Ground-state Baryons in a consistent quark model with the Coulomb plus linear potential

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A Thesis

in

The Department

of

Physics

Presented in Partial Fulfillment of the Requirements for the Degree of Master of Science at Concordia University Montréal, Québec, Canada

March 1985

ABSTRACT

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The method of using a harmonic oscillator basis to examine a specific type of potential and the quadratic approximation method are evaluated. Masses of the ground-state baryons are calculated using the above methods and the hypothesis that the potential between quarks is the sum of a Coulomb and a linear potential. Mixings caused by hyperfine interactions and the anharmonic potential are also taken into account. The results are in good agreement with experiment.

ACKNOWLEDGEMENT

Special thanks to Dr. C. S. Kalman, my supervisor, for suggesting the problem and his guidance during the period of preparation of this thesis.

Thanks also to the Concordia university Physics Department for providing financial assistance.

TABLE OF CONTENTS

INTRODUCTION

At one time the world seemed to consist of very few elementary particles. The proton, the neutron, the electron, the neutrino, and the photon. However, this aspect turned out to be deceptive. There were several indications that the situation is more involved.

The observed interaction between protons and neutrons- the nuclear force-did not appear to be as simple and fundamental as the electromagnetic forces between charges. Being repulsive at short range and attractive at longer range and being dependent on the spin and the symmetry of the quantum state of the partners, it resembles the chemical force between the atoms. The chemical force derives from electric forces between the constituents of the atoms. The analogy with the chemical force suggests the possibility that the nuclear force also is a relatively complicated manifestation of some more fundamental forces acting within the nucleon and connected with its internal structure.

The idea of the internal structure of the nucleon became more evident when Fermi and his collaborators found a short-lived excited state of the proton and the neutron, the so-called Δ particle. The nucleon appears to be excitable into different quantum states. It therefore could no longer be considered as elementary; it has to have some internal structure. Indeed there is good evidence that the nucleons and mesons are composite particles, made of a new kind of "elementary" particle: the Quark.

The idea of the quark was introduced in 1964 independently by Murray Gell-Mann and George Zweig[1] in an effort to summarize and systematize the great proliferation of nuclear particles that were being produced by accelerators on the high-energy frontier of the 1950s. Regularities were perceived in the masses of these particles as well as in the characteristics of their creation, their interaction, and their decays. Gell-Mann and Zweig showed that these regularities could be accounted for in terms of the simple motions and interactions of just three different kinds of fractionally charged spin-1/2 quarks. The discoveries of new particles led to the introduction of new quarks to describe them. Until now we have a family of six different kinds of quark which can give us a fairly clear look into the changing picture of high energy physics.

Chapter I: QUARK

1.1 The internal quantum numbers of quarks

High energy scattering experiments reveal a richness in the spectrum of the strongly interacting particles. This gives rise to a new branch of spectroscopy known as Hadron spectroscopy. The hadron spectroscopy shows a striking similarity to the spectra in atomic physics. The hadrons with half-integer spin are usually called baryons,while those with integer spin are called mesons. Baryons consist of three quarks and mesons contain one quark and one antiquark. This implies that quarks must be fermions with spin 1/2. An odd number of them give half integer spin,an even number give integral spin. To every quark there is,of course,an anti-quark.

Because three quarks make up a baryon,one can ascribe a baryon number B=1/3 to a quark;then we get B=1 for baryons and B=0 for mesons.

It is necessary to ascribe to quark isospin and all addititive quantum numbers such as strangeness s,charm c,beauty b,t quantum number and any similar new quantum numbers of hadrons which may still be discovered. Until now we have six types (or flavors) of quarks:

a) Two types of ordinary quarks called up and down symbolized by letters u and d.

b) Strange quarks referred to with letter s.

c) Charm quarks referred to with letter c.

d) Bottom (or beauty) and top quarks called b-quarks and t-quarks. Quarks and their quamtum numbers are described in table 1.

In order to obtain the isospin of hadrons, one considers the two types u,d of the ordinary quarks as part of an isospin doublet (I=1/2);u refers to the spin up with I_3 =+1/2,d to the spin down with I_3=-1/2. The other flavors are assumed to be isospin singlets,I=0. This means that all isospin properties must come from the ordinary quark types u,d contained in hadrons.s-quark,c-quark,b-quark,t-quark are the carriers of s,c,b,t quantum numbers respectively.

The generalized Gell-Mann-Nishijima relation relates the electric charge to these quantum numbers:

$$Q/e = I_3 + (B+s+c+b+t)/2 \qquad (1.1)$$

One can define the hypercharge by:

$$Y = s+B-(c-b+t)/3 \qquad (1.2)$$

Insert in (1.1) we arrive at

$$Q/e = I_3+Y/2+2c/3+b/3+2t/3 \qquad (1.3)$$

For up and down quarks $B=1/3, I_3=+1/2, -1/2, c=s=b=t=0$, then from (1.1) we get

Table 1
Quantum numbers of the six quarks and the six antiquarks

	J^P	B	Q/e	I	I_3	Y	s	c	t	b
u	$1/2^+$	1/3	2/3	1/2	1/2	1/3	0	0	0	0
d	$1/2^+$	1/3	-1/3	1/2	-1/2	1/3	0	0	0	0
s	$1/2^+$	1/3	-1/3	0	0	-2/3	-1	0	0	0
c	$1/2^+$	1/3	2/3	0	0	0	0	1	0	0
t	$1/2^+$	1/3	2/3	0	0	0	0	0	1	0
b	$1/2^+$	1/3	-1/3	0	0	0	0	0	0	-1
$\bar{u}$	$1/2^-$	-1/3	-2/3	1/2	-1/2	-1/3	0	0	0	0
$\bar{d}$	$1/2^-$	-1/3	1/3	1/2	1/2	-1/3	0	0	0	0
$\bar{s}$	$1/2^-$	-1/3	1/3	0	0	2/3	1	0	0	0
$\bar{c}$	$1/2^-$	-1/3	-2/3	0	0	0	0	-1	0	0
$\bar{t}$	$1/2^-$	-1/3	-2/3	0	0	0	0	0	-1	0
$\bar{b}$	$1/2^-$	-1/3	1/3	0	0	0	0	0	0	1

$$Q/e = \pm 1/2 + 1/6 = \begin{cases} 2/3 & \text{for u-quark} \\ -1/3 & \text{for d-quark} \end{cases}$$

The charges of s-, c-, b-, t-quarks are also determined by (1.1) with $I_3 = 0, B = 1/3$ and $c = 1, s = -1, b = -1, t = 1$ respectively. This way we arrive at the quantum numbers given in table 1.

It is surprising to encounter fractional charges. But any combination of three quarks or of quarks and antiquarks with those fractional charges yield hadrons having values of electric charge.

1.2 Color

The low-lying baryon states are symmetric with respect to the interchange of quark flavor and spin indices. Consider, for example, the Δ^{++} and Ω^- particles. For $J_3 = 3/2$ state, it is easily seen that the spin wave function is symmetric, and the flavors of three quarks are the same. Thus the combined spin-flavor wave function

$$| \Delta^{++}, J_3 = 3/2 > = | u\uparrow u\uparrow u\uparrow >$$

and

$$| \Omega^-, J_3 = 3/2 > = | s\uparrow s\uparrow s\uparrow >$$

are obviously symmetric. Furthermore, Δ^{++} and Ω^{-} are the lowest mass states and consequently correspond to the three-quark ground state which has total angular momentum L=0 and symmetric. This means the total spin,flavor and space wave function is also symmetric with respect to the interchange of the quarks. But if quarks are spin-1/2 objects they must obey Fermi-Dirac statistics and their total wave function must be antisymmetric. One way out of this dilemma is the introduction of new quantum numbers.

