DESIGN, ANALYSIS, AND PERFORMANCE EVALUATION OF A POLAR BASED CONTOURING SYSTEM

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ABSTRACT

DESIGN, ANALYSIS,

AND PERFORMANCE EVALUATION

OF A POLAR BASED CONTOURING SYSTEM

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This thesis presents the design, analysis, and performance evaluation of a polar based contouring system. The system consists of a contour digitizer and an NC machine. The contour digitizer may be a template digitizer or a software digitizer. The combination of the template digitizer and NC machine (TD/NC machine) operates as a copying machine. While the combination of the software digitizer and NC machine (SD/NC machine) resembles in operation a conventional NC system.

The polar contouring system employs an open loop digital control scheme that does not require an on-board computer.

The system offers a number of desirable features over both conventional copying machines and NC systems. Speed scaling, and scaling of contouring speed without the need of reprocessing the contour information are among these features. Furthermore, when operated as an SD/NC machine, the system is capable of tracking a contour with constant speed.

A detailed description of the polar contouring system and the salient features of the prototypes of the template and software digitizers, and of the NC machine that has been built is presented. An analysis of the contouring error due to the incremental approximation of the contour is carried out. An optimization procedure that locates the contour pole with respect to the machine pole for a minimum area error is outlined. Test results of locating the poles of three test contours for a minimum area error are also presented. A dynamic analysis of the NC machine that includes the type of contour, the type and magnitude of the load, and the contouring speed is presented. The results of the analysis indicate that in general, increasing the magnitude of the load or the contouring speed increases the torque required from the axes-drives. It is also shown that the torque increases rapidly with the decrease of the radius of curvature of the contour. The maximum allowable load and contouring speed are shown to be a function of the location of the contour pole with respect to the machine pole. An optimization procedure to locate the contour pole relative to the machine pole is presented for maximizing the allowable limits on load and on contouring speed. A procedure using composite graphs is provided for rapid calculation of the maximum torque required and available from the axes-drives in different contouring operations.

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NOMENCLATURE

a_t	acceleration of the NC machine carriage along the tangent to the contour.
$a^{(i)}$	area error between the contour and its approximating curve in the i-th angular resolution.
a_c	acceleration of the NC machine carriage.
a_{cn} , a_{cr}	acceleration of the NC machine carriage normal to the arm and along the arm respectively.
C	radial displacement of the machine pole from the contour pole.
c	projected length of a line along the radial vector.
F_1 , F_2	objective functions for optmization.
F_f	friction force normal to the NC machine arm and due to its support.
F_{cr} , F_{cn}	components of the force opposing the tool, radial and normal to the arm respectively.
F_{sum} , F_{dif}	dimensionless functions made up of N_R and N_Θ .
H	template thickness
J_A'	mass moment of inertia of the NC machine arm.
J_A	equivalent mass moment of inertia of the

NC machine arm and angular axis-drive.

K_1 torque transformation factor for the worm worm-gear reducer, including friction losses.

K_2 torque to force transformation factor for the power screw and nut assembly, including friction losses.

K_c force factor for the grinding operation.

K_T , K_V torque and voltage constants of the stepping motors.

k_R angular to linear transformation factor for the power screw and nut assembly.

k_θ speed reduction factor for the worm worm-gear reducer.

L_A distance between the NC machine pole and the arm support.

L_a inductance of the stepping motor coils.

ℓ linear segment of an approximation to a contour.

M'_c mass of the NC machine carriage.

M_c equivalent mass of the NC machine carriage, with the power screw and radial axis-drive inertias reflected on it.

N_R , N_θ number of radial and angular steps representing a line respectively.

N_s number of teeth on the stepping motor rotor.

R_a	total resistance of the stepping motor coil and drive circuit.
R_r , R_r^*	tracking roller or tool-radius and its nondimensional value with respect to the thickness of the template.
r	radial displacement of the center of the carriage from the machine pole.
r_1	radial displacement of a point (P) on the contour from the NC machine pole.
r_c	radial displacement of the center of the carriage from the contour pole.
T_p	pull-out torque of a stepping motor.
T_{PR} , $T_{P\Theta}$	pull-out torque of the radial and angular stepping motors respectively.
T_R , T_Θ	torque required from the radial and angular axes respectively.
t	time.
V	voltage.
V_t	constant velocity of a point along the tangent to the contour.
V_x , V_y	velocity of the machine tool along the X- and Y-axis respectively.
v, v^*	velocity of the tracking roller at the center of the carriage and its nondimensional value with respect to $2V_t$, respectively.
$v_{G'}$	velocity of the pressure roller

tangential to the contour.

α angle between the radial vector from the contour pole and the tangent to the contour, at a point (P).

α_c inclination of the grinding force to the work-piece surface.

β anglular displacement of the machine pole from the contour pole.

γ angle between the machine arm and the tangential vector at a point on the contour; measured from the arm.

ΔA template digitizer and NC machine angular resolution

ΔR, $\Delta\theta$ template digitizer and NC machine radial and angular stepping increments, respectively.

Δs length on the contour corresponding to an angular or radial step.

ε_a area error between the contour and its approximating curve.

ε_d deviation error between the contour and its approximating curve.

θ angular displacement of the template digitizer or the NC machine arm.

θ_1 angular displacement of a point on the contour with respect to the NC machine pole.

θ_c	angular displacement of the center of the carriage with respect to the contour pole.
ℓ	lead of the power screw helix.
Λ_1 , Λ_2 , ...	equation simplification parameters, grouping parameters.
ρ	radius of curvature at a point on the contour.
τ_1 , τ_2 , ...	equation simplification parameters, grouping parameters.
T_R , T_θ	torque ratio, defined as the ratio of the torque required to that available for the radial and angular axes respectively.
ϕ	angle of inclination of the velocity vector tangential to the contour, or of the tangent to the contour.
ω_R , ω_θ	angular velocity of the radial and angular axes-drives respectively, measured in electric degrees.

Note:
- Superscripts (I) and (G) indicate inertial and grinding cases, respectively.
- Subscripts R and θ refer to the radial

and angular axes respectively.

- A "^" on top of a parameter indicates the maximum value of that parameter along the contour.
- A "*" on top of a parameter indicates a nondimensional value of that parameter.

CHAPTER 1

INTRODUCTION

1.1 General

Contouring is defined as an operation of following or tracing the silhouette of an object. This operation is frequently used in many aspects of engineering and manufacturing practices.

Contouring systems are found incorporated into machines that are intended for a wide variety of applications. Some examples where such systems are utilized are:

- **Material removal:** In the manufacturing of turbine blades, cams and a host of similar products turning, milling, and grinding machines are equipped with contouring systems that guide the tool over the required contour.
- **Non-contact cutting:** Various non-contact cutting methods like flame or plasma cutting rely on a contouring system to guide the cutting nozzle over the cutting line.
- **Drafting:** The use of contouring systems in automatic drafting machines is relatively new. With the advent of computer-aided-design (CAD) these machines have become a necessity in modern design offices. In these machines the contouring system guides the drafting pen to produce the required drawing.
- **Clothing industry:** In the clothing industry, sewing ma-

chines are equipped with contouring systems to guide the sewing head over the sewing line (e.g. sewing of a shirt collar).

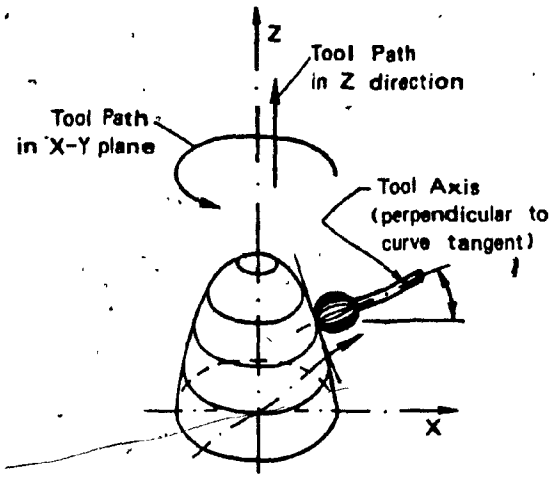
In general, contouring can be classified as:

a) spacial (3D/2½D), where a set of three axes is required to define any point on the contour, and

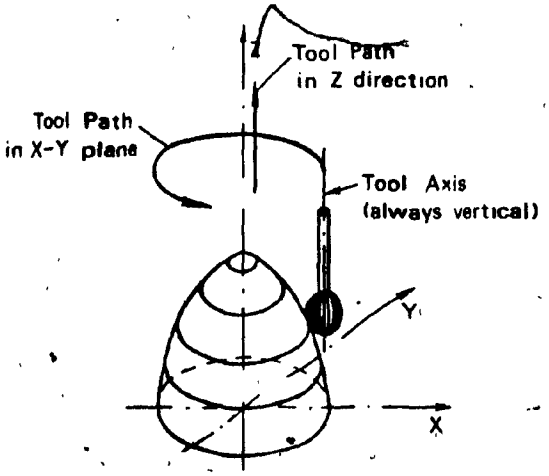
b) planar (2D), where a set of only two axes is needed.

Spacial contours may be produced by a series of parallel planar contours each incremented from the previous one along the normal (third axis) to the planes with the tool tilted such that its tangent plane at the cutting point coincides with the tangent plane of the contour. This type of operation, shown in Figure 1.1.a is known as 3D (three dimensional) contouring. Warped and double curved surfaces are examples that require such contouring operations. A less complex operation which approximates the 3D is commonly known as 2½ D, and is shown in Figure 1.1.b. In this operation the tool is indexed along the third axis with no tilt adjustment. This operation can be easily performed using planar contouring (2D, shown in Figure 1.1.c) by providing it with an indexing drive along the third axis. The latter three of the above examples are typical of planar contouring.

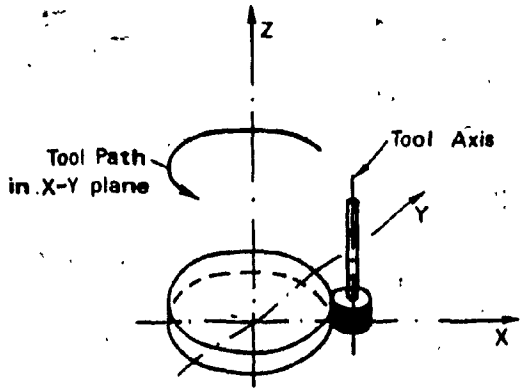
Some 2D contours can be produced by means of linkage mechanisms. Circles, for example, can be produced by one end of a rod constrained to move with the other end fixed at a point in the plane. References [1] and [2] present a



(a) 3D



(b) $2\frac{1}{2}$D



(c) 2D

Figure 1.1: 2D, $2\frac{1}{2}$D, and 3D Contouring Operations.

large number of such mechanisms for producing a variety of contours. In general, however, such mechanisms are very complicated in construction. Furthermore, each of these mechanisms is specialized to produce a single family of contours and in general cannot be easily adapted to produce other families. Thus, for a single system to produce any arbitrary contour, it cannot make use of these linkage mechanisms.

A large number of general purpose contouring machines have evolved over the years. These machines require as input some form of information about the contour they have to produce. Depending on the form of input, these machines can be categorized as: a) Copying machines, and b) Numerical control (NC) machines.

1.2 Copying machines

These machines utilize a template of the required contour for their operation. The pantograph copier shown in Figure 1.2 is a pioneer system belonging to this category. With reference to the figure, in its primitive form, the tracer pin is manually guided over a template. By virtue of the design of the pantograph links, the tool traces a curve geometrically similar to the template. The reproduction scale depends upon the position of the tool on the tool arm. The main application of the pantograph copier is in engraving. When large forces are encountered by the tool, as in metal removal applications, high power electric, hydraulic,

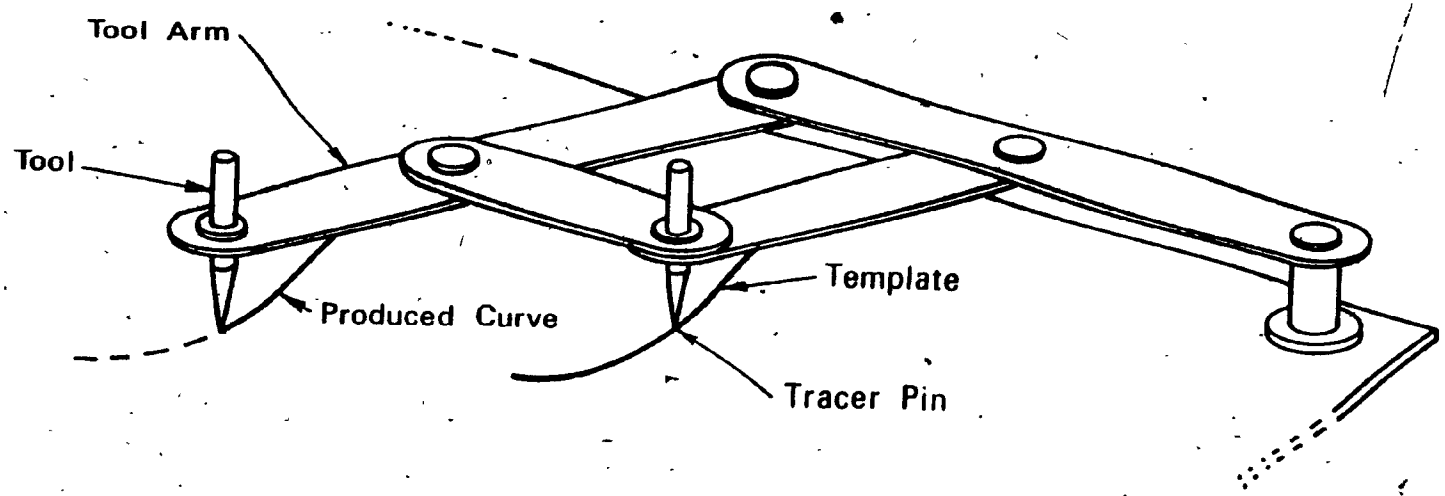


Figure 1.2: Pantograph Copier.

or electro-hydraulic copying machines have to be used. Figure 1.3.a shows a schematic diagram of such a machine. The machine consists of two machine beds, one sliding along the X-axis, the other along the Y-axis. The machine tool is linked to a template follower either directly or through a force amplifier. With reference to Figure 1.3, and with no loss of generality, the template follower may be one of the following types:

a) Passive-type, as shown in Figure 1.3.b, in which the template follower is simply preloaded against the template along the Y-axis. Moving the template along the X-axis causes the Y-axis bed to move back and forth in accordance with the template profile thus producing the copying action. Passive followers can be used only within a small range of template slope angles, the range of angles being a function of the speed of template travel along the X-axis. Reference [3] describes 2D and 2½ D copying machines utilizing such followers. The machine described in the reference utilizes hydraulic servo-controlled amplifier between the tool and the template follower.

b) Active-type followers, shown schematically in Figure 1.3.c, carry a set of two force transducers arranged normal to each other. The transducer signals are utilized to control the X- and Y-axis bed movements. The beds are controlled in such a way that the follower is always kept in contact with the template.

c) A third type of follower utilizes an opto-electronic

sensor to follow a line or edge (silhouette) type template. Reference [4] describes a flame cutting copying machine that uses such a follower. One such sensor is detailed in Reference [5] and shown schematically in Figure 1.3,d. The sensor consists of a circular array of photo-diodes and a single diode at the centre. The control element of the copying machine uses the information from the circular array of sensors to guide the central photo-diode over the silhouette.

d) Powered-type followers are mainly employed in applications where the force opposing the tracking action is low. The clothes manufacturing industry is the largest user of this type of follower. An industrial sewing machine utilizing a powered follower is described in Reference [6]. Figure 1.3.e shows a schematic of a powered follower-cam arrangement. The follower is the shaft of an electric motor that is kept in contact with a magnetized steel template.

References [7] to [10] describe a gyroscopic tracking mechanism where the gyro spin axis acts as a powered follower. The spin axis adheres to the template due to the gyroscopic action. Detailed analysis of the mechanism is given in the references. References [9] and [10] report prototypes that has been satisfactorily tested.

1.3 NC machines

In its broad meaning, a system is termed numerically controlled if it operates automatically according to a set

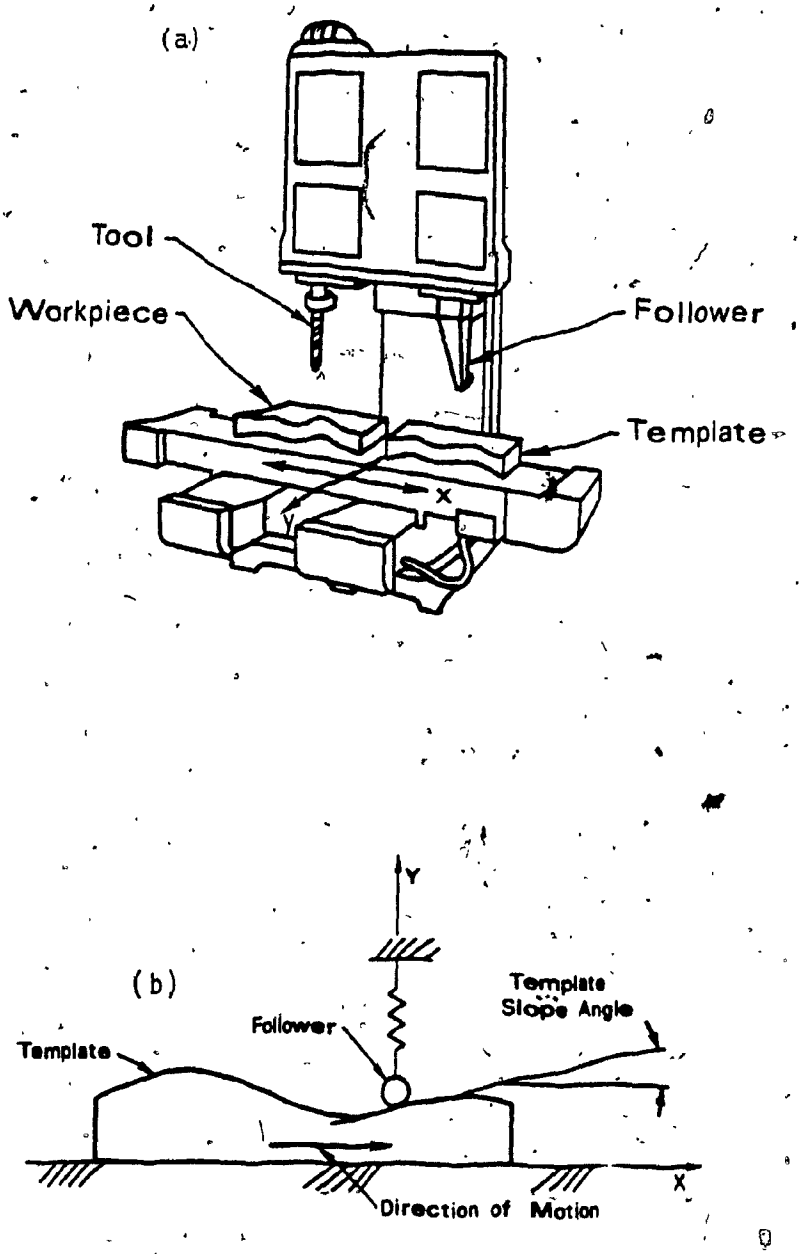


Figure 1.3: Schematic Diagram of a Template Copying Machine/Template Configuration.

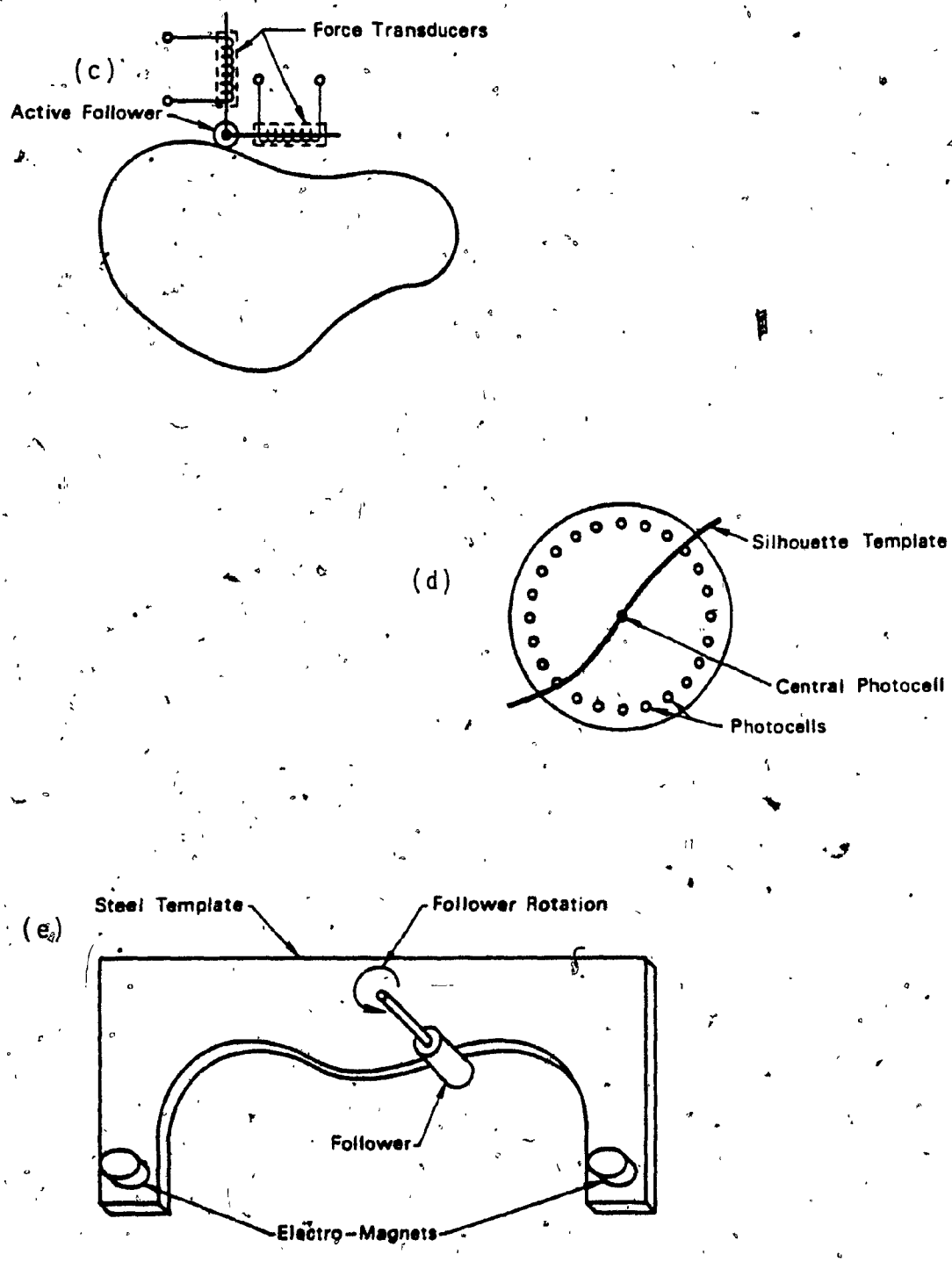


Figure 1.3: Schematic Diagram of a Template Copying Machine/Template Configuration.

of coded instructions fed to it. References [11] and [12] report the first NC machines to have appeared during the 18th century : the Falcon (1728) and Jacquard (1800) looms. According to Reference [11], for the next 150 years the only descendants of these machines were the piano and the barrel organ player. Modern NC machines first appeared around the year 1942, and were developed simultaneously but separately by the Bendix corporation, and by Massachusetts Institute of Technology in collaboration with I.B.M. and Cincinnati Milling Company. The 1942 machines were computer controlled. In the last three decades, due to the rapid development in computers and related technologies, NC machines have proliferated all over the U.S.A. Europe, and Japan, and the term "numerically controlled" has become synonymous with computer controlled machines. Figure 1.4 shows a schematic of an NC machine. These machines look similar to copying machines with the exception of the absence of the template follower, and the presence of a controlling computer. Literature on NC machines is abundant. References [13] to [16] are conference proceedings of engineering societies devoted to the subject. These references report advances in both the hardware and software of NC machines.

NC machines differ from servo controlled machines in that they move in preset increments or resolutions on each of their axes. The axes-drives, however, may themselves be controlled through analog servos. The machine can index

only to those points on its axes that are multiples of the resolutions. Servo controlled machines, however, have an infinite resolution and can move to any loction on the axes.

Depending on the capability of NC machines they are classified as: a) Point-to-point, b) Straight path, or c) Contouring NC machines.

a) Point-to-point NC machines are by far the simplest and the most common of the three classes. Here the tool cannot operate while the machine beds are in motion. The machine acts as a positioning table for the work piece and also performs some control functions like tool change and operation. Point-to-point NC machines are mainly used in applications like drilling, tapping, reaming and the like.

b) The tool of a straight path NC machine is in continual operation while the machine beds are in motion. The controller for this type of machine is more complex than that for the previous type, and is capable of coordinating the axes-motors so that the work-piece can move in a straight line in any direction. Recent improvements on these machines allow them to follow circular paths, Reference [11].

c) In the case of NC contouring machines the contour information is in the form of coordinate sets of points on the curve. A large memory is required to store this information. The curve distance between the given points on the contour are much larger than the machine resolution. The controller has to interpolate between the supplied points in

order to smoothen the curve. Furthermore, the controller has to coordinate the velocity of the axes-drives in order to produce constant contouring speed. These two mathematical tasks together with auxiliary control functions, like tool change and operation, necessitate a powerful controller. These controllers are usually built around powerful microprocessor units, and although the ratio of cost to power of microprocessors is dropping rapidly, the cost of controllers is not decreasing appreciably yet. This is due to the high development cost, the small number of units built, and the expense of good control software.

In almost all contouring applications, constant speed of travel along the contour is a highly desirable feature. Reasons for such preference are specific to each application. In flame or plasma cutting applications, for example, constant speeds of travel along the cutting line allow precise gauging of the power required, resulting in savings in cutting energy. For metal removal applications like milling or grinding, the constant contouring speeds would result in a better surface finish and extended tool life. In some contouring applications, as in the clothing industry, speeds of travel along the sewing line must be constant to insure equal stitch lengths. Such a homogeneous stitch is essential both for the strength, as well as for the aesthetic appearance of the product. Some other contouring applications are rather insensitive to the speed of travel along the contour. Drafting pens for example have

reached such a maturity of design that allows them to operate over a wide range of travel speeds. Consequently, there is no constraint of constant contouring speed along contours in drafting applications.

All the commercially available contour following systems utilize a Cartesian system of coordinates for their operation. Referring to Figure 1.5, constant contouring speeds would be achieved if the following mathematical relation is satisfied at every point on the contour:

$$V_t = \left[V_y^2 + V_x^2\right]^{\frac{1}{2}} = \text{constant} \qquad 1.1$$

To satisfy the above relation, the tangent to the contour at every point must be known. In the case of copying machines, finding the tangent direction involves complicated instrumentation which substantially increases the cost of the system. For NC machines, however, the controller approximates the velocity relation to keep the tracking velocity within acceptable tolerance. In the powered type follower systems, the follower is driven at a constant rotational speed and supplies the motive power. Under ideal conditions of no slip, this translates to constant tracking speed along the magnetized template.

Reference [17] describes the prototype of a plotter that utilizes polar coordinates. A photograph of the prototype is shown in Figure 1.6. The plotter consists of a semicircular bed (A). An arm (B) hinged at the centre of

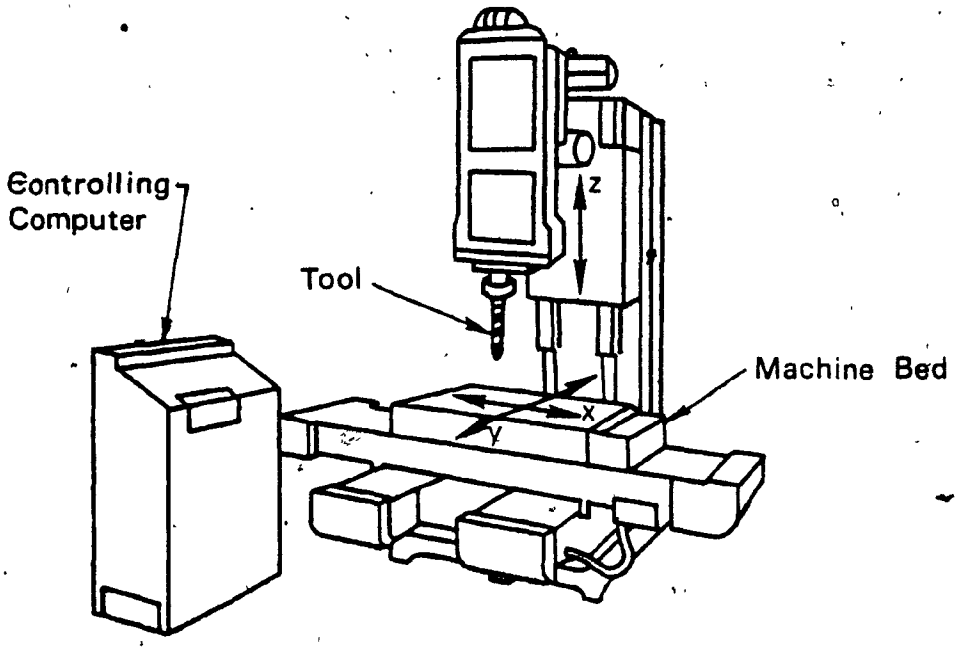


Figure 1.4: Schematic Diagram of an NC Machine.

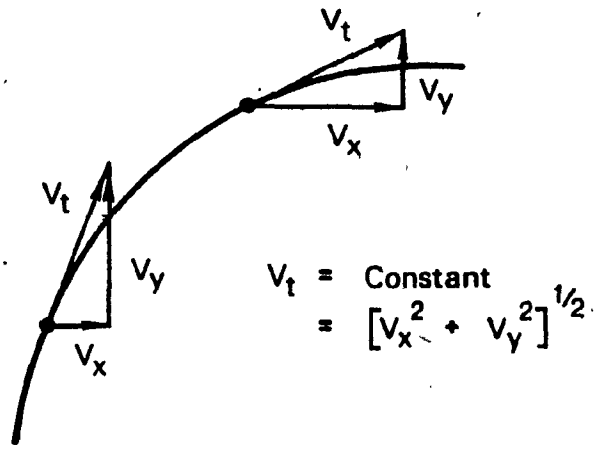


Figure 1.5: Velocity Diagram in Contouring Operations.

the semicircle is powered by means of a cable (C) and pulley arrangement. The arm swings around the hinge (D). A stepping motor (E) drives the cable (C). The arm carries a carriage (F) fitted with the plotting pen and driven by another stepping motor (G) through a cable and pully system (H). The configuration of this system is shown to be suitable only for plotting applications. No analysis or study of the mechanism is reported.

This thesis is concerned with the planar class of contouring systems, and for the purpose of brevity the terms "contours" and "contouring" will refer to the planar class throughout the text, unless otherwise specified.

The thesis presents a contouring system which utilizes polar coordinates for its operation. The system is comprised of a contour digitizer and an NC machine. Two types of digitizers are discussed: a physical template digitizer, a prototype of which has been built and tested, and a software digitizer which requires as input a mathematical representation or points on the contour. The software digitizer uses a computer which is not on-board the NC machine and does not control its operation on real time basis. The output of both digitizers can either be transmitted directly to the NC machine or can be recorded on magnetic tape for repetitive, or later, use. A prototype of the NC machine has also been built. The machine controller is simple and does not utilize any microprocessors for its operation. The machine is dedicated for contouring appli-

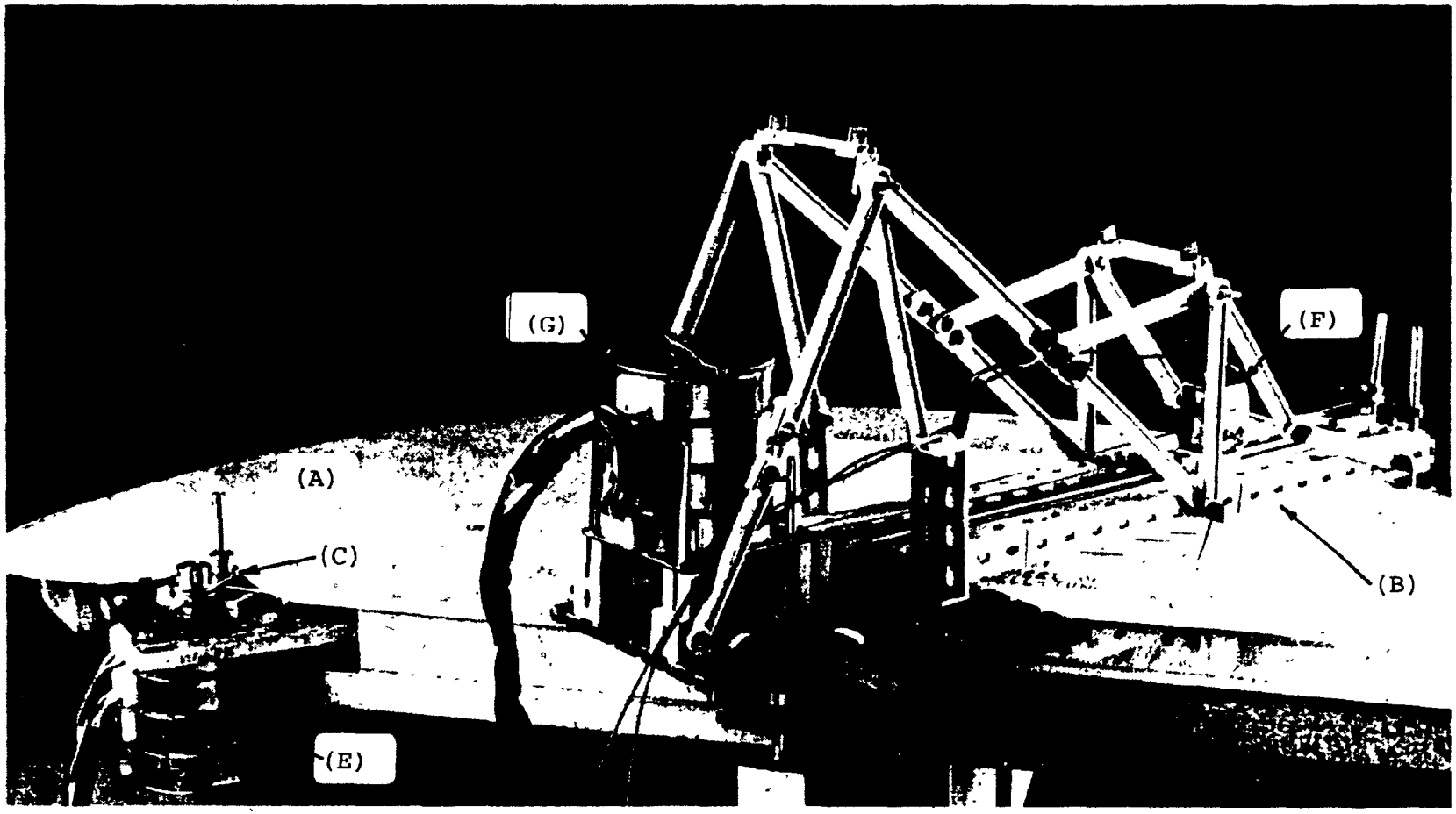


Figure 1.6: Polar Plotter.

cations. Some auxiliary control functions, however, can be added without complication.

1.4 Thesis outline

The thesis discusses the overall philosophy of a polar coordinate based contouring system. The main operational aspects of the system are studied in detail. The study includes synthesis, analysis, and design of the contouring system. A detailed study of the system under kinematic and dynamic conditions is given. Two methods are presented, one for optimally locating a contour within the work space in order to minimize the digitization error, and the other to evaluate the limit of the contouring speed of the system under load. Other studies, concerned with the structural rigidity, vibration, and similar aspects have not been included.

Chapter 2 of the thesis presents first the different configurations of the system. This is followed by a description of two different methods used to digitize contours: the template digitizer, and the software digitizer. Prototypes of a template digitizer and an NC machine have been built. Detailed description of both prototypes, as well as the software digitizer that has been constructed is also presented.

Chapter 3 discusses the main features of the polar contouring system. These features as well as the advantages and shortcomings of the system are itemized and compared

with those of commercially available contouring machines. A summary of the salient points is given in the form of a tabulated comparison at the end of the chapter. Due to the nature of polar coordinates, the system resolution is not homogeneous within the operating area, as a result, an error analysis which is of paramount importance is studied in this chapter.

In Chapter 4 it is shown that an area error criterion is more stringent than the traditional deviation error criterion. A study of this error shows that its magnitude depends upon the position of the contour with respect to the machine pole. The calculation of the area error is too lengthy a process to be used as an objective function in an optimization routine for minimization. An alternate objective function, formulated in Chapter 4, is fast to calculate and is suitable to incorporate in an optimization routine. The results of optimization and some sample calculations of area errors for three test curves are also presented in this chapter.

Chapter 5 deals exclusively with the kinematics and dynamics of the NC machine. The dynamic study is used to determine the load capacity of the machine. The load on the axes-drives depends upon the machining forces as well as on the contour being processed and the contouring speed. The effect of these operational parameters on the torque requirement of the axes-drives is also studied in this chapter. Furthermore it is shown that for a given set of

operating parameters the load on the axes-drives varies with the position of the contour pole relative to the machine pole.

In Chapter 6 a detailed analytical study is conducted to evaluate the effect of variation of operating parameters on the torque of the NC machine axes. An objective function is formulated for minimizing the total torque of the NC machine and equalizing the load imposed on the two axis drives. The objective function is incorporated within an optimization algorithm to locate the relative location of the machine pole from the contour pole. Based on the results of the optimization, a procedure is outlined to increase either the contouring speed or the allowable load on the machine.

In Chapter 7 a graphical procedure is also presented by which both the torques required and available from the two drives can be easily found.

The thesis concludes by outlining the salient features of the contouring system. A summary of the investigations as well as the important findings are given. Recommendations on the future studies towards development of a second generation prototype and eventually, of an industrial system are outlined.

CHAPTER 2

CONCEPT AND OPERATION OF THE CONTOURING SYSTEM

2.1 Introduction

The configuration of the polar contouring system allows it to operate as a copying system or as an NC system that has very little on-board signal processing. Figure 2.1 shows a block diagram of the contouring system in its different configurations as well as the flow of signals between the various components. The system is composed of two parts, a contour digitizer and an NC machine. As shown in the figure, the digitizer can be either a template digitizer or a software digitizer. When the contouring system is used for copying, it is configured as a template digitizer/NC machine (TD/NC machine), while when used as an NC system it is configured as a software digitizer/NC machine (SD/NC machine). Contour information for the template digitizer is provided by means of a template of special form. Two forms of templates intended for precision and non-precision work will be discussed later. The software digitizer utilizes a computer to digitize the contour, with the information being in equation form or as a number of discrete points on the contour. The control signals from a digitizer can be directly channeled to the NC machine or recorded on magnetic tape for later use. Recording the control signals prior to use has a number of advantages that will be discussed in

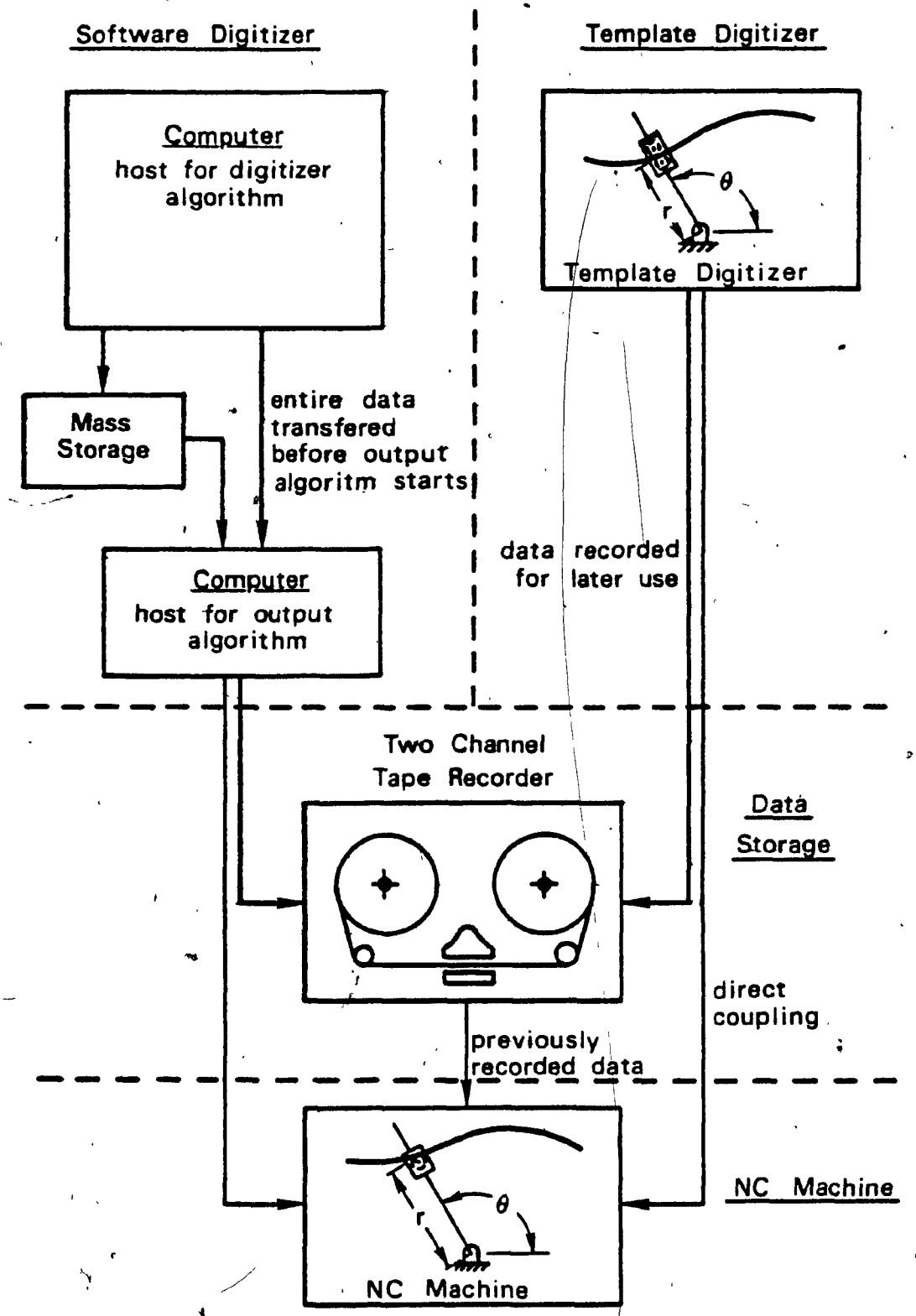


Figure 2.1: Block Diagram of a Polar Contouring System.

detail later. Some of these advantages are:

a) Ease of scaling the contouring speed by altering the tape playback speed,

b) A single template digitizer can be used to prepare recorded control signals for more than one NC machine, and

c) Recording control signals rather than directly channelling them to the NC machine allow software digitizers to run in BATCH mode on computers. In addition, recording the control signals allow small computers to be used as hosts for the digitizer since no speed constraint is imposed on the computation. Furthermore, computer resources are not tied down executing the same commands repeatedly, and can be available for other processes.

For recording and playback of the control signals, a conventional stereo, reel or cassette tape recorder can be used. The speed control circuit on the tape recorder can be modified to allow for variable speed and thus providing speed scaling. Some commercially available recorders are built with such a feature. The output signals from the two recorder channels are conditioned before feeding to the NC machine.

The NC machine accepts the control signals and uses a simple hardware logic circuit to direct them to the appropriate axis-drive.

Prototypes of the various components of the system shown in Figure 2.1 have been built and tested. Detailed descriptions of prototypes of the three main components in

the figure, namely, the template digitizer, the software digitizer, and the NC machine are presented below.

2.2 Prototype template digitizer

A schematic diagram of the prototype template digitizer and correspondingly labelled photographs are shown in Figures 2.2 and 2.3 respectively. With reference to the figures, the platform (A) constitutes the working area of the digitizer. An arm (B) is keyed to a shaft (S) at one end and free to swing within the limits marked by the stops (C) and (C') (not shown on the photographs). The stop (C) on the right, marks the arm's home position or the reference of the angular axis. A digital shaft encoder (D) which translates shaft positions to electric signals is fixed on the shaft in order to indicate the angular position of the arm. A carriage (E) is mounted on the arm and can slide over it. The fully retracted position of the carriage corresponds to its home position, or the reference position of the radial axis. The carriage carries another shaft encoder (F) and a pair of tracking rollers (G) and (G'), of equal radii. These rollers are designed to provide high friction between themselves and the template. Both rollers are powered in order to equally distribute the driving force and reduce the possibility of slip while rolling over the template. The rollers are driven through a power train by two precision, variable speed, DC motors (H) and (H') that are mounted on the carriage. The motors are wired in such a

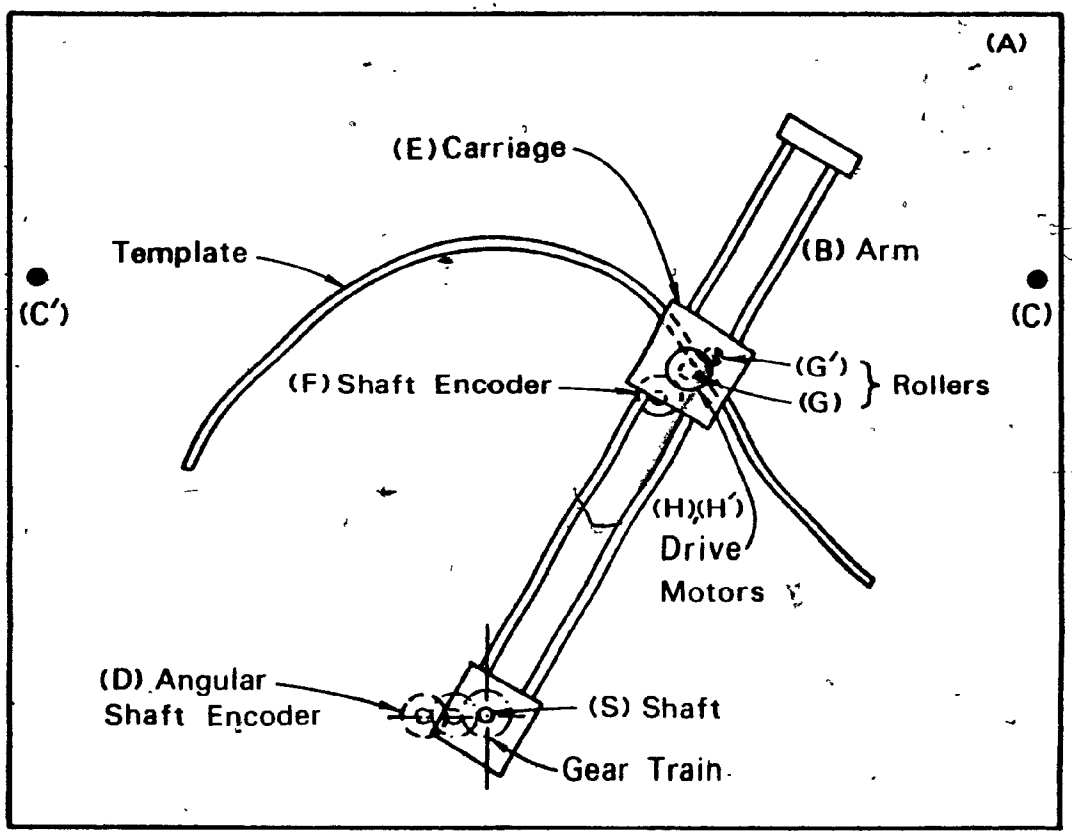


Figure 2.2: Schematic Diagram of the Template Digitizer.

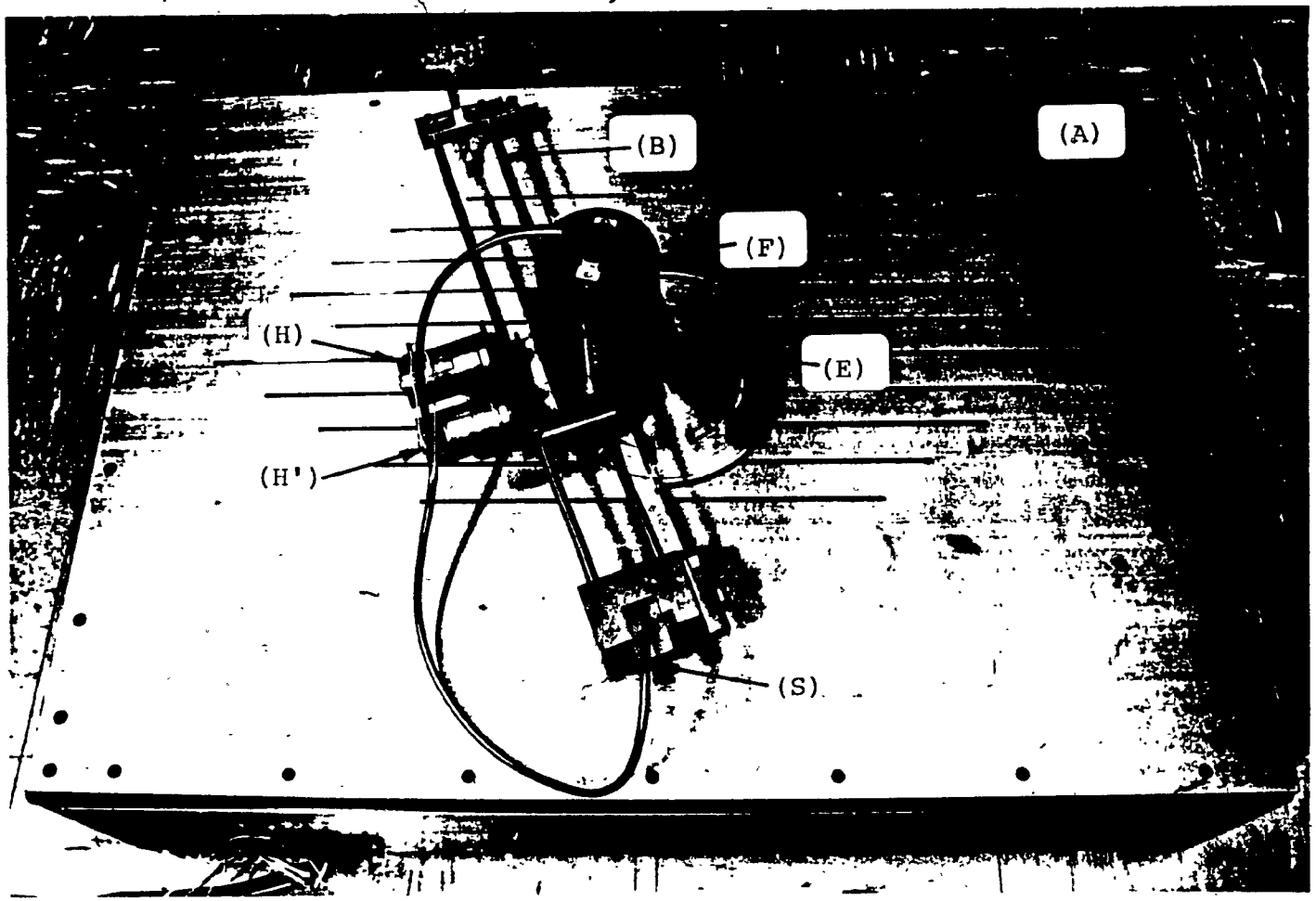


Figure 2.3.a: Photograph of the Prototype Template Digitizer.

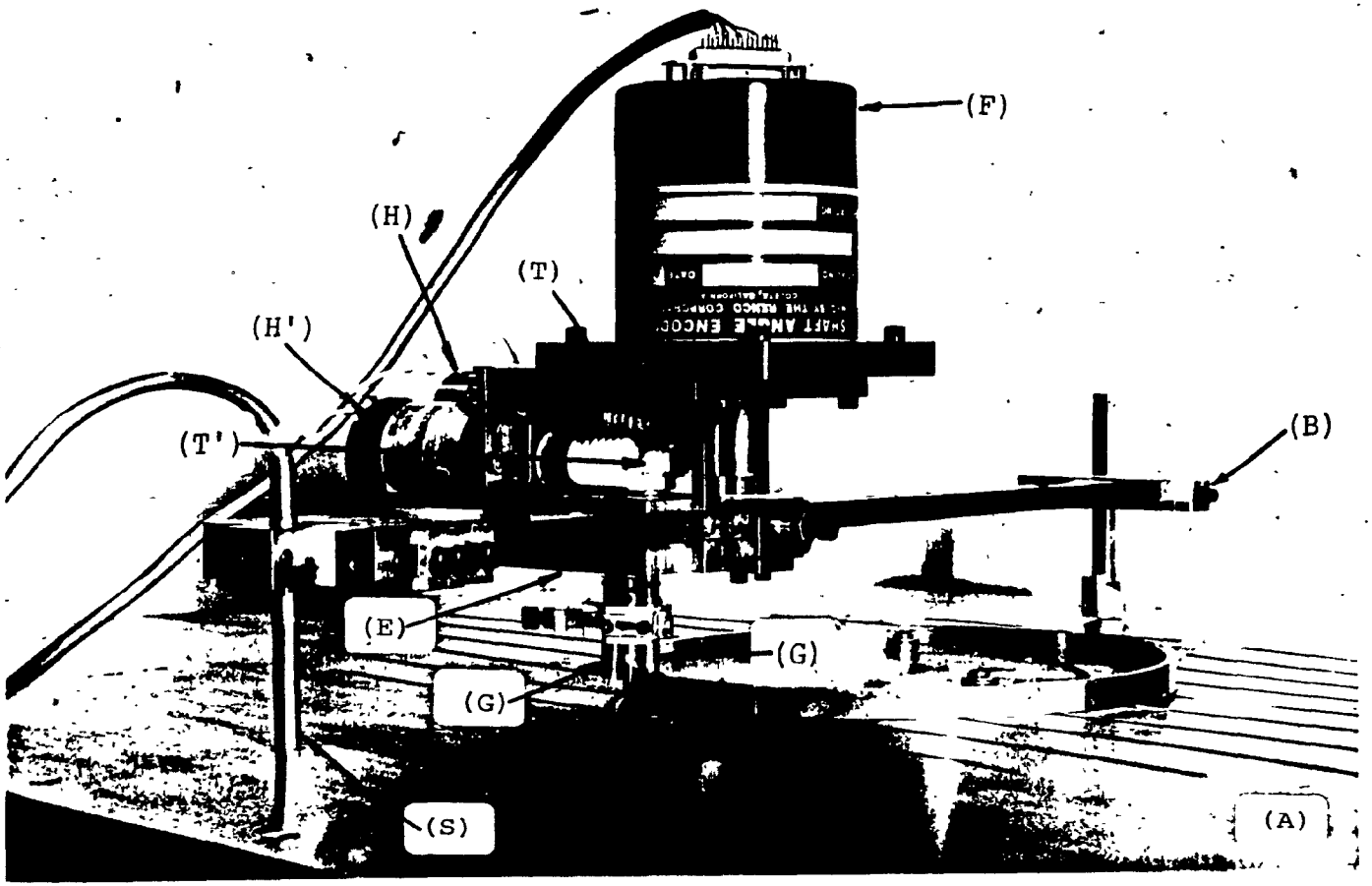


Figure 2.3.b: Template Digitizer Arm Assembly.

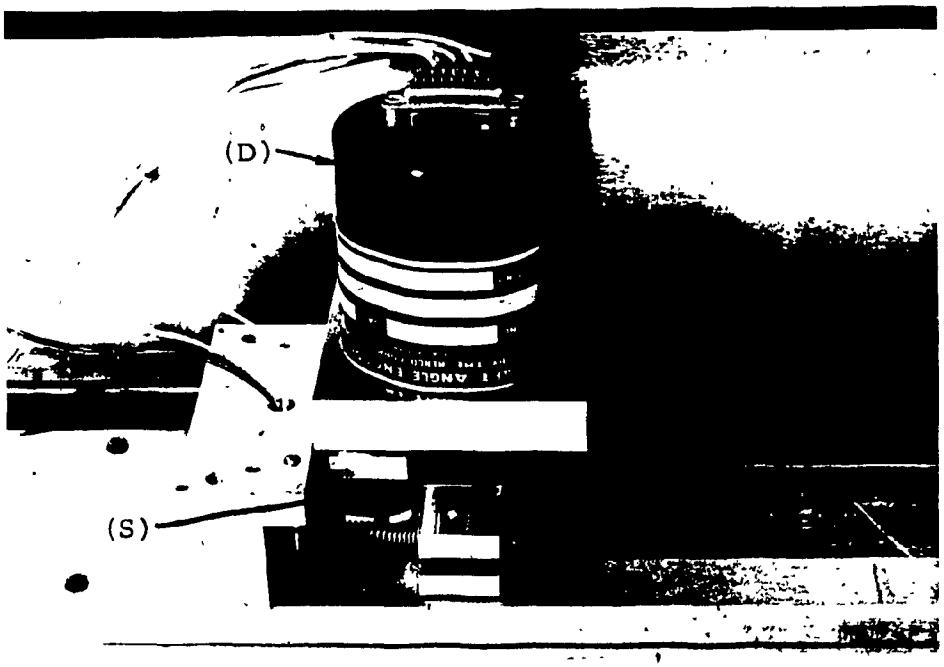


Figure 2.3.c: Template Digitizer Angular Shaft Encoder.

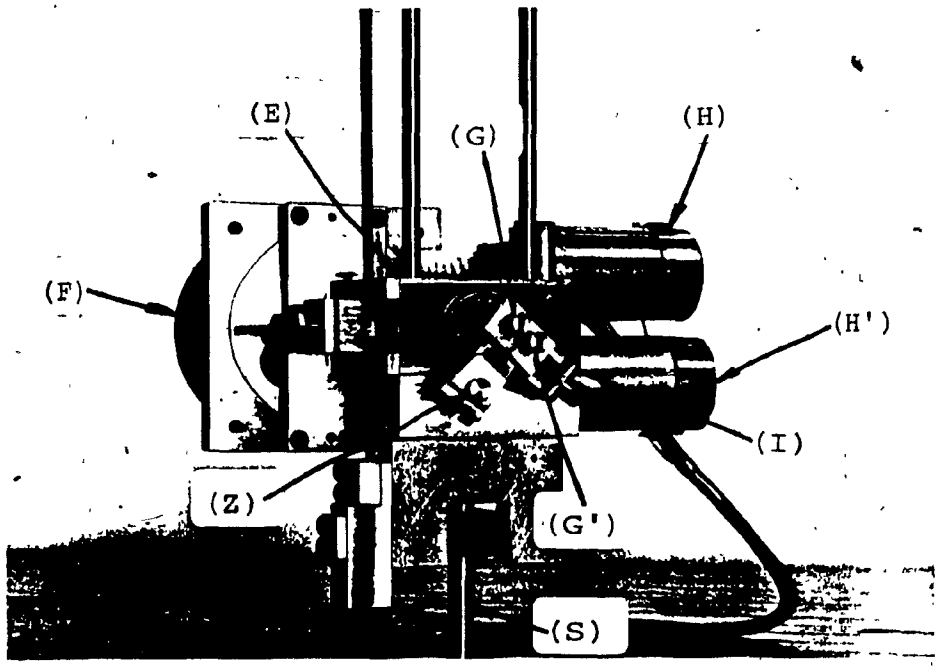


Figure 2.3.d: Template Digitizer Tracking Roller Mechanism.

way so as to produce a mechanical differential effect. The shaft encoder (F) has a friction roller mounted on its shaft. The roller rolls on the arm, and the signals from this encoder indicates the position of the carriage along the arm.

Assembly drawings of the major components of the template digitizer are included in Appendix [A-1].

Figure 2.4 shows two types of templates. The template shown in Figure 2.4.a is formed out of a flexible L-section strip which is fixed along its edge to the working area of the digitizer by means of screws. This type of template is less suited for precision work. The template shown in Figure 2.4.b consists of a raised track machined out of a solid plate, made of metal or otherwise. That side of the track over which the roller (G) rolls is the actual contour that will be produced by the NC machine and must be accurately formed. For no slip between the template and the roller (G), the contouring speed should be directly proportional to the rotational speed of (G).

The rotational speed of the rollers (G) and (G') may be controlled in either of the following two ways:

a) the rotational speed of (G) can be kept constant through the use of a speed sensor and a feed-back control loop, or

b) the sum of the two rollers speed is kept constant through the use of a feed-back controller.

Although the former case is preferable, the controller is complex. The controller in the latter case is simple, how-

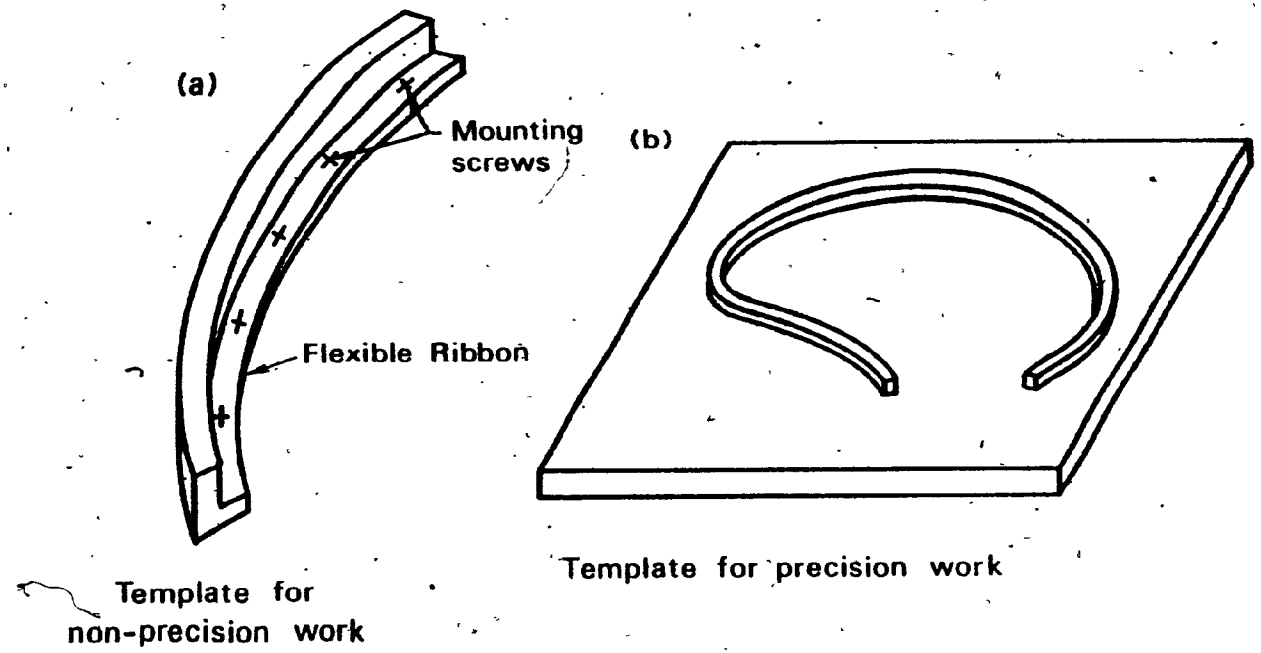


Figure 2.4: Sample Template Configurations.

ever, depending on the contour being tracked, the contouring speed may vary slightly. Detailed analysis of the variation in the contouring speed and the parameters contributing to it are discussed in Section 3.5 of the thesis.

The arrangement of the two rollers is shown schematically in Figure 2.5. Roller (G) is located at the centre of the carriage. An arm (I) swings about the axis of (G) and carries the second roller (G'). Roller (G') can slide on (I) and is spring-loaded against the roller (G) by a spring (J). The tracking roller (G) offsets the centre of the carriage from the template contour by a distance equal to its radius. If the template contour is to be reproduced, the diameter of the roller (G), and (G'), must be identical to that of the working diameter of the tool used on the NC machine.

In order for the carriage to be able to follow closed contours, the arm (I) has to be able to rotate through a full revolution around the axis of roller (G). The power train for the rollers is designed so as not to interfere with arm (I). A perspective drawing of the roller and power train arrangement is shown in Figure 2.6. The arrangement consists of two concentric shafts located at the centre of the carriage. The inner shaft extends beyond the outer one on both ends and has a worm gear (T) and the roller (G) fixed to its upper and lower ends respectively. The outer shaft also has a worm gear (T') fixed to its upper end. On

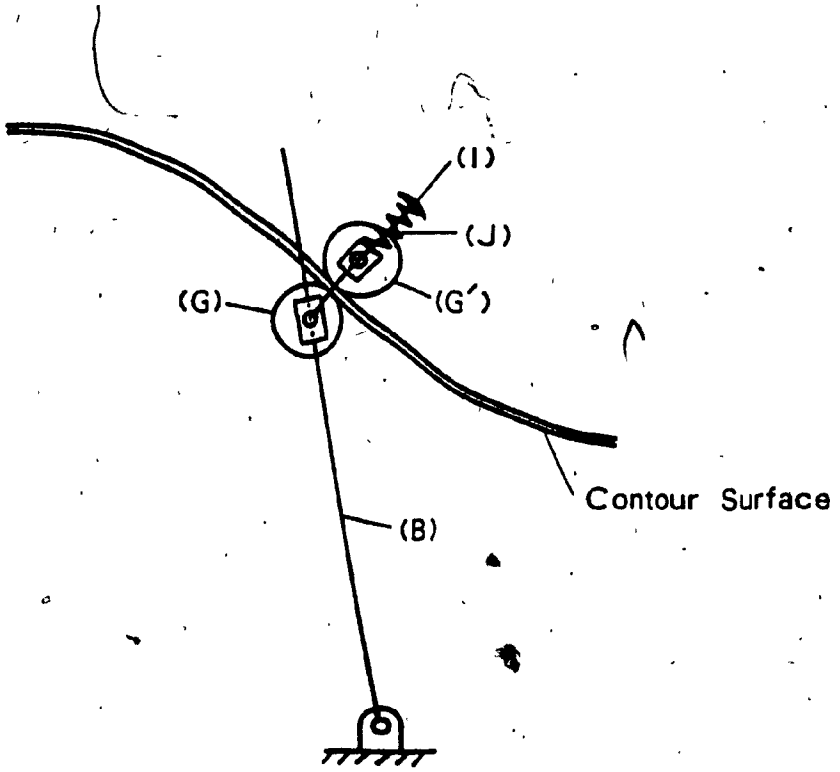


Figure 2.5: Schematic Diagram of the Roller Mechanism.

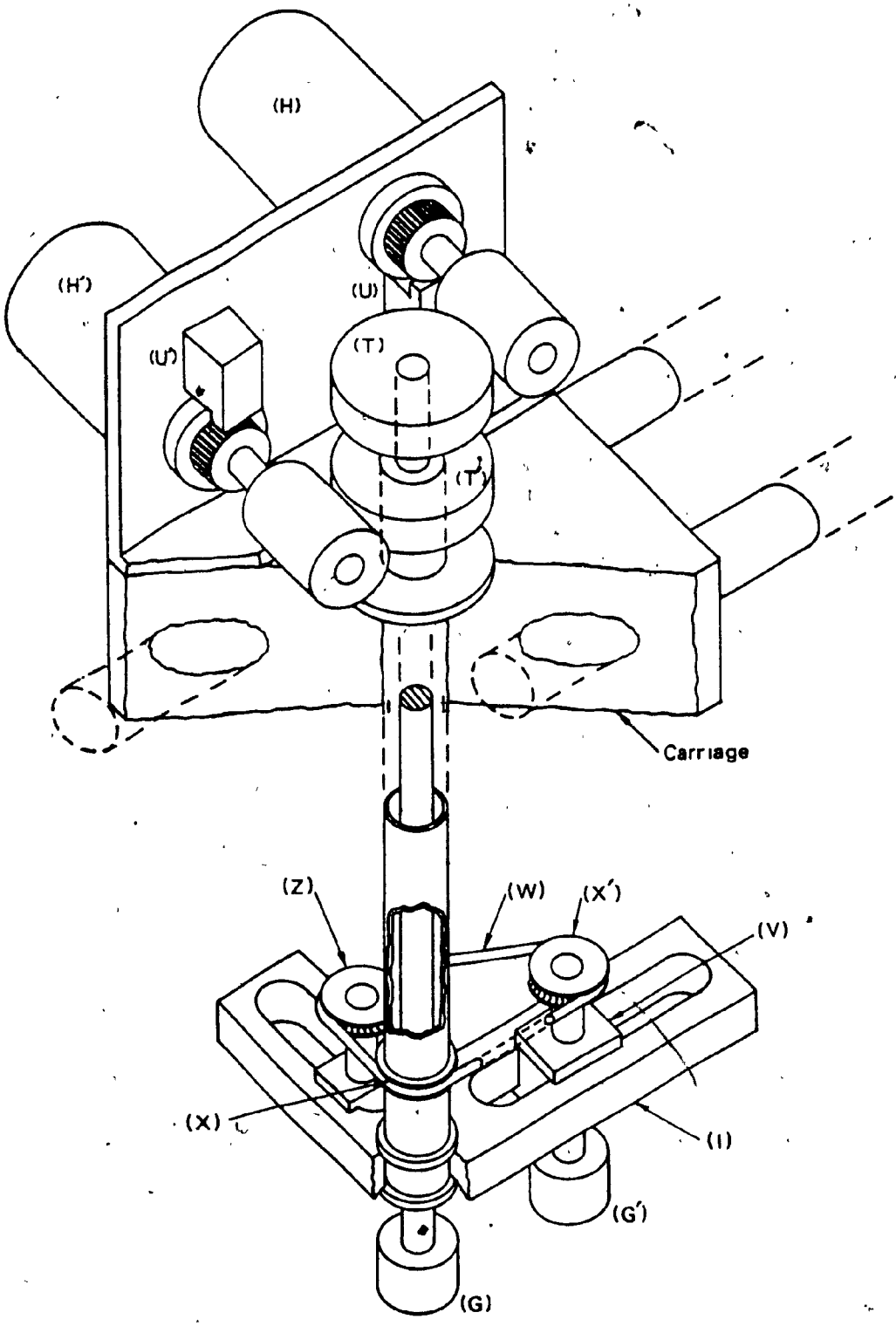


Figure 2.6: Pictorial Diagram of the Roller Mechanism.

its lower end, however, a pulley (X) is mounted followed by the arm (I) as shown. The arm is free to rotate around the outer shaft. A slider (V) slides on (I) and carries another pulley (X') on top and the roller (G') on its bottom side. The roller (G') and the pulley are fixed to rotate together by a short shaft (not shown). The slider is spring-loaded so as to bring the two rollers together. A belt (W) transmits motion between the pulleys (X) and (X'). An idler roller assembly (Z) provide tension on the belt to avoid slip. The worm gears (T) and (T') are meshed to two worms which are driven by two DC motors as shown in the figure. The motor shafts carry two optical shaft encoders (U) and (U') which are used for speed feedback of the rollers (G) and (G').

The two DC motors are series-connected and are powered by a variable voltage power supply. The electro-mechanical relationship of the two motor system is derived in Appendix [B] and resembles in function a conventional gear differential. The importance of driving the rollers through a differential arrangement can be best demonstrated by means of a schematic diagram. Figure 2.7 shows the rollers tracking two possible templates. In Figure 2.7.a the path of roller (G) in the section labelled a-b on the template is shorter than the path of roller (G'). Since both rollers are constrained to move together, the difference in speed resulting from the unequal path lengths is taken up in the differential arrangement thus preventing the rollers from

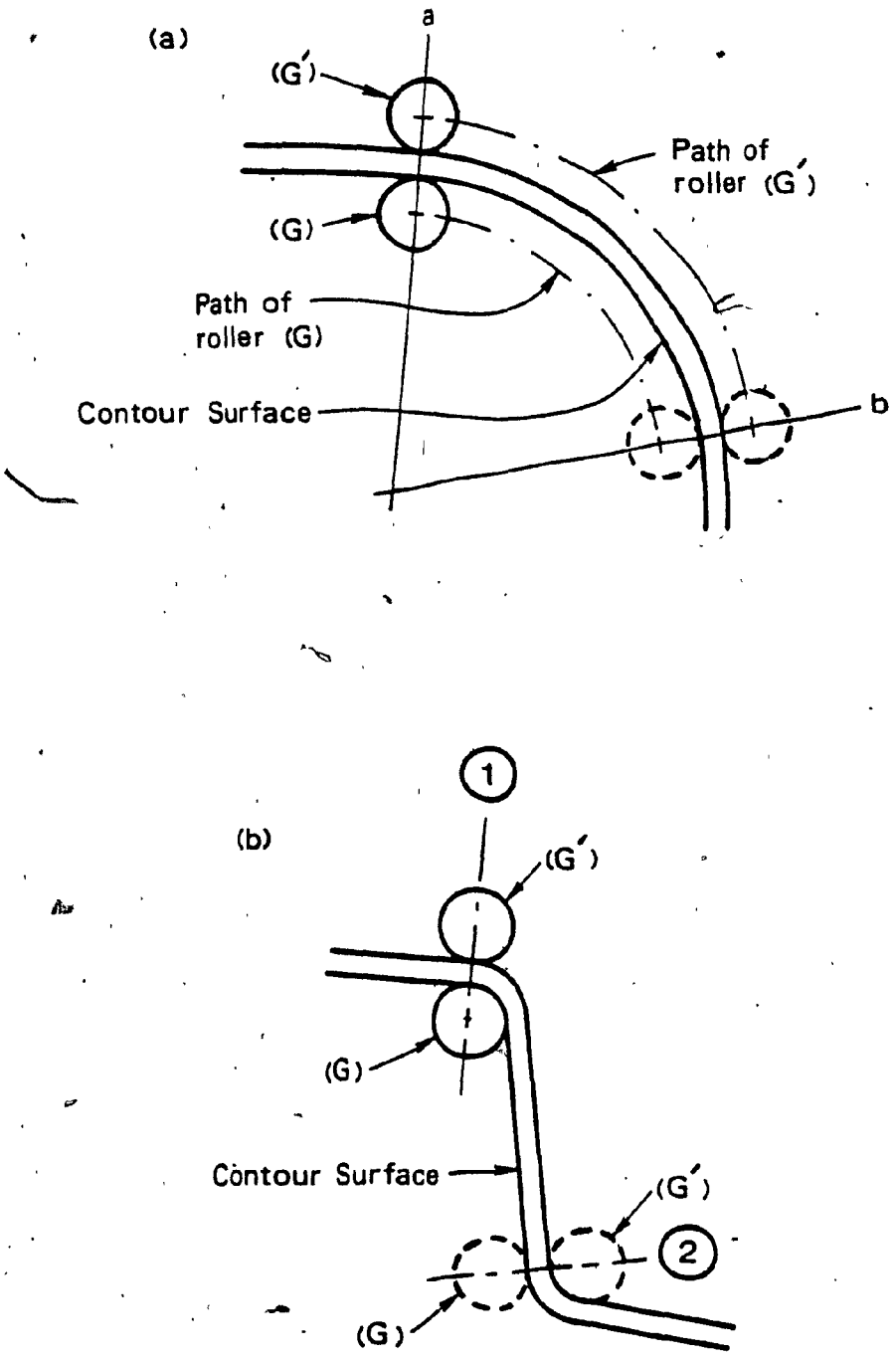


Figure 2.7: Path of the Roller Mechanism for Curved Templates.

slipping. The need for the differential here is analogous to that in the case of the driving wheels of a car when manoeuvring curves. For the template shown in Figure 2.7.b, assuming (G) alone was powered, when the rollers reach the position indicated as (1), the direction of the driving force is such that it does not allow (G') to manoeuvre around the curve and result in wedging and slipping of the roller system. In the event of (G') alone being powered, a similar situation would occur at the position marked (2), with (G) and (G') interchanging their roles. The presence of the differential arrangement allows (G) at position (1) and (G') at position (2) to come to a full-stop while the other roller negotiates the curve.

Each of the shaft encoders used in the prototype generate two logic signals as shown in Figure 2.8. The horizontal axis of the traces represent shaft rotation. With reference to the figure the upper and lower traces indicate Clockwise (CW) and Counter clockwise (CCW) indexations respectively of the encoder shaft. These two signals are combined into one as shown in the figure in order to permit recording the signals from one encoder onto one of the two channels of a commercial stereo tape recorder. The encoders are arranged such that the motion of the carriage away from the pole and rotation of the arm in the CCW direction generate positive going pulses as shown in the figure. The reverse of the above mentioned directions of motion result in negative pulses. The time rate at which the pulses are

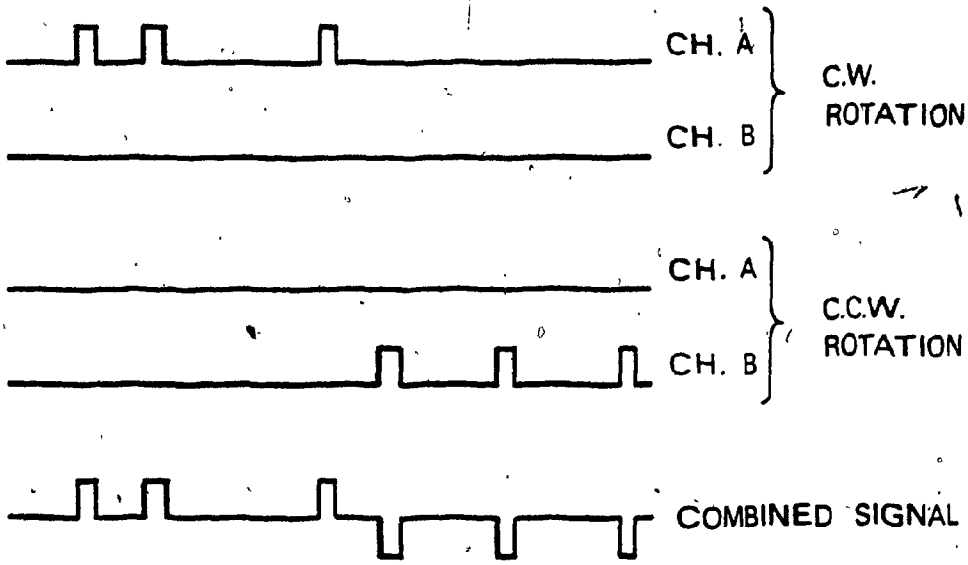


Figure 2.8: Sample Shaft Encoder Signals.

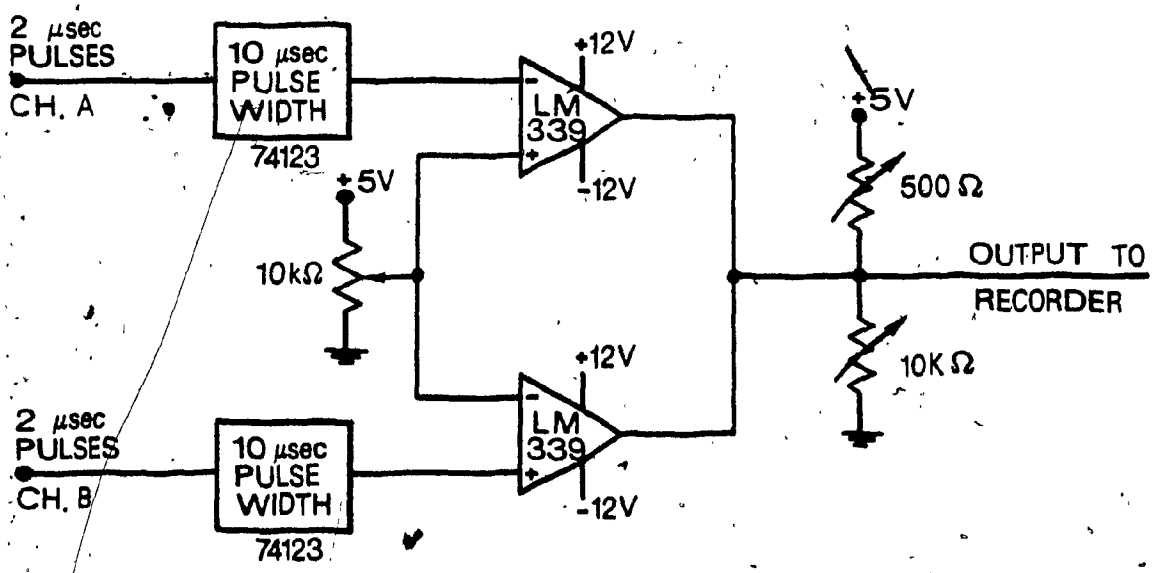


Figure 2.9: Conditioning/Combining Circuit for Shaft Encoder Signals.

generated for the trace in Figure 2.8 corresponds to the speed of motion of the axis. Since the encoder output signals have a pulse width of 2 μs, which is unsuited for tape recording, the pulse width is first increased to 10 μs using monostable multivibrators before processing. An operational amplifier circuit shown in Figure 2.9 is used to carry-out this process of signal addition. One of the encoder channels is connected to an inverting amplifier and the other to a non-inverting one. Since the CW and CCW signals cannot occur simultaneously the output of both amplifiers are simply connected together as shown in the figure. One such circuit is provided per axis-drive.

The operation of the digitizer is as follows: the rollers (G) and (G') are forced apart and then released so as to squeeze the raised track of the template in between them. Powering the DC motors through a speed regulator circuit causes the rollers, and hence the carriage, to track the template. As the carriage moves along the template, the shaft encoders generate two series of signals through their associated circuitry, each series corresponding to an axis.

The specifications of the template digitizer prototype are as follows:

Radial resolution : $R=2.54\times10^{-3}$ cm. (0.001 inch)

Angular step : $=3.927\times10^{-4}$ radians

Angular resolution : varies linearly with the radial displacement from the machine pole, and given by:

$\Delta A = r \Delta \theta$

Work area : is a section of an annulus subtended by an angle of 2.8 radians, and inner and outer radii of 7.62 cm. (3 inch) and 33.02 cm. (13 inches) respectively.

2.3 Software digitizer

The software digitizer, shown in Figure 2.1, is made up of two computer algorithms. The first one is a digitizing algorithm written in the FORTRAN language. The second, an output algorithm, is an ASSEMBLER routine responsible for transfer of the data generated by the digitizing algorithm to the output device. As shown in Figure 2.1, the digitizing algorithm requires the contour information, the tracking speed, as well as information about the tool to be used on the NC machine. The algorithm generates a file that is a numerical image of the signals that would be generated by the template digitizer if a template of the same contour was being tracked. This file can be either transferred to a mass storage device or stored in data memory.

When the file representing the whole contour has been generated, the output algorithm is initiated. The output algorithm receives the information from the data file in memory and synchronizes their delivery to the output port with time signals provided by the real time clock of the computer. An electronic circuit is used to combine the in-

formation from two bits on the data lines of the output port into two signals similar to that shown in Figure 2.8.

The digitizer algorithm runs under BATCH mode on time-shared computers, as well as on small and slow computers. The output routine, however, runs under REAL TIME conditions, thus precluding time-shared computers from hosting the algorithm. The file generated by the digitizer can be accessed from the data memory of the time-shared machine through a direct memory access device (DMA), thus eliminating physical memory duplication. Alternatively, the file can be transferred entirely to the memory of the computer hosting the output algorithm. Description of the two algorithms are given below.

