ESSAYS ON ASSET PRICING MODELS

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ABSTRACT

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The central paradigm in Finance is the equilibrium pricing of risky assets. Recent studies have questioned the appropriateness of the two principal models of asset pricing, namely the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Model (APM). The purpose of this dissertation is to investigate empirically both the CAPM and the APM, and to propose an alternative model which may be preferable to the CAPM and the APM at both a theoretical and empirical level. The dissertation contains six (somewhat independent) essays which have been grouped under three asset pricing approaches: the CAPM (chapter two), asset pricing at the individual investor level (chapter three), and the APM (chapter four).

The overall conclusions to this dissertation are as follows:

(i) The traditional CAPM contains at least two empirical hurdles ($\underline{E} - \underline{V}$ and β instability) which prevent it from being empirically tested using traditional econometric procedures.

(ii) Individual investors (as proxied by Canadian households) appear to hold efficient portfolios in a mean-variance sense, although such portfolios are not the traditional market portfolio. Furthermore, the expected returns on these individual portfolios are linearly related to their variances.

(iii) While factor analytic techniques are generally used to test the APM, they imply serious deficiencies in test design, and better procedures for the empirical validation of the APM need to be found. Using standard procedures, the APM predicted equilibrium relationship is not consistent with the data. Thus, whether the APM is an inappropriate model or not is still an unresolved issue.

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CHAPTER ONE

INTRODUCTION

As has been shown by Rubinstein (1973) and by Pogue and Lall (1974), one of the central paradigms in Finance is the equilibrium pricing of risky assets (i.e., contingent claims). Concepts derived from this paradigm have had a number of theoretical and empirical applications, such as the examination of the informational efficiency of capital markets, the determination of the required rate of return on investment projects, the evaluation of the performance of managed portfolios, and the estimation of the fair return on stockholdings in regulated industries.

Although the pricing of assets in a world with perfect capital markets and certainty is relatively straightforward, the equilibrium pricing of contingent claims in a world of uncertainty, with or without market imperfections, has long been a major challenge to financial economists. Of the numerous models of asset pricing that are available, the capital asset pricing model (CAPM), which was developed by Sharpe (1973), Lintner (1965) and Mossin (1966), has gained general acceptance by both academicians and practitioners. The reasons for this general acceptance of the CAPM include the simplicity of its theoretical derivation, its intuitive appeal in terms of its explanations of concepts such as risk and return, and its relative ease of application.

However, the CAPM has not been unambiguously supported empirically. For example, tests conducted by Blume (1968) and by Friend and Blume (1970) concluded that the Sharpe-Lintner-Mossin version of the CAPM is not supported empirically. On the other hand, tests by Black, Jensen and Scholes (1972) and by Fama and MacBeth (1973) supported the Black (1972) version of the CAPM, while tests by Blume and Friend (1973) did not support that version of the CAPM. Recently, Roll (1977,1978) has shown that the CAPM can never be tested unambiguously because it is not possible to observe the "true" market portfolio. He also has shown that past tests, which relied on the linear relationship between risk and return, were ambiguous, because the mean-variance efficiency of the market portfolio implied necessarily and sufficiently the linearity between return and risk. However, if a linear relationship was found for a proxy market portfolio, this did not imply that the true market portfolio was mean-variance efficient. Thus, it is the mean-variance efficiency of the market portfolio which is the central tenet of the CAPM, and not the linearity of the relationship between the returns and risks of securities.

While empirical evidence that was unsupportive of the CAPM accumulated, researchers developed and tested alternative types of asset pricing models. These include, amongst others, the arbitrage pricing model (APM) of Ross (1976), the inter-temporal asset pricing models of Merton (1973) and of Breeden (1979), the two-risk-attribute (or three moment) model by Arditti (1967) and Kraus and Litzenberger (1976), the two-index model by Stone (1974), and the individual level version of the CAPM by Mao (1971) and by Levy (1978). Among these replacements for the CAPM, Ross's APM is the front-runner in terms of theoretical development and promise for empirical investigation.

As was reviewed in detail in the dissertation proposal, models other than the CAPM and the APM are generally considered to be inferior to either the CAPM or the APM. For example, the multi-moment models [Arditti (1967), Arditti and Levy (1973), Kraus and Litzenberger (1976)] and the multi-index models [Stone (1974), Lee and Lloyd (1976)] are not based on a basic and well-developed economic rationale. More specifically, while the multi-moment models cannot explain why moments beyond a given order are neglected, the multi-index models cannot explain either why the market is segmented or into how many segments the market is divided. The models based on the stochastic dominance criteria [Hader and Russell (1969), Whitmore (1970)] are also not satisfactory because they do not provide an ordering which is complete and compatible with the ranking given by the class of concave utility functions which exhibit decreasing absolute risk aversion. Although Hakansson's (1971) capital growth model initially appeared appropriate, it is merely a re-interpretation of the CAPM if the period studied is defined as being a period of sufficient length to always imply a positive return on wealth. The most promising of these alternate models is Breeden's (1979) extension of Merton's (1971, 1973) intertemporal asset pricing model. Unfortunately, Cornell (1981) has shown that the model cannot be tested because the consumption betas (i.e., the risk measures in the Breeden model) are not intertemporally stationary, except for trivial variants of the model.

Thus, given the present state of the field, the purpose of this dissertation is to investigate empirically both the CAPM and the APM, and to propose an alternative model of asset pricing which could, on both a theoretical and empirical basis, be preferable to the CAPM and the APM. The dissertation contains six somewhat independent essays of unequal length which have been grouped under three asset pricing approaches: the CAPM (chapter two), asset pricing at the individual investor level (chapter three), and the APM (chapter four).

CHAPTER TWO

THE CAPM: SOME THEORETICAL AND EMPIRICAL ISSUES

The capital asset pricing model (CAPM) was derived from the portfolio selection model developed by Markowitz (1959). Using the principle of maximization of expected utility which was first proposed by Von Neumann and Morgenstern (1944), Markowitz devised a parametric quadratic programming algorithm to obtain a general solution to the portfolio selection problem. If quadratic utility was assumed, or if the investor's terminal wealth followed a distribution characterized only by its mean and variance, Tobin (1958) showed that the Markowitz model implied that the process of investment choice can be broken down into two independent decisions: first, the choice of the best unique combination of risky assets; and second, the allocation of wealth between a riskless asset and the best combination of risky assets. This important result, known as Tobin's separation theorem, was also shown by Hicks (1972). It was also found earlier by Gordon and Gangolli (1962) in the context of the theory of choice among different lotteries, and by Roy (1952) in the context of the "safety first" investment criterion.

The Sharpe (1963) version of the CAPM was developed at about the same time as the Lintner (1965a) - Mossin (1966) versions. This version of the CAPM states that, given the information available at the end of period t-1, all assets will be priced according to their appropriately measured risk. Thus, in equilibrium, each asset is priced such that its probability distribution of yield over the immediately ensuing period, t, is exactly the distribution uniformly expected by investors. In other words, the expected returns on common shares with similar risks are equalized.

Thus, for asset i, i= 1,...,n:

$$E(\tilde{r}_i | \phi) = E(\tilde{r}_i | \psi) , \tag{0.1}$$

where $\tilde{r}_i$ is the random return on security i in period t;

ϕ is the available information set at the end of period t-1 regarding the probability distribution of returns on all assets during period t;

ψ is a measure of the risk (however defined) of returns for asset i during period t; and

$E(\cdot)$ is the expectation operator.

The CAPM also maintains that the market portfolio (i.e., the portfolio which includes all assets each weighted by its relative market value to the total market value of all assets) is mean-variance efficient. Thus, in equilibrium, each asset will be priced in such a manner that its one-period expected return is determined by the attempts of investors to hold mean-variance efficient portfolios.

In the Black (1972) version of the CAPM, the equilibrium relationship between risk and return is as follows:

$$E(\tilde{r}_i) = E(\tilde{r}_z) + \beta_i \, [E(\tilde{r}_m) - E(\tilde{r}_z)] , \tag{0.2}$$

where $E(\tilde{r}_i)$ is the expected rate of return on asset i in period t given asset i's risk of return in period t;

$E(\tilde{r}_z)$ is the expected return in period t on the asset that is uncorrelated with the market portfolio and has minimum variance among such assets;

$E(\tilde{r}_m)$ is the expected return on the market portfolio in period t; and

β_i is a measure of the risk of asset i relative to the total risk of the market portfolio.

Thus, the Black version reduces to the Sharpe version if the return on the so-called "zero-beta" portfolio is constant and equal to the risk-free rate, r_f.

The CAPM implies that the relationship between the expected return on an asset and its risk relative to the market portfolio's risk (i.e., its systematic risk) is linear. It should be noted that the model does not imply that any unsystematic or specific asset risk is rewarded, nor does it imply the intertemporal stationarity of β_i (see Essay 2).

The problem of relating expected ex-ante asset returns with realized ex-post returns can be solved by using a specific stochastic return generating model or by using techniques such as multidimensional scaling [Gooding (1975), Slovic (1972)].[1] Two stochastic return-generating models, the single- and the two-factor market models, are consistent with the Sharpe and the Black formulations of the CAPM, respectively. These models have been used to provide testable implications of the CAPM.

The single factor market model (SFM) was first proposed by Sharpe. The model assumes that the rate of return $\tilde{r}_{it}$ of asset i in period t is linearly related to the rate of return r_{mt} of the market portfolio in period t as follows:

$$\tilde{r}_{it} = r_{ft} + \beta_{it}(\tilde{r}_{mt} - r_{ft}) + \tilde{\varepsilon}_{it} \quad , \qquad (0.3)$$

where $\tilde{r}_{it}$, r_{ft}, β_{it}, $\tilde{r}_{mt}$ are as defined earlier (but are now assumed to be potentially variable from time period to time period); and

$\tilde{\varepsilon}_{it}$ is the disturbance term.

The SFM implies that the systematic (or non-diversifiable) part of a security's return is captured by its estimated linear relationship with the return on the market portfolio. Any returns not accounted for by this relationship will be reflected in the disturbance term $\tilde{\varepsilon}_{it}$. Therefore, this term captures the effects of non-market influences, such as industry and specific company influences. Thus, unlike the CAPM, non-systematic risk can exist in the SFM in a specific time period because it is usually assumed that $\tilde{\varepsilon}_{it}$ has a constant positive variance. It is also assumed that $E(\tilde{\varepsilon}_{it}) = 0$ and that the $\tilde{\varepsilon}_{it}$ are not correlated pairwise across i, nor serially across t. The SFM was first tested by Blume (1968). His results did not support the model as an explanation of the relationship between ex-ante expected returns and ex-post returns, and thus, by way of implication, did not support the Sharpe version of the CAPM. Morin (1976) found similar results for Canadian securities.

The two factor market model (TFM), or zero-beta market model, is consistent with the Black (1972) version of the CAPM. The TFM is given as follows:

$$\tilde{r}_{it} = \tilde{r}_{zt} + \beta_{it} (\tilde{r}_{mt} - \tilde{r}_{zt}) + \tilde{\xi}_{it} \quad , \qquad (0.4)$$

where $\tilde{r}_{it}$, $\tilde{r}_{zt}$, β_{it}, $\tilde{r}_{mt}$ are as defined earlier (but are now assumed to be potentially variable from time period to time period); and

$\tilde{\xi}_{it}$ is the stochastic disturbance term of the return on security i in period t (with an assumed mean of zero, a positive constant variance, independent of $\tilde{r}_{mt}$ and uncorrelated pairwise across i).

In this model, the return on a security is assumed to be a function of the general market variables, $\tilde{r}_{zt}$ and $\tilde{r}_{mt}$, and of the firm-specific variables, β_{it} and $\tilde{\varepsilon}_{it}$. Generally, it is also assumed that β_{it} is constant over time. Extensive empirical testing of the model by Black, Jensen and Scholes (1972) and by Fama and MacBeth (1973) has found the model to be a reasonable description of the stochastic process generating asset returns. However, tests conducted by Blume and Friend (1973) and Morin (1976) have found conflicting evidence.

Recently, Roll (1977) has shown that past tests of the CAPM are ambiguous, because tests using the SFM and TFM are joint tests of the CAPM and of the SFM and TFM, respectively. He notes that the only hypothesis that can be tested unambiguously is the mean-variance efficiency of the market portfolio.

Two CAPM related issues will be dealt with in the two essays of this chapter. In the first essay, the assumption of the stationarity of the mean vector and of the variance-covariance matrix of security returns will be empirically tested. In the second essay, it is demonstrated that the beta estimates from time series data are endogenous variables, since they are functions of past security returns. Thus, time-series estimates of security betas will behave as random variables.

FOOTNOTES

1. The multidimensional scaling technique was used by Gooding (1975) and Slovic (1972) to try to unravel how investors/portfolio managers appraise financial assets. This technique is basically a type of psychometric factor analysis.

ESSAY 1

THE STATIONARITY OF THE VECTOR OF MEAN RETURNS, E, AND OF THE VARIANCE-COVARIANCE MATRIX OF RETURNS, V

Since researchers have generally used time series of asset returns in empirically testing asset pricing models, they have implicitly or explicitly invoked the assumption that the applicable parameters of the security return distributions being studied are intertemporally stationary. In a mean-variance framework, this assumption means that the expected security return vector, E, and the variance-covariance matrix of security returns, V, are both intertemporally stationary. Although this assumption is not essential to the theoretical development of such one-period asset pricing models as the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Model (APM), it is necessary when time series return data are used to test either these asset pricing models or the relationships based on these models (such as the excess returns earned from exploiting nonpublic information).

While a number of authors have acknowledged the importance of the E - V stationarity assumption, few have attempted to test its validity. Thus, the purpose of this section of the thesis is to empirically test the intertemporal stationarity of E, V and R (the correlation matrix of security returns) using statistics proposed by Box (1949) and Jennrich (1970). These tests are not only conducted on samples of randomly-selected securities but also on samples of securities selected according to their beta values, their industry classifications and their market values.

The remaining part of this essay is organized as follows. In the next section, the literature on the stationarity of E and V is reviewed. As will be shown below, much of that literature only indirectly tests for the intertemporal stationarity of E and V. In the second and third sections, the statistical tests and sampling procedures, respectively, are presented. In the fourth section, the empirical results for the intertemporal stationarity of E and V for the various samples, using both nominal and real security returns, are presented and analyzed. In the fifth section, the empirical results for the intertemporal stationarity of R are presented and analyzed. In the sixth and last section, some concluding remarks are offered.

II Review of the literature:

Indirect tests

Blume (1968,1970) and King (1966) were two of the first authors to note that the variability of security returns on the New York Stock Exchange decreased from the prewar to the postwar period. Both authors used security return variances as the appropriate measure of variability (dispersion). Subsequently, Officer (1970) found that the variance of the NYSE index was higher in the 1930's than in either the pre-depression or postwar years. Thus all three authors provided empirical evidence that return variances were not intertemporally stationary since they were influenced by general uncertainty about business conditions.

A number of studies have supported the notion that security returns are heteroscedastic. For example, Blattberg and Gonedes (1974) found that the variances of individual security returns changed over time. Their results suggested that observed rates of return on common shares can be characterized as independent drawings from a normal population, with presumably a constant mean and a changing variance.

There appear to be several plausible explanations for the intertemporal non-stationarity of security return variances. First, technological innovations, business combinations and/or divestitures can be expected to change the return distribution of a firm's common shares. Second, based on multi-period consumption-investment theory, Rubinstein (1974b) and Fama (1970) have shown that, if in each period consumers and investors plan their consumption and investment over multiple future periods, then the variances for securities may change over time as new information enters the market and/or new individuals with new preferences bid for risky assets.

In their test of their option pricing model, Black and Scholes (1972) discussed the potential deficiencies of security return variances which have been estimated using ex-post time series data. More specifically, they showed that using past data to estimate the variances of security returns caused their option pricing model to overprice options on high variance securities and to underprice options on low-variance securities. Black and Scholes suggested that the "inaccurate" estimates of asset return variances were due to the well-known problem of errors-in-measurement, and to the evidence of non-stationarity in the variances.

Subsequently, Johnson (1979) proposed an option pricing model which allowed for systematic changes in the variances of asset returns. Although Johnson's model is a significant advancement in the incorporation of stochastic variances in an option pricing framework, his model is still unable to deal with the difficult problem of non-systematic movements in the variances of security returns. For as has been aptly described by Barry (1978, p.422), non-stationarity (or stochastic parameter variation) is the condition where "the characteristics of random processes change through time in a non-systematic way".

Schmalensee (1976) attempted to observe and explain the changes in the variances of subjective distributions by using an experimental approach. Like Fisher (1962), Schmalensee found that the logarithmic adaptive expectation models appear to be more descriptive of actual individual behaviour rather than the technical expectation formation mechanisms which are assumed to be appropriate in almost all empirical work. Schmalensee's findings implicitly support the notion that individuals do account for parametric changes in asset return distributions in their decision-making process. In addition, Schmalensee showed that turning points are more important in the formation of expectations than are the trends of economic variables.

Some of the more recent studies have also attempted to provide a rationale for the intertemporal non-stationarity of estimated security betas. For example, Fabozzi and Francis (1978) used a number of underlying explanatory factors (including firm-specific factors, macroeconomic factors, political factors and market-related factors) in order to explain the observation that "many stocks' beta coefficients move randomly through time rather than remain stable". Scott and Brown (1980) showed that the simultaneous violation of two OLS assumptions (namely, autocorrelation and a leading dependent variable) can imply biased and unstable beta estimates even when the true betas are stable.[1] Riding (1982) argued that information flows are responsible for both random and structural shifts in beta values.

Direct tests

Because asset pricing models deal essentially with the interdependence structure among assets, and not overly with the means and variances of individual asset or portfolio returns (especially in large markets), it is essential to test the assumption that the mean vectors, $\underline{E}$, and variance-covariance matrices, $\underline{V}$, of security returns are intertemporally stationary. With regard to the intertemporal stationarity of the means and variances of security returns, none of the literature reviewed above contains a direct test of the equality of the mean vectors, $\underline{E}$, and variance-covariance matrices, $\underline{V}$, of security returns. In a paper designed to examine the general factor model underlying the APM, Gibbons (1981) provided a direct test of the intertemporal stationarity of $\underline{V}$. As a preliminary step before using factor analytic techniques to explore the interdependence structure of 41 industry portfolios, Gibbons tested the validity of the assumption that the covariance matrix was stationary. More specifically, on a sample which was divided into two sub-periods, Gibbons used Box's χ^2 approximation to the likelihood ratio statistic in order to test for the equality of two covariance matrices,[2] and Jennrich's χ^2 statistic in order to test for the equality of two correlation matrices.[3] While Gibbons found that the covariance matrix was not stationary at the .0001 level, he also found that the same covariance matrix, if it is standardized to yield a correlation matrix, is stationary with almost certainty across the two sub-periods in his sample. Thus, he concluded that while the variances are not stationary, the correlation coefficients are. Consequently, the use of factor analytic techniques is feasible provided that the asset returns are first standardized.

Statistical tests

The likelihood ratio statistic developed by Morrison (1976, pp. 136-138) for testing for the equality of two mean vectors was used in order to test for the intemporal stationarity of $\underline{E}$. More specifically, Morrison has shown that the quantity Q_1 in equation (1.1) follows an F-distribution with N and $(T_1 + T_2 - N - 1)$ degrees of freedom:

$$Q_1 = \frac{T_1 T_2 (T_1 + T_2 - N - 1)}{(T_1 + T_2)(T_1 + T_2 - 2)} \; (\underline{E}_1 - \underline{E}_2)' \; (\underline{S}_1 + \underline{S}_2)^{-1} \; (\underline{E}_1 - \underline{E}_2) \quad , \qquad (1.1)$$

where $\underline{E}_1$ and $\underline{E}_2$ are the N-element (security) vectors of sample mean returns computed for the N (N=50) selected securities over $t_1 = 1, \ldots, T_1$ and $t_2 = 1, \ldots, T_2$ return observations for the first (t_1) and second (t_2) contiguous sub-periods, respectively;

$\underline{S}_1$ and $\underline{S}_2$ are the NxN matrices of sums-of-squares and product-moments of the sample of N securities computed for the contiguous sub-periods t1 and t2, respectively, and

T_1 and T_2 are the length of the first and second contiguous sub-periods of observations, respectively.

The null hypothesis is that the two mean vectors are equal (i.e., that $\underline{E}$ is stationary). It is accepted if $Q_1 \leq F_{\alpha;\ N,\ T_1+T_2-N-1}$; it is rejected otherwise.

Box's F-approximation to the likelihood ratio statistic for testing for the equality of two or more covariance matrices was used in order to test for the intertemporal stationarity of the covariance matrix, $\underline{V}$. More specifically, as shown by Box (1949) and Pearson (1969), the quantity Q2 in equation (1.2) approximately follows an F-distribution with $d_1 = N(N + 1)/2$ and $d_2 = [12\ T_1^2\ (N+1)^2]\ [N(N + 1) + 4] / [7(N + 1)^2\ (N - 1)\ (N + 2) - 6(2N^2 + 3N - 1)]$ degrees of freedom for the case where there are two covariance matrices and T1 equals T2. Equation (1.2) is given by:

$$Q_2 = (T_1+T_2-2)\ \mathrm{Ln}|\underline{S}| - (T_1-1)\ \mathrm{Ln}|\underline{S}_1| - (T_2-1)\ \mathrm{Ln}|\underline{S}_2|\ , \qquad (1.2)$$

where $\underline{S} = \frac{1}{T_1+T_2-2}\ (T_1-1)\underline{S}_1 + (T_2-1)\underline{S}_2$; and

all the other variables are as defined earlier.

The null hypothesis is that the two covariance matrices are equal (i.e., that $\underline{V}$ is intertemporally stationary). It is accepted if $Q_2 \leq F_{\alpha};d_1,d_2$; it is rejected otherwise.

Box's χ^2 approximation to the test statistic for the equality of two covariance matrices is not deemed to be appropriate because of the relatively large dimensionality (N=50) of the covariance matrices. The χ^2 approximation is adequate only for relatively small ($N \leq 5$) covariance matrices [see Morrison (1966, p.252)].

Both of the statistics, Q_1 and Q_2, assume that the sampled security returns are distributed according to a N-variate normal distribution. As originally developed, the statistic Q_1 is based on the assumption that the two independent random samples of observations of sizes T_1 and T_2 have the same, although unknown, covariance matrix $\underline{V}$ which is of full rank N. Fortunately, Ito and Schull (1964) have demonstrated that if the sample sizes T_1 and T_2 are equal, then unequal covariance matrices have no effect upon the size of the type 1 error probability. Thus, in this essay where T_1 is equal to T_2, a test of Q_1 is **not** a joint test of the equality of the mean vectors and of the covariance matrices, but a test of the equality of the mean vectors independently of the covariance matrices. In addition, a formal test of the homogeneity of the mean vectors, given that the unequal covariance matrices are available, can be used for those cases there T_1 is not equal to T_2. This formal test is based on the multivariate analog of the Behren-Fisher problem [see Giri (1977, pp. 171-172)]. On the other hand, Q_2 is an independent test of the homogeneity of the covariance matrices since it assumes unknown (and potentially unequal) mean vectors [see Giri (1977), pp. 223-232)].

Both $\underline{E}$ and $\underline{V}$ must be stationary in order to validate the use of time series returns in empirically testing (or applying) asset pricing models such as the CAPM or the APM. In other words, the intertemporal stationarity of one or the other (i.e., $\underline{E}$ or $\underline{V}$) is not sufficient for such testing, since **both** $\underline{E}$ and $\underline{V}$ must be intertemporally stationary. Although a joint test of the equality of both the mean vectors and covariance matrices is available [see Giri (1977), pp. 232-233)], it has not been used herein because independent tests of the homogeneity of the mean vectors and the covariance matrices provide more detailed results than a joint test. Also, the joint test simply results from the intersection of the independent test of the homogeneity of the mean vectors given equal covariance matrices, and of the independent test of the homogeneity of the covariance matrices. Thus, it does not add to the power of the independent tests.

Jennrich's (1970) χ^2 test procedure, as summarized in (1.3), was used to replicate Gibbon's test of the stationarity of the correlation matrices of security returns, $\underline{R}$. Q_3 has an asymptotic χ^2 distribution, with $N(N-1)/2$ degrees of freedom. Q_3 is given by:

$$Q_3 = \tfrac{1}{2}\, tr(\underline{Z}^2) - dg'(\underline{Z})\, \underline{U}^{-1}\, dg(\underline{Z}) \quad , \qquad (1.3)$$

Where $\underline{Z}$ = $c\ \underline{R}^{-1} (\underline{R}1 - \underline{R}2)$;

c = $T1\ T2 / (T1 + T2)$;

$\underline{R}$ = $(T_1\underline{R}1 + T_2\underline{R}2) = \{r_{ij}\}$;

$\underline{R}1$ and $\underline{R}2$ are the N x N matrices of sample correlations between the N securities computed for the first and second contiguous subperiods, respectively;

$dg(.)$ denotes the elements in the principal diagonal expressed as a column vector;

$tr(.)$ denotes the trace;

$\underline{U}$ = $\{\delta_{ij} + r_{ij}\, r^{ij}\}$;

δ_{ij} is the Kronecker delta;

r^{ij} is the i-th line, j-th column element of R^{-1}; and

all the other variables are as defined earlier.

The null hypothesis is that $\underline{R}_1$ and $\underline{R}_2$ are equal (i.e., that $\underline{R}$ is intertemporally stationary). It is accepted if $Q3 \le \chi^2_{\alpha;\frac{N(N-1)}{2}}$; it is rejected otherwise.

III. Sampling procedures

In order to test for the intertemporal stationarity of $\underline{E}$, $\underline{V}$ and $\underline{R}$ for contiguous time periods, security samples were formed by (1) random sampling, (2) selecting securities according to their beta values, (3) selecting securities according to their size (as measured by the total market value of their common equity), and (4) selecting securities according to their industry classification. These sampling procedures were chosen because they are the portfolio formation methods most commonly used in empirical tests of asset pricing models such as the CAPM and the APM.

The sampling procedure used for the randomly selected samples was as follows. First, eleven samples (each consisting of 50 securities) were randomly drawn from those securities that were included on the CRSP monthly tapes over the 360-month period from January 1948 to December 1977.[4] Second, each of these eleven random samples was divided into eight pairs of contiguous subperiods (as given in Table 1.1) to yield 88 sample-pairs. This subdivision was designed to facilitate the examination of the intertemporal stationarity of $\underline{E}$ and $\underline{V}$ for various contiguous time periods of equal length.

The selection of samples according to the beta values of individual securities proceeded as follows. First, the beta estimates for all 456 continuously listed securities on the CRSP tape were calculated using all 360 months of data for the period from January 1948 through December 1977. The beta estimates were derived using the standard single factor market model, where the market proxy was the value-weighted (cum-dividend) New York Stock Exchange Index. The beta values varied

TABLE 1.1
DESCRIPTION OF THE EIGHT PAIRS OF CONTIGUOUS SUBPERIODS FOR EACH OF THE ELEVEN SAMPLES

Length of each Subperiod[1]	First Subperiod starts	First Subperiod ends	Second Subperiod starts	Second Subperiod ends	Pair Identifier
60 months	Jan. 1948	Dec. 1952	Jan. 1953	Dec. 1957	1
	Jan. 1953	Dec. 1957	Jan. 1958	Dec. 1962	2
	Jan. 1958	Dec. 1962	Jan. 1963	Dec. 1967	3
	Jan. 1963	Dec. 1967	Jan. 1968	Dec. 1972	4
	Jan. 1968	Dec. 1972	Jan. 1973	Dec. 1977	5
120 months	Jan. 1948	Dec. 1957	Jan. 1958	Dec. 1967	6
	Jan. 1958	Dec. 1967	Jan. 1968	Dec. 1977	7
180 months	Jan. 1948	Dec. 1962	Jan. 1963	Dec. 1977	8

[1] $T_1 = T_2$, where T_1 and T_2 are the length of the first and second subperiods, respectively, of each pair of contiguous subperiods.

from 1.917 for the security with the highest beta to 0.522 for the 450th ranked security to 0.212 for the security with the lowest beta. Second, beginning with the security with the highest beta, securities were grouped into nine samples, each consisting of 50 securities. Thus, the six securities with the smallest beta values were not used. Third, each of these nine beta-based samples was divided into eight pairs of contiguous sub-periods to yield 72 sample-pairs.