Today,the hidden quantum numbers which distinguish the three quark multiplets are usually called the three colors:red,blue and green. Each quark can take on any value of these three. The baryons wave function are required to be antisymmetric in color and then the Pauli principle is saved.

The three colors of quarks form the color group SU(3). For the system of three quarks,one can construct a singlet,two octets and a decuplet.

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

Out of quark and anti-quark we have

$$3 \otimes \bar{3} = 1 \oplus 8$$

But until now,one has observed only singlet states of color. Baryons with three quarks with different colors and mesons with color-anticolor pairs. This leads to the confinement postulate which states:All hadrons and all physical observables (current,energy-momentum tensor etc.) are color singlets. This is also an explanation why free quarks have never been observed experimentally.

Another strong support for the existence of color quantum numbers is the measurement of R-the ratio of the cross section for an electron-positron pair to annihilate to hadrons,summed over all configurations,to the cross section to annihilate to a pair of muons. The former process is described by the diagram shown in figure 1.1. The annihilation of the electron-positron system yields a virtual photon,which in turn creates a quark-antiquark pair. This process is quite analogous to electron -

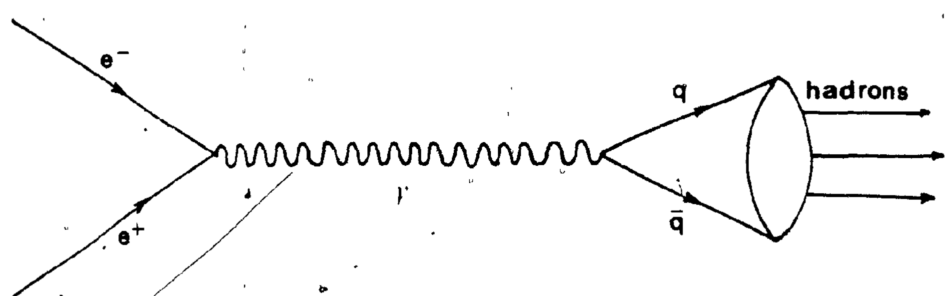

Figure 1.1 Electron-positron annihilate to hadrons

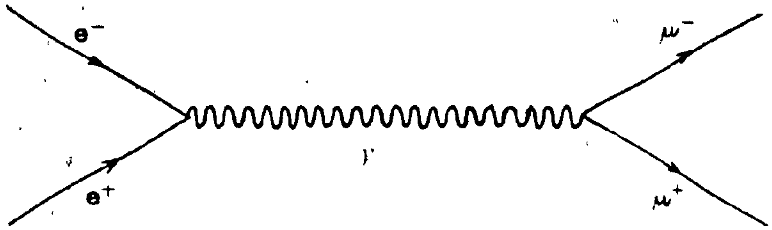

Figure 1.2 Electron-positron annihilate into a muon pair

positron annihilation into a muon pair.(Figure 1.2). Muons like electrons are pointlike members of the other family of Fermions known as Leptons,that is,particles that do not experience the strong nuclear forces at all. The muons are charged and,of course,interact through the well-tested and well-understood electromagnetic forces. If the quarks are pointlike their contribution should exhibit the same energy dependence as found in pair production of pointlike muons and the ratio of cross sections should measure the sum of the squares of the quark charges as well as approximately energy independent. So we have

$$R = \frac{\sigma(e^+ e^- \longrightarrow \text{hadrons})}{\sigma(e^+ e^- \longrightarrow \mu^+\mu^-)} = \sum_{q's} Q^2/e^2 \qquad (1.4)$$

Figure 1.3 shows the measured ratio, in which, there are three regions where R is approximately constant. Below the charm threshhold, i.e below E ~ 3.5 Gev only the first three quark flavors u, d and s appear. Thus we have

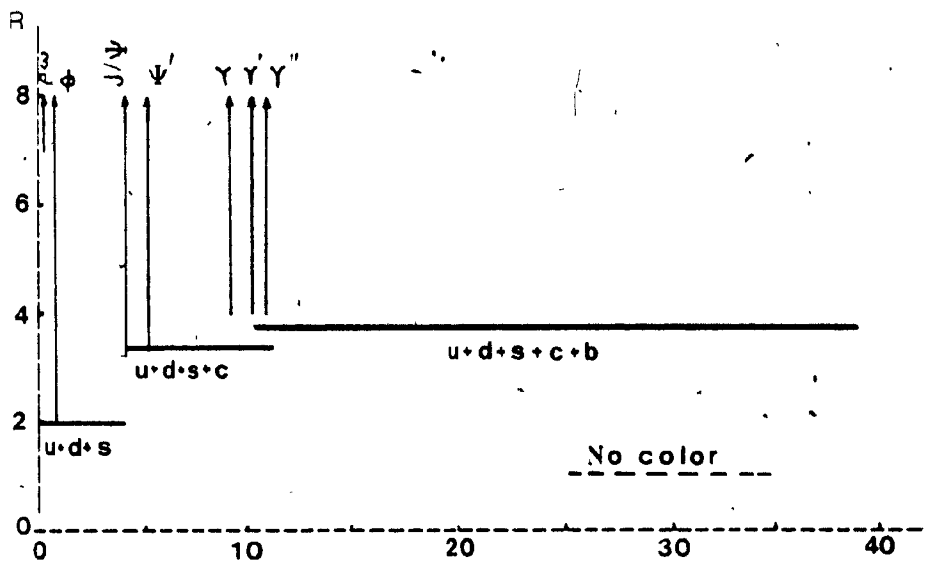

Figure 1.3_ Experimental measurement of R.

$$R = n[(Q_u/e)^2 + (Q_d/e)^2 + (Q_s/e)^2]$$

$$= n[(2/3)^2 + (-1/3)^2 + (-1/3)^2] = 2n/3 \qquad (1.5)$$

Where n is the number of colors. The experimental value R ≃ 2- 2.5 for energy between 2 and 3.8 Gev is in good agreement with eq.(1.5) for three colors (n=3).

Otherwise, without colors, there would be a discrepancy of a factor of three. This result thus represents a triumph for the hypothesis of color triplets of quarks.

Another place where colors can be seen indirectly is any transition of a single meson to a nonhadronic state. (Figure 1.4). One example is the two-photon decay of neutral mesons. In general, the rate of decay depends on the relative momentum dependence of the qq wave function. For the pion we have:

$$\Gamma(\pi^0 \longrightarrow 2\gamma) = \frac{m^3}{32F_\pi^2}\left(\frac{\alpha}{\pi}\right)^2\left(\frac{n}{3}\right)^2 = 7.87(n/3)^2 \text{ eV} \qquad (1.6)$$

Where F = 0.96 m is the pion decay constant and n is the number of colors. For n=3, (1.6) is in excellent agreement with experimental value.

$$\Gamma(\pi^0 \longrightarrow 2\gamma) = 7.95 + .55 \text{ eV} \qquad (1.7)$$

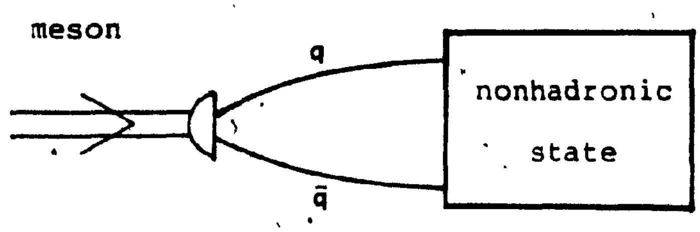

Fig 1.4 Quark diagram for the transition of a meson to a nonhadronic state

CHAPTER II

Forces between quarks or quark-antiquark

2.1 Introduction

The photons,which are vector quanta of QED,are themselves electrically neutral. This is characteristic of an abelian gauge theory. A non-abelian gauge theory represents the generalization of QED to the theory of strong interactions in which a triplet of colored quarks interact through an octet of colored gluons. Gluons which are vector quanta of this theory known as quantum chromodynamics,or QCD,themselves carry the charge. The color quantum number plays the same role in QCD as does the electric charge in QED. The Lagrangian density for QCD[2] is:

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \bar{q}_{a\alpha} (i\partial_\mu \gamma^\mu \delta_{\alpha\beta} + \frac{g}{2} \lambda^i_{\alpha\beta} A^i_\mu \gamma^\mu - m_a \delta_{ab}\delta_{\alpha\beta}) q_{b\beta} \qquad (2.1)$$

Where

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g f^{ijk} A_{\mu j} A_{\nu k}$$

$q_{a\alpha}$ are the quark fields with color indices $\alpha = \{1,2,3\}$ and flavor a

A^i_μ are an octet of vector mesons (gluons)

m_a is the mass of quark a

g is the bare coupling

$\lambda^i/2$ are the Gell-Mann generators of SU(3) which satisfy the following commutation relations

$$[\lambda^i/2, \lambda^j/2] = i f^{ijk} \lambda^k/2$$

$$[\lambda^i/2, \lambda^j/2] = \delta^{ij}/3 + d^{ijk} \lambda^k/2$$

Where f^{ijk}, d^{ijk} are structure constants of SU(3).

The major difference between QED and QCD is that because of QCD's non-abelian nature gluons also couple to themselves. As first pointed out by t'Hooft[3] and later shown by Gross and Wilczek and by Politzer[4] this leads to the property of "asymptotic fredom". That is, at large momentum transfer or at short distance the coupling becomes small and quarks inside hadrons behave as weakly bound particles. On the other hand, at large distances lattice gauge theory[5] and the string model[6] lead one to expect a linear confining potential

$$V_{conf}(r) = ar$$

From these properties, one can assume that the potential between quarks or quark and antiquark is a combination of a Coulomb and a linear potential. This chapter is devoted to calculate the energy spectra for a potential which is a combination of a Coulomb and a linear potential by using harmonic oscillator wave functions. For simplicity,we solve the one-particle problem. Which includes two-particle systems such as mesons,where the mass m will play the role of a reduced mass.

2.2 General problem

The method employs the Hamiltonian of the form

$$H = m + P^2/2m + V, \tag{2.2}$$

where V is sum of coulomb and linear

$$V(r) = -a/r + br \tag{2.3}$$

where a,b are constants. Following Gromes and Stamatescu[7] we rewrite the potential

$$V(r) = (1/2)Kr^2 + [V(r) - (1/2)Kr^2]$$

$$\equiv V_0(r) + U(r) \qquad (2.4)$$

Then the Hamiltonian becomes

$$H = m + p^2/2m + 1/2Kr^2 + U(r)$$

The contribution of the harmonic-oscillator potential to the energy of each state is given by

$$E_0 = (n + 3/2)w$$

Where $w^2 = K/m$

The anharmonic part of the potential U(r) is treated as perturbation of the harmonic-oscillator potential. Finally the energy of each state is given by

$$E = m + (n + 3/2)w + \langle U(r) \rangle \qquad (2.5)$$

The harmonic-oscillator wave functions for n=0 to n=6 are given as follows:

$$\Psi_{000} = \frac{\beta^{3/2}}{\pi^{3/4}} \exp(-1/2\,\beta^2 r^2) \qquad (2.6)$$

$$\Psi_{11m} = (8/3)^{1/2} \frac{\beta^{5/2}}{\pi^{1/4}}\, r \exp(-1/2\,\beta^2 r^2) Y_{1m}(\theta,\phi) \qquad (2.7)$$

$$\Psi_{200} = (2/3)^{1/2} \frac{\beta^{3/2}}{\pi^{3/4}} (3/2 - \beta^2 r^2) \exp(-1/2\,\beta^2 r^2) \qquad (2.8)$$

$$\Psi_{22m} = (16/15)^{1/2} \frac{\beta^{7/2}}{\pi^{1/4}}\, r^2 \exp(-1/2\,\beta^2 r^2)\, Y_{2m}(\theta,\phi) \qquad (2.9)$$

$$\Psi_{31m} = (16/15)^{1/2} \frac{\beta^{5/2}}{\pi^{1/4}}\, r\, (5/2 - \beta^2 r^2) \exp(-1/2\,\beta^2 r^2)\, Y_{1m}(\theta,\phi) \qquad (2.10)$$

$$\Psi_{33m} = (32/105)^{1/2} \frac{\beta^{9/2}}{\pi^{1/4}} r^3 \exp(-1/2\,\beta^2 r^2)\, Y_{3m}(\theta,\phi) \qquad (2.11)$$

$$\Psi_{400} = (2/15)^{1/2} \frac{\beta^{3/2}}{\pi^{3/4}} (15/4 - 5\beta^2 r^2 + \beta^4 r^4) \exp(-1/2\,\beta^2 r^2) \qquad (2.12)$$

$$\Psi_{42m} = (32/105)^{1/2} \frac{\beta^{7/2}}{\pi^{1/4}} r^2 (7/2 - \beta^2 r^2) \exp(-1/2\,\beta^2 r^2)\, Y_{2m}(\theta,\phi) \qquad (2.13)$$

$$\Psi_{44m} = (64/945)^{-1/2} \frac{\beta^{11/2}}{\pi^{1/4}}\, r^4 \exp(-1/2\,\beta^2 r^2)\, Y_{4m}(\theta,\phi) \qquad (2.14)$$

$$\Psi_{51m} = (16/105)^{1/2} \frac{\beta^{5/2}}{\pi^{1/4}} (35/4 - 7\beta^2 r^2 + \beta^4 r^4) r \exp(-1/2\beta^2 r^2) Y_{1m}(\theta,\phi) \quad (2.15)$$

$$\Psi_{53m} = (64/945)^{1/2} \frac{\beta^{9/2}}{\pi^{1/4}} (9/2 - \beta^2 r^2) r^3 \exp(-1/2\beta^2 r^2) Y_{3m}(\theta,\phi) \quad (2.16)$$

$$\Psi_{55m} = (128/10395)^{1/2} \frac{\beta^{13/2}}{\pi^{1/4}} r^5 \exp(-1/2\beta^2 r^2) Y_{5m}(\theta,\phi) \quad (2.17)$$

$$\Psi_{600} = (35/16)^{1/2} \frac{\beta^{3/2}}{\pi^{3/4}} (1 - 2\beta^2 r^2 + 12/15\,\beta^4 r^4 - 8/105\,\beta^6 r^6) \exp(-1/2\beta^2 r^2) \quad (2.18)$$

$$\Psi_{62m} = (42/5)^{1/2} \frac{\beta^{7/2}}{\pi^{1/4}} (1 - 4/7\,\beta^2 r^2 + 4/63\,\beta^4 r^4)\, r^2 \exp(-1/2\beta^2 r^2) Y_{2m}(\theta,\phi) \quad (2.19)$$

$$\Psi_{64m} = (128/10395)^{1/2} \frac{\beta^{11/2}}{\pi^{1/4}} (11/2 - \beta^2 r^2)\, r^4 \exp(-1/2\beta^2 r^2) Y_{4m}(\theta,\phi) \quad (2.20)$$

$$\Psi_{66m} = (256/135135)^{1/2} \frac{\beta^{13/2}}{\pi^{1/4}} r^6 \exp(-1/2\beta^2 r^2) Y_{6m}(\theta,\phi) \quad (2.21)$$

Where

$$\beta^4 = Km$$

The energies are calculated for m=0.5 and is determined by the variational method. The results shown in tables 2.1-2.4. are compared with correct values produced by an exact method.[17] The result is fairly good when the linear part becomes larger.

