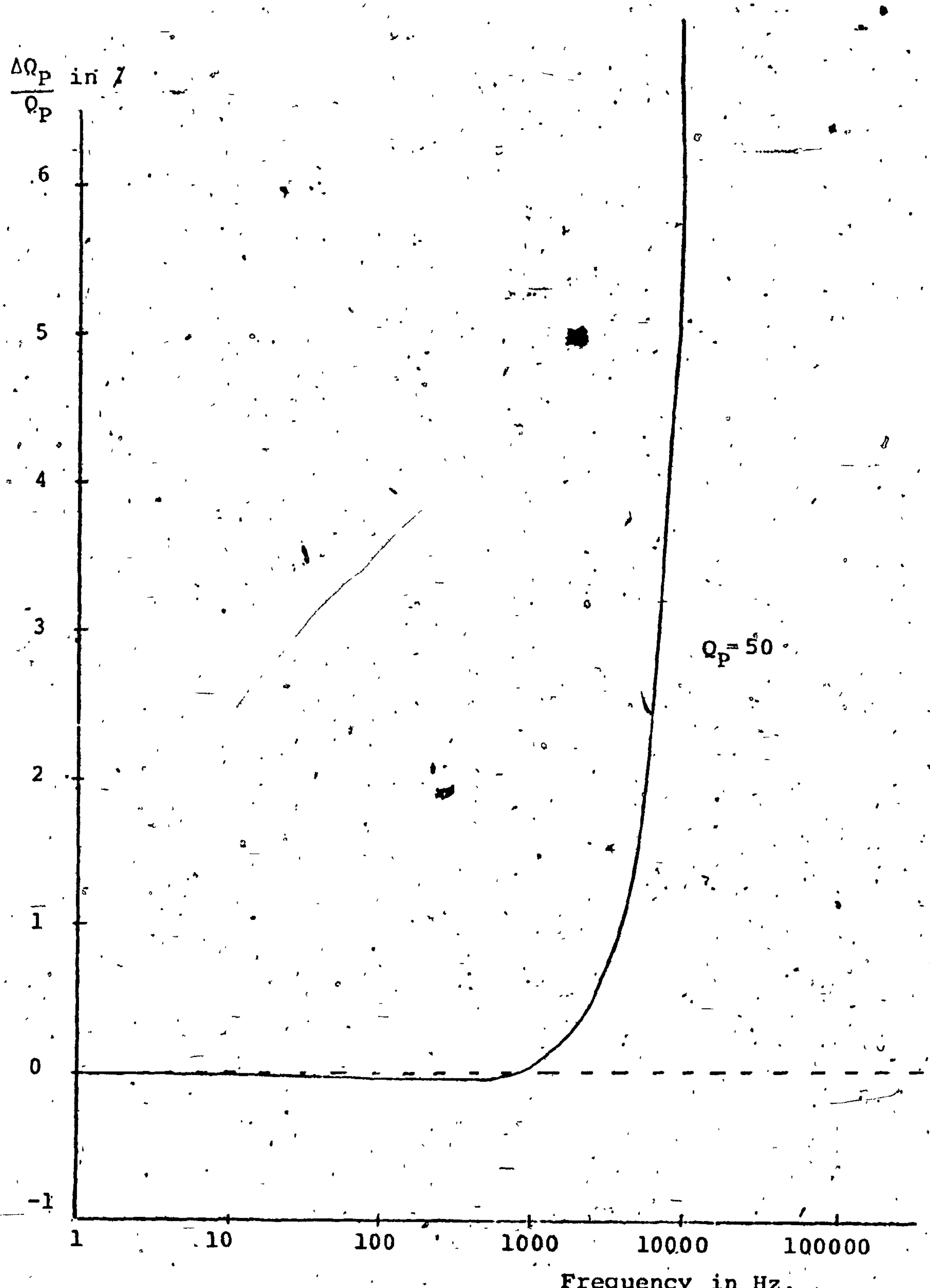


FIGURE 4.3.10 THE EFFECT OF THE OAs POLES ON Q_P FOR THE TYPE 3 QMC

$$\frac{\omega_P}{Q_{P0}}\left[\frac{1+K(1+\frac{Y}{x})}{B} + \frac{2K(1+\frac{Y}{x})}{AB}\right] + \frac{K(1+\frac{Y}{x})\omega_P^2}{B^2}] +$$

$$S[\frac{\omega_P}{Q_{P0}}[1 + \frac{1+K(1+\frac{Y}{x})}{A} + \frac{K(1+\frac{Y}{x})}{A^2}] +$$

$$\omega_P^2 \frac{1+K(1+\frac{Y}{x})}{B} + \frac{2K(1+\frac{Y}{x})}{AB}]] +$$

$$[1 + \frac{K^2}{x} + \frac{1+K(1+\frac{Y}{x})}{A} + \frac{K(1+\frac{Y}{x})}{A^2}]\omega_P^2 \qquad (4.3.22)$$

Using Eqn. 4.3.36 the values of $\frac{\Delta\omega_P}{\omega_P}$ and $\frac{\Delta Q_P}{Q_P}$ due to the effect of the pole of the amplifier have been obtained for various values of ω_P and are given in Fig. 4.3.9 and 4.3.10 respectively. Use of O.A. SN72.301 properly compensated has been assumed in the computations.

4.4 A QMC Realization Obtained Using An RII Generating Circuit

The overall realization consisting of two amplifiers as shown in Fig. 4.4.1a (Figs. 4.4.1b and 4.4.1c show the actual realizations of the two amplifiers contained in Fig. 4.3.1a). It consists of a single amplifier optimized network [17, 18] constituting an RII generating circuit having a t_v of type N1 used in the configuration of Fig. 4.2.1 with $G_F = 0$. The entire network corresponds to a null filter having a denominator decomposition of type 6. Its transfer function is given by

$$T_6 = \frac{\frac{K_m K_0}{x} \frac{G_0}{G_P} [S^2 + \frac{G_1 G_2 (C_2+C_3)}{C_1 C_2 C_3}]}{S^2 [1+\frac{K_m K_0}{x}] + S [\frac{G_3}{C_3}(1-K_0) + \frac{G_2 (C_2+C_3)}{C_2 C_3}] S + \frac{G_1 G_2 (C_2+C_3)}{C_1 C_2 C_3} [1+\frac{K_m K_0}{x}]} \quad (4.4.1)$$

$$x = 1 + \frac{G_0+G_A}{G_H} \quad (4.4.1b)$$

$$K_m = \frac{G_A}{G_B} \quad (4.4.1c)$$

$$K_0 = 1 + \frac{G_{A'}}{G_{B'}} \quad (4.4.1d)$$

This takes care of the loading effect of the amplifier. The parameter x has a minimum value of one which can be obtained for an unrestricted resistive spread only. The design equations of the generating circuit have been given in Table 4.2.3. It is known[17,18] that for this generating circuit the minimum capacitive spread which is smaller than two occurs for the following values of the design parameters $b = 3$, $c = 0.71$, $\epsilon_1 = 1.65$ and that for these values $bK_0 <$ 3.91. We will use this set of design parameters values wherever needed in our subsequent work.

From the transfer function given in Eqn. 3.4.1, we have

$$\omega_P = \omega_{P0} = \{\frac{G_1 G_2 (C_2+C_3)}{C_1 C_2 C_3}\}^{\frac{1}{2}} \quad (4.4.2)$$

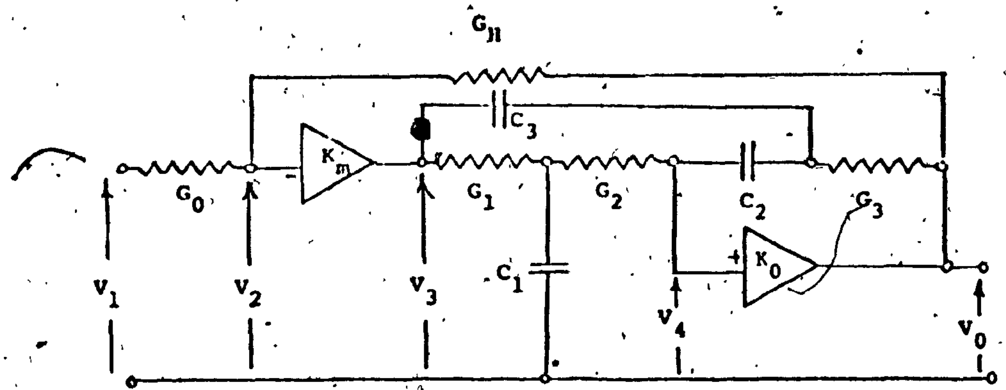


FIGURE 4.4.1a

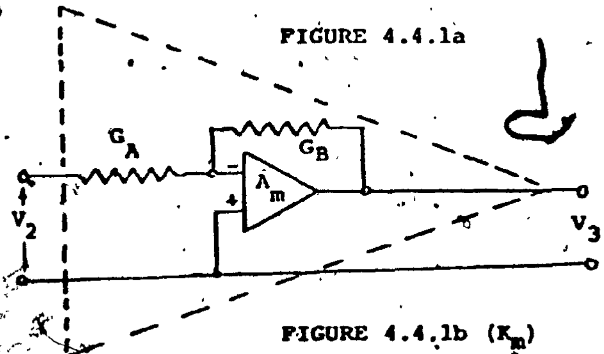


FIGURE 4.4.1b (K_m)

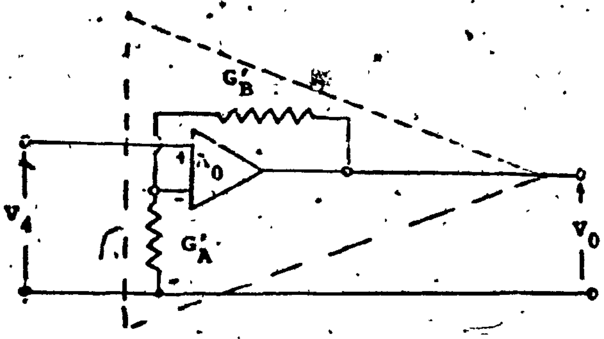


FIGURE 4.4.1c (K_0)

FIGURE 4.4.1 A NULL QMC REALIZATION OBTAINED FROM AN RII GENERATING CIRCUIT.

<u>A SET OF DESIGN EQUATIONS</u>

$G_1, G_2, G_3, C_1, C_2, C_3$ are given in Table 4.2.3

$G_H/G_A = 1$

$K_0 = G'_A/G'_B + 1$

$K_m = G_A/G_B = G_H/G_0$

K_m, K_0 are obtained from Fig.4.4.6 for specified Q_P

$$S_{K_0}^{\omega_P} = S_{K_0}^{\omega_P} = 0 \tag{4.4.3}$$

$$Q_P = Q_{P0}\left[1+\frac{K_m K_0}{x}\right] \tag{4.4.4a}$$

where

$$Q_{P0} = \frac{\left[\frac{G_1 G_2 (C_2+C_3)}{C_1 C_2 C_3}\right]^{\frac{1}{2}}}{\left[\frac{G_3}{C_3}(1-K_0)+\frac{G_2(C_2+C_3)}{C_2 C_3}\right]} = \frac{1}{c-b(1-\frac{1}{K_0})} \tag{4.4.4b}$$

$$S_{K_m}^{Q_P} = \frac{\frac{K_m K_0}{x}}{1+\frac{K_m K_0}{x}} \tag{4.4.5}$$

$$S_{K_0}^{Q_P} = S_{K_0}^{Q_{P0}} + S_{K_0}^{\rho} \tag{4.4.6a}$$

However it is known from [17] that

$$S_{K_0}^{Q_{P0}} = bQ_{P0} \tag{4.4.6b}$$

Hence

$$S_{K_0}^{Q_P} = \frac{bQ_P+\frac{K_m K_0}{x}}{1+\frac{K_m K_0}{x}} \tag{4.4.6c}$$

$$|K_m S_{K_m}^{Q_P}| = \frac{\frac{K_m^2 K_0}{x}}{1+\frac{K_m K_0}{x}} \tag{4.4.7}$$

$$|K_0 S_{K_0}^{Q_P}| = \frac{K_0 b Q_P + \frac{K_m K_\theta^2}{x}}{1+\frac{K_m K_0}{x}} \tag{4.4.8}$$

Using Eqn. 4.4.7 and 4.4.8 we get

$$F = \frac{K_0 b Q_P + \frac{K_m K_0}{x}(K_m + K_0)}{1+\frac{K_m K_0}{x}} \tag{4.4.9}$$

Using the expression of K_0 given in the generating function's design equations

$$K_0 = \frac{b}{b-c+\frac{1}{Q_{P0}}} \tag{4.4.10}$$

and Eqn. 4.4.4a it is possible to express K_m and F respectively as:

$$K_m = \frac{x}{K_0^2}[\beta - \alpha K_0] \tag{4.4.11}$$

$$K_0 < \frac{\beta}{\alpha} \tag{4.4.11a}$$

$$F = \frac{\frac{x}{K_0^2}[\beta-\alpha K_0]^2 + K_0[\beta-\alpha K_0] + \beta K_0^2}{K_0 + (\beta - \alpha K_0)} \tag{3.4.12}$$

where

$$\beta \stackrel{\Delta}{=} bQ_P \tag{4.4.13a}$$

$$\alpha \triangleq (b-c)Q_P+1 \qquad (4.4.13b)$$

Minimization of F for various values of x using K_0 as a variable has been carried and values of $\frac{F_m}{Q_P}$ versus Q_P and the corresponding values of K_0, K_m and Q_{P0} are given in Fig. 4.4.2 to Fig. 4.4.5.

From Eqn. 4.4.12 as well as from the curves shown in Fig. 4.4.2 it is clear that it is desirable to reduce as much as possible the value of the parameter x in order to reduce the value of F_m corresponding to any prescribed value of Q_P. Also the curves of Figs. 4.4.3, 4.4.4, and 4.4.5 show that the reduction of x yields reductions in the corresponding values of K_m, K_0, and Q_{P0} which are desirable because of the following reasons.

(a) The reduction in the values of K_0, and Q_{P0} decreases the possibility of instability caused by the presence of negative terms in the S coefficient of the denominators especially for high values of Q_{P0}.

