HEDGING CANADIAN SHORT-TERM INTEREST RATES: THE BAX MARKET

JOHN J. SIAM

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Abstract

Hedging Canadian Short-term Interest Rates:
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John J. Siam, Ph.D.
Concordia University, 2000

This thesis adds to the body of literature seeking to improve the estimation of the optimal hedge ratio used in hedging money market and fixed income securities in Canada. A more accurate or improved depiction of the hedge ratio is of considerable importance and is the primary goal of this thesis.

The specific futures contract analyzed in this thesis is the Canadian Bankers' Acceptance Futures contract, the BAX, which was introduced on the Montreal Exchange in the early eighties as an instrument to manage interest-rate risk. Institutional features of the BAX market and the growth of this market, particularly in the second part of the 1990s, are described in Chapter 2. The efficiency of the BAX market is also addressed in this chapter where cointegration analysis is used to investigate the unbiasedness hypothesis.

Chapter 3 presents a univariate analysis of the BAX, and its underlying instrument the Canadian Bankers' Acceptance or BA, in a general framework that permits the statistical evaluation of myriad dynamic volatility models which have been used in such contexts.

The successful estimation of these general models requires a considerable amount of data and necessitates the daily sampling frequency which is used in this thesis.

Chapter 4 presents a brief background and motivation for the hedge ratio, surveys early attempts at characterizing the hedge ratio and presents the bivariate models needed to estimate the hedge ratio. Most importantly, the general univariate framework used extensively in the previous chapter is extended to the bivariate case. The time-varying BA/BAX Hedge Ratio is then estimated. The hedging performance of these models is then evaluated. We also discuss whether there is any practical value in using daily data, versus weekly data, in the determination of the hedge ratio.

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Chapter One Introduction

This thesis adds to the body of literature seeking to improve the estimation of the optimal hedge ratio used in hedging money market and fixed income securities in Canada. There are immediate gains that result from selecting a hedge ratio that more accurately represents the correlation between cash and futures prices. Accordingly, a more accurate or improved depiction of the hedge ratio is of considerable importance and is the primary goal of this thesis.

Chapter 2 deals with the origin, development, and growth of the most-popular traded Canadian instrument, the Canadian Bankers' Acceptance Futures contract, the BAX, which was introduced on the Montreal Exchange in the early eighties as an instrument to manage interest-rate risk. The underlying asset for the BAX is the Canadian Bankers' Acceptance or the BA. Institutional features of the BAX market and the growth of this market, particularly in the second part of the 1990s, are described in Chapter 2. The efficiency of the BAX market is also addressed in this chapter where cointegration analysis is used to investigate the unbiasedness hypothesis.

Chapter 3 undertakes a univariate analysis of the two underlying series, the BA and the BAX, in a general framework that permits the statistical evaluation of myriad dynamic volatility models. The framework is general along two dimensions. Disturbances are permitted to have an asymmetric impact on the latent volatility process; here again the parameterization is flexible, permitting non-uniform patterns of asymmetry. As well, the evolution of the volatility process itself is analyzed in a more general fashion. This characterization allows us to test whether the usual analysis of volatility dynamics is correct. The successful estimation of these models requires a considerable amount of data and necessitates the daily sampling frequency which is used in this thesis.

Chapter 4 presents a brief background and motivation for the hedge ratio, surveys early attempts at characterizing the hedge ratio and presents the bivariate models needed to estimate the hedge ratio. Most importantly, the general univariate framework used extensively in the previous chapter is extended to the bivariate case. The time-varying BA/BAX Hedge Ratio is then estimated. These models are then evaluated based on their hedging performance. We also discuss whether there is any practical value in using daily data, versus weekly data, in the determination of the hedge ratio.

1.1 Data and Statistical Methodology

The data considered in this paper involve the closing daily prices of BA (Bankers' Acceptance) and futures contract daily settlement prices BAX (Bankers' Acceptance Futures) series, both provided by the Montreal Exchange. The data consist of three-month daily BA prices and daily settlement of BAX prices both taken at 3:00 p.m. The time span of the data used in the univariate analysis (Chapter 3) starts with January 3rd 1995 and ends on June 30th 1999, representing a total of 1,133 daily observations for both series. The first 1,069 observations ending March 31st, 1999 are used to model the series. The remaining 64 observations (for the months April 1999, May 1999, and June 1999) are used to test out-of-sample performance of the models. The reasons given for selecting this period are outlined in detail Chapter 2. This data set focuses on the time period when the BAX market attained Canadian market prominence and achieved international exposure. Also this is the first time, to our knowledge, that daily data are used to model the BA and BAX series.

The time span of data used for the bivariate analysis (Chapter 4), starts with January 3rd 1995 and ends on December 30th 1999. The first 1,068 observations ending March 30th, 1999 are used to model the bivariate models. The remaining 190 observations were divided into three samples to reflect a 3-month forecasting horizon ending June 14, 1999, a 6-month horizon ending September 13, 1999, and a 9-month horizon ending December 30th 1999.

The statistical methodology used in Chapter 3 uses the Hentschel (1995) framework to analyze a variety of GARCH-type models. In this framework the volatility dynamics are specified as follows:

$$\frac{\sigma_t^\lambda - 1}{\lambda} = \omega + \alpha\sigma_{t-1}^\lambda f^\nu(\varepsilon_{t-1}) + \delta\frac{\sigma_{t-1}^\lambda - 1}{\lambda} \quad , \tag{1.1}$$

where

$$f(\varepsilon_t) = |\varepsilon_t - S| - R(\varepsilon_t - S) \quad . \tag{1.2}$$

It is clear that this characterization is more general than the standard GARCH(1, 1) model. There are two additional asymmetry parameters denoted R and S which mediate the impact of the basic shocks to the volatility process in equation (1.2). These are incorporated into the model's dynamics in a more general manner than the usual quadratic approach via the parameter ν. Moreover, equation (1.1) reflects a Box-Cox transformation of the basic structure of the dynamics involving an additional parameter λ.

In Chapter 3, models of this form are estimated for both the BA and BAX series; the underlying disturbances are assumed to be normal in one set of estimations, student-t in another, and GED (Generalized Error Distribution) on a third.

This framework nests not only the most popular traditional GARCH models, but also introduces some new types of GARCH models. Accordingly, we are able to use the likelihood ratio test to discriminate among different classes of simpler GARCH models.

Chapter 4 presents a brief survey of bivariate GARCH models that have appeared in the literature; these include the Bollerslev Constant Correlation model. This model provides a framework within which it is straightforward to extend the univariate analysis (undertaken in Chapter 3) to the multivariate case. In all, we focus on six bivariate GARCH models, and estimate these using both the normal and t-distribution. For each model so estimated, a time-varying hedge parameter is determined. Chapter 4 takes up the issue of estimating the hedging performance of these various candidates with regard to two simple benchmarks, including one based on the constant hedge ratio determined by OLS. In total, eight models are used in the evaluation of the hedging process. The within and out-of-sample performance of these models are compared in this Chapter for their ability to reduce risk exposure.

1.2 Findings

Here we focus on presenting the major themes in the empirical studies of Chapters 3 and 4. With regard to the univariate analysis in Chapter 3, the data supports strongly a more general specification of GARCH models than is usually adopted along both of the dimensions discussed above. The important Box-Cox parameter [λ] is found to be close to neither 2 or 1 or even 0; as well, the transformation parameter [ν] is not 2, as is it is specified in usual GARCH studies. Moreover, there is considerable evidence of asymmetry in the data; in some instances, both the shift parameter S and the rotation parameter R are found to be significant, using the usual t-test. Both these findings are confirmed using likelihood ratio tests.

A second theme that emerges in the univariate analysis of Chapter 3 concerns the nature of the underlying disturbances. On the assumption of normality, some estimation results did not satisfy the requirement of covariance stationarity. In general, we found that the results improved when the models are estimated using the student-t distribution.

The models are also evaluated in Chapter 3 on the basis of their performance in within-sample and out-of-sample predictive power. A "best" model emerges from all this analysis. This is the model termed Free GARCH-t; that is the model obtained when the four parameters λ, ν, R and S are estimated freely using maximum likelihood based on student-t disturbances.

Chapter 4 takes up the challenge of finding the best Bivariate GARCH model of the BA and BAX series. Using the time varying variances and covariances from the estimates of these models, the time-varying hedge ratio can be defined. Hedging performance based on the different hedge ratios are then used to evaluate the models.

Chapter 4 incorporates the framework of Chapter 3 by extending the univariate Free GARCH analysis to the bivariate setting. This model is one of three Bivariate GARCH models estimated in the chapter that are estimated under both normal and t-distributions. The hedging performance of the six hedge ratios so obtained are then compared with standard approaches, including that based on OLS estimation of a constant hedge ratio. Two performance measures are considered. One is the usual one based on the idea of variance reduction; the other focuses by contrast on the second moment.

The empirical analysis shows that there are significant gains to be had in hedging performance in moving to the Bivariate Free GARCH model which determined a hedge ratio that performed well over three forecast horizons.

One important aspect of this finding is that such a model cannot be estimated using weekly data. There is not sufficient data to carry out the estimation in a reliable manner using likelihood techniques. The finer sampling frequency permits the estimation of a more sophisticated model that has important practical consequences. Moreover, we indicate in Chapter 4 that the gain in hedging performance is not tied to the fact that the portfolio is re-balanced more frequently. Rather the model permits a better estimation of the hedge ratio that can be used as the hedger finds appropriate. In fact, re-balancing on average less than once a week based on hedge ratios obtained from daily data leads to better hedging performance than based on weekly re-balancing using hedge ratios obtained from weekly estimates.

Chapter Two The BAX Market

The primary aim of this chapter is to describe the Canadian Bankers' Acceptance Futures market, or the BAX market. Several perspectives are taken. First, the historical context of the inception of the market in the early 1980s and its subsequent explosive development is surveyed. We then indicate how interest rate futures contracts can be used to manage interest-rate exposure, and determine the institutional and economic features of the BAX instrument that give it an edge over other alternatives in managing interest-rate risk. The relationship between the BAX and its underlying, the Canadian Bankers' Acceptance, or BA, is considered from a statistical perspective. In particular, a cointegration relationship is established between the two series. This statistical feature is exploited in Chapter 4. In the present context, it is simply interpreted as indicating the BAX rate is an unbiased predictor of future BA interest rates.

2.1 The Canadian Bankers' Acceptance Futures: Origins and Development

Futures contracts are standardized agreements to buy or sell the underlying item on a specified future date at a predetermined price. Financial futures contracts are derivatives based on an underlying financial instrument such as a bond or market index. Financial futures represent a relatively recent innovation and are traditionally traded on exchanges.

Money market futures are futures contracts based on short-term interest rate instruments. The most popular world-wide futures contract is the Eurodollar future which was introduced in 1981. The Eurodollar trades twenty hours per day on three different exchanges on three different continents. The Canadian financial futures market is also relatively new. Trading started in 1983 when the Toronto Exchange launched the T-bill futures contract similar to the one that existed in the United States. The first attempt to launch a Canadian futures contract failed, as T-bills futures were delisted only 6-months after their inception. Traders attribute this striking failure to several reasons that are important in the context of the subsequent success of the BAX contract. First, the Canadian T-bills futures required physical delivery similar to its U.S. counterpart, an undesirable trading characteristic for a small, thinly-traded market where the possibility of a credit squeeze is always present. Second, financial derivatives were fairly new at the time and traditional money market traders did not understand the complexity of derivatives. Moreover, there was no institutional support to educate traders in these new products. Finally, the launching of T-bills futures occurred before the takeover of major brokers by Canadian banks when the two groups of financial institutions were in direct competition with each other. The Banks preferred to sell their own FRAs and Swap contracts, the traditional money market derivatives, and did not push the T-bill futures which were perceived as a brokers' product.

The advent of the BAX contract five years later in April 1988 was more readily accepted. Futures trading on the Montreal Exchange began in 1988 with the introduction of the three-month Canadian Bankers' Acceptance futures contract (BAX) in response to the need for market participants to manage interest-rate risk. The market took several years to develop. By 1995, the BAX was established as one of the main contracts to hedge Canadian short-term interest rates. A decade after the introduction of the BAX contract, the annual volume of the BAX contract was estimated to be over 6.3 million contracts per year with a daily average of 26,092 (each contract represents an underlying value of one million dollars). In 1997 outstanding market value (stock) of the BAX contract surpassed the most popular Canadian money market instrument, the T-bill. Today, the BAX market value (stock outstanding) is over two and half times the size of the Canadian T-bill market.

In sharp contrast to the previous experience with T-bill futures, the popularity of the BAX contracts with Canadian financial institutions (amounting to about 40% of all BAX trading) was not an accident, but was orchestrated by the Montreal Exchange. The Exchange wanted a contract that would be used by the financial community, especially by the Canadian Banks. Originally the Exchange wanted to model a contract based on the Canadian Interbank Offer Rate, the Canadian version of the British LIBOR (similar to the Eurodollar, the Sterling, and the PIB, which are based on Interbank Offered Rates). However, after the Exchange surveyed financial institutions, it recommended the use of the Canadian Bankers' Acceptance as the underlying asset of the proposed future contract. It was all in all a straightforward decision, as the banks themselves use the BA rate as their Interbank Offer Rate (the Canadian LIBOR was only used by London-based market makers).

Over the years, four other financial futures were introduced on the Montreal Exchange: the Ten-year Government of Canada Bond Futures (CGB, introduced 1989), the Five-year Government of Canada Bond Futures (CGF, introduced 1995), and the newly-introduced S&P60 futures contract (originally sold on the TSE), the BAR contract (one-month 3-million Canadian Bankers' Acceptance futures) introduced in 1992 was discontinued in 1997. In addition, there exist two options on futures contracts: the Option on Three-month Canadian Bankers' Acceptance Futures (OBX), and Options on Ten-year Government of Canada Bond Futures (OGB).

The BAX was introduced in 1988 in an economic period that was characterized by increased volatility of short-term interest rates and a strong market demand for financial instruments designed to manage interest rate risk. Table 2.1 compares the annual trading volume and open interest with other futures traded on the Montreal Exchange. All financial futures showed a continual increase in trading volume (except the BAR contract which was de-listed in 1997). Total trading volume has increased by 1400 percent since 1991 to a total of 8,690,396 contracts in 1998. Clearly the BAX contract has the lion's share of this market. It represents over 78 percent of the total trading volume of all futures contracts that trade on the Montreal Exchange, and is the only money market future that remains after the failure of the BAR contract. Open interest numbers recap similar progress: 79% of all futures-contracts' open interest on the Montreal Exchange are BAX contracts. The BAX market share represents over 3.7 times the volume of the nearest rival the CGB (Ten-year Government bond futures contract).

The phenomenal growth of Bankers' Acceptance futures is reflected in monthly trading volume. Figure 2.1 presents the history of this growth. The BAX contract began with a modest monthly volume of 210 contracts in April 1988; in June 1999 the monthly volume was just under 600,000 contracts, the highest volume (over 866,000 contracts) occurred in August 1998. These numbers involve enormous amounts of capital, as each BAX contract represents an underlying value of one million dollars. Therefore, the June 1999 monthly volume represents an underlying value of 600 billion dollars, an impressive amount for any Canadian instrument trading in the Canadian market.

A closer examination of Figure 2.1 also reveals that a fundamental change in the total turnover of the BAX contracts occurred during the last five years. The average monthly volume in 1993 amounted to about 60,000 contracts compared to an average of 161,000 in 1994, an increase of 268 percent. This increase represents a fundamental shift in the development of the BAX market. In 1995 an average of 200,000 contracts per month were traded, an increase of over 21 percent over the 1994 volume. In 1996, this growth was maintained with an average of 201,000 contracts, followed by an explosion in monthly volume to 345,000 contracts in 1997, an increase over the pervious year of over 70 percent. Another plateau was reached in 1998, when the monthly volume exceeded 500,000 contracts per month. The BAX average traded in 1998 was 567,000 contracts, a growth of 65 percent from the previous year. This level was maintained in 1999 with an estimated monthly volume of 531,000 contracts.

The 1994 period has been identified as the dollar crisis period, when interest rates were increasingly volatile. Due to pre-election jitters, the dollar lost 1.37 cents in value in less than one week. The dollar slide continued until the it to its lowest level in 5 years. Even Central Bank intervention in the currency markets did not save the Canadian dollar from further depreciation. As a result, T-bill rates rose sharply and the increased volatility pointed to further expected rate jumps of the Central Bank rate. The BAX contract played an important hedging role during this period. In addition to money market traders, foreign exchange traders also participated in the BAX Market (the price correlation of the BAX contract and the Canadian dollar is quite high). Foreign exchange traders who were caught long with the Canadian dollar offset their position in the BAX market. New market participants saw the value of the BAX market, and these new players were impressed by the depth and liquidity of the BAX market. They subsequently remained as players in the BAX market when the crisis was over. In short, the BAX contract came to establish itself as an effective hedging tool. It should be mentioned that at this time a co-ordinated effort was spearheaded by the Montreal Exchange, the Bank of Canada, the Department of Finance and the financial community to promote the use of futures contracts in Canada. All of those events came together to contribute to the extraordinary growth of the contract.

The popularity of the BAX contract is also represented by the growth of open interest outstanding (open BAX contracts) depicted in Figure 2.2. Open interest is an important feature of the market. It depicts increased market attention to the contract and is a reflection of increased market depth and liquidity. Open interest has grown from 600 contracts per month in April 1988 to over 4.5 million contracts in June 1999. The open interest high of over 6.8 million contract was registered in August 1998.

In the early 1990s, the Bank of Canada and the Montreal Exchange launched a co-ordinated effort to market Canadian futures internationally. The effort appears to have been successful. Today, foreign investors hold about 43 percent of all open interest, a strong indication that the BAX is on its way to becoming an international futures contract. Certainly the similarity between the BAX contract and the Eurodollar contract has given the institutional trader an efficient way to arbitrage/hedge domestic and international short-term interest rates as evidenced by the increased interest in the BAX/Euro spread.

Another point worth noting is the increase in the number of monthly transactions. Monthly transactions have grown from 37 in April 1988 to over 30,000 in January 1999 (see Figure 2.3). The high of over 37,000 was registered in August 1998. The significance of this number is to be found in conjunction with the open interest number. The BAX market appears to be of increasing liquidity and depth. Market interest in the BAX contract is not restricted to a select few market participants such as institutional traders but to an increasing number and variety of market participants.

To conclude this section, we now summarize the main institutional features of the BAX. The BAX is a futures contract traded on the floor of the Montreal Exchange between 8:00 a.m. and 3:00 p.m. It is based on an investment of $1,000,000 in the three-month Bankers' Acceptance (BA). The BAX trades on an index basis. The price is calculated by subtracting the annualized implied yield on the Bankers' Acceptance from 100. For example, if the March-00 contract is trading at 94.95 on the floor of the Exchange, this price implies a 5.05 per cent (or 100.00 B 94.95) annual yield for BAs issued in March 2000. The BAX contracts mature two business days prior to the third Wednesday of the month in March, June, September and December over a three-year period.

Contracts are identified by their delivery month; the first contract has the nearest delivery, while the twelfth contract has the furthest. As with other futures markets, the first BAX contract is the most widely used and therefore the most liquid. However, trading activity and positions start shifting toward the second contract about a month before the first contract expires. The higher liquidity of the first contract is reflected in the narrower spread between its bid and asked price. The standard bid/ask spread for the first contract is just one basis point (the smallest trading increment), about two basis points for the next two contracts, and can reach 5 basis points for the remaining contracts.

The delivery dates correspond to the delivery dates of Eurodollar futures contract traded on the Chicago Mercantile Exchange, which helps create arbitrage opportunities between the BAX and the Eurodollar futures markets. Table 2.2 exhibits the similarity not only between the BAX and the Eurodollar but as well between the French PIB and the British

Sterling contract. Increased international markets integration is forcing the standardizing of futures contracts to enhance their liquidity, depth, and international exposure.

BAX contracts are marked-to-market daily. Resulting profits or losses are credited or charged to the investor's margin account. If these daily adjustments result in the margin account falling below a pre-specified level, the investor must make an incremental deposit to bring it back to the desired level, or the futures position will be liquidated. When the contract expires, outstanding positions are liquidated. Although most investors are not interested in acquiring the underlying instrument of the contract, they must abide by certain rules. For instance, when they liquidate their position prior to the delivery date of the futures contract, they must buy an offsetting position on the floor of the exchange.

2.2 Users and Uses of the BAX Contract

The primary use of BAX contracts is to manage short-term interest rate exposure; the contract is used as well to speculate and to arbitrage. The BAX contracts' major participants can be divided into two main groups: financial and non-financial. Some of those users trade for themselves and others on behalf of clients. About 70 percent of all trades are done on behalf of clients. Professional traders and locals (independent traders on the Montreal Exchange) account for the balance.

Financial institutions mainly act as intermediaries in the derivatives market, extracting revenues from fees, their dealers' bid/ask spreads and trading profits. However, in some cases they are forced to take one side of the transaction (ie, they cannot find a customer for the other side). They use the BAX contract to shed away some of the risks inherent in these transactions and to hedge their own books (inventories of short-term paper). They also use the BAX contract to hedge risks associated with their banking operations; for example, banks can hedge their demand deposits with offsetting positions in the BAX contacts. In addition, financial institutions-- banks in particular-- are concerned with the impact that interest rate fluctuations might have on their assets and liabilities. Canadian banks have set up risk-management departments to measure and manage interest rate exposure resulting from their intermediation role. In general, they try to cover any undesirable Forward Rate Agreement (FRA) position through offsetting it in the BAX market. For example, if an FRA trader is caught long (short) as interest rates are rising (falling) he can offset his position in the BAX market by selling (buying) the equivalent numbers of BAX contracts. Financial institutions thus carry out a significant portion of their interest rate management through the BAX market. It is estimated (by the Montreal Exchange) that about 40 percent of all BAX volume is a direct result of trading by financial institutions.

However, the major users of the BAX are non-financial entities that manage heavy short-term debt commitments. Non-financial institutions that issue short-term debt and use credit lines to finance their day-to-day operations are exposed to short-term interest fluctuations. Therefore, the most common use of the BAX contract is to minimize the impact of short-term interest rate fluctuations on the firm's financing strategy. These institutions use BAXs to manage their exposure to interest-rate risk. Treasurers use BAXs to achieve a more desirable matching of current assets and liabilities. They can also protect themselves against the risk of unfavourable interest rate movements by selling or buying BAX contracts. The important point is that BAXs allow their users to lock in their short-term financing rate; a detailed example is given below.

According to data published by the Montreal Exchange over half of the transactions in the BAX market are undertaken for hedging purposes (the remainder includes 25% for speculation and 25% for arbitrage and yield-investment strategies). Further hedging examples can be readily found. Banks hedge some of their BAs issues. Bank traders who manage the swap book hedge their exposures with BAXs. Most money market traders have used the BA futures to hedge their positions in one way or another.

It should be mentioned that non-residents trade BAXs to hedge or take advantages of the spread, or interest rate differential, between Canada and other countries. As a consequence of the existence of these international contracts, traders can speculate not only on the direction of short-term interest rates, but also on volatility spillovers, movements, and spreads differentials between short-term domestic and short-term international rates. One of the popular spreads to emerge is what has become known in the industry as the "BED" spread. The BED spread is the difference between the Canadian BAX contract price and the U.S. Eurodollar futures price. Depending on whether the BED spread narrows or widens with respect to some benchmark selected by the trader, the trader will take offsetting positions in the hope that the spread will "normalize" in the future.

For individual investors the main rationale for using BAX contracts is to hedge the returns of their investments. A detailed example of the use of the BAX contract to hedge returns is now given. The example is termed a *long naive pure perfect hedge*: it is a long hedge, because the user is buying the future (and selling the spot); naive because it is constructed on a one-to-one ratio between the cash and its future contract; pure, because it is performed between the BAX and the underlying BA, and not some other asset; and (it turns out to be) perfect, because the basis lost is equal to the basis gained.

In this example, a firm has a policy to use the BA market to lend excess cash in its inventory. However, the firm's financial manager is facing declining interest rates. She expects to receive $10M in three month's time. She plans to lend the money out for three months using today's lending rate:

Market Conditions	Today	In Three Months	Change in Basis
3-months BA (implied price)	5 % ($95.00)	4.5 % ($95.50)	-50 (50)
3-month BAX (implied rate)	$94.50 (5.5 %)	$95.00 (5.0 %)	50 (-50)
Basis	-50	-50	0

Her objective to lock in today's interest rate is achieved by buying 10 BAX contracts; each BAX contract is worth $1 million. The firm's manager will experience an opportunity loss of $12,500 (= $10,000,000 × .005 × 3/12) by investing in the rate available three months from now (4.5%). Having sold 10 BAX contracts the firm's manger will show a gain of $12,500 [= (95.00 - 94.50) × 10 × $25 × 100], which will help to offset the opportunity lost as a result of the lower investment rate. Note that in this calculation, $25 is the value of one basis point. We see here the basis (ie, the difference between cash prices, cash futures prices or yields) lost is equal to basis gained. This is known as a perfect hedge. Perfect hedges are rare and slippage is common in hedges. However, this simple example demonstrates how a BAX contract can be used to minimize interest-rate exposure.

The previous example involved what is termed a naïve hedge. Here there is a one-to-one relationship between the cash position and the futures position. In general, however, it is not clear that this relationship is the optimal one for the manager to undertake. The appropriate relationship, called the hedge ratio, is taken up in Chapter 4.

2.3 The BAX Advantage

To evaluate the advantage of the BAX contract, other available alternatives in managing interest-rate risk must be examined. The best alternative to the BAX contract is found in over-the-counter money market instruments (OTCs). These are off-exchange money market derivatives that had an earlier start than the BAX contract and for a time were the only Canadian product available to manage interest-rate exposure (short of going to the U.S. market). These OTC products include forward rate agreements (FRAs), interest-rate swaps and over-the-counter options. Interest-rate forward rate agreements are contracts that set the rate of interest to be paid (or received) over a predetermined period of time. Interest-rate swaps are agreements whereby two parties (known as the counter-parties) agree to exchange short-term floating interest rate payments for a longer-term fixed interest rate payments or vice versa. Interest-rate options are provisions to pay or receive a specific interest rate on a predetermined principle for a pre-set period of time.

When the Bank of Canada conducted a survey of the Canadian foreign-exchange market in April 1995, the survey included for the first time questions concerning over-the-counter derivatives. The survey was repeated in April 1998. The daily turnover of OTC interest rate derivatives amounted to 9 billion Canadian dollars, an increase of over 48 percent from April 1995. The size of the Canadian derivatives market ranked eighth worldwide (unchanged from 1995). Interest-rate swaps are closely linked to borrowing/lending operations. They are by far the largest segment of interest rate derivatives (close to $2,000 billion). Swaps contracts are denominated in many foreign currencies (only about a third are in Canadian dollars), and these contracts usually have maturities that are of one year and longer. Since swaps are agreements that allow individuals to swap interest payments from short-term interest-rate payments to longer-term interest payments, a direct comparison between the swaps and the BAX contract is inappropriate. BAX contracts allow the hedger to fix the rate of interest for a specified term in a future period and are used primarily to manage interest-rate risk.

Two other contracts were reported in the Bank of Canada survey. Interest-rate forward rate agreements and OTC interest-rate options. The later accounted for less than one-sixth of the total amount outstanding of interest-rate derivatives (the smallest segment of this market) and are used primarily to hedge long-term debt instruments. The OTC interest rate options represent an entirely different hedging instrument than futures contracts such as the BAX. Whereas futures contracts are obligations (the buyer must accept delivery and the seller must deliver), options give the holder choices (the right not obligations). Options are instrument specific and price specific (with an exercise price) and are not usually used in cross hedging which involves different instruments.

The only appropriate comparison of the BAX contract is with the OTC interest-rate FRAs. Figure 2.4 exhibits the total value of FRAs, BAXs and Bankers' Acceptances (included for comparison, since both use the 3-month BA rate for settlement purposes) for 1992, 1995 and 1998, a period of remarkable growth for both contracts. The FRAs have grown over 98 percent since 1992, and 20 percent since 1995, while the BAs have grown by about 79 percent since 1992, and about 30 percent since 1995. The BAXs experienced growth of over 800 percent since 1992, and 130 percent since 1995. The figure also shows that the market size of the FRAs is over three times the size of the BAX contract. However, a qualification is in order. The difference between the amount of the FRA outstanding and BAX open interest is not a pure reflection of the relative liquidity of the two markets. Open interest is measured by the total net position on the Montreal Exchange, while outstanding FRAs represent the gross of the notional amounts held by the banks. Any offsetting position on a contract thus reduces the net amount of open interest in the BAX, while for FRAs, it is added to outstanding. The FRAs are distributed over a much broader range of maturities than the open interest in the BAX. Only half of the FRAs have a remaining term to maturity of one year and less, while the majority of the BAX contracts (about 85%) have maturities of one-year and less. A number of FRA contracts are denominated in other currencies, while the BAXs are denominated only in Canadian dollars. If we compare the BAX contract with Canadian dollars dominated FRAs of one-year maturity and less, the markets are of equivalent size.

The relatively higher growth of the BAX contract is due to a number of advantages of the BAX contract over OTC contracts; these include:

- Although OTC products such as the FRAs offer customized flexible protection, they involve considerable negotiation. The only item that the client has to negotiate with standardized BAX contract is its price.

- The BAX daily volume of over 26,000 contracts, and the tight bid/ask spreads ensure great liquidity. Usually the BAX bid/ask spread is about one basis point apart ($25), while the OTC contracts range from about four to ten basis points ($100-$250)). A wide bid and ask spread adds additional costs to entry and exits from the positions established. Figure 2.5 demonstrates the progress of the BAX liquidity, today the BAX contract bid/ask spread is down to the minimum increment possible of one-tick (one basis point).

- One of the strongest advantages of the BAX contract is its creditworthiness. When entering into OTC contract the creditworthiness of the contract is only as good as the parties involved, and some defaults have been encountered in this area. However, the CDCC (Canadian Derivatives Clearing Corporation) guarantees both sides of the BAX contract, thereby eliminating default risk.

- Prices and price quotes are another advantage of the BAX contract. The BAX contract is a publicly-traded contract and is traded on a single competitive market. Therefore, BAX prices are determined by national and international supply and demand of Canadian short-term credit. Although this is also true in principal for OTC products, market participants must generally "shop around", so price competitiveness may be significantly reduced.

- Margin requirements and transaction costs are lower for exchange-traded products than for off-exchange products. To trade OTC products a credit line is generally required which bars entry for some market participants. Margin requirements for the BAX contract, represent about 0.001 % of the underlying value. This margin is lower than any other traded instrument in Canada. The BAX contract transaction costs have been declining ever since the contract was lunched in 1988. Today, a round-trip fee can be as low as $7.5 per BAX contract (a 50% reduction from 1988). By comparison, the commission charged for a one million dollar OTC contract is one basis point ($25), four times the commission for BAXs.

In this part of the Chapter, we have described the growth of the BAX contract from its inception in 1988. The BAX contract has met many domestic and international needs in providing one of the main hedging tools for hedging Canadian short-term interest rates. It also provides for both domestic and international investors a speculation and arbitrage instrument that responds more quickly to the arrival of new information than other more traditional instruments. The BAX contract today is seen not only as a Canadian benchmark to price interest rates derivatives, but the benchmark to price all Canadian money market instruments including Treasury Bills. Moreover, as Table 2.3 clearly indicates, the Canadian futures market is well behind other international markets, a position that suggests that the BAX market will continue to grow and figure prominently among Canadian money-market derivatives.

2.4 The Canadian BAX Market: Unbiasedness, Efficiency and Cointegration

At any time, there are twelve BAX contracts that trade, corresponding to four different delivery months--March, June, September, and December-- over a three-year period. The contracts within the second set of maturities (second-year delivery) are known as reds; the contracts within the third set of maturities (third-year delivery), as greens. Researchers have traditionally used the first-delivery month of futures contracts to model the underlying process of a particular futures time series. The reason is that the first month is usually more liquid than any other contract. In addition, the first month BAX contract reflects the implied three-month Canadian short-term forward interest rate and will be used in the this thesis to represent the Canadian short-term forward rate.

In Figure 2.6, the daily volume growth of the all BAX contracts is depicted along with the volume of the first month contract. As the figure indicates, trading volume picked up dramatically in 1995, when the average daily volume was up about 25 percent from the pervious year's daily volume. In the following year a slightly higher number traded with a yearly average of over 300,00 contracts. The trend has continued upwards through the late nineties. Figure 2.7, representing the number of daily transactions, tells a complimentary story. The increasing number of daily transactions is an assurance of increased liquidity and depth of the BAX market. Moreover, as Figures 2.6 and 2.7 clearly indicate, the first-month contract is responsible for about 40 to 50% of all BAX contracts.

The data considered in this thesis is the daily closing three-month BA price (taken at 3 p.m.) and the daily settlement price of the BAX first-month contract (also at 3 p.m.). The Montreal Exchange provided both sets of data. The time period under consideration begins with the fourth day of January 1995 and ends on June 30 1999, representing a total of 1133 daily observations for both series.

The major reason for choosing the sample period 1995 - 1999 reflects the concern for sufficient daily trading volume. Newly listed instruments need time to develop. This process usually takes from five to ten years, as it did with the BAX contract. When the BAX contract was first introduced, there were days when the first-month contract did not trade. However, in subsequent years the volume picked up dramatically. Based on Figures 2.6 and 2.7 we can clearly see that 1995 represents the year in which the BAX market attained prominence in terms of depth and liquidity. Moreover, no other research on the BAX has focused on this time period. An earlier study, Gagnon and Lypny (1995) examined the hedge implication of the BAX, using weekly data from 1990 to 1994 period; see Chapter 4 for further details.

As the BAX contract approaches its maturity, the BAX process and the BA process will converge. Expiration effects may come to dominate the market. Accordingly, researchers in the analysis of futures process rollover the contract some time before its expiration. In this thesis, the BAX contract is rolled one month before its expiration to the nearest subsequent contract. For example, in the middle of February the contract for March delivery (expiration date of the BAX contract occurs on the second Monday of the delivery month) is rolled over into the BAX June contract.

The intuition that in properly functioning markets speculators cannot expect to make excess returns is central to financial economics and is known as the unbiasedness hypothesis. There is a considerable literature on the unbiasedness hypothesis; see Boothe and Longworth (1986) for a survey of the results. Typically, OLS has been used to estimate the following regression:

$$s_t = \alpha + \beta f_{t-m} + \varepsilon_t, \qquad (2.1)$$

where f_{t-m} is the (log) of the futures price observed at t-m for a bond to be delivered m periods ahead at time t when the (log) of the spot price is s_t ; in this context, we test whether $(\alpha, \beta) = (0, 1)$, corresponding to the joint assumptions of risk neutrality and rationality, or whether $\beta = 1$, corresponding to the assumptions of constant risk premium and rationality .

The choice of estimation method is crucial in this context, since it is likely that the variables contain a stochastic trend (unit root) which renders OLS inappropriate and a cointegration approach more suitable; for an early analysis of this problem see Patel and Zeckhauser (1988), and for an account of the direction of incorrect inference when cointegration is overlooked in regression-based testing see Brenner and Kroner (1995).

In what follows, we side step the problems involved in the two-step approach of first estimating the cointegrating vector and subsequently testing whether the cointegrating vector assumes specific values. Rather we simply impose the intuition that the cointegrating vector relating the two nonstationary series assumes the form (1, -1) and test whether the new series generated by imposing this relationship is stationary.

Let s_t denote the log of the BA rate at time t and let f_t denote the log of the BAX rate at time t for the closest contract at least one month away. We also consider two other variables: $Basis_t$ defined as s_t - f_t , and $Prem_t$ which is the difference between the realized spot rate $s_{t'}$ and f_t . The graphs of these series are given in Figures 2.8, 2.9 and 2.10.

As indicated in Figure 2.8. the BA series appears to track the BAX series, while neither exhibits mean-reverting behaviour. This feature is at the heart of the statistical cointegration analysis that follows. The next two series appear to move around a mean. The movement of Prem [the futures premium series] around a mean between 0.00 and -0.01 (Figure 2.10) is related both to the cointegration of the BA and BAX series as well as to the unbiasedness hypothesis.

Table 2.4 reports the results of the Dickey-Fuller tests and Phillips-Perron tests for a unit root in each of the four variables. The tests are based on the value of the autoregressive coefficient in the usual regression [rho test] or the value of the t-statistic; see Hamilton (1994) for a description of the tests. The lag length refers to the order of first differences of the variable included in the regression; these terms are included to capture the effects of the serial correlation. The statistical analysis readily confirms the message conveyed by the graphical analysis: both the BA series and the BAX series are non-stationary, while the other three series appear stationary. For the two series for which the null of a unit root cannot be rejected, Table 2.4 also reports the results of the unit root tests applied to the first differences of the variables [i.e. tests whether the variable is integrated of order 2]. In both instances, the null hypothesis of a unit root in the differences is rejected. The analysis indicates that BA and BAX are both integrated of order 1.

The analysis also indicates that the Premium, or difference between the futures price [ie, the BAX] and the realized spot price [ie, the BA] is stationary while the components of the Premium are non-stationary. We interpret this result, which establishes that the cointegrating vector relating the BA and the BAX is given by (1, -1), as supporting the unbiasedness hypothesis as described above. This approach is taken by Hakkio and Rush (1989) among others.

We now explore whether the BAX market is efficient. More specifically, do BAX prices incorporate all readily available information concerning future spot prices? The usual approach to evaluate this claim is to analyse whether the implied forecast errors, i.e. the difference between the BAX forward price and the associated realized price, are in any way systematic. Any correlation between these errors and the information contained in another variable known when the BAX price was determined would count against the efficiency of the BAX market; at least, in so far as the relation could be exploited. In this section, the relation between the forecast errors and the information contained in the basis (the difference between current spot and BAX prices) is investigated; a similar analysis is also applied to the information contained in revisions of the BAX price.

First, it should be emphasized that correlations around the mean of the forecast errors are what count against the efficiency of the BAX market. The mean is simply the premium. Second, by interpreting the difference between realized spot and BAX as an error, it is implicitly assumed that the unbiasedness hypothesis is true. In short, it is assumed that the cointegration vector between the two variables is (1., -1.), an assumption which is supported by the previous results. Consider then the following regression:

$$Premium_t \;=\; \beta_0 \;+\beta_1\, Revisions \;+ \ldots + \beta_k\, Revisions_{t-k} + u_t \quad . \qquad (2.2)$$

The usual approach is simply to consider an F-test determining whether all the slope coefficients are 0. Since on the null of market efficiency the errors may be heteroscedastic, a Wald test based on the robustified covariance matrix is also considered.

The results in Table 2.5 indicate that there is no information contained in the manner that BA or BAX revisions are made at the time of the forecast that improves the forecast performance of the BAX series. However, it appears that there may be some relevant information in the pattern of revisions over a two-week period, or even in the current basis or difference between the current spot and future. Whether this information can be exploited is debatable. And it should be kept in mind that these are tests of the joint hypothesis of efficiency and constant premium. We may prefer to interpret these results as suggesting that the risk premium in holding the future is varying over time, a position that is more in the spirit of the analysis of the Chapters that follow.

In this section, statistical procedures suitable for series that contains a unit root have been applied to the BA and BAX series. We have found that these two series are indeed cointegrated. There exists a stable linear relationship between the implied forward rate of the BAX contract, and the future spot rate implied by the BA paper. This relationship is used in Chapter 4 as part of the estimation procedure that determines the time-varying hedge ratio.