Digitizer algorithm:

This algorithm is written in FORTRAN and thus is machine independent. The algorithm solves the geometric relationship of the NC machine as the axes are indexed by one resolution at a time, subject to the constraints imposed by the contour $r_1 = r_1\{\theta_1\}$ and contour pole location $(C, \underline{/\beta})$. The time interval required to cover one resolution on either axis is calculated from the contouring speed and the path length corresponding to the resolution. Figure 2.10 shows a schematic diagram of a tool of radius R_r tracking a curve $r_1 = r_1\{\theta_1\}$ whose pole (0_1) is related to the machine pole (0) by vector $(C, \underline{/\beta})$. Two geometric expressions can be formulated, for the point of contact (P) between the tool and the curve, to relate the angular orientation of the arm

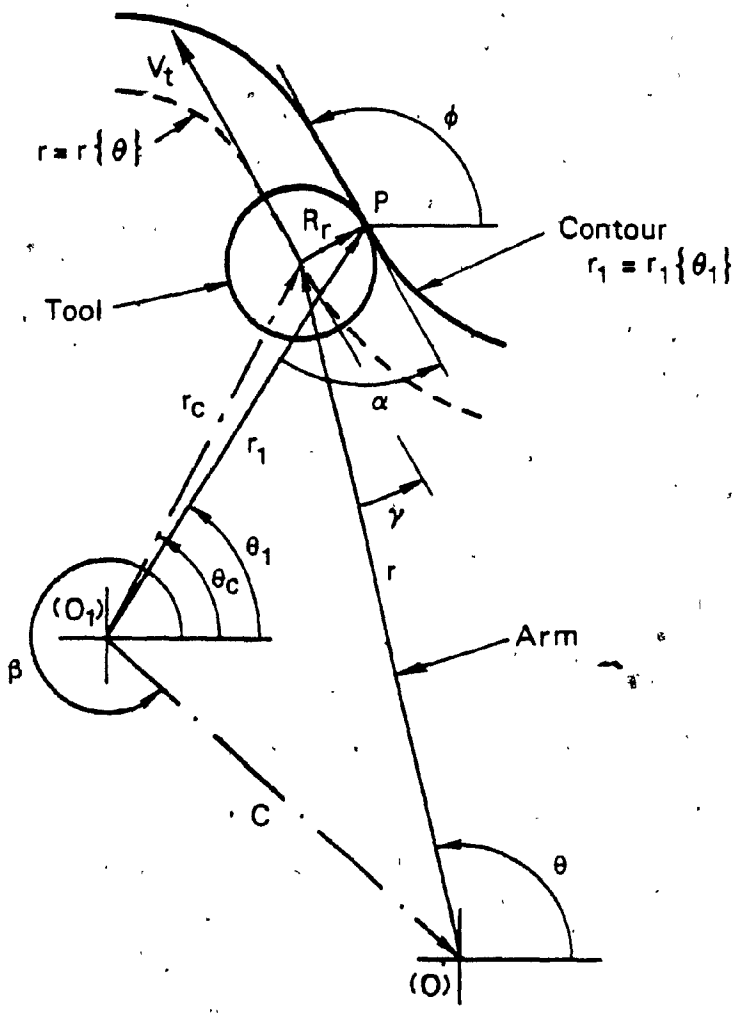


Figure 2.10: Vector Representation of the Polar System in Contouring.

and the radial displacement of the center of the tool r to the angular position of (P) as follows:

$$r_1 \sin(\theta - \theta_1) - C \sin(\theta - \beta) - R_r \cos(\alpha - \theta + \theta_1) = 0 \qquad 2.1$$

$$r - \left[C^2 + r_c^2 - 2Cr_c \cos(\theta_c - \beta)\right]^{\frac{1}{2}} = 0 \qquad 2.2$$

where,

$$\alpha = \cos^{-1}\left[\frac{dr_1}{d\theta_1} \Big/ r_1\right]$$

$$r_c = \left[r_1^2 + R_r^2 - 2\, r_1\, R_r \cos\alpha\right]^{\frac{1}{2}}$$

and,

$$\theta_c = \theta_1 - \sin^{-1}\left[R_r \cos\alpha / r_c\right]$$

The distance travelled on the contour (Δs) that results due to indexing of either of the two axes by one resolution is given by:

$$\Delta s = \left\{ \begin{array}{l} \displaystyle\int_{\theta_1\ (\text{initial})}^{\theta_1\ (\text{final})} r_1\ \{\theta_1\}\ d\theta_1 \quad \text{For R-axis movement} \\ \\ \displaystyle\int_{r_1\ (\text{initial})}^{r_1\ (\text{final})} \theta_1\ \{r_1\}\ dr_1 \quad \text{For } \theta\text{-axis movement} \end{array} \right\} \qquad 2.3$$

The time required to cover this length is given by:

$$t = \Delta s / V_t \qquad 2.4$$

A simplified flow-chart of the digitizer algorithm is shown in Figure 2.11, and a computer listing of a software implementing the flowchart is given in Appendix [C-1]. With reference to the flow-chart, the algorithm consists essentially of two loops, each corresponding to an axis in the polar system. Starting with one of the axes as determined by IFLAG; say θ, it is indexed by $\Delta\theta$ in a direction determined by the required direction of rotation. Using the new value θ Equation 2.1 is iterated to solve for θ_1. If no convergence is achieved, the direction of indexation of θ is reversed and the iteration process restarted. Once θ_1 is found, the value of r is calculated from Equation 2.2. The change in r is evaluated, and if found greater in magnitude than the ΔR, the θ loop is abandoned and the radial axis loop is started. For a change less than ΔR, the loop continues by calculating the length of the contour curve in the interval $\Delta\theta$ using Equation 2.3. The time required to cover this interval is then calculated using Equation 2.4. Information about the time, the direction of indexation and the axis indexed are written into the output file. Conditions for the end of the contour, i.e., the stop criterion, are then evaluated and the loop is restarted if the end of the contour is not reached. The loop for the radial axis is exactly similar to that of the angular axis with the axial parameters interchanged as can be seen from

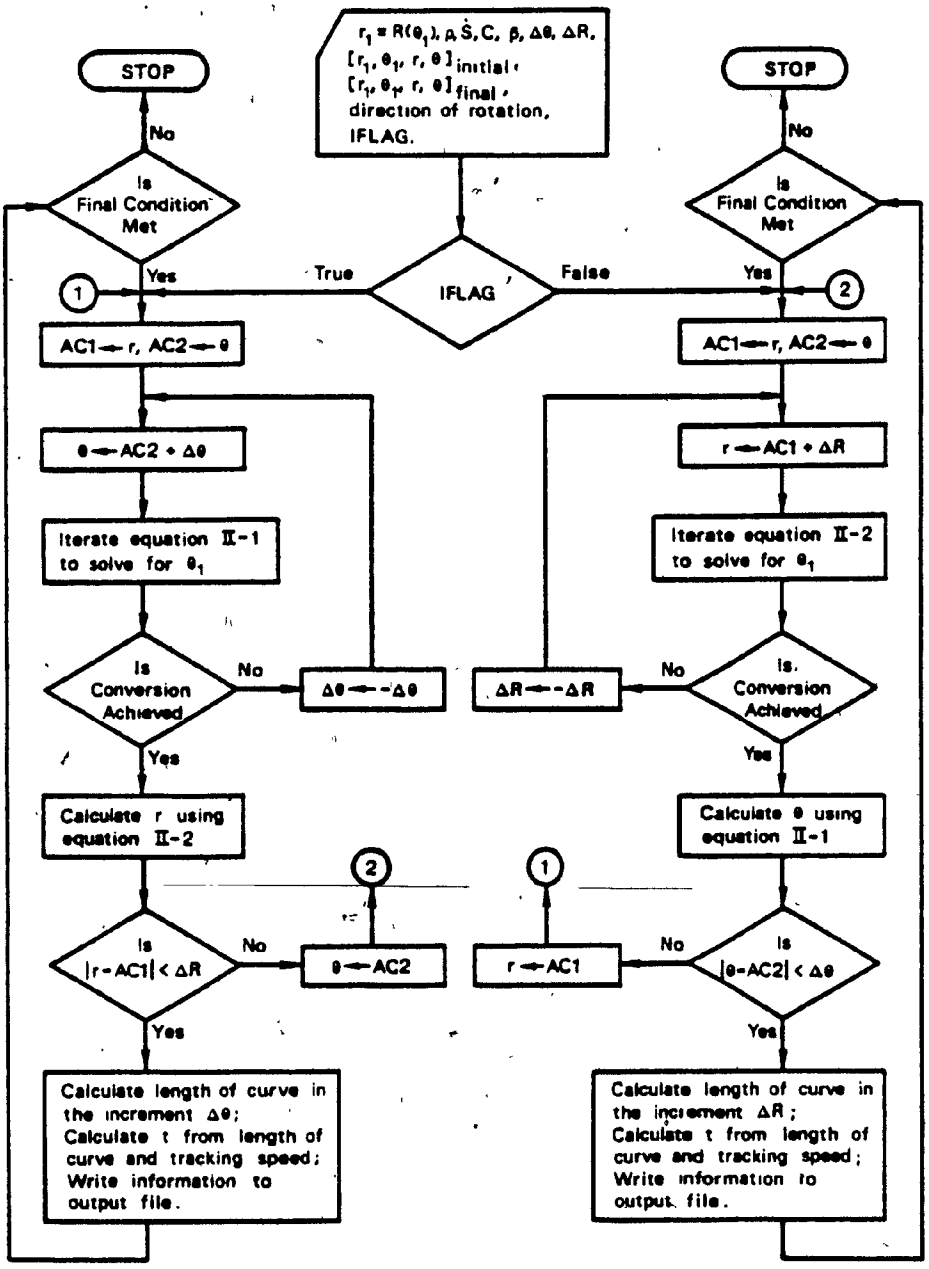


Figure 2.11: Flowchart of the Digitizer Algorithm.

the flow-chart. In this loop, however, the radial displacement r is indexed by ΔR and Equation 2.2 is iterated to solve for θ.

In this algorithm, the contour information can be supplied in equation form or as discrete points on the contour. In the latter case, more points are generated in between those given by using the Cubic Spline interpolator. A description of this interpolation technique is given in Reference [18]. If the analytical expression of the contour $r_1\{\theta_1\}$ and the first derivative of r_1 with respect to θ_1 are known, then an iterative procedure for finding the solution of Equations 2.1 and 2.2 are carried out using the Newton-Raphson root finding technique [18]. If the first derivative cannot be obtained easily, then the Secant root finding technique [18] is used. Both the above-mentioned root finding techniques require initial guesses to start the iteration process. The immediate previous values of points on the contour are used for such guesses. The integration of Equation 2.3 required to evaluate the length Δs is carried out numerically using a Guassian Quadrature, Reference [18]. The integrand is approximated by a Laguerre Polynomial, whose weights and roots are given in Reference [19].

The generated output file consists of multiple records of two entries each. The first entry is the time required to cover the resolution, and the second indicates the axis indexed and the direction of indexation. The second entry

in the record occupies only two bits in a computer word, with the least significant bit indicating a radial or angular resolution if it is 1 or 0 respectively. The other bit indicates positive or negative direction of indexation, if it is 1 or 0 respectively. Simultaneous indexation of both axes is entered as two successive records with zero time period in between them. For excessively long contours the generated data file is divided into a number of smaller ones with cumulative time entries inserted at their boundaries. These small files are processed one at a time. The cumulative time signals are used to synchronize the appending of these files together.

Output algorithm:

Figure 2.12 shows a flow-chart of this algorithm. The ASSEMBLER code is best suited here due to the high execution speed required and to the simplicity of the algorithm. Computers that can host this algorithm should have access to a real time clock. As shown in Figure 2.12 the algorithm is composed of one simple loop. The loop starts by reading the first entry, which is a time entry, from a record in the data file generated by the digitizing algorithm. The time value is transferred to the clock register. The clock is started, and interrupts the central processor when the time period loaded into its register lapses. The processor then transfers the second entry in the record, axes information, to the output port. The end of file condition is checked and if records still exist on file the loop is re-

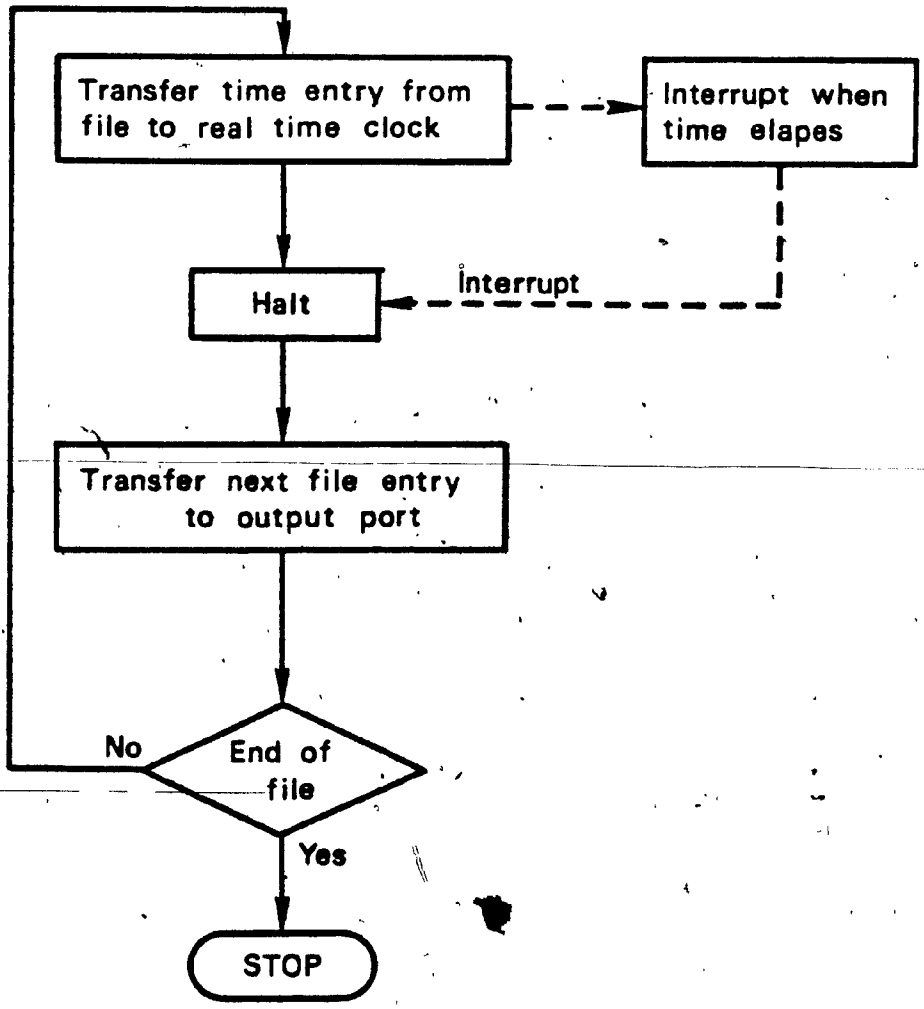


Figure 2.12: Flowchart of the Output Algorithm.

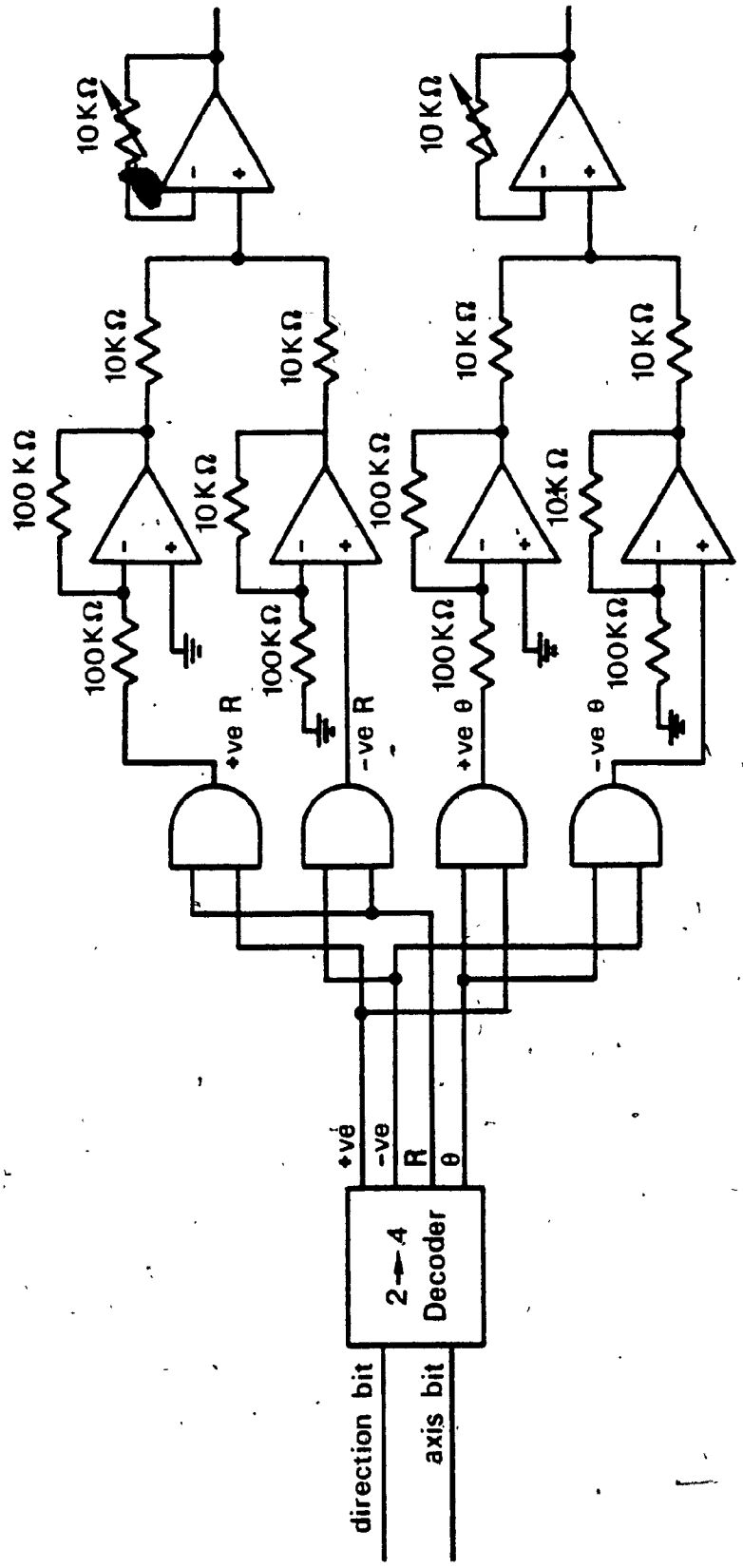


Figure 2.13: Circuit for Decoding the Software Digitizer Signals.

started.

Appendix [C-2] lists a fully documented sample program that has been implemented and tested on a TM-990/189 single board microcomputer. The program is based on the above-illustrated flowchart. A full description of the microcomputer is given in References [20] and [21]. The computer is a 16-bit machine, which allows packing of the time and axis entries in a record into one word, thus making full use of the available memory. With 14-bits available for a time entry, a time period ranging from 0 seconds up to approximately 0.5 sec in steps of 33 μsec can be achieved. The axes information in the two least significant bits are decoded using the circuit shown in Figure 2.13 into two signals similar to those of Figure 2.8. The circuit uses a 2-to-4 decoder and four AND gates to trigger a bank of four operational amplifiers as shown. The outputs from each pair of amplifiers corresponding to one axis are then summed.

2.4 Prototype polar NC machine

A schematic diagram and labelled photographs of the prototype NC machine are shown in Figures 2.14 and 2.15. With reference to the figures, the prototype resembles the template digitizer described earlier. It consists mainly of a work area (A) and an arm (B) keyed at one end to a shaft (S). The shaft is driven by a stepping motor (C) via a worm-and-gear train (D). The arm is essentially a cantilever pivoted at one end and supported by a unit (E) at the

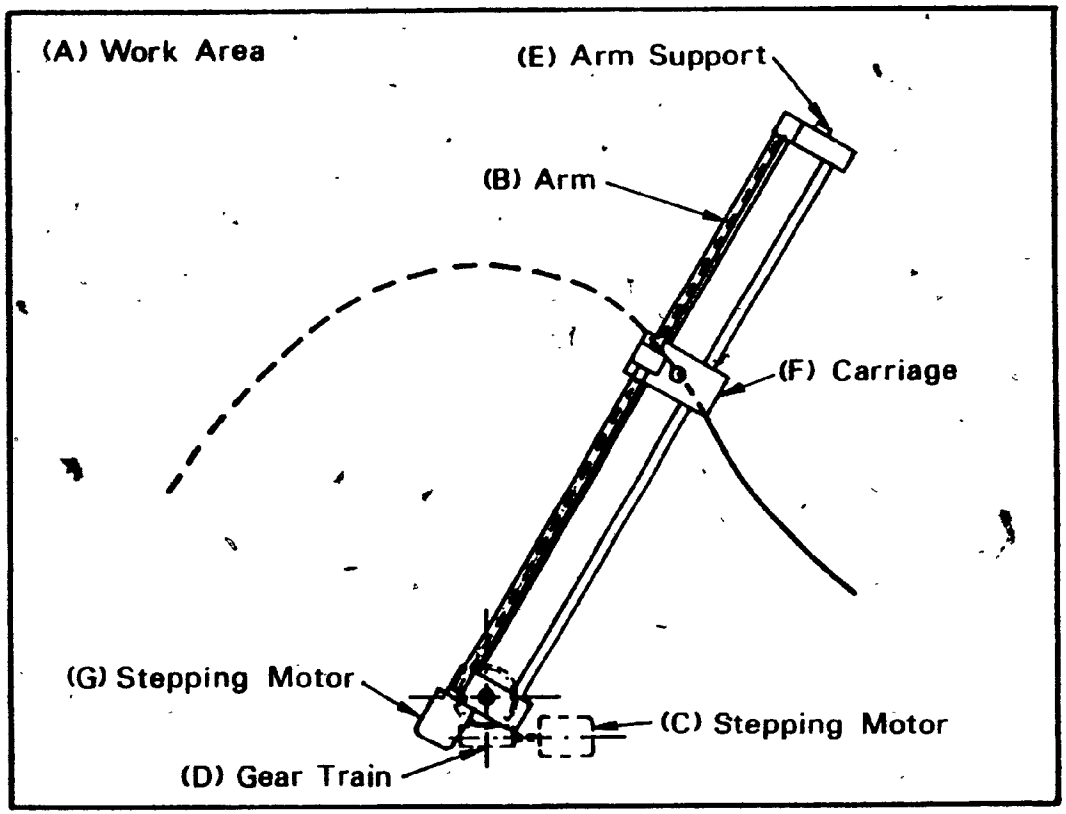


Figure 2.14: Schematic Diagram of the Prototype NC Machine.

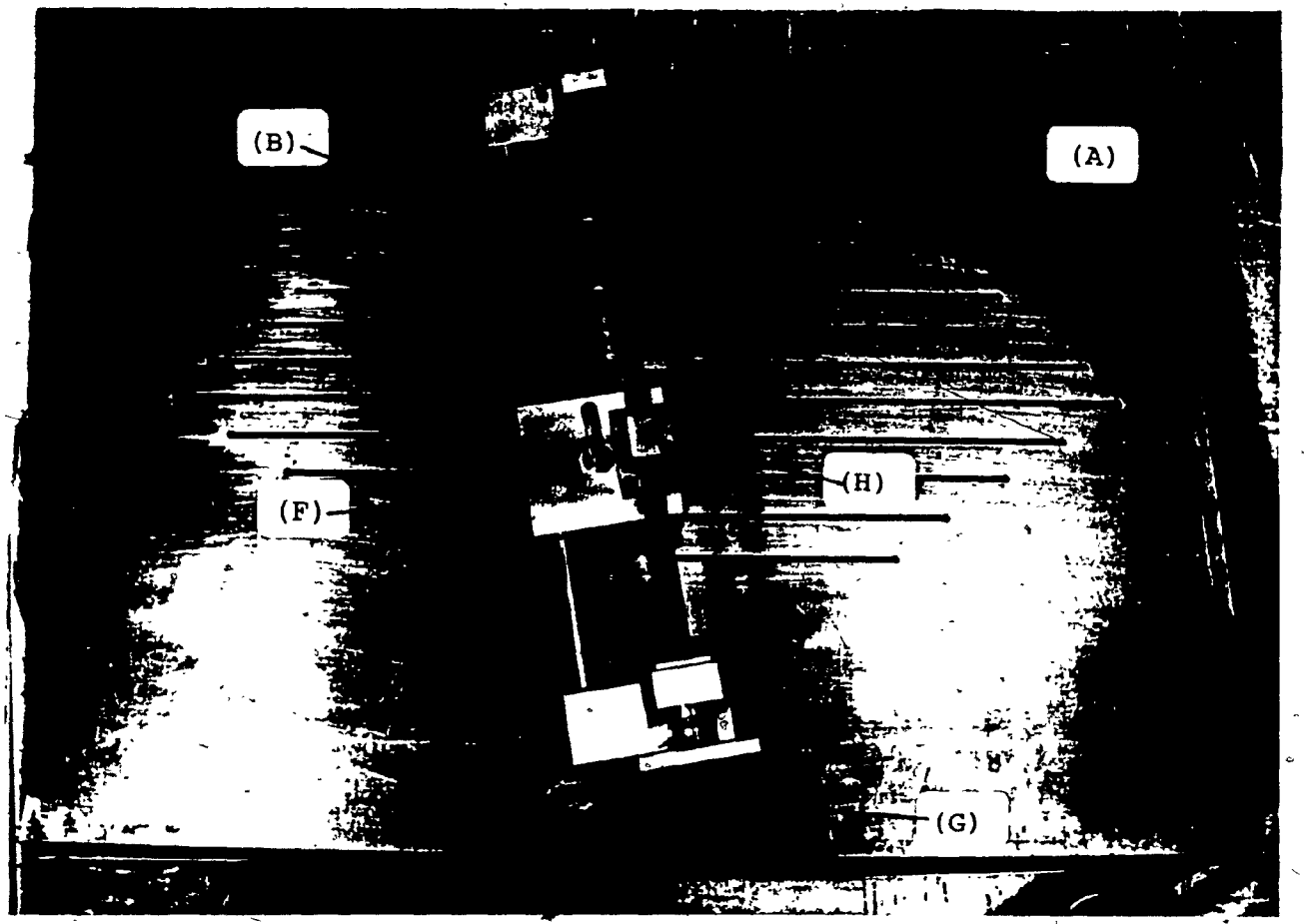


Figure 2.15.a: Photograph of the Prototype NC Machine.

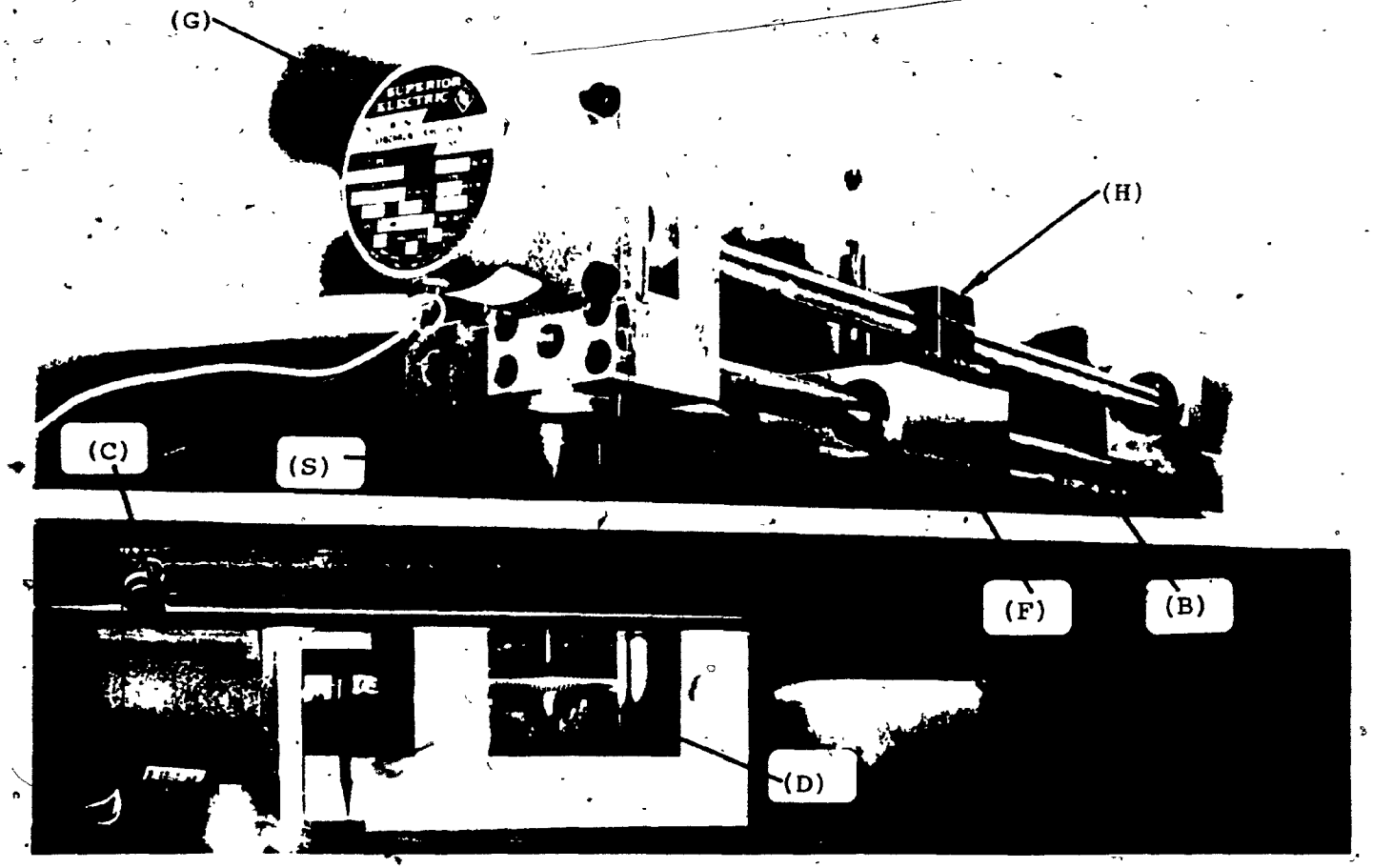


Figure 2.15.c: NC Machine Axes-Drives Arrangement.

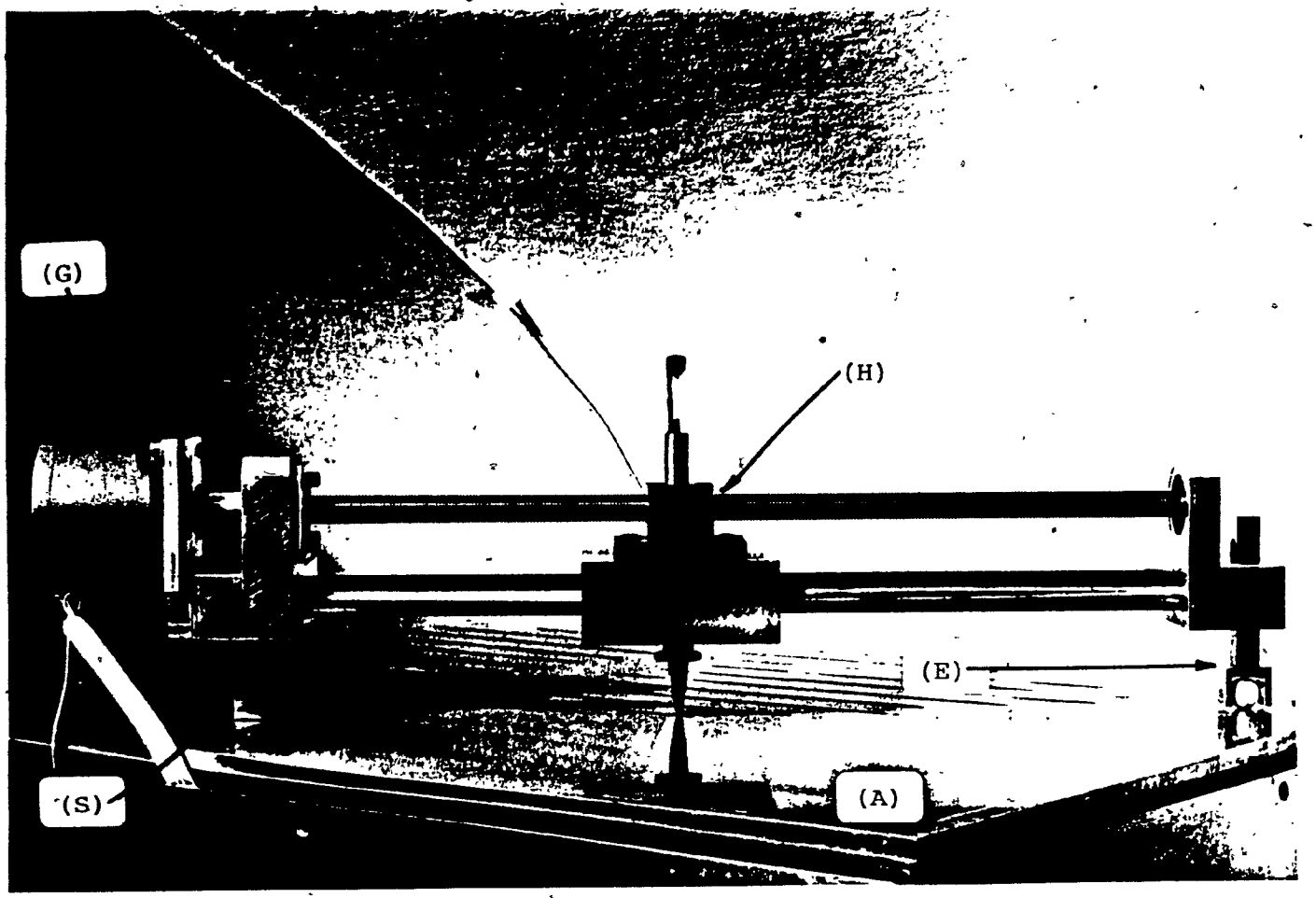


Figure 2.15.b: NC Machine Arm and Carriage Assemblies.

overhanging end. A carriage (F) travels on linear bearings along the length of the arm, being driven by another stepping motor (G) mounted on the arm. The drive force is transmitted to the carriage via a split nut and lead-screw arrangement (H). Within the context of polar coordinates the arm position provides the angular axis while the displacement of the carriage along the arm provides the radial position. The carriage carries the tool. The tool is a pen in the demonstrated prototype. The motion of the arm is limited by the two stops (I) and (I') (not shown on the photographs). The right stop (I) marks the home position of the arm or the angular axis reference. The location of (I) corresponds to the home position of the arm of the template digitizer. The position at which the carriage is at its minimum displacement from the shaft (S) marks its home position, or the radial axis reference. This reference also is identical with the corresponding reference on the template digitizer.

Subassembly drawings of the prototype NC machine are included in Appendix [A-2]. A hardware logic circuit (not shown) uses threshold comparators to decode the two composite incoming signals before channelling them to the two motor drive circuits. Figure 2.16 shows a schematic of the logic circuit used. The two stepping motors can also be driven directly from the encoder signals.

The stepping motors are energized in a "wave" scheme, also known as "half step" scheme, as opposed to the commonly

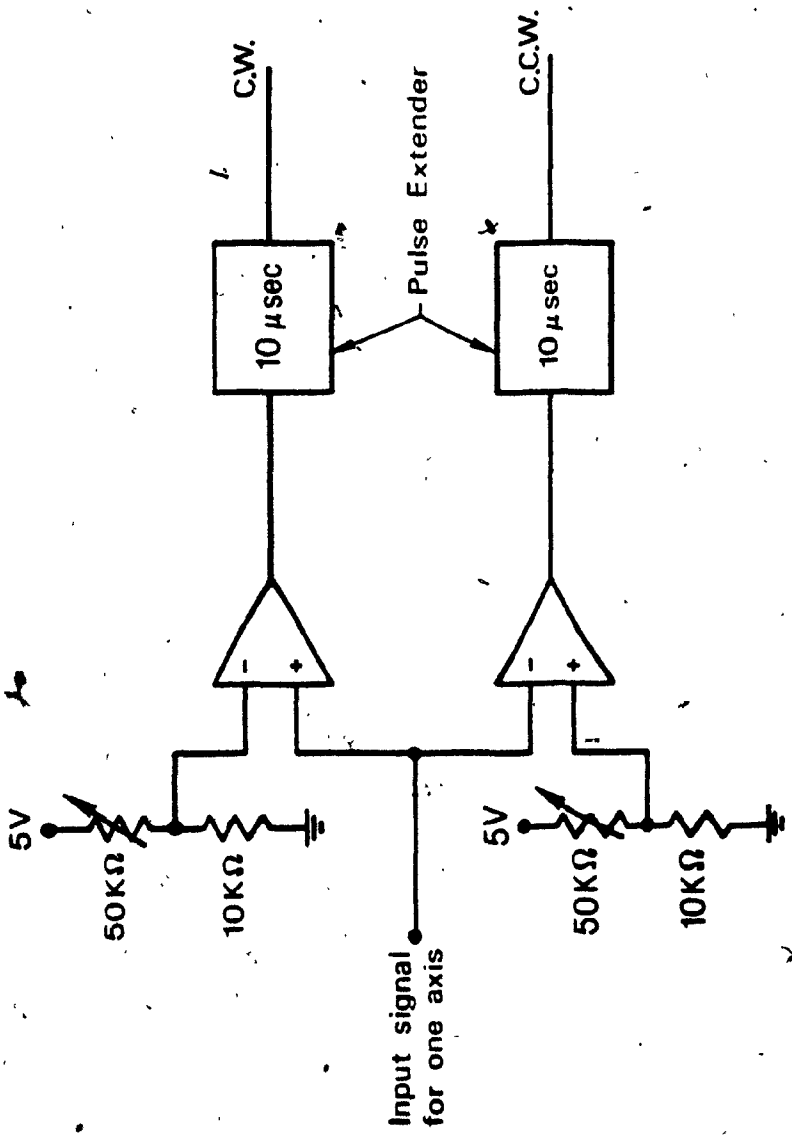


Figure 2.16: Decoding Circuit for NC Machine Control Signals.

used single and double phase schemes. Description of this mode of operation as well as the general description, operation, and characteristics of electric stepping motors can be found in References [22] to [29]. A brief description of the motor is outlined in Appendix [F.1]. The drive circuits required for the wave scheme are more complex than those required for the other two schemes. Further, the output torque of the motor is only 60% to 70% of that attainable when employing two phase energization. The wave mode of operation is used, however, since it offers two important advantages over the other two: the step angle of the motor is half its normal one thus resulting in finer machine resolutions, and secondly, resonance problems inherent to stepping motors are drastically reduced, Reference [22] (pp.228).

The resolution and the size of work area of the NC machine are identical to those of the template digitizer.

CHAPTER 3

SALIENT FEATURES, ADVANTAGES AND DISADVANTAGES OF THE POLAR CONTOURING SYSTEM

3.1 Introduction

Commercially available contouring systems are either copying machines having analog servos as their controlling element and using a template as a source of contour information or they can be NC machines relying totally on an on-board computer for control and utilizing information in the form of numerical data. The system proposed in this thesis occupies a distinct space in-between these two types of machines. It is numerically controlled, but rather than using an on-board computer for control, it uses a simple hardware logic circuit and timing signals implicit within the contour data. The contour information can originate from a template or from an off-board computer. In the latter case the information is recorded on magnetic tape and then supplied to the machine as described earlier in Chapter 2. Since the system is digitally based, it makes use of recent advances in digital electronics to handle and manipulate the flow of information between its various components. The configuration of the system allows it to be more versatile than copying machines. The system may be equipped with an on-board computer which would make it as versatile and flexible as conventional NC machines in general applications.

In strict contouring applications, the system offers a number of advantages over both types of conventional machines. It should be noted also that the system has some drawbacks, but the advantages outweigh these drawbacks. An itemized comparison between the proposed contouring system and the conventional machines follows. At the end of this chapter a tabulated summary of this comparison is provided in Table 3.11.

3.2 Method of supplying contour information

The method by which the contour information is supplied to the proposed system is among its most outstanding features. As mentioned earlier the system can operate both as a copying machine or as an NC machine. This flexibility allows the system to be used as an NC machine, in applications where the required contour is too complex to be put in template form or where a high degree of accuracy is required. If a template is available, the system may be used as a copying machine. This flexibility is not shared by available systems.

Another outstanding feature of the system is the ease with which digitized contour signals can be recorded and then played back to produce duplicates of the digitized contour. This feature is particularly useful in large volume production where one taped set of data can be duplicated so as to operate several polar contouring machines. Although this ability is shared by commercial NC machines, they have

the drawback of requiring one computer per machine in order to reprocess the data supplied to produce a single contour.

3.3 Complexity of controller

Although the complexity of the controller is transparent to the user it is reflected both in the cost of the system and in its serviceability.

Analog servos in copying machines use complicated precision components for their operation. These servos have reached such a maturity of design that their performance and reliability can be fairly well predicted. Because of their complexity only experienced and well trained personnel can service servo systems. NC machines are by far the most complex contouring machines available. The controlling computer is specially designed for high instruction execution speed and large data storage space. They are also equiped in the majority of cases, with hardware arithmetic units and similar hardware devices to expedite the execution of the interpolation and axes control tasks.

The proposed system-controller is less complex than those available on commercial contouring machines as evident from the descriptions given in the previous chapter. Although no experience has yet been accumulated concerning its reliability and serviceability it is believed that its simplicity should give it an advantage in these areas.

3.4 Versatility

NC machines can in general perform more varied tasks than available copying machines or the proposed contouring system. Tool change, control of cooling fluid in metal removal applications, set-up of the work-piece, and the like are examples of such tasks that can be carried out. More advanced NC machines are even capable of adaptive control, that is, the modification of parameters in the control programs to compensate for changing conditions like tool wear or work-piece temperature rise.

Both conventional copying machines and the proposed contouring system are confined only to contouring applications. Since the proposed system is digitally based it can be equipped with a small computer controller to carry out auxiliary tasks similar to those mentioned above.

3.5 Contouring speed

Most of the available copying machines cannot track a contour at a constant speed. This is mainly due to the lack of information about the direction of the tangent at the point of contact between the template and the tool. An exception to this is the type of copying machines employing powered followers which are mainly used in the clothing industry. NC machines track a contour at approximately constant speed, Reference [12] (pp.25). Depending upon the

percentage of speed regulation, this control process could be the most time demanding task that engages the machine controller.

The proposed system has the capability of tracking a contour at a constant speed when operated with a software digitizer. Since digitization and imbedding of the time-reference into the signal is done prior to the contouring operation itself, the digitizing software is not affected by production time constraints. When these data are recorded and played back on commercial, stereo, magnetic tape recorders with typical tape speed regulation of 0.07%, the maximum contouring speed fluctuation resulting will not exceed 0.14% peak to peak.

For the TD/NC machine, the contouring speed depends upon the control scheme of the tracking rollers (G) and (G') as mentioned in Section 2.2. Using the control scheme whereby the sum of the speeds of the two rollers is kept constant, results in possible variation in the contouring speed as the roller mechanism tracks a contour. This variation depends upon the radius of curvature of the contour at the point of contact with the roller, the radius of the roller, and the thickness of the template. The relation between these parameters is derived as follows: Figure 3.1 shows two rollers each of radius R_r joined together by the link (I) and tracking a contour of uniform thickness H. The rollers contact the two sides of the template at (P) and (P') and are spring loaded against the template. Assuming

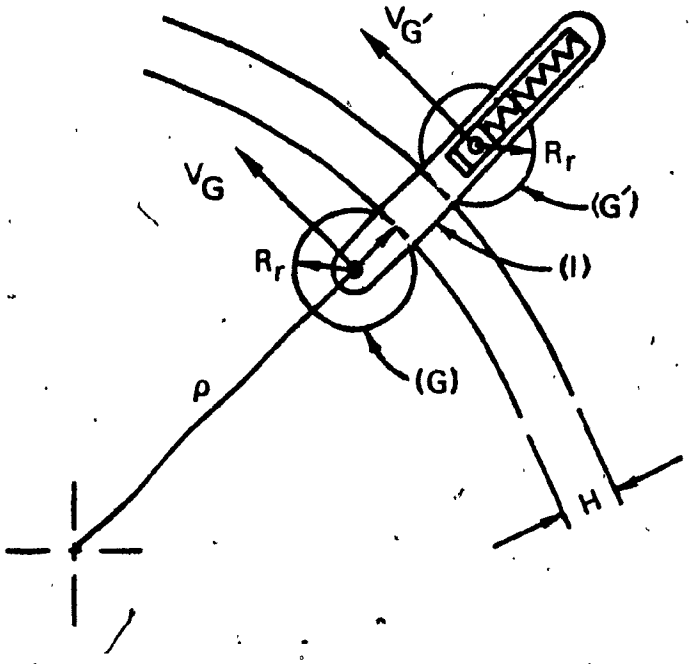


Figure 3.1: Schematic Diagram of the Roller Mechanism in Contouring.

negligible friction in the roller bearings, the equilibrium position of the link (I) is along the normal to the tangent at (P) and (P'). This position is also collinear with the direction of the radius of curvature ρ at (P). The instantaneous direction of motion of the rollers is around the centre of curvature. With reference to the figure the following kinematic relation can be derived:

$$v_{G'} = \frac{v(\rho + H + R_r)}{\rho - R_r} \qquad 3.1$$

where v and $v_{G'}$ are the velocities of the rollers (G) and (G') respectively. The roller differential arrangement is driven at a constant angular velocity that would result in the following equation:

$$2V_t = v + v_{G'} \qquad 3.2$$

Substituting Equation 3.1 into Equation 3.2 and rearranging yields:

$$v^* = \frac{\rho^* - R_r^*}{2\rho^* + 1} \qquad 3.3a$$

$$v^* = \frac{\rho^* + R_r}{2\rho^* - 1} \qquad 3.3b$$

Equations 3.3.a and 3.3.b are for the cases where the center of curvature of the contour and the roller (G) are on the same side of the contour or on opposite sides respec-

tively.

where v^* is the nondimensional contouring speed with respect to the set speed V_t and is given by: $v^*=v/V_t$,

R^* is the nondimensional roller radius with respect to the template thickness and is given by: $R_r^*=R_r/H$,

and ρ^* is the nondimensional radius of curvature with respect to the template thichness and is given by; $\rho^* = \rho/H$. ρ^* has a minimum value of R_r^*.

Reference [30], (pp.17-1-5), gives the radius of curvature ρ for a curve $r_1=r_1\{\theta_1\}$ in polar coordinates as:

$$\rho = \frac{\left[r_1^2 + (dr_1/d\theta_1)^2\right]^{3/2}}{\left[r_1^2 + 2(dr_1/d\theta_1)^2 - r_1(d^2r_1/(d\theta_1))^2\right]} \qquad 3.4$$

Figure 3.2 shows a plot of the percentage variation in the contouring speeds for different values of R_r^* and ρ^*. The figure shows a high rate of decrease in the percentage error in contouring speed with the increase of ρ^*, particularly so at small R_r^* values. The figure also shows that when the radius of curvature of the contour equals the rollers diameter, the error in contouring speed increase to 100% and 200% for the cases when the center of curvature and the roller (G) are on the same side of the contour and when they are on opposite sides respectively.

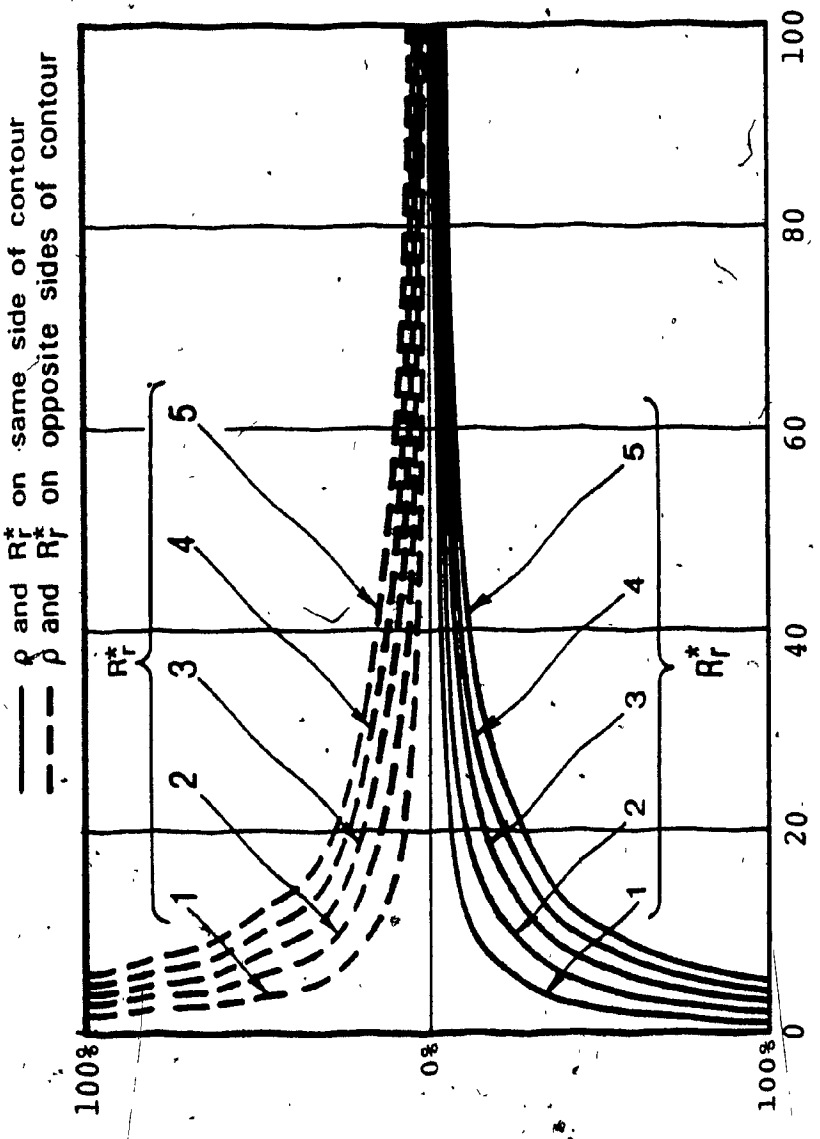


Figure 3,2: Contouring Speed Error Versus ρ*

For moderately undulating contours, and practical rollers diameter and template thickness , the variation in contouring speed is within acceptable limits. For example, in the case of the prototype template digitizer with rollers diameter and template thickness of 0.3175 cm (1/8 inch), speed varies by less than 10% for a radius of curvature as low as 5 cm (2 inches).

Three templates were digitized on the prototype template digitizer. The signals from the encoders and the time period between them were collected using a hybrid computer as explained in Appendix [D]. The three templates are shown in a photograph in Figure 3.3. The circular template represents the case of constant radius of curvature, while the line template has an infinite radius of curvature. The rectangular template has an infinite radius of curvature along its linear segments and a radius of curvature very close to the radius of the rollers at the corners. An analysis of the collected data showed that upon switching the power supply on, that is to say applying a step velocity input, the contouring speed oscillates for a short length on the template before stabilizing. Imperfections, in the form of indentations, on the rectangular template cause the contouring speed to oscillate again for a short length on the template.

3.6 Scaling of contouring speed

When a contour is digitized, the generated signals are

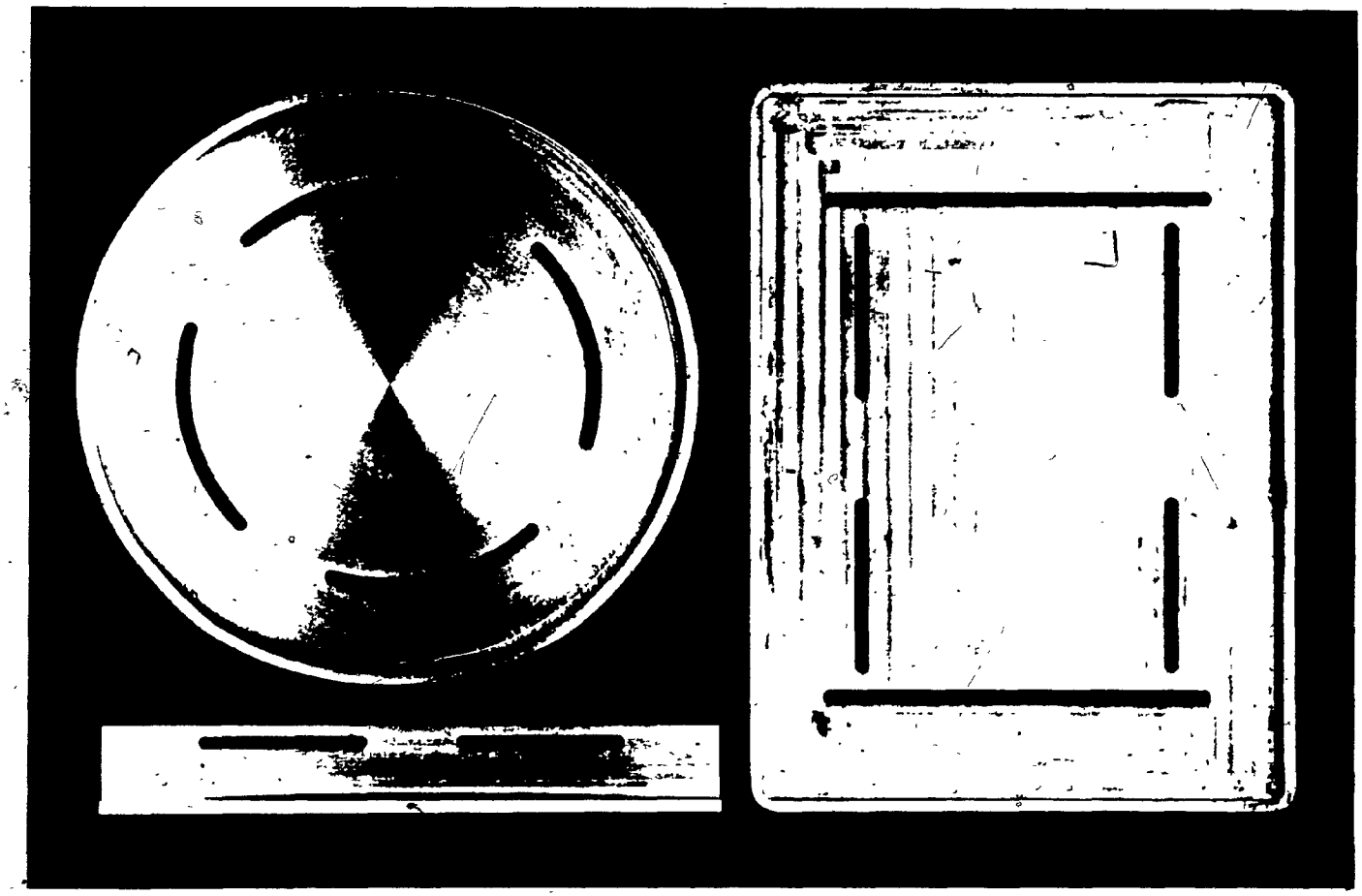


Figure 3.3: Photograph of Templates Used in Testing.

timed so as to reflect a constant contouring speed. Recording these signals on magnetic tape, and then using a different playback speed other than that used for recording, results in a proportional change in the time periods between the signals, and hence in scaling the contouring speed. Scaling of the contouring speed has practical applications in cases where the contouring speed needs to be changed after a contour has been digitized. It also can be made use of as explained in Appendix [D] to collect time period information between the two encoder signals. The procedure consists of recording the signals at a high tape speed and then playing it back at a slow speed. This allows more accurate measuring of the time periods between the signals.

3.7 Size scaling

The majority of commercial copying machines are not capable of producing a contour of different size other than that of the generating template. Copiers that are capable of size scaling are expensive. Available NC machines can generally be programmed to track a contour of different scale than that of the supplied information. In the proposed machine, however, up or down-scaling of the size can be very easily achieved by simply scaling the radial axis only. The angular resolution in polar coordinates is given by:

$$\Delta A = r \Delta \Theta = n_R \Delta R \Delta \Theta \qquad 3.5$$

where ΔA is the angular resolution,

$\Delta \theta$ is the angular step,

ΔR is the radial step (also the radial resolution),

and n_R is an integer equal to the number of radial resolutions from the machine pole to the carriage position.

The simplicity of scaling here is attributed to the fact that the angular resolution is proportional to the radial resolution, and scaling the radial resolution causes an equivalent scaling of the angular one.

Scaling the radial resolution is achieved by designing the system such that one radial resolution on the NC machine is equivalent to m resolutions on the digitizer. Using a presettable logic counter to intercept the radial signals to the NC machine, and setting it so as to signal the radial axis after every k incoming signals instead of after the usual m, where both m and k are integers, results in scaling the contour by a factor of m/k. Figure 3.4.a shows a simple example where the digitizer radial resolution is four times smaller than that of the NC machine. The counter signals that would result in doubling the contour size as well as those that would halve it are also shown.

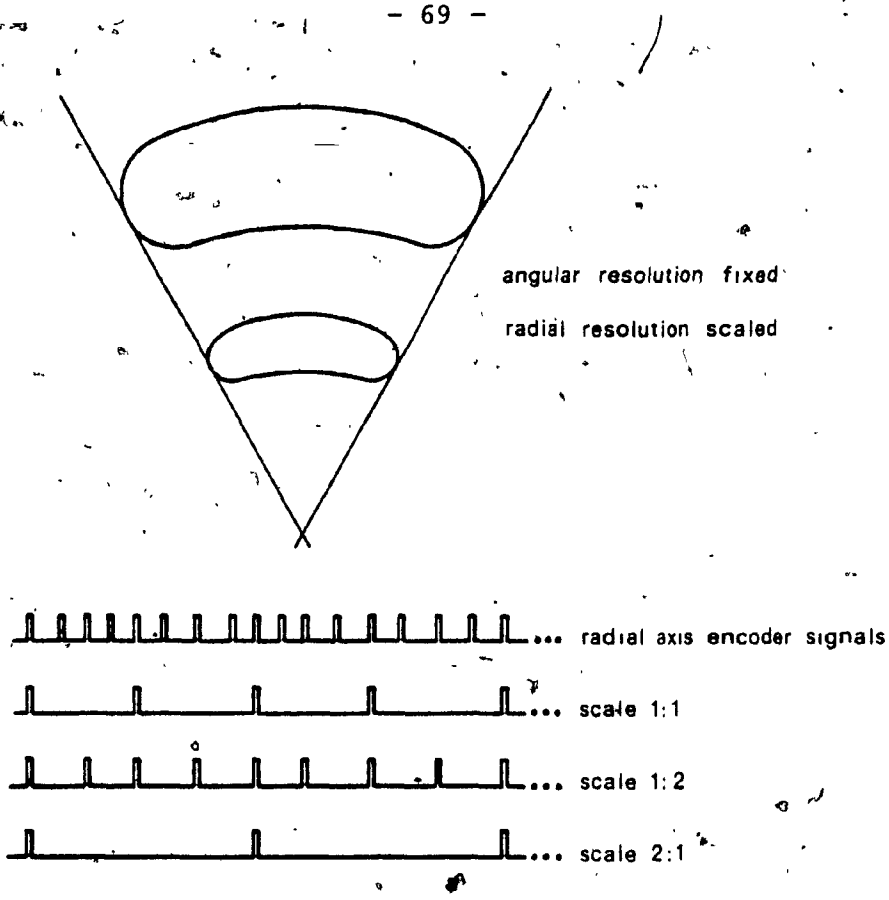


Figure 3.4.a: Size Scaling on the Polar Contouring System.

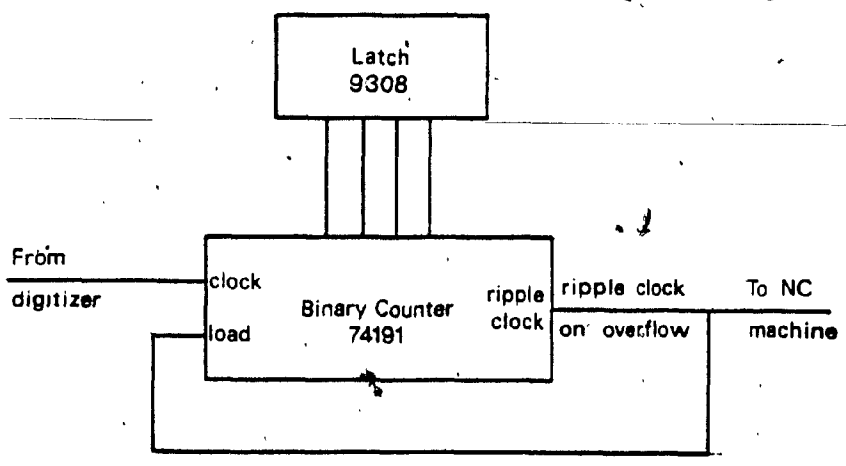


Figure 3.4.b: Logic Circuit for Size Scaling.

Figure 3.4.b shows a single counter chip connected so as to produce scaling factors ranging from 1:4 down to 4:1 when used with the above mentioned example. The circuit is introduced here to emphasize the simplicity of achieving size scaling.

Size scaling is a very practical feature particularly in metal removal applications. The original contour is scaled down after every pass to compensate for the metal removed during that pass. Furthermore, when combined with speed scaling, higher machining speeds can be used for rough cutting passes and lower ones for finishing cuts. These speeds can be produced with the same taped signals.

3.8 Accuracy of contouring

The accuracy of copying machines depends upon the analog servo used to control them, as well as on the load on the axes-drives. NC machines have preselected resolutions, and when operated within their load carrying capacity, the accuracy is limited to one resolution.

Cartesian based NC machines usually have equal resolutions along both axes. The resolution is homogeneous within the work space. The proposed contouring machine has a constant resolution along the R-axis. However, the angular axis resolution varies linearly with the radial position within the work space. This variable angular resolution, although a drawback simplifies size scaling as mentioned earlier.

The upper accuracy bound of a contour produced on the polar NC machine is equal to the maximum angular resolution, and occurs if the contour is machined at the extremity of the work area.

3.9 Work area

Cartesian coordinate based NC machines allow more work area than those using polar coordinates for the same overall machine size.

3.10 Power utilization of the axes-drives

Available copying machines as well as the majority of NC machines utilize feedback loops to guard against errors. This allows the machines to operate at near full torque capacity of the axes-drives. The proposed contouring system operates with an open loop, and thus a reasonable factor of safety on loading must be imposed. Alternately, a method for determining the exact torque required for a given application may be used. By varying the machining parameters, the torque required by the axes-drives is kept below that available from them. A simple method for determining such a required torque will be presented in a subsequent chapter.

3.11 Reference position

The homogeneity of the resolutions in Cartesian based systems allow a contour to be generated anywhere on the work

space from a given set of information. The reference position is simply transferred to the new position. This floating reference feature simplifies setting up of a work-piece for machining.

In the proposed polar system, a contour can be produced only from the same radial displacement used for digitizing it. The reference position of the angular axis, however, can be floating. Setting up of a work piece at the radial reference is a simple task and can be automated.

3.12 Cost

Present trends towards digital controls, inspired by rapid advances in the field of digital electronics are continually reducing the costs of digital systems hardware. This trend, however, is counterbalanced by an ever increasing cost of driver software required for these controllers. The type of software required for these controllers is very specialized due to the complexity of the control task and the high execution speed required to keep up with contouring speeds dictated by production practices. The proposed contouring system eliminates the need for such software since the axes-drives are not controlled directly by a computer even when a software digitizer is employed. This results in a lower cost system. Contouring systems utilizing analog controls are inherently more expensive than the proposed system, owing to the cost of the high precision components required.

A comprehensive cost study would have to include other economic aspects such as production time savings, and maintenance costs. For the production of a large batch of the same contour, the production times for the three systems is expected to be identical. However, as both the ratio of the contouring time to the setting-up time and the number of required parts in a production batch decrease, conventional NC machines would prove more economical. Production time savings by factors as high as 4 or 5 times for such applications have been reported in Reference [11].

The cost of maintenance of conventional NC machines would be higher than those of the other two systems due to the complexity of their controllers. The maintenance costs of available copying machines would be still higher than those for the proposed system, due to the complexity and precision of servo controllers.

TABLE 3.1: Comparison between existing contouring systems and the polar contouring system.

ATTRIBUTES	2,2½D COPYING MACHINES	2,2½D NC MACHINES	PROPOSED SYSTEM
•METHOD OF SUPPLYING CONTOUR INFORMATION	CONTOUR INFORMATION IN THE FORM OF A TEMPLATE	CONTOUR INFORMATION IN THE FORM OF NUMERICAL DATA	CONTOUR INFORMATION IN THE FORM OF A TEMPLATE OR NUMERICAL DATA
•SIMULTANEOUS USE OF THE CONTOUR INFORMATION ON OTHER MACHINES	NOT POSSIBLE	CONTOUR INFORMATION CAN BE SUPPLIED ON PUNCHED, OR MAGNETIC TAPE TO OTHER MACHINES	DIGITIZED CONTOUR SIGNALS CAN BE RECORDED, DUPLICATED AND REUSED ON OTHER MACHINES
•COMPLEXITY OF CONTROLLER	PRECISION SERVO CONTROLLER	COMPLEX COMPUTER BASED CONTROLLER	SIMPLE HARDWARE LOGIC CONTROLLER
•VERSATILITY	CONFINED TO CONTOURING APPLICATIONS	VERSATILE; CAPABLE OF TOOL CHANGE; INITIAL SET UP OF WORK-PIECE AND THE LIKE	CONFINED TO CONTOURING APPLICATION; EASY TO EQUIP WITH COMPUTER CONTROLLER TO CARRY OUT AUXILLIARY CONTROL TASKS
•CONTOURING SPEED	NOT CONSTANT	SMALL VARIATION; CAN BE MADE CONSTANT FOR SLOW CONTOURING SPEEDS	CONSTANT IN CASE OF SOFTWARE DIGITIZER; SMALL VARIATIONS SUBJECT TO CONTOUR IN CASE OF TEMPLATE DIGITIZER
•CONTOURING SPEED SCALING	THE CONTOURING PROCESS HAS TO BE INITIATED AT THE NEW SPEED	THE COMPUTER CONTROLLER HAS TO REPROCESS THE CONTOUR INFORMATION SUBJECT TO THE NEW REQUIRED SPEED	CONTOURING SPEED CAN BE SCALED UP OR DOWN BY PLAYING BACK A RECORDED SIGNAL AT AN APPROPRIATE SPEED OTHER THAN THE RECORDED ONE
•SIZE SCALING	POSSIBLE ON MORE EXPENSIVE COPIERS	EASY TO IMPLEMENT BECAUSE OF AVAILABLE COMPUTER POWER	EASY TO IMPLEMENT; REQUIRES SCALING OF THE RADIAL AXIS ONLY
•ACCURACY	DEPENDS ON THE ACCURACY OF THE SERVO CONTROLLER	RESOLUTION KNOWN AND HOMOGENEOUS WITHIN THE WORK SPACE	RADIAL RESOLUTION, CONSTANT; ANGULAR RESOLUTION, NOT HOMOGENEOUS WITHIN THE WORK SPACE; ACCURACY WITH RESPECT TO WORST CASE RESOLUTION EXPRESSED
•WORK AREA	RECTANGULAR OR SQUARE, RESULTING IN BETTER UTILIZATION OF THE MACHINE SPACE	RECTANGULAR OR SQUARE; RESULTING IN BETTER UTILIZATION OF THE MACHINE SPACE	CIRCULAR; GENERALLY SMALLER THAN CARTESIAN COORDINATE BASED MACHINES
•POWER UTILIZATION OF THE DRIVE MOTION	LOW FACTOR OF SAFETY ON TORQUE IS USED DUE TO PRESENCE OF FEEDBACK LOOP	LOW FACTOR OF SAFETY ON TORQUE IS USED DUE TO PRESENCE OF FEEDBACK TORQUE	REASONABLE FACTOR OF SAFETY ON TORQUE HAS TO BE IMPOSED; ALTERNATELY A METHOD CAN BE USED TO ACCURATELY GAGE THE TORQUE REQUIRED FOR A PARTICULAR OPERATION
•REFERENCE POSITION	BOTH AXES LOCATION FOR THE REFERENCE POINT ARE FLOATING	BOTH AXES LOCATION FOR THE REFERENCE POINT ARE FLOATING	THE ANGULAR LOCATION OF THE REFERENCE POINT IS FLOATING, THE RADIAL REFERENCE HAS TO BE SET AT THE SAME LOCATION USED ON THE DIGITIZER
•COST	MEDIUM COST	HIGH COST	LOW COST

CHAPTER 4

CONTOUR LOCATION FOR MINIMUM CONTOURING ERRORS ON THE POLAR MACHINE

4.1 Introduction

The digitizer signals received by the polar NC machine causes the axes-drives to move and trace a curve which approximates the digitized contour. The rate of incoming signals determines the contouring speed. If this rate is slowed down such that the response time of the axes-drives (that is to say, the time required to execute a signal) became comparatively negligible, the traced curve would be composed of linear segments along the radial axis joined by short circular arcs using the machine pole as a center. Such a contouring process is termed quasi-static. Figure 4.1 shows a contour and a curve approximating it produced by a point tool, R=0. Referring to the figure, there is a discrepency between the contour and the approximating curve produced under quasi-static conditions. This discrepency is indicative of the contouring accuracy.

This chapter will show that the accuracy of the contouring process in the polar contouring system depends upon the relative location between the contour pole and the machine pole. A method for increasing the contouring accuracy will be also presented.

4.2 Contouring error criterion

The absolute value of the maximum deviation between the machine generated contour and the reference contour can be used as an error criterion.

For NC machines, the deviation error is generally limited to one resolution. In the case of the proposed NC machine also the deviation error is limited to one resolution. However, due to the increase of the angular resolution with radial displacement, the deviation error is given as:

$$\varepsilon_d = \begin{cases} \Delta R & \text{for} \quad r \leq \frac{\Delta R}{\Delta \Theta} \\ \Delta A & \text{for} \quad r > \frac{\Delta R}{\Delta \Theta} \end{cases} \qquad 4.1$$

where ΔA is the angular resolution.

Another way of expressing the contouring error is to use the sum of the absolute values of the areas bounded by the contour and its approximation curve. This error is indicated in Figure 4.1 by the hatched areas. The error area criterion is more stringent than the one based on the deviation error. This can be typically portrayed as shown in Figure 4.1 where a small deviation error, sustained over a long length of the contour, results in a large area error. The area error criterion will be used in this thesis as the basis for estimating the contouring errors. Figure 4.2 is a

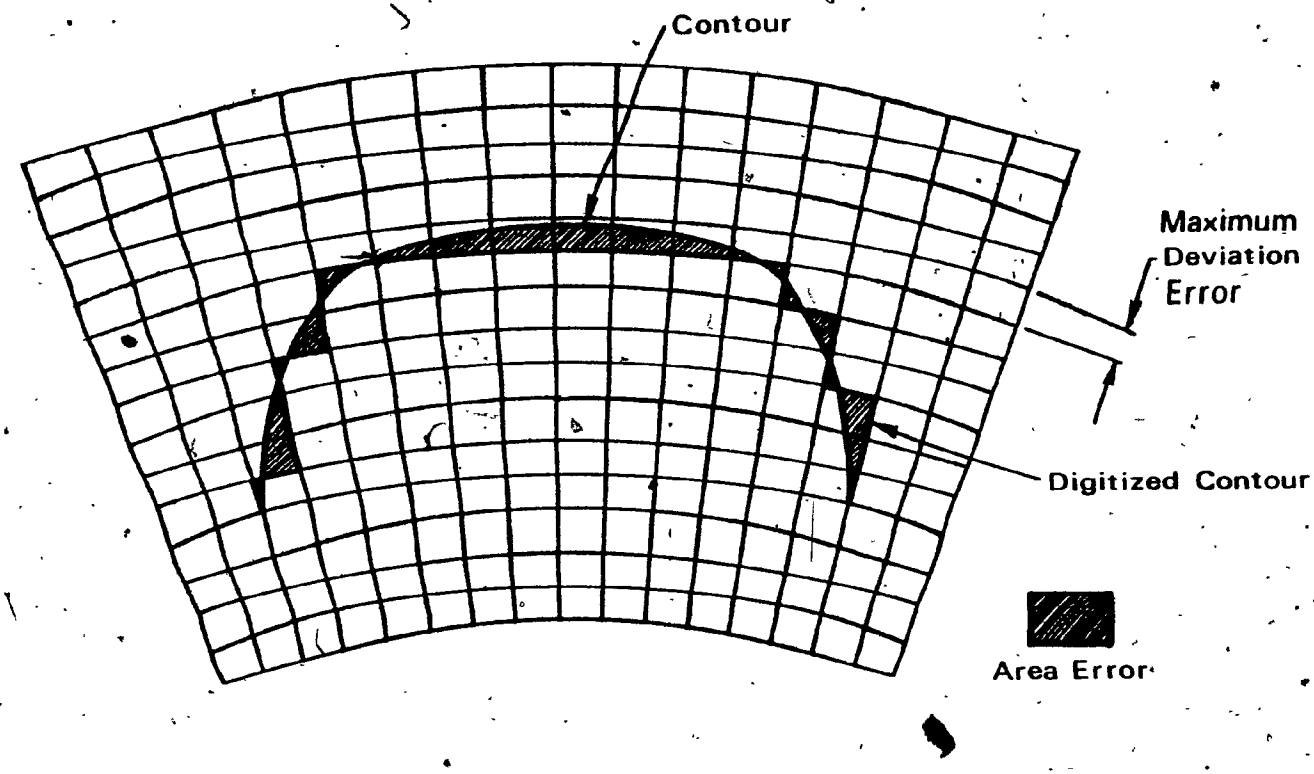


Figure 4.1: Error Criteria in the Contouring Operation.

magnification of one part of Figure 4.1 with some salient geometric parameters indicated. Referring to Figure 4.2, the total area error is given by:

$$\varepsilon_a = \sum_{i=1}^{n} a^{(i)} \qquad 4.2$$

where n is the number of the angular steps required to represent the contour.

The individual area errors $a^{(i)}$ fora contour $r=r\{\theta\}$ can be calculated from Figure 4.2 as follows:

$$a^{(i)} = \frac{1}{2}\left\{ \left| \int_{\theta^{(i)}}^{\nu^{(i)}} \left[r\{\theta\}\right]^2 d\theta - \left[r^{(i)}\right]^2 \left(\nu^{(i)} - \theta^{(i)}\right) \right| + \right.$$

$$\left. \left| \int_{\nu^{(i)}}^{\theta^{(i+1)}} \left[r\{\theta\}\right]^2 d\theta - \left[r^{(i)}\right]^2 \left(\theta^{(i+1)} - \nu^{(i)}\right) \right| \right\} \quad i = 1,2,\ldots n \qquad 4.3$$

where,

$$\theta^{(i)} = \theta^{(0)} + (i-1)\,\Delta\theta$$

$$r^{(i)} = r^{(i-1)} + k\,\Delta R$$

$$k = \text{integer}\left[\frac{r\{\theta^{(i+1)}\} - r^{(i-1)}}{\Delta R}\right]$$

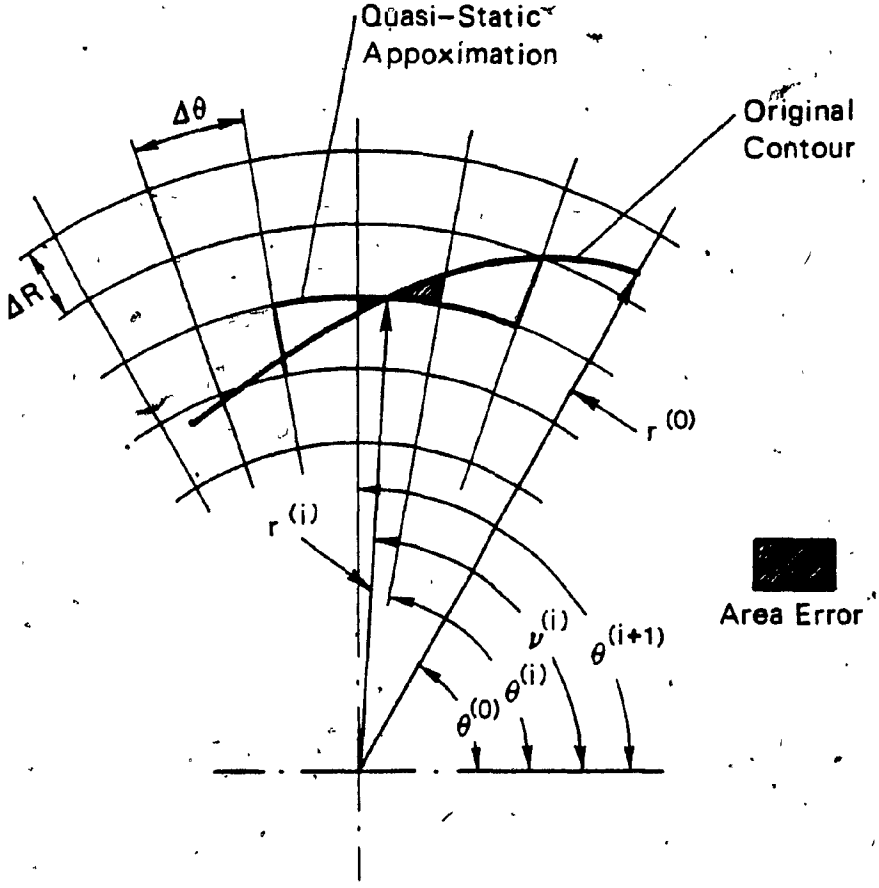


Figure 4.2: Salient Geometric Parameters for the Area Error Calculation.

$$\nu^{(i)} = \begin{cases} \theta^{(i)} & \text{for } \theta^{(i)} < \theta = \theta\{r^{(i)}\} < \theta^{(i+1)} \\ \theta^{(i+1)} & \text{for } \theta = \theta\{r^{(i)}\} \geq \theta^{(i+1)} \end{cases}$$

and,

$$I = \begin{cases} 0 & \text{for } \nu^{(i)} = \theta^{(i+1)} \\ 1 & \text{for } \theta^{(i)} < \nu^{(i)} < \theta^{(i+1)} \end{cases}$$

$r(0)$ and $\theta(0)$ are the first radial and angular grid lines intersected by the contour. Equation 4.3 and consequently Equation 4.2 are very difficult to evaluate analytically, and thus numerical evaluation has to be employed.

4.3 Factors affecting the area error

Perhaps the easiest method to reduce the area error and contouring errors in general, is to reduce the machine resolution. However, beyond a certain limit, the cost of reducing the resolution escalates faster than the benefit of error reduction. A further ill-effect of this reduction would be to reduce the maximum contouring speed that can be attained.

A close examination of the equation for the area error shows that this error depends upon the contour representation with respect to the machine pole, or in other words, on

the location of the contour within the work area. For the same resolution sizes, a reduction in the area error could be achieved by a careful selection of contour location.

4.4 An optmization problem for area error minimization

The evaluation of the exact path of the approximating contour is, in normal operation, a dynamic problem where factors such as resisting forces, inertias, and contouring speed would be involved. For the purpose of finding the optimal location of the contour for minimum reproduction error, the assumption of quasi-static operation is justified due to the following reasons:

a) Such a condition results in the maximum deviation of the approximating curve from the original contour and thus results in an upper bound for error.

b) A contour location that would minimize the error for quasi-static operation would also entail minimizing the error for conditions of normal operation. This point can be further explained with the aid of Figure 4.3. The figure shows four curves: (A) the original contour, (B) an approximating curve that would be produced under quasi-static conditions, and two dynamic curves (C) and (D) that would be produced in normal operation. The underdamped oscillatory trace (C) represents the case of a predominantly inertial load, and the overdamped trace (D) represents an application where large dissipative forces, such as in metal grinding, are involved. The area error bound by trace (C)

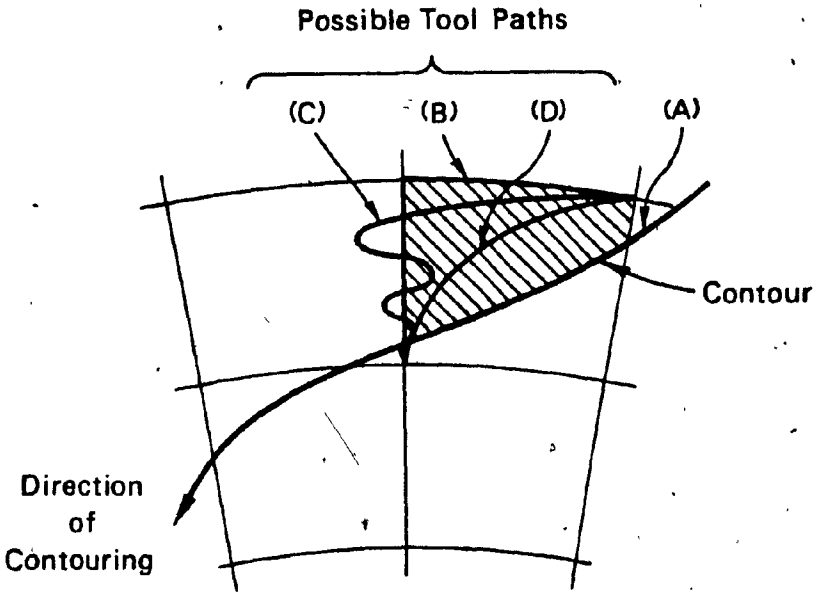


Figure 4.3: Different Tool Paths When Tracking a Contour.

and the original contour can be shown to be equal for all practical purposes to the area error for the quasi-static case. The area error for trace (D), however, is smaller than that for the quasi-static case. This discussion can be carried further to include other stepping situations.

As outlined previously, the area error varies with the location of the contour and can be calculated from Equation 4.3. Due to the complexity and discontinuous nature of the area error expression, analytic methods cannot be used to locate the contour within the work area for minimum error. Hence, numerical optimization techniques have to be employed. For the problem of minimizing the area error, the normal approach would be to use the area error expression from Equation 4.2 as an objective function. The most efficient and commonly used optimization techniques belong to either of the following two categories [31]: direct search techniques and gradient search techniques. Although gradient techniques are the more efficient of the two schemes, they require the derivative of the function with respect to the optimization parameters. For the area error minimization, such a derivative cannot be evaluated. Hence, the less efficient direct search techniques have to be used. For this problem, the use of direct search techniques requires large computer time, (several thousands of central processor seconds for one run using a CDC CYBER 835) due to the time required to evaluate the area error and the large number of function evaluations required. Furthermore,

the problem becomes unmanageable in cases where a narrow ridge is encountered in the objective function and the number of function evaluations required to yield a solution become prohibitively large.

4.5 Selection of an alternate objective function

Another approach to deal with this optimization problem is to find an alternative objective function that is faster to evaluate and at the same time quantitatively duplicates the relationship of the area error function with respect to the machine pole. That is to say, the extrema of the alternative objective function would coincide in location with those of the area error function. In the search for an alternate objective function, one approach that would result in a reduction of the area error, is to reduce the resolution of the polar machine. This approach in effect increases the number of radial and angular steps representing the contour. It is possible, however, to alter the number of radial and angular steps representing the contour by simply changing the location of the contour pole with respect to the machine pole.

To study the relationship between the number of steps and the area error, an algorithm is formulated to alter the location of a contour and to calculate the area error, and the radial and angular steps representing the contour. A straight line path is chosen as a study case. The reasons for this choice are three-folds: first, the calculation of

the area error for a straight line is much simpler than that for any other contour, a factor of paramount importance due to the large number of evaluations required to study the desired relationship; secondly, a straight line template can be accurately and easily machined for digitization and testing on the prototype template digitizer; and lastly, any contour could be approximated by a number of straight line segments, an approach which is practical in dealing with a general curve. Proper choice of the line segments representing the contour can result in an advantage of simplicity which would outweigh the disadvantages of resultant error due to the approximation. This piecewise linear approximation is used later in this chapter.

The number of radial and angular steps representing a straight line are combined together to form two functions which are formulated as follows:

Consider a straight line of length L which is divided into n number of small segments of length ℓ, thus

$$L = \sum_{i=1}^{n} \ell^{(i)} \qquad 4.4$$

Let $N_R^{(i)}$ and $N_\theta^{(i)}$ be the number of radial and angular steps registered in tracing the ith segment. Then, the first function F_{sum} is formulated as:

$$F_{sum} = \begin{cases} \sum_{i=1}^{n} \left| N_R^{(i)} + N_\theta^{(i)} \right| & \text{for } N_R^{(i)} \leq N_\theta^{(i)} \\ \sum_{i=1}^{n} \left| N_R^{(i)} + N_\theta^{(i)} \frac{K}{R_m} \right| & \text{for } N_R^{(i)} > N_\theta^{(i)} \end{cases} \qquad 4.5$$

where K is the ratio of the radial to angular step size, $\Delta R/\Delta\theta$, and has the dimension of length,

and R_m is the radial displacement of the mid-point of the line.

This function needs to be maximized in order to increase the number of steps representing the contour.

The second function is formulated with the objective to prevent the dominance of either the radial or angular steps over the other, and is given by:

$$F_{dif} = \begin{cases} \sum_{i=1}^{n} \left| N_R^{(i)} - N_\theta^{(i)} \right| & \text{for } N_R^{(i)} \leq N_\theta^{(i)} \\ \sum_{i=1}^{n} \left| N_R^{(i)} - N_\theta^{(i)} \frac{K}{R_m} \right| & \text{for } N_R^{(i)} > N_\theta^{(i)} \end{cases} \qquad 4.6$$

Minimizing F_{dif} causes the number of steps taken by both

axes-drives to be close to each other.