The selection of samples according to the market values of the individual securities proceeded as follows. First, the market values of all 456 securities were computed as of December 1977.[5] The market values varied from 40.2 billion dollars for the largest market value to 11.3 million dollars for the 450th ranked security to 5.1 million dollars for the smallest market value. Second, beginning with the security with the highest market value, securities were grouped into nine samples, each consisting of 50 securities. Thus, the six securities with the least market values were not used. Third, each of the nine market-valued-based samples was divided into eight pairs of contiguous sub-periods to yield 72 sample-pairs.

The selection of samples according to the industry classification of individual securities proceeded as follows. First, each of the 456 securities in the basic sample was identified by its two-digit SIC code. This level of industrial classification was chosen in order to ensure that the number of securities in each industry was adequate. Second, using the two-digit SIC code, securities were assigned to 17 samples, where each sample contained from 9 to 44 securities. Third, each of the 17 industry-based samples was divided into eight pairs of contiguous sub-periods to yield 136 sample-pairs.

In all sample selections, the holding period was one month. Also the time periods used in computing $\underline{E}$ coincided exactly with those used to compute $\underline{V}$ for the same sample-pair.

IV Empirical results for E and V

Nominal returns

The empirical results for the tests of the intertemporal stationarity of E and V for the 88 randomly selected sample-pairs are given in Table 1.2. For sub-periods with equal lengths of 60, 120 and 180 months, the Q_1 statistics for the intertemporal stationarity of E were statistically significant for 18%, 100% and 100% of the applicable sample-pairs, respectively. For sub-periods with equal lengths of 60, 120 and 180 months, the Q_2 statistics for the intertemporal stationarity of V were statistically significant for 31, 23 and 18 percent of the applicable sample-pairs, respectively. For sub-periods with equal lengths of 60, 120 and 180 months, the Q_1 and Q_2 statistics for the intertemporal stationarity of E and V were both statistically significant for 7, 23 and 18 percent of the applicable sample-pairs, respectively. These empirical results are in general not consistent with the hypothesis that both E and V are intertemporally stationary, nor are they consistent with the hypothesis that V is intertemporally stationary. They are consistent with the hypothesis that E is intertemporally stationary for sub-periods with equal lengths of either 120 or 180 months. The intertemporal non-stationarity of E for sub-periods of 60 months is probably due to the fact that Q_1 for relatively small sub-periods has a small sample distribution which is not well approximated by the asymptotic distribution.

The empirical results for the tests of the intertemporal stationarity of E and V for the samples selected according to beta values, market values and industry classifications are summarized in Tables 1.3, 1.4 and 1.5, respectively. With few exceptions, these empirical results are basically the same as those obtained earlier for the randomly selected samples. The exceptions include: (i) E is basically intertemporally stationary for all industry-based samples, with the exception of those for industry 35 "machinery, except electrical", for sub-periods with equal lengths of 60 months; and (ii) E is intertemporally

Table 1.2

E-V STATIONARITY OF THE RANDOMLY-SELECTED SAMPLES USING NOMINAL SECURITY RETURNS

Panel A - F-Test for the Stationarity of E

Pair of Subperiods Identifier	Sample										
	1	2	3	4	5	6	7	8	9	10	11
1 (60 mos)	2.800	3.843	2.972	2.285	4.446	2.940	2.550	2.360	3.413	2.018	2.023
2	3.524	1.773*	3.033	2.039	3.979	2.066	2.491	2.025	2.929	3.123	3.134
3	1.939*	3.289	2.070	3.305	2.316	3.022	1.597*	1.993	3.027	2.553	4.249
4	1.772*	1.579*	1.961	2.773	3.243	2.156	2.078	2.025	2.998	2.364	2.205
5	1.953*	2.143*	1.779*	2.092	1.739*	1.910	1.997	2.135	1.599*	1.899	1.847
6 (120 mos)	1.825*	1.167*	1.027*	0.950*	1.529*	1.085*	1.167*	1.053*	1.146*	0.971*	1.032*
7	0.396*	0.686*	0.765*	0.668*	0.838*	0.549*	0.523*	0.450*	0.824*	0.766*	0.522*
8 (180 mos)	0.479*	0.741*	0.508*	0.454*	0.532*	0.692*	0.528*	0.578*	0.722*	0.437*	0.549*

Panel B - F-Test for the Stationarity of V

Pair of Subperiods Identifier	Sample										
	1	2	3	4	5	6	7	8	9	10	11
1 (60 mos)	2.885	0.218*	3.376	1.005	4.634	3.571	30.076	0.052*	1.572	0.088*	0.103*
2	15.872	0.262*	0.219*	1.659	2.293	11.402	7.982	0.023*	5.729	5.740	10.891
3	4.487	0.726*	0.943*	4.134	10.933	1.227	0.011*	0.748*	5.740	1.450	0.000*
4	0.002*	2.007	3.634	3.758	1.042	0.212*	39.996	0.512*	4.459	10.319	7.514
5	25.250	0.909*	1.711	0.336*	13.640	1.375	2.172	2.054	2.269	3.472	1.535
6 (120 mos)	9.452	0.235*	2.100	0.041*	1.025	6.977	1.691	0.516*	15.477	5.879	20.051
7	10.833	4.341	8.841	2.453	1.025	0.477*	68.046	2.191	0.000*	25.819	23.863
8 (180 mos)	14.774	3.177	1.033	0.021*	14.770	3.741	53.246	0.010*	22.036	35.532	57.130

* Not significantly different from zero at the 0.05 level.

Table 1.3

E-V STATIONARITY OF THE BETA-BASED SAMPLES USING NOMINAL SECURITY RETURNS

Panel A - F-Test for the Stationarity of E

Pair of Subperiods Identifier	Sample**								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	2.121	2.470	3.561	3.191	2.406	2.599	1.893	1.473*	2.186
2	2.940	2.744	4.878	2.890	2.538	2.586	2.089	1.928	1.385*
3	3.039	2.912	2.236	3.285	1.700	3.130	2.723	2.544	2.303
4	3.237	2.659	3.346	2.999	2.968	2.212	2.069	1.508*	1.304*
5	2.322	2.131	2.098	2.317	1.591	2.815	2.712	1.723	0.994*
6 (120 mos)	1.158*	1.170*	1.814	1.158*	1.215*	0.893*	1.549	1.103*	0.495*
7	0.684*	1.035*	0.813*	0.712*	0.799*	0.802*	0.628*	0.426*	0.337*
8 (180 mos)	0.498*	0.574*	0.503*	0.620*	0.443*	0.633*	0.675*	0.647*	0.440*

Panel B - F-Test for the Stationarity of V

Pair of Subperiods Identifier	Sample**								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	0.015*	1.343	0.219*	2.600	0.424*	8.321	10.445	0.574*	1.295
2	4.839	5.175	0.905*	0.000*	0.055*	1.597	3.922	5.093	6.070
3	1.046	2.901	5.857	0.556*	0.499*	2.716	0.012*	28.567	0.104*
4	9.195	0.970*	4.508	3.723	1.904	3.689	0.002*	15.507	0.776*
5	3.443	6.365	1.486	1.684	0.968*	21.288	8.532	0.264*	1.082
6 (120 mos)	5.623	1.757	6.036	3.229	0.092*	5.220	0.004*	73.176	16.069
7	24.750	4.901	3.934	10.105	4.718	0.152*	5.434	58.815	0.497*
8 (180 mos)	34.914	4.155	1.940	0.988*	3.257	1.051	3.673	242.507	4.190

* Not significantly different from zero at the 0.05 level.

** Sample 1 contains the 50 securities with the highest betas, while sample 9 contains the 50 securities with the lowest betas.

Table 1.4

E-V STATIONARITY OF THE MARKET-VALUE-BASED SAMPLES USING NOMINAL SECURITY RETURNS

Panel A - F-Test for the Stationarity of E

Pair of Subperiods Identifier	Sample**								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	3.444	1.853	2.392	3.635	3.877	2.849	2.799	2.270	2.321
2	2.225	2.278	3.530	3.761	2.541	2.868	2.523	2.536	2.237
3	3.285	2.017	2.556	2.386	3.511	2.370	2.527	2.035	2.967
4	2.792	1.912	1.716	1.920	3.080	3.772	2.293	2.236	3.244
5	1.984	1.610	1.787	2.624	2.057	2.439	2.562	2.286	1.761
6 (120 mos)	1.674	0.985*	1.273*	1.451*	1.040*	1.122*	1.445*	0.908*	1.099*
7	0.916*	0.500*	0.683*	0.578*	0.767*	0.795*	0.746*	0.583*	0.674*
8 (180 mos)	0.795*	0.465*	0.508*	0.665*	0.535*	0.602*	0.555*	0.550*	0.557*

Panel B - F-Test for the Stationarity of V

Pair of Subperiods Identifier	Sample**								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	10.445	1.223	5.961	1.504	2.600	2.434	0.015*	0.424*	0.574*
2	3.922	0.659*	1.780	3.810	0.000*	1.962	4.839	0.055*	5.093
3	0.012*	0.394*	0.000*	1.613	0.556*	10.331	1.046	0.499*	28.567
4	0.002*	9.627	5.484	4.043	3.723	26.568	9.195	1.904	15.507
5	8.532	0.001*	2.790	0.072*	1.684	2.508	3.443	0.968*	0.264*
6 (120 mos)	0.004*	2.024	0.006*	8.011	3.229	24.828	5.623	0.092*	73.176
7	5.434	15.666	20.428	15.587	10.105	34.858	24.750	4.718	58.815
8 (180 mos)	3.673	19.316	17.247	40.653	0.988*	0.036*	34.914	3.257	242.507

* Not significantly different from zéro at the 0.05 level.

** Sample 1 contains the 50 securities with the highest market values, while sample 9 contains the 50 securities with the lowest market values.

Table 1.5

E-V STATIONARITY OF THE INDUSTRY-BASED SAMPLES USING NOMINAL SECURITY RETURNS

Panel A – F-Test for the Stationarity of E

Pair of Subperiods Identifier	SIC code**																
	10	20	26	28	29	30	32	33	34	35	36	37	40	45	49	53	67
1 (60 mos)	0.364*	1.993	0.440*	1.313*	0.403*	0.285*	0.503*	0.636*	0.234*	1.565	0.814*	1.556*	0.881*	0.165*	1.565*	0.406*	1.100*
2	0.650*	1.183*	0.357*	2.338	0.701*	0.344*	0.490*	1.674	0.668*	2.035	0.950*	1.607*	0.866*	0.324*	1.535*	0.459*	0.515*
3	0.384*	1.413*	0.206*	1.154*	0.721*	0.371*	0.259*	1.138*	0.730*	2.009	0.612*	1.597*	0.870*	0.156*	1.604	0.862*	0.713*
4	0.335*	0.957*	0.185*	0.974*	0.552*	0.327*	0.674*	1.157*	0.215*	1.988	1.194*	1.322*	1.042*	0.214*	1.248*	0.320*	0.889*
5	0.213*	1.400*	0.375*	1.208*	0.805*	0.085*	0.704*	0.669*	0.432*	2.029	0.767*	0.992*	0.306*	0.387*	0.859*	0.607*	0.526*
6 (120 mos)	0.195*	0.771*	0.369*	1.271*	0.413*	0.286*	0.279*	0.694*	0.347*	0.642*	0.315*	0.660*	0.263*	0.142*	0.924*	0.352*	0.303*
7	0.159*	0.471*	0.108*	0.372*	0.103*	0.096*	0.202*	0.380*	0.226*	0.427*	0.289*	0.342*	0.147*	0.157*	0.562*	0.117*	0.183*
8 (180 mos)	0.169*	0.393*	0.155*	0.394*	0.191*	0.150*	0.113*	0.235*	0.196*	0.511*	0.163*	0.283*	0.160*	0.061*	0.329*	0.329*	0.173*

Panel B – F-Test for the Stationarity of V

Pair of Subperiods Identifier	SIC code**																
	10	20	26	28	29	30	32	33	34	35	36	37	40	45	49	53	67
1 (60 mos)	0.012*	3.786	0.224*	10.445	3.554	18.496	0.940*	0.219*	2.246	0.424*	0.023*	2.600	6.168	0.334*	5.961	4.186	2.434
2	0.957*	10.444	2.579*	3.922	1.623	40.247	2.028	0.905*	4.951	0.055*	0.099*	0.000*	5.718	1.912	1.780	1.108*	1.962
3	0.094*	11.296	4.768	0.012*	1.664	3.121	0.550*	5.857	1.203*	0.499*	0.006*	0.556*	3.962	0.561*	0.000*	4.160	10.331
4	0.260*	13.879	11.727	0.002*	41.532	0.113*	9.482	4.508	10.811	1.904	7.825	3.723	5.350	5.652	5.484	10.215	26.568
5	2.672	0.518*	0.233*	8.532	11.330	12.709	0.009*	1.486	5.287	0.968*	0.026*	1.684	1.038*	0.375*	2.790	0.016*	2.508
6 (120 mos)	1.554	35.856	10.613	0.004*	17.233	18.916	4.688	6.036	8.450	0.092*	0.156*	3.229	15.553	4.476	0.006*	2.153	24.828
7	0.601*	56.052	7.804	5.434	43.151	1.599	14.252	3.934	6.091	4.718	15.345	10.105	14.199	18.187	20.428	31.869	34.858
8 (180 mos)	0.059*	161.222	1.637	3.673	5.292	7.962	21.181	1.940	0.769*	3.257	15.304	0.988*	0.248*	35.626	17.247	54.486	0.036*

* Not significantly different from zero at the 0.05 level.

** SIC code and industry names (sample sizes):

10 Metal Mining (12)
20 Food & Kindred Products (35)
26 Paper & Allied Products (14)
28 Chemical & Allied Products (32)
29 Petroleum & Coal Products (16)
30 Rubber & Miscellaneous Products (10)
32 Stone, Clay & Glass Products (17)
33 Primary Metal Industries (24)
34 Fabricated Metal Products (15)
35 Machinery, Except Electrical (38)
36 Electric & Electronic Equipment (21)
37 Transportation Equipment (31)
40 Railroad Transportation (11)
45 Transportation by Air (9)
49 Electric, Gas & Sanitary Services (44)
53 General Merchandise Stores (13)
67 Holding & Other Investment Services (17)

non-stationary for all market-value-based samples for sub-periods with equal lengths of 60 months.

Real returns

Since the intertemporal non-stationarity detected in E and V may be due to the use of nominal returns,[6] the values of Q_1 and Q_2 were recalculated for the beta-based and market-value-based samples using real returns.[7] All returns were adjusted for the all-item cost-of-living index published by the Department of Commerce in **Business Statistics.** The results for the beta-based and market-value-based samples using real returns, which are summarized in Tables 1.6 and 1.7, respectively, are basically the same as those reported earlier for the nominal returns. More specifically, only the hypothesis that E is intertemporally stationary is consistent with the data for sub-periods with equal lengths of 120 and 180 months.

Empirical results for R

In order to replicate Gibbon's (1981) test for the intertemporal stationarity of the correlation matrix, R, the quantity Q_3 was estimated for the beta-based and market-value-based samples of securities. The results for these samples for nominal and real security returns are summarized in Tables 1.8 and 1.9, respectively. Since none of the observed Q_3 values are significantly different from zero at the 0.0001 level, the data is consistent with the null hypothesis that the correlation matrices of security returns are intertemporally stationary. Thus, our findings for R are identical to those of Gibbons (1981).

Table 1.6

E–V STATIONARITY OF THE BETA-BASED SAMPLES USING REAL SECURITY RETURNS

Panel A – F-Test for the Stationarity of E

Pair of Subperiods Identifier	Sample**								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	2.102	2.538	3.565	3.198	2.422	2.604	1.921	1.473*	2.303
2	2.924	2.719	4.869	2.757	2.539	2.603	2.095	1.905	1.387*
3	3.049	2.321	2.252	3.620	1.704	3.099	2.712	2.503	2.340
4	3.228	2.663	3.333	2.985	2.974	2.272	2.039	1.434*	1.239*
5	2.362	2.099	2.112	2.756	1.623	2.794	2.796	1.732	1.004*
6 (120 mos)	1.157*	1.241*	1.812	1.086*	1.214*	0.902*	1.551	1.062*	0.497*
7	0.685*	1.028*	0.843*	0.739*	0.833*	0.848*	0.687*	0.453*	0.365*
8 (180 mos)	0.508*	0.596*	0.519*	0.650*	0.460*	0.655*	0.704*	0.618*	0.487*

Panel B – F-Test for the Stationarity of V

Pair of Subperiods Identifier	Sample**								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	0.015*	1.276	0.247*	2.341	0.503*	7.420	10.904	0.765*	1.380
2	4.854	4.995	0.854*	0.000*	0.044*	1.628	3.892	5.578	5.766
3	1.068	2.856	5.928	0.545*	0.407*	2.559	0.020*	28.271	0.065*
4	9.274	0.947*	4.619	3.458	1.799	3.923	0.002*	15.313	0.842*
5	3.563	6.424	1.533	1.768	1.075	21.151	8.735	0.238*	1.216
6 (120 mos)	5.613	1.698	5.817	2.864	0.122*	5.017	0.029*	76.112	15.870
7	25.089	4.991	4.048	9.681	4.905	0.125*	5.559	58.424	0.424*
8 (180 mos)	35.389	4.202	1.694	1.073	3.740	1.039	3.423	244.053	4.509

* Not significantly different from zero at the 0.05 level.

** See Table 1.3

Table 1.7

E-V STATIONARITY OF THE MARKET-VALUE-BASED SAMPLES USING REAL SECURITY RETURNS

Panel A - F-Test for the Stationarity of E

Pair of Subperiods Identifier	Sample**								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	3.495	1.845	2.407	3.636	3.876	2.912	2.572	1.905	2.336
2	2.221	2.756	3.521	3.766	2.544	2.814	2.372	2.579	2.226
3	3.281	1.943	2.553	2.177	3.537	2.496	2.572	2.204	3.021
4	2.774	1.838	1.694	1.969	3.138	4.022	2.387	2.209	3.290
5	2.065	1.576	1.750	2.665	2.019	2.469	2.700	2.201	1.766
6 (120 mos)	1.676	0.983*	1.276*	1.456*	1.042*	1.125*	1.460*	0.921*	1.100*
7	0.954*	0.538*	0.689*	0.605*	0.832*	0.737*	0.811*	0.611*	0.683*
8 (180 mos)	0.823*	0.505*	0.545*	0.707*	0.568*	0.541*	0.624*	0.560*	0.578*

Panel B - F-Test for the Stationarity of V

Pair of Subperiods Identifier	Sample**								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	10.904	1.420	5.595	1.219	2.341	2.148	0.015*	0.503*	0.765*
2	3.892	0.577*	1.590	3.441	0.000*	1.818	4.854	0.044*	5.578
3	0.020*	0.333*	0.006*	1.577	0.545*	10.399	1.068	0.407*	28.271
4	0.002*	9.381	5.443	3.976	3.458	26.279	9.274	1.799	15.313
5	8.735	0.000*	2.860	0.078*	1.768	2.615	3.563	1.075	0.238*
6 (120 mos)	0.029*	2.087	0.002*	7.732	2.864	23.382	5.613	0.122*	76.112
7	5.559	15.506	20.745	15.281	9.681	34.568	25.090	4.905	58.424
8 (180 mos)	3.423	19.468	17.936	39.775	1.073	0.106*	35.390	3.740	244.053

* Not significantly different from zero at the 0.05 level.

** See Table 1.4

Table 1.8

$\underline{R}$ STATIONARITY OF THE BETA-BASED AND MARKET-VALUE-BASED SAMPLES USING NOMINAL SECURITY RETURNS *

Panel A – Beta-Based Samples

Pair of Subperiods Identifier	Sample (See**, Table 1.3)								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	10.557	7.621	5.402	6.080	3.999	4.855	3.675	2.994	3.390
2	11.638	7.205	4.891	5.175	4.700	5.170	3.959	3.408	3.752
3	12.113	7.875	5.866	5.704	5.542	5.171	4.850	3.951	3.316
4	13.561	9.358	8.989	6.551	7.203	8.408	6.330	5.952	4.405
5	14.830	9.912	10.401	10.402	9.684	10.939	9.059	7.135	6.331
6 (120 mos)	13.163	9.412	7.066	6.315	6.371	5.706	5.070	3.821	3.486
7	23.228	15.667	13.169	10.755	10.950	11.648	10.837	9.892	6.791
8 (180 mos)	23.686	14.833	12.676	10.182	11.450	11.040	11.130	9.699	6.715

Panel B – Market-Value-Based Samples

Pair of Subperiods Identifier	Sample (See**, Table 1.4)								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	2.992	4.055	4.356	4.344	5.266	5.848	6.863	6.500	7.355
2	3.586	4.955	4.843	4.444	5.868	6.359	6.777	6.341	6.371
3	4.234	5.967	4.227	5.420	5.620	7.879	7.308	6.818	7.864
4	5.283	8.113	5.213	7.400	6.896	9.829	9.148	9.115	11.464
5	6.630	8.273	7.785	6.994	9.664	11.591	9.426	12.328	15.875
6 (120 mos)	5.956	5.930	5.594	4.958	6.646	8.175	7.796	8.349	8.146
7	7.959	10.478	8.796	10.739	11.673	16.032	14.678	15.661	19.799
8 (180 mos)	6.836	9.594	7.715	9.648	11.600	16.105	14.421	16.423	20.269

* All coefficients are not significantly different from zero at the 0.0001 level.

Table 1.9

R STATIONARITY OF THE BETA-BASED AND MARKET-VALUE-BASED SAMPLES USING REAL SECURITY RETURNS *

Panel A - Beta-Based Samples

Pair of Subperiods Identifier	Sample (See**, Table 1.3)								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	10.348	7.606	5.306	5.896	3.862	4.586	3.646	2.911	3.335
2	11.597	7.165	4.894	5.142	4.644	5.069	3.941	3.386	3.651
3	12.027	7.839	5.834	5.634	5.458	5.064	4.803	3.888	3.261
4	13.507	9.292	8.895	6.468	7.108	8.307	6.231	5.849	4.358
5	14.715	9.873	9.937	10.279	9.521	10.863	9.028	7.099	6.356
6 (120 mos)	13.063	9.372	6.937	6.202	6.189	5.580	4.964	3.796	3.418
7	23.238	15.744	13.161	10.757	10.925	11.718	10.842	9.917	6.775
8 (180 mos)	23.915	15.023	12.746	10.184	11.465	11.221	11.245	9.844	6.794

Panel B - Market-Value-Based Samples

Pair of Subperiods Identifier	Sample (See**, Table 1.4)								
	1	2	3	4	5	6	7	8	9
1 (60 mos)	2.993	3.929	4.144	4.269	5.141	5.718	6.803	6.436	7.100
2	3.575	4.978	4.801	4.413	5.826	6.243	6.762	6.295	6.230
3	4.198	5.907	4.164	5.344	5.589	7.777	7.199	6.783	7.799
4	5.232	8.052	5.112	7.287	6.808	9.749	9.053	9.066	11.399
5	6.601	8.250	7.776	6.979	9.557	11.522	9.350	12.273	15.725
6 (120 mos)	5.803	5.860	5.541	4.897	6.566	8.086	7.705	8.257	8.105
7	7.994	10.555	8.805	10.727	11.649	16.127	14.778	15.670	19.792
8 (180 mos)	6.818	9.819	7.900	9.766	11.674	16.378	16.623	16.601	20.535

* All coefficients are not significantly different from zero at the 0.0001 level.

VI Concluding remarks

Based on the empirical findings presented above, some conclusions can be drawn. Firstly, the joint hypothesis that both E and V are intertemporally stationary is not consistent with the monthly return data for securities included on the CRSP file. This finding was robust in that the intertemporal non-stationarity of both E and V appears to be invariant to the nature of the sampling procedure used and to whether nominal or real security return data were used. Secondly, only the intertemporal stationarity of E was empirically supported for contiguous sub-periods with equal lengths of ten and fifteen years. This finding was also robust in that is was invariant to the nature of the sampling procedures used and to whether nominal or real security return data were used. Thirdly, the hypothesis that R is intertemporally stationary was empirically supported. This finding is consistent with the findings of Gibbons (1981). Therefore, like Gibbons, one is tempted to conclude that factor analysis can be used to unravel the factor structure of security returns, provided that the matrix of co-variations among security returns is first standardized. Fourthly, since some of the contiguous sub-periods exhibited the required intertemporal stationarity in the E and V, it is still possible to empirically test an asset pricing model such as the CAPM or the APM if the assumption that E and V is intertemporally stationary is first empirically verified for the sample (or samples) to be used in the empirical tests.

FOOTNOTES

1 A more complete review of the literature on the stationarity of security betas is presented in Essay 2.

2 The theory underlying this test statistic is presented in Morrison (1976, pp. 252-253). Box (1949) provided the initial χ^2 and the F approximations to the distribution of the likelihood ratio statistics for testing the equality of a number of covariance matrices. Box's approximations were later refined by Pearson (1969). Korin (1969) provided tables of the upper .05 critical values for the likelihood ratio statistic.

3 See Jennrich (1970).

4 Because the test statistics used in this paper involve matrix inversions, the size of the samples had to be severely constrained. The effect of this constraint is compensated for somewhat by using eleven samples of data.

5 When either the security price or the number of shares outstanding was not available, the available data for the month after December 1977 which is closest to December 1977 was used.

6 Fama and MacBeth (1973) demonstrated that nominal returns were not intertemporally stationary and that real returns seemed to be independent and stationary through time.

7 Industry- and randomly-based samples were not used because of budgetary limitations and their limited additional contribution to the understanding of the E - V stationarity phenomenon.

ESSAY 2

INTERTEMPORAL INSTABILITY OF THE TIME-SERIES ESTIMATES OF SECURITY BETAS

I Introduction

Estimating and analyzing the stability of security betas is important both for theory and practice. First, betas are important for understanding risk-return relationships in capital market theory. Second, they are important in investment management applications of the Capital Asset Pricing Model (CAPM), such as asset selection and portfolio performance evaluation.[1] Third, they are being increasingly used in the determination of rates for regulated industries such as public utilities.[2] Since an ex-post return series is the usual input for measuring systematic risk, it is important to determine whether or not, and under what conditions, security betas can be expected to be intertemporally stable.

With the exception of the study by Riding (1982), there has been little research on the theoretical causes of the intertemporal instability of beta. Much effort has been directed to the empirical testing of the stability of the systematic risk of common shares.[3] The methodology generally used in these studies involves cross-sectional correlation and other forms of the OLS regression estimates of systematic risk for both individual securities and portfolios of securities over two or more contiguous time periods. Most (if not all) of these studies implicitly assume that beta instability only results when a leading assumption of the CAPM is violated.[4] Merton (1971, 1973) has found that the requirement of a constant investment opportunity set (which requires that security returns are serially independent and obey a stationary distribution over time) is a critical and simplifying assumption in the context of a dynamic (i.e., multiperiod) CAPM.[5]

Therefore, the purpose of this essay is to show that even if ex-ante security returns are assumed to be serially independent and obey a stationary distribution over time (i.e., the mean return vector and the variance-covariance matrix of returns for securities are assumed to be constant over time), the beta of individual securities will vary over time whenever the betas are estimated using a time series of ex-post returns. In the next section, a review of the literature is presented. In the third section, a simple model of the intertemporal instability of the time-series estimates of the security betas is presented. In the fourth and final section, some implications of the intertemporal instability of security betas are presented.

Before proceeding, the terms **"stability"**, **"stationarity"** and **"randomness"** must be defined. In this essay, the term **stability** will refer to the "sameness" of a sample coefficient or estimate, which has been obtained for different time intervals, or different sample sizes, or different estimating techniques. As in the first essay, the term **stationarity** will refer to the time-related stability of the estimated parameters of the distribution of a random variable. The term **randomness** will refer to the following property of a variable and, of course, of an estimated sample coefficient; namely, that it is known through a probability distribution or a density function. A variable, an estimate, or a coefficient, which is written as a function of a random variable, is itself a random variable.