(b) The reduction of K_m contributes to the reduction of the resistive spread of the circuit.

However, decreasing the value of x is associated with an increase in the resistive spread γ. Let

$$x = 1 + \frac{1}{\gamma_A} + \frac{1}{\gamma_0} \qquad (4.4.14a)$$

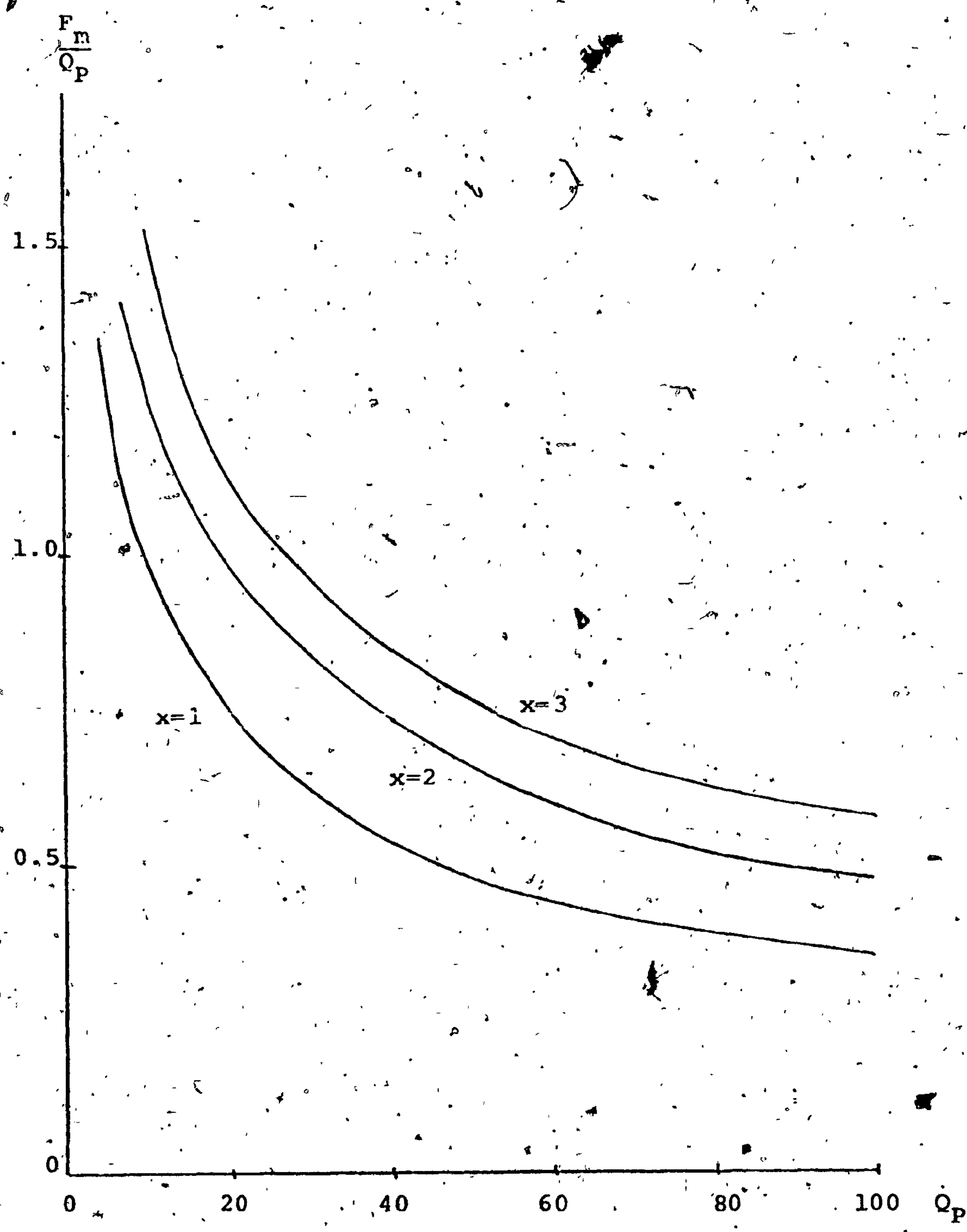


FIGURE 4.4.2 F_m/Q_P OF THE NULL TYPE 6 QMC FOR SEVERAL VALUES OF x VERSUS Q_P.