When a straight line passes through the machine pole, and the pole lies at either end or beyond the length of the line, no angular steps would be required to represent the line, and the magnitudes of the two functions reach an extreme value:

$$F_{sum} = F_{dif} = L/\Delta R \qquad 4.7$$

When the machine pole lies somewhere on the line, in between its two ends, the two function values from Equation 4.5 and Equation 4.6 respectively become:

$$F_{sum} = L/\Delta R + 2\pi/\Delta\theta \qquad 4.8$$

$$F_{dif} = L/\Delta R - 2\pi/\Delta\theta \qquad 4.9$$

Figure 4.4.b and Figure 4.5.b show plots of F_{sum} and F_{dif} versus the angular orientation of a straight line for different minimum radii R_{min}. The plots have been normalized with respect to the values F_{sum} and F_{dif} obtained when the line forms an equilateral triangle with respect to the machine pole and has a height equal to 7.62 cm. (3 inches). Plots of the actual area error for the straight line calculated based on Equation 4.2 and normalized in a manner similar to that described above are shown in Figures 4.4.a and 4.5.a for comparison purposes. In the plot of the area error, it should be noted that at γ equals zero and π radians the area error is zero. However, for a small perturbation in γ about zero or π radians the area error

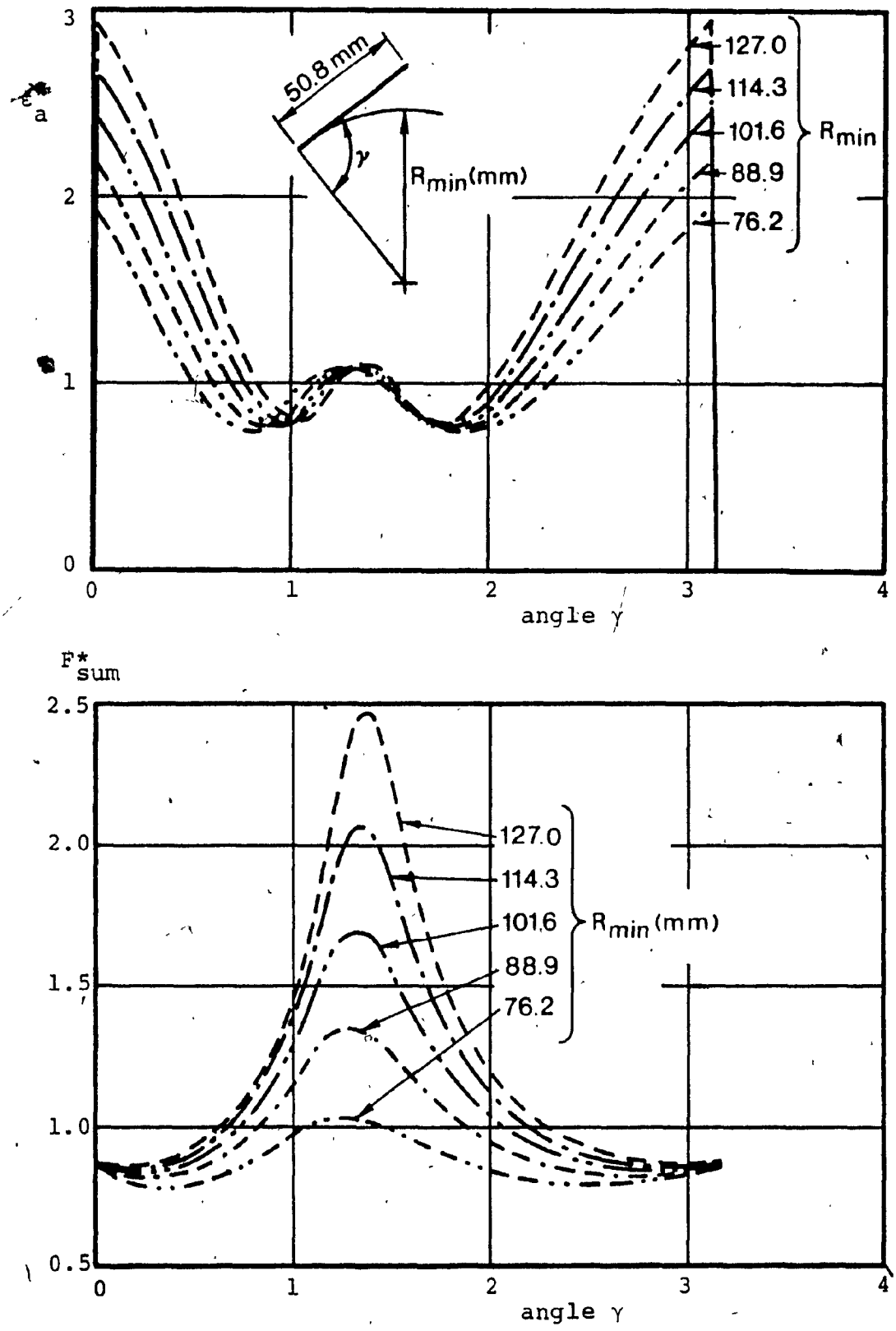


Figure 4.4: Normalized F_{sum} and ε_a Versus γ.

reaches a maximum value. The method used for calculating the two functions and the area error, as well as the algorithm implementing the method are given in Appendix [E].

Figure 4.4 shows that the F_{sum} function does not duplicate the relationship of the area error, although both have their maxima at approximately 1.3 radians. The F_{sum} function also exhibits two regions of minima on either side of its maxima, similar to the area error function. The location of the two minima, however, do not coincide in location with those of the area error function.

The behavior of F_{dif} as portrayed in Figure 4.5 is similar to that of the area error. However, comparing the minima for different values of R_{min}, there is a considerable spread in the function value F_{dif} in contrast to the area error. Also the value of the function F_{dif} at the minima, decreases with increasing R_{min}, and exhibits a reverse trend to that of the area error. Because of these factors, other types of objective functions which are combinations of F_{sum} and F_{dif} where examined for their effectiveness in emulating the area error plots.

Two such functions that are combinations of F_{sum} and F_{dif} are:

$$F_1 = F_{dif}/F_{sum} \qquad 4.10$$

$$F_2 = F_{dif} + 1/F_{sum} \qquad 4.11$$

Experimentation with both functions showed that the function in Equation 4.11 becomes progressively insensitive

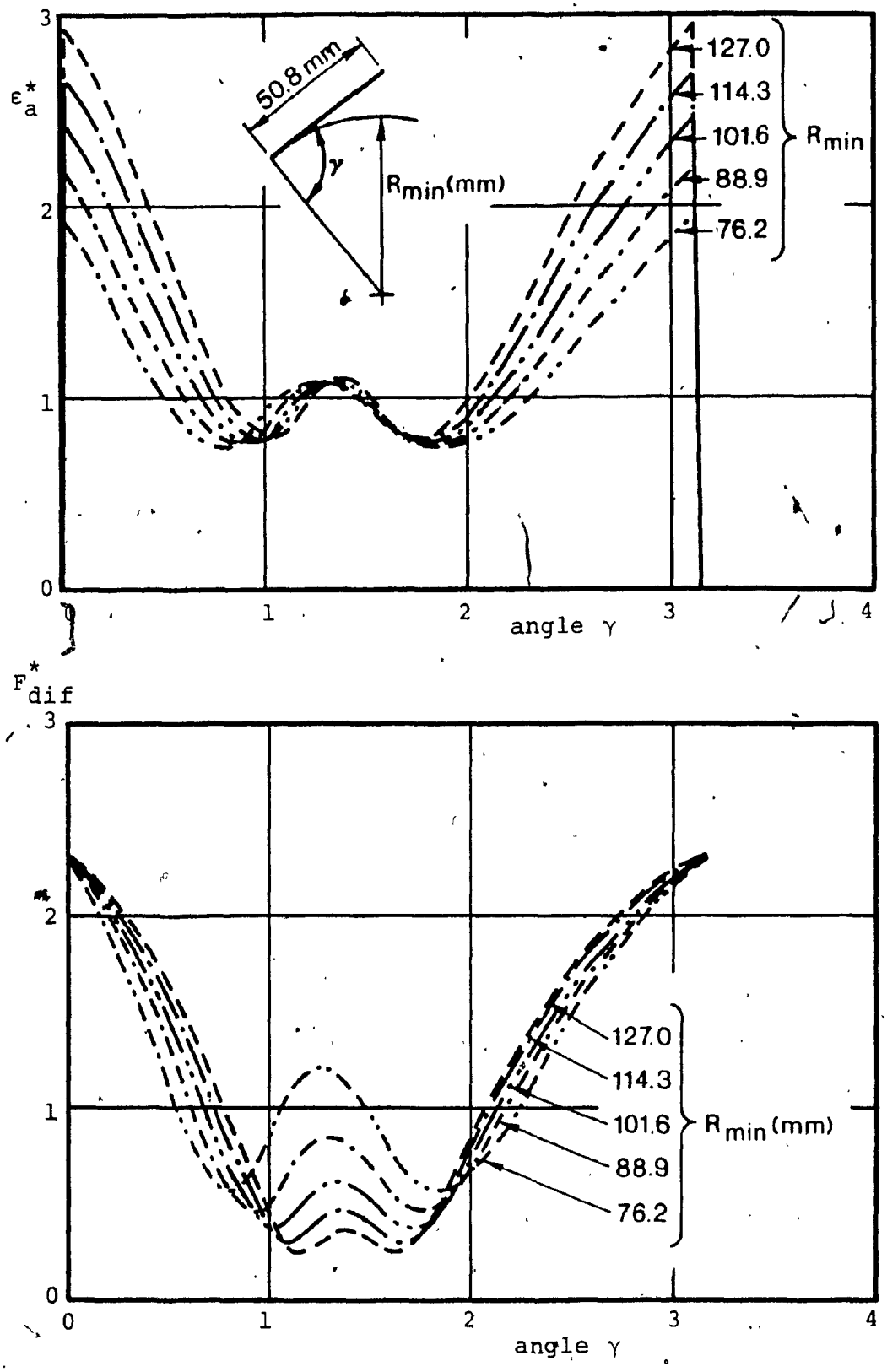


Figure 4.5: Normalized F_{dif} and ε_a Versus γ.

as F_{sum} increases and F_{dif} becomes dominant. The function in Equation 4.10 was found suitable and hence is chosen as the objective function for optimally locating the contour pole with respect to the machine pole.

Figure 4.6 shows a plot of this objective function corresponding to the functions F_{sum} and F_{dif} of Figure 4.4 and Figure 4.5 respectively. Again, here, the area error is provided for comparison. Figure 4.6 shows that the objective function F_1 behaves very similarly to the area error, and has its maxima and minima coinciding in location with those of the area error, and thus can be used as an objective function to predict the optimal location for minimum error.

It should be mentioned here that the factor K/R_m is introduced in F_{sum} and F_{dif} as a penalty factor. Figure 4.7.a shows the effect of this penalty factor on the function F_1 in contouring a line segment. The line segment under consideration is shown in Figure 4.7.b with some salient geometric parameters indicated. Refering to Figure 4.7.b, the ratio of the number of angular to radial steps is given by:

$$\frac{N_\theta}{N_R} = \cos^{-1}\left[\frac{(c^*)^2 - 1 + 2(r^*)^2 + 2\,r^*c^*}{2\,r^*c^* + 2(r^*)^2}\right]\frac{K}{c^*L} \qquad 4.12$$

where c^* is a nondimensional parameter,

and r^* is the nondimensional radial displacement.

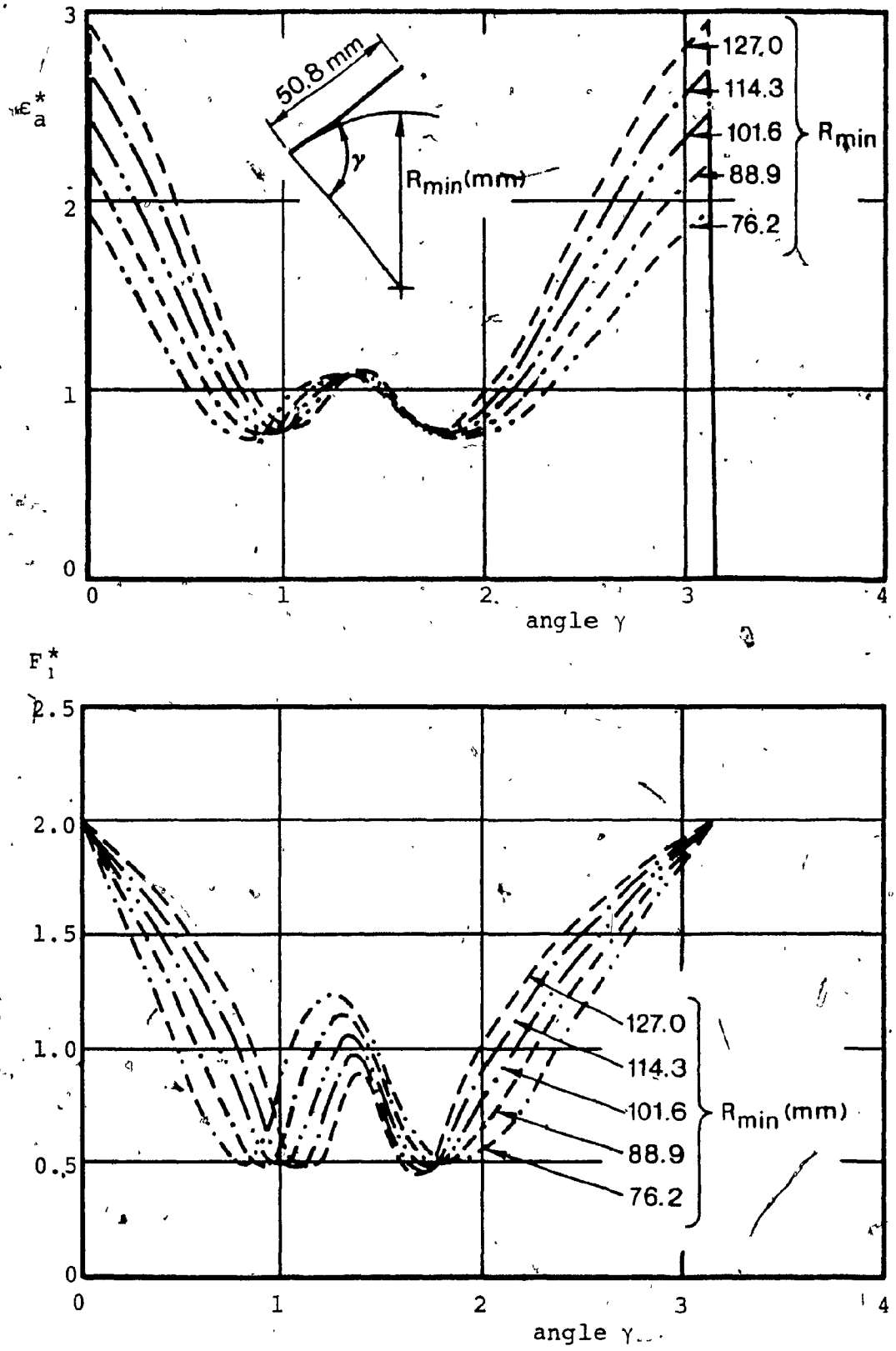


Figure 4.6: Normalized Objective Function F_1 Versus γ.

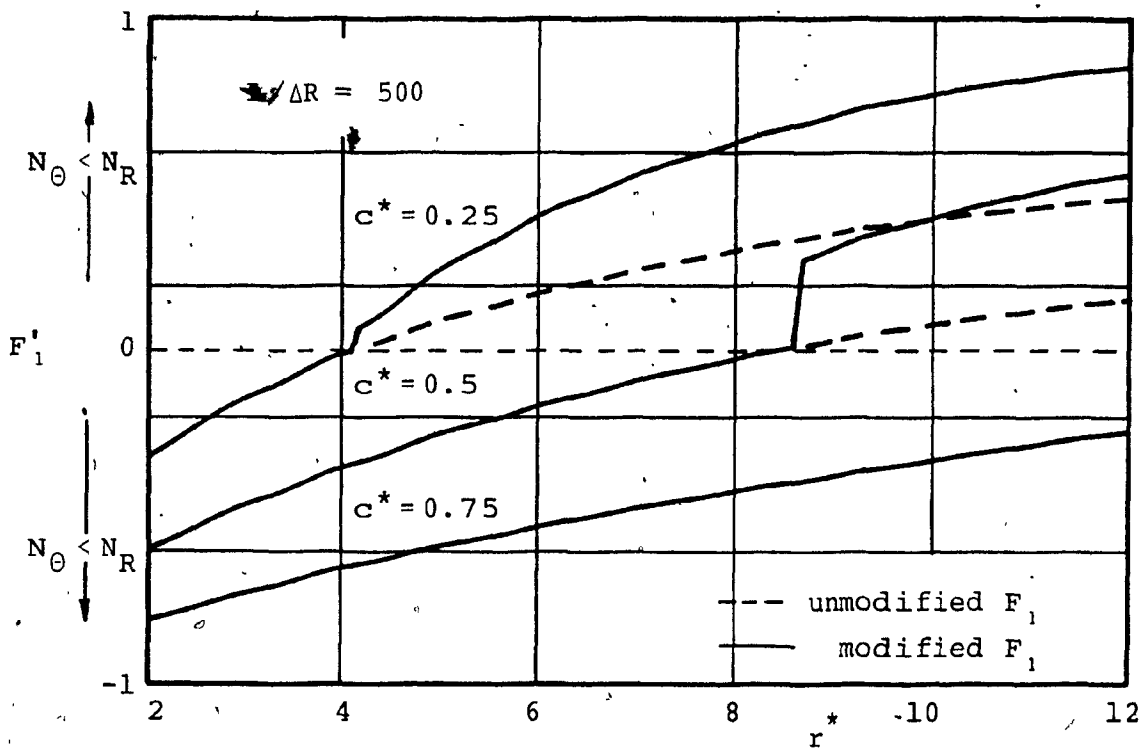


Figure 4.7.a

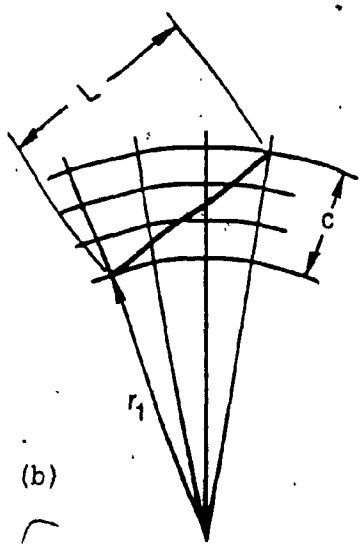


Figure 4.7: Effect of the Penalty Factor K/R_m on the Objective Factor F_j.

F_j' is the sign-ed value of F_j.

The parameter c^* is indicative of the inclination of the line on the radial axis. At $c^*=1$, the line segment is collinear with the axis. The plots in Figure 4.7.a show that below $N_\theta/N_R=1$ and as c^* tends towards unity, the slope of the function F_1 decreases rapidly, especially at large radial displacements. For small radial displacements, the slope increases slightly in the region very close to $c^*=1$. The solid lines on the curves show the objective function F_1 including the effect of using the above-mentioned penalty factor. The dashed lines show the trend in F_1 if the penalty factor was not used. With reference to the figure the penalty factor steepens the plots in the region where it operates. This action causes the function F_1 to react faster to small changes in the position of the line within this region. The parameters r^* and c^* are functions of the relative location between the contour pole and the machine pole, $(C,/\underline{\beta})$.

4.6 Experimental verification of the objective function F_1

In order to verify the calculated objective function and to evaluate the performance of the template digitizer, a template of a straight line has been manufactured and was digitized on the prototype template digitizer. The data were collected as described in Appendix [D], using a hybrid computer, for five different pole locations of the template contour. Substituting the experimental data in Equations 4.5, 4.6, and 4.10, the objective function for the

five locations is calculated. These experimental results are shown in Figure 4.8 together with the theoretically calculated objective function.

The plot shows that, for such a line at the given radial displacement, only one minimum exists and is at an angular orientation of 1.35 radians (77,3 deg.). The figure also shows correspondance between the theoretical and experimental results.

The good correlation between the experimental and theoretical results also provides implicit verification of the performance of the template digitizer.

4.7 Formulation of the optimization problem

The evaluation of the function F_1 at every iterative step in the optimization requires the calculation of the number of radial and angular steps representing the contour segments. For contours that has equations characterizing their geometry, the function F_1 can be simplified by substituting;

$$N_R = \int dr/\Delta R \qquad 4.13$$

$$N_\theta = \int d\theta/\Delta\theta \qquad 4.14$$

into Equation 4.10.

In general, if a contour is represented by a piecewise linear approximation, such that the line segments join successively all the stationary points on the contour, at the least, then the objective function F_1 calculated by

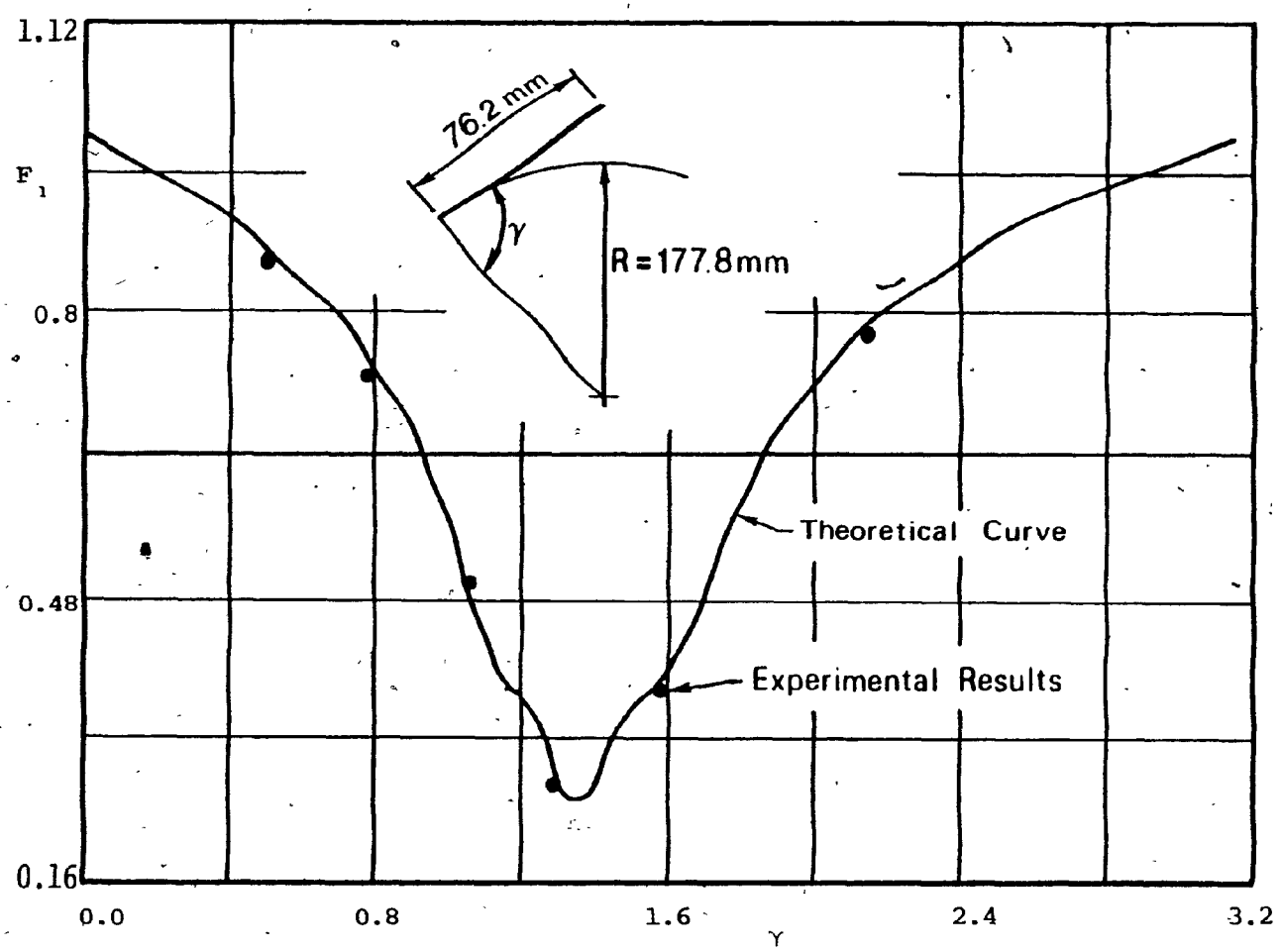


Figure 4.8: Theoretical and Experimental Results of the Objective Function F_1 for a Straight Line.

combining Equations 4.10, 4.13, and 4.14 reduces to:

$$F_1 = \begin{cases} \dfrac{\sum_{i=1}^{n} \left| \dfrac{|\int dr|}{\Delta R} - \dfrac{|\int d\theta|}{\Delta\Theta} \right|}{\sum_{i=1}^{n} \left\{ \dfrac{|\int dr|}{\Delta R} + \dfrac{|\int d\theta|}{\Delta\Theta} \right\}} & \text{for } \dfrac{|\int dr|}{\Delta R} < \dfrac{|\int d\theta|}{\Delta\Theta} \\ \dfrac{\sum_{i=1}^{n} \left| \dfrac{|\int dr|}{\Delta R} - \dfrac{|\int d\theta|}{\Delta\Theta} \dfrac{K}{R_m} \right|}{\sum_{i=1}^{n} \left\{ \dfrac{|\int dr|}{\Delta R} + \dfrac{|\int d\theta|}{\Delta\Theta} \dfrac{K}{R_m} \right\}} & \text{for } \dfrac{|\int dr|}{\Delta R} > \dfrac{|\int d\theta|}{\Delta\Theta} \end{cases} \qquad 4.15$$

where n is the number of line segments representing the contour,

and

$$\frac{dr}{d\theta} = - \frac{b[\cos\theta + m \sin\theta]}{[\sin\theta - m \cos\theta]^2} \qquad 4.16$$

calculated from the equation of a straight line in polar coordinates:

$$r = b/[\sin\theta - m \cos\theta] \qquad 4.17$$

where b is the intercept of the line in the Y-axis,

and m is the slope of the line.

The optimization problem for locating the contour with respect to the machine pole can be mathematically represented as:

minimize F_1 (Equation 4.15)

subject to;

a) Work area constraint of the template digitizer or NC machine and given by:

$$7.62 \text{ cm. (3 inch)} < r < 33.02 \text{ cm. (13 inch)}$$
$$0 < \theta < 2.8 \text{ (in radians)}$$

b) A constraint that would not allow the placement of the machine pole inside a closed contour due to geometrical limits imposed by the construction of the template digitizer.

4.8 Optimization algorithm

An optmization routine having as objective function F_1 of Equation 4.15 was constructed. The routine is based on the direct search algorithm proposed by Hooke and Jeeves. The algorithm is composed of two sections, an exploratory move section, and a pattern move section. In the exploratory move, the objective function is evaluated at equal and orthogonal vectors that originate in all directions from a base point. The resultant vector that cause an improvement in the objective function value is then calculated from these vectors. If no improvement is found, the algorithm scales down the orthogonal vectors, and the exploratory process restarted. In the pattern move, the algorithm transports the base point along the resultant

vector and beyond its length. A detailed description of this algorithm can be found in Reference [31] (pp.69).

4.9 Case study results on the suitability of the objective function in predicting the location for minimum area error

In order to evaluate the effectiveness of using the above mentioned objective function, a number of contours were selected and their positions within the polar coordinate grid was optimized to give minimum area error. To verify how closely the objective function F_1 simulate the trends of the area error, another computer program was constructed to digitize the contours and the actual area error was calculated. Listings of both algorithms are given in Appendix [E]. Three curves are chosen as test contours. These curves represent open, closed, and sharply undulating types, and are as follows:

1) The Witch of Agnesi given by:

$$y = 8a^3/[x^2 + 4a^2] \qquad 4.18$$

2) The Limacon of Pascal given by:

$$r = b + 2a\cos\theta \qquad 4.19$$

3) The Serpentine curve given by:

$$y = abx/[a^2 + x^2] \qquad 4.20$$

Figures 4.9, 4.10, and 4.11 show these curves as well as their topographical contours of the objective function.

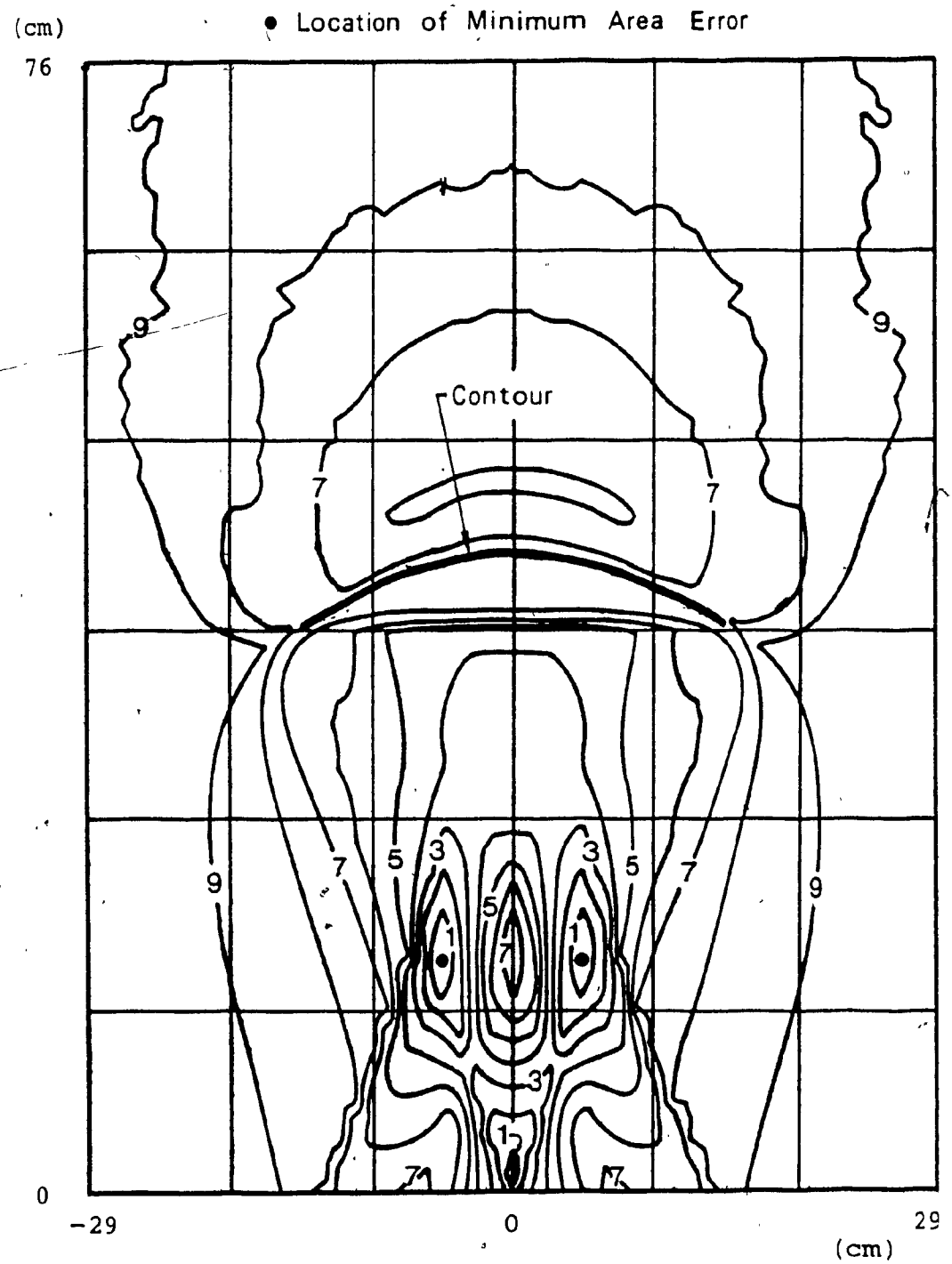


Figure 4.9: Topographical Plots of the Objective Function F_1 for the Witch of Agnesi Contour.

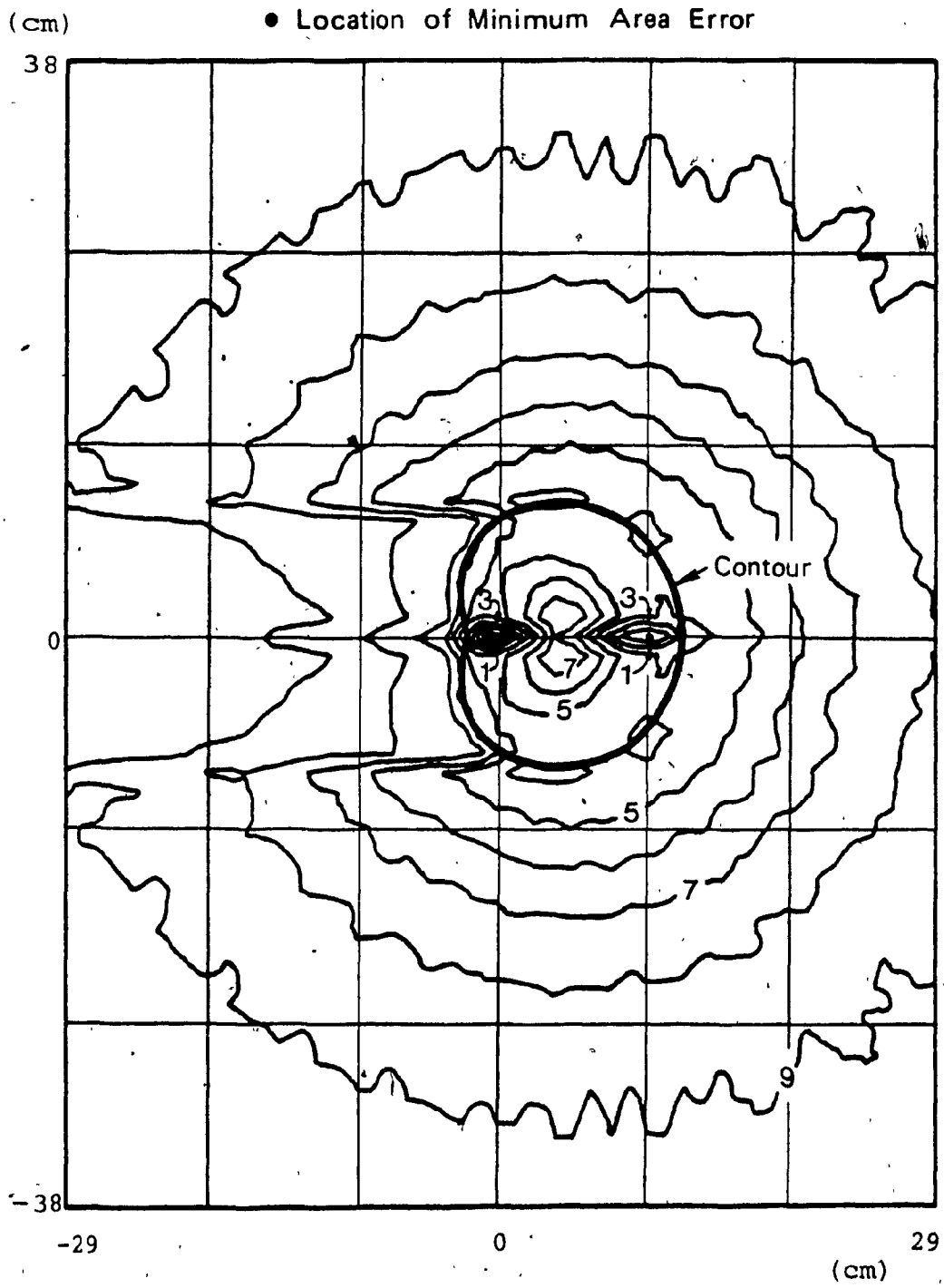


Figure 4.10: Topographical Plots of the Objective Function F_1 for the Limacon of Pascal Contour.

The topographical contours in these figures represent machine pole locations where the values of the objective functions are equal. The topographical contours numbered from 1 to 9, with higher numbers indicating larger objective function F_1 value. The topographical contours are not part of the optimization procedure and are introduced here to give a visual image of the behavior of the objective function, and hence of the area error. The procedure consisted of running the optimization routine with the objective function (Equation 4.15) and finding the location of the pole for minimum area error. The algorithm that calculates the exact area error (Equation 4.2) was then used to evaluate the error for a number of randomly selected locations within the neighborhood of the optimally located pole position based on the objective function F_1. The locations of the pole for the minimum area error found using the randomly selected locations are also shown on the topographical plots.

The test contours parameters, as well as numerical results of the study are tabulated in Table 4.1.

For the "Witch of Agnesi" and the "Serpentine" curves the optimization routine was able to identify the location of the pole for minimum error accurately, as can be seen from Figures 4.9 and 4.11. For the "Limacon of Pascal" the optimization routine identified a point very close to that of the actual minimum area error. The percentage difference in area error between the identified and the actual locations

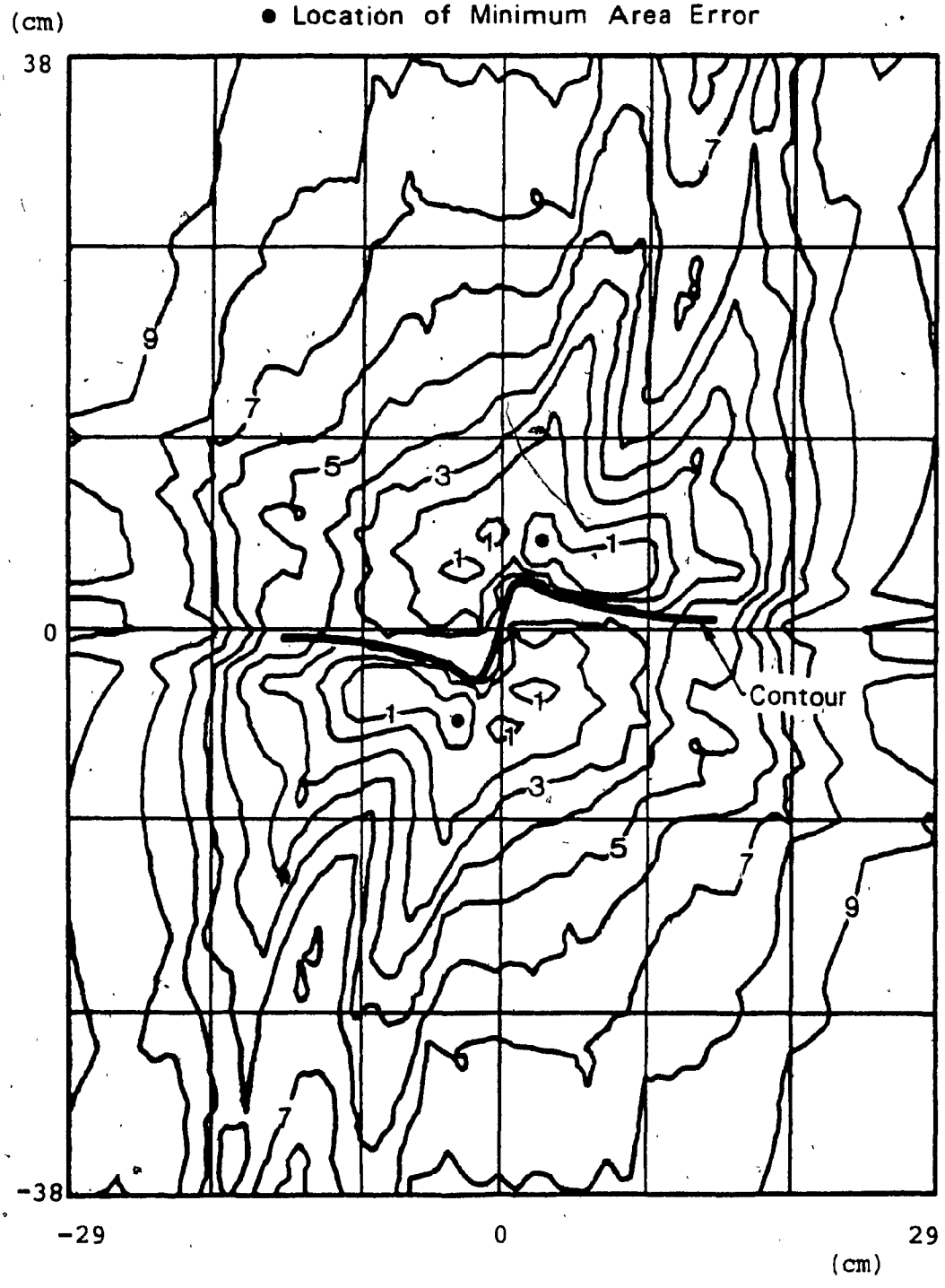


Figure 4.11: Topographical Plots of the Objective Function F_1 for the Serpentine Contour.

Table 4.1: Test Contour Parameters and Results of Optimization for Area Error Minimization

Test Contour / Parameter Results	Witch of Agnesi	Limacon of Pascal	Serpentine
Mathematical representation	$y = 8a^3/(x^2 + 4a^2)$	$r_1 = b + a\cos\theta_1$	$y = abx/(a^2 + x^2)$
Contour constants	a = 21.59 cm (8.5 inch)	a = 5.08 cm (2 inch) b = 7.62 cm (3 inch)	a = 1.27 cm (0.5 inch) b = 7.62 cm (3 inch)
Optimal location of machine pole for F_1 junction	C = 15.46 cm (6.09 inch) β = 1.236 radians, or = 1.905 radians	C = 0.86 cm (0.34 inch) $\beta = 2\pi$ radians	C = 5.37 cm (2.11 inch) β = 1.105 radians, or = 4.247 radians
Area error value	ε_a = 2.59 sq.mm $(0.409\times10^{-2}$ sq.inch)	ε_a = 4.05 sq.mm $(0.628\times10^{-2}$sq.inch)	ε_a = 4.563 sq.mm $(0.707\times10^{-2}$sq.inch)
Optimal location of machine pole for area error	same location as for F_1 function	C = 1.27 cm (0.5 inch) $\beta = 2\pi$ radians	same location as for F_1 function
Area error value	same as for optimal location of F_1 function	ε_a = 4.01 sq.mm $(0.6216\times10^{-2}$sq.inch)	same as for optimal location F_1 function
Percentage difference in area error value	0%	0.97%	0%

in this particular case is 0.96 %.

4.10 Summary

In this chapter it was shown that for evaluating the performance of the contouring process, an area error criterion is more stringent than the deviation error. It was also shown that the area error is a function of the relative position of the contour pole from the machine pole. Thus by varying the location of the contour pole, the area error can be minimized. The calculation of the area error is lengthy and cumbersome, and using it as an objective function for optimization proves impractical.

An alternate objective function is formulated based upon maximizing the ratio of F_{dif}, defined as the difference between the number of radial and angular steps representing the contour, to F_{sum}, defined as the sum of the radial and angular steps representing the contour. It is shown in this chapter that the alternate objective function duplicates the trends of the area error, and is easy to use. The alternate objective function is incorporated in an optimization routine to locate the contour pole for minimum area error. Test results show that the optimization algorithm is able to accurately identify the minimum area location.

This chapter deals exclusively with the case of a point tool. And although it is difficult to formulate analytically the objective function in cases where the tool has a finite diameter, reason suggests that the technique

presented in this chapter is equally applicable to these cases. This is because the formulated area error minimization method is based on the number of radial and angular steps representing the contour and is independant of the tool diameter.

CHAPTER 5

DYNAMIC ANALYSIS OF THE POLAR NC MACHINE

5.1 Introduction

The NC machine operates in an open loop control scheme as has been described earlier. This scheme has the advantage of reduced complexity of the NC controller. Open loop control, however, does not have the capability of detecting the failure of an axis-drive to respond to a command (loss of synchronism) and compensating for the failure. The loss of synchronism would occur if the torque imposed by the load exceed the torque capacity of the drive.

The upper limit of the torque that can be imposed on a stepping motor without loss of synchronism decreases as the rate of incoming step signals increases. The torque as well as the rate of step signals of each of the radial and angular axes stepping motors depend on the contour as well as such operating conditions as machining forces and contouring speed. Changes in the torque required and torque available from the axes-drives as the machining conditions change should be thoroughly understood. This knowledge would be used to identify the operating limits of the NC machine for a given set of machining conditions beyond which synchronism failure may occur.

The study of the torque changes in the NC machine axes-drives is a dynamics problem that deals with the

dynamics of motion along the contour and not with the dynamics of a single step. In this study it is assumed that the signals generated by the digitizer have been properly processed to insure constant contouring speed as has been discussed in Chapter 2. The kinematics of the arm and carriage mechanisms is studied with a tool moving at a constant speed and producing a contour according to the signals received from the digitizer. The instantaneous velocity and acceleration of each axis are calculated and used in the dynamic analysis in order to determine the torque required from the stepping motors. The velocity of each axis is further used to determine the limiting torque available from the corresponding stepping motor.

5.2 Kinematic analysis of the NC machine arm and carriage mechanism

Figure 5.1 shows a vector repesentation of the prototype NC machine. The figure shows the machine pole (O) displaced from the contour pole (O_1) by a vector ($C, \underline{/\beta}$) and a tool of radius R_r touching the required contour $r_1 = r_1\{\theta_1\}$ at a point (P).

With reference to the figure, for machining operations requiring no tool offset (i.e. $R_r = 0$) like plasma or flame cutting, the tool centre coincides with (P) and moves along the contour. This case of no tool offset represents the simplest configuration of the vector diagram and will be studied first.

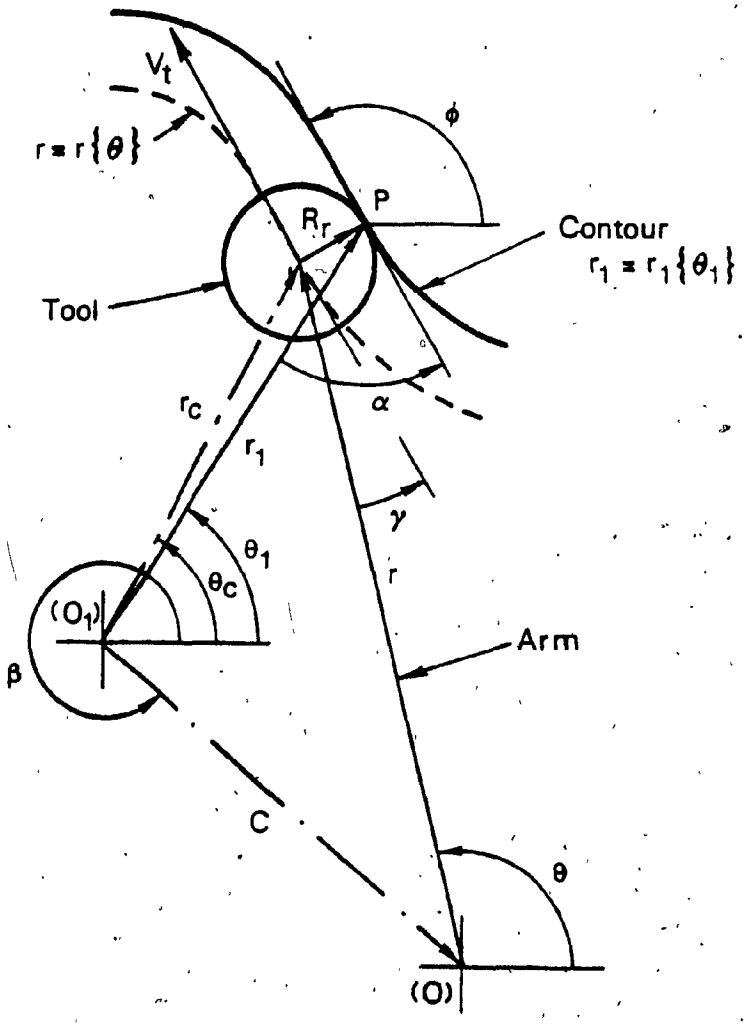


Figure 5.1: Vector Representation of the Polar System in Contouring.

5.2.1 Case 1: Tool with no offset (zero radius)

With reference to Figure 5.1, for the case of no tool offset, the vector $r_1 \angle\theta_1$ coincide with $(r, \angle\theta)$. Hence, the velocity vector of the centre of the tool and the carriage can be expressed with respect to the machine pole (O) by:

$$V_t\, e^{j\phi} = \dot{r}\, e^{j\theta} + j\, r\, \dot{\theta}\, e^{j\theta} \qquad 5.1$$

Resolving the velocity equation in the radial and tangential directions give:

$$\dot{r} = V_t \cos\gamma \qquad 5.2$$

$$\dot{\theta} = (V_t \sin\gamma)/r \qquad 5.3$$

where,

$$\gamma = \cos^{-1}\left[\frac{\Lambda_1\, \Lambda_3}{r^2\Lambda_2 - r\, \Lambda_1\Lambda_4}\right] \qquad 5.4$$

$$\Lambda_1 = \left[r_C - C\cos(\theta_C - \beta)\right]\frac{dr_C}{d\theta_C} + r_C\, C \sin(\theta_C - \beta)$$

$$\Lambda_2 = C^2 + 2r\, C \cos(\theta - \beta) + r^2$$

$$\Lambda_3 = r + C\cos(\theta - \beta)$$

$$\Lambda_4 = C \sin(\theta - \beta)$$

The acceleration of point (P) is obtained by differentiating Equation 5.1 with respect to time as follows:

$$a_c = a_t\, e^{j\phi} + j\, V_t\, \dot{\phi}\, e^{j\phi}$$

$$= \ddot{r}\, e^{j\theta} + 2\, j\, \dot{r}\, \dot{\theta}\, e^{j\theta} + j\, r\, \ddot{\theta}\, e^{j\theta} - r(\dot{\theta})^2\, e^{j\theta} \qquad 5.5$$

Since it is desired to maintain a constant contouring speed along the contour, it follows that the acceleration a_c of point (P) tangential to the contour is identically zero.

Resolving Equation 5.5 to its components along the arm and normal to it, yields:

$$a_{cr} = V_t\, \dot{\phi}\, \sin\gamma$$

$$= \ddot{r} - r(\dot{\theta})^2 \qquad 5.6$$

$$a_{cn} = V_t\, \dot{\phi}\, \cos\gamma$$

$$= r\,\ddot{\theta} + 2\dot{r}\,\dot{\theta} \qquad 5.7$$

In Equation 5.6 the term $r(\dot{\theta})^2$ is the centripetal acceleration of the carriage and its direction is always towards the pole. The term $2\dot{r}\dot{\theta}$ of Equation 5.7 is the Coriolis acceleration and has a direction normal to the arm in the sense of the angular velocity or opposing it for cases of positive or negative radial velocities respectively.

The time rate of change of the velocity vector direction $\dot{\phi}$ is given by:

$$\dot{\phi} = V_t/\rho \qquad 5.8$$

where ρ is the radius of curvature at point (P),

and is calculated as shown in Equation 3.4.

5.2.2 Case 2: Tool with finite radius

In contouring applications where a tool of finite radius is used, as in grinding and milling operations, the tool offsets the centre of the carriage from the required contour by a distance equal to its radius. Let the loci of the tool centre be designated $r=r\{\theta\}$. With reference to Figure 5.1, for a point (P) on the contour $r_1\{\theta_1\}$, given r_1 or θ_1, r and θ can be solved for implicitly from the following equations:

$$r = \left[r_c^2 + C^2 - 2r_c\, C \cos(\theta_c - \beta)\right]^{\frac{1}{2}}$$

$$r_c = \left[r_1^2 + R_r^2 - 2r_1 R_r \sin\alpha\right]^{\frac{1}{2}} \qquad 5.9$$

$$\theta_c = \theta - \sin^{-1}\left[\frac{C}{r_c}\sin(\theta - \beta)\right]$$

The kinematic relations for this case can now be formulated by substituting for r, r_c, θ_c and $dr_c/d\theta$ from Equation 5.9 into the kinematic relationship for the case of no tool offset. The derivative $dr_c/d\theta_c$ is given as follows:

$$\frac{dr_c}{d\theta_c} = \frac{\Lambda_5\Lambda_6}{1 - \Lambda_5\Lambda_7} \qquad 5.10$$

where,

$$\Lambda_5 = \left[r_1 \frac{dr_1}{d\theta_1} - \frac{dr_1}{d\theta_1} R_r \sin\alpha + R_r r_1 \cos\alpha \frac{d\alpha}{d\theta_1}\right] \Big/ r_c$$

$$\Lambda_6 = \left[r_c r_1 - r_c R_r \sin\alpha\right] \Big/ \left[r_c r_1 - r_c R_r \sin\alpha + R_r r_c \sin\alpha \frac{d\alpha}{d\theta_1}\right]$$

$$\Lambda_7 = R_r \cos\alpha \Big/ \left[r_c r_1 - r_c R_r \sin\alpha + R_r r_c \sin\alpha \frac{d\alpha}{d\theta_1}\right]$$

and,

$$\frac{d\alpha}{d\theta_1} = \left[r_1 \frac{d^2 r_1}{d(\theta_1)^2} - \left(\frac{dr_1}{d\theta_1}\right)^2\right] \Big/ \left[r_1^2 + \left(\frac{dr_1}{d\theta_1}\right)^2\right]$$

5.3 Dynamic analysis of the polar NC machine

The focus here is on the calculation of the dynamic forces that must be overcome by the stepping motors. All components of the NC machine will be considered as rigid bodies.

a) The swinging arm,

b) The worm and worm-gear reducer,

c) The carriage, and

d) The power screw and nut assembly.

Figure 5.2 shows pictorially the arm and carriage mechanisms of the polar NC machine with salient parameters indicated.

The free body diagram of these components and the forces acting on them are shown in Figure 5.3. In devel-

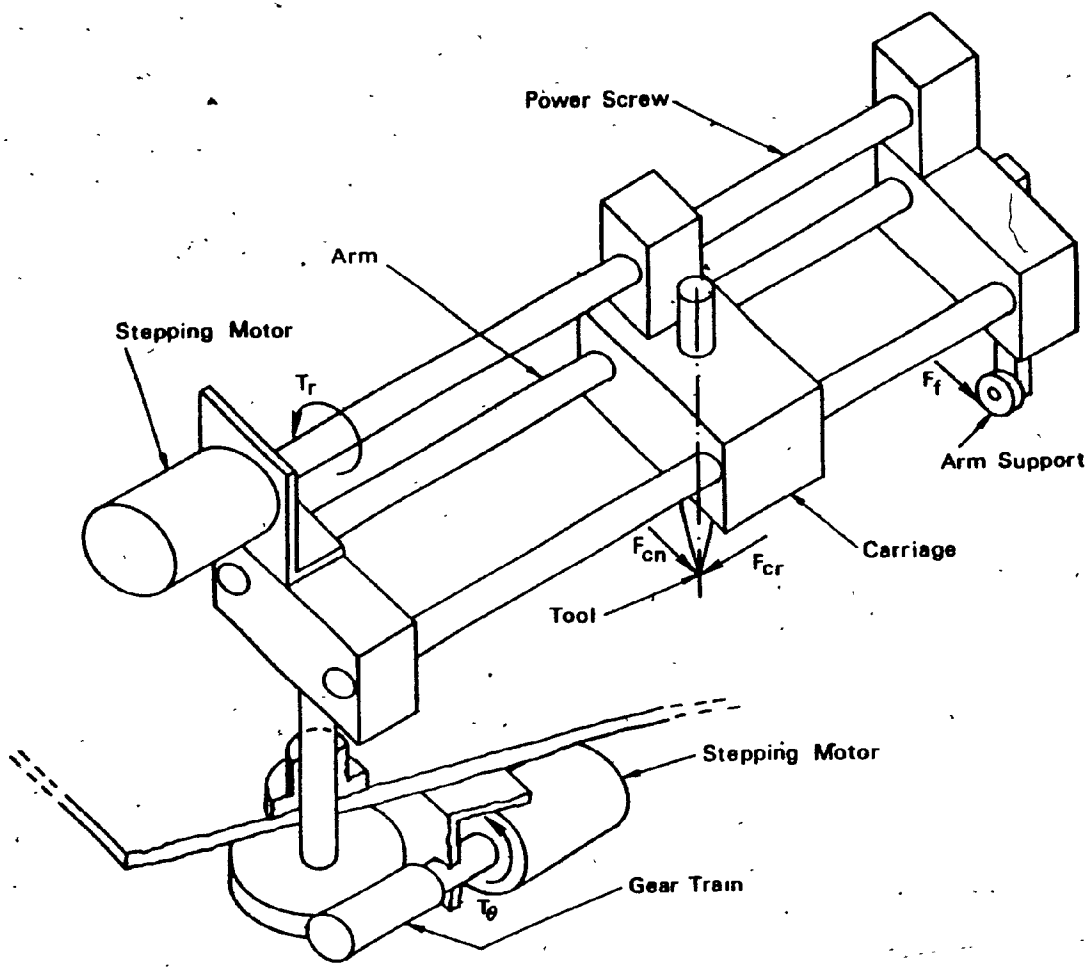


Figure 5.2: Pictorial Diagram of the NC Machine.

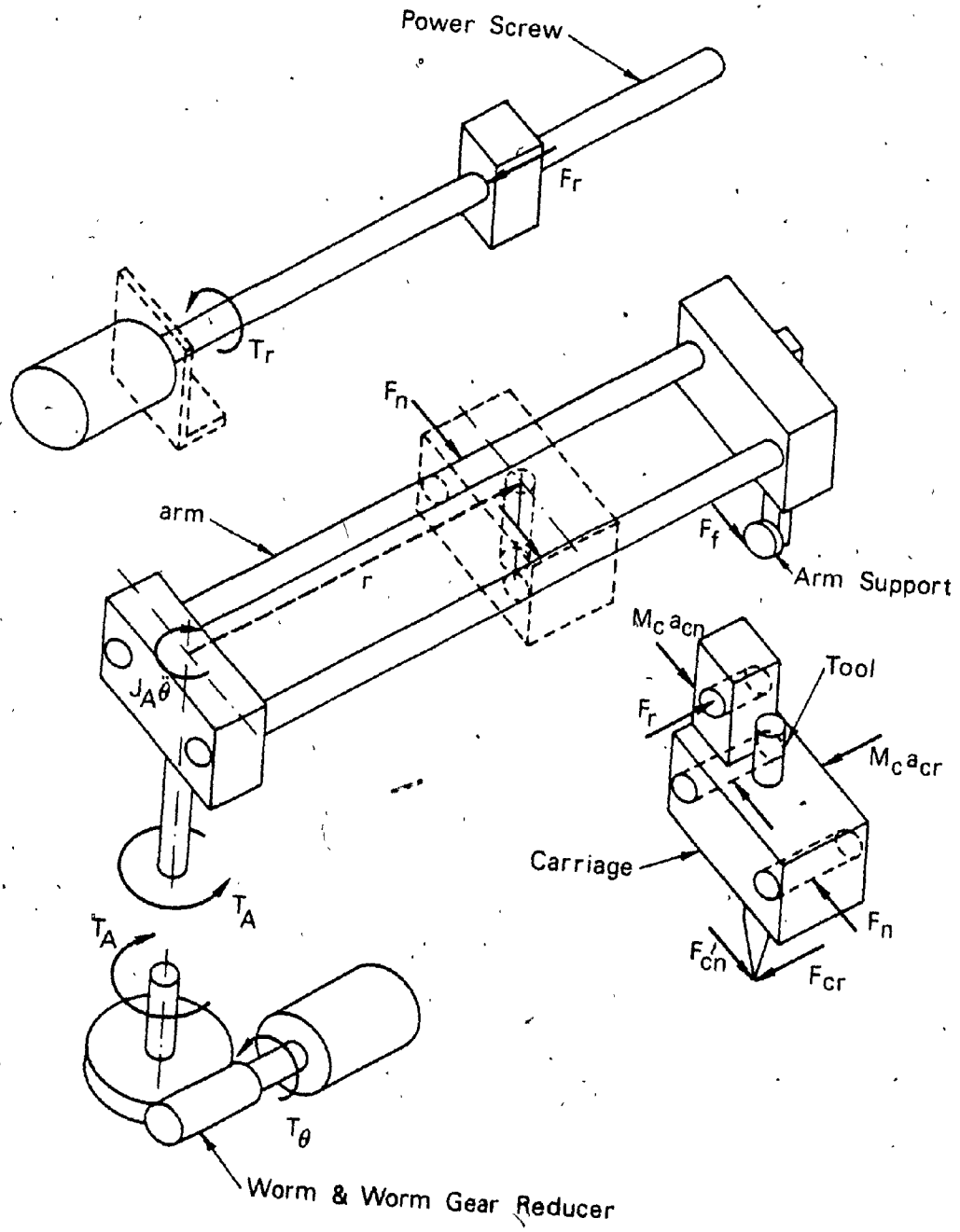


Figure 5.3: Free Body Diagram of the NC Machine.

oping the free body diagram, the mass moment of inertia of the worm-gear, the lead screw, and the radial axis stepping motor are lumped with that of the arm. Also, the mass moments of inertia of the power screw and the radial axis stepping motor are reflected to the mass of the carriage-nut assembly. The power screw and nut assembly is considered to be a transformer with friction losses only. Reaction forces to ground and machine body are not shown since they do not contribute to the torque equations. With reference to the free body diagram, the balance of forces and moments equations for the four components of the NC machine are as follows:

For the arm:

$$T_A = J_A' \ddot{\theta} + F_m r + F_f L_A \qquad 5.11$$

where $J_{A'}$ is the equivalent mass moment of inertia of the arm.

T_A is the sum of all the torques that act on the arm,

F_m is the force transmitted from the carriage normal to the arm,

and F_f is the friction force of the arm support.

For the worm and worm-gear reducer:

$$T_\theta = K_1 T_A + J_M \frac{R_g}{R_w} \ddot{\theta}$$

$$= K_1 \left[J_A \ddot{\theta} + F_{cn} r + F_f L_A \right] \qquad 5.12$$

where,

$$J_A = J'_A + \frac{J_M R_g}{R_w K_1}$$

where J_M is the mass moment of inertia of the angular axis motor and the worm combined,

K_1 is a factor that combines both the torque magnification due to the gearing, and the friction losses between the worm and worm-gear, and is given by Reference [32] (pp.545) as:

$$K_1 = \frac{R_w}{R_g} \frac{[\cos\zeta \sin\lambda + \mu_1 \cos\lambda]}{[\mu_1 \sin\lambda - \cos\zeta \cos\lambda]}$$

where R_w, R_g are the pitch radii of the worm and worm-gear respectively,

ξ is the pressure angle of the gear system,

λ is the lead angle of the worm-gear,

and μ_1 is the friction coefficient between the worm and the worm-gear.

For the carriage:

$$\left.\begin{aligned} F_r &= M_c' a_{cr} + F_{cr} \\ F_n &= M_c' a_{cn} + F_{cn} \end{aligned}\right\} \qquad 5.13$$

where F_m ,F_m is the sum of all the radial and normal, forces acting on the carriage respectively

For the power screw and nut assembly:

$$T_R = J_M a_{cr} \frac{2\pi}{l} + K_2 F_r$$

$$= K_2 \left[M_c a_{cr} + F_{cr} \right] \qquad 5.14$$

where,

$$M_c = M_c' + \frac{2\pi J_M}{l K_2}$$

K_2 is the torque to force transformation factor of the power screw and nut assembly. This factor also takes into account the friction losses. K_2 is given in Reference [33] (pp.3-42) by:

$$K_2 = R_s \frac{\left[l + 2\pi R_s \mu_2 \sec\xi \, (1 - \sin\psi \sin\xi)^{\frac{1}{2}} \right]}{\left[2\pi R_s - l \mu_2 \sec\xi \, (1 - \sin\psi \sin\xi)^{\frac{1}{2}} \right]}$$

where R_s is the mean radius of the thread

l is the lead,

μ_2 is the friction coefficient between the nut and power screw,

and ψ, ξ are the helix and thread angles respectively.

Combining Equations 5.11 to 5.14 yield the torques required from the angular and radial axes stepping motors as follows:

$$T_R = K_2 \left[M_c\, a_{cr} + F_{cr}\right] \qquad 5.15$$

$$T_\theta = K_1 \left[J_A \ddot{\theta} + F_{cn} r + F_f L_A + M_c r\, a_{cn}\right] \qquad 5.16$$

As the tool travels along the contour, the torque required from the stepping motors as given by Equations 5.15 and 5.16 varies. Each stepping motor has to be able to provide a torque at least equal to the maximum required torque.

5.4 Torque speed relationship of a stepping motor

As mentioned in Section 2.4, the wave energization scheme is used for driving both the radial and angular stepping motors. Although this scheme is not commonly used, the justification for using it here has been given earlier. Pull-out torque-speed characteristics of stepping motors operating with wave energization are not commonly available from the manufacturers. Pull-out torque curves (T_p versus steps/s) show for a given speed (steps/second), the limit torque at which missing of steps start to occur. Testing

the motors in order to obtain these characteristics is a lengthy and complicated procedure. A description of the procedure is given in Appendix [G]. Appendix [G] presents a derivation of the characteristics for the pull-out torque of a stepping motor operating with the wave energization scheme. The relation is given by:

$$T_P = \frac{1.17632\ V\ K_T}{\left[R_a^2 + \omega^2 L_a^2\right]^{\frac{1}{2}}} - \frac{K_V\ \omega\ R_a\ K_T}{N_s (R_a^2 + \omega^2 L_a^2)} \qquad 5.17$$

where K_T, K_V are the torque and voltage constants of the stepping motor respectively,

R_a, L_a are the motor winding resistance and inductance respectively,

V is the power supply voltage,

and ω is the angular velocity of the motor (in electric radians/s).

The angular velocity of each of the stepping motors can be found from the respective axes velocity $\dot{r}$ and $\dot{\theta}$ given in Equations 5.2 and 5.3 as follows:

$$\omega_R = 2\pi N_s\ k_R\ \dot{r} \qquad 5.18$$

$$\omega_\theta = 2\pi N_s\ k_\theta\ \dot{\theta} \qquad 5.19$$

Substituting Equations 5.18 and 5.19 in Equation 5.17 result in the pull-out torques T_{PR} and $T_{P\theta}$ for the radial and angular axes as follows:

$$T_{PR} = \frac{1.17632\ V\ K_T}{\left[R_a^2 + \omega_R^2 L_a^2\right]^{\frac{1}{2}}} - \frac{K_V\ \omega_R\ R_a\ K_T}{N_s (R_a^2 + \omega_R^2 L_a^2)} \qquad 5.20$$

$$T_{P\Theta} = \frac{1.17632\ V\ K_T}{\left[R_a^2 + \omega_\Theta^2 L_a^2\right]^{\frac{1}{2}}} - \frac{K_V\ \omega_\Theta\ R_a\ K_T}{N_s\ (R_a^2 + \omega_\Theta^2 L_a^2)} \qquad 5.21$$

5.5 Effect of the contouring speed, load, and contour on the axes-drives torque

A parameter called "torque ratio", and defined as the quotient of the torque required from an axis-drive to the pull-out torque available from the respective drive, is used as a measure of the success or failure of a contouring operation. A torque ratio larger than unity means that the torque required from a stepping motor exceedes that which it is able to deliver, and hence missing of steps.

Equations 5.15 and 5.16 show that the torques required from the two axes-drives are function of the load as well as on the kinematic relations for the axes. Equations 5.1 to 5.10 show explicitly the dependence of these kinematic relations on the contouring speed as well as on the contour, and its location from the machine pole. Furthermore, the pull-out torque available from the axes-drives, and given by Equations 5.20 and 5.21, are also functions of these kinematic relations.

The load on the NC machine consists of the inertia of the tool and its accessories, and the force acting on the tool. This load varies from one application to another. For example in the case of metal removal by grinding, these loads are predominantly dissipative, while in the case of plasma or flame cutting it is predominantly inertial, where the cutting nozzle and assocciated metering instruments provide the added mass on the carriage.

A computer algorithm implementing the dynamic analysis has been formulated in order to evaluate the effect of different loading conditions and contours on the torque ratio. Four curves shown in Figures 5.4 to 5.7 are used as test contours, these are:

a) a circle,

b) a Witch of Agnasi,

c) a Limacon of Pascal, and

d) a Serpentine curve.

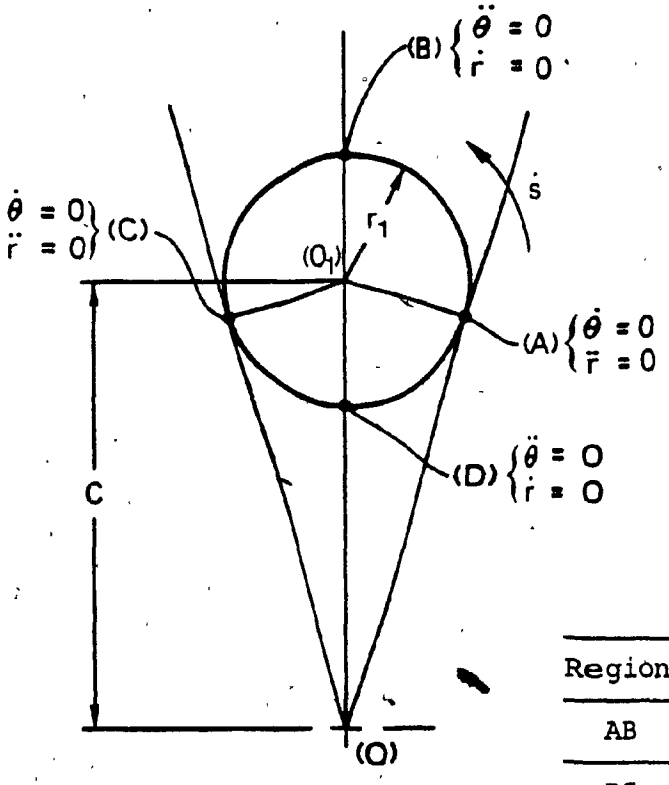
These curves are chosen to cover cases of profiles having symmetric, open, and closed contours. A straight line, however, was not chosen as a test curve since its radius of curvature is at infinity, and with reference to Equations 5.6, 5.7, 5.8, 5.15, and 5.16 inertial load on the mechanism has no effect on the axes-drives when the radius of curvature is of infinite length.

Inertial as well as grinding operations are considered for the above-mentioned test contours in order to study the effect of the load, the contouring speed, and the contour on

r_1 = constant
= 5.08 (2 inch)

C = 17.78 cm (7 inch)

β = 3 /4



Region	$\dot{r}$	$\ddot{r}$	$\dot{\theta}$	$\ddot{\theta}$
AB	+	−	+	+
BC	−	−	+	−
CD	−	+	−	−
DA	+	+	−	+

Figure 5.4: Test Contour : Circle.

$$y = \frac{8a^3}{x^2 + 4a^2}$$

a = 21.59 cm (8.5 inch)

C = 25.4 cm (10.0 inch)

$\beta = \pi/2$

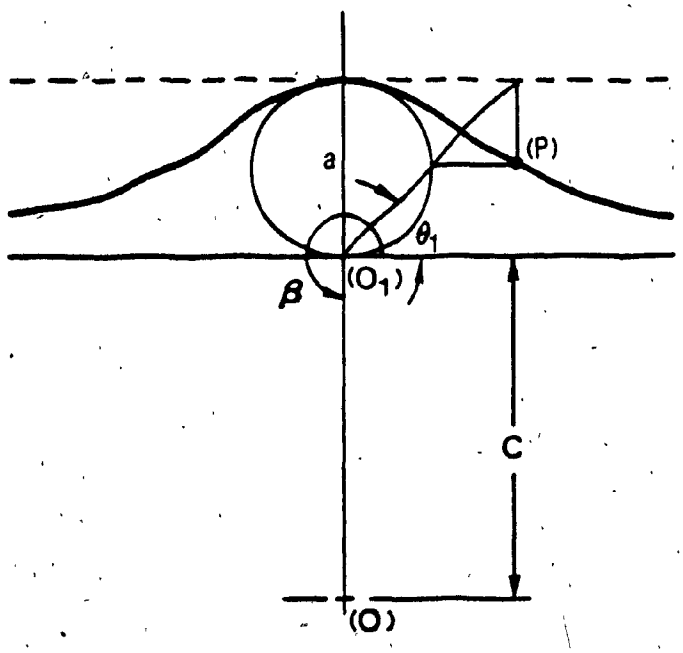


Figure 5.5: Test Contour : Witch of Agnesi.

$r_1 = b + a \cos\theta_1$

a = 5.08 cm (2 inch)

b = 7.62 cm (3 inch)

C = 17.78 cm (7 inch)

$\beta = 3/4$

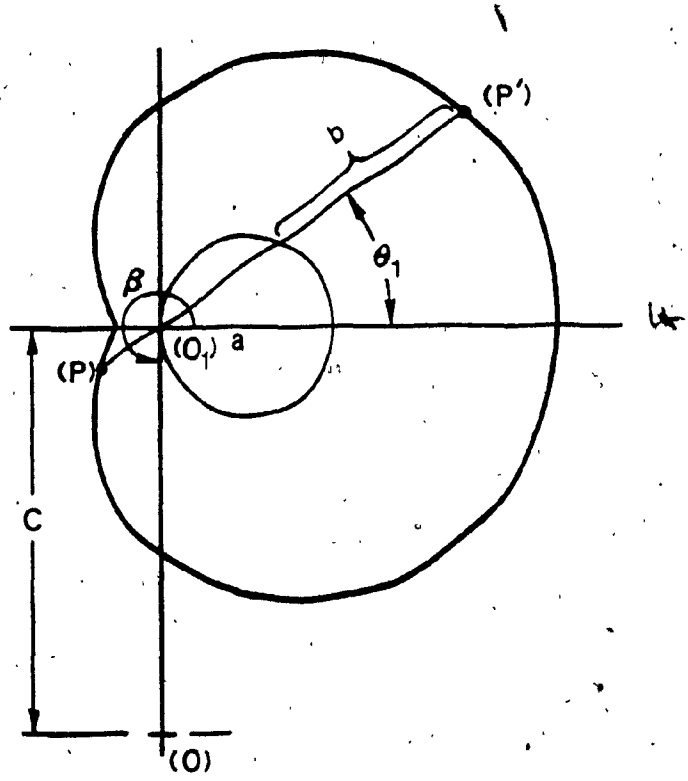


Figure 5.6: Test Contour : Limacon of Pascal.

$$y = \frac{abx}{a^2 + x^2}$$

a = 1.27 cm (0.5 inch)

b = 7.62 cm (3 inch)

C = 17.78 cm (7 inch)

$\beta = 3\pi/4$

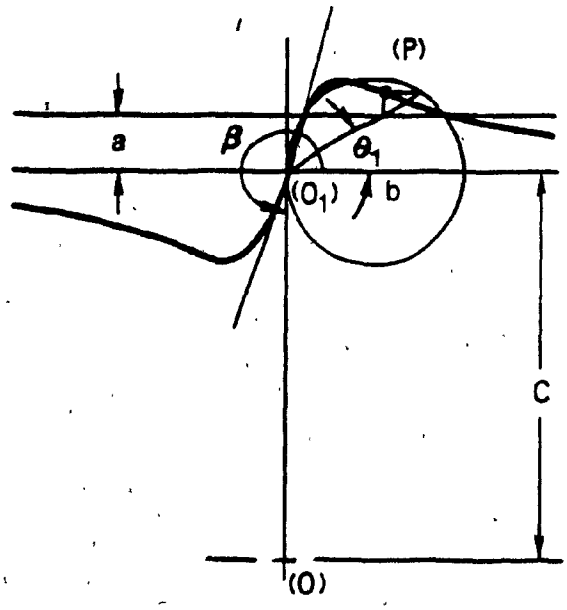


Figure 5.7: Test Contour : Serpentine.

the axes-drives torque. The system parameters and operating conditions are listed in Table 5.1.

5.5.1 Inertial load

For the case of inertial loads, Figures 5.8 to 5.15 show plots of the torque ratios versus the tool location on the contour for different values of mass on the carriage. Plots for two contouring speeds are also shown for each contour. The machine pole in each case is situated as shown in Figures 5.4 to 5.7. Plots for the circular contour will be analyzed first, the reason being that the contour is symmetric around all its axes and has a constant radius of curvature. Because of the aforementioned two features of the circular contour, it is possible to study the isolated effect of the machine load on the torque ratio. All observations concerning the torque ratio plots for the circular contour are common to all the other contours.

In Figure 5.4, the regions of positive and negative values of axes accelerations and velocities for the circular profile are identified and will be frequently referred to subsequently.

Case (a): Circular contour

The plots in Figures 5.8.a and 5.8.b for the circular contour show that for the radial axis, the torque ratios are symmetric about $\theta=\pi/2$. The reduction in the magnitude of the torque ratio while the tool is tracking the part of the

Table 5.1: System/Operating Parameter

System Parameters		
Equivalent Mass of the Carriage	$M_C = 0.9$ kg	0.061 slugs
Equivalent Inertia of the Arm	$J_A = 0.217$ kg m^2	0.16 slug ft^2
Friction Torque due to Arm Support	$F_f L_A = 0.127$ Nm	0.094 lbf.ft
Worm Worm-Gear Factor	$K_1 = 2.78 \times 10^{-2}$	-
Power Screw Split Nut Factor	$K_2 = 2.25 \times 10^{-2}$ m	7.38×10^{-2} ft
Worm Worm-Gear Reduction	$k_\theta = 40:1$	-
Power Screw Split Nut Reduction	$k_R = 7.874$ Rev/cm	20 Revs/in
Stepping Motor Power Supply	$V \triangleq 24$ Volts	-
Stepping Motor Voltage Constant	$K_V = 0.068 \frac{\text{Volts/winding}}{\text{rad/s}}$	-
Stepping Motor Torque Constant	$K_T = 0.0759$ Nm	0.056 lbf ft/amp
Number of Teeth on Stepping Motor Rotter	$N_s = 50$	-
Resistance of Stepping Motor Coil and Drive Circuit	$R_a = 6.5\ \Omega$	-
Inductance of Stepping Motor Coil	$L_a = 6.3 \times 10^{-4}$ Henry	-
Maximum Contouring Speed	$\hat{V}_t = 1.746$ cm/s	3.438 ft/min

Operating Parameters

Inertial Load:

Inertial Load Added to the Mass of the Carriage	M_c = 14.6 kg	1 slug
	29.2 kg	2 slugs
	43.8 kg	3 slugs
Contouring Speed	V_t = 0.51 cm/s	1 ft/min
	1.51 cm/s	3 ft/min

Grinding Application

Grinding Tool Peripheral Speed	V_{tool} = 16.62 m/s	54.5 ft/s
Width of cut	w = 0.635 cm	0.25 inch
Grinding Wheel-Workpiece Force Factor	$k_c = 6.46 \times 10^7$ KN/m^2	1.35×10^9 lbf/ft^2
Normal to Tangential Grinding Force Ratio	c_1 = 1.66	-
Depth of Cut	$d = 0.254 \times 10^{-2}$ mm	0.001 in
	0.508×10^{-2} mm	0.002 in

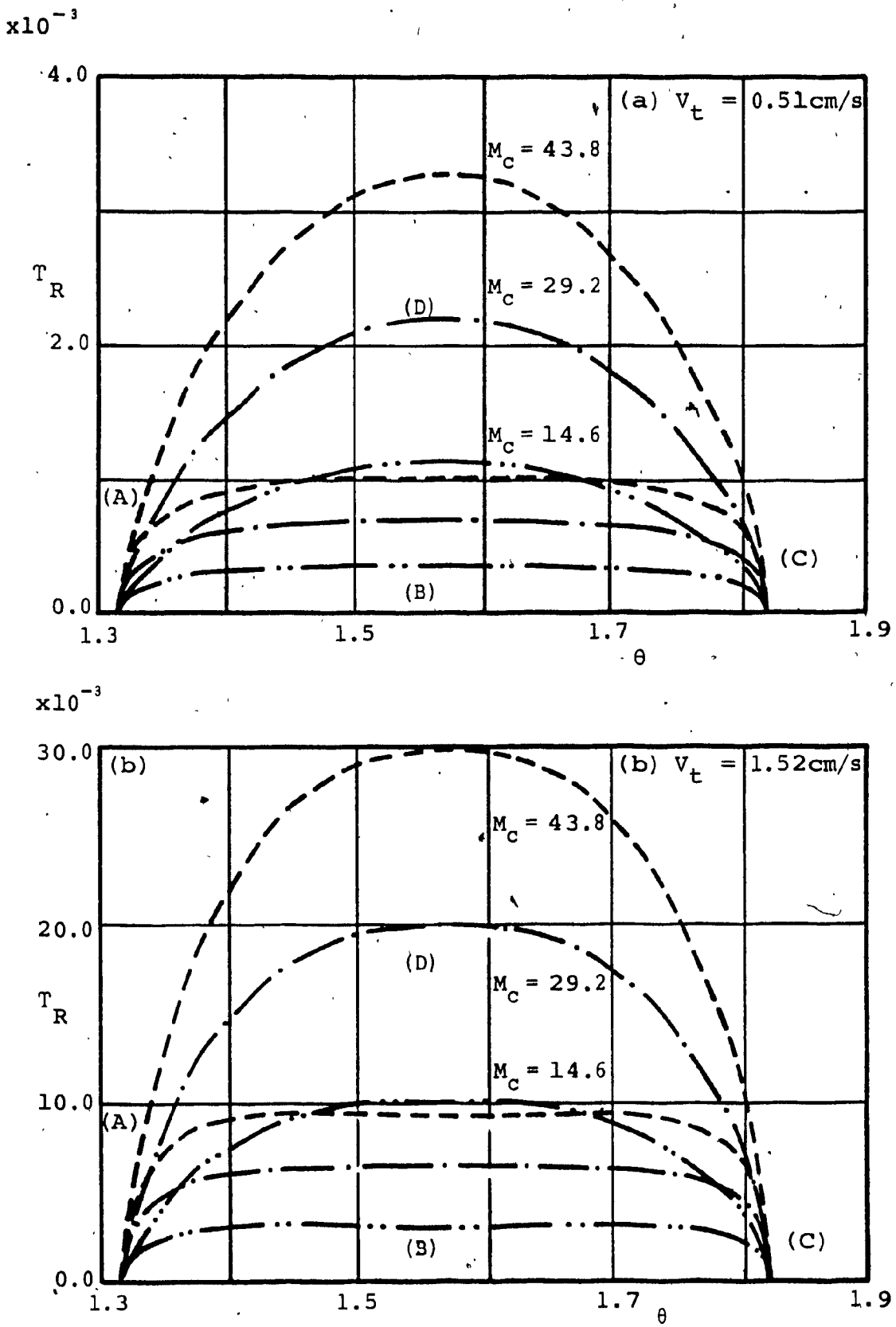


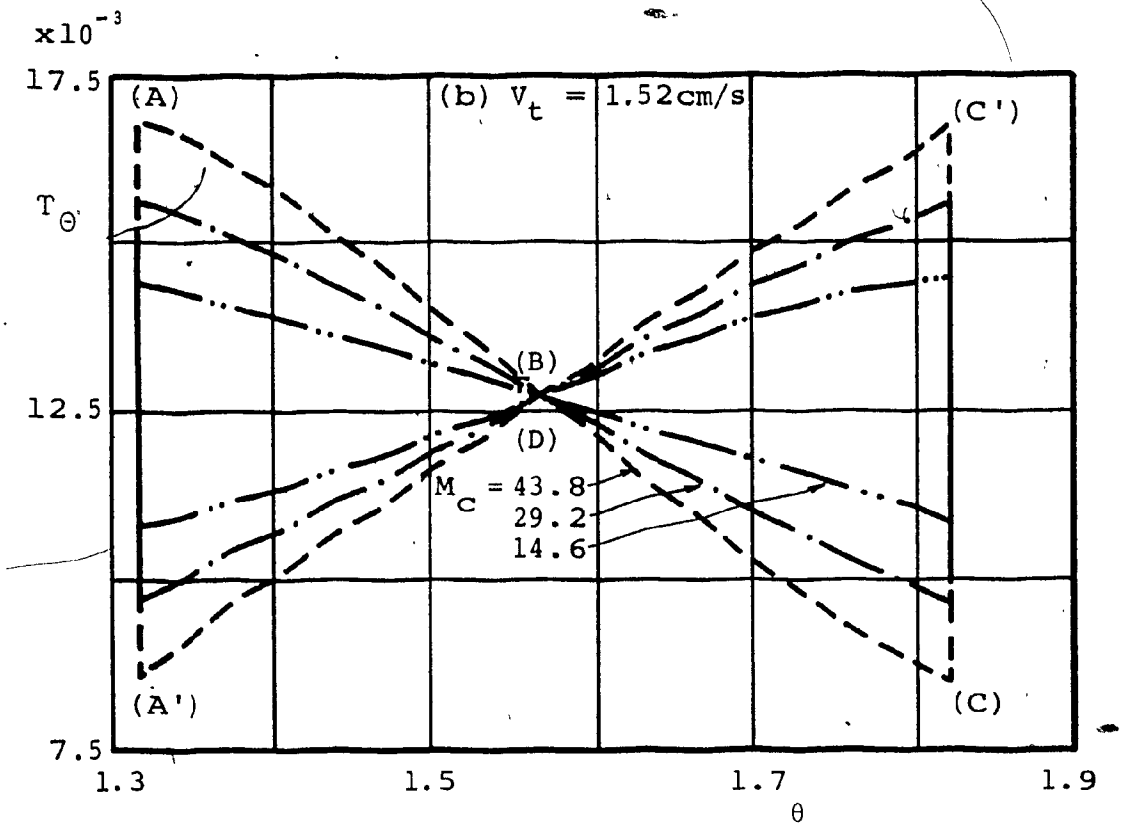
Figure 5.8: Radial Axis Torque Ratio Plots for Circular Contour (Inertial Load). M_c is in kgs.

circle furthest away from the machine pole, ABC, is due to the fact that the inertial force due to the radial acceleration and the centrifugal force are in the same direction as the force due to radial axis stepping motor and thus reduces the torque demanded of the motor (refer to the part ABC of the circle in Figure 5.4). For the part CDA of the circle, the force due to the radial acceleration and the centrifugal force oppose each other, and depending on their magnitudes, they may either reduce or increase the torque demand from the radial axis motor.