FIGURE 2.1: Bankers' Acceptance Futures [BAX]
Monthly Volume

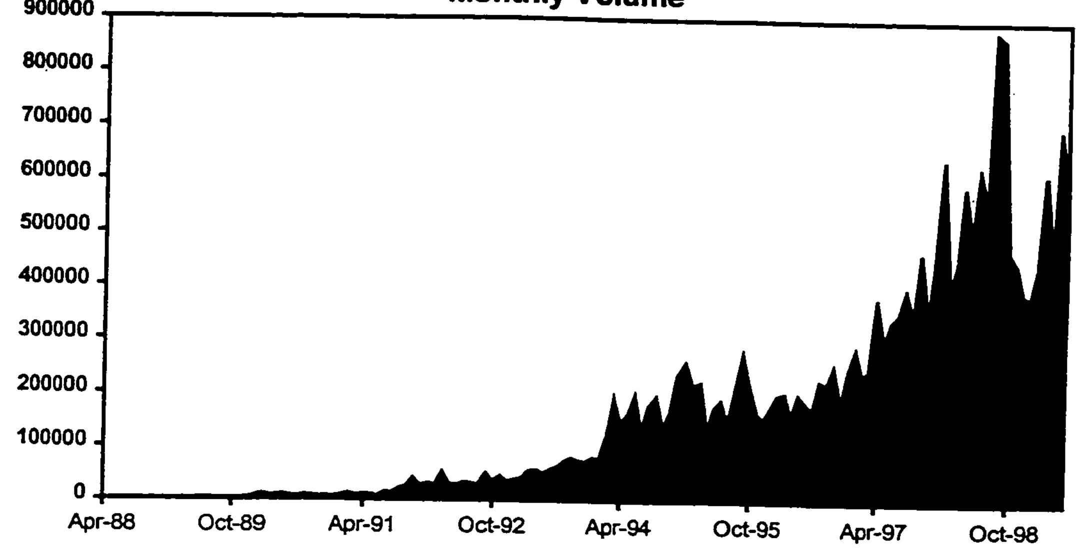


FIGURE 2.2:Bankers' Acceptance Futures [BAX]
Monthly Open Interest

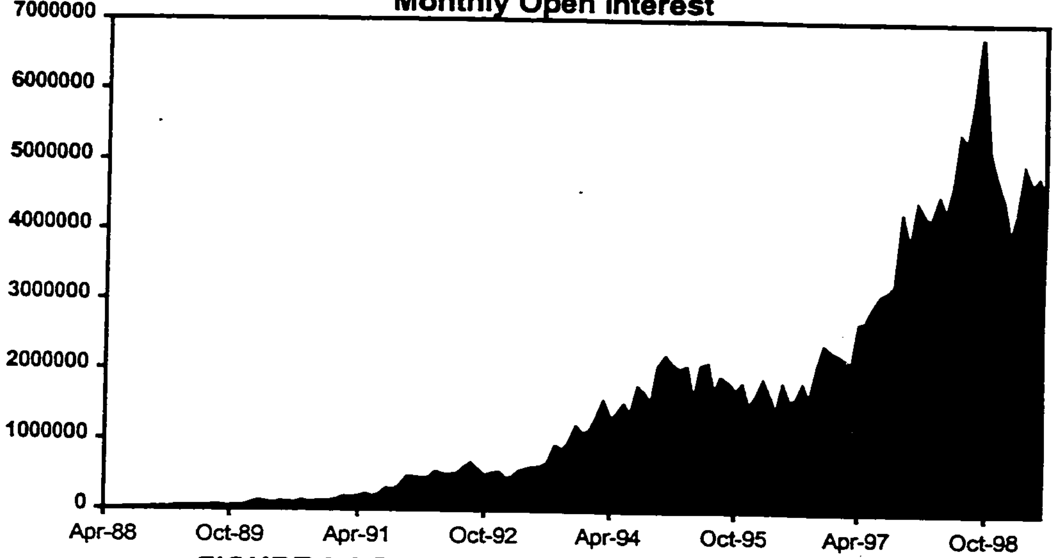


FIGURE 2.3:Bankers' Acceptance Futures [BAX]
Monthly Number of Transactions

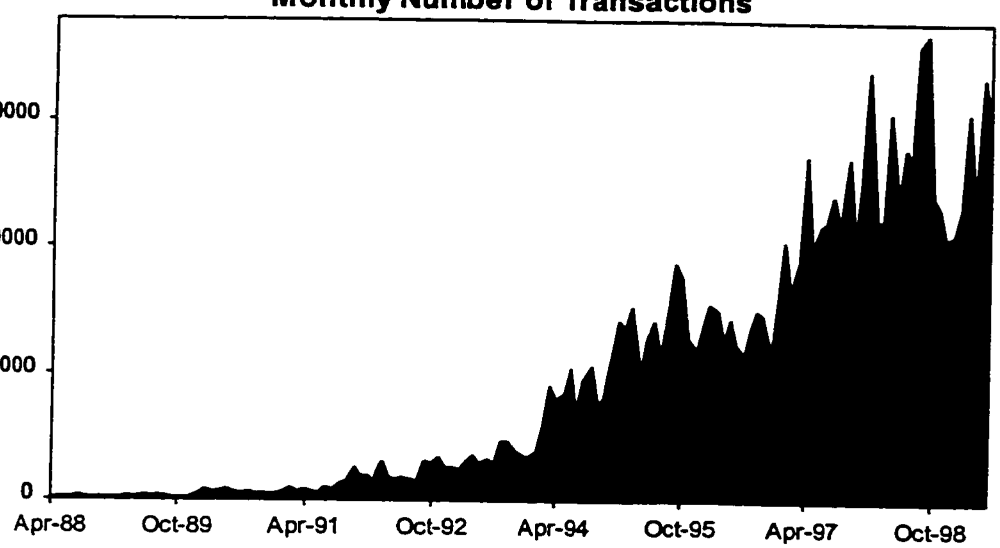


Figure 2.4: Bankers' Acceptance [BA], Bankers' Acceptance Futures [BAX] and Forward Rate Agreement Total Outstanding Numbers

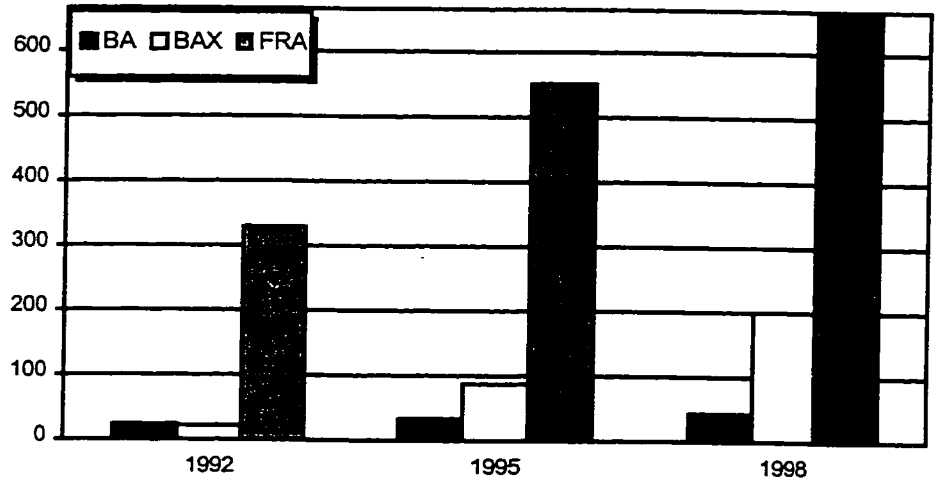


Figure 2.5: Bankers' Acceptance Futures [BAX] Bid-Ask Spread

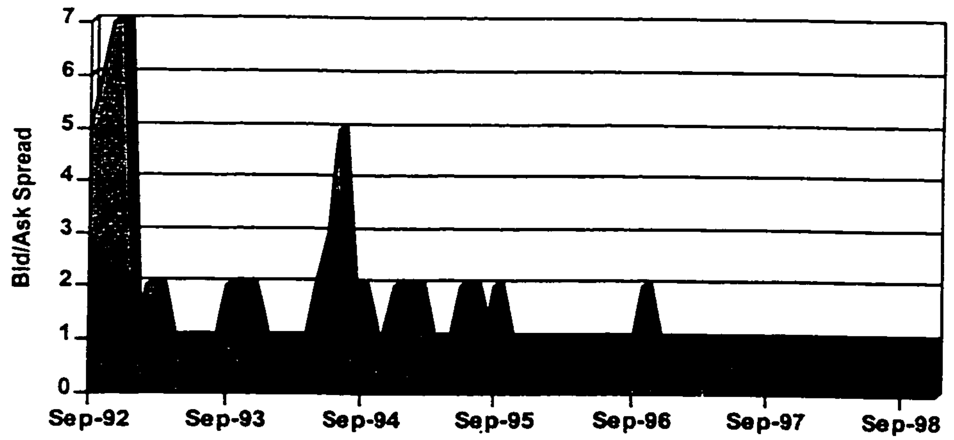


FIGURE 2.6: Bankers' Acceptance Futures [BAX]
Daily Volume

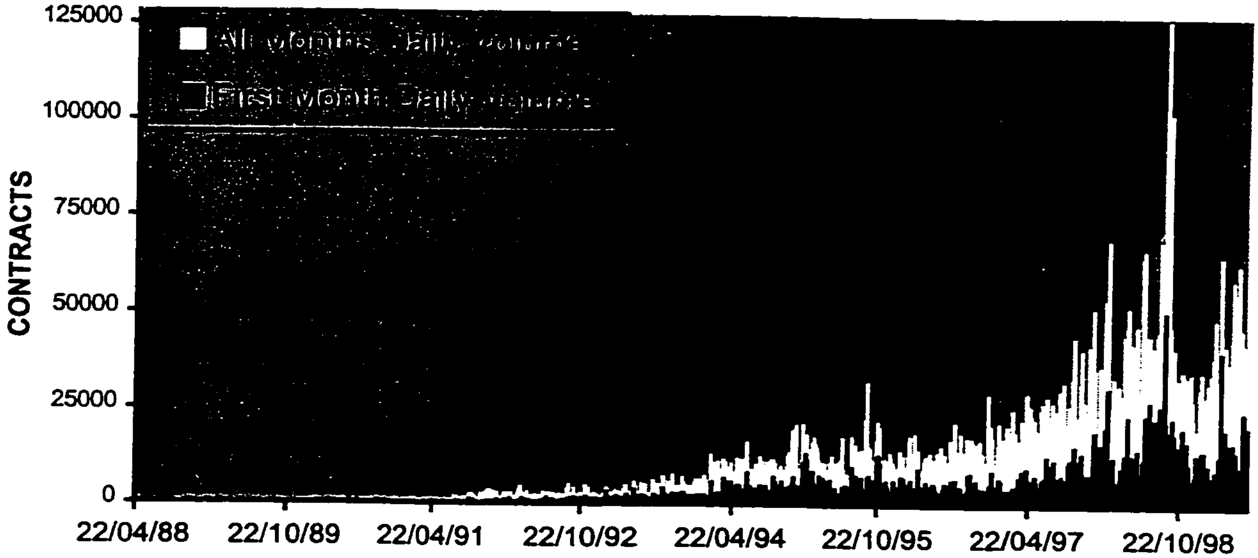


FIGURE 2.7:Bankers' Acceptance Futures [BAX]
Daily Number of Transaction

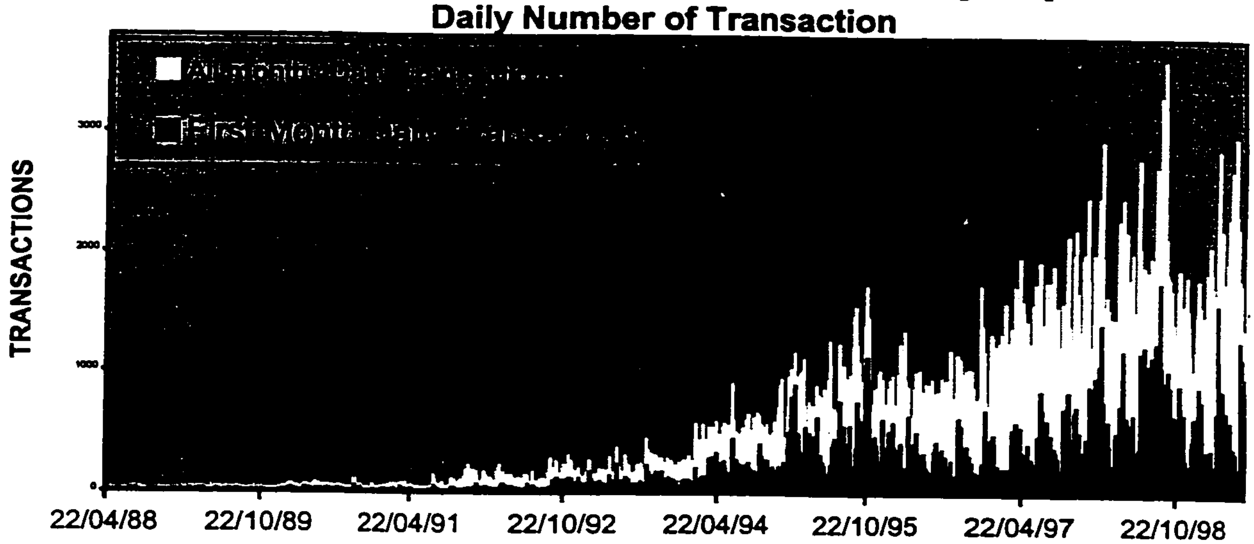


Figure 2.8: Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX] Daily Log Prices

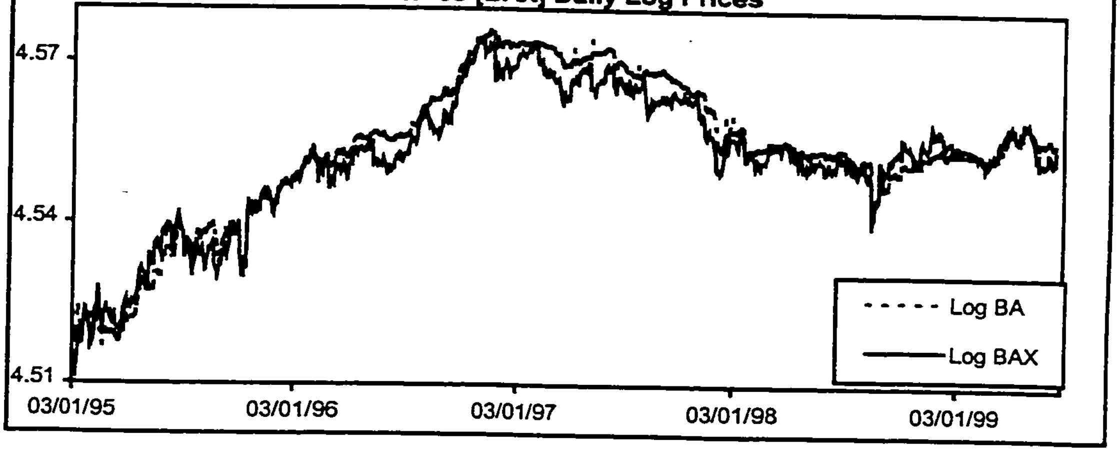


Figure 2.9: Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX] Daily Basis

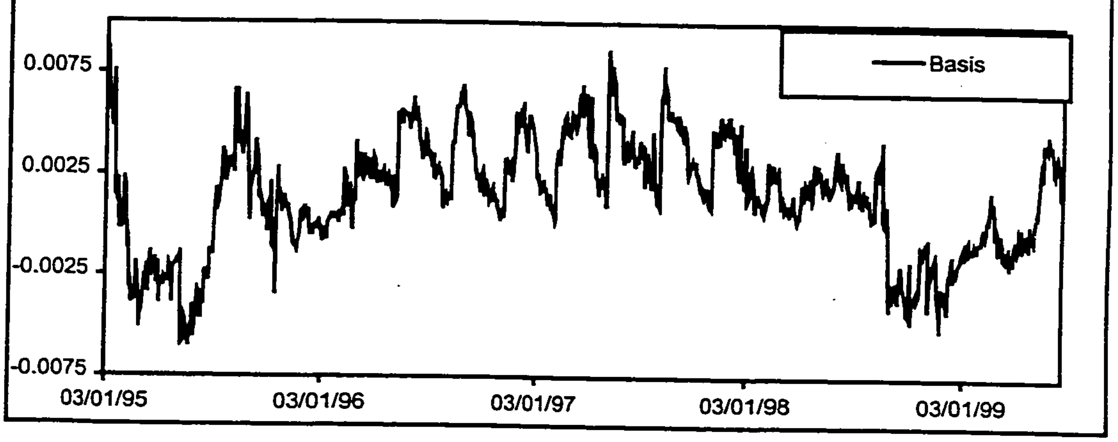
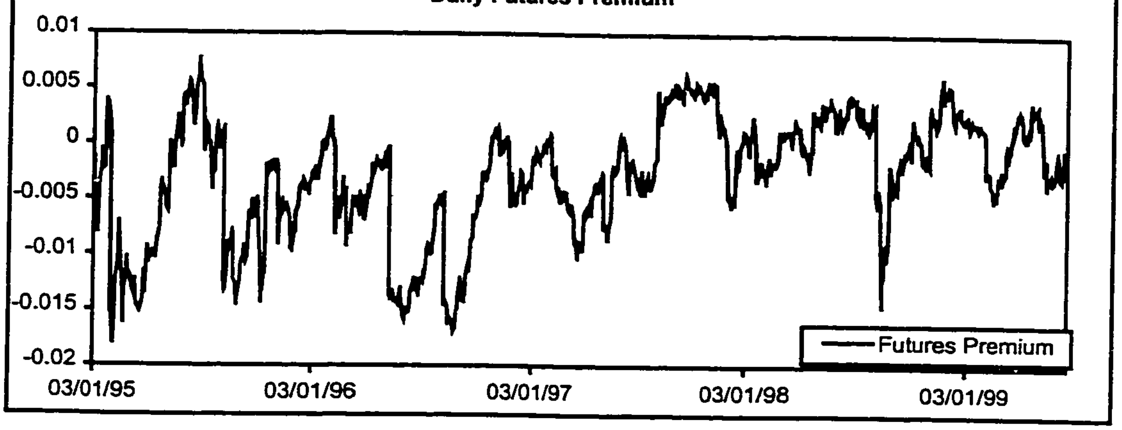


Figure 2.10: Bankers' Acceptance Futures [BAX] and Realized Spot Daily Futures Premium



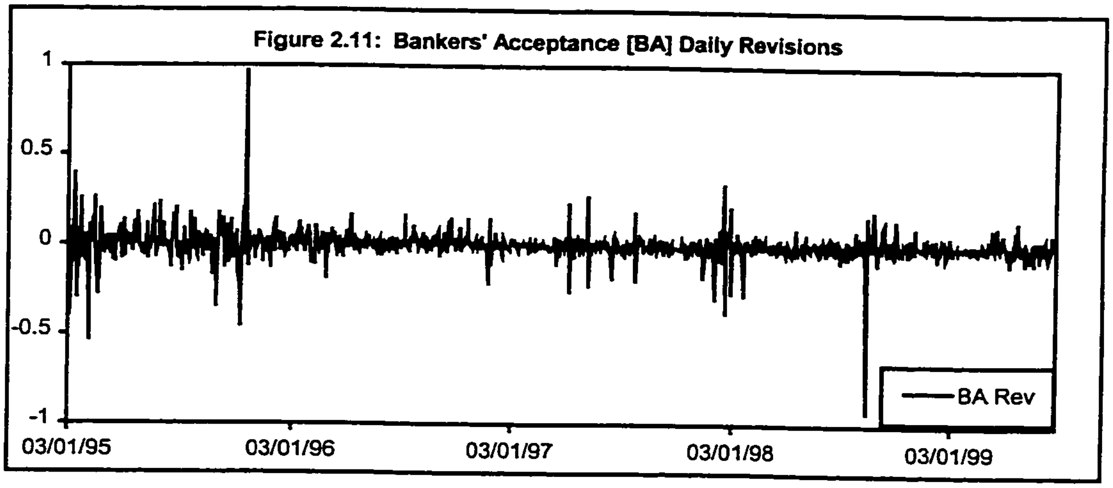


Figure 2.11: Bankers' Acceptance [BA] Daily Revisions
1
0.5
0
-0.5
-1
03/01/95
03/01/96
03/01/97
03/01/98
03/01/99
BA Rev

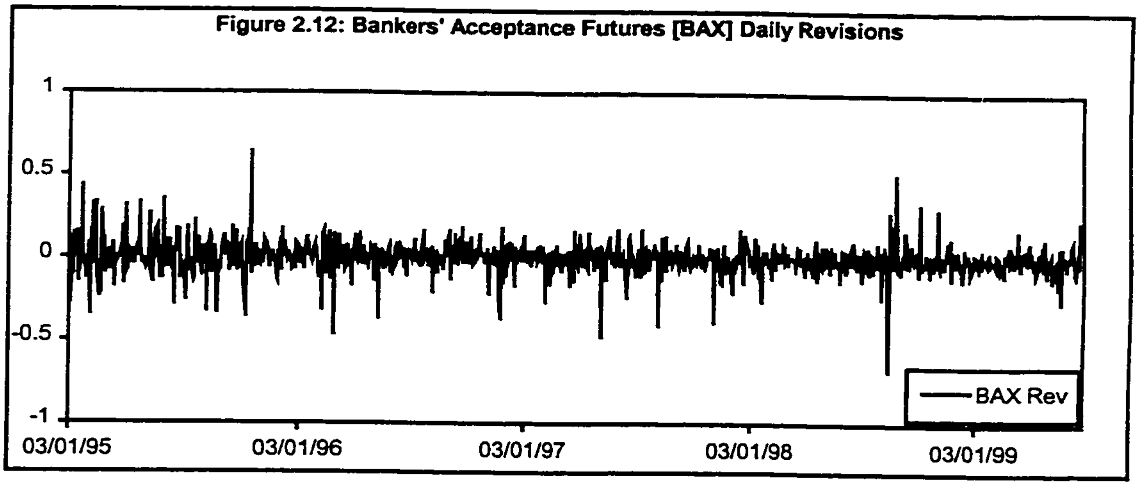


Figure 2.12: Bankers' Acceptance Futures [BAX] Daily Revisions
1
0.5
0
-0.5
-1
03/01/95
03/01/96
03/01/97
03/01/98
03/01/99
BAX Rev

Table 2.1

Growth of Financial Futures

	BAR Volume Open Interest	BAX Volume Open Interest	CGF Volume Open Interest	CGB Volume Open Interest	Total
1998	0	6,805,304 171,354	45,113 1,479	1,836,979 42,626	8,690,396 215,459
1997	0	4,1329,777 186,535	50,944 3,576	1,272,970 36,285	5,463,691 226,396
1996	314 15	2,415,563 99,564	35,649 2,799	1,072,111 19,784	3,523,637 122,162
1995	7,225 0	2,326,709 67,255	63,842 2,171	1,026,854 15,368	3,424,630 84,794
1994	12,172 1,718	1,918,976 83,837	N/A N/A	1,496,543 20,740	3,427,691 106,330
1993	24,552 1,312	724,158 49,882	N/A N/A	895,047 15,789	1,643,757 66,983
1992	23,502 596	419,765 12,749	N/A N/A	515,732 3,673	958,999 26,018
1991	N/A N/A	194,071 21,874	N/A N/A	421,493 3,713	615,564 25,587

The Table exhibits the monthly trading volume (first line) and open interest (second line) of the contracts that are listed on the Montreal Exchange. BAR- One-month Canadian Banker's Acceptance Futures; BAX- Three-month Canadian Bankers' Acceptance Futures; CGF- Five-year Government of Canada Bond Futures; CGB-Ten-year Government of Canada Bond Futures. Source: The Montreal Exchange.

Table 2.2

Comparison of International Futures Contracts

	BAX	ED	STERLING	PIB
Unit of Trading	$1,000,000 CDN	$1,000,000 US	500,000 sterling	5,000,000 FRF
Settlement	Cash	Cash	Cash	Cash
Underlying	CDN Bas	ED Deposits	SD	PIBOR
Quotation	100 minus rate of interest	100 minus rate of interest	100 minus rate of interest	100 minus rate of interest
Min price Fluctuation	0.01 $25 CDN	0.005 $12.5 US	0.01 12.5 Sterling	0.01 125 FRF
Delivery Months	Mar., Jan. Sept., Dec.	Mar., Jan. Sept., Dec.	Mar., Jan. Sept., Dec.	Mar., Jan. Sept., Dec.

The Table lists some of the most popular international short-term interest rates futures, Canadian Bankers' Acceptance futures [BAX] contract, the British three-month sterling deposits [SD], and the French Paris inter-bank offer rate on three-month deposits [PIBOR]. Source: The Montreal Exchange.

Table 2.3

International Money Market Futures Contracts
Trading Volumes: 1995

	Underlying Instrument	Percent of GDP
Canada	BANKERS' ACCEPTANCE	2.60
United States	Treasury Bills and European Deposits (90 Days)	15.40
France	PIBOR	8.99
United Kingdom	Short-tem Sterling	12.05

Source: The Bank of Canada Review, Autumn 1996.

Table 2.4
Unit Root Tests

	s	f	Basis	Prem	Δs	Δf
DF rho test						
Lag-5	-2.97	-5.51	-36.90*	-53.47*	-4046*	-3161*
Lag-10	-2.43	-5.57	-26.79*	-40.95*	-2907*	-2581*
Lag-20	-2.76	-5.07	-30.10*	-73.77*	-1900*	-4183*
Lag-30	-3.44	-4.53	-41.37	-81.03*	-657*	-372*
DF t-test						
Lag-5	-1.24	-1.9	-4.79*	-4.98*	-16.43*	-15.54*
Lag-10	-1.39	-2.21	-3.65*	-4.17*	-12.81*	-11.84*
Lag-20	-1.74	-2.18	-3.47*	-4.73*	-8.34*	-7.95*
Lag-30	-1.75	-1.86	-3.66*	-4.39*	-6.33*	-6.82*
PP rho test						
Lag-5	-3.02	-5.99	-41.91*	-50.99*	-1035*	-961*
Lag-10	-2.47	-5.24	-42.44*	-47.44*	-936*	-872*
Lag-20	-2.2	-4.78	-50.80*	-48.15*	-882*	-814*
Lag-30	-2.24	-4.55	-57.09*	-49.55*	-882*	-783*
PP t-test						
Lag-5	-1.2	-1.43	-5.19*	-5.14*	-31.78*	-30.32*
Lag-10	-1.09	-1.84	-5.21	-4.97*	-31.88*	-30.34*
Lag-20	-1.02	-1.78	-5.57*	-5.00*	-32.12*	-30.55*
Lag-30	-1.03	-1.75	-5.84	-5.07*	-32.12*	-30.76*

A trend term is included is the regression estimating the autoregressive coefficient. The variables are defined in the text. The 5% critical value for the rho test is -21.8; the 5% critical value for the t-test is -3.41. Starred entries are significant at the 5% level.

Table 2.5
Efficiency Tests

Estimation of Premium s.e. White s.e.	0.26 (0.015) (0.015)
Efficiency with regard to BA Revisions	
Current Period F-test [p-value] Wald test [p-value]	 0.058 0.079
With 9 lags F-test [p-value] Wald test [p-value]	 0.002 0.002
Efficiency with regard to BAX Revisions	
Current Period F-test [p-value] Wald test [p-value]	 0.087 0.122
With 9 lags F-test [p-value] Wald test [p-value]	 0.042 0.002
Efficiency with regard to Basis	
Current Period F-test [p-value] Wald test [p-value]	 0.000 0.000

The premium is the estimate of the mean of $Y_t = (SPOT_t - BAX_t)$. The efficiency tests report a test that the slope coefficients are all 0 in a regression of Y_t on a constant and the variable, or on a constant with the variable along with 9 lags. The OLS procedure reports a standard F-statistic. The second procedure builds the usual Wald test based on the robustified covariance matrix; here the test statistic is χ^2 with degrees of freedom equal to the number of explanatory variables. P-values are given in parentheses.

Chapter Three Modelling Canadian Bankers' Acceptance Futures

Traditional statistical analysis of the distributional properties of financial series was originally challenged in the important papers of Mandelbrot (1963) and Fama (1965) where it was shown that (the first differences of the logs of) common stock prices have fatter tails than those associated with the normal distribution. Mandelbrot also observed: " large changes tend to be followed by large changes-of either sign- and small changes tend to be followed by small changes ..." Considerable research effort has been exercised in attempting to design models that accommodate these empirical regularities that hold as well for the BAX series. Bankers' Acceptance is the rate that Canadian banks use to lend to each other. The BA rate is comparable to the London Inter-Bank offer rate (LIBOR), and the BAX is the futures contract on Canadian Interbank Offer Rate (CIBOR). It was at the suggestion of the financial institutions (when surveyed by the Montreal Exchange) to use Bankers' Acceptance as the underlying instrument for the proposed BAX contract; for further information on the evolution of the BAX contract, see Chapter Two. In this chapter, a model is sought to describe the evolution of the BA and BAX series.

This chapter works in the tradition of searching for a model that captures the distributional properties of the BA and BAX series. Although the focus is ultimately on the BAX series and the determination of its volatility dynamics, the BA series is modelled as well for the following reasons: first, it is the underlying series of the BAX contract; second, the BA is needed in Chapter 4 as part of the hedge ratio; and third, for comparison purposes. A univariate specification for the two series is sought within the ARCH class of models which have been advanced to capture the dynamics of the serial dependence described by Mandelbrot; for a survey of these models see Bollerslev, Chou and Kroner (1992) and, more recently, Gouriéroux (1997). The chapter is organized as follows. Section 3.2 describes the basic features of the data which support the search for an ARCH representation and presents the general framework of the ARCH methodology for representing the volatility dynamics of the BA and BAX series. Several methods of discriminating between various ARCH models, using the nested GARCH approach which originated with Hentschel (1995), are also surveyed in this section. Section 3.3 presents the estimation results for a variety of GARCH specifications, while Section 3.4 analyses the results of different likelihood ratio tests which serve to distinguish among families of GARCH specifications. Section 3.5 assesses the predictive power of the models by testing the within-sample performance and out-of-sample performance of the estimated models in predicting volatility. Section 3.6 concludes.

3.1 General ARCH Framework for Analysing Volatility

3.1.1 Descriptive Statistics

The data considered in this chapter involve the BA daily prices (Bankers' Acceptance) and the settlement prices the BAX (Bankers' Acceptance Futures) contract. Both are provided by the Montreal Exchange. The data consists of 3-month daily BA prices and the daily settlement of BAX prices, each taken at 3:00 p.m. The time span under consideration begins with January 3rd 1995 and ends on June 30th 1999, representing a total of 1,133 daily observations for both series. The first 1,069 observations ending March 31, 1999 are used to model the series. The remaining 64 observations (for the months April 1999, May 1999, and June 1999) are used to test out-of-sample performance of the models; the 3-month duration represents the life span of one BAX contract. The reasons given for selecting this period concern the liquidity and depth of the market, as described in Chapter 2 where it is argued that the last four and a half years better represents the evolution of the BA and BAX markets than do earlier periods, when the market for the BAX was being established. For a more complete analysis of this view, see Chapter 2.

Figures 3.1 and 3.2 present the log of the BA series first-differenced and the log of the BAX first-differenced. The data under consideration are mean non-stationary. However, the first differences of the series are stationary (see Chapter 2). As with most financial time series data, it is reasonable to base inference on the change in the logarithm of price. A quick look at the graphs confirms Mandelbrot's general observation that there are clusters of high and low volatility. The data also appear non-normal (there are many outliers) and asymmetric. However, the BA series appears to be more problematic than in the BAX series. The range of the changes are more extreme in the BA case than the BAX case, with a lot more outliers. However, the BAX series seems to be more asymmetric than the BA. A simple examination of the two graphs shows that there are considerably more small changes in the BAX market than in the BA market. One explanation is that the BAX market is a more liquid and has greater depth. The BAX market reflects the fast influx of information faster and quicker, whereas the BA market's response to the influx of information may be delayed, producing sharper changes than those in the BAX market. These observations are now pursued more formally.

Table 3.1 presents a variety of descriptive statistics with the results of some tests. This preliminary analysis confirms the initial observations made from examining Figures 3.1 and 3.2. A formal test for skewness rejects the null of symmetry with low p-values. As well, the data are fat-tailed, as is confirmed by the test for zero excess kurtosis. The Bera-Jarque test for normality, which combines both the skewness and kurtosis tests is highly significant, indicating that normality is rejected; this is consistent with the two pervious results.

A straightforward way to test for ARCH effects is to regress a series containing the squared difference of the elements of the BA and BAX series on a constant and k lags of the series. The Langrange multiplier test is based on the statistic defined as TR^2, where T is the number of observations and R^2 is the square of the multiple correlation coefficient. This statistic is distributed $\chi^2(k)$. A lag length of 10 was used . The statistic is quite large, indicating that past values of the square of the BAX series are useful in predicting current volatility; in short, there is clustering.

The Table also confirms that the BA series is more problematic than the BAX series. There is a greater departure from normality for BA first-differenced series than the BAX's. Over five times the kurtosis is present in the BA series than in the BAX: there is greater skewness and these are more pronounced ARCH effects. We should not forget that the BA is far less liquid than the BAX (the volume is less than 20% of the BAX market) and the BA market is "managed" by Canadian banks; for instance, one can not "short" the BAs. The only section of the market that can take advantage of potential "arbitrage conditions" arising from a "reverse Repo" (selling of BAs and buying BAXs) are the Canadian banks, since they are the only ones that keep an inventory of BAs. Therefore, we expect the BA series to be a more "difficult" series to model than the BAX.

3.1.2 The GARCH-M Modelling Framework

The purpose of this section is to study the evolution of the conditional variance of the BAX series within the GARCH framework. One problem that arises with different models that fall within this framework within that they do not display obvious links to one another. It is difficult, as a consequence, to discriminate between them. Hentschel (1995) provides a unifying framework which such models can be viewed and tested. The proposed framework nests all of the popular GARCH models plus a host of other new specifications. In this section, we will briefly review the ARCH-M framework and then introduce the nested approach developed by Hentschel.

In what follows Y_t is taken to represent the first differences of the log of the BA or BAX price at time t (multiplied by 1000). We begin with a simple univariate representation of Y_t described by the following three equations:

$$Y_t = \mu_t + u_t \ , \tag{3.1}$$

$$\mu_t = \gamma + \beta_1 \sigma_t^2 \ , \tag{3.2}$$

$$u_t = \sigma_t \, \varepsilon_t \ , \quad u_t | I_t \sim N(0,\sigma_t^2) \ . \tag{3.3}$$

where σ_t^2 is the time-varying variance described below. One of the most common specifications for the mean equation (3.1) used in empirical studies is the GARCH-in-mean model of Engle, Lilien, and Robins (1987). According to finance theory, assets that provide higher returns may be associated with higher risk. This suggests that asset prices must reflect a time-varying risk premium related to their variances (σ_t^2). This specification has been widely used [e.g. Bollerslev, 1987; French, Schwert, and Stambaugh, 1987; Nelson 1991; Sentana 1991] to model time-varying risk premium and the behaviour of stock return variances. The specification expressed in equation (3.2) has been used to reflect the impact of higher perceived variability of u_t on the level of Y_t, via the parameter β_1.

Just over fifteen years ago, the focus of most financial time series analysis was centred on the conditional first moments with any temporal dependencies in higher moments treated as a nuisance. The increased importance played by risk and uncertainty considerations has necessitated the development of new econometric time series techniques that allow for modelling of time-varying variances and covariances. The popularity of the Generalized Autoregressive Conditional Heteroskedasticity model [GARCH} was due to the model's ability to capture volatility persistence in a simple and flexible way. Since then GARCH models have been used extensively in macroeconomics and finance literature. However, these models do not arise directly from economic theory. Their primary motivation is that they provide a flexible and parsimonious approximation of the conditional variance dynamics in exactly the same way the ARMA models provide a flexible and parsimonious approximation to the conditional mean dynamics.

The ARCH process introduced by Engle (1982) explicitly recognizes the difference between the unconditional and the conditional variance, allowing the latter to change over time as a function of past errors. The GARCH(p, q) process under normal disturbances is governed by:

$$u_t | I_{t-1} \sim N(0, \sigma_t^2) \quad , \tag{3.4}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \delta_i \sigma_{t-i}^2 \quad , \tag{3.5}$$

where I_t is the information available at time t. The simplest and often the most useful is the GARCH (1, 1) specification that treats the evolution of the conditional variance as

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \delta \sigma_{t-1}^2 \quad , \tag{3.6}$$

where it is generally assumed that ω, α, and δ are positive constants. The δ parameter is the key persistence parameter: a high δ implies a high carryover effect of past to future volatility, while a low δ implies a damped dependence of past volatility. All of the parameters can be estimated directly from the data.

In the traditional GARCH model (3.6), the variance equation is symmetric with respect to past disturbances. A variety of modifications to this framework has been proposed to allow for the empirical regularity that negative returns are followed by larger increases in volatility than numerically equal positive returns. This phenomenon, commonly referred to as the "leverage effect", was first identified by Black (1976). The second empirical finding is that stock returns are fat-tailed. This leptokurtosis is reduced when returns are normalized by the time-varying variances of GARCH models, but it is by no means eliminated. An important modification to the GARCH framework is the Nelson (1991) Exponential GARCH model which incorporates an asymmetry parameter to account for the "leverage effect". In addition to Nelson, other researchers have modified the general framework of GARCH to include parameters that account for asymmetry resulting from the "leverage effect" and the fact that returns are fat-tailed. These include the Zakoian (1991) Threshold GARCH model; Higgins and Bera (1992) Nonlinear ARCH; Glosten, Jagannathan, Runkle (1993) GJR-GARCH,; Engle and Ng (1993) Nonlinear-asymmetric GARCH, Ding, Granger, Engle (1993) Asymmetric Power ARCH, among others.

The above GARCH models effectively impose restrictions on possible transformations of the conditional variance and on possible transformations of lagged squared errors. Exceptions to this approach are the Nonlinear Model proposed by Higgins and Bera (1992) and the Asymmetric Power Model proposed by Ding, Granger, and Engle (1993). In the Asymmetric Power Model, for example, the transformation of the GARCH variance is freely estimated; however, the transformation of the squared errors is restricted to have the same form (i.e. $\lambda = \nu$ discussed below).

We now describe the main lines of a general family of models introduced by Hentschel (1995) which imposes no restrictions on the transformation of the conditional variance, nor does it restrict the evaluation of the lagged squared errors. Following Hentschel, this model will be termed the FREE GARCH class of models. The evolution of the nested GARCH variance is governed by the following equations:

$$\frac{\sigma_t^{\lambda}-1}{\lambda} = \omega + \alpha\sigma_{t-1}^{\lambda} f^{\nu}(\varepsilon_{t-1}) + \delta\frac{\sigma_{t-1}^{\lambda}-1}{\lambda} , \tag{3.7}$$

$$f(\varepsilon_t) = |\varepsilon_t - S| - R(\varepsilon_t - S) . \tag{3.8}$$

The above framework incorporates two departures from the standard GARCH model. First, Equation (3.8) generalises the GARCH model to express potential asymmetry. As pointed out earlier, previous research has found that returns are strongly asymmetric: negative returns are followed by larger increases in volatility numerically equal positive returns (see Figures 3.1 and 3.2). The standard GARCH model developed by Bollerslev (1986) cannot reflect such asymmetry, while the previous examples of asymmetric GARCH models have largely concentrated on one feature of asymmetry while ignoring others.

Hentschel uses the "news impact curve" introduced by Pagan and Schwert (1990) to illustrate the role played by the parameters R and S in equation (3.8) in determining volatility. Figure 3.3 examines the relative impact of positive and negative disturbances on $f(\varepsilon)$ for different values of R and S. Figure3.3a shows the impact news curve with the asymmetry parameters set to 0. Here disturbances are symmetric around 0. A non-zero value of S, with $R = 0$, effectively moves the point of symmetry away from the origin as in Figure 3.3b. A non-zero value of R, with $S = 0$, rotates the impact about the origin. There is asymmetry in either scenario. But the shift model describes an environment where no news is bad news, and the impact of small negative shocks is much greater than small positive shocks; here the impact of large negative shocks is similar to the impact of large positive shocks. The rotation model captures this difference in impact involving large shocks more effectively.

The type of asymmetry that rotates the news impact curve is caused by the "leverage effect" discussed above. The "leverage effect" is the result of impact of large shocks on the system. By the introduction of the rotation parameter R in the variance equation, one can measure the asymmetric variance responses that caused a rotation by allowing slopes of different magnitude on either side from the origin. The impact of different types of shocks that produce asymmetric responses can be viewed and measured. A positive value of R corresponds to a clockwise rotation of the news impact curve, as shown in Figure (3.3c); a negative value for R corresponds to a anti-clockwise rotation of the news impact curve, as shown in Figure (3.3d) which also includes a positive shift. Models that incorporate a rotation parameter such as R include the Nelson (1991) E-GARCH model and those considered by Glosten, Jagannathan, and Runkle (1993). A number of models have incorporated an analogue to the rotation parameter in the variance equation presented in a GARCH setting. Examples include the Quadratic GARCH Model of Sentana (1991) and Engle (1990), and the Nonlinear-Asymmetric ARCH Model of Engle and Ng (1993).

In equation (3.7) the parameter ν serves to transforms the potentially shifted and/or rotated disturbances that appear in the function $f(\varepsilon_t)$. Figure 3.4 exhibits the transformation of functions of ε_t for different values of ν. When $\nu = 1$ the transformation is linear; when $\nu > 1$, a convex function of ε_t is driving the variance equation, and for values $0 < \nu < 1$ the transformation is concave on either side of the point of symmetry.