In this essay, the **stability** of the time-series estimates of security betas will first be discussed in general terms. Then, it will be shown that these betas are **unstable** because they are **random** (that is, functions of random security returns, which themselves may have **stationary** distributions). However, no conjectures are offered with regard to the stationarity of the distributions of the beta estimates.

II Literature review

As was noted earlier, the intertemporal stability of the beta coefficients of common shares has received considerable attention in the recent finance literature. A brief review of that literature will now be presented. Blume (1971) examined the long-term stability of the beta coefficient using monthly prices and successive seven-year estimation periods. He concluded that while portfolio betas are very stable, individual betas are highly unstable. R. Levy (1971) reported similar conclusions based on weekly data and shorter time periods. More specifically, Levy used 52-week base periods with 52-, 26- and 13-week subsequent periods. After detailed testing, Bogue (1973) reached similar conclusions about individual beta instability. Meyers (1973) found that the assumption that beta was stable for individual securities over time was unwarranted for the 1950-67 time period. Gonedes (1973) and Fisher (1970,1971) found that the regression parameters of the market model were non-stable at the level of individual assets, and that the optimal estimation interval was seven years. Levitz (1974) found that portfolio betas were stable when calculated using three-year base periods and one-year subsequent periods.

Klemkosky and Martin (1975) investigated the source of forecast errors of extrapolated beta coefficients and examined the efficacy of three adaptive procedures recommended by others for improving beta forecasts. They concluded that the accuracy of the simple no-change extrapolative beta forecast could be improved by using a procedure which combined a Bayesian predictor and a reasonable portfolio size.

Porter and Ezzell (1975) questioned two of the major conclusions of the previous studies of beta stability; that is, that the beta coefficient shows remarkable stability for portfolios containing large numbers of securities, and that the demonstrated stability is a positive function of the number of securities in the portfolio. After critically evaluating the portfolio "selection" procedures used in past studies such as those of Blume (1968) and R. Levy (1971), Porter and Ezzel (1978) state:

> Evidence presented in this note suggest that the intertemporal stability of the beta coefficient ... is sensitive to the procedure used to select portfolios. In particular, it was shown that if portfolios are randomly selected, the time-stability of beta is relatively slight and is totally unrelated to the number of securities in the portfolio [Porter and Ezzell (1978, p. 369)].

Blume (1975) provided some new empirical evidence which showed that a major reason for the observed regression of the estimated betas towards the grand mean of all betas is real non-stability in the underlying values of beta and not the so-called "selection bias". More recently, Pettway (1978), using an elaborate empirical procedure, found that long periods of unstable betas seem to be interspersed with periods of stable betas. He concluded that these periods of instability were transitory because the betas tended to return to their previous levels after leaving a period of disturbance.

Baesel (1974) estimated betas using various estimation intervals (one year, two years, four years, six years and nine years) to determine the impact of the length of the estimation period on the stability of the estimated betas. He concluded that the stability of beta is dependent upon the beta value. Furthermore, unlike Gonedes, Baesel concluded that the longest of the estimation intervals evaluated (i.e., nine years) was optimal. Altman et al. (1974), using data for the French stock market, also concluded that for single securities, the longer the estimation period, the higher the period-to-period correlation. Although Baesel and Altman et al. indicate that increasing the estimation period tends to increase the stability of individual security betas, they both used initial and subsequent estimation periods that were of the same length. Roenfeldt et al. (1978) studied whether the stability of individual security betas was affected by shortening the length of the second sub-period. They found that if a one-year forecast is desired, it is better to base it on a four-year previous period than simply to use the immediately preceding year.

In a recent article, Alexander and Chervany (1980) discussed the optimal estimation period discrepancy between Baesel and Gonedes, and concluded that the conclusions of Baesel were improperly drawn [Alexander and Chervany (1980, p. 123)]. More specifically, using mean absolute deviation as a measure of beta stability, they presented evidence showing that extreme betas are less stable than interior betas and that the optimal estimation interval was generally four-to-six years. Alexander and Chervany also suggested that the magnitude of intertemporal changes in portfolio beta coefficients decreases as the number of securities in the portfolio rises, regardless of how the portfolios are formed. They pointed out that the discrepancy in the results of Porter and Ezzell with those of Blume and Levy are caused by the fact that attempts to measure the stability of portfolio beta coefficients by the use of correlation coefficients masks the relationship between beta stability and portfolio size, for portfolios formed by random selection.

Fabozzi and Francis (1977) used standard econometric significance tests to determine whether the betas for a sample of 700 NYSE stocks differed significantly when measured over bull and bear market conditions. They concluded that neither the alpha nor the beta statistics in the single-index model appear to be significantly affected by the alternating forces of bull and bear markets. In a subsequent article, Kim and Zumwalt (1979) extended the design of Fabozzi and Francis to include an analysis of the variation of returns of securities and portfolios in up-and-down markets. Kim and Zumwalt found that:

> The results indicate (1) that unlike the Fabozzi and Francis study, more securities exhibited statistically significant differences between up-market and down-market betas than would occur by chance, and (2) that it appears that investors do require a premium for taking on downside variation and do pay a premium for upside variation [Kim and Zumwalt (1979, p. 1016)].

Brenner and Smidt (1977) tested a specific model of beta instability (that was based on the relation between the risk of the security and the risk of the underlying real assets) against a model of beta stability. Based on empirical tests conducted on a sample of 762 NYSE listed securities, Brenner and Smidt found that the evidence tends to favour the hypothesis of constant beta coefficients.

Fisher and Kamin (1978) conducted tests of the stability hypothesis and concluded that the behaviour of market risk is best described by a random walk or a first-order autoregressive process with a serial correlation very close to one. Sunder (1980) used a random coefficients test of the stability hypothesis of market risk of individual stocks and groups of stocks. He found that the estimates of variance of the market risk of portfolios and tests of significance of such estimates indicate that when the risk of individual stocks is non-stable, diversification does not diminish the statistical significance of the non-stability in spite of a decrease in the step variance (i.e., the variance of the residual errors) of portfolio risk.

Some of the more recent papers have attempted to provide an explanation for the instability of estimated security betas. For example, Fabozzi and Francis (1978) attempted to explain the observation that for "many stocks beta coefficients move randomly through time rather than remain stable" by using a number of underlying explanatory factors (including firm-related factors, macroeconomic factors, political factors and market-phenomena-related factors). In a similar vein, Scott and Brown (1980) show that simultaneous violations of two OLS assumptions can lead to beta estimates that are both biased and unstable, even when the true betas are stable. More specifically, they demonstrate that a combination of autocorrelated residuals and security returns that lead measured market returns yield biased beta estimates. Scott and Brown conclude that autocorrelation and security returns that lead market returns may continue to produce the unstable betas that Blume, Brenner and Smidt, Fabozzi and Francis, and Levy have noted, because these studies used monthly data and short time periods to estimate betas.

Riding (1982) proposed that information flows are the causes of structural as well as random shifts in beta coefficients. Using a model which incorporated information flows in a state-preference framework, Riding showed that changes in beta values are caused by variations in the expected terminal values of the securities, and (to a lesser extent) by variations in the initial wealths or the expected terminal wealths of investors. Using different types of information flows (global, industry or firm-specific, and transient or long-lasting), the random coefficient model and five recursive models, Riding demonstrated the existence of significant changes in beta values (both random and structural). Nevertheless, there are many other possible explanations for shifts in beta values, one of which might be the working mechanism of the capital market itself.

III A simple model of the effect of past return realizations on current ex-post beta estimates

A simple model will suffice to show that security betas estimated using ex-post return data cannot be intertemporally stable, even when security returns are assumed to be serially independent and have a stationary distribution over time. That is, estimated security betas can be expected to exhibit intertemporal instability even when the vector of ex-ante expected payoffs (i.e., $\underline{E}$) and the variance-covariance matrix of payoffs (i.e., $\underline{V} = [\sigma_{ij} = \mathrm{cov}(\tilde{r}_i, \tilde{r}_j)]$) are assumed to have an intertemporally stationary distribution. The only assumption which is needed to obtain such a result is that there is no rebalancing of the market portfolio, or no additional supply, or withdrawal, of new or old securities, so that the market weights are constant from time period to time period.

Assume that at time zero, investors have only cash and that they are offered various financial assets. If the market cleared, prices, a market portfolio and market weights, x_{io} where i denotes the asset i, i = 1,...,n and o denotes the initial time period (t = o) are obtained. Now assume that $\underline{E}$ and $\underline{V}$ are stationary. Then, the expected payoff and the variance of the payoff on the market portfolio can be written as:

$$E_{mo} = \underline{X}_o' \underline{E} \quad , \tag{2.1}$$

and

$$\sigma^2_{mo} = \underline{X}_o' \underline{V} \underline{X}_o \, , \tag{2.2}$$

where $\underline{X}_o = \{x_{io}\}$ or the column vector of market weights in time period o. Assume now that one period has passed. That is, assume that the roulette wheel has spun and that a state of nature along with its associated payoffs (i.e., price relatives), $\tilde{r}_{i1}$, has been obtained for each asset i. If one assumes further that no new securities were offered to the investors over the period and that there was no increase or decrease in the supply of the existing assets, then the total market wealth has changed as follows:

$$\tilde{r}_{m1} = \underline{X}_o' \underline{R}_1 \quad , \tag{2.3}$$

where $\underline{R}_1 = \{\tilde{r}_{i1}\}$ is the column vector of payoffs in time period one.

The market value of each asset i has also increased by $\tilde{r}_{i1}$ (i.e., the respective payoff on asset i). Therefore, the market weight of asset i has increased from x_{io} to x_{i1}, such that:

$$x_{i1} = x_{io} \frac{\tilde{r}_{i1}}{\tilde{r}_{m1}} \quad , \tag{2.4}$$

or in vector form:

$$\underline{X}_1 = \tilde{r}_{m1}^{-1} \; \underline{P}_1 \, \underline{X}_o \quad , \tag{2.5}$$

where: $\underline{P}_1$ is a diagonal n x n matrix, where the diagonal elements are the respective elements of $\underline{R}_1$ and all cross-elements are equal to zero.

To examine the effect of the shift in market weights, obtain the expression for the column-vector $\underline{B}_1 = \{\beta_{i1}\}$, where β_{i1} is the measure of systematic risk for asset i in time 1. β_{i1} is given by:

$$\beta_{i1} = \frac{\mathrm{cov}(\tilde{r}_{i1}, \tilde{r}_{m1})}{\mathrm{var}(\tilde{r}_{m1})} \quad . \tag{2.6}$$

$\underline{B}_1$ is given by:

$$\underline{B}_1 = \frac{\tilde{r}_{m1}^{-1} \, \underline{V} \; \underline{X}_1}{\underline{X}_1' \, \underline{V} \, \underline{X}_1} \quad . \tag{2.7}$$

Using (2.5), (2.7) can be written as:

$$\underline{B}_1 = \frac{\tilde{r}_{m1}^{-1} \, \underline{V} \, (\underline{P}_1 \, \underline{X}_o)}{\tilde{r}_{m1}^{-1} \, (\underline{P}_1 \, \underline{X}_o)' \, \underline{V} \, \tilde{r}_{m1}^{-1} \, (\underline{P}_1 \, \underline{X}_o)} \quad . \tag{2.8}$$

Because both $\underline{V}$ and $\underline{P}_1$ are symmetric matrices, their pre- or post-multiplication yields the same matrix. Thus, (2.8) can be rewritten as:

$$\underline{B}_1 = \tilde{r}_{m1} \, \frac{\underline{P}_1 \, \underline{V} \, \underline{X}_o}{\underline{X}_o' \, \underline{V} \, \underline{P}_1' \underline{P}_1 \, \underline{X}_o} \quad . \tag{2.9}$$

Now note that the matrix $\underline{P}'_1 \, \underline{P}_1$ is a n x n diagonal matrix, where the diagonal elements are $\tilde{r}^2_{i1}$ and all cross-elements are zero, that $\sigma^2_{mo} = \underline{X}'_o \, \underline{V} \cdot \cdot \underline{X}_o$ from (2.2), and that $\underline{B}_o = \underline{V} \, \underline{X}_o \; / \; \underline{X}_o' \, \underline{V} \, \underline{X}_o$ from (2.7). Therefore, using (2.9), each element of $\underline{B}_1$ can be expressed as:

$$\beta_{i1} = \frac{\tilde{r}_{m1} \, \tilde{r}_{i1} \, \mathrm{cov}(\tilde{r}_{io}, \tilde{r}_{mo})}{\tilde{r}^2_{i1} \, \mathrm{var}(\tilde{r}_{mo})} \tag{2.10}$$

or

$$\beta_{i1} = \frac{\tilde{r}_{m1}}{\tilde{r}_{i1}} \, \beta_{io} \quad . \tag{2.11}$$

Based on (2.11), it is obvious that β_{i1} will be different from β_{io}, whatever i, if $\tilde{r}_{i1} \neq \tilde{r}_{j1}$ for at least one i or j, i $\neq$ j. If $\tilde{r}_{i1} = \tilde{r}_{j1}$ for all i and j, then of course $\tilde{r}_{m1} = \tilde{r}_{i1} = \tilde{r}_{j1}$; and thus β_{i1} will be equal to β_{io} for all i. In other words, the measure of systematic risk will be stationary over time only if the assets' relative prices are the same from one time period to the next. From the development of this model, it is also clear that such a result is obtained because the price relatives affect the market weights. Thus, it follows that if other influences in the capital markets counter the shifts in market weights (i.e., induce a rebalancing of the market portfolio), then the measure of systematic risk will be stable over time.

The most likely candidate for a counterforce is the possibility that all investors not only hold the market portfolio in time period zero but also exhibit constant absolute risk aversion (CARA). More specifically, if all investors hold the market portfolio from time period zero to time period one, then the wealth of each and every investor will increase by a factor of $\tilde{r}_{m1}$. Since each investor will be offered the same investment opportunity set in time period one, each will choose the same market portfolio as he or she did in time period zero, given CARA. In other words, even though each investor is initially offered a market portfolio with different $\underline{E} - \underline{V}$ attributes, each investor (given CARA) will attempt to maintain the same market portfolio from time period zero to time period one (that is, the portfolio with attributes $E_{m1}=E_{mo}$ and $\sigma^2_{m1}=\sigma^2_{mo}$).

Unfortunately, the existence of constant absolute risk aversion is not consistent with the hypothesis that all investors hold the same market portfolio. This follows because the latter hypothesis is derived from the CAPM, which itself follows from the assumption of quadratic utility. In turn, quadratic utility implies increasing absolute risk aversion. Moreover, the assumption of constant absolute risk aversion is neither theoretically defensible [see Arrow (1971)] nor empirically supported [see Blume and Friend (1975)]. Furthermore, if investors do not all hold the market portfolio, their respective wealth will increase (or decrease) at varying rates. Thus, whether or not they exhibit constant absolute risk aversion, investors will adjust their portfolios in such a manner that the beta values will shift [see Riding (1982)].

Thus, unless a strong counterforce exists, such as the one described above, the model presented above demonstrates that the beta values will change by the sheer return generating mechanism of the capital market. Thus, fluctuations of security betas are a very pervasive phenomenon, which are caused by influences such as information flows [see Riding (1982)], by shifts in market weights induced by the random payoffs, and by counterforces to such shifts. Moreover, while the model deals with the instability of beta from a theoretical standpoint, other sources of empirical instability, such as measurement error and mispecification of the estimation procedures, are likely.

Before proceeding to the implications of the above, it should be noted that no specific type of return distribution was assumed for each of the securities in the above model. All that was assumed was that: (a) each security's return distribution could be adequately characterized by its mean and variance, and (b) the mean and variance of return for each security was intertemporally stationary.[6]

IV Implications of the instability of the time-series estimates of security betas

Given the above findings, it is not surprising that most authors have found that the betas of individual securities are **not** stable intertemporally. While the instability of the betas in practice may be caused by a number of factors that affect the intertemporal stationarity of $\underline{E}$ and $\underline{V}$ [for example, as has been proposed in Fabozzi and Francis (1978) and Riding (1982)], it has been shown that estimated betas will not be stable even if $\underline{E}$ and $\underline{V}$ are intertemporally stationary, because the estimated betas are cumulative functions of the past series of (random) realized security returns.

Before proceeding, it should be noted that some of the findings reported above have been implicitly anticipated in the literature. More specifically, Cheng and Grauer (1980) have also noted that since betas are ex-post endogenously determined, they are unlikely to be constant, and thus they cannot be estimated as regression coefficients using ordinary or generalized least squares techniques. More specifically, Cheng and Grauer (1980, p. 661) state that for the betas to be intertemporally stable, "must also necessarily imply that the equilibrium firm values must vary through time in equal proportions".[7] Unfortunately, they did not recognize that the estimated betas are ultimately functions of the security returns and thus are themselves random variables. Instead, Cheng and Grauer argued (correctly) that the betas are functions of the expected return, $E_{zt} = E(\tilde{r}_{zt})$, on the zero-beta portfolio; a value which varies from period to period even in the case when $\underline{E}$ is intertemporally stationary.

The above findings on the intertemporal instability of estimated betas have some important implications for the empirical testing of the CAPM. The regression techniques used in most (if not all) past empirical tests of the CAPM to estimate betas now appear dubious. In the words of Cheng and Grauer (1980, p. 662) past empirical tests assumed "a CAPM disconcertingly different from the CAPM being tested". A feel for this contradiction between purpose and procedure can be obtained by examining Figure 2.1, which depicts the CAPM implied by past tests of the CAPM, and Figure 2.2, which depicts that implied by the theory. Although the CAPM, as generally formulated, is a one-period model, past empirical tests of the CAPM nonetheless imply that the linear relationship between E_i and β_i has an intertemporally stable slope.[8] Thus, these empirical tests have implicitly assumed that in the three-dimensional space spanned by E_i, β_i and t, the CAPM is depicted by a plane (see Figure 2.1). In theory, there is no requirement in the CAPM that the linear relationship between E_i and β_i have an intertempo-

Figure 2.1

The Type of Multi-Period CAPM Relationship Generally Empirically Tested

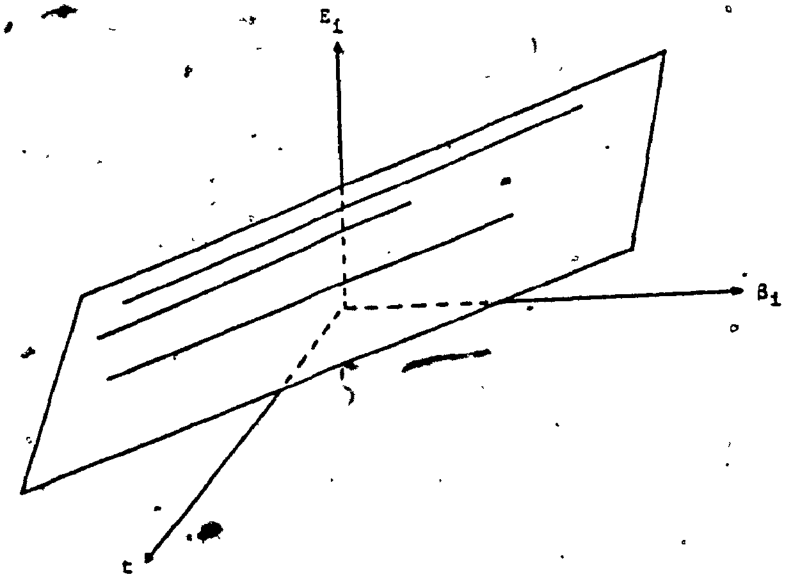
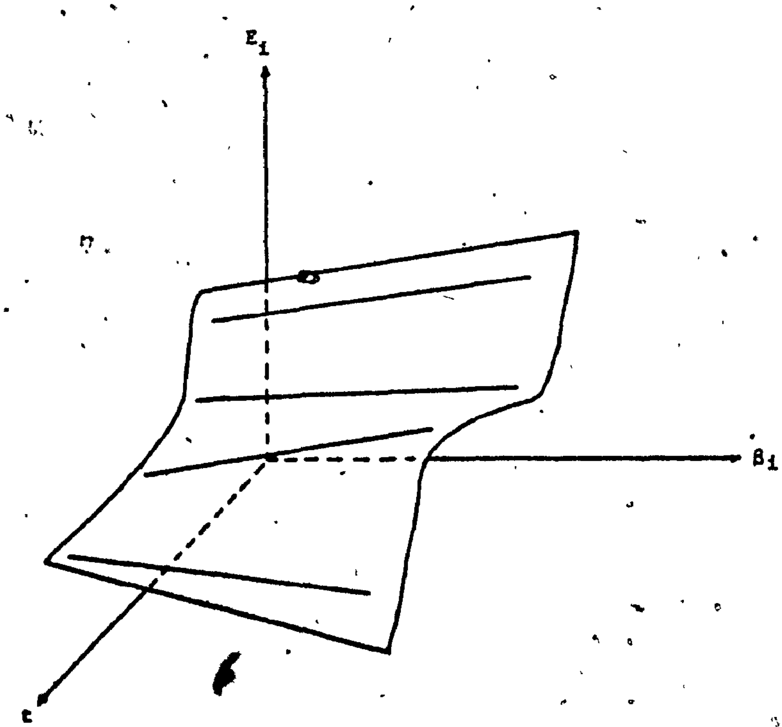


Figure 2.2

The Expected Multi-Period CAPM Relationship Assuming Intertemporal Stationarity of $\underline{E}$ and $\underline{V}$



rally-stable slope. Thus, based on the findings of this study, in the space spanned by E_i, β_i and t, the CAPM asset-pricing relationship is more likely to resemble the curved plane in Figure 2.2 than the plane in Figure 2.1.

Furthermore, the random coefficients model developed by Theil (1971) and used by Fabozzi and Francis (1978), assumes in its tests of statistical significance that the coefficients are normally distributed with stationary parameters. Thus, it can potentially be used if it is known that the beta distributions are stationary and normal. Unfortunately, no study has yet appeared on the nature and the stationarity of the beta distributions. However, if the number of variations in beta values is finite, recursive techniques can be used to estimate the true beta value [See Riding (1982)].

The above commentary on the past empirical tests of the CAPM does not imply that beta instability makes the CAPM untestable. More specifically, if $\underline{E}$ and $\underline{V}$ are assumed to be intertemporally stationary, then a (somewhat tedious) procedure to estimate the beta values for each security (or portfolio of securities) for each time period can be developed. These values can then be used to run a series of cross-sectional tests involving $\underline{E}$ and $\underline{\beta}$ for a specified time period.

FOOTNOTES

1 For a review and discussion of the use of betas in portfolio selection, see Elton, Gruber and Urich (1978).

2 For a review and discussion of the use of betas in regulatory proceedings, see Pettway (1978).

3 This literature will be reviewed in the next section of this essay.

4 In the single-period CAPM, the individual security betas are by definition constant. However, in a multiperiod framework, the individual security betas are endogenously determined for each t. Thus, in both theory and practice, they could vary from period to period. This has been noted by Rubinstein (1973) who discusses how ex-ante betas are endogenous in a multi-period context; by Galai and Masulis (1976) who develop theoretical arguments for the randomness of ex-ante betas; and by Sunder (1980, p. 866) who maintains that there are both theoretical and empirical reasons why the distribution of β is **not** stationary.

5 See Merton (1972, p. 17).

6 The stationarity properties of beta using simulation are presented in Kryzanowski and To (1984).

7 Rosenberg and Ohlson (1976) reached a similar conclusion in a different context. More specifically, they noted that:

> Thus, portfolio separation across any subset of assets, together with the stationary-distribution-of-returns hypothesis, essentially implies a constant shares of wealth property across those assets, which, in turn, implies a fundamental degeneracy in the derived behavior of security prices. The result takes on significance in that so much research in modern capital theory depends crucially on either (or both) of these properties... Upon reflection, the described contradiction is not the least surprising. Separation induces irrelevance with respect to parameters unique to the investors, so if the economy repeats itself over time, then the same structure of asset prices must prevail if the return-generating process is stationary. [Rosenberg and Ohlson (1976, p. 400)]

8 For example, Black, Jensen and Scholes (1972, p. 86) assumed that the betas were stationary, based on evidence presented by Blume (1968), and thus used an equally-weighted market index.

CHAPTER THREE

A CLINICAL-LEVEL APPROACH TO ASSET PRICING

Although the standard form of the CAPM presents a number of important theoretical and empirical deficiencies, it has been reformulated into a number of testable variants. Since these variants are disaggregated to individual investor level, they generally focus on the equilibrium of individual investors rather than on the general equilibrium of the capital markets. As a result, these models are referred to in this dissertation as being "clinical" models.

The models developed by Mao (1971) and by Levy (1978) have been compared and reconciled in Kryzanowski and To (1982). In Essay 3, a clinical-level variant of the CAPM for imperfect markets, which is more suitable for empirical testing than the Mao-Levy models, will be presented and tested using data on the portfolios of individual investors. In Essay 4, an alternate clinical-level asset pricing model, which invokes no assumption about the functional form of the utility functions of individual investors, is developed and tested. This "clinical-level asset pricing theory" is tested using the same panel data as was used in the empirical tests presented in Essay 3.

ESSAY 3

SOME CLINICAL-LEVEL TESTS OF THE CAPM IN IMPERFECT MARKETS

I Introduction

In a paper published in 1978, Levy (1978) proposed a general capital asset pricing model (GCAPM), which he obtained by maximizing investors' utility when the number of securities held in each investor's portfolio was contrained. In a recent paper, Kryzanowski and To (1982b) demonstrated that Levy's GCAPM was similar to a model developed earlier by Mao (1971). Mao had earlier developed his model by modifying the familiar Sharpe-Lintner-Mossin capital asset pricing model (CAPM) to reflect the implications of market imperfections.

Kryzanowski and To (1982b) also noted that the Mao and Levy versions of the CAPM for imperfect markets (unlike the standard CAPM) could be tested using data on the portfolios of individual investors (i.e., "clinical-level" or also "personalized" data). In fact, such empirical tests would avoid the ambiguous nature of the past joint tests of the standard CAPM that were noted by Roll (1979). More specifically, since in the standard CAPM all investors will invest their funds at risk in the market portfolio, an empirical test of the standard CAPM at the clinical level requires that the unobservable market portfolio be measured. On the other hand, this is not the case in a Mao- or Levy- type of CAPM for imperfect markets. Since investors in such a market will invest their funds at risk in equilibrium in mean-variance efficient constrained portfolios (i.e., portfolios that contain a relatively small subset of all the available assets), an empirical test of the CAPM in imperfect markets at the clinical level does **not** require that the unobservable market portfolio be measured.

In this essay, a model derived from the Mao-Levy propositions is first formulated and then tested using data on the portfolios of individual investors. The form of these empirical tests is similar, but not identical to the two tests proposed by Kryzanowski and To (1982b).

The remainder of this essay is organized as follows. In the next section, the CAPM relationship in imperfect markets for the portfolio of an individual investor in equilibrium is derived. In section III, the data sources and the empirical procedures used to test the clinical CAPM are described. In section IV, the empirical findings for the initial test of the derived CAPM are presented and discussed. In section V, an additional test of the derived CAPM is presented. In section VI, some concluding comments to this essay are offered.

II. One specific clinical-level ("personalized") form of the CAPM in Imperfect Markets

The following pricing relationship can be derived from Rubinstein's (1973b) or Levy's (1978) development of the CAPM:[1]

$$E(\tilde{r}_i) - r_f = \lambda_k \operatorname{cov}(\tilde{r}_i, \tilde{r}_k) \qquad \forall_i , \tag{3.1}$$

where $\tilde{r}_i$ is the random return on risky asset i, $i = 1, \ldots, n_k$ where $n_k < n$ because of market imperfections and investor myopia;

r_f is the known return on the risk-free asset;

λ_k is the intra-portfolio price of risk which is equal to $[E(\tilde{r}_k - r_f)] / \operatorname{var}(\tilde{r}_k)$; and

$\tilde{r}_k$ is the random return on the portfolio held by investor k.