At the two extreme positions of the arm, at $\theta = 1.32$ and $\theta = 1.82$ radians, the radial axis torque is zero, since $\ddot{r} = 0$ and $\dot{\theta} = 0$ at the positions indicated A and C as shown in Figure 5.4.

The plots also show that the torque ratio increase with either the increase of the contouring speed or the mass on the carriage.

The plots of the torque ratio for the angular axis, shown in Figure 5.9.a and 5.9.b exhibit similar characteristics of an increase in the peak value of the torque ratio for the increase of either the contourng speed or the mass on the carriage. The lack of symmetry about $\theta = \pi/2$ is attributed to the change in the direction of the Coriolis acceleration due to the change in the direction of the radial velocity in the four segments of the circle. Furthermore, in both figures the torque ratio plots are biased away in the torque ratio axis by a constant amount. This shift is



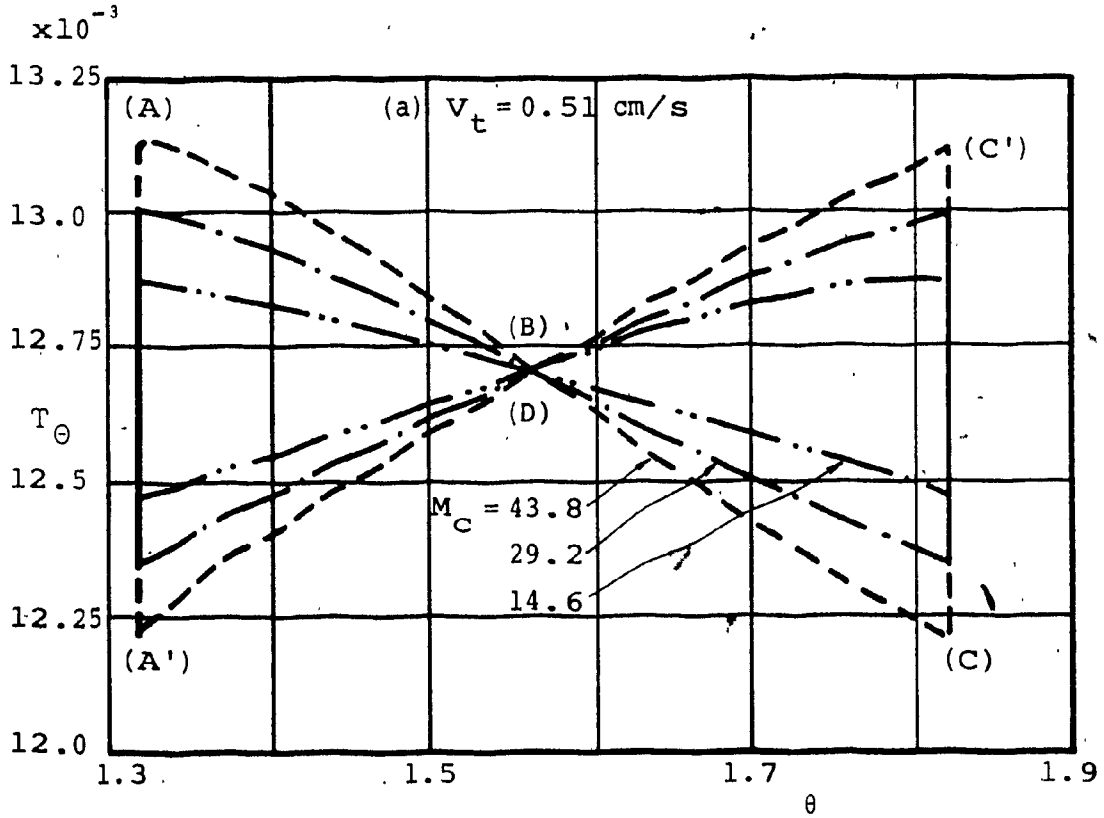


Figure 5.9: Angular Axis Torque Ratio Plots for Circular Contour (Inertial Load). M_c is in kgs.

due to the friction torque resulting from the arm support and is comparable to the inertial torque at low speeds. This bias causes the torque ratio for the angular axis to decrease for the circular parts BC and DA and increase for the parts AB and CD with the increase of either the contouring speed or the mass on the carriage. The "node" seen on the plots at $\theta = \pi/2$ occurs due to zero inertial torque at that point. This is due to vanishing transverse acceleration of the carriage at that point resulting from zero values for the radial velocity and angular acceleration. For both low and high contouring speeds, the magnitude of the torque ratio is the same at the location of the node. The sudden change in the magnitude of the torque ratios at the extreme positions of the arm results due to the Coulomb friction force of the arm support when the arm changes direction of motion.

Case (b): Witch of Agnesi contour

The torque ratio plots for the Witch of Agnesi in Figures 5.10 and 5.11 are symmetric about $\theta = \pi/2$ for the radial axis, but are asymmetric for the angular axis. This asymmetry is due to the change in the direction of the Coriolis acceleration. The vanishing effect of the inertial torque on the angular axis can be seen at $\theta = \pi/2$ in Figure 5.11, and results in a node at which all plots for different masses on the carriage pass. The magnitude of the torque ratio at the node corresponds to the friction torque

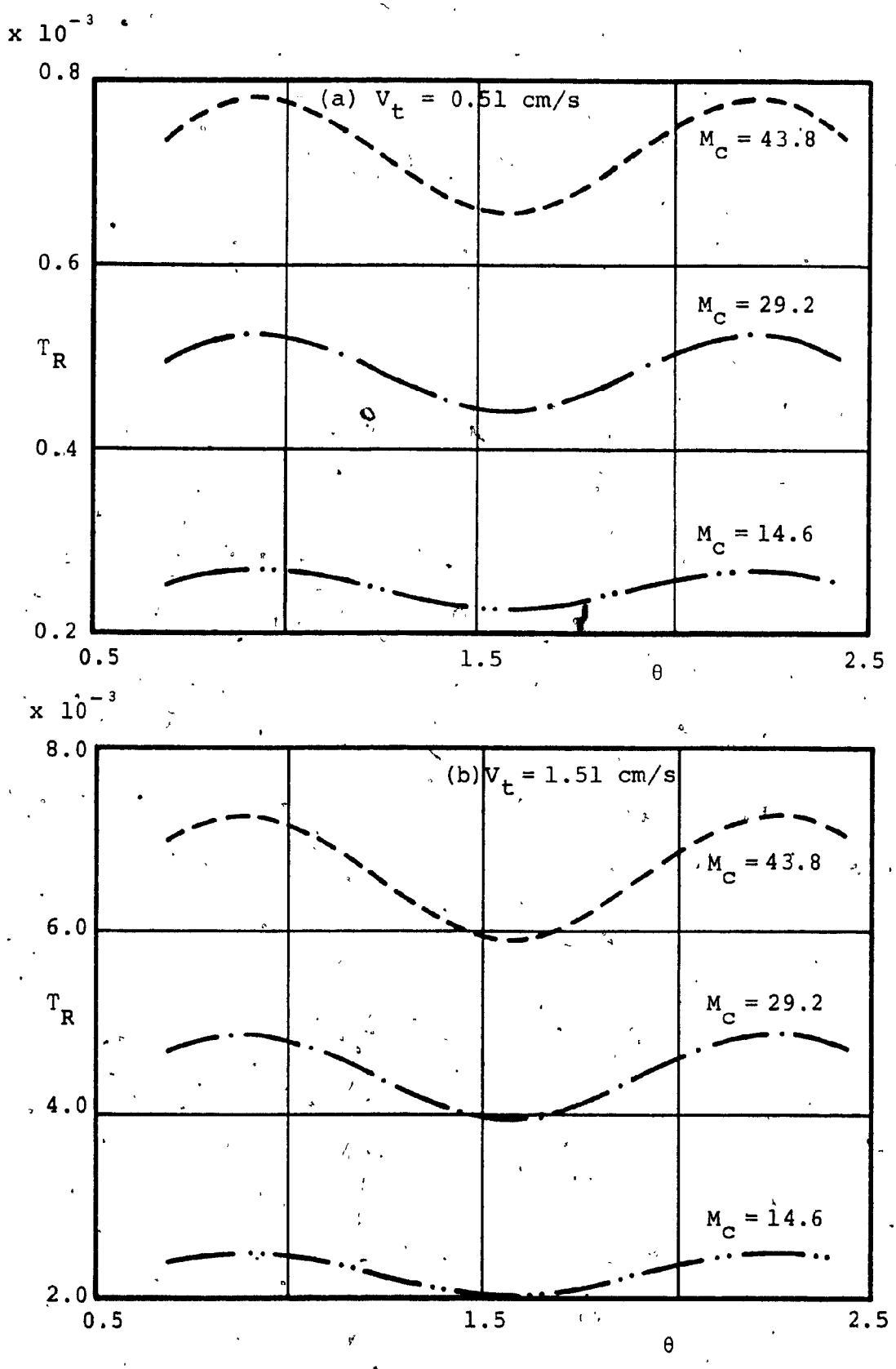


Figure 5.10: Radial Axis Torque Ratio Plots for Witch of Agnesi Contour (Inertial Load). M_C is in kgs.

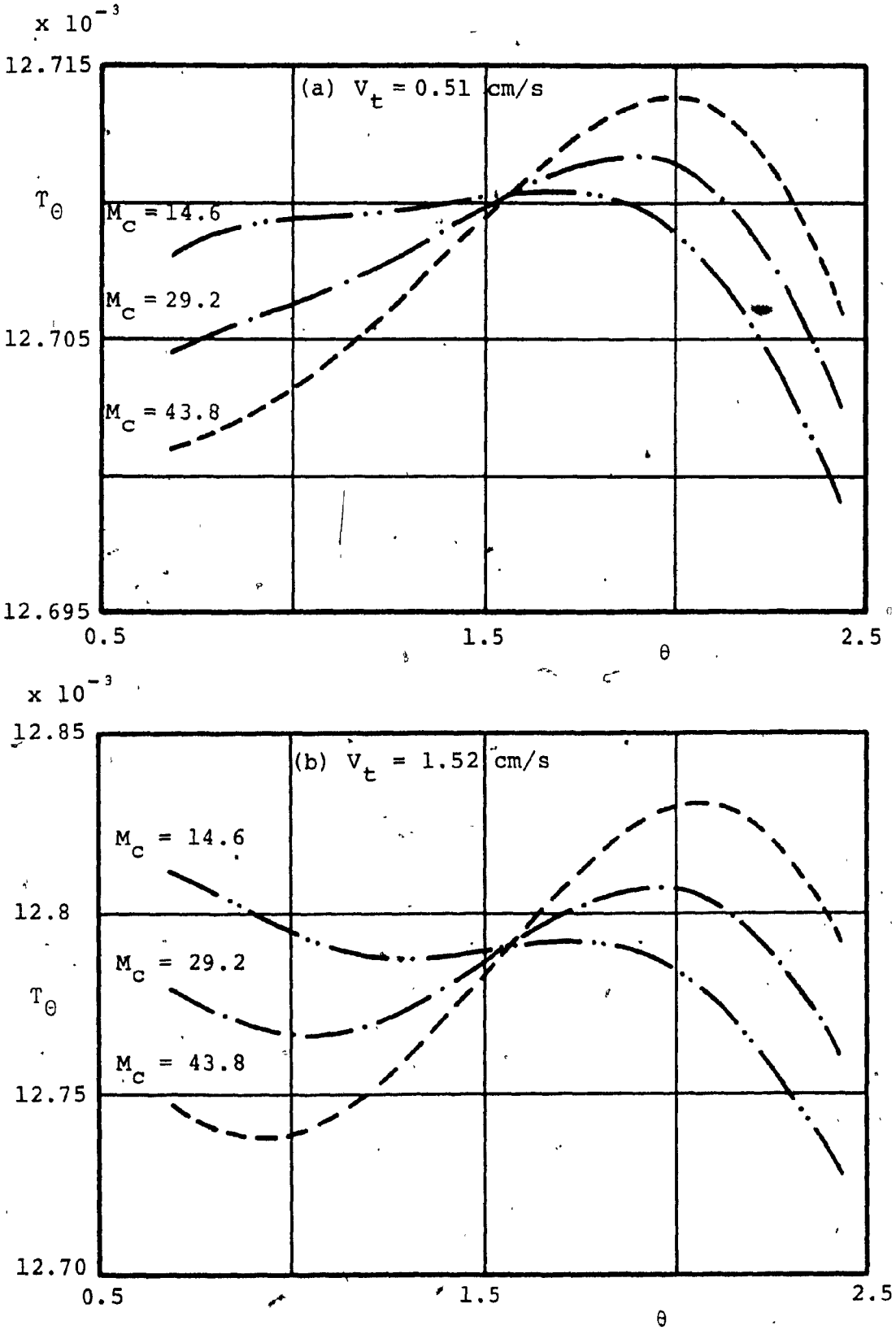


Figure 5.11: Angular Axis Torque Ratio Plots for Witch of Agnesi Contour (Inertial Load). M_c is in kgs.

of the arm support.

The torque ratios for this contour are smooth due to the slow and gradual change of both γ and the radius of curvature along the contour. Since the arm did not change direction while the tool is tracking the contour, there is no sudden jump in the torque ratios that would result due to the Coulomb friction in the arm support.

Case (c): Limacon of Pascal contour

In the case of the Limacon of Pascal, the plots shown in Figures 5.12 and 5.13 are similar to those of the circular contour. Both the radial and angular axes plots, however, show some small jumps between $\theta = 1.67$ and $\theta = 1.72$ radians due to the change in radial acceleration which results from the indentation in the curve at $\theta_1 = \pi$ (refer to Figure 5.6). The angular axis plots show the same disturbances at the above mentioned angles, the disturbances here however, are more accented by the Coulomb effect of the support friction force. The plots here are slightly lopsided and shifted from $\theta = \pi/2$ as a result of the relative location between the machine pole and the contour pole as can be seen in Figure 5.6.

Case (d): Serpentine contour

For the Serpentine profile, the torque ratios experience a sharp change in magnitudes at approximately $\theta = 1.5$ and $\theta = 1.66$ radians as can be seen from the plots in

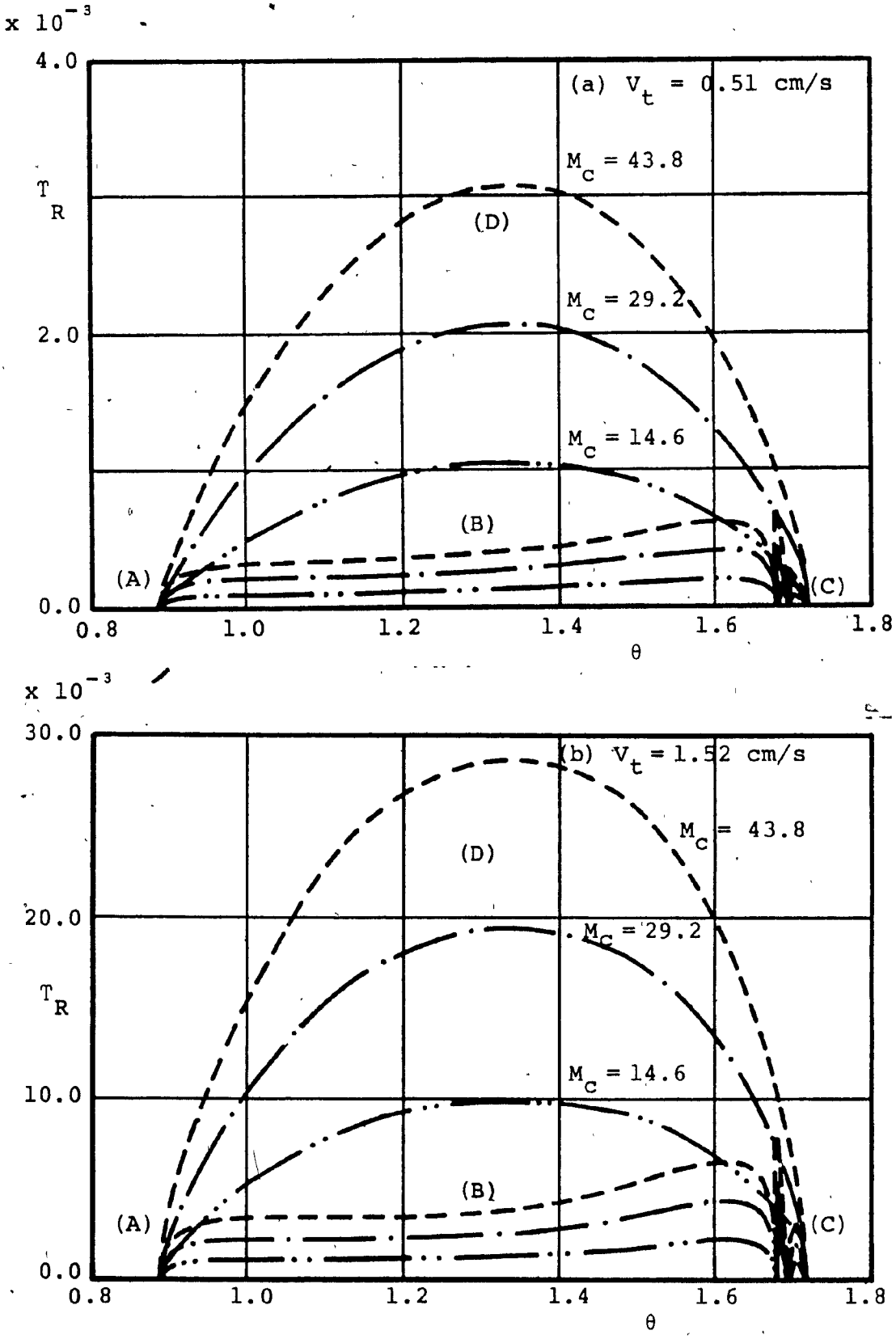


Figure 5.12: Radial Axis Torque Ratio Plots for Limacon of Pascal Contour (Inertial Load). M_c is in kgs.

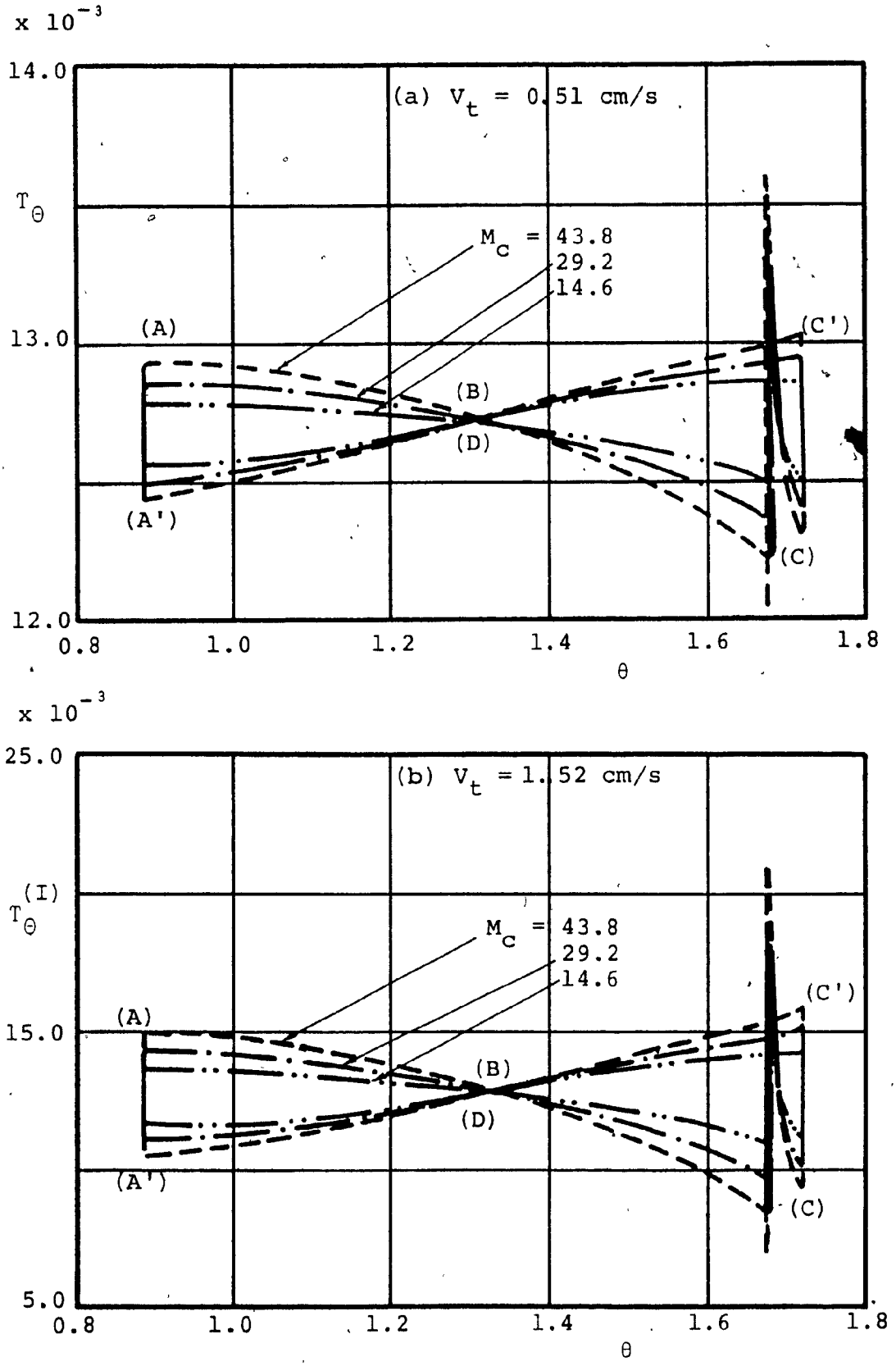


Figure 5.13: Angular Axis Torque Ratio Plots for Limacon of Pascal Contour (Inertial Load). M_c is in kgs.

Figures 5.14 and 5.15. The sharp change results when the tool leaves the smooth part of the curve and starts tracking that part of the curve that has an asymptote inclined at approximately 1.4 radians to the contour axis. The transition between one part of the curve to the other is fast, that is to say, the change in γ and in the radius of curvature is fast, and causes a sharp increase in the magnitude of both the radial and angular accelerations, and hence the required torque.

The torque ratios for both axes increase with either the increase in contouring speed or mass on the carriage. The increase due to the mass on the carriage, however, is not apparent from the plots in Figures 5.14 and 5.15 since all the curves for the different cases coincide.

The torque ratio plots, in Figures 5.15.a and 5.15.b, for the angular axis show that for the major portion of the contour the torque ratio is more or less the same. The torque ratio at each of the two spikes (at θ=1.5 and θ=1.66 radians) changes magnitude from above the nominal value to below it, when the motor decelerates the arm and carriage rapidly. This is caused by the decrease of the angular acceleration when the tool is tracking the asymptote.

5.5.2 Grinding application

For the case of a grinding application the magnitude and direction of the cutting force F_c and α_c respectively are given in References [34] and [35] by:

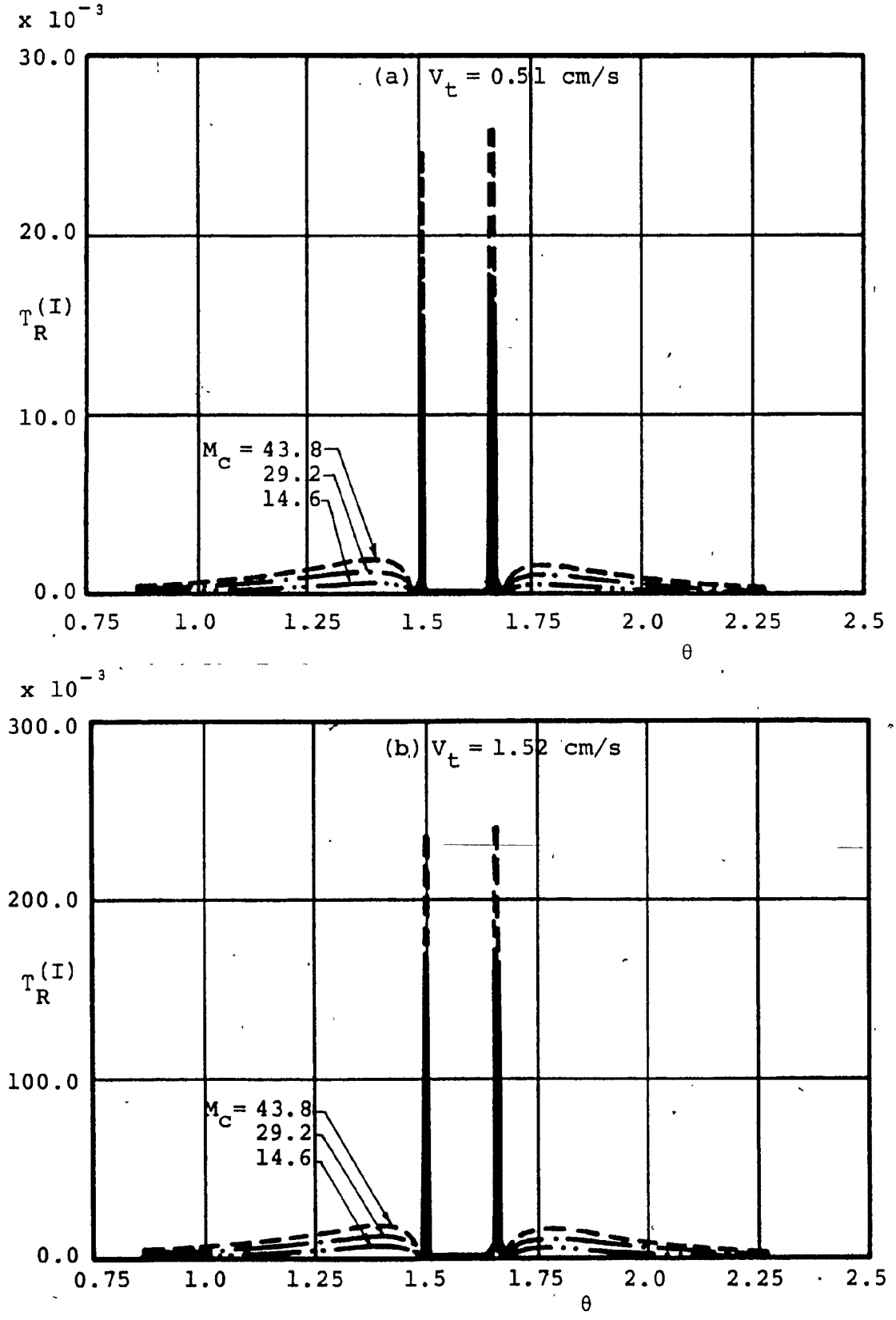


Figure 5.14: Radial Axis Torque Ratio Plots for Serpentine Contour (Inertial Load). M_c is in kgs.

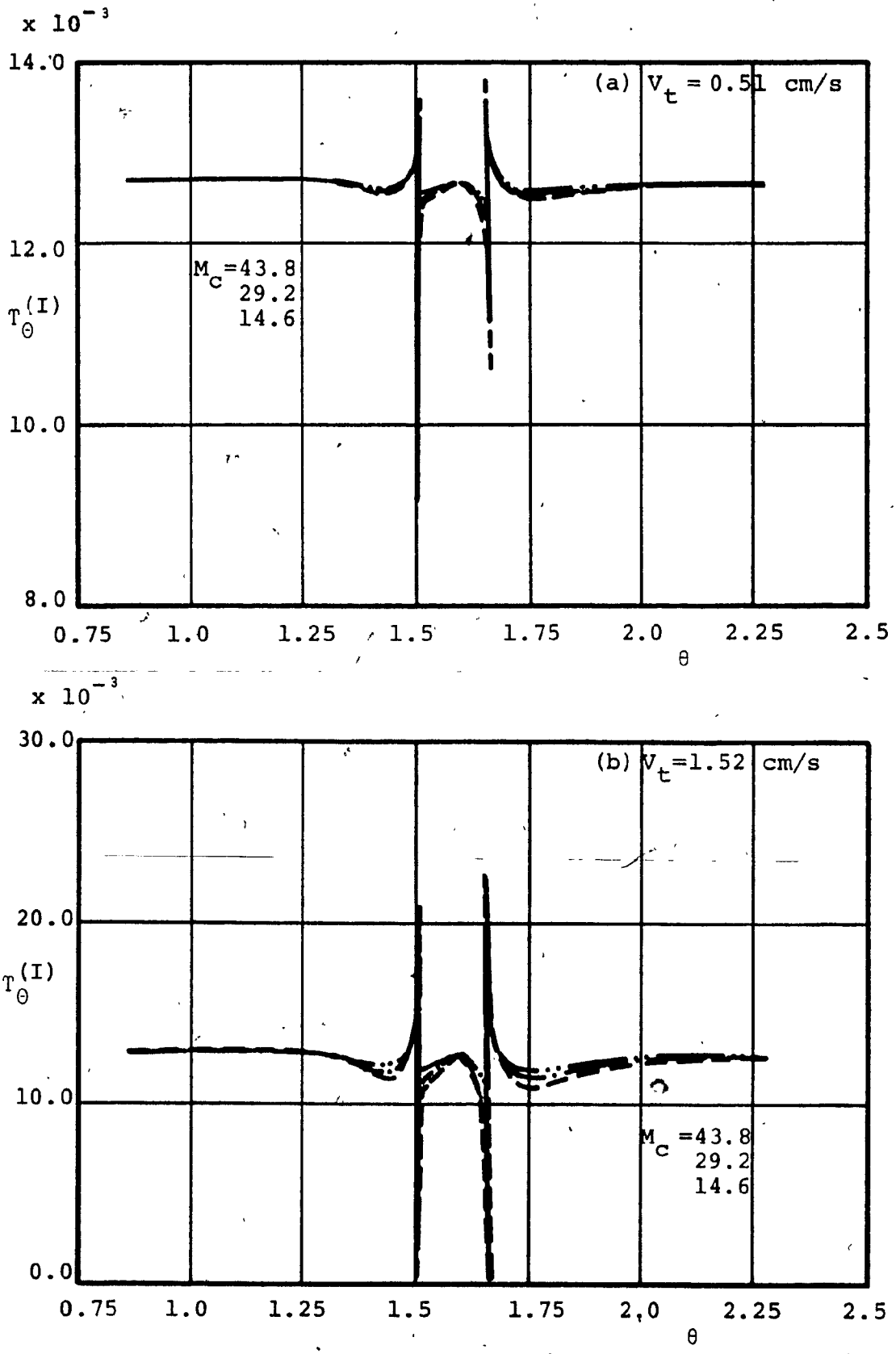


Figure 5.15: Angular Axis Torque Ratio Plots for Serpentine Contour (Inertial Load). M_c is in kgs.

$$F_c = K_c V_t \qquad 5.22$$

$$\alpha_c = \tan^{-1} (c_1) \qquad 5.23$$

$$K_c = k_c [1 + c_1^2]^{\frac{1}{2}} \frac{dw}{V_{tool}}$$

where α_c is the angle of inclination of the cutting force to the surface,

c_1 is the ratio of the normal to tangential forces to the surface,

k_c is a constant that depends on the materials of the workpiece and the grinding wheel,

d is the depth of cut,

w is the width of cut,

and V_{tool} is the peripheral velocity of the grinding wheel.

The ratio c_1 varies between 1.5 and 3 [34] (pp. 548), with larger values corresponding to larger depth of cut and higher grinding wheel peripheral velocity.

The plots for the torque ratios while grinding the four contours are shown in Figures 5.16 to 5.19. In each figure, the torque ratios are plotted for different feed rates.

Case (a): Circular contour

Figure 5.16.a shows that for a circular contour, the

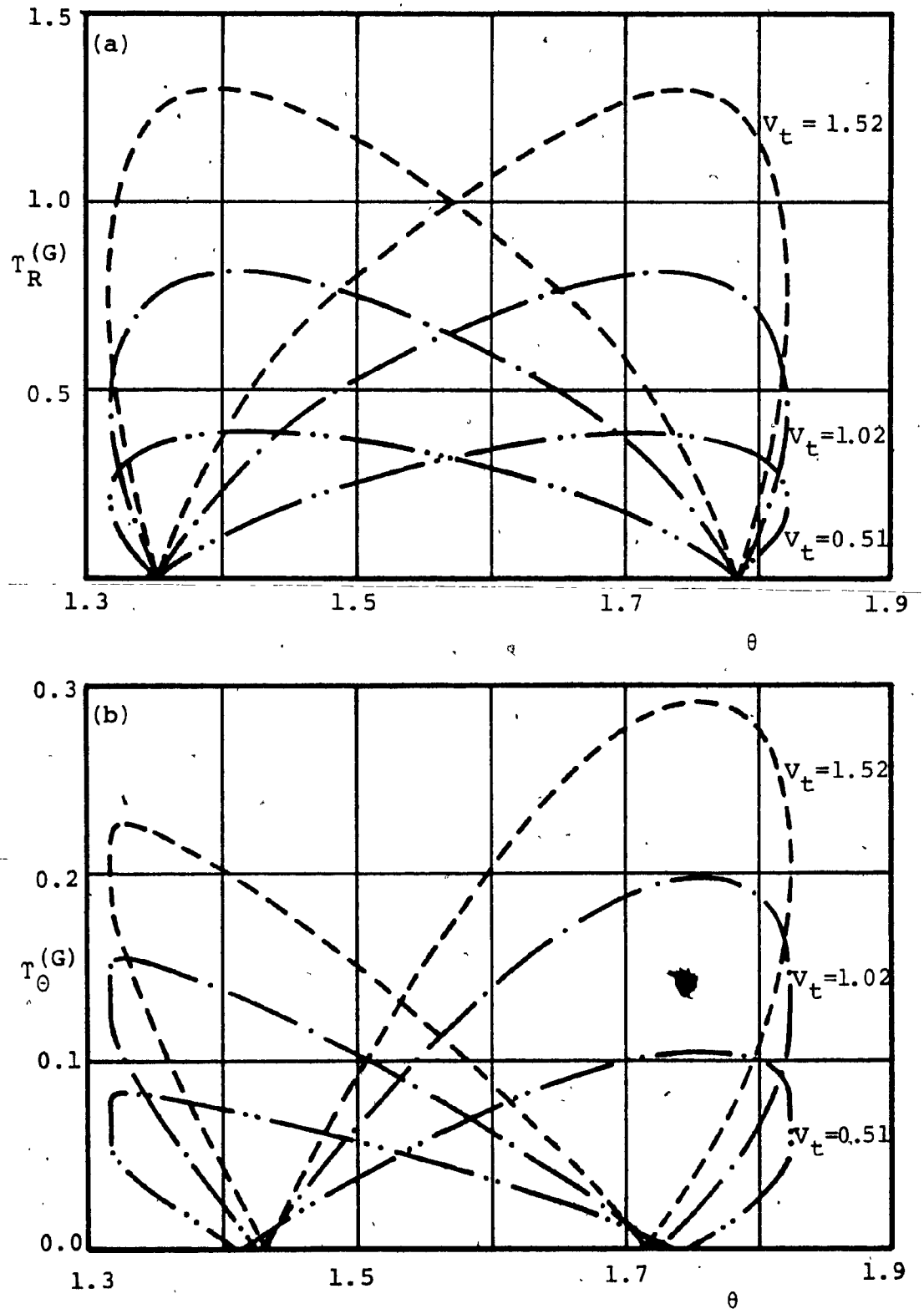


Figure 5.16: Torque Ratio Plots for the Circular Contour. (Grinding Application). V_t is in cm/s.

torque ratio in the radial axis have no symmetry similar to the case of the inertial load. This is due to the dominance of the cutting force over the inertial force acting on the mechanism, and due to the inclination α_c of this force on the tangent to the contour. The locations where the torque ratios are equal to zero are shifted from the maximum swing angles of the arm at θ=1.327 and θ=1.824 as can be seen from the figure. This shift which does not occur in the case of inertial application, Figures 5.8.a and 5.8.b, results due to the inclination α_c of the cutting force on the contour tangent. For a contouring speed of 1.52 cm/s (3 ft/min) the torque ratio exceeds unity and indicates that for the machine settings corresponding to this plot, the contouring operation will fail.

For the angular axis plots in Figure 5.16.b, the torque resulting from the cutting force dominates over both the inertial and friction torques. The friction torque however, can still be seen at θ=1.327 and θ=1.824 radians as a small jump in the torque ratio values.

The positions where the angular axis torque ratio is equal to zero, shift along the θ-axis with the change in the contouring speed as can be observed around θ=1.46 and θ=1.67 radians in Figure 5.16.b. This occurs at the points of equilibrium between the inertial force and the sum of the cutting and frictional forces.

The plots in Figure 5.16 also show that the maxima of the radial axis torque ratio and the minima of the angular

axis torque ratio, and vice-versa, occur at almost the same angular locations. At these locations, the dominant cutting force is either aligned with the arm (for maximum radial torque ratio) or is perpendicular to it (for maximum angular torque ratio).

Case (b): Witch of Agnesi contour

For the Witch of Agnesi contour, Figure 5.17.a shows that for the radial axis plots, the torque ratios are asymmetric about $\theta=\pi/2$ (unlike the case of the inertial load in Figure 5.11). This is due to the inclination α_c of the cutting force to the contour tangent. For a contouring speed of 1.52 cm/s (3 ft/min), the torque ratio exceeds unity, thus indicating contouring failure. The absence of the node (refer to the case of an inertial load) is also attributed to the dominance of the cutting force over the inertial and friction forces and due to its inclination to the contour surface.

The plots in Figure 5.17 show that the peak value of the torque ratio in one axis occurs at the same location where the minimum value of the torque ratio for the other axis occurs.

Case (c): Limacon of Pascal contour

The Limacon of Pascal plots in Figure 5.18 bear no resemblance to those of the inertial application in Figures 5.12 and 5.13. The plots of the angular axis show

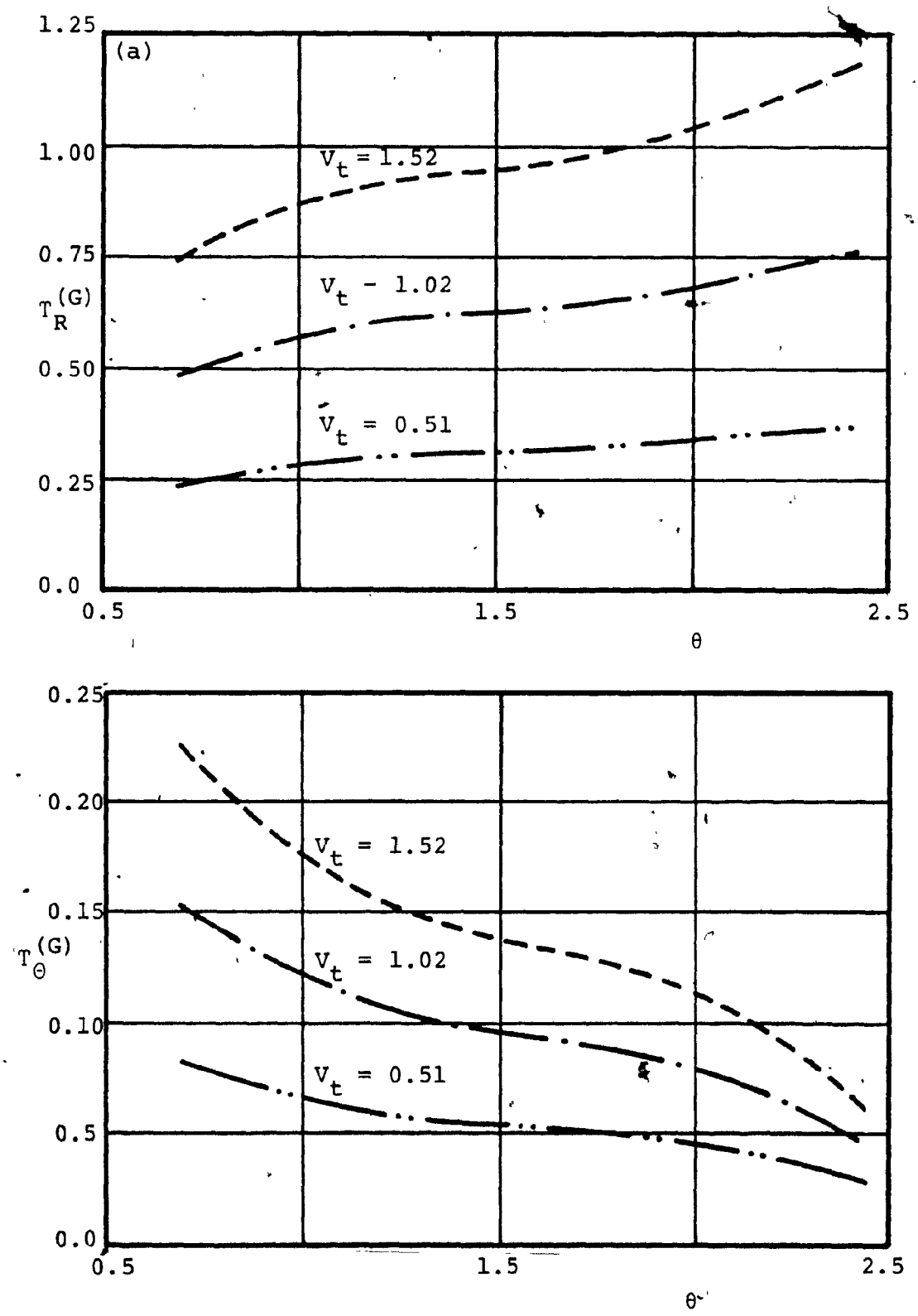


Figure 5.17: Torque Ratio Plots for the Witch of Agnesi. (Grinding Application). V_t is in cm/s.

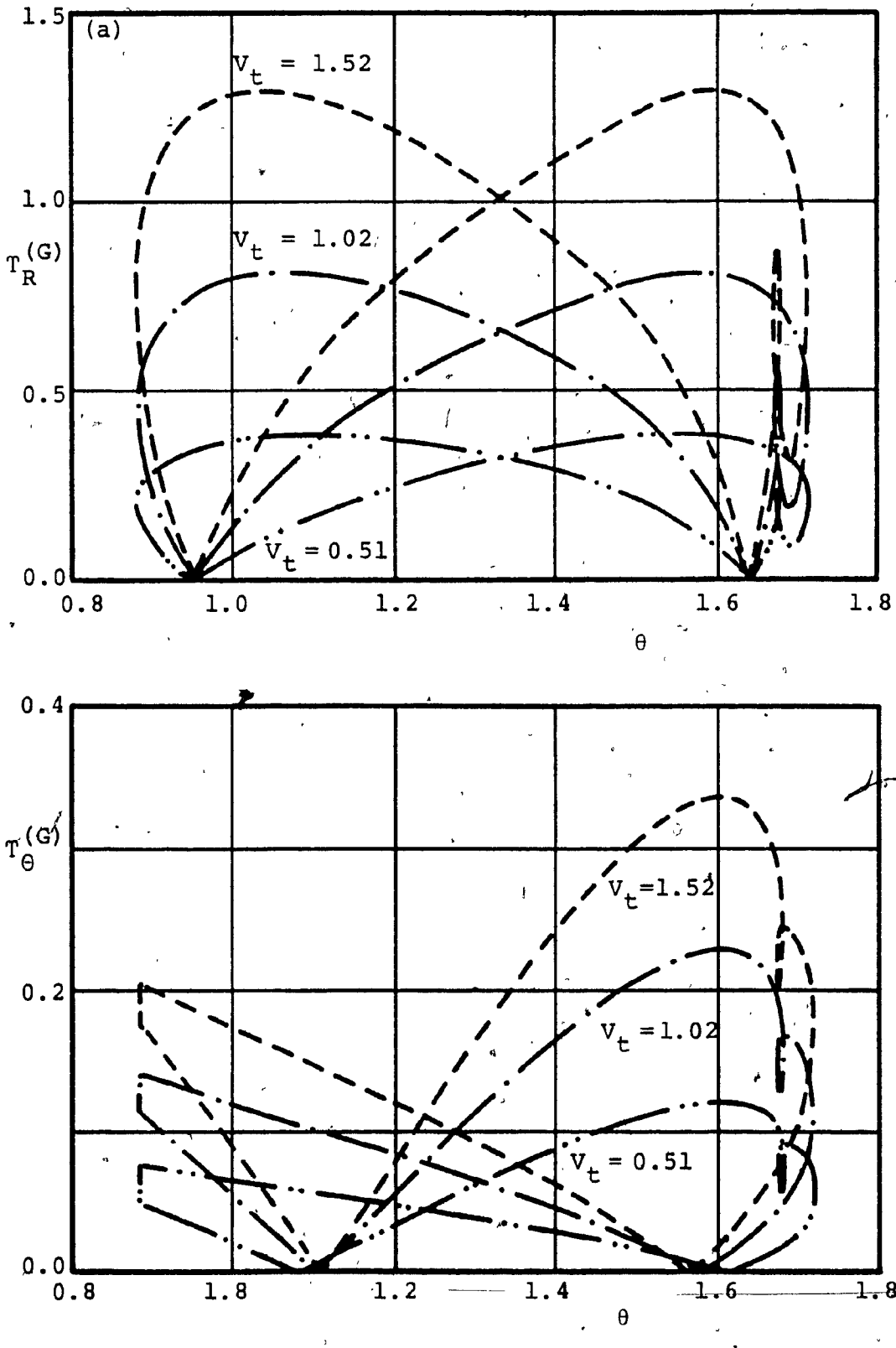


Figure 5.18: Torque Ratio Plots for Limacon of Pascal. (Grinding Application). V_t is in cm/s.

the effect of the Coulomb friction at θ=0.88, θ=1.67, and θ=1.72. Also the positions where the torque ratio is equal to zero, change with the change in contouring speed. The effect of the indentation in the contour is apparent in both plots between θ=1.67 and θ=1.72.

Similar to the previous two cases, the torque ratio plots show that the peaks on one axis coincide in location with that of the minimum values on the other axis.

Case (d): Serpentine contour

Figure 5.19 show the torque ratio plots for the Serpentine curve. Both plots for the radial and angular axes show large jump in the magnitudes of the torque ratio at θ=1.5 and θ=1.66 radians. These jumps occur at the same angular orientation as in the case of the inertial load.

For this contour also, the peaks of the torque ratio on one axis coincide in location with that of the minimum values on the other axis.

5.6 Effect of relative location between the contour pole and the machine pole on the torque ratios

In the study outlined in Section 5.5 the results are presented for several contours with the contour pole located at a particular point with respect to the machine pole. The radial and angular kinematic relations, and consequently the axes torques, have been shown to depend upon the contour as well as the relative location between the contour pole and

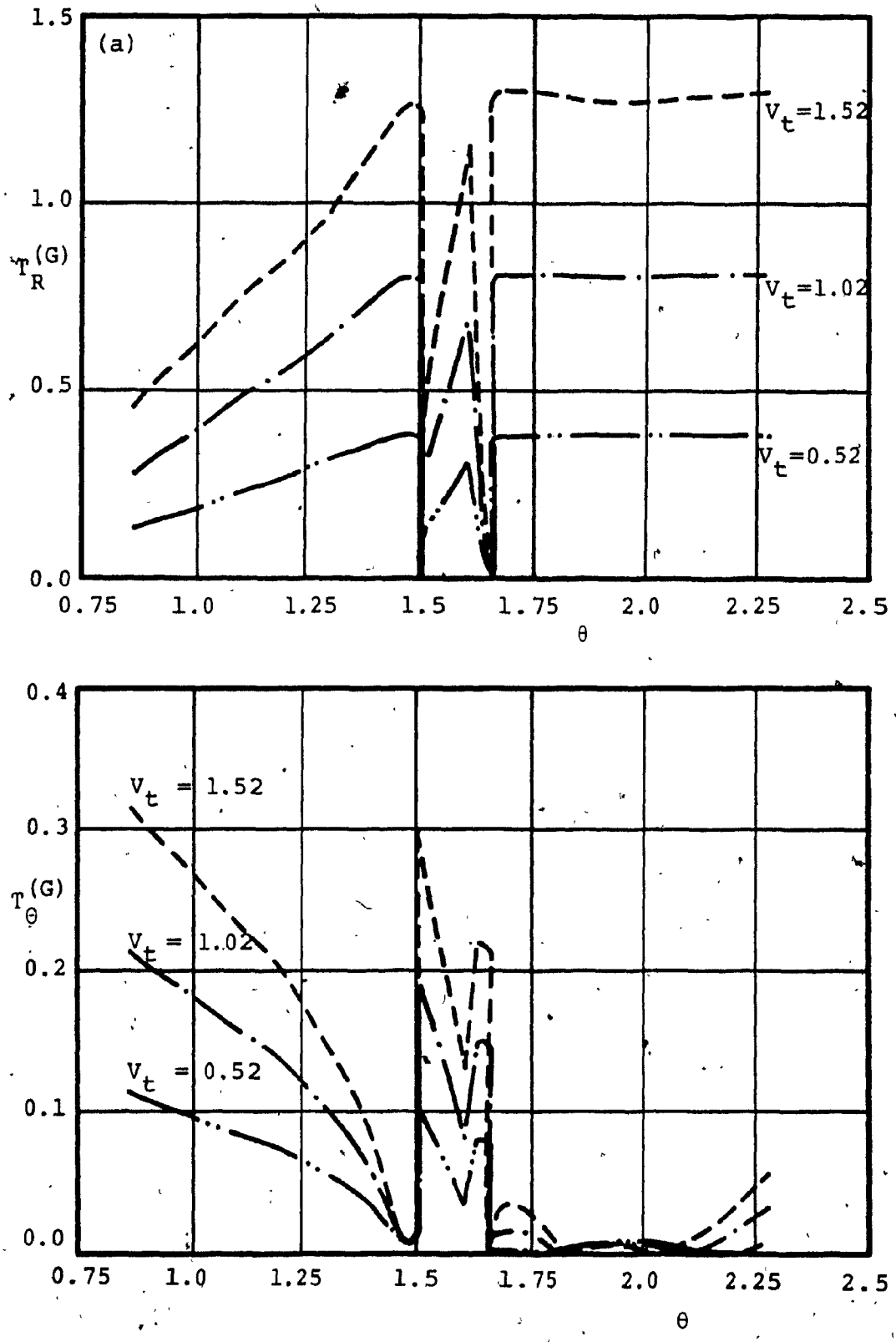


Figure 5.19: Torque Ratio Plots for Serpentine. (Grinding Application). V_t is in cm/s.

the machine pole as evident from Equations 5.2 to 5.10. Hence it is necessary to study the effect of the relative location between the contour pole and the machine pole on the torque ratios. For this purpose, the analysis was carried out for both the inertial load and the grinding operation with the machine pole located at a point where the contouring error is minimum. It should be noted that this location for each contour is the same as that found in the study in Chapter 5. This study is conducted only for the Witch of Agnesi, the Limacon of Pascal, and the Serpentine contours.

5.6.1 Inertial load

Figures 5.20 to 5.22 show the results for the case of inertial load.

For the Witch of Agnesi contour, Figure 5.20, the radial axis plots show some resemblance to those of Figure 5.10.b. The magnitude of the torque ratio, however, is lower. The angular axis plots show that the torque ratio decreases with the increase of mass on the carriage. The relative position of the contour pole with respect to the machine pole is such that the torque due to the carriage acceleration in the transverse direction, a_t, decreases with the increase in the mass on the carriage.

The plots in Figure 5.21 show that for the Limacon of Pascal, the torque ratios are considerably larger with the machine pole at the minimum area error location than those

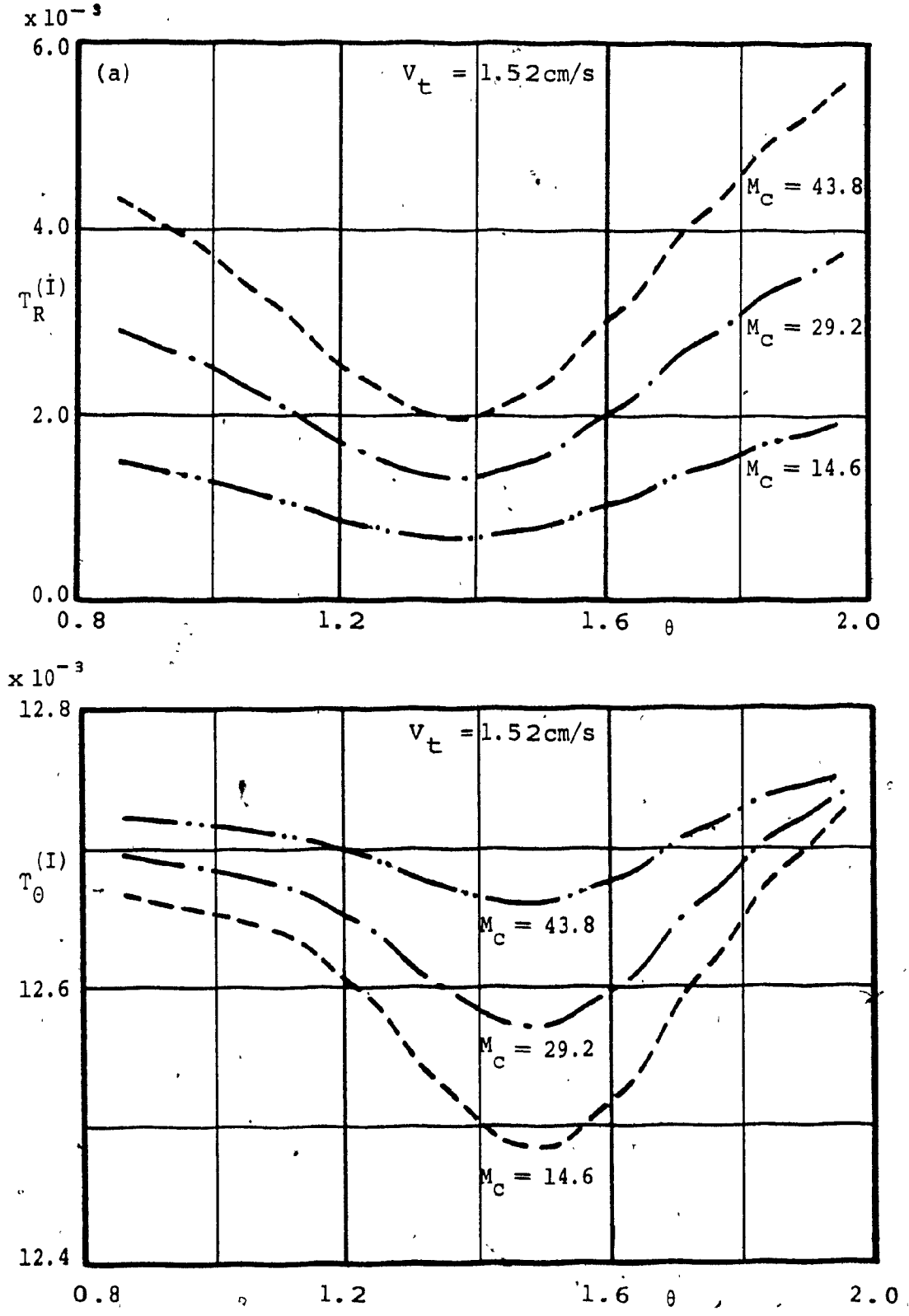


Figure 5.20: Torque Ratio Plots for Machine Pole at the Location of Minimum Area Error: Witch of Agnesi Contour (Inertial Load). M_c is in kgs.

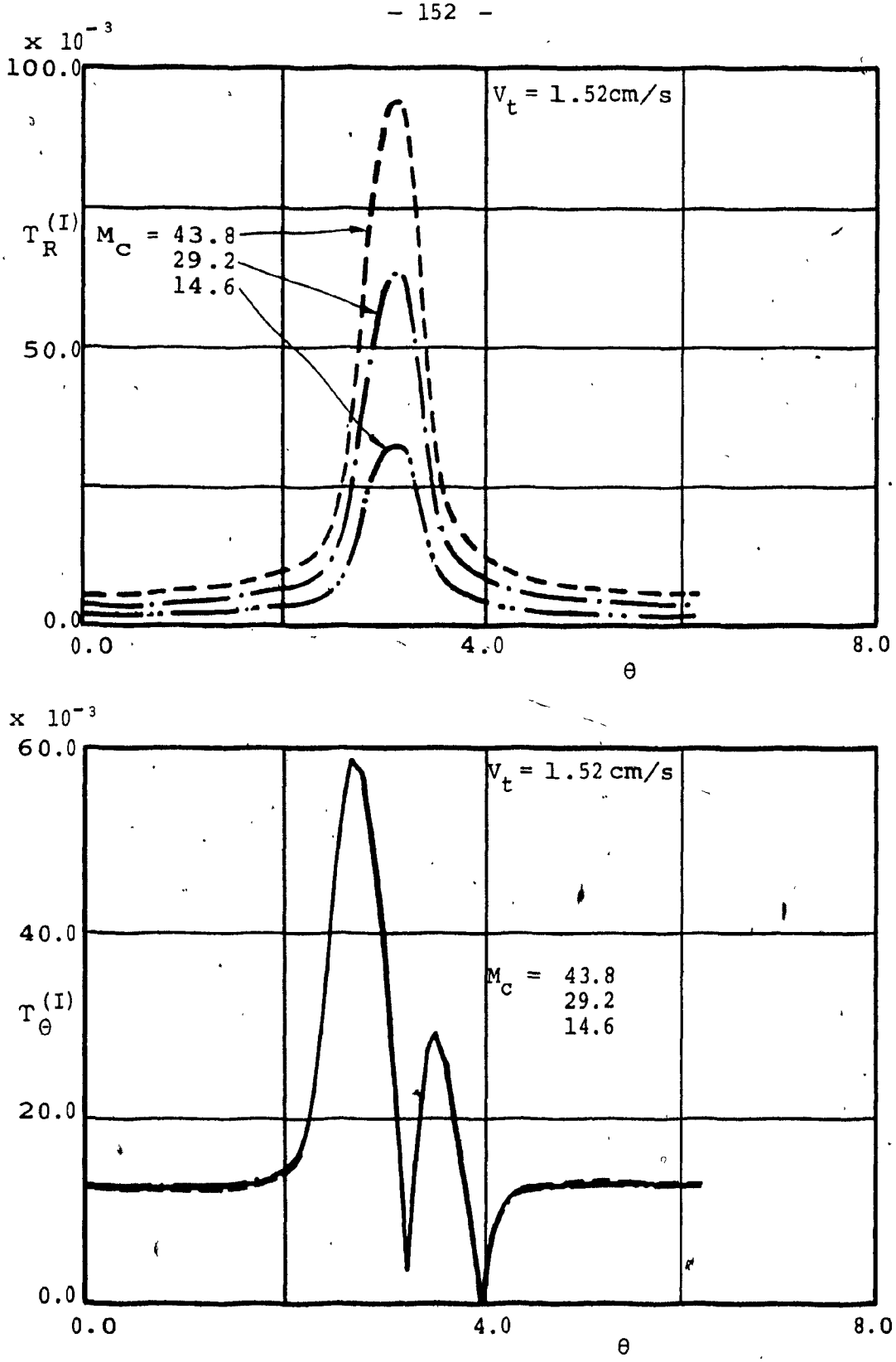


Figure 5.21: Torque Ratio Plots for Machine Pole at the Location of Minimum Area Error: Limacon of Pascal Contour (Inertial Load). M_c is in kgs.

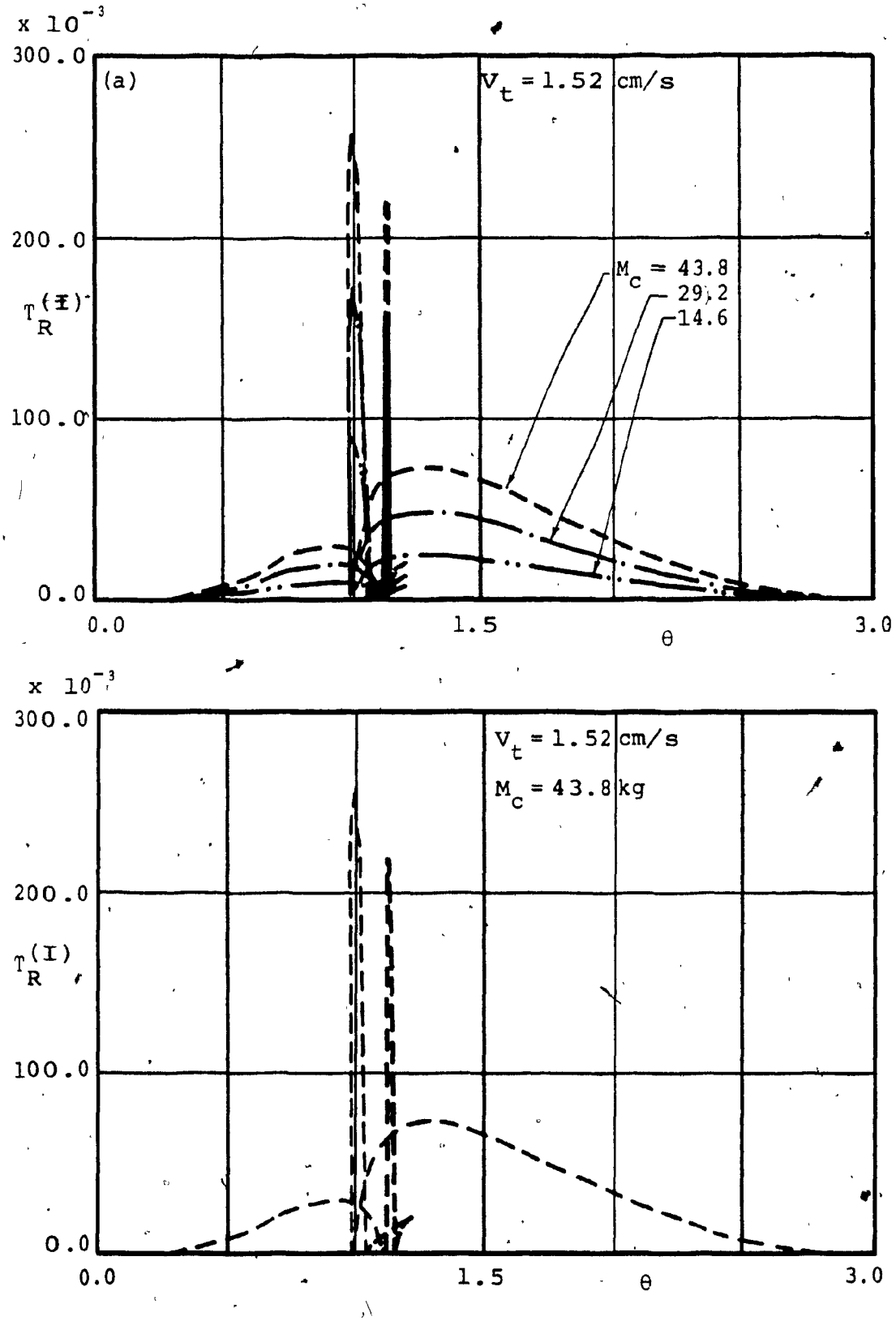


Figure 5.22: Torque Ratio Plots for Machine Pole at the Location of Minimum Area Error: Serpentine Contour (Inertial Load). M_c is in kgs.

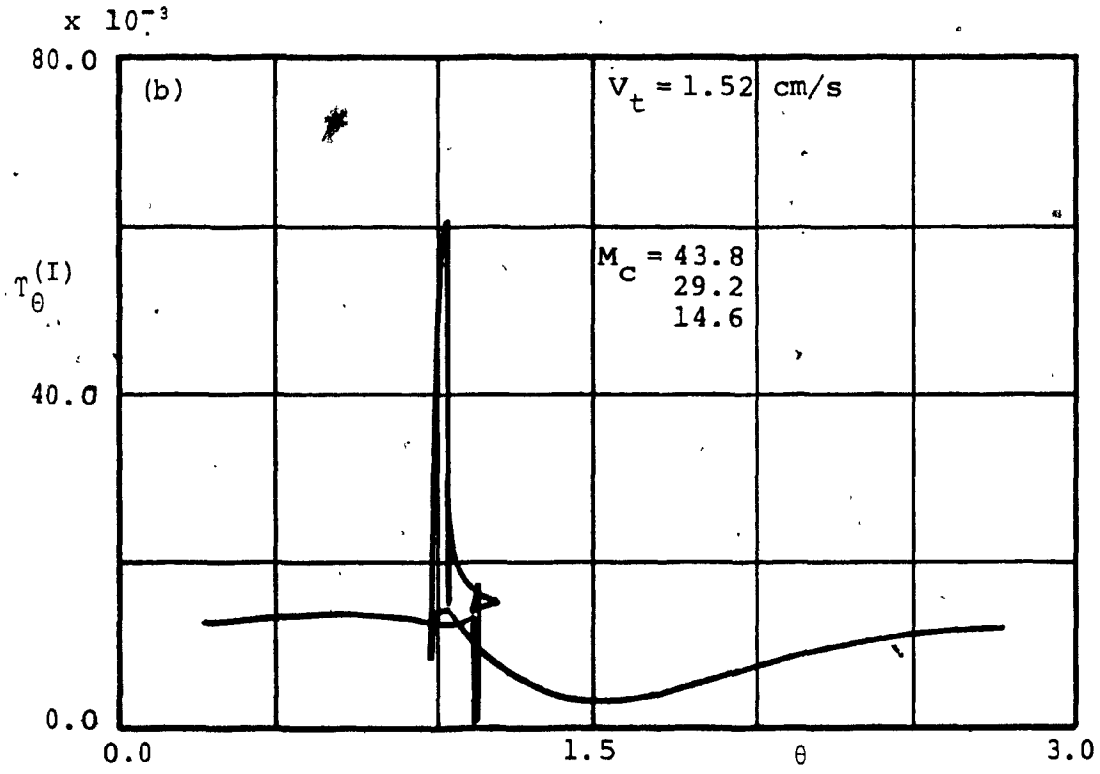


Figure 5.22: Torque Ratio Plots for Machine Pole at the Location of Minimum Area Error: Serpentine Contour (Inertial Load). M_c is in kgs.

in Figures 5.12.b and 5.12.d. The machine pole in this case is located within the contour and hence the angular orientation in Figure 5.21 extend from $\theta=0$ to $\theta=2\pi$. Furthermore, the shape of the plots bears no resemblance to those in the previous study.

The Serpentine contour plots in Figure 5.22 show that the radial axis torque ratio does not seem to be affected by the change in mass on the carriage. However, the angular axis torque ratio is considerably larger than that in Figure 5.15.b.

5.6.2 Grinding application

For the case of a grinding application the torque ratio plots are shown in Figures 5.23 to 5.25 for the three contours.

For the Witch of Agnesi contour, Figure 5.23, locating the machine pole at the position of minimum area error causes the maximum value of the radial axis torque ratio not to exceed unity for the contouring speed of 1.52 cm/s (3 ft/min), contrary to the earlier case presented in Figure 5.17.a. The maximum value of the angular axis torque ratio, however, has a larger value than that of the previous case (Figure 5.17.b).

The torque ratio plots in Figure 5.24 for the Limacon of Pascal contour show that the magnitude of the maximum radial axis torque ratio remains essentially unchanged from those in the previous case shown in Figure 5.18.a. However,

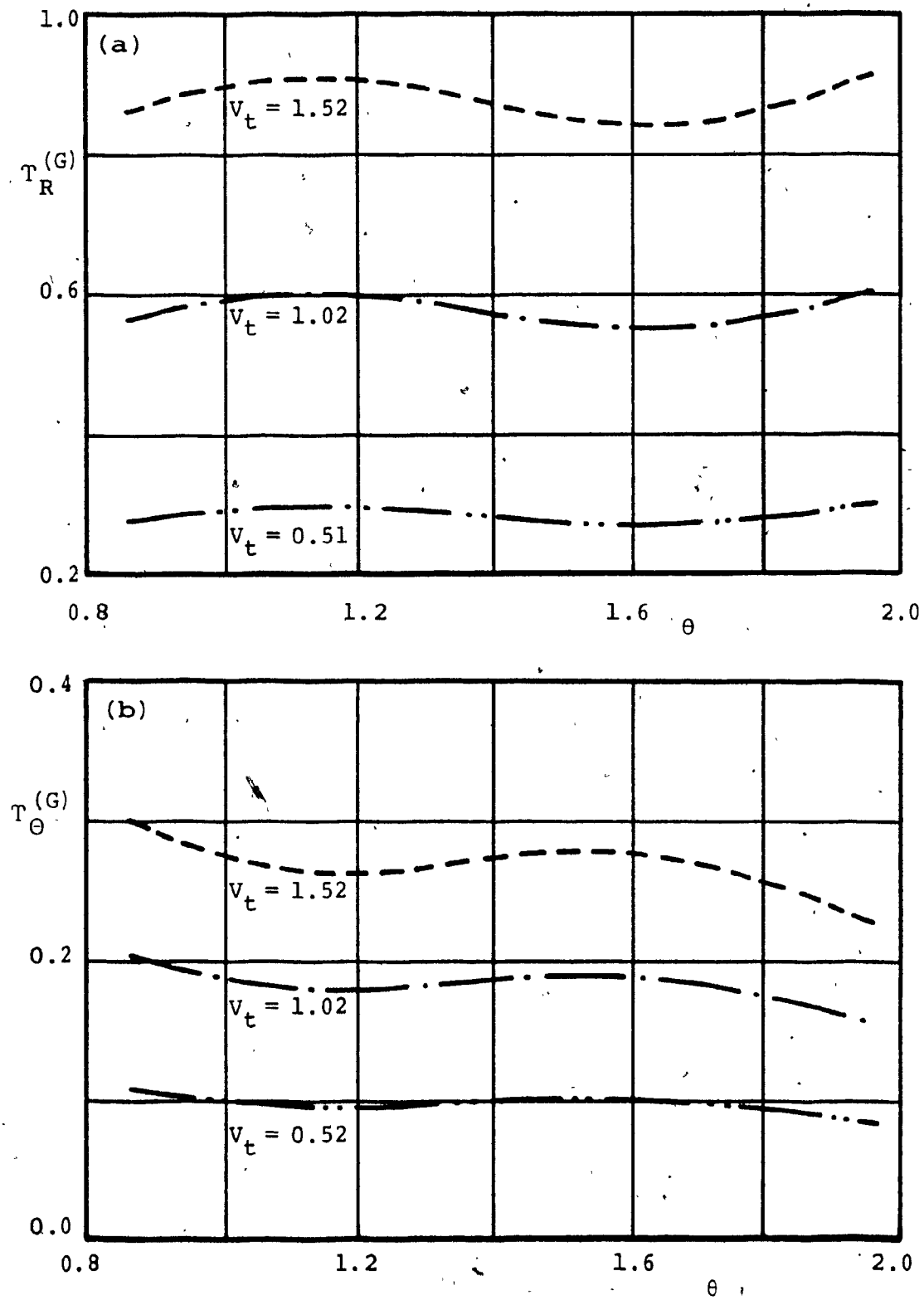


Figure 5.23: Torque Ratio Plots for Machine Pole at the Location of Minimum Area Error: Witch of Agnesi Contour (Grinding Application). V_t is in cm/s.

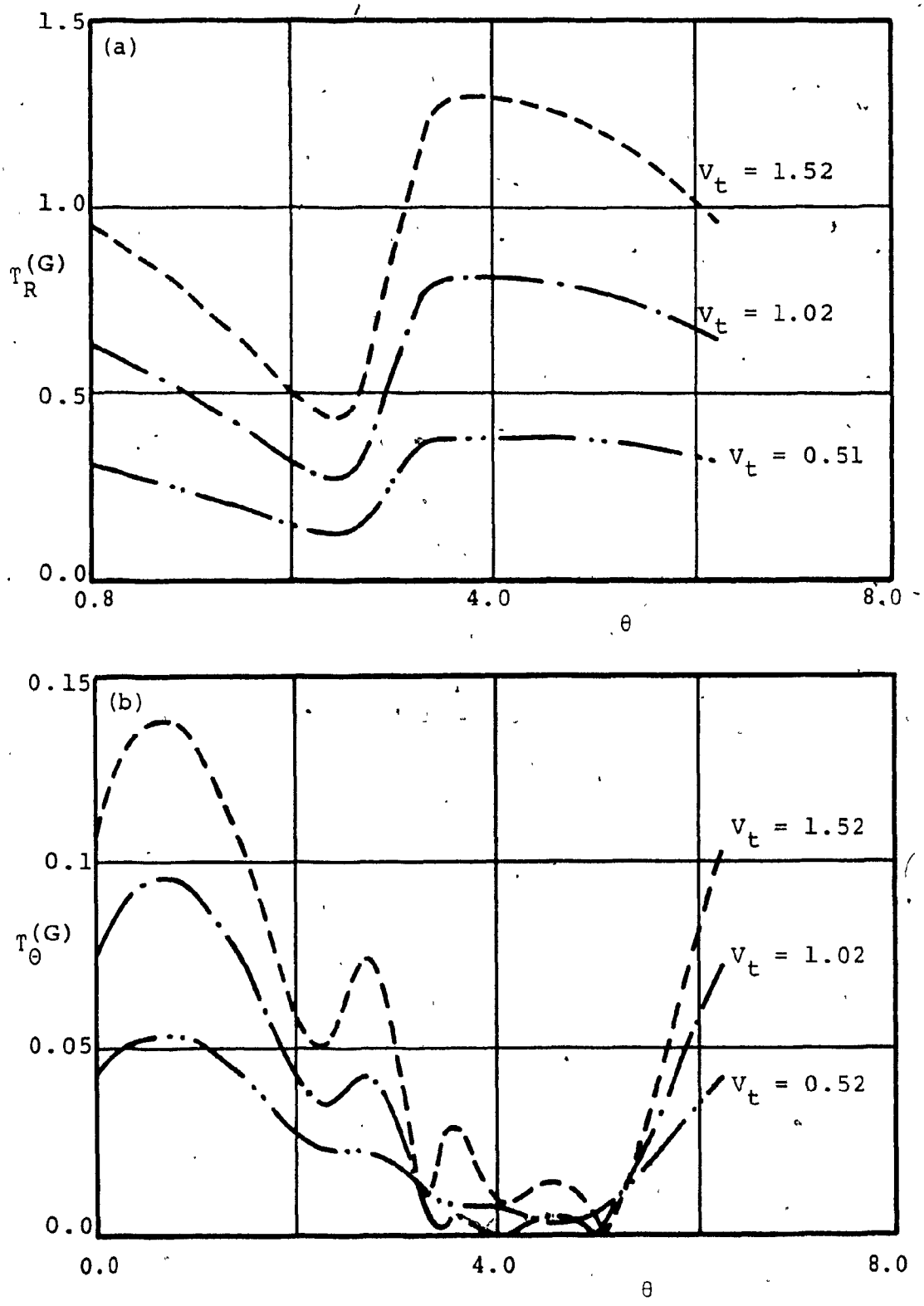


Figure 5.24: Torque Ratio Plots for Machine Pole at the Location of Minimum Area Error: Limacon of Pascal Contour (Grinding Application). V_t is in cm/s.

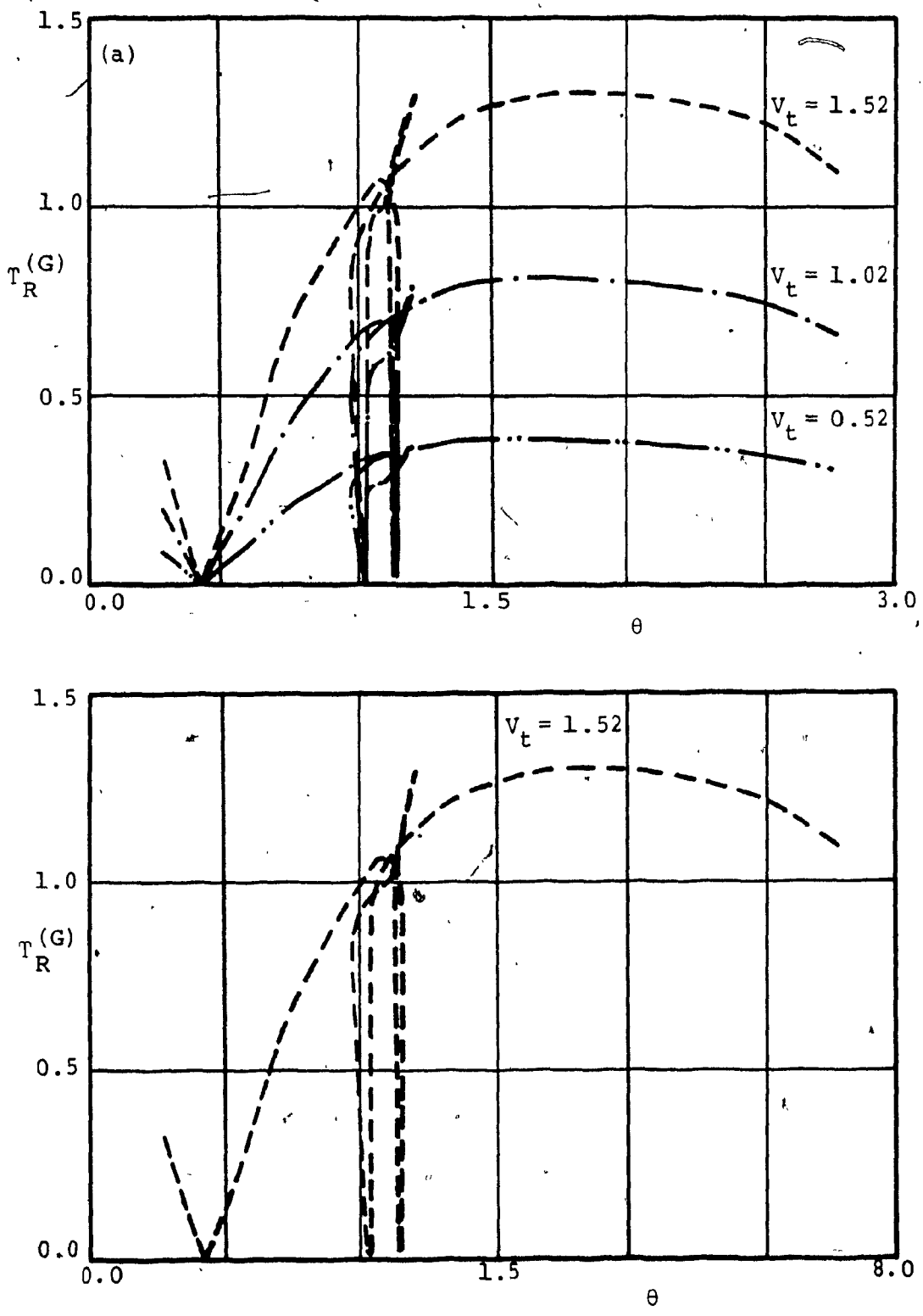


Figure 5.25: Torque Ratio Plots for Machine Pole at the Location of Minimum Area Error: Serpentine Contour (Grinding Application). V_t is in cm/s.

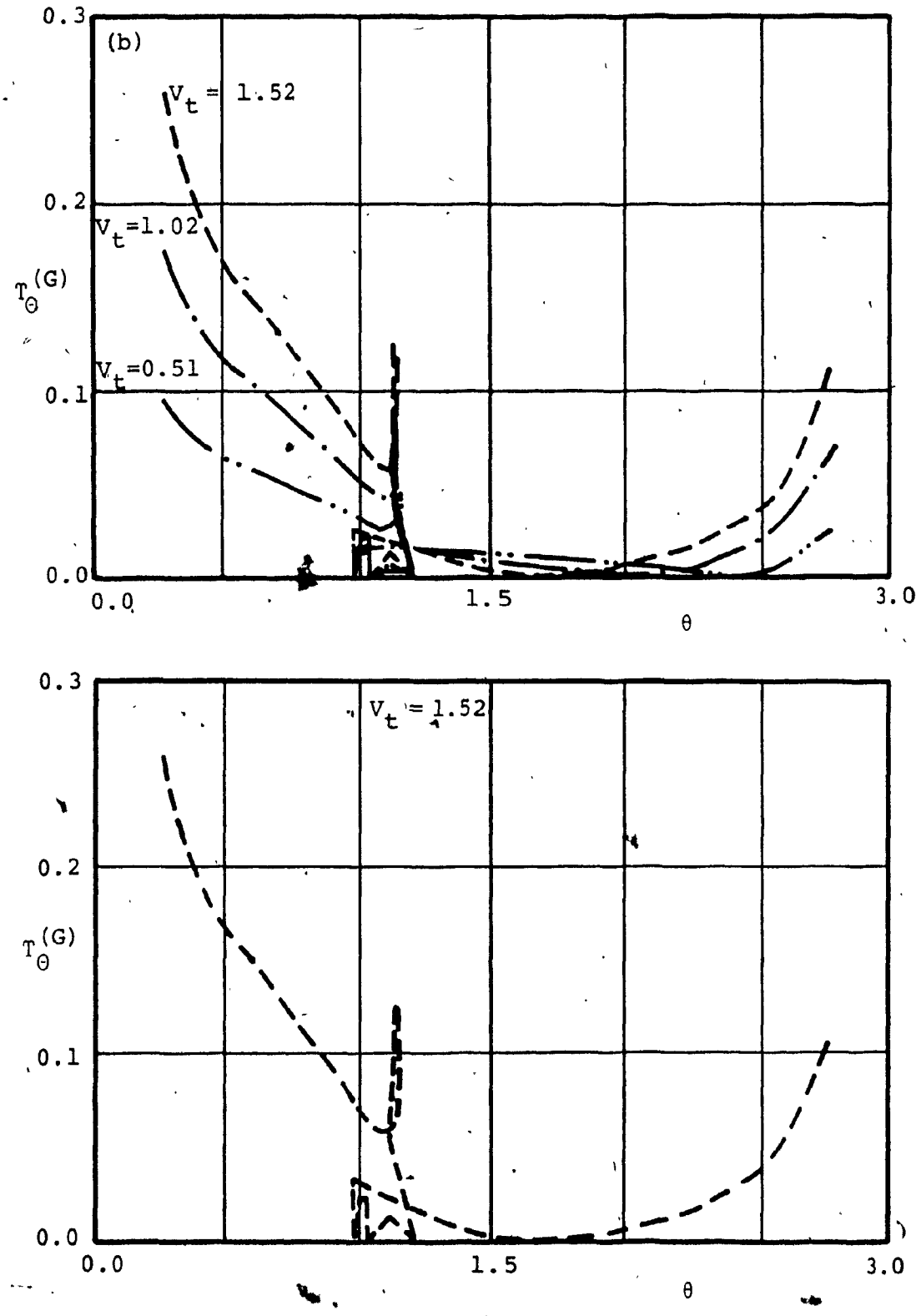


Figure 5.25: Torque Ratio Plots for Machine Pole at the Location of Minimum Area Error: Serpentine Contour (Grinding Application). V_t is in cm/s.

the maximum value of the angular axis torque ratio decreased by approximately 50% in comparison to the torque ratio in Figure 5.18.b. By comparing Figures 5.24.b and 5.18.b, it can be seen that the angular axis plots do not exhibit the sudden jump in the torque ratio resulting from the Coulomb friction. This is due to the fact that the machine pole is located within the contour, which causes the angular axis to move only in one direction.

The plots for the Serpentine contour in Figure 5.25 show that at the new machine pole location, the radial axis torque ratio is comparable to that of the previous pole location (Figure 5.19.a). The angular axis torque ratio, however, is marginally lower than that of the previous case shown in Figure 5.19.b.

5.7 Summary

This chapter deals with the dynamics of the polar NC machine. The kinematic analysis shows that the velocities of the arm and the carriage at any point on the contour are functions of the contouring speed V_t and the inclination of the contour tangent to the arm γ. The acceleration of the arm and the carriage are also functions of V_t and γ, however, are inversely proportional to the radius of curvature ρ of the contour at the operating point. The role of γ in the kinematic relationships is to proportion the total velocity and acceleration over the two machine axes.

In the dynamic analysis presented in this chapter, it

is assumed that the components involved are rigid. The analysis results in relationships for the torque required from the radial and angular axes-drives of the NC machine for inertial and grinding type loads.

The stepping motors used in the prototype NC machine are energized using the wave energization scheme. Wave energization results in a finer resolution and less resonance during operation than the other energization schemes. The pull-out torque-speed relationship for stepping motors operating with the wave energization scheme is derived in this chapter.

The quotient of the torque required from an axis-drive to the pull-out torque available from that drive is defined as the torque ratio. This ratio determines the success or failure of a contouring operation. A ratio larger than unity indicates more torque required from the axis-drive than the torque available from that drive, hence contouring failure.

The effects of the nature of the contour, the type and the magnitude of the load on the NC machine, the contouring speed, and the relative location between the contour pole and the machine pole on the torque ratios are also studied in this chapter. The study shows that in general, an increase in the contouring speed or the magnitude of the machining load causes an increase in the torque ratio. Furthermore, sharply undulating contours result in a higher torque ratio than smooth contours. The loads in a grinding

application have more effect on the torque ratio of both axes than inertial loads. The effect of the contour radius of curvature on the torque ratios is more apparent in an inertial type application than in the grinding application (Refer to the cases of the Serpentine contour).