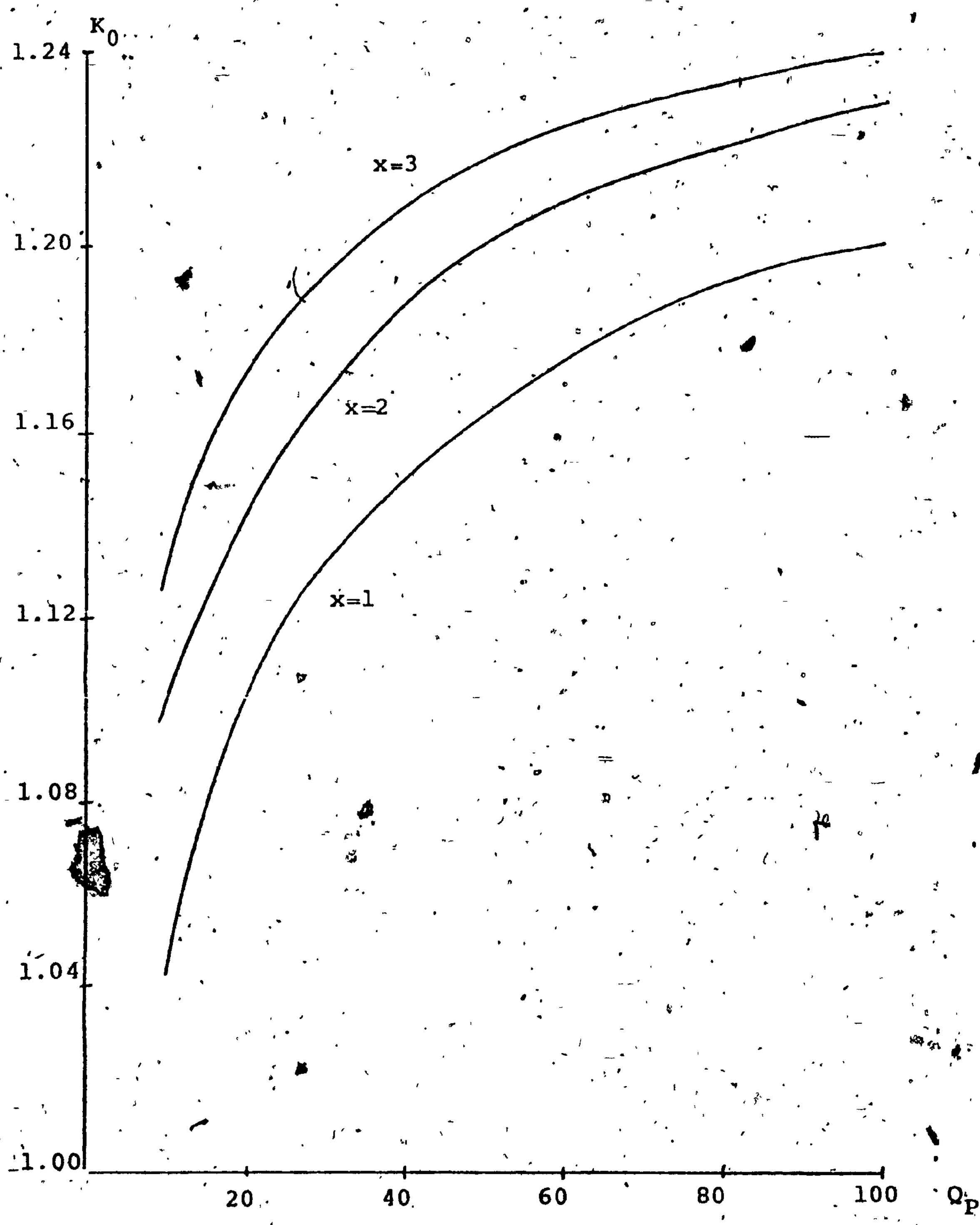


FIGURE 4.4.3 K_0 OF THE NULL TYPE 6 QMC FOR SEVERAL VALUES OF x AND MINIMIZED F VERSUS Q_P

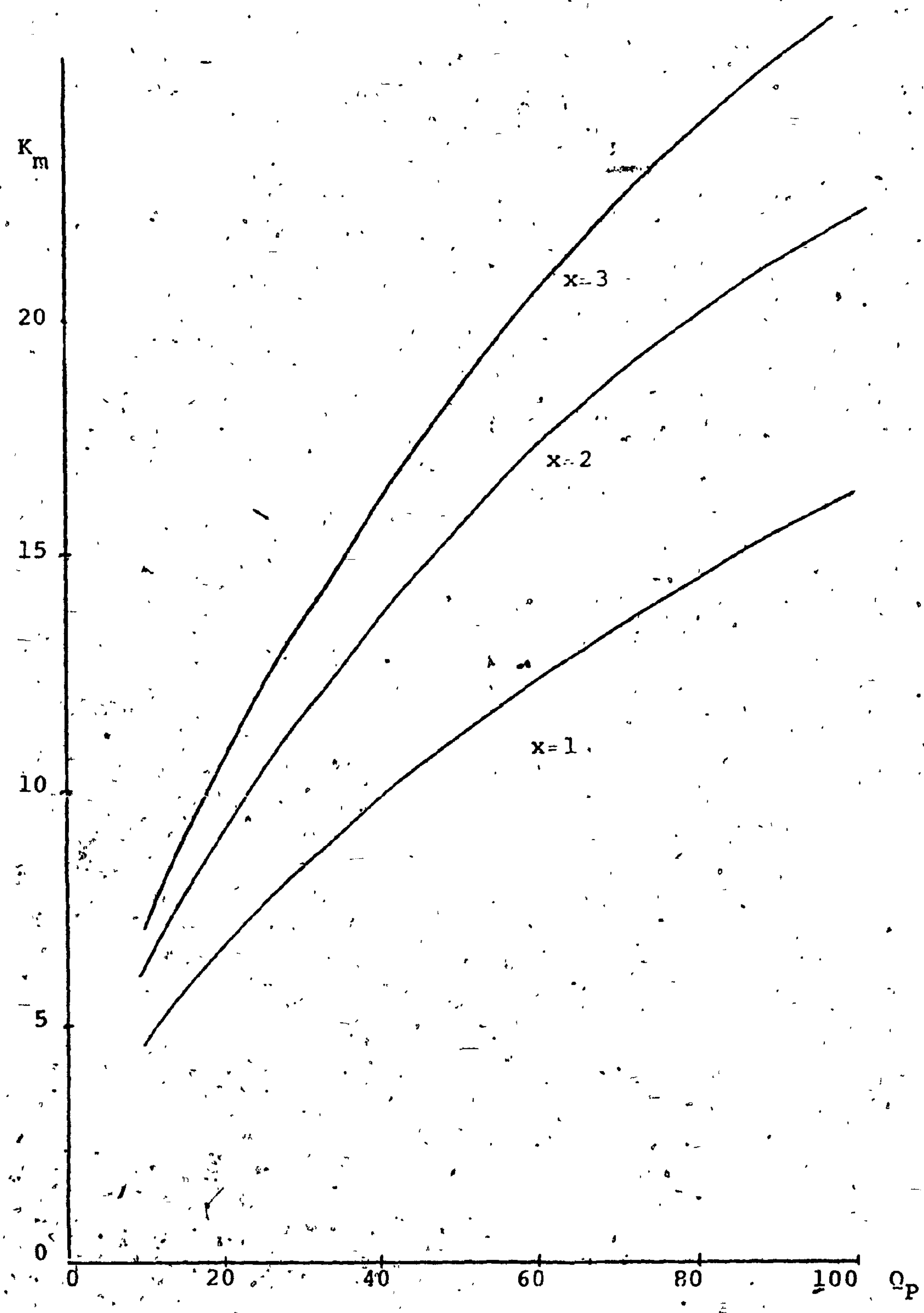


FIGURE 4[illegible]4 K_m OF THE NULL TYPE 6 QMC FOR SEVERAL VALUES OF x AND MINIMIZED F VERSUS ρ_P

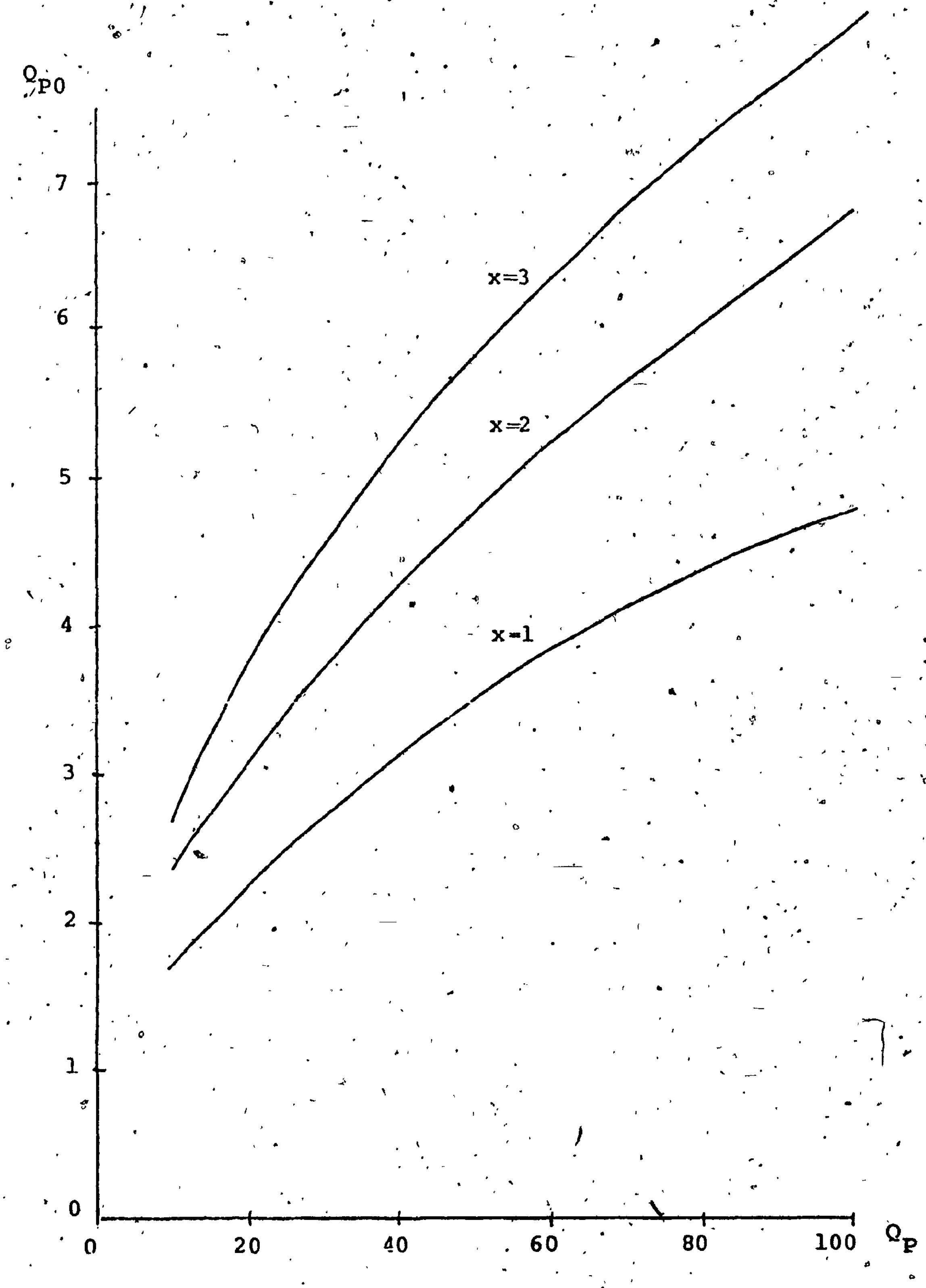


FIGURE 4.4.5 Q_{P0} OF THE NULL TYPE 6 QMC FOR SEVERAL VALUES OF x AND MINIMIZED F VERSUS Q_P

$$\frac{1}{\gamma_A} = \frac{G_A}{G_H} \qquad (4.4.14b)$$

$$\frac{1}{\gamma_0} = \frac{G_0}{G_H} \qquad (4.4.14c)$$

where for the purpose of minimizing x it is assumed that $\gamma_A > 1$, $\gamma_0 > 1$.

With these assumptions,

$$\frac{G_H}{G_B} = \gamma_A K_m \qquad (4.4.15)$$

is the quantity which for low values of Q_{P0} and high values of Q_P determines the resistive spread γ, the following set of design equations appears to be a good compromise able to achieve both low values of F_m and of γ

$$\gamma_A = 1 \qquad (4.4.16a)$$

$$\gamma_0 = K_m \qquad (4.4.16b)$$

Hence

$$x = 2 + \frac{1}{K_m} \qquad (4.4.17a)$$

and using Eqns. (4.4.11), (4.4.16), and 4.4.17a

$$F = \frac{[1+(1+\frac{K_0^2}{\beta-\alpha K_0})]^{\frac{1}{2}}[\frac{\beta-\alpha K_0}{K_0}]^2 + K_0(\beta-\alpha K_0)+\beta K_0^2}{(\beta-\alpha K_0)+K_0} \qquad (4.4.17b)$$

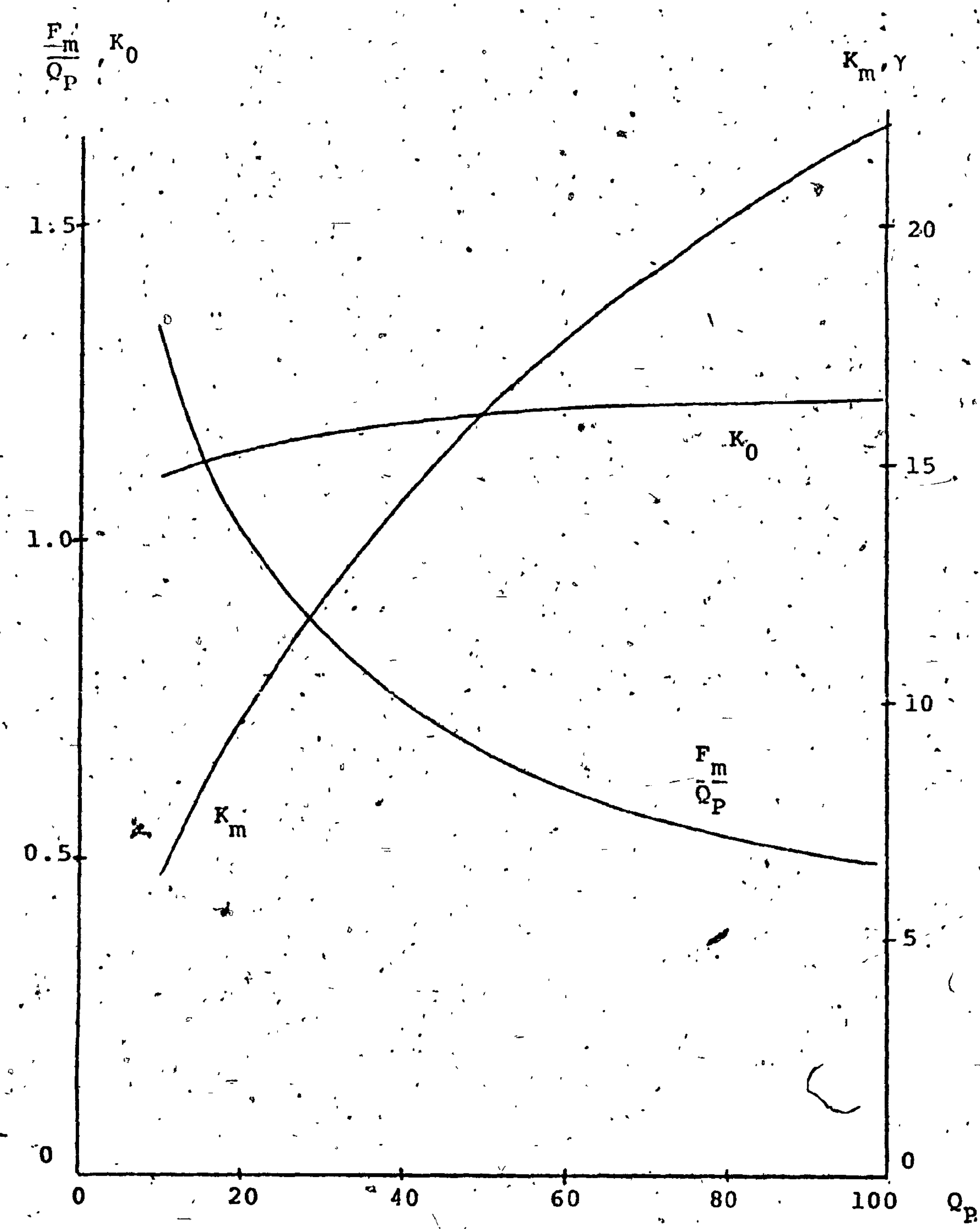


FIGURE 4.4.6 F_m/Q_P, K_0, K_m, γ OF THE NULL TYPE 6 QMC WHEN $x=2+1/K_m$ VERSUS Q_P

A set of curves giving F_m/Q_p, K_0 obtained from the minimization of F as expressed in Eqn. 4.4.17b, K_m obtained using Eqn. 4.4.11 and γ, versus Q_p are shown in Fig. 4.4.6.

For the Null filter network shown in Fig. 4.4.1 grounding the input end of G_0 and lifting the capacitor C_3 off the ground and connecting it to the input terminal as shown in Table 4.2.3 we get a Band Pass filter network of type 6 having the same denominator as that of the Null network. For the same element values this network shall have all the properties, namely, sensitivities, gain sensitivity products, F, Q-multiplication, etc. of the Null network and as such no further detailed discussion of this network is necessary.

4.4.1 Experimental Results

The network shown in Fig. 4.3.1 was built up using discrete elements and tested. The results are summarized below:

Designed values:

ω_P = 6283.185 rad/sec. = 1000 Hz.

Q_P = 50

Q_{PO} = 4.858

ρ = 10.292

K_m = 15.946

K_0 = 1.202

x = 2.063

C_1 = 9.356 KPF

$C_2 = 4.93$ KPF

$C_3 = 8.984$ KPF

$R_1 = 12.077$ KΩ

$R_2 = 70.422$ KΩ

$R_3 = R_F = R_A = 6.933$ KΩ

$R_B = R_0 = 110.554$ KΩ

$R_A = 100$ KΩ

$R_B = 20.2$ KΩ

$K_0 S_{K_0}^{\omega_P} = K_m S_{K_m}^{\omega_P} = 0$

$|K_0 S_{K_0}^{Q_P}|/Q_P = 0.37$

$|K_m S_{K_m}^{Q_P}|/Q_P = 0.29$

$\frac{F_m}{Q_P} = 0.66$

Element spread:

Capacitive 1.898:1

Resistive 15.946:1

The 3dB frequencies are:

$F_{C1} = 990.05$ Hz. F_{C2} 1010.05 Hz.

Any or both of K_m and K_0 can be adjusted in order to achieve the tuning of Q_P without affecting ω_P.

The circuit has been implemented using 1% tolerance resistors and 2% tolerance condensers with values chosen to be, within the range of currently available elements, as close as possible to the designed ones (Trim pots have been used wherever necessary. The OAs have been used where SN72301 compensated according to the current specifications of the manufacturer.

Actual values obtained:

ω_P = 999.4 Hz.

F_{C1} = 990.1 Hz. F_{C2} = 1010.4 Hz.

Q_P = 49.23

The depth of the notch has been found to be -70dB. It is found that the circuit was stable during activation. Q_P and ω_P variations from their designed values are 1.53% and 0.06% respectively, This is due to the small differences between the designed values and the real ones as well as to the tolerance of the elements. The power supply voltages used were ∓10V and ∓15V. Also the OAs were heated and their temperature controlled. The response was experimentally measured and plotted at 22°C (room temperature) and 70°C for both power supplies voltages. Only the OAs were heated; the passive elements were not heated in order to simulate Hybrid Integrated Circuit technology.

As shown in Fig. 4.4.7 no appreciable change in the response was observed; thus the experimental results confirm the theoretical predictions.

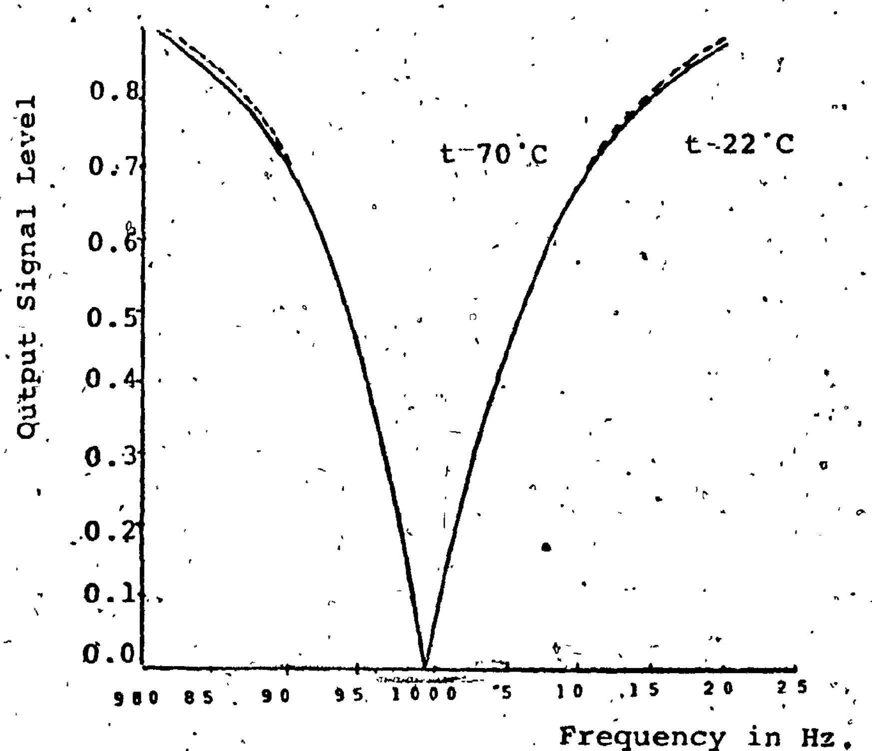


FIGURE 4.4.7a (±10V)

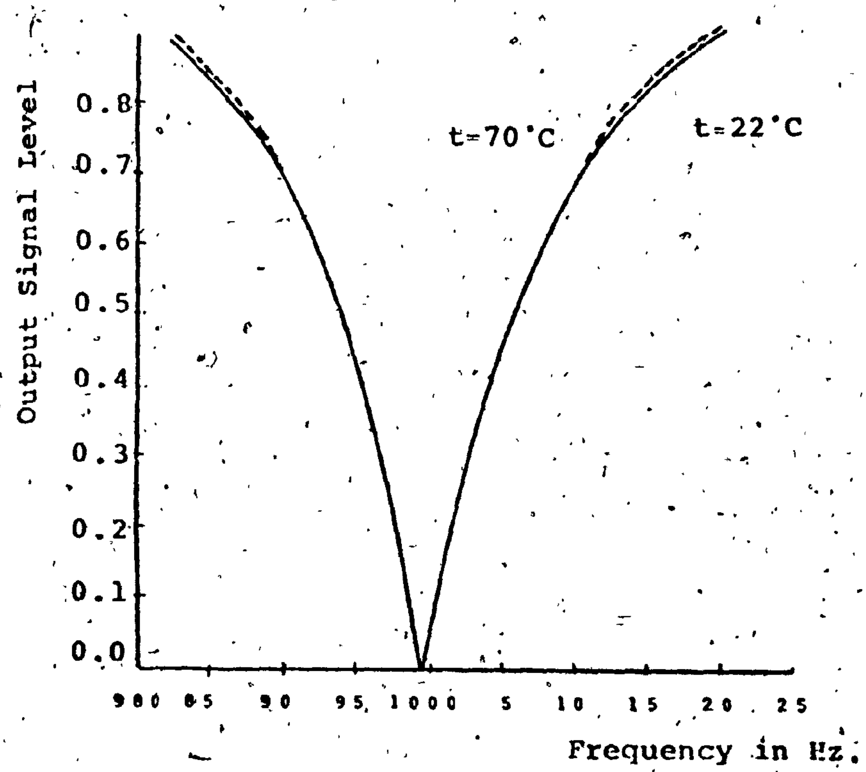


FIGURE 4.4.7b (±15V)