If a frictionless, purely competitive and perfect capital market is assumed, and the condition that every available asset is held is satisfied, equation (4.1) can be easily aggregated to either the individual investor or market levels to obtain two testable linear relationships. The first is between the expected return and "own variance" of investor k's portfolio [i.e., Lintner's (1965a) so-called market opportunity line or MOL for investor k]; the second is between the expected return and variance of the market portfolio (i.e., the so-called capital market line or CML). Both of these linear relationships deal with efficient portfolios (the investor's portfolio in the first relationship and the market portfolio in the second relationship). Therefore, the linear relationships are between the expected returns on the (efficient) portfolios and the variances of return on these portfolios, and not with some measures of systematic risk. It is interesting to note that in equilibrium, $CML=MOL_k$, $k=1,\ldots,K$, so that neither hypothesis can be actually tested because the "true" market portfolio is itself unobservable [Roll (1977)]. Of course, one could argue that the empirically verifiable evidence [see, for example, Blume and Friend (1975) and Schlarbaum, Lewellen and Lease (1978b)] that no one holds all available assets is (in and of itself), sufficient evidence to reject one of the testable implications of the CAPM in perfect markets.

Since portfolio diversification is costly (due to market imperfections such as taxes, transaction costs, information costs and portfolio management costs) and most of the benefits of diversification can be attained with a relatively small number of "well chosen" assets [see Evans and Archer (1968), Mao (1971), amongst others], it seems reasonable to assume that investors will attain their optimal MOLs by investing their funds at risk in a constrained (or non-market) portfolio of assets. In other words, it seems reasonable to assume that, given the existence of market imperfections, few (if any) investors will hold the market portfolio or even a portfolio containing a substantial number of assets. Furthermore, it would be expected that different investors would hold different portfolios.[2]

If a purely competitive (but imperfect) capital market is assumed, and the condition that every available asset is held is satisfied, equation (3.1) can be aggregated to either the individual investor or market levels to obtain two types of testable linear relationships. The first are the MOLs for all K investors; the second is the CML. Each of these aggregations will now be dealt with in turn.

Since (3.1) holds for each asset held by investor k, (3.1) can be aggregated over the n_k assets held by investor k in order to derive the optimal equilibrium relationship for investor k's own portfolio. Thus, by weighting each asset i in investor k's portfolio by the relative proportion of investor k's total wealth invested in asset i, one obtains:

$$E(\tilde{r}_k) = r_f + \lambda_k \, var(\tilde{r}_k) \; . \qquad (3.2)$$

Equation (3.2) states that: (i) the expected return on the portfolio of investor k is positively related to its variance; (ii) the intercept of this linear relationship is the risk-free return; and (iii) the slope of the linear relationship is a measure of the intra-portfolio price of risk. Furthermore, as shown by Roll (1977, p. 159), equation (3.2) provides both the necessary and the sufficient condition for establishing that the portfolio held by investor k is locally efficient.[3] Thus, while the standard CAPM implies that each (and every) investor will invest his or her funds at risk in the market portfolio, the CAPM in imperfect markets implies that each investor will invest his or her funds at risk in some (unique) subset of all the available assets. This subset will be a constrained E-V efficient portfolio.

Using Roll's (1977) efficient set mathematics, it can readily be shown that (3.2) is replaced by (3.3) when no risk-free asset is available to investor k:

$$E(\tilde{r}_k) = E(\tilde{r}_{zk}) + \lambda_k \, var(\tilde{r}_k) \quad , \qquad (3.3)$$

where: $E(\tilde{r}_{zk})$ is the expected return on the efficient portfolio which is orthogonal to K (i.e., the portfolio held by investor k); and

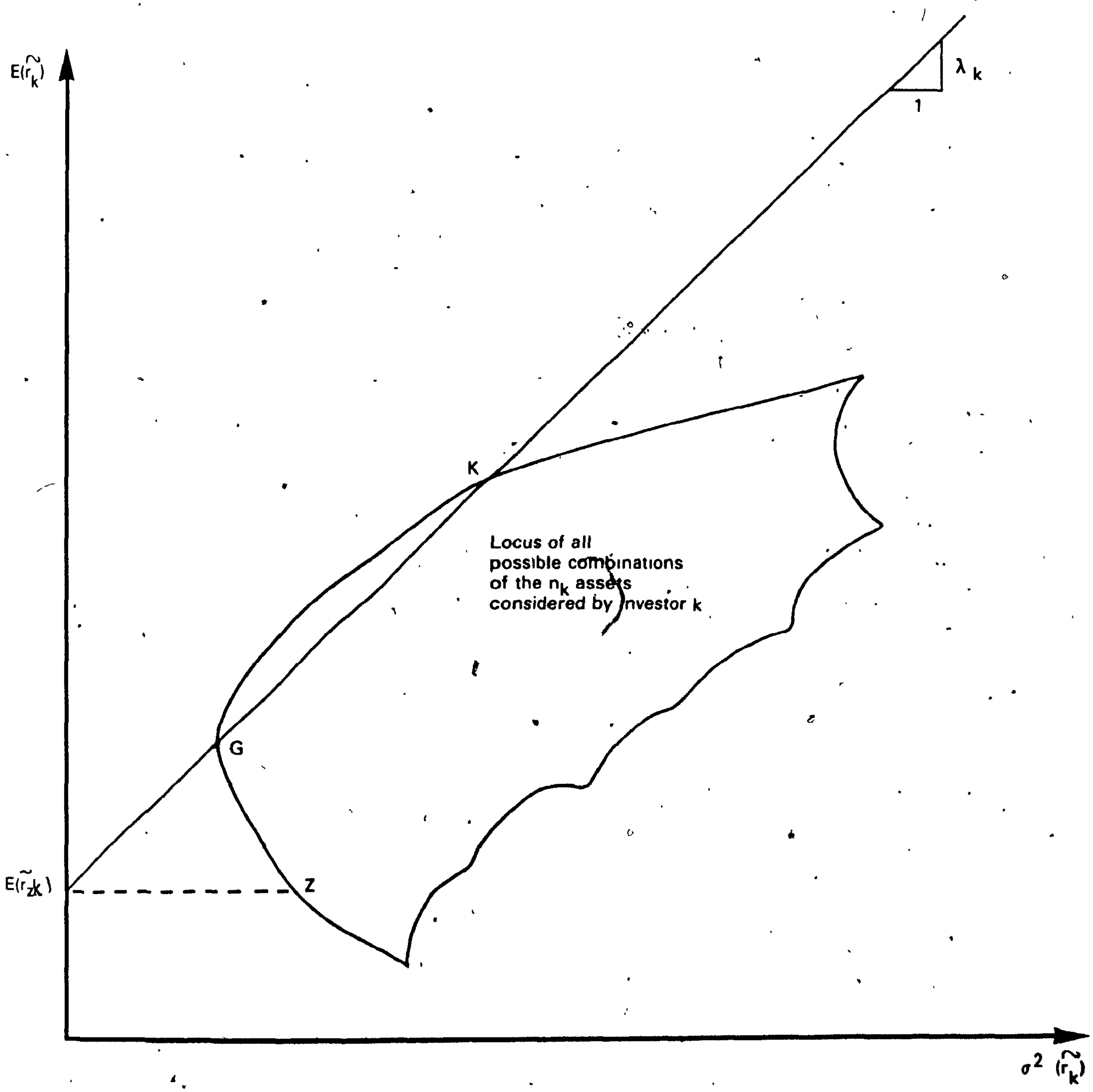
all the other variables are as defined earlier.

Under these conditions, investor k's intra-portfolio price of risk and risk premium are given by $[E(\tilde{r}_k) - E(\tilde{r}_{zk})] / var(\tilde{r}_k)$ and $E(\tilde{r}_i) - E(\tilde{r}_{zk})$, respectively.[4]

Using the general formulation (3.3), the equilibrium for investor k can be graphically depicted as in Figure 3.1. It is apparent from the figure that the optimal holdings for investor k in equilibrium (i.e., portfolio K) denotes: (i) the revealed risk preferences of investor k, which are given by the slope of the line joining G and K (i.e., λ_k) and (ii) investor k's perceived set of investment opportunities since portfolio K lies on the mean-variance efficient frontier for the n_k assets. In other words, equation (3.3) reflects both the parameters of investor k's utility function of wealth and the parameters of investor k's personal market opportunity line. In addition, it is now apparent that the intercept of equation (3.3) (i.e., the intercept of the line joining G and K in Figure 3.1) is investor-specific and thus is **not** likely to be constant across all investors.

While (3.3) denotes optimality at the individual investor's level, does it also denote equilibrium at the market level? If one assumes that no individual investor can influence security prices, equation (3.1) shows that individual investors will attain their optimal conditions by adjusting their portfolio compositions. Therefore, price-taking behaviour is necessary to move from clinical optimality to market equilibrium. In effect, the market mechanism acts as if the set of prices were collectively determined by all individual investors and not by any

FIGURE 3.1
A REPRESENTATION OF THE CLINICAL EQUILIBRIUM FOR INVESTOR k



Where G: Global minimum variance portfolio;
K: Investor k's portfolio; and
Z: The portfolio orthogonal to K.

particular investor individually. However, this adjustment could result in some securities not being exhaustively held. To guarantee market clearing, one must have a type of tatonnement process where different schedules of prices are tried until each investor holds his or her optimal portfolio and the market clears itself at the same time. In this sense, the prices of securities are co-determined. Note, however, that this co-determination does not necessarily imply that beliefs are homogeneous. All the tatonnement process does is simply search out the market clearing prices. While a market pricing equation cannot be attributed to the market clearing prices, equation (3.1) will hold with respect to each investor's beliefs, risk preference and opportunity set.

Therefore, this market setting is compatible with perpetual and continuous trading where market clearance can be adequately characterized by a tatonnement process.[6] In this essay, homogeneity of beliefs will be assumed only for testing purposes, because it is possible to derive only a single measure of $E(\tilde{r}_i)$, and thus the same estimate must be used for each and every investor. However, the assumption of homogeneity of beliefs is not instrumental in the theoretical development of (3.3).

III. Data sources and empirical procedures

There are three testable hypotheses associated with equation (3.3).[7] They are: (i) linearity between $E(\tilde{r}_k)$ and $var(\tilde{r}_k)$, (ii) a positive risk-return tradeoff (i.e., $\lambda_k \geq 0$), and (iii) investors hold optimal portfolios while engaging in riskless borrowing and lending (i.e., a joint hypothesis).

There are a number of alternative procedures for testing for linearity. Fama and MacBeth (1973) and Grauer (1978b) used a quadratic risk term (and also the variance of the disturbance term). Their test for linearity was based on the statistical and/or economic significance of the estimated regression coefficient of the squared risk term. Stambaugh (1982) used a Lagrangian multiplier (χ^2) test of linearity. Stambaugh preferred the Lagrangian multiplier test to both the Wald and the likelihood ratio tests because based on Monte Carlo simulations the Wald test rejected the linearity hypothesis too often while the likelihood ratio test did not reject it often enough. However, like MacBeth's (1975) test, Stambaugh's test was conditional upon previous estimates of the regression coefficients. More specifically, while MacBeth assumed that the estimates were made without error, Stambaugh assumed that they were made with a disturbance term. A Box-Cox transformation of the var($\tilde{r}_k$) term can also be used to test for linearity.

However, none of these tests are as satisfactory as one would like. Use of a quadratic risk term (or other ad hoc variables) is somewhat deficient in that one is never sure whether the use of other ad hoc variables, which generally have some economic content, would have supported non-linearity. Use of the Lagrangian multiplier test of linearity is also somewhat deficient in that the test is not independent of the estimates of the regression coefficients, which are themselves only meaningful if it is assumed that the estimated relationship is linear. Furthermore, it appears that the Lagrangian multiplier test is dependent upon the rank of the subset of linearly independent asset returns in the vector of returns used to compute the market index. Finally, the Box-Cox transformation is somewhat deficient in that it implies a greater chance of not accepting the linearity hypothesis (that is, that $\nu \neq 1$ in $[\mathrm{var}(\tilde{r}_k)^{\nu}]$) than of accepting it; that is, that $\nu = 1$.

Since none of these alternative tests results in an adequate test of the linearity hypothesis, it can be argued that a minimalist approach should be followed. That approach is merely to reject (or not to reject) the linearity hypothesis based on: (i) the economic and statistical significance of the regression coefficient associated with var ($\tilde{r}_k$), (ii) the F-value test of the linear regression in (3.3); and (iii) the significance of the determination coefficient of that regression. Nevertheless, to be consistent with the literature, the traditional quadratic risk term is also used herein to provide a further test of linearity.

The hypothesis of a positive risk-return tradeoff can be tested by examining the sign of the regression coefficient associated with $var(\tilde{r}_k)$. As in Grauer (1978b), the third (joint) hypothesis can be tested by comparing the estimated intercept term of the regression in the form of (3.3) with an a priori estimate of $E(\tilde{r}_{zk})$, and by testing if the estimate of $E(\tilde{r}_{zk})$ is statistically different from zero.

Following the tradition established by Fama and MacBeth (1973), Grauer (1978b), amongst others, our tests are conducted with portfolios of assets and not with individual assets. However, unlike other authors who grouped assets into portfolios based on the size of their risk terms in order to alleviate an errors-in-measurement problem, the grouping criterion used herein is that dictated by the theory and is observable in the market (i.e., the portfolios of individual investors).

Machine-readable data from the 1977 "Survey of Consumer Finances", conducted by Statistics Canada,[8] was used to empirically test equation (3.3). The data included the 1976 income, various asset and liability category values as of May 31, 1977, and various additional socio-economic characteristics, for a sample of 12,734 Canadian economic households. Since the data was for economic units (i.e., households) and not for individuals, it was necessary to assume that each household acted as if it were a single (or homogeneous) investor.

Statistics Canada used a multi-stage, stratified, clustered, probability sampling technique to select an initial sample of 17,000 economic units. The sample design purports to cover Canadian economic units of all trades, regions and wealth categories, with the exception of "marginal" units such as those confined to hospitals, prison inmates, residents of Indian reserves, and residents of the Northwest and Yukon Territories. In order "to improve the representation of the asset and debt holdings of Canadians at the upper end of the income and wealth distribution",[9] a special supplementary sample of 500 economic units was selected from the original sample of 17,000. These "special economic units" accounted for 139 of the 12,734 households in the final sample. Since not all households responded to all the questions in the questionnaire, imputed householder data was used for from 2 to 18.1 percent of the responses for any specific question. This imputed data was generally based upon extraneous sources of information, and on the averages computed from households of similar socio-economic characteristics which had responded to the appropriate questions. According to Statistics Canada the reliability of the survey data is "quite good and encouraging".[10]

The information which is available on each household's assets and liabilities are given in Table 3.1. To test equation (3.3), it is necessary to assume that the return distributions for each asset and liability category listed in Table 3.1 are homogeneous (i.e., the same) across households, and then determine the $\underline{E}$ and $\underline{V}$ for the asset and liability categories. Then given the $\underline{E}$ and $\underline{V}$ data and the structure (category compositions) of each household's portfolio, it is fairly easy to calculate the $E(\tilde{r}_k)$ and $\sigma^2(\tilde{r}_k)$ values needed in order to test equation (3.3).

The assumption of homogeneous return distributions across households (HHRD) is subject to criticism. However, it has been shown by Mao (1971) that a large percentage of the potential benefits of diversification is attained with a relatively small number of assets. For example, up to 50 percent of the potential benefits of diversification are attainable with only three securities. Schlarbaum, Lewellen and Lease (1978b, p. 400) found that the average performance of the portfolios of 2,500 investors was not "distinguishable statistically--at any of the commonly employed levels of significance--from (that) ... available from a passive buy-the-market-and-hold investment strategy, nor from the returns generated by the mutual funds". This was the case even though the "average" investor in their sample held from 3.6 to 6.1

Table 3.1

Asset and Liability Categories for each Investor's Portfolio

Asset Categories

ALC 1 Cash on hand, including outstanding cheques.

ALC 2 Bank deposits and all other deposits in financial institutions.

ALC 3 Investment in registered retirement savings plans (RRSP) and in registered homeownership savings plans (RHOSP), at face value plus accrued interest.

ALC 4 Bonds, at face value, including federal government bonds, provincial government bonds, foreign government bonds and industrial corporation bonds.

ALC 5 Stocks and shares, at market value, including those of public corporations, listed and unlisted, of mutual funds, of investment funds and clubs, and also rights and warrants of these stocks and shares.

ALC 6 Equity in business, farm, or profession, computed as the difference between market or book value of total assets, and book value of total debts.

ALC 7 Investment in other non-liquid financial assets, at market value or outstanding amount, including mortgage loans, loans to individuals and businesses (including one's own), interests in trusts and estates, royalties from mines, oil wells, copyrights, and future contracts.

ALC 8 Cars for personal use, at market value.

ALC 9 Owner-occupied home, and other real estate held for personal use, at market value, including vacation home.

Liability Categories

ALC 10 Consumer debts, including loans from chartered banks, sales finance and consumer loan companies, credit unions and caisses populaires, and charge accounts and installment debt; and other personal debt, including loans guaranteed by stocks and bonds, student loans, miscellaneous loans from financial institutions, unpaid bills and taxes, and loans from individuals outside the household, from business firm (including one's own); at outstanding amount.

ALC 11 Mortgage debt on the owner-occupied home, and on other real estate held for personal use, at outstanding amount.

securities at any given point in time over the period from 1964 to 1970. Similarly, Blume and Friend (1975) found that investors, on the average, held 3.4 dividend-paying securities.

Thus, even if the households in the current sample held ALC's which differ in their compositions, one can assume that within each of the 12 broad ALC's the return distributions do not differ materially across households, and can be proxied by a "basket" of the available assets (or liabilities) within each ALC. While this assumption is subject to criticism, it can be argued that it is no more unpalatable than the assumption of homogeneous expectations which has been invoked in all past empirical tests of asset pricing models.

The asset and liability categories (ALC's) in Table 3.1 do not include those that are non-marketable. Since the non-marketable asset, human capital, constitutes a significant portion of the wealth held by most households, an attempt was made to incorporate an estimate of each household's human wealth into its portfolio of assets.

To incorporate human wealth in the investor's portfolio, one is faced with two measurement problems: (i) the measurement of the return on human wealth, and (ii) the measurement of the value of human wealth itself.

Mayers (1972) and Fama and Schwert (1977) were two of the first studies which proposed measurements of the return on human wealth. Mayers (1972) defined the total payoff (income) on all non-marketable assets, or human wealth, as the aggregate income received by the labor force extant at the beginning of the time period. Fama and Schwert (1977) correctly argued that such a measurement ignored the effect of variations in the size of the labor force. Therefore, they used the "income per capital of the labor force to measure the variation through time in the payoff to a unit of human capital." Mayers (1972), as well as Fama and Schwert (1977) were primarily interested in estimating the covariances between income and returns from time series data. Therefore, they had to assume that the bivariate distribution of the income and return variables was stationary through time. However, Fama and Schwert (1977) found that the distribution of per capita income could not be stationary, since it has an upward trend (at least in the aggregate) and it has autocorrelations that were close to one for many lags. They proposed a "standard cure for that type of non-stationarity" by using percentage changes in the per capita income to estimate the covariances between income and return variables.

For all practical purposes, Fama and Schwert (1977) defined income as "wage and salary disbursements plus the proprietors' income portion of seasonally adjusted personal income." However, they also tried other definitions of income such as "net transfer payments" and "total personal income". According to Fama and Schwert, these definitions produced "results similar to those reported...". Fama and Schwert (1977) acknowledged that "there are many legitimate quarrels with the way we measure the return to human capital." They quoted their ignorance of the maintenance cost of the human asset in computing a net payoff to human capital, their disregard for the potential disparity in quality of human wealth, and their assumption that human wealth is a non-marketable asset.

However, Fama and Schwert (1977) did not specifically compute a return on human wealth, but rather the payoff on each unit of human wealth. In other words, they only computed the numerator of what could be a measure of the return on human wealth. Their use of the percent change in the income per capita of the labor force was strictly designed to avoid the autocorrelation problem in directly using the payoff per unit of the labor force.

Nonetheless, Fama and Schwert (1977) appear to have interpreted the percentage change in per capita income as a partial return on human wealth. Discussing the "missing assets" model noted by Mayers (1972), Fama and Schwert (1977) concluded that the percentage change in per capita income is "also the percent capital gain return on human capital".

They reached this conclusion by assuming that the percentage change in per capita income follows a random walk, that the beta of human wealth is constant through time, and that the expected return-risk relationship of the CAPM is also constant through time. Under these conditions, "the discount rate applied to expected future incomes to get the market value of human capital is constant through time, and the market value of human capital is proportional to and thus perfectly correlated with income." Fama and Schwert (1977) used this conclusion to assert:

> the fact that in the missing assets model the percent change in per capital income corresponds only to the capital gain portion of the percent return on human capital is unimportant. In the present scenario, income and the market value of human capital are perfectly correlated. If the capital gain return on human capital is unrelated to the returns on marketable assets, so is the "dividend" return.

The last sentence of the above quote is the principal conclusion drawn by Fama and Schwert (1977) from their paper. However, in the context of this essay, given that income and the market value of human wealth are perfectly correlated, any relationship observed between the returns on marketable assets and the capital gain portion of the return on human wealth would also be true should the total return on human wealth be used. This is the case because the dividend portion of the return on human wealth is proportional to the market value of human wealth, and thus a non-random variable.

In this essay, as well as in the next one, the measurement of the market value of human wealth and of the capital gain portion of the return on human wealth are such that one can invoke the same procedural qualifications that were invoked by Fama and Schwert (1977).

First, the return to human wealth is defined as total personal income per unit of the labor force in Canada. More specifically, total aggregate personal income reported in the Canadian national accounts is divided by the total employed labour force reported by Statistics Canada in order to obtain per capita personal income. Percentage changes of per capita personal income are then construed as the capital gain portion of the return on human wealth.

Second, the market value of human wealth is defined, as suggested by Fama and Schwert (1977), as the present value of all future incomes from the present age up to the mandatory retirement age of 65. This definition implies two further empirical measurement procedures. The first involves the discount rate, and the second involves the estimation of the stream of annual incomes from the present age up to age 65 for each individual in the sample.

While it is difficult to choose a suitable discount rate for future incomes, it seems reasonable to assume that in the long run any increase in the real income from labour will come from an increase in productivity. Therefore, it can be assumed that the riskiness of future incomes is dependent upon the variance of the inflation rate, the variance of the rate of productivity gain, and the covariance of both rates, if any. This riskiness of future incomes should also be reflected in the discount rate. A very conservative measure of the rate of productivity gain is zero.[12] If real incomes are used, then one can abstract from the influence of the inflation rate. Thus, in this essay, the market value of human wealth is defined as the "undiscounted" sum of real incomes from the present age up to age 65, or, alternatively as the present value of all future nominal incomes from the present age up to age 65, where the discount rates are the periodic inflation rates.

This definition of the market value of human wealth implies that the "dividend" portion of the return on human wealth is proportional to the market value of human wealth, although the specific proportion differs from individual-to-individual. (In fact, it equals the reciprocal of the difference between age 65 and the present age, given that real income does not increase with age.) Therefore, for each individual, the covariances between the capital gain portion of the return on human wealth and the returns on marketable assets are also the covariances between the total return on human wealth and the returns on marketable assets.

In order to implement this measure of the market value of human wealth, a procedure must be designed to estimate each year's income from the present age up to the retirement age for each individual. Friedman (1957) has argued that a consumer's behavior is based on permanent income and not on measured income. That is, the decisions of consumers are related not to current incomes (which might include transient components) but on incomes which are considered to be stable. Thus, in order to aggregate an investor's income stream over his or her life cycle in order to obtain a market value estimate of his or her human wealth, one needs to estimate the investor's permanent income. One procedure for doing so is to assume that individuals, who share the same relevant income determinants, will also have the same permanent (but not necessarily measured) incomes. The differences between their measured and their assumed permanent incomes will represent the transient income components of their current incomes. In other words, in the regression of current income on the income determiants, the points on the regression plane are measures of permanent income, while the distances between the regression plane and the actual observations (i.e., error terms) are the measures of transient incomes.

In turn, this suggests that a multiple regression between current before tax income (BTI) against a number of relevant income determinants will yield an estimate of annual permanent income. The income determinants which were chosen herein are: household size (HS) [because BTI is measured for the whole household]; net tangible wealth (NTW); schooling of household head (SHH); occupation of household head (OHH); city size (CS); regional location (RL); and age of household head (AHH). The regression results obtained are:

$$\begin{aligned} BTI = {} & 3950.4 + 2350.6\ HS + 0.05\ NTW + 1342\ SHH - 519.4\ OHH \\ & - 473.4\ CS + 1189\ RL + 35.7\ AHH \end{aligned} \tag{3.4}$$

where the Adj. R = .404, F-value = 589.8, and the sample size = 6080.

All the coefficients are significant at the .001 level, and the adjusted R is reasonable given the cross-sectional nature of the multiple regression. Since equation (3.4) is used structly for predictive purposes, and only a good estimator of annual permanent income (i.e., one with the smallest confidence interval) was desired, some compromises had to be made with econometric orthodoxy. Notably, the indices, occupation of the head of household (OHH) and regional location (RL), were used as if they were numeric variables in estimating (3.4). Nonetheless, for all other income determinants, the sign and magnitude of the coefficients seem plausible. More specifically, equation (3.4) implied that a minimum annual income of $3,950.40 existed for each Canadian household independently of all other income determinants.[13] The larger the household, the larger the income (the rate of growth was $2,350.60 for each additional person in the household). The yield on net tangible wealth is a credible five percent per annum. Each additional 3-to-4 years of schooling of the household head increased annual income by $1,342.00 for the household. Each year of age of the household ~~head increased the~~ annual income of the household by $35.70. Household income decreased with city size (CS is larger for smaller cities), with less socially desirable occupations of the household head (OHH is 1 for managers, ..., 5 for unskilled workers), and increased with westward location of the household (RL is 1 for Maritimes, 2 for Québec, 3 for Ontario, 4 for Prairies and 5 for British Columbia).

The aggregation of permanent income from a household head's present age to the retirement age of 65 was carried out using equation (3.4), where BTI increased with AHH at the annual rate of $35.70. Using the formula for the sum of an arithmetic progression from the present age (i.e., AHH) to the retirement age of 65, with an argument of 35.7, the market value of human wealth is given by:

$$HW = (65 - AHH)\ BTI + 35.7\ (65 - AHH + 1)(65 - AHH)/2 \qquad (3.5)$$

Equation (3.5) is exactly the life-cycle permanent income concept advocated by Friedman (1957), when the rate of growth of permanent income is arithmetic and not geometric, and the sum of an arithmetic progression, and not an integral of an exponential function of time, is used.

The special households were not used in estimating (3.4), because in order to ensure anonimity for these households no values were contained on the tape for RL and AHH. The HW of the special households was estimated in (3.5) by using the mid-range sample values for RL and AHH of 2.5 and 40, respectively. Furthermore, the HW of all households, whose head was more than 65 years old, was arbitrarily set to zero. While it can be argued that these households are likely to have some positive human wealth, the amount must not only be small relative to their net tangible wealth but it is difficult to estimate because of the uncertain date of eventual (and certain) death of each household head.