The second departure from the standard GARCH involves the introduction of a Box-Cox transformation of the variances as given in equation (3.7). When $\lambda = 2$, the process is driven by the conditional variance (standard GARCH); when $\lambda = 1$, the driving force is of the conditional standard deviation (Absolute Value GARCH); and when the parameter $\lambda = 0$, the transformation is driven by the log of the conditional variance (Exponential GARCH). Therefore, λ determines the shape of the transformation. For $\lambda > 1$ the transformation of the conditional standard deviation is convex, while for $\lambda < 1$ the transformation is concave.

In the next section, we will see that the Hentschel (1995) framework nests all of the popular GARCH within the specification of what he terms the Free-GARCH model. More precisely, these models are special cases of Equations (3.7) and (3.8) that can be obtained by choosing the parameters λ, v, S, and R appropriately. For example, if we set $\lambda = v = 2$, and R=S=0 we get the standard GARCH class of models.

3.1.3 The Nested GARCH Models

This section will identify all the popular GARCH models that are nested within the FREE GARCH framework. These special cases of the FREE GARCH specification are obtained by appropriately choosing the parameters λ, v, S, and R, and will carry the designation of the original authors. However, as discussed in the previous section, the FREE GARCH framework permits several variations on these models. These variation will be identified by their special features; for example, we can estimate Bollerslev (1986) Standard GARCH with both a shift and rotation parameters to capture asymmetry. This model will be identified as Asymmetric GARCH (AS-GARCH). This section will follow the Hentschel specifications very closely. Hentschel identified twelve models. In this chapter, we will explore the full range of the FREE GARCH framework in presenting twenty models per distribution per series. As well, we will estimate these models under different distributions: normal, t-distribution and GED distributions (Hentschel estimated his models under the assumption of normality). Although the focus is the BAX series, we will also attempt to estimate the BA series for completeness and as a basis for comparison. A total of one hundred and twenty different models for both the BAX and BA series will be estimated. The remainder of this section will outline the models to be estimated in Section 3.2.

(1) The most commonly used GARCH model is the Standard GARCH(1,1) where the underlying disturbances are independent standard normals. The Bollerslev (1986) Standard-GARCH model is obtained by restricting the parameters of the FREE GARCH specification with $\lambda = \nu = 2$, $S = R = 0$, and restricting $f(\varepsilon_t)$ to be the simple absolute value $|\varepsilon_t|$. The variance equation then reduces to:

$$\sigma_t^2 = \omega + \delta_1 \sigma_{t-1}^2 + 2\alpha_1 \sigma_{t-1}^2 (\varepsilon_{t-1})^2 \quad . \tag{3.9}$$

(2) The Nonlinear-Asymmetric-GARCH of Engle and Ng (1993) introduces a new asymmetry parameter similar to the shift parameter in the FREE GARCH specification. This additional parameter S is introduced in the variance equation to capture asymmetry caused by small shocks to the system Accordingly, the Nonlinear-Asymmetric-GARCH is obtained when we restrict the general equation parameters with $\lambda = \nu = 2$, $R = 0$, and S is freely estimated, as in (3.10). Engle and Ng found this asymmetry parameter to be significant using (CRSP) daily data on stock index returns.

$$\sigma_t^2 = \omega + \delta_1 \sigma_{t-1}^2 + 2\alpha_1 \sigma_{t-1}^2 (|\varepsilon_{t-1} - S|)^2 \quad . \tag{3.10}$$

(3) The GJR-GARCH(1,1). Some researchers have concluded that negative surprises seem to increase volatility more than positive disturbances, attributable perhaps to a leverage effect in the equity market. A model developed by Glosten, Jagannathan and Runkle (1993) works within the GARCH framework to include a leverage effect as follows:

$$\sigma_t^2 = \omega + \delta_1 \sigma_{t-1}^2 + 2\alpha_1 \sigma_{t-1}^2 [|\varepsilon_{t-1}| - R\varepsilon_{t-1}]^2 \ . \tag{3.11}$$

The FREE-GARCH model reduces to GJR GARCH when we impose the restrictions that $\lambda = \nu = 2$, $S = 0$, and R is estimated freely to capture the effect of large shocks to the system. Glosten, Jagannathan and Runkle, using monthly excess returns of (CRSP) value-weighted stock index portfolio from 1954 to 1989, found that the standard GARCH-M model to be misspecified. However, when the model is modified to allow positive and negative unanticipated returns to have different impacts on the conditional variance, they find a negative significant relation between volatility and expected returns.

(4) The above models differ only in the way they treat the shocks in the variance equation. An additional GARCH(1,1) model, the Asymmetry-GARCH (AS-GARCH) will be estimated with both S and R parameters included in the variance

equation.

$$\sigma_t^2 = \omega + \delta_1\sigma_{t-1}^2 + 2\alpha_1\sigma_{t-1}^2(|\varepsilon_{t-1}-S|-R|\varepsilon_{t-1}-S|)^2 . \tag{3.12}$$

(5) A new class of models is estimated by setting $\lambda = \nu = 1$. This class of models treats the conditional standard deviation as a linear function of shocks and lagged standard deviations. We consider first Symmetric Absolute Value GARCH (SA-GARCH, obtained by setting $S = R = 0$.

$$\sigma_t = \omega + \delta_1\sigma_{t-1} + \alpha_1\sigma_{t-1}|\varepsilon_{t-1}| . \tag{3.13}$$

(6) An extension of the above model is to introduce a parameter that permits shift of the news impact curve; this model is called Asymmetric-Nonlinear-GARCH (AN-GARCH).

$$\sigma_t = \omega + \delta_1\sigma_{t-1} + \alpha_1\sigma_{t-1}|\varepsilon_{t-1} - S| . \tag{3.14}$$

(7) A model that permits rotation of the news impact curve and treats the conditional standard deviation as a linear function of shocks and lagged standard deviations was introduced by Zakoian (1991) and named Threshold GARCH. The FREE GARCH framework is reduced to Threshold GARCH by setting $\lambda = \nu = 1$, $S = 0$, and estimating R freely.

$$\sigma_t = \omega + \delta_1\sigma_{t-1} + \alpha_1\sigma_{t-1}[|\varepsilon_{t-1}| - R\varepsilon_{t-1}] \quad . \tag{3.15}$$

(8) A model that permits both a shift and rotation of the news impact curve is the Sentana (1991) Q-GARCH Model. Engle and Ng (1993) named a variance equation that specifies the conditional variance as a function of a shifted parabola the Absolute-Value-GARCH (A-GARCH). The A-GARCH model incorporates both of the previous approaches (shift and rotation) to permit asymmetry in the variance equation.

$$\sigma_t = \omega + \delta_1\sigma_{t-1} + \alpha_1\sigma_{t-1}[(|\varepsilon_{t-1}| - S) - R(\varepsilon_{t-1} - S)] \quad . \tag{3.16}$$

(9) A different class of Nonlinear-GARCH models can be estimated with FREE-GARCH model by setting $\lambda = \nu$. Higgins and Bera (1992) introduced the ARCH version of this model. A GARCH extension can be obtained from the FREE GARCH specification by setting $\lambda = \nu$, and S=R=0.

$$\sigma_t^\lambda = \omega + \delta_1\sigma_{t-1}^\lambda + \alpha_1\lambda\sigma_{t-1}^\lambda|\varepsilon_{t-1}|^\lambda \quad . \tag{3.17}$$

Higgins and Bera (1992) introduced a new class of nonlinear ARCH models that encompasses several functional forms for ARCH based on the Box-Cox transformation of the variance. N-ARCH was used to model several weekly exchange rates for the period 1973 to 1985; the model demonstrated superior performance over the standard linear ARCH model.

(10) A non-linear GARCH model that permits a shift in the news curve with the above specification can be obtained by estimating S freely, called Nonlinear-Power-GARCH (NP-GARCH).

$$\sigma_t^{\lambda} = \omega + \delta_1 \sigma_{t-1}^{\lambda} + \alpha_1 \lambda \sigma_{t-1}^{\lambda} |\varepsilon_{t-1} - S|^{\lambda} \quad . \tag{3.18}$$

(11) By setting S = 0, $\lambda = \nu$, and estimating R freely we get the GARCH version of an ARCH model developed earlier by Ding, Granger, and Engle (1993), termed Asymmetric Power ARCH. The Asymmetric-Power-GARCH (AP-GARCH) specification includes a rotation parameter.

$$\sigma_t^{\lambda} = \omega + \delta_1 \sigma_{t-1}^{\lambda} + \alpha_1 \lambda \sigma_{t-1}^{\lambda} [|\varepsilon_{t-1}| - R\varepsilon_{t-1}]^{\lambda} \quad . \tag{3.19}$$

(12) A specification similar to the above model that includes both shift and rotation of the news curve is identified as Full Power GARCH (FP-GARCH).

$$\sigma_t^{\lambda} = \omega + \delta_1 \sigma_{t-1}^{\lambda} + \alpha_1 \lambda \sigma_{t-1}^{\lambda} [|\varepsilon_{t-1} - S| - R(\varepsilon_{t-1} - S)]^{\lambda} \quad . \tag{3.20}$$

(13) The Free-GARCH class of models is determined by estimating λ, and ν freely. We first set the S and R parameters to zero to get the Symmetric-Free-GARCH (SF-

GARCH).

$$\frac{\sigma_t^{\lambda}-1}{\lambda} = \omega + \alpha\sigma_{t-1}^{\lambda}[|\varepsilon_t|]^{\nu} + \beta\frac{\sigma_{t-1}^{\lambda}-1}{\lambda} \quad . \tag{3.21}$$

(14) As in previous models, we now add a shift parameter to the variance equation and obtain Asymmetric-Free-GARCH (AF-GARCH).

$$\frac{\sigma_t^{\lambda}-1}{\lambda} = \omega + \alpha\sigma_{t-1}^{\lambda}[|\varepsilon_t - S|]^{\nu} + \beta\frac{\sigma_{t-1}^{\lambda}-1}{\lambda} \quad . \tag{3.22}$$

(15) A model that estimates λ, and ν freely, but includes a rotation parameter is the Threshold-Free-GARCH (TF-GARCH).

$$\frac{\sigma_t^{\lambda}-1}{\lambda} = \omega + \alpha\sigma_{t-1}^{\lambda}[|\varepsilon_t| - R(\varepsilon_t)]^{\nu} + \beta\frac{\sigma_{t-1}^{\lambda}-1}{\lambda} \quad . \tag{3.23}$$

(16) By lifting the restrictions on S and R and estimating λ, and ν freely, we obtain Free-Absolute-Value-GARCH (Free-GARCH). This is the most general (free from any restrictions) GARCH model estimated within this framework.

$$\frac{\sigma_t^{\lambda}-1}{\lambda} = \omega + \alpha\sigma_{t-1}^{\lambda}[(|\varepsilon_t - S| - R(\varepsilon_t - S)]^{\nu} + \beta\frac{\sigma_{t-1}^{\lambda}-1}{\lambda} \quad . \tag{3.24}$$

Hentschel estimated this equation using daily stock returns (CRSP) that spanned the period January 2, 1926 to December 31, 1990 [17,486 observations]. The shift and rotation parameters were quite significant and tests reject all standard GARCH models in favour of this less restrictive framework.

(17) Another class of models can be estimated by setting $\lambda = 0$, and $\nu = 1$. The Box-Cox transformation converges to the natural logrithm as λ goes to zero:

$$\lim_{\lambda \to 0} \frac{(\sigma_t^{\lambda} - 1)}{\lambda} = \ln \sigma_t^2 \quad .$$

If we also set $\nu = 1$ within the general specification, we obtain the Nelson (1991) Exponential-GARCH model. By setting S and R equal to zero we get the symmetric version of Exponential GARCH (SE-GARCH). When the constant, unconditional mean of $f(\varepsilon_t)$ is subtracted from $f(\varepsilon_t)$ and added to the intercept, the variance equation becomes:

$$\ln \sigma_t^2 = \omega + \delta_1 \ln \sigma_{t-1}^2 + \alpha_1 |\varepsilon_{t-1}| - E|\varepsilon_{t-1}| \quad . \tag{3.25}$$

(18) An exponential GARCH that allows a shift parameter is termed the Asymmetric-Exponential-GARCH (AE-GARCH).

$$\ln \sigma_t^2 = \omega + \delta_1 \ln \sigma_{t-1}^2 + \alpha_1 [|\varepsilon_{t-1} - S| - E|\varepsilon_{t-1} - S|] \quad . \tag{3.26}$$

(19) Nelson (1991) Exponential-GARCH can be obtained from the FREE GARCH model by setting $\lambda = S = 0$, and $\nu = 1$, and estimating R freely.

$$\ln\sigma_t^2 = \omega + \delta_1 \ln\sigma_{t-1}^2 + \alpha_1[|\varepsilon_{t-1}| - E|\varepsilon_{t-1}| - R(\varepsilon_{t-1})] \quad . \tag{3.27}$$

In Nelson (1991), this framework was used to estimate a model using the(CRSP) Value-weighted Market index from 1962 to 1987. Nelson found the asymmetric relation between returns and changes in volatility, as captured by the asymmetry parameter (R in our case), to be highly significant, indicating that volatility tends to rise (fall) when returns surprises are negative (positive).

(20) One can see that Nelson Exponential-GARCH allows only rotation of the news impact curve in response to shocks. Another version of Exponential-GARCH can be estimated allowing for both shifts and rotation of the news impact curve, or Free-Exponential-GARCH (FE-GARCH).

$$\ln\sigma_t^2 = \omega + \delta_1 \ln\sigma_{t-1}^2 + \alpha_1[|\varepsilon_{t-1} - S| - E|\varepsilon_{t-1} - S| - R(\varepsilon_{t-1} - S)] \quad . \tag{3.28}$$

The Free-GARCH family of models nests important examples of symmetric and asymmetric GARCH and provides a unifying framework in which the models can be viewed and tested (see Section 3.3). Table 3.2 summarizes the models described in this section. These models are all nested within the FREE GARCH framework and are obtained by appropriately choosing the parameters λ, v, S, and R as indicated in the Table. The specific parameter values are given in columns two through five. Column six names the model. This identification is either given by the original authors, or by their special features. We turn to the estimation of these models.

3.2 Estimation of the GARCH Models

The models presented in the last section are now estimated for the BAX series (the futures contract on Canadian Interbank Offer Rate, the main subject of this paper) and the BA series (Canadian Interbank Offer Rate, the Bankers' Acceptance). The models have been estimated with three types of distributions; normal, student t- and the general error distribution (GED). A total of 120 models is estimated using numerical maximum likelihood based on the algorithm due to Broyden, Fletcher, Goldfarb and Shanno (BFGS). The simplex method is used to determine initial values for the parameters to be estimated.

The extension of the positivity and stationarity conditions from the usual framework to the nested version was developed by Hentschel (1995). More specifically, the family of GARCH equations in this paper uses equation (3.6), (3.7) and (3.8) to describe the evolution of the variance:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \delta\sigma_{t-1}^2 \ ,$$

$$\frac{\sigma_t^\lambda - 1}{\lambda} = \omega + \alpha\sigma_{t-1}^\lambda f^\nu(\varepsilon_{t-1}) + \delta\frac{\sigma_{t-1}^\lambda - 1}{\lambda} \ ,$$

$$f(\varepsilon_t) = |\varepsilon_t - S| - R(\varepsilon_t - S) \ .$$

The sufficient conditions to insure the positivity of the conditional variance are analogous to the standard GARCH model. First, It should be noted that for $\lambda = 0$ (E-GARCH model) or even integers $2/\lambda$, the conditional variance is found by exponentiation, or raising σ_t^λ to an even power; either of these operations guarantees that the conditional variance is nonnegative. Hence, positivity does not impose any restrictions on these models. Next, for v not an even number, the conditional variance is greater than 0, if

$$\omega > 0 \;, \alpha \geq 0, \quad \delta \geq 0, \quad \text{and} \quad |R| \leq 1 \;, \tag{3.29}$$

where $\omega = \lambda\zeta - \delta + 1$. The restriction $|R| \leq 1$ is sufficient to guarantee that $f^{v}(\varepsilon_t)$ is nonnegative, the other restriction ensures that σ_t^λ is the sum of positive elements and is therefore itself positive.

Next we examine the stationarity of the conditional variance. Sufficient conditions for covariance stationarity of the family of GARCH(1,1) processes studied by Hentschel are extension of Nelson's (1990b) conditions for the standard GARCH model.

$$\mathrm{Var}(\varepsilon_t) < \infty, \quad \omega > 0, \quad \text{and} \quad E[(\alpha\lambda f^{v}(\varepsilon_t) + \delta)^{2/\lambda}] < 1 \quad . \tag{3.30}$$

Under the distributional assumptions used in this thesis, the first condition is automatically satisfied. The second and third conditions are verified once the parameters are estimated. For the Exponential GARCH class where λ equals zero the condition for stationarity reduces to $\delta < 1$.

The organization of Tables 3.3a to 3.8a which present the estimation results is as follows. Each class of GARCH models is displayed in one panel. The class is identified in the first column; the top panel displays the standard GARCH family where $\lambda = \nu = 2$. The second panel shows the Absolute Value GARCH family where $\lambda = \nu = 1$. The third panel contains results for the Exponential GARCH family where $\lambda = 0$, and $\nu = 1$. The fourth panel concerns the Nonlinear Power GARCH family where $\lambda = \nu$. And finally the last panel displays the family of FREE GARCH models where λ and ν are estimated freely.

The second column indicates the name of the models in each class. The third and fourth columns contain estimates of the values of λ and ν. The fifth and sixth columns exhibit the values of the parameters in the estimation of the mean equation; B_0 is the constant term and B_1 is the M-GARCH term representing the time-varying component of the risk premium. The seventh, eighth and ninth columns contain estimates of the parameters of the variance equation. To obtain the constant term for the variance equation we compute $\omega = \lambda\xi - \beta + 1$. The nested model transformation estimates the α parameter divided by λ; to obtain the true value of α we must multiply the estimated coefficient by λ. The asymmetry parameters are given in columns ten and eleven. Tables 3.7a and 3.8a contain an extra column labelled η which is the estimate of the thickness parameter of the GED. Standard errors (se) are shown in parentheses.

Tables 3.3b to 3.8b are organized as follows. The first column identifies the class of models, the second names the model within each class. The third column contains the adjusted values of the intercept. The fourth presents the result of the calculation $E[(\alpha\lambda f^{v}(\varepsilon_t) + \delta)^{2/\lambda}]$ under the appropriate distributional assumption and the relevant parameter estimates. Skewness and kurtosis of the underlying disturbances are given in columns five and six.

3.2.1 Estimation Results under Normality

The estimation results under the assumption of underlying normally-distributed disturbances are presented in Tables 3.3a, and Table 3.4a for the BA and BAX series respectively; the analysis of the conditions for positivity and covariance stationarity are given in Tables 3.3b and 3.4b.

First consider the results for the BA series. A first look at the stationarity condition in Table 3.b suggests that only models within the Exponential class and Free-Garch classes are stationary. So we focus our discussion of the parameter estimates to models in these two classes which also satisfy the positivity conditions since ω, α and δ are positive and R is less than 1 in absolute value in each instance.

Examining the estimates for the third panel, the E-GARCH class, we see that the constant in the mean equation is not significant except for the specification where symmetry is assumed. However, in both specifications where the rotation parameter R is included, its estimate is significant. The E-GARCH specifications find only the rotation parameter significant, suggesting that asymmetry is more pronounced for large shocks. These results contrast with the parameter estimates in the Free-GARCH class. Here both shift and rotation parameters are significant, a result which points to asymmetric volatility response no matter the size of the shock. Notice also that the shift parameter is negative, while the rotation parameter is positive; this pattern appears frequently in the estimations and will be discussed further below.

It is also noteworthy that in the specification which includes the two asymmetry parameters, the estimates of B_0 and B_1 in the mean equation are both significant and positive. Notice as well that both skewness and kurtosis are both smaller for the Free-GARCH specifications than for the E-GARCH models. In the most general specification in the Free-GARCH class, we see that the estimates of λ and v are respectively 0 .1 and 0.54, which suggest strongly that the traditional GARCH framework is too restrictive.

We turn to Tables 3.4a and 3.4b which present results for estimates of models of the BAX series under the assumption of normal disturbances. In contrast with the previous analysis for the BA series, a quick examination of Table 3.4b indicates that all the models estimated for the BAX series satisfy both the positivity constraint and the stationarity requirements. Notice that although skewness and kurtosis are present, there is less departure from normality than in the models for the BA.

We consider the estimates in the first panel in Tables 3.4a; in particular, the estimates in columns ten and eleven for the asymmetry parameters. The first impression we get indicates the presence of asymmetry in the BAX series, these values are statistically significant only for the rotation parameter in the case of GJR-GARCH and in the case of AS-GARCH. In fact, there is no improvement in the standard GARCH class a by adding a shift parameter. Moving down to the other panels we get a somewhat different story. Both asymmetry parameters are found to be significant when estimated together. These results indicate that there is asymmetric volatility responses to shocks for both small and large values of shocks. The sole anomaly is that whereas the rotation and shift parameters re-enforce each other in FE-GARCH (ie, they both have the same sign),. the shift parameter is negative in the Free-GARCH and FP-GARCH specifications.

When R and S are both positive, there is greater volatility associated with negative shocks than with positive shocks. A negative shock to the price of the BAX entails a positive shock to its yield. Accordingly, positive estimates of R and S suggest that greater volatility is associated with increases in yields than with decreases in yields. Such increased activity may be associated with the idea of a flight to quality to the bond market as yields increase. In the Free-GARCH specification, the estimate of the shift parameter S is negative while the estimate of the rotation parameter R is positive. In this context, small positive shocks smaller than a ceratin value will lead to greater volatility than equally-sized negative shocks as the shift parameter effects dominate the rotation parameter. For shocks greater than this value the reverse is true, as the rotation effect dominates. The crossover point is given by the value -S/R. In the case of the Free-GARCH estimates this value is given by is 0.62 (ie, 0.499/0.81).

These results show that for relatively small shocks greater volatility follows a positive shock to prices than for negative shocks; in other words, greater volatility is associated with negative shocks to interest rates than with positive shocks. Notice first that this phenomenon is not captured when the S parameter is estimated alone in the AF-GARCH specification. It may be the case that when the S parameter is estimated with R as in Free-GARCH, the value of S becomes significant in conjunction with a higher estimate of R (relative to TF-GARCH). Simply, the extra parameter improves the fit of the estimate in a counterbalancing manner. Is it possible to attach additional economic significance to this phenomenon? A possible explanation focuses on the fact that BAX yields were low during the estimation period. In this context, a positive shock to prices–a negative shock to interest rates–could actually lead to greater market volatility as investors move from the bond market to the stock market than would be associated with a small increase in bond yields. But, the explanation continues, the asymmetry works in the oppositive direction for shocks with a magnitude greater than a certain value as the quality phenomenon begins to dominate.

One early conclusion can be derived from these results: we can reject the standard symmetric GARCH models in favour of models that can capture asymmetry. Furthermore, we seem to be able to reject models that favour one type of asymmetry over another in favour of models that are able to reflect both types of asymmetry. A formal test to distinguish between the models in each class and between the classes will be presented below.

In Table 3.4a, the transformation parameters λ and v within the first three panels are restricted to be either 2, 1, or zero. In the last two panels these parameters are estimated freely. One striking observation is that the freely estimated parameters are neither 2, 1, or zero making the traditional GARCH models inappropriate for the BAX series. The range for the estimated λ's for the BAX series is between 0.35 to 0.45, and the range of v's is between 0.20 to 0.50. In fact, all the values of the λ's and v's are below one, and suggest concave transformations for both the conditional variance and a concave transformation for the absolute value function.

The results for the estimation of the models under normality may be summarized:

(i) Asymmetry is evident in both series which makes the symmetric GARCH models inappropriate.

(ii) Asymmetry appears to be present in two ways as reflected by the statistical and significance of the values obtained for both the "rotation" and the "shift" parameters.

(iii) The values of the λ's and v's are neither two, one, or zero. The traditional GARCH models are inappropriate; in fact, the values for these parameters are below one which suggest a concave transformation for both $f(\varepsilon_t)$ and σ.

(iv) All of the models estimated for the BAX series under normal distributions were found to be stationary and satisfied the positivity condition. For the BA series, only specifications within the E-GARCH and Free-GARCH families are covariance stationary.

3.2.2 Estimation Results under the Student t-Distribution

The second set of estimation results, which are exhibited in Tables 3.5a, and 3.6a for the BA and the BAX series respectively, assumes that the error terms are drawn from the student t-distribution with five degrees of freedom. The positivity and stationarity analysis for the t-distribution models are presented in Tables 3.5b and 3.6b.

When models with the degrees of freedom were estimated freely, the estimates of the degrees of freedom were about 2.7, which is not consistent with the existence of a finite fourth moment. Five degrees of freedom were imposed on the estimation of the various models.

Table 3.5b confirms the earlier result that the stationarity of the BA series can not be taken for granted. As with the estimates under the assumption of normality, the stationarity conditions are met only for the E-GARCH class and the FREE GARCH class (estimates in the Nonlinear Power Class are very close to 1). In clear contrast, the results of Table 3.6b confirm that all of the models estimated for the BAX series satisfy the positivity and the stationarity conditions, with the estimates for specifications in the standard class of GARCH models close to 1.

Another difference between the estimation results for the two series concerns the asymmetry parameters which are almost always insignificant in results for the BA. Only the shift parameter is significant in the AF-GARCH model and the Free-GARCH model and, as the likelihood value attests, the estimates of these models are very close. With regard to the BAX, in most of the models presented in Table 3.6a, one or other of the asymmetry parameters is statistically significant from zero. The shift parameter S appears to have somewhat greater prominence in these estimations under the t-distribution than previous estimates under normality. Moreover, both R and S are significant in the FP-GARCH and Free-GARCH models with the recurring phenomenon that the shift parameter is negative and the rotation parameter is positive. In the latter model, both parameters in the mean equation are significant.

The BAX values of the estimated λ's under the t-distribution range from 0.24 to 0.71, and for the vs range from 0.58 to 0.66.; and the values for the BAs of the estimated λs range from 0.15 to 0.75, and for the vs range from 0.53 to 0.74. Again we are moving away from the traditional GARCH models where both parameters are assumed to be 2.

Skewness and kurtosis measurements for the various models are presented in Table 3.5b and Table 3.6b. Since the theoretical kurtosis of the student-t distribution is given by the expression 3(v - 2)/ (v - 4), where v's are degrees of freedom. The kurtosis implied by the distribution with 5 degrees of freedom is 9. The estimate of the kurtosis for the various models of the BAX series is around 9, so on this measure the models with t-distribution appears to provide a better fit for the BAX series. The same cannot be said for the models for the BA where the kurtosis remains uniformly high due to the presence of several very large outliers.

The results for the estimation of the models under the t-distribution may be summarized:

(i) Asymmetry is more pronounced for the BAX than for the BA. The rotation parameter is more important than the shift parameter, although both are significant in the Free-GARCH specification.

(ii) Again the values for the transformation parameters, the λs and the vs, are neither two, one, nor zero. The values under the student-t distribution support the conclusion that the transformations associated with these parameters are concave.

(iii) The models estimated with the student-t distribution appear to "fit" the BAX better than those estimated under the normal.

3.2.3 Estimation Results under the GED Distribution

The GED distribution, used by Nelson (1991), has a parameter η which measures the thickness of the tails of the distribution. For $\eta = 2$, we have a normal distribution; the density has tails thicker than the normal when $\eta < 2$; and for $\eta > 2$ the density has thinner tails. Estimation results for the various GARCH models estimated using the GED are presented in Table 3.7a and Table 3.8a, while Tables 3.7b and 3.8b present the positivity and stationarity conditions for the GED models for both the BA and BAX respectively.

Examining Table 3.7b, we find for the first time that all of the models estimated for the BA series under the GED distribution satisfy the positivity and stationarity constraints. Moreover, skewness seems to be less pronounced under GED than under either the normal or the student-t distribution.

On the other hand, the estimation results for the BA series presented in Table 3.7a appear somewhat erratic. Neither of the asymmetry parameters is significant for 3 of the 5 classes of models, including the Free-GARCH class. In the E-GARCH class, the rotation parameter R is significant and negative when estimated alone, but positive when estimated with the shift parameter; the latter is of no statistical significance and the difference in likelihood between the models is small. The sign of the estimate of the R parameter also changes sign in the standard GARCH class. It is difficult to make economic sense of these results.

The situation improves with the estimation of the BAX series. Table 3.8b confirms that the models estimated for the BAX under the GED satisfy both the positivity and stationarity conditions. In Table 3.8a we see that the estimate of the thickness parameter ranges from 0.89 to 0.93 for the BAX; all of the values are below 1, supporting the fact that the distribution process underlying these series is fat tailed. Asymmetry in these estimates appears to be captured solely by the rotation parameter, with no estimate of the S parameters significant in any model. However, these R values are lower here than in the corresponding estimates using other distributions.

Even in these estimates of the model parameters for the BAX series there is some indication that the results may be unreliable. The likelihood function is very flat, and there appears to be very little gain in adding additional parameters. Moreover, convergence in the numerical estimates of these models was extremely slow. In sum, we are not particularly confident in whatever message may be conveyed by these results.

3.2.4 Conclusions

The richness of the above FREE GARCH framework is evident from the range of specifications obtained using equations (3.6), (3.7) and equation (3.8) which have lead to a host of "new" GARCH models. Moreover, there is the added advantage in having all of these models nested in one framework. The main results of this section may be usefully summarized.

(i) One immediate conclusion concerning the value of λs, and vs, is that their values do not appear to be either zero, one, or two. This conclusion will be tested and confirmed in the next section and , accordingly, we can reject most of the existing popular GARCH models, like the standard GARCH, Threshold and nonlinear GARCH, and Nelson exponential GARCH. The values obtained for the λs, and the vs are lower than one, suggesting a concave transformation of the conditional standard deviation, and a concave transformation of the curve $f(\varepsilon_t)$.

(ii) All of the models considered for the BAX are stationary under the three distributions. However, estimation of the BA series proved to be more problematic in this regard; for example, only the E-GARCH class models and FREE GARCH class models are found to be stationary under normal distributions. However, more models for the BA were found to be stationary under non-normal distributions. All FREE GARCH models were found to be stationary for both series, under all distributions.

(iii) There is evidence of asymmetry in the impact of disturbances on volatility. The rotation parameter appears to play a bigger role than does the shift parameter. With regard to the BAX, when both parameters are significant, the shift parameter is generally negative while the rotation parameter is positive. In this situation, small positive shocks lead to greater volatility than small negative shocks; whereas the opposite is the case for larger shocks.

(iv) Judging by the value of the likelihood function, we see that the least restricted GARCH models proved a better fit for the two series. This intuition will be confirmed more formally in the next section.

(v) Estimation under the student-t distribution appears to yield a better fit than under the normal, while estimation under GED seems to pose numerical problems and yield erratic results.

3.3 Likelihood Ratio Tests

The advantage of the Hentschel nesting models lies not only in the richness of the models presented, but also in the ability to discriminate among them. The nesting framework permits a simple means to test the fitness of the models. We have estimated the restricted and unrestricted models in Section 3.2. In this section, we test the quality of the fit of the GARCH models against one another. In effect, we are testing whether a linear combinations of the parameters λ, v, S, and R are significantly different from zero. This can be conducted through the Likelihood Ratio Test of parameter restrictions. Two types of tests will be implemented. The first test will be used to discriminate among the particular models within each class of GARCH models; in particular, we will be testing to determine the best specification of asymmetry. The second group of tests will be used to discriminate among the different functional forms of the general GARCH model. Both groups of tests are conducted within specific distributional assumptions.

3.3.1 Likelihood Ratio Tests for Asymmetry

Tables 3.9a, 3.10a and 3.11a present the results of likelihood ratio tests for asymmetry for the BA; while Tables 3.9b, 3.10b and 3.11b present the results of likelihood ratio tests for asymmetry for the BAX series estimated under normal, student-t and GED distributions. The first column in these tables identifies the class of model and lists the restrictions on λ and v that characterize the maintained hypothesis for each panel. The second column displays the three of four possible types of asymmetry that form the null hypothesis for each model. If there were no asymmetries, the S's and the R's would equal to zero, as indicated in the first row. This possibility can now be tested against the three alternative hypotheses listed in the last columns of the table. The asymmetry could be caused by a shift of the news impact curve, as shown in the third column; it could be caused by a rotation of the news impact curve, as shown in the fourth column; or it could be caused by both a shift and a rotation, as shown in the fifth column. The likelihood ratio statistics have a χ^2 distribution with one or two degrees of freedom depending on the number of constraints involving R and S; critical values are presented at the bottom of the tables. Significance levels for each are shown in parentheses. The purpose is to test whether the inclusion of the shift and rotation parameters improves the overall fit of the model.

The set of results for the BA series estimated under normality (Table 3.9a) involving the Exponential Class and the Free GARCH class are fairly conclusive with regard to the importance of the asymmetry parameters. All the symmetric specifications within these two classes can be rejected in favour of some alternative involving non-zero asymmetric parameters R and S. For example, in the Free GARCH class, the symmetric null is rejected in favour of any of the three possible alternatives involving non-zero R and S. But the results also reveal the importance of the rotation parameter relative to the shift parameter. The null that R is 0 with S free is rejected in favour of the alternative that R is non-zero and S is free. By contrast, the null that fixes S at 0 and lets R be free, is not rejected in favour of the alternative that both parameters are free.

In the case of the BAX models estimated with normal disturbances, the results are conclusive that all of the symmetric models can be rejected in favour of alternatives that include both a shift and rotation parameters. It should be noted that the models that include only a shift parameter do not show an improvement over the symmetric models. Nevertheless, all of the models that include both a shift and rotation parameters show a noteworthy improvement, not only over the symmetric models but also over other models that include only a shift or a rotation parameter.

Tables 3.10a and 3.10b present the results for the asymmetry test for the BA and the BAX respectively under the student-t distribution. The results for the BA series are fairly weak. All of the symmetric models are not rejected in favour of any asymmetric alternative, with the sole exception of the specification in the Free GARCH class which is weakly rejected in favour of the alternative that includes both asymmetry parameters. The results in Table 3.10b suggest the pattern of asymmetry that was found when the models were estimated under normal disturbances, particularly for the Free GARCH class, where symmetry is conclusively rejected in favour of asymmetry with the hint that the rotation parameter is doing most of the work in capturing the asymmetry.

Even though the estimation results for the parameters of the models estimated with the GED disturbances are tentative, the likelihood ratio tests for asymmetry are presented for completeness' sake in Table 3.11a for the BA and in Table 3.11b for the BAX. With regard to the BA, models specified under the Free GARCH exhibit the presence of asymmetry most dramatically. Here we have the strong suggestion that both the shift and rotation parameters play an important role: any null hypothesis that fixes one or both the parameters is rejected in favour of an alternative that frees a parameter. A similar pattern is found in models of the Standard Class. It is somewhat odd that symmetry can not be rejected in the Exponential Class. The results vary for the BAX series. Asymmetry is most pronounced for models estimated within the Standard class. There is a suggestion of the earlier pattern of asymmetry within the Free GARCH class, where the rotation parameter appears to be the asymmetric pattern of choice.

3.3.2 Likelihood Ratio Tests for Functional Form

Tables 3.12 to 3.14 present the results of likelihood ratio test regarding the functional form of the specifications. More specifically, the tests involve hypotheses concerning the values of λ and ν. Four basic null hypotheses are tested against various alternatives. In the first, we test the null of Standard GARCH where $\lambda = \nu = 2$ against the alternative $\lambda = \nu$ or the alternative where λ and ν are estimated freely. In the second test, Nonlinear GARCH where $\lambda = \nu = 1$ is tested against the same two alternatives. In the third, the null of Exponential GARCH where $\lambda = 0$ and $\nu = 1$ is tested against the general alternative where λ and ν are estimated freely. Finally, the specification of Nonlinear Power GARCH ($\lambda = \nu$) is tested against the Free GARCH alternative. In all tests in this section, the asymmetry parameters are estimated freely both under the null and the alternative.

The results of the functional form test can readily be summarized. For the BA series under any assumption concerning the underlying distribution of disturbances, the likelihood ratio test rejects any null imposing constraints on λ and v in favour of the alternative that these parameters are freely estimated. The results are similar for the BAX series with the exception of the models estimated under GED disturbances where it is found that the Nonlinear GARCH model where $\lambda = 1$ and $v = 1$ can not be rejected in favour of either the Nonlinear Power model nor the Free GARCH model.

The general conclusion is that the usual functional forms for GARCH modelling are too restrictive and that different forms should be employed. The tests, however, do not indicate the extent to which the models are different. To evaluate these differences, we focus on three models of the BAX series estimated under student-t disturbances with the asymmetry parameters estimated freely: Standard GARCH, Exponential GARCH and Free GARCH. Figure 3.5 presents graphs of the estimates of the conditional standard deviations for the three models.

An inspection of these graphs suggests that the Standard GARCH and E-GARCH display similar patterns, while the Free-GARCH pattern is relatively mute and more jagged than the other two depictions of conditional volatility. It would appear that large shocks have a greater impact in the first two graphs, while there is greater response to smaller shocks in the Free-GARCH model. These suggestions are confirmed in the next Figures 3.6 which displays the differences in conditional volatility among the series. Except for two periods the volatility differences between Standard GARCH and E-GARCH are not particularly pronounced, while there is greater movement in the bottom two graphs. The next Figure 3.7 exhibits these differences in percentage terms. Whereas the percentage differences between the Standard GARCH and E-GARCH rarely exceed 10% and are for the most part small, there is much greater percentage difference in the bottom two graphs. In short, conditional volatility differences in the more general GARCH model are quite systematically different than in the usual specifications.

3.4 Predictive Power

In this section we evaluate the various GARCH models with regard to their within- and out-of-sample predictive power of the conditional variance. The sample of data at our disposal is 1,133 daily observations; 1,069 daily observations were used for model estimation and within-sample comparison and the remaining 64 daily observations are used to test the out-of-sample performance of the models. The 64 observations are the daily settlement prices for the BA and the BAX series for the months April, May, June of 1999 a three-month period which is the duration of a BAX first-month contract. Again for the purpose of richness and completeness predictive power was assessed for all of the models estimated.

3.4.1 Background

One of the popular methods for evaluating predictive power uses regressions involving the ex-post squared errors over a relevant horizon. The procedure used in this paper follows Pagan and Schwert (1990) who consider the following ex-post squared errors-volatility regressions:

$$\hat{u}_t^2 = \alpha + \beta\hat{\sigma}_t^2 + n_t \ , \tag{3.31}$$

$$\ln\hat{u}_t^2 = \alpha + \beta\ln\hat{\sigma}_t^2 + n_t \ . \tag{3.32}$$

Here the estimate of the conditional variance is determined by the model under consideration. This regression is an analog to a common procedure for evaluating forecasts for the conditional mean. Unbiased forecasts entail that $\alpha = 0$ and $\beta = 1$; in which case, the estimates that are generated by a GARCH process are consistent with the squared errors of the model. The coefficient of determination will also be computed as a measure of goodness of fit. The second regression in logs is also considered; in computing the coefficient of determination for this regression, errors in predicting small variances are given more weight than similar errors in the first equation. A quadratic loss function is implicit in the first regression whereas a proportional loss function is associated with the second.

The regressions in (3.31) and (3.32) rely on the observed squared errors as a measure of realized volatility. This is justified to the extent that they provide unbiased estimators of the day-by-day latent volatility. However, the values of the coefficients of determination as determined in a variety of studies are typically low, and have been the subject of concern. Many studies have found that standard volatility models explain little of the variability in ex-post squared errors; see, for example, Cumby et al,(1993), Figlewski (1997), and Jorion (1995, 1996). In recent literature Chirstofferson (1998) and Anderson and Bollerslev (1998) have disputed the suggestion that these low R^2s reported in the literature are signals that ARCH models may be seriously mis-specified and, consequently, provide poor volatility forecasts and are of limited practical use. Anderson and Bollerslev (1998) argue that by increasing the frequency of the sampled data a more accurate ex-post volatility measurements is achieved. By moving away from monthly, weekly, and even daily data to tick data, the R^2 associated with forecast regressions such as are considered in this Section have increased by about 7 to 8 times. They have demonstrated that ARCH and stochastic volatility models do provide good volatility forecasts.

Different assessment procedures are used in this line of research . Anderson and Bollerslev (1998) test the predictive power by regressing the squared returns on the conditional volatility, whereas Jorion (1996) tests the predictive power by regressing implied realized volatility over the remaining life of the option contract on the forecasted volatility of the remaining life of the option contract. We are following the methodology where we regress the square of the estimated residuals on the estimated conditional volatility. What is relevant to our research is that α, β, and R^2 (even low ones) do provide us with reasonable methods of evaluating different types of models and that by using daily data our models capture more of the volatility dynamics than those estimated using monthly or even weekly data.