Table 2.1
Energy levels of U(r) = r

State	Correct value	Calculated	% difference
1S	2.8381	2.8448	0.2
1P	3.8613	3.8678	0.2
2S	4.5879	4.5753	0.3
1D	4.7482	4.7544	0.1
2P	5.3845	5.3798	0.1
1F		5.5569	
3S	6.0206	5.9979	0.4
2D	6.1297	6.1297	0.0
1G		6.3001	
3P	6.7076	6.6928	0.2
3D	7.3689	7.3602	0.1
4S	7.2867	7.2565	0.4
4P	7.9057	7.8834	0.3
4D	8.5097	8.4929	0.2

Table 2.2
Energy levels of $U(r) = -1/r + 100r$

State	Correct value	Calculated	% difference
1S	46.902	47.164	0.6
1P	70.516	70.698	0.3
2S	85.839	85.558	0.3
1D	90.215	90.372	0.2
2P	103.81	103.73	0.08
1F		107.98	
3S	117.23	117.70	0.4
2D	120.25	120.27	0.0
1G		124.20	
3P	132.59	132.28	0.2
3D	147.12	146.95	0.1
4S	144.82	144.11	0.5
4P	158.59	158.10	0.3
4D	171.83	171.47	0.2

Table 2.3
Energy levels of U(r) = -1/r +r

State	Correct value	Calculated	% difference
1S	1.8979	1.9506	2.8
1P	3.3256	3.3439	0.6
2S	3.9751	3.9674	0.2
1D	4.3506	4.3625	0.3
2P	4.9619	4.9640	0.04
1F		5.2362	
3S	5.5329	5.5050	0.5
2D	5.7930	5.7985	0.1
1G		6.0251	
3P	6.3476	6.3350	0.2
3D	7.0716	7.0663	0.1
4S	6.8701	6.8292	0.6
4P	7.5869	7.5636	0.3
4D	8.2406	8.2253	0.2

Table 2.4
Energy levels of $U(r) = -1/r + 0.01r$

State	Correct value	Calculated	% difference
1S	0.27897	0.31686	13.6
1P	0.51740	0.52381	1.2
2S	0.53472	0.53651	0.3
1D	0.60247	0.60500	0.4
2P	0.62571	0.62912	0.5
1F		0.66127	
3S	0.64191	0.63921	0.4
2D	0.68358	0.68580	0.3
1G		0.70722	
3P	0.70487	0.70595	0.2
3D	0.75135	0.75248	0.2
4S	0.72029	0.71575	0.6
4P	0.77129	0.77092	0.05
4D	0.81134	0.81151	0.0

CHAPTER III

Quadratic approximation method in a consistent quark model

3.1 Introduction

In theoretical calculations of the masses of mesons and baryons,the quadratic approximation method has been widely used in various papers. The purpose of this chapter is to evaluate this method. Three terms a(t), b(t), c(t) are introduced in (3.3). These terms are determined by using some values of low-level energies and then will be used to calculate the energies of higher levels. The calculation is expanded to five terms a(t), b(t), c(t), d(t), e(t) in (3.4). In (3.5),d(t) and e(t) are calculated from a(t), b(t), c(t) by using the Gaussian quadrature formula instead of employing more values of energies as in (3.4). The values calculated are compared with those from chapter II.

3.2 Meson Hamiltonian

We will restrict ourselves to the case of mesons (two-body system) with $m_1 = m_2 = m$. The model employs the hamiltonian of the form

$$H = 2m + P^2/4m + (V + H_{hyp} + H_{so})\sum_i \lambda_1^i \lambda_2^i \qquad (3.1)$$

Where m is the mass of the quark; for instance, the c quark or ψ system and the b quark for Υ system, and also

$$H_{hyp} = \frac{\alpha_s}{2m^2}\left\{\frac{8\pi}{3}(\vec{S}_1 \cdot \vec{S}_2)\delta^3(\vec{r}) + \frac{1}{r^3}\left[\frac{3(\vec{S}_1 \cdot \vec{r})(\vec{S}_2 \cdot \vec{r})}{r^2} - \vec{S}_1 \cdot \vec{S}_2\right]\right\} \quad (3.2a)$$

$$H_{so} = H_{so(1g)} + H_{so(ho)} \quad (3.2b)$$

$$H_{so(1g)} = (\alpha_s / m^3 r^3)\, \vec{L} \cdot \vec{S} \quad (3.2c)$$

$$H_{so(ho)} = -(k/m^2)\, \vec{L} \cdot \vec{S} \quad (3.2d)$$

Where $\vec{r}$, $\vec{P}$, $\vec{L}$, $\vec{S}$ are the interquark distance, momentum, angular momentum and spin respectively, $\vec{S}_i$, λ_i^{α} $(-\lambda_i^{*\alpha})$ are the spin and color vectors of quark (antiquark), α_s is the quark-gluon fine structure constant. Finally

$$V = 1/2\, Kr^2 + U(r) \quad (3.2e)$$

Where U(r) is some unknown potential which incorporates attractive potential at short range (a coulomb-type piece derived from QCD) and deviations from the harmonic-oscillator interaction at large distances.

In application to the baryons, the spin-orbit force is neglected from the beginning. This is based on the calculation by Isgur and Karl[8] which indicates "that spin-orbit forces, if present at all, are at a level much reduced over naive expectation". Isgur and Karl[8] suggest that this result is due in part to a cancellation between that part of the spin-orbit interaction arising from one-gluon exchange (3.2c) and that arising from the harmonic potential (3.2d). This suggestion is considered in detail by Schnitzer[10]. He suggests also that the spin-orbit term is negligible for baryons, weakly attractive for ordinary mesons, and strongly attractive for charmonium.

3.3 Quadratic approximation calculation of energy up to D-state

The contribution of harmonic-oscillator potential to the energy of the state is given by

$$E_0 = (n + 3/2)w \tag{3.3}$$

where

$$w^2 = 4k/m \tag{3.4}$$

Calculation of the anharmonic part of the potential has been discussed by Kalman, Hall and Misra[9]. Following this work we set

$$a(t) = \frac{\beta^3 t^{3/2}}{\pi^{3/2}} \int d^3r \; U(r) \exp(-t\beta^2 r^2) \qquad (3.5a)$$

$$b(t) = \frac{\beta^5 t^{5/2}}{\pi^{3/2}} \int d^3r \; U(r) r^2 \exp(-t\beta^2 r^2) \qquad (3.5b)$$

$$c(t) = \frac{\beta^7 t^{7/2}}{\pi^{3/2}} \int d^3r \; U(r) r^4 \exp(-t\beta^2 r^2) \qquad (3.5c)$$

and we construct the quadratic approximation

$$a(t) = A + Bt + Ct^2 \qquad (3.6a)$$

Then from (3.5a),(3.5b),(3.5c) we have

$$b(t) = (3A + Bt - Ct^2)/2 \qquad (3.6b)$$

$$c(t) = (15A + 3Bt - Ct^2)/4 \qquad (3.6c)$$

The value of A, B, C are obtained from a, b and c by setting t=1 in eqs(3.6). Then from (2.6),(2.7),(2.8),(2.9) we get the total energy excluding mixing,hyperfine and spin-orbit effects as

$$E\,(S) = 2m + 3/2w + a(t) \tag{3.7}$$

$$E\,(P) = 2m + 5/2w + 2/3b(t) \tag{3.8}$$

$$E\,(S') = 2m + 7/2w + 3/2a(t) - 2b(t) + 2/3c(t) \tag{3.9}$$

$$E\,(D) = 2m + 7/2w + 4/15c(t) \tag{3.10}$$

In addition to mixing caused by the hyperfine interaction, the anharmonic potential U itself has an off diagonal contribution

$$U = \langle S|U|S'\rangle = \langle S'|U|S\rangle$$

$$=(3/2)^{1/2}\,a(t) - (2/3)^{1/2}\,b(t) \tag{3.11}$$

The calculation is carried by using values of E (S), E (P) and E (S') of two systems. From these six values, a system of six equations is constructed to find m ,t,w ,A,B,C. We set t=1 for one system and m' is determined by

$$t = (m' / m)^{1/2} \qquad (3.12)$$

The results are then used to calculate E (D) and E'(D) for two systems. The result shown in tables 3.1 to 3.4