Data to compute the holding period returns for each of the twelve ALC's were gathered as the annual averages of the monthly one-year holding period returns for the 25-year period from 1952 to 1976.[14] The specific return computation procedure used for each ALC is given in Table 3.2.[15]

Since λ_k is a function of the utility parameters in equation (3.3), it has to be estimated for homogeneous household groupings (that is, households which have identical utility functions). Seven criteria were initially chosen in order to stratify the sample into the required homogeneous household groupings. These criteria were household size, household wealth, schooling of the household head, household location, total household after-tax income, age of household head and occupation of household head. Since the use of all seven criteria would have resulted in stratified sub-samples with few households, two stratification criteria (specifically, total household after-tax income and occupation of household head) were not used because of their high colinearity with the other criteria.[16] Based on the remaining five cri-

Table 3.2

Procedures for the Calculation of Holding Period Returns for the Twelve Asset and Liability Categories (ALC's)

Identifier	ALC	Calculation Procedure
ALC 1	Cash on hand	Return is set to zero.
ALC 2	Bank and other deposits	Arithmetic average of returns on 3-month Treasury Bills, 90-day financial paper, 5-year Guaranteed Deposit Certificates, and saving deposits without chequing privileges.
ALC 3	RRSPs and RHOSPs	10% by the arithmetic average of returns on 5-year Canada's, long-term Canada's, and McLeod, Young & Weir 10 Provincial Bonds; plus 10% by McLeod, Young & Weir 10 Industrial Bonds; plus 15% by returns on stock (see ALC 5); plus 59% by return on mortgage (see ALC 11); and plus 6% by the arithmetic average of returns on 3-month Treasury Bills and 90 day financial paper.[a]
ALC 4	Bonds	Arithmetic average of returns on 5-year Canada's, long-term Canada's, McLeod, Young & Weir 10 Provincial Bonds, and McLeod, Young and Weir 10 Industrial Bonds.
ALC 5	Stocks and shares	Annual average of monthly one-year holding period returns on the Toronto Stock Exchange 300 Index, including dividends.
ALC 6	Equity in Business, Farm or Profession	Before-tax returns on equity for all industries.
ALC 7	Non-liquid financial assets	Arithmetic average of the returns on mortgage loans plus 300 basis points, and on the returns on ALC 6 minus 200 basis points.[b]
ALC 8	Market value of cars	Relative changes in the consumer price index for new car purchases.[c]
ALC 9	Real estate	Relative changes in one of six regional price indexes for housing.[d]

Table 3.2 Continued

Identifier	ALC	Calculation Procedure
ALC 10	Consumer debt	Annual average of the interest rates on prime business loans plus 106 basis points.[e]
ALC 11	Mortgage debt	Annual average of monthly holding period returns on mortgage loans, plus 50 basis points for service fees.
ALC 12	Human wealth	Relative change in per capita personal income.

Notes:

[a]In 1976, the RRSP funds were invested according to the following proportions: 10% in government bonds, 10% in corporate bonds, 15% in stocks, 59% in mortgage loans and 6% in other financial assets, mostly short-term deposits.

[b]It is assumed that non-liquid financial assets are composed equally of second mortgage loans and loans to private firms.

[c]Because a decision to purchase a car (or a home or vacation real estate) is both a consumption and an investment decision, the current market value of a car or house is a reflection of a household's optimal decision to invest in a capital asset which acts as the storage of future consumption for the good "transportation" in the case of a car, or for the good "housing" in the case of real estate. Thus, at the beginning of any period, households can either realize the storage value of the capital assets by selling them, or forego an amount equal to the rate of price increase on similar cars or homes, if the cars or homes are retained. In other words, the storage values of the cars or homes can be viewed as capital assets which provide an annual return which is equal to the relative price changes on similar assets.

[d]The six regional indexes for housing are the Atlantic (St. John's, Halifax and St. John), Québec (Montréal), Ontario (Ottawa and Toronto), Prairies (Winnipeg, Saskatoon/Regina and Edmonton/Calgary) and British Columbia (Vancouver) for households which live in the above five regions and ~~in~~ cities of 100,000 or more inhabitants. All other households were taken as having real estate with a return equal to the relative change on the composite housing price index for Canada. Note that the housing price index was a rent index for 1952/53, and a shelter index for 1953/60. Adjustments were made to insure homogeneity from one index to the next.

[e]During the three-year period, 1975-1977, the rate on new consumer loans was, on average, 1.06 percent over and above the prime rate on business loans (see the Canadian Bankers Institute Magazine, 1978).

teria, the total sample of households was grouped into 432 stratified sub-samples (strata) of supposedly homogeneous households. However, only 33 of these 432 stratified sub-samples consisted of more than one hundred households, and thus were used to test equation (3.3). Some of the characteristics of these 33 stratified subsamples are summarized in Table 3.3.

IV The clinical-level tests of the CAPM in imperfect markets

Using the E and V values, $E(\tilde{r}_k)$ and $\sigma^2(\tilde{r}_k)$ were calculated for the portfolios of the households in each of the 33 retained strata. Then, using the OLS technique, $E(\tilde{r}_k)$ was cross-sectionally regressed on $\sigma^2(\tilde{r}_k)$ to obtain estimates of $E(\tilde{r}_{zk})$ and λ_k for each of the 33 strata of households.

The empirical findings obtained from applying the empirical procedure described earlier are presented in Table 3.4.

Although it will be further tested in the next section of the essay, the findings generally support the first hypothesis that $E(\tilde{r}_k)$ and $\sigma^2(\tilde{r}_k)$ are linearly related. More specifically, not only do the R^2-values compare very favourably with those generally reported for empirical tests of the standard CAPM but the values are significant for all but seven strata for both the nominal and the real returns.

Although the second hypothesis implies that the estimated λ_k should be positive and significant for all 33 strata, this was the case for approximately one-half the strata (namely, sixteen of the strata using nominal returns and sixteen of the strata using real returns). Thus, the number of non-statistically significant and non-positive estimates of the intra-portfolio price of risk for the 33 strata suggests that the second hypothesis is not empirically supported. This finding is however consistent with a number of possible explanations such as the

TABLE 3.3

CHARACTERISTICS OF THE 33 STRATIFIED SUBSAMPLES USED IN EMPIRICALLY TESTING THE CLINICAL CAPM

Wealth	Age of Household Head	Schooling of Household Head	Size of Household	Location	Stratum Identifier
<0	≤35 yrs	9-12 yrs	1-2	Ontario	1
$0 to $24,999	≤35 yrs	9-12 yrs	1-2	Québec	2
				Ontario	3
				Prairies	4
				B.C.	5
			≥3	Atlantic	6
				Québec	7
				Ontario	8
				Prairies	9
		≥13 yrs	1-2	Québec	10
				Ontario	11
	36-65 yrs	≤8 yrs	1-2	Québec	12
			≥3	Atlantic	13
				Québec	14
		9-12 yrs	1-2	Ontario	15
			≥3	Atlantic	16
				Québec	17
	>65 yrs	≤8 yrs	1-2	Atlantic	18
				Québec	19
				Prairies	20
$25,000 to $49,999	36-65 yrs	≤8 yrs	≥3	Atlantic	21
				Québec	22
		9-12 yrs	≥3	Atlantic	23
				Québec	24
				Ontario	25
≥$50,000	≤35 yrs	≤8 yrs	≥3	Ontario	26
				Prairies	27
	36-65 yrs	9-12 yrs	1-2	Ontario	28
				Prairies	29
			≥3	Ontario	30
				Prairies	31
		≥13 yrs	≥3	Ontario	32
	Special	–	–	Special	33

TABLE 3.4

Empirical Results for the Clinical-Level Tests of Equation (3.3)

Stratum Identifier	Nominal Returns $\hat{E}(\tilde{r}_{zk})$	Nominal Returns $\hat{\lambda}_k$	Nominal Returns Adj. R^2	Nominal Returns F-Value	Real Returns $\hat{E}(\tilde{r}_{zk})$	Real Returns $\hat{\lambda}_k$	Real Returns Adj. R^2	Real Returns F-Value
1	6.441	-0.142	.850	800.759	6.019	-0.132	.834	711.323
2	2.123	0.904	.592	239.177	1.881	1.038	.629	279.177
3	2.017	0.924	.327	102.426	1.659	1.095	.371	123.558
4	3.706	0.517	.144	29.912	3.523	0.569	.136	28.169
5	3.648	0.520	.139	14.672	3.095	0.680	.183	19.988
6	3.289	0.602	.312	70.540	3.030	0.692	.329	76.163
7	3.392	0.574	.161	32.245	3.103	0.669	.172	34.801
8	3.045	0.648	.170	32.134	2.521	0.824	.217	43.137
9	4.866	0.211*	.006	1.588*	4.342	0.313*	.016	2.788*
10	2.186	0.892	.572	135.961	2.043	0.994	.575	137.720
11	2.274	0.862	.270	50.643	1.681	1.091	.336	68.937
12	4.995	-0.034*	.0	0.124*	4.820	-0.062*	.0	0.360*
13	5.469	-0.086*	.0	0.472*	5.499	-0.196*	.067	2.176*
14	2.399	0.809	.257	60.785	2.212	0.913	.249	58.318
15	6.709	-0.362	.100	11.921	6.393	-0.403	.116	13.858
16	1.551	1.029	.333	62.367	1.371	1.166	.330	61.647
17	2.379	0.814	.344	67.486	2.189	0.920	.342	67.104
18	1.378	-0.433	.175	22.879	1.293	-0.456	.186	24.526
19	1.439	-0.223	.567	155.664	1.383	-0.234	.596	175.353
20	0.592	-0.014*	.0	0.211*	4.285	.0*	.0	.0*
21	2.156	0.921	.289	43.671	2.022	1.015	.272	40.247
22	2.801	0.736	.159	23.085	2.532	0.858	.161	23.488
23	1.446	1.140	.504	95.631	1.217	1.307	.512	98.550
24	6.534	-0.433	.141	17.293	6.261	-0.500	.158	19.515
25	2.455	0.797	.569	144.820	2.232	0.911	.572	146.913
26	5.145	-0.126	.022	5.075	4.864	-0.126	.018	4.282
27	9.099	-1.266	.220	37.688	8.470	-1.274	.161	25.949
28	4.128	-0.172*	.007	1.822*	3.824	-0.132*	.0	0.942*
29	6.381	-0.812	.174	25.508	5.931	-0.790	.136	19.227
30	4.761	.0*	.0	.0*	4.521	.0*	.0	.0*
31	8.720	-1.136	.297	99.963	8.402	-1.289	.284	94.175
32	5.180	-0.095*	.0	0.889*	4.917	-0.103*	.0	0.888*
33	6.957	-0.368	.282	50.502	6.508	-0.360	.268	47.026

*Not significant at the 0.05 level.

existence of systematic errors-in-measurement in determining $E(\tilde{r}_k)$ and $\sigma^2(\tilde{r}_k)$ and with the likelihood that some groups of investors are not risk averse. Systematic errors-in-measurement are possible because observed before-tax returns were used in this study. Investors (especially those in higher marginal tax brackets) may be making investment decisions based on after-tax returns, and thus may appear to be more risk-seeking on a before-tax basis.

The empirical findings appear to be consistent with the third hypothesis, since all of the estimates of $E(\tilde{r}_{zk})$ are statistically and economically significant. The reasonableness of the estimates of $E(\tilde{r}_{zk})$ are based on the observation that the average annual nominal rate of return on treasury bills for the sampled period was 4.35 percent.

V. A further test of linearity

As noted earlier, while none of the available alternatives leads to an adequate test of the linearity hypothesis, it was arbitrarily decided to follow the tradition established by Fama and MacBeth (1973) for conducting such a test. Thus, a multiple cross-sectional regression of the form (3.6) was run for each strata:

$$E(\tilde{r}_k) = E(\tilde{r}_{zk}) + \lambda_k\ \sigma^2(\tilde{r}_k) + \phi_k\ \sigma^4(\tilde{r}_k)\ , \tag{3.6}$$

where ϕ_k is the coefficient on the quadratic risk term; and

all the other terms are as previously defined.

The summary statistics for these cross-sectional multiple regressions are presented in Table 3.5.

TABLE 3.5

Empirical Results for the Clinical-Level Tests of Equation (3.6)

Stratum identifier	Nominal Returns $\hat{E}(\tilde{r}_{zk})$	Nominal Returns $\hat{\lambda}_k$	Nominal Returns $\hat{\phi}_k$	Nominal Returns Adj. R^2	Nominal Returns F-Value	Real Returns $\hat{E}(\tilde{r}_{zk})$	Real Returns $\hat{\lambda}_k$	Real Returns $\hat{\phi}_k$	Real Returns Adj. R^2	Real Returns F-Value
1	5.275	0.224	-0.021	.866	464.7	5.075	0.207	-0.021	.849	397.8
2	-4.237*	4.162*	-0.416*	.592	120.2	-2.860*	3.869*	-0.422*	.628	139.7
3	5.814*	-1.004*	0.244*	.325	51.1	9.660*	-3.643*	0.701*	.370	62.1
4	-17.182*	11.145	-1.[illegible]50	.165	17.9	-14.112*	11.024	-1.547*	.150	16.1
5	-11.329*	8.105*	-0.959*	.138	7.8	-1.342*	3.296*	-0.385*	.174	9.9
6	11.095*	-3.409*	0.515*	.312	35.7	9.825*	-3.378*	0.609*	.329	38.4
7	39.111	-17.577	2.303	.207	22.3	39.309	-20.795	3.178	.218	23.7
8	4.970*	-0.327*	0.123*	.165	15.9	11.283*	-4.353*	0.764*	.214	21.7
9	45.603	-20.351*	2.592*	.027	2.4	64.113	-34.899	5.182	.078	5.5
10	-21.416	12.789	-1.498	.596	75.4	-23.281	15.875	-2.184	.605	78.4
11	-7.959*	5.984*	-0.641*	.266	25.2	-17.866*	12.489*	-1.661*	.335	34.8
12	1.257	1.570	-0.137	.369	33.5	1.255	1.663	-0.162	.356	31.7
13	-0.352*	2.833	-0.346*	.421	64.9	0.175*	2.718	-0.366	.412	62.7
14	-0.826*	2.943	-0.333	.317	41.1	-1.173*	3.482	-0.463	.309	39.7
15	1.739*	1.577	-0.161	.264	18.5	1.892*	1.582	-0.182	.255	17.8
16	1.206*	1.297*	-0.046*	.328	31.0	0.969*	1.521*	-0.071*	.325	30.7
17	-7.030	6.133	-0.742	.462	55.5	-7.602	7.336	-1.039	.469	57.2
18	-2.024	1.244	-0.167	.548	63.5	-1.833	1.264	-0.183	.576	70.8
19	- .047*	0.141	-0.008	.821	268.8	0.100*	0.114	-0.008	.827	281.4
20	0.941	-0.114*	0.003*	.0	0.[illegible]*	0.716	-0.058*	0.002*	.0	0.2*
21	-1.109*	3.805	-0.577	.331	27.0	-1.685*	4.725	-0.848	.325	26.3
22	4.159	-0.427*	0.228*	.161	12.1	3.672	-0.245*	0.247*	.159	12.1
23	1.996*	0.692*	0.084*	.500	47.5	1.754*	0.809*	0.108*	.507	48.9
24	0.343*	2.337	-0.261	.796	194.2	0.216*	2.603	-0.330	.798	196.3
25	0.264*	2.313	-0.254	.587	78.4	0.058*	2.645	-0.336	.589	79.1
26	4.838	0.017*	-0.005*	.019	2.8*	4.360	0.141*	-0.010*	.019	2.8*
27	7.508	0.118*	-0.274*	.231	20.5	7.006	0.118*	-0.307*	.166	13.9
28	5.080	-0.738	0.045*	.020	2.3*	4.507	-0.581*	0.039*	.003	1.2*
29	7.472	-1.561	0.092*	.194	15.0	6.880	-1.498	0.095*	.147	11.0
30	3.291	1.036	-0.169	.044	7.5	2.857	1.309	-0.240	.055	8.8
31	9.597	-1.904	0.152*	.300	51.2	9.601	-2.425	0.247	.294	49.6
32	4.795	0.130*	-0.029*	.0	0.8*	4.461	0.192*	-0.041*	.0	0.9*
33	9.469	-1.671	-0.152	.405	43.9	8.833	-1.716	0.175	.404	43.7

*Not significant at the 0.05 level.

The results can be summarized as follows: (i) Eighteen and seventeen of the λ_k estimates are not statistically significant for nominal and real return data, respectively; (ii) the number of statistically significant positive λ_k estimates has been reduced slightly from that found earlier (see Table 3.4); and (iii) approximately two-thirds of the estimates of $E(\tilde{r}_{zk})$ are no longer statistically significant, positive and reasonable. However, the adjusted R^2 did increase on average across the strata when the quadratic term is included. Furthermore, the partial correlation coefficients associated with $\sigma^4(\tilde{r}_k)$ are not positive and statistically significant for most of the 33 strata. Thus, the results imply the rejection of the hypothesis that a quadratic relationship exists between $E(\tilde{r}_k)$ and $\sigma^2(\tilde{r}_k)$. However, as noted earlier, one cannot assert unambiguously (or even with much confidence) that the findings are consistent with linearity based on this test alone.

VI. Concluding remarks

The primary purpose of this essay was to empirically test a CAPM in imperfect markets using clinical-level data. At the clinical level, such a model asserts that the expected return on each investor's optimal portfolio is linearly related to its variance of return (as depicted, for example, for investor k in Figure 3.1). Thus, the CAPM in imperfect markets would be empirically validated if the linear relationship between expected return and risk was statistically significant, if the estimated intercept was found to be equal to $E(\tilde{r}_{zk}) > 0$, and the estimated slope, λ_k, was positive.

The OLS technique was used to cross-sectionally estimate $E(\tilde{r}_{zk})$ and λ_k for 33 strata of Canadian households using both nominal and real return data. The findings were somewhat supportive of the CAPM in imperfect markets at the clinical level in that all of the estimates of $E(\tilde{r}_{zk})$ (in the linear models) were statistically significant, positive and reasonable, and almost all of the linear adjustments were statistically significant. The empirical findings were somewhat mixed with regard to the reward for bearing intraportfolio risk, since such estimates were positive and significant for only about one-half of the strata. The introduction of a quadratic term to further test for linearity did not unambiguously support or reject the proposed linear model. While the explanatory power of the model increased, only one-half of the estimated coefficients of the quadratic term were statistically significant. Also, this latter test resulted in a slight reduction in the number of statisticaly significant positive estimates, and in a drastic reduction in the number of $E(\tilde{r}_{zk})$ estimates which were statistically significant, positive and reasonable. However, the last result itself could be ambiguous, because non-significant intercept terms generally imply that the model was initially misspecified.

The major potential deficiency of this research is the HHRD assumption; that is, that the composition of holdings within an ALC is relatively unimportant in that it does not cause the distribution of returns for that ALC for an individual portfolio to differ materially from the distribution of returns for the universe of assets or liabilities in that ALC. In other words, whether an individual actually holds one or all of the available assets or liabilities in that ALC, it was implicitly assumed that the investor held a sufficiently well-diversified portfolio of the assets (or liabilities) in that ALC.

While it is easy to envision how this potential deficiency can be remedied, this remedy is extremely difficult to implement. More specifically, to remedy the potential deficiency, the actual holdings of each household's portfolio for all asset/liability categories (ALC's) and reliable estimates of the means and covariances of all these individual asset/liability returns, are required.[17] However, such data is presently costly, if not impossible, to obtain.

A number of arguments were advanced earlier to justify the palability of the HHRD assumption for each ALC for the purposes of empirical testing. If those arguments suffice, then it can be concluded that many of the households in the 33 strata held optimal portfolios in the sense of equation (3.3). If those arguments do not suffice, then the empirical tests presented herein are in fact joint tests of the validity of equation (3.3) and of the HHRD assumption for the 12 broad ALC's.

FOOTNOTES

1 This is identical to Levy's (1971) equation (6'). In deriving (6'), Levy made no assumption about the nature of aggregation to the market level (i.e., aggregation across both assets and investors), and no assumption about the specific form of investor k's utility function, u_k, except that it was concave and twice differentiable with respect to investor k's wealth.

2 Mayshar (1981) recently proposed a formal formulation of the CAPM which takes into account transaction costs. While transaction costs can be incorporated into the derivation of an asset pricing model, other market imperfections such as the indivisibility of assets, and externalities such as investors' myopia and inertia are more difficult to incorporate into a formal model. Thus, while market imperfections and investors' externalities are invoked in this paper as an explanation for the constrained number of risky assets held in the portfolios of individual investors, they are not formally incorporated into the derived equilibrium pricing relationships.

3 Since $\text{var}(\tilde{r}_k)$ is always a positive quantity, the full-rank variance-covariance matrix of security returns is always positive definite. That is,

$$\underline{x}'\,\underline{V}\,\underline{x} \geq 0 \;\forall_{\underline{x}},\; \underline{x} \neq \underline{0},\; \underline{x}'\underline{I} = 1,$$

where $\underline{x}$ is the column vector of the proportions of wealth invested in the n_k assets considered by investor k;
$\underline{V}$ is the variance-covariance matrix of the n_k assets;
$\underline{0}$ is the null vector; and
$\underline{I}$ is the identity vector.

4 While Levy (1971, p. 653) may have foreshadowed the existence of a relationship such as (3.3) when he stated that "it is obvious ... that variance plays a central role in explaining the risk-return relationship," it appears that he was concerned with the relative impact of a risky asset's variance versus its covariances on its expected return, and not with the effect of "own" variance on the expected return of an investor's optimal portfolio of assets. Nonetheless, the seminal papers by both Levy (1978) and Mao (1971) have directly inspired the research contained herein.

5 The intercept and the slope of the line are obtained simultaneously by drawing the line joining G, i.e., the global-minimum-variance portfolio, to k (i.e., k's dominant portfolio).

6 For a discussion of the equilibrium in an imperfect market when the number of risky assets held is constrained, one is referred to Mao (1971), Levy (1978), Lintner (1965b) or Mayshar (1981). While the knowledge of the conditions of market equilibrium is desirable, it will not be examined in this paper because it does not affect condition (4.3) for individual equilibria. Unfortunately, as is the case for the standard CAPM, both the Mao and Levy models cannot be

tested at the market level because the formulation at that level includes the expected return on the market portfolio.

Hadar (1971, pp. 289-293) described a tatonnement process for the clearing of the market which is similar to that assumed in this essay. In Hadar's mechanism, an auctioneer announces price schedules for all securities, and no securities are exchanged unless a particular schedule clears the market simultaneously for all securities. Hadar showed that, under certain conditions, this tatonnement process will converge to a unique and stable equilibrium.

7 Similar hypotheses have been tested by Fama and MacBeth (1973), MacBeth (1975), Grauer (1978b) and Stambaugh (1982).

8 This data is contained on a microdata tape, "Income (1976), Assets and Debts (1977) of Economic Families and Unattached Individuals," that is available from Statistics Canada. Specific information on the survey, the sampling design and the questionnaire used in collecting the interview data, and on the data base itself, are available from a number of sources. These include: the Microdata Tape User's Manual; Statistics Canada, "Evaluation of Data on Family Assets and Debts, 1977", Publication Cat. no. 13-571, December 1979; and Statistics Canada, "Methodology of the Canadian Labour Force Survey, 1976," Publications Cat. no. 71-526.

Furthermore, various characteristics and descriptive statistics of the households' incomes, assets, debts and portfolio compositions are presented and discussed in Statistics Canada, "Income, Assets and Indebtedness of Families in Canada, 1977," Publication Cat. no. 13-572 (occasional); and in Statistics Canada, "The Distribution of Income and Wealth in Canada, 1977," Publication Cat. no. 13-570 (occasional).

9 Statistics Canada, Microdata Tape User's Manual, p. 79.

10 Statistics Canada, "Evaluation of Data on Family Assets and Debts, 1977," Publication Cat. no. 13-571, December 1979, p. 72.

11 Fama and Schwert (1977) used various definitions of the capital gain portion of the return on human wealth with similar results for the covariances between the returns on marketable assets and the return on human wealth.

12 A long-run rate of growth in productivity of about 1.5 to 2 percent is more probable.

13 The annual welfare benefits for a single adult in 1977 were about 3 600 \$, and the maximum annual unemployment benefits in that year were 3 900 \$.

14 Except for returns on ALC 2, ALC 6 to ALC 10, and ALC 12, all the return data were drawn from Appendix 10, Volume II, "The Retirement Income System in Canada," Task Force on Retirement Income Policy, Department of Finance, 1979. Returns on ALC 2, ALC 7 and ALC 10 were computed using various series drawn from the "Monthly Review,"

Bank of Canada. Returns on ALC 8 and ALC 9 were computed from various series in "Prices and Price Indexes," Statistics Canada, Publications Catalogue no. 62-002. Returns on ALC 6 were computed from "Rate of Return and Investment Profitability," Department of Finance, Long Range and Structural Analysis Division, Catalogue no. F2-481 1980 E. Finally, returns on ALC 12 were computed using various series given in the "Monthly Review," Statistics Canada, Publications Catalogue no. 11-003 F.

15 Real returns were calculated using nominal returns by dividing (1 + the nominal return) by (1 + the relative change in the all item consumer price index from December of one year to December of the next year). For more details on this procedure, see Appendix 10, Vol. II, "The Retirement Income System...," op.cit.

16 Note that because of the definitions of human wealth used in (4.10), the use of total wealth (which includes human wealth) should result in approximately the same stratification.

17 Expectational investor data would also be required to avoid the assumption required for empirical testing of the CAPM; that is, that investors share the same beliefs with respect to the distribution of asset returns.

ESSAY 4

A CLINICAL-LEVEL ASSET PRICING THEORY

1. Introduction

Based on the pioneering work by Markowitz (1959) and Tobin (1958), financial economists have developed a number of different models to depict the process for pricing assets in both perfect and imperfect capital markets. The most prominent of these models during the past decade have been the Sharpe-Lintner-Mossin and the Black versions of the capital asset pricing model (CAPM). Empirical tests of the CAPM have been somewhat unfavourable. While empirical tests by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973) have supported the CAPM; tests by Blume (1968), Friend and Blume (1970), Blume and Friend (1973), Cheng and Grauer (1980), Banz (1981) and Reinganum (1981a, 1981b) have not supported the CAPM. Roll (1977) has shown that all of these past tests have been ambiguous, since they have incorrectly tested the linear relationship between ex-ante return and risk and not the mean-variance efficiency of the market portfolio. Furthermore, since the market portfolio can not be observed (and thus measured), Roll asserts that it is not possible to unambiguously test the CAPM.

As the empirical and theoretical evidence against the CAPM mounted, the search for a more adequate asset pricing model intensified. Although many replacements for the CAPM have been proposed, the most promising ones appear to be Ross's (1976a, 1976b) arbitrage pricing model and Breeden's (1979) consumption-based intertemporal model.[1]

In a similar vein, this essay attempts to formulate a model that is more acceptable in terms of its predictions than the basic CAPM. The model presented herein follows from the discussion in the preceeding essay on individual investor equilibria. In turn, that study was based on the seminal studies by Lintner (1965b), Mao (1971), Rubinstein

(1973b) and Levy (1978) on asset pricing in imperfect markets (more specifically, the case where the number of assets held by each investor is constrained to some number which is less than the number of assets in the universe). Although the theory proposed herein assumes the existence of capital market imperfections (such as transaction costs and portfolio management costs) and that investors have homogeneous expectations with respect to the means, variances and covariances of returns for all securities, it makes no assumption with regard to the form of each investor's utility function, and it allows these utility functions to differ from investor to investor. In fact, unlike modern portfolio theory, the theory proposed herein is consistent with traditional portfolio management practice that maintains that portfolio selection is not independent of individual preferences and circumstances. Because the orientation of this theory is with the equilibria of individual investors, we propose that it can appropriately be referred to as the "clinical-level asset pricing theory (CAPT)".

The remainder of this essay is organized as follows: In the next section, the clinical-level asset pricing theory is formulated. In section III, a salient feature of the CAPT is discussed. In section IV, a procedure for testing the CAPT is outlined. In section V, the empirical results are presented and discussed. And finally, in section VI, some concluding remarks are offered.

II. The clinical-level asset pricing theory (CAPT)

Assume that investor k attempts to maximize his or her expected utility of wealth and consumption in (4.1), subject to his or her budget constraint (4.2):

$$E[u_k(\tilde{W}_k, c_k)] \quad , \tag{4.1}$$

where u_k is the monotonically increasing and strictly concave utility function of investor k;

$\tilde{w}_k$ is the random end-of-period wealth of investor k;

c_k is the consumption by investor k during the period; and

E(.) is the expectations operator.

$$y_k = c_k + \Sigma_j w_{kj} + w_{kf} \quad , \tag{4.2}$$

where y_k is the income of investor k for the period;

w_{kj} is the amount of investor k's wealth invested in risky asset j, $j= 1,\ldots,J_k$ (where J_k is the number of risky assets out of the n available risky assets that are held by investor k);

w_{kf} is the amount of investor k's wealth invested in the risk-free asset; and

all the other variables are as defined earlier.