3.4.2 Within-Sample Predictive Power

The results for the within-sample predictive power are presented in the left columns of Tables 3.15a,b to 3.17a,b; in particular, the results of the OLS regression estimates of equation (3.31) and (3.32). There are 1,069 daily observations starting on January 3, 1995 and ending March 31, 1999. The first column of the first compartment (column three) exhibits the intercept value of the regression, the fourth column displays the slope, and columns five and six present R^2 and R^2 for logs, respectively the coefficients of determination associated with regression estimates of equations (3.31) and (3.32).

The within-sample predictive power under normal distributions for both the BA and the BAX series are reported in Tables 3.15a and 3.15b. Consider first the Exponential class and the Free GARCH class for the BA series, the sole groups for which the models are stationary. In all specifications within these classes the estimates of the intercept are all significantly different from 0 and range from 0.49 to 0.88; the estimates for the slope coefficient are all significantly different from 1, and range from 0.02 to 0.35. Matters improve considerably when we turn to the results for the BAX. The estimates for α are not significantly different from 0 except for the Standard class, and the estimates for β are not different from 1 with the exception again of specifications within the Standard class. Moreover, the estimates for α and β are particularly close to 0 and 1 for the Free GARCH models. The Free GARCH specification has the highest R^2 and R^2 for logs of all the specifications considered.

Tables 3.16a and 3.16b present results for the BA and BAX series under the student-t distribution. The predictive power for models of the BA series improves somewhat when the underlying disturbances are assumed to be student- t. For models estimated within both the Nonlinear Power class and Free GARCH classes, the estimate of the slope coefficient in equation (3.31) are all close to 1; the intercepts have estimates around .37, significantly different from 0 at 5%. And the Free GARCH specification has the highest R^2 and R^2 for logs of all the specifications considered. A similar message emerges when the models of the BAX series are estimated using student-t. Now all the estimates of α are insignificantly different from 0 and all the estimates of β are insignificantly different from 1. Moreover, the Free GARCH specification has the highest R^2 and R^2 for logs of all the specifications considered.

Tables 3.17a and 3.17b present results for the BA and BAX series under the GED distribution. Regarding the BA series, the first striking observation is the extreme departure of the estimates for α from zero with values as small as -2.89 and few positive values. The standard errors are considerably higher in these estimates than in the previous ones using normal or student-t distributions. For example, in the Free GARCH specification the estimate for β is 0.99 but it is insignificantly different from 0 as well as from 1. The models estimated under the GED distribution fared much better in modeling the BAX series with intercept coefficients close to 0 and slope coefficients close to 1. Moreover, R^2 and R^2 in logs for the Free GARCH specification is comparable to that of the same specification using student-t.

From a consideration of the relative predictive in-sample performance of the various GARCH models, it may be concluded that the Free-GARCH specification under both student-t and GED performs best for the BAX, but only this model estimated under student-t does an adequate job in dealing with the BA series.

3.4.3 Out-of-Sample Predictive Power

We return to Tables 3.15a,b through 3.17a,b to assess out-of-sample predictive power. The out-of-sample size is 64 observations for the months of April, May, and June of 1999. As the results in Table 3.15a indicate, the out-of-sample estimates for the BA under normality are poor. For the most part, the estimates of the slope coefficient are significantly negative. The models under normality fare better for the BAX series where for all models except those in the Standard Class have estimates of α and β that are not significantly different from 0 and 1. But the standard errors of the estimates of the slope coefficient are large, and the estimates are not significantly different from 0 either.

The results regarding out-of-sample predictive power for the BA series do not improve when models are estimated under the student-t distribution. The estimates of the slope coefficient are closer to -1 than they are to 1. As under normality, the results for the BAX series are better than those for the BA. But again the standard errors associated with estimates of the slope coefficient are large. The best predictive results are obtained by the Free-GARCH specification; this model leads to estimates of α and β of 0.35 and 0.97 respectively. The results for this model are comparable to those for the same specification under normal disturbances. R^2 for logs is higher under normal than under student-t; the reverse is true for R^2 .

Finally, the analysis of out-of-sample predictive results for the BA series under GED presented in Table 3.17a suggests that the models estimated under GED are no better than those considered previously. Equally, it is difficult to pick the best among the Free-GARCH specifications under normality, student-t and GED in the context of out-of-sample predictive power.

To conclude the section: the Free-GARCH specification under student-t is able best to forecast movements in volatility for the BA series. The same specification estimated under normal, student-t and GED does best with regard to in-sample and out-of-sample prediction.

3.5 Conclusions

This chapter presented an investigation of the volatility dynamics of BA and BAX prices within an extended GARCH modeling framework. Within this framework, there have been two concerns: the role of asymmetry parameters relating to the shifting and rotation of the news impact curve, and the general function form of the volatility equation. As well, the impact of different distributions on the estimation of the various models has been investigated. The results of an extensive and comprehensive set of estimations may be concisely summarized as follows:

(i) With regard to the positivity and covariance stationarity constraints, almost all models estimated under normal distribution were found to be non-stationary for the BA series, except for the E-GARCH and the Free-GARCH classes. Also, some of the restrictive models were found to be non-stationary when estimated under student-t, such as the standard GARCH class and the Absolute Value GARCH class. The remaining models satisfy both the positivity and covariance stationarity conditions.

(ii) All the models under all three types of distributions estimated were found to satisfy both the positivity and stationarity constraints for the BAX series.

(iii) The significance of the asymmetry parameters emerged in the estimations performed in this chapter. The rotation parameter appears to play a more prominent role with the implication that the asymmetric impact of shocks is particularly pronounced for large shocks. In some estimations, particularly in the Free-GARCH specification, the estimate of the shift parameter is actually negative. This result is new and suggestive that small positive shocks have greater impact on volatility than small negative shocks, whereas the reverse is true for large shocks.

(iv) These intuitions gained from the estimation results were confirmed to some extent by likelihood ratio tests of asymmetry.

(v) Further likelihood ratio testing suggested that the functional form parameters λ and v are different from two, one, or even zero. These results indicate that the underlying volatility dynamics are not correctly modeled within either the standard GARCH class, the Absolute Value GARCH class, or even the Exponential GARCH class of models. Similar tests also confirm the superiority of the FREE GARCH class of models over the Nonlinear Power GARCH.

(vi) The chapter concluded with an exploration of the in-sample and out-of-sample predictive power of the various models. This analysis confirmed the superiority of the Free-GARCH specification, particularly for the BAX series and the BA series under student-t.

(vii) Among the models considered, the specification within the FREE-GARCH class that includes both a shift and rotation of the news impact curve estimated with the t-distribution emerges appears to be the most successful in modeling both the BA and BAX series.

There is one caveat to this general assessment. The modelling of the BA series has not been particularly successful. One explanation for the relatively poor performance of the models in dealing with the BA series is that the BA market is less liquid and less deep compared to the market for the BAX. Participants prefer to interact in a liquid and deep market in order to place "bets" on the direction of interest rates. Accordingly, the market may be better behaved and easier to model. Moreover, only the Canadian Banks (the only issuers of the BA) can fully take advantage of any arbitrage opportunities between these two instruments. Participants do not have the option of what is referred to in the market as "reverse repo" (selling the BA short and buying the BAX). Participants are not able borrow the BA for delivery for the duration of the reserve repo, so there exists only the possibility of a one-sided arbitrage for market participants. So the small size of the BA market, the inability to fully take advantage of arbitrage opportunities, and the lack of depth in the BA market may account for the difficulties encountered in modeling the BA series.

Figure 3.1: Bankers' Acceptance [BA]
First Differences of Logs

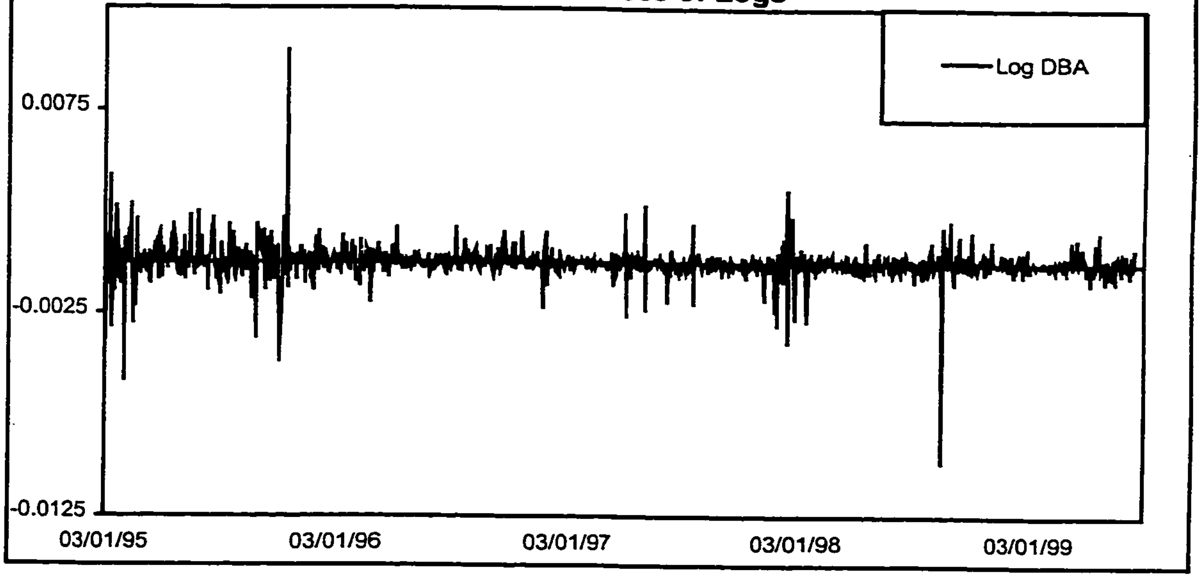


Figure 3.2: Bankers' Acceptance Futures [BAX]
First Differences of Logs

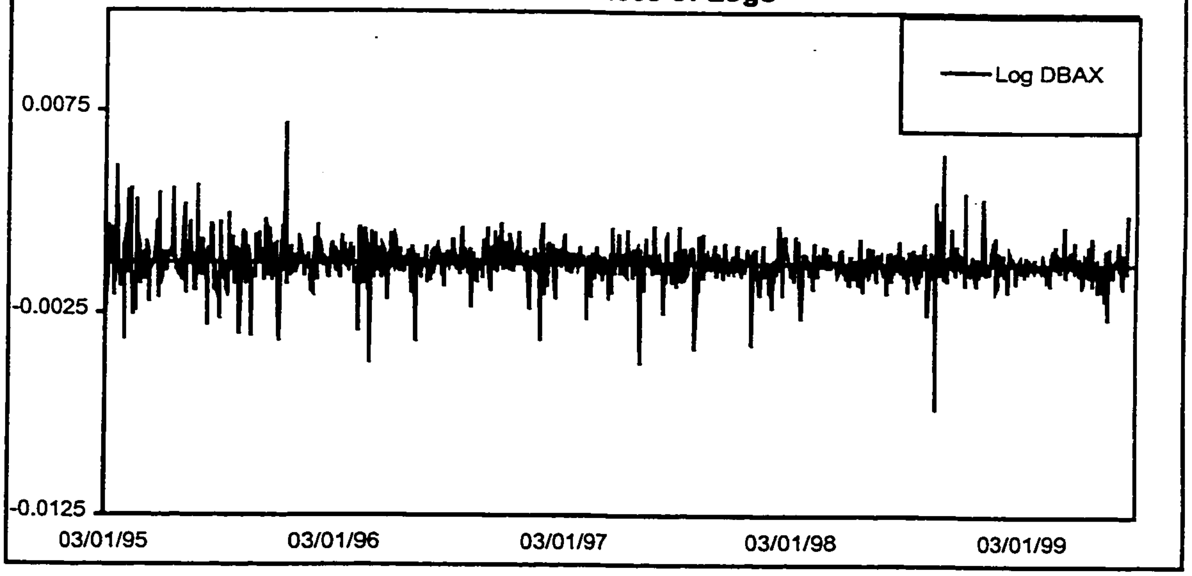
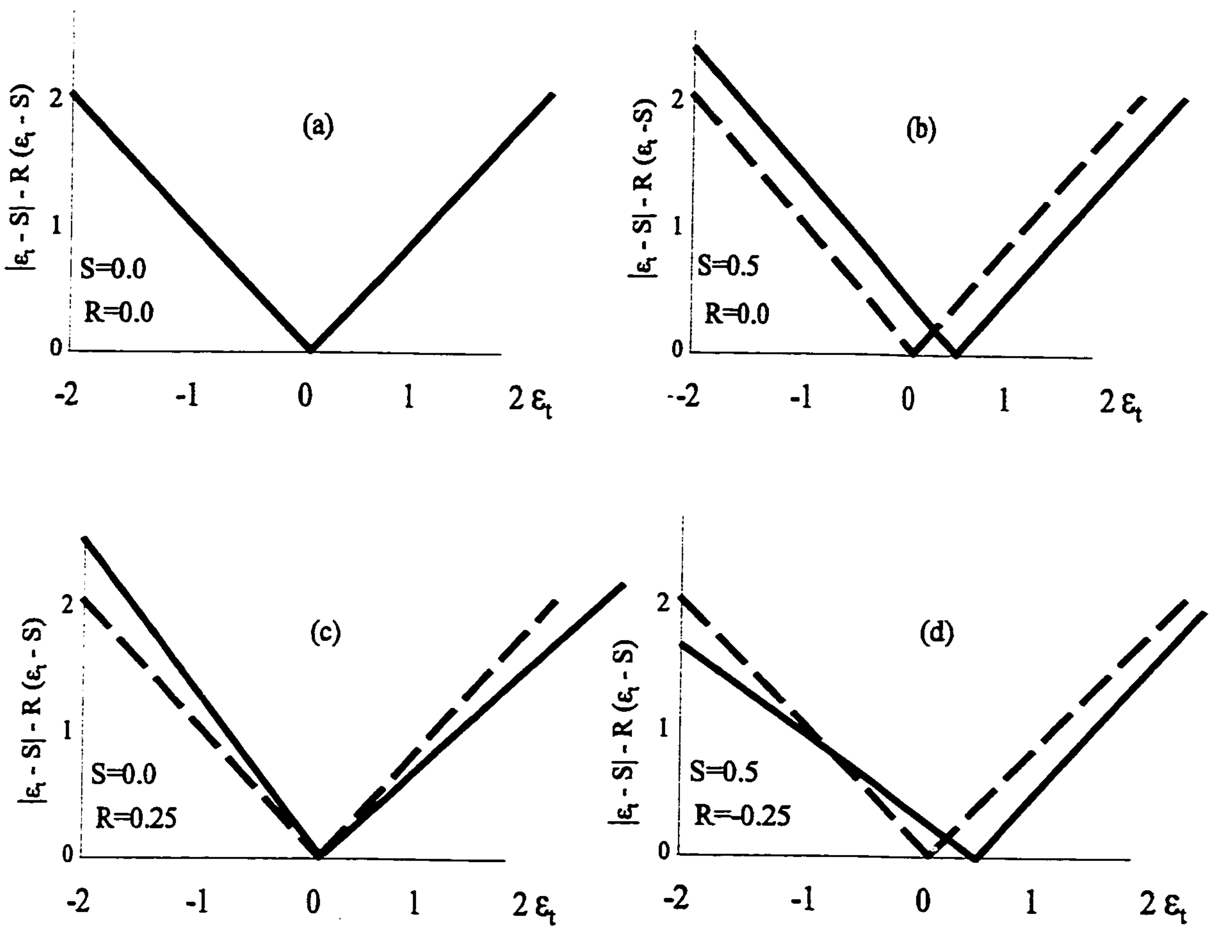
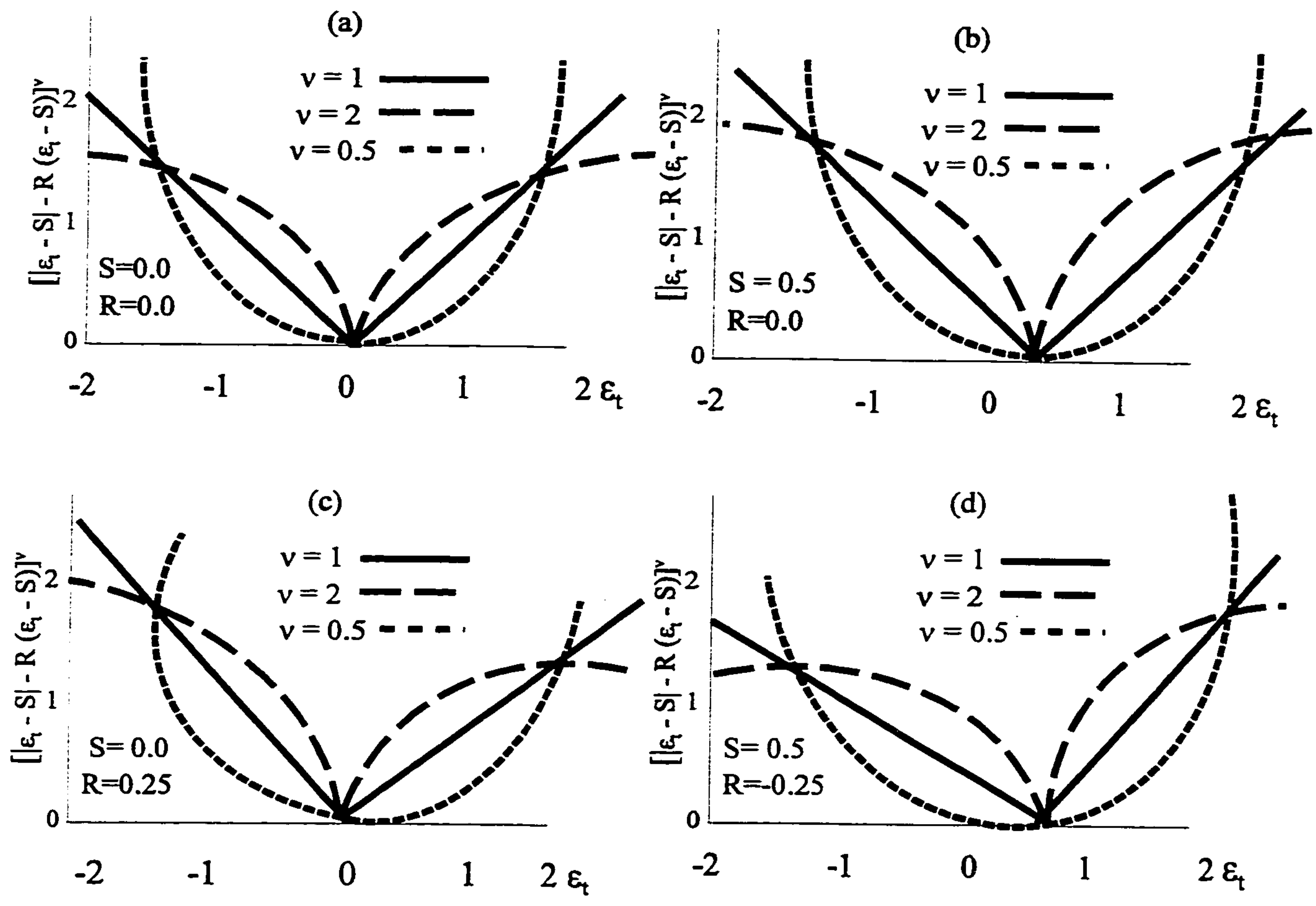


Figure 3.3
The News Impact Curve
Shift and Rotation



Each panel shows the shifting and rotation of the absolute value function. Panel (a) represents a symmetric news impact curve. While panel (b) shows asymmetry caused by a shift in the news impact curve, and panel (c) shows asymmetry caused by a rotation in the news impact curve. Panel (d) shows the power of the Free-GARCH model in measuring asymmetry caused by both a shift and rotation in the news impact curve.

Figure 3.4
The News Impact Curve
The Transformation $f^{v}(\varepsilon_t)$



By setting different values for v, S and R the transformation $f^{v}(\varepsilon_t)$ controls the impact of shocks, ε_t, on the transformed conditional volatility, σ_t.

Figure 3.5: Estimates of Conditional Standard Deviation
Bankers' Acceptance Futures [BAX]

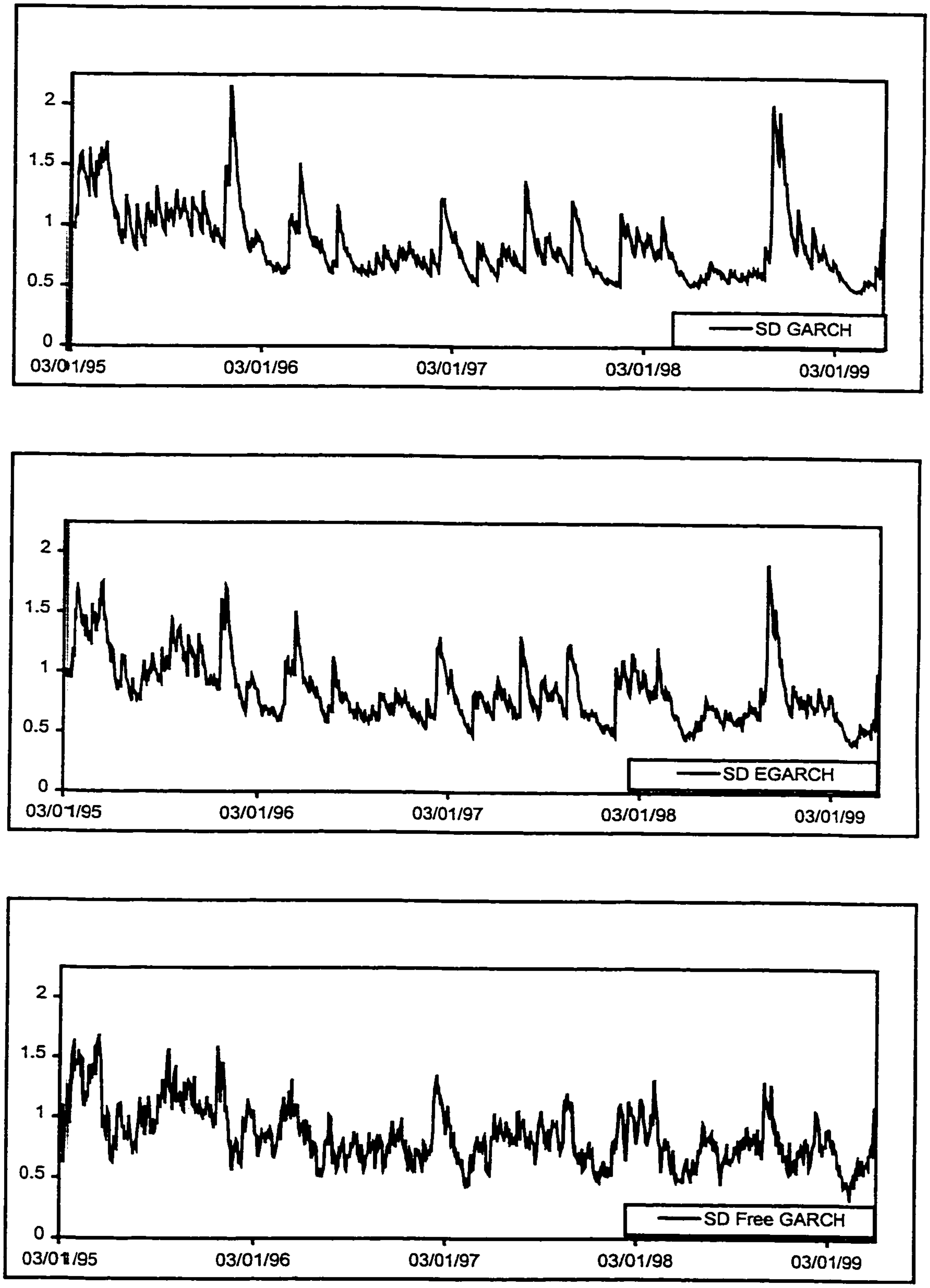


Figure 3.6: Absolute Difference between SD Models Estimates Bankers' Acceptance Futures [BAX]

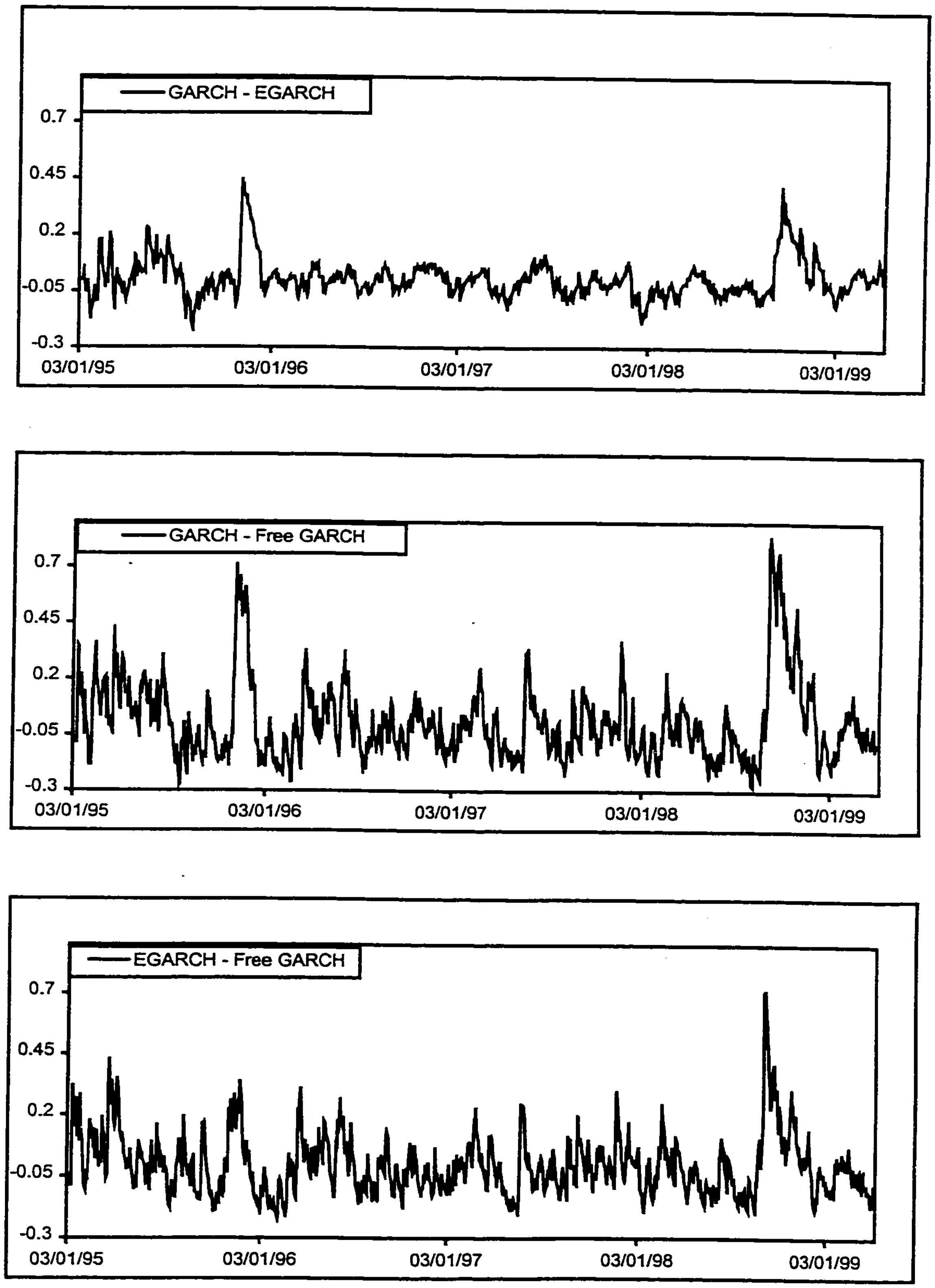


Figure 3.7: Percentage Difference between Models Volatility Estimates Bankers' Acceptance Futures [BAX]

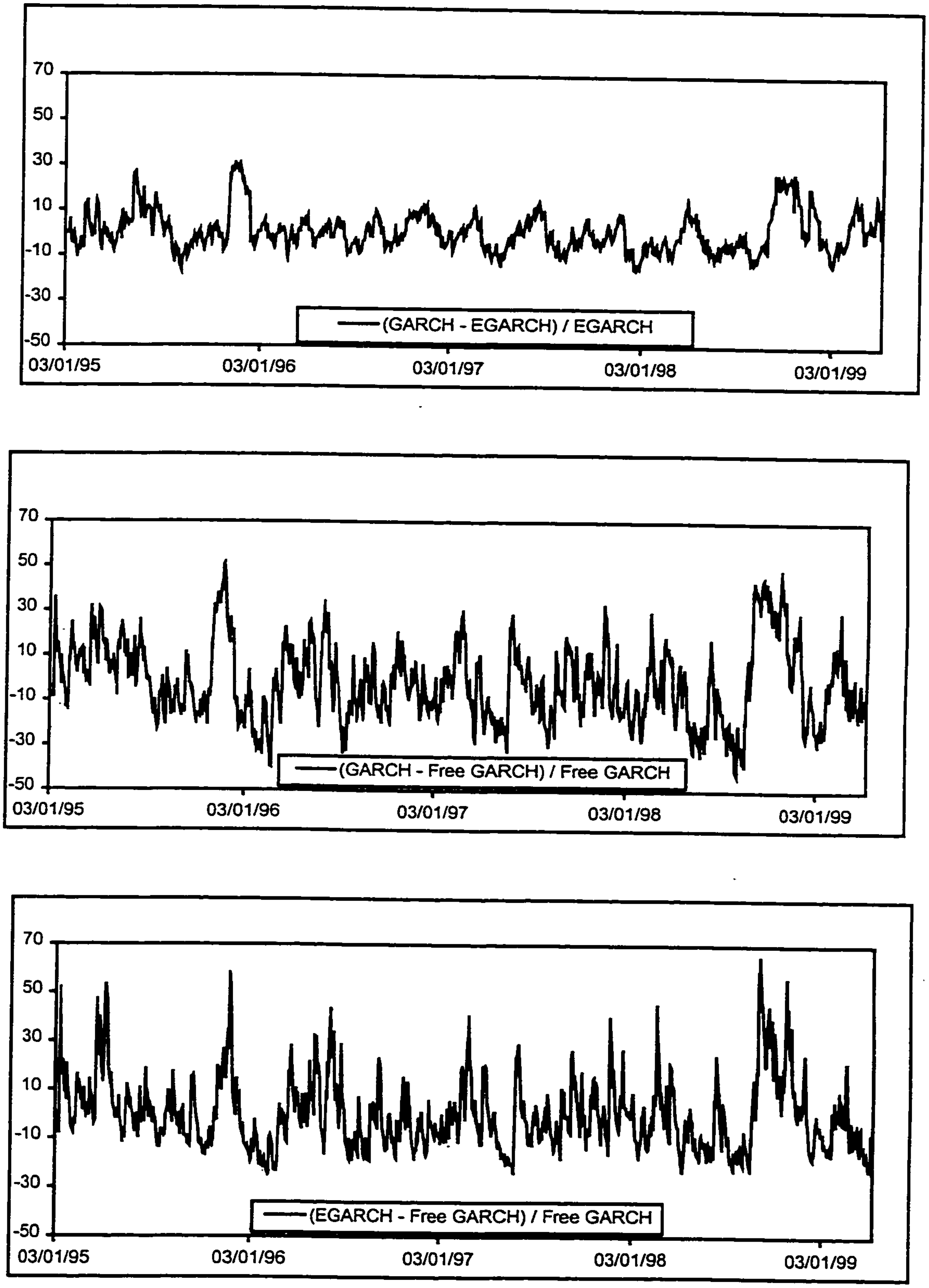


Table 3.1
The Bankers' Acceptance [BA] and the
Bankers' Acceptance Futures [BAX] Descriptive Statistics

	BA	BA Diff.	BAX	BAX Diff.
Mean	4.553	0	4.552	0
Maximum	4.575	0.0104	4.574	0.0069
Minimum	4.516	-0.0098	4.511	-0.0072
Skewness	-0.69 (0.00)	-1.22 (0.00)	-0.81 (0.00)	-0.30 (0.00)
Kurtosis	0.05 (0.00)	44.75 (0.00)	0.32 (0.00)	8.54 (0.00)
BJ test for Normality	91.66 (0.00)	94,902 (0.00)	130.6 (0.00)	3,467 (0.00)
Test for ARCH effects		37.15 (0.00)		32.48 (0.00)

The BJ [Berra-Jarque] test for normality is distributed $\chi^2(2)$. The test for ARCH is described in the text; it is distributed $\chi^2(2)$. P-values are given in parentheses.

Table 3.2
The Nested GARCH Models

#	λ	v	S	R	MODEL
1	2	2	0	0	Bollerslev Standard Garch
2	2	2	free	0	Engle, and Ng Nonlinear-asymmetry Garch
3	2	2	0	free	Glosten, Jannathan and Runkle Garch
4	2	2	free	free	Asymmetry Garch
5	1	1	0	0	Symmetric Absolute Value Garch
6	1	1	free	0	Asymmetric Nonlinear Garch
7	1	1	0	$\|R\| \leq 1$	Zakoian Threshold Garch
8	1	1	free	$\|R\| \leq 1$	Engle and Ng Absolute Value Garch
9	free	λ	0	0	Higgins, and Bera Nonlinear Garch
10	free	λ	1	0	Nonlinear Power Garch
11	free	λ	0	1	Ding, Granger, and Engle Asymmetric Power Garch
12	free	λ	free	free	Full Power Garch
13	free	free	0	0	Symmetric Free Garch
14	free	free	free	0	Asymmetric Free Garch
15	free	free	0	free	Threshold Free Garch
16	free	free	free	free	Hentschel FREE Garch
17	0	1	0	0	Symmetric Exponential Garch
18	0	1	free	0	Asymmetric Exponential Garch
19	0	1	0	free	Nelson Exponential Garch
20	0	1	free	free	Free Exponential Garch

Columns 2- 5 list the restrictions applied to Equations 3.7 and 3.8 to arrive at the desired model. The last column identifies the models, either by their author's original designations or by their special features (new specifications achieved through Hentschel framework). All the models are nested except for the ones that set $\lambda = 0$.

Table 3.3a
Bankers' Acceptance [BA]
Estimation of Nested GARCH Models under Normality

Class	Model (likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)
Standard Class	Garch (-117.28)	2	2	.000 (.003)	-.016 (.000)	-.395 (.001)	1.17 (.014)	.11 (.003)	0	0
	NA-Garch (-115.41)	2	2	.012 (.023)	-.014 (.013)	-.389 (.056)	1.11 (.057)	.125 (.132)	-.055 (.094)	0
	GJR-Garch (-114.18)	2	2	.019 (.003)	-.013 (.000)	-.399 (.001)	1.19 (.015)	.10 (.003)	0	-.106 (.007)
	AS-Garch (-114.10)	2	2	.019 (.003)	-.013 (.000)	-.401 (.001)	1.21 (.016)	.10 (.003)	.0154 (.005)	-.126 (.007)
Nonlinear Class	SA-Garch (-98.21)	1	1	-.000 (.003)	-.013 (.004)	-.335 (.034)	.59 (.064)	.59 (.049)	0	0
	AN-Garch (-98.18)	1	1	.002 (.003)	-.012 (.005)	-.337 (.001)	.60 (.004)	.59 (.003)	-.008 (.005)	0
	T-Garch (-97.54)	1	1	.007 (.004)	-.012 (.007)	-.286 (.001)	.50 (.003)	.66 (.002)	0	.079 (.011)
	A-Garch (-97.48)	1	1	.005 (.003)	-.014 (.007)	-.289 (.001)	.51 (.003)	.66 (.002)	.008 (.006)	.071 (.010)
Exponential Class	SE-Garch (-123.44)	0	1	.012 (.004)	.001 (.012)	-.369 (.002)	.54 (.003)	.93 (.002)	0	0
	AE-Garch (-120.93)	0	1	.000 (.004)	-.002 (.015)	-.333 (.002)	.49 (.002)	.94 (.002)	.097 (.087)	0
	E-Garch (-112.81)	0	1	.000 (.005)	.000 (.010)	-.368 (.002)	.54 (.003)	.93 (.002)	0	.231 (.012)
	FE-Garch (-112.80)	0	1	-.000 (.004)	.000 (.010)	-.367 (.002)	.54 (.003)	.94 (.002)	.002 (.007)	.230 (.012)

Class	Model (likelihood)	λ (se)	v (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)
Nonlinear Power Class	N-Garch (-93.06)	.65 (.004)	λ	.027 (.002)	-.023 (.005)	-.444 (.001)	.68 (.003)	.65 (.004)	0	0
	NP-Garch (-91.87)	.64 (.004)	λ	-.009 (.002)	-.032 (.004)	-.451 (.001)	.69 (.003)	.63 (.003)	.05 (.005)	0
	AP-Garch (-91.25)	.57 (.003)	λ	.037 (.002)	-.022 (.004)	-.567 (.001)	.84 (.003)	.58 (.003)	0	-.071 (.010)
	FP-Garch (-89.17)	.53 (.003)	λ	.008 (.002)	-.009 (.003)	-.664 (.002)	.98 (.004)	.53 (.003)	.097 (.004)	-.214 (.009)
Free-Garch Class	SF-Garch (-91.38)	.41 (.007)	.58 (.007)	.027 (.002)	-.022 (.005)	-.510 (.001)	.75 (.003)	.58 (.007)	0	0
	AF-Garch (-85.56)	.42 (.007)	.57 (.007)	.022 (.002)	-.028 (.005)	-.512 (.001)	0.75 (.003)	.70 (.003)	.029 (.005)	0
	TF-Garch (-78.09)	.39 (.007)	.66 (.009)	-.005 (.002)	-.020 (.007)	-.363 (.001)	.56 (.002)	.79 (.002)	0	.133 (.013)
	Free-Garch (-76.54)	.10 (.007)	.54 (.006)	.007 (.002)	.035 (.006)	-.411 (.001)	.60 (.002)	.89 (.002)	-.087 (.005)	.196 (.014)

The table presents the estimates of the nested GARCH model for the Bankers' Acceptance (BA) time series. The parameters estimates are obtained by estimating the mean equation (3.1), and the variance equations (3.7) or (3.8). The sample period starts on January 3, 1995 and ends on March 31 , 1999 for a total of 1,069 observations. The numbers in parentheses are asymptotic standard errors.

Table 3.3b
Bankers' Acceptance [BA]
Positivity and Stationarity of the Estimated Covariance Matrix under Normality

Class	Model	ω	α/λ	δ	|R|	$E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}]$	skewness	Kurtosis
		Positivity and Stationarity Conditions						
Standard Class	Garch	0.10	1.17	0.11	0	2.4631	1.6	28.39
Standard Class	NA-Garch	0.09	1.11	0.12	0	2.3567	1.44	28.13
Standard Class	GJR-Garch	0.1	1.19	0.1	0.1	2.5156	1.28	27.35
Standard Class	AS - Garch	0.1	1.21	0.1	0.1	2.5626	1.27	27.21
Nonlinear Class	SA-Garch	0.07	0.59	0.59	0	1.2727	0.57	33
Nonlinear Class	AN-Garch	0.07	0.60	0.59	0	1.2755	0.55	33
Nonlinear Class	T-Garch	0.05	0.50	0.66	0.1	1.2293	0.64	33.68
Nonlinear Class	A-Garch	0.05	0.51	0.66	0.1	1.2336	0.65	33.67
Exponential Class	SE-Garch	na	na	na	na	0.9302	-1.1	44.7
Exponential Class	AE-Garch	na	na	na	na	0.9437	-1.02	43.93
Exponential Class	E-Garch	na	na	na	na	0.9316	-0.36	36.63
Exponential Class	FE-Garch	na	na	na	na	0.9318	-0.36	36.64

Model		Positivity and Stationarity Conditions					skewness	Kurtosis
		ω	α/λ	δ	$\lvert R \rvert$	$E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}]$		
Nonlinear Power Class	N-Garch	0.08	0.68	0.65	0	1.0936	0.83	31.89
	NP-Garch	0.08	0.69	0.63	0	1.08667	0.95	31.97
	AP-Garch	0.09	0.84	0.58	0	1.05211	0.88	29.94
	FP-Garch	0.112	0.98	0.53	0.2	1.03831	0.95	28.81
Free-Garch Class	SF-Garch	0.09	0.75	0.58	0	0.89279	0.94	30.87
	AF-Garch	0.08	0.75	0.7	0	0.93233	0.9	29.62
	TF-Garch	0.07	0.56	0.79	0.13	0.9382	0.24	27.57
	FREE-Garch	0.07	0.6	0.89	0.19	0.34876	0.26	27.8

The table presents results regrading positivity and stationarity conditions. The positivity conditions are $\omega > 0$, $\alpha > 0$, $\delta > 0$, and $|R| < 1$. The values exhibited in column three are the results of transformation $\omega = \lambda\zeta - \delta + 1$ to arrive at the variance constant. The stationarity condition requires that $E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}] < 1$. For the E-GARCH class of models the conditional variance is found by exponentiation, this operation guarantees that the conditional variance is nonnegative, the restriction that guarantees stationarity, is $\delta < 1$, all skewness and kurtosis values are significant.