3.4 Quartic approximation calculation of energy up to G-state

Apart from (3.5a),(3.5b) and (3.5c) we set

$$d(t) = \frac{\beta^9 t^{9/2}}{\pi^{3/2}} \int d^3r\ U(r) r^6 \exp(-t\beta^2 r^2) \qquad (3.13a)$$

$$e(t) = \frac{\beta^{11} t^{11/2}}{\pi^{3/2}} \int d^3r\ U(r) r^8 \exp(-t\beta^2 r^2) \qquad (3.13b)$$

and constructing quartic approximation about t=1

$$a(t) = A + Bt + Ct^2 + Dt^3 + Et^4 \tag{3.14}$$

From (3.5a),(3.5b),(3.5c),(3.13a),(3.13b) we get

$$b(t) = (3A + Bt - Ct^2 - 3Dt^3 - 5Et^4)/2 \tag{3.15}$$

$$c(t) = (15A + 3Bt - Ct^2 + 3Dt^3 + 15Et^4)/4 \tag{3.16}$$

$$d(t) = (105A + 15Bt - 3Ct^2 + 3Dt^3 - 15Et^4)/8 \tag{3.17}$$

$$e(t) = (945A + 105Bt - 15Ct^2 + 9Dt^3 - 15Et^4)/16 \tag{3.18}$$

Besides (3.7) to (3.10), the total energy excluding mixing,hyperfine and spin-orbit effect is then

$$E(P') = 2m + 9/2w + 5/3b(t) - 4/3c(t) + 4/15d(t) \tag{3.19}$$

$$E\,(F) = 2m + 9/2w + 8/105d(t) \qquad (3.20)$$

$$E\,(S'') = 2m + 11/2w + 15/8a(t) - 5b(t) + 13/3c(t)$$
$$-4/3d(t) + 2/15e(t) \qquad (3.21)$$

$$E\,(D') = 2m + 11/2w + 14/15c(t) - 8/15d(t) + 8/105e(t) \qquad (3.22)$$

$$E\,(G) = 2m + 11/2w + 16/945e(t) \qquad (3.23)$$

And the mixings by the anharmonic potential U itself are given by:

$$\langle S|U|S''\rangle = \langle S''|U|S\rangle$$
$$= (15/8)^{1/2}\,a(t) - (10/3)^{1/2}\,b(t) + (2/15)^{1/2}\,c(t) \qquad (3.24)$$

$$\langle S'|U|S''\rangle = \langle S''|U|S'\rangle$$

$$= (45/16)^{1/2}\, a(t) - (45/4)^{1/2}\, b(t)$$

$$+ (169/45)^{1/2}\, c(t) - (4/45)^{1/2}\, d(t) \qquad (3.25)$$

$$\langle P|U|P'\rangle = \langle P'|U|P\rangle$$

$$= (10/9)^{1/2}\, b(t) - (8/45)^{1/2}\, c(t) \qquad (3.26)$$

$$\langle D|U|D'\rangle = \langle D'|U|D\rangle$$

$$= (112/450)^{1/2}\, c(t) - (32/1575)^{1/2}\, d(t)$$

$$(3.27)$$

The calculation is then carried out by using E(S),E(P), E(S') and E(S'') of two systems. The eight quantities to be determined are m,t,w,A,B,C,D,E. The energy states E(P'),E(D),E(D') are then calculated from these values. The results are shown in tables 3.5 to 3.8;the values used for the calculation are underlined.

3.5 Gaussian quadrature formula

There is one way to obtain d(t), e(t) directly from a(t), b(t), c(t). From

$$a_n(t) = \frac{(t^{1/2}\beta)^{2n+1}}{\Pi^{3/2}} \int d^3r\, U(r)\, \exp(-t\beta^2 r^2)\, r^{2n-2} \qquad (3.28)$$

$a_n(t)\,(n=1,2,3,4,5) \equiv a(t), b(t), c(t), d(t), e(t)$

The equations can be rewritten as follow:

$$\int dx\ x^2 U(\rho x)\ \exp(-x^2) = \sqrt{\pi}\ a(t)/4 \qquad (3.29)$$

$$\int dx\ x^4 U(\rho x)\ \exp(-x^2) = \sqrt{\pi}\ b(t)/4 \qquad (3.30)$$

$$\int dx\ x^6 U(\rho X)\ \exp(-x^2) = \sqrt{\pi}\ c(t)/4 \qquad (3.31)$$

$$\int dx\ x^8 U(\rho x)\ \exp(-x^2) = \sqrt{\pi}\ d(t)/4 \qquad (3.32)$$

$$\int dx\ x^{10} U(\rho x)\ \exp(-x^2) = \sqrt{\pi}\ e(t)/4 \qquad (3.33)$$

where

$$x = \beta t^{1/2} r \tag{3.34}$$

$$\rho = (\beta t^{1/2})^{-1} \tag{3.35}$$

From numerical analysis[12] we have

$$\int_0^\infty dx\ y(x)\ \exp(-x^2) \simeq \sum_{i=1}^{3} A_i\ y(x_i) \tag{3.36}$$

with $y(x)$ is even function and A_i, x_i have known numerical values.

writing

$$U_i = U(x_i \rho) \quad i=1,2,3 \tag{3.37}$$

and

$$\alpha_1(t) = \sqrt{\pi}\ a(t)/4,\ \alpha_2(t) = \sqrt{\pi}\ b(t)/4\ \ldots \tag{3.38}$$

Equations (3.29)-(3.33) have the form

$$\sum_i A_i X_i^{2n} U_i \simeq \alpha_n(t) \quad n=1,2,3,4,5 \qquad (3.39)$$

when knowing a(t),b(t),c(t) or $\alpha_n(t)$ n=1,2,3 the first three equations can be used to determine U . Then d(t),e(t) can be obtained from these values.

The calculation is carried out following this method and the results are displayed in tables 3.9-3.12 in the same pattern as was used in the previous part. The underlined are values used for the calculation

3.6 Conclusion

The result when the $a_n(t)$ are calculated directly from the values of the energy states is fairly good. Again,it becomes much better when the potential is dominated by the linear part. The result when d(t) and e(t) are calculated from a(t),b(t),c(t) is not as good as the direct calculation. The reason may lie at the difference between the approximation property of $a_n(t)$ and the perfect solution for a system of equations. Nevertheless,the method still can give us some predictions which can serve as guidance for later calculations.

