# A dynamical test for terrestrial planets in the habitable zone of HD 204313 

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#### Abstract

Summary: With improvements in exoplanet detection techniques, the number of multiple planet systems discovered is increasing, while the detection of potentially habitable Earth-mass planets remains complicated and thus requires new search strategies. Dynamical studies of known multiple planet systems are therefore a vital tool in the search for stable and habitable planet candidates.


Here, we present a dynamical study of the three-planet system HD 204313 to determine whether it could harbour an Earth-like planet within its habitable zone for a sufficient time to develop life. We found two semi-stable regions in the system, but neither prove stable for long enough for a terrestrial planet to develop life. Our investigations suggest that overlapping weak and high order resonances may be responsible for these semi-stable regions. This study established a framework for a larger project that will study the dynamical stability of the habitable zone of multiple planet systems, providing a list of interesting targets for future habitable low-mass planet searches.

Keywords: Planetary systems, Numerical methods: N-body simulation, Planetary systems: dynamical evolution and stability, Exoplanets, Habitability, Astrobiology

## Introduction

Since the discovery of the first exoplanet, more than a thousand exoplanets have been confirmed, including 170 multiple planet systems. With improvements in exoplanet detection techniques, new discoveries are expected and will continue to enhance our understanding of planetary systems and the planet formation process. However the detection of habitable Earthmass planets remains complicated, and only 12 super-Earth planets are currently known to orbit within the habitable zone of their host star [1], while 35 additional Kepler candidates $\sqrt[2]{ }$ are still waiting to be confirmed.

[^0]The habitable zone $(\mathrm{HZ})$ is defined as the region around a star where liquid water could be present on the surface of rocky planets [2]. The presence of liquid water on the surface depends on a broad range of parameters, such as the stellar incident energy, spectral distribution, radiative properties of the planetary atmosphere (absorption, diffusion, emission, clouds) and the reflectivity of its surface (albedo) [2]. Earth is the only habitable planet of which we currently know, and it appears that life took $\sim 1$ Gyr to develop a significant atmospheric signature that would be detectable [3]. Using the Earth as a baseline, we will therefore work on the assumption that a planet would need to survive for at least 1 Gyr within a star's habitable zone to detect any life there. This therefore requires additional constraints to be placed on the orbit of the planet for it to be deemed detectably habitable - not only must it currently orbit within the habitable zone, but it must also maintain an orbit which is simultaneously dynamically stable and confined within the HZ (i.e. a small eccentricity) for this entire duration.

Dynamical studies thus play a crucial role in the hunt for habitable exoplanets, and the evaluation of the long-term stability of the HZ of known multiple exoplanet systems can help focus future observations. There are a number of examples in the literature of multiple planet systems which turn out to be dynamically unstable on short timescales. For example, the two planet system orbiting the catacylsmic variable HU Aquarii announced by [4] was shown by [5] and [6] to be unstable on timescales of order $10^{4}$ years, strongly suggesting that the multiplanet system is not real or that the orbital parameters must be different from those reported in the literature. While numerical simulations have been commonly used and are necessary to understand the dynamics of multiple systems (e.g. [5], [7], [8]), analytical stability criteria tools can be used to provide a quick assessment of the stable regions of a multiplanet system (e.g [9], [10]).

We aim to establish a new framework to systematically investigate the stability of HZs of multiple planetary systems. Firstly, the HZ of such systems must be located using atmospheric model criteria, and then a numerical search for dynamically stable regions within the HZ can be conducted. This requires the integration of the orbits of massless test particles randomly distributed inside the HZ . Once stable regions (where test particles survive at least 1 Gyr ) are identified, we can then explore their long-term stability by (i) investigating their resonance state, and (ii) producing maps of their stability as a function of orbital elements by varying the stable test particles initial conditions around the stable known initial parameters. Finally, to ensure stability of a potential Earth-mass planets in these stable and habitable zones, numerical simulations must be run by substituting the stable test particles with Earth-mass planets.

In this paper we present a study of the dynamical stability of test particles in the habitable zone of the star HD 204313. Using the N-body integrator SWIFT [11], we first isolate regions of stability within the habitable zone of HD 204313 using massless test particles. We then numerically investigate the long-term stability of test particles in the identified dynamically stable regions of the HZ and compare these locations with the stable regions derived using the analytical stability criteria of [9]. The paper is organized as follows: first, we describe the HD 204313 system, and the location of its habitable zone, in the following section. We then present the analytical and numerical methods used to investigate the stability of the system's habitable zone. We then report our findings in section 3 before discussing the potential origin of two identified semi-stable zones in section 4.