Table 3.4a
Bankers' Acceptance Futures [BAX]
Estimation of Nested GARCH Models under Normality

Class	Model (Likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)
Standard Class	Garch (-493.25)	2	2	.013 (.023)	.006 (.022)	-.123 (.004)	.15 (.008)	.42 (.032)	0	0
	NA-Garch (-493.18)	2	2	.011 (.023)	.006 (.022)	-.122 (.004)	.15 (.008)	.42 (.050)	.036 (.050)	0
	GJR-Garch (-484.61)	2	2	-.087 (.024)	.122 (.019)	-.048 (.001)	.03 (.007)	.81 (.047)	0	.814 (.047)
	AS-Garch (-484.60)	2	2	-.088 (.024)	.122 (.019)	-.048 (.001)	.03 (.001)	.78 (.007)	.007 (.036)	.835 (.047)
Nonlinear Class	SA-Garch (-479.17)	1	1	-.096 (.019)	.119 (.060)	-.075 (.022)	.11 (.032)	.86 (.048)	0	0
	AN-Garch (-479.12)	1	1	-.093 (.018)	.118 (.023)	-.077 (.001)	.11 (.002)	.86 (.005)	-.032 (.039)	0
	T-Garch (-477.16)	1	1	-.130 (.022)	.149 (.023)	-.073 (.015)	.10 (.022)	.86 (.035)	0	.263 (.052)
	A-Garch (-470.73)	1	1	-.053 (.060)	.092 (.000)	-.096 (.001)	.17 (.004)	.80 (.010)	-.416 (.052)	.539 (.023)
Exponential Class	SE-Garch (-480.53)	0	1	-.121 (.020)	.148 (.024)	-.121 (.002)	.17 (.003)	.95 (.004)	0	0
	AE-Garch (-480.52)	0	1	-.113 (.011)	.139 (.022)	-.121 (.000)	.17 (.001)	.95 (.003)	-.019 (.054)	0
	E-Garch (-479.11)	0	1	-.146 (.022)	.170 (.024)	-.114 (.002)	.16 (.003)	.94 (.004)	0	.208 (.054)
	FE-Garch (-475.83)	0	1	-.095 (.019)	.125 (.025)	-.135 (.002)	.20 (.003)	.93 (.004)	.289 (.046)	.372 (.028)

Class	Model (Likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)
Nonlinear Power Class	N-Garch (-478.12)	0.56 (.015)	λ	-.004 (.014)	.012 (.018)	-.171 (.001)	.24 (.002)	.80 (.006)	0	0
	NP-Garch (-478.00)	0.55 (.015)	λ	.006 (.007)	.010 (.009)	-.177 (.001)	.24 (.002)	.80 (.006)	-.018 (.010)	0
	AP-Garch (-476.49)	0.53 (.014)	λ	-.003 (.010)	.009 (.033)	-.181 (.065)	.25 (.079)	.80 .(058)	0	.066 (.122)
	FP-Garch (-465.03)	0.66 (.014)	λ	-.027 (.018)	.055 (.022)	-.136 (.001)	.24 (.003)	.81 (.005)	-.451 (.032)	.613 (.014)
Free-Garch Class	SF-Garch (-473.42)	0.36 (.021)	0.322 (.008)	-.100 (.014)	.132 (.014)	-.263 (.001)	.33 (.001)	.83 (.005)	0	0
	AF-Garch (-473.29)	0.36 (.013)	0.35 (.186)	-.101 (.018)	.130 (.027)	-.240 (.201)	.30 (.228)	.84 (.342)	.003 (.041)	0
	TF-Garch (-473.13)	0.35 (.033)	0.50 (.017)	-.132 (.014)	.154 (.016)	-.136 (.001)	.18 (.001)	.89 (.028)	0	.130 (.050)
	Free-Garch (-459.06)	0.45 (.012)	0.20 (.0 02)	.020 (.021)	.009 (.023)	-.371 (.001)	.51 (.004)	.77 (.004)	-.499 (.011)	.81 (.004)

The table presents the estimates of the nested GARCH model for the Bankers' Acceptance (BA) time series. The parameters estimates are obtained by estimating the mean equation (3.1), and the variance equations (3.7) or (3.8). The sample period starts on January 3, 1995 and ends on March 31 , 1999 for a total of 1,069 observations. The numbers in parentheses are the asymptotic standard errors.

Table 3.4b
Bankers' Acceptance Futures [BAX]
Positivity and Stationarity of the Estimated Covariance Matrix under Normality

Model		Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	$\|R\|$	$E[(\alpha\lambda f^{\nu}(\varepsilon_t)+\delta)^{2/\lambda}]$		
Standard Class	Garch	0.3308	0.15	0.42	0	0.73356	-0.68	7.07
	NA-Garch	0.3321	0.15	0.42	0	0.7321	-0.68	7.09
	GJR-Garch	0.1257	0	0.81	0.81	0.87674	-0.71	6.86
	AS - Garch	0.1256	0	0.78	0.83	0.87689	-0.71	6.86
Nonlinear Class	SA-Garch	0.058	0.11	0.86	0	0.91469	-0.98	7.9
	AN-Garch	0.058	0.11	0.86	0	0.9152	-0.98	7.88
	T-Garch	0.069	0.1	0.86	0.26	0.89195	-0.94	7.75
	A-Garch	0.1022	0.17	0.8	0.53	0.8415	-0.77	6.91
Exponential Class	SE-Garch	na	na	na	na	0.9525	-1.03	8.03
	AE-Garch	na	na	na	na	0.9528	-1.03	8.04
	E-Garch	na	na	na	na	0.9488	-0.99	7.86
	FE-Garch	na	na	na	na	0.9308	-0.86	7.37

Model		Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	$\|R\|$	$E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}]$		
Nonlinear Power Class	N-Garch	0.095	0.24	0.8	0	0.74705	-0.93	7.7
	NP-Garch	0.098	0.24	0.8	0	0.7347	-0.93	7.67
	AP-Garch	0.097	0.25	0.8	0.10	0.72479	-0.94	7.83
	FP-Garch	0.091	0.24	0.81	0.61	0.79264	-0.78	7.03
Free-Garch Class	SF-Garch	0.073	0.33	0.83	0	0.69796	-0.99	8.03
	AF-Garch	0.071	0.3	0.84	0	0.69756	-0.99	8.09
	TF-Garch	0.057	0.18	0.89	0.13	0.74482	-1.03	8.41
	FREE-Garch	0.059	0.51	0.77	0.81	0.81675	-0.85	7.73

The table presents results regrading positivity and stationarity conditions. The positivity conditions are $\omega > 0$, $\alpha > 0$, $\delta > 0$, and $|R| < 1$. The values exhibited in column three are the results of transformation $\omega = \lambda\zeta - \delta + 1$ to arrive at the variance constant. The stationarity condition requires that $E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}] < 1$. For the E-GARCH class of models, the conditional variance is found by exponentiation. This operation guarantees that the conditional variance is nonnegative, the restriction that guarantees stationarity is, $\delta < 1$, all skewness and kurtosis values are significant.

Table 3.5a
Bankers' Acceptance [BA]
Estimation of Nested GARCH Models under t-Distribution

Class	Model (Likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)
Standard Class	Garch (-174.59)	2	2	.004 (.011)	.002 (.026)	-.268 (.001)	.28 (.016)	.43 (.004)	0	0
	NA-Garch (-174.43)	2	2	.005 (.011)	.005 (.026)	-.271 (.002)	.28 (.017)	.43 (.004)	-.038 (.052)	0
	GJR-Garch (-174.58)	2	2	.044 (.011)	.002 (.026)	-.269 (.001)	.28 (.019)	.43 (.003)	0	-.001 (.084)
	AS-Garch (-174.24)	2	2	.004 (.001)	.007 (.026)	-.261 (.001)	.27 (.016)	.45 (.004)	-.089 (.053)	.060 (.044)
Nonlinear Class	SA-Garch (-157.26)	1	1	-.002 (.010)	.049 (.039)	-.180 (.001)	.21 (.006)	.79 (.004)	0	0
	AN-Garch (-156.91)	1	1	-.006 (.011)	.054 (.038)	-.176 (.001)	.21 (.006)	.80 (.002)	.054 (.069)	0
	T-Garch (-157.04)	1	1	-.002 (.010)	.049 (.039)	-.175 (.001)	.21 (.006)	.80 (.002)	0	.056 (.063)
	A-Garch (-156.87)	1	1	-.006 (.009)	.055 (.032)	-.176 (.001)	.21 (.006)	0.80 (.002)	.053 (.031)	.022 (.064)
Exponential Class	SE-Garch (-167.29)	0	1	-.002 (.004)	.034 (.024)	-.285 (.008)	.27 (.009)	.95 (.004)	0	0
	AE-Garch (-166.98)	0	1	-.004 (.012)	.031 (.035)	-.282 (.006)	.27 (.008)	.95 (.003)	.066 (.098)	0
	E-Garch (-167.28)	0	1	-.002 (.012)	.035 (.042)	-.285 (.008)	.27 (.009)	.95 (.004)	0	-.002 (.073)
	FE-Garch (-166.91)	0	1	-.004 (.012)	.032 (.040)	-.281 (.008)	.27 (.009)	.95 (.004)	.070 (.101)	.028 (.072)

Class	Model (Likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)
Nonlinear Power Class	N-Garch (-155.68)	.62 (.011)	λ	.003 (.006)	.054 (.034)	-.237 (.002)	.29 (.005)	.83 (.003)	0	0
	NP-Garch (-154.81)	.67 (.012)	λ	-.007 (.007)	.062 (.032)	-.233 (.002)	.28 (.006)	.82 (.001)	-.027 (.027)	0
	AP-Garch (-154.75)	.71 (.027)	λ	-.000 (.005)	.056 (.037)	-.208 (.001)	.25 (.012)	.82 (.001)	0	.067 (.075)
	FP-Garch (-154.59)	.70 (.013)	λ	-.008 (.009)	.067 (.034)	-.211 (.002)	.26 (.005)	.83 (.003)	-.038 (.033)	.099 (.066)
Free-Garch Class	SF-Garch (-155.40)	.43 (.033)	.70 (.149)	-.003 (.002)	.070 (.024)	-.254 (.006)	.30 (.017)	.85 (.014)	0	0
	AF-Garch (-152.75)	.51 (.013)	.63 (.039)	-.006 (.008)	.076 (.031)	-.237 (.002)	.29 (.003)	.85 (.003)	.09 (.036)	0
	TF-Garch (-154.64)	.75 (.015)	.74 (.051)	-.011 (.008)	.069 (.039)	-.181 (.001)	.22 (.005)	.84 (.002)	0	.092 (.070)
	Free-Garch (-152.40)	.15 (.118)	.53 (.089)	-.006 (.007)	.084 (.027)	-.275 (.099)	.33 (.137)	.93 (.221)	.100 (.038)	-.025 (.098)

The table presents the estimates of the nested GARCH model for the Bankers' Acceptance (BA) time series. The parameters estimates are obtained by estimating the mean equation (3.1), and the variance equations (3.7) or (3.8). The degrees of freedom were set at five to insure a sensible distribution. The sample period starts on January 3, 1995 and ends on March 31, 1999 for a total of 1,069 observations. The numbers in parentheses are asymptotic standard errors.

Table 3.5b
Bankers' Acceptance [BA]
Positivity and Stationarity of the Estimated Covariance Matrix under the t-Distribution

	Model	Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	|R|	$E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}]$		
Standard Class	Garch	0.031	0.28	0.43	0	1.36786	0.56	36.07
	NA-Garch	0.032	0.28	0.43	0	1.36768	0.43	36.1
	GJR-Garch	0.031	0.28	0.43	0	1.36882	0.56	36.11
	AS - Garch	0.03	0.27	0.45	0.1	1.34604	0.35	34.84
Nonlinear Class	SA-Garch	0.022	0.21	0.79	0	1.03519	-1.52	42.53
	AN-Garch	0.02	0.21	0.8	0	1.03827	-1.4	41.24
	T-Garch	0.021	0.21	0.8	0.1	1.03416	-1.41	40.91
	A-Garch	0.02	0.21	0.8	0	1.03801	-1.34	40.51
Exponential Class	SE-Garch	na	na	na	na	0.9526	-2.4	52.32
	AE-Garch	na	na	na	na	0.9555	-2.26	50.31
	E-Garch	na	na	na	na	0.9527	-2.41	52.45
	FE-Garch	na	na	na	na	0.9552	-2.32	51.32

Model		Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	$\lvert R\rvert$	$E[(\alpha\lambda f^{v}(\varepsilon_t)+\delta)^{2/\lambda}]$		
Nonlinear Power Class	N-Garch	0.025	0.29	0.83	0	0.9853	-1.18	37.55
	NP-Garch	0.026	0.28	0.82	0	0.99694	-1.48	39.78
	AP-Garch	0.024	0.25	0.82	0.1	0.99571	-1.3	37.15
	FP-Garch	0.022	0.26	0.83	0.1	1.00654	-1.31	38.48
Free-Garch Class	SF-Garch	0.041	0.3	0.85	0	0.88563	-0.91	36.6
	AF-Garch	0.024	0.29	0.85	0	0.97689	-1.11	34.66
	TF-Garch	0.019	0.22	0.84	0.1	1.00685	-1.65	39.76
	FREE-Garch	0.0314	0.33	0.93	0	0.70604	-1.03	33.61

The table presents results regrading positivity and stationarity conditions. The positivity conditions are $\omega > 0$, $\alpha > 0$, $\delta > 0$, and $|R| < 1$. The values exhibited in column three are the results of transformation $\omega = \lambda\zeta - \delta + 1$ to arrive at the variance constant. The stationarity condition requires that $E[(\alpha\lambda f^{v}(\varepsilon_t) + \delta)^{2/\lambda}] < 1$.For the E-GARCH class of models, the conditional variance is found by exponentiation. This operation guarantees that the conditional variance is nonnegative, the restriction that guarantees stationarity is, $\delta < 1$.

Table 3.6a
Bankers' Acceptance Futures [BAX]
Estimation of Nested GARCH Models under the t-Distribution

	Model (Likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)
Standard Class	Garch (-703.01)	2	2	.009 (.022)	.060 (.029)	-.038 (.001)	.03 (.002)	.90 (.004)	0	0
	NA-Garch (-701.72)	2	2	.002 (.022)	.069 (.027)	-.042 (.001)	.03 (.000)	.90 (.004)	.375 (.105)	0
	GJR-Garch (-701.38)	2	2	.001 (.022)	.072 (.028)	-.041 (.001)	.03 (.005)	.89 (.005)	0	.202 (.099)
	AS-Garch (-700.86)	2	2	-.003 (.037)	.077 (.054)	-.044 (.010)	.03 (.007)	.88 (.030)	.262 (.261)	.172 (.071)
Nonlinear Class	SA-Garch (-693.73)	1	1	-.013 (.022)	.089 (.052)	-.081 (.002)	.10 (.003)	.89 (.006)	0	0
	AN-Garch (-693.08)	1	1	-.016 (.021)	.090 (.026)	-.084 (.002)	.10 (.003)	.89 (.006)	.182 (.093)	0
	T-Garch (-690.61)	1	1	-.024 (.021)	.100 (.026)	-.078 (.001)	.09 (.003)	.89 (.006)	0	.333 (.114)
	A-Garch (-690.39)	1	1	-.018 (.021)	.095 (.026)	-.075 (.014)	.09 (.018)	.89 (.022)	-.083 (.081)	.384 (.101)
Exponential Class	SE-Garch (-694.86)	0	1	-.013 (.022)	.091 (.030)	-.145 (.004)	.17 (.006)	.97 (.006)	0	0
	AE-Garch (-694.26)	0	1	-.018 (.021)	.094 (.027)	-.147 (.004)	.17 (.006)	.97 (.006)	.176 (.104)	0
	E-Garch (-692.54)	0	1	-.025 (.021)	.104 (.027)	-.136 (.004)	.15 (.005)	.97 (.006)	0	.262 (.116)
	FE-Garch (-692.43)	0	1	-.019 (.037)	.097 (.055)	-.133 (.023)	.15 (.027)	.97 (.012)	-.056 (.104)	.289 (.112)

Class	Model (Likelihood)	λ (se)	v (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)
Nonlinear Power Class	N-Garch (-692.69)	.66 (.036)	λ	-.018 (.020)	.104 (.028)	-.128 (.002)	.16 (.003)	.88 (.007)	0	0
	NP-Garch (-692.36)	.60 (.032)	λ	-.030 (.021)	.106 (.028)	-.138 (.002)	.17 (.003)	.88 (.007)	.028 (.066)	0
	AP-Garch (-689.46)	.71 (.100)	λ	-.032 (.000)	.114 (.005)	-.110 (.028)	.14 (.037)	.89 (.019)	0	.383 (.100)
	FP-Garch (-689.22)	.69 (.037)	λ	-.020 (.032)	.098 (.048)	-.106 (.030)	.14 (.039)	.89 (.021)	-.116 (.050)	.466 (.141)
Free-Garch Class	SF-Garch (-692.66)	.69 (.057)	.66 (.077)	-.018 (.020)	.104 (.028)	-.128 (.002)	.16 (.003)	.88 (.007)	0	0
	AF-Garch (-692.20)	.51 (.662)	.59 (.108)	-.030 (.009)	.106 (.028)	-.134 (.031)	.17 (.041)	.89 (.092)	.029 (.023)	0
	TF-Garch (-688.75)	.57 (.067)	.63 (.067)	-.038 (.018)	.131 (.024)	-.122 (.002)	.15 (.003)	.89 (.006)	0	.387 (.091)
	Free-Garch (-686.24)	.41 (.048)	.45 (.024)	-.029 (.016)	.108 (.002)	-.147 (.002)	.21 (.004)	.90 (.006)	-.391 (.043)	.728 (.048)

The table presents the estimates of the nested GARCH model for the Bankers' Acceptance (BA) time series. The parameters estimates are obtained by estimating the mean equation (3.1), and the variance equations (3.7) or (3.8). The degrees of freedom were set at five to insure a sensible distribution. The sample period starts on January 3, 1995 and ends on March 31, 1999 for a total of 1,069 observations. The numbers in parentheses are asymptotic standard errors.

Table 3.6b
Bankers' Acceptance Futures [BAX]
Positivity and Stationarity of the Estimated Covariance Matrix
under the t-Distribution

Model		Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	$\|R\|$	$E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}]$		
Standard Class	Garch	0.02	0	0.9	0	0.99831	-1.3	9.21
	NA-Garch	0.02	0	0.9	0	0.99423	-1.3	9.35
	GJR-Garch	0.02	0	0.89	0.2	0.9879	-1.18	8.83
	AS - Garch	0.02	0	0.88	0.2	0.98684	-1.19	8.98
Nonlinear Class	SA-Garch	0.03	0.1	0.89	0	0.98039	-1.23	9.37
	AN-Garch	0.03	0.1	0.89	0	0.97871	-1.26	9.67
	T-Garch	0.03	0.1	0.89	0.3	0.96878	-1.23	9.57
	A-Garch	0.03	0.1	0.89	0.4	0.96939	-1.22	9.54
Exponential Class	SE-Garch	na	na	na	na	0.9689	-1.24	9.37
	AE-Garch	na	na	na	na	0.9693	-1.27	9.37
	E-Garch	na	na	na	na	0.9667	-1.22	9.42
	FE-Garch	na	na	na	na	0.9673	-1.22	9.42

Model		Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	$\|R\|$	$E[(\alpha\lambda f^{v}(\varepsilon_t) + \delta)^{2/\lambda}]$		
Nonlinear Power Class	N-Garch	0.03	0.16	0.88	0	0.93955	-1.19	9.24
	NP-Garch	0.03	0.17	0.88	0	0.92966	-1.18	9.2
	AP-Garch	0.03	0.14	0.89	0.4	0.92724	-1.24	9.74
	FP-Garch	0.03	0.14	0.89	0.5	0.94124	-1.25	9.83
Free-Garch Class	SF-Garch	0.03	0.16	0.88	0	0.94482	-1.19	9.24
	AF-Garch	0.03	0.17	0.89	0	0.90951	-1.18	9.23
	TF-Garch	0.04	0.15	0.89	0.4	0.91092	-1.25	9.86
	FREE-Garch	0.04	0.21	0.9	0.7	0.84286	-1.06	9.25

The table presents results regrading positivity and stationarity conditions. The positivity conditions are $\omega > 0$, $\alpha > 0$, $\delta > 0$, and $|R| < 1$. The values exhibited in column three are the results of transformation $\omega = \lambda\zeta - \delta + 1$ to arrive at the variance constant. The stationarity condition requires that $E[(\alpha\lambda f^{v}(\varepsilon_t) + \delta)^{2/\lambda}] < 1$. For the E-GARCH class of models, the conditional variance is found by exponentiation. This operation guarantees that the conditional variance is nonnegative, the restriction that guarantees stationarity is, $\delta < 1$.

Table 3.7a
Bankers' Acceptance [BA]
Estimation of Nested GARCH Models under GED

Class	Model (Likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)	η (se)
Standard Class	Garch (-769.21)	2	2	.024 (.011)	-.022 (.009)	.036 (.176)	.22 (.522)	.32 (2.45)	0	0	.35 (.012)
	NA-Garch (-764.36)	2	2	.019 (.000)	-.014 (.000)	.110 (.000)	.23 (.002)	.38 (.000)	.007 (.011)	0	.33 (.011)
	GJR-Garch (-760.60)	2	2	.018 (.000)	-.011 (.000)	.159 (.000)	.16 (.000)	.50 (.000)	0	.055 (.000)	.31 (.010)
	AS-Garch (-748.41)	2	2	.017 (.000)	-.014 (.000)	.106 (.000)	.14 (.038)	.10 (.000)	.014 (.001)	-.070 (.027)	.33 (.009)
Nonlinear Class	SA-Garch (-687.88)	1	1	.000 (.000)	.000 (.000)	.171 (.040)	.15 (.065)	.55 (.076)	0	0	.31 (.000)
	AN-Garch (-672.09)	1	1	.000 (.000)	.000 (.000)	.026 (.063)	.23 (.118)	.41 (.262)	.007 (.243)	0	.31 (.000)
	T-Garch (-672.21)	1	1	.000 (.000)	.000 (.000)	.075 (.037)	.27 (.075)	.54 (.084)	0	-.017 (.367)	.31 (.004)
	A-Garch (-670.88)	1	1	.000 (.000)	.000 (.000)	.181 (.081)	.25 (.138)	.45 (.151)	.026 (.252)	.001 (.422)	.30 (.006)
Exponential Class	SE-Garch (-781.73)	0	1	-.024 (.000)	-.019 (.000)	.081 (.000)	.24 (.000)	.52 (.000)	0	0	.35 (.012)
	AE-Garch (-780.57)	0	1	.023 (.000)	-.017 (.000)	.124 (.000)	.27 (.000)	.45 (.000)	-.020 (.000)	0	.35 (.008)
	E-Garch (-780.81)	0	1	.023 (.000)	-.017 (.000)	.124 (.000)	.27 (.000)	.45 (.000)	0	-.020 (.001)	.35 (.012)
	FE-Garch (-779.83)	0	1	.021 (.000)	-.016 (.000)	.116 (.000)	.33 (.001)	.40 (.000)	.000 (.000)	.023 (.001)	.35 (.012s)

Class	Model (Likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)	η (se)
Nonlinear Power Class	N-Garch (-669.35)	.95 (.006)	λ	.000 (.000)	-.000 (.000)	.123 (.071)	.21 (.124)	.44 (.185)	0	0	.30 (.006)
	NP-Garch (-667.89)	.88 (.413)	λ	.000 (.000)	.000 (.000)	.164 (.080)	.25 (.137)	.40 (.173)	-.004 (.218)	0	.30 (.000)
	AP-Garch (-666.00)	.99 (.557)	λ	.000 (.000)	.000 (.000)	.163 (.075)	.21 (.129)	.47 (.155)	0	.075 (.448)	.30 (.000)
	FP-Garch (-656.58)	.83 (.427)	λ	-.000 (.000)	.000 (.000)	.153 (.044)	.15 (.066)	.40 (.114)	-.015 (.261)	.030 (.634)	.28 (.000)
Free-Garch Class	SF-Garch (-666.75)	.92 (1.80)	1.12 (1.48)	-.000 (.000)	.000 (.000)	.185 (.082)	.20 (.137)	.39 (.191)	0	0	.30 (.006)
	AF-Garch (-659.81)	0.90 (1.30)	1.04 (.449)	-.000 (.000)	.000 (.000)	.182 (.043)	.19 (.076)	.49 (.083)	.026 (.266)	0	.29 (.004)
	TF-Garch (-655.47)	.41 (1.12)	1.10 (.409)	-.000 (.000)	.000 (.000)	.168 (.063)	.23 (.138)	.45 (.145)	0	-.057 (.346)	.29 (.005)
	Free-Garch (-641.61)	.27 (1.07)	0.96 (.344)	.000 (.000)	.000 (.000)	.186 (.069)	.24 (.145)	.36 (.172)	-.002 (.207)	.003 (.407)	.27 (.003)

The table presents the estimates of the nested GARCH model for the Bankers' Acceptance (BA) time series. The parameters estimates are obtained by estimating the mean equation (3.1), and the variance equations (3.7) or (3.8). η is the thickness parameter The sample period starts on January 3, 1995 and ends on March 31 , 1999 for a total of 1,069 observations. The numbers in parentheses are asymptotic standard errors.

Table 3.7b
Bankers' Acceptance [BA]
Positivity and Stationarity of the Estimated Covariance Matrix under GED

Model		Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	$\lvert R\rvert$	$E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}]$		
Standard Class	Garch	0.7478	0.2	0.3	0	0.77987	0.25	50.26
	NA-Garch	0.8405	0.2	0.4	0	0.86906	0.13	48.91
	GJR-Garch	0.8369	0.2	0.5	0.05	0.8803	-0.1	46.09
	AS - Garch	1.1156	0.1	0.1	0.07	0.37347	0.42	49.1
Nonlinear Class	SA-Garch	0.6238	0.2	0.6	0	0.4415	-0.25	40.48
	AN-Garch	0.6167	0.2	0.4	0	0.4452	0.15	42.73
	T-Garch	0.5319	0.3	0.5	0.01	0.54122	-0.08	41.43
	A-Garch	0.7261	0.3	0.5	0.00	0.6009	-0.09	40.72
Exponential Class	SE-Garch	na	na	na	na	0.5204	-0.06	42.62
	AE-Garch	na	na	na	na	0.4546	0.02	42.96
	E-Garch	na	na	na	na	0.4545	0.04	42.88
	FE-Garch	na	na	na	na	0.4002	0.21	43.51

Model		Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	$\|R\|$	$E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}]$		
Nonlinear Power Class	N-Garch	0.6816	0.2	0.4	0	0.4503	0	41.14
	NP-Garch	0.746	0.3	0.4	0	0.5252	0.04	40.44
	AP-Garch	0.6911	0.2	0.5	0.07	0.5490	0	40.5
	FP-Garch	0.7237	0.2	0.4	0.03	0.4814	-0.21	40.14
Free-Garch Class	SF-Garch	0.7843	0.2	0.4	0	0.5225	0.12	42.26
	AF-Garch	0.6685	0.2	0.5	0	0.6143	0.00	40.59
	TF-Garch	0.6189	0.2	0.5	0.05	0.4964	0.19	41.16
	FREE-Garch	0.6867	0.2	0.4	0.00	0.3867	0.33	33.15

The table presents results regrading positivity and stationarity conditions. The positivity conditions are $\omega > 0$, $\alpha > 0$, $\delta > 0$, and $|R| < 1$. The values exhibited in column three are the results of transformation $\omega = \lambda\zeta - \delta + 1$ to arrive at the variance constant. The stationarity condition requires that $E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}] < 1$. For the E-GARCH class of models, the conditional variance is found by exponentiation. This operation guarantees that the conditional variance is nonnegative, the restriction that guarantees stationarity is, $\delta < 1$.

Table 3.8a
Bankers' Acceptance Futures [BAX]
Estimation of Nested GARCH Models under GED

	Model (Likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)	η (se)
Standard Class	Garch (-1322.69)	2	2	.000 (.006)	-.001 (.009)	-.127 (.019)	.15 (.035)	.53 (.096)	0	0	.89 (.034)
	NA-Garch (-1322.39)	2	2	.000 (.016)	-.002 (.014)	-.127 (.009)	.14 (.018)	.54 (.050)	.126 (.134)	0	.90 (.044)
	GJR-Garch (-1322.39)	2	2	.000 (.016)	.000 (.014)	-.147 (.011)	.17 (.023)	.45 (.062)	0	.128 (.087)	.90 (.044)
	AS-Garch (-1317.44)	2	2	.000 (.017)	.000 (.000)	-.031 (.003)	.03 (.003)	.90 (.007)	.169 (.139)	.166 (.110)	.91 (.042)
Nonlinear Class	SA-Garch (-1312.14)	1	1	-.004 (.019)	.006 (.019)	-.078 (.003)	.11 (.005)	.88 (.011)	0	0	.93 (.041)
	AN-Garch (-1312.00)	1	1	-.003 (.016)	.009 (.018)	-.083 (.002)	.11 (.005)	.88 (.010)	.103 (.145)	0	.92 (.040)
	T-Garch (-1310.14)	1	1	-.006 (.016)	.014 (.018)	-.079 (.002)	.11 (.005)	.88 (.010)	0	.306 (.139)	.92 (.040)
	A-Garch (-1309.92)	1	1	-.008 (.015)	.018 (.017)	-.075 (.003)	.11 (.005)	.88 (.009)	-.093 (.101)	.384 (.107)	.92 (.041)
Exponential Class	SE-Garch (-1312.59)	0	1	-.000 (.015)	.000 (.015)	-.143 (.006)	.19 (.009)	.96 (.010)	0	0	.91 (.040)
	AE-Garch (-1312.56)	0	1	-.000 (.015)	.000 (.016)	-.143 (.006)	.19 (.009)	.96 (.010)	.029 (.153)	0	.91 (.039)
	E-Garch (-1311.37)	0	1	.000 (.014)	.000 (.016)	-.129 (.005)	.17 (.008)	.96 (.010)	0	.237 (.150)	.91 (.039)
	FE-Garch (-1311.36)	0	1	.000 (.015)	.000 (.017)	-.132 (.005)	.18 (.008)	.96 (.009)	-.004 (.140)	.232 (.148)	.91 (.040)

	Model (Likelihood)	λ (se)	ν (se)	B_0 (se)	B_1 (se)	ζ (se)	α/λ (se)	δ (se)	S (se)	R (se)	η (se)
Nonlinear Power Class	N-Garch (-1311.69)	.48 (.029)	λ	-.001 (.012)	.003 (.014)	-.191 (.003)	.25 (.005)	.86 (.012)	0	0	.92 (.038)
	NP-Garch (-1311.08)	.60 (.040)	λ	-.000 (.013)	.000 (.017)	-.127 (.003)	.17 (.004)	.88 (.010)	-.004 (.034)	0	.93 (.038)
	AP-Garch (-1309.91)	.41 (.022)	λ	-.005 (.009)	.015 (.014)	-.168 (.002)	.22 (.004)	.89 (.009)	0	.31 (.103)	.93 (.038)
	FP-Garch (-1309.70)	.49 (.026)	λ	-.017 (.014)	.035 (.016)	-.190 (.003)	.25 (.005)	.86 (.011)	.018 (.043)	.486 (.026)	.93 (.038)
Free-Garch Class	SF-Garch (-1311.60)	.50 (.323)	.41 (.044)	-.002 (.220)	.002 (.156)	-.207 (.019)	.27 (.016)	.84 (.059)	0	0	.93 (.711)
	AF-Garch (-1310.43)	.15 (.086)	.61 (.072)	-.000 (.013)	.000 (.016)	-.125 (.003)	.17 (.004)	.94 (.010)	-.006 (.041)	0	.90 (.038)
	TF-Garch (-1309.20)	.26 (.066)	.46 (.040)	-.011 (.012)	.030 (.016)	-.163 (.003)	.21 (.004)	.91 (.010)	0	.303 (.103)	.93 (.038)
	Free-Garch (-1308.70)	.68 (.076)	.57 (.057)	-.006 (.014)	.015 (.017)	-.130 (.002)	.18 (.004)	.87 (.009)	-.010 (.049)	.315 (.104)	.93 (.038)

The table presents the estimates of the nested GARCH model for the Bankers' Acceptance (BA) time series. The parameters estimates are obtained by estimating the mean equation (3.1), and the variance equations (3.7) or (3.8). η is the thickness parameter. The sample period starts on January 3, 1995 and ends on March 31, 1999 for a total of 1,069 observations. The numbers in parentheses are asymptotic standard errors.

Table 3.8b
Bankers' Acceptance Futures [BAX]
Positivity and Stationarity of the Estimated Covariance Matrix under GED

Class	Model	Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	$\lvert R\rvert$	$E[(\alpha\lambda f^{v}(\varepsilon_t)+\delta)^{2/\lambda}]$		
Standard Class	Garch	0.209	0.2	0.5	0	0.82742	-0.79	7.29
	NA-Garch	0.201	0.1	0.5	0	0.82695	-0.78	7.48
	GJR-Garch	0.251	0.2	0.5	0.1	0.81214	-0.74	7.49
	AS - Garch	0.03	0	0.9	0.2	0.97048	-1.2	8.87
Nonlinear Class	SA-Garch	0.04	0.1	0.9	0	0.92509	-1.13	8.73
	AN-Garch	0.04	0.1	0.9	0	0.92796	-1.16	8.99
	T-Garch	0.04	0.1	0.9	0.3	0.91993	-1.13	8.95
	A-Garch	0.04	0.1	0.9	0.4	0.92591	-1.13	8.96
Exponential Class	SE-Garch	na	na	na	na	0.9626	-1.18	8.98
	AE-Garch	na	na	na	na	0.9629	-1.18	9.03
	E-Garch	na	na	na	na	0.9638	-1.17	9.05
	FE-Garch	na	na	na	na	0.9629	-1.17	9.05

Model		Positivity and Stationarity					skewness	kurtosis
		ω	α/λ	δ	$\|R\|$	$E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}]$		
Nonlinear Power Class	N-Garch	0.05	0.3	0.9	0	0.80194	-1.13	8.73
	NP-Garch	0.04	0.2	0.9	0	0.82845	-1.15	8.93
	AP-Garch	0.04	0.2	0.9	0.3	0.82309	-1.2	9.36
	FP-Garch	0.05	0.3	0.9	0.5	0.81611	-1.17	9.37
Free-Garch Class	SF-Garch	0.05	0.3	0.8	0	0.80652	-1.12	8.66
	AF-Garch	0.04	0.2	0.9	0	0.67015	-1.2	9.35
	TF-Garch	0.05	0.2	0.9	0.3	0.69985	-1.17	9.36
	FREE-Garch	0.04	0.2	0.9	0.3	0.89357	-1.2	9.39

The table presents results regrading positivity and stationarity conditions. The positivity conditions are $\omega > 0$, $\alpha > 0$, $\delta > 0$, and $|R| < 1$. The values exhibited in column three are the results of transformation $\omega = \lambda\zeta - \delta + 1$ to arrive at the variance constant. The stationarity condition requires that $E[(\alpha\lambda f^{\nu}(\varepsilon_t) + \delta)^{2/\lambda}] < 1$. For the E-GARCH class of models, the conditional variance is found by exponentiation. This operation guarantees that the conditional variance is nonnegative, the restriction that guarantees stationarity is, $\delta < 1$.