FIGURE 4.4.7 FREQUENCY RESPONSE OF THE NULL TYPE 6 QMC FOR $Q_p = 50$

4.4.2 Effect of the pole of the O.A. on Q and ω [33, 17]

In order to study the effect of the pole of the OA on Q_P or ω_P it is assumed that the desired elements are available and therefore one pole of the passive circuit always cancels with one of its zeros. Replacing the amplifiers of Fig. 4.4.1a by the networks of Fig. 4.4.1b, and 4.4.1c the analysis gives:

$$D(S) = S^2[1+\frac{K_mK_0}{x}+\frac{K_0}{A_0(S)}+\frac{1}{A_m(S)}(1+K_m\frac{Y}{x})+\frac{K_0}{A_m(S)A_0(S)}(1+K_m\frac{Y}{x})]+$$

$$S\{\frac{1}{Q_{P0}}+\frac{K_0}{A_0(S)}(b+\frac{1}{Q_{P0}})\}[1+\frac{1}{A_m(S)}(1+K_m\frac{Y}{x})]+$$

$$[1+\frac{K_mK_0}{x}+\frac{K_0}{A_0(S)}+\frac{1}{A_m(S)}(1+K_m\frac{Y}{x})+\frac{K_0}{A_m(S)A_0(S)}(1+K_m\frac{Y}{x})] \quad (4.4.18a)$$

where

$$y = 1 + \frac{G_0}{G_F} \quad (4.4.18b)$$

Using

$$A_0(S) = \frac{B_0}{\omega_P S+\omega_0} \quad (4.4.19a)$$

$$A_m(S) = \frac{B_m}{\omega_P S+\omega_m} \quad (4.4.19b)$$

where B_0, B_m are the unity gain bandwidth of the OAs. ω_0, ω_m are the poles of the OAs. A_0, A_m are the D-C gains of the OAs, we get

$$D(S) =$$

$$s^4\left[\frac{\omega_P^2 K_0 (1+K_m \frac{Y}{x})}{B_0 B_m}\right] +$$

$$s^3\left[\frac{K_0\omega_P}{B_0} + (1+K_m \frac{Y}{x})\frac{\omega_P}{B_m} + \frac{K_0(1+K_m \frac{Y}{x})}{B_0 B_m}(\omega_P\omega_m+\omega_P\omega_0) + K_0(b+\frac{1}{Q_{P0}})(1+K_m \frac{Y}{x})\frac{\omega_P^2}{B_0 B_m}\right]$$

$$+ s^2\left[1 + \frac{K_m K_0}{x} + K_0\frac{\omega_0}{B_0} + (1+K_m \frac{Y}{x})\frac{\omega_m}{B_m} + K_0(1+K_m \frac{Y}{x})\frac{\omega_0\omega_m}{B_0 B_m} + K_0(b+\frac{1}{Q_{P0}})\frac{\omega_P}{B_0} + (1+K_m \frac{Y}{x})\frac{\omega_P}{B_m Q_{P0}} + \frac{K_0(1+K_m \frac{Y}{x})\omega_P^2}{B_0 B_m} + \frac{K_0(b+\frac{1}{Q_{P0}})(1+K_m \frac{Y}{x})}{B_0 B_m}(\omega_P\omega_m+\omega_P\omega_0)\right] +$$

$$+ S\left[\frac{1}{Q_{P0}} + \frac{\omega_0}{B_0}K_0(b+\frac{1}{Q_{P0}}) + \frac{\omega_m}{B_m}\frac{(1+K_m \frac{Y}{x})}{Q_{P0}} + K_0(b+\frac{1}{Q_{P0}})(1+K_m \frac{Y}{x})\frac{\omega_0\omega_m}{B_0 B_m} + \frac{K_0\omega_P}{B_0} + (1+K_m \frac{Y}{x})\frac{\omega_P}{B_m} + \frac{K_0(1+K_m \frac{Y}{x})(\omega_P\omega_m+\omega_P\omega_0)}{B_0 B_m}\right]$$

$$+ \left[1 + \frac{K_0 K_m}{x} + \frac{K_0\omega_0}{B_0} + (1+K_m \frac{Y}{x})\frac{\omega_m}{B_m} + K_0(1+K_m \frac{Y}{x})\frac{\omega_0\omega_m}{B_0 B_m}\right] \tag{4.4.20}$$

If we let $B_m = B_0 \triangleq B$, $\omega_m = \omega_0 = \omega_c$ and use $B = \omega_c A$ we get

$$D(S) =$$

$$S^4 [\frac{\omega_P^2 K_0 (1+K_m \frac{Y}{X})}{B^2}] +$$

$$S^3 [\frac{\omega_P}{B}(1+K_0+K_m \frac{Y}{X}) + 2 \frac{\omega_P}{AB} K_0 (1+K_m \frac{Y}{X}) +$$

$$+ \frac{\omega_P^2}{B^2}(b+\frac{1}{Q_{P0}}) K_0 (1+K_m \frac{Y}{X})]$$

$$S^2 [1 + \frac{K_m K_0}{X} + \frac{[1+K_0+ \frac{K_m K_0}{X}]}{A} + \frac{K_0}{A}(1+K_m \frac{Y}{X})$$

$$+ \frac{\omega_P}{BQ_{P0}}[1+K_0+K_m \frac{Y}{X}] + \frac{\omega_P}{B} K_0 B +$$

$$+ 2 \frac{\omega_P}{AB}(b+\frac{1}{Q_{P0}}) K_0 (1+K_m \frac{Y}{X}) + \frac{\omega_P^2}{B^2} K_0 (1+K_m \frac{Y}{X})]$$

$$+ S[\frac{1}{Q_{P0}} + \frac{1}{AQ_{P0}}[1+K_0+K_m \frac{Y}{X}] + \frac{1}{A} K_0 b +$$

$$(b+\frac{1}{Q_{P0}}) K_0 (1+K_m \frac{Y}{X}) + \frac{\omega_P}{B}(1+K_0+K_m \frac{Y}{X}) + 2 \frac{\omega_P}{AB}(1+K_m \frac{Y}{X}) K_0]$$

$$[1 + \frac{K_0 K_m}{X} + \frac{1}{A}[1+K_0+K_m \frac{Y}{X}] + K_0 \frac{(1+K_m \frac{Y}{X})}{A^2}]$$

(4.4.21)

Using Eqn. 4.4.21 the values of $\frac{\Delta\omega_P}{\omega_P}$ and of $\frac{\Delta Q_P}{Q_P}$ due to the effect of the poles of the amplifiers have been obtained for various values of ω_P and are given in Fig. 4.4.8 and 4.4.9 respectively. The use of SN72301 OAs compensated according to the current specifications of the manufacturer has been assumed in the computations.

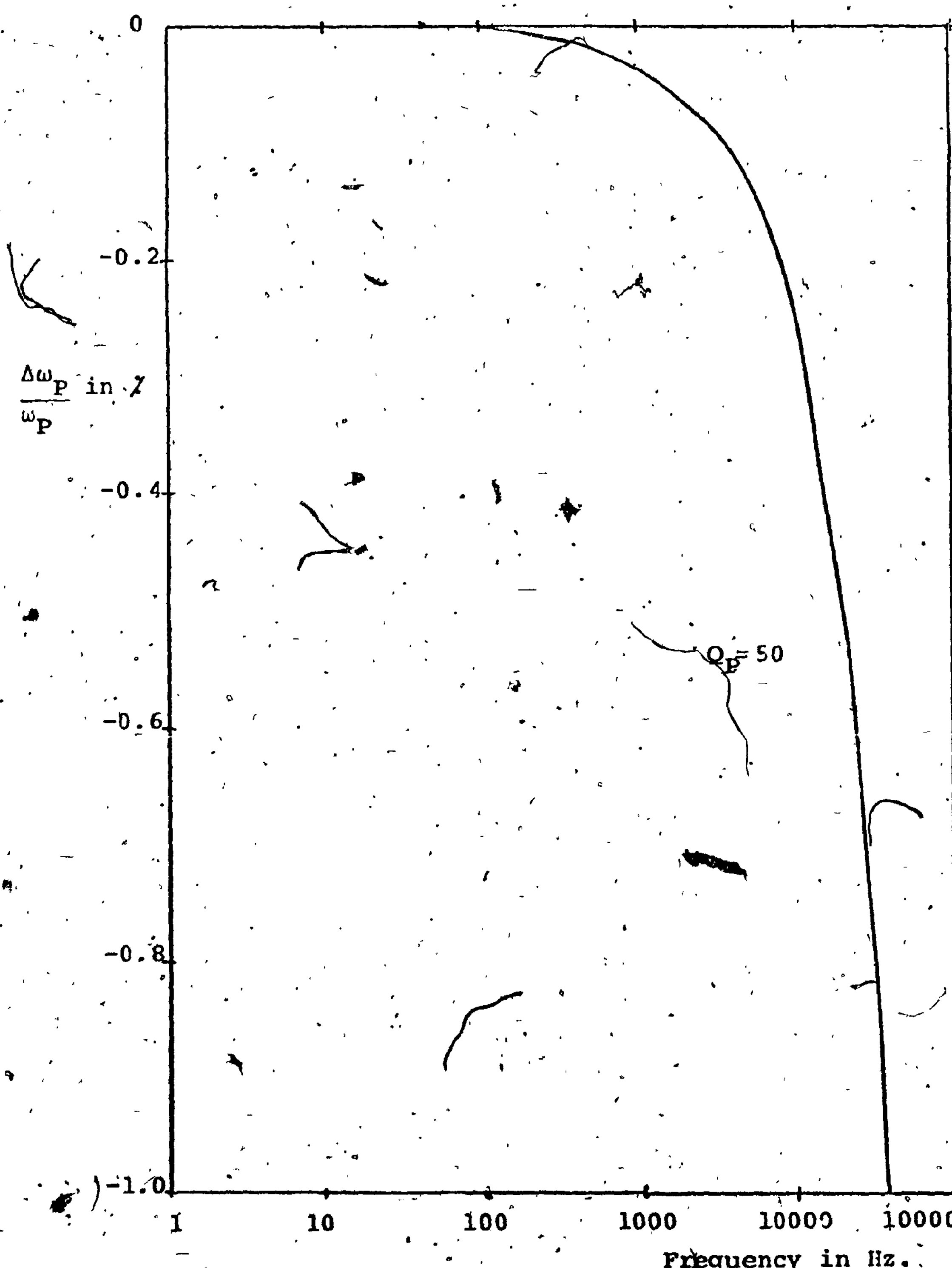


FIGURE 4.4.8 THE EFFECT OF THE OAs POLES ON ω_P FOR THE TYPE 6 QMC

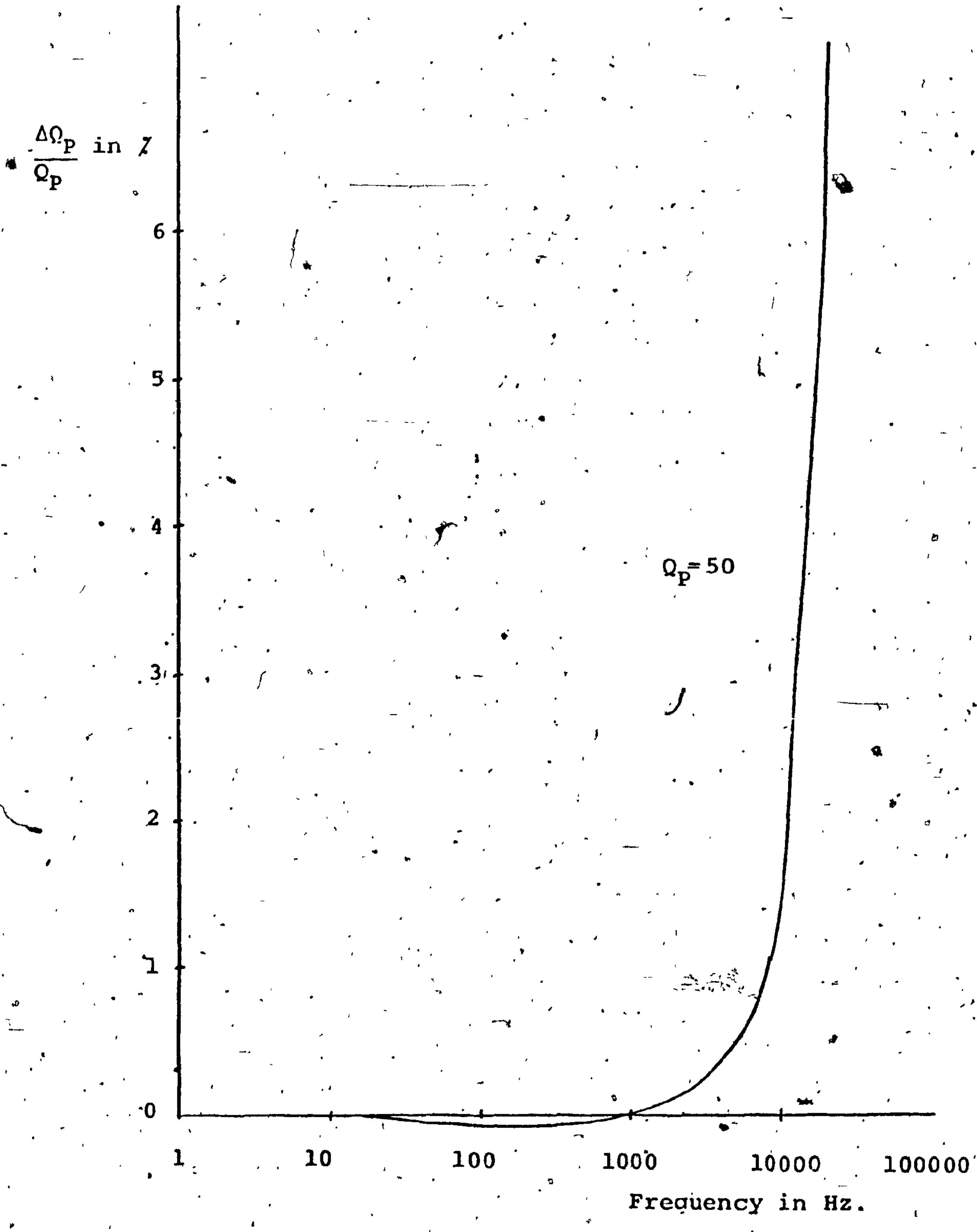


FIGURE 4.4.9 THE EFFECT OF THE OAs POLES ON Q_P FOR THE TYPE 6 QMC

4.5 A QMC Realization Obtained Using An RIII Generating Circuit

The overall realization consisting of two amplifiers is as shown in Fig. 3.5.1a (Figs. 4.5.1b and 4.5.1c shows the actual realizations of the two amplifers contained in Fig. 4.5.1a). It consists of a differential amplifier optimized network [17, 18] constituting an RIII generating circuit having a t_v of type N2 used in the configuration of Fig. 4.2.1 with $G_F = 0$. The entire network corresponds to a null filter having a denominator decomposition of type 7. Its transfer function is given by:

$$T_7 = \frac{\frac{K_m K_D}{x} \cdot \frac{G_0}{G_H}\left[S^2 + \frac{G_1 G_3 (C_1 + C_3)}{C_1 C_2 C_3}\right]}{S^2\left[1 + K_D + \frac{K_m K_0}{x}\right] + S\left[\frac{G_2}{C_1} + \frac{G_3 (C_1 + C_3)}{C_1 C_2 C_3}\right] + \frac{G_1 G_3 (C_1 + C_3)}{C_1 C_2 C_3}\left[1 + K_D + \frac{K_m}{x} K_D\right]} \quad (4.5.1a)$$

$$x = 1 + \frac{G_0 + G_A}{G_H} \quad (4.5.1b)$$

$$K_m = \frac{G_A}{G_B} \quad (4.5.1c)$$

and

$$\frac{G_{A0}}{G_{B0}} = \frac{G_{A0'}}{G_{B0'}} = K_D \quad (4.5.1d)$$

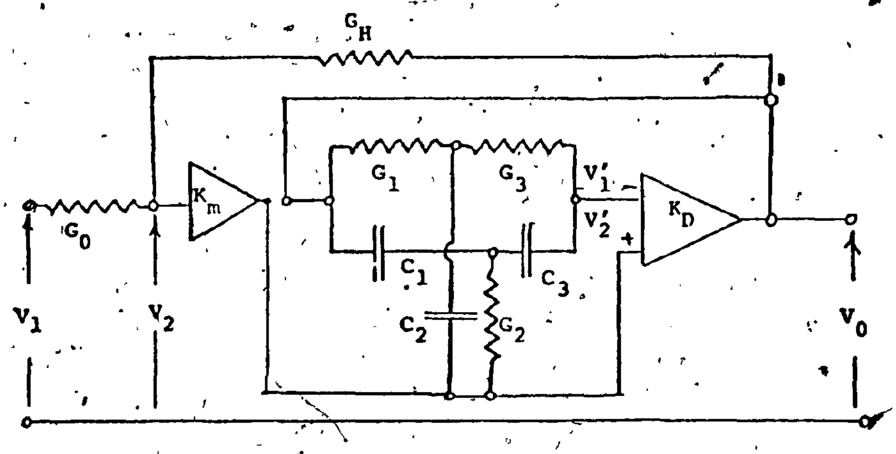


FIGURE 4.5.1a

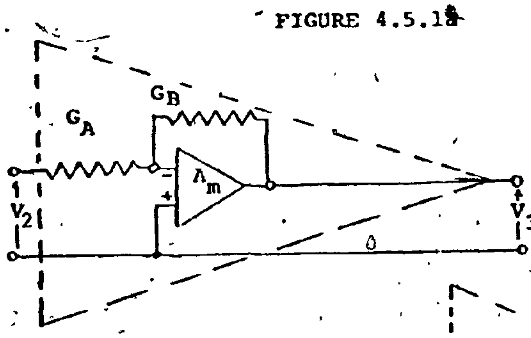


FIGURE 4.5.1b (K_m)

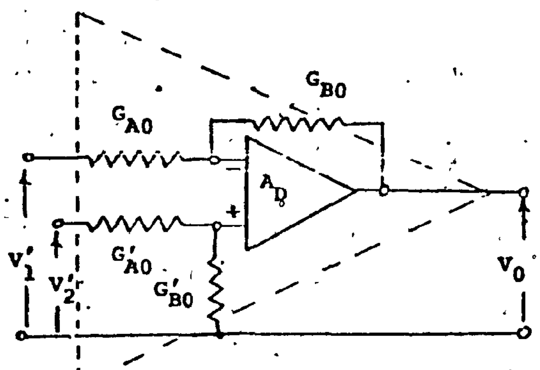


FIGURE 4.5.1c (K_D)

FIGURE 4.5.1 A NULL QMC REALIZATION OBTAINED FROM AN RIII GENERATING CIRCUIT.

A SET OF DESIGN EQUATIONS

$G_1, G_2, G_3, C_1, C_2, C_3$ are given in Table 4.2.3

$G_H/G_A = 1$

$K_D = G_{A0}/G_{B0} = G'_{A0}/G'_{B0} = G_H/G_0$

$K_m = G_A/G_B$

K_m, K_D are obtained from Fig.4.5.5 for specified Q_P

The parameter x has a minimum value of one which can be obtained for an unrestricted resistive spread only. The design equations of the generating circuit have been given in Table 4.2.3. It is known [17] that for this generating circuit the minimum capacitive spread which is smaller than 2.5 occur for the following values of the design parameters b = 3, η = 2.3. We will use this set of design parameters values wherever needed in our subsequent work.

For the transfer function given in Eqn. 4.4.1 we have

$$\omega_{P0} = \omega_P = \left\{\frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}\right\}^{\frac{1}{2}} \qquad (4.5.2)$$

$$S_{K_D}^{\omega_P} = S_{K_m}^{\omega_P} = 0 \qquad (4.5.3)$$

$$Q_P = Q_{P0}\left[1 + \frac{K_DK_m}{x(K_D+1)}\right] \qquad (4.5.4a)$$

where

$$Q_{P0} = \frac{\{C_1C_2C_3G_1G_3(C_1+C_3)\}^{\frac{1}{2}}}{C_2C_3G_2+C_2G_3(C_1+C_3)}(K_D+1) = \frac{K_D+1}{b} \qquad (4.5.4b)$$

Hence

$$Q_P = \frac{1}{b}\left[1+K_D+\frac{K_DK_m}{x}\right] \qquad (4.5.4c)$$

$$S_{K_m}^{Q_P} = \frac{\frac{K_DK_m}{x}}{1+K_D+\frac{K_DK_m}{x}} \qquad (4.5.5a)$$

This could be expressed using Eqn. 4.5.4c as

$$S_{K_m}^{Q_P} = \frac{1}{bQ_P}\frac{K_m K_D}{x} \quad (4.5.5b)$$

$$S_{K_D}^{Q_P} = \frac{K_D + \frac{K_D K_m}{x}}{1+K_D+\frac{K_D K_m}{x}} \quad (4.5.6a)$$

This could be expressed using Eqn. 4.5.5a

$$S_{K_D}^{Q_P} = \frac{K_D}{bQ_P}(1+\frac{K_m}{x}) \quad (4.5.6b)$$

Hence

$$|K_m S_{K_m}^{Q_P}| = \frac{1}{bQ_P}\frac{K_m^2 K_D}{x} \quad (4.5.7)$$

$$|K_P S_{K_D}^{Q_P}| = \frac{K_D^2}{bQ_P}(1+\frac{K_m}{x}) \quad (4.5.8)$$

and

$$F = \frac{K_D}{bQ_P}[\frac{K_m^2}{x} + K_D(1+\frac{K_m}{x})] \quad (4.5.9)$$

Minimization of F for given values of Q_P and of the parameter x which is independent of both K_m and K_D can be achieved in the following way : Using Eqns. 4.5.4c, 4.5.7 , and 4.5.9 it is possible to express K_m and F respectively as :

$$K_m = x[\frac{bQ_P-1}{K_D} - 1] \quad (4.5.10)$$

$$F = \frac{K_D}{bQ_P}\left[x\left[\frac{bQ_P-1}{K_D} - 1\right]^2 + (bQ_P-1)\right] \qquad (4.5.11)$$

Setting the partial derivative of F with respect to K_D equal to zero yields:

$$K_D = (bQ_P-1)\left[\frac{x}{bQ_P+x-1}\right]^{\frac{1}{2}} \qquad (4.5.12)$$

Hence using this value of K_D in Eqn. 4.5.9 and 4.5.11 we get respectively the value of F (called F_m) for prescribed values of b, x, and Q_P and the corresponding value of K_m respectively as:

$$F_m = \frac{x(bQ_P-1)}{bQ_P}\left[\frac{x}{bQ_P+x-1}\right]^{\frac{1}{2}}\left[\left(\frac{bQ_P+x-1}{x}\right)^{\frac{1}{2}} - 1\right]^2 + \frac{(bQ_P-1)^2}{bQ_P}\left[\frac{x}{bQ_P+x-1}\right]^{\frac{1}{2}} \qquad (4.5.13a)$$

and

$$K_m = x\left[\left(\frac{bQ_P+x-1}{x}\right)^{\frac{1}{2}} - 1\right] \qquad (4.5.13b)$$

The result of minimization of F for several values of x as well as the corresponding values of K_D, and K_m versus Q_P are shown in Fig. 4.5.2 to Fig. 4.5.4. From Eqn. 4.5.11 as well as from the curves shown in Fig. 4.5.2 it is clear that it is desirable to reduce the value of the parameter x in order to reduce as much as possible the value of F_m