Table 3.1

Calculation for $c\bar{c}$ and $b\bar{b}$ systems[11] which incorporate the potential of the form $U(r) = -K/r + r/a^2$ where $K = 0.52$, $a = 2.34\ Gev^{-1}$ and $m_c=1.84$ GeV, $m_b=5.17$ GeV

State	Energy(Gev)	Calculated(Gev)	% difference
m=1.84			
1S	3.67		
1P	4.12		
2S	4.24		
1D	4.38	4.43	1.1
m=5.17			
1S	9.74		
1P	10.43		
2S	10.49		
1D	10.69	10.87	1.7

Table 3.2
Calculation for two systems of masses m=1 and m=0.5
with the potential U(r) = -1/r + 100r

State	Energy	Calculated	% difference
m=1			
1S	36.90		
1P	56.44		
2S	68.44		
1D	72.36	72.85	0.7
m=0.5			
1S	47.40		
1P	71.40		
2S	86.34		
1D	90.72	91.31	0.7

Table 3.3
Calculation for two systems of masses m=1 and m=0.5
with the potential U(r) = -1/r + r

State	Energy	Calculated	% difference
m=1			
1S	2.58		
1P	3.97		
2S	4.45		
1D	4.86	4.91	1.0
m=0.5			
1S	2.40		
1P	3.83		
2S	4.48		
1D	4.85	4.94	1.9

Table 3.4
Calculation for two systems of masses m=1 and m=0.5
with the potential $U(r) = -1/r + 0.01r$

state	Energy	Calculated	% difference
m=1			
1S	1.51		
1P	1.92		
2S	1.93		
1D	2.03	2.16	6.4
m=0.5			
1S	0.78		
1P	1.02		
2S	1.03		
1D	1.10	1.17	6.4

Table 3.5

Calculation for $c\bar{c}$ and $b\bar{b}$ systems[11] which incorporate the potential of the form $U(r) = -k/r + r/a$ where $k=0.52$, $a=2.34$ Gev^{-1} and $m_c=1.84$ Gev, $m_b=5.17$ Gev

State	Energy(Gev)	Calculated(Gev)	% difference
m=1.84			
1S	3.67		
1P	4.12		
2S	4.24		
1D	4.38	4.65	6.2
2P	4.50	4.56	1.3
3S	4.60		
2D	4.70	4.84	3.0
m=5.17			
1S	9.74		
1P	10.43		
2S	10.49		
1D	10.69	10.70	0.1
2P	10.76	10.67	0.8
3S	10.82		
2D	10.94	10.90	0.4

Table 3.6
Calculation for two systems of masses m=1 and m=0.5
with the potential U(r) = -1/r + 100r

State	Energy	Calculated	% difference
m=1			
1S	<u>36.90</u>		
1P	<u>56.44</u>		
2S	<u>68.44</u>		
1D	72.36	72.96	0.8
2P	83.10	82.96	0.2
3S	<u>93.61</u>		
2D	96.32	98.19	1.9
m=0.5			
1S	<u>47.40</u>		
1P	<u>71.02</u>		
2S	<u>86.34</u>		
1D	90.74	91.31	0.6
2P	104.31	102.07	2.1
3S	<u>118.23</u>		
2D	120.75	118.35	2.0

Table 3.7
Calculation for two systems of masses m=1 and m=0.5
with the potential U(r) = -1/r + r

State	Energy	Calculated	% difference
m=1			
1S	2.58		
1P	3.97		
2S	4.45		
1D	4.86	5.10	4.9
2P	5.34	5.26	1.7
3S	5.76		
2D	6.04	6.07	0.5
m=0.5			
1S	2.40		
1P	3.83		
2S	4.48		
1D	4.85	4.91	1.2
2P	5.46	5.39	1.3
3S	6.03		
2D	6.29	6.28	0.2

Table 3.8
Calculation for two systems of masses m=1 and m=0.5
with the potential U(r) = -1/r + 0.01r

State	Energy	Calculated	% difference
m=1			
1S	1.51		
1P	1.92		
2S	1.93		
1D	2.03	2.06	1.5
2P	2.04	2.00	2.0
3S	2.05		
2D	2.10	2.09	0.5
m=0.5			
1S	0.78		
1P	1.02		
2S	1.03		
1D	1.10	1.33	20.9
2P	1.13	1.18	4.4
3S	1.14		
2D	1.18	1.31	11

Table 3.9

Calculation for $c\bar{c}$ and $b\bar{b}$ systems[11] which incorporate the potential of the form $U(r) = -k/r + r/a$ where $k=0.52, a=2.34$ Gev^{-1} and m =1.84 Gev, m =5.17 Gev

State	Energy(Gev)	Calculated(Gev)	% difference
m=1.84			
1S	3.67		
1P	4.12		
2S	4.24		
1D	4.38	4.43	1.1
2P	4.50	4.69	4.2
3S	4.60	4.80	4.3
2D	4.70	4.99	6.2
m=5.17			
1S	9.74		
1P	10.43		
2S	10.49		
1D	10.69	10.87	1.7
2P	10.76	11.18	3.9
3S	10.82	10.95	1.2
2D	10.94	11.46	4.8

Table 3.10
Calculation for two systems of masses m=1 and m=0.5
with the potential U(r) = -1/r + 100r

State	Energy	Calculated	% difference
m=1			
1S	36.90		
1P	56.44		
2S	68.44		
1D	72.36	72.85	0.7
2P	83.10	87.97	5.9
3S	93.61	99.30	6.1
2D	96.32	104.0	8.0
m=0.5			
1S	47.40		
1P	71.02		
2S	86.34		
1D	90.74	90.31	0.5
2P	104.3	109.9	5.4
3S	118.2	113.4	4.1
2D	120.7	123.5	2.3

Table 3.11
Calculation for two systems of masses m=1 and m=0.5 with the potential U(r) = -1/r + r

State	Energy	Calculated	% difference
m=1			
1S	<u>2.58</u>		
1P	<u>3.97</u>		
2S	<u>4.45</u>		
1D	4.86	4.91	1.0
2P	5.34	5.85	9.5
3S	5.76	6.49	13
2D	6.04	6.86	14
m=0.5			
1S	<u>2.40</u>		
1P	<u>3.83</u>		
2S	<u>4.48</u>		
1D	4.85	4.94	1.9
2P	5.46	5.90	8.0
3S	6.03	6.35	5.3
2D	6.29	6.91	9.9

Table 3.12
Calculation for two systems of masses m=1 and m=0.5
with the potential U(r) = -1/r + 0.01r

State	Energy	Calculated	% difference
m=1			
1S	1.51		
1P	1.92		
2S	1.93		
1D	2.03	2.17	6.9
2P	2.04	2.33	14
3S	2.05	2.09	2.0
2D	2.10	2.44	16
m=0.5			
1S	0.78		
1P	1.02		
2S	1.03		
1D	1.10	1.16	5.4
2P	1.13	1.27	12
3S	1.14	1.28	12
2D	1.18	1.41	19

CHAPTER IV

Masses of the Ground-state Baryons

4.1 Introduction

In this chapter, the masses of the ground-state baryons are calculated by using the hypothesis that the potential between quarks is a combination of a Coulomb and a linear potential. The model employs an harmonic-oscillator basis. Hyperfine interaction and the mixing caused by the anharmonic potential are also taken into account.