In the case of a grinding application, peak values of the torque ratio in one axis coincide in location with the minima of the torque ratio in the other axis.

The relative location between the contour pole and the machine pole was shown to affect the magnitudes of the torque ratios.

CHAPTER 6

ANALYTICAL STUDY FOR PARAMETRIC VARIATION AND OPTIMUM LOADING OF THE NC MACHINE

6.1 Introduction

In Chapter 5, the effects of the type of load and its magnitude, the contouring speed, the type of contour, as well as the relative location between the contour pole and the machine pole on the torque ratios have been studied. In this chapter, an analytical study is carried out to evaluate the isolated effect of each of the above parameters on the magnitude of the torque ratios. The effect of these parameters on the location of the maximum value of the torque along the contour is also studied. The study in Chapter 5 also showed that the magnitude of the maximum axes torques change with the change in the relative location between the contour pole and the machine pole. Based on this fact, an objective function for optimization is then formulated which, when minimized, results in a location for the contour pole where the peak load on the axes drives is minimum. The results of the parametric study are then utilized to selectively increase the machine settings (i.e contouring speed and loads), so as to utilize the mechanism to its full capacity.

Throughout this chapter, a "^" will be used to indicate the maximum values of torque and torque ratio along a

contour. Furthermore, the superscripts (I) and (G) will be used to indicate inertial and grinding cases respectively.

6.2 Effect of inertial load

6.2.1 Variable inertial loads with constant contouring speed

For the case of pure inertial load there is no force acting on the tool. However, extra mass is added and is lumped with that of the carriage. The torque ratios are hence given by:

$$T_R^{(I)} = M_c \frac{a_{cr} K_2}{T_{PR}} \qquad 6.1$$

$$T_\theta^{(I)} = \frac{\left(J_A \ddot{\theta} + F_f L_A\right) K_1}{T_{P\theta}} + M_c \frac{a_{cn} r K_1}{T_{P\theta}} \qquad 6.2$$

Equations 6.1 and 6.2 show that the torque ratios are directly proportional to the mass on the carriage.

The radial axis torque ratio plots for the inertial case shown in Chapter 5 verifies this proportionality. As an example the Limacon of Pascal plots in Figure 5.11.b show that at $\theta=\pi/2$, the torque ratio is linearly proportional to the mass on the carriage.

For the angular axis, the results in Chapter 5 do not readily show the proportionality as per Equation 6.2. This is due to the intercept in the torque ratio axis

(Equation 6.2) given by the first term in the R.H.S.

6.2.2 Constant inertial load with variable contouring speeds

Isolating the contouring speed in Equations 6.1 and 6.2 yields:

$$T_R^{(I)} = \frac{\Lambda_8 V_t^2 + \Lambda_9 V_t^4}{\left[\Lambda_{10}^2 + \Lambda_{11}^2 V_t^2\right]^{\frac{1}{2}} - \Lambda_{12} V_t} \qquad 6.3$$

$$T_\Theta^{(I)} = \frac{\Lambda_{14} + \Lambda_{15} V_t^2 + \Lambda_{16} V_t^4}{\left[\Lambda_{10}^2 + \Lambda_{17}^2 V_t^2\right]^{\frac{1}{2}} - \Lambda_{18} V_t} \qquad 6.4$$

where,

$$\Lambda_8 = \left[M_c\ K_2\ R_a^2\ \sin\gamma\right]/\rho$$

$$\Lambda_9 = \Lambda_8\left[2\pi N_s\ k_R\ L_a\ \cos\gamma\right]^2/R_a^2$$

$$\Lambda_{10} = 1.17632\ V\ K_T\ R_a$$

$$\Lambda_{11} = \Lambda_{10}\left[2\pi N_s\ k_R\ L_a\ \cos\gamma\right]/R_a$$

$$\Lambda_{12} = K_V\ 2\pi R_a\ K_T\ k_R\ \cos\gamma$$

$$\Lambda_{19} = \left[\Lambda_{10}^2 + \Lambda_{11}^2 V_t^2\right]^{\frac{1}{2}} - \Lambda_{12} V_t$$

$$\Lambda_{14} = R_a^2\ F_f\ L_a\ K_1$$

$$\Lambda_{15} = R_a^2 K_1\left[J_A\left(\frac{\cos\gamma}{r\rho} - \frac{\sin 2\gamma}{r^2}\right) + M_c\frac{r\ \cos\gamma}{\rho}\right] +$$

$$K_1\left[\left(F_f L_A\ 2\pi N_s\ k_\Theta\ L_a\ \sin\gamma\right)/r\right]$$

$$\Lambda_{16} = K_1 \left[J_A \left(\frac{\cos\gamma}{r\rho} - \frac{\sin 2\gamma}{t^2} \right) + M_C \frac{r \cos\gamma}{\rho} \right] x$$

$$\left[\left(2N_s k_\Theta L_a \sin\gamma \right) / r \right]$$

$$\Lambda_{17} = \Lambda_{10} \left(2\pi N_s k_\Theta L_a \sin\gamma \right) / \left(R_a r \right)$$

$$\Lambda_{18} = \left(K_V R_a K_T 2\pi k_\Theta \sin\gamma \right) / r$$

$$\Lambda_{19} = \left[\Lambda_{10}^2 + \Lambda_{17}^2 V_t^2 \right]^{\frac{1}{2}} - \Lambda_{18} V_t$$

Equations 6.3 and 6.4 show that the torque ratio for both axes is a quartic polynomial in the contouring speed, V_t. Differentiating Equations 6.3 and 6.4 partially with respect to the contouring speed gives the sensitivities $\Gamma_R^{(I)}$ and $\Gamma_\Theta^{(I)}$ as:

$$\Gamma_R^{(I)} = \frac{\partial T_R}{\partial V_t} = \left[2 \Lambda_8 V_t + 4 \Lambda_9 V_t^3 - T_R \left(\partial \Lambda_{13} / \partial V_t \right) \right] / \Lambda_{13} \qquad 6.5$$

$$\Gamma_\Theta^{(I)} = \frac{\partial T_\Theta}{\partial V_t} = \left[2 \Lambda_{15} V_t + 4 \Lambda_{16} V_t^3 - T_\Theta \left(\partial \Lambda_{19} / \partial V_t \right) \right] / \Lambda_{19} \qquad 6.6$$

where;

$$\frac{\partial \Lambda_{13}}{\partial V_t} = \left[\Lambda_{10}^2 + \Lambda_{11}^2 V_t^2\right]^{-\frac{1}{2}} \Lambda_{10}^2 V_t - \Lambda_{12}$$

$$\frac{\partial \Lambda_{19}}{\partial V_t} = \left[\Lambda_{10}^2 + \Lambda_{17}^2 V_t^2\right]^{-\frac{1}{2}} \Lambda_{17}^2 V_t - \Lambda_{18}$$

Figures 6.1 and 6.2 show plots of the sensitivity parameters $\Gamma_R^{(I)}$ and $\Gamma_\theta^{(I)}$ respectively. The plots are obtained for values of $\gamma=\pi/4$, $r=7.62$ cm. (3 inch) and $\rho=2.54$ cm. (1 inch). The plots indicate that both torque ratios increase monotonically and rapidly with the increase in contouring speed. For example, consider the torque ratio plots for the circular contour. At $\gamma=\pi/4$ the value of $\theta=1.77$, and occurs on the lower part of the circle, CDA in Figure 5.4. For the radial axis referring to, Figure 5.8, at $\theta=1.77$ on the path CDA the torque ratio increases by approximately 10 times for an increase of 3 times in the contouring speed.

For the angular axis, Figure 5.9, at $\theta=1.77$ on the path CDA, the torque ratio increases by approximately 23% for an increase of 200% in the contouring speed. However, at the same angular location on the path ABC of the circle, Figure 5.9 shows a decrease in the torque ratio with an increase in the contouring speed. This is due to the fact that part of the inertial force acting on the carriage resulting from the Coriolis acceleration, helps the angular axis stepping motor to overcome the frictional force of the arm support and the inertial force resulting from the

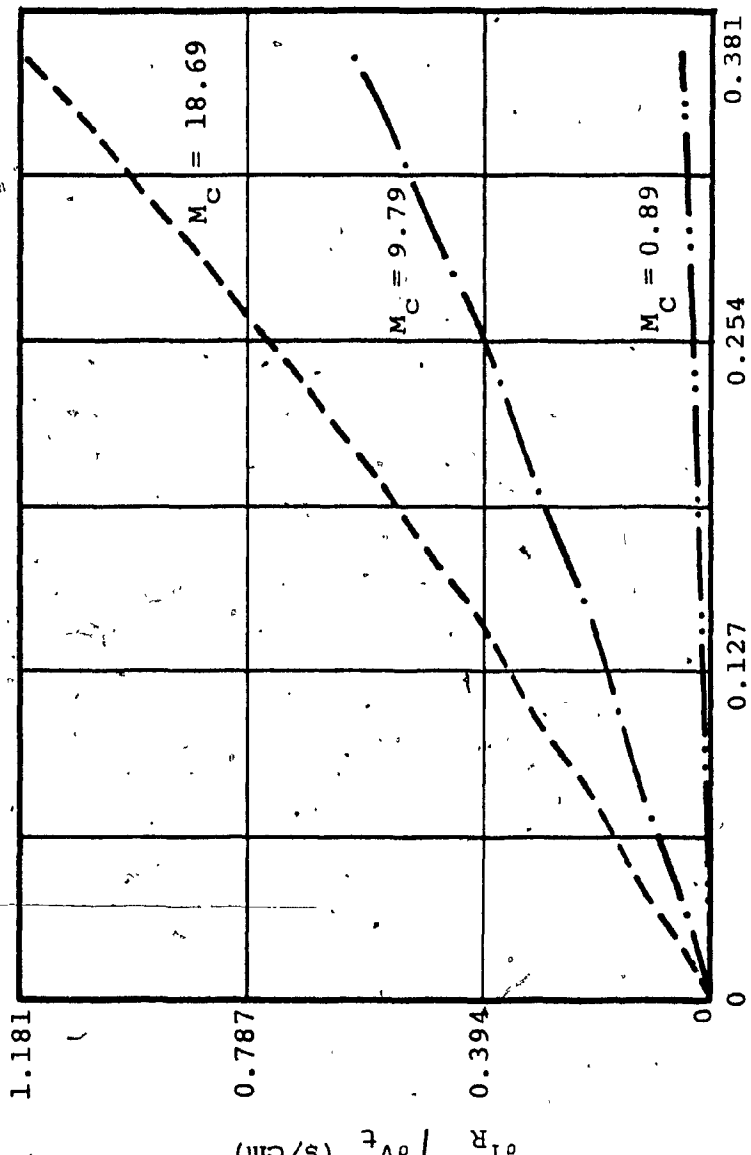


Figure 6.1: Sensitivity Parameter $\Gamma_\Theta^{(I)}$ Versus V_t.

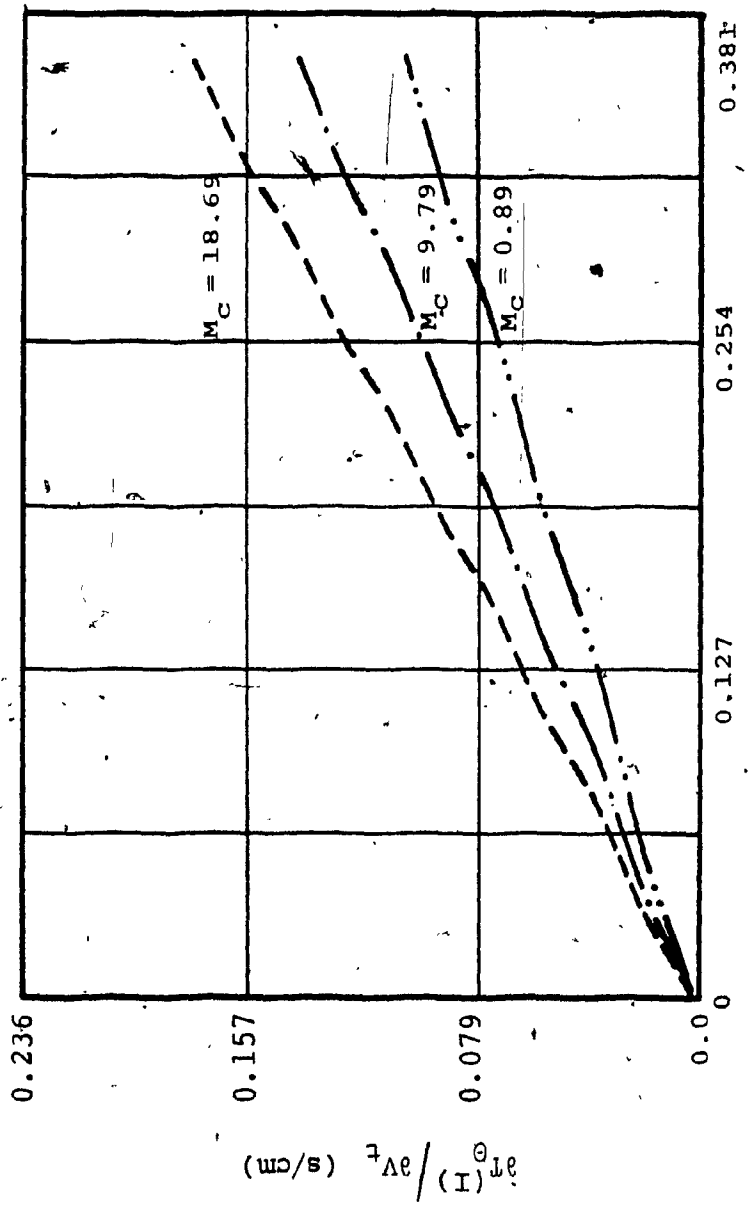


Figure 6.2: Sensitivity Parameter $\Gamma_\Theta^{(I)}$ Versus V_t.

angular acceleration of the arm.

6.3 Effect of dissipative load (Grinding application)

6.3.1 Variable dissipative loads with constant contouring speed

For the case of the grinding application, the cutting force given by Equations 5.22 and 5.23 can be incorporatd in the torque ratio relationships in Equations 5.15 and 5.16. The factor K_c which determines the magnitude of the cutting force can be isolated out to give:

$$T_R^{(G)} = \frac{M_c\, a_{cr}\, K_2}{T_{PR}} + K_c\, \frac{K_2 V_t \cos(\gamma - \alpha_c)}{T_{PR}} \qquad 6.7$$

$$T_\theta^{(G)} = \frac{(J_A \ddot{\theta} + M_c r\, a_{cn} + F_f\, L_A) K_1}{T_{P\theta}} + K_c\, \frac{K_1 V_t\, r \sin(\gamma - \alpha_c)}{T_{P\theta}} \qquad 6.8$$

In grinding applications, the machining force dominates over the inertial and the frictional forces. Thus with reference to Equations 6.7 and 6.8, the first two terms in the right hand side of both equations can be neglected. Then, both equations become linearly proportional to the machining force. This proportionality can be readily seen in the torque ratio plots for the four test contours, Figures 5.16 to 5.19.

6.3.2 Constant dissipative load with variable contouring speeds

Rewriting Equations 6.7 and 6.8 so that the contouring speed can be isolated, yields:

$$T_R^{(G)} = \frac{\left[\Lambda_{20} V_t + \Lambda_8 V_t^2 + \Lambda_{21} V_t^3 + \Lambda_9 V_t^4\right]}{\left[\Lambda_{10}^2 + \Lambda_{11}^2 V_t^2\right]^{\frac{1}{2}} - \Lambda_{12} V_t} \qquad 6.9$$

$$T_\Theta^{(G)} = \frac{\left[\Lambda_{14} + \Lambda_{22} V_t + \Lambda_{15} V_t^2 + \Lambda_{23} V_t^3 + \Lambda_{16} V_t^4\right]}{\left[\Lambda_{10}^2 + \Lambda_{17}^2 V_t^2\right]^{\frac{1}{2}} - \Lambda_{18} V_t} \qquad 6.10$$

where,

$$\Lambda_{20} = R_a^2 K_c K_2 \cos(\gamma - \alpha_c)$$

$$\Lambda_{21} = K_c K_2 \left[2\pi N_s k_R L_a \cos\gamma\right]^2 \cos(\gamma - \alpha_c)$$

$$\Lambda_{22} = R_a^2 K_1 K_c r \sin(\gamma - \alpha_c)$$

$$\Lambda_{23} = \Lambda_{22} \left[2\pi N_s k_R L_a \cos\gamma\right]^2 / R_a^2$$

Equations 6.9 and 6.10 show that the torque ratios in this case are also represented by a quartic polynomial in the contouring speed, V_t. The sensitivities, $\Gamma_R^{(G)}$ and $\Gamma_\Theta^{(G)}$ are calculated by taking the partial derivatives of Equations 6.9 and 6.10 with respect to V_t, and are given by:

$$\Gamma_R^{(G)} = \frac{\partial T_R}{\partial V_t} = \left[\Lambda_{20} + 2\Lambda_8 V_t + 3\Lambda_{21} V_t^2 + 4\Lambda_9 V_t^3 - T_R\left(\partial\Lambda_{13}/\partial V_t\right)\right]/\Lambda_{13} \qquad 6.11$$

$$\Gamma_{\theta}^{(G)} = \frac{\partial T_{\theta}}{\partial V_t} = \left[\Lambda_{22} + 2\Lambda_{15}V_t + 3\Lambda_{23}V_t^3 + 4\Lambda_{16}V_t^3 - T_{\theta}\left(\partial\Lambda_{19}/\partial V_t\right)\right]/\Lambda_{19} \qquad 6.12$$

Figures 6.3 and 6.4 show plots of Equations 6.11 and 6.12 respectively, for $\gamma = \pi/4$, $r = 7.62$ cm. (3 inch), and $\rho = 2.54$ cm. (1 inch). The plots indicate that, in this case, the torque ratios increase almost linearly and at a very slow rate with the increase in the contouring speed. experimentation with different values of γ, as well as all the angular axis torque ratio plots in Chapter 5, indicate that the above mentioned trend is true for all values of γ.

6.4 Effect of changes in the parameters on the location of the maximum torques and torque ratios along the contour

In this study the location of the maximum torque and torque ratios along the contour is given in terms of the angle γ. The angle γ is useful as a parameter for solving the equation set, 5.4, 5.9, and 5.10 in Chapter 5 in order to obtain θ.

Substituting Equations 5.6, 5.7, and 5.8 in Equations 5.15 and 5.16 yields the torque expressions for both axes as:

$$T_R = M_c K_2 V_t^2 \frac{\sin\gamma}{\rho} + K_c K_2 V_t \cos(\gamma - \alpha_c) \qquad 6.13$$

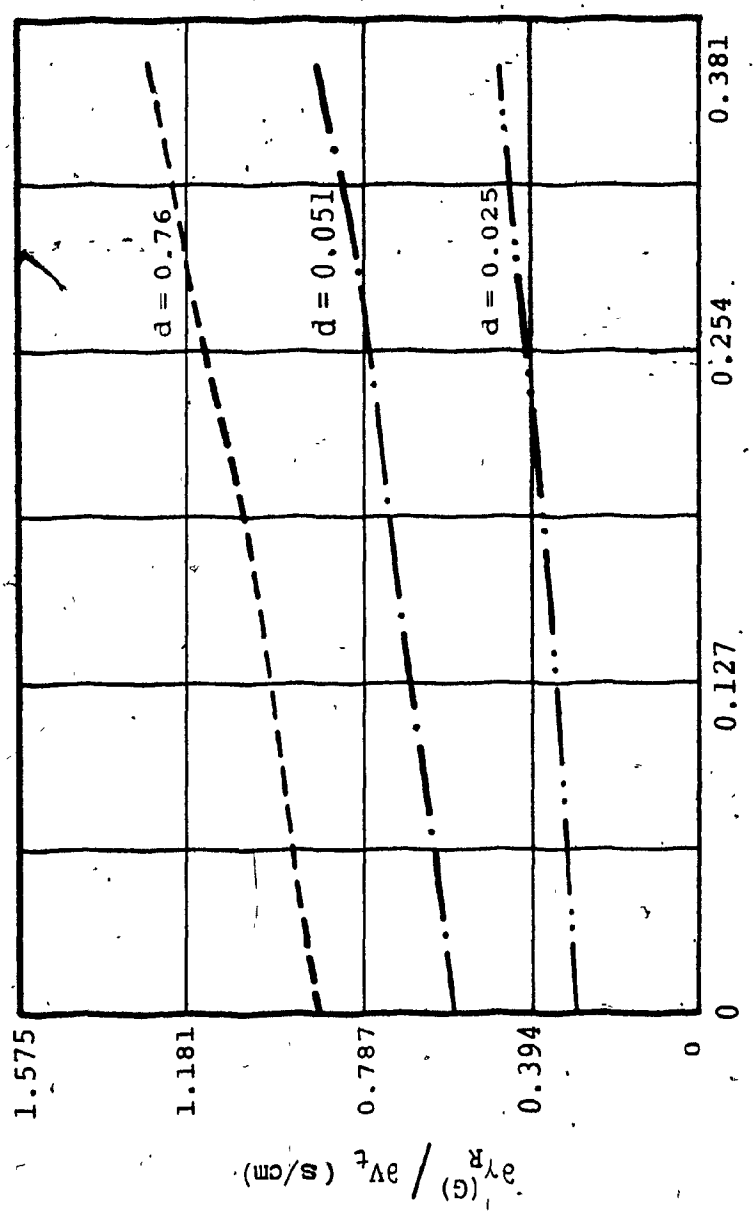


Figure 6.3: Sensitivity Parameter $\Gamma_R^{(G)}$ Versus V_t

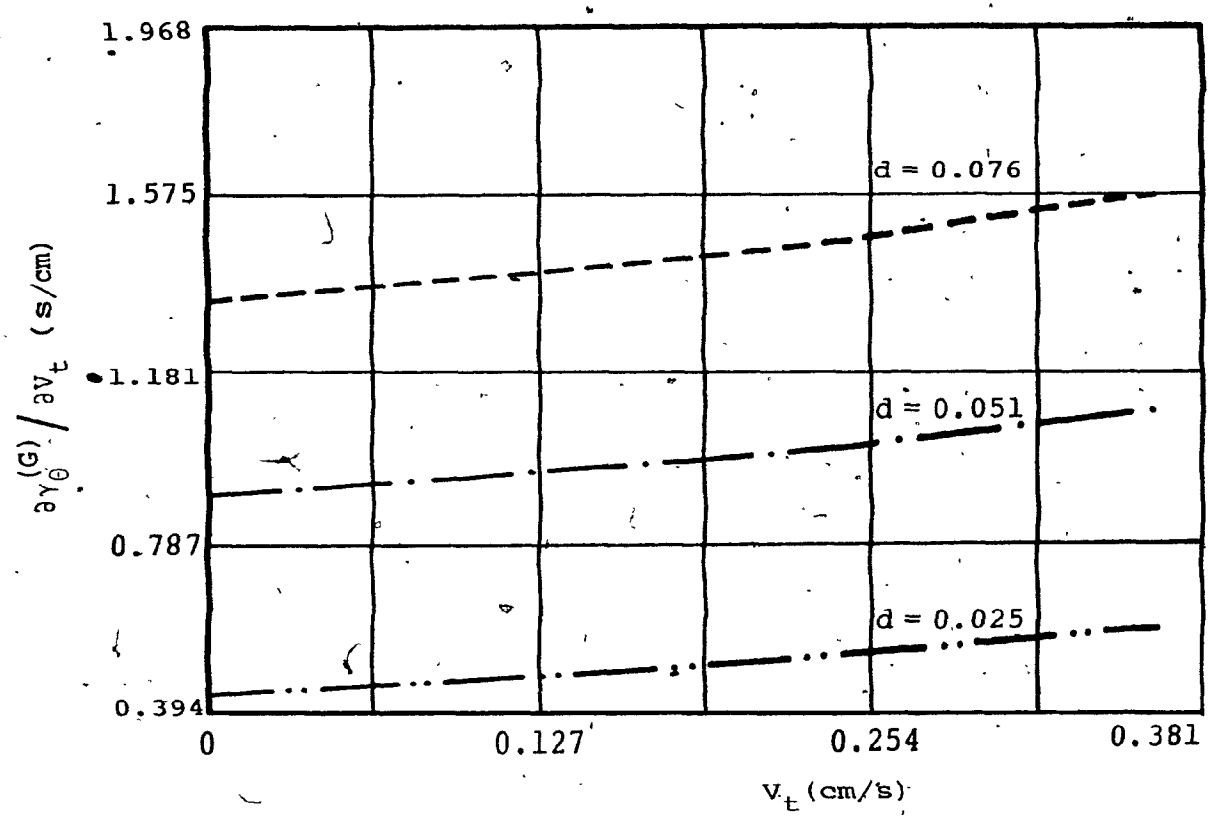


Figure 6.4: Sensitivity Parameter $\Gamma_\theta^{(G)}$ Versus V_t.

$$T_\theta = \frac{K_1 V_t^2}{\rho} \cos\gamma \left[M_c r + J_A \ddot{r}\right] - \frac{2K_1 V_t^2}{r^2} \sin\gamma \cos\gamma +$$

$$K_1 F_f L_A + K_c K_1 V_t \sin(\gamma - \alpha_c) \qquad 6.14$$

6.4.1 Inertial load

In the case of inertial loads, the machining force components given by the last term in Equations 6.13 and 6.14 are identically zero. In order to find the location along the contour, of the maximum radial axis torque, Equation 6.13 is differentiated partially with respect to γ and the derivative is equated to zero. The result of this operation shows that the value of $\gamma_R^{(I)}$ for maximum torque in the radial axis is a constant and equal to $\pi/2$. That is to say, the maximum radial axis torque, and hence its maximum torque ratio, occur always at the same location θ on the contour irrespective of the changes in any of the parameters. This conclusion can be verified from the torque ratio plots of the radial axis for the case of inertial load shown in Sections 5.5.1 and 5.6.1.

For the angular axis, differentiating Equation 6.14 partially with respect to γ and equating the result to zero yields a quadratic equation in $\sin\gamma$. Solving for the value of $\gamma_\theta^{(I)}$ that gives the maximum angular axis torque, yields:

$$\gamma_\theta^{(I)} = \sin^{-1}\left\{\frac{\left[\Lambda_{24} - \left(\Lambda_{24}^2 + 0.5\rho^2\right)^{\frac{1}{2}}\right]}{\rho}\right\}$$

$$\frac{3\pi}{2} < \gamma \leq 2\pi \qquad 6.15$$

$$\Lambda_{24} = \left[M_C r^3 + J_A r\right] / 8$$

Equation 6.15 shows that $\gamma_R^{(I)}$ changes with the change in the inertial load M_C. The contouring speed, however, has no effect on the location of the maximum torque along the contour. In order to find the sensitivity of $\gamma_\theta^{(I)}$ to changes in M_C, Equation 6.15 can be differentiated with respect to M_C and gives:

$$\frac{\partial \gamma_\theta^{(I)}}{\partial M_C} = \left\{\left[\Lambda_{24}^2 + 0.5\rho^2\right]^{\frac{1}{2}} - \Lambda_{24}^2\right\} r^3 / \Lambda_{25}$$

$$\Lambda_{25} = 8\left\{\left[\Lambda_{24}^2 + 0.5\rho^2\right]\left[-2\Lambda_{24}^2 + 2\Lambda_{24}\left(\Lambda_{24}^2 + 0.5\rho^2\right)^{\frac{1}{2}} + 0.5\rho^2\right]\right\}^{\frac{1}{2}} \qquad 6.16$$

The radius of curvature, ρ in Equation 6.16 can be expressed as [30]:

$$\frac{d\gamma}{d\theta} = \frac{ds/d\theta}{\rho} - 1 \qquad 6.17$$

Equation 6.17 shows that in general the sensitivity of γ with respect to θ is inversely proportional to ρ and is directly proportional to $ds/d\theta$. The derivative $d\gamma/d\theta$ is a function of the inclination γ of the tangent on the radial vector, and has a value of zero for cases of $\gamma = \pi/2$ or $\gamma = 3\pi/2$, and a value of infinity for cases of $\gamma = 0$ or $\gamma = 2\pi$.

Solving Equations 6.16 and 6.17, for the angular axis,

the rate of change in angular location of the maximum torque on the contour, due to a change in the magnitude of the mass on the carriage is given by:

$$\frac{\partial \theta_{\theta}^{(I)}}{\partial M_c} = \frac{\left\{\left[\Lambda_{24}^2 + 0.5\rho^2\right]^{\frac{1}{2}} - \Lambda_{24}^2\right\} r^3 \rho}{\Lambda_{25}\,(ds/d\theta - \rho)} \qquad 6.18$$

Figure 6.5 shows plots of $d\theta/dM_c$ versus ρ for different values of the mass on the carriage. The plots are obtained for typical values of the prototype parameters, given in Table 5.1, and for r=20.32 cm. (8 inch) and $ds/d\theta$=2.54 cm/rad (1 inch/rad). With reference to Figure 6.5, for all values of the mass on the carriage, $d\theta/dM$ approaches zero for either small or large values of ρ. This indicates that for small or large values of ρ, the maximum torque will occur at the same angular location on the contour irrespective of the magnitude of the mass on the carriage. This conclusion can be confirmed from the results shown in Figures 5.13.a and 5.13.b at θ=1.67 radians when the mechanism is tracking the indentation in the Limacon of Pascal, and in Figures 5.15.a and 5.15.b at θ=1.5 and θ=1.66 when the mechanism is negotiating the tight curve at both ends of the asymptote of the Serpentine curve.

For values of ρ in a narrow region around ρ=27 cm, the value of $d\theta_{\theta}^{(I)}/dM_c$ deviates from zero depending upon the magnitude of M_c. This region occurs when the term $ds/d\theta-\rho$ in the denominator has a magnitude in the neighborhood of

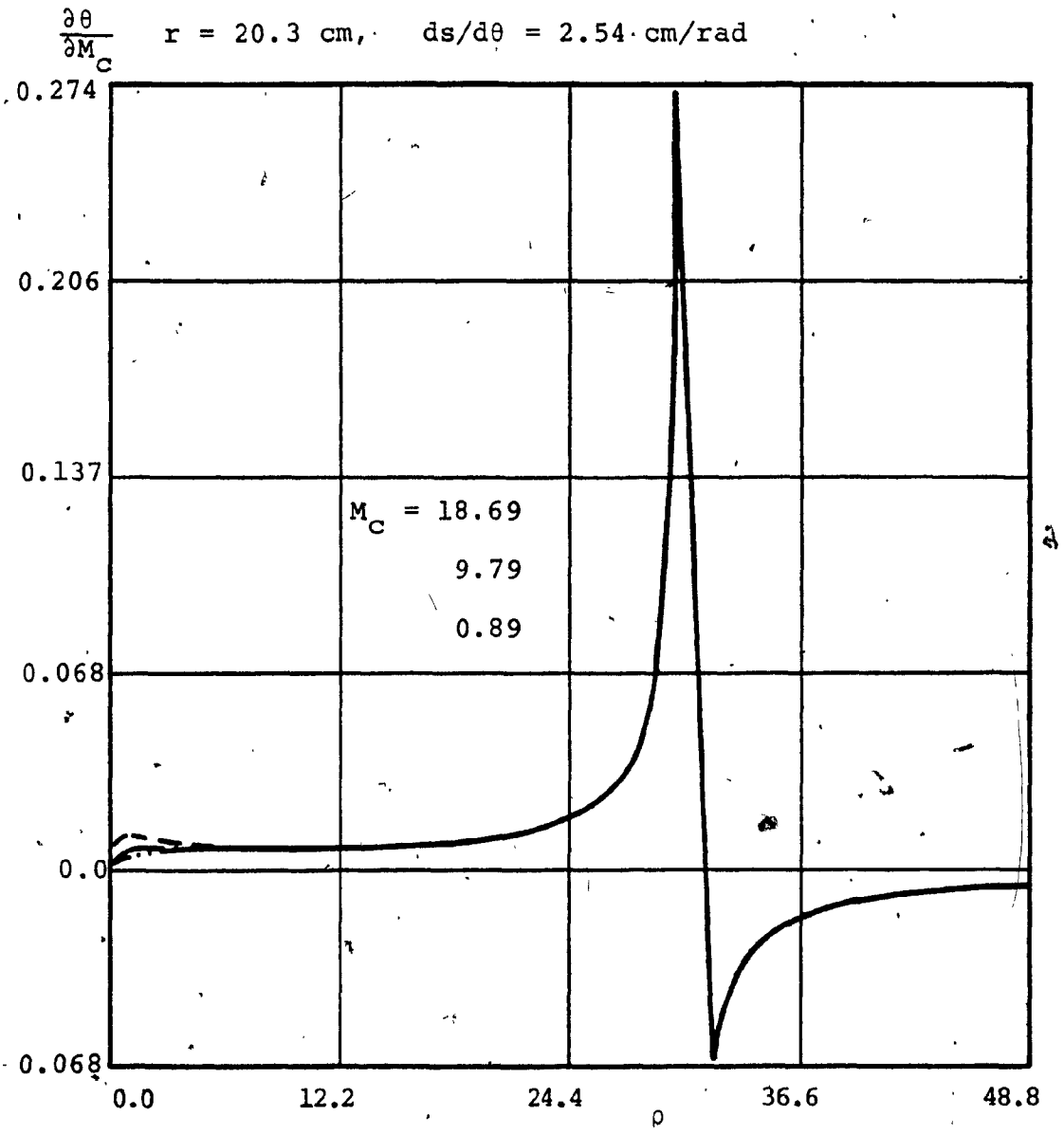


Figure 6.5: $d\theta/dM_c$ Versus ρ for the Maximum Angular Axis Torque for an Inertial Load.

zero. However with reference to Equation 6.17, this condition occurs when changes in γ are negligible due to changes in θ, and results in a wide region in which the magnitude of the torque changes marginally. The change in magnitude of $d\theta_{\theta}^{(I)}/dM_c$ indicates that the maximum torque shifts its location along the contour within this region of ρ values, as the magnitude of the mass on the carriage changes. This shift can be seen in the case of the Witch of Agnesi contour (Figure 5.11 a and 5.11.b) because of its radius of curvature falls within this region. The magnitude of the change, however, is small.

6.4.2 Grinding application

For the case of a grinding application, the second term in Equation 6.14 is insignificant in comparison with the torque due to the dissipative force and can be ignored. The location on the contour at which the maximum torque for the two axes occurs, is found by differentiating Equations 6.13 and 6.14 partially with respect to γ and equating the result to zero. Thus:

$$\gamma_R^{(G)} = \tan^{-1}\left[\frac{M_c V_t}{\rho K_c \cos_c} + \tan\alpha_c\right] \qquad 6.19$$

$$\gamma_\theta^{(G)} = \cot^{-1}\left[\frac{V_t\left[M_c r + J_A/r\right]}{\rho K_c \cos\alpha_c} + \tan\alpha_c\right] \qquad 6.20$$

To find the sensitivity of $\gamma^{(G)}$ and $\gamma_\theta^{(G)}$ to changes in

the contouring speed and the machining factor K_c, Equations 6.19 and 6.20 are differentiated partially with respect to V_t and K_c. The results of this operation gives:

For V_t;

$$\frac{\partial \gamma_R^{(G)}}{\partial V_t} = \frac{M_c \rho K_c \cos \alpha_c}{\left(M_c V_t\right)^2 + 2\rho K_c M_c V_t \sin\alpha_c + \left(\rho K_c\right)^2} \qquad 6.21$$

$$\frac{\partial \gamma_\Theta^{(G)}}{\partial V_t} = -\rho K_c \left[M_c r + J_A/r\right] \cos\alpha_c / \Lambda_{26}$$

$$\Lambda_{26} = \left[V_t - \left(M_c r + J_A/r\right)\right]^2 - 2\rho K_c V_t \left(M_c r + J_A/r\right) \sin\alpha_c + \left(\rho K_c\right)^2 \qquad 6.22$$

For K_c;

$$\frac{\partial \gamma_R^{(G)}}{\partial K_c} = -\frac{M_c V_t \rho \cos\alpha_c}{\left(M_c V_t\right)^2 + 2\rho K_c M_c V_t \sin\alpha_c + \left(\rho K_c\right)^2} \qquad 6.23$$

$$\frac{\partial \gamma_\Theta^{(G)}}{\partial K_c} = V_t \rho \left[M_c r + J_A/r\right] \cos\alpha_c / \Lambda_{26} \qquad 6.24$$

Combining Equation 6.17 with each of Equations 6.21 to 6.24, gives a set of equations that relate the rate of change of the maximum torque location on the contour to the change in V_t and K_c. These are:

For V_t;

$$\frac{\partial \theta_R^{(G)}}{\partial V_t} = \frac{M_c \rho^2 K_c \cos\alpha_c}{\left[(M_c V_t)^2 + 2\rho K_c M_c V_t \sin\alpha_c + (\rho K_c)^2\right]\left[ds/d\theta - \rho\right]} \quad 6.25$$

$$\frac{\partial \theta_\theta^{(G)}}{\partial V_t} = -\frac{\rho^2 K_c \left[M_c r + J_A/r\right]}{\Lambda_{26}\left[ds/d\theta - \rho\right]} \quad 6.26$$

For K_c;

$$\frac{\partial \theta_R^{(G)}}{\partial K_c} = -\frac{M_c V_t \rho^2 \cos\alpha_c}{\left[(M_c V_t)^2 + 2\rho K_c M_c V_t \sin\alpha_c + (\rho K_c)^2\right]\left[ds/d\theta - \rho\right]} \quad 6.27$$

$$\frac{\partial \theta_\theta^{(G)}}{\partial K_c} = -\frac{V_t \rho^2 \left[M_c r + J_A r\right]}{\Lambda_{26}\left[ds/d\theta - \rho\right]} \quad 6.28$$

In Equations 6.25 and 6.26 the denominator is a quadratic polynomial in V_t. The constant term $(\rho K_c)^2$ in the polynomial contains the grinding force factor and is typically much larger in comparision to all the other terms in general grinding applications (an approximate ratio of one million in the grinding case under consideration). Consequently changes in the contouring speed do not affect the location where the maximum torque occurs on the contour. This conclusion is confirmed from all the torque ratio plots for the grinding application, shown in Chapter 5 The effect of the term $ds/d\theta-\rho$ in this case is similar to that in the case of the inertial load discussed above, with the exception that the region in which $d\theta_R^{(G)}/dV_t$ and $d\theta_\theta^{(G)}/dV_t$ change is very narrow.

Equation 6.27 and 6.28 shows that the change in location of the maximum torque along the contour is inversely proportional to the square of the machining force factor K_c for both the radial and angular axes. Since typical values of the force factor K_c in grinding applications are large, it can be seen from Equations 6.19 and 6.20 that only marginal change in the location of the maximum torque along the contour will occur.

6.5 Highlights of the analytical parametric study

The above study shows the following:

6.5.1 Inertial load

- The radial and angular axes torque ratios vary linearly with the change in magnitude of the mass on the carriage.
- The radial axis torque ratio vary as a quartic polynomial in V_t and always increases as V_t increases.
- The angular axis torque ratio vary as a quartic polynomial in V_t. It may increase or decrease depending upon the magnitude and direction of motion of the carriage.
- The location on the contour where the maximum radial axis torque, and hence the radial axis torque ratio, occurs does not change with the change in the value of the mass on the carriage, or the contouring speed.
- For the angular axis, the location on the contour where

the maximum torque, and hence its torque ratio, varies with the change in the magnitude of the mass on the carriage. The contouring speed V_t has no effect on this location.

6.5.2 Grinding application

- The radial and angular torque ratios vary linearly with the variations in the machining force factor K_c.
- The radial and angular torque ratios vary as a quartic polynomial in the contouring speed. The variation, however, approaches a linear relationship.
- The location on the contour where the maximum torque, and hence torque ratio, does not change with either the change of the contouring speed or the mass on the carriage.

6.6 Optimum loading of the polar NC machine

In the previous chapter it was shown that a change in the relative location of the contour pole from the machine pole results in a change in the shape and magnitude of the torque ratio plots. This feature is used here to find a location for the contour pole that would give a minimum value for the total torque on both axes for a given machining operation. The minimization process in effect increases the load carrying capacity of the NC machine. After carrying out the minimization process, the load imposed on the machine can be increased further up to the limit when either

of the torque ratios have a value of unity.

6.6.1 Total torque required on both axes in contouring operation

The torque required by the two axes during a machining operation is given by:

$$T_T = T_R + T_\theta \qquad 6.29$$

where T_T is the total torque of the two axes

This total torque varies as the tool travels along the contour. It is important to know its exact characteristics, and change in magnitude for parameter variations. Figures 6.6 to 6.19 show plots of the sum of torques T_T versus angular location on the contour.

For an inertial load

The figures are organized as follows:

Figures 6.6 to 6.9 are for the case of the contour poles located as shown in Figures 5.4 to 5.7,

Figures 6.10 to 6.12 are for the case of the contour pole location coinciding with that of minimum area error given in Chapter 5. The plots are given for one contouring speed of 1.52 cm/s (3 ft/min).

From the figures, it can be seen that for all the test contours and for the two cases of contour pole locations, the maximum total torque occur at exactly the same location on the contour irrespective of the contouring speed or the

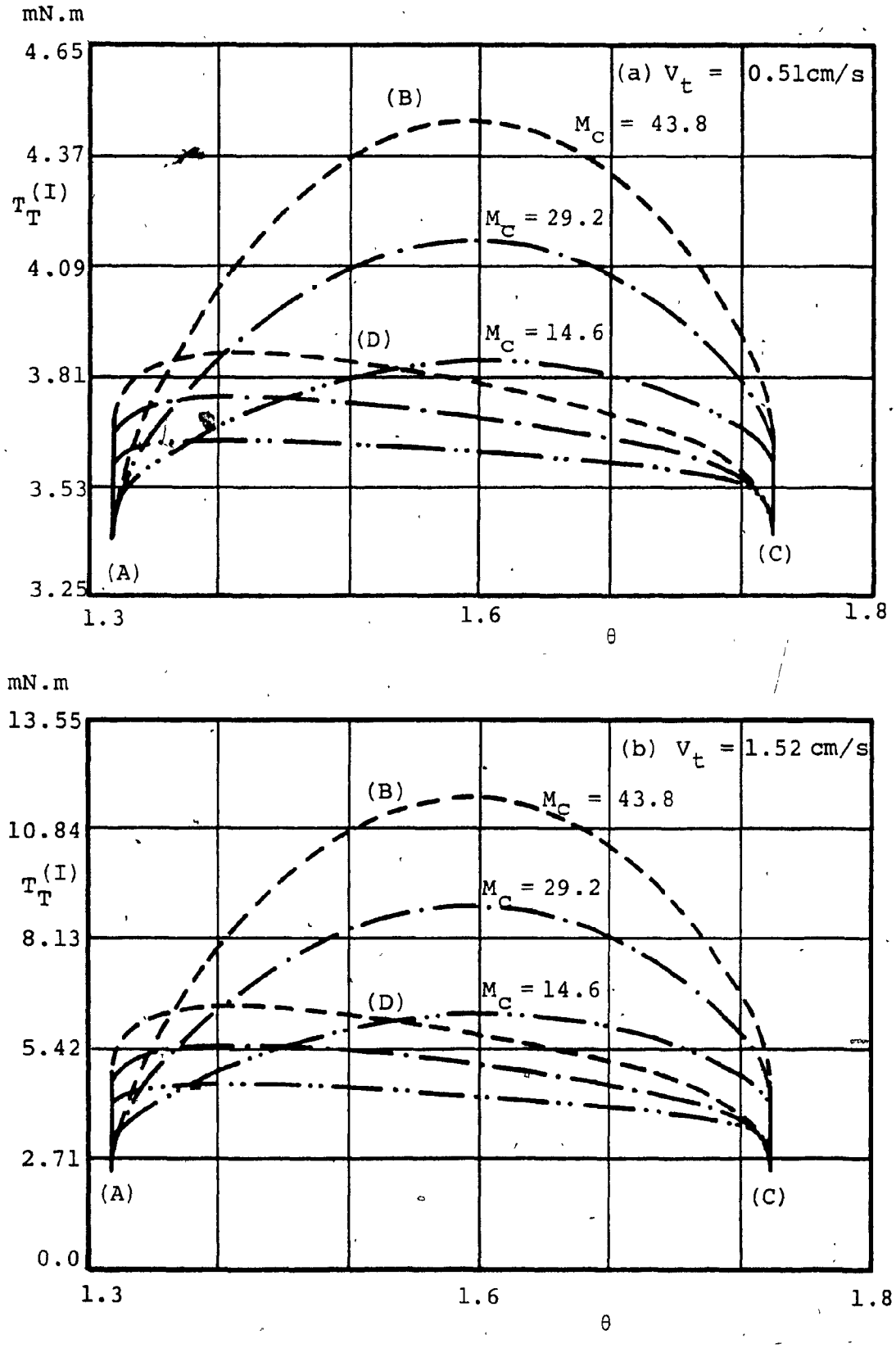


Figure 6.6: Sum of Torques Versus Angular Location for the Circular Contour (Inertial Load). M_C is in kgs.

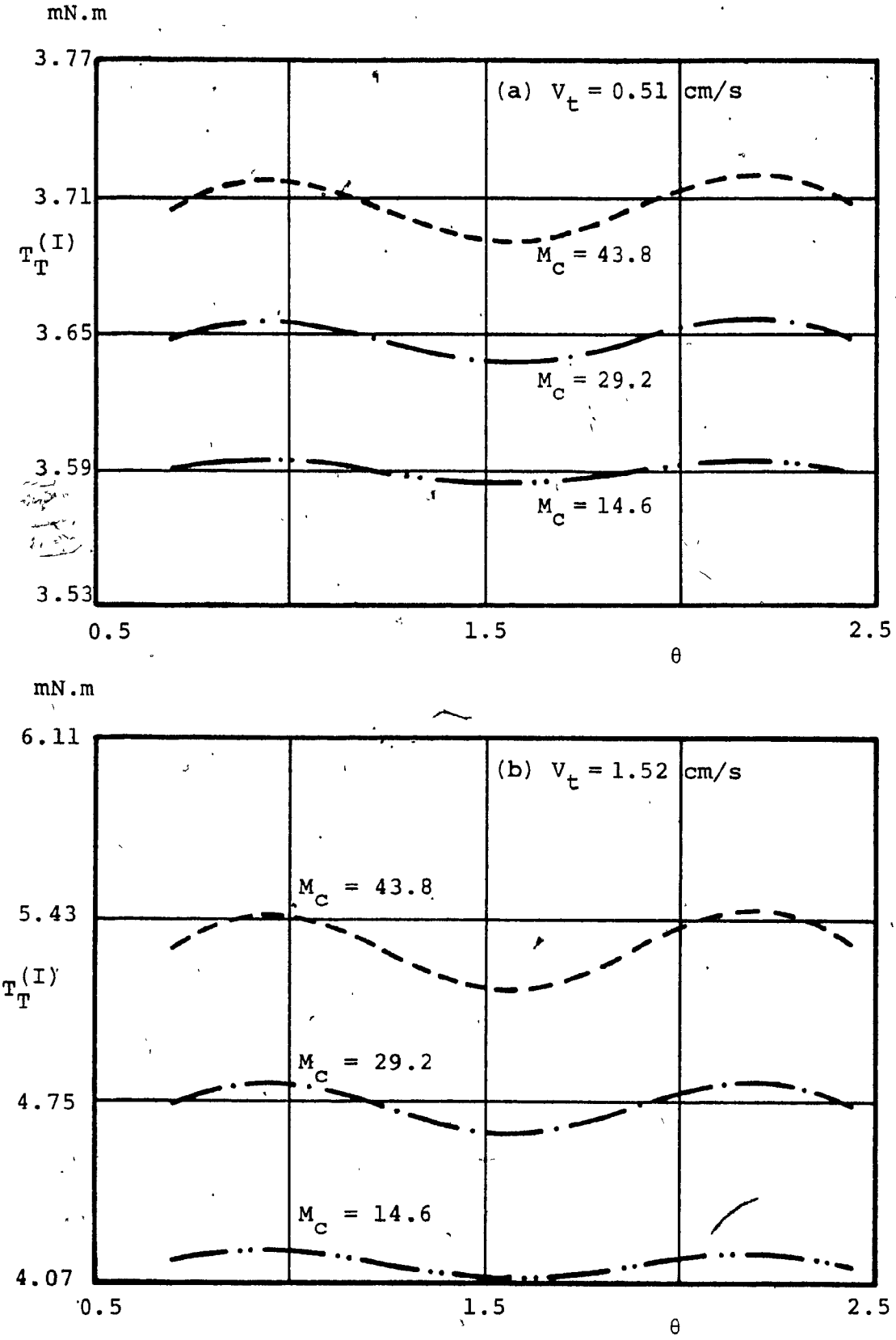


Figure 6.7: Sum of Torque Versus Angular Location for the Witch of Agnesi Contour (Inertial Load). M_c is in kgs.

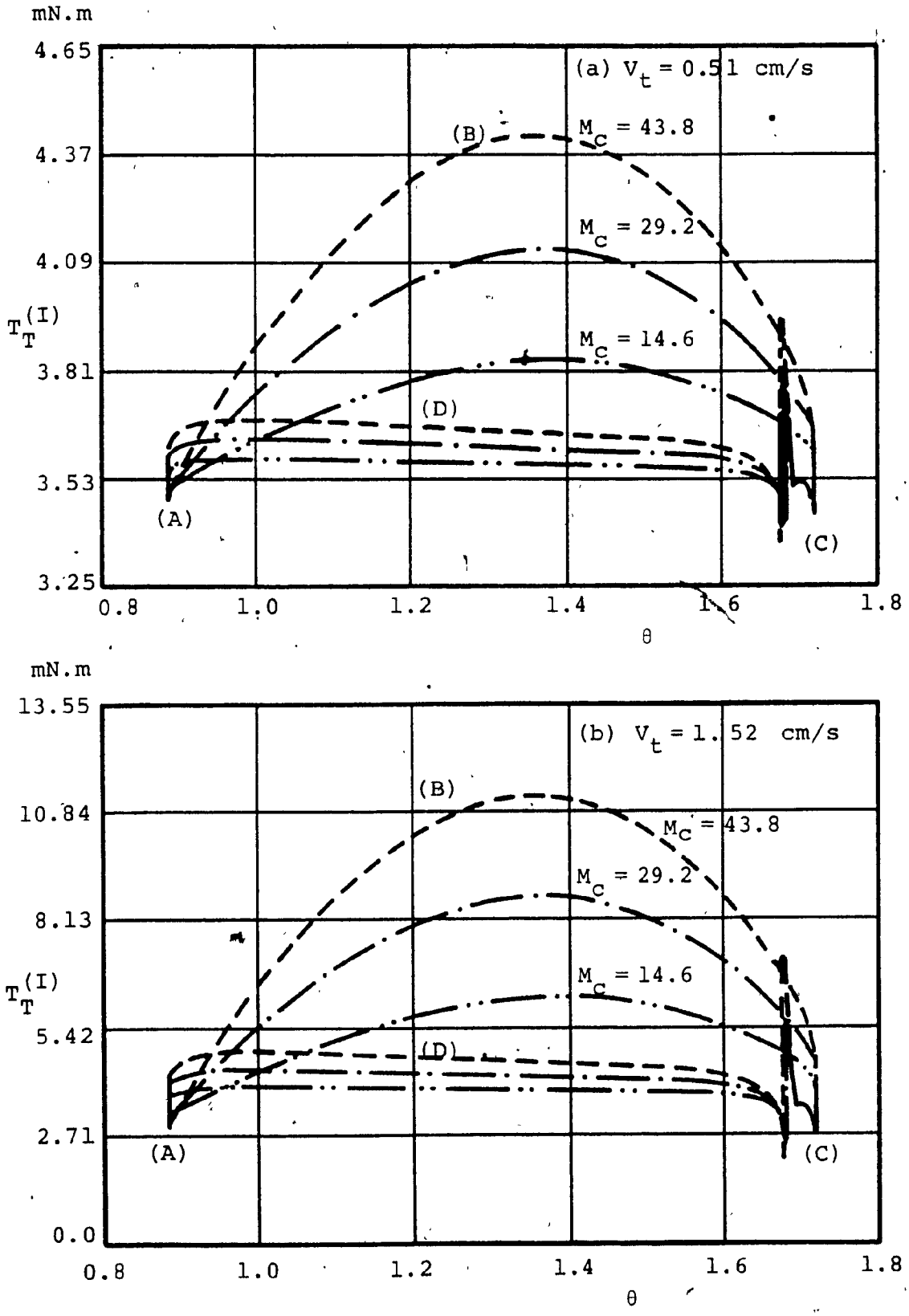


Figure 6.8: Sum of Torque Versus Angular Location for the Limacon of Pascal Contour (Inertial Load). M_c is in kgs.

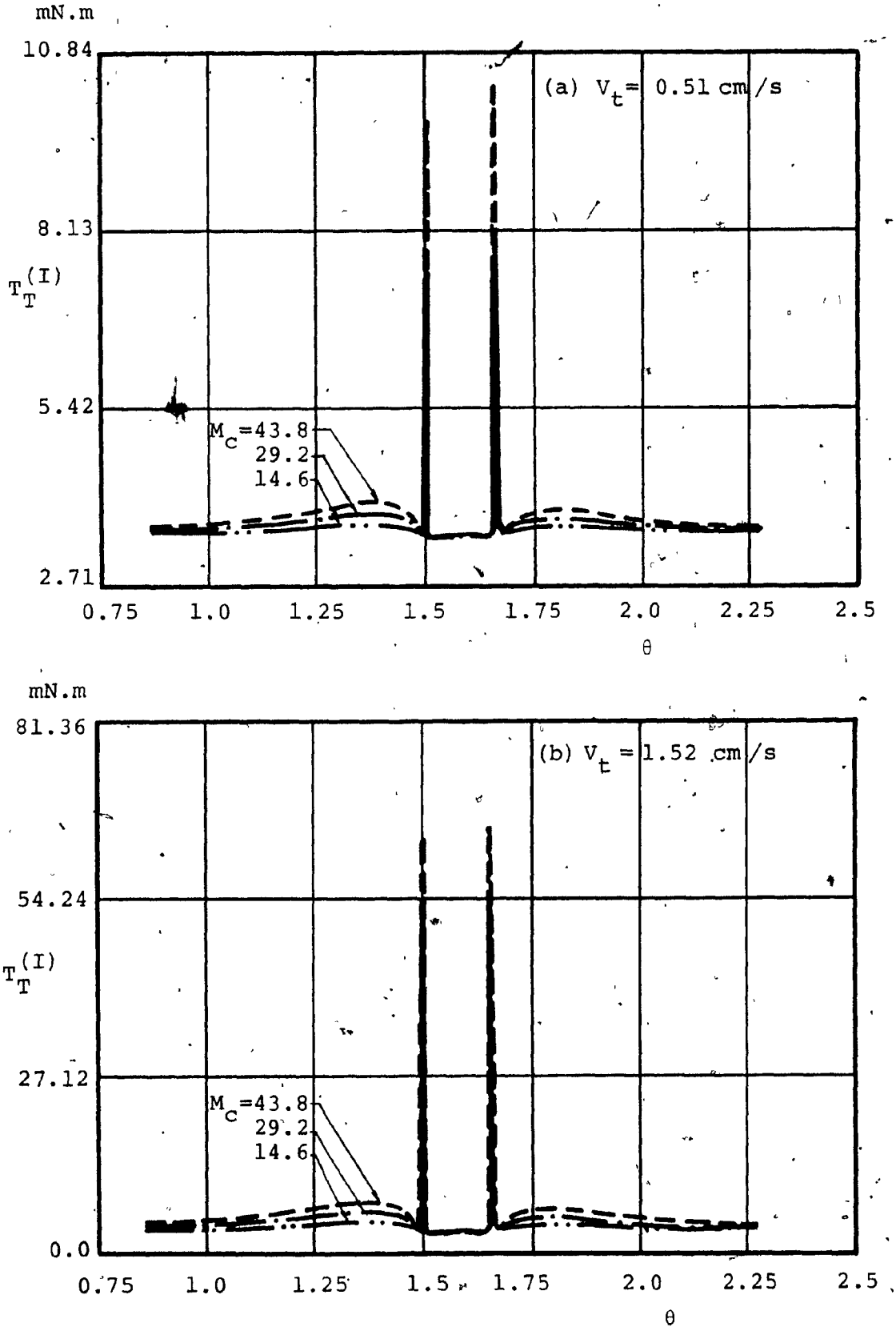


Figure 6.9: Sum of Torque Versus Angular Location for the Serpentine Contour (Inertial Load). M_c is in kgs.

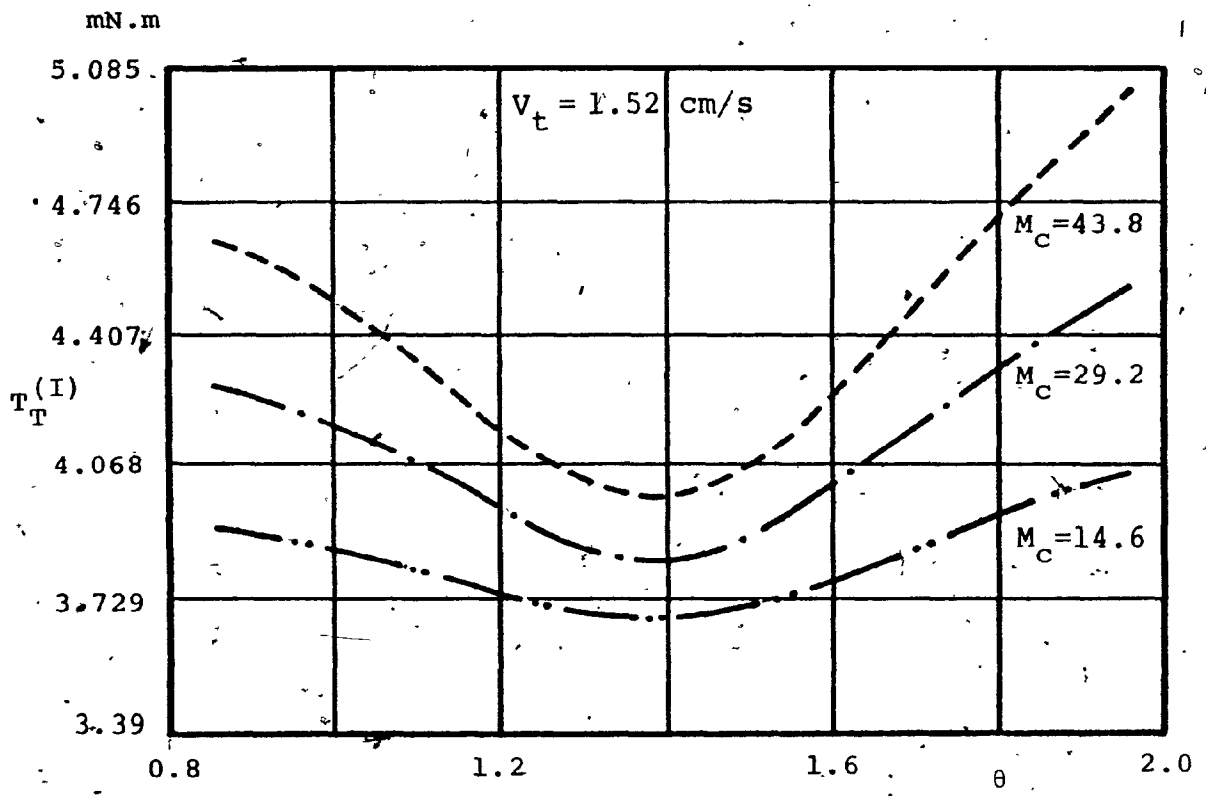


Figure 6.10: Sum of Torque Versus Angular Location for the Witch of Agnesi Contour, Pole at Location of Minimum Area Error (Inertial Load). M_c is in kgs.

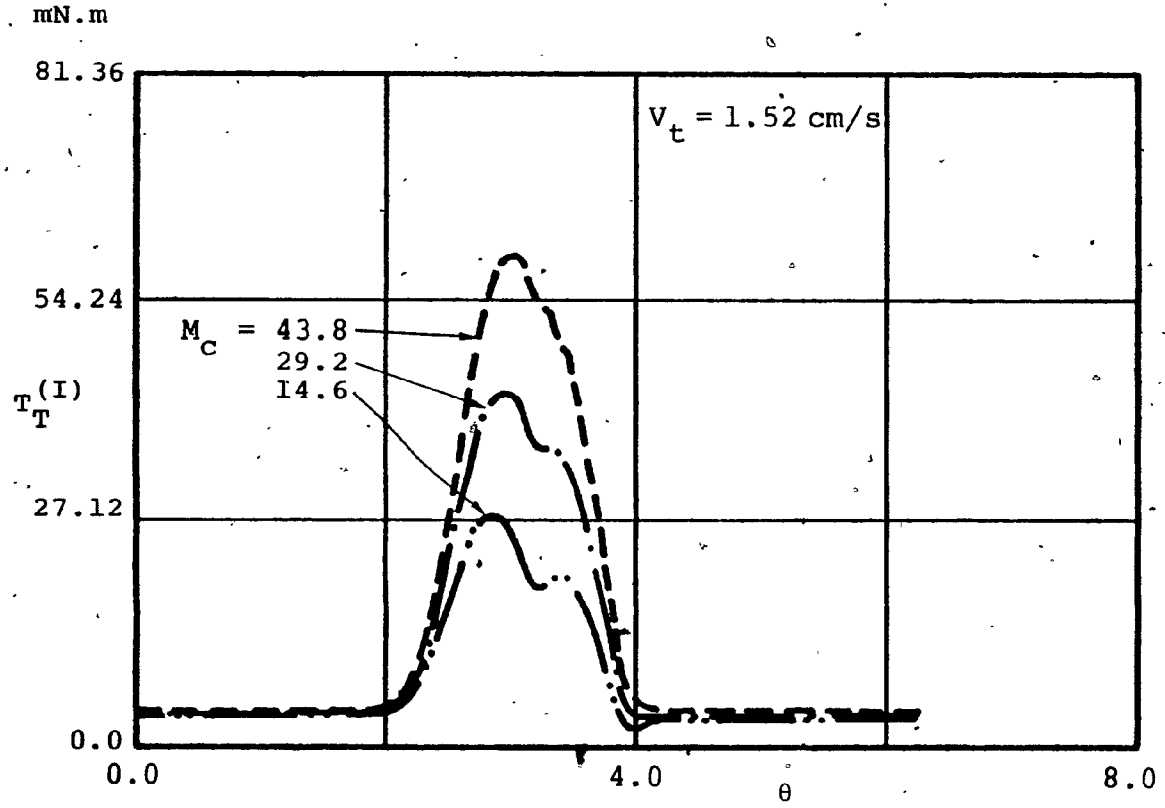


Figure 6.11: Sum of Torque Versus Angular Location for the Limacon of Pascal-Contour, Pole at the Location of Minimum Area Error (Inertial Load). M_c is in kgs.

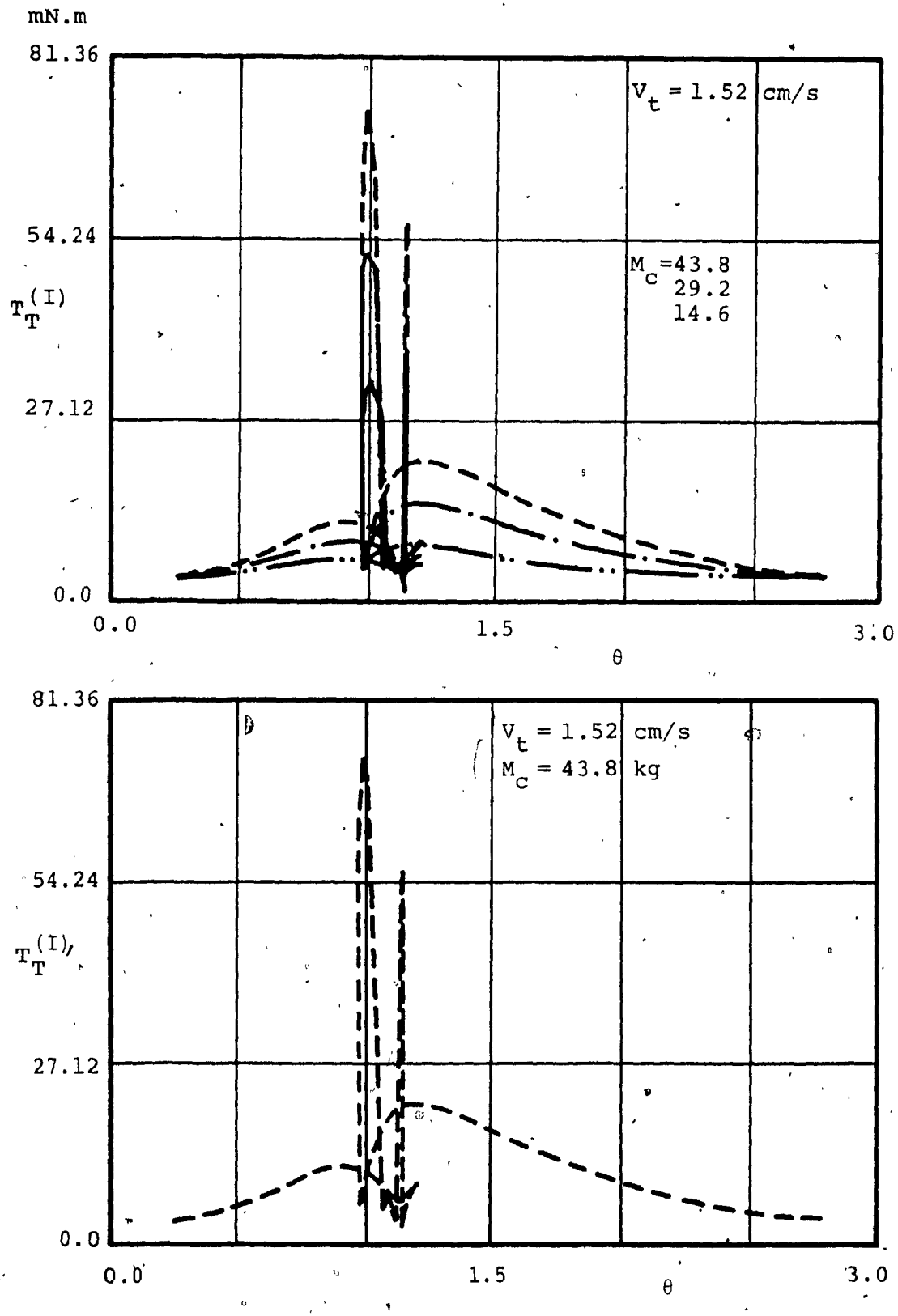


Figure 6.12: Sum of Torque Versus Angular Location for the Serpentine Contour, Pole at the Location for Minimum Area Error (Inertial Load). M_c is in kgs.

mass on the carriage. This group of figures also show that the magnitude of the total torque changes with the change in the contour pole location. For example, the total torque for the Limacon of Pascal changes from T_T=1.15 N.cm (0.85 lbf.ft) to T_T=6.1 N.cm (3.24 oz.in) for a change of contour pole location from that shown in Figure 5.6 to that of minimum area error respectively.

For a grinding application

The figures are organized as follows:

Figures 6.13 to 6.16 are for cases of the contour pole location as shown in Figures 5.4 to 5.7.

Figures 6.17 to 6.19 are for the cases of the contour pole location

The plots in Figures 6.13 to 6.19 show that for all the test contours, the total torque increases linearly with the increase in the contouring speed. Test results (not presented) show that T_T also varies linearly with variation in the machining force factor K_c. Furthermore, the location of the maximum total torque along the contour does not change with the change in contouring speed or with K_c.

The plots also show that the change in contour pole location results in a change of the magnitude of the maximum total torque.

6.6.2 Formulation of the optimization problem

The objective here is to find the location of the

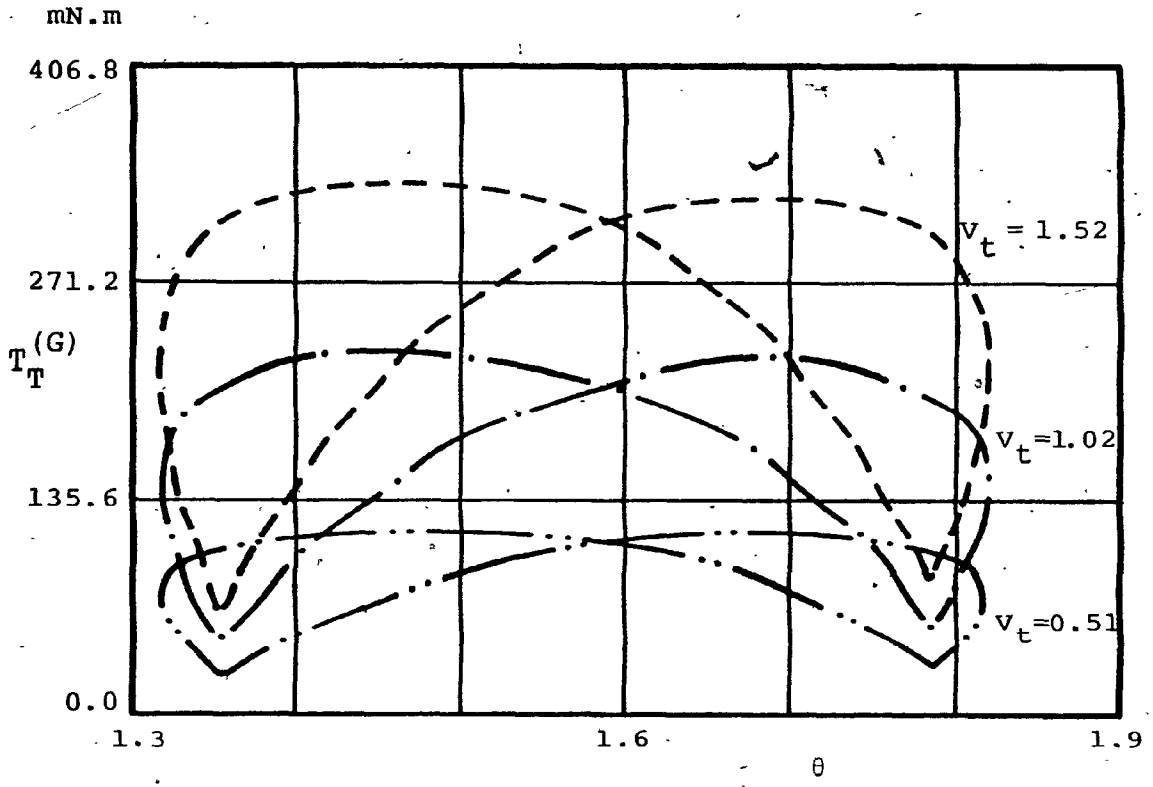


Figure 6.13: Sum of Torque Versus Angular Location for the Circular Contour (Grinding Application). V_t is in cm/s.

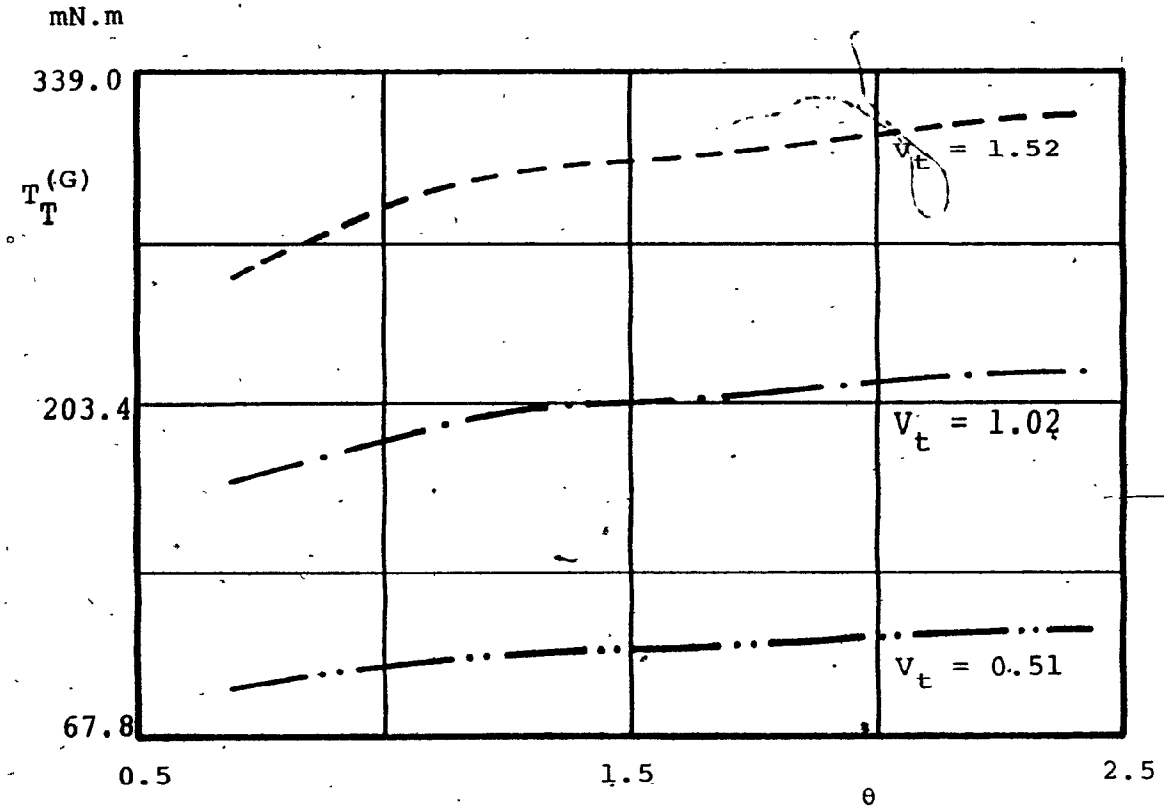


Figure 6.14: Sum of Torque Versus Angular Location for the Witch of Agnesi Contour (Grinding Application). V_t is in cm/s.

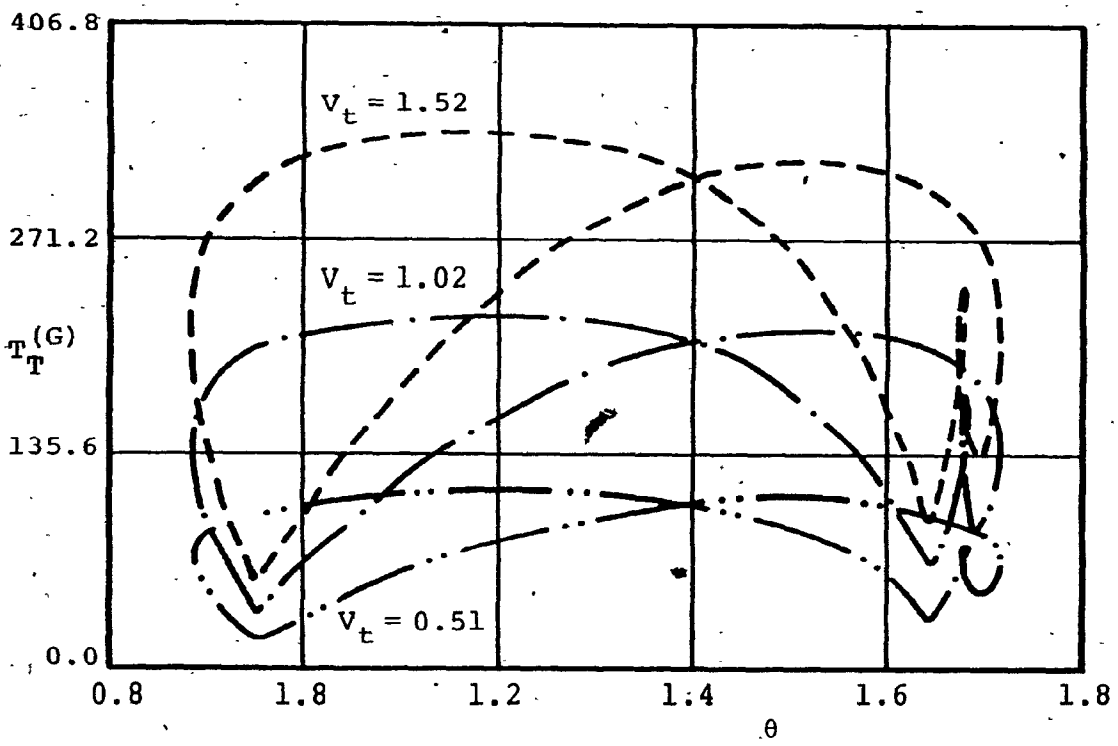


Figure 6.15: Sum of Torque Versus Angular Location for the Limacon of Pascal Contour (Grinding Application). V_t is in cm/s.

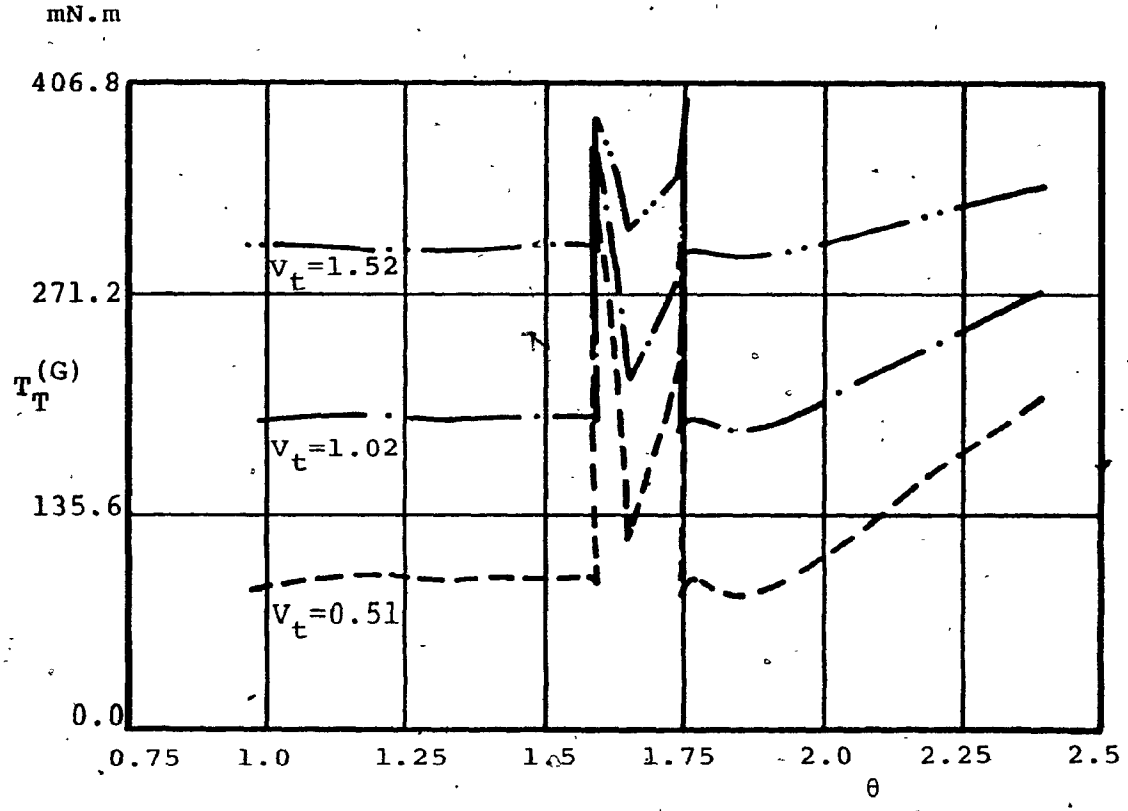


Figure 6.16: Sum of Torque Versus Angular Location for the Serpentine Contour (Grinding Application). V_t is in cm/s.

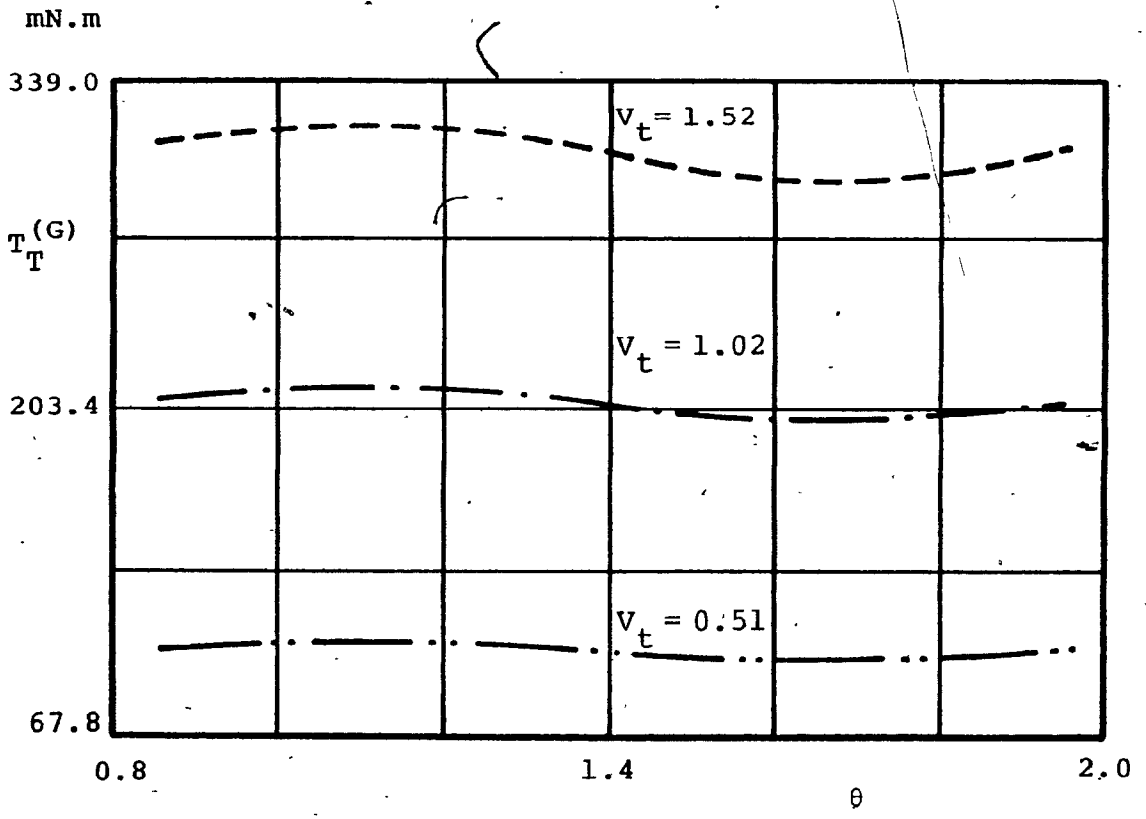


Figure 6.17: Sum of Torque Versus Angular Location for the Witch of Agnesi Contour, Pole at Location of Minimum Area Error (Grinding Application). V_t is in cm/s.

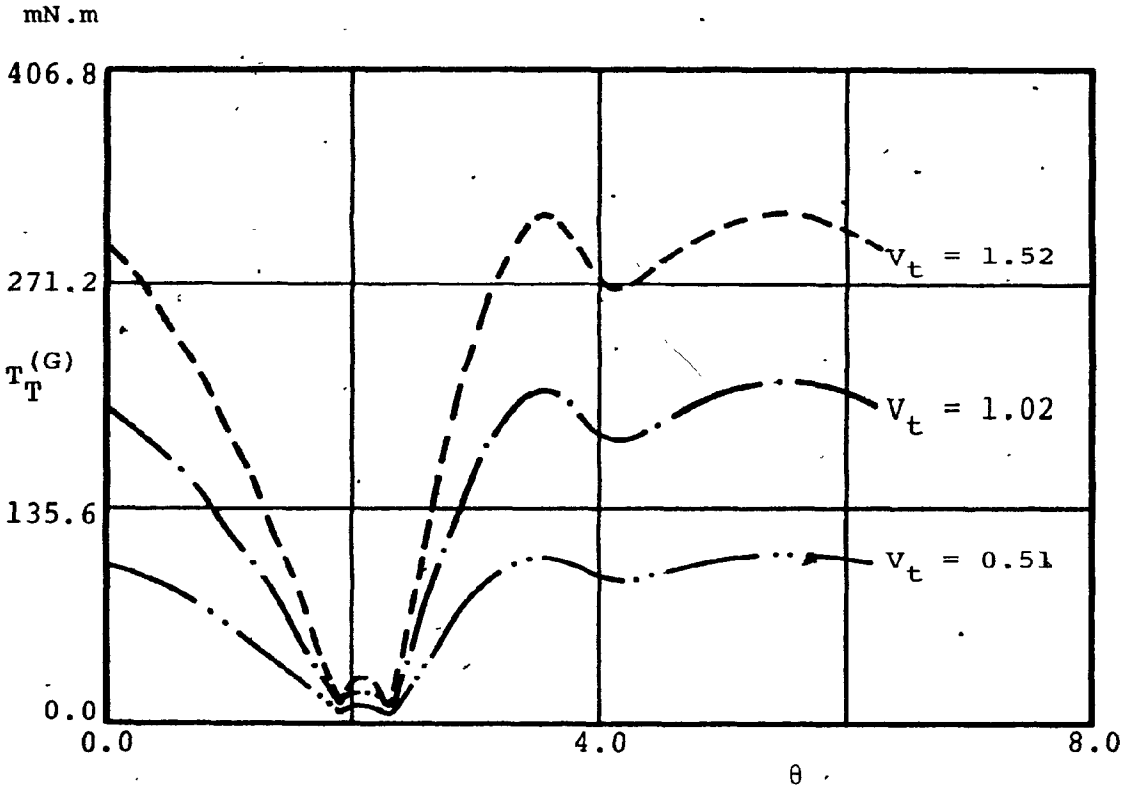


Figure 6.18: Sum of Torque Versus Angular Location for the Limacon of Pascal Contour, Pole at the Location of Minimum Area Error (Grinding Application). V_t is in cm/s.