## The HD 204313 System and its Habitable Zone

HD 204313 is a metal-rich Sun-like star of spectral type G5 [12], with a mass of 1.02 $M_{\odot}$ and age of 7.2 Gyrs [13]. It has been the target of multiple radial velocity surveys and was observed by [14] with the 2.7 m Smith telescope over 8 years to obtain Radial Velocity measurements and a stellar spectrum. Using the MOOG local thermodynamic equilibrium line analysis and spectral analysis program [15], they determined the effective temperature, $T_{\text {eff }}$ and by matching the abundances of the Fe I and Fe II lines, they determined the metallicity of HD 204313, [Fe/H]. The star's distance and absolute magnitude, $m_{V}$, were also derived by [14] and all their stellar properties are summarized in Table 1 .

Table 1: Stellar properties for HD 204313 from [14].

| Property | Value |
| :---: | :---: |
| Spectral type ${ }^{a}$ | G5 V |
| Age ${ }^{\text {b }}$ | 7.2 Gyrs |
| Mass ${ }^{\text {b }}$ | $1.02 M_{\odot}$ |
| Distance | $47.0 \pm 0.3 \mathrm{pc}$ |
| $T_{\text {eff }}$ | $5760 \pm 100 \mathrm{~K}$ |
| $m_{V}$ | $4.63 \pm 0.03$ |
| [Fe/H] | $0.24 \pm 0.06$ |

${ }^{b}$ From [13], maximum likelihood estimate.

This system is known to harbour at least three exoplanets: [16] presented the discovery of HD 204313 b , a super-Jupiter planet ( $M \sin (i) \sim 3.5 M_{J}$ ) with a period $P \sim 5$ years with the CORALIE Echelle spectrograph, while [17] revealed an interior ( $P \sim 35$ days) Neptune-mass planet HD 204313 c with the HARPS spectrograph. [14] simultaneously fitted the data from the CORALIE Echelle spectrograph in conjunction with their observations from the Harlan J. Smith Telescope at McDonald Observatory. The residuals from their fully generalised LombScargle periodogram for a one-planet model (HD 204313 b) showed significant peaks, notably around $P \sim 2700$ days. By fitting an additional planet at this period, their fitting routine converged for an additional Jupiter-mass planet: HD 204313 d with mass $M \sin (i) \sim 1.68$ $M_{J}$, and a period of 2831 days. They did not include the planet HD 204313 c in their study since its radial velocity amplitude as detected by [17] was below the sensitivity limit of their data, but that planet could be responsible for the remaining scatter in their two planet fit. We will be using the orbital parameters derived by [14] for the two planets, HD 204313 b and d, in our study - see Table 2. Their best-fit orbits reveal that HD 204313 b and d are trapped in a mutual 3:2 mean-motion resonance (MMR). By performing further dynamical analysis, the authors found that outside this MMR, there is no stable configuration for these two planets, and that fine tuning of orbital elements of the two planets was required in order for this system to survive. Moreover they confirmed that adding the inner planet, HD 204313 c, to their dynamical analysis does not affect the behaviour of HD 204313 b and d.

As discussed earlier, the location of the HZ around a given star depends both on the nature of the star itself (parameters such as its luminosity and temperature) and the atmosphere of the planet in question (absorption, diffusion, albedo, emission efficiency and cloud coverage) [2].

Thus the location of the HZ is expected to migrate outward as the star evolves on the main sequence and its effective temperature and luminosity increase. Therefore the time available for a planet to develop life in its HZ depends on (i) the width of this zone and (ii) the lifetime of a star on its main sequence (which depends of the stellar mass and metallicity) [18]. There are several ways to define the inner and outer boundary of the HZ (as discussed in [19], [20], [18] and [21]). We select the inner boundary as the maximum distance from the star at which a 'runaway greenhouse effect' would lead to the evaporation of all surface water, while for the outer boundary we chose the 'maximum greenhouse effect', which defines the distance at which a cloud-free $\mathrm{CO}_{2}$ atmosphere could maintain a surface temperature of 273 K .