4.2 The Isgur-Karl model

The model employs a Hamiltonian of the form

$$H = \sum_{i=1}^{3} m_i + H_0 + H_{hyp} \qquad (4.1)$$

where m_i is the quark mass, and

$$H_{hyp} = \sum_{i<j} \frac{2\alpha_s}{3m_i m_j} \left\{ \frac{8\pi}{3}\delta^3(\vec{r}_{ij})(\vec{S}_i \cdot \vec{S}_j) + \frac{1}{r_{ij}^3}\left[\frac{3(\vec{S}_i \cdot \vec{r}_{ij})(\vec{S}_j \cdot \vec{r}_{ij})}{r_{ij}^2} - \vec{S}_i \cdot \vec{S}_j\right]\right\} \qquad (4.2)$$

where α_s is the quark-gluon fine structure constant,

$\vec{r}_{ij}$ is the separation between a pair of quarks and $\vec{S}_i$ is the spin of i^{th} quark

$$H_0 = \sum_i \frac{P_i^2}{2m_i} + \sum_{i<j} V(r_{ij}) - \frac{(\sum_i P_i)^2}{2\sum_i m_i} \tag{4.3a}$$

$$V(r_{ij}) = -\alpha_s/r_{ij} + ar_{ij} \tag{4.3b}$$

Following Gromes and Stamatescu[7] we rewrite (4.3b) as:

$$V(r_{ij}) = (1/2)Kr_{ij}^2 + [V(r_{ij}) - (1/2)Kr_{ij}^2]$$

$$= V_0(r_{ij}) + U(r_{ij}) \tag{4.4}$$

No spin-orbit terms are included for the reason mentioned in the previous chapter.

Consider the U=0 and H_{hyp} =0 limit. In terms of the Jacobi relative coordinates,

$$\vec{\rho} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2), \tag{4.5}$$

$$\vec{\lambda} = \frac{1}{\sqrt{6}} (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3), \tag{4.6}$$

the Hamiltonian becomes the sum of two independent harmonic oscillators with the same spring constant K:

$$H = P_\rho^2/2m_\rho + P_\lambda^2/2m_\lambda + 3/2K(\rho^2 + \lambda^2) \qquad (4.7)$$

The contribution of this hamiltonian to the total energy of the ground state (N=0) is then

$$E_0 = 3(w_\rho + w_\lambda)/2 \qquad (4.8)$$

where

$$w_\rho^2 = 3K/m_\rho \qquad (4.9a)$$

and

$$w_\lambda^2 = 3K/m_\lambda \qquad (4.9b)$$

m_ρ and m_λ are the reduced masses of the ρ and λ oscillators, respectively. Now in the approximation $m_u \simeq m_d$ at least two of the quark masses are equal. We will employ the convention that in all cases the quarks comprising the ρ

oscillator have the same mass. Hence

$$m_\rho = m_1 \tag{4.10a}$$

$$m_\lambda = 3m_1 m_3/(2m_1 + m_3) \tag{4.10b}$$

The particles which we are going to discuss here are elements of the following strangeness sectors

$$m_1 = m_u, s = 0,-1 \tag{4.11a}$$

$$m_1 = m_s, s = -2,-3 \tag{4.11b}$$

$$m_3 = m_u, s = 0,-2 \tag{4.12a}$$

$$m_3 = m_s, s = -1,-3 \tag{4.12b}$$

s=0 sector includes N, Δ particles; s=-1 Λ, Σ particles; s=-2 and s=-3 are corresponding to Ξ and Ω particles respectively.

Applying eqs.(4.9)-(4.12) to (4.8), we see that for the ground state in case s=0 $w_\rho = w_\lambda = w$ and $E_0 = 3w$. For the ground state of the other strangeness sectors using the notation $x = m_u/m_s$ we get

$$E_0 = \begin{cases} 3/2w\,\{1 + [(2x+1)/3]^{1/2}\}\ , s=-1 & (4.13a) \\ 3/2w\,\{x + [(2 + x/3)]^{1/2}\}\ , s=-2 & (4.13b) \\ 3wx^{1/2}\ , s=-3 & (4.13c) \end{cases}$$

For N = 1 (negative parity) baryons, the ρ and λ oscillators can be separately excited. Thus in the s=-1,-2 sectors the degeneracy is lifted. The exact energies of the first excited states are:

$$E_0 = 4w\ , \quad s=0 \qquad (4.14a)$$

$$E_0^\rho = w\{5/2 + 3/2[(2x + 1)/3]^{1/2}\}\ , \quad s=-1 \qquad (4.14b)$$

$$E_0^{\lambda} = w\{3/2 + 5/2[(2x + 1)/3]^{1/2}\} \quad , s=-1 \qquad (4.14c)$$

$$E_0^{\rho} = w\{5/2x + 3/2[(2 + x)/3]^{1/2}\}, \ s=-2 \qquad (4.14d)$$

$$E_0^{\lambda} = w\{3/2x + 5/2[(2 + x)/3]^{1/2}\}, \ s=-2 \qquad (4.14e)$$

$$E_0 = 4wx^{1/2}, \quad , s=-3 \qquad (4.14f)$$

The splitting in energy in the s=-1 sector, eqs. (4.14b) and (4.14c), contributes to making the $\Lambda\frac{5}{2}^-$ heavier than $\Sigma\frac{5}{2}^-$ in reversal of the situation in the ground state. In accordance with eqs. (4.14d) and (4.14e) a similar splitting occurs in the s=-2 sector. Similar equations can be given for the energies corresponding to the N=2 (positive-parity) baryons.

The eigenfunctions of the Hamiltonian (4.7) are given by:

$$\psi_{mn}^{ab} = \phi_{mn}^{ab}\exp(-1/2\,\alpha_\rho^2\rho^2 - 1/2\,\alpha_\lambda^2\lambda^2) \qquad (4.15)$$

$$\phi_{00}^{\lambda\lambda} = \left(\frac{2}{3}\right)^{1/2}\frac{\alpha_\rho^{3/2}\,\alpha_\lambda^{7/2}}{\pi^{3/2}}\left(\lambda^2 - \frac{3}{2}\alpha_\lambda^{-2}\right) \qquad (4.16)$$

$$\phi_{00}^{\rho\lambda} = \frac{2}{\sqrt{3}} \frac{\alpha_\rho^{5/2} \alpha_\lambda^{5/2}}{\pi^{3/2}} \vec{\rho}\cdot\vec{\lambda} \tag{4.17}$$

$$\phi_{00}^{\rho\rho} = \left(\frac{2}{3}\right)^{1/2} \frac{\alpha_\rho^{7/2} \alpha_\lambda^{3/2}}{\pi^{3/2}} \left(\rho^2 - \frac{3}{2}\alpha_\rho^{-2}\right) \tag{4.18}$$

$$\phi_{22}^{\lambda\lambda} = \frac{1}{\sqrt{2}} \frac{\alpha_\rho^{3/2} \alpha_\lambda^{7/2}}{\pi^{3/2}} \lambda_+\lambda_+ \tag{4.19}$$

$$\phi_{22}^{\rho\lambda} = \frac{\alpha_\rho^{5/2} \alpha_\lambda^{5/2}}{\pi^{3/2}} \rho_+ \lambda_+ \tag{4.20}$$

$$\phi_{22}^{\rho\rho} = \frac{1}{\sqrt{2}} \frac{\alpha_\rho^{7/2} \alpha_\lambda^{3/2}}{\pi^{3/2}} \rho_+ \rho_+ \tag{4.21}$$

$$\phi_{11}^{\rho\lambda} = \frac{\alpha_\rho^{5/2} \alpha_\lambda^{5/2}}{\pi^{3/2}} (\rho_+\lambda_3 - \rho_3\lambda_+) \tag{4.22}$$