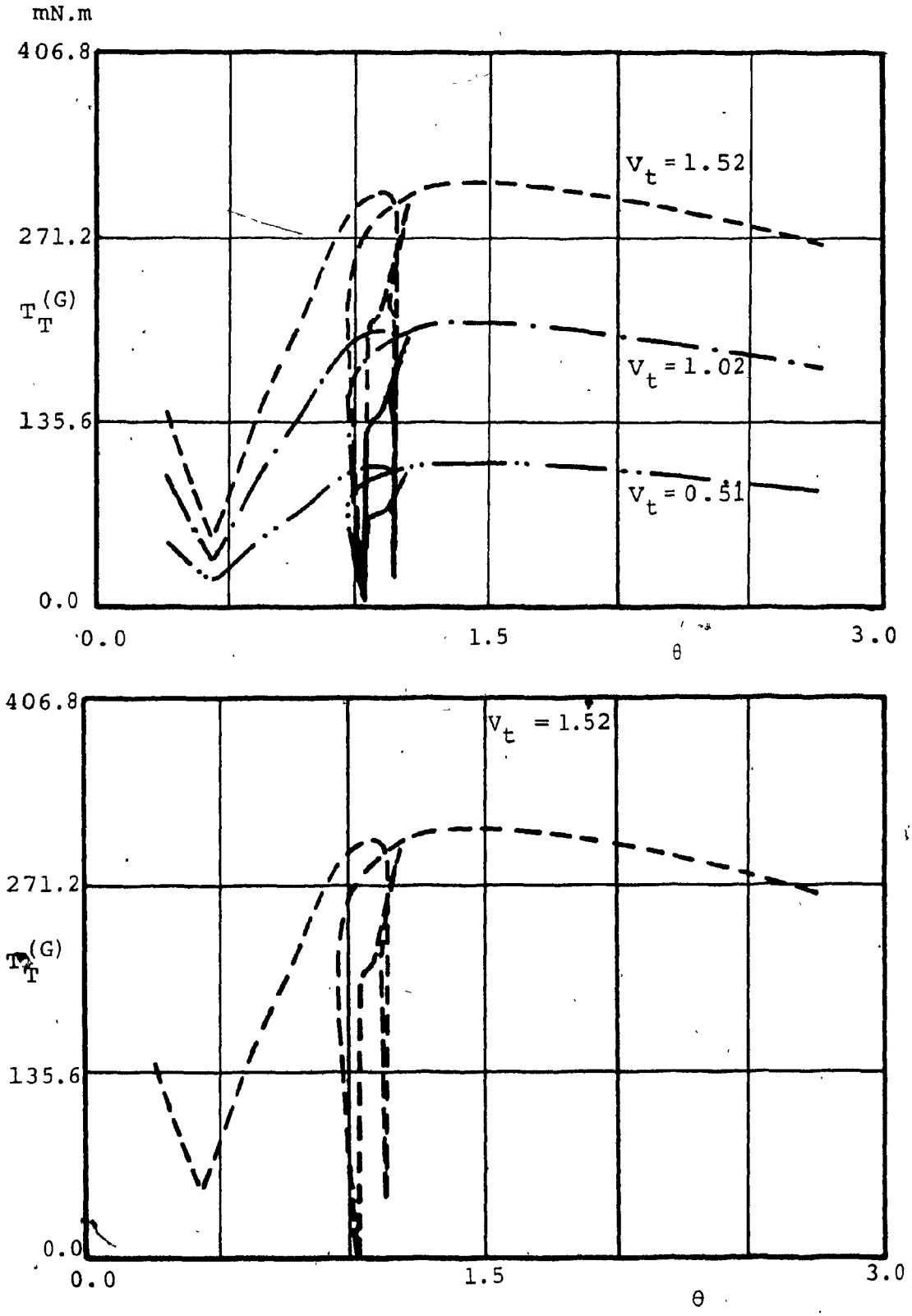


Figure 6.19: Sum of Torque Versus Angular Location for the Serpentine Contour, Pole at the Location for Minimum Area Error (Grinding Application). V_t is in cm/s.

contour pole ($C, \underline{/\beta}$) for a given contour, that will give a minimum value for the maximum total torque. Normalizing the value of axes torques by the available torque from the respective stepping motors, the objective function for minimization can be formulated as:

$$T_T = T_R + T_\theta \qquad \text{maximum along the contour} \qquad 6.30$$

In order that both stepping motors carry equal shares of the load, the objective function in Equation 6.30 can be modified and the optimization problem can be stated as follows:

Minimize;

$$\frac{T_T}{\left|\{1 - |(T_R - T_\theta)|\}\right|} \qquad 6.31$$

Subject to;

a) Work area constraint of the NC machine and is given by:

7.62 cm. (3 inch) $< r <$ 33.02 cm. (13 inch)

$0 < \theta < 2.8$ (in radians)

b) A constraint that would not allow the placement of the machine pole inside a closed contour due to geometrical limits imposed by the construction of the NC machine.

The Hooke and Jeeves optimization technique used earlier, and described in Chapter 4 is also used here. Upon convergence of the optimization, the program returns the maximum values of torque ratios found along the contour.

During the optimization process no constraint is

imposed on the magnitude of the radial and angular torque ratios and their values can exceed unity. Eliminating this constraint was found to speed up the optimization process dramatically. The torque ratio values returned by the optimization routine are subsequently scaled to unity by modifying the load on the mechanism or the contouring speed. Equations 6.1 to 6.12 are used to carry out the scaling. Changing the load on the NC machine or the contouring speed was shown, earlier in this chapter, to have no effect on the location where the maximum torques occur. The scaling procedure in effect increases the load carrying capacity of the NC machine.

6.6.3 Results of loading optimization for four test contours

The algorithm is tested on the four test curves used earlier namely, a) a circle, b) a Witch of Agnesi, c) a Limacon of Pascal, and d) a Serpentine curve.

Results for two inertial loads of 43.77 kg (3 slug) and 145.9 kg (10 slug) are given in Table 6.1. The results show that the magnitude of the inertial load has no bearing on the pole location for optimal loading, and an extremely large mass can be accommodated before the mechanism is overloaded. However, the contouring speed can be increased only up to 1.74 cm/s (3.44 ft/min). This is due to the fact that the mechanism is geared towards dissipative type applications and the axes drives reach their maximum stepping

Table 6.1: Results of Optimization for Loading Conditions (Inertial Load)

For contouring speed of 1.53 cm/s (3ft/min), and		Circle		Witch of Agnesi		Limacon of Pascal		Serpentine	
Initial mass on carriage	kg (slug)	43.77 (3.0)	145.9 (10.0)	43.77 (3.0)	145.9 (10.0)	43.77 (3.0)	145.9 (10.0)	43.77 (3.0)	145.9 (10.0)
Initial machine pole location:	r cm (in)	53.8 (21.213)	-	0.0 (0.0)	-	53.8 (21.213)	-	17.8 (7.0)	-
	θ rad	3.927	-	0.0	-	3.927	-	4.712	-
Results from the optimization program,									
Optimum machine pole location:	r cm (in)	0.0 (0.0)	-	299.2 (117.8)	-	4.06 (1.6)	-	21.6 (8.5)	-
	θ rad	0.0	-	3.528	-	0.588	-	4.242	-
Objective function		0.0136	-	0.0126	0.0127	0.0136	-	0.0146	0.0145
Maximum torque ratios:	T_R	0.0132	-	0.0126	0.0126	0.0133	-	0.0139	0.0144
	T_Θ	4.2×10^{-4}	-	4.9×10^{-5}	-	1.2×10^{-3}	-	4.2×10^{-3}	4.2×10^{-3}
To achieve maximum allowable torque ratios, $T_R = 1$ or $T_\Theta = 1$, set M_c or V_t to:									
Maximum mass on carriage	kg (slug)	3316.2 (227.3)	-	3473.9 (238.1)	-	3291.5 (225.6)	-	3148.5 (215.8)	-
Maximum contouring speed	cm (fpm)	1.75 3.4375	-	1.75 3.4375	-	1.75 3.4375	-	1.75 3.4375	-

rate before the contouring speed is high enough to cause large inertial torques.

For dissipative type loads the grinding application is again used as an example. Results for loads corresponding to depth of cuts of 0.254 mm (0.01 inch) and 0.508 mm (0.02 inch) are shown in Table 6.2. Here also the magnitude of the load has no effect on the position of the contour pole for optimal loading. For the contours (a), (b), and (d) above, the results show that with a contouring speed of 1.016 cm/s (2 ft/min) the depth of cut can be increased beyond 0.508 mm (0.02 inch), while for the Limacon of Pascal a depth of cut of 0.457 mm (0.018 inch) cannot be exceeded. The table also shows that for the smaller load the contouring speed for all four contours can be increased to 1.74 cm/s (3.44 ft/min), which corresponds to the maximum stepping rate of the axes drives. For the larger load the contouring speed can be increased for contours (a), (b), and (d) but must be decreased for contour (c).

Table 6.2: Results of Optimization for Loading Conditions (Grinding Application)

For contouring speed of 1.02 cm/s (2ft/min), and

		Circle		Witch of Agnesi		Limacon of Pascal		Serpentine	
Initial depth of cut	mm (in)	0.254 (0.01)	0.508 (0.02)	0.254 (0.01)	0.508 (0.02)	0.254 (0.01)	0.508 (0.02)	0.254 (0.01)	0.508 (0.02)
Initial machine pole location:	r cm (in)	53.8 (21.2)	-	0.0 (0.0)	-	53.8 (21.2)	-	17.8 (7.0)	-
	θ rad	3.927	-	0.0	-	3.927	-	4.712	-
Results from the optimization program,									
Optimum machine pole location:	r cm (in)	0.0 (0.0)	-	38.91 (15.32)	-	0.747 (0.294)	-	0.0 (0.0)	-
	θ rad	0.0	-	1.971	-	2.642	-	0.0	-
Objective function		0.454	0.894	0.217	0.421	0.550	1.088	0.490	0.967
Maximum torque ratios	T_R	0.274	0.042	0.198	0.383	0.075	0.121	0.094	0.176
	T_θ	0.426	0.823	0.186	0.376	0.543	1.086	0.469	0.937
To achieve maximum allowable torque ratios, $T_R = 1$ or $T_\theta = 1$, set d or V_t to:									
Depth of cut d	mm (in)	0.58 (0.023)	-	1.346 (0.053)	-	0.5 (0.019)	-	0.541 (0.0213)	-
Maximum contouring speed V_t,	cm/s (fpm)	1.75 (3.437)	(2.33)	1.18 (3.437)	-	1.75 (3.437)	0.94 (1.86)	1.05 (2.06)	1.02 (2.0)

CHAPTER 7

GRAPHICAL PROCEDURE FOR EVALUATING THE LOAD AND THE LOAD CARRYING CAPACITY OF THE NC MACHINE

7.1 Introduction

In a practical environment, where production constraints such as shorter machining time and optimal machine load are considered, it is desirable to utilize the machinery to its full capability. With an open loop controller used on the NC machine, it is necessary to know the magnitude and type of the machining load, as well as the capability of the machine in order to avoid overloading the NC machine.

In a controlled and automated environment where computer capability is available, the analyses presented in Chapters 4, 5, and 6, may be used to prevent overloading the NC machine. In cases where decisions concerning the operating parameters have to be made on a machine shop floor, or in the absence of computer capability, a simple method to determine the load imposed by the machining operation and the load carrying capacity of the NC machine would prove helpful.

In this chapter, a graphical procedure is outlined for ease of evaluation of the load and the maximum load carrying capacity of the NC machine. The procedure is essentially a graphical solution of the torque equations, Equations 6.13

and 6.14, of the two axes. The graphical solution is presented in the form of composite plots of the torques versus the different contour and the operating parameters.

To use the graphical procedure, a user must know the minimum radius of curvature of the contour being tracked, and the relative location between the radial displacement of the point of minimum radius of curvature from the machine pole. Using this information, together with the information on the desired operating parameters, the maximum torque imposed by the machining operation on both axes and the respective stepping motors pull-out torque can be found.

7.2 Description and organization of the composite graphs

7.2.1 Inertial load

For the radial axis:

The maximum radial axis torque for the case of an inertial load was shown in Chapter 6 to occur at $\gamma_R^{(I)} = \pi/2$. Substituting for γ in Equation 6.13 and setting the force factor K_c to zero yields:

$$\hat{T}_R^{(I)} = M_c \tau_1 \qquad 7.1$$

$$\tau_1 = K_2 V_t^2/\rho$$

The torque available from the stepping motor is found by substituting $\gamma_R^{(I)} = \pi/2$ in Equation 5.20. Figure 7.1 shows

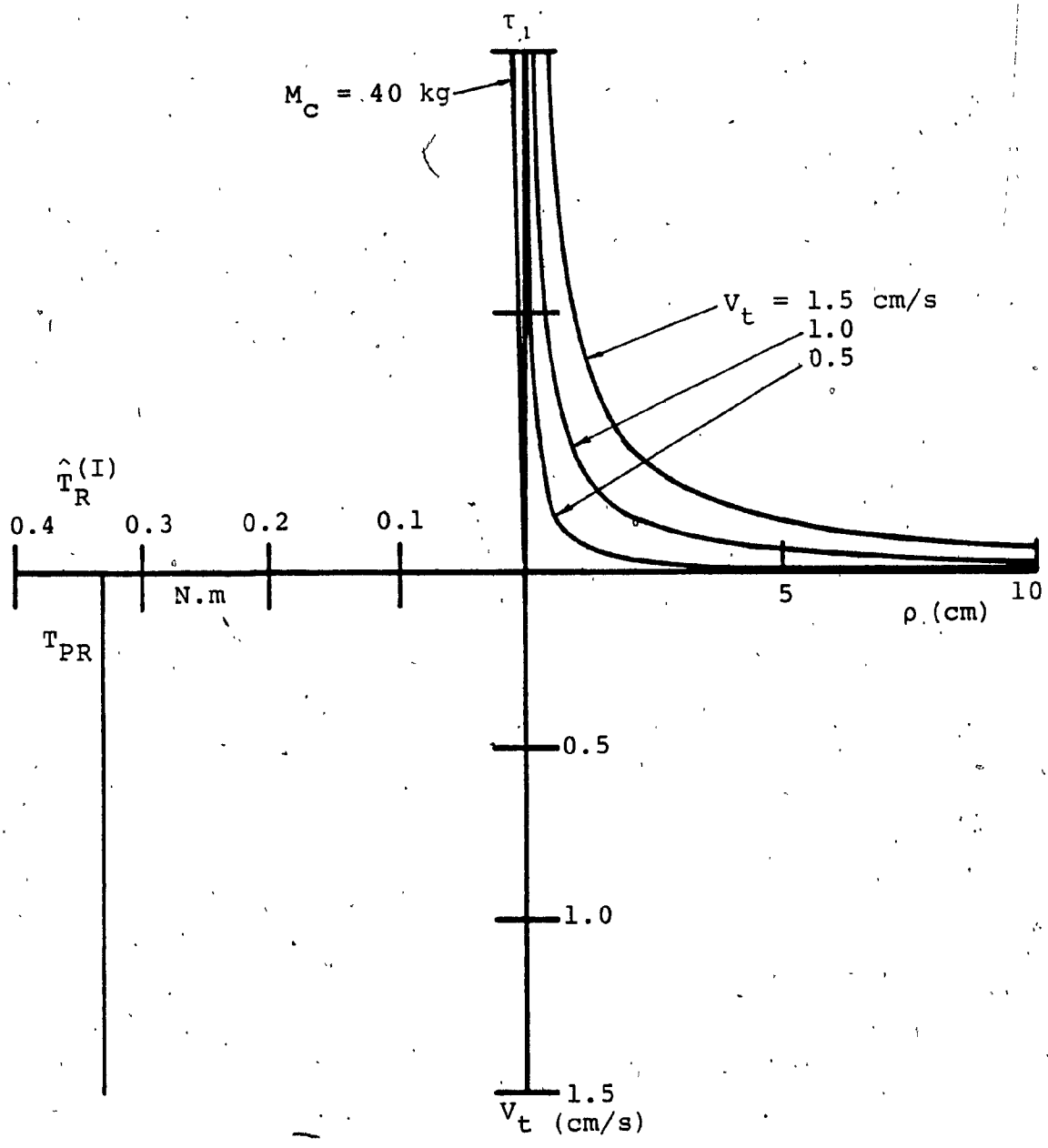


Figure 7.1: Torque Required and Available from the Radial Axis-Drive (Inertial Load).

the composite graph used to solve for $\hat{T}_R^{(I)}$, and for the torque available from the stepping motor T_{PR}. This graph is set up as follows:

Quadrant I: τ_1 versus ρ for different V_t.

Quadrant II: $\hat{T}_R^{(I)}$ versus τ_2 for different M_c.

Quadrant III: T_{PR} versus V_t.

For the angular axis

The angle γ at which the maximum angular axis torque occurs is given by Equation 6.15. This value $\gamma_\theta^{(I)}$ is substituted in Equation 6.14 so as to obtain the maximum torque on the angular axis. The constant torque of the arm support, given by $K_c F_f L_A$, is transferred with a negative sign to the stepping motor pull-out torque equation. Furthermore, the force factor K_c is set to zero. Thus:

$$\hat{T}_\theta^{(I)} = 2K_1 V_t^2 \tau_4 \qquad 7.2$$

where;

$$\tau_4 = \left[5\tau_3 - \sqrt{\tau_3^2 + 0.5}\right] x \left[0.5 - 2\tau_3^2 + 2\tau_3\sqrt{\tau_3^2 + 0.5}\right]^{\frac{1}{2}}$$

$$\tau_3 = \tau_2/\rho$$

$$\tau_2 = \Lambda_{24} = \left[M_c r^3 + J_A r\right]/8$$

The torque available from the angular axis stepping motor is found by substituting for $\gamma_R^{(I)}$ in Equation 5.21.

Figures 7.2 and 7.3 are used to solve for $\hat{T}_{\theta}^{(I)}$ and $T_{PR}^{(I)}$. The figures are organized as follows:

Figure 7.2:

Quadrant I: τ_2 versus r for different M_c.

Quadrant II: τ_3 versus τ_2 for different ρ.

Quadrant III: τ_4 versus τ_3 for different r.

Quadrant IV: $\hat{T}_{\theta}^{(I)}$ versus τ_4 for different V_t

Figure 7.3:

Quadrant I: τ_5 versus τ_3 for different r_c.

$$\tau_5 = \sin \gamma_{\theta}^{(I)}/r$$

$$= \frac{\tau_3 - \left[\tau_3^2 + 0.5\right]^{\frac{1}{2}}}{r}$$

Quadrant II: $T_{P\theta}$ versus τ_5 for different V_t.

7.2.2 Grinding application

The maximum torque for the radial and angular drives occurs at angles $\gamma_R^{(G)}$ and $\gamma_{\theta}^{(G)}$ given by Equations 6.19 and 6.20, respectively. Substituting these values in Equations 6.13 and 6.14, respectively, results in the maximum torques required from the axes drives while tracking a contour.

For the radial axis

The maximum torque is given by:

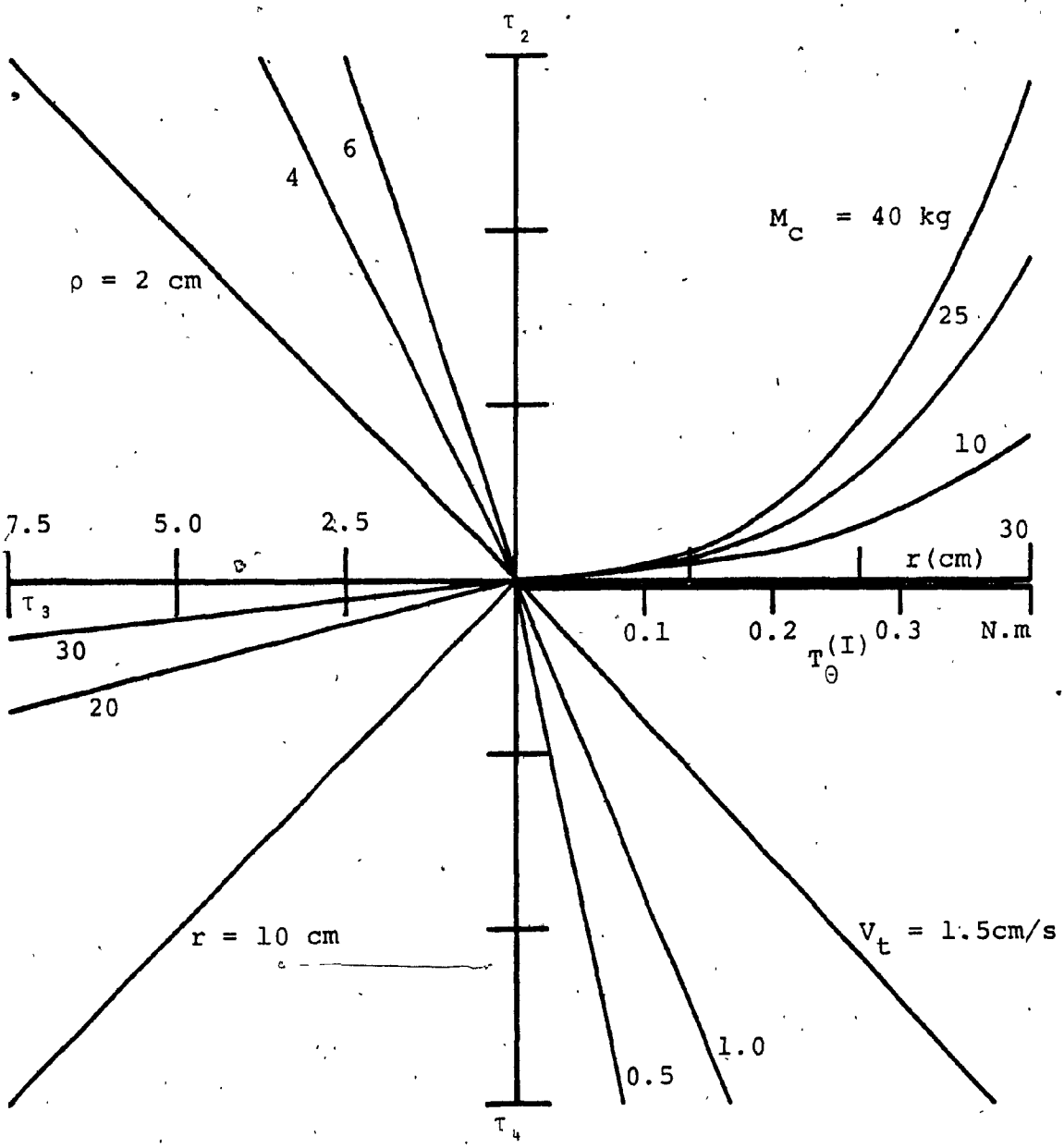


Figure 7.2: Torque Required and Available from Angular Axis-Drive (Inertial Load). Part 1 of 2.

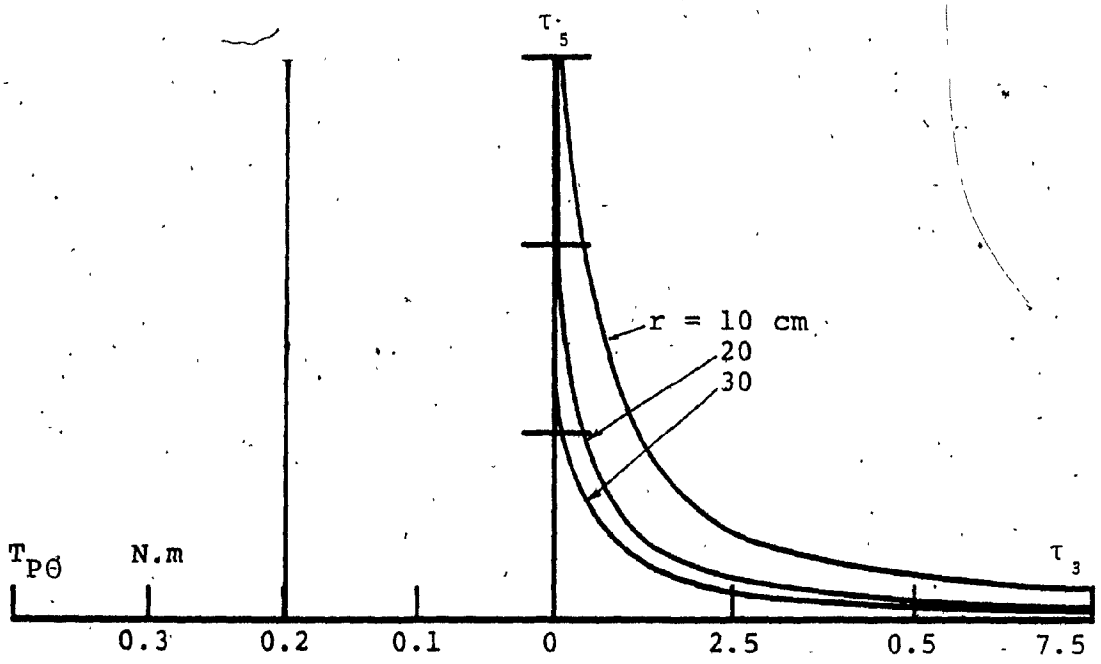


Figure 7.3: Torque Required and Available from the Angular Axis-Drive (Inertial Load). Part 2 of 2.

$$\hat{T}_R^{(G)} = V_t \tau_9 \qquad 7.3$$

where;

$$\tau_9 = K_c \tau_8$$

$$\tau_8 = K_2 \left[M_c \tau_7 \sin\gamma + \cos(\gamma - \alpha_c) \right]$$

$$\tau_7 = V_t \tau_6$$

$$\tau_6 = 1/(K_c \rho)$$

The pull-out torque of the radial axis stepping motor is found by substituting $\gamma_R^{(G)}$ from Equation 5.19 into Equation 5.20.

The graphs to solve for the maximum torque required from the radial axis-drive, and the pull-out torque of the stepping motor are organized in Figures 7.4 and 7.5 as follows:

Figure 7.4:

Quadrant I: τ_6 versus ρ for different K_c.

Quadrant II: τ_7 versus τ_6 for different V_t

Quadrant III: τ_8 versus τ_7 for different α_c.

Quadrant IV: τ_9 versus τ_8 for different K_c.

Figure 7.5

Quadrant I: $\gamma_R^{(G)}$ versus τ_7 for different α_c.

Quadrant II T_{PR} versus $\gamma_R^{(G)}$ for different V_t.

Quadrant III $\hat{T}_R^{(G)}$ versus τ_9 for different V_t.

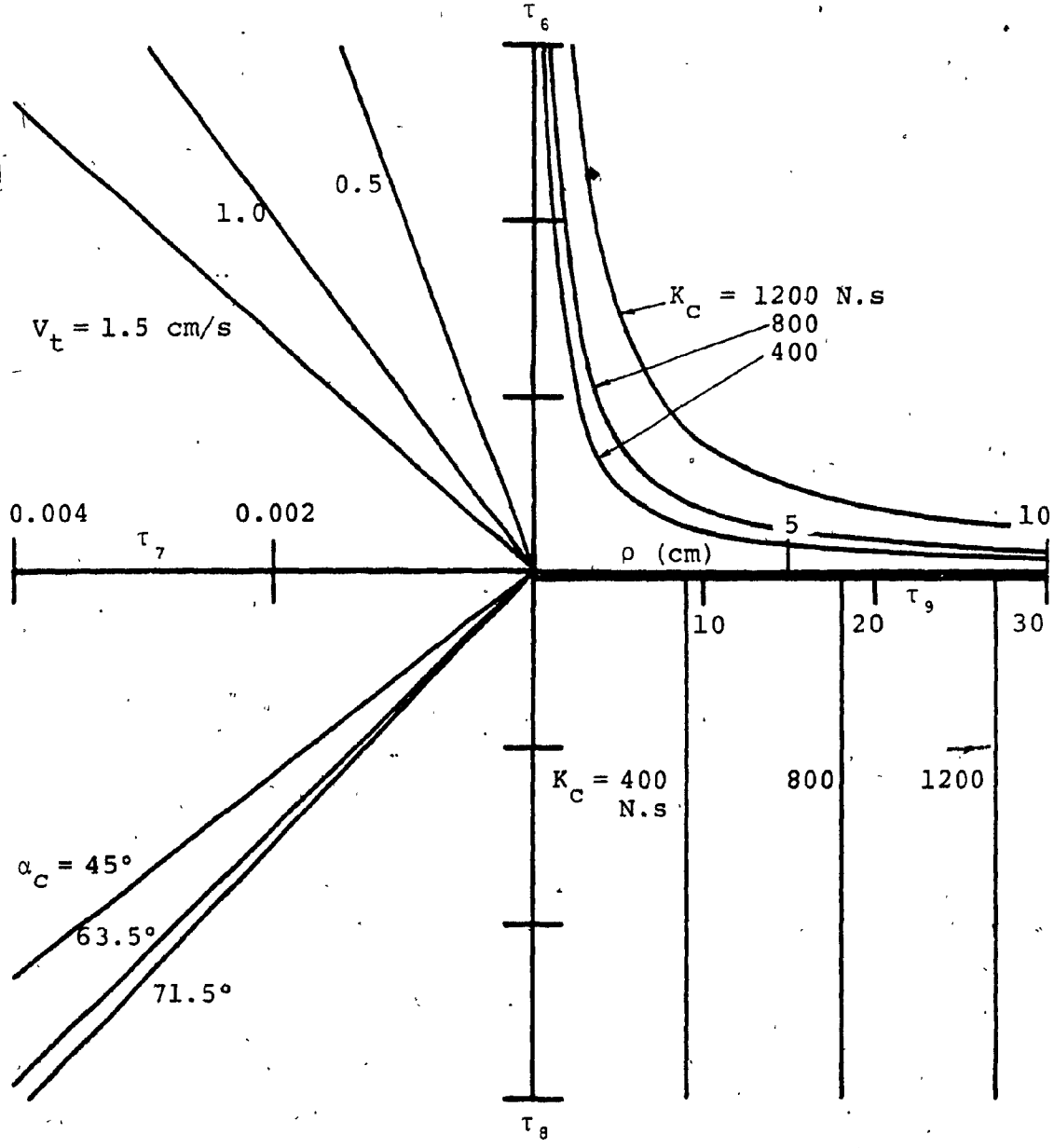


Figure 7.4: Torque Required and Available from the Radial Axis-Drive (Grinding Application). Part 1 of 2.

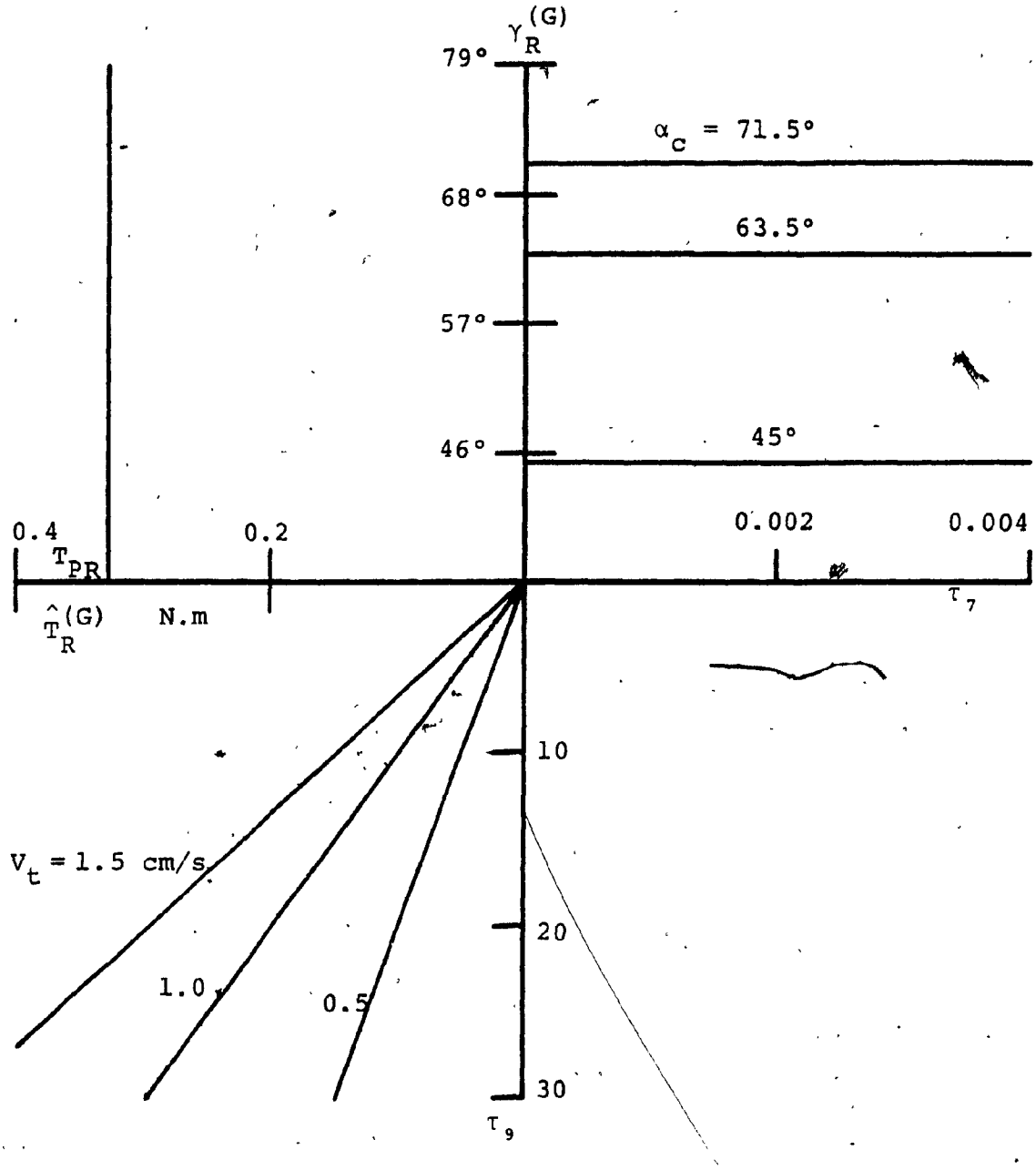


Figure 7.5: Torque Required and Available from the Radial Axis-Drive (Grinding Application). Part 2 of 2.

For the angular axis

The constant torque due to the friction force of the arm support in Equation 6.14 is transferred with a negative sign to the relation of the pull-out torque of the angular axis stepping motor. Furthermore, for a grinding application the second term of Equation 6.14 is negligible in comparison with the other two terms as mentioned previously.

The equation for the maximum torque acting on the angular axis is hence given by:

$$\hat{T}_{\theta}^{(G)} = V_t \, \tau_{12} \qquad 7.4$$

where;

$$\tau_{12} = K_c \, K_1 \, \tau_{11}$$

$$\tau_{11} = \tau_{10} \cos\gamma + \sin(\gamma - \alpha_c)$$

$$\tau_{10} = \tau_6 \left[M_c r + J_A / r \right]$$

The pull-out torque of the angular axis stepping motor is found by substituting $\gamma_{\theta}^{(G)}$ from Equation 6.20 into Equation 5.21.

The graphs for this axis are organized in Figures 7.6 and 7.7 as follows:

Figure 7.6:

Quadrant I: τ_{10} versus τ_7 for different r.

Quadrant II: τ_{11} versus τ_{10} for different α_c.

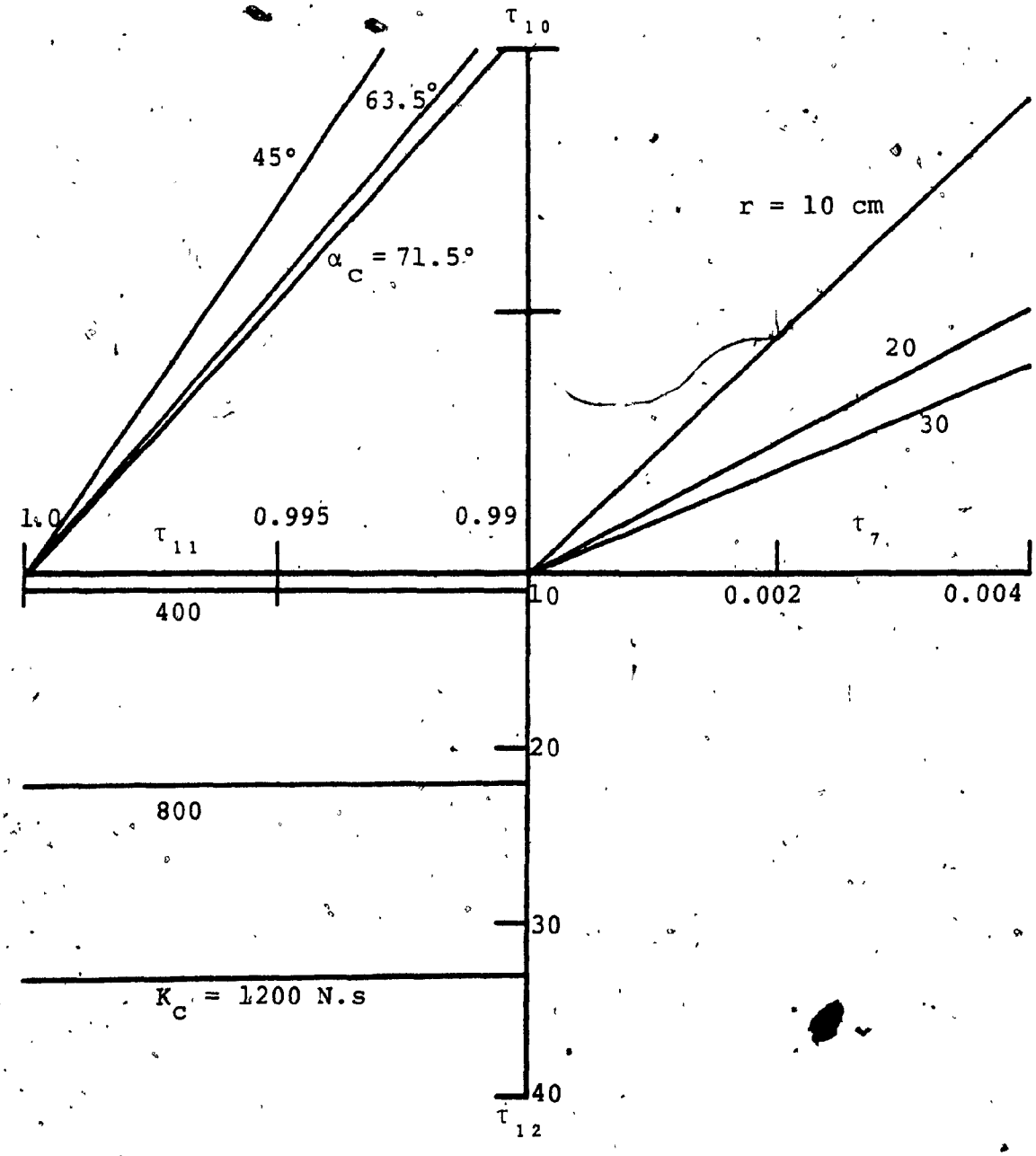


Figure 7.6: Torque Required and Available from the Angular Axis-Drive (Grinding Application). Part 1 of 2.

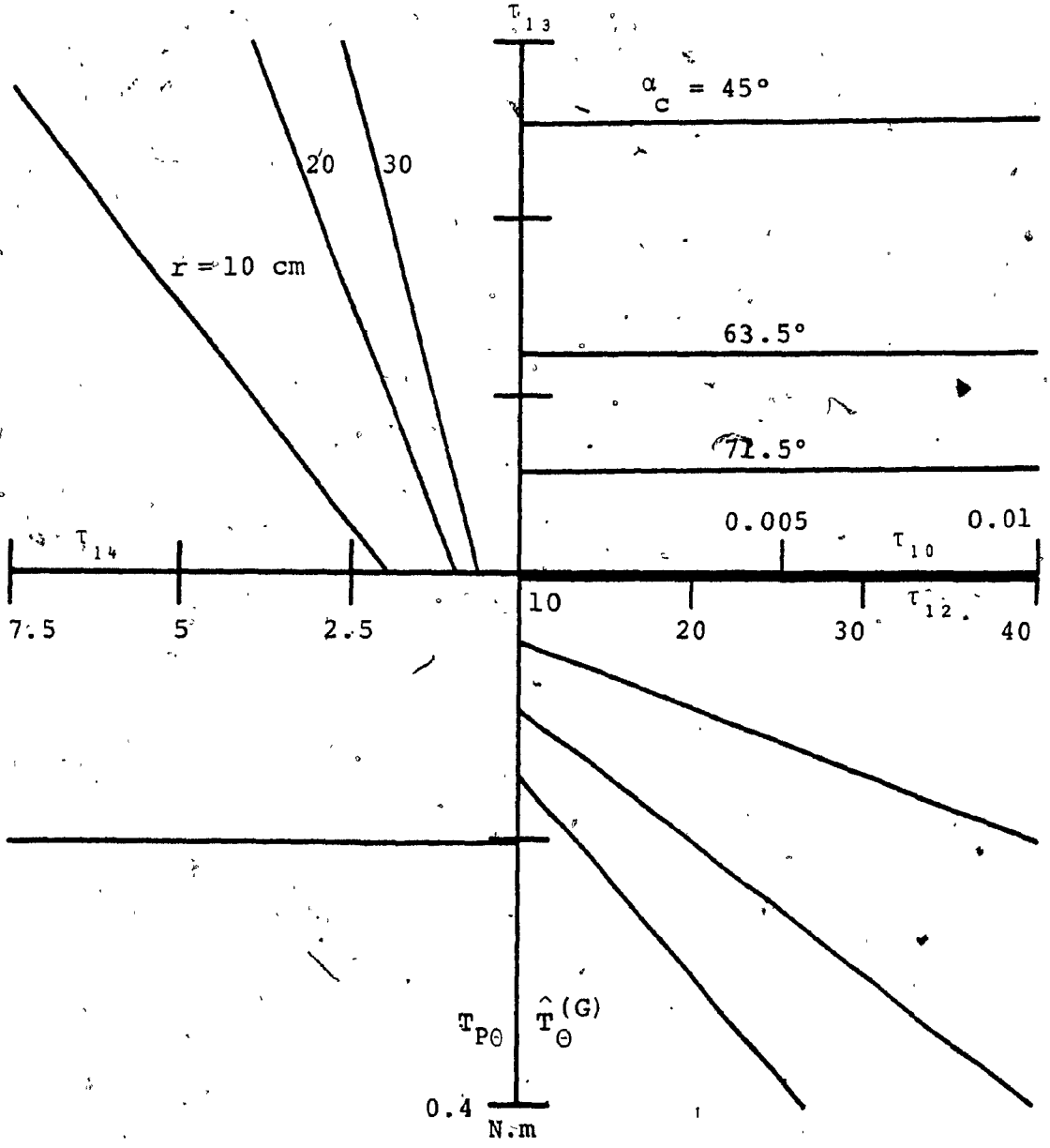


Figure 7.7: Torque Required and Available from the Angular Axis-Drive (Grinding Application). Part 2 of 2.

Quadrant III: τ_{12} versus τ_{11} for different K_c.

Figure 7.7:

Quadrant I: τ_{13} versus τ_{10} for different α_c.

$\tau_{13} = \sin\gamma_{\Theta}^{(G)}$

Quadrant II: τ_{14} versus τ_{13} for different r.

$\tau_{14} = \tau_{13}/r$

Quadrant III: $T_{P\Theta}$ versus τ_{14} for different V_t

Quadrant IV: $\hat{T}_{\Theta}^{(G)}$ versus τ_{12} for different V_t.

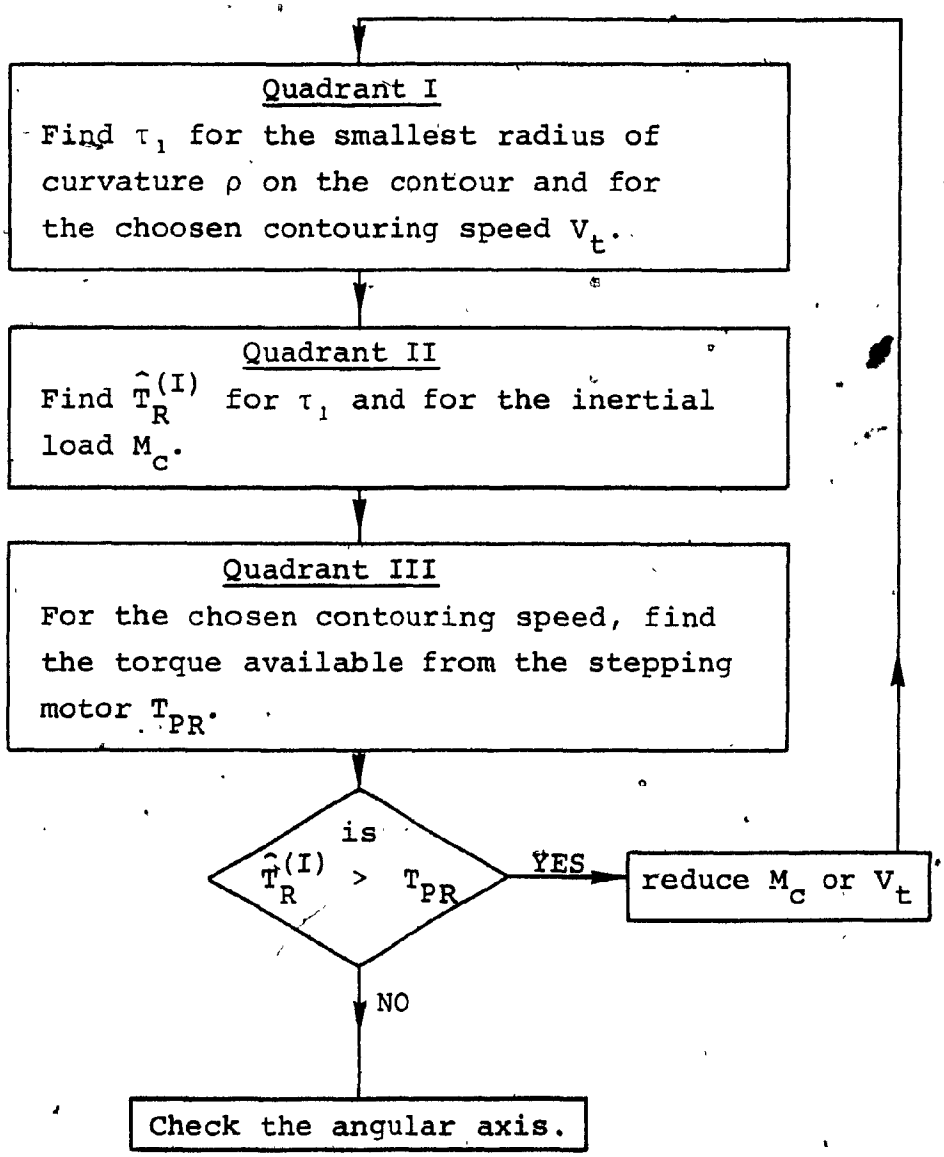
7.3 Description of the graphical procedure

The solution procedures for the load and the load carrying capacity of the NC machine axes are presented in flowchart form as follows.

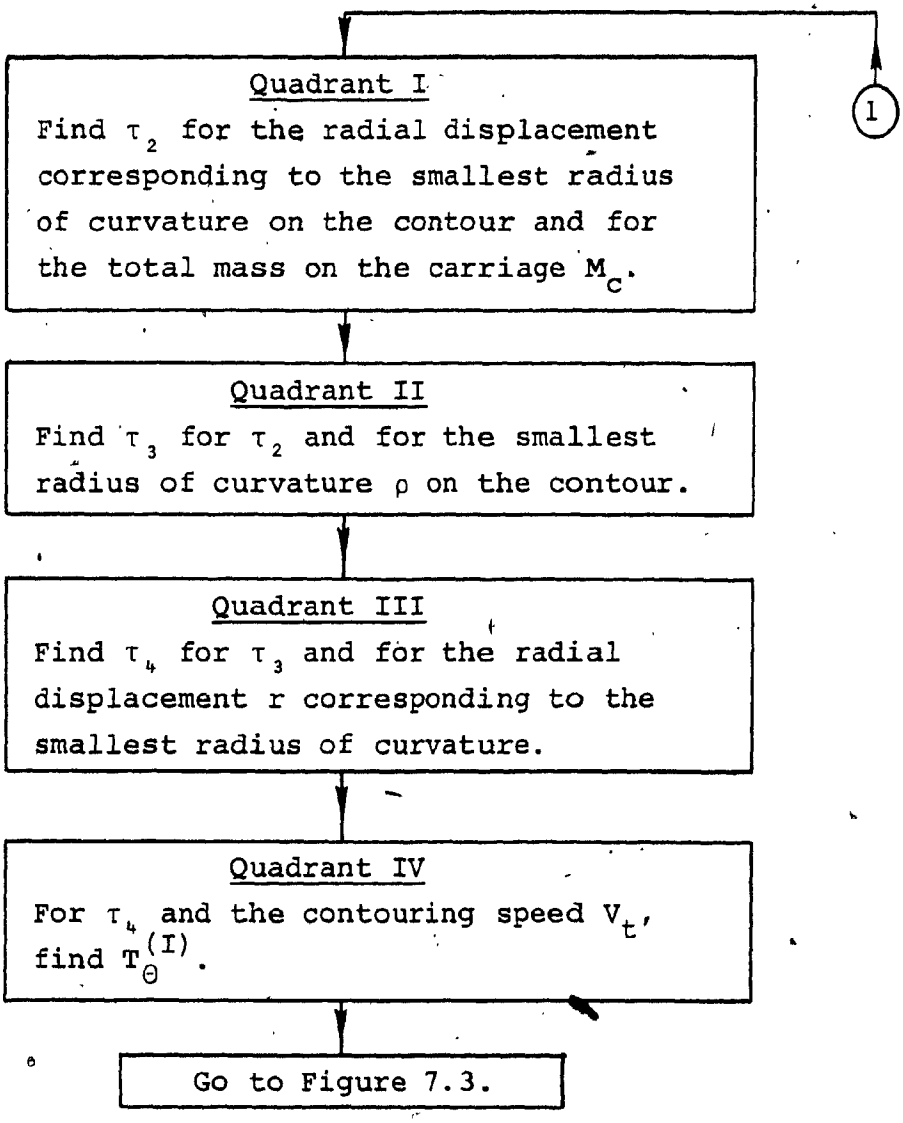
7.3.1 Inertial Load

For the radial axis

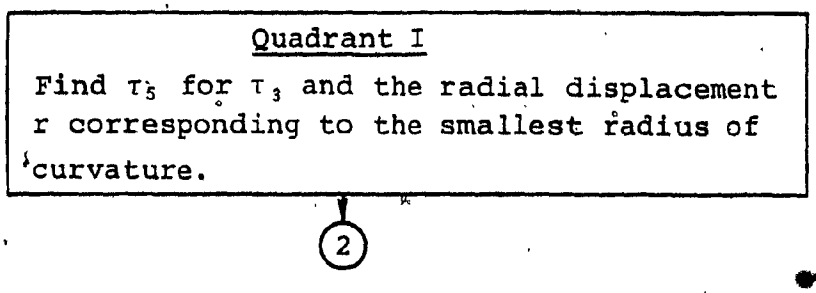
With reference to Figure 7.1

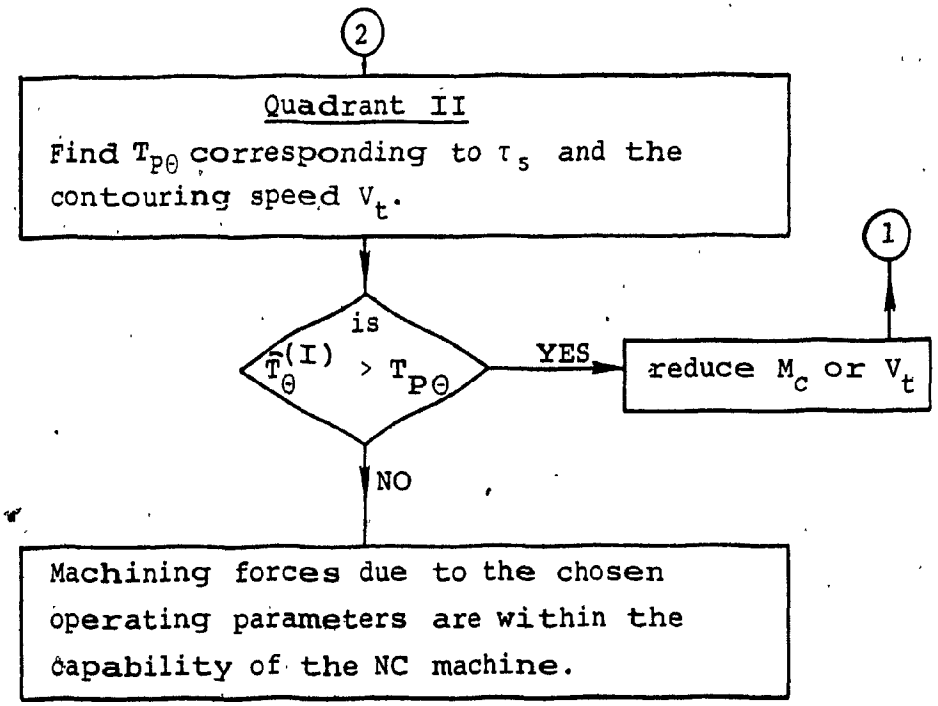


For the angular axis
With reference to Figure 7.2



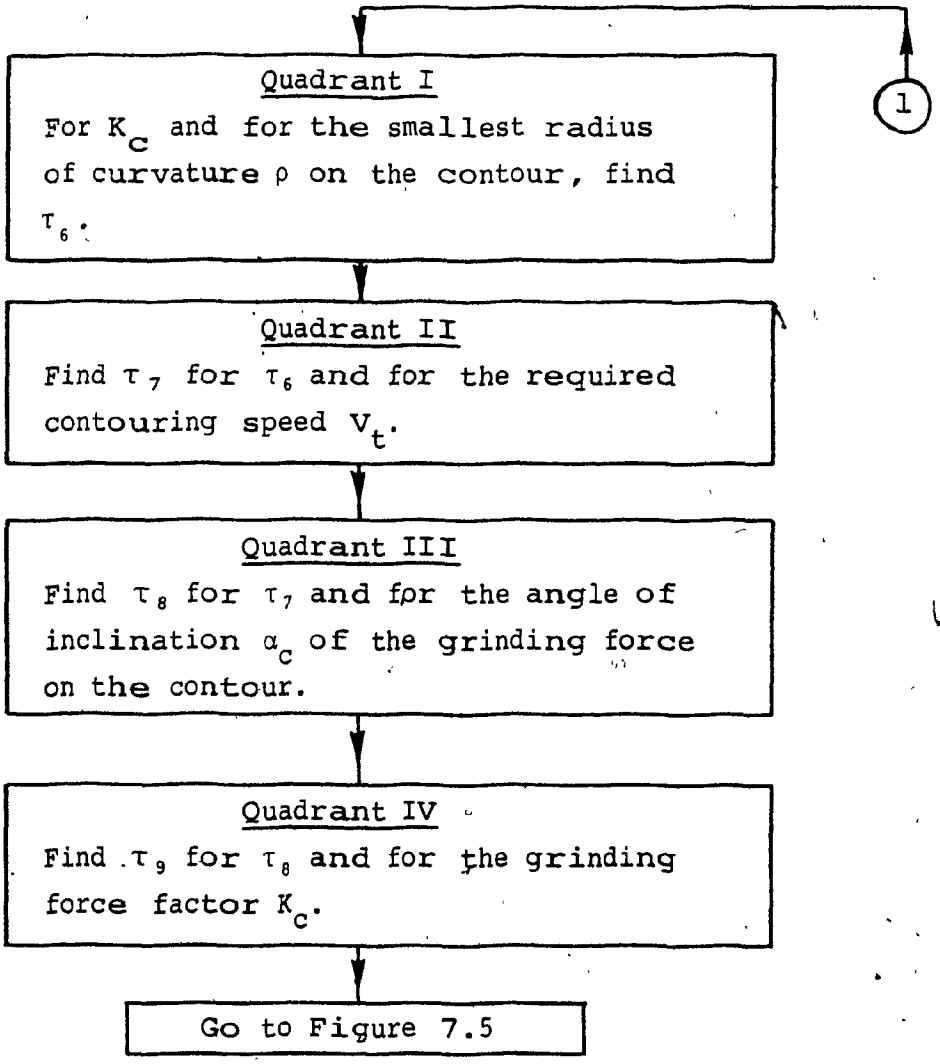
With reference to Figure 7.3



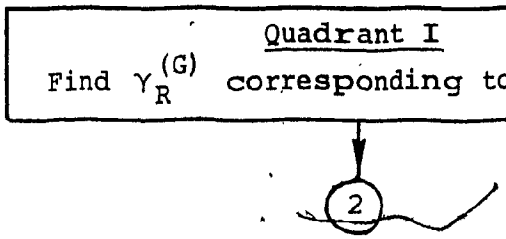

2
Quadrant II
Find $T_{P\theta}$ corresponding to τ_s and the contouring speed V_t.
is $\bar{T}_\theta^{(I)} > T_{P\theta}$
YES
reduce M_c or V_t
1
NO
Machining forces due to the chosen operating parameters are within the capability of the NC machine.

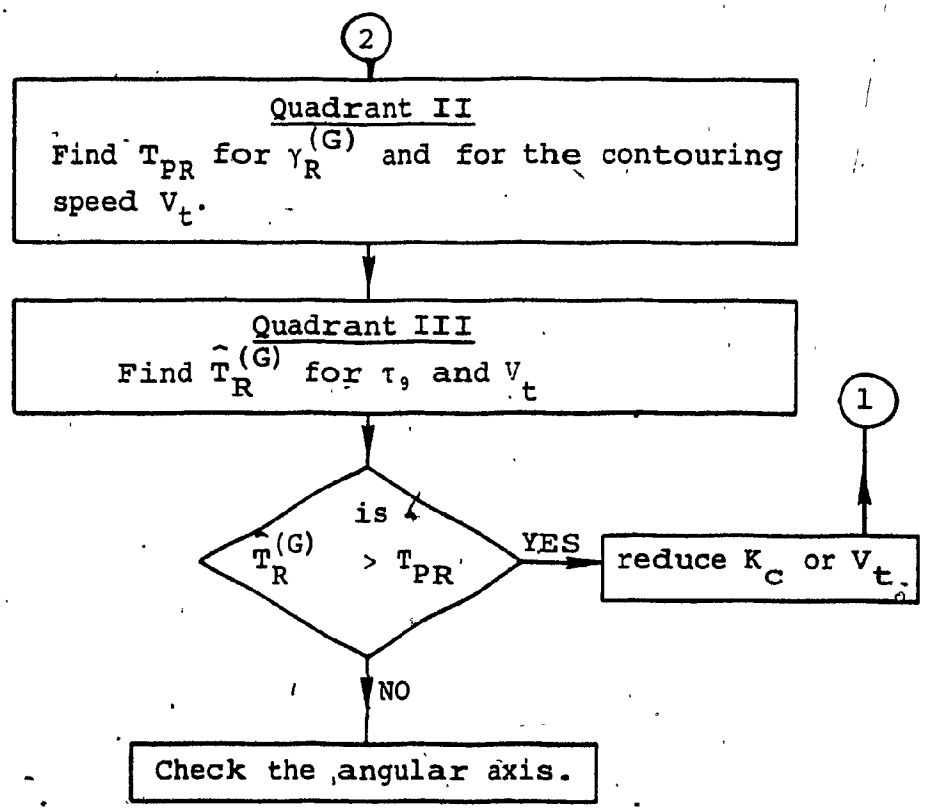
7.3.2 Grinding Application

For the radial axis
With reference to Figure 7.4



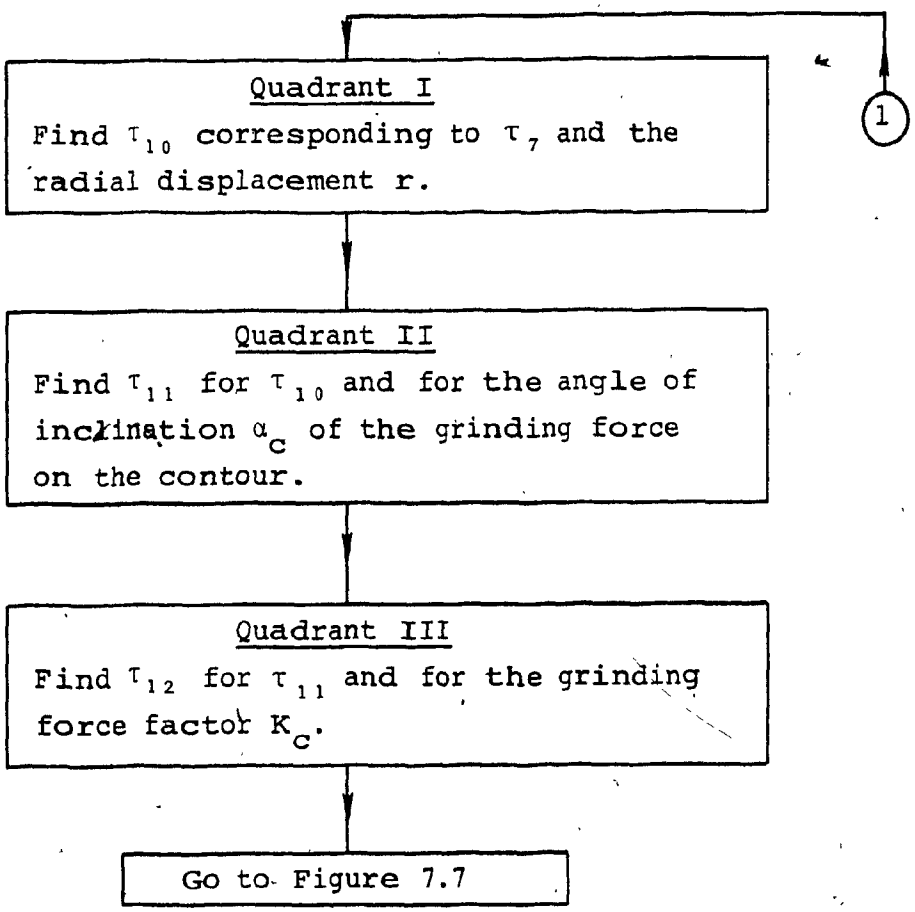
With reference to Figure 7.5



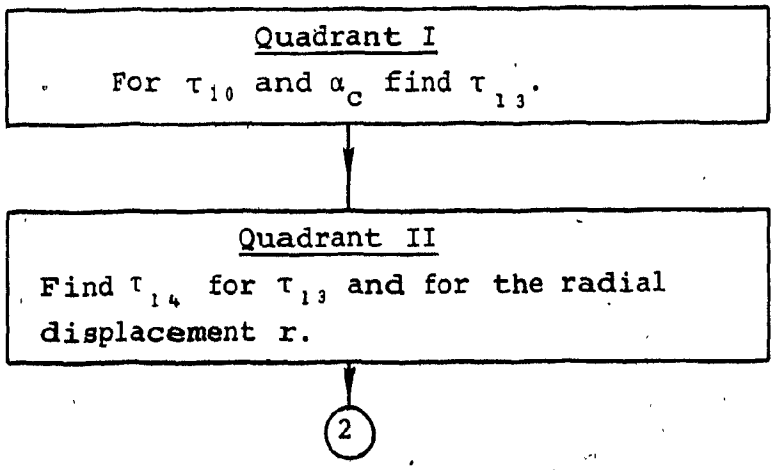

2
Quadrant II
Find T_{PR} for $\gamma_R^{(G)}$ and for the contouring speed V_t.
Quadrant III
Find $\hat{T}_R^{(G)}$ for τ_9 and V_t
1
is $\hat{T}_R^{(G)} > T_{PR}$
YES
reduce K_c or V_t.
NO
Check the angular axis.

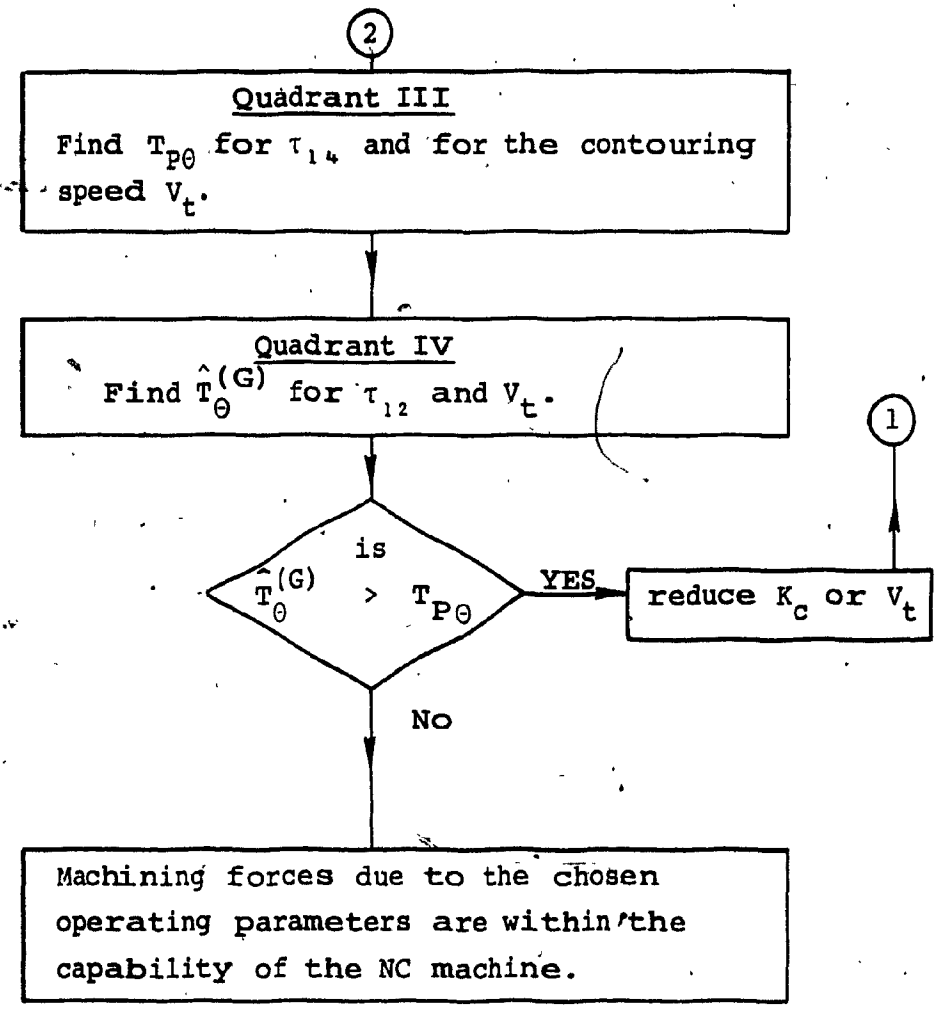
For the angular axis

With reference to Figure 7.6



With reference to Figure 7.7




2
Quadrant III
Find $T_{P\theta}$ for τ_{14} and for the contouring speed V_t.
Quadrant IV
Find $\hat{T}_{\theta}^{(G)}$ for τ_{12} and V_t.
1
is $\hat{T}_{\theta}^{(G)} > T_{P\theta}$
YES
reduce K_c or V_t
No
Machining forces due to the chosen operating parameters are within the capability of the NC machine.

CHAPTER 8

CONCLUSION

AND RECOMMENDATION FOR FUTURE WORK

8.1 Conclusion and major highlights

This thesis presents the design, analysis and performance evaluation of a polar based contouring system. The system is composed of a digitizer and an NC machine. The digitizer can be either a template digitizer (TD) or a software digitizer (SD). The system can thus be operated as a TD/NC machine or a SD/NC machine.

The template digitizer uses a unique method of tracking a template. The tracking mechanism consists of two rollers powered by electric motors which are connected in such a way so as to produce a mechanical differential effect. The two rollers are preloaded against each other and together track a ribbon-like template which is positioned in between them. The rollers are mounted on a carriage which in turn is mounted on an arm. The arm swings around a pivot and represents the angular axis of the polar set. The position of the carriage on the arm represents the radial axis. Two encoders, mounted on the arm-pivot and on the carriage, supply data on the angular and radial position. Due to the roller-differential arrangement, nearly constant tracking speeds are possible. Two types of template intended for accurate and non-accurate contouring have been described.

The software digitizer is composed of two parts, a

digitizer algorithm and an output algorithm. The digitizer algorithm can run under BATCH mode on time-shared computers, and it generates the positional and time data required for contouring. The output algorithm is short and simple, and its function is to transfer the data generated by the digitizer algorithm to the output device. This algorithm require real-time computing. The accuracy of the tracking speed for the software digitizer depend upon the resolution of the real-time clock of the output computer. Typical resolutions for such clocks are within 10 to 30 sec. The control signals from either digitizer can be recorded on commercial stereo tape recorders for later use or may be supplied directly to the NC machine.

The NC machine is composed essentially of an arm and carriage arrangement similar to that of the template digitizer. The machine operates with an open loop controller. Stepping motors operating with a wave energization scheme are used to drive both the radial and angular axes.

Prototypes of both digitizers as well as of the NC machine have been built and tested. The configuration of the polar based contouring system offers a number of operational features as well as some advantages over commercially available copying or NC machines.

One of the important features of the system is that it can operate as a copier (TD/NC machine) or as an NC system (SD/NC machine). When operating as a copier, the contour

information is in the form of a physical template, and the template digitizer is used to translate the given template into numerical data. When a software digitizer is used, the system resembles the commercially available NC systems but with no on-board computer.

An important advantage of this system over a Cartesian-based system is the ease with which size scaling is achieved. For a polar-based system, scaling is accomplished solely by radial axis scaling. A simple method for carrying out the scaling has been described in Section 3.7. When the NC machine is operated with prerecorded digitizer data, speed scaling can be achieved by altering the play-back speed. Chapter 3 discusses and tabulates a number of other features and advantages of the proposed system over the commercially available systems.

The major drawback of the polar system is the dependence of the angular resolution on the radial displacement. The variation of the angular resolution necessitates optimal location of the contour with respect to the machine pole in order to reduce the errors due to the digitization of the contour. An area error criterion has been developed in Chapter 4 and has been shown to be more stringent than the traditionally used deviation error. It was also shown that the area error generated under quasi-static conditions is a conservative representation of the error generated during actual operation. It was also shown that the area error is a function of the relative

location of the contour pole from the machine pole. Thus by varying the location of the contour pole, the area error can be minimized. The calculation of the area error is lengthy and cumbersome, and using it as an objective function for optimization proves impractical. An alternate objective function has been developed and was shown to duplicate the behavior of the area error. This alternate function can be easily and rapidly calculated. Comparative tests carried out on three test contours show that the alternate objective function can be used to accurately determine the contour location for minimum area error.

A dynamic analysis of the NC machine has been presented. The analysis takes into account the contour being tracked and the diameter of the tool used on the NC machine. The analysis is used to calculate the torque required from the axes-drives during the contouring process. These torques have to be less than the pull-out torques available from the stepping motors powering the respective axes. The torque ratio is defined as the ratio of the torque required to the pull-out torque of the stepping motors.

The effect of the type and magnitude of the load and of the contouring speed and of the type of contour has been studied. In general, an increase in the magnitude of the load or of the tracking speed causes an increase in the torque ratio for all contours. Furthermore, it was shown that the torque required from an axis-drive, and

consequently the torque ratio, increases rapidly with the decrease of the radius of curvature of the contour. The study also showed that the magnitude of the load or of the contouring speed has no effect on the location, along the contour, of the maximum value of the sum of torques required from both axes.

During the course of the study it was observed that the change in the location of the contour causes a change in the torque ratio. This phenomenon is utilized as shown in Chapter 6, in an optimization routine to locate the contour relative to the machine pole such that the maximum value of the sum of the torque ratios is a minimum. Results of a parametric study, conducted in Chapter 6, are subsequently utilized in order to find the limiting machine setting beyond which failure would occur.

In Chapter 7, a procedure using composite graphs is provided for rapid calculation of the maximum torque required and available from the axes-drives in different contouring operations.

8.2 Recommendation for future work

The studies conducted in this thesis prove the feasibility of the polar-based contouring system. They also point out to a number of desirable features inherent to the polar axis system, as well as others that result from the configuration of the system.

In order to advance the concept to a commercially

viable stage, further studies should be conducted. A dynamic analysis of the template digitizer mechanism should be carried-out in order to improve the design. Such a study would also prove helpful in eliminating or reducing the oscillations of the digitizer rollers that have been found to occur at the beginning of the tracking of a template.

The effect of size scaling on the contour pole location for minimum area error should also be investigated.

The dynamic analysis that has been conducted on the NC machine is geared towards estimating the load on the machine axes. The scope of the study should be expanded to include such areas as rigidity and vibration effects of the arm and carriage mechanism.

A control concept of modifying the timing of the original control data in order to compensate for positional errors at higher contouring speeds or loads should be studied. To perform such a modification, a dynamic model that includes the behaviour of the system within a step must be developed. Finally, to enhance the performance of the contouring system, a small on-board microcomputer may be employed to carry out the auxiliary functions that may be required during a machining operation. The interaction between these functions and the contouring process should be carefully coordinated.

The possibility of implementing the proposed concept in a $2\frac{1}{2}$D and 3D contouring system should be considered.

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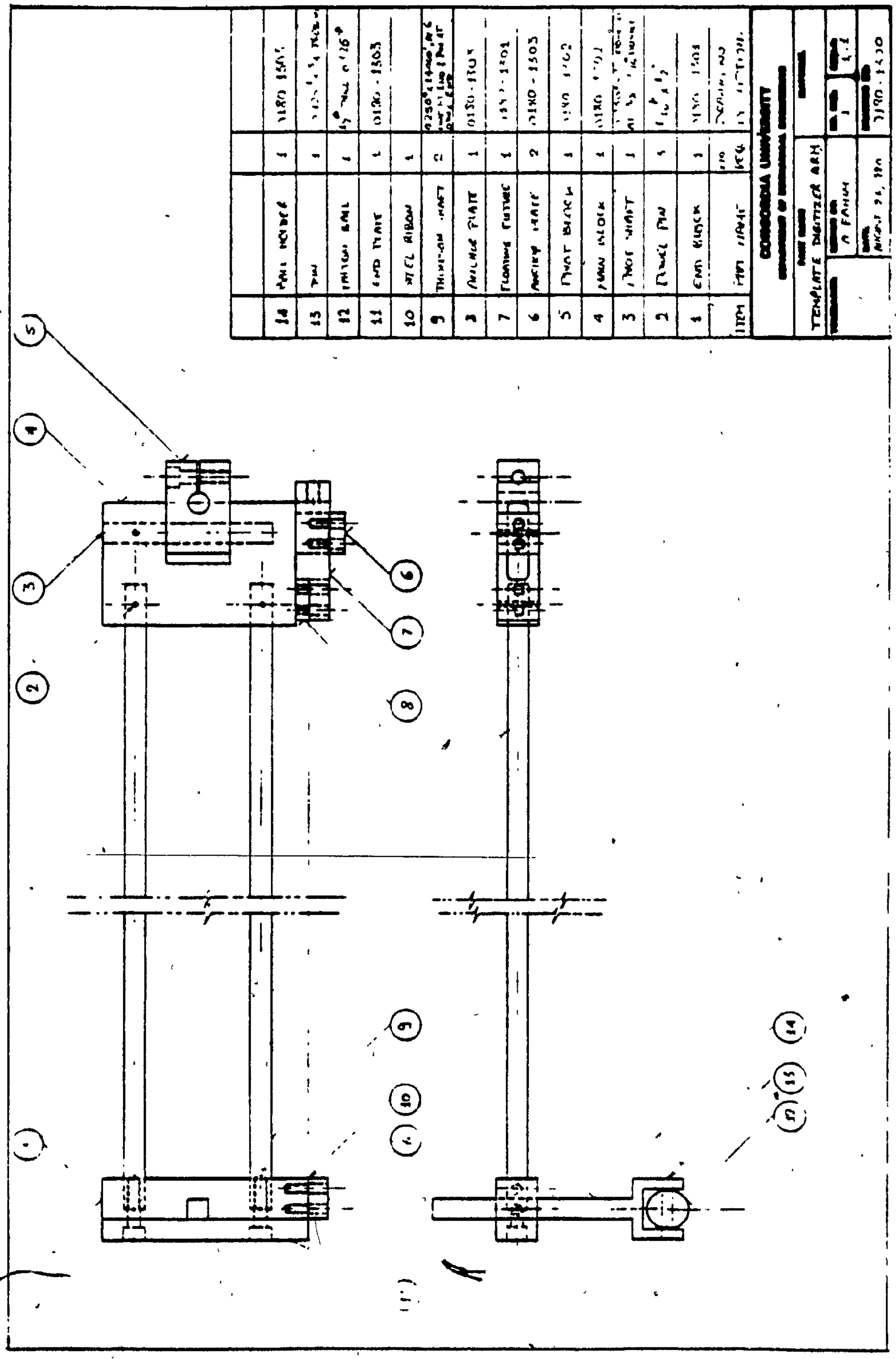
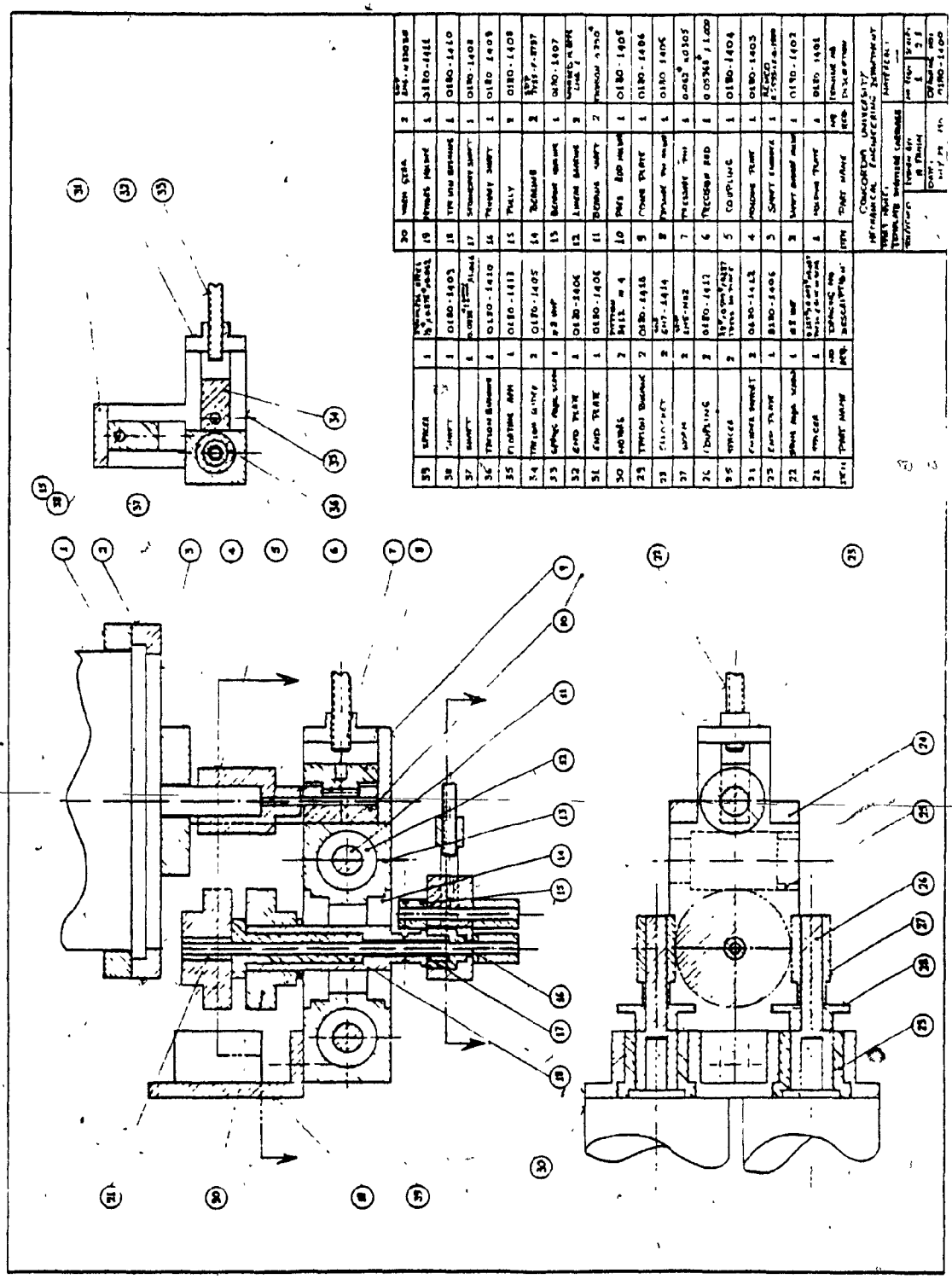
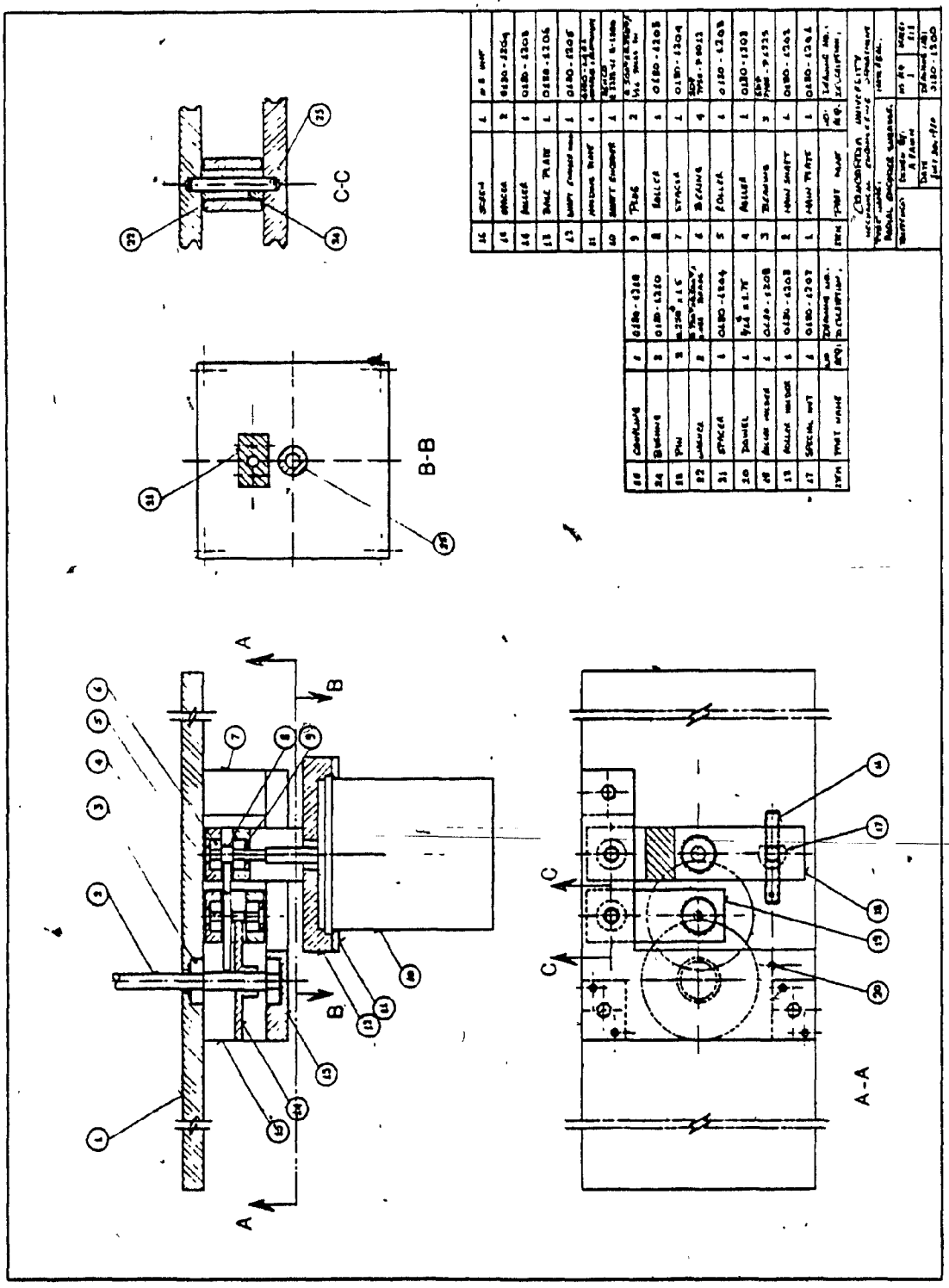
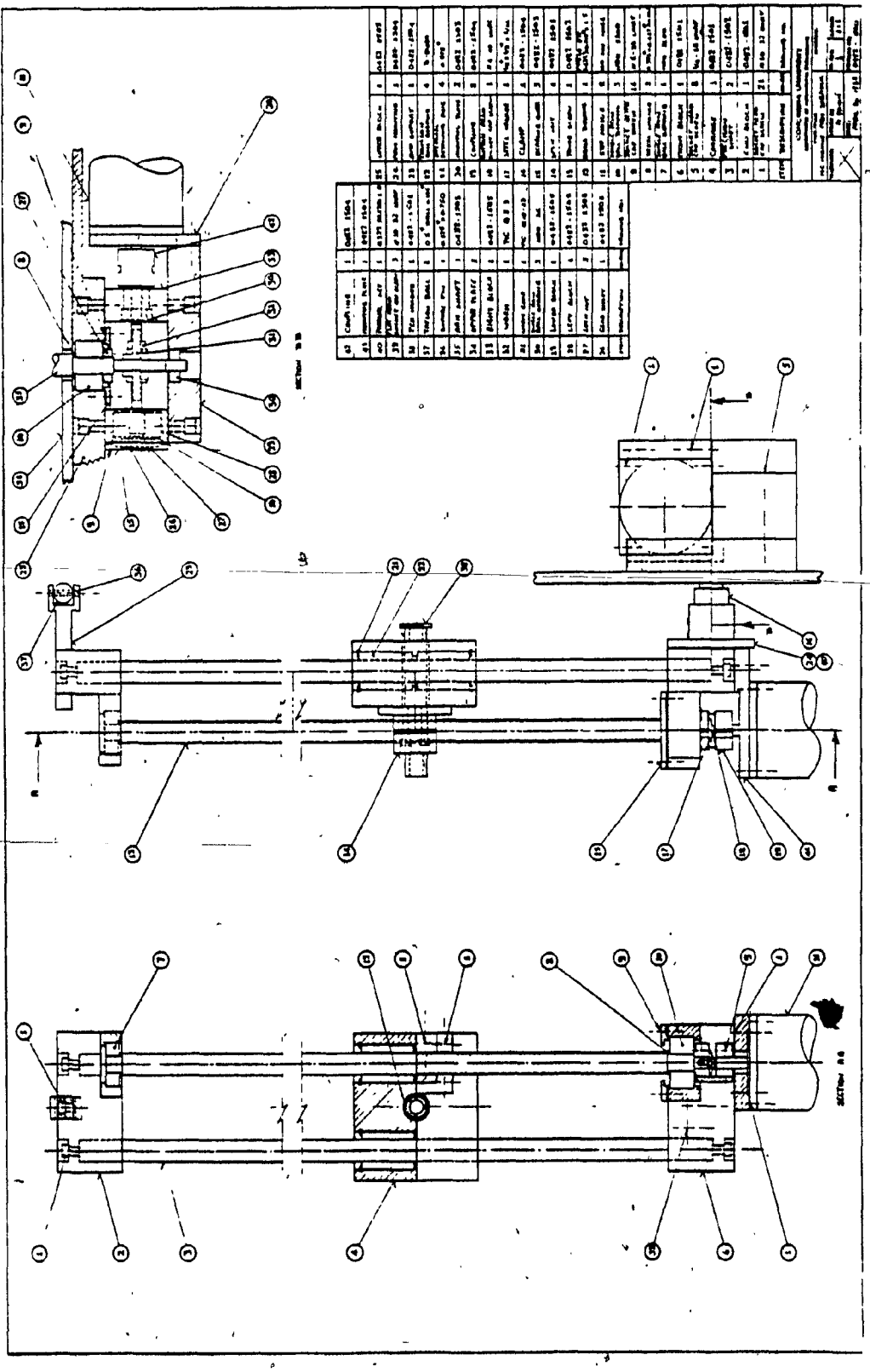
APPENDIX A

ASSEMBLY DRAWINGS OF THE TEMPLATE DIGITIZER AND THE NC MACHINE


CONCORDIA UNIVERSITY
TEMPLATE DIGITIZER ARM
STEEL RIBBON
END PLATE
END BLOCK
ITEM


CONCORDIA UNIVERSITY
MECHANICAL ENGINEERING DEPARTMENT


C-C
B-B
A
B
C
A-A



APPENDIX B

ELECTRIC DIFFERENTIAL MECHANISM

B.1 Introduction

A mechanical differential is a power transmission mechanism that is used to distributee power from one input shaft to two output shafts. This is done in such a way that the sum of the rotational speeds ω_{10} and ω_{20} of the two output shafts is equal to the rotational speed of ω_i of the input shaft, or a constant multiplier of that speed, that is to say

$$\omega_i = \omega_{10} + \omega_{20} \qquad \text{B.1}$$

Furthermore the torque T_i of the input shaft is equally distributeed to the two output shafts, thus:

$$T_i = T_{10} + T_{20} \qquad \text{B.2}$$

where T_{10}, T_{20} are the torques of the two output shafts.