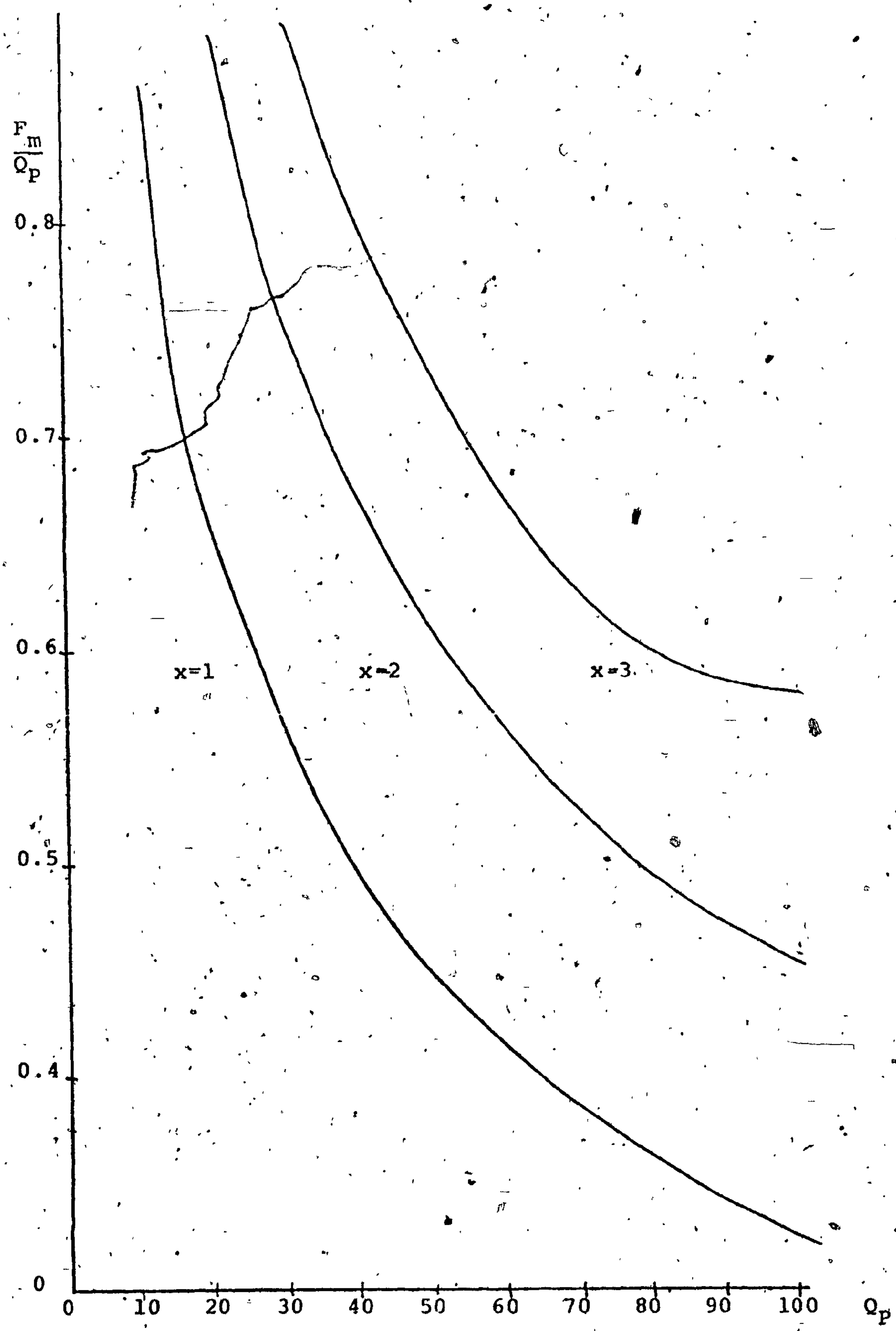


FIGURE 4.5.2 F_m/Q_P VERSUS Q_P OF THE NULL TYPE 7 QMC FOR SEVERAL VALUES OF x

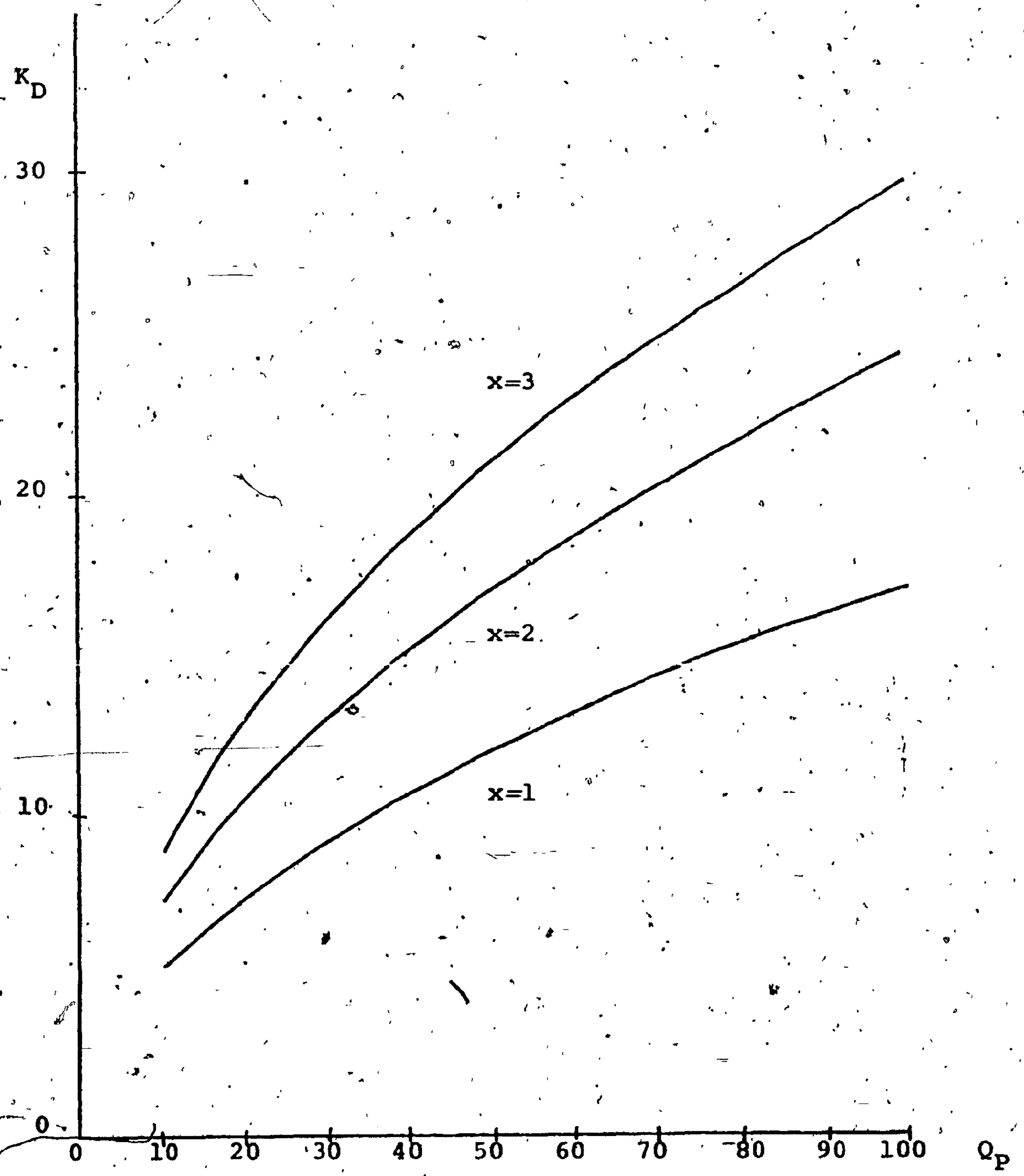


FIGURE 4.5.3 K_D OF THE NULL TYPE 7 QMC FOR SEVERAL VALUES OF x, AND MINIMIZED F, VERSUS Q_P.

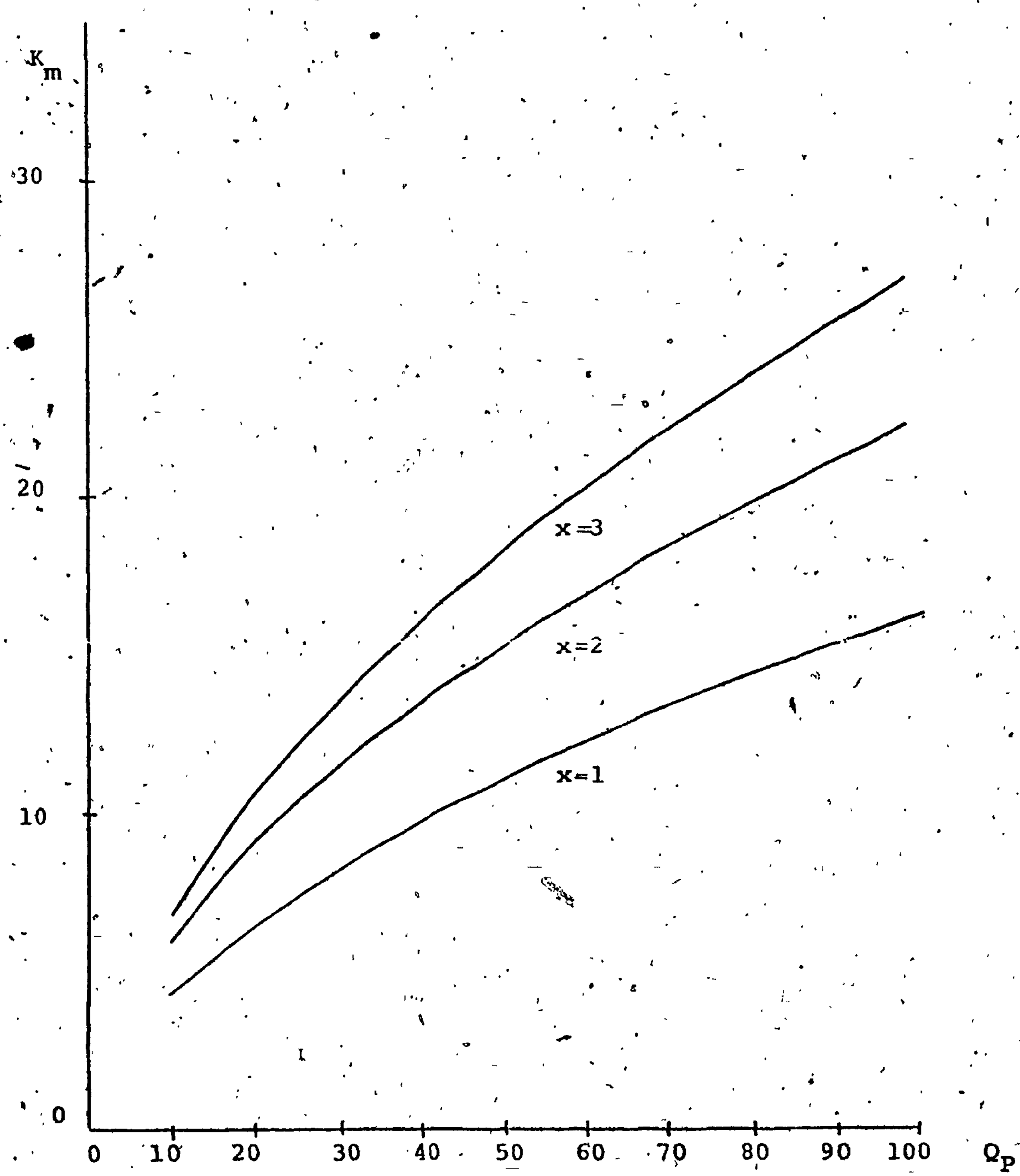


FIGURE 4.5.4 K_m OF THE NULL TYPE 7 QMC FOR SEVERAL VALUES OF x, AND MINIMIZED F VERSUS Q_P.

corresponding to any prescribed value of $Q_P^{\&}$. Also inspection of the curves shown in Fig. 4.5.3 and Fig. 4.5.4 shows that the reduction of x contributes to a decrease in the resistive spread γ. However decreasing the value of x is associated with an increment in the resistive spread γ. Hence we can follow the same procedure used previously. Let

$$x = 1 + \frac{1}{\gamma_A} + \frac{1}{\gamma_0} \tag{4.5.14a}$$

$$\frac{1}{\gamma_A} = \frac{G_A}{G_H} \tag{4.5.14b}$$

$$\frac{1}{\gamma_0} = \frac{G_0}{G_H} \tag{4.5.14c}$$

where for the purpose of minimizing x it is assumed that $\gamma_A \geq 1$, $\gamma_0 \geq 1$ with these assumptions.

$$\frac{G_H}{G_B} = \gamma_A K_m \tag{4.5.15}$$

is the quantity which for low values of Q_{P0} and high values of Q_P determines the resistive spread γ.* The following

---

*The resistive spread γ to which it is referred here does not take into account the scaling up of the resistances of the generating circuit which may be necessary to avoid the loading of the Twin-T if the differential amplifiers is realized using a single O.A. as shown in Fig. 4.5.1c.

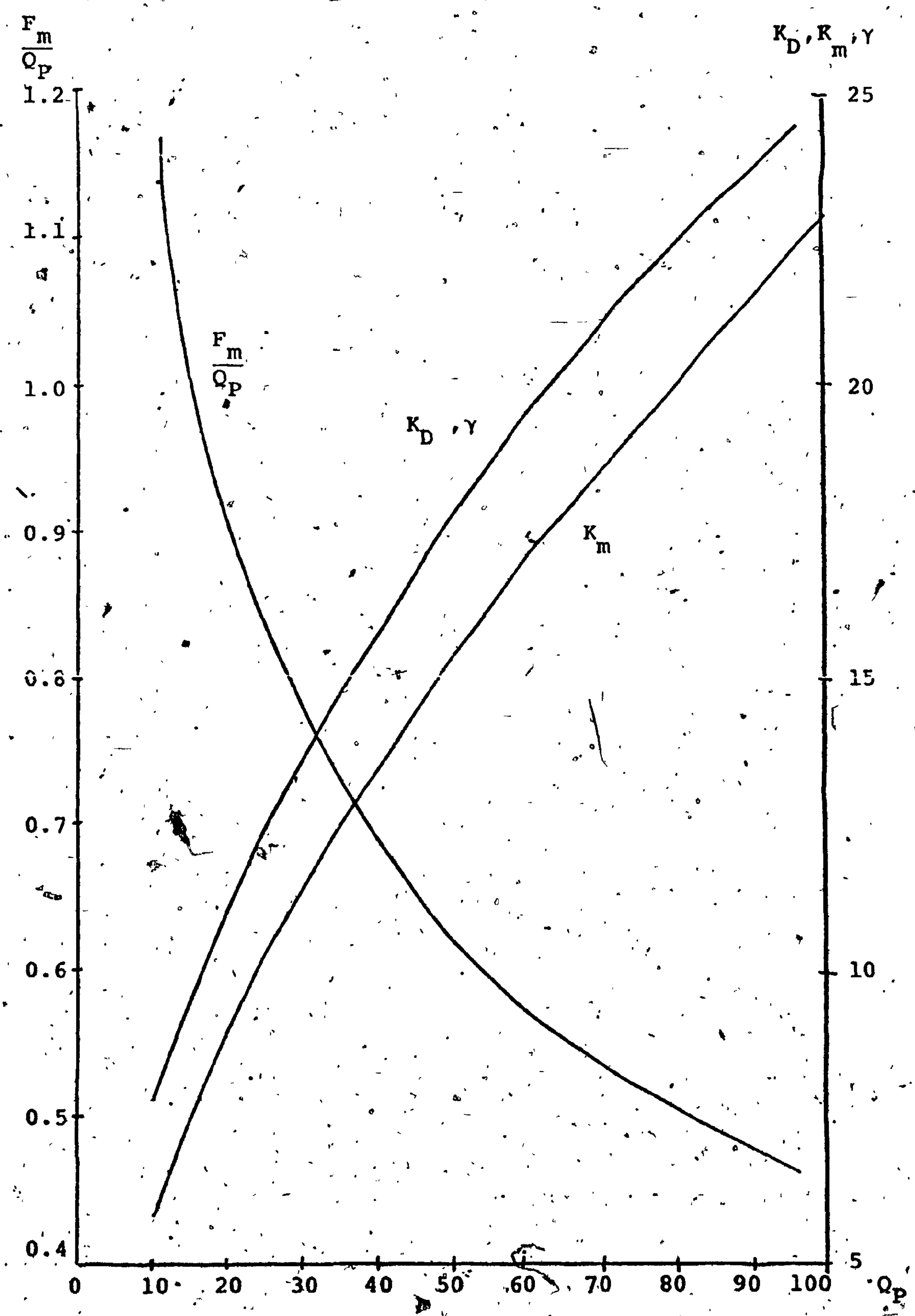


FIGURE 4.5.5 F_m/Q_P, K_D, K_m, γ, OF THE NULL TYPE 7 QMC WHEN $x = 2 + 1/K_D$ VERSUS Q_P

set of design equations appears to be a good compromise able to achieve low values of F_m and γ

$$\gamma_A = 1 \tag{4.5.16a}$$

$$\gamma_D = K_D \tag{4.5.16b}$$

Hence

$$\alpha = 2 + \frac{1}{K_D} \tag{4.5.17a}$$

and using Eqns. 4.5.11, 4.5.16 and 4.5.17a

$$F = \frac{K_0}{bQ_P}\left[(2 + \frac{1}{K_D})\left[\frac{bQ_P-1}{K_D} - 1\right]^2 + (bQ_P-1)\right] \tag{4.5.17b}$$

A set of curves giving $\frac{F_m}{Q_P}$, K_D obtained from the minimization of F as expressed in Eqn. 4.5.17b. K_m obtained using Eqn. 4.5.10 and γ versus Q_P are shown in Fig. 4.5.5.

4.5.1 Experimental results

The network shown in Fig. 4.5.1 was built up using discrete elements and tested the results are summarized below.

Designed values:

$\omega_P = 6283.185$ rad/sec. = 1000 Hz.