It can be noted that the initial endowment is thus: $\Sigma_j w_{kj} + w_{kf}$.

Furthermore, while c_k is a flow and w_k is a stock, they are proportionally related in Samuelson's (1969) and Merton's (1969) multiperiod models. Therefore, the optimization problem (1)-(2) can be construed as the one-period version of the multiperiod models where the consumer-investor attempts to maximize the expected utility of the sole variable c_k over his or her lifetime.

Let:

$$\tilde{w}_k = \Sigma_j w_{kj} \tilde{r}_j + w_{kf} r_f \quad , \tag{4.3}$$

where $\tilde{r}_j$ is the random return on risky asset j;

r_f is the known return on the risk-free asset held by investor k; and

all the other variables are as defined earlier.

Thus, investor k's problem is to maximize the Langrángian L_k:

$$L_k = E[u_k(\tilde{w}_k, c_k)] + \mu(y_k - c_k - \Sigma_j w_{kj} - w_{kf}) , \qquad (4.4)$$

where μ is the Langrangian multiplier.

Two of the first order conditions for the optimal solution of (4.4) are:

$$\partial L_k / \partial w_{kj} = E[(u_k') \tilde{r}_j] - \mu = 0 \quad \forall_j,\ j=1,\ldots,J_k; \text{ and} \qquad (4.5)$$

$$\partial L_k / \partial w_{kf} = E[(u_k') r_f] - \mu = 0, \qquad (4.6)$$

where $u_k' = \partial u_k / \partial \tilde{w}_k$.

Using (4.5) and (4.6), one obtains:

$$E[u_k' (\tilde{r}_j - r_f)] = 0. \qquad (4.7)$$

Or, using basic statistical calculus, one obtains:

$$\text{cov}(u_k', \tilde{r}_j - r_f) + E(u_k') E(\tilde{r}_j - r_f) = 0. \qquad (4.8)$$

Rewriting (4.8), one obtains:

$$\text{cov}(u_k', \tilde{r}_j) + E(u_k') [E(\tilde{r}_j) - r_f] = 0. \qquad (4.9)$$

Because (4.9) holds for each risky asset j, $j = 1,\ldots, J_k$, one can aggregate (4.9) over the J_k risky assets held by investor k. In this aggregation, each risky asset j is weighted by its value relative to the total value of all J_k risky assets in investor k's portfolio; that is, the weight for risky asset j is x_{kj} which, in turn, is equal to w_{kj}/w_k. Such an aggregation for investor k gives:

$$\text{cov}(u_k', \Sigma_j\, x_{kj}\tilde{r}_j) + E(u_k')[E(\Sigma_j\, x_{kj}\tilde{r}_j) - r_f] = 0 \,. \tag{4.10}$$

or , if $\tilde{r}_k = \Sigma_j\, x_{kj}\, \tilde{r}_j$:

$$\text{cov}\,(u_k', \tilde{r}_k) + E(u_k')\,[E(\tilde{r}_k) - r_f] = 0. \tag{4.11}$$

Replacing $E(u'_k)$ in (4.9) by its implied value in (4.11) yields:

$$E(\tilde{r}_j) - r_f = \text{cov}(u_k', \tilde{r}_j)\, \frac{E(\tilde{r}_k) - r_f}{\text{cov}(u_k', \tilde{r}_k)} \,. \tag{4.12}$$

If (as was previously assumed) investor k's utility function is monotonically increasing and strictly concave, both $\text{cov}(u'_k, \tilde{r}_j)$ and $\text{cov}(u'_k, \tilde{r}_k)$ will be strictly negative. This is because $u''_k = \partial u'_k/\partial \tilde{w}_k < 0$ and $\partial w_k/\partial \tilde{r}_k$ (or $\partial \tilde{w}_k/\partial \tilde{r}_j$)>0. Therefore, $\partial u'_k/\partial \tilde{r}_k$ (or $\partial u'_k/\partial \tilde{r}_j$) is strictly negative. Thus, equation (4.12) can be rewritten in either of two more familiar forms. The first is:

$$E(\tilde{r}_j) - r_f = \lambda_k\, \text{cov}(u_k', \tilde{r}_j) \tag{4.13}$$

where

$$\lambda_k = \frac{E(\tilde{r}_k) - r_f}{\text{cov}(u_k', \tilde{r}_k)} \;; \quad \lambda_k \leq 0 \;\forall_k$$

The second is:

$$E(\tilde{r}_j) - r_f = \theta_{kj}\, [E(\tilde{r}_k) - r_f] \,, \tag{4.14}$$

where

$$\theta_{kj} = \frac{\text{cov}(u_k', \tilde{r}_j)}{\text{cov}(u_k', \tilde{r}_k)} \;; \quad \theta_{kj} \geq 0 \;\; \forall_k \,, \forall_j \,.$$

It should be noted that r_f can be redefined as the return on that asset the covariance of which with u''_k is null. Then that asset will take on the same meaning and economic significance as in the CAPM or in the clinical variant of the CAPM discussed in the previous essay. From section III on, one will use that generalization of the asset considered "risk-free" by each investor individually.

III. A utility-free asset pricing model

A salient feature of the CAPT can be readily drawn from equation (4.14). The equilibrium relationship between the expected return and the systematic risk of each asset as denoted by equation (4.14) is less restrictive than that given by the CAPM. In particular, the measure of systematic risk in the CAPT, namely θ_{kj}, is more "general" or "personalized" than the corresponding measure in the CAPM, namely β_j, j= 1,..., n. However, it should be noted that that generalization is obtained at a cost. The cost is the possibility of having a capital market where trading occurs perpetually and continuously and where clearance is a tatonnement process.

In the CAPM, the equilibrium pricing relationship for an individual asset j is given by equation (0.2). Although the CAPM-implied equation (0.2) and the CAPT-implied equation (4.14) are similar, their two differences are readily apparent. The first difference is that the expected return of the portfolio of investor k, and not the expected return on the market portfolio, is on the RHS of (4.14). This difference is due to the different levels of aggregation used in both approaches; while the CAPM aggregated to the market level, the CAPT only aggregates to the individual investor level. Although in the CAPT it is possible to aggregate to the market level, the resulting market equilibrium would not be testable because it would require that the unobservable market portfolio be measured.[3] While the existence of an equilibrium at the market level is necessary for equation (4.14) to have any economic meaning, it has been shown in the previous essay that optimality at the individual investor level is compatible with a market setting where trading is perpetual and continuous and where clearance is a tatonnement process, provided that all investors are price takers and hold homogeneous expectations.[4] Thus, while the exact specification of the market equilibrium condition is both desirable and interesting in its own right, it is not necessary in order to validate equation (4.14) and thus the CAPT. The second difference is that the general formulation of the systematic risk of security j, namely θ_{kj}, and not the familiar β_j, is on the RHS of (4.14).

Since it can be assumed that each investor has a unique utility function, θ_{kj} is essentially a "personalized" and thus less restrictive measure of the systematic risk of asset j.[5] As a result, in the CAPT, there is no single market price of risk but rather what Mao (1971) has referred to as "intra-portfolio" prices of risk. Thus, in a CAPT framework, investors measure the systematic risk of any asset j relative to their own portfolios and their own individual utility functions and not relative to the market portfolio.

IV. A procedure for testing the CAPT

If the required panel data on the composition of individual (or institutional) investor portfolios are available, equation (4.14) can be empirically tested. One such test would be essentially patterned after the two-step procedure commonly used to test the CAPM. In step one, it is assumed that the stochastic return generating process can be adequately depicted by (and estimated from) a single-factor market model (SFM) such as a CAPT variant of the SFM model given in chapter two. In step two, two predictive relationships derived from (4.14) could then be tested using a hold-out sample.

A CAPT SFM for depicting the stochastic return generating process for securities might take the form:

$$\tilde{r}_{jt} = \tilde{r}_{zkt} + \theta_{kj}\,[\tilde{r}_{kt} - \tilde{r}_{zkt}], \qquad (4.15)$$

where $\tilde{r}_{zkt}$ denotes the return on the investor-specific zero-beta asset;

t denotes the time period; and

all other variables are as previously defined.

However, if regression techniques are used to estimate θ_{kj}, this would imply that:

$$\theta_{kj} = \text{cov}(\tilde{r}_j, \tilde{r}_k) \,/\, \text{var}(\tilde{r}_k) . \qquad (4.16)$$

Thus, if tested in this form, the CAPT would in essence be another clinical-level version of the CAPM, which has already been tested in the preceeding essay.

Therefore, it is necessary to estimate θ_{kj} from (4.14) without invoking the existence of a specific stochastic return generating model. Since it has been assumed that investors hold homogeneous expectations with respect to the return distributions of assets, θ_{kj} cannot be estimated directly from (4.14). To estimate θ_{kj}, it is first necessary to obtain a modified version of that equilibrium relationship. From (4.14) where r_f will now be replaced by $E_{zk}=E(\tilde{r}_{zk})$, write:

$$E(\tilde{r}_j) - E_{zk} = \theta_{kj}\,[\textstyle\sum_i x_{ki}\, E(\tilde{r}_i) - E_{zk}] . \qquad (4.17)$$

Using (4.17), write:

$$E(\tilde{r}_j) - E_{zk} = \theta_{kj} x_{kj} E(\tilde{r}_j) + \theta_{kj} \sum_{i \neq j} x_{ki}\, E(\tilde{r}_i) - \theta_{kj} E_{zk} ; \qquad (4.18)$$

or:

$$\theta_{kj} x_{kj} E(\tilde{r}_j) = E(\tilde{r}_j) - E_{zk} - \theta_{kj} E(\tilde{r}_{k:j}) + \theta_{kj} E_{zk} , \quad (4.19)$$

where $E(\tilde{r}_{k:j}) = \Sigma_{i \neq j} x_{ki} E(\tilde{r}_i)$.

With some re-arrangement of (4.19), one obtains:

$$x_{kj} = \frac{E(\tilde{r}_j) - E_{zk}(1 - \theta_{kj})}{\theta_{kj} E(\tilde{r}_j)} - E^{-1}(\tilde{r}_j) E(\tilde{r}_{k:j}) . \quad (4.20)$$

Since equation (4.20) is a re-arrangement of equation (4.14) it has the same economic interpretation. However, unlike (4.14) no colinearity between the dependent variable , $E(\tilde{r}_j)$, and the independent variable, $E(\tilde{r}_k)$ in equation (4:20) exists because $E(\tilde{r}_j)$ has been excluded from $E(\tilde{r}_k)$ in (4.20). This has the effect of showing that the weight that investor k would, at the optimum, give to asset j is in proportion to the ratio $E(\tilde{r}_{k:j})$ to $E(\tilde{r}_j)$, that is to the expected return on the portfolio, of which the asset j is excluded, relative to the expected return on asset j itself.

Using (4.20), θ_{kj} can be estimated by regressing x_{kj} against $E(\tilde{r}_{k:j})$. The estimated slope of such a regression line is an estimate of $-E^{-1}(\tilde{r}_j)$ and the estimated intercept, A, is an estimate of $\theta_{kj}^{-1} - \theta_{kj}^{-1} . E^{-1}(\tilde{r}_j) E_{zk}(1 - \theta_{kj})$. Therefore, an estimate of θ_{kj} is: $[E(\tilde{r}_j) - E_{zk}] / [A_{kj} E(\tilde{r}_j) - E_{zk}]$.

Given an estimate of θ_{kj}, the second step of the procedure for testing the predictions of (4.14) could be implemented by either regressing $E(\tilde{r}_j)$ against θ_{kj}, or by regressing $E(\tilde{r}_k)$ against θ_{kj}^{-1}. The regression of $E(\tilde{r}_j)$ against θ_{kj} is done for each stratum (there are thus 33 such regressions) and involves only 12 observations, one for each asset. The regression of $E(\tilde{r}_k)$ against θ_{kj}^{-1} is done for each asset (there are thus 12 such regressions) and involves 33 observations, one for each stratum. Equation (4.14) would then be vindicated if in the first regression the estimated intercept is equal to E_{zk} and the estimated slope is equal to $E(\tilde{r}_k) - E_{zk}$, and if in the second regression the estimated intercept is equal to E_{zk} and the estimated slope is equal to $E(\tilde{r}_j) - E_{zk}$.

It is important to note that since the θ_{kj} values are estimated cross-sectionally, they are stationary if it is assumed that the asset return distributions are intertemporally stationary.[6] This latter assumption was invoked earlier in order to derive the mean vector and the covariance matrix of asset returns.

The same data base and stratification scheme as was used in Essay 3 is used to test equation (4.14). However, because of the different empirical procedures used in this essay, the sample sizes in each stratum have changed somewhat. As in essay 3, there are three testable hypotheses associated with equation (4.14). They are: (i) linearity between $E(\tilde{r}_j)$ and θ_{kj} (or between $E(\tilde{r}_k)$ and θ_{kj}^{-1}), (ii) a positive risk-return tradeoff (i.e. $E(\tilde{r}_k) \geq E_{zk}$ or $E(\tilde{r}_j) \geq E_{zk}$, and (iii) investors hold optimal portfolios while engaging in riskless borrowing and lending (i.e. $\tilde{r}_{zk}$ should be proxied by traditionally known "risk-free" returns). All three hypotheses are tested next.

V. Empirical results

In the first step of the empirical procedure, regressions of the form (4.20) were run for each household stratum and for each asset and liability category. This provided estimates of the intercept term, A_{kj}, for each household strata, and for each asset and liability category. Using these A_{kj} estimates, estimates of θ_{kj} were derived using the procedures outlined in the previous section of the essay.

The 396 (that is, 33 strata by 12 asset and liability categories) regressions were generally statistically significant, especially for strata of households which were wealthier, better educated and in their "productive" years (i.e., between 35 and 65 years old), and for such asset and liability categories as cash (ALC1), bank and other deposits (ALC2), real estate (ALC9), consumer debt (ALC10), mortgage debt (ALC11) and human capital (ALC12).

In estimating θ_{kj} using A_{kj}, it was necessary to estimate E_{zk}. Since the zero-beta asset might be endogeneous and investor specific, the expected return on a portfolio orthogonal to investor k's portfolio was calculated for each household stratum and used herein.

In the second step of the empirical procedure, $E(\tilde{r}_j)$ was regressed against θ_{kj} across the 33 household strata for each of the 12 asset and liability categories, and $E(\tilde{r}_k)$ was regressed against θ_{kj}^{-1} across the 12 asset and liability categories for each of the 33 household strata. These empirical results are presented in Tables 4.1 and 4.2.

While the results given in Tables 4.1 and 4.2 do not provide overwhelming support for or against the linearity hypothesis, there is some evidence that for some groupings of economic units, and for some types of assets, the CAPT linear relationship as depicted by (4.14) holds. For example, for RRSPs and RHOSPs (ALC3), bonds (ALC4, nominal returns), human wealth (ALC12, nominal returns), stocks (ALC5, real returns), non-liquid financial assets (ALC7, real returns), cars (ALC8, real returns), equation (4.14) seems to hold. Also, strata 1 (negative net tangible wealth), 3, 4, 5, 13, 19, 21 and 23 all hold portfolios which can be characterized by (5.14), both in nominal and real terms. However, because 57% of the regressions of $E(\tilde{r}_j)$ against θ_{kj}, both using nominal and real returns, are not significant, and because 58.1% and 41.5% of the regressions of $E(\tilde{r}_k)$ against θ_{kj}^{-1}, using nominal

Table 4.1

Empirical Results for the Regressions of

$E(\bar{r}_j)$ Against $\bar{\theta}_{kj}$

Stratum Identifier	Nominal Returns				Real Returns			
	E_{zK}	$E(\bar{r}_k) - E_{zK}$	Adj. R^2	F-Value	E_{zK}	$E(\bar{r}_k) - E_{zK}$	Adj. R^2	F-Value
1	8.487	-4.37	.447	9.07	5.122	-2.865	.516	11.67
2	7.179	.028*	0.	.03*	2.247*	.825*	.158	2.87*
3	7.233	-1.55	.455	9.35	3.793	-2.384	.753	34.56
4	7.316	.095*	0.	.04*	2.422	- .145	.639	18.71
5	8.703	-5.394	.612	15.39	4.65	-3.619	.8	37.21
6	5.847	1.537	.418	7.47	3.605	- .686	.327	5.37
7	7.639	- .861*	0.	.31*	2.367*	.964*	0.	.99*
8	7.225	-1.004*	.057	1.66*	3.833	-3.353	.808	43.02
9	IND	IND	0.	IND	2.717*	.607*	0.	.33*
10	6.802	.027*	.033	1.38*	2.8	-2.189	.805	46.29
11	6.345	-2.359	.749	33.88	IND	IND	0.	IND
12	9.224	-3 195*	.047	1.34*	4.412*	-1 154*	0	.39*
13	6.572	-1 062	.543	12.9	2.786	-1.245	.726	30.21
14	7.423	- .233*	0.	.01*	4.394*	-1.132*	0.	28*
15	5.805	1.313	.268	5.03	1.802*	1.107	.327	6.35
16	6 893	-3.676	.635	20.13	3.041*	.342*	0.	.13*
17	6.933	-2.033	.671	19.39	4.305*	-1.331*	0	.28*
18	7.047	-3.18	.535	9.06	1.149*	- .32	.844	33.52
19	10.342	-6.848	.741	15.34	5.632	-3.151	.956	88.64
20	7.921	-1.792*	0.	.59*	3.432*	- .231*	0	.05*
21	6.309	- .733	.636	18.49	2.624	- 673	.729	27.88
22	IND	IND	0.	IND	3.38	- .118*	0.	.02*
23	7.74	-4.631	.753	31.47	3.722	-2 508	.795	39.86
24	7.399	- .335*	0.	.05*	2.665	.611*	.083	1.99*
25	6.586	.676*	.126	2.59	3.231	-2.823	.85	57.71
26	7.285	- .814*	0.	.16*	3.514	- .297*	0.	.28*
27	8.691	-3.293*	.011	1.12*	4.659	-2.1*	-.026	1.24*
28	7.029	- .455*	0.	.38*	3.276	- .452*	0.	.95*
29	7.667	-2.127*	.001	1.02*	3.314	- .643*	0.	.71*
30	7.364	- .149*	0.	.03*	3.082	.314*	0.	.32*
31	6 277	1.286*	.015	1.17*	3.993	-1.727*	054	1.63*
32	7.528	- .575*	0.	.26*	3.579	- .61*	0.	.48*
33	IND	IND	0.	IND	2.827*	.648*	0.	.55*

* Not significant at the .05 level

IND: Indeterminate

Table 4.2
Empirical Results for the Regressions of
$E(\tilde{r}_k)$ Against θ_{kj}^{-1}

ALC Identifier	Nominal returns				Real returns			
	E_{zk}	$E(\tilde{r}_j)-E_{zk}$	Adj. R^2	F-Value	E_{zk}	$E(\tilde{r}_j)-E_{zk}$	Adj. R^2	F-Value
ALC2	7.159	.002*	0.	.12*	2.92	.498*	.08	3.52
ALC3	7.238	.045	.257	10.32	3.393	.005	.15	5.42
ALC4	8.35	-.538	.249	10.94	3.598	-.071*	0.	.14*
ALC5	7.203	.008*	0.	.04*	2.76	-.349	.33	12.85
ALC6	7.097	.339*	0.	.99*	3.485	.206*	0.	.22*
ALC7	6.825	-.585*	.132	4.94	2.954	-1.061	.261	9.48
ALC8	16.706	-7.679*	.064	3.17*	-2.839	10.987	.791	118.49
ALC9	6.67	.411*	0.	.66*	-2.563*	6.386*	.08	3.79*
ALC10	7.095	-.024*	0.	.45*	IND	IND	0.	IND
ALC11	7.196	.003*	.001	1.04*	IND	IND	0.	IND
ALC12	7.065	.254	.148	6.05	3.407	.202*	-.002	1.06

* Not significant at the .05 level.
IND: Indeterminate

and real returns, respectively, are not significant, one cannot assert that the linearity hypothesis should be accepted.

With regard to the riskless borrowing and lending hypothesis and the positive risk-return tradeoff hypothesis (i.e. the estimates of E_{zk} and $E(\tilde{r}_j) - E_{zk}$, respectively), it is interesting to note the following. First, the estimates of r_f appear to be within the 6%-10% range in nominal terms, and the 1%-5% range in real terms, and are generally significant at the .05 level. Second, because of the predominant negative sign of $E(\tilde{r}_k) - E_{zk}$, it would seem that investors are not positively rewarded for bearing greater intra-portfolio risk. However, if investors make decisions based on after-tax returns and the tax "bite" per dollar of return is lower on more risky assets than less risky due, for example, to favourable tax treatment of certain stocks and bonds (such as movies, drilling funds and low dividend-paying stocks), equity in business, farm or profession (such as the lower tax rate on small business income), market value of cars, and real estate, then this anomaly is explainable and is probably due to an omitted variable, personal taxes.

Since the majority of the E_{zk} estimates are economically and statistically significant, both in nominal and real terms, one could accept the hypothesis that investors do borrow and/or lend at a riskless rate while they engage in the process of finding their optimal portfolio. However, the relatively large number of non-significant regressions also implies that the estimated intercept might be an estimate of the mean portfolio return, from which the E_{zk} estimates might not be distinguishable. Therefore, the evidence of the third hypothesis is not clearly supportive. Moreover, since most of the estimates of $[E(\tilde{r}_k) - E_{zk}]$ are negative and/or statistically non-significant, one cannot accept the hypothesis of a positive risk-return trade-off. It should be noted nonetheless that the rejection of this hypothesis might be caused by an omitted variable, personal taxes. Furthermore, this result is very similar to that obtained in Essay 3 and might be explained by the same systematic errors-in-measurement problem.

VI. Concluding remarks

In this essay, an asset pricing theory that is more acceptable in terms of its predictions than the basic CAPM was formulated and discussed. The theory incorporated the empirical observation (and theoretical demonstration) that in imperfect capital markets individual investors attain individual (and unique) equilibria. Such equilibria are characterized by differences in both the number and types of assets held by each investor, and by the strong possibility that the number of assets held by each investor is less than the number of assets in the universe. While the proposed theory assumed the existence of capital market imperfections (such as transaction costs and portfolio management costs) and that investors have homogeneous expectations with respect to asset return distributions, it made no assumption with regard to the form of each investor's utility function and it allowed for the possibility that these utility functions could differ from investor to investor. Because of its preoccupation with the equilibria of individual investors, the model was labeled as being the "clinical-level asset pricing theory (CAPT)".

Three of the important predictions of the model are as follows. Firstly, in equilibrium, there is no single market price of risk but rather various "intra-portfolio" prices of risk. Secondly, in equilibrium, investors measure the systematic risk of any asset relative to their own portfolio and their own (and possibily idiosyncratic) utility functions and not relative to some market portfolio. Thirdly, in a CAPT world, the "risk-free" asset can be expected to be investor-specific and thus endogenous because it is the asset for which the correlation coefficient between that asset's return and an investor's marginal utility is nil.

A two-step empirical procedure was used to test the CAPT. The empirical results were mixed in that the linear relationship was empirically supported only for some types of assets and for some groups of investors. In addition, while the estimates of the zero beta expected returns were generally both statistically and economically significant, the reward for risk bearing at the clinical level was negative when personal taxes were unaccounted for.

FOOTNOTES

1 Breeden's model is an extension of the intertemporal CAPM that was formulated by Merton (1973).

2 If such a constraint does not exist, the proposed model will be identical to the CAPM.

3 The market equilibrium condition can be obtained by aggregating a re-arranged version of (5.14) across all K individual investors. In this aggregation, each individual investor's portfolio is given a weight of x_k, where x_k is found by taking the investor's wealth at risk relative to the total collective wealth at risk. Such an aggregation gives:

$$\Sigma_k \, x_k \, \theta_{kj}^{-1} \, [E(\tilde{r}_j) - r_f] = E[\Sigma_k \, x_k \, \tilde{r}_k] - r_f;$$

or

$$E(\tilde{r}_j) - r_f = \psi_j \, [E(\tilde{r}_m) - r_f] \, ,$$

where

$\psi_j = 1/[\Sigma_k x_k \theta_{kj}^{-1}]$ and is the reciprocal of the market average of the reciprocals of $\theta_{kj} = 1,\ldots,k$.

4 See Hadar (1971) or the discussion in essay 3 about the market clearance mechanism as a tatonnement process.

5 To empirically test (4.14), this assumption has to be relaxed somewhat. More specifically, if panel data on individual investors is used, it is necessary to assume that individual investors that have been grouped into homogeneous socio-economic strata will share the same utility function.

6 It has been shown in Essay 2 that time-series estimates of β_j cannot be intertemporally stable even when the return distributions are assumed to be intertemporally stationary.

CHAPTER FOUR

THE ARBITRAGE PRICING THEORY

Due to a number of serious theoretical and empirical difficulties involved with the CAPM, there has been renewed interest in deriving and testing alternative asset pricing models. Of these models, the arbitrage pricing model (APM) proposed by Ross (1976a, 1976b) has attracted the most interest because of the elegant simplicity of its theoretical development, its promise for practical application and its potential for unambiguous empirical validation. To date, the empirical testing of the APM has attempted to answer two questions: First, how many factors are "priced" in the market? This is an important question because the APM would be considered an unsatisfactory model if the number of factors was equal (or nearly equal) to the number of assets in the universe, or if the number was so large that it severely inhibited the empirical testing and the application of the model. Second, how closely does the ex-post market pricing relationship conform to the ex-ante theoretically-derived (linear) functional relationship between security returns and the underlying return-generating factors? The findings of the initial empirical tests of the APM are generally supportive of the model; studies by Gehr (1975), Roll and Ross (1980), Chen (1981), Hughes-Brennan (1981), Berges (1982) and Brown and Weinstein (1982) support the validity of the APM, while a study by Reinganum (1981) does not support the APM.

While these empirical studies have made an important contribution to the testing of the APM, they have (understandably given the amount of statistical analysis to be carried out) been only partial (incomplete) tests. In each of the studies, one or more of the assumptions, which must necessarily be empirically verified in order to ensure that the APM can be tested unambiguously using time-series data, have been left untested or insufficiently tested. Before proceeding, it must be emphasized that while these assumptions are not required in the theoretical derivation of the APM, they are necessary in order to empirically test that model. There are four assumptions which should be empirically validated before the APM can be unambiguously tested using time-series data. The first assumption is that the mean vector, $\underline{E}$, and the variance covariance matrix, $\underline{V}$, of security returns is intertemporally stationary. Although all one-period asset pricing models are derived without invoking the assumption that the $\underline{E}$ and $\underline{V}$ of individual assets are intertemporally stationary, such an assumption is necessary when these models are empirically tested using time-series data. Furthermore, while a number of authors [see, for example, Roll (1977)] have pointed out that few empirical tests of asset pricing models would be feasible without invoking such an assumption, it appears that no direct test of the stationarity of $\underline{E}$ and $\underline{V}$ has yet appeared in the published literature.

In this thesis, direct tests of the stationarity of $\underline{E}$ and $\underline{V}$ were presented in Essay 1. In that essay, it was found that only 11 random sample pairs among the 88 random sample pairs studied exhibited stationarity in $\underline{E}$ and $\underline{V}$. Therefore, any test of the APM (or any traditional test of the CAPM), which uses time series return data could be inappropriate if the sample of security returns was not first checked for stationarity in $\underline{E}$ and $\underline{V}$. The reason is that the risk measures implied by the CAPM and the APM are results of the decomposition of the $\underline{V}$ matrix. If such a matrix was not stationary, there is no reason to believe that the risk measures would be so, and thus any tests, based on time-series data, using them would be self-defeating by construction. Thus, the test of the APM reported in Essay 6 only uses the 11 sample-pairs for which $\underline{E}$ and $\underline{V}$ were found to be stationary in Essay 1.