A mechanical differential is essentially composed of linkages and gears. In some cases, physical constraints do not permit the use of a mechanical differential. Also when a mechanical differential is to be used, and the output shafts have to be far apart, a large number of additional power transmission components have to be utilized to transmit power.

Two direct current (DC), permanent magnet (PM), electric motors connected in series were found to duplicate

the function of a differential mechanism. This arrangement will be referred to as an "electric differential mechanism". In applications where a mechanical differential is cumbersome, or impossible to use, an electric differential mechanism can be used. When an electric motor is utilized as the power source, the use of an electric differential mechanism would simplify the design considerably. A typical example of such a case is the driving mechanism of the roller arrangement on the prototype template digitizer. In this case the two output shafts have to be concentric, and electric motors have to be used as the power source.

B.2 Analysis of the electric differential mechanism

For a DC-PM motor operating below its magnetic saturation level, the voltage relation can be approximated by:

$$V_s = R_a i + L_a \frac{di}{dt} + K_v \omega \qquad \text{B.3}$$

where V_s is the applied voltage,

L_a, R_a are the motor winding inductance and resistance respectively,

i is the current passing in the motor winding,

K_V is the motor voltage constant,

and ω is the motor shaft rotational speed.

The relation between the torque generated by the motor and the windings current is given by:

$$T = K_T i \qquad \text{B.4}$$

where T is the torque generated,

and K_T is the motor torque constant.

Figure B.1 shows a schematic diagram of the circuit of the electric differential mechanism. With reference to the figure, the equations of the circuit are derived as follows: Subscripts 1 and 2 refer to the first and second motor respectively.

$$V_s = V_1 + V_2 \qquad \text{B.5}$$

$$T_t = T_1 + T_2 \qquad \text{B.6}$$

$$i = i_1 + i_2 \qquad \text{B.7}$$

where V_s is the power supply voltage,

T_t is the total torque of the motors,

and i is the current passing through the motors.

For identical motors, the equations are given by:

$$V_1 = R_a i + L_a \frac{di}{dt} + K_v \omega_1 \qquad \text{B.8}$$

$$T_1 = K_T i \qquad \text{B.9}$$

$$V_2 = R_a i + L_a \frac{di}{dt} + K_v \omega_2 \qquad \text{B.10}$$

$$T_2 = K_T i \qquad \text{B.11}$$

Solving Equations B.5 to B.11 simultaneously give:

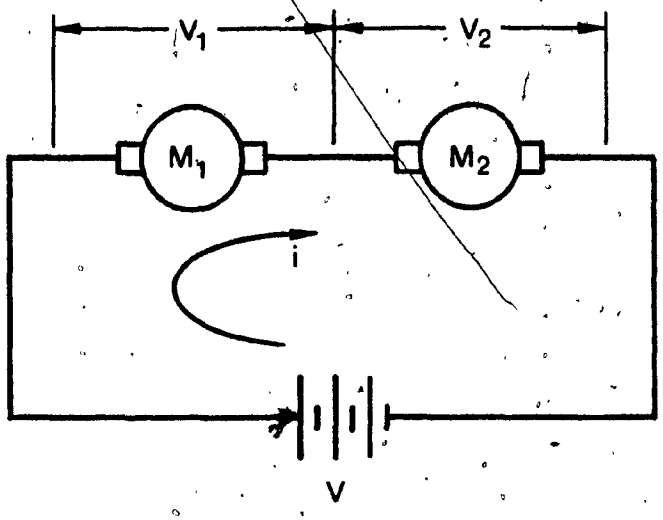


Figure B.1: Circuit Diagram of an Electric Differential Mechanism.

$$V_s = (2R_a)i + (2L_a)\frac{di}{dt} + K_v(\omega_1 + \omega_2) \qquad \text{B.12}$$

$$T_1 = T_2 = T_t/2 \qquad \text{B.13}$$

Equations B.12 and B.13 resembles those of an equivalent motor with the following parameters;

Winding resistance $=2R_a$

Winding inductance $=2L_a$

Voltage constant $=K_V$

Torque constant $=2K_T$

The equivalent motor has a rotational speed given by:

$$\omega_i = \omega_1 + \omega_2 \qquad \text{B.14}$$

Equations B.13 and B.14 of the motor arrangement in Figure B.1 resembles those of the mechanical differential given by Equations B.2 and B.1 respectively.

B.3 Performance of the electric differential

The torques generated by the two motors in Figure B.1 are equated to the load torques on the motors as follows:

$$T_1 = T_{L1} + J\dot{\omega}_1 + T_F + B\omega_1 \qquad \text{B.15}$$

$$T_2 = T_{L2} + J\dot{\omega}_2 + T_F + B\omega_2 \qquad \text{B.16}$$

where T_{L1}, T_{L2} are the load torques on the two motors respectivel,

J is the combined load and motor inertia,

T_F is the frictional torque of the motor,

and B is the motor damping.

Solving Equations B.15 and B.16 with Equations B.9, B.11, and B.12 simultaneously give:

At steady state di/dt=0, thus;

$$\left[\frac{K_V K_T + 2R_a + B}{K_V K_T}\right] \omega_1 + \omega_2 = \frac{V_s}{K_v} - \frac{2R_a (T_{L1} + T_F)}{K_V K_T} \qquad B.17$$

$$\omega_1 + \left[\frac{K_V K_T + 2R_a + B}{K_V K_T}\right] \omega_2 = \frac{V_s}{K_v} - \frac{2R_a (T_{L2} + T_F)}{K_V K_T} \qquad B.18$$

Equations B.17 and B.18 are solved simultaneously to evaluate the two output shafts speeds of the electric differential mechanism at different loading conditions.

The measured and calculated parameters of the PM-DC motors used to drive the template digitizer rollers are listed in Table B.1.

Figure B.2 shows the theoretical and the experimental torque-speed characteristics of the electric differential mechanism. The experimental results where obtained by loading both motors to their total torque setting such that they are operating at different speeds. The rotational speeds of the two motors as well as the sum of the speeds are indicated on the figure. The figure shows an excellent agreement between the theoretical and the experimental results.

B.4 Control scheme for the electric differential mechanism

The rollers (G) and (G') of the template digitizer are

Table B.1: Data of the Motors Used in the Electric Differential Mechanism

No Load Speed		=	804 rad/s	—
Stall Torque		=	35.79 mN.m	5.07 oz in
Friction Torque	T_F	=	2.47 mN.m	0.35 oz in
Inertia	J	=	4.66 gm cm^2	3.44×10^{-7} slug ft^2
Maximum Voltage	V	=	24 V	—
Stall Current	I	=	1.4 A	—
Voltage Constant	K_V	=	0.0273 V/rad/s	—
Torque Constant	K_T	=	27.32 mN.m/A	3.87 oz in/A
Coil Resistance	R_a	=	17.15 Ω	—
Coil Inductance	L_a	=	8.62 mH	—
No Load Current		=	0.113 A	—

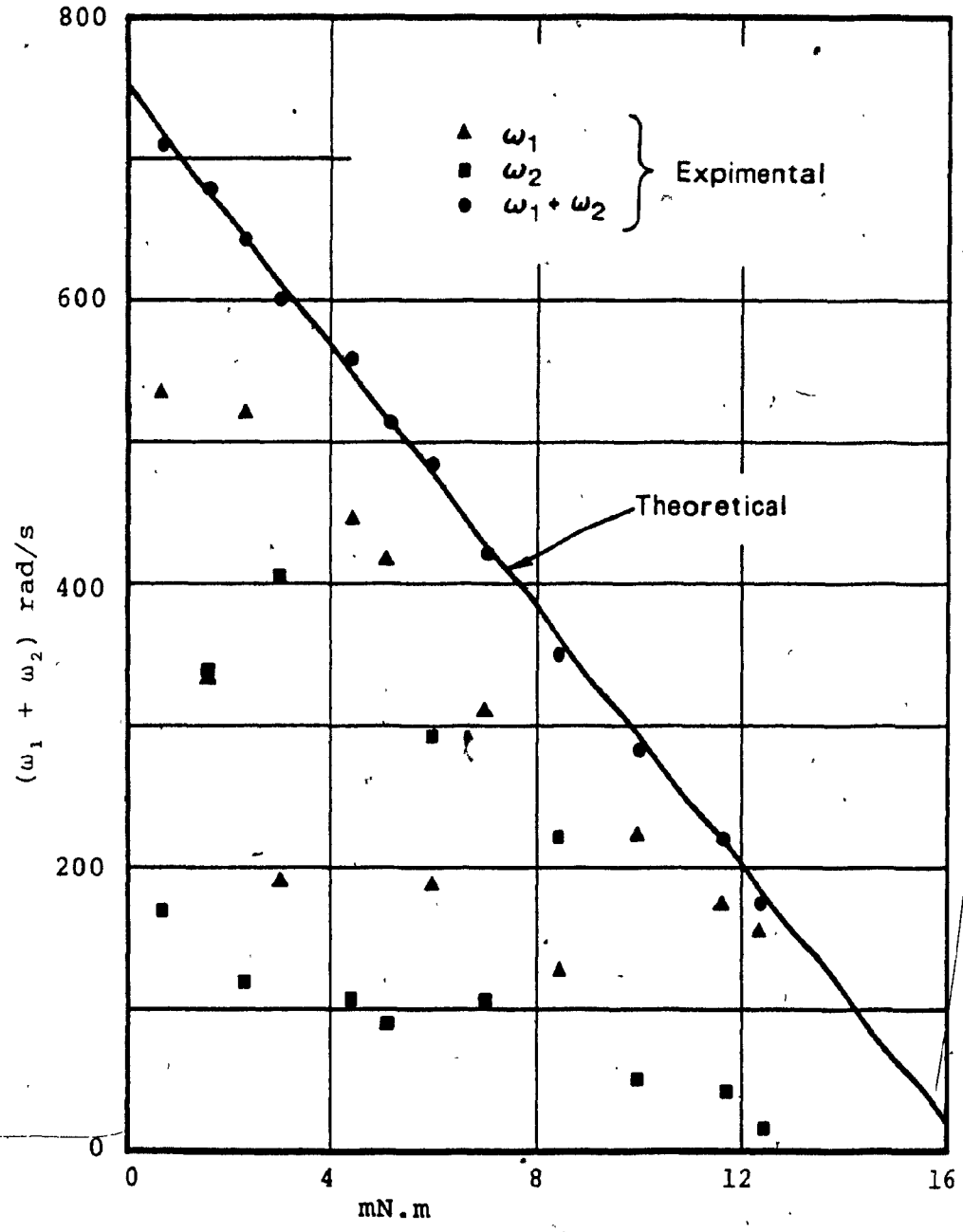


Figure B.2: Torque-Speed Characteristics of an Electric Differential Mechanism.

powered through the electric differential mechanism as described in Section 2.2. Furthermore, the analysis in Section 3.5 shows that in order to maintain a constant contouring speed, a controller utilizing speed feedback from roller (G) has to be used. The controller required to regulate the speed of (G) is complicated in design. This is because it has to be able to vary the speed of roller (G') from infinity to zero while the roller mechanism is tracking certain contours as shown in Figure 3.7.b at sections (1) and (2) respectively

For moderately undulating contours, such a control scheme can be implemented successfully by utilizing conventional feedback controllers, or phase lock loop controllers.

A much simpler control scheme to implement, would entail holding the sum of the two motors speeds constant. This can be implemented by utilizing speed feedback from both rollers. Alternatively, negative impedence circuits, with impedences matched to those of the equivalent motor shown in Section B.2, may be used. This control scheme result in small fluctuations in the contouring speed as shown in Section 3.4. The magnitude of the fluctuations is subject to the nature of the contour.

APPENDIX C

SOFTWARE

DIGITIZER ALGORITHMS

```
C=====================================================================      100
C                                                                          110
C THIS PROGRAM KINEMATICALLY RESEMBLES THE INPUT PART                      120
C OF THE POLAR TEMPLATE DIGITIZED SYSTEM.                                  130
C THE PROGRAM PRODUCES A FILE CONSISTING OF AXES                           140
C INCREMENTATION VERSUS TIME.                                              150
C THE OUTPUT FILE CAN BE TRANSFERED ON MAGNETIC                            160
C TAPE USING THE OUTPUT ROUTINE WHICH IS PROVIDED                          170
C FOLLOWING THIS PROGRAM.                                                  180
C                                                                          190
C=====================================================================      200
C                                                                          210
      COMMON/CURVE/ RI,EPS,RR,BB,AA                                        220
      COMMON/PAR/ PY,ROOT(4),WE(4),NN                                      230
      COMMON/OPR/ FUN                                                      240
      DATA KKK/1/                                                          250
      DATA RI,EPS,RR,AA,BB/5.0,70.0,0.125,2.0,3.0/                         260
      DATA ROOT/2*0.3399810435849,2*0.861136311594/                        270
      DATA WE/2*0.652145154862 5,2*0.347854845137 4/,IK/1/,IL/1/           280
C---------------------------------------------------------------------     290
C ENTER CONTOURING SYSTEM RESOLUTIONS, RADIAL THEN ANGULAR                 300
      PRINT *,'ENTER RADIAL THEN ANGULAR RESOLUTIONS'                      310
      READ *,DELR,DELTH                                                    320
C---------------------------------------------------------------------     330
C ENTER NUMBER OF ITERATIONS BEYOUND WHICH SEARCH OR THE                   340
C ROOT IS INITIATED IN THE REVERSE DIRECTION                               350
      PRINT *,'ENTER MAXIMUM ALLOWABLE NUMBER OF ITTERATIONS'              360
      READ *,NN                                                            370
C---------------------------------------------------------------------     380
C ENTER OPERATING PARAMETERS:                                              390
C     RI =CONTOUR POLE RADIAL DISPLACEMENT                                 400
C     EPS=CONTOUR POLE ANGULAR DISPLACEMENT                                410
C     RR =TOOL RADIUS.                                                     420
      PRINT *,'ENTER CONTOUR POLE RADIAL AND ANGULAR DISPLACEMENTS'        430
      READ *,RI,EPS                                                        440
      PRINT *,'ENTER TOOL RADIUS'                                          450
      READ *,RR                                                            460
C---------------------------------------------------------------------     470
C ENTER CONTOUR PARAMETERS                                                 480
      PRINT *,'ENTER CONTOUR PARAMETERS'                                   490
      READ *,AA,BB                                                         500
C IF MORE PARAMETERS ARE REQUIRED TO EXPRESS THE CONTOUR, ADD              510
C THEM TO THE COMMON BLOCK "CURVE".                                        520
C---------------------------------------------------------------------     530
      ROOT(2)=-ROOT(2)                                                     540
      ROOT(4)=-ROOT(4)                                                     550
      PY=2*ASIN(1.0)                                                       560
      THE=34.50471682                                                      570
      GAM=GAMM=0.0                                                         580
      AL=8.294224337                                                       590
      EPS=EPS*PY/180.0                                                     600
      DELTH=DELTH*PY/180.0                                                 610
      THE=THE*PY/180.0                                                     620
      GAM=GAMM=GAM*PY/180.0                                                630
      ALL=INT(AL/DELR)*DELR                                                640
 10   CONTINUE                                                             650
      T=TEMP                                                               660
      NM=NM+1                                                              670
      GAMM=GAM                                                             680
      TT=TT+T                                                              690
      PRINT 1000, NM,T,LLL,KKK,THE,ALL,GAM,FUN,TT,AL                       700
      IF(GAM .GT. 2*PY) GO TO 100                                          710
 31   CONTINUE                                                             720
      THE=THE+IK*DELTH                                                     730
      GAM=GAMA(GAM,THE,AL,0,N)                                             740
      IF(N .LT. NN) GO TO 30                                               750
      IK=-IK                                                               760
      THE=THE+IK*DELTH                                                     770
      GO TO 31                                                             780
 30   CONTINUE                                                             790
      SD=SR(GAMM,GAM)                                                      800
      T=TEMP=SD/VEL                                                        810
```

```
      M=ABS(ALL-AL)/DELR                                                820
      IF(M .LT. 1) GO TO 10                                             830
      TEMPAC=0.0                                                        840
      DO 20 I=1,M                                                       850
 41   CONTINUE                                                          860
      ALL=ALL+IL*DELR                                                   870
      GAM=GAMA(GAMM,THE,ALL,1,N)                                        880
      IF(N .LT. NN) GO TO 40                                            890
      IL=-IL                                                            900
      ALL=ALL+IL*DELR                                                   910
      GO TO 41                                                          920
 40   CONTINUE                                                          930
      SD=SR(GAMM,GAM)                                                   940
      GAMM=GAM                                                          950
      T=SD/VEL                                                          960
      TEMPAC=TEMPAC+T                                                   970
      NM=NM+1                                                           980
      TT=TT+T                                                           990
      PRINT 1000, NM,T,KKK,LLL,THE,ALL,GAM,FUN,TT                      1000
 20   CONTINUE                                                         1010
      TEMP=TEMP-TEMPAC                                                 1020
      GO TO 10                                                         1030
 100  CONTINUE                                                         1040
 1000 FORMAT(1X,I6,2X,E18.10,2X,2(I1,2X),6(F10.6,2X))                  1050
      STOP                                                             1060
      END                                                              1070
C                                                                      1080
C                                                                      1090
C                                                                      1100
      FUNCTION GAMA(GAA,THE,AL,IFL1,N)                                 1110
C=====================================================================1120
C                                                                      1130
C THIS ROUTINE ITTERATIVELY FINDS THE ANGULAR AND RADIAL               1140
C DISPLACEMENTS OF A POINT ON THE CONTOUR WITH RESPECT                 1150
C TO THE MACHINE POLE.                                                 1160
C THE ROUTINE USES NEWTON RAPHSON UNMODIIED ITTERATIVE                 1170
C METHOD TO SOLVE FOR THE ROOTS OF THE EQUATIONS                       1180
C DESCRIBING THE SYSTEM                                                1190
C                                                                      1200
C=====================================================================1210
C                                                                      1220
      COMMON/CURVE/ RI,EPS,RR,BB,AA                                    1230
      COMMON/PAR/ PY,ROOT(4),WE(4),NN                                  1240
      COMMON/OPR/ FUN                                                  1250
C---------------------------------------------------------------------1260
C FOR THE FLAG "IFL"                                                   1270
      IFL=0 EXECUTE ANGULAR LOOP                                       1280
      IFL=1 EXECUTE RADIAL LOOP                                        1290
C---------------------------------------------------------------------1300
      GMA=GAA                                                          1310
      DO 40 K=1,NN                                                     1320
      GAMA=GMA                                                         1330
      CALL EQUA(GAMA,FUN,DFUN,DDFUN)                                   1340
      IF(ABS(DFUN) .LT. 1.0E+99) GO TO 10                              1350
      PHI=0                                                            1360
      GO TO 20                                                         1370
 10   CONTINUE                                                         1380
      IF(ABS(DFUN) .GE. 1.0E-99) GO TO 30                              1390
      PHI=PY/2.0                                                       1400
      GO TO 20                                                         1410
 30   CONTINUE                                                         1420
      PHI=PY/2.0-ATAN(DFUN/FUN)                                        1430
 20   CONTINUE                                                         1440
      DPHI=(DFUN**2-FUN*DDFUN)/(DFUN**2+FUN**2)                        1450
      IF(IFL1 .EQ. 1) GO TO 50                                         1460
      SINE=SIN(THE-GAMA)                                               1470
      F=RI*SIN(EPS-THE)-RR*COS(THE-GAMA-PHI)-FUN*SINE                  1480
      DF=-RR*SIN(THE-GAMA-PHI)*(1+DPHI)-DFUN*SINE+FUN*COS(THE-PHI)     1490
      GO TO 60                                                         1500
 50   CONTINUE                                                         1510
      COSPH=COS(PHI)                                                   1520
      SINPH=SIN(PHI)                                                   1530
```

```
      Z2=FUN**2+RR**2+2*FUN*RR*SINPH                                    1540
      Z=SQRT(Z2)                                                        1550
      ALAM=ASIN(RR*COSPH/Z)                                             1560
      COSCO=COS(EPS+ALAM-GAMA)                                          1570
      DZ2=2*FUN*DFUN+2*RR*DFUN*SINPH+2*FUN*RR*COSPH*DPHI                1580
      DALAM=-(Z*RR*SINPH*DPHI+RR*COSPH*DZ2/(2*Z))/                      1590
     +(Z*SQRT(Z2-(RR*COSPH)**2))                                        1600
      F=Z2+RI**2+2*Z*RI*COSCO-AL**2                                     1610
      DF=DZ2+RI*COSCO*DZ2/Z-2*Z*RI*SIN(EPS+ALAM-GAMA)*(DALAM-1)         1620
 60   CONTINUE                                                          1630
      IF(ABS(DF) .GT. 1.0E-99 .OR. ABS(F) .LT. 1.0E+99) GO TO 80        1640
      N=NN                                                              1650
      GO TO 90                                                          1660
 80   CONTINUE                                                          1670
      GMA=GAMA-F/DF                                                     1680
C     PRINT 1000,K,GMA,GAMA,F,DF,DPHI                                   1690
      IF(ABS(GMA-GAMA) .LE. 1.0E-6) GO TO 41                            1700
 40   CONTINUE                                                          1710
 41   CONTINUE                                                          1720
      N=K                                                               1730
 90   CONTINUE                                                          1740
      GAMA=GMA                                                          1750
      IF(IFL1 .EQ. 1) GO TO 70                                          1760
      Z=SQRT(FUN**2+RR**2+2*FUN*RR*SIN(PHI))                            1770
      ALAM=ASIN(RR*COS(PHI)/Z)                                          1780
      AL=Z*SIN(EPS+ALAM-GAMA)/SIN(EPS-THE)                              1790
 70   CONTINUE                                                          1800
 1000 FORMAT(1X,I2,5(2X,F10.6))                                         1810
      RETURN                                                            1820
      END                                                               1830
C                                                                       1840
C                                                                       1850
C                                                                       1860
      SUBROUTINE EQUA(GMA,FUN,DFUN,DDFUN)                               1870
C=======================================================================1880
C                                                                       1890
C THIS ROUTINE CONTAINS THE EQUATION OF THE TEMPLATE,                   1900
C AND THE FIRST AND SECOND DERIVATIVES OF THE EQUATION.                 1910
C FOR DESCRETE POINT DATA USE AN INTERPOLATOR.                          1920
C                                                                       1930
C=======================================================================1940
C                                                                       1950
C                                                                       1960
C                                                                       1970
C-----------------------------------------------------------------------1980
C                                                                       1990
C THE TEMPLATE USED NOW IS A LIMACON OF PASCAL                          2000
C                                                                       2010
C-----------------------------------------------------------------------2020
      COMMON/CURVE/ RI,EPS,RR,BB,AA                                     2030
      COSG=COS(GMA)                                                     2040
      FUN=BB+AA*COSG                                                    2050
      DFUN=-AA*SIN(GMA)                                                 2060
      DDFUN=-AA*COSG                                                    2070
      RETURN                                                            2080
      END                                                               2090
C                                                                       2100
C                                                                       2110
C                                                                       2120
      FUNCTION SR(GAMM,GAM)                                             2130
C=======================================================================2140
C                                                                       2150
C THIS ROUTINE CALCULATES THE LENGTH ON THE CONTOUR CORRESPONDING        2160
C TO ONE RADIAL OR ANGULAR RESOLUTION.                                  2170
C THE ROUTINE USES GAUSS-LEGENDRE QUADRATURE FOR INTEGRATING            2180
C THE LENGTH ON THE CONTOUR.                                            2190
C A 4 POINT POLYNOMIAL IS USED.                                         2200
C                                                                       2210
C=======================================================================2220
C                                                                       2230
      COMMON/PAR/ PY,ROOT(4),WE(4),NN                                   2240
      DIF=(GAMM-GAM)/2.0                                                2250
```

```
      SUM=(GAMM+GAM)/2.0                                                2260
      AI=0.0                                                            2270
      DO 10 I=1,4                                                       2280
      X=SUM+DIF*ROOT(I)                                                 2290
      CALL EQUA(X,FUN,DFUN,DDFUN)                                       2300
      AI=AI+WE(I)*FUN                                                   2310
10    CONTINUE                                                          2320
      SR=ABS(DIF*AI)                                                    2330
      RETURN                                                            2340
      END                                                               2350
EOI ENCOUNTERED.
```

```
*                                                                       100
OUTPUT PROGRAM FOR THE SOTWARE DIGITIZER.                               110
PROGRAM IN TI-990/980 MICRO COMPUTER ASSEMBLER CODE                     120
DATA STORED AS FOLLOWS:                                                 130
BIT 0 =1 TO ALLOW REAL TIME CLOCK OPERATION                             140
BITS 1 TO 14 CONTAIN THE TIME PERIOD BETWEEN SIGNALS                    150
BITS 14 AND 15 CONTAIN THE AXES INFORMATION                             160
*                                                                       170
*                                                                       180
*                                                                       190
 200 02E0    LWPI >1E0          INITIALISE WORK SPACE COUNTER           200
 202 01E0                                                               210
 204 0202    LI   R2,>300       DATA START AT LOCATION                  220
 206 0300                       300 IN MEMORY                           230
           *                    START OF THE OUTPUT LOOP                240
 208 020C ST LI   R12,>00       SET BASE ADRESS TO U10 9901             250
 20A 0000                                                               260
 20C 3392    LDCR *R2,14        LOAD FIRST 14 BITS INTO CLOCK REGISTER  270
                                AND START THE CLOCK                     280
 20E 1E00    SBZ  0             SWITCH 9901 TO INTERUPT MODE            290
 210 1D03    SBO  3             ENABLE U10 9901 TO INTERUPT ON LEVEL 3  300
 212 020C    LI   R12>03C       SET POINTER TO BIT 14 OF THE CURRENT    310
 214 003C                       DATA WORD                               320
 216 0340    IDLE               PUT COMPUTER IN IDLE STATE, TO BE       330
           *                    AWAKENED BY INTERUPT                    340
 218 0340    LWPI >1E0          INITIALIZE WORK SPACE, USED HERE        350
 220 01E0                       TO BALANCE TIMING OF THE LOOP TO MINIMUM 360
 222 3062    LDCR *R2+,2        OUTPUT 2 BITS STARTING FROM THE         370
           *                    LOCATION OF THE POINTER IN THE DATA WORD 380
 21A 10F6    JMP  ST            GO TO BEGINING OF OUTPUT LOOP           390
           *                    IF DATA STILL EXIST IN MEMORY           400
           *                    PERIOD OF THE REAL TIMIE CLOCK          410
```

APPENDIX D

PERFORMANCE EVALUATION OF THE TEMPLATE DIGITIZER

D.1 Introduction

The template digitizer is tested to evaluate both the digitization accuracy and the error in contouring speed. The hybrid computer EAI 680/640 is used as a means of collecting and preprocessing the radial and angular shaft encoder signals. The sequence of the radial and angular signals, as well as the time periods between these signals, are recorded. The sequence of the signals is used to evaluate the accuracy of the digitization process. The time periods between the signals are used to calculate the contouring speed.

The data collected on the hybrid computer is subsequently transmitted to the CDC CYBER 835 for processing.

D.2 Testing circuit

Figure D.1 shows a block diagram of the different components used to extract the required information and the flow of signals between these components. With reference to the figure, the four outputs generated by the digitizer encoders are combined into two signals using the recording circuit. The two signals are then recorded independently on two channels of an FM magnetic tape recorder. A high tape

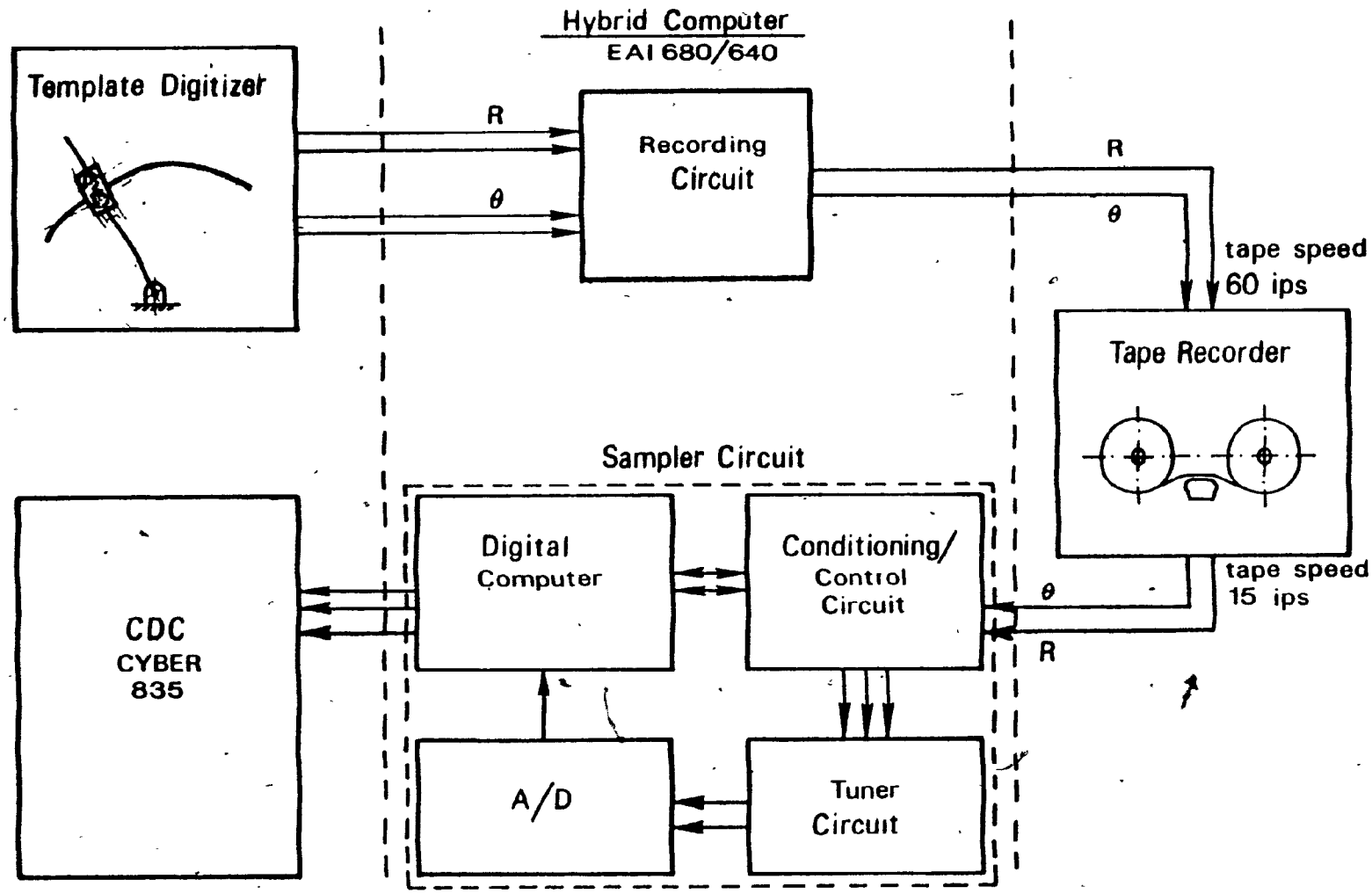


Figure D.1: Block Diagram of the Circuits Used in the Performance Evaluation of the Template Digitizer.

speed of 152.4 cm/s is used for recording. The recorded signals are then played back at a low tape speed of 38.1 cm/s and fed to the sampler circuit. Recording at high tape speeds and then playing back at low speeds scales up the time period between the signals, and allows more accurate time measurment. The sampler circuit processes the two incoming signals and generates two sets of digital output that feed into the digital computer part of the hybrid computer. One of these sets corresponds to the sequence of radial and angular signals, while the other set corresponds to the time period between the signals. The digital computer accumulates the data in a file until the end of the run. The file is then transferred to the CDC CYBER 835 for analysis.

The recording circuit is shown in Figure D.2. The circuit is a direct implementation of the one in Figure 2.9. The output of the circuit is scaled to 2 volts (p.p.) to match the maximum input signals level of the tape recorder. The digital components on the analog computer are syncronized to the computer clock, consequently the accuracy of timing between the recorded encoder signals is within one clock period, 1 μs. At the beginning of each recording, an accurate reference signal is taped and is used later for calibration.

The recorded signals are slowed down when played back as has been mentioned earlier. The played back signals are fed to the sampler circuit. The circuit is shown in

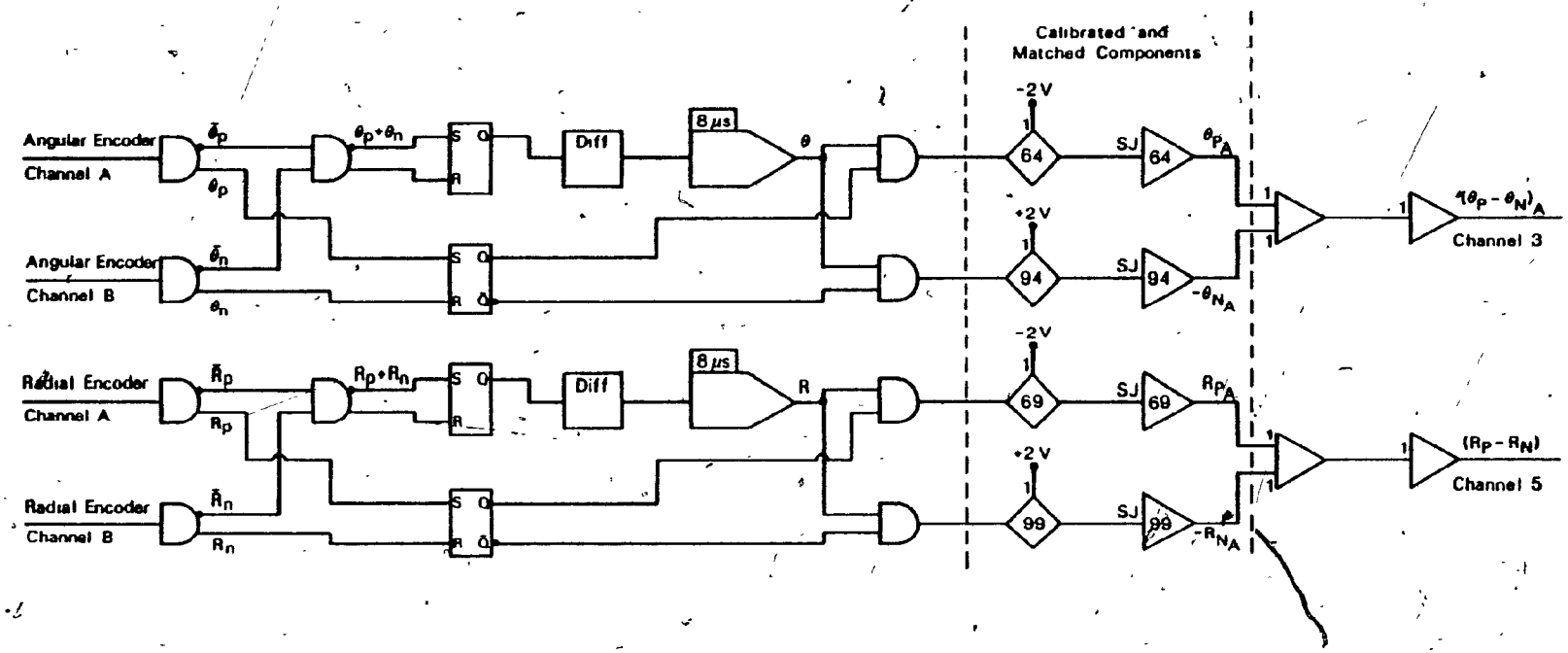


Figure D.2: Recording Circuit for the Encoder Signals.

Figure D.3. It consists essentially of two parts, these are:

Input part; this part uses threshold comparators to restore the played back signals to digital logic level and shape. The reconstructed signals are then processed by a digital part to give four logic signals. These signals indicate either the radial or angular axis indexed and its direction of motion. The four signals are stored temporarily in a register.

Timing part; the timing part of the circuit uses three operational amplifiers connected as integrators. The input to the integrators is a constant voltage. When the circuit is allowed to integrate, the voltage on the output of the integrator is proportional to time. Three integrators are used to circumvent the problem of the finite time required to sample an integrator output, and reset the integrator. The three integrators are operated in a cyclic order. While one integrator is integrating, the second is being sampled, and the third is being reset. The function of the three integrators is controlled by a logic circuit based upon the four signals generated by the input part of the sampler circuit. An integrator starts integrating the constant input voltage at the beginning of any transient indicating indexation, and is put in a hold state at the occurance of the next such transient. The output of the integrator is converted into a digital equivalance using an A/D converter. The digital equivalence of time as well as the four signals

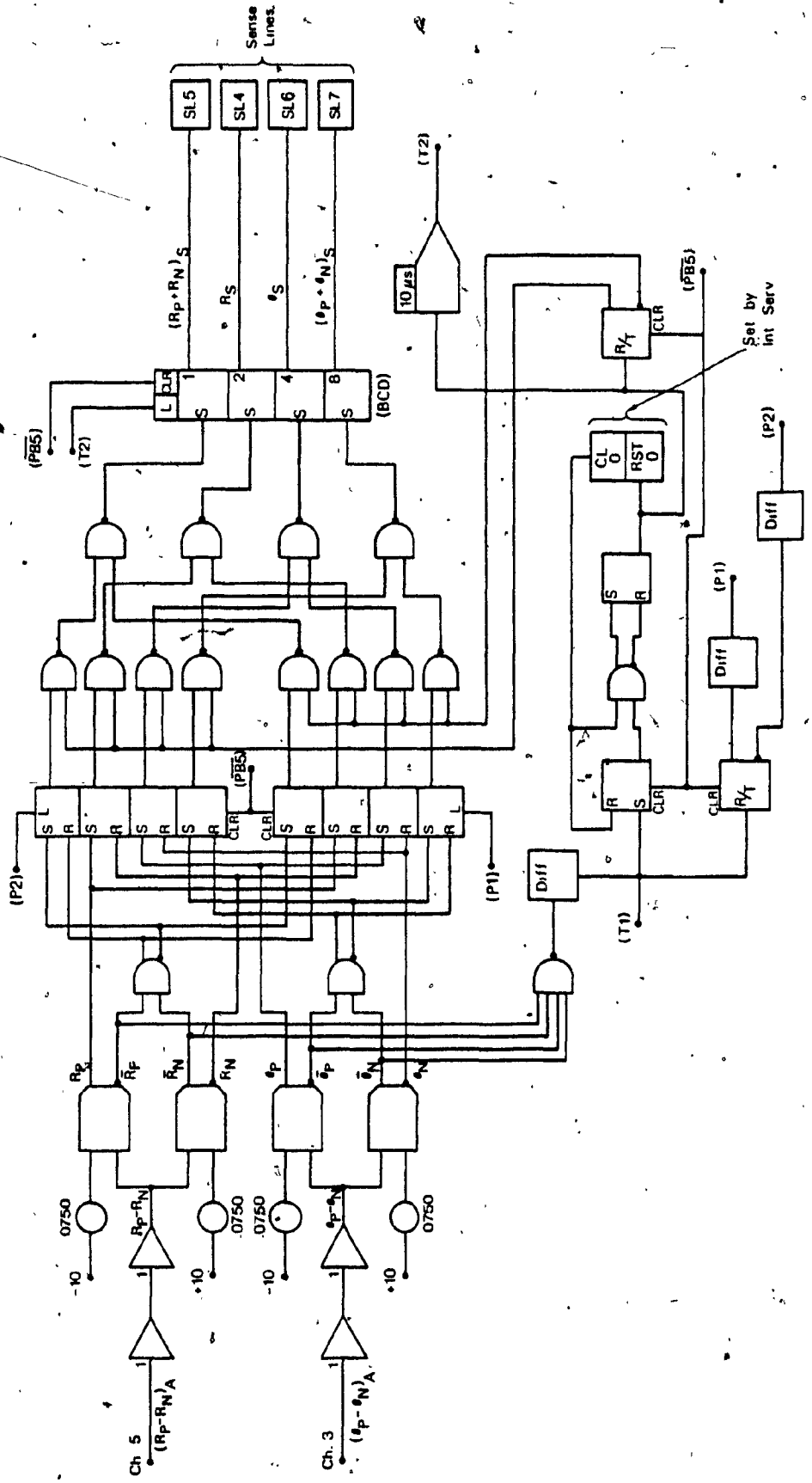


Figure D.3: Sampler Circuit for the Recorded Encoder Signals. (continued on D.7).

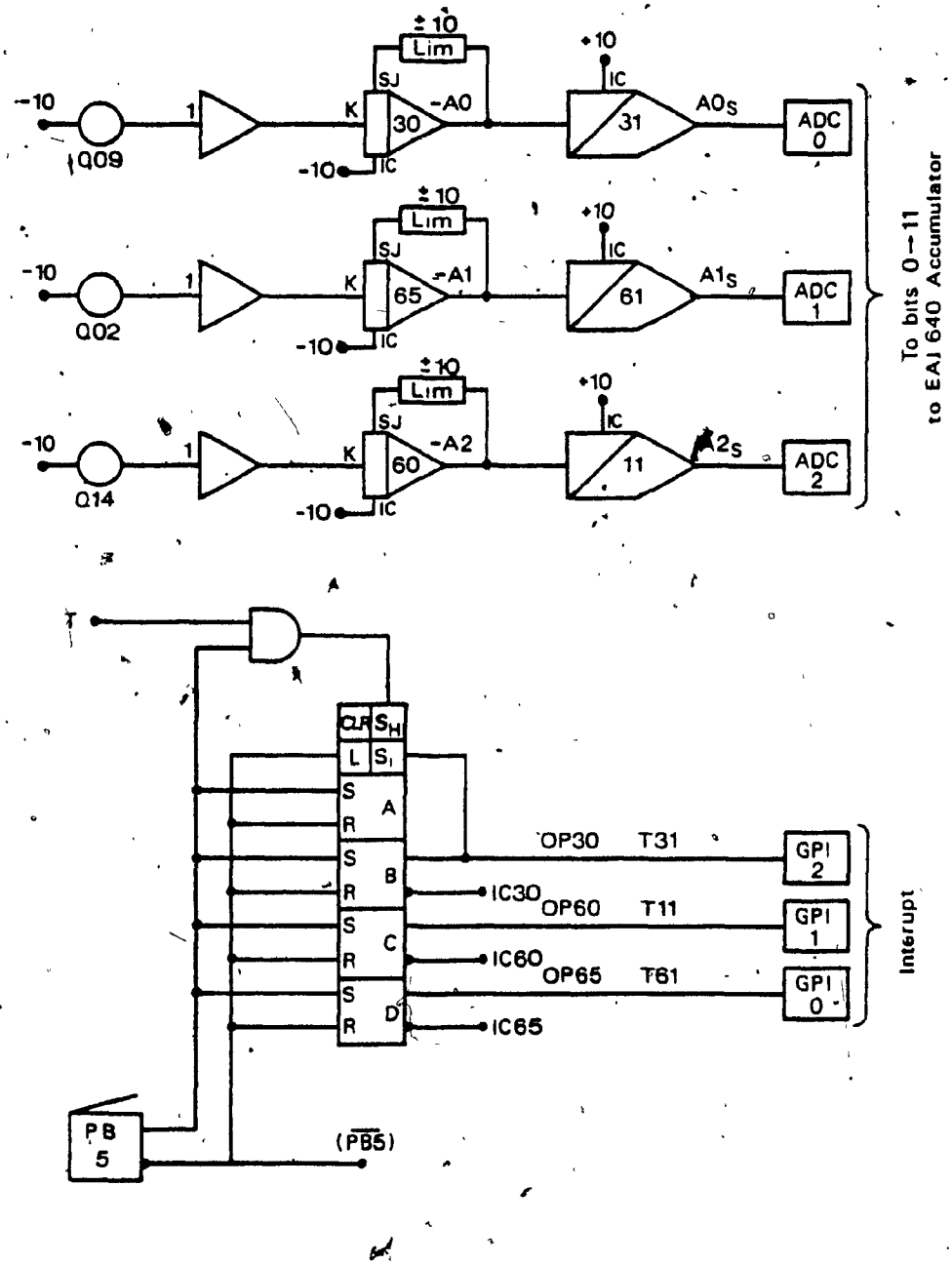


Figure D.3: Sampler Circuit for the Recorded Encoder Signals.

from the input part are compiled into one 16 bit computer word and transferred to the digital computer memory. The data are transferred from the analog part of the computer to the digital part in INTERUPT mode of operation. Interupts are generated when either the radial or angular template digitizer axes index.

An ASSEMBLER program executes on the EAI 640 and controls the hybrid computer operation to carry out the testing. A listing of this program is provided at the end of this appendix.

The data collected by the hybrid computer are subsequently transferred in ASCII code to the CYBER 835 for analysis. Two FORTRAN programs carry out a two stage decoding of the collected data and prepare them for analysis. Listings of the decoding programs are also provided at the end of this appendix.

D.3 Results of testing

An analysis of the collected data showed that the template digitizer perform the contour digitization process accurately for the three templates shown in the photograph in Figure 3.3. The data collected in testing of the line template were also used in verifying the objective function for area error minimization and are shown in Figure 4.8. The agreement between the theoretically and experimentally calculated objective functions serve as an implicit verification of the accuracy of digitization.

The analysis of the time signals collected, shows that for all practical purposes the contouring speed is found to be constant. At the beginning of motion, however, when power is switched on for the the roller mechanism motors, the contouring speed was found to oscillate for a short time. Furthermore, whenever the roller mechanism motion is disturbed by template imperfections in the form of indentations, oscillation starts again but lasts only or a short time.

```
*****************************************************
*
*      ACQUISITION SYSTEM MAIN PROGRAM.
*
*       -----  ATEF  ------
*
*    VER. 1.4                APR. 28/1981 DRH
*
*    ORIGINATED FROM ACQ VER. 3.2
*
*
*   AT PAUSE ENTER A=#OF POINTS.
*
       NAME    ACQUIR
       ABS     '1000
*
*
ACQUIR EQU     *
       P       0        SET UP REGISTER OPTIONS.
       STA     LENTH
       EX
       LA      FWAD
       EQ
       L,I     ASAMPL
       LX      LENTH
       LA      FWAD
       EQ
       L,I     ASENDR
       J       ACQUIR
*
LENTH  OCT     0
FWAD   OCT     2000
ASAMPL ADR     SAMPLE
ASENDR ADR     SENDER
       BSS     3        KEEP BUFF ADD. AT 1030.
BUFF   BSS     '634
BUFO   BSS     82
EACQ   BSS     1
**
*
*****************************************************
*
*         SUBROUTINE SAMPLE
*
*
*   USED TO SAMPLE ADC AT SPECIFIED RATES WITH
*   SPECIFIED AMOUNTS OF DATA,
*   DIRECTED TO A SPECIFIED BUFFER AREA.
*
*    FIRST 12 BITS OF WORD IS ADC VALUE,
*    LAST 4 BITS IS SL4,SL5,SL6,SL7.
*
*
*   TO USE:
*
*      L SAMPLE
*
*
*      -X- CONTAINS NUMBER OF POINTS TO SAMPLE,
*       RANGE IS FROM 1 TO 15000.
*
*      -B- CONTAINS DATA BUFFER START ADDRESS,
*       NORMALLY '2000.
*
*     SAMPLER CLOCK FROM EAI680 - GP INTERRUPT 0.
*
**
**
       ABS     '10
SAMPLE ADR     0
       EQ               BUFFER FWA TO -A-
       STA     FFWA
```

```
         EQ                                                          820
         STX     LENGTH   SAVE BUFFER LENGTH.                       830
         EX                                                          840
         A       FFWA     ADD FWA TO LENGTH TO FIND END.            850
         STA     BFPNT     SAVE IN BUFFER POINTER.                  860
         LA      LENGTH   GET BACK LENGTH.                          870
         TCA                                                         880
         SSP              FOR NEG VALUE IN -X-                       890
         STA     LENGTH   AND SAVE IT.                               900
         EX               NEG BUFF LENGTH INTO -X-.                  910
         LA      DF680                                               920
         DO      '53      SELECT ANALOG CONSOLE.                     930
         LA      CLOT8                                               940
         DF      '43      RESET CL-0,CL2,CL4,CL6,CL8                 950
         LA      AGPI0                                               960
         STA     '705     SET UP INT. POINTERS.                      970
         LA      AGPI1                                               980
         STA     '706                                                990
         LA      AGPI2                                              1000
         STA     '707                                               1010
         LA      NADC                                               1020
         A       BFPNT    WILL IGNORE 1ST SAMPLE.                   1030
         STA     BFPNT                                              1040
         LA      SLMASK   INITIALIZE FIRST SL READING.              1050
         STA     OLDSL                                              1060
         J       SAM      START SAMPLING.                           1070
*                                                                   1080
*************************************                               1090
*                                                                   1100
NADC     OCT     177777                                            1110
MSKGPI   OCT     160000                                            1120
LENGTH   OCT     0                                                 1130
FFWA     OCT     0                                                 1140
BFPNT    OCT     0                                                 1150
DF680    OCT     40                                                1160
CLO      OCT     100000                                            1170
INT680   OCT     100000                                            1180
KREG     OCT     1000                                              1190
AGPI0    ADR     SERV0                                             1200
AGPI1    ADR     SERV1                                             1210
AGPI2    ADR     SERV2                                             1220
CLOT8    OCT     125200                                            1230
*                                                                  1240
*****************************************************              1250
*                                                                  1260
SAM      EQU     *                                                 1270
         LA      KREG                                              1280
         ES                                                        1290
         LA      INT680   ENABLE CONSOLE 1 INTERRUPT.              1300
         DO      '54                                               1310
         LA      MSKGPI                                            1320
         DO      '46      ENABLE GPO-2 INTERRUPTS.                 1330
         CLR                                                       1340
         DF      '64      SELECT ADC-0 AND CONVERT.                1350
         LA      CLO                                               1360
         DF      '41      SET CL-0.                                1370
WWAIT    SMI                                                       1380
         J       *        WAIT FOR INTERRUPT.                      1390
*                                                                  1400
*****************************************************              1410
*                                                                  1420
*       SERVICE ROUTINES     SERV0,SERV1,SERV2                     1430
*                                                                  1440
*       ADC CHANNELS 0,1,2 ONLY.                                   1450
*                                                                  1460
SERV0    OCT     0                                                 1470
         OCT     0                                                 1480
         CLR              SET UP AD CHAN 0.                        1490
         J       SAMP                                              1500
SERV1    OCT     0                                                 1510
         OCT     0                                                 1520
         LA      ADC1                                              1530
```

```
         J       SAMP                                                    1540
SERV2    OCT     0                                                       1550
         OCT     0                                                       1560
         LA      ADC2                                                    1570
         J       SAMP                                                    1580
ADC1     OCT     1                                                       1590
ADC2     OCT     2                                                       1600
*                                                                        1610
*   SAMPLE SELECTED ADC AND COMBINE WITH SL4-7.                          1620
*                                                                        1630
SAMP     EQ                      SAVE ADC CHANNEL NO.                    1640
         SI      '44             READ SL'S 4,5,6,7 FOR                   1650
         SI      '45             RESET TO ZERO.                          1660
         SI      '46                                                     1670
         SI      '47                                                     1680
         DI      '65             READ CURRENT ADC VALUE(PREVIOUS CONVERT)1690
         EQ                                                              1700
         DF      '64             START NEXT CONVERSION.                  1710
         DI      '42             READ ALL 8 SENSE LINES.                 1720
         EQ                                                              1730
         AND     ADMASK                                                  1740
         OR      OLDSL           COMBINE DATA WITH PREVIOUS SENSE LINES. 1750
         STA,IX  BFPNT           STORE LATEST VALUE IN BUFFER.           1760
         EQ                                                              1770
         AND     SLMASK                                                  1780
         STA     OLDSL           SAVE NEW SENSE LINES.                   1790
         LA      CLOT8                                                   1800
         DF      '41             TURN ON CL#0,CL#2,CL#4,CL#6,CL#8        1810
         ICX     1                                                       1820
         J       *+2                                                     1830
         J       *+2             FINISHED SAMPLING.                      1840
         J       WWAIT           WAIT FOR NEXT INTERRUPT.                1850
         LA      INT680                                                  1860
         DO      '53             DISABLE 680 INT.                        1870
         LA      MSKGPI                                                  1880
         DO      '40             DISABLE GP  INT.                        1890
         J,I     SAMPLE          RETURN TO CALLER AT END OF BUFFER.      1900
SLMASK   OCT     17                                                      1910
ADMASK   OCT     177760                                                  1920
OLDSL    OCT     17                                                      1930
*                                                                        1940
************************************************************           1950
*                                                                        1960
****************************************************************       1970
*                                                                        1980
*         SUBROUTINE SENDER                                              1990
*                                                                        2000
*   SENDS A SPECIFIC MEMORY BUFFER OF SPECIFIED                          2010
*   LENGTH TO DATA PHONE CONTROLLER.                                     2020
*                                                                        2030
*   ASSUMES THAT PROTOCOL IS 6000 TEXT MODE,                             2040
*   THEREFOR SENDS 80 CHARS PLUS CR                                      2050
*   WAITS FOR LF AND SENDS AGAIN.                                        2060
*                                                                        2070
*   THE MOST SIGNIFICANT 12 BITS OF EACH MEMORY                          2080
*   IS THE ADC VALUE, WHILE THE LAST 4 BITS                              2090
*   REPRESENT THE RELATED SL SETTINGS (SL4-SL7).                         2100
*   THE COMPLETE 16-BIT WORD IS SENT AS 4 ASCII CHARS                    2110
*   IN THE RANGE '301 TO '320 (A TO P).                                  2120
*                                                                        2130
*   TO RECOVER THE 12 BIT PORTION OF THE WORD, SUBTRACT 1                2140
*   FROM EACH OF 1ST 3 CHARS IN EACH GROUP OF 4 AND                      2150
*   ASSEMBLE THE LEAST SIGNIFICANT 4 BITS OF                             2160
*   EACH OF 3 CHARS INTO A 12 BIT WORD.                                  2170
*                                                                        2180
*   TO RECOVER THE 4-BIT SENSE-LINE WORD, SUBTRACT 1                     2190
*   FROM LAST CHAR IN EACH GROUP OF 4 AND KEEP LEAST                     2200
*   SIGNIFICANT 4 BITS OF THE CHARACTER.                                 2210
*                                                                        2220
*                                                                        2230
*     TO USE:                                                            2240
*                                                                        2250
```

```
*     L SENDER                                                    2260
*                                                                 2270
*     -X- CONTAINS BUFFER LENGTH                                  2280
*                                                                 2290
*     -B- CONTAINS BUFFER FIRST WORD ADDRESS.                     2300
*                                                                 2310
*                                                                 2320
*       WHEN IN TEXT MODE, START SENDING DATA                     2330
*       BY ENTERING  CNTRL R   ON TTY.                            2340
*                                                                 2350
*                                                                 2360
*     SSW A  WILL SEND S CHAR TO COMM. MODULE.                    2370
*                                                                 2380
*                                                                 2390
ACC   OCT   0                                                     2400
PSW   OCT   100                                                   2410
FWA   ADR   BUFF                                                  2420
INP   OCT   0                                                     2430
OUTP  OCT   0                                                     2440
*                                                                 2450
*************************************************               2460
*                                                                 2470
ROC   OCT   0                                                     2480
      STA   ACC                                                   2490
      L     FSTAT   WAIT FOR WREG FROM COMM. MODULE.              2500
      LA,I  ACC                                                   2510
      OR    FT                                                    2520
      DF    '30     SEND OUT ONE LINE OF DATA.                    2530
      AOM   ACC                                                   2540
      C     CR                                                    2550
      SE                                                          2560
      J     *-7                                                   2570
      LA    FLAG                                                  2580
      SAE           WAIT FOR LAST OUTPUT                        2590
      SKU           INTERRUPT FLAG - COMM. MODULE.                2600
      J     *-3                                                   2610
      J,I   ROC                                                   2620
*                                                                 2630
*************************************************               2640
*                                                                 2650
INITC OCT   0                                                     2660
      LA    SFLAG   RESET OUTPUT INTERRUPT                        2670
      STA   FLAG    FLAG FOR COMM. MODULE.                        2680
      LA    DFON                                                  2690
      DF    '30                                                   2700
      LA    AINTC                                                 2710
      STA   '674                                                  2720
      LA    ASRV                                                  2730
      STA   '646                                                  2740
      J,I   INITC                                                 2750
SFLAG OCT   0                                                     2760
*                                                                 2770
*****************************************************             2780
*                                                                 2790
DIAL  OCT   0                                                     2800
      LA    TTO                                                   2810
      DF    1                                                     2820
      L     STAT                                                  2830
      LX    D24                                                   2840
DL1   LA,X  MSG                                                   2850
      DO    1                                                     2860
      L     STAT                                                  2870
      DCX   1                                                     2880
      J     DL1                                                   2890
      LA    TTO                                                   2900
      DF    2                                                     2910
      DI    2                                                     2920
      L     SONE    SEND ONE TTY CHAR TO COMM. MODULE.            2930
      J,I   DIAL                                                  2940
MSG   OCT   212,215                                               2950
      DATA  S\NI GOL ,MEDOM NO NRUT\                              2960
      OCT   212,215                                               2970
```

```
Q24    DEC    24                                                    2980
*                                                                   2990
*********************************************************           3000
*                                                                   3010
STAT   OCT    0                                                     3020
       SI     0                                                     3030
       ADA                                                          3040
       SAE                                                          3050
       J      *-3                                                   3060
       J,I    STAT                                                  3070
TTO    OCT    100012                                                3080
TTI    OCT    100016                                                3090
CR     OCT    50015                                                 3100
EOT    OCT    204                                                   3110
ESC    OCT    33                                                    3120
*                                                                   3130
ASRV   ADR    SERV                                                  3140
DFON   OCT    130620                                                3150
AINTC  ADR    INTRC                                                 3160
*                                                                   3170
**********  SENDER  ***************************************          3180
*                                                                   3190
SENDER EQU    *                                                     3200
       OCT    0                                                     3210
       EQ                  GET BUFFER FIRST ADDRESS,                3220
       STA    MP           AND PUT IN MEMORY POINTER.               3230
       EX                  GET BUFFER LENGTH                        3240
       A      MP           ADD FIRST ADDRESS                        3250
       STA    ME           AND SET INTO MEMORY END POINTER.         3260
       L      INITC                                                 3270
       LA     FWA                                                   3280
       STA    INP          RESET IN BUFF POINTER.                   3290
       STA    OUTP         RESET TTY BUFF POINTER.                  3300
       L      DIAL                                                  3310
       LA     TTI                                                   3320
       DF     2                                                     3330
LOGIN  L      BRIC                                                  3340
       SSW    A                                                     3350
       SKU                                                          3360
       J      SENDS                                                 3370
       LA     OUTP                                                  3380
       C      INP                                                   3390
       SGE                                                          3400
       J      TYPE                                                  3410
       P      0            WAIT FOR, INTERRUPT OUT OF P.            3420
       J      LOGIN+1                                               3430
*                                                                   3440
*********************************************************           3450
*                                                                   3460
SERV   OCT    0                                                     3470
       OCT    0                                                     3480
       DI     2                                                     3490
       C      EOT                                                   3500
       SNE                                                          3510
       LA     ESC                                                   3520
       C      CNTR                                                  3530
       SNE                                                          3540
       J      RREAD        CODE AND SEND LINE IF CNTRL-R-.          3550
DOUT   EQ                                                           3560
       L      FSTAT        WAIT FOR COMM.MODULE WREQ.               3570
       EQ                                                           3580
       OR     FT                                                    3590
       DF     '30          OUT CHAR TO COMM. MODULE.                3600
       LA     FWA                                                   3610
       STA    INP                                                   3620
       STA    OUTP                                                  3630
       J      LOGIN                                                 3640
CNTR   OCT    222                                                   3650
FLAG   BSS    1                                                     3660
FT     OCT    50000                                                 3670
*                                                                   3680
*********************************************************           3690
```

```
*
TYPE    LA      TTO
        DF      3
        L       STAT
        LA,I    OUTP
        DO      3
        L       STAT
        AOM     OUTP
        LA      TTI
        DF      2
        J       LOGIN+1
*
*************************************************************
*
FSTAT   OCT     0
        STA     TEMP
        LA      FLAG
        SAE                     COMM. MODULE READY FOR OUTPUT?
        SKU
        J       *-3
        CLR
        STA     FLAG            RESET OUTPUT FLAG.
        LA      TEMP
        J,I     FSTAT
TEMP    BSS     1
*
*************************************************************
*   INITIALIZE COMM. MODULE FOR INTERRUPTS.
*
*
BRIC    OCT     0
        ES
        OR      PSW             SET UP INT. MASK.
        ES
        LA      FWA             INITIALIZE INPUT
        STA     INP             BUFFER POINTER.
        SMI
        J,I     BRIC
*
*************************************************************
*   INTERRUPT SERVICE ROUTINE, COMM. MODULE.
*
*
INTRC   OCT     0
        OCT     0
        SI      '30
        SKP                     IS RREG ON ?
        J       READ
        LLS     1
        SKN                     IS WREG ON ?
        J       ERROR
        LA      DFWR            RESET WREG
        DF      '30
        LA      ONE             TURN ON OUTPUT
        STA     FLAG            INTERRUPT FLAG.
        LA      INTRC+1
        SMI
        J,I     INTRC
READ    AND     RMASK           GET INPUT CHAR.
        OR      BIT8            TURN ON PARITY BIT.
        STA     TEMP1
        LA      DFWC            RESET RREG.
        DF      '30
        LA      TEMP1
        STA,I   INP             SAVE IN CHAR. BUFFER
        AOM     INP
        C       BPNT            CODING BUFFER ADDRESS
        SGE                     EXCEEDED ?
        J       *+4
        LA      FWA
        STA     INP             RESET IN BUFF POINTER.
        STA     OUTP            RESET TTY BUFF POINTER.
```

```
         LA       INTRC+1                                          4420
         SMI                                                       4430
         J,I      INTRC                                            4440
*                                                                  4450
ERROR    LRS      1                                                4460
         P        '77        NOT WREG OR RREG.                     4470
*                                                                  4480
DFWR     OCT      10620                                            4490
RMASK    OCT      377                                              4500
TEMP1    BSS      1                                                4510
DFWC     OCT      100620                                           4520
BIT8     OCT      200                                              4530
ONE      OCT      1                                                4540
*                                                                  4550
DOC      EQU      ROC                                              4560
*                                                                  4570
*****************************************************************4580
*                                                                  4590
RREAD    SMI                                                       4600
         LA       BPNT                                             4610
         STA      BI,BO                                            4620
         LX       BPNT                                             4630
         L        ROR                                              4640
         LA       BPNT                                             4650
         L        DOC                                              4660
         L        WAIT                                             4670
         J        RREAD+1                                          4680
*                                                                  4690
*****************************************************************4700
*                                                                  4710
WAIT     OCT      0                                                4720
         LA       FWA                                              4730
         STA      INP,OUTP                                         4740
PAS      P        1          WAIT FOR INTERRUPT.                   4750
         LA,I     OUTP                                             4760
         AOM      OUTP                                             4770
         C        LF                                               4780
         SE                                                        4790
         J        PAS                                              4800
         L        DELAY      ,DELAY 0.3 SEC.                       4810
         J,I      WAIT                                             4820
LF       OCT      212                                              4830
SAVE     OCT      0                                                4840
*                                                                  4850
*****************************************************************4860
*                                                                  4870
DELAY    OCT      0                                                4880
         LA       NEG        GIVES 0.3 SEC DELAY.                  4890
         STA      NCNT                                             4900
         LRS      '16                                              4910
         NOP                                                       4920
         NOP                                                       4930
         AOM      NCNT                                             4940
         J        *-4                                              4950
         J,I      DELAY                                            4960
NEG      DEC      -16384                                           4970
NCNT     BSS      1                                                4980
*                                                                  4990
*****************************************************************5000
*                                                                  5010
ROR      OCT      0                                                5020
         STX      SAVE                                             5030
         STX      BP                                               5040
         LX       DM20                                             5050
AGN      LA       DM4                                              5060
         STA      COUNT                                            5070
         LA       MP         1ST WRD ADDRESS OF DATA BUFF.         5080
         C        ME         LAST WRD ADD. OF DATA BUFFER.         5090
         SNE                                                       5100
         J        EXIT                                             5110
         LA,I     MP                                               5120
         AOM      MP                                               5130
```

```
         EQ                                                          5140
NXT      CLR                                                         5150
         LLD     4                                                   5160
         A       K301                                                5170
         STA,I   BP        SET UP TEMP. 80 CHAR BUFF.                5180
         AOM     BP                                                  5190
         AOM     COUNT                                               5200
         J       NXT                                                 5210
         ICX     1                                                   5220
         J       AGN                                                 5230
         LA      MCR                                                 5240
         STA,I   BP        PUTS GR AT END OF BP BUFF.                5250
         LX      SAVE                                                5260
         J,I     ROR                                                 5270
*                                                                    5280
*                                                                    5290
EXIT     EQU     *                                                   5300
         LA      MCR                                                 5310
         STA,I   BP                                                  5320
         LA      BPNT                                                5330
         L       DOC                                                 5340
         L       WAIT                                                5350
         LA      METX      THIS ETX CHAR, 'CNTRL-C-',                5360
         DF      '30       GETS OUT OF TEXT MODE.                    5370
         J       LOGIN                                               5380
*                                                                    5390
*********************************************************************5400
*                                                                    5410
SENDS    LA      SS                                                  5420
         J       DOUT      SEND S TO COMM. MODULE.                   5430
SS       OCT     323                                                 5440
*                                                                    5450
*********************************************************************5460
*   SENDS FIRST CHAR TYPED AFTER DIAL.                               5470
*   INITIALIZES COMM.MODULE.                                         5480
*                                                                    5490
*                                                                    5500
SONE     OCT     0                                                   5510
         STA     ACC                                                 5520
         LA      DFON                                                5530
         DF      '30       INITIALIZE COMM.MODULE.                   5540
         LA      ACC                                                 5550
         OR      FT                                                  5560
         DF      '30       OUT 1ST CHAR TO COMM. MODULE.             5570
         J,I     SONE                                                5580
*                                                                    5590
****** MISC INTEGERS  ***************                               5600
*                                                                    5610
METX     OCT     50003                                               5620
BI       OCT     0                                                   5630
DM20     DEX     -20                                                 5640
DM4      DEC     -4                                                  5650
COUNT    OCT     0                                                   5660
MP       OCT     0                                                   5670
ME       OCT     0                                                   5680
K301     OCT     301                                                 5690
BP       OCT     0                                                   5700
MCR      OCT     15                                                  5710
BO       OCT     0                                                   5720
BPNT     ADR     BUFO                                                5730
SENDXX   END     ACQUIR                                              5740
```

```
      PROGRAM DECODE2(INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT)
C=====================================================================
C     VER 1.4       APR 13/82 DRH
C
C     DECODE VER 2.7 USED AS STARTING BASIS.
C
C  DECODE 1ST 3 ASCII CHARS INTO 12 BIT INTEGER,
C  EACH ASCII CHAR CONTAINS FOUR BITS OF THE INTEGER,
C  SUBTRACT ONE FROM CHAR BEFORE DECODING,(F='306=5).
C
C  NOTE THAT CHARS ARE IN DISPLAY CODE (F='006=5).
C
C   NOTE THAT NEGATIVE INTEGERS FORMED ARE IN 2'S COMPLEMENT,
C   MUST BE CONVERTED TO ONE'S COMPLEMENT.
C
C    RESULTING SAMPLES ON TAPE2 IN VOLTS.
C    VOLTAGE READING REPRESENTS TIME BETWEEN PULSES.
C         (EXAMPLE:  IN APR. 1982,   1 MICROSEC. = .01 VOLTS)
C
C    DECODE 4TH ASCII CHAR INTO POSITIVE INTEGER,
C    RESULT ON TAPE IN INTEGER RANGE 0-15.
C
C=====================================================================
      DIMENSION CODE(80),DATA(20),IDAT(20)
      INTEGER CODE
C
C   WILL READ LINES FROM TAPE1 UNTIL EOF.
C
      OLDAT=0.0
  3   CONTINUE
      READ(1,120) (CODE(J),J=1,80)
 120  FORMAT(80R1)
      IF(EOF(1).GT.0) GO TO 80
C
      DO 5 I=1,80
 5    CODE(I)=CODE(I)-1
      JJ=0
      DO 20 II=1,77,4
      JJ=JJ+1
      IDAT(JJ)=CODE(II+3)
      DO 10 J=1,2
      CODE(II) = CODE(II)*16 + CODE(II+J)
 10   CONTINUE
C
C   CONVERT FROM TWO'S TO ONE'S COMPLEMENT.
C
C   FORM NUMBER EXPRESSED IN VOLTAGE.
C
      IF(CODE(II).LE.2047) GO TO 15
      CODE(II)=CODE(II).OR.7777777777777770000B
      CODE(II)=CODE(II)-1
 15   R=CODE(II)
      DATA(JJ)=10.*R/2048.
      IF(DATA(JJ).GE.(9.999)) DATA(JJ)=9.999
      IF(DATA(JJ).LE.-9.999) DATA(JJ)=-9.999
      DATA(JJ)=DATA(JJ)+10.0
 20   CONTINUE
C
C   SHIFT DATA TO MATCH WITH PROPER SENSE-LINES.
C
      TEMP=DATA(20)
      KK=20
      DO 30 I=1,19
      KK=KK-1
      DATA(KK+1)=DATA(KK)
 30   CONTINUE
      DATA(1)=OLDAT
      OLDAT=TEMP
      WRITE(2,130) (IDAT(I),DATA(I),I=1,10)
      WRITE(2,130) (IDAT(I),DATA(I),I=11,20)
 130  FORMAT(10(1X,I2,F5.2))
      GO TO 3
 80   STOP
      END
```

```
      PROGRAM DECODE2(INPUT,TAPE2,TAPE3)                                100
C=====================================================================  110
C                                                                       120
C THIS PROGRAM GENERATES A FILE OF TIME, RADIAL, AND ANGULAR            130
C DISPLACEMENTS FROM THE DATA PREVIOUSLY DECODED BY DECODE1.            140
C THESE DATA HAVE BEEN COLLECTED BY THE HYBRID COMPUTER                 150
C FROM THE TEMPLATE DIGITIZER.                                          160
C THIS FILE IS COMPARED WITH THAT PRODUCED BY THE SOFTWARE              170
C DIGITIZER FOR A SIMILAR CONTOUR, AND SERVES AS A MEANS OF             180
C EVALUATING THE PERFORMANCE OF THE TEMPLATE DIGITIZER.                 190
C                                                                       200
C=====================================================================  210
C                                                                       220
      DIMENSION TIME(10),IRT(10),STO(336),IRS(336),ITHS(336)            230
      DATA KK,IRP,IRN,ITHP,ITHN/5*0/                                    240
      TIM=0.0                                                           250
 50   CONTINUE                                                          260
      READ(2,1000)(TIME(I),IRT(I),I=1,10)                               270
 1000 FORMAT(10(F5.2,I3))                                               280
      IF(EOF(2).GT.0)GO TO 100                                          290
      DO 20 J=1,10                                                      300
      IF(IRT(J) .EQ. 0) GO TO 20                                        310
      IF(IRT(J).LT.1.OR.IRT(J).GT.15)IRT(J)=2                           320
      TIM=TIM+(TIME(J)+10)/10000.0                                      330
      GO TO(1,2,3,4,5,4,7,2,1,2,3,12,13,12,15),IRT(J)                   340
 1    IR=0                                                              350
      ITHETA=-1                                                         360
      ITHN=ITHN+1                                                       370
      GO TO 200                                                         380
 2    WRITE(3,110)                                                      390
 110  FORMAT(10X,*ILLEGAL VALUE FOR IRT(J)*)                            400
      GO TO 200                                                         410
 3    IR=0                                                              420
      ITHETA=1                                                          430
      ITHP=ITHP+1                                                       440
      GO TO 200                                                         450
 4    IR=-1                                                             460
      ITHETA=0                                                          470
      IRN=IRN+1                                                         480
      GO TO 200                                                         490
 5    IR=-1                                                             500
      ITHETA=-1                                                         510
      IRN=IRN+1                                                         520
      ITHN=ITHN+1                                                       530
      GO TO 200                                                         540
 7    IR=-1                                                             550
      ITHETA=1                                                          560
      IRN=IRN+1                                                         570
      ITHP=ITHP+1                                                       580
      GO TO 200                                                         590
 12   IR=1                                                              600
      ITHETA=0                                                          610
      IRP=IRP+1                                                         620
      GO TO 200                                                         630
 13   IR=1                                                              640
      ITHETA=-1                                                         650
      IRP=IRP+1                                                         660
      ITHN=ITHN+1                                                       670
      GO TO 200                                                         680
 15   IR=1                                                              690
      ITHETA=1                                                          700
      IRP=IRP+1                                                         710
      ITHP=ITHP+1                                                       720
 200  CONTINUE                                                          730
      KK=KK+1                                                           740
      IF(KK .GT.336) GO TO 300                                          750
      STO(KK)=TIM                                                       760
      IRS(KK)=IR                                                        770
      ITHS(KK)=ITHETA                                                   780
      GO TO 20                                                          790
 300  CONTINUE                                                          800
      KK=0                                                              810
```

```
      DO 400 JJJ=1,56                                                    820
      WRITE(3,2000) (STO(JJ),IRS(JJ),ITHS(JJ),JJ=JJJ,336,56)             830
2000  FORMAT(1X,6(F10.6,2I4,4X))                                         840
 400  CONTINUE                                                           850
      GO TO 200                                                          860
 20   CONTINUE                                                           870
      GO TO 50                                                           880
 100  WRITE(3,130)IRN,IRP                                                890
 130  FORMAT(1HO,10X,*TOTAL R    NEG= *,I5,*TOTAL R     POS= *,I5)      900
      WRITE(3,140)ITHN,ITHP                                              910
 140  FORMAT(1HO,10X,*TOTAL THETA NEG= *,I5,*TOTAL THETA POS= *,I5)      920
      STOP                                                               930
      END                                                                940
```