Table 3.9a
Bankers' Acceptance [BA]
Likelihood Ration Test for Asymmetry under Normality

Maintained Hypothesis	H_0	H_a S = free, R = 0	H_a S = 0, R = free	H_a S = free, R = free
Standard Class $\lambda = 2, \nu = 2$	S = R = 0	3.74 (sg.lv. = .053)	6.20 (sg.lv. = .012)	6.36 (sg.lv. = .041)
	S = free, R = 0			2.62 (sg.lv. = .105)
	S = 0, R = free			0.16 (sg.lv. = .689)
Nonlinear Class $\lambda = 1, \nu = 1$	S = R = 0	0.08 (sg.lv. = .777)	1.36 (sg.lv. = .243)	1.48 (sg.lv. = .477)
	S = free, R = 0			1.40 (sg.lv. = .236)
	S = 0, R = free			0.12 (sg.lv. = .729)
Exponential Class $\lambda = 0, \nu = 1$	S = R = 0	5.00 (sg.lv. = .025)	21.24 (sg.lv. = .000)	21.26 (sg.lv. = .000)
	S = free, R = 0			16.26 (sg.lv. = .000)
	S = 0, R = free			0.02 (sg.lv. = .887)
Nonlinear Power Class $\lambda = \nu$	S = R = 0	2.38 (sg.lv. = .122)	3.60 (sg.lv. = .057)	7.78 (sg.lv. = .020)
	S = free, R = 0			5.40 (sg.lv. =.020)
	S = 0, R = free			4.81 (sg.lv. = .040)
Free-GARCH Class λ free, ν free	S = R = 0	11.66 (sg.lv. =.000)	26.56 (sg.lv. = .000)	29.68 (sg.lv. = .000)
	S = free, R = 0			18.04 (sg.lv. = .000)
	S = 0, R = free			3.12 (sg.lv. = .077)

Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.9b
Bankers' Acceptance Futures [BAX]
Likelihood Ration Test for Asymmetry under Normality

Maintained Hypothesis	H_0	H_a S = free, R= 0	H_a S = 0, R = free	H_a S = free, R = free
Standard Class $\lambda = 2, \nu = 2$	S = R = 0	0.14 (sg.lv. = .708)	17.28 (sg.lv. = .000)	17.30 (sg.lv. = .000)
	S = free, R = 0			17.16 (sg.lv. = .000)
	S = 0, R = free			0.02 (sg.lv. = .887)
Nonlinear Class $\lambda = 1, \nu = 1$	S = R = 0	0.12 (sg.lv. = .729)	4.02 (sg.lv. = .044)	16.90 (sg.lv. = .000)
	S = free, R = 0			16.78 (sg.lv. = .000)
	S = 0, R = free			12.88 (sg.lv. = .000)
Exponential Class $\lambda = 0, \nu = 1$	S = R = 0	0.02 (sg.lv. = .887)	2.84 (sg.lv. = .091)	9.40 (sg.lv. = .009)
	S = free, R = 0			9.38 (sg.lv. = .002)
	S = 0, R = free			6.56 (sg.lv. = .010)
Nonlinear Power Class $\lambda = \nu$	S = R = 0	0.24 (sg.lv. = .624)	3.26 (sg.lv. = .070)	26.18 (sg.lv. = .000)
	S = free, R = 0			25.94 (sg.lv. =.000)
	S = 0, R = free			22.92 (sg.lv. = .000)
Free-GARCH Class λ free, ν free	S = R = 0	0.26 (sg.lv. =.610)	0.60 (sg.lv. = .438)	28.74 (sg.lv. = .000)
	S = free, R = 0			28.46 (sg.lv. = .000)
	S = 0, R = free			28.14 (sg.lv. = .000)

Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.10a
Bankers' Acceptance [BA]
Likelihood Ration Test for Asymmetry under the t-Distribution

Maintained Hypothesis	H_0	H_a S = free, R = 0	H_a S = 0, R = free	H_a S = free, R = free
Standard Class $\lambda = 2, \nu = 2$	S = R = 0 S = free, R = 0 S = 0, R = free	0.32 (sg.lv. = .571)	0.02 (sg.lv. = .887)	0.70 (sg.lv. = .704) 0.38 (sg.lv. = .708) 0.68 (sg.lv. = .409)
Nonlinear Class $\lambda = 1, \nu = 1$	S = R = 0 S = free, R = 0 S = 0, R = free	0.74 (sg.lv. = .389)	0.48 (sg.lv. = .488)	0.84 (sg.lv. = .657) 0.10 (sg.lv. = .751) 0.36 (sg.lv. = .548)
Exponential Class $\lambda = 0, \nu = 1$	S = R = 0 S = free, R = 0 S = 0, R = free	0.62 (sg.lv. = .431)	0.02 (sg.lv. = .887)	0.76 (sg.lv. = .683) 0.14 (sg.lv. = .708) 0.74 (sg.lv. = .389)
Nonlinear Power Class $\lambda = \nu$	S = R = 0 S = free, R = 0 S = 0, R = free	1.70 (sg.lv. = .184)	1.92 (sg.lv. = .165)	2.20 (sg.lv. = .138) 0.44 (sg.lv. =.507) 0.28 (sg.lv. =.593)
Free-GARCH Class λ free, ν free	S = R = 0 S = free, R = 0 S = 0, R = free	5.30 (sg.lv. =.021)	1.50 (sg.lv. = .220)	6.00 (sg.lv. = .049) 0.70 (sg.lv. = .402) 4.48 (sg.lv. = .034)

Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.10b
Bankers' Acceptance Futures [BAX]
Likelihood Ration Test for Asymmetry under the t-Distribution

Maintained Hypothesis	H_0	H_a S = free, R = 0	H_a S = 0, R = free	H_a S = free, R = free
Standard Class	S = R = 0	2.60 (sg.lv. = .106)	3.28 (sg.lv. = .070)	4.32 (sg.lv. = .037)
$\lambda = 2, v = 2$	S = free, R = 0			1.72 (sg.lv. = .189)
	S = 0, R = free			1.04 (sg.lv. = .307)
Nonlinear Class	S = R = 0	1.30 (sg.lv. = .254)	6.24 (sg.lv. = .012)	6.68 (sg.lv. = .035)
$\lambda = 1, v = 1$	S = free, R = 0			5.38 (sg.lv. = .002)
	S = 0, R = free			0.44 (sg.lv. = .507)
Exponential Class	S = R = 0	1.20 (sg.lv. = .273)	4.64 (sg.lv. = .031)	4.86 (sg.lv. = .088)
$\lambda = 0, v = 1$	S = free, R = 0			3.66 (sg.lv. = .055)
	S = 0, R = free			0.22 (sg.lv. = .639)
Nonlinear Power Class	S = R = 0	0.66 (sg.lv. = .416)	6.46 (sg.lv. = .011)	6.94 (sg.lv. = .031)
$\lambda = v$	S = free, R = 0			6.28 (sg.lv. =.001)
	S = 0, R = free			0.48 (sg.lv. =.488)
Free-GARCH Class	S = R = 0	0.92 (sg.lv. =.337)	7.82 (sg.lv. = .051)	12.84 (sg.lv. = .001)
λ free, v free	S = free, R = 0			11.92 (sg.lv. = .000)
	S = 0, R = free			5.02 (sg.lv. = .025)

Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.11a
Bankers' Acceptance [BA]
Likelihood Ration Test for Asymmetry under GED

Maintained Hypothesis	H_0	H_a S = free, R = 0	H_a S = 0, R = free	H_a S = free, R = free
Standard Class λ = 2, ν = 2	S = R = 0 S = free, R = 0 S = 0, R = free	9.70 (sg.lv. = .001)	17.24 (sg.lv. = .000)	41.60 (sg.lv. = .000) 31.78 (sg.lv. = .000) 24.36 (sg.lv. = .000)
Nonlinear Class λ = 1, ν =1	S = R = 0 S = free, R = 0 S = 0, R = free	31.60 (sg.lv. = .000)	31.34 (sg.lv. = .000)	34.02 (sg.lv. = .000) 2.42 (sg.lv. = .119) 2.68 (sg.lv. = .101)
Exponential Class λ = 0, ν =1	S = R = 0 S = free, R = 0 S = 0, R = free	2.60 (sg.lv. = .106)	2.10 (sg.lv. = .147)	4.08 (sg.lv. = .130) 1.48 (sg.lv. = .223) 1.98 (sg.lv. = .159)
Nonlinear Power Class λ = ν	S = R = 0 S = free, R = 0 S = 0, R = free	2.92 (sg.lv. = .087)	6.70 (sg.lv. = .009)	25.54 (sg.lv. = .000) 22.62 (sg.lv. =.001) 18.84 (sg.lv. =.030)
Free-GARCH Class λ free, ν free	S = R = 0 S = free, R = 0 S = 0, R = free	13.88 (sg.lv. =.000)	22.52 (sg.lv. = .000)	50.26 (sg.lv. = .000) 36.38 (sg.lv. = .000) 27.74 (sg.lv. = .000)

Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.11b
Bankers' Acceptance Futures [BAX]
Likelihood Ration Test for Asymmetry under GED

Maintained Hypothesis	H_0	H_a S = free, R = 0	H_a S = 0, R = free	H_a S = free, R = free
Standard Class λ = 2, ν = 2	S = R = 0 S = free, R = 0 S = 0, R = free	0.60 (sg.lv. = .438)	0.58 (sg.lv. = .446)	10.48 (sg.lv. = .005) 9.88 (sg.lv. = .001) 9.90 (sg.lv. = .001)
Nonlinear Class λ = 1, ν =1	S = R = 0 S = free, R = 0 S = 0, R = free	0.12 (sg.lv. = .729)	4.00 (sg.lv. = .045)	4.46 (sg.lv. = .107) 4.34 (sg.lv. = .037) 0.46 (sg.lv. = .497)
Exponential Class λ = 0, ν =1	S = R = 0 S = free, R = 0 S = 0, R = free	0.60 (sg.lv. = .806)	2.44 (sg.lv. = .118)	2.64 (sg.lv. = .292) 2.40 (sg.lv. = .121) 0.02 (sg.lv. = .887)
Nonlinear Power Class λ = ν	S = R = 0 S = free, R = 0 S = 0, R = free	1.22 (sg.lv. = .269)	3.58 (sg.lv. = .058)	3.98 (sg.lv. = .136) 2.76 (sg.lv. =.096) 0.40 (sg.lv. =.527)
Free-GARCH Class λ free, ν free	S = R = 0 S = free, R = 0 S = 0, R = free	2.34 (sg.lv. =.126)	4.80 (sg.lv. = .028)	5.80 (sg.lv. = .055) 3.46 (sg.lv. = .062) 1.00 (sg.lv. = .317)

Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.12a
Bankers' Acceptance [BA]
Likelihood Ration Test for Functional Form under Normality

H_0:	H_A: Nonlinear Power $\lambda = v$	H_A: Free Class λ = free, v = free
Standard Garch $\lambda = 2, v = 2$	49.86 (sg.lv. = .000)	75.12 (sg.lv. = .000)
Nonlinear Garch $\lambda = 1, v = 1$	16.62 (sg.lv. = .000)	41.88 (sg.lv. = .000)
Exponential Garch $\lambda = 0, v = 1$	Does not apply	72.52 (sg.lv. = .000)
Nonlinear Power Garch $\lambda = v$	Does not apply	25.26 (sg.lv. = .000)

In all of the models tested above, the asymmetry parameters S and R were freely estimated. Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.12b
Bankers' Acceptance Futures [BAX]
Likelihood Ration Test for Functional Form under Normality

H_0:	H_A: Nonlinear Power $\lambda = v$	H_A: Free Class λ = free, v = free
Standard Garch $\lambda = 2, v = 2$	19.14 (sg.lv. = .000)	51.08 (sg.lv. = .000)
Nonlinear Garch $\lambda = 1, v = 1$	11.40 (sg.lv. = .000)	23.34 (sg.lv. = .000)
Exponential Garch $\lambda = 0, v = 1$	Does not apply	33.54 (sg.lv. = .000)
Nonlinear Power Garch $\lambda = v$	Does not apply	11.94 (sg.lv. = .000)

In all of the models tested above, the asymmetry parameters S and R were freely estimated. Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.13a
Bankers' Acceptance [BA]
Likelihood Ration Test for Functional Form under the t-Distribution

H_0:	H_A: Nonlinear Power $\lambda = v$	H_A: Free Class λ = free, v = free
Standard Garch $\lambda = 2, v = 2$	39.66 (sg.lv. = .000)	44.36 (sg.lv. = .000)
Nonlinear Garch $\lambda = 1, v = 1$	4.56 (sg.lv. = .032)	8.94 (sg.lv. = .002)
Exponential Garch $\lambda = 0, v = 1$	Does not Apply	28.50 (sg.lv. = .001)
Nonlinear Power Garch $\lambda = v$	Does not Apply	4.38 (sg.lv. = .036)

In all of the models tested above, the asymmetry parameters S and R were freely estimated. Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.13b
Bankers' Acceptance Futures [BAX]
Likelihood Ration Test for Functional Form under the t-Distribution

H_0:	H_A: Nonlinear Power $\lambda = v$	H_A: Free Class λ = free, v = free
Standard Garch $\lambda = 2, v = 2$	23.28 (sg.lv. = .000)	29.24 (sg.lv. = .000)
Nonlinear Garch $\lambda = 1, v = 1$	2.34 (sg.lv. = .126)	8.30 (sg.lv. = .003)
Exponential Garch $\lambda = 0, v = 1$	Does not Apply	12.38 (sg.lv. = .000)
Nonlinear Power Garch $\lambda = v$	Does not Apply	5.96 (sg.lv. = .014)

In all of the models tested above, the asymmetry parameters S and R were freely estimated. Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.14a
Bankers' Acceptance [BA]
Likelihood Ration Test for Functional Form under GED

H_0:	H_A: Nonlinear Power $\lambda = v$	H_A: Free Class λ = free, v = free
Standard Garch $\lambda = 2, v = 2$	183.68 (sg.lv. = .000)	213.60 (sg. lv. = .000)
Nonlinear Garch $\lambda = 1, v = 1$	28.60 (sg.lv. = .000)	58.54 (sg. lv. = .000)
Exponential Garch $\lambda = 0, v = 1$	Does not Apply	276.83 (sg.lv. = .000)
Nonlinear Power Garch $\lambda = v$	Does not Apply	29.92 (sg. lv. = .000)

In all of the models tested above, the asymmetry parameters S and R were freely estimated. Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.14b
Bankers' Acceptance Futures [BAX]
Likelihood Ration Test for Functional Form under GED

H_0:	H_A: Nonlinear Power $\lambda = v$	H_A: Free Class λ = free, v = free
Standard Garch $\lambda = 2, v = 2$	15.48 (sg.lv. = .000)	17.48 (sg. lv. = .000)
Nonlinear Garch $\lambda = 1, v = 1$	0.44 (sg.lv. = .516)	2.44 (sg. lv. = .118)
Exponential Garch $\lambda = 0, v = 1$	Does not Apply	5.32 (sg.lv. = .021)
Nonlinear Power Garch $\lambda = v$	Does not Apply	2.00 (sg. lv. = .157)

In all of the models tested above, the asymmetry parameters S and R were freely estimated. Significance level at 5%, $X^2(1)$ =3.84; significance level at 2.5%, $X^2(1)$ =5.02; significance level at 1%, $X^2(1)$ =6.63. Significance level at 5%, $X^2(2)$ =5.99; significance level at 2.5%, $X^2(2)$ =7.38; significance level at 1%, $X^2(2)$ =9.21.

Table 3.15a
Bankers' Acceptance [BA]
Within and Out-of-Sample Predictive Power under Normality

	Model	Within-Sample				Out-of-Sample			
		α (SE)	β (SE)	R^2	R^2 for Logs	α^o (SE)	β^o (SE)	R^2	R^2 for Logs
Standard Class	Garch	0.72 (.164)	0.08 (.049)	0.03589	0.13636	0.22 (.065)	-0.05 (.038)	0.0153	0.0000
Standard Class	NA-Garch	0.69 (.166)	0.09 (.055)	0.03710	0.11625	0.23 (.066)	-0.05 (.040)	0.0179	0.0000
Standard Class	GJR-Garch	0.69 (.175)	0.08 (.062)	0.04106	0.13138	0.22 (.061)	-0.04 (.031)	0.0132	0.0001
Standard Class	AS - Garch	0.69 (.176)	0.08 (.062)	0.04144	0.13371	0.22 (.061)	-0.03 (.029)	0.0122	0.0001
Nonlinear Class	SA-Garch	0.56 (.157)	0.24 (.093)	0.03207	0.16039	0.32 (.102)	-0.30 (.159)	0.0518	0.0068
Nonlinear Class	AN-Garch	0.56 (.158)	0.24 (.094)	0.03212	0.13864	0.32 (.101)	-0.30 (.157)	0.0524	0.0037
Nonlinear Class	T-Garch	0.57 (.156)	0.24 (.088)	0.02723	0.12354	0.36 (.113)	-0.42 (.195)	0.0626	0.0075
Nonlinear Class	A-Garch	0.57 (.156)	0.24 (.087)	0.02761	0.13342	0.35 (.112)	-0.41 (.193)	0.0611	0.0004
Exponential Class	SE-Garch	0.84 (.178)	0.02 (.016)	0.00490	0.12962	0.40 (.126)	-0.52 (.240)	0.0512	0.0259
Exponential Class	AE-Garch	0.80 (.166)	0.05 (.036)	0.00864	0.12256	0.40 (.130)	-0.54 (.263)	0.0396	0.0041
Exponential Class	E-Garch	0.87 (.183)	0.00 (.000)	0.00034	0.09265	0.43 (.133)	-0.61 (.252)	0.0588	0.0026
Exponential Class	FE-Garch	0.88 (.181)	0.00 (.000)	0.00034	0.07356	0.43 (.133)	-0.61 (.252)	0.0585	0.0033

Class	Model	Within-Sample α (SE)	Within-Sample β (SE)	Within-Sample R^2	Within-Sample R^2 for Logs	Out-of-Sample α^O (SE)	Out-of-Sample β^O (SE)	Out-of-Sample R^2	Out-of-Sample R^2 for Logs
Nonlinear Power Class	N-Garch	0.51 (.155)	0.33 (.116)	0.02847	0.13477	0.36 (.114)	-0.45 (.198)	0.0745	0.0218
	NP-Garch	0.52 (.153)	0.34 (.111)	0.02891	0.14774	0.35 (.114)	-0.42 (.195)	0.0638	0.0015
	AP-Garch	0.49 (.157)	0.34 .(125)	0.03118	0.13302	0.35 (.108)	-0.38 (.174)	0.0732	0.0242
	FP-Garch	0.48 (.177)	0.35 (.158)	0.03594	0.11381	0.28 (.091)	-0.21 (.126)	0.0368	0.0030
Free-Garch Class	SF-Garch	0.49 (.158)	0.35 (.129)	0.03232	0.13523	0.36 (.112)	-0.41 (.185)	0.0746	0.0219
	AF-Garch	0.50 (.155)	0.34 (.121)	0.03159	0.13971	0.35 (.111)	-0.41 (.185)	0.0669	0.0137
	TF-Garch	0.52 (.155)	0.32 (.110)	0.02698	0.18553	0.38 (.124)	-0.51 (.226)	0.0714	0.0171
	FREE-Garch	0.50 (.160)	0.37 (.128)	0.02784	0.18603	0.40 (.183)	-0.77 (.347)	0.0810	0.0272

The within sample period is from January 3rd, 1995 to March 31 1999 a sample size of 1,069 observations. The out-of-sample period starts from April1st, 1999 to June 30th, 1999 for out-of-sample size of 64 observations. OLS estimates for α and β, and R^2, are reported for equation (3.31); *R^2 for logs* comes from the regression estimation of equation (3.32).

Table 3.15b
Bankers' Acceptance Futures [BAX]
Within and Out-of-Sample Predictive Power under Normality

Class	Model	Within-Sample α (SE)	Within-Sample β (SE)	Within-Sample R^2	Within-Sample R^2 for Logs	Out-of-Sample α^o (SE)	Out-of-Sample β^o (SE)	Out-of-Sample R^2	Out-of-Sample R^2 for Logs
Standard Class	Garch	0.64 (.120)	0.35 (.117)	0.01646	0.05537	0.53 (.221)	0.08 (.173)	0.00059	0.02036
	NA-Garch	0.63 (.119)	0.36 (.116)	0.01681	0.05521	0.53 (.220)	0.08 (.165)	0.00053	0.02138
	GJR-Garch	0.42 (.114)	0.56 (.128)	0.02343	0.05421	0.49 (.251)	0.13 (.328)	0.00142	0.00196
	AS - Garch	0.42 (.115)	0.57 (.128)	0.02347	0.05428	0.49 (.252)	0.13 (.330)	0.00147	0.00179
Nonlinear Class	SA-Garch	0.09 (.198)	0.91 (.229)	0.02966	0.06428	0.26 (.506)	0.46 (.790)	0.00631	0.01269
	AN-Garch	0.10 (.198)	0.90 (.227)	0.02948	0.06807	0.30 (.504)	0.40 (.779)	0.00473	0.01499
	T-Garch	0.03 (.189)	0.98 (.225)	0.03256	0.06763	0.24 (.416)	0.47 (.642)	0.01012	0.00521
	A-Garch	0.27 (.138)	0.72 (.165)	0.03077	0.07413	0.49 (.278)	0.43 (.366)	0.00965	0.01575
Exponential Class	SE-Garch	0.20 (.173)	1.03 (.256)	0.02908	0.02214	0.32 (.394)	0.42 (.714)	0.00574	0.00395
	AE-Garch	0.18 (.187)	0.86 (.226)	0.02914	0.02739	0.30 (.391)	0.46 (.716)	0.00705	0.00451
	E-Garch	0.10 (.185)	0.96 (.231)	0.03313	0.03018	0.30 (.306)	0.45 (.572)	0.01033	0.01284
	FE-Garch	0.11 (.184)	0.95 (.228)	0.03304	0.02834	0.30 (.307)	0.44 (.568)	0.01004	0.01120

Model		Within-Sample				Out-of-Sample			
		α (SE)	β (SE)	R^2	R^2 for Logs	α^O (SE)	β^O (SE)	R^2	R^2 for Logs
Nonlinear Power Class	N-Garch	0.06 (.205)	0.94 (.241)	0.03039	0.07399	0.12 (.436)	0.61 (.664)	0.02093	0.00219
	NP-Garch	0.06 (.206)	0.94 (.241)	0.03027	0.07120	0.13 (.432)	0.59 (.655)	0.02021	0.00068
	AP-Garch	0.03 (.201)	0.97 (.238)	0.03209	0.07200	0.10 (.417)	0.62 (.635)	0.02411	0.00478
	FP-Garch	0.14 (.157)	0.85 (.192)	0.03506	0.08405	0.46 (.304)	0.65 (.391)	0.02703	0.00637
Free-Garch Class	SF-Garch	0.01 (.197)	0.99 (.231)	0.03357	0.04644	-0.03 (.515)	0.88 (.859)	0.03250	0.00344
	AF-Garch	0.00 (.198)	1.00 (.232)	0.03383	0.06165	-0.02 (.522)	0.88 (.867)	0.03112	0.00586
	TF-Garch	-0.02 (.194)	1.03 (.228)	0.03553	0.06491	0.00 (.562)	0.83 (.909)	0.02315	0.00581
	FREE-Garch	0.05 (.119)	1.01 (.232)	0.03763	0.08451	-0.03 (.476)	0.89 (.820)	0.03527	0.00919

The within sample period is from January 3rd, 1995 to March 31 1999 a sample size of 1,069 observations. The out-of-sample period starts from April1st, 1999 to June 30th, 1999 for out-of-sample size of 64 observations. OLS estimates for α and β, and R^2, are reported for equation (3.31); *R^2 for logs* comes from the regression estimation of equation (3.32).

Table 3.16a
Bankers' Acceptance [BA]
Within and Out-of-Sample Predictive Power under the t-Distribution

Class	Model	Within-Sample				Out-of-Sample			
		α (SE)	β (SE)	R^2	R^2 for Logs	α^O (SE)	β^O (SE)	R^2	R^2 for Logs
Standard Class	Garch	0.62 (.164)	0.28 (.144)	0.03768	0.11812	0.26 (.076)	-0.25 (.161)	0.02519	0.00197
	NA-Garch	0.62 (.166)	0.29 (.150)	0.03868	0.12634	0.25 (.076)	-0.25 (.158)	0.02525	0.00270
	GJR-Garch	0.62 (.164)	0.28 (.145)	0.03781	0.11825	0.25 (.076)	-0.25 (.161)	0.02504	0.00198
	AS - Garch	0.62 (.162)	0.28 (.134)	0.03499	0.13051	0.27 (.080)	-0.30 (.177)	0.03037	0.00343
Nonlinear Class	SA-Garch	0.43 (.154)	0.81 (.273)	0.03241	0.17381	0.45 (.143)	-1.37 (.582)	0.06878	0.01932
	AN-Garch	0.44 (.153)	0.79 (.259)	0.03211	0.14354	0.44 (.140)	-1.31 (.566)	0.05941	0.00964
	T-Garch	0.44 (.153)	0.79 (.256)	0.03077	0.17167	0.46 (.148)	-1.46 (.607)	0.06940	0.01506
	A-Garch	0.44 (.152)	0.78 (.252)	0.03159	0.14689	0.44 (.141)	-1.33 (.571)	0.05958	0.00857
Exponential Class	SE-Garch	0.73 (.164)	0.25 (.143)	0.00953	0.15449	0.51 (.170)	-1.70 (.724)	0.06206	0.01783
	AE-Garch	0.72 (.163)	0.27 (.153)	0.01021	0.12532	0.50 (.164)	-1.60 (.696)	0.05265	0.00237
	E-Garch	0.73 (.163)	0.26 (.816)	0.00974	0.15445	0.51 (.170)	-1.69 (.723)	0.06192	0.01799
	FE-Garch	0.69 (.162)	0.33 (.186)	0.01254	0.12609	0.49 (.160)	-1.52 (.674)	0.05067	0.00684

Class	Model	Within-Sample α (SE)	β (SE)	R^2	R^2 for Logs	Out-of-Sample α^O (SE)	β^O (SE)	R^2	R^2 for Logs
Nonlinear Power Class	N-Garch	0.37 (.159)	1.04 (.322)	0.02942	0.18577	0.48 (.157)	-1.66 (.667)	0.08211	0.01702
	NP-Garch	0.36 (.147)	1.05 (.324)	0.02861	0.15557	0.49 (.159)	-1.78 (.704)	0.09265	0.01088
	AP-Garch	0.38 (.150)	1.02 (.324)	0.02861	0.15557	0.49 (.159)	-1.78 (.704)	0.07329	0.01162
	FP-Garch	0.39 (.147)	0.97 (.294)	0.02688	0.15716	0.49 (.160)	-1.74 (.716)	0.08260	0.00551
Free-Garch Class	SF-Garch	0.38 (.147)	1.02 (.313)	0.03178	0.18484	0.46 (.148)	-1.53 (.616)	0.08023	0.02114
	AF-Garch	0.38 (.146)	1.02 (.313)	0.02813	0.17254	0.44 (.142)	-1.33 (.572)	0.05464	0.00198
	TF-Garch	0.38 (.145)	1.02 (.305)	0.02804	0.14685	0.49 (.165)	-1.81 (.768)	0.07621	0.00490
	FREE-Garch	0.34 (.147)	1.14 (.350)	0.02965	0.17128	0.44 (.141)	-1.29 (.557)	0.05462	0.00245

The within sample period is from January 3rd, 1995 to March 31 1999 a sample size of 1,069 observations. The out-of-sample period starts from April 1st, 1999 to June 30th, 1999 for out-of-sample size of 64 observations. OLS estimates for α and β, and R^2, are reported for equation (3.31); *R^2 for logs* comes from the regression estimation of equation (3.32).

Table 3.16b
Bankers' Acceptance Futures [BAX]
Within and Out-of-Sample Predictive Power under the t-Distribution

Class	Model	Within-Sample α (SE)	Within-Sample β (SE)	Within-Sample R^2	Within-Sample R^2 for Logs	Out-of-Sample α^O (SE)	Out-of-Sample β^O (SE)	Out-of-Sample R^2	Out-of-Sample R^2 for Logs
Standard Class	Garch	0.37 (.162)	0.81 (.228)	0.02135	0.06907	0.56 (.311)	0.08 (.717)	0.00010	0.01704
	NA-Garch	0.32 (.159)	0.89 (.231)	0.02341	0.07116	0.42 (.270)	0.33 (.649)	0.00282	0.00622
	GJR-Garch	0.31 (.160)	0.89 (.235)	0.02581	0.06659	0.51 (.249)	0.17 (.551)	0.00085	0.02001
	AS - Garch	0.28 (.159)	0.93 (.934)	0.02654	0.06812	0.44 (.244)	0.28 (.559)	0.00273	0.01657
Nonlinear Class	SA-Garch	0.25 (.163)	0.96 (.237)	0.02958	0.08598	0.28 (.400)	0.63 (.942)	0.00866	0.01932
	AN-Garch	0.26 (.159)	0.95 (.232)	0.02959	0.08744	0.26 (.342)	0.66 (.817)	0.01221	0.00983
	T-Garch	0.21 (.152)	1.01 (.230)	0.03277	0.08832	0.30 (.287)	0.56 (.678)	0.01352	0.00414
	A-Garch	0.21 (.151)	1.02 (.229)	0.03311	0.08626	0.33 (.279)	0.65 (.649)	0.01117	0.00838
Exponential Class	SE-Garch	0.20 (.173)	1.03 (.256)	0.02891	0.08427	0.29 (.395)	0.61 (.944)	0.00740	0.01552
	AE-Garch	0.21 (.168)	1.03 (.250)	0.02898	0.08796	0.26 (.340)	0.70 (.837)	0.01084	0.00698
	E-Garch	0.16 (.166)	1.10 (.255)	0.03176	0.08537	0.30 (.297)	0.59 (.733)	0.01151	0.00790
	FE-Garch	0.15 (.167)	1.11 (.255)	0.03190	0.08709	0.31 (.292)	0.62 (.714)	0.01001	0.01295

Class	Model	Within-Sample				Out-of-Sample			
		α (SE)	β (SE)	R^2	R^2 for Logs	α^O (SE)	β^O (SE)	R^2	R^2 for Logs
Nonlinear Power Class	N-Garch	0.23 (.164)	0.99 (0.240)	0.03097	0.09031	0.15 (.440)	0.89 (1.04)	0.01967	0.02089
	NP-Garch	0.22 (.166)	1.01 (.244)	0.03144	0.08618	0.12 (.441)	0.90 (1.05)	0.02234	0.01595
	AP-Garch	0.22 (.150)	1.01 (.226)	0.03259	0.07711	0.21 (.309)	0.84 (.263)	0.02497	0.00559
	FP-Garch	0.22 (.145)	1.01 (.220)	0.03249	0.08876	0.26 (.299)	0.92 (.698)	0.02461	0.01505
Free-Garch Class	SF-Garch	0.23 (.163)	0.99 (.240)	0.03098	0.09008	0.15 (.440)	0.89 (1.04)	0.01972	0.02106
	AF-Garch	.21 (.166)	1.02 (.245)	0.03161	0.08641	0.12 (.441)	0.95 (1.05)	0.02231	0.01674
	TF-Garch	0.21 (.150)	1.02 (.225)	0.03314	0.07925	0.20 (.310)	0.87 (.758)	0.02584	0.00955
	FREE-Garch	0.20 (.144)	1.02 (.217)	0.03376	0.09181	0.35 (.264)	0.97 (.564)	0.02401	0.01477

The within sample period is from January 3rd, 1995 to March 31 1999 a sample size of 1,069 observations. The out-of-sample period starts from April 1st, 1999 to June 30th, 1999 for out-of-sample size of 64 observations. OLS estimates for α and β, and R^2, are reported for equation (3.31); *R^2 for logs* comes from the regression estimation of equation (3.32).

Table 3.17a
Bankers' Acceptance [BA]
Within and Out-of-Sample Predictive Power under GED

	Model	Within-Sample				Out-of-Sample			
		α (SE)	β (SE)	R^2	R^2 for Logs	α^O (SE)	β^O (SE)	R^2	R^2 for Logs
Standard Class	Garch	0.24 (.315)	0.38 (0.210)	0.03657	0.06335	0.57 (.288)	-0.30 (.210)	0.02181	0.00077
	NA-Garch	0.18 (.332)	0.35 (.186)	0.03608	0.06104	0.65 (.330)	-0.30 (.201)	0.02337	0.00213
	GJR-Garch	-0.05 (.381)	0.43 (.199)	0.03151	0.05484	1.11 (.561)	-0.52 (.306)	0.03017	0.00057
	AS - Garch	-0.23 (.688)	0.74 (.498)	0.04473	0.03266	0.64 (.406)	-0.34 (.291)	0.01281	0.00567
Nonlinear Class	SA-Garch	-2.89 (1.33)	1.59 (.605)	0.03971	0.02939	1.71 (.694)	-0.69 (.301)	0.07696	0.00365
	AN-Garch	-1.15 (.794)	1.30 (.560)	0.04472	0.02681	0.91 (.376)	-0.53 (.251)	0.06648	0.00942
	T-Garch	-0.79 (.594)	0.77 (.308)	0.04026	0.03298	0.91 (.356)	-0.39 (.177)	0.00322	0.00322
	A-Garch	-1.53 (.915)	0.99 (.410)	0.04270	0.02390	1.06 (.436)	-0.39 (.184)	0.06193	0.00918
Exponential Class	SE-Garch	-2.65 (1.83)	2.44 (1.31)	0.04587	0.07616	1.60 (.676)	-1.04 (.607)	0.07052	0.00056
	AE-Garch	-2.18 (1.78)	1.99 (1.20)	0.04446	0.07003	1.40 (.612)	-0.84 (.404)	0.06346	0.00110
	E-Garch	-2.21 (1.83)	2.00 (1.24)	0.04568	0.07223	1.39 (.610)	-0.83 (.404)	0.06473	0.00086
	FE-Garch	-1.02 (1.24)	1.24 (.857)	0.03640	0.06947	1.16 (.507)	-0.69 (.337)	0.06263	0.00036

	Model	Within-Sample				Out-of-Sample			
		α (SE)	β (SE)	R^2	R^2 for Logs	α^O (SE)	β^O (SE)	R^2	R^2 for Logs
Nonlinear Power Class	N-Garch	-2.00 (1.09)	1.45 (.597)	0.04461	0.02748	1.16 (.477)	-0.54 (.246)	0.07462	0.00769
	NP-Garch	-2.02 (1.10)	1.30 (.535)	0.04607	0.02750	1.07 (.434)	-0.43 (.198)	0.07708	0.00755
	AP-Garch	-1.73 (.026)	1.15 (.446)	0.03880	0.02539	1.30 (.518)	-0.54 (.235)	0.07715	0.00848
	FP-Garch	-1.42 (.948)	1.15 (.473)	0.04482	0.02121	1.65 (.681)	-0.79 (.351)	0.08303	0.00749
Free-Garch Class	SF-Garch	-1.73 (1.17)	1.22 (.583)	0.04662	0.02166	1.16 (.514)	-0.50 (.247)	0.05841	0.01087
	AF-Garch	-2.13 (1.17)	1.26 (.525)	0.04280	0.02529	1.33 (.554)	-0.51 (.238)	0.06121	0.00929
	TF-Garch	-0.81 (1.08)	0.71 (.480)	0.04592	0.02250	0.94 (.421)	-0.35 (.185)	0.05322	0.00803
	FREE-Garch	-1.38 (1.33)	0.99 (.613)	0.04673	0.02130	1.00 (.434)	-0.41 (.238)	0.06640	0.00943

The within sample period is from January 3rd, 1995 to March 31 1999 a sample size of 1,069 observations. The out-of-sample period starts from April1st, 1999 to June 30th, 1999 for out-of-sample size of 64 observations. OLS estimates for α and β, and R^2, are reported for equation (3.31); *R^2 for logs* comes from the regression estimation of equation (3.32).

Table 3.17b
Bankers' Acceptance Futures [BAX]
Within and Out-of-Sample Predictive Power under GED

Model		Within-Sample				Out-of-Sample			
		α (SE)	β (SE)	R^2	R^2 for Logs	α^O (SE)	β^O (SE)	R^2	R^2 for Logs
Standard Class	Garch	0.66 (.115)	0.34 (.112)	0.01647	0.03441	0.52 (.214)	0.10 (.220)	0.00094	0.02038
	NA-Garch	0.64 (.112)	0.37 (.111)	0.01759	0.04155	0.52 (.205)	0.10 (.201)	0.00094	0.02336
	GJR-Garch	0.69 (.095)	0.30 (.081)	0.01870	0.03644	0.55 (.186)	0.06 (.120)	0.00055	0.02751
	AS - Garch	0.26 (.175)	0.78 (.211)	0.02721	0.04163	0.46 (.247)	0.21 (.454)	0.00201	0.00978
Nonlinear Class	SA-Garch	0.20 (.185)	0.85 (.225)	0.02979	0.05586	0.29 (.419)	0.45 (.743)	0.00679	0.00034
	AN-Garch	0.22 (.175)	0.83 (.214)	0.03007	0.07301	0.24 (.384)	0.55 (.709)	0.01222	0.00313
	T-Garch	0.15 (.168)	0.90 (.213)	0.03429	0.07503	0.29 (.299)	0.43 (.535)	0.01268	0.00735
	A-Garch	0.15 (.167)	0.90 (.211)	0.03507	0.07604	0.35 (.283)	0.56 (.495)	0.02910	0.00341
Exponential Class	SE-Garch	0.18 (.189)	0.87 (.229)	0.02908	0.02214	0.32 (.394)	0.42 (.714)	0.00574	0.00395
	AE-Garch	0.18 (.187)	0.86 (.226)	0.02914	0.02739	0.30 (.391)	0.46 (.716)	0.00705	0.00451
	E-Garch	0.10 (.185)	0.96 (.231)	0.03313	0.03018	0.30 (.306)	0.45 (.572)	0.01033	0.01284
	FE-Garch	0.11 (.184)	0.95 (.228)	0.03304	0.02834	0.30 (.307)	0.44 (.568)	0.01004	0.01120

Model		Within-Sample				Out-of-Sample			
		α (SE)	β (SE)	R^2	R^2 for Logs	$α^O$ (SE)	$β^O$ (SE)	R^2	R^2 for Logs
Nonlinear Power Class	N-Garch	0.21 (.171)	0.84 (.210)	0.03076	0.06045	0.16 (.294)	0.63 (.696)	0.02082	0.00164
	NP-Garch	0.18 (.179)	0.89 (.223)	0.03097	0.05090	0.17 (.435)	0.65 (.798)	0.01516	0.00159
	AP-Garch	0.14 (.161)	0.93 (.206)	0.03246	0.07838	0.09 (.364)	0.76 (.592)	0.04715	0.00879
	FP-Garch	0.21 (.152)	0.93 (.189)	0.03235	0.07410	0.15 (.327)	0.77 (.613)	0.04632	0.00961
Free-Garch Class	SF-Garch	0.17 (.172)	0.87 (.212)	0.03087	0.06559	0.12 (.417)	0.68 (.729)	0.02324	0.00153
	AF-Garch	0.18 (.176)	0.85 (.212)	0.03182	0.03897	0.21 (.407)	0.57 (.735)	0.01356	0.00269
	TF-Garch	0.12 (.161)	0.94 (.205)	0.03313	0.08502	0.11 (.299)	0.72 (.568)	0.03231	0.00513
	FREE-Garch	0.16 (.158)	0.96 (.200)	0.03387	0.08458	0.17 (.340)	0.76 (.631)	0.05332	0.00945

The within sample period is from January 3rd, 1995 to March 31 1999 a sample size of 1,069 observations. The out-of-sample period starts from April1st, 1999 to June 30th, 1999 for out-of-sample size of 64 observations. OLS estimates for α and β, and R^2, are reported for equation (3.31); *R^2 for logs* comes from the regression estimation of equation (3.32).

Chapter Four Hedging Canadian Short-Term Interest Rates

Future contracts enable market participants to alter the risk they face from unexpected price changes. In this context, a hedge is an attempt to reduce the price exposure associated with possession of an underlying asset. Typically, users of futures contracts exchange one type of risk for another: hedgers exchange price risk of the underlying for basis risk; ie, the changing difference between the price of the underlying and futures price. The fundamental decision made by the hedger is the number of futures contracts required to offset expected changes in the price of the underlying. This number is the hedge ratio.

The optimal hedge ratio is associated with the covariance between the spot price of the underlying and the current futures price. In early studies, the covariance was estimated using traditional regression procedures. In effect, the covariance was assumed to be constant over some period of time. Given the evolving nature of market volatility, the assumption of constant covariance is dubious and more sophisticated estimation procedures have been adopted. In particular, GARCH-type models of changing volatility have been extended to a bivariate framework to permit the estimation of time-varying covariance.

The primary goal of this chapter is to extend the methodology used in the previous chapter to a bivariate framework in order to provide better estimates of the time-varying hedge ratio.

The Chapter proceeds as follows. Section 4.1 presents the basic motivation for the hedge ratio, and surveys earlier estimates of the hedge ratio. These include the naive hedge ratio and the OLS estimate of the hedge ratio. A brief account of bivariate GARCH models is also included, along with the extension of the Free GARCH univariate models to the bivariate setting. In Section 4.2, the models introduced in the previous section are estimated using the BA and BAX series. Section 4.3 compares the hedging performance of the hedge ratio associated with each of the various models. Section 4.4 establishes that the use of daily versus weekly data in the estimation of the hedge ratio improves significantly the performance of the hedging portfolio. Section 4.5 concludes.

4.1 The Hedge Ratio

This section begins with a more complete description of how the hedge ratio is used in practice. The relationship between price risk and basis risk in hedging short-term interest rates is explicitly drawn, and a detailed example detailing the use of the BAX in reducing exposure to interest rate changes is presented in section 4.1.1. The hedge ratio is a precise measure of how the BAX is employed in such examples. The next sub-section concerns the determination of the optimal hedge ratio in simple contexts. A simple regression estimate of the optimal hedge ratio is then obtained. Section 4.1.3 presents more elaborate estimates of the hedge ratio in contexts where it is time varying. More specifically, the ratio is estimated using bivariate GARCH models which include specifications used previously in the empirical literature, as well as a new specification which extends the Free GARCH analysis presented in Chapter 3 to the bivariate context.

4.1.1 Motivation For the Hedge Ratio

Hedging enables market participants to alter the risk they face from unexpected price changes. The fact that futures contracts are a low-cost effective way to transfer price risk is one of the main reasons for the existence of futures markets. Hedgers sell futures when they are long the "cash asset" and buy futures when they are short the "cash asset". Any loss (gain) resulting from the cash asset is offset by gain (loss) in the futures contact.

An effective application of hedging requires a way to manage the difference between cash and futures price movements (basis- risk minimization). The hedger must be confident that the price changes of both the spot and the futures contract will move together. This correlation motivates the concept of hedging and basis risk minimization, which is now elaborated. The concept of basis risk involves the difference between the price of a cash asset and the price of the futures contract. The convention is to place the price of spot first (which involves immediate delivery) and the future price second (later delivery). We have:

$$Basis = Cashprice - Futuresprice \quad . \tag{4.1}$$

Essentially, a hedger exchanges price risk for basis risk. For example, suppose the current spot price of an asset is S_0. The price risk from owning the asset from t = 0 to t = 1 is $\hat{S}_1 - S_0 = \Delta\hat{S}$, where $\hat{S}_1$ is the unknown spot price at the time of sale at time $t = 1$. A hedger's risk is the changing basis risk:

$$
\begin{aligned}
&(\hat{S}_0 - S_0) - (\hat{F}_1 - F_0) \quad , \\
&(\hat{S}_1 - \hat{F}_1) - (S_0 - F_0) \quad , \\
&(\hat{basis}_1 - \hat{basis}_0) \;=\; \Delta \hat{basis} \quad ,
\end{aligned}
\tag{4.2}
$$

where F_0 is the current futures price, and $\hat{F}_1$ is the unknown futures price at the time of sale. An important feature of this analysis is that the spot and the future must be sold simultaneously. The $\hat{basis}_1$ is unknown at time t = 0, and accordingly $\Delta\hat{basis}$ is unknown at this time. Basically, the hedger tries to manage the unexpected changes in the basis throughout the duration of the hedge. This implies, based on the cost of carry model which we now introduce, that the hedger is trying to manage or forecast the cost of carry minus any carry return that will occur in the next period.

$$
\begin{aligned}
F &= S + CC - CR \quad , \\
F - S &= CC - CR \quad , \\
F - S &= CC - CR = basis \quad .
\end{aligned}
\tag{4.3}
$$

F is the futures price, S is the spot price, CC is the cost of carry of the spot for the duration of the futures contract, and CR is the carry return (benefits) from the spot for the duration of the futures contract. The cost of carry model is an arbitrage model. Equation (4.3) equates the spot and futures prices incorporating all of the relevant costs of holding on to the underlying asset minus any revenues (benefits) from owning the asset for the duration of the hedging period.