$Q_P = 50$

$Q_{P0} = 6.19$

$K_m = 15.376$

$K_D = 17.58$

$R_1 = 1K\Omega$

$R_2 = 0.533\ K\Omega$

$R_3 = 1.76\ K\Omega$

$C_1 = 0.199\ \mu F$

$C_2 = 0.239\ \mu F$

$C_3 = 0.08649\ \mu F$

$x = 2.0569$

$R_H = R_A = 0.533\ K\Omega$

$R_B = R_0 = 8.195\ K\Omega$

$R_{A0} = R'_{A0} = 5.668\ K\Omega$

$R_{B0} = R'_{B0} = 100\ K\Omega$

Capacitive spread 2.77:1

$$S_{K_0}^{\omega_P} = S_{K_m}^{\omega_P} = 0$$

$$|K_0 S_{K_0}^{Q_P}|/Q_P = 0.3492$$

$$|K_m S_{K_m}^{Q_P}|/Q_P = 0.2694$$

$$F_m/Q_P = 0.619$$

Capacitive spread 2.76:1

The 3 dB frequencies are:

$F_{C1} = 990.05$ Hz. $F_{C2} = 10110.05$ Hz.

Any or both of K_m and K_D can be adjusted in order to achieve the tuning of Q_P without affecting ω_P.

The circuit has been implemented using 1% tolerance resistors and 2% tolerance condensers with values chosen to be, within the range of currently available elements, as close as possible to the designed ones (Trim pots have been used wherever necessary). The OAs which have been used were internally compensated ML741. Actual values obtained

ω_P 1000.6 Hz.

F_{C1} = 989.8 Hz. F_{C2} = 1010.2 Hz.

Q_P = 49.044

The depth of the notch has been found to be -70 dB. It is found that the circuit was stable during activation. Q_P and ω_P variations from their designed values are respectively 1.9% and 0.06%. This is due to the small differences between the designed values and the used ones as well as to the tolerances of the elements. Power supply voltage used were ±10V and ±15V. Also the OAs were heated and their temperature controlled. The response was experimentally measured and plotted at 22°C (room temperature) and 70°C for both power supplies voltages. Only the OAs were heated. The passive elements were not heated in order to simulate Hybrid Integrated technology.

As shown in Fig. 4.5.8 no appreciable change in the response was observed; thus the experimental results confirm the theoretical predictions.

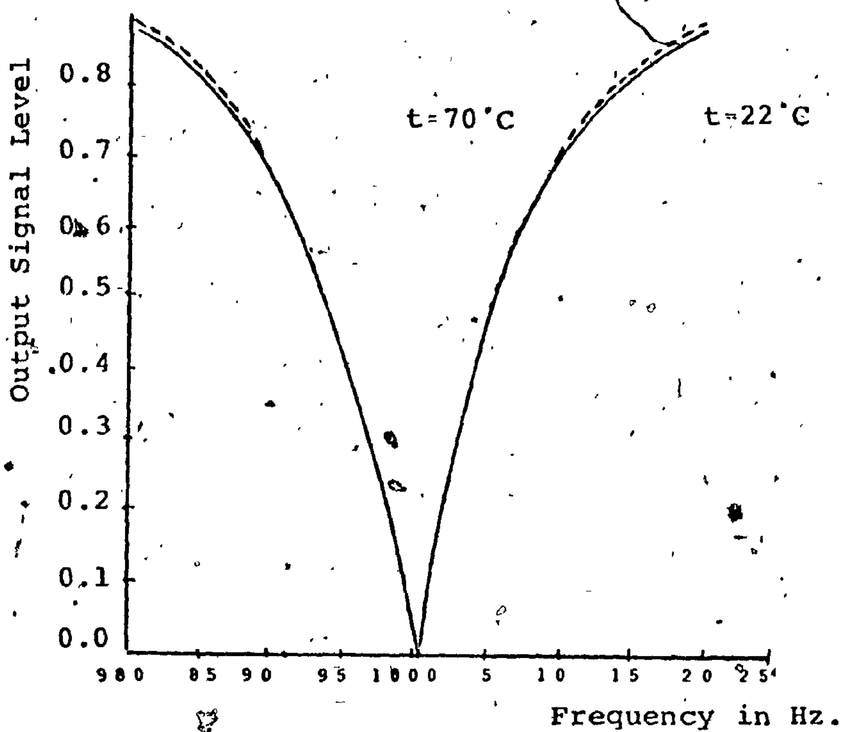


FIGURE 4.5.6a (±10V)

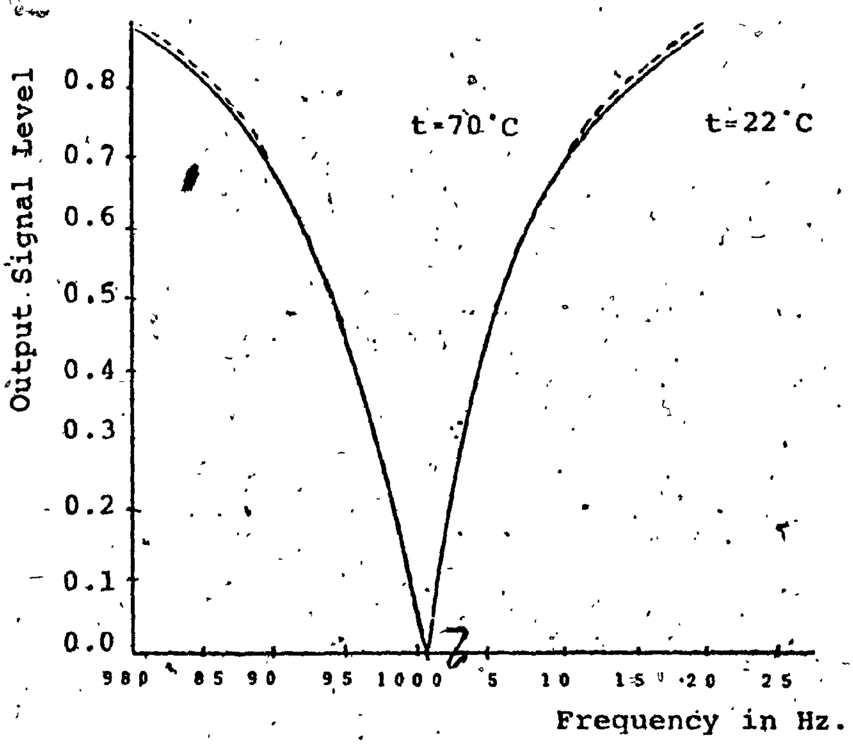


FIGURE 4.5.6b (±15V)

FIGURE 4.5.6 FREQUENCY RESPONSE OF THE NULL TYPE 7 QMC FOR $Q_P = 50$

4.5.2 Effect of the poles of the amplifiers on Q_P and ω_P [17, 33]

In order to study the effect of the pole of the amplifiers on Q_P and ω_P it is assumed that the desired elements are available and that the input impedance of the differential amplifier is high enough to avoid any appreciable loading of the twin-T. Therefore one pole of the passive circuit always cancels with one of its zeros. Replacing the amplifiers of Fig. 3.5.1a by the networks represented in Fig. 3.5.1b and 3.5.1c the analysis gives

$$D(S) =$$

$$S^2\left[K'_D + \frac{K_m K_D}{x} + \frac{1}{A_m(S)} K'_D(1+K_m \frac{y}{x}) + \frac{K'_D}{A_D(S)} + K' \frac{(1+K_m \frac{y}{x})}{A_m(S)A_D(S)}\right]$$

$$+ bS\left[1 + \frac{K'_D}{A_D(S)} + \frac{(1+K_m \frac{y}{x})}{A_m(S)} + K'_D \frac{(1+K_m \frac{y}{x})}{A_m(S)A_D(S)}\right] +$$

$$\left[K'_D + \frac{K_m K_D}{x} + \frac{1}{A_m(S)} K'_D (1+K_m \frac{y}{x}) + \frac{K'_D}{A_D(S)} + K'_D \frac{(1+K_m \frac{y}{x})}{A_m(S)A_D(S)}\right] \quad (4.5.17a)$$

$$K'_D = 1 + K_D$$

$$y = 1 + \frac{G_0}{G_H} \quad (4.5.17c)$$

Using

$$A_D(S) = \frac{B_D}{\omega_P S+\omega_D} \quad (4.5.18a)$$

$$A_m(S) = \frac{B_m}{\omega_P S+\omega_m} \qquad (4.5.18b)$$

where

B_D, B_m are the unit gain bandwidth of the OAs; B_D, B_m are the poles of the OAs; A_D, A_m are the D-C gains of the OAs we get:

$$D(S) =$$

$$S^4\left[\frac{\omega_P^2 K_D(1+K'_m \frac{y}{x})}{B_m B_D}\right] +$$

$$S^3\left[K'_D(1+K_m \frac{y}{x}) \frac{\omega_P}{B_m} + K'_D \frac{\omega_P}{B_D} + K'_D(1+K_m \frac{y}{x})(\omega_m+\omega_0)\omega_P \frac{K'_D(1+K \frac{y}{x})}{B_D B_m} b\omega_P^2\right]$$

$$+ S^2\left[K'_D + \frac{K_m K_D}{x} + K'_D(1+K_m \frac{y}{x}) \frac{\omega_m}{B_m} + K'_D \frac{\omega_D}{B_D} + K'_D(1+K_m \frac{y}{x}) \frac{\omega_D\omega_m}{B_D B_m} + \frac{K'_D}{B_D}\omega_P b + \frac{(1+K_m \frac{y}{x})}{B_m} b\omega_P + \frac{K'_D(1+K_m \frac{y}{x})(\omega_D+\omega_m)\omega_P b}{B_D B_m} + \frac{K_D(1+K_m \frac{y}{x})\omega_P^2}{B_D B_m}\right]$$

$$+ S\left[b(1+K'_D \frac{\omega}{B_D} + \frac{\omega_m}{B_m}(1+K_m \frac{y}{x}) + K'_D(1+K_m \frac{y}{x}) \frac{\omega_D\omega_m}{B_D B_m} + K'_D \frac{\omega_P}{B_D} + \frac{K'_D(1+K_m \frac{y}{x})\omega_P}{B_m} + \frac{K'_D(1+K_m \frac{y}{x})(\omega_D+\omega_m)\omega_P}{B_D B_m}\right]$$

$$+\left[K'_D + \frac{K_m K_D}{x} + \frac{K'_D\omega_D}{B_D} + \frac{K'_D(1+\frac{y}{x})\omega_m}{B_m} + K'_D(1+K_m \frac{y}{x}) \frac{\omega_D\omega_m}{B_D B_m}\right] \qquad (4.5.19)$$

If we let $B_m = B_D = B$, $\omega_m = \omega_0 = \omega_c$ and use $B = \omega_c A$ we get:

$$
\begin{aligned}
D(S) = \; & \\
& s^4\left[\frac{\omega_P^2 K'_D(1+K_m \frac{Y}{x})}{B^2}\right] + \\
& s^3\left[\frac{\omega_P K'_D(2+K_m \frac{Y}{x})}{B} + \frac{K'_D(1+K_m \frac{Y}{x})\omega_P^2 b}{B^2} + \frac{2K'_D(1+K_m \frac{Y}{x})\omega_P}{AB}\right] \\
& + s^2\left[K'_D + \frac{K_m K_D}{x} + \frac{K'_D(2+K_m \frac{Y}{x})}{A} + \frac{K'_D(1+K_m \frac{Y}{x})}{A^2} + \right. \\
& \qquad \left. b\omega_P\left[\frac{1+K'_D+K_m \frac{Y}{x}}{B} + \frac{2K'_D(1+K_m \frac{Y}{x})}{AB}\right] + \frac{K'_D(1+K_m \frac{Y}{x})\omega_P^2}{B^2}\right] \\
& + s\left[b\left[1+\frac{(1+K'_D+K_m \frac{Y}{x})}{A} + \frac{K'_D(1+K_m \frac{Y}{x})}{A^2}\right] + \frac{\omega_P K'_D(2+K_m \frac{Y}{x})}{B} \right. \\
& \qquad \left. + \frac{2K'_D(1+K_m \frac{Y}{x})\omega_P}{AB}\right] \\
& \left[K'_D + \frac{K_m K_D}{x} + \frac{K'_D(2+K_m \frac{Y}{x})}{A} + \frac{K'_D(1+K_m \frac{Y}{x})}{A^2}\right]
\end{aligned}
\tag{4.5.20}
$$

Using Eqn. 4.5.19 the values of $\frac{\Delta\omega_P}{\omega_P}$ and of $\frac{\Delta Q_P}{Q_P}$ due to the poles of the amplifiers have been obtained for various values of ω_P and are given in Fig. 4.5.7 and Fig. 4.5.8 respectively. The use of LM741 internally compensated OAs has been assumed in the computations.