To calculate the distance of these boundaries from the star, we followed the work of [22], who express the effective flux received at the HZ boundaries, $S_{\text {eff }}$, as a quadratic function of the difference between the effective temperature of the star and that of the Sun, $T_{e f f}-T_{\odot}$. The slope of this function is given by coefficients that depend on the boundary criteria of the HZ, with all coefficients for each boundary given in [22]. We selected the coefficients for a 'runaway greenhouse effect' as inner boundary and the coefficients for a 'maximum greenhouse effect' as an outer boundary. The ratio between the luminosity of the star and our Sun, $L / L_{\odot}$, is then deduced from its apparent visual magnitude, $m_{V}$, and from its distance to our Sun, $D$, in pc. Finally, the corresponding distance, $d_{\mathrm{HZ}}$, of the limits of the HZ is given by:

$$
\begin{equation*}
d_{\mathrm{HZ}}=\left(\frac{1}{S_{e f f}} \frac{L}{L_{\odot}}\right)^{1 / 2} \tag{1}
\end{equation*}
$$

Using the stellar parameters from Table 1, we find the HZ boundaries for HD 204313 at:

$$
\begin{gathered}
H Z_{\text {inner }}(\text { runaway greenhouse })=1.1 \mathrm{AU} \\
H Z_{\text {outer }}(\text { maximum greenhouse })=1.9 \mathrm{AU}
\end{gathered}
$$

## Numerical and Analytic methods

We expand on the work of [14], who conducted a dynamical stability study of the HD 204313 system to constrain the orbital parameters of the two outer planets using the N -body dynamics package MERCURY [23]. Because of its negligible mass compared to the other two planets and its short period, HD 204313 c was found to have little impact on the system's dynamics: its Hill radius, which defines its zone of gravitational influence, is $\sim 0.005 \mathrm{AU}$ which means that it is dynamically well separated from the two outer planets and the HZ, and therefore

Table 2: Orbital parameters of HD 204313 planets from [14]

| Elements | HD 204313 ${ }^{\text {a }}$ c | HD 204313 b | HD 204313 d |
| :---: | :---: | :---: | :---: |
| $M \sin (i)\left(M_{J}\right)$ | $0.054 \pm 0.005$ | $3.55 \pm 0.2$ | $1.68 \pm 0.3$ |
| $a$ ( AU ) | $0.2103 \pm 0.0035$ | $3.04 \pm 0.06$ | $3.93 \pm 0.14$ |
| e | $0.17 \pm 0.09$ | $0.23 \pm 0.04$ | $0.28 \pm 0.09$ |
| $P$ (days) | $34.88 \pm 0.03$ | $1920 \pm 25$ | $2830 \pm 150$ |
| $M\left({ }^{\circ}\right.$ ) |  | $300 \pm 0.4$ | $137 \pm 2$ |
| $\omega\left({ }^{\circ}\right)$ |  | $298 \pm 6$ | $247 \pm 16$ |

does not significantly perturb them. As a result, those authors did not include that planet in their long-term simluations. In their study, the orbital parameters of HD 204313 b were kept fixed, while those of HD 204313 d were varied: the eccentricity, e, semi-major axis, $a$, longitude of periastron, $\omega$, and mean anomaly, $M$, of HD 204313 d were systematically varied by $\pm 3 \sigma$ around their best fit values, with simulations run for a maximum $10^{8}$ years. The orbits of the two giant planets are assumed to be coplanar. They derived two stability maps displaying the mean dynamical lifetime of the HD 204313 b and d planetary system as (i) a function of the initial semi-major axis and eccentricity of planet d, and (ii) as a function of the initial semimajor axis and longitude of periastron of planet $d-$ see their Figure 5 [14].

We used the SwIFT integration software package to numerically integrate the orbits of HD 204313 b and HD 204313 d over a similar duration ( $10^{8}$ years). We used the N-body symplectic RMVS integrator of SWIFT, a well-tested integrator first computed by [11] which conserves the Hamiltonian during the integration and integrates close encounters between planets and test particles. Their goal was to model the orbit of short-period comets under the gravitational influence of all the Solar system's planets except Mercury. To ensure that Swift gives similar results to MERCURY, we repeated a subsample of the simulations of [14] with SWIFT, selecting 13 simulations along the edge of the stable zone in both $a, e$ and $a, \omega$ space - see Table 3. We found that $11 / 13$ of the simulations were stable for $10^{8}$ years, with the

Table 3: Initial orbital parameters of HD 204313 d for the 13 simulations taken from [14]]