APPENDIX E

ALGORITHMS USED FOR AREA ERROR MINIMIZATION

```
      BLOCK DATA                                                        100
C=====================================================================  110
C ASSIGNMENT OF INPUT AND OUTPUT FILES                                  120
C=====================================================================  130
      COMMON /IO/ IN,IOUT,IDATA                                         140
      DATA IN,IOUT,IDATA /5,6,7/                                        150
      END                                                               160
C                                                                       170
C                                                                       180
C                                                                       190
      PROGRAM CONTOUR(INPUT,OUTPUT=100B,TAPE7,TAPE5=INPUT,TAPE6=OUTPUT, 200
C=====================================================================  210
C MAIN PROGRAM TO CALCULATE AREA ERROR                                  220
C=====================================================================  230
     + DEBUG=OUTPUT)                                                    240
      COMMON /IO/ IN,IOUT,IDATA                                         250
      COMMON /PIE/ TWOPI,PI                                             260
      COMMON /PATH/ JROUTA,JROUTB                                       270
      COMMON /SYSPAR/ DR,DTHETA,ORIGINR,ORIGINT,TMIN,TMAX,              280
     1                XMIN,XMAX,TYPE,DT,DX,PX,PTHETA,ISECT,            290
     2                TOTAL,ORIGINX,ORIGINY                            300
C                                                                       310
      INTEGER TYPE                                                      320
      LOGICAL ENDFLG,BWT                                                330
      DIMENSION RADIUS(40),THETA(40)                                    340
      DATA ESCAPE,EPSI /1.0E-06,0.6E-12/                                350
C                                                                       360
C  BEGIN (* MAIN *)                                                     370
C                                                                       380
C$    STORES(THE2,THE1)                                                 390
C     ARRAYS(RADIUS,THETA)                                              400
      PI=4.0*ATAN(1.)                                                   410
      TWOPI=2.0*PI                                                      420
      TOTAL=0.0                                                         430
      JROUTA=0                                                          440
      JROUTB=0                                                          450
C-------------------------------------------------------------------    460
C INITIALIZE DATA AND GET INORMATION REQUIRED TO                        470
C CALCULATE THE AREA ERROR                                              480
C-------------------------------------------------------------------    490
      CALL INIT                                                         500
      CALL SET(RADIUS(1),THETA(1),TYPE)                                 510
      IF (TYPE.EQ.2) CALL FINDDX(THETA(1))                              520
      IF (TYPE.EQ.1) CALL FINDDT(THETA(1))                              530
      I=1                                                               540
C-------------------------------------------------------------------    550
C OPERATING LOOP                                                        560
C-------------------------------------------------------------------    570
110   I=I+1                                                             580
      IF (I .GT. 40) GOTO 9999                                          590
      CALL POINT(RADIUS(I),THETA(I),ENDFLG)                             600
      IF (ENDFLG)  GOTO 200                                             610
120   DDTHE=DIFF(THETA(I),THETA(1))                                     620
      IF (DDTHE .LT. DTHETA) GOTO 110                                   630
      THE1=THETA(I)                                                     640
      THE2=THETA(I)                                                     650
      IF (BWT(THETA(I),0.0,THETA(1))) GOTO 10                           660
      OMEGA=THETA(1)-DTHETA                                             670
      IF (THETA(I) .GT. THETA(1))                                       680
     1                        OMEGA=THETA(1)+DTHETA                     690
      GOTO 20                                                           700
10    IF (THETA(I) .GT. THETA(1)) GOTO 30                               710
      OMEGA=THETA(1)+DTHETA                                             720
      IF (COSINE(THETA(I)) .GE. 0.0) OMEGA=THETA(1)-DTHETA              730
      GOTO 20                                                           740
30    OMEGA=THETA(1)-DTHETA                                             750
      IF (COSINE(THETA(1)) .GE. 0.0) OMEGA=THETA(1)+DTHETA              760
20    CALL BIND(OMEGA,RADIUS(I-1),THETA(I-1),RADIUS(I),                 770
     1                  THETA(I),RTEMP,TTEMP)                          780
      IF (ABS(THETA(I)) .LT. EPSI) THETA(I)=0.0                         790
      CALL FINDBAS(RBASE,RADIUS,THETA,I,RNXT,TNXT)                      800
      CALL FINDERR(RADIUS,THETA,I,RNXT,ERROR)                           810
```

```
      IF (ERROR .LT. 1.0E-02) GOTO 240                                  820
      IF (TYPE .EQ. 2) GOTO 800                                         830
      WRITE(6,230) ERROR,PTHETA                                         840
230   FORMAT(" ERROR FOR SECTOR ",G14.8,5X,"PTHETA =",G14.8)            850
      GOTO 820                                                          860
800   WRITE(IOUT,810) ERROR,PX                                          870
810   FORMAT(" ERROR FOR SECTOR ",G14.8,5X,"PX     =",G14.8)           880
820   CALL PRNT(RADIUS,THETA,RNXT,TNXT,I)                               890
240   TOTAL=TOTAL+ERROR                                                 900
      GOTO(440,550,660,660),TYPE                                        910
440   IF ((ABS(PTHETA-TMAX).LE.ESCAPE).OR.(PTHETA .GE. TMAX)) GOTO 200  920
      GOTO 300                                                          930
550   IF (PX.GT.XMAX) GOTO 200                                          940
      GOTO 300                                                          950
660   CONTINUE                                                          960
300   RBASE=RNXT                                                        970
      RADIUS(1)=RADIUS(I)                                               980
      THETA(1)=THETA(I)                                                 990
      RADIUS(2)=RTEMP                                                  1000
      THETA(2)=TTEMP                                                   1010
      I=2                                                              1020
      GOTO(700,701,120,120),TYPE                                       1030
700   CALL FINDDT(THETA(1))                                            1040
      GOTO 702                                                         1050
701   CALL FINDDX(THETA(1))                                            1060
702   I=1                                                              1070
      GOTO 110                                                         1080
200   WRITE(6,220) TOTAL                                               1090
220   FORMAT(20H THE TOTAL ERROR IS , G14.8)                           1100
      WRITE(IOUT,290) JROUTA,JROUTB                                    1110
290   FORMAT(1X,"JROUTA =",I7,5X,"JROUTB =",I7)                        1120
      STOP                                                             1130
9999  CALL PRNT(RADIUS,THETA,RNXT,TNXT,40)                             1140
      WRITE(IOUT,9998)  PX,DX                                          1150
9998  FORMAT(5X,"PX     =",G14.8,5X,"DX     =",G14.8)                1160
      STOP                                                             1170
      END                                                              1180
C                                                                      1190
C                                                                      1200
C                                                                      1210
      SUBROUTINE SET(RADIUS,THETA,ITYPE)                               1220
C=====================================================================1230
C SET STARTING POINT AT NEAREST GRID POINT OF R AND THETA              1240
C=====================================================================1250
C                                                                      1260
      COMMON /IO/ IN,IOUT,IDATA                                        1270
      COMMON /SYSPAR/ DR,DTHETA,ORIGINR,ORIGINT,TMIN,TMAX,             1280
     1                XMIN,XMAX,TYPE,DT,DX,PX,PTHETA,ISECT,            1290
     2                TOTAL,ORIGINX,ORIGINY                            1300
      LOGICAL ENDFLG                                                   1310
C                                                                      1320
      GOTO(100,200,300,300),ITYPE                                      1330
100   PTHETA=TMIN-DT                                                   1340
      GOTO 400                                                         1350
200   PX=XMIN-DX                                                       1360
      GOTO 400                                                         1370
300   REWIND IDATA                                                     1380
400   THETA=0.0                                                        1390
      CALL POINT(R1,T1,ENDFLG)                                         1400
      IF (ENDFLG) GOTO 5000                                            1410
      CALL POINT(R2,T2,ENDFLG)                                         1420
      IF (ENDFLG) GOTO 5000                                            1430
      IF (T2 .LT. T1) THETA=DTHETA                                     1440
      THETA=THETA+INT(T1/DTHETA)*DTHETA                                1450
      RADIUS=INT(R1/DR)*DR                                             1460
      GOTO(1000,2000,3000,3000),ITYPE                                  1470
1000  PTHETA=TMIN+DT                                                   1480
      RETURN                                                           1490
2000  PX=XMIN+DX                                                       1500
      RETURN                                                           1510
3000  REWIND IDATA                                                     1520
      READ(IDATA,*) SKP1,SKP2                                          1530
```

```
      RETURN                                                            1540
5000  WRITE(IOUT,5010)                                                  1550
5010  FORMAT("   *** INSUFFICIENT DATA ***   ")                         1560
      STOP                                                              1570
      END                                                               1580
C                                                                       1590
C                                                                       1600
C                                                                       1610
      SUBROUTINE POINT(RADIUS,THETA,ENDFLG)                             1620
C=======================================================================1630
C SUBROUTINE TO GENERATE THE CONTOUR DATA FROM EQUATION                 1640
C OR DESCRETE POINT INPUT.                                              1650
C=======================================================================1660
      COMMON /IO/ IN,IOUT,IDATA                                         1670
      COMMON /SYSPAR/ DR,DTHETA,ORIGINR,ORIGINT,TMIN,TMAX,              1680
     1                XMIN,XMAX,TYPE,DT,DX,PX,PTHETA,ISECT,            1690
     2                TOTAL,ORIGINX,ORIGINY                            1700
      LOGICAL ENDFLG                                                    1710
      INTEGER TYPE                                                      1720
      ENDFLG=.FALSE.                                                    1730
      GOTO(100,200,300,400),TYPE                                        1740
100   PTHETA=PTHETA+DT                                                  1750
      CALL GET1(RADIUS,THETA,PTHETA)                                    1760
      GOTO 700                                                          1770
200   PX=PX+DX                                                          1780
      CALL GET2(RADIUS,THETA,PX)                                        1790
      GOTO 700                                                          1800
300   READ(IDATA,*) RADIUS,THETA                                        1810
      IF (EOF(IDATA).NE.0.0) GOTO 500                                   1820
      GOTO 700                                                          1830
400   READ(IDATA,*) RADIUS,THETA                                        1840
      IF (EOF(IDATA).NE.0.0) GOTO 500                                   1850
700   CALL ADJUST(RADIUS,THETA,ORIGINX,ORIGINY,TYPE)                    1860
      RETURN                                                            1870
500   ENDFLG=.TRUE.                                                     1880
      RETURN                                                            1890
      END                                                               1900
C                                                                       1910
C                                                                       1920
C                                                                       1930
      SUBROUTINE FINDDX(ALPHA)                                          1940
C=======================================================================1950
C FIND THE ANGLE WITH RESPECT TO THE MACHINE POLE                       1960
C FOR THE INCREMENTAL DISPLACEMENT DX.                                  1970
C=======================================================================1980
      COMMON /IO/ IN,IOUT,IDATA                                         1990
      COMMON /PIE/ TWOPI,PI                                             2000
      COMMON /SYSPAR/ DR,DTHETA,ORIGINR,ORIGINT,TMIN,TMAX,              2010
     1                XMIN,XMAX,TYPE,DT,DX,PX,PTHETA,ISECT,            2020
     2                TOTAL,ORIGINX,ORIGINY                            2030
      INTEGER TYPE                                                      2040
C                                                                       2050
      DATA EPSI/1.0E-06/                                                2060
C$    STORES(ALPHA1,PXDBG)                                              2070
      ALPHA1=ALPHA * 180.0 / PI                                         2080
      PXDBG=PX                                                          2090
      CALL SECOND(TSTRT)                                                2100
100   CALL SECOND(TEND)                                                 2110
      IF ((TEND-TSTRT) .GT. 5.0) GOTO 500                               2120
      II=0                                                              2130
      XVAL = PX + (ISECT-1) * DX                                        2140
200   CALL GET2(RADIUS,THETA,XVAL)                                      2150
      CALL ADJUST(RADIUS,THETA,ORIGINX,ORIGINY,TYPE)                    2160
      DDTHE = DIFF(THETA,ALPHA)                                         2170
      IF (DDTHE .GE. DTHETA) GOTO 300                                   2180
      XVAL = XVAL + DX                                                  2190
      II = II + 1                                                       2200
      IF (II .LE. 10 ) GOTO 200                                         2210
      IF (DDTHE.LT.EPSI) DDTHE=EPSI                                     2220
      DX = DX * (6 * DTHETA) / (5 * DDTHE)                              2230
      GOTO 100                                                          2240
300   RJJ = DDTHE/DTHETA                                                2250
```

```
      JJ=RJJ                                                            2260
      IF (JJ .EQ. 1) GOTO 400                                           2270
      IF (JJ .NE. 2) GOTO 350                                           2280
      IF (II .LE. 2) DX=DX/10.0                                         2290
      DX=DX*FLOAT(JJ)/3.0                                               2300
      GOTO 100                                                          2310
350   DX = DX/RJJ                                                       2320
      GOTO 100                                                          2330
400   DX = ABS(XVAL-PX)/FLOAT(ISECT)                                    2340
*      WRITE(IOUT,450) ALPHA1,PX,XVAL,DX                                2350
*450   FORMAT(5X,"ALPHA1 =",G14.8,2X,"PX      =",G14.8,                2360
+     1       2X,"XVAL   =",G14.8,2X,"DX      =",G14.8)                 2370
      RETURN                                                            2380
500   WRITE(IOUT,510) ALPHA1,PXDBG,DDTHE,XVAL,RADIUS                    2390
     1               ,THETA,DX                                          2400
510   FORMAT(15X,"ALPHA1 =",G14.8,15X,"PX      =",G14.8,/,              2410
     1       15X,"DDTHE  =",G14.8,15X,"XVAL    =",G14.8,/,             2420
     2       15X,"RADIUS =",G14.8,15X,"THETA   =",G14.8,/,             2430
     3       15X,"DX     =",G14.8)                                     2440
      WRITE(6,530) TOTAL                                                2450
530   FORMAT(20H THE TOTAL ERROR IS , G14.8)                            2460
      STOP                                                              2470
      END                                                               2480
C                                                                       2490
C                                                                       2500
C                                                                       2510
      SUBROUTINE FINDDT(ALPHA)                                          2520
C=======================================================================2530
C FIND THE ANGLE THETA WITH RESPECT TO THE MACHINE POLE                 2540
C FOR THE INCREMENTAL ANGULAR DISPLACEMENT WITH RESPECT                 2550
C TO THE CONTOUR POLE.                                                  2560
C=======================================================================2570
      COMMON /IO/ IN,IOUT,IDATA                                         2580
C                                                                       2590
      COMMON /SYSPAR/ DR,DTHETA,ORIGINR,ORIGINT,TMIN,TMAX,              2600
     1                XMIN,XMAX,TYPE,DT,DX,PX,PTHETA,ISECT,             2610
     2                TOTAL,ORIGINX,ORIGINY                             2620
      INTEGER TYPE                                                      2630
C                                                                       2640
      DATA EPSI/1.0E-06/                                                2650
      PI=2 * ASIN(1.)                                                   2660
C     STORES(ALPHA1,RJJ,DT,PTHDBG,DDTDBG)                               2670
      CALL SECOND(TSTRT)                                                2680
      ALPHA1=ALPHA * 180.0 / PI                                         2690
      PTHDBG=PTHETA * 180.0 / PI                                        2700
100   CALL SECOND(TEND)                                                 2710
      IF ((TEND-TSTRT) .GT. 5.0) GOTO 500                               2720
      II=0                                                              2730
      OMEGA = PTHETA + (ISECT-1) * DT                                   2740
200   CALL GET1(RADIUS,THETA,OMEGA)                                     2750
      CALL ADJUST(RADIUS,THETA,ORIGINX,ORIGINY,TYPE)                    2760
      DDTHE = DIFF(THETA,ALPHA)                                         2770
      IF (DDTHE .GE. DTHETA) GOTO 300                                   2780
      OMEGA = OMEGA + DT                                                2790
      II = II + 1                                                       2800
      IF (II .LE. 10 ) GOTO 200                                         2810
      IF (DDTHE.LT.EPSI) DDTHE=EPSI                                     2820
      DT = DT * (6 * DTHETA) / (5 * DDTHE)                              2830
      GOTO 100                                                          2840
300   RJJ = DDTHE/DTHETA                                                2850
      JJ=RJJ                                                            2860
      IF (JJ .EQ. 1) GOTO 400                                           2870
      IF (JJ .NE. 2) GOTO 350                                           2880
      IF (II .LE. 2) DT=DT/10.0                                         2890
      DT=DT*FLOAT(JJ)/3.0                                               2900
      GOTO 100                                                          2910
350   DT = DT/RJJ                                                       2920
      GOTO 100                                                          2930
400   DT = ABS(ABS(OMEGA)-ABS(PTHETA))/FLOAT(ISECT)                     2940
      RETURN                                                            2950
500   WRITE(IOUT,510) ALPHA1,PTHDBG,DDTHE,OMEGA,RADIUS                  2960
     1               ,THETA,DT                                          2970
```

```
510   FORMAT(15X,"ALPHA1 =",G14.8,15X,"PTHETA =",G14.8,/,                    2980
     1       15X,"DDTHE  =",G14.8,15X,"OMEGA  =",G14.8,/,                    2990
     2       15X,"RADIUS =",G14.8,15X,"THETA  =",G14.8,/,                    3000
     3       15X,"DT     =",G14.8)                                           3010
52    WRITE(6,530) TOTAL                                                     3020
530   FORMAT(20H THE TOTAL ERROR IS , G14.8)                                 3030
      STOP                                                                   3040
      END                                                                    3050
C                                                                            3060
C                                                                            3070
C                                                                            3080
      SUBROUTINE INIT                                                        3090
C=============================================================================3100
C INITIALIZATION AND INFORMATION INPUT SUBROUTINE.                           3110
C NOTE "TYPE" IS PARAMETER THAT DEFINES TYPE OF CONTOUR                      3120
C INFORMATION GIVEN, AND IS DEFINED AS FOLLOWS:                              3130
C     TYPE = 1 CONTOUR IS GIVEN IN POLAR COORDINATES                         3140
C     TYPE = 2 CONTOUR IS GIVEN IN CARTEZIEN COORDINATES                     3150
C     TYPE = 3 CONTOUR IS GIVEN AS DESCRETE POINTS IN POLAR                  3160
C              COORDINATES                                                   3170
C     TYPE = 4 CONTOUR IS GIVEN AS DESCRETE POINTS IN CARTEZIEN              3180
C              COORDINATES                                                   3190
C=============================================================================3200
C                                                                            3210
C                                                                            3220
C-----------------------------------------------------------------------------3230
C THE CONTOUR PARAMETERS ARE DEINED AS FOLLOWS                               3240
C     DR      = SYSTEM RADIAL RESOLUTION                                     3250
C     DTHETA = SYSTEM ANGULAR RESOLUTION                                     3260
C     ORIGINX= MACHINE POLE LOCATION X AXIS                                  3270
C     ORIGINY= MACHINE POLE LOCATION Y AXIS                                  3280
C     TMIN    = STARTING ANGULAR DISPLACEMENT OF THE CONTOUR                 3290
C               WITH RESPECT TO THE CONTOUR POLE                             3300
C     TMAX    = MAXIMUM ANGULAR DISPLACEMENT OF THE CONTOUR                  3310
C               WITH RESPECT TO THE CONTOUR POLE                             3320
C     ISECT   = NUMBER OF POINTS WITHIN ONE RESOLUTION TO                    3330
C               APPROXIMATE THE CONTOUR                                      3340
      COMMON /IO/ IN,IOUT,IDATA                                              3350
      COMMON /SYSPAR/ DR,DTHETA,ORIGINR,ORIGINT,TMIN,TMAX,                   3360
     1                XMIN,XMAX,TYPE,DT,DX,PX,PTHETA,ISECT,                  3370
     2                TOTAL,ORIGINX,ORIGINY                                  3380
      INTEGER TYPE                                                           3390
      PI=4.0 * ATAN(1.)                                                      3400
      READ(IN,*) TYPE                                                        3410
      WRITE(IOUT,*) TYPE                                                     3420
      GOTO(100,200,300,400),TYPE                                             3430
C                                                                            3440
C-----------------------------------------------------------------------------3450
C FOR A CONTOUR IN POLAR COORDINATES                                         3460
100   READ(IN,*) DR,DTHETA,ORIGINX,ORIGINY,TMIN,TMAX,ISECT                   3470
      TMIN=TMIN * PI / 180.0                                                 3480
      TMAX=TMAX * PI / 180.0                                                 3490
      ORIGINR=SQRT(ORIGINX**2+ORIGINY**2)                                    3500
      ORIGINT=0.0                                                            3510
      IF ((ORIGINX.NE.0.0) .OR. (ORIGINY.NE.0.0))                            3520
     1            ORIGINT=ATAN2(ORIGINY,ORIGINX)                           3530
      IF (ORIGINT .LT. -1.0E-200) ORIGINT=2.0*PI+ORIGINT                     3540
      DTHETA=DTHETA * PI / 180.0                                             3550
      DT=DTHETA/FLOAT(ISECT)                                                 3560
      PTHETA=TMIN                                                            3570
      WRITE(IOUT,110) DR,DTHETA,ORIGINR,ORIGINT,ORIGINX,ORIGINY,             3580
     1                TMIN,TMAX,ISECT                                        3590
110   FORMAT(15X,"DR      =",G14.8,15X,"DTHETA  =",G14.8,/,                  3600
     1       15X,"ORIGINR =",G14.8,15X,"ORIGINT =",G14.8,/,                  3610
     1       15X,"ORIGINX =",G14.8,15X,"ORIGINY =",G14.8,/,                  3620
     2       15X,"TMIN    =",G14.8,15X,"TMAX    =",G14.8,15X,/,            3630
     3       15X,"ISECT   =",I2)                                             3640
      GOTO 500                                                               3650
C                                                                            3660
C-----------------------------------------------------------------------------3670
C                                                                            3680
C                                                                            3690
```

```
C-------------------------------------------------------------------------      3700
C OR A CONTOUR EQUATION IN CARTEZIEN COORDINATES                               3710
200   READ(IN,*) DR,DTHETA,ORIGINX,ORIGINY,XMIN,XMAX,DX,ISECT                 3720
      PX=XMIN                                                                  3730
      DTHETA=DTHETA * PI / 180.0                                               3740
      WRITE(IOUT,210) DR,DTHETA,ORIGINX,ORIGINY,XMIN,XMAX,ISECT               3750
210   FORMAT(15X,"DR      =",G14.8,15X,"DTHETA  =",G14.8,/,                    3760
     1       15X,"ORIGINX =",G14.8,15X,"ORIGINY =",G14.8,/,                    3770
     2       15X,"XMIN    =",G14.8,15X,"XMAX    =",G14.8,15X,/,              3780
     3       15X,"ISECT   =",I2)                                               3790
      GOTO 500                                                                 3800
C                                                                              3810
C-------------------------------------------------------------------------     3820
C                                                                              3830
C                                                                              3840
C-------------------------------------------------------------------------     3850
C OR A CONTOUR IN DESCRETE POINT FORM GIVEN IN POLAR                           3860
C COORDINATES                                                                  3870
300   READ(IN,*) DR,DTHETA,ORIGINX,ORIGINY                                     3880
      ORIGINR=SQRT(ORIGINX**2+ORIGINY**2)                                      3890
      ORIGINT=0.0                                                              3900
      IF ((ORIGINX.NE.0.0) .OR. (ORIGINY.NE.0.0))                              3910
     1             ORIGINT=ATAN2(ORIGINY,ORIGINX)                            3920
      IF (ORIGINT .LT. -1.0E-200) ORIGINT=2.0*PI+ORIGINT                       3930
      DTHETA=DTHETA * PI / 180.0                                               3940
      WRITE(IOUT,*) DR,DTHETA,ORIGINR,ORIGINT                                  3950
      GOTO 500                                                                 3960
C                                                                              3970
C- -------------------------------------------------------------------------   3980
C                                                                              3990
C                                                                              4000
C- -------------------------------------------------------------------------   4010
C FOR CONTOUR INORMATION IN DESCRETE POINT FORM GIVEN                          4020
C IN CARTEZIEN COORDINATES                                                     4030
400   READ(IN,*) DR,DTHETA,ORIGINX,ORIGINY                                     4040
      DTHETA=DTHETA * PI / 180.0                                               4050
      WRITE(IOUT,*) DR,DTHETA,ORIGINX,ORIGINY                                  4060
C-- -------------------------------------------------------------------------  4070
500   RETURN                                                                   4080
      END                                                                      4090
C                                                                              4100
C                                                                              4110
C                                                                              4120
      SUBROUTINE GET1(R,THETA,PHI)                                             4130
C=========================================================================     4140
C SUBROUTINE TO GENERATE THE CONTOUR FROM AN EQUATION IN                       4150
C POLAR COORDINATES                                                            4160
C=========================================================================     4170
      THETA=PHI                                                                4180
      R=3.0+2.0*COS(THETA)                                                     4190
      RETURN                                                                   4200
      END                                                                      4210
C                                                                              4220
C                                                                              4230
C                                                                              4240
      SUBROUTINE GET2(X,Y,XVAL)                                                4250
C=========================================================================     4260
C SUBROUTINE TO GENERATE THE CONTOUR FROM AN EQUATION IN CARTEZIEN              4270
C COORDINATES.                                                                 4280
      DATA A,B /0.5,3.0/                                                       4290
      X=XVAL                                                                   4300
      Y=A*B*X/(A**2+X**2)                                                      4310
      RETURN                                                                   4320
      END                                                                      4330
C                                                                              4340
C                                                                              4350
C                                                                              4360
      SUBROUTINE BIND(PHI,A,ALPHA,B,BETA,C,GAMMA)                              4370
C=========================================================================     4380
C SUBROUTINE TO FIND THE INTERSECTION BETWEEN THE CONTQUR                       4390
C AND A GRID LINE WITHIN ONE INCREMENT.                                        4400
C=========================================================================     4410
```

```
      COMMON /PIE/ TWOPI,PI                                                  4420
C     STORES(AX,BX,ALX,BEX,PHIX,T1,T2,T3,C)                                  4430
      IF (ABS(PHI) .LT. PI) GOTO 10                                          4440
      IF (PHI .GT. PI) GOTO 20                                               4450
      IF (PHI .LT. -PI) PHI=TWOPI+PHI                                        4460
      GOTO 10                                                                4470
20    PHI=PHI-TWOPI                                                          4480
10    C= (A * B * SIN(DIFF(ALPHA,BETA)))/                                    4490
     1       (A * SIN(DIFF(ALPHA,PHI)) + B * SIN(DIFF(PHI,BETA)))           4500
      GAMMA=PHI                                                              4510
      TMP=B                                                                  4520
      B=C                                                                    4530
      C=TMP                                                                  4540
      TMP=GAMMA                                                              4550
      GAMMA=BETA                                                             4560
      BETA=TMP                                                               4570
      RETURN                                                                 4580
      END                                                                    4590
C                                                                            4600
C                                                                            4610
C                                                                            4620
      SUBROUTINE FINDBAS(A,RADIUS,THETA,LIMIT,C,GAMMA)                       4630
C=============================================================================4640
C FIND NEAREST BASE POINT ON THE POLAR GRID                                  4650
C=============================================================================4660
C                                                                            4670
      COMMON /SYSPAR/ DR,DTHETA,ORIGINR,ORIGINT,TMIN,TMAX,                   4680
     1                XMIN,XMAX,TYPE,DT,DX,PX,PTHETA,ISECT,                  4690
     2                TOTAL,ORIGINX,ORIGINY                                  4700
      INTEGER TYPE                                                           4710
      DIMENSION RADIUS(LIMIT),THETA(LIMIT)                                   4720
      C=A                                                                    4730
      DO 10 I=1,LIMIT                                                        4740
        C = C + INT((RADIUS(I)-C)/DR)*DR                                     4750
10    CONTINUE                                                               4760
      GAMMA=THETA(LIMIT)                                                     4770
      RETURN                                                                 4780
      END                                                                    4790
C                                                                            4800
C                                                                            4810
C                                                                            4820
      SUBROUTINE FINDERR(RADIUS,THETA,LIMIT,B,ERROR)                         4830
C=============================================================================4840
C SUBROUTINE TO SET UP THE CONDITIONS AND ZONES TO                           4850
C CALCULATE THE AREA ERROR.                                                  4860
C=============================================================================4870
C                                                                            4880
      COMMON /PATH/ JROUTA,JROUTB                                            4890
      LOGICAL BWT,FLAG1                                                      4900
      DIMENSION RADIUS(LIMIT),THETA(LIMIT)                                   4910
C                                                                            4920
      AREA=0.0                                                               4930
      ERROR=0.0                                                              4940
      ANGINIT=THETA(1)                                                       4950
      LIM=LIMIT-1                                                            4960
      FLAG1=(THETA(1) .GT. THETA(2))                                         4970
      DO 300 K=2,LIM                                                         4980
        IF (FLAG1) GOTO 310                                                  4990
        IF (.NOT.(THETA(K).GT.THETA(K+1))) GOTO 300                          5000
        GOTO 350                                                             5010
310     IF ((THETA(K).GT.THETA(K+1))) GOTO 300                               5020
        GOTO 350                                                             5030
300   CONTINUE                                                               5040
      JROUTA=JROUTA+1                                                        5050
      DO 100 K=1,LIM                                                         5060
        IF (BWT(RADIUS(K),B,RADIUS(K+1))) GOTO 200                           5070
        AREA=AREA+RADIUS(K)*RADIUS(K+1) * SIN(DIFF(THETA(K),THETA(K+1)))     5080
        GOTO 100                                                             5090
200     CALL CALCANG(B,RADIUS(K),THETA(K),RADIUS(K+1),THETA(K+1),PHI)        5100
        ARSCTR = DIFF(PHI,ANGINIT) * B * B                                   5110
        AREA = AREA + RADIUS(K) * B * SIN(DIFF(THETA(K),PHI))                5120
        ERROR = ERROR + ABS(AREA-ARSCTR)/2.0                                 5130
```

```
        ANGINIT=PHI                                                          5140
        AREA = B * RADIUS(K+1) * SIN(DIFF(PHI,THETA(K+1)))                   5150
100     CONTINUE                                                             5160
        ARSCTR=DIFF(THETA(LIMIT),ANGINIT) * B * B                            5170
        ERROR = ERROR + ABS(AREA-ARSCTR)/2.0                                 5180
        RETURN                                                               5190
350     JROUTB=JROUTB+1                                                      5200
        DO 400 K=1,LIM                                                       5210
          IF (BWT(RADIUS(K),B,RADIUS(K+1))) GOTO 500                         5220
          ERROR=ERROR+CALCERR(B,RADIUS(K),THETA(K),RADIUS(K+1),THETA(K+1))   5230
          GOTO 400                                                           5240
500       CONTINUE                                                           5250
          CALL CALCANG(B,RADIUS(K),THETA(K),RADIUS(K+1),THETA(K+1),PHI)      5260
          ERROR=ERROR+CALCERR(B,RADIUS(K),THETA(K),B,PHI)                    5270
          ERROR=ERROR+CALCERR(B,B,PHI,RADIUS(K+1),THETA(K+1))                5280
400     CONTINUE                                                             5290
        RETURN                                                               5300
        END                                                                  5310
C                                                                            5320
C                                                                            5330
C                                                                            5340
        FUNCTION CALCERR(BASE,R,T,S,Q)                                       5350
C=============================================================================5360
C FUNCTION TO CALCULATE THE AREA ERROR WITHIN A SMALL SEGMENT                5370
C=============================================================================5380
C                                                                            5390
        ARSCTR=DIFF(Q,T)*BASE*BASE                                           5400
        ARTRI=R*S*SIN(DIFF(Q,T))                                             5410
        CALCERR=ABS(ARSCTR-ARTRI)/2.0                                        5420
        RETURN                                                               5430
        END                                                                  5440
C                                                                            5450
C                                                                            5460
C                                                                            5470
        SUBROUTINE CALCANG(BASE,A,ALPHA,B,BETA,PHI)                          5480
C=============================================================================5490
C THIS SUBROUTINE FINDS THE ANGLE WITH RESPECT TO THE                        5500
C MACHINE POLE, FOR THE INTERSECTION BETWEEN THE CONTOUR                     5510
C AND THE GRID LINE WITHIN THE INCREMENT.                                    5520
C=============================================================================5530
C=============================================================================5540
        REAL IOTA,KAPPA                                                      5550
        DATA EPSI /1.0E-12/                                                  5560
C                                                                            5570
        IF (ABS(BASE) .LT. EPSI) GOTO 300                                    5580
        IF (DIFF(BETA,ALPHA) .GT. 1.0E-90) GOTO 200                          5590
        DELTA=0.0                                                            5600
        GOTO 10                                                              5610
200     C=SQRT(A**2+B**2-2*A*B*COS(DIFF(BETA,ALPHA)))                        5620
        TMP=B/C*SIN(DIFF(BETA,ALPHA))                                        5630
        IF (ABS(TMP).LT. 1.0) GOTO 100                                       5640
300     DELTA=DIFF(ALPHA,BETA)/2.0                                           5650
        GOTO 110                                                             5660
100     IOTA=ASIN(TMP)                                                       5670
        KAPPA=ASIN(A/BASE*TMP)                                               5680
        DELTA=DIFF(IOTA,KAPPA)                                               5690
110     IF (ALPHA.GT.BETA) GOTO 10                                           5700
        PHI=ALPHA+DELTA                                                      5710
        RETURN                                                               5720
10      PHI=ALPHA-DELTA                                                      5730
        RETURN                                                               5740
        END                                                                  5750
C                                                                            5760
C                                                                            5770
C                                                                            5780
        SUBROUTINE ADJUST(A,ALPHA,B,BETA,ITYPE)                              5790
C=============================================================================5800
C COORDINATE TRANSFORMATION SUBROUTINE.                                      5810
C TRANSORMS THE COORDINATES O A POINT ON THE CONTOUR                         5820
C TO THE MACHINE POLE.                                                       5830
C=============================================================================5840
C   , STORES(ALPHA2,ALPDBG2,XA,YA)                                           5850
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      IF ((B.EQ.0.0).AND.(BETA.EQ.0.0)) GOTO 300                        5860
      IF ((ITYPE.NE.1) .AND. (ITYPE.NE.3)) GOTO 100                     5870
      XA=A*COSINE(ALPHA)+B                                              5880
      YA=A*SINE(ALPHA)+BETA                                             5890
      GOTO 200                                                          5900
100   XA=A+B                                                            5910
      YA=ALPHA+BETA                                                     5920
200   A=SQRT(XA**2+YA**2)                                               5930
      ALPHA = 0.0                                                       5940
      IF ((XA.NE.0.0) .OR. (YA.NE.0.0)) ALPHA=ATAN2(YA,XA)              5950
300   RETURN                                                            5960
      END                                                               5970
C                                                                       5980
C                                                                       5990
C                                                                       6000
      FUNCTION SINE(ALPHA)                                              6010
C=======================================================================6020
C FUNCTION TO HELP IN ELEMINATING OVERLOW OF COMPUTER                   6030
C WORD FOR SMALL VALUES OF THE INTRINSIC FUNCTIONS                      6040
C=======================================================================6050
      DATA EPSI /6.87E-15/                                              6060
      SINE=SIN(ALPHA)                                                   6070
      IF (ABS(SINE) .LT. EPSI) SINE=0.0                                 6080
      RETURN                                                            6090
      END                                                               6100
C                                                                       6110
C                                                                       6120
C                                                                       6130
      FUNCTION COSINE(ALPHA)                                            6140
C=======================================================================6150
C FUNCTION TO HELP IN ELEMINATING OVERLOW OF COMPUTER                   6160
C WORD FOR SMALL VALUES OF THE INTRINSIC FUNCTIONS                      6170
C=======================================================================6180
      DATA EPSI /1.62E-14/                                              6190
      COSINE=COS(ALPHA)                                                 6200
      IF (ABS(COSINE) .LT. EPSI) COSINE=0.0                             6210
      RETURN                                                            6220
      END                                                               6230
C                                                                       6240
C                                                                       6250
C                                                                       6260
      SUBROUTINE PRNT(RADIUS,THETA,A,ALPHA,LIMIT)                       6270
C=======================================================================6280
C PRINTING SUBROUTINE                                                   6290
C=======================================================================6300
C                                                                       6310
      COMMON /IO/ IN,IOUT,IDATA                                         6320
      DIMENSION RADIUS(LIMIT),THETA(LIMIT)                              6330
C                                                                       6340
      DO 100 I=1,LIMIT                                                  6350
      WRITE(IOUT,110) I,RADIUS(I),THETA(I)                              6360
110   FORMAT(10X,I2,5X,"RADIUS =",F11.7,10X,"THETA =",F11.7)            6370
100   CONTINUE                                                          6380
      WRITE(IOUT,120) A,ALPHA                                           6390
120   FORMAT(10X,"RBASE=",F11.7,10X,"TBASE =",F11.7)                    6400
      RETURN                                                            6410
      END                                                               6420
C                                                                       6430
C                                                                       6440
C                                                                       6450
      LOGICAL FUNCTION BWT(A,B,C)                                       6460
C=======================================================================6470
C RETURNS A TRUE OR FALSE VALUE DEPENDING ON WIETHER                    6480
C A GIVEN VALUE LIES INBETWEEN TWO POINTS OR NOT                        6490
C=======================================================================6500
C     BWT IS TRUE IF B IS BETWEEN A AND C                               6510
      IF (A .GT. B) GOTO 10                                             6520
      IF (B. LT. C) GOTO 20                                             6530
      GOTO 30                                                           6540
10    IF (B .GT. C) GOTO 20                                             6550
30    BWT=.FALSE.                                                       6560
      RETURN                                                            6570
```

```
C
20    BWT=.TRUE.                                                    6580
      RETURN                                                        6590
      END                                                           6600
C                                                                   6610
C                                                                   6620
C                                                                   6630
      FUNCTION DIFF(KAPPA,LAMBDA)                                   6640
C=====================================================================6650
C FUNCTION RETURNS THE VALUE OF THE DIFFERENCE BETWEEN              6660
C TWO ANGLES IN THE ZONE FROM 0 TO 180 DEGREES.                     6670
C=====================================================================6680
      COMMON /PIE/ TWOPI,PI                                         6690
      REAL KAPPA,LAMBDA                                             6700
      DIFF=ABS(KAPPA-LAMBDA)                                        6710
      IF (DIFF .GT. PI) DIFF=TWOPI-DIFF                             6720
      RETURN                                                        6730
      END                                                           6740
```

```
      PROGRAM HOKJEV(TAPE5,INPUT=TAPE5,TAPE6,OUTPUT=TAPE6)                  100
C=====================================================================     110
C                                                                         120
C THIS PROGRAM FINDS THE RELATIVE LOCATION BETWEEN THE CONTOUR            130
C POLE AND THE MACHINE POLE, THAT WOULD RESULT IN A MINIMUM               140
C AREA ERROR.                                                             150
C THE PROGRAM USES THE ALTERNATE OBJECTIVE FUNCTION BASED                 160
C ON FSUM AND FDIF.                                                       170
C HOOKE AND JEEVES OPTIMISATION ALGORITHM IS USED FOR THE                 180
C MINIMIZATION.                                                           190
C                                                                         200
C=====================================================================     210
C                                                                         220
C-------------------------------------------------------------------      230
C THE PROGRAM IS SET CURRENTLY TO OPERATE ON THREE TEST                   240
C CONTOURS.                                                               250
C-------------------------------------------------------------------      260
      COMMON PI,NTP                                                       270
     'DIMENSION EPS(2),RK(2),B(2),BB(2),W(2),PC(1000,2),URK(2),SRK(2)     280
C-------------------------------------------------------------------      290
C "NTP" IS AN INTEGER. IT DEFINES THE NUMBER OF DESCRETE DATA POINTS      300
C ON THE CONTOUR.                                                         310
C ALSO USED HERE TO DEFINE THE TEST CONTOUR IN USE.                       320
C                                                                         330
C             1=WHITCH OF AGNASI                                          340
C             2=SEPENTINE                                                 350
C             3=LIMACON OF PASCAL                                         360
      READ *,NTP                                                          370
C-------------------------------------------------------------------      380
      PI=2*ASIN(1.0)                                                      390
C-------------------------------------------------------------------      400
C ENTER CONTOURING SYSTEM RESOLUTION (R=0.001,THETA=0.0225)               410
C FOR PROTOTYPE SYSTEM                                                    420
C     R     =0.001                                                        430
C     THETA=0.0225 DEGREES                                                440
      PRINT *,'ENTER SYSTEM RESOLUTIONS, RADIAL THEN ANGULAR'             450
      READ *,DELR,DELTH                                                   460
C-------------------------------------------------------------------      470
      DELTH=DELTH*ASIN(1.0)/90.0                                          480
C-------------------------------------------------------------------      490
C ENTER CONTOUR PARAMETERS.                                               500
C FOR TEST CONTOURS THE                                                   510
C CURVE PARAMETERS ARE (1: A=8.5                                          520
C                       2: A=0.5 D=3.0                                    530
C                       3: A=2.0 B=3.0)                                   540
C N = NUMBER OF SEGMENTS APPROXIMATING THE CONTOUR                        550
      READ *,N,K,L,A,B,C,D                                                560
C-------------------------------------------------------------------      570
      CALL CUREVA(N,K,L,A,B,C,D,PC,NP)                                    580
C-------------------------------------------------------------------      590
C ENTER MAXIMUM NUMBER OF ITERATIONS AND FUNCTION EVALUATIONS             600
C SUGGESTED VALUES:                                                       610
C     NUMBER OF ITTERATIONS       = 200                                   620
C     NUMBER OF FUNC. EVALUATIONS = 50                                    630
      PRINT *,'ENTER MAXIMUM NUMBER OF ITTERATIONS'                       640
      PRINT *,'THEN NUMBER OF FUNCTION EVALUATIONS'                       650
      READ *,ITMAX,NKAT                                                   660
C-------------------------------------------------------------------      670
C                                                                         680
C-------------------------------------------------------------------      690
C ENTER INITIAL GUESS (Y THEN X)                                          700
      PRINT *,'ENTER INITIAL GUESS Y, THEN X'                             710
      READ *,(RK(II),II=1,2)                                              720
C-------------------------------------------------------------------      730
C                                                                         740
C-------------------------------------------------------------------      750
C ENTER INITIAL MOVE SIZE FOR EACH DIRECTION (Y, THEN X)                  760
C SUGGESTED VALUES:                                                       770
C     Y = 0.125                                                           780
C     X = 0.125                                                           790
      PRINT *,'ENTER INITIAL MOVE SIZE (Y, THEN X)'                       800
      READ *,(EPS(JJ),JJ=1,2)                                             810
```

```
C-----------------------------------------------------------------------  820
C                                                                        830
C-----------------------------------------------------------------------  840
C ENTER SCALING FACTOR FOR MOVE SIZE, INCREASE THEN REDUCTION            850
C AND THE ERROR OR STOP CRITERION                                        860
C SUGGESTED VALUES:                                                      870
C     ALPHA = 1.0                                                        880
C     BETA  = 0.5                                                        890
C     EPSY  = 1.0E-20                                                    900
      PRINT *,'ENTER SCALING FACTORS, FOR INCREASE THEN DECRESE'         910
      PRINT *,'AND THE ERROR FOR STOPPING'                               920
      READ *,ALPHA,BETA,EPSY                                             930
C-----------------------------------------------------------------------  940
C                                                                        950
C-----------------------------------------------------------------------  960
C ENTER PARAMETRIC LIMITS, Y THEN X                                      970
C PARAMETRIC LIMITS ARE GOVERNED BY THE SIZE OF THE WORK AREA            980
      PRINT *,'ENTER SIZE OF THE WORK AREA, XMAX, XMIN, YMAX, YMIN'      990
      READ *,((URK(I),SRK(I)),I=1,2)                                    1000
C-----------------------------------------------------------------------  1010
C                                                                        1020
C-----------------------------------------------------------------------  1030
C ENTER CONTOUR DATA IF CONTOUR IS IN DESCRETE FORM                      1040
C FIRST X THE Y COORDINATES                                              1050
C     READ *,((PC(J,I),I=1,2),J=1,NP)                                    1060
C READ INSTRUCTION IS COMMENTED OUT FOR THIS RUN                         1070
C-----------------------------------------------------------------------  1080
C                                                                        1090
      PRINT 001                                                          1100
 001  FORMAT(1H1,//,20X,*PROGRAM INPUT DATA*,/20X,18(*-*))              1110
      PRINT 002                                                          1120
 002  FORMAT(//,5X,*STRAIGHT LINE APPROXIMATION OF CURVE*,/,5X,37(*-*))  1130
      PRINT 003                                                          1140
 003  FORMAT(/,5X,*POINTS ORDINATES*,5X,*POINTS ABSCISSAE*,/,2(5X,16(*-*  1150
     $)))                                                                1160
      DO 10 I=1,NP                                                       1170
 10   PRINT 004,(PC(I,J),J=1,2)                                          1180
 004  FORMAT(2(5X,F16.8))                                                1190
 100  CONTINUE                                                           1200
      PRINT 005,ITMAX,NKAT                                               1210
 005  FORMAT(/,5X,*MAXIMUM NOMBER OF ITERATIONS AND STEP SIZE REDUCTIONS  1220
     $ ALLOWED RESPECTIVELY,ARE,*,5X,I4,*,*,I4)                          1230
      PRINT 006,(RK(II),II=1,2)                                          1240
 006  FORMAT(/,5X,*INITIAL GUESS,ORDINATES AND ABSCISSA RESPECTIVELY,ARE  1250
     $*,2(5X,F16.8))                                                     1260
      QD=0.0                                                             1270
      CALL HOOKE(RK,EPS,ITMAX,NKAT,EPSY,ALPHA,BETA,QD,Q,QQ,W,            1280
     $NP,PC,URK,SRK,DELR,DELTH)                                          1290
      STOP                                                               1300
      END                                                                1310
C                                                                        1320
C                                                                        1330
C                                                                        1340
      SUBROUTINE HOOKE(RK,EPS,MAXK,NKAT,EPSY,ALPHA,BETA,QD,Q,QQ,         1350
     $W,NP,PC,URK,SRK,DELR,DELTH)                                        1360
C========================================================================  1370
C                                                                        1380
C THIS OPTIMIZATION ROUTINE IS BASED ON THE HOOKE AND JEEVES             1390
C ALGORITHM.                                                             1400
C                                                                        1410
C========================================================================  1420
      DIMENSION RK(2),EPS(2),Q(2),QQ(2),W(2),PC(1000,2),URK(2),SRK(2)    1430
      PRINT 001                                                          1440
 001  FORMAT(1H1,//,10X,*OPTIMUM POSITION OF CURVE W.R.T. ORIGEN FOR POL  1450
     $AR SCANING*,/,9X,57(*-*))                                          1460
      PRINT 002                                                          1470
 002  FORMAT(///,3X,*NUMBER OF*,6X,*ORIGEN ORDINATE*,7X,*ORIGEN ABSCISSA  1480
     $*,7X,*FUNCTION VALUE*,/,3X,*FUNCTION*,/,3X,*EVALUATIONS*,/,3       1490
     $X,11(*-*),4X,2(15(*-*),7X),14(*-*))                                1500
      KFLAG=0                                                            1510
      DO 601 I=1,2                                                       1520
      Q(I)=RK(I)                                                         1530
```

```
      W(I)=0.0                                                          1540
601   CONTINUE                                                          1550
      KAT=0.0                                                           1560
      KK1=0                                                             1570
 70   KCOUNT=0                                                          1580
      WBEST=W(2)                                                        1590
      CALL OBJECT(SUM,SUM1,SUM2,RK,NP,PC,DELR,DELTH)                    1600
      KK1=KK1+1                                                         1610
      IF(KK1 .EQ. 1) PRINT 100,KK1,(RK(I),I=1,2),SUM,SUM1,SUM2          1620
      BO=SUM                                                            1630
      IF(KK1 .NE. 1) GO TO 10                                           1640
      GD=SUM                                                            1650
      ACC1=SUM1                                                         1660
      ACC2=SUM2                                                         1670
      GO TO 201                                                         1680
10    CONTINUE                                                          1690
      IF(BO-GD) 11,201,12                                               1700
11    CONTINUE                                                          1710
      GD=BO                                                             1720
      ACC1=SUM1                                                         1730
      ACC2=SUM2                                                         1740
      GO TO 201                                                         1750
12    CONTINUE                                                          1760
      KFLAG=1                                                           1770
201   DO 55 I=1,2                                                       1780
      GG(I)=RK(I)                                                       1790
      TSRK=RK(I)                                                        1800
      RK(I)=TSRK+EPS(I)                                                 1810
      CALL OBJECT(SUM,SUM1,SUM2,RK,NP,PC,DELR,DELTH)                    1820
      KK1=KK1+1                                                         1830
      W(I)=SUM                                                          1840
      IF(W(I) .LT. GD) GO TO 58                                         1850
      RK(I)=TSRK-EPS(I)                                                 1860
      CALL OBJECT(SUM,SUM1,SUM2,RK,NP,PC,DELR,DELTH)                    1870
      KK1=KK1+1                                                         1880
      W(I)=SUM                                                          1890
      IF(W(I) .LT. GD) GO TO 58                                         1900
      RK(I)=TSRK                                                        1910
      IF(I .EQ. 1)GO TO 513                                             1920
      W(I)=W(I-1)                                                       1930
      GO TO 613                                                         1940
513   W(I)=BO                                                           1950
613   CONTINUE                                                          1960
      KCOUNT=1+KCOUNT                                                   1970
      GO TO 55                                                          1980
 58   GD=W(I)                                                           1990
      ACC1=SUM1                                                         2000
      ACC2=SUM2                                                         2010
      GG(I)=RK(I)                                                       2020
 55   CONTINUE                                                          2030
      PRINT 100,KK1,(RK(I),I=1,2),GD,ACC1,ACC2                          2040
100   FORMAT(/,5X,I5,5(5X,E16.8))                                       2050
      IF(KK1 .GT. MAXK) GO TO 94                                        2060
      IF(KAT .GE. NKAT) GO TO 94                                        2070
      IF(ABS(W(2)-WBEST) .LE. EPSY) GO TO 94                            2080
      IF(KCOUNT .GE. 2) GO TO 28                                        2090
      DO 26 I=1,2                                                       2100
      RK(I)=RK(I)+ALPHA*(RK(I)-G(I))                                    2110
 26   CONTINUE                                                          2120
      DO 25 I=1,2                                                       2130
      G(I)=GG(I)                                                        2140
 25   CONTINUE                                                          2150
      GO TO 70                                                          2160
 28   KAT=KAT+1                                                         2170
      IF(KFLAG .EQ. 1) GO TO 202                                        2180
      GO TO 204                                                         2190
202   KFLAG=0                                                           2200
      DO 203 I=1,2                                                      2210
      RK(I)=G(I)                                                        2220
203   CONTINUE                                                          2230
204   DO 80 I=1,2                                                       2240
      EPS(I)=EPS(I)*BETA                                                2250
```

```
 80   CONTINUE                                                          2260
      PRINT 101,KAT                                                     2270
 101  FORMAT(/,I4,1X,*REDUCTION IN STEP SIZE UP TO TIME*)               2280
      GO TO 70                                                          2290
 94   PRINT 460,(EPS(I),I=1,2)                                          2300
 460  FORMAT(///,5X,*FINAL STEP SIZES USED,ARE*,5X,*Y=*,F16.8,5X,*X=*,F1 2310
     $6.8)                                                              2320
      PRINT 103,(RK(I),I=1,2)                                           2330
 103  FORMAT(///,5X,*ORDINATE AND ABSCISSA FOR MINIMUM FUNCTION VALUE AR 2340
     $E,*,5X,*Y=*,F16.8,5X,*X=*,F16.8)                                  2350
      PRINT 462,GD,ACC1,ACC2                                            2360
 462  FORMAT(///,5X,*THE MINIMUM FUNCTION VALUE IS,*,5X,F16.8,5X,2      2370
     $(3X,F16.8))                                                       2380
      RETURN                                                            2390
      END                                                               2400
C                                                                       2410
C                                                                       2420
C                                                                       2430
      SUBROUTINE OBJECT(SUMN,SUMF1,SUMF12,RKK,NP,PC,DELR,DELTH)         2440
C=======================================================================2450
C                                                                       2460
C THIS ROUTINE SETS UP THE PARAMETERS FOR EVALUATING THE                2470
C OBJECTIVE FUNCION OF THE LINE SEGMENTS APPROXIMATING                  2480
C THE CONTOUR.                                                          2490
C THE ROUTINE ALSO EVALUATES THE TOTAL OBJECTIVE FUNCTION               2500
C FOR THE CONTOUR.                                                      2510
C                                                                       2520
C=======================================================================2530
      DIMENSION RKK(2),PC(1000,2)                                       2540
      PI=2.0*ASIN(1.0)                                                  2550
      NNP=NP-1                                                          2560
      SUMF1=SUMF2=0.0                                                   2570
      K=1                                                               2580
      DO 10 J=1,NNP                                                     2590
      Y1=PC(K,1)-RKK(1)                                                 2600
      Y2=PC(J+1,1)-RKK(1)                                               2610
      X1=PC(K,2)-RKK(2)                                                 2620
      X2=PC(J+1,2)-RKK(2)                                               2630
      XX=X2-X1                                                          2640
      IF(XX .EQ. 0.0) XX=10.0**(-60)                                    2650
      SLOPE=(Y2-Y1)/XX                                                  2660
      BINT=Y2-X2*SLOPE                                                  2670
      THETU=ANGLE(X2,Y2)                                                2680
      THETL=ANGLE(X1,Y1)                                                2690
      IF(ABS(THETU)-ABS(THETL)) 11,10,12                                2700
 011  ACC=THETU                                                         2710
      THETU=THETL                                                       2720
      THETL=ACC                                                         2730
 012  CONTINUE                                                          2740
      CALL XXINTT(SLOPE,BINT,THETU,THETL,SUMX1,SUMX2,DELR,DELTH,IFL)    2750
      IF(IFL .EQ. 1) GO TO 10                                           2760
      SUMF1=SUMF1+ABS(SUMX1)                                            2770
      SUMF2=SUMF2+ABS(SUMX2)                                            2780
      K=J+1                                                             2790
 010  CONTINUE                                                          2800
      SUMN=SUMF1/SUMF2                                                  2810
      SUMF12=SUMF2                                                      2820
      RETURN                                                            2830
      END                                                               2840
C                                                                       2850
C                                                                       2860
C                                                                       2870
      SUBROUTINE XXINTT(SLOPE,BINT,THETU,THETL,SUMX1,SUMX2,DELR,DELTH   2880
     $,IFL)                                                             2890
C=======================================================================2900
C                                                                       2910
C THIS SUBROUTINE CALCULATES THE OBJECTIVE FUNCTION FOR THE             2920
C INDIVIDUAL LINE SEGMENTS APPROXIMATING THE CONTOUR.                   2930
C                                                                       2940
C=======================================================================2950
      SINU=SIN(THETU)                                                   2960
      SINL=SIN(THETL)                                                   2970
```

```
      COSU=COS(THETU)                                                    2980
      COSL=COS(THETL)                                                    2990
      AU1=BINT/(SINU-SLOPE*COSU)                                         3000
      AL1=BINT/(SINL-SLOPE*COSL)                                         3010
      AU2=(COSU+SLOPE*SINU)/BINT                                         3020
      AL2=(COSL+SLOPE*SINL)/BINT                                         3030
      A1=ABS((AU1-AL1)/DELR)                                             3040
      A2=ABS(ABS(THETU)-ABS(THETL))/DELTH                                3050
      IF(A1 .GE. A2) A2=ABS((AU2-AL2)*DELR/DELTH**2)                     3060
      IFL=0                                                              3070
      IF(A1 .EQ. 0.0 .OR. A2 .EQ. 0.0)  IFL=1                            3080
      SUMX1=A1-A2                                                        3090
      SUMX2=A1+A2                                                        3100
      RETURN                                                             3110
      END                                                                3120
C                                                                        3130
C                                                                        3140
C                                                                        3150
      FUNCTION  ANGLE(XXX,YYY)                                           3160
C========================================================================3170
C                                                                        3180
C FUNCTION TO CALCULATE AN ANGLE BETWEEN 0 AND 360 DEGREAS,              3190
C GIVEN THE TANGENT OF  THE ANGLE.                                       3200
C                                                                        3210
C========================================================================3220
      PY=2.0*ASIN(1.0)                                                   3230
      IF(YYY) 1,2,3                                                      3240
 1    CONTINUE                                                           3250
      ANGLE=3.0*PY/2.0-ATAN(XXX/YYY)                                     3260
      RETURN                                                             3270
 2    CONTINUE                                                           3280
      ANGLE=0.0                                                          3290
      IF(XXX .LT. 0.0)  ANGLE=PY                                         3300
      RETURN                                                             3310
 3    CONTINUE                                                           3320
      ANGLE=PY/2.0-ATAN(XXX/YYY)                                         3330
      RETURN                                                             3340
      END                                                                3350
C                                                                        3360
C                                                                        3370
C                                                                        3380
      SUBROUTINE XINTT(SLOPE,BINT,THETU,THETL,SUMX1,SUMX2,DELR,DELTH)    3390
C========================================================================3400
C                                                                        3410
C INTEGRATION ROUTINE  TO BE USED IN PLACE OF XXINTT SUBROUTINE,         3420
C AND  CALLED FROM SUBROUTINE "OBJECT".                                  3430
C THE  INTEGRATION IS CARRIED OUT USING GUASS-LEGENDRE QUADRATURE.       3440
C                                                                        3450
C========================================================================3460
      DIMENSION EPPS(12),WE(12)                                          3470
C     DATA (EPPS(I),I=1,12)                                              3480
C    $/0.0640568926,0.1911188675,0.3150426797,0.4337935076,0.5454214714,3490
C    $0.6480936519,0.7401241916,0.8200019860,0.8864155270,0.9382745520,0 3500
C    $.9747285560,0.9951872200/                                          3510
C     DATA (WE(I),I=1,12)                                                3520
C    $/0.1279381953,0.1258374563,0.1216704729,0.1155056681,0.1074442701,3530
C    $0.0976186521,0.0861901615,0.0733464814,0.0592985849,0.0442774388,0 3540
C    $.0285313886,0.0123412298/                                          3550
      DATA (EPPS(I),I=1,3)                                               3560
     $/0.2386191860,0.6612093864,0.9324691542/                           3570
      DATA (WE(I),I=1,3)                                                 3580
     $/0.4679139345,0.3607615730,0.1713244923/                           3590
      X1=(THETU+THETL)/2.0                                               3600
      X2=(THETU-THETL)/2.0                                               3610
      SUMX1=0.0                                                          3620
      SUMX2=0.0                                                          3630
C     DO 10 I=1,12                                                       3640
      DO 10 I=1,3                                                        3650
      XX1=X1+X2*EPPS(I)                                                  3660
      CALL FUNC(XX1,BINT,SLOPE,FUN11,FUN21,DELR,DELTH)                   3670
      XX2=X1-X2*EPPS(I)                                                  3680
      CALL FUNC(XX2,BINT,SLOPE,FUN12,FUN22,DELR,DELTH)                   3690
```

```
      SUMX1=SUMX1+(FUN11+FUN12)*WE(I)                                    3700
 10   SUMX2=SUMX2+(FUN21+FUN22)*WE(I)                                    3710
      SUMX1=SUMX1*X2                                                     3720
      SUMX2=SUMX2*X2                                                     3730
      RETURN                                                             3740
      END                                                                3750
C                                                                        3760
CC                                                                       3770
C                                                                        3780
      SUBROUTINE FUNC(X,BINT,SLOPE,FUN1,FUN2,DELR,DELTH)                 3790
C========================================================================3800
C                                                                        3810
C SUBROUTINE TO CALCULATE THE OBJECTIVE FUNCTION,                        3820
C AND IS CALLED ONLY WHEN "XINTT" IS USED.                               3830
C                                                                        3840
C========================================================================3850
      R11=ABS(SIN(X)-SLOPE*COS(X))                                       3860
      R1=R11**2                                                          3870
      R2=ABS(BINT*(COS(X)+SLOPE*SIN(X)))                                 3880
      R=R2/R1                                                            3890
      FUNY1=R/DELR                                                       3900
      FUNY2=DELR*R11/(DELTH**2*ABS(BINT))                                3910
      FUN1=ABS(FUNY1-FUNY2)                                              3920
      FUN2=FUNY1+FUNY2                                                   3930
      RETURN                                                             3940
      END                                                                3950
C                                                                        3960
CC                                                                       3970
C                                                                        3980
      SUBROUTINE CUREVA(N,K,L,A,B,C,D,PC,NP)                             3990
C========================================================================4000
C SUBROUTINE TO CALCULATE THE LINEAR SEGMENTS THAT APPROXIMATE           4010
C THE CONTOUR.                                                           4020
C                                                                        4030
C========================================================================4040
      COMMON PI,NTP                                                      4050
      DIMENSION PC(1000,2)                                               4060
      NP=K+1                                                             4070
      IF(NTP .EQ. 3) GO TO 100                                           4080
      DX=(C-B)/FLOAT(K)                                                  4090
      DO 10 I=1,NP                                                       4100
      PC(I,2)=XX=B+(I-1)*DX                                              4110
      IF(NTP .EQ. 1) PC(I,1)=8.0*A**3.0/(XX*XX+4.0*A*A)                  4120
      IF(NTP .EQ. 2) PC(I,1)=A*D*XX/(A**2+XX**2)                         4130
 10   CONTINUE                                                           4140
      RETURN                                                             4150
 100  CONTINUE                                                           4160
      THE=2*PI/FLOAT(K)                                                  4170
      DO 11 I=1,NP                                                       4180
      TH=THE*(I-1)                                                       4190
      COSS=COS(TH)                                                       4200
      R=B+A*COSS                                                         4210
      PC(I,2)=R*COSS                                                     4220
      PC(I,1)=R*SIN(TH)                                                  4230
 11   CONTINUE                                                           4240
      RETURN                                                             4250
      END                                                                4260
```

APPENDIX F

STEPPING MOTOR

PULL-OUT TORQUE SPEED CHARACTERISTICS

F.1 Introduction

Electric stepping motors are electro magnetic actuators which convert digital pulse inputs to output shaft angular incrementations of equal magnitudes. Stepping motors are gaining ground in control applications over conventional AC and DC motors due to the following three reasons:

a) Stepping motors are inherently discrete motion devices and are more compatible with modern digital control techniques.

b) Positional errors of stepping motors are non-cumulative

c) Control systems employ stepping motors in an open loop configuration, yet achieve accurate position and speed control. In open loop control, costly transducers and feedback components are avoided. Furthermore, instability problems inherent in closed loop control systems are eliminateted.

The stepping motors utilized in the prototype NC machine are of the DC permanent magnet type. Axial and sectional views of such motors are shown in Figures F.1 and F.2, respectively. The most common configuration of the motor consists of a toothed stator with teeth spaced at a 1/48 pitch. The stator has eight salient poles with two-phase four-pole bifilar windings. Bifilar windings are windings

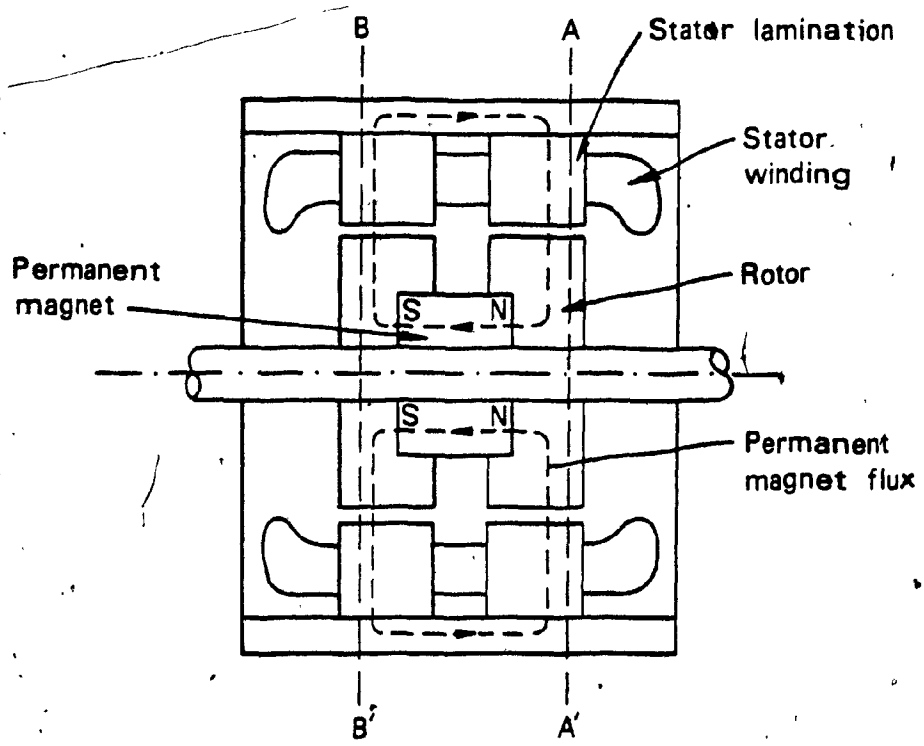

Figure F.1: Axial View of a D-C P-M Stepping Motor.

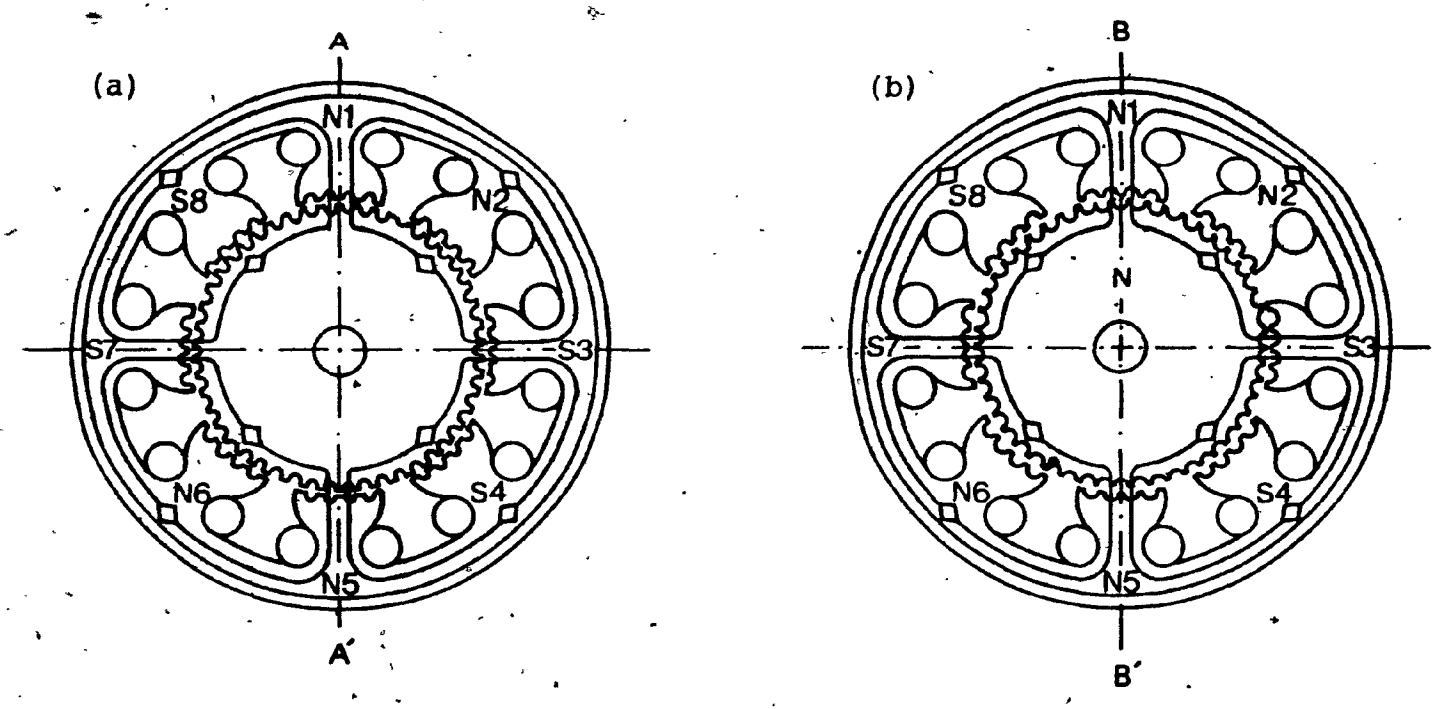

Figure F.2: Section View of a D-C P-M Stepping Motor.

wound in opposite direction to each other on the same pole. Thus, instead of reversing the direction of the current in a winding, current in a bifilar winding is simply switched ON and OFF.

The stator houses a toothed rotor composed of two toothed plates each having 50 teeth. The plates sandwich a permanent magnet between them and are displaced angularly from each other by half a tooth as can be seen in Figures F.2.a and F.2.b.

Figure F.3 shows a simplified version of the motor with a four tooth stator and a 5 tooth rotor. A wiring diagram of the windings of the stepping motor and its drive circuit is shown Figure F.4. Torque develops in a stepping motor based upon the principle that, when the stator windings are energized, a magnetic flux pattern is set which interacts with the permanent magnet field so as to move the motor and line up the two fields.

Three energization (switching) schemes are used to drive stepping motors. These are: full step two windings ON, full step one winding ON, and half step (also known as wave) schemes. Table F.1 shows the states of the four switches in Figure F.3 that are required to drive the motor using these three schemes.

Figures F.5.a to F.5.e show the alignment of the rotor as current is sequenced according to the most commonly used full step two windings ON scheme. The polarity of the stator poles is indicated on each of the figures. The se-

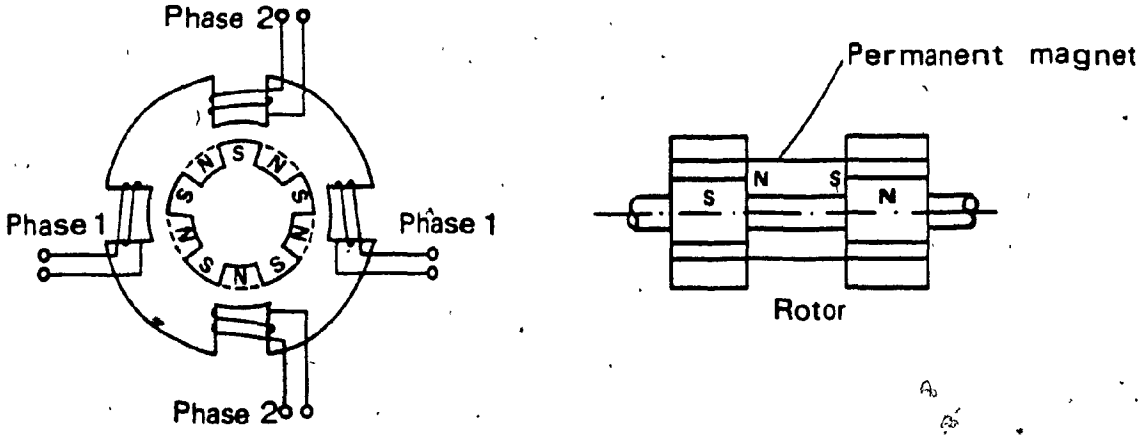

Figure F.3: Stepping Motor with 5 Teeth on the Rotor.

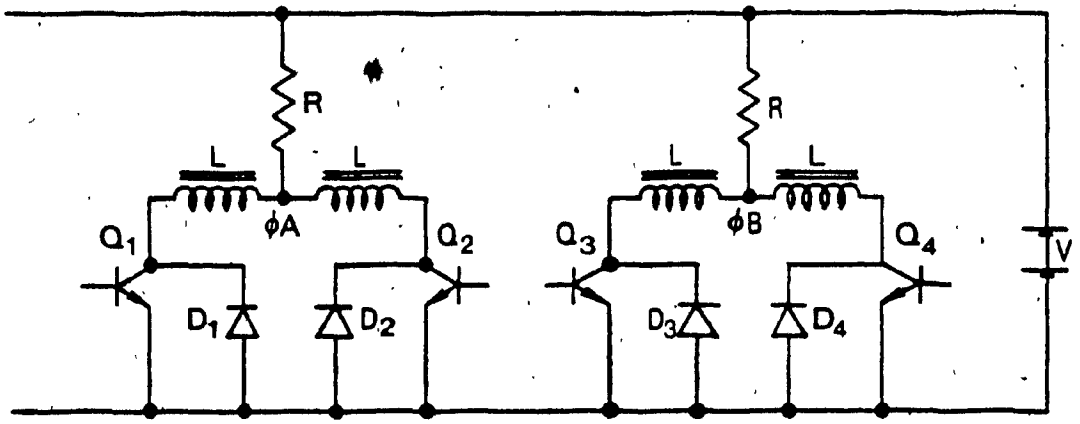

Figure F.4: Drive Circuit for Stepping Motor.

Table F.1: Switching Sequences for D-C P-M Stepping Motors

Full-Step, One Winding On

Step Number	Motor Coil Switch			
	SW1	SW2	SW3	SW4
1	OFF	OFF	OFF	ON
2	ON	OFF	OFF	OFF
3	OFF	OFF	ON	OFF
4	OFF	ON	OFF	OFF
1	OFF	OFF	OFF	ON

Full-Step, Two Windings On

Step Number	Motor Coil Switch			
	SW1	SW2	SW3	SW4
1	ON	OFF	OFF	ON
2	ON	OFF	ON	OFF
3	OFF	ON	ON	OFF
4	OFF	ON	OFF	ON
1	ON	OFF	OFF	ON

Half Step

Step Number	Motor Coil Switch			
	SW1	SW2	SW3	SW4
1	OFF	OFF	OFF	ON
2	ON	OFF	OFF	ON
3	ON	OFF	OFF	OFF
4	ON	OFF	ON	OFF
5	OFF	OFF	ON	OFF
6	OFF	ON	ON	OFF
7	OFF	ON	OFF	OFF
8	OFF	ON	OFF	ON
1	OFF	OFF	OFF	ON

quence of switching shown in Figures F.5.a to F.5.d, respectively, results in a clockwise rotation of the rotor. Counter-clockwise rotation is achieved by reversing the sequence shown in the figure.

The full step one winding ON scheme results in similar stepping sequence to that shown in Figure F.5. The control logic required for such an energization scheme is less complex than the one required for the full step two windings ON scheme. The output torque is 60% to 70% of that provided by the two windings ON scheme, and step accuracy may be slightly degraded. This mode however, requires only 50% of the power required for the full step two windings ON scheme.

The half step scheme requires a more complex control logic than that required by the other two schemes. Energization of the motor windings alternates between two windings ON and one winding ON, hence the name wave energization scheme. As a result of the wave energization, the motor output torque alternates between "strong steps", when two windings are ON, and "weak steps", when one winding only is ON. This energization scheme provides a finer resolution, half the step size obtained from the other schemes. In addition it permits starting at higher stepping rates. The alternating torque values, strong and weak steps, result in better resonance characteristics of the stepping motor [21].

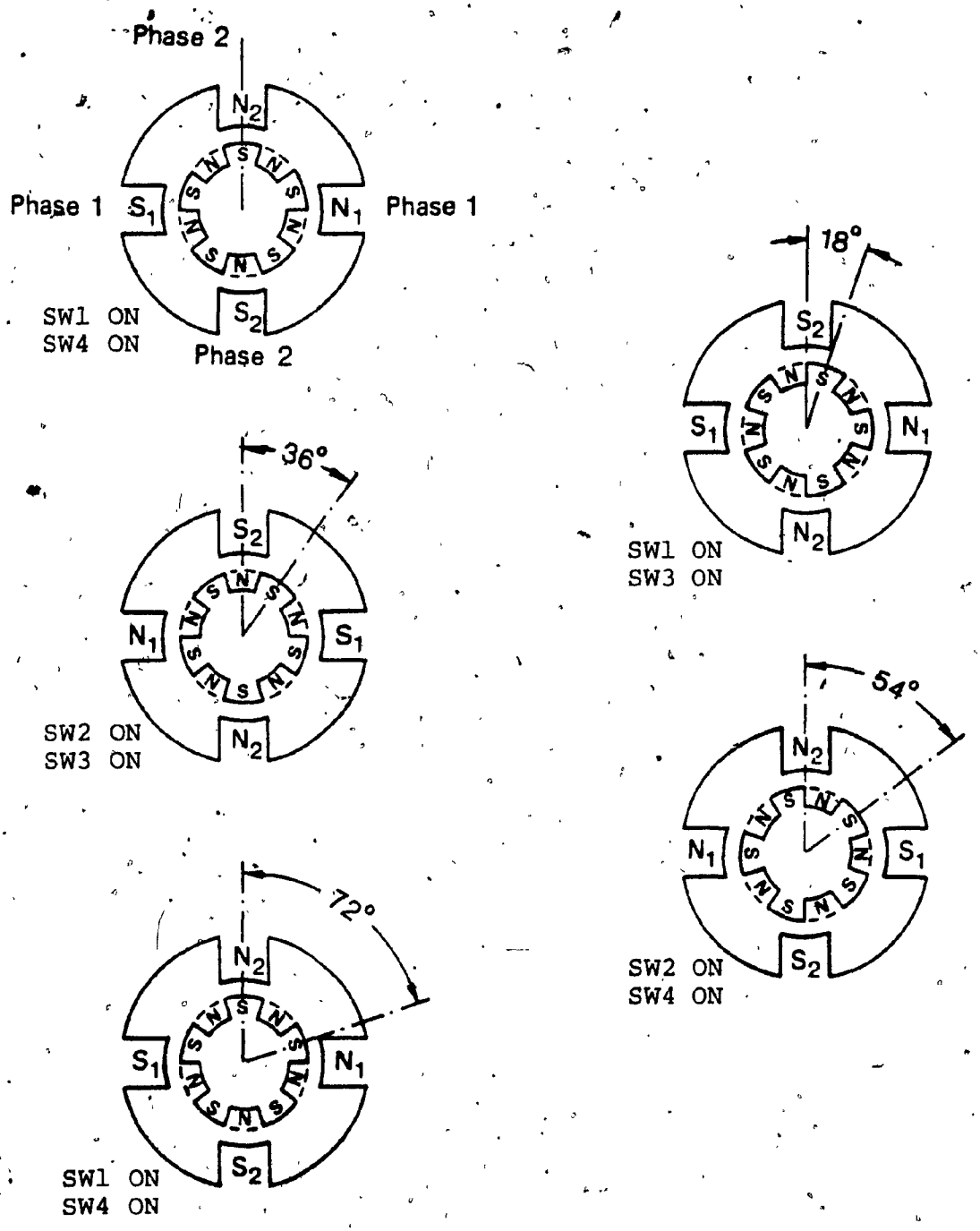

Figure F.5: Stepping Motor Rotor Positions in Sequential Stepping.

F.2 Derivation of the pull-out torque speed relationship

The phase voltage pattern for the wave energization scheme is shown in Figure F.6. This voltage pattern can be decomposed*, using Fourier Series, to its sinusoidal constituents and are given by:

$$V(t) = \frac{2V}{(2n+1)} \cdot \sum_{n=0}^{\infty} \left\{ K_n \cos\left[(2n+1)\,(\omega t-\varepsilon_1)\right] + \left(1+k_n\right) \sin\left[(2n+1)\,(\omega t-\varepsilon_1)\right] \right\} \qquad \text{F.1}$$

where,

$$K_n = (-1)^{\text{Int}\left(\frac{2n+1}{4}\right)} \Big/ \sqrt{2}$$

$$\varepsilon_1 \begin{cases} = 0 & \text{for phase A} \\ = \frac{3\pi}{2} & \text{for phase B} \end{cases}$$

where ω is the frequency of the stepping signals.

The drive circuit shown in Figure F.4 is a unipolar, low L/R type. The circuit used to drive the motor is similar to the one shown in the figure. With reference to Figure F.4 the voltage-time relation for the phases is given by:

$$V(t) = i(t)R_a + L_a \frac{di(t)}{dt} - K_V \sin\left(N_s\sigma - \varepsilon_2\right) \qquad \text{F.2}$$

where,

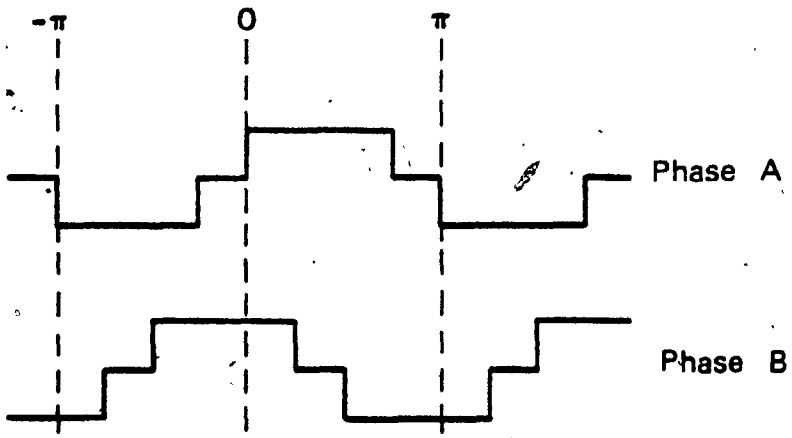

Figure F.6: Stepping Voltage Pattern for Wave Energization.

$$\varepsilon_2 \begin{cases} = \frac{\pi}{4} & \text{for phase A} \\ = -\frac{\pi}{4} & \text{for phase B} \end{cases}$$

where L_a is the motor winding inductance,

N_s is the number of teeth on the motor rotor,

K_V is the voltage constant of the motor winding,

and σ is the angular displacement of the motor shaft measured from a stable position.

Stepping motors are essentially ssynchronous machines, thus when operated at a constant angular velocity ω the angle σ can be expressed as:

$$\sigma = \omega_o t - \Phi$$

where Φ is the load angle,

and ω_o is the angular velocity of the motor shaft.

Combining Equations F.1 and F.2 and solving the resulting differential equation yields:

$$i(t) = C_0 \, e^{-R_a t/L_a} + \sum_{n=0}^{\infty} \frac{2V\sqrt{2K_n + 2k_n + 1}}{(2n+1)\,\pi\sqrt{R_a^2 + \left[(2n+1)\,\omega L_a\right]^2}}$$

$$\cos\left[(2n+1)\,(\omega t - \varepsilon_1) + \lambda_n - \Delta\right] +$$

$$\frac{K_v \omega_o}{\sqrt{R_a^2 + L_a^2 \omega^2}} \cdot \cos\left[\omega t + N_s \phi - \varepsilon_2 + \lambda_1\right] \qquad \text{F.3}$$

where,

$$\omega = N_s \omega_o$$

$$\lambda_n = \tan^{-1}\left[\frac{R_a}{(2n+1)\,L_a \omega}\right]$$

$$\Delta = \tan^{-1}\left[\frac{K_n}{1+K_n}\right]$$

The constant of integration C_0 is evaluated at t=0 where;

$$i(0) = \frac{V}{R_a} + \frac{K_V}{R_a}\,\omega_o \sin\left(N_s \Phi + \varepsilon_2\right)$$

and is given by:

$$C_0 = \frac{V}{R_a} + \frac{K_V \omega_o}{R_a} \sin\left(N_s \Phi + \varepsilon_2\right) - \sum_{n=0}^{\infty} \frac{2V\sqrt{2K_n^2 + 2K_n + 1}}{(2n+1)\,\pi\sqrt{R_a^2 + \left[(2n+1)\,\omega L_a\right]^2}}$$

$$\cos\left[\lambda_n - (2n+1)\varepsilon_1 - \Delta\right] + \frac{K_V \omega_o}{\sqrt{R_a^2 + \omega^2 L_a^2}} \cos\left(N_s \Phi + \varepsilon_2 + \lambda_1\right)$$

The torque equation for permanent magnet stepping mo-

tor is given [24] by:

$$T = -K_T\, i(t)\, \sin\left(\omega t - N_S\phi - \varepsilon_2\right) \qquad F.4$$

where K_T is the torque constant of the stepping motor.

Solving Equations F.3 and F.4 results in a torque expression which consists of the following groups of terms:

a) Non-sinusoidal decaying components resulting from terms in C_0 with n=0,

b) Decaying sinusoidal components resulting from the remaining terms in C_0,

c) Constant components resulting from terms with n=0 excluding those in C_0; time does not appear in these components and

d) Sinusoidal components resulting from all the remaining terms in the torque expression.

It is of interest in this analysis to calculate the pull-out torque, i.e. the maximum average torque [24], that the motor can develop at different speeds. Due to the small time constant of the drive circuit used, the exponential terms decay rapidly and can be ignored. Furthermore the sinusoidal terms have an average torque of zero and can also be ignored. Thus the remaining terms, having constant magnitude, are:

$$T_{const.} = \frac{(1.17632)\ V\ K_T}{\sqrt{R_a^2 + \omega^2 L_a^2}} \sin\left(N_s\phi - \lambda_1 + \Delta + \frac{3\pi}{4}\right) -$$

$$\frac{K_V\ \omega_o\ K_T}{\sqrt{R_a^2 + \omega^2 L_a^2}} \sin\lambda_1 \qquad \text{F.5}$$

The pull-out torque is the maximum of this constant torque and is thus given by:

$$T_P = \frac{(1.17632)\ V\ K_T}{\sqrt{R_a^2 + \omega^2 L_a^2}} - \frac{K_V\ \omega\ R\ K_T}{N_s\left(R_a^2 + \omega^2 L_a^2\right)} \qquad \text{F.6}$$

At any given stepping frequency the pull-out torque can be calculated using this equation.

F.3 Experimental verification of the pull-out torque relationship

The pull-out torque speed characteristics of the stepping motors used in the prototype NC machine were measured and compared to the published and to the calculated data. Two tests were conducted. In the first, the motor losses due to friction, windage, and damping were measured, using a torque transducer, while the motor is being driven by another motor.

In the second test the stepping motor is energized at a constant rate. The load on the motor is then gradually increased untill failure to register step commands starts to

occur. The load is varied by means of a friction brake and flywheel discs. The torque delivered by the motor is measured using a torque transducer inserted between the motor and the load. Failure to register step commands is detected by calculating the theoretical motor shaft position for a preset number of stepping commands, and comparing that position with the actual position of the motor shaft. A logic counter is used to mark the theoretical position on an oscilloscope screen. The actual motor shaft position is displayed on the oscilloscope screen by means of a rotational variable differential transformer (RVDT) coupled to the stepping motor shaft.

Figure F.7 shows the experimental results, the theoretical results, the manufacturer published data, as well as the measured motor losses. The figure shows that the theoretical torque value have a maximum deviation of 10% from the measured torque values at very low stepping speeds. This is mainly due to the inaccuracy in measuring the motor losses at such low speeds, and the use of a linear regression to approximate the losses. At high stepping speeds there is good correlation between the theoretical and measured data.

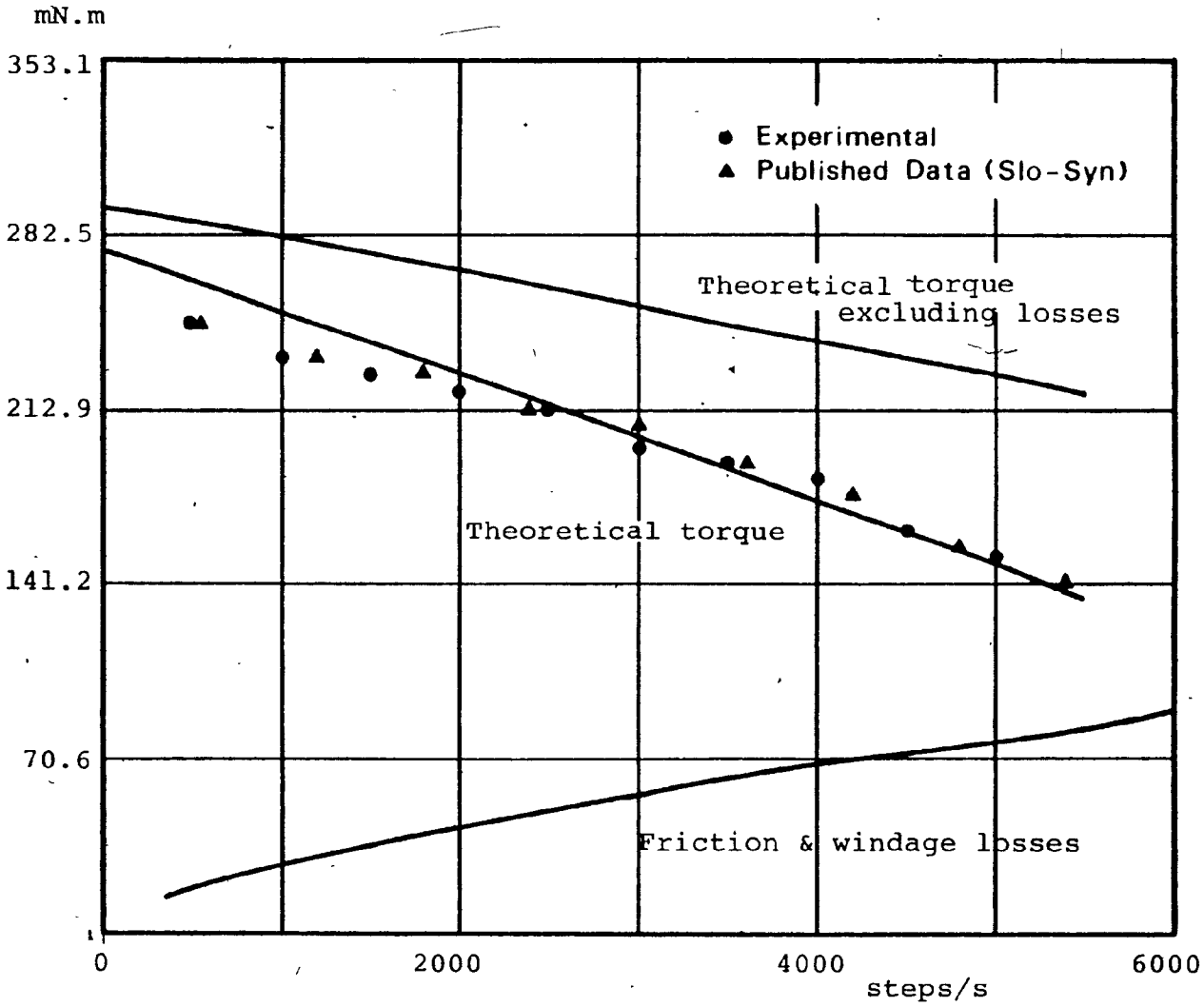

Figure F.7: Experimental and Theoretical Results for the Pull-Out Torque of a Stepping Motor.