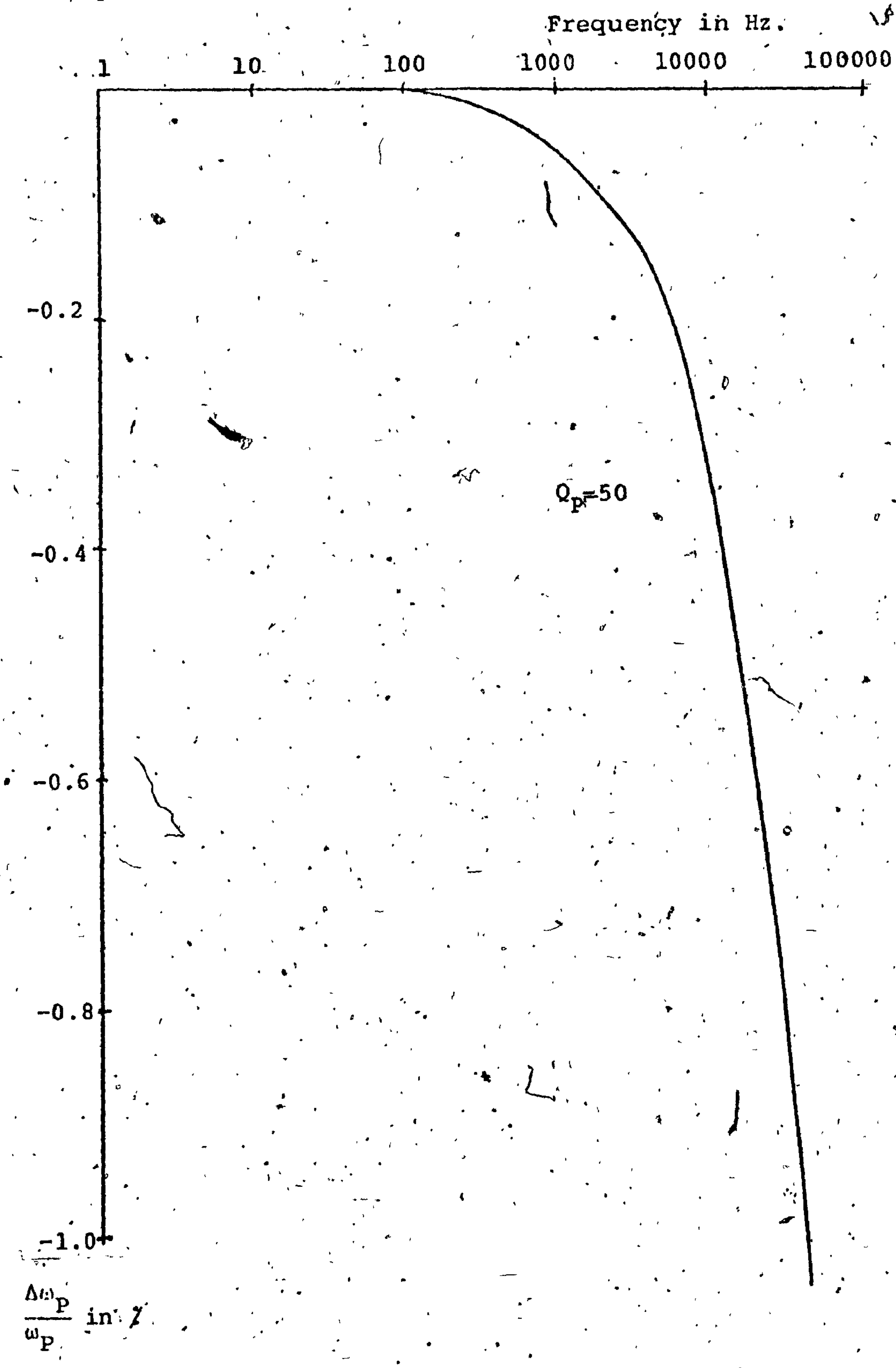


FIGURE 4.5.7 THE EFFECT OF THE OAs POLES ON ω_p FOR THE TYPE 7 QMC.

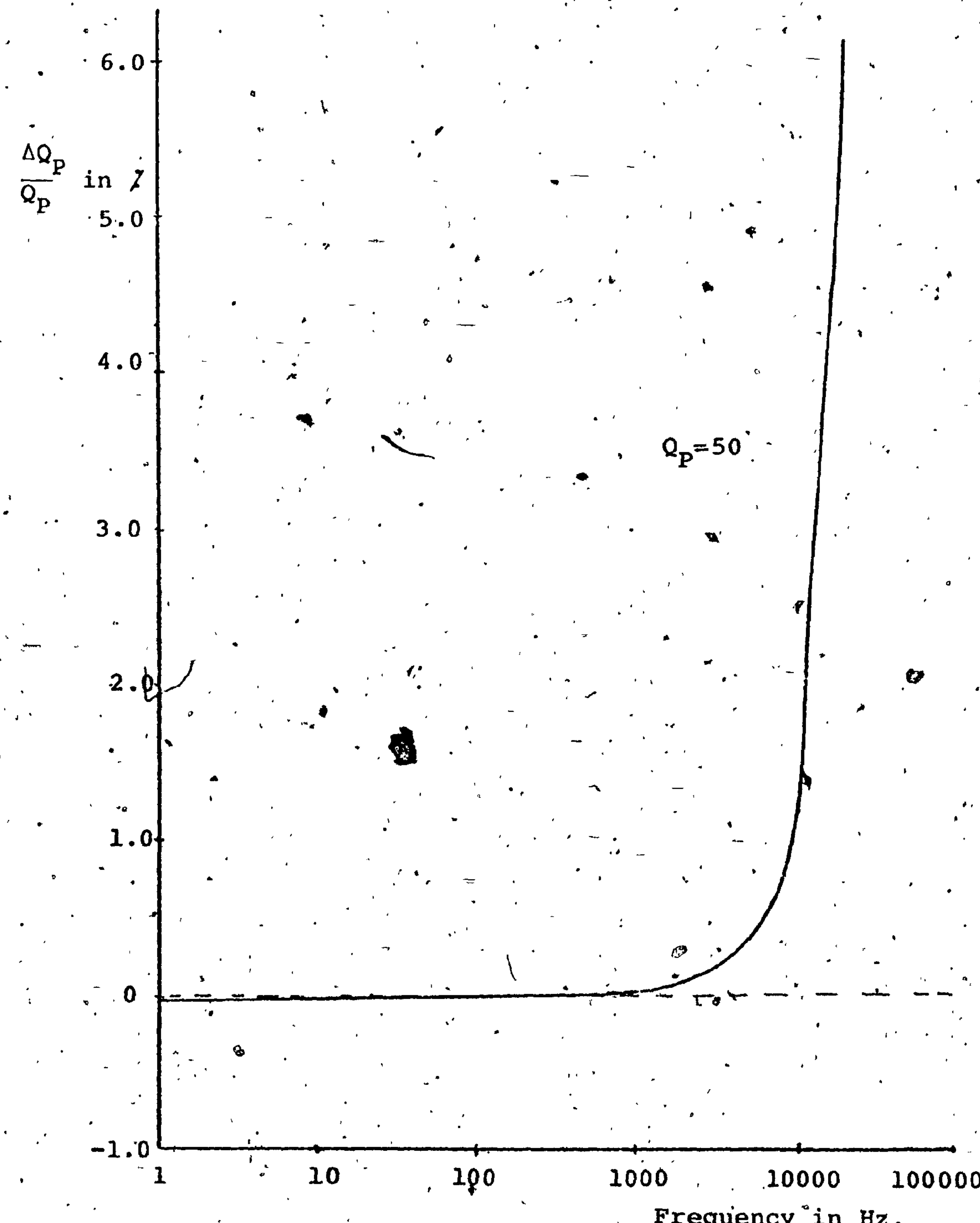


FIGURE 4.5.8 THE EFFECT OF THE OAs POLES ON Q_P FOR THE TYPE 7 QMC

4.6 Summary and Discussion

In this chapter we have considered QMCs which are obtained starting from a second order generating function t_v. The general TAC discussed in Chapter II has been adapted to suit the use of a second order t_v which requires that all the impedances be resistive. It is shown that only twelve types of generating functions satisfying Eqn. 2.6.4 are needed to obtain all the possible denominator polynomial decompositions of T_v.

It has also been shown that only Band Pass Null or All Pass filters are obtainable. Only SACs having zero GωSP and minimized GQSP have been used. The resulting QMCs retain the zero GωSP property and have been optimized to minimize F. Hence QMCs having minimized $\left|\frac{\Delta T_v}{T_v}\right|_{S=\delta\omega_p}$ are obtained.

The QMCs have been classified into three categories: RI, RII, and RIII according to the generating SAC. One representative circuit of each kind has been studied in detail. Table 4.6.1 gives a comparison between the three given realizations. It has been found that $\frac{F_m}{Q_p}$ of the order of 0.5 or lower is achievable and that $\frac{F_m}{Q_p}$ decreases as Q_p increases. Experimental results confirm the theoretical studies.

TABLE 4.6.1

COMPARISON BETWEEN THE THREE GIVEN REALIZATIONS

TABLE 4.6.1

1st Realization	2nd Realization	3rd Realization
$\omega_P = \omega_{P0}$	$\omega_P = \omega_{P0}$	$\omega_P = \omega_{P0}$
$S_{K_m}^{\omega_P} = S_{K_0}^{\omega_P} = 0$	$S_{K_m}^{\omega_P} = S_{K_0}^{\omega_P} = 0$	$S_{K_m}^{\omega_P} = S_{K_D}^{\omega_P} = 0$
$Q_P = Q_{P0}[1+K_mK_0/x]$	$Q_P = Q_{P0}[1+K_mK_0/x]$	$Q_P = Q_{P0}[1+\frac{K_mK_D}{x(K_D+1)}]$
$S_{K_m}^{Q_P} = \frac{K_mK_0/x}{1+K_mK_0/x} < 1$	$S_{K_m}^{Q_P} = \frac{K_mK_0/x}{1+K_mK_0/x} < 1$	$S_{K_m}^{Q_P} = \frac{1}{bQ_P}\left(\frac{K_mK_D}{x}\right) < 1$
$S_{K_0}^{Q_P} = \frac{K_mK_0/x}{1+K_mK_0/x} < 1$	$S_{K_0}^{Q_P} = \frac{bQ_P+K_mK_0/x}{1+K_mK_0/x}$	$S_{K_D}^{Q_P} = \frac{K_D}{bQ_P}[1+\frac{K_m}{x}] < 1$
For $x = 1$ and $\frac{C_{max}}{C_{min}} = 2$ $\frac{F}{Q_P} = 1.28 \mid Q_P=10$ $\frac{F}{Q_P} = 0.56 \mid Q_P=50$ $\frac{F}{Q_P} = 0.40 \mid Q_P=100$	For $x = 1$ and $\frac{C_{max}}{C_{min}} = 1.9$ $\frac{F}{Q_P} = 1.0 \mid Q_P=10$ $\frac{F}{Q_P} = 0.48 \mid Q_P=50$ $\frac{F}{Q_P} = 0.34 \mid Q_P=100$	For $x = 1$ and $\frac{C_{max}}{C_{min}} = 2.75$ $\frac{F}{Q_P} = 0.87 \mid Q_P=10$ $\frac{F}{Q_P} = 0.45 \mid Q_P=50$ $\frac{F}{Q_P} = 0.33 \mid Q_P=100$
OAs should be properly compensated to avoid high frequency oscillations due to excessive phase shift.		The generating circuit may need scaling of resistances and hence the resistive spread could be high.

CHAPTER V

SUMMARY AND DISCUSSIONS

CHAPTER V

SUMMARY AND DISCUSSIONS

This thesis discusses realizations of second order transfer functions by two-amplifier active RC networks. The realizations of the five commonly occuring types of filters, namely LP, HP, BP, N, and AP are studied. The filters should be implemented using hybrid IC technology. Assuming this technology, designs have been obtained which yield filters with zero $\frac{\Delta\omega_p}{\omega_p}$ (at least for first order variations) and minimized $\frac{\Delta Q_p}{Q_p}$ which is equivalent to minimized F (the sum of the magnitudes of the gain-Q-sensitivity products).

Chapter II discusses the various possible denominator polynomial decompositions having the property of zero GωSP. It is found that twenty-four such decompositions are possible. A general two amplifier configuration is proposed which will realize all the possible polynomial decompositions. The configuration is not unique and some variations of this configuration are given. The overall realization requires the use of "generating functions" which can be of three types, namely zero order, first order and second order. These generating functions have to be realized by single-amplifier RC networks having their outputs taken from the output of the amplifier. The case of the zero order is not emphasized further in this thesis, as it has been found that it is not possible to design filters in a relatively simple manner.

In Chapter III, the realizations using first order generating functions are discussed. It is shown that only low-pass, high-pass, and band-pass filters are obtainable when the admittances are of the form $Y = CS + G$. Also, these filters realize eight polynomial decompositions, starting from only two types of generating functions. They are designed to yield zero GωSP and minimized F. A value of $\frac{F}{Q_p}$ of the order of 6.4 (for $Q_p = 50$) has been shown to be obtainable for LP, HP, and BP corresponding to the polynomial decomposition of type 1 with a unity capacitive spread and a moderate resistive spread. For the BP corresponding to the polynomial decomposition of type 22b, $\frac{F}{Q_p}$ is also of the order of 6.4 with a capacitive spread of 5. Lower values of $\frac{F}{Q_p}$ can be achieved if the capacitive and/or resistive spreads are allowed to increase. It is quite possible that some of the circuits generated might have been discussed in the literature; nevertheless a systematic method of obtaining two amplifier circuits starting from first order generating functions is given as shown in Tables 3.2.2, 3.2.3, 3.2.4.

In Chapter IV "Q-multiplier circuits" are discussed in detail. This "Q-multiplication" requires a second order generating function t_V realized by a single amplifier circuit. It is shown that all the possible twenty-four polynomial decompositions can be obtained by the use of twelve types of generating functions. Only SACs having zero GωSP and minimized

GQSP have been used. The overall QMCs preserve the property of zero GωSP and have been optimized to minimize F.

It has been found that since the numerator of the QMCs transfer function is the same as the one of its generating SAC only BP, N and AP filters are obtainable. The QMCs are further classified according to the realization of the generating SAC. Three realizations, one of each kind, have been studied in detail and compared. The comparison shown in Table 4.6.1 gives the different features of each realization and shows that for all of them a value of $\frac{F}{Q_P}$ of the order of 0.5 (for $Q_P = 50$) is obtainable while the capacitive spread is kept below three and that $\frac{F_m}{Q_P}$ decreases as Q_P increases.

Three null QMCs and two band pass filters obtained from first order generating circuits all for $Q_P = 50$ were built and tested in the laboratory at DC supply voltages of ±10V and ±15V and with the OAs at temperatures of 22° C and 70° C. The test results confirm the theoretical predictions.

Thus this study clearly establishes that:

(i) Two-amplifier RC filter networks having zero GωSP and minimized F (sum of the magnitudes of the GQSPs) can be obtained.

(ii) All the possible twenty-four polynomial decompositions having the property of zero GωSP can be realized by one general configuration.

(iii) Starting from low-Q single amplifier circuits with zero GωSP, Q-multiplication permits us to obtain high-Q circuits with low F and preserving the property of zero GωSP.

In this thesis, zero GωSP has been the starting condition. It is desirable to conduct investigations regarding TAC realizations that will minimize a weighted sum of $\frac{\Delta\omega_P}{\omega_P}$ and $\frac{\Delta Q_P}{Q_P}$. Also the zero order and first order cases are not exhaustive. It is worthwhile to study filter realizations when the passive admittances are realized in various forms. Furthermore only open circuit voltage transfer functions have been considered and realized in this thesis, but the same procedure could be readily extended to realize current transfer functions using the property of network transposition [39].

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