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | $9^{a}$ | 10 | 11 | 12 | $13^{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a(\mathrm{AU})$ | 3.90 | 3.93 | 3.93 | 3.98 | 3.98 | 3.98 | 3.98 | 3.98 | 4.01 | 4.01 | 4.01 | 4.01 | 4.07 |
| $e$ | 0.41 | 0.12 | 0.3 | 0.05 | 0.05 | 0.12 | 0.12 | 0.30 | 0.05 | 0.05 | 0.24 | 0.24 | 0.21 |
| $\omega\left({ }^{\circ}\right)$ | 292 | 256 | 283 | 238 | 247 | 247 | 256 | 292 | 238 | 247 | 283 | 292 | 274 |
| $M\left({ }^{\circ}\right)$ | 137 | 137 | 137 | 137 | 137 | 137 | 137 | 137 | 137 | 137 | 137 | 137 | 131 |
| Models 9 and 13 were found to be unstable. |  |  |  |  |  |  |  |  |  |  |  |  |  |

two unstable models originating at the extreme edge of the stability zone of [14] (see their Figure 5). Given that Mercury and Swift do not handle close encounters in exactly the same manner, it is not surprising to see some differences for critical cases. Whilst Mercury treats a close encounter between a planet and test particle by switching from a sympletic integrator to a classic Bulirsch-Stoer integrator to ensure the precision set by the user is reached, SWIFT treats the close encounter using the same symplectic RMVS integrator but switching to the planet as the barycenter of the system instead of the star during the encounter.

Using these 11 stable models, we investigated the system's suitability for harboring additional stable bodies by randomly distributing 2000 massless test particles in the HZ of HD 204313 and integrating for $10^{8}$ years. The test particles had initial eccentricities $e_{T P}$ ranging from 0.0 to 0.3 , inclinations $i_{T P}$ of 0 degrees, and random values of longitude of periastron, $\omega_{T P}$, and mean anomaly, $M_{T P}$. The outcome of these simulations allowed us to identify stable regions within the HZ. We then inspected the long-term stability of these zones by determining the resonance state of the stable particles, and produced maps of the test particle lifetimes as a function of their initial orbital elements. To create these maps, we ran additional simulations with test particle orbital elements ( $a_{T P}, e_{T P}, \omega_{T P}, M_{T P}$ ) distributed over a specific range of values: we examined 51 values of $a_{T P}$ in a radius of 0.6 AU around the stable zone; 51 values of $e_{T P}$ between $0.0-0.3$ for each $a_{T P} ; 19$ values of $\omega_{T P}$ between $0.0-360^{\circ}$ for each $e_{T P}$ value and 19 values of $M_{T P}$ between $0.0-360^{\circ}$ for each $\omega_{T P}$ value. Thus more than $10^{6}$ test particles with unique initial conditions were generated to derive a
high resolution stability map as a function of orbital elements inside the HZ .
By following this method, the numerical simulations can provide a complete picture of the stability state of test particles in the HZ. However, to ensure no dependence on exact initial conditions, this method requires a large number of simulations ( $10^{6}$ in our experiment). Analytical criteria are another way to quickly appraise the location of stable regions. By generalizing the resonance overlap criterion for the onset of stochastic behaviour in the planar circular restricted three-body problem from [24] to multiplanetary systems, [9] provide analytical expressions that can put constraints on the location of stable zones. Their criterion is based on that the fact that if the orbits of planets in a multiplanetary system cross and are not protected by some resonant mechanism, this could lead to the planets in the system colliding with one another, or being ejected from the system as a result of their mutual interactions. More precisely, around each planet $j$ exists a region of instability, $\delta_{j}$, where a close encounter with another body will perturb the dynamics enough to lead to chaotic diffusion of the eccentricity and semi-major axis, resulting in either collision or escape. The width of this region depends on the mass, $m$, eccentricity, $e$, and semi-major axis, $a$, of the planet, and on the mass of the encountering body. In our case, we will apply this criterion of the two planets HD 204313 b and HD 204313 d encountering an Earth-mass planet. Thus $\delta_{j}$ is given by [9]:

$$
\begin{equation*}
\delta_{j}=1.57 \times a(j) \times\left(\left(\frac{m(j)}{m_{\text {star }}}\right)^{2 / 7}+\left(\frac{m_{\text {earth }}}{m_{\text {star }}}\right)^{2 / 7}\right) . \tag{2}
\end{equation*}
$$

In the simple case of a planet $j$ on a circular orbit, as long as no other bodies enter the region $a_{j} \pm \delta_{j}$ around the planet, the bodies will be stable with respect to planet $j$. For eccentric planets such as HD 204313 b and d , this region is extended to $q_{j}-\delta_{j}$ and $Q_{j}+\delta_{j}$, where $q_{j}$ is the periastron and $Q_{j}$ the apastron of planet. We use the expressions derived by [9] to assess the position of the interior and exterior limit of the unstable zone around HD 204313 b and d, and compare it to the stability zone of our test particles in the HZ from our numerical simulations.