where

$$\rho_+ = \rho_1 + i\rho_2 \tag{4.23a}$$

$$\lambda_+ = \lambda_1 + i\lambda_2 \tag{4.23b}$$

and

$$\alpha_\rho^4 = 3Km_\rho \tag{4.24a}$$

$$\alpha_\lambda^4 = 3Km_\lambda \tag{4.24b}$$

4.3 Anharmonic oscillator terms

Following Isgur and Karl[13], we write

$$a_n\sqrt{\pi} = 12\int_0^{\infty} U(x/\sigma)\ x^{2n}e^{-x^2}\,dx \ , \ n=1,2,3 \qquad (4.25)$$

$$\sigma = (wm_u/2)^{1/2} \qquad (4.26)$$

Where m_u is the mass of u quark. In terms of a_n, the contributions in the first-order perturbation of the Coulomb, linear and simple harmonic oscillator terms contained in the anharmonic potential

$$U(r_{ij}) = -\alpha_s/r_{ij} + ar_{ij} + br_{ij} \qquad (4.27)$$

are given by:

$$\begin{pmatrix} -\langle\Sigma\alpha_s/r_{ij}\rangle \\ \langle\Sigma ar_{ij}\rangle \\ \langle\Sigma br_{ij}^2\rangle \end{pmatrix} = M \begin{pmatrix} 2a_1 \\ 2a_2 \\ 4a_3 \end{pmatrix} \qquad (4.28)$$

where

$$6M = \begin{pmatrix} 10 & -11 & 2 \\ -15 & 27 & -6 \\ 4 & -8 & 2 \end{pmatrix}$$

Note that the result of this calculation is independent of w. Using values of the parameters obtained by Isgur and Karl[9], namely m_u=350 Mev, w=250 Mev, a_1=-650 Mev, a_2=-405 Mev and a_3=-908 Mev, we find that

$$-\langle \Sigma \alpha_s / r_{ij} \rangle = -1287 \text{ MeV}$$

$$\langle \Sigma a r_{ij} \rangle = 1421 \text{ MeV}$$

$$\langle \Sigma b r_{ij}^2 \rangle = -784 \text{ MeV} \qquad (4.29)$$

$$\langle \Sigma 1/2Kr_{ij}^2 \rangle = 375 \text{ MeV}$$

Thus the harmonic oscillator term $1/2Kr_{ij}^2$ and the harmonic oscillator contribution contained within the anharmonic potential U(r) do not cancel.

A major triumph of the model of Isgur and Karl is the correct prediction in sign and magnitude of $\Lambda \frac{5}{2}^-$ (1830) and $\Sigma \frac{5}{2}^-$ (1765) relative to the ground state. However, if these parameters obtained in their fit to the positive-parity baryons, are applied to the problem of negative-parity baryons, the mass difference between $\Lambda \frac{5}{2}^-$ and $\Sigma \frac{5}{2}^-$ will be reduced from 50 to 15 Mev; this is no longer consistent with experiment. Kalman and Hall[14] noted that the resolution of this difficulty is the method of calculation of the non-harmonic part of the potential. Isgur and Karl[13] obtain the value of the contribution of this term in the SU(3) limit ($m_s=m_u$). Kalman, Hall and Misra[9] instead generalize eqs.(4.25) and (4.26) to the case of three -body problem, in which the mass difference is indicated by the coefficient t, writing

$$a_n(t)\sqrt{\tau} = 12\int_0^\infty U[x/\sigma(t)]x^{2n}\exp(-x^2)dx \qquad (4.30)$$

$n=1,2,3$

$$\sigma(t) = (wm_u t/2)^{1/2} \quad , \; t = 4[1 + 3(m_\rho/m_\lambda)^{1/2}]^{-1} \qquad (4.31)$$

Kalman and Hall[14] show that, in such a consistent model, the mass difference between $\Lambda\frac{5^-}{2}$ and $\Sigma\frac{5^-}{2}$ is restored to a value in agreement with experiment.

As noted by Isgur and Karl[15], mixing between the ground state and the first excited positive- parity baryons (caused by hyperfine interaction) is quite important. In this chapter, the element of the mixing matrix corresponding to the first excited-parity baryons are calculated based on eqs.(4.30) and (4.31). Kalman and Mukerji[16] in their fit to the ψ and Υ spectrum note that in addition to mixing caused by hyperfine interaction, the anharmonic potential U also has an off- diagonal contribution. Similar contributions to the mixing matrix are also included here. If we now force the exact cancellation of the harmonic oscillator terms, that is we explicitly take the anharmonic term to have the form given by (4.3b) and (4.4), then the $a_n(t)$ n=1,2,3 defined by eqs.(4.30) and (4.31) can be approximately evaluated in term of α_s, w and one free parameter a. This is done following Kalman, Hall and Misra[9], by constructing quadratic approximations about t=1 for $a_n(t)$, n=1,2,3

$$a_1(t) \simeq A + Bt + Ct^2 \tag{4.32}$$

$$a_2(t) \simeq (3A + Bt - Ct^2)/2 \tag{4.33}$$

$$a_3(t) \simeq (15A + 3Bt - Ct^2)/4 \tag{4.34}$$

where A,B,C are evaluated from the values of $a_n = a_n(1)$ n=1,2,3 given by following equations

$$a_1 = D - 3w/2 - E \tag{4.35}$$

$$a_2 = 2D - 15w/4 - E \quad (4.36)$$

$$a_3 = 6D - 105w/8 - 2E \quad (4.37)$$

where

$$D = 6\sqrt{2}a/(m_u w \pi)^{1/2} \quad (4.38)$$

and

$$E = [4\alpha_s(m_u w/2\pi)]^{1/2} \quad (4.39)$$

The parameters to be determined are m_u, $x=m_u/m_s$, α_s, a and w. This calculation has been done earlier by Kalman and is recalculated here by using different values of w's for different bands. Besides, an arbitrary constant is included in the potential. The best fitting of the masses occurs at m_u =904 Mev, x=0.74, α_s =0.39, a=0.48 Gev^2, w=1.2 and 3 Gev for N=0 and N=2 respectively; the value of the arbitrary constant used is -4.97 Gev. The masses of the ground-state baryons calculated are shown in table 4.1. The highest deviation from experimental values is 2.6% and it is much better than the earlier calculation using a single value of w for every band. The above numerical results of α_s and a show that the Coulomb and the linear part of the potential have the same sizes. In this case, from chapter II and III we see that the errors when using the harmonic-oscillator basis are less than 1% except for the first one is 2.8% (Table 2.3); and the errors caused by the quadratic approximation method are 1% and 1.9% (Table 3.3). This is consistent with the error here for baryons.

The method of using an harmonic-oscillator basis to examine the potential between quarks or quark and antiquark and the quadratic approximation method have been used by Isgur and Karl[13,15] and Kalman[9,16] in their calculations for mesons and baryons; but the accuracy of these methods has never been tested. Chapter II and III of this thesis serve as a test of these methods and we can see that they work well provided that the Coulomb part is not too big compared to the linear one.

The quark model which employs an attractive potential at short range (a Coulomb-type piece derived from QCD) and deviations from the harmonic-oscillator potential at large distances has been working well for mesons but the situation is still not clear for baryons. The agreement between experimental values and calculation for baryons here enables us to conclude that the interquark potential may be closely approximated by a Coulomb plus a linear potential.

Table 4.1
Calculation of the masses of the ground-state baryons

Particle	Experiment (MeV)	Calculation (MeV)	% difference
N	939	960	2.2
Δ	1236	1235	0.08
Λ	1116	1114	0.18
$\Sigma\ \frac{1}{2}$	1193	1162	2.6
$\Sigma\ \frac{3}{2}$	1385	1385	0.0
$\Xi\ \frac{1}{2}$	1318	1288	2.3
$\Xi\ \frac{3}{2}$	1533	1559	1.7
Ω	1672	1690	1.1

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