The second assumption is that security returns are characterized by an explicit and unique underlying factor structure composed of one or more general or common factors.[1] In all past tests of the CAPM the importance of this assumption has been recognized. More specifically, while the CAPM does not assume any specific stochastic return-generating model (i.e., no specific underlying factor structure), all past empirical tests of the CAPM have used either the single-factor market (SFM) model or the two-factor market (TFM) model.[2]

On the other hand, no specific number of factors have been assumed in past empirical tests of the APM, although the factor structure assumption has been tested by Gehr (1975) and by Roll and Ross (1980). Since the major purpose of the paper by Roll and Ross was to empirically validate the APM, they did not, a priori, propose that the factor structure would consist of a fixed number of priced factors, nor did they attempt to identify the general factors in the APM.[3] While these considerations may not be essential for testing the APM, a complete understanding of the basic underlying structure of security returns (especially for practical applications) requires answers to the following three questions: (i) How many general factors impact upon security returns? (ii) Are two of these general factors the so-called "market" factor (i.e., the market portfolio) and its mean-variance efficient orthogonal portfolio (i.e., the zero-beta portfolio)? (iii) What is the identity of each of the relevant general factors? Preliminary answers to the first of these questions have been given in Kryzanowski and To (1983). The remaining two questions are in themselves the subject of lengthy research and will not be dealt with in this dissertation.

The third assumption is that the underlying factor structure of security returns is congruent. Thus, it is important to determine if the underlying factor structure is exactly replicable across various asset subsets of the asset universe for the same time period, and across various time periods for the same asset subset. This is important because the APM (or any other asset pricing model) would not be considered a "satisfactory" theory if it resulted in different (essentially ex-post) explanations for different samples of the same (homogeneous) population of assets or for the same sample of assets for different time periods. This problem was only partially and indirectly dealt with by Gehr (1975), Roll and Ross (1980), Hughes (1981) and Brown and Weinstein (1983). Gehr used returns for two samples, one consisting of 24 indices and the other of 41 securities, for the same time period;

Roll and Ross used returns for 42 samples, each consisting of 30 securities, for the same time period.[4] However, in both studies, the replicability of the factor structure for the same asset sample across various time periods was not examined. Hughes tested for the equality, across two subsamples of securities, of the expected return on the portfolio orthogonal to all general factors. Brown and Weinstein tested for the equality across two subsamples of securities of all risk premia associated with the general factors and of the expected return on their orthogonal portfolio. In a sense, if the factors were congruent, then Hughes and Brown and Weinstein would not be able to reject their tested hypothesis.

To further these studies, the congruence of the factor structure of security returns will be examined in Essay 5.

The fourth assumption is that the volatility coefficients of the relevant underlying factors are intertemporally stationary. In other words, the coefficients which relate the expected return of any given security i, $E_i = E(\tilde{r}_i)$, with the expected realization of any given factor k, $E(\tilde{\delta}_k)$, are usually assumed to be constant in both the APM and the CAPM. However, the random nature of the volatility coefficient has been empirically supported by a number of studies. Furthermore, an analytical demonstration that ex post estimates of the volatility coefficients can be expected to be unstable over time was provided in Essay 2.

Thus, prudence would suggest that each of these assumptions should be subjected to a careful validation before the APM (and some of the other asset pricing models) can be unambiguously tested using time-series data. Such a test of the APM is attempted in Essay 6.

FOOTNOTES

1 Furthermore, as noted by Huberman (1980), the number of factors, m, in the underlying factor structure can be assumed to be a fixed number. In such a case, the number of factors, m, should not change as the number of risky assets, n, increases.

2 The internal inconsistency in both the SFM and the TFM was shown in Kryzanowski and To (1982a).

3 While such identification is necessary before the APM can be used by practitioners, it seem reasonable to agree with Roll and Ross (1980), p. 1077) that the identification of the factors "...is an area that can be investigated separately from testing asset pricing theories." In effect, identifying each general factor "...is equivalent to asking what causes the particular values of covariance terms in the CAPM..." and is thus "...no more appropriate... than it would be for tests of the CAPM to examine what, if anything, causes returns to be multivariate normal." [Roll and Ross (1980, p.1077)]. The identification and economic interpretation of the factors has been the subject of at least two studies. Rosenberg and McKibben (1973) made an unsuccessful attempt to find the macroeconomic correlates for the common factors. More recently, Rosenberg and Marathe (1976) were more successful in correlating the factors with microeconomic descriptors. It is interesting to note that both of these studies first assumed that the CAPM was valid, and then attempted to decompose the beta derived from the single factor market model into its constituent parts. On the other hand, Fogler, John and Tipton (1980) obtained positive results by correlating stock returns with three macroeconomic descriptors: the CRSP value weighted index, the long term Aa utility bond index, and the three month U.S. Treasury bill rate.

4 For example, in the Roll and Ross study, the 42 groups of assets contained from 1445 to almost 2619 daily trading returns.

ESSAY 5

THE FACTOR STRUCTURE OF SECURITY RETURNS: SOME TESTS OF INTERTEMPORAL AND CONTEMPORANEOUS CONGRUENCE

I. Introduction

This essay deals with tests of the third assumption among the four discussed in the introduction to chapter four of the thesis; namely, the essay deals with whether or not the underlying factor structure of security returns is congruent intertemporally (i.e., across time) and contemporaneously (i.e., across subsets of securities for the same time period).

The remainder of this essay is organized as follows: In the next section, the relevent literature is reviewed. In sections III and IV, the data and the empirical procedure, respectively, are discussed. In section V, the empirical results are presented and analyzed. And finally, in section VI, some concluding remarks are presented.

II. Review of the literature

Due to the dimensionality constraint,[1] researchers have tested the APM using a number of subsets of securities. For example, Roll and Ross, and Brown and Weinstein used 42 groups, each containing 30 securities; Gehr used two samples, one containing 24 indices and the other containing 41 securities; Hughes used two groups, each containing 110 securities; and Reinganum used 30 groups, containing from a minimum of 50 to a maximum of 80 securities. However, with the exception of the studies by Brown and Weinstein (1982) and Hughes (1981), none of these studies have tested whether or not the factor structure of security returns replicates itself contemporaneously across the groups of securities. However, even in the Brown and Weinstein and Hughes studies, the test of congruence was only partial and indirect, because only the contemporaneous equality of the risk-free estimates and of the risk premia were tested across the sample of securities. The satisfaction of the congruence assumption is however important, because the APM (or any other asset pricing model) would not be considered a satisfactory theory if it resulted in different ex-post explanations for different samples of assets, or for the same sample of assets for different time periods.

The studies by Kryzanowski and To (1983) and Gibbons (1981) are also somewhat related to the issue of factor congruence, since they tested for the stationarity of the variance-covariance matrix of security returns. For as noted by Jorëskog (1971), a test of the equality of the covariance matrices is the first step in testing the hypothesis that the factor structure is congruent.

III. Data

Tests of the stationarity of the mean vector, E, and of the variance-covariance matrix, V, of security returns are presented in Essay 1. Eleven basic random samples, each consisting of 50 securities, were drawn from the CRSP monthly tapes. All selected securities had to be listed on both January 1948 and December 1977.[2] Each of the eleven random samples were then divided into eight pairs of contiguous subperiods of equal length, as was shown earlier in Table 1.1.

It was found in Essay 1 that only 11 of the 88 random sample-pairs exhibited intertemporal stationarity in **both E and V**. These eleven random sample-pairs consisted of: four sample-pairs with equal contiguous subperiods with a 60-month length (hereafter referred to as **intertemporal couples 1 to 4**), five sample pairs with equal contiguous subperiods with a 120-month length (hereafter referred to as **intertemporal couples 5 to 9**), and two sample pairs with equal contiguous subperiods with a 180-month length (hereafter referred to as **intertemporal couples 10 and 11**). These eleven **intertemporal couples** are used in this paper to test for factor congruence across time.

Of the eleven sample pairs that exhibited stationarity in both E and V, only five sample pairs could be combined so that they covered identical calendar time periods. They consisted of four sample pairs with contemporaneous subperiod lengths of 240 months (hereafter referred to as **contemporaneous couples 1 to 4**), and one sample pair with a contemporaneous subperiod length of 360 months (hereafter referred to as **contemporaneous couple 5**). These five **contemporaneous couples** are used to test for factor congruence across samples.

IV. Empirical procedure

Measurement of Intertemporal Congruence

In factor analysis parlance, testing for intertemporal factor congruence is similar to testing for the replicability of the factor structure. Three such measures, for two factor structures, where the variables (securities) are the same but the observations (time periods) are different, have been proposed in Anderson and Engledown (1980), Harman (1967) and Veldman (1967). The first measure is the root-mean-square deviation [See Harman (1967, eq.15.7)]. It measures the extent of agreement between corresponding factor weights in the first subperiod with those in the second subperiod, as follows:

$$rms_{pq} = \sqrt{\sum_{j=1}^{n} [({}_1a_{jp} - {}_2a_{jq})^2] / n} \, , \qquad (5.1)$$

where

rms_{pq} is the root-mean-square deviation between the factor p in subperiod 1 and the factor q in subperiod 2;

${}_1a_{jp}$ is the factor loading of security j on factor p in subperiod 1;

$_2a_{jq}$ is the factor loading of security j on factor q in subperiod 2; and

n is the number of securities.

Perfect agreement between factor p in subperiod 1 and factor q in subperiod 2 would be implied if rms_{pq} had a null value.

A second measure of intertemporal congruence is the degree of factorial similarity, that is, the coefficient of congruence [see Harman (1967, eq. 15.8)]. This measure is given by:

$$\phi_{pq} = \frac{\sum_{j=1}^{n} ({}_1a_{jp})\,({}_2a_{jq})}{\sqrt{\{\sum_{j=1}^{n} {}_1a_{jp}^2\}\;\{\sum_{j=1}^{n} {}_2a_{jq}^2\}}}, \qquad (5.2)$$

where

ϕ_{pq} is the degree of factorial similarity between factor p in subperiod 1 and factor q in subperiod 2; and

all the other variables are as defined earlier.

While equation (5.2) seems to be similar to that for the correlation coefficient, it is different because the loadings are not mean deviations. Possible values of the coefficient of congruence range from +1 (perfect agreement) to zero (no agreement) to -1 (perfect inverse agreement).

The third measure of intertemporal congruence is based on Veldman's proposal to compute the degree of rotation necessary to maximize the degree of overlap between two factor structures. Although Veldman proposed to measure this required degree of rotation as a matrix of cosines of the angles between all the pairs of factor axes in the two structures, the slopes, or the tangents of the angles between the two factor axes, must be a more readily interpretable measure of the degree of rotation than the cosines. The slopes can be computed as the regression coefficients, when the factor loadings of one rotated factor axis are regressed against the factor loadings of the other unrotated factor axis. Thus, since the loadings need to be normalized to equate the origins of the two axes structures, the degree of congruence between the two factor structures can be effectively measured by the following correlation coefficients:

$$\rho_{pq} = \beta_{pq} \frac{\sigma({}_1a_p)}{\sigma({}_2a_q)} = \frac{\mathrm{cov}({}_1a_p, {}_2a_q)}{\sigma({}_1a_p)\ \sigma({}_2a_q)} \qquad (5.3)$$

where:

ρ_{pq} is the correlation coefficient between factor p in subperiod 1 and factor q in subperiod 2;

β_{pq} is the regression coefficient when factor p in subperiod 1 is regressed on factor q in subperiod 2;

${}_1a_p$ is the set of factor loadings of the n securities on factor p in subperiod 1; and

${}_2a_q$ is the set of factor loadings of the n securities on factor q in subperiod 2.

Since we are not dealing here with prescribed factor structures, the factorial solution obtained for each of the intertemporal couples are only determined up to a matrix transformation. Therefore, the three measures of intertemporal congruence were calculated using the following three steps.

First, for each subperiod of the eleven intertemporal couples, Rao factor analysis was used to compute the initial factor structure of the security returns.[3] This factor structure was then rotated to minimize factorial complexity by using the quartimax technique.[4]

Second, the quartimax-rotated factor structure of the first subperiod is further rotated in order to maximize the overlap with the quartimax-rotated factor structure of the second subperiod.[5] This rotation technique, which is based on the criterion of least squares fit, is known as the Procrustes transformation. It has been proposed by Green (1952), Schönemann (1966) and Cliff (1966).

Third, the three intertemporal congruence measures were computed for all statistically-significant factors, then for only the first five factors, and finally for only the first two factors. The number of statistically-significant factors was determined using the X^2 statistic for the exact number of factors, with the lower level of accounted for common variance set at two percent (i.e., 100% or 50 securities). The five-factor structure was used because of the empirical findings by Roll and Ross (1980) that security returns are spanned by about five factors (and 3 or 4 "priced" factors only). And finally, the two-factor structure was used because of the empirical findings by Kryzanowski and To (1983) that one and perhaps two factors are general (i.e., loaded on by a majority of the securities) and generalizable.

Measurement of Contemporaneous Congruence

Testing for congruence across samples within the same time period is somewhat easier than testing for congruence for the same sample across time. In fact, as has been suggested by Veldman, one could simply estimate the factor scores from one factor structure and then regress them against the estimated factor scores for the other factor structure.[6] If the factors are the same for both samples, then these factor scores (e.g., the returns on a general or "market" portfolio) would be perfectly correlated.

A second measure of contemporaneous congruence was proposed by Harman (1967, eq. 15.9). It is similar to Veldman's measure, except that in theory it is not a correlation coefficient. The measure is given by:

$$\Omega_{pq} = \frac{\sum_{i=1}^{t} ({}_1f_{pi}) \cdot ({}_2f_{qi})}{\{\sum_{i=1}^{t} {}_1f_{pi}^2\} \quad \{\sum_{i=1}^{t} {}_2f_{qi}^2\}} \tag{5.4}$$

where

Ω_{pq} is the contemporaneous congruence between factor p of sample 1 and factor q of sample 2;

${}_1f_{pi}$ is the factor score in observation (time period) i of factor p in sample 1;

${}_2f_{qi}$ is the factor score in observation (time period) i of factor q in sample 2; and

t is the number of observations (time periods).

Before proceeding, it should be noted that in practice both of these contemporaneous measures of congruence will yield exactly the same results because the factor loadings are estimated in such a manner that the factor scores are normalized. Therefore, the means of the factor scores are null and equation (5.4) becomes an equation for calculating the correlation coefficient between factors p and q.

As for the intertemporal testing, the factor structure in each contemporaneous couple was first quartimax rotated. They were not transformed using the Procrustes routine because the securities in the members of each couple were not the same. Thus, the axes in each pair of factor structures were not compatible.

V. The empirical results

Intertemporal congruence

The empirical results averaged over the eleven intertemporal couples of security groupings are generally dismal for the all significant factor structures of factor returns. More specifically, none of the (ten to fourteen) statistically significant factors are intertemporally congruent for all of the three measures of congruence. For example, the root mean squares deviations vary from 0.2 to 0.6 (that is, an average loading of about 0.5 for the first factor to less than 0.1 for all the remaining factors). The absolute values of the intertemporal correlation coefficients and the coefficients of congruence are all less than 0.5. While there are no tests of statistical significance for either the root mean square deviations, or the degree of factorial similarity, Harman (1967, p. 153) has noted that values upward of 0.94 denote congruent factors when testing for the (intertemporal) congruence of factor structures.

Such results might be caused by the possibility that the Procrustes routine cannot effectively achieve a "maximum" overlapping of the factor structures of the intertemporal couples, because of the relatively large number of factors being retained. This possibility has some support, since the measures of intertemporal congruence for the first factor increased strikingly when the number of retained factors decreased to five and two factors (see Tables 5.1 and 5.2, respectively).

Based on examinations of Tables 5.1 and 5.2, one can note that the rms are smallest in the diagonal of panel A, while ϕ and ρ are largest in the diagonal of panels B and C, respectively.

While the average loading on the first factor remains near 0.5 for each of the three factor structures (i.e., all statistically significant factors, five-factors and two-factors), it is less than 0.1 for all factors beyond the first in each of the three factor structures. Fur-

TABLE 5.1

AVERAGE MEASURES OF INTERTEMPORAL CONGRUENCE FOR THE ELEVEN COUPLES OF FIVE-FACTOR STRUCTURES*

Panel A -- Root-Mean-Square Deviations

Procrustes-Rotated Factor -- First Subperiod	Factor -- Second Subperiod				
	1	2	3	4	5
1	.14 (.02)	.55 (.03)	.55 (.03)	.54 (.02)	.55 (.02)
2	.55 (.03)	.20 (.03)	.28 (.03)	.26 (.03)	.26 (.02)
3	.56 (.03)	.29 (.03)	.21 (.04)	.25 (.03)	.24 (.02)
4	.55 (.03)	.29 (.02)	.26 (.03)	.20 (.04)	.24 (.03)
5	.56 (.03)	.29 (.04)	.26 (.03)	.24 (.03)	.20 (.02)

Panel B -- Degrees of Factorial Similarity

	1	2	3	4	5
1	.97 (.01)	.16 (.07)	.14 (.06)	.17 (.05)	.12 (.03)
2	.17 (.07)	.60 (.09)	.08 (.12)	.14 (.14)	.07 (.17)
3	.14 (.05)	.09 (.10)	.44 (.21)	.20 (.13)	.16 (.07)
4	.16 (.04)	.12 (.12)	.19 (.12)	.43 (.22)	.20 (.07)
5	.11 (.03)	.08 (.15)	.14 (.07)	.19 (.08)	.40 (.11)

Panel C -- Correlation Coefficients (Factor Loadings)

	1	2	3	4	5
1	.33 (.22)	-.02 (.24)	-.13 (.14)	-.03 (.21)	-.08 (.22)
2	-.11 (.17)	.59 (.09)	.05 (.13)	.10 (.15)	.04 (.18)
3	-.13 (.17)	.05 (.11)	.43 (.21)	.17 (.13)	.13 (.08)
4	-.17 (.21)	.09 (.12)	.16 (.13)	.41 (.22)	.18 (.08)
5	-.04 (.18)	.06 (.15)	.12 (.08)	.17 (.08)	.39 (.11)

*Mean values and standard deviations (in parentheses) are only reported for each factor for each congruence measure.

TABLE 5.2

AVERAGE MEASURES OF INTERTEMPORAL CONGRUENCE FOR THE ELEVEN COUPLES OF TWO-FACTOR STRUCTURES*

Panel A -- Root-Mean-Square Deviations

Procrustes-Rotated Factor -- First Subperiod	Factor -- Second Subperiod 1	2
1	.14 (.02)	.55 (.03)
2	.56 (.03)	.24 (.03)

Panel B -- Degrees of Factorial Similarity

	1	2
1	.97 (.01)	.14 (.08)
2	.14 (.08)	.45 (.12)

Panel C -- Correlation Coefficients (Factor Loadings)

	1	2
1	.36 (.21)	-.10 (.21)
2	-.01 (.22)	.43 (.11)

*Mean values and standard deviations (in parentheses) are only reported for each factor for each congruence measure.

thermore, based on the diagonal values in the matrices of the three measures of intertemporal congruence, it is obvious that congruence declines as one moves from factors with high accounted for common variance to factors with low accounted for common variance. In addition, the measures of intertemporal congruence for the first factor, both in terms of average values and standard deviations, are relatively more stable than those for any of the other factors for the eleven intertemporal couples.

As noted earlier, there are no statistical tests for two of the measures -- root mean square deviations and the degree of factorial similarity. However, the statistical significance of the correlation coefficients can be assessed using a t-test. While all the diagonal elements of the matrix of correlation coefficients are found to be statistically significant at the 0.05 level based on the t-test, all non-diagonal elements are not statistically significant at the 0.05 level. Such results imply that factors are congruent one-to-one (or on a hierarchical basis) and are orthogonal even when the time framework is changed.

Contemporaneous Congruence

The empirical results derived from conducting the tests of the contemporaneous congruence for the five and two factor structures averaged over the five contemporaneous couples of security groupings are summarized in Table 5.3. (The results for the all significant factor structures are not reported herein because they are similar to those reported in Table 5.3).

The major findings drawn from Table 5.3 can be summarized as follows: First, the coefficients of congruence (or correlation coefficients) for the first factor are almost identical for both factor structures for each of the five contemporaneous couples of security groupings. Second, only the diagonal elements of the correlation matrix are significant at the 0.05 level. Third, while it is not self-evident from the table, the correlation coefficient for the second factor was over 0.95 for two of the couples and less than 0.5 for the remaining three cou-

TABLE 5.3

AVERAGE COEFFICIENTS OF CONGRUENCE (CORRELATION COEFFICIENTS)
FOR THE FIVE CONTEMPORANEOUS COUPLES OF FIVE- AND TWO-FACTOR STRUCTURES*

Panel A -- Five-Factor Structure

Factor - Sample 2	Factor -- Sample 1: 1	2	3	4	5
1	.55 (.38)	-.06 (.10)	.08 (.06)	.05 (.11)	.05 (.03)
2	.02 (.06)	.18 (.34)	-.02 (.04)	.07 (.19)	-.04 (.06)
3	.02 (.06)	-.02 (.10)	-.07 (.11)	.02 (.21)	-.02 (.07)
4	-.01 (.08)	.00 (.10)	.05 (.06)	.04 (.19)	.08 (.16)
5	.04 (.05)	.13 (.32)	.10 (.20)	.00 (.20)	-.05 (.16)

Panel B -- Two-Factor Structure

Factor -- Sample 2	Factor -- Sample 1: 1	2
1	.55 (.38)	-.02 (.07)
2	-.08 (.11)	.15 (.32)

*Average values and standard deviations (in parentheses) are only reported for each factor for the five- and two-factor structures.

ples. Therefore, although only the first factor is consistently congruent contemporaneously across security groupings, the second factor can also be expected to be contemporaneously congruent for some security groupings.

VI. Concluding comments

In this essay, both the intertemporal and contemporaneous congruence of the factor structure of security returns were tested. Although a test of the APM is neither intended nor presented herein, tests of factor congruence are an extremely important step in devising acceptable tests of the APM using factor analytic techniques.

It was found that only the first factor (and perhaps the second factor) was congruent across time for the same sample of securities, and only the first factor was somewhat congruent contemporaneously across different samples of securities. These results are consistent with previous findings by Kryzanowski and To (1983) that only the first factor (and perhaps the second factor) is general and generalizable to the population of securities.. Thus, it appears that in empirical tests of the APM, the number of factors should be restricted to a maximum of two when using a two-step test procedure that assumes time stationarity.

Since the results on contemporaneous congruence are not conclusive, more research needs to be done on the contemporaneous replicability of the factor structure from sample to sample before the APM can be tested or used in practical applications with time-series data.

FOOTNOTES

1 Because the determination of the factor structure requires that the correlation or variance-covariance matrix of security returns be inverted, computer capacity limitations are a matrix rank of about 60 for a reasonably sized computer to a maximum of about 110 for the largest computers.

2 The geometric monthly mean return was used for the (few) months which had missing return data.

3 Rao or canonical factor analysis is based on the maximum likelihood principle. It is one of the few factor analytic techniques which has a test of statistical significance. See Harman (1967).

4 The quartimax rotation criterion is one of many such criteria. However, it is one of the best for achieving a parsimonious factor structure which not only allows for the reduction of the complexity of the factorial representation of the securities but also for the possibility of having many securities loaded on the same factor.

5 The quartimax rotated factor structure of the second subperiod was rotated in order to maximize its overlap with the quartimax rotated factor structure of the first subperiod using the Procrustes transformation. For all practical purposes, the results were similar to those obtained for the reverse procedure. The results for the non-Procrustes transformed loading matrices were also only marginally less congruent than the results which are presented in this essay. Thus, it seems reasonable to conclude that the Procrustes transformation has a very marginal impact on the factor axes orientation conducted herein.

6 The factor scores are the values of the factors when the factor score coefficient matrix is vector multiplied by the values of the variables (securities).

ESSAY 6

SOME EMPIRICAL TESTS OF THE ARBITRAGE PRICING MODEL

I. Introduction

Based on Markowitz's (1959) pioneering study, Sharpe (1963) and Lintner (1965a) advanced the first positivist formulations of the capital asset pricing model (CAPM). Their models were subsequently refined by Mossin (1966), Fama (1968), Black (1970) and others.[1] Even though the CAPM has been studied extensively, it has not been empirically validated.[2] According to Roll (1979), the CAPM cannot be tested in an unambiguous fashion because of a number of intractable measurement and computational difficulties, and the joint nature of the hypotheses to be tested.

A number of other asset pricing models have appeared in the literature.[3] Of these models, the arbitrage pricing model (APM) proposed by Ross (1976b) has attracted increased interest because of the elegant simplicity of its theoretical development, its promise for practical application and its potential for unambiguous, empirical validation.[4] To date, the empirical testing of the APM has essentially been limited to two issues: (i) the number of factors that exist in the return-generating process for assets in the economy,[5] and (ii) on how closely the ex-post market pricing relationship conforms to the ex-ante theoretically derived (approximately linear) functional relationship between security returns and the underlying return-generating factors. The findings of the empirical tests of the APM are generally supportive of the model; studies by Gehr (1975), Roll and Ross (1980), Chen (1982), Hughes (1981), Berges (1982) and Brown and Weinstein (1983) support the validity of the APM, while the study by Reinganum (1981b) does not support the APM.

While these empirical studies have made an important contribution to the testing of the APM, they have (understandably given the amount of statistical analysis to be carried out) been only partial (incomplete) tests. In each of the studies, one or more of the assumptions, which must necessarily be empirically verified in order to ensure that the APM can be tested unambiguously using time-series data, have been left untested or insufficiently tested.[6]

Before proceeding, it must be emphasized that while these assumptions are not required in the theoretical derivation of the APM, they are necessary in order to empirically test the APM using time series data. Thus, a validation of the APM would have to be based on its predictions [see Roll and Ross (1980)]. Nevertheless, prudence would suggest that each of these assumptions should be subjected to a careful validation before the APM (and some of the other asset pricing models) can be unambiguously tested using time-series data. Therefore, as a series of pre-requisite steps to a direct test of the APM, a validation of each of the four assumptions has been conducted and reported separately [see Essays 1, 2 and 4 and Kryzanowski and To (1983)].

These assumptions, as presented in Essay 5, can be summarized as follows: (1) the mean vector, $\underline{E}$, and the variance covariance matrix, $\underline{V}$, of security returns is intertemporally stationary; (2) Security returns are characterized by an explicit underlying factor structure composed of one or more general or common factors (where a general or common factor is a factor which impacts on all security returns); (3) The underlying factor structure of security returns is congruent; that is, the underlying factor structure is exactly replicable (the same) across various asset subsets of the asset universe and across various time periods for the same asset subset; and (4) The volatility coefficients, β_{ik}, of the relevant underlying factors are intertemporally stationary.

As the last essay in a series devoted to the APM, the specific purpose of this essay is to present some empirical tests of the APM itself. To this end, the validity of the APM is tested using a two-step procedure on eleven intertemporally stationary sample pairs (of 50 securities each), which were drawn from the CRSP monthly tapes for the period from January 1948 to December 1977.

The remainder of this essay is organized as follows. In the next section, a brief review of the literature on the empirical tests of the APM is presented. In the third section, the data sources and the empirical procedures used are discussed. In the fourth section, the empirical results are presented and analyzed. And finally, in the fifth section, some concluding remarks are offered.

II. A brief review of the literature

Due to the dimensionality constraint,[7] researchers have tested the APM using a number of subsets of securities. For example, Roll and Ross, and Brown and Weinstein used 42 groups, each containing 30 securities; Gehr used two samples, one containing 24 indices and the other containing 41 securities; Hughes used two groups, each containing 110 securities; and Reinganum used 30 groups, each containing from a minimum of 50 to a maximum of 80 securities.

Basically, four different empirical procedures have been used to date to test the APM. Gehr (1975), Roll and Ross (1980) and Hughes (1981) used a two-step procedure to test the APM implied equilibrium relationship (6.2), which is obtained from the return model (6.1):

$$\tilde{r}_i = E_i + \beta_{i1}\tilde{\delta}_1 + \beta_{i2}\tilde{\delta}_2 + \dots + \beta_{ij}\tilde{\delta}_j \quad , \tag{6.1}$$

where:

$\tilde{r}_i$ is the return on security i;

$\tilde{\delta}_j$ is the realization of factor j, j=1,...,k;

β_{ij} is the relative risk of asset i relative to factor j.

$$E_i = E^0 + \beta_{i1}(E^1-E^0) + \beta_{i2}(E^2-E^0) + \ldots + \beta_{ij}(E^j-E^0) \,, \qquad (6.2)$$

where:

E^0 is the expected return on the risk-free portfolio; that is, the portfolio which is orthogonal to all the factors, $\tilde{\delta}_j$, j = 1, ..., k;

E^j is the expected return on the portfolio associated with general factor j, $\tilde{\delta}_j$, j = 1, ..., k; and

all the other variables are as defined earlier.