In the case of the BA contract, the carry return (CR) drops out, since the BA is a discount bond and caries no coupon. The cost of carry (CC) is simply the cost of borrowing to buy and hold the cash instrument for the duration; i.e., the short-term borrowing rate.

$$\begin{aligned} F &= S + CC \ , \\ F &= S + r \times S \ , \\ S - F &= -rR = basis \ . \end{aligned} \tag{4.4}$$

The basis risk here is the short-term interest rate exposure throughout the duration of the hedging period. The rate implied by buying the cash and selling the futures is the interest costs associated with the underlying asset (the cash). This cost of carry rate from t = 0 to t = 1 is also known as the repo rate

$$r_{0,1} = \frac{F - S}{S} \ . \tag{4.5}$$

To minimize the basis risk, the hedger hopes that price changes of the cash asset and the futures price will be highly correlated. The higher the correlation between the price changes of the cash asset and those of the futures contract, the lower is the basis risk. Theoretically, the basis can not stray too far from the repo rate outlined in Equation (4.5), because of the "no arbitrage condition" implied by the cost of carry model. As time passes, the basis shrinks and the cost of carry is reduced, so that at maturity the spot price and the futures price converge and the basis equals zero. A perfect hedge is one where basis lost equals basis gained. As outlined in Chapter Two a perfect hedge is rare, since some basis slippage is expected. In fact, some market participants speculate on the changes in the basis. If the basis is too "wide", they would sell it and if it is too "narrow" they would buy it. Therefore, managing the basis is part of the hedging process and is accomplished through the use of a cash-futures equivalency ratio which is known as the hedge ratio. The hedge ratio (HR) is the number of futures contracts sold per spot contract purchased. These ideas are now illustrated via an example.

Consider a corporate treasurer who wants to hedge against a rise in short-term interest rates between Sept. 10^{th} and Dec. 10^{th} when the firm wishes to raise $10 million by issuing Three-month prime rate commercial paper. Due to the fact that the commercial paper is very closely correlated with BAs, the treasurer decides to sell 3-month BAX futures. Given that the price of the BAX contract is inversely related to interest rates, if interest rates rise during the hedging period, the price of Dec. BAX futures will fall. By selling BAX futures today and buying them back at a lower price on December 10, the treasurer will profit and reduce borrowing costs for the next period. By hedging with BAX futures, the corporate treasurer will have locked-in the forward borrowing rate implied by the current price of the December BAX contract. To summarize:

- Objective: lock in today's borrowing rate (or a close approximation).

- Strategy: use the BAX market by selling 9 BAX Dec.99 contracts to offset any interest rate increases during the period. The treasurer is effectively agreeing to supply $9 million of 3-month BAs in 3 months.

- Result: by selling futures contracts the treasurer has effectively guaranteed the selling price of the BA.

Some numbers may be useful. First, we assume that the hedge ratio(HR) is 0.92

$$Number\ of\ Contracts\ to\ Sell = \frac{Amount\ to\ Sell}{Futures\ Face\ Value} \times HR$$

$$= \frac{\$10{,}000{,}000}{\$1{,}000{,}000} \times 0.92 = 9.2 \approx 9$$

Market Conditions	September 10th	December 10th	Change in Basis
3-months Commercial Paper (implied price)	4.85 % ($95.15)	5.27 % ($94.73)	42 (-42)
3-months BA (implied price)	4.82 % ($95.18)	5.24 % ($94.76)	42 (-42)
3-month Dec. BAX (implied BA Futures rate)	$94.72 (5.280%)	$94.40 (5.60%)	-32 (32)
Basis at time t	-43	-33	10

$$\textit{Each basis point} = (\$1,000,000 \times 0.01\%)\left[{}^{3\ months}\!\!/_{12\ months}\right] = \$25$$

$$\textit{Gain on the BAX} = 9 \times (94.72 - 94.40) \times 100 \times 25 = \$7,200$$

$$\textit{The Interest Paid on CP} = \$10,000,000\left[1 - {}^{1}\!\!/_{1 + (.0527 \times {}^{90}\!/_{365})}\right] = \$128,278$$

$$\textit{Net Interest Cost} = \$128,278 - \$7,200 = \$121,078$$

$$\textit{Effective Borrowing Rate} = \left[\frac{\$121,278}{\$10,000,000} \times \frac{365}{90}\right] \times 100 = 4.91\%$$

The effective borrowing rate represents a saving of 36 basis points over the current rate that existed in December 10th. However, there is slippage of 6 basis points (effective borrowing rate is 4.91% instead of 4.85%). Therefore, this cross hedge is not perfect.

The aim of a fully-hedged position is to have the cash price changes offset by changes in the futures price, so that in the previous example the treasurer would have locked in the earlier rate of 4.85%. In practice, the hedger wishes to be as close to the current rate as possible. In other words, the objective is to obtain minimal variance between the gain on the cash position and the loss on the futures position or visa versa. The closer is the price sensitivity of the futures position to the price sensitivity of the cash position, the more effective the hedge will be. Unfortunately, a perfect one-to-one price correlation between futures contracts and cash instruments is rare. Therefore, the optimal hedge ratio (HR) indicates the proper number of futures contracts required to compensate for the price movement of the cash.

The hedge ratio then, is the number of future contracts required to sell (buy) per unit of the cash position. In the above example the hedge ratio is 92%. The hedge ratio is the position taken in the futures contracts in an attempt to minimize basis risk exposure. By entering simultaneously into two offsetting positions the hedger is in fact creating a portfolio with lower risk than results from a naked position in either futures or cash.

4.1.2 Hedging and Early Methods of Hedging

As stated earlier, the purpose of hedging is the elimination of price risk, and we assume that the hedger is interested in risk minimization. Traditionally the inherited risk is measured by the variance of a portfolio of spot units and futures contracts. We assume that the hedger is long one unit of cash (3-months 1-Million Dollar BA) and short (h) units of BAX (the underlying value of a BAX contract is 1-Million Dollar BA value). The expected changes in the value of the portfolio from time t = 0 to time t = 1:

$$(\hat{S}_1 - S_0) - h(\hat{F}_1 - F_0) \quad , \qquad (4.6)$$
$$\Delta\hat{S}_1 - h\Delta\hat{F}_1$$

The risk of the portfolio is reflected by its variance. In order to minimize risk, we optimize with respect to the choice variable, in this instance the number h of BAX contracts to sell; a textbook account of the procedure can be found in Hull(1989). The optimum hedge ratio is given by:

$$h^* = \frac{cov(\Delta\hat{S},\Delta\hat{F})}{var(\Delta\hat{F})} = \rho\frac{\sigma(\Delta\hat{S})}{\sigma(\Delta\hat{F})} \quad , \qquad (4.7)$$
$$where \quad \rho = \frac{cov(\Delta\hat{S},\Delta\hat{F})}{\sigma(\Delta\hat{S})\sigma(\Delta\hat{F})} .$$

ρ is the coefficient of correlation between the $\Delta\hat{S}$ and $\Delta\hat{F}$.Therefore, the optimal hedge ratio (h*) is the product of the correlation coefficient (ρ) and the ratio of the standard deviations of the cash and the futures contract ($\Delta\hat{S}$, $\Delta\hat{F}$).

An early approach to dealing with risk involved basis hedging (or naive hedging). Cash and future prices have a tendency to move together maintain a fairly "predictable" relationship. The basic assumption of basis hedging is that cash and futures prices respond identically to interest rate fluctuations (i.e. $\Delta S = \Delta F$). On this assumption, the naive hedge ratio is a special case of the optimal hedge ratio. If we impose $\rho = 1$ and assume that $(\sigma(\Delta\hat{S}) = \sigma(\Delta\hat{F}))$ the optimal hedge ratio collapses to h* = 1.0. We have seen an example in Chapter Two involving a naive hedge ratio.

A further generalization of the optimal hedge ratio is to estimate ρ from the historical association between the cash and the futures contract as we have implicitly done in an example in Section 4.1.0, where ρ is the optimal hedge ratio and was set at 0.92. From equation (4.6), assuming that the variances are identical,

$$h^{*} = 0.92 \frac{\sigma(\Delta S)}{\sigma(\Delta F)} = 0.92 \quad .$$

An OLS regression framework provides a direct calculation of the ratio between the sample covariance and the sample variance. Its use in this context originated in the portfolio selection theory developed by Markowtiz (1952). Subsequent work adopted this framework to estimate the hedge ratio; for examples, see Johnson (1960), Stein (1961), and Ederington (1979). In this regression approach, (ΔS) is the dependent variable, (ΔF) is the independent variable in the model:

$$\Delta S_t \;=\; \alpha + \beta \Delta F_t + \varepsilon_t \quad . \tag{4.8}$$

The estimated slope coefficient is the estimate of the optimal hedge ratio, $\hat{h}^* \;=\; b$. The resulting estimated slope coefficient defines how many futures contracts to trade, per unit of cash position, in an attempt to minimize price risk.

The OLS estimation assumes that the variances of price changes, both spot and futures, and the covariance between them is constant throughout the sample and into the future. However, if the volatility of asset prices changes is time-varying the OLS approach is inappropriate. We follow the tradition in the empirical literature and define the time-varying hedge ratio as the natural extension of the hdege ratio given above:

$$h_t = \frac{Cov(\Delta(S), \Delta(F) | \Omega_{t-1})}{Var(\Delta(F) | \Omega_{t-1})} \quad . \tag{4.9}$$

The optimality property of the hedge ratio so defined can be derived under certain assumptions concerning the hedger's utility preferences and the martingale property of futures prices; see, for example; Gagnon and Lypny (1997). In what follows, we simply assume that the appropriate hedge ratio is as defined in Equation (4.8) and adopt different estimation procedures to determine the time-varying variances and covariances.

Researchers such as Bollerslev (1987), Ceccetti, Cumby, and Figlewski (1988), Baillie and Myers (1991) first adopted a (G)ARCH methodology to model the time-varying nature of the volatility of stock and bond prices in the context of hedging. We now turn to this modeling framework

4.1.3 Bivariate GARCH Models

In this chapter we will estimate a number of bivariate GARCH models of the joint distribution of changes in the BA log-prices and changes in the BAX log-prices. This section surveys the models that will be estimated in the next section. These include the an extension of the Free GARCH univariate framework used extensively in Chapter 3 to the bivariate context.

A general form of the bivariate GARCH (1,1) specification may be written in vector form as follows:

$$\begin{bmatrix} \Delta LBA_t \\ \Delta LBAX_t \end{bmatrix} = \begin{bmatrix} B_{01} \\ B_{02} \end{bmatrix} + \begin{bmatrix} \varepsilon_{BA,t} \\ \varepsilon_{BAX,t} \end{bmatrix}$$

$$\Sigma_t = \begin{bmatrix} C_{01} \\ C_{02} \\ C_{03} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{BA,t-1} \\ \varepsilon_{BA,t-1}\varepsilon_{BAX,t-1} \\ \varepsilon^2_{BAX,t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{23} & b_{33} \end{bmatrix} \begin{bmatrix} \sigma^2_{11,t-1} \\ \sigma^2_{12,t-1} \\ \sigma^2_{22,t-1} \end{bmatrix}, \tag{4.10}$$

where [ΔBA, ΔBAX]' is a (2 × 1) vector containing the BA first-differenced log-spot prices and the BAX first differenced log-futures prices; $[B_{01}, B_{02}]'$ is a (2 ×1) vector containing a vector of constants for the BA and BAX series respectively, and $\varepsilon_t = (\varepsilon_{BA,,t}, \varepsilon_{BAX,t})'$ is a (2×1) vector of disturbances with $\Sigma_t = E(\varepsilon_t \varepsilon'_t \mid \Omega_{t-1})$ and Σ_t = vec σ_t^2 which omits redundant terms. Accordingly, C is a (3 ×1) vector containing the variance constants. A and B are (3×3) matrices containing the ARCH variance elements (the a's) and the autoregressive variance elements (the b's) respectively. Time-varying Σ_t is modeled by an ARMA process, where the dynamic hedge ratio, denoted h_t, is estimated by using elements of the conditional variance-covariance matrix Σ_t. The mean equation will be given a more elaborate formulation in the next section; here the focus is on the formulation of the conditional covariance matrix.

The estimation of Σ_t so parameterized is not a trivial matter. Equation (4.9) poses at least two problems. First, the conditional variance equation has twenty-one parameters, a considerable number to estimate in empirical situations. Second, on estimation, Σ_t is not guaranteed to be positive semi-definite–the positivity requirement-and there are no simple conditions that may be imposed on the parameters during estimation to guarantee that the positivity requirement is fulfilled.

Accordingly, for empirical implementation, it is desirable to restrict the parameterization of Equation (4.9). One natural simplification is the "diagonal representation" which was first proposed in the context of GARCH specifications by Bollerslev, Engle, and Wooldridge (1988). The motivating idea is that each element of the covariance matrix $\Sigma_{jk,t}$ depends solely on past values of itself $\Sigma_{jk,t-1}$ and on the product of its past residuals values $\varepsilon_{j,t-1}\,\varepsilon_{k,t-1}$. According to this formulation,

$$\Sigma_t = \begin{bmatrix} C_{01} \\ C_{02} \\ C_{03} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{BA,t-1} \\ \varepsilon_{BA,t-1}\varepsilon_{BAX,t-1} \\ \varepsilon^2_{BAX,t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} \sigma^2_{11,t-1} \\ \sigma^2_{12,t-1} \\ \sigma^2_{22,t-1} \end{bmatrix} . \tag{4.11}$$

Here the off-diagonal elements of the matrices A and B are set to zero. This formulation is too restrictive as emphasized by Gouriéroux (1997); cross disturbance effects are ignored and it is difficult to characterize in a coherent way the positivity requirement. Accordingly, despite its frequent use in empirical studies, this formulation is not adopted in the next section where the hedge ratios are computed for the BA and BAX.

The BEKK specification , described in Engle and Kroner(1995), was designed to ensure positivity under weak conditions. The simplest version for the GARCH(1,1) context may be written with A, B and C (triangular) 2 by 2 matrices:

$$\Sigma_t = C'C + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{BA,t-1} & \varepsilon_{BA,t-1}\varepsilon_{BAX,t-1} \\ \varepsilon_{BAX,t-1}\varepsilon_{BA,t-1} & \varepsilon^2_{BAX,t-1} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \Sigma_{t-1} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} , \tag{4.12}$$

or, written out for comparison with the previous vector formulation,

$$
\begin{aligned}
\sigma^2_{11,t} &= C_{11} + a^2_{11}\varepsilon^2_{BA,t-1} + 2a_{11}a_{21}\varepsilon_{BA,t-1}\varepsilon_{BAX,t-1} + a^2_{21}\varepsilon^2_{BAX,t-1} \\
&\quad + b^2_{11}\sigma^2_{11,t-1} + 2b_{11}b_{21}\sigma^2_{12,t-1} + b^2_{21}\sigma^2_{22,t-1} \\
\sigma^2_{12,t} &= C_{12} + a_{11}a_{12}\varepsilon^2_{BA,t-1} + (a_{21}a_{12} + a_{11}a_{22})\varepsilon_{BA,t-1}\varepsilon_{BAX,t-1} + a_{21}a_{22}\varepsilon^2_{BAX,t-1} \\
&\quad + b_{11}b_{12}\sigma^2_{11,t-1} + (b_{21}b_{12} + b_{11}b_{22})\sigma^2_{12,t-1} + b_{21}b_{22}\sigma^2_{22,t-1} \\
\sigma^2_{22,t} &= C_{13} + a^2_{12}\varepsilon^2_{BA,t-1} + 2a_{12}a_{22}\varepsilon_{BA,t-1}\varepsilon_{BAX,t-1} + a^2_{22}\varepsilon^2_{BAX,t-1} \\
&\quad + b^2_{12}\sigma^2_{11,t-1} + 2b_{12}b_{22}\sigma^2_{12,t-1} + b^2_{22}\sigma^2_{22,t-1} \quad .
\end{aligned}
\tag{4.13}
$$

As is clear, the BEKK formulation is designed to guarantee positivity. Moreover, compared with the full vector formulation, there is some reduction in the number of parameters to be estimated. Whereas, the previous formulation had 21 parameters, the BEKK formulation leaves 11 to be estimated. Finally, Engle and Kroner(1995) give a simple characterization of covariance stationary.

First, define AA = A⊗A and BB = B⊗B, where A and B are given in Equation (4.9). Then the process {ε } is covariance stationary if and only if all the eigenvalues of the matrix (AA + BB) are less than 1 in modulus.

A third form of the multivariate GARCH specification is due to Bollerslev (1990). In this form of multivariate GARCH, the assumption is that the conditional correlation between the elements of ε_t are constant over time. The Bollerslev specification is given by:

$$\sigma_{11,t}^2 = C_{11} + a_{11}\varepsilon_{BA,t-1}^2 + b_{11}\sigma_{11,t-1}^2$$
$$\sigma_{22,t}^2 = C_{22} + a_{22}\varepsilon_{BAX,t-1}^2 + b_{22}\sigma_{22,t-1}^2$$
$$\sigma_{12,t}^2 = \rho_{BA,BAX} \times \sigma_{11,t-1}\sigma_{22,t-1} \quad . \tag{4.14}$$

This specification reduces the complexity of estimation and is quite tractable. The number of parameters to be estimated in the variance-covariance equations is 7 as compared to 11 in the BEKK formulation. As well, the conditional variance-covariance matrix in this formulation may be written:

$$\Sigma_t = diag(\sigma_{ii,t}) R\, diag(\sigma_{ii,t}) \quad , \tag{4.15}$$

where R is the correlation matrix. So it is clear that the positivity condition for the joint process reduces to conditions on the univariate processes, as does the condition for covariance stationarity; see Gouriéroux (1997).

The Bollerslev specification also provides a natural bivariate environment to incorporate the generalizations to univariate GARCH processes introduced in Chapter 3. It should be recalled that the Hentschel model introduces two asymmetry parameters R and S that displace the news impact curve, and two parameters λ and ν that affect its shape. These parameters are introduced into the Bollerslev bivariate setting as follows:

$$\frac{\sigma_{11,t}^{\lambda}-1}{\lambda}=C_{11}+a_{11}\sigma_{11,t-1}^{\lambda}[|\varepsilon_{BA,t-1}-S_1|-R_1(\varepsilon_{BA,t-1}-S_1)]^{v}+b_{11}\frac{\sigma_{11,t-1}^{\lambda}-1}{\lambda}$$

$$\frac{\sigma_{22,t}^{\lambda}-1}{\lambda}=C_{22}+a_{22}\sigma_{22,t-1}^{\lambda}[|\varepsilon_{BAX,t-1}-S_2|-R_2(\varepsilon_{BAX,t-1}-S_2)]^{v}+b_{22}\frac{\sigma_{22,t-1}^{\lambda}-1}{\lambda}$$

$$\sigma_{12,t}=\rho_{BA,BAX}\times\sigma_{11,t-1}\sigma_{22,t-1} \quad . \tag{4.16}$$

Several remarks are in order. The asymmetry parameters are allowed to change for both the BA and BAX series to permit a completely flexible framework for dealing with asymmetry. Accordingly, four new parameters R_1 , S_1 , R_2 and S_2 are introduced within the Bollerslev framework. These parameters indicate whether the pattern of asymmetry is more strongly present for small shocks or large shocks. The shape parameters λ and v are identified across the two variance equations. Here the intuition is that they are deeper structural parameters reflecting underlying preferences. On the other hand, the specification does not impose the constraint that they assume fixed values such as 0, 1 or 2. We have seen, in the univariate case at least, that such assumptions are overly restrictive. In sum, the Bollerslev constant correlation model permits a ready generalization that focuses on the separate univariate parameterization in a natural manner. The assumption of constant correlation entails that the positivity as well as the stationarity conditions for the bivariate process $\{\varepsilon_t\}$reduces to separate conditions on $\{\varepsilon_{1t}\}$ and $\{\varepsilon_{2t}\}$. These conditions are given in the previous Chapter and are summarized here. For positivity, we need for i = 1, 2.

$$c_{ii} > 0, \quad a_{ii} \geq 0, \quad b_{ii} \geq 0, \quad |R_i| \leq 1 \quad . \tag{4.17}$$

For covariance stationarity we need for i = 1, 2:

$$\mathrm{var}(\varepsilon_{ii,t}) < \infty, \mathrm{c_i} > 0, \text{and } \mathrm{E}[(\mathrm{a_{ii}}\lambda_i \mathrm{f}^{\nu}(\varepsilon_{ii,t}) + \mathrm{b_{ii}})^{2/\lambda}] < 1 \quad . \tag{4.18}$$

This account of bivariate GARCH models has focused on three specifications, all of which will be estimated in the following section for the BA and BAX processes. Several considerations have motivated the choice of these specifications. Fully-general bivariate models involve a considerable number of parameters which may be difficult to estimate in practice. Moreover, even if such models are successfully estimated, coherency conditions such as positivity are difficult to verify. Finally, we are looking for a framework within which the extension of the Free-GARCH univariate approach is natural. In this context, we have determined three appropriate bivariate specifications. The first is the simplest version of the BEKK model presented in Engle and Kroner(1995). The second is the constant-correlation Bollerslev model. The third is the Bollerslev model extended to include the Free GARCH parameterization of the variance equations.

4.2 Estimation of the Hedge Ratio

A number of researchers have estimated the hedge ratio in the context of interest rate futures. An early work in this area is Cecchetti, Cumby and Figlewski (1988) which used an ARCH methodology to estimate the optimal hedge ratio between U.S. Treasury bonds and T-bond futures. Of particular relevance to our work is Gagnon and Lypny (1995) which evaluated the hedge ratio for the same instruments, the BA and the BAX contract, which are the focus of interest in this thesis. However, this paper uses weekly data, whereas our paper uses daily data. As well, the time period analyzed is different; Gagnon and Lypny estimate models using data beginning in March 7, 1990 and ending March 30, 1994, for a total of 211 observations. Our sample of daily data starts January 3, 1995, and ends Dec 14, 1999 for a total of 1,258 observations available for in-sample and out-of-sample analysis. We begin with the simple OLS estimate of the hedge ratio and then turn to the estimation of the bivariate GARCH models.

Table 4.1 exhibits the results of estimating the usual linear model:

$$\Delta LBA_t = \alpha + \beta \Delta LBAX_t + \varepsilon_t \quad , \qquad (4.19)$$

where ΔLBA_t is differences in log prices of the BA series and $\Delta LBAX_t$ is the BAX log prices differences. According to this model, the estimated slope coefficient from the regression is the estimate of the optimal hedge ratio which is assumed to be constant over the sample. The hedge ratio implied by the OLS estimation is b = 0.558; that is, it is optimal to construct a hedge portfolio long one-million dollar BA and short 0.558 BAX (the underlying is one-million dollar BA). Robust standard errors were also estimated. The standard error under robust estimation is over three times the usual standard error; a 95% confidence interval is given by (0.420, 0.696). The hedge ratio appears to be imprecisely estimated.

We turn to GARCH models of a time-varying hedge ratio. As indicated in the previous section, the estimation focuses on three specifications: BEKK, Bollerslev constant correlation, and Free GARCH with constant correlation. The specification of the conditional variance matrix was given explicitly in the previous section. It remains to give an account of the mean equations that are estimated in the three specifications:

$$\begin{bmatrix} \Delta LBA_t \\ \Delta LBAX_t \end{bmatrix} = \begin{bmatrix} B_{01} \\ B_{02} \end{bmatrix} + \begin{bmatrix} d_1(LBA_{t-1} - LBAX_{t-1}) \\ d_2(LBA_{t-1} - LBAX_{t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon_{BA,t} \\ \varepsilon_{BAX,t} \end{bmatrix} , \tag{4.20}$$

where LBA_t and the $LBAX_t$ are the logarithms of the daily spot (BA's) and the daily futures (BAX's) prices in levels. A number of previous research have found that many of the cash and futures time series to be cointegrated [Engle and Granger(1987), Brenner and Korner(1995)]. If a long-run relationship does exist between these two variables, then its omission in model estimation leads to inconsistent estimates. Our own results from Chapter Two indicate that the BA and BAX prices are indeed cointergrated with cointegrating vector (1, -1). The parameters d_1 and d_2 are the error correction coefficients for the two mean equations.

The three bivariate GARCH models were estimated with error-correction terms for both multivariate normal and multivariate-t distributions. The estimates for the bivariate GARCH models with normal distribution are displayed in Tables 4.2a and the estimates of the bivariate GARCH models with multivariate-t distribution are presented in Tables 4.3a. Tables 4.2a and 4.3a are organized in the following fashion; column one exhibits the parameters of the models estimated. Column two presents the results of the BEKK bivariate GARCH model, while the third column presents the results of the Bollerslev Constant Correlation bivariate GARCH model. The fourth column contains the estimation results of the Bollerslev Constant Correlation Free GARCH model. Tables 4.2b and 4.3b present results regarding positivity and stationarity conditions.

Before the parameter estimates are analyzed in any detail, it is particularly important to verify the positivity and stationarity conditions. In the case of estimation with normal disturbances the BEKK model can be ruled out as inappropriate, since the stationarity condition is not met. As Table 4.2b indicates, one of the eigenvalues of the matrix $A \otimes A$ and $B \otimes B$, where A and B are given in Equation (4.12), is greater than 1. The Constant Correlation model is also ruled out, since the stationarity condition is not met as well; here the model for the underlying BA process is not stationary, a result which echoes difficulties in modeling the BA series by itself in Chapter 2. The Free GARCH model with constant correlation estimated under normality does meet both the positivity and stationarity conditions as indicated in Table 4.2b; the estimated coefficients of the variance series for both the BA and BAX series are all positive, with the values of the rotation asymmetry parameters both less than 1 in absolute value; the expectation conditions for stationarity are also satisfied.

Several features of the estimates of the latter model merit remark. First, both error-correction parameters are strongly significant. Several of the asymmetry parameters are significant.

Consistent with previous estimates of the univariate processes the parameters λ and ν are significantly different from the values imposed in standard GARCH estimation, both being significantly less than 1and greater than 0 . The constant correlation in this environment is estimated to be 0.62 which indicates a strong correlation between shocks to BA prices and BAX prices.

We turn now to the estimates of the GARCH models under the mutivariate-t distribution. In sharp contrast to the previous estimations under normality, the estimates of the three bivariate GARCH models all satisfy the positivity and covariance stationarity conditions. It is difficult to compare parameter values between BEKK on the one hand and the two constant correlation models on the other. We will see how the models compare in practice with regard to reducing uncertainty of the hedging portfolio in the next section.

Several conclusions may be drawn in comparing the standard correlation model with the Free GARCH version. First, the constant correlation coefficient is virtually identical under both estimations (and very close to the value obtained under normal disturbances). Again the parameters λ and ν are significantly different from the values imposed in standard GARCH estimation, both being significantly less than 1. Finally, 2 out of the 4 asymmetry parameters appear significant in the Free estimation. The rotation parameter appears to play a greater role in capturing asymmetry in the BAX series; only the rotation parameter is of interest in the BA variance process. From the perspective of parameter estimation, it is clear that the Free GARCH approach differs from the standard specification. It remains to determine whether the difference has practical significance.

4.3 Evaluation of Hedging Performance

We now turn our attention to the evaluation of the hedging performance of the models estimated in the previous section. The objective is to determine whether the dynamic specification of the hedge ratio yields improved hedging performance, as earlier research suggests, and whether the Free-GARCH specification yields improved hedging performance over standard bivariate GARCH approaches. We begin with a discussion of the framework for evaluating hedging performance.

4.3.1 General Issues

The objective of hedging remains the elimination of price-risk inherent in owning (buying) the underlying asset. The elimination of price-risk (capital gain or loss) implies a zero change in the basis between the cash and its futures at the time the hedge is constructed and the time the position is lifted. Any change in the price of one instrument is off-set by the price change in the other. The model that provides the closest performance to a zero-basis change during the hedging period is deemed best.

Within- and out-of-sample performance of the models are used to discriminate further among the models. The performance is based on the ability of the models to reduce the variance of a portfolio constructed of cash and futures prices, using a hedge ratio obtained from the estimation of the appropriate models to indicate the numbers of futures contract to sell visa vie the cash instrument. The model that delivers the largest variance reduction of the constructed portfolio (termed the hedging portfolio) with respect to the unhedged position is chosen as the best performer.

According to the finance literature, total return is measured by the income generated from the asset plus any capital gain/loss accrued during the holding period. For a fixed income security total return is given by:

$$\text{TR} = \text{Coupon Payment} + \text{Capital Gain}. \tag{4.21}$$

As indicated in Chapter Two, the BA is a money market instrument and carries no coupon; the BA is bought at a discount and redeemed at face value (in one-year). Therefore, BA return arises from the second part of the above equation. Adverse price fluctuation can erode any return promised by the asset. The hedging process eliminates any price loss /gain that might arise during the length of the hedge. From this viewpoint, the hedger's primary concern is the preservation of both the initial value of the asset (wealth preservation) and the return promised by the asset (return guarantee). Wealth preservation and return guarantee is achieved by having the variance of the hedged portfolio as small as possible. Zero variance would be ideal. The hedging portfolio is constructed by buying one BA and selling the appropriate number of BAX contracts implied by the hedge ratio. At the current period the value of the hedging portfolio is given by:

$$V_t = BA_t - b_t BAX_t \ , \tag{4.22}$$

where b_t is the hedge ratio implied by the appropriate model. The change in the value of the hedged portfolio from one period to the next is given by:

$$\Delta V_{t+1} = \Delta BA_{t+1} - b_t \Delta BAX_{t+1} \quad . \tag{4.23}$$

A perfect hedge implies that the change in the value of the hedging portfolio from one period to the next is zero, i.e. the hedged portfolio must have a zero mean and a zero variance. Actually the appropriate measure of the effectiveness of the hedging portfolio in this case is not the variance but the second moment. Recall that the hedger's objective is to be in a position where any BA price movement is offset. Accordingly, the objective then is to have the mean of the constructed hedged portfolio to be zero. Any model that attains a mean different from zero (both negative or positive) must be penalized. The appropriate measure in this context is the second moment. By contrast, the variance measures the variability of return around a particular mean of return; this measure is appropriate when dealing with the performance of stock index futures, for example. For comparability and completeness both the variance and the second moment of the hedging portfolio are reported in the analysis that follows. To illustrate the point further, an example of a constructed hedged portfolio where the outcome is a perfect hedge (the basis at the beginning of the hedge equals to the basis when the hedge is lifted) is given below.

A hedger is long one BA and short one BAX contract (for simplicity we construct a naive hedge similar to the example in Chapter Two). The annual return promised by the BA is 5% (the price implied is 100 - the annual yield), to preserve the return of the BA, the hedger will short one BAX contract. Market conditions are summarized in the following Table:

Market Conditions	Today	In three months	Change in basis
3-months BA (implied price)	5 % ($95.00)	5.20 % ($94.80)	20 (-20)
3-month BAX (implied rate)	$94.50 (5.5 0%)	$94.30 (5.70 %)	-20 (20)
Basis	-50	-50	0

In three months, the BA lost 20 basis point (because of rise in interest rates), however , the BAX also lost 20 basis points, which give the hedger a net gain/loss on the price movements of zero, without the hedge the adverse price movements would have eroded the return promised by the BA. By entering into a hedged position the hedger in the above example achieved the goal of zero return on the constructed portfolio and has attained the promised BA return .

We will used the variance and the second moment of the series of changes in the value of the hedging portfolio as defined by Equation (4.23) to measure the within and out-of-sample hedging performance of the models estimated. The first 1,068 observations were used for within-sample estimation, a period which covers January 3, 1995 to March 30, 1999. The remaining 190 observations were used for out-of-sample estimations, spanning the period March 31, 1999 to December 30, 1999. These 190 observations were divided into three forecast horizons corresponding to 3 months, 6 months and 9 months.

4.3.2 Within- and Out-of-Sample Hedging Performance

This section evaluates the hedging performance of the models previously estimated. To confirm whether there is any gain in hedging versus no hedging at all, we compare each model's hedging performance with that of an unhedged position. In particular, we compare the improvement in the constructed portfolio's variance and second moment with respect to the unhedged cash position.

The unhedged position is simply being long the cash; ie, long the BAs. The hedged position is constructed by being long one cash unit and short the appropriate number of futures contracts implied by the hedge ratio associated with the accompanying model. Along with this hedged portfolio, a naive hedge (similar to Example 1 in Chapter 2) is also constructed. The naive hedge ratio simply equals one; we are long one million dollar BA and short one three-month BAX contract.

The within sample estimates of the hedging ratio are obtained from estimations of the model using the first 1,068 observations of the data. The out-of-sample estimation of the hedge ratio are obtained from the last 190 observations of the data using the parameter estimates from the estimation. Here the (H_t) matrix is updated and a hedge ratio is re-computed. The procedure is then repeated for the duration of the three out-of-sample periods, 3-month horizon (53 times), 6-month horizon (115 times), and 9-month horizon (190 times).

Table 4.4 presents the results of the within-sample hedging performance of various models. In evaluating the hedging performance of the models, the unhedged position is used as the first benchmark to evaluate the models. These measurements are presented in column three; the top number in each cell is the variance measurement and the bottom measurement is the second moment. Columns four and five report the percentage reduction in variance (respectively, second moment) is computed both with respect to the variance (respectively, second moment) associated with the unhedged position and the portfolio constructed using the constant hedge ratio [0.558; see Table 1]determined by OLS.

Table 4.4 also gives the rankings of the models based on variance and the second moment. The general pattern of within-sample results is readily discernible. The Free GARCH models along with the BEEK model estimated using the t-distribution are at the top of the class, the three portfolios based on a constant hedge ratio are at the bottom of the class. When the second moment is used to measure performance, OLS registers a 56% gain [ie, percentage loss]; the BEEK model estimated using the t-distribution [or BEKK-t], a 70% gain and the Free Garch again estimated using the t-distribution [or Free Garch-t], a 71% gain. The best GARCH models show a 30-35% gain over OLS using either variance or the second moment as the measure of performance. It should be noted that all the models estimated with the t-distribution have outperformed their counterparts estimated with normal distribution.

Gagnon and Lypny (1995) also reports an improvement in the within-sample performance of the Bivariate BEKK-t GARCH model over Naive and OLS hedge. Their gains using weekly data include a 44 % improvement over the unhedged position within sample but only a 9 % improvement over OLS [their performance measure is relative to variance].

Now we turn our attention to out-of-sample estimation. The out-of-sample period was divided in three to reflect 3-month, 6-month, and 9-month forecast horizons. Table 4.5 displays the results of 3-month out-of-sample horizon. Taking the variance as the criteria for hedging evaluation, the first striking observation is that the OLS model outperforms the rest of the models, with OLS ranked first and Free-GARCH with t-distribution ranked second. Taking the second moment as the performance criterion, the second striking feature of the results is that in this period all hedging approaches have under performed the naked position. The estimated mean of the unhedged position shows the least departure from zero. A situation where a naked position can outperform a hedged one can occur in the market for a short period of time. What is striking in this case is that basis risk is higher than price risk. The two underlying instruments the BA and the BAX pulled away from each other, as is confirmed by an examination of Figure 4.7 which displays the log daily prices of both the BA and BAX for the out-of-sample period. We can see for the first hedging horizon (ending June 14) the BA series (the unhedged position) stayed relatively stable compared to the BAX series which moved in a volatile and erratic way. Figure 4.8 plots the out-of-sample basis.

Within the first 3-month hedging horizon the basis moved from negative to positive. The basis (the difference between the BA and the BAX) widened considerably, while the price movement of the BA (price risk) stayed relatively flat; so here a hedge technique based on basis minimization is ineffective.

Among the GARCH models, the Free GARCH models perform well, as does the Constant-Correlation-t when a smaller second moment is the performance criterion and BEKK-t when a smaller variance is the performance criterion.

Table 4.6 presents the results of the second out-of-sample period, the six-month horizon. If we use reduced variance as the criterion for evaluating hedging performance, the more sophisticated models outperform the simple ones. The highest ranking under this criteria goes to the Free GARCH model with t-distribution. Another interesting result is that the second ranking goes to the constant correlation model with t-distribution outperforming both the normal Free GARCH and the BEKK t-distribution models. The third ranging goes to the normal Free GARCH and the fourth spot goes to the BEKK model with t-distribution. OLS takes the eight spot just before the Naive hedge.

If we use the second moment as the performance criterion, a similar story to Table 4.5 emerges, where the unhedged position outperforms the hedged position. Examining Figure 4.7 we see a similar path for both the BA and BAX, where the BA prices are relatively stable and the BAX prices move widely. With the largest one-day BAX move in the out-of-sample period, the BAX price dropped 53 basis point on August 17th 1999, from 94.89 to 94.36, where the BA actually went up one basis point from 94.99 to 95.00. Typically this is reflected in the basis as we can clearly see the basis change in Figure 4.8. The basis were 10 basis points (94.99 - 94.89) on August 16th and the increased to 64 basis points (95.00 - 94.36). Why did the BAX contract drop in price while the BA went up one point?

It should be kept in mind that the BAX market is a more liquid market and reacts to information more readily than the BA. As mentioned in Chapter Two, an increasingly important phenomena is emerging in the BAX market: F/X traders aware of the high correlation between the Canadian dollar and the BAX contract often use the BAX contract as a hedge when ever they are caught long or short the dollar. Given this environment and a hedging technique of minimizing basis risk, it is not surprising to find that a hedged portfolio may underperform an unhedged position.

Nevertheless, we are concerned with the hedging performance of the models. Under the second moment criterion, the best hedging model is clearly the Free GARCH model with t-distribution. The constant correlation model with t-distribution holds second place, with the normal Free GARCH model slightly behind, and the BEKK-t ranked fifth. The OLS model holds eighth place, as it did using variance as the performance criterion. and dead last is the Naive hedge.

Table 4.7 presents the results of the last hedging horizon which covers 9 months. A different scenario emerges than the previous ones. Based on reduced variance as the hedging performance criterion, the best model is the Free GARCH with t-distribution with over 5% improvement in the variance over its closest rival, the constant correlation with t-distribution. Moreover, the model exhibits clear superiority over the OLS model with a of 35 % reduction invariance. The third rank goes to the BEKK with t-distribution. In this period, the models that show inferior performance along with the unhedged position include the OLS model and the Naive hedge. The ranking according to the second performance criterion confirms these results. The Free GARCH model with t-distribution is the best, a 3% improvement over its nearest rival, the constant correlation model with t-distribution. Finally, both the OLS model and Naive model underperform the unhedged position.

The following conclusions may be drawn from this assessment of the within sample and out-of-sample hedging performance where the hedge ratio is model dependent:

(iv) The within-sample analysis of the results in this section supports previous research that hedging does eliminate some risk exposure resulting from adverse price movements.

(v) Investors, however, are still exposed to basis risk. Nevertheless, there is support for the position there is considerable improvement in reducing risk when using hedge ratios derived from more sophisticated hedging models.

(vi) Figures 4.1 to Figure 4.6 illustrate the time-varying nature of the hedge ratio with respect to the constant OLS hedge of 0.559. The movement in the hedge ratio corresponds to potential reduction in basis risk. The movement in the hedge ratio is smoother when the models are estimated under the t-distribution as opposed to under the normal.

(vii) The bivariate Free GARCH model with t-distribution is deemed best. It accounts for over 71 % in-sample reduction second moment relative to the unhedged position and about 35 % improvement over the OLS model; out of sample, the respective gains for the 9-months horizon are 30% and 35%.

4.4 The Issue of Daily Verses Weekly Data

The trend in the last decade is for researchers to increase the frequency of the data used in time series analysis. In fact, there is much recent research that uses tick data. In the context of estimating a dynamic hedge ratio, is there any gain in increasing the frequency of the data? More precisely: is there any gain attained from increasing the frequency from weekly to daily data? This section answers this question definitively in the affirmative

The weekly data set consists of Wednesday settlement prices that span the same period as the original daily series, from January 3, 1995 to March 30, 1999 for both the BA and the BAX . This gives a within-sample size of 220 observations The total out-of-sample period spans from March 31, 1999 to December 30, 1999 for a sample size of 41 observation; the total size of the within- and out-of-sample is 261 observation. For the four holidays occurring on Wednesday, Thursday prices are used. The out-of-sample period is divided into three forecast horizons similar to the daily data, a 3-month, a 6-month, and a 9-month horizons.

Four bivariate GARCH models were estimated using weekly data for the purpose of this analysis: the Constant Correlation and BEKK both under normality and the t-distribution. The first important result in this analysis is that the Free GARCH class of models could not be estimated with such a small data set. No sensible estimation results could be found. So the additional data gained by increasing the frequency of the sampling permits the estimation of more complicated models.

The model estimates based on the weekly model permit the determination of a new hedge ratio each Wednesday. Next, it is assumed that the hedging portfolio based on these estimates is fixed from Wednesday to Wednesday, so that the hedging performance of portfolios based on weekly estimates of the hedge ratio can be compared with those based on daily estimates.