Both Gehr (1975) and Roll and Ross (1980) first estimated β_{ij} by the use of a factor analytic technique.[8] In factor analytic parlance, the β_{ij} are referred to as the factor loadings. They measure the degree of associability between each asset and the factor. When the security returns are standardized, the β_{ij} are simple correlation coefficients between $\tilde{r}_i$ and $\tilde{\delta}_j$. In a second step, these authors cross-sectionally regressed the E^i over the β_{ij}. This second step involved the estimation of the factor score coefficients, or risk premia associated with the $\tilde{\delta}_j$. Thus, it should be evident that this two-step procedure is a factor analytic adaptation of the two-step procedure used by Fama and MacBeth (1973) to test the CAPM.

The second (and very direct) procedure for testing the APM was used by Reinganum. This procedure attempts to determine if arbitrage opportunities exist in the capital markets that are unexplainable by the APM. Thus, to test for the existence of arbitrage opportunities due to the "small firm" effect, Reinganum first estimated the factor loadings, β_{ij}. Then, using ten portfolios formed on the basis of market values, he estimated the size-effect arbitrage opportunities after controlling for the β_{ij}'s (i.e., the APM risk measures). Reinganum found that the arbitrage returns associated with the size effect were such that the APM should be rejected.

The third empirical procedure was proposed by Jobson (1982).[9] Basically, Jobson showed that testing for the validity of equation (6.2) is equivalent to testing the hypothesis that the constant term in the n regressions $E[(\tilde{r}_i - E^o) \mid (\tilde{\delta}_j - E^o), j=1,\ldots k]$, for $i = 1,\ldots,n$, is null for all assets.

The fourth empirical procedure is the bilinear paradigm proposed by Brown and Weinstein (1983) and also used by Hughes (1981).

In this essay, only the first empirical procedure will be used. While the second procedure is interesting, it is not used herein. More specifically, if the size effect is indeed systematic, it should be accounted for in the revealed factor structure for the sample(s) being studied. If it is not so revealed, this pinpoints a weakness in the specific statistical technique used to identify the factor structure of security returns.[10] (One such possibility would be the manipulative difficulties encountered in the use of factor analytic techniques.) Thus, it does not seem prudent to test the APM further using the second procedure, at least when factor analytic techniques have first been used to unravel the underlying factor structure of security returns.

While the third empirical procedure appears to be very simple to use, Dhrymes (1982) showed that its apparent simplicity is deceptive for a number of reasons. The first and most important one is the circularity of Jobson's procedure which requires that the return premium vectors be estimated for all the securities in the sample. Unfortunately, these return premium vectors can only be estimated if the β_{ij}'s are known. If a zero-beta portfolio return, or a constant risk free rate, is used to estimate the security return premia, Dhrymes has shown that any non-singular sub-matrix of the factor loadings matrix could be used in the Jobson procedure. This virtually ensures that the APT would always be validated because a suitable sub-matrix will always be found given enough computer time. Also, according to Dhrymes, Jobson's procedure is not applicable to individual assets. However, if the likelihood ratio test proposed by Jobson is adapted to portfolios of securities, Dhrymes showed that this implied the singularity of the regression coefficient matrix, and is therefore ambiguous in an econometric sense.

The fourth procedure is the bilinear paradigm used by Hughes (1981) and Brown and Weinstein (1983). Hughes (1981) tested the hypothesis that E^{0} is constant for all assets in her sample. Brown and Weinstein (1983) tested the hypothesis that when a sample is split into two sub-samples, then each sub-sample should yield the same vector of risk premia. The deficiency of the bilinear paradigm is that there is no apparent justification for splitting each sample into two sub-samples. In fact, it is easy to see that if the samples were split into a larger number of sub-samples, the larger are the odds against finding that the vector premia would be equal among the sub-samples. So, the bilinear procedure is not an unambiguous empirical procedure for testing the APM.

III. Data sources and empirical procedures

Data sources

Eleven basic samples were drawn, each consisting of 50 securities, from the CRSP monthly tapes.[11] All selected securities had to be listed on both January 1948 and December 1977. Each of the eleven samples were then divided into eight pairs of contiguous subperiods of equal length, as is shown in Table 1.1.

As was found in Essay 1, only 11 of the 88 sample-pairs exhibited intertemporal stationarity in both E and V. These eleven sample-pairs consisted of: four sample pairs with equal contiguous subperiods with a 60-month length (hereafter referred to as **intertemporal couples 1 to 4**), five sample pairs with equal contiguous subperiods with a 120-month length (hereafter referred to as **intertemporal couples 5 to 9**), and two sample pairs with equal contiguous subperiods with a 180-month length (hereafter referred to as **intertemporal couples 10 and 11**). These eleven intertemporal couples are used in this paper in a two-step procedure designed to test the validity of equation 6.1.

Empirical procedure

As noted earlier, a two-step test procedure is used in this paper. In the first step, the factor loadings, β_{ij}, were computed using the returns in the first subperiod for each intertemporal couple.[12] In the second step, the expected returns computed from the returns in the second subperiod were cross-sectionally regressed on the factor loadings of the first subperiod. In order to provide for a standard of comparison, an "own"-variance model was also tested by cross-sectionally regressing the expected return in the second subperiod against the estimated variance of returns for the first subperiod for each intertemporal couple. This procedure not only eliminates the ambiguity created by the assumption of stationarity in E and V but it also avoids the difficulties caused by applying the two-step procedure to the same set of data. As a test of sensitivity, the procedure was also applied in reverse; that is, the expected returns in the first subperiod were cross-sectionally regressed on the factor loadings, and the variances, estimated from returns in the second subperiod.

The procedure was applied to four different types of security factor structures: an all-statistically-significant factor structure,[13] a five factor structure, a two factor structure and a one factor structure. The all-statistically-significant factor structures, which involved from 10 to 13 factors depending upon the intertemporal couple, were chosen for obvious reasons. The five-factor structure was chosen because of the empirical findings by Roll and Ross that security returns are spanned by about five "priced" factors. And finally, the one and two factor structures were chosen because of the empirical findings by Kryzanowski and To (1983) that a one (or at most a two) factor structure is common to all securities, and is also intertemporally congruent.

Since no prescribed factor structures are being used herein, the factorial solution obtained for each of the intertemporal couples is only determined up to a matrix transformation. Therefore, the factor loadings (or β_{ij}) were calculated using the following two steps. First, for each subperiod of the eleven intertemporal couples, Rao factor analysis was used to compute the initial factor structure of the security returns.[14] This factor structure was then rotated to minimize factorial complexity by using the quartimax technique.[15] Second, the quartimax-rotated factor structure of the first subperiod was further rotated in order to maximize the overlap with the quartimax rotated factor structure of the second subperiod.[16] This rotation technique, which is based on the criterion of least squares fit, is known as the Procrustes transformation. It has been proposed by Green (1973), Schönemann (1966) and Cliff (1966).

IV. Empirical results

Using the procedure discussed in the previous section, the sample mean returns of the second subperiod were regressed on the various factor loading structures and return variances of the first subperiod for each of the eleven intertemporal couples. These results are summarized in Tables 6.1 and 6.2.

From Tables 6.1 and 6.2, it appears that the "own"-variance model has more explanatory power than the "best" APM (i.e., the five-factor

TABLE 6.1

Summary Statistics for the Regressions of the Expected Returns for the Second Subperiod Against the Estimated Loadings and Variances from the First Subperiod for the Eleven Intertemporal Couples

Couple	"Own"-Variance		One-Factor		Two-Factor		Five-Factor		All-Sign.-Factors	
	Adj. R^2	F-Value	Adj. R^2	F-Value	Adj. R^2	F-Value	Adj. R^2	F-Value	Adj. R^2	F-Value
1	.329	21.058	.000	0.506*	.015	1.633*	.395	6.491	.074	1.327*
2	.014	1.612*	.000	1.010*	.003	1.065*	.221	3.438	.040	1.159*
3	.521	46.654	.021	1.890*	.041	1.930*	.472	10.611	.000	0.953*
4	.491	41.568	.232	13.656	.284	9.516	.276	4.281	.107	1.939*
5	.299	18.899	.017	1.710*	.057	2.309*	.473	8.726	.000	0.698*
6	.743	122.251	.000	0.175*	.086	3.017*	.524	10.467	.000	0.853*
7	.765	137.832	.140	7.823	.259	8.531	.468	8.563	.000	0.754*
8	.899	374.111	.120	6.718	.027	2.202*	.295	4.606	.000	0.743*
9	.727	112.819	.188	10.710	.070	2.626*	.379	6.241	.127	1.595*
10	.799	168.353	.029	2.248	.196	6.257	.508	9.892	.037	1.189*
11	.902	388.246	.109	6.129	.033	1.741*	.310	4.865	.003	1.014*

* Not statistically significant at the 0.05 level.

TABLE 6.2

Regression Coefficients for the Regressions of the Expected Returns from the Second Subperiod Against the Estimated Loadings and Variances from the First Subperiod for the Eleven Intertemporal Couples

Couple	"Own"-Variance		One-Factor			Two-Factor			
	Const.	Var.	Const.	Fac. 1	No.[a]	Const.	Fac. 1	Fac. 2	No.[a]
1	-.000*	.144	.013	-.097*	0	.011	IND	.010*	0
2	.070*	.087*	.022	-.016*	0	.026	-.022*	.005*	0
3	-.023	[illegible]	.036	-.038*	0	.043	-.048*	.017*	0
4	.008	.111	.044	-.051	1	.047	-.056	.016*	1
5	-.000*	.198	.026	-.019*	0	.032	-.028*	.010*	0
6	.004	.137	.018	-.006*	0	.277	-.230*	.023*	0
7	-.014	.313	.043	-.065	1	.062	-.094	-.035	2
8	-.003	.191	.047	-.060	1	.014	IND	-.017*	0
9	.001*	.109	.029	-.037	1	.025	-.023*	-.027*	0
10	-.000*	.146	.027	-.031*	0	.050	-.069	-.026*	1
11	-.002*	.198	.045	-.060	1	.023*	-.016*	-.022*	0

Couple	Five-Factor						
	Const.	Fac. 1	Fac. 2	Fac. 3	Fac. 4	Fac. 5	No.[a]
1	.001*	.018*	.016	.016*	.006*	.036	2
2	.008*	.007*	.024	.005*	.017*	.008*	1
3	-.016*	.040	.006*	.015*	.038	IND	[illegible]
4	.014*	.004*	.014*	.014*	.019*	.019*	[illegible]
5	.017*	-.007*	.003*	.023	.025	.022	3
6	-.002*	.023*	.022	.019*	.038	.043	3
7	-.001*	.019*	.032*	-.021*	.037	.072	2
8	-.005*	.029*	.008*	.036	[illegible]	.027	2
9	-.004*	.022*	.025*	.028	.048	.021*	2
10	-.005*	.030*	.030	.065	.033	.029*	3
11	-.004*	.028*	.018*	.046	.030	.011*	2

* Not significant at the 0.05 level.

"IND" Indeterminate.

[a] The number of factors with significant coefficients.

structure APM). More specifically, while the adjusted R^2 values of both models are reasonably high, the "own"-variance model has the higher values for eight of the intertemporal couples. (This is not totally unexpected given that the $\underline{V}$ for each of the intertemporal couples was intertemporally stationary by empirical design). Nevertheless, while all but one of the estimated volatility coefficients for the "own"-variance model was positive and statistically significant, the estimated intercepts for the "own"-variance model were generally either negative or not significant. In only two out of eleven cases were they both positive and statistically significant.

The regression coefficients for the all-significant-factor structure APM were not presented in Table 6.2 because they were generally not statistically significant. Moreover, it seems that while the adjusted R^2 values increased from the one- to the two- to the five-factor structures, they decreased from the five to the all-significant-factor structures. This can be explained by the fact that the adjusted R^2 value usually increases with the number of explanatory variables. However, beyond a certain threshold the adjusted R^2 value will decrease significantly due to the addition of non-significant variables and the resultant decrease in the number of degrees of freedom.

From Table 6.2, it appears that the constant term is not significantly different from zero for all the intertemporal couples for the five factor structure APM, while it is generally positive and significant for the one and two factor structures. This observation is difficult to explain and might result from a procedural problem encountered in estimating the factor loadings (that is, in the first step of the empirical procedure followed herein).

From Table 6.2, it appears that a large number of the volatility coefficients for the one, two and five factor structure APMs are not statistically significant. More specifically, for the five factor structure APM, the volatility coefficients of the first, second, ..., fifth factors were not statistically significant for 10, 7, 6, 4 and 5 intertemporal couples, respectively. In addition, it appears that, as the factor structure increases from one to five factors, the first factor is less likely to be significant ("priced") and the most recently added factor (e.g., the fifth factor) is more likely to be "priced".

When the expected returns in the first subperiod were regressed over the loadings and variances estimated from the second subperiod, the results were less supportive of the APM (see Tables 6.3 and 6.4). More specifically, the "own"-variance model clearly dominated all the four variants of the APM (see Table 6.3), and the volatility coefficients of the first, second, ..., fifth factors were not statistically significant (or were indeterminate) for 11, 11, 10, 10 and 11 intertemporal couples, respectively. However, the previous results for the E^0 values were again replicated. The E^0 values were found not to be significantly different from zero for all eleven intertemporal couples in the five factor structure, all positive in the one factor structure, and positive and significant or negative and non-significant in the two factor structure (see Table 6.4). This phenomenon might constitute a procedural problem inherent to factor analytic techniques.

V Concluding remarks

This essay is the last in a series of essays, where the first ones dealt with various specific prerequisite steps necessary for an "unambiguous" test of the APM. Based on the first step, eleven samples which had intertemporally stationary $\underline{E}$'s and $\underline{V}$'s were used herein to test the APM. The empirical procedure used in this paper, based on the Fama and MacBeth procedure, had previously been used by Gehr, Roll and Ross, and Hughes. This procedure was used on a split sample in order to conform to the original intent of Fama and MacBeth.

TABLE 6.3

Summary Statistics for the Regressions of the Expected Returns for the Second Subperiod Against the Estimated Loadings and Variances from the First Subperiod for the Eleven Intertemporal Couples

Couple	"Own"-Variance		One-Factor		Two-Factor		Five-Factor		All-Sign.-Factors	
	Adj. R^2	F-Value	Adj. R^2	F-Value	Adj. R^2	F-Value	Adj. R^2	F-Value	Adj. R^2	F-Value
1	.301	13.627	.000	IND	.026	1.558*	.000	0.934*	.000	0.945*
2	.255	15.395	.013	1.556*	.098	3.340	.000	0.597*	.000	0.797*
3	.695	96.502	.000	0.240*	.000	.939*	.020	1.224*	.000	0.855*
4	.579	58.824	.026	2.106*	.000	.105*	.073	1.675*	.012	1.047*
5	.355	24.111	.120	6.737	.000	.291*	.009	1.074*	.000	0.967*
6	.608	93.576	.053	3.350	.033	1.738*	.000	0.605*	.212	2.096
7	.797	165.415	.154	8.647	.048	2.089*	.031	1.347*	.052	1.243*
8	.680	90.096	.057	3.521	.190	6.027	.049	1.439*	.000	1.001*
9	.590	61.521	.049	3.143	.011	1.481*	.224	4.106	.128	1.602*
10	.795	163.842	.000	0.328*	.000	.290*	.000	0.893	.000	0.642
11	.776	146.236	.076	4.470	.075	2.733*	.105	2.011	.076	1.406*

* Not significant at the 0.05 level.

TABLE 6.4

Regression Coefficients for the Regressions of the Expected Returns from the Second Subperiod Against the Estimated Loadings and Variances from the First Subperiod for the Eleven Intertemporal Couples

	"Own"-Variance		One-Factor			Two-Factor				Five-Factor						
Couple	Const.	Var.	Const.	Fac. 1	No.[a]	Const.	Fac. 1	Fac. 2	No.[a]	Const.	Fac. 1	Fac. 2	Fac. 3	Fac. 4	Fac. 5	No.[a]
1	.006	.084	IND	IND	0	-.046*	.120	.296	2	.129*	-.060*	-.386*	.340*	IND	.238*	0
2	.005*	.104	.023	-.017*	0	-.006*	.032*	.429	1	-.424*	.476*	.341*	-.267*	.079*	.114*	0
3	-.013	.196	.314*	-.008*	0	.248*	-.377*	-.102*	0	-.427*	.543*	IND	.351*	.273*	-.462*	0
4	.003*	.152	.023	-.020*	0	.068*	-.075*	-.091*	0	.090*	-.464*	.210*	-.260*	.402*	.592*	0
5	-.000*	.188	.028	-.026	1	.061*	-.043*	-.144*	0	-.002*	-.184*	.475*	-.391*	.038*	.051*	0
6	.005	.101	.029	-.031*	0	-.188*	.403*	.389*	0	.224*	.224*	.045*	-.088*	-.446*	-.142*	0
7	-.002*	.187	.065	-.081	1	.477*	-.818*	-.023*	0	.503*	-.749*	-.180*	.697*	-.210*	IND	0
8	.004	.134	.033	-.035*	0	-.594	1.073	.295*	1	.374*	-.819*	-.515*	-.370*	-.257*	.449*	0
9	.005	.107	.030	-.026*	0	-.006	IND	-.190*	0	.090*	-.478*	-.186*	.865	IND	-.278*	1
10	.003	.114	.021*	-.012*	0	-.014*	IND	.106*	0	-.563*	.771*	.427*	-.223*	-.208*	-.060*	0
11	-.001*	.175	.046	-.061	1	-.310*	.512*	.413	1	.797*	-1.476*	-.290*	-.550*	.748	.224*	1

* Not significant at the 0.05 level.

"IND" Indeterminate.

[a] The number of factors with significant coefficients.

The empirical evidence would seem to confirm the findings of Roll and Ross (1980) that a five-factor structure provides the highest explanatory power in terms of asset pricing. However, contrary to the findings of Roll and Ross, it was found that: (i) no more than three factors are ever "priced" in the market; and (ii) the "own"-variance model is preferable to the APM, for whatever factor structure model is chosen, for tests on groupings of securities with intertemporally stationary E and V.[17]

Since the findings presented herein are mixed in terms of their support of the APM, they suggest that further research on the empirical validity of the APM is warranted. Furthermore, since Reinganum (1981b) has shown that factor analysis is unable to "pick up" the "small firm effect" factor, it appears that statistical techniques other than factor analysis need to be identified and used to unravel the underlying factor structure of security returns. For example, Brock (1979, 1982) may have pioneered a very promising avenue for future research. He attempted to develop general equilibrium models of asset pricing by using the recent advances in stochastic calculus and control theory. Such models would presumably have the advantage of making the uncertain factors endogeneous to the asset pricing process. Thus, the common factors could be identified theoretically (or even, definitionally, as in Brock's models). However, these models need to be recast in a positivist format for them to be more valuable to both theoreticians and practitioners.

FOOTNOTES

1 For a theoretical discussion of the one factor and two factor formulations of the CAPM, see Fama (1976, chapter 7).

2 For a review of the empirical tests of the CAPM, see Fama (1976, chapters 4 and 9), and Jensen (1972).

3 Studies dealing with other asset pricing models include, amongst others, Hakansson (1971), Merton (1973), Kraus and Litzenberger (1976) and Breeden (1979).

4 Shanken (1982) has recently objected to the potential testability of the APM. However, his criticisms suffer from serious deficiencies, as is documented in To, Kryzanowski and Parienté (1983).

5 Chen (1982) has shown that if k is the number of factors that exist in the market, then the number of "priced" factors can be any number between 1 and k. Thus, if at least one factor is found to exist in the return-generating process of assets, then the number of factors that are priced is somewhat irrelevant.

6 Some of these assumptions, which are required to unambiguously test the CAPM and other asset pricing models, also remain untested.

7 Because the determination of the factor structure requires that the correlation or variance-covariance matrix of security returns be inverted, computer capacity limitations are a matrix rank of about 60 for a reasonably sized computer to a maximum of about 110 for the largest computers.

8 Factor analysis includes a large variety of very different techniques. The interested reader is referred to Harman (1967).

9 Jobson's procedure has not yet been used to test the APM.

10 In other words, this would only show that factor analytic techniques were unable to identify a factor (i.e., market value) that was important in the pricing of assets.

11 Most empirical studies used daily data by invoking the assumption that daily returns are more likely to be stationary than monthly returns. None of these studies, except Gibbon's, explicitly tested for that hypothesis as a preliminary step.

12 Roll and Ross (1980) claimed that by estimating the β_{ij}'s and the (E^j-E^o)'s simultaneously, the measurement error problem is somehow alleviated. However, the measurement error problem exists as soon as the security returns are measured with error, and could not be alleviated by increasing the sample size used in estimating the β_{ij}'s and the risk premia, whether the estimation procedure is simultaneous or step-by-step. However, from simulation results (obtainable from the author), for a sample size of 50 securities, the estimation bias of the β_{ij}'s could be less than 1%, for β_{ij}'s in the range 0.5 to 3.0, if it is assumed that the measurement error of security returns is homoscedastic, that the induced factors imply equal weighting of the securities, and that the variance of the error term is equal to 1% of the variance of the security return.

13 The number of statistically significant factors was determined using the X^2 statistic, with the lower level of accounted-for common variance set at two percent (i.e. 1 divided by 50 securities).

14 Rao or canonical factor analysis is based on the maximum likelihood principle. It is one of the few factor analytic techniques which has a test of statistical significance. See Harman (1967).

15 The quartimax rotation criterion is one of many such criteria. However, it is one of the best for achieving a parsimonious factor structure which not only allows for the reduction of the complexity of the factorial representation of the securities but also for the possibility of having many securities loaded on the same factor.

16 The quartimax rotated factor structure of the second subperiod was rotated in order to maximize its overlap with the quartimax rotated factor structure of the first subperiod using the Procrustes transformation. For all practical purposes, the results were similar to those obtained for the reverse procedure. The results for the non Procrustes transformed loading matrices were also only marginally less congruent than the results which are presented in this paper. Thus, it seems reasonable to conclude that the Procrustes transformation had a very marginal impact on the factor axes orientation conducted herein.

17 Although all one period asset pricing models are derived without invoking the assumption that the $\underline{E}$ and $\underline{V}$ of individual assets are intemporally stationary, such an assumption is necessary when these models are empirically tested using time-series data. For example, Roll (1979) has pointed out that few empirical tests of asset pricing models would be feasible without invoking such an assumption.

CHAPTER FIVE
CONCLUSION

In the six essays presented in this thesis, the CAPM, the APM and the clinical-level approach to asset pricing were investigated.

In Essay 1, an important consideration in empirical tests of the CAPM (and in other applications using time series of returns on financial assets) is studied. It was found that various subsamples of securities recorded on the CRSP tapes are generally not consistent with the hypothesis of stationary mean return vector ($\underline{E}$) and variance-covariance matrix ($\underline{V}$). The only exception occurs when subperiods of 120-months and of 180-months are used to test for stationarity of $\underline{E}$. However, the data seems to be consistent with the hypothesis of stationary correlation matrices of security returns, whatever security groupings are used and whatever the subperiod length. While these results further question the appropriateness of past empirical tests of the CAPM, they support Gibbons's (1981) claim that factor analysis can be used to unravel the factor structure of security returns, provided that standardized returns are used to obtain a correlation matrix.

A further problem encountered in the empirical tests and the applications of the CAPM is dealt with in Essay 2. More specifically, using a simple illustration of a closed two-security market, it was shown that the time-series estimates of security betas are random variables. The underlying intuition is as follows: given a stationary $\underline{V}$, the time-series estimates of beta are formed by applying market weights to $\underline{V}$. These market weights however shift from period to period because of the relative magnitude of individual security return realizations to market return realizations. Therefore, OLS estimates of beta will vary when different time periods and different calendar dates are used in selecting the applicable input return series. Although it was shown in Essay 2 that the time-series estimates of beta are random, it is not known whether or not the distribution of these beta estimates are stationary (i.e., have constant distribution parameters across time periods) and it is not known what type of distribution best describes the beta distributions. An examination of the nature of beta research distribution is probably the next logical step in the research on beta, since such knowledge is a prerequisite to the use of Theil's (1971) random coefficient model.

In Chapter 3, two empirical tests are conducted on the clinical-level approach to asset pricing. In Essay 3, a clinical-level version of the CAPM which is similar to the Mao (1971) and Levy (1978) models is obtained. At the individual investor level, the investor's portfolio is mean-variance efficient. This relationship holds whether or not the investor holds the market portfolio or an optimal subset of securities, and it has been tested in Essay 3, using a large body of data on Canadian households. The results are consistent in general with the two hypotheses associated with the clinical-level version of the CAPM: (i) linearity between risk and return and (ii) the existence of riskless borrowing and lending. The results seem to indicate that a third hypothesis (i.e., a positive risk-return trade-off) is not supported, at least for wealthier households. A possible explanation for such a result is that an important variable, personal taxes, has not be reflected in the empirical tests due to a lack of data. Although a number of compromises were used in the data transformations, the results presented in Essay 3 seem to imply that the CAPM might be a good description of individual investor's behavior in capital markets. Specifically, one could relax the perfect market assumption and still work with a simple, intuitively appealing paradigm such as the CAPM. Unfortunately, a body of data superior to that used in Essay 3, or presently existent, is needed.

In Essay 4, a model, which is more general than the clinical-level version of the CAPM, is proposed. This model invokes no assumptions about the distribution of returns or any functional form of utility. However, empirical tests of the model using the same body of data used in

Essay 3 are not in general consistent with the model's predictions. More specifically, of the three hypotheses tested (i.e., linearity of the risk- return relationship, a positive risk-return trade-off, and riskless borrowing and lending), only the third is somewhat supported empirically.

In Essays 5 and 6, the APM is empirically examined. Since an empirical test of the APM must be preceeded by an investigation of a number of issues, Essay 5 attempts to test one such issue ; namely, the intertemporal and the contemporaneous congruence of the factor structure of security returns. Other issues (such as E - V stationarity, the stability of the volatility coefficients, and the underlying factor structure of security returns) have already been investigated in Essays 1 and 2, and Kryzanowski and To (1983), respectively. To this end, the eleven random sub-samples which were shown to be endowed with E - V stationarity in Essay 1, were submitted to the traditional tests of factor structure congruence. The results are consistent with only the congruence of the first and (to a much lesser degree) the second factor for various factor structures of security returns.

In Essay 6, empirical tests of the APM are presented. Using the same data set as in Essay 5, these tests do not support the APM equilibrium predictions. In fact, the APM predicted equilibrium relationship is statistically dominated by the "own"-variance model.

Thus, while the APM is theoretically appealing, its test procedures must rest on econometric techniques other than factor analytic techniques, or Jobson's procedure which also involves such techniques in its first step. A very promising potential research avenue has been pioneered by Brock (1979, 1982) who advanced the formulation of general equilibrium models using stochastic growth theory (i.e., stochastic calculus and control theory). Not only do Brock's models incorporate random factors as endogeneous variables and encompass both the APM and the CAPM as simplifying variants, but they also theoretically identify the factors. However, since Brock's models are not presently positivist constructs, they ~~are not~~ empirically testable.

The overall conclusions to this dissertation are as follows:

(i) The traditional CAPM contains at least two empirical hurdles ($\underline{E} - \underline{V}$ and β instability) which prevent it from being empirically tested using traditional econometric procedures.

(ii) Individual investors (as proxied by Canadian households) appear to hold efficient portfolios in a mean-variance sense, although such portfolios are not the traditional market portfolio. Furthermore, the expected returns on these individual portfolios are linearly related to their variances.

(iii) While factor analytic techniques are generally used to test the APM, they imply serious deficiencies in test design, and better procedures for empirical validation of the APM need to be found. Using standard procedures, the APM predicted equilibrium relationship is not consistent with the data. Thus, whether the APM is an inappropriate model or not is still an unresolved question.

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