Table 4.8 presents the results of the hedging performance of the GARCH models for three out-of-sample periods. In the top panel of the table is organized in the following way, column one identifies the models, column two presents the results of the 3-month hedging horizon, column three presets the results of 6-month hedging horizon, and the last column presets the results of the 9-month hedging horizon. Within each column the hedging performance of the model with weekly portfolio re-balancing is compared to the same model based on daily re-balancing. The top line in each cell, presents the variance in changes in the value of the hedging portfolio; the bottom gives the second moment. The numbers in square brackets are percentage improvements in the variance and the second moment.

The results in the top panel of Table 4.8 are conclusive. There is considerable gain to be had in moving from weekly to daily data. For the 6-month and 9-month horizons the percentage gain is at least 20% on either performance criterion and in some instances over 30%. These gains for models within the standard GARCH models are impressive indeed.

Comparisons between the BEKK-t model estimated with weekly data with the Free GARCH model estimated using the t-distribution on daily data are presented in the second panel of Table 4.8. The message is clear and sound. There is a clear benefit from going from the restrictive traditional models that uses weekly data to a Free GARCH model with no restrictions on the functional form and employing asymmetry parameters estimated of necessity on daily data. The gains run from 30 to 35%.

In sum, there is a 20% gain in hedging performance in moving from weekly to daily data, and a further 10 to 15% gain in moving to the more complicated Free GARCH model.

To conclude this Chapter, one further issue is addressed. We have assumed that the the hedger must re-balance the hedged portfolio daily in response to the daily calculation of the hedge ratio. But the hedger need not to be so active. Consider a re-balancing rule that says that the hedger must maintain the current proportion of futures in the hedging portfolio unless the new hedge ratio surpasses a certain fraction (up or down) of the current ratio. To what extent would such rules affect the performance of the hedging portfolio determined by the Free GARCH-t model?

The answers are provided in Table 4.9. The Table presents the results of imposing a re-balancing bound on the path of the hedge ratio. The bounds are: 0%, corresponding to daily re-balancing; 5 %; 10 %; 25 %; and 50 %, corresponding to infrequent re-balancing. The number of times the portfolio is re-balanced is also indicated. The first observation is that a 25% bound reduces considerably the number of re-balancings to less than a weekly frequency on average. On the other hand, the performance deterioration is not considerable relative to the daily re-balancing.

In this section, we have demonstrated there is a clear advantage for hedging performance in using daily data over weekly data. All of the models estimated using daily data show a superior performance over the same models that uses weekly data. Moreover the daily data allows for the estimation of more complicated models which further improve hedging performance. Finally, this hedging advantage is robust to the number of re-balancings undertaken by the hedger.

4.5 Conclusions

This Chapter extended the analysis in Chapter Three to define the bivariate version of the dynamic Free GARCH models. Such models can be used to determine hedge ratios. The chapter presented a brief description of the history and theory of hedging, and surveyed a variety of hedging techniques. Six bivariate GARCH models were estimated and the estimation results were presented in Tables 4.2a and 4.3a. All of the standard bivariate models with normal distribution failed to satisfy the stationarity conditions; the only model under normality to pass the stationarity requirement was the bivariate Free GARCH model. All the models estimated under t-distribution satisfied the positivity and stationarity conditions, a result similar to the univariate analysis attained in Chapter Three.

The Chapter also evaluated the within-sample and out-of-sample hedging performance using a hedge ratio determined by the estimated parameters of the GARCH models. The within-sample period (Table 4.4) and the 9-month out-of sample period (Table 4.7) both confirm the popular notion that a hedged position is certainly better than no hedge at all.

The main focus of the thesis has been the evaluation of the hedging performance of the models estimated. Here we have reached a clear conclusion: the more complicated models delivered a superior hedging performance over the simpler models for both the within and out-of-sample periods. The Free-GARCH under t-distribution clearly outperformed the rest of the models estimated, based on its ability to further reduce uncertainty as measures by both the variance and the second moment of the constructed hedged portfolio.

An additional issue was tackled concerning the advantage in using daily data compared to weekly data. Here the conclusion is again very strong: there is a significant advantage resulting from the use of daily data in so far as daily re-balancing permits a further reduction in uncertainty and further in so far as the daily data permits the estimation of more complicated models of the hedge ratio.

Figure 4.1:Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX]
Time-Varying vs. OLS Hedge Ratio Constant- Correlation Model under Normality

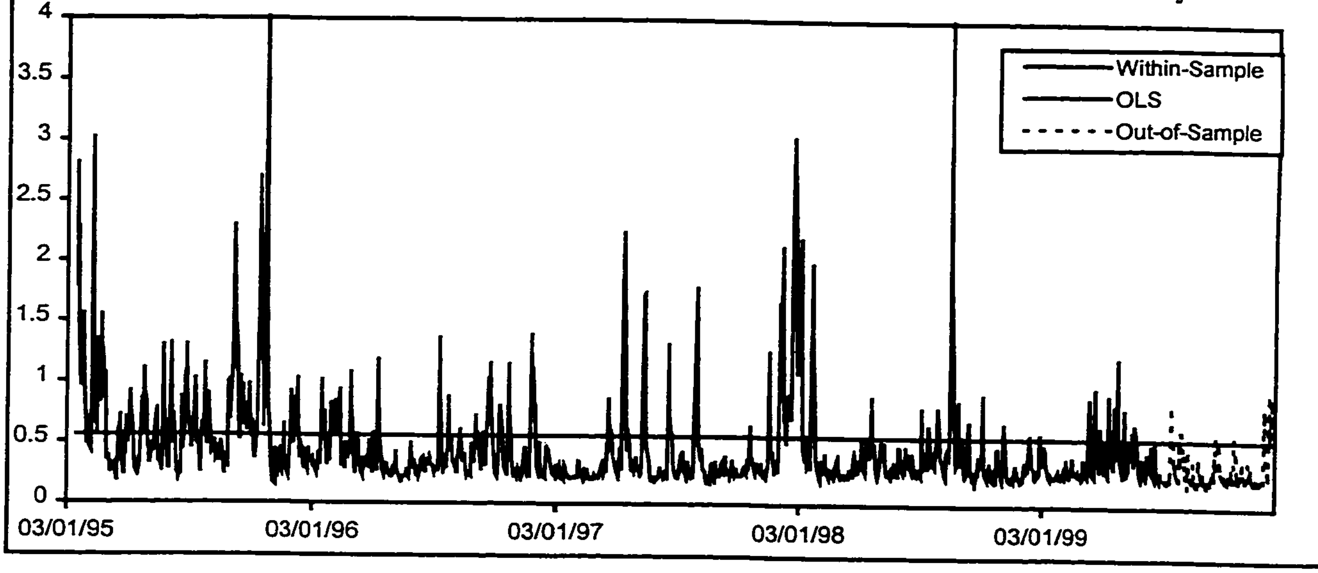


Figure 4.2: Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX]
Time-Varying vs. OLS Hedge Ratio: BEKK Model under Normality

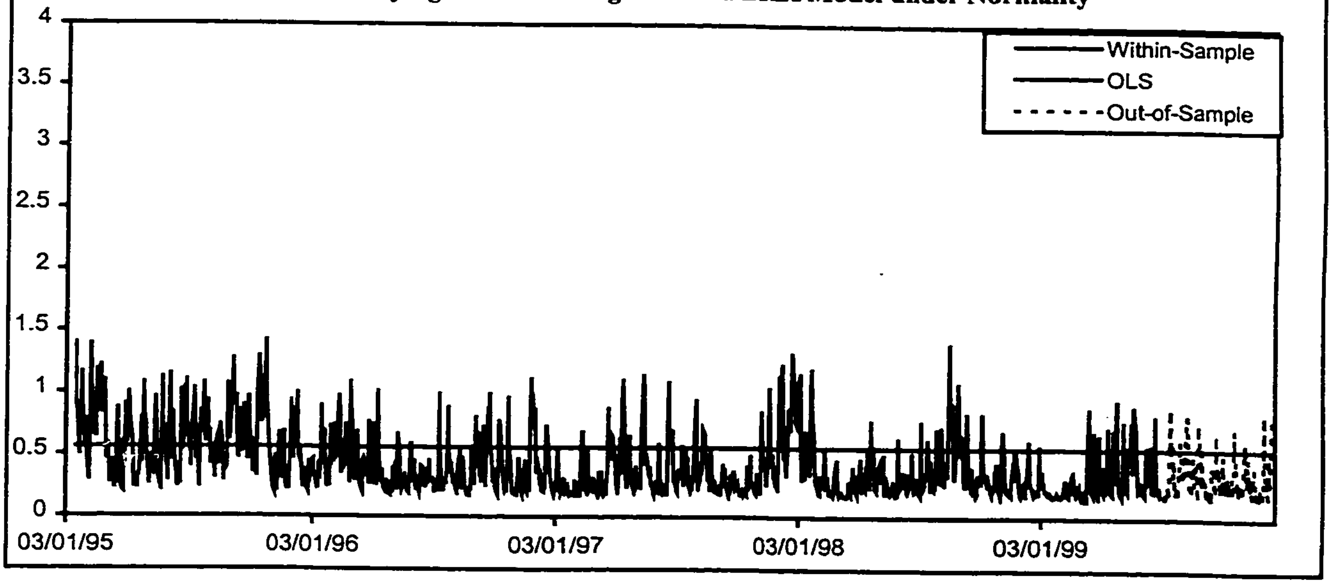


Figure 4.3: Bankers' Acceptance[BA] and Bankers' Acceptance Futures [BAX] Time-Varying vs. OLS Hedge Ratio Constant Correlation Model under t-dist.

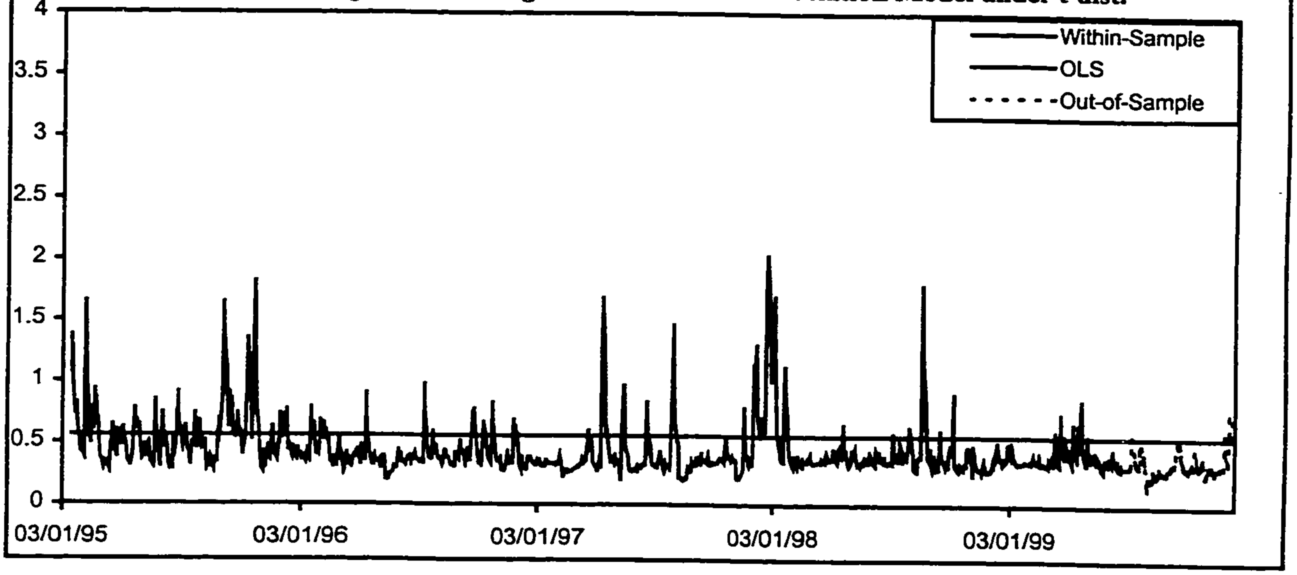
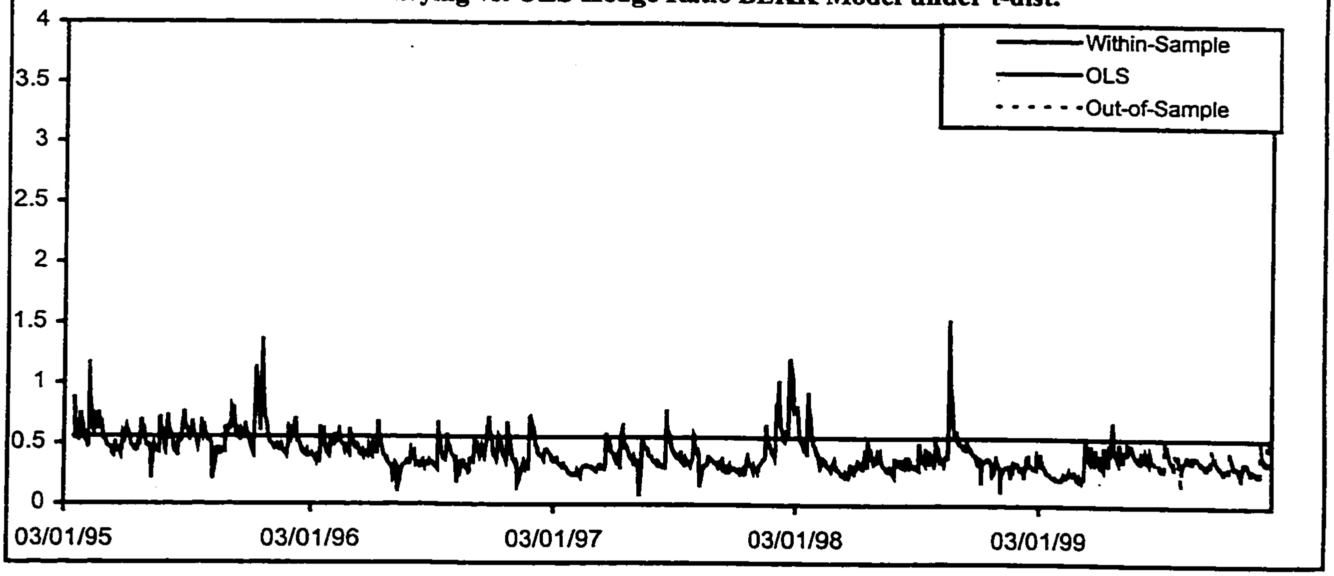


Figure 4.4: Bankers' Acceptance[BA] and Bankers' Acceptance Futures [BAX] Time-Varying vs. OLS Hedge Ratio BEKK Model under t-dist.



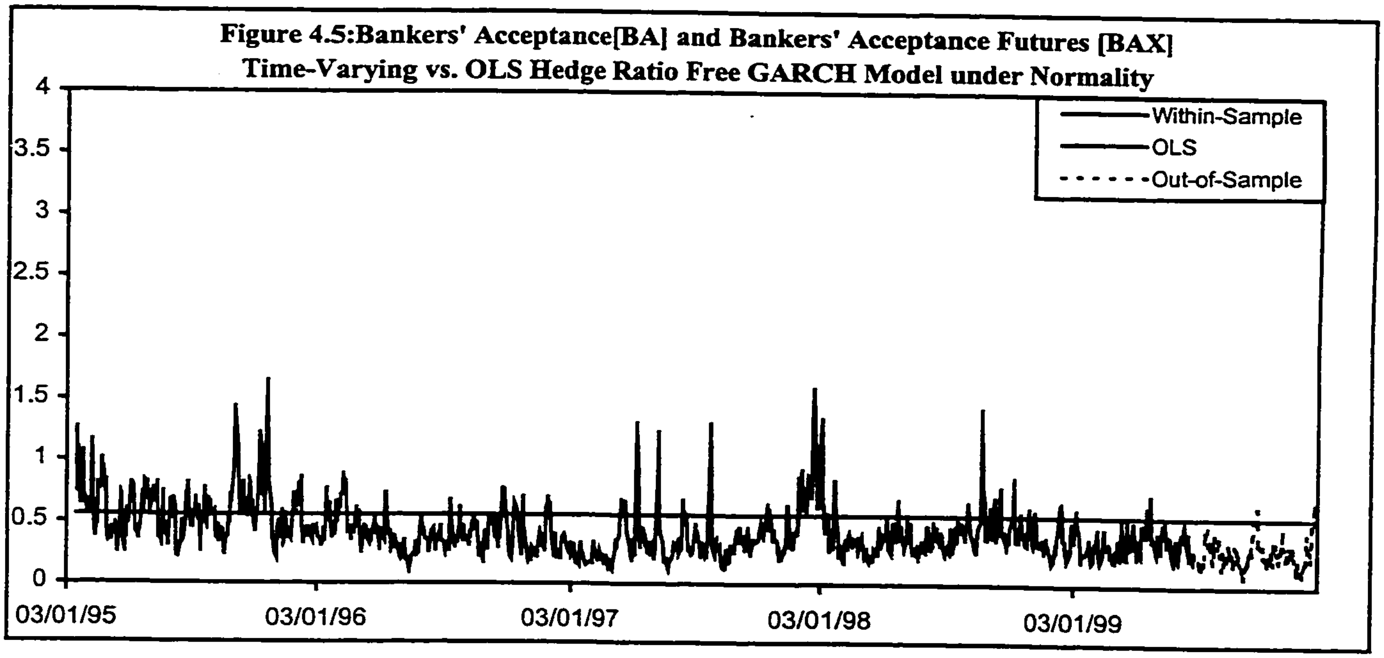


Figure 4.5:Bankers' Acceptance[BA] and Bankers' Acceptance Futures [BAX]
Time-Varying vs. OLS Hedge Ratio Free GARCH Model under Normality
Within-Sample
OLS
Out-of-Sample
4
3.5
3
2.5
2
1.5
1
0.5
0
03/01/95
03/01/96
03/01/97
03/01/98
03/01/99

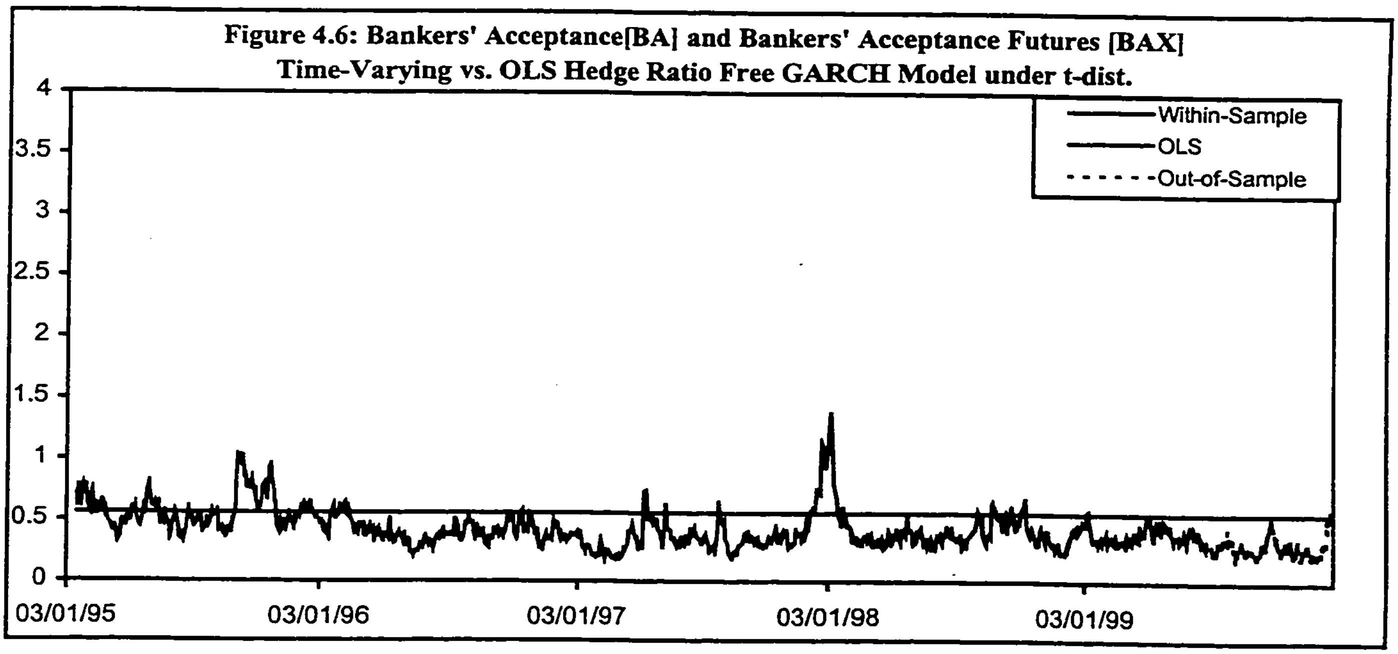


Figure 4.6: Bankers' Acceptance[BA] and Bankers' Acceptance Futures [BAX]
Time-Varying vs. OLS Hedge Ratio Free GARCH Model under t-dist.
Within-Sample
OLS
Out-of-Sample
4
3.5
3
2.5
2
1.5
1
0.5
0
03/01/95
03/01/96
03/01/97
03/01/98
03/01/99

Figure 4.7: Out-of-SampleBankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX] Daily Log Prices

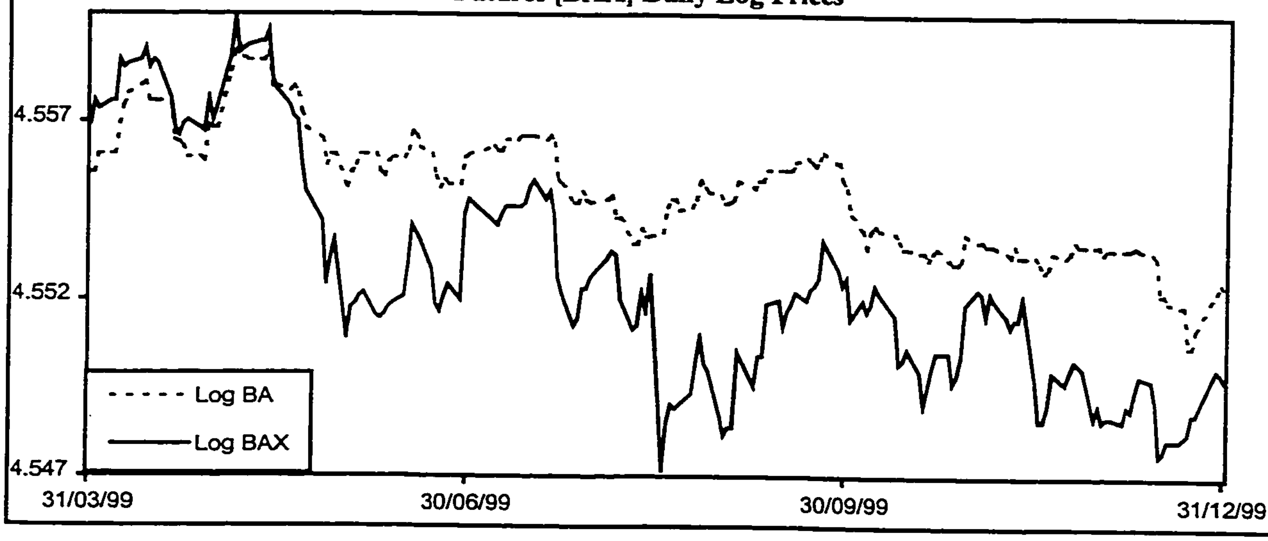


Figure 4.8: Out-of-SampleBankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX] Daily Log Basis

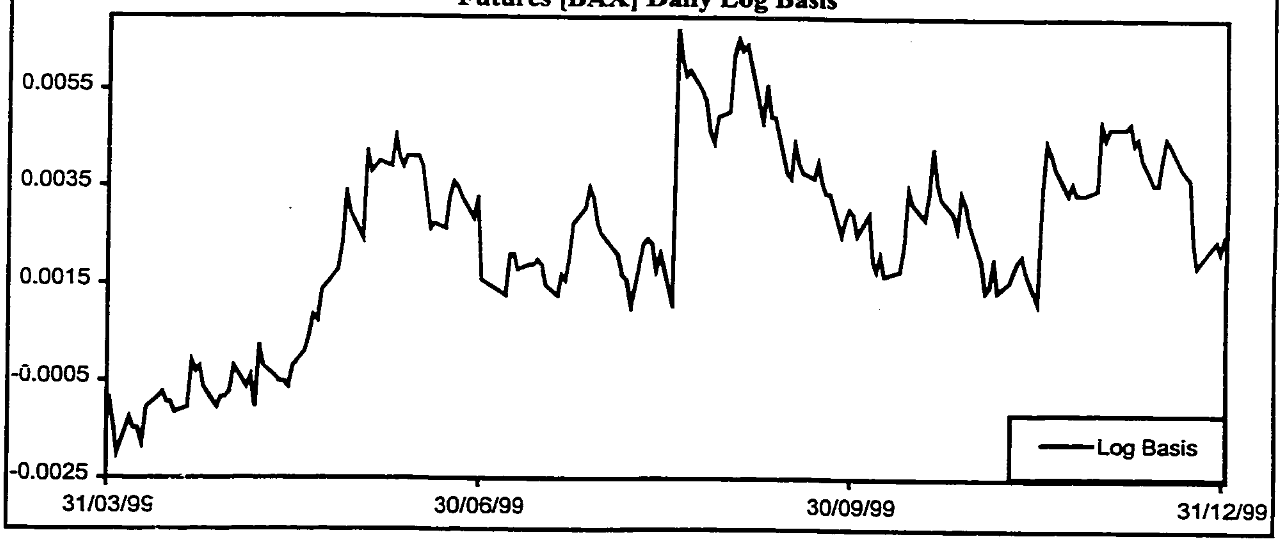


TABLE 4.1
Bankers' Acceptance [BA]
OLS Estimate of the Hedge Ratio

	α	β
OLS (se)	0.000 (.023)	0.558 (.022)
Robust OLS (Robust se)	0.000 (.023)	0.558 (.069)

Equation (4.19) is estimated over the sample January 3, 1995 to March 30, 1999. Standard errors are in parentheses; White standard errors are given in the second row.

Table 4.2a
Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX]
Bivariate GARCH under Normality

Model Parameters		BEKK	Constant Correlation	Free-GARCH
1	λ (SE)	2	2	0.18 (.025)
2	ν (SE)	2	2	0.40 (.033)
3	S_1 (SE)	0	0	0.116 (.006)
4	S_2 (SE)	0	0	-0.446 (.030)
5	R_1 (SE)	0	0	-0.141 (.146)
6	R_2 (SE)	0	0	0.329 (.049)
7	d_1 (SE)	0.015 (.003)	0.018 (.005)	0.016 (.000)
8	d_2 (SE)	-0.012 (.004)	-0.004 (.006)	-0.015 (.000)
9	c_{12} [ρ] (SE)	0.035 (.002)	0.620 (.021)	0.620 (.024)
10	α_{11} [α_{11}/λ] (SE)	0.623 (.013)	0.544 (.009)	0.904 (.021)
11	α_{12} (SE)	0.344 (.012)	---	---
12	α_{21} (SE)	0.387 (.005)	---	---
13	α_{22} [α_{22}/λ] (SE)	0.375 (0.015)	0.012 (.000)	0.225 (.005)
14	β_{11} (SE)	0.294 (.007)	0.397 (.004)	0.761 (.020)
15	β_{12} (SE)	0.197 (.006)	---	---
16	β_{21} (SE)	0.238 (.003)	---	---
17	β_{22} (SE)	0.371 (0.012)	0.946 (.005)	0.888 (.008)

The Table presents the estimates of the Bivariate GARCH models and Free Bivariate GARCH models under normality. The mean equation (4.20).Sample: January 3, 1995 to March 30, 1999 for a total of 1,069 observations. The numbers in parentheses are robust asymptotic standard errors.

Table 4.2b
Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX]
Positivity and Stationarity Conditions for Bivariate GARCH under Normality

Model		Positivity and Stationarity				
		c_{ii}	α_{ii}	β_{ii}	$\|R_i\|$	$E[(\alpha_{ii}\lambda f^{v}(\varepsilon_t)+\beta_{ii})^{2/\lambda}]$ Eigenvalues for the BEKK model
Normal BEKK	i=1 i=2	na	na	na	na	1.11 0.16 0.14 0.02
Normal Constant Correlation	i=1 i=2	0.048 0.027	1.08 0.02	0.39 0.94	0	1.48 0.97
Normal Free-GARCH	i=1 i=2	0.11 0.08	0.16 0.04	0.76 0.88	0.14 0.33	0.36 0.41

The Table presents results regarding positivity and stationarity conditions. For the constant correlation models the positivity conditions are c_{ii}, a_{ii} and $b_{ii} > 0$, $|R_i| < 1$ $(i = 1, 2)$; the stationarity conditions are and $E[(\alpha_{ii}\lambda f^{v}(\varepsilon_t) + \beta_{ii})^{2/\lambda}] < 1$ $(i = 1,2)$. For the BEKK model, the stationarity condition is that all the eigenvalues of the matrix $A \otimes A + B \otimes B$ are less than one in modulus; see the text for details. Sample: January 3, 1995 to March 30, 1999 for a total of 1,069 observations.

Table 4.3a
Bankers' Acceptance[BA] and Bankers' Acceptance Futures [BAX]
Bivariate GARCH under the t-Distribution

Model Parameters		BEKK	Constant Correlation	Free-GARCH
1	λ (SE)	2	2	0.801 (.007)
2	ν (SE)	2	2	0.591 (.042)
3	S_1 (SE)	0	0	-0.001 (.019)
4	S_2 (SE)	0	0	0.050 (.031)
5	R_1 (SE)	0	0	0.127 (.069)
6	R_2 (SE)	0	0	0.381 (.105)
7	d_1 (SE)	0.009 (.002)	0.008 (.003)	0.010 (.002)
8	d_2 (SE)	-0.020 (.004)	-0.021 (.006)	-0.018 (.004)
9	c_{12} [ρ] (SE)	-0.002 (.001)	0.700 (.014)	0.698 (.014)
10	α_{11} [α_{11}/λ](SE)	0.249 (.023)	0.084 (.007)	0.215 (.003)
11	α_{12} (SE)	0.028 (.008)	---	---
12	α_{21} (SE)	0.074 (.007)	---	---
13	α_{22} [α_{22}/λ] (SE)	0.175 (.010)	0.019 (.002)	0.122 (.002)
14	β_{11} (SE)	0.736 (.005)	0.527 (.002)	0.798 (.001)
15	β_{12} (SE)	0.083 (.003)	---	---
16	β_{21} (SE)	0.120 (.004)	---	---
17	β_{22} (SE)	0.888 (.003)	0.823 (.004)	0.863 (.004)

The Table presents the estimates of the Bivariate GARCH models and Free Bivariate GARCH models under the t-distribution. The mean equation (4.20).Sample: January 3, 1995 to March 30, 1999 for a total of 1,068 observations. The numbers in parentheses are asymptotic standard errors.

Table 4.3b
Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX]
Positivity and Stationarity Conditions for Bivariate GARCH
under the t-Distribution

Model		Positivity and Stationarity				
		c_{ii}	α_{ii}	β_{ii}	$\lvert R_i \rvert$	$E[(\alpha_{ii}\lambda f^{\nu}(\varepsilon_t)+\beta_{ii})^{2/\lambda}]$ / Eigenvalues for the BEKK model
t-distribution BEKK	i=1 i=2	na	na	na	na	0.93 0.69 0.68 0.51
t-distribution Con. Corr.	i=1 i=2	0.02 0.03	0.16 0.04	0.53 0.82	0	0.71 0.86
t-distribution Free-GARCH	i=1 i=2	0.02 0.03	0.17 0.10	0.79 0.86	0.12 0.39	0.86 0.87

The Table presents results regarding positivity and stationarity conditions. For the constant correlation models the positivity conditions are c_{ii}, a_{ii} and $b_{ii} > 0$, $|R_i| < 1$ $(i = 1, 2)$; the stationarity conditions are and $E[(\alpha_{ii}\lambda f^{\nu}(\varepsilon_t)+\beta_{ii})^{2/\lambda}] < 1$ $(i = 1,2)$. For the BEKK model, the stationarity condition is that all the eigenvalues of the matrix $A \otimes A + B \otimes B$ are less than one in modulus; see the text for details. Sample: January 3, 1995 to March 30, 1999 for a total of 1,069 observations.

Table 4.4
Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX]
Bivariate GARCH Models
Within Sample Hedging Performance

	Mean	Variance Second Moment	% Reduction Unhedged	% Reduction OLS
Unhedged	0.01926	.88583 [9] 1.28114 [9]	----- -----	-57.8 -128.2
Naive	-0.01458	.76456 [8] .99103 [7]	13.7 22.6	-36.2 -76.5
OLS	0.00040	.56134 [7] .56148 [6]	36.6 56.1	----- -----
Constant Correlation normal	-0.02169	.51489 [6] 1.00933 [8]	41.8 21.2	8.3 -79.7
BEKK normal	-0.00392	.40623 [5] .42238 [4]	54.1 67.0	27.6 24.8
Constant Correlation t-dist.	-0.00828	.39827 [4] .47032 [5]	55.0 63.3	29.0 16.2
BEKK t-dist.	-0.00329	.37164 [3] .38301 [2]	58.0 70.1	33.8 31.8
Free-GRACH normal	-0.00497	.36455 [1] .39051 [3]	58.8 69.5	35.0 30.4
Free-GARCH t-dist.	0.00105	.36460 [2] .36575 [1]	58.8 71.4	35.0 34.9

The models are ranked with regard to the variance and the second moment of the values of the hedging portfolio given by Equations (4.22). Overall percentage changes for the two measures are given in the last two columns with the unhedged position as the benchmark in column 4 and OLS position in the last column. Sample: January 3, 1995 to March 30, 1999 for a total of 1,069 observations.

Table 4.5
Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX]
Bivariate GARCH Models
Out of Sample [3-month] Hedging Performance

	Mean	Variance Second Moment	% Reduction Unhedged	% Reduction OLS
Unhedged	0.03372	.23763 [8] .29779 [1]	----- -----	-111.5 27.1
Naive	0.10721	.25240 [9] .86161 [9]	-6.2 -189.3	-125.0 -110.9
OLS	0.07475	.11233 [1] .40845 [7]	52.7 -37.2	----- -----
Constant Correlation normal	0.06486	.16923 [6] .39219 [6]	28.8 -31.7	-50.6 4.0
BEKK normal	0.06884	.17396 [7] .42513 [8]	26.8 -42.8	-54.9 -4.1
Constant Correlation t-dist.	0.05998	.13366 [5] .32431 [2]	43.7 -8.9	-19.0 20.6
BEKK t-dist.	0.06744	.12741 [3] .36845 [5]	46.4 -23.7	-13.4 9.8
Free-GARCH normal	0.06069	.13289 [4] .32813 [3]	44.0 -10.2	-18.3 19.7
Free-GARCH t-dist.	0.06511	.11522 [2] .33987 [4]	51.5 -14.1	-2.6 16.8

The table presents the out-of-sample [3-month] hedging effectiveness. Overall percentage changes for the two measures are given in the last two columns with the unhedged position as the benchmark in column 4 and OLS position in the last column. Sample: March 31, 1999 to June 14, 1999 for a total of 53 observations.

Table 4.6
Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX]
Bivariate GARCH Models
Out of Sample [6-month] Hedging Performance

	Mean	Variance Second Moment	% Reduction Unhedged	% Reduction OLS
Unhedged	0.01280	.16275 [7] .18159 [1]	----- -----	11.4 43.7
Naive	0.05218	.57430 [9] .88736 [9]	-252.3 -388.6	-212.5 -174.8
OLS	0.03478	.18376 [8] .32287 [8]	-12.9 -77.8	----- -----
Constant Correlation normal	0.0346	.14288 [5] .28042 [6]	12.2 -54.4	22.2 13.1
BEKK normal	0.0329	.15835 [6] .28297 [7]	2.7 -55.8	13.8 12.3
Constant Correlation t-dist.	0.0316	.12673 [2] .24129 [3]	22.1 -32.8	31.0 25.3
BEKK t-dist.	0.031	.13384 [4] .24440 [5]	17.76 -34.6	27.2 24.3
Free-GARCH normal	0.0307	.13366 [3] .24186 [4]	17.87 -33.2	27.3 25.1
Free-GARCH t-dist.	0.0301	.11369 [1] .21806 [2]	30.1 -20.0	38.1 32.5

The table presents the out-of-sample [6-month] hedging effectiveness. Overall percentage changes for the two measures are given in the last two columns with the unhedged position as the benchmark in column 4 and OLS position in the last column. Sample: March 31, 1999 to September 13, 1999 for a total of 115 observations.

Table 4.7
Bankers' Acceptance [BA] and Bankers' Acceptance Futures [BAX]
Bivariate GARCH Models
Out of Sample [9-month] Hedging Performance

	Mean	Variance Second Moment	% Reduction Unhedged	% Reduction OLS
Unhedged	-0.00998	.13823 [7] .15715 [7]	----- -----	12.1 6.7
Naive	0.02164	.46684 [9] .55578 [9]	-237.7 -253.7	-196.8 -229.9
OLS	0.00767	.15729 [8] .16847 [8]	-13.8 -7.2	----- -----
Constant Correlation normal	0.00644	.11760 [5] .12549 [4]	14.9 20.1	25.2 25.5
BEKK normal	0.00352	.12577 [6] .12812 [5]	9.0 18.5	20.0 23.9
Constant Correlation t-dist.	0.00546	.10906 [2] .11474 [2]	21.1 27.0	30.6 31.9
BEKK t-dist.	0.00365	.11337 [3] .11590 [3]	18.0 26.2	27.9 31.2
Free-GARCH normal	0.00817	.11637 [4] .12905 [6]	15.8 17.9	26.0 23.4
Free-GARCH t-dist.	0.00632	.10174 [1] .10934 [1]	26.4 30.4	35.3 35.1

The table presents the out-of-sample [9-month] hedging effectiveness. Overall percentage changes for the two measures are given in the last two columns with the unhedged position as the benchmark in column 4 and OLS position in the last column. Sample: March 31, 1999 to December 30th, 1999 for a total of 190 observations.

TABLE 4.8
Out-of-Sample Hedging Performance
WEEKLY GARCH MODELS vs. DAILY GARCH MODELS

Models	3-Month Weekly Daily Variance [%Red] 2nd Mom [%Red]		6-Month Weekly Daily Variance [%Red] 2nd Mom [%Red]		9-Month Weekly Daily Variance [%Red] 2nd Mom [%Red]	
Constant Correlation normal	.11935	.16923[-42]	.19453	.14288[26]	.16398	.11760[28]
	.46634	.39219[16]	.35648	.28042[21]	.17649	.12549[29]
BEKK normal	.14257	.17396[-22]	.19832	.15835[20]	.16608	.12577[24]
	.61545	.42513[31]	.44955	.28297[37]	.20949	.12812[39]
Constant Correlation t-dist.	.11952	.13366[-12]	.16489	.12673[23]	.14136	.10906[23]
	.47397	.32431[31]	.32122	.24129[25]	.15122	.11474[24]
BEKK t-dist.	.13786	.12741[7.6]	.17232	.13384[22]	.14870	.11337[24]
	.58688	.36845[37]	.34614	.24440[29]	.16451	.11590[29]
Potential Gain Daily Free-GARCH-t dist. vs. Weekly BEKK-t dist.						
Free-GARCH t-dist.	.11522 [16]		.11369 [34]		.10174 [31]	
	.33987 [42]		.21806 [37]		.10934 [33]	

The Table presents a hedging comparison between models using weekly observations and models using daily observation. See text for details.

TABLE 4.9

Impact of Re-balancing Constraints on Portfolio Performance

Daily Free GARCH t-dist. Model

Re-Balancing Bounds	Number of Re-Balancings	Variance	2^{nd} Moment	% Reduction Weekly BEKK t.-dist. Variance	2^{nd} Moment
Three-Month Horizon					
0 %	53	.11522	.33987	16	42
5 %	36	.11505	.33987	16	42
10 %	24	.11550	.32774	16	44
25 %	5	.11195	.36119	19	38
50%	2	.11209	.48019	19	18
Six-Month Horizon					
0 %	115	.11369	.21806	34	37
5 %	73	.11588	.22349	33	35
10 %	44	.11754	.22116	32	36
25 %	14	.10668	.20552	38	40
50%	6	.13572	.26947	21	22
Nine-Month Horizon					
0 %	190	.10174	.10934	31	33
5 %	127	.10324	.11099	30	32
10 %	84	.10396	.11135	30	32
25 %	28	.09785	.10031	34	39
50%	10	.11557	.12094	22	26

The Table presents a comparison between the weekly BEKK t-dist. model and the Free-GARCH t-dist. model. A Re-balancing bound of 5, 10, 25, and 50 percent are set on the hedge ratio estimate of the Free-GARCH model. See text for further details.

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