## Results \& Discussion

In each of our simulations, all of the test particles were removed from the HZ on timescales far shorter than the maximum possible run time ( $10^{8}$ years). As a result, the integrations were terminated when the final test particle was removed - and so no simulation actually completed its full 100 million year run time. Figure 1 shows the time evolution of the semi-major axis, $a$, for the test particles and the two planets for the longest lasting model in which the final test particle was ejected after a period of $\sim 9 \times 10^{6}$ years (model 4 in Table 3). In all other models, the time at which the final test particle was removed was always significantly shorter. Figure 2 zooms into the HZ region of model 4, in which it is apparent that all test particles in the HZ are rapidly removed by the gravitational influence of the two outer giant planets, aside from two areas located around $a \sim 1.5 \mathrm{AU}$ and $a \sim 1.4 \mathrm{AU}$. We observed the same stable regions in each of our 11 simulations, and those regions were always located at the same distance from the central star. We therefore carried out a more detailed investigation in order to determine why test particles in these two regions displayed semi-stable behaviour across our entire suite of simulations.

One potential explanation for the behaviour is resonance trapping (e.g. [25]), and so we examined the potential mean motion resonances (MMR) located between 1.4 and 1.6 AU


Fig. 1: Evolution of the semi-major axis, a, for the test particles and planets in model 4. The dotted lines represent the location of the habitable zone.


Fig. 2: Zoom of the HZ region of model 4, following the evolution of the semi-major axis, a, for the test particles. The dashed lines represent the location of the habitable zone.
resulting from the two giant planets, HD 204313 b and d. Particles orbiting around $a \sim 1.5 \mathrm{AU}$ have a period $P_{T P_{1.5}} \sim 667$ days, while particles orbiting around $a \sim 1.4 \mathrm{AU}$ have a period $P_{T P_{1.4}} \sim 601$ days. Thus the ratio between the orbital periods of the particles trapped at 1.5 AU and HD 204313 d is $P_{d} / P_{T P_{1.5}}=4.2$, suggesting that test particles may possibly be trapped in a 1:4 MMR with HD 204313 d. Similarly, the ratio between the orbital periods of the particles trapped at 1.4 AU and HD 204313 b is $P_{b} / P_{T P_{1.4}}=3.2$, suggesting that test particles may possibly be trapped in a 1:3 MMR with HD 204313 bl ${ }^{3}$. To test this scenario, we plotted the evolution of the resonant argument, $\phi$, for those MMRs:

$$
\begin{align*}
& \phi_{d}=\lambda_{T P}-4 \lambda_{d}+3 \omega_{T P}  \tag{3}\\
& \phi_{b}=\lambda_{T P}-3 \lambda_{b}+2 \omega_{T P} \tag{4}
\end{align*}
$$

[^1]

Fig. 3: Variation in the resonant argument $\phi_{b}$ given by Eq. 3 for the longest living test particle at 1.4 AU of model 4.


Fig. 4: Variation in the resonant argument $\phi_{d}$ given by Eq. 4 for the longest living test particle at 1.5 AU of model 4.
where $\lambda$ is the mean longitude and $\omega$ the argument of the periastron. However we found that these resonant arguments did not exhibit libration or any periodic behaviour for any of the test particles tested - as can be seen in Figures 3 and 4 ,

Since the locations of the semi-stability regions do not match any strong MMRs with HD 204313 b or HD 204313 d, we examined whether the test particles might be trapped in regions where a number of overlapping weak and/or high order MMRs could be combining to create a region of "sticky chaos", acting to stabilise the test particles motion [25]. We calculated all possible resonances with HD 204313 b or HD 204313 d up to order 51 and plotted their individual resonant arguments with respect to both planets. However, we were unable to identify any clear resonant behaviour. We therefore conclude that no single weak/high order resonance is responsible for these semi-stable regions, although we note that test particles may be captured and stabilised as a result of the overlap between a number of these high-order resonances. In such a scenario, test particles would chaotically hop from one resonance to another, without spending any significant period of time trapped in any specific resonance. Such behaviour has been invoked in the past to explain regions of stability for


Fig. 5: The mean dynamical lifetime of the test particles as a function of (a) the initial semimajor axis, a, and eccentricity, $e$, around 1.4 AU, and around (b) 1.5 AU . The lifetime in years is plotted with a logarithmic color scale.
other exoplanetary systems [27].
In order to better define the two semi-stable regions, we produced high resolution maps of the mean dynamical test particle lifetime as a function of their initial orbital elements by running additional simulations with $10^{6}$ test particles distributed around 1.4 AU and 1.5 AU . Figure 5 shows the mean dynamical lifetime of the test particles as a function of the initial semi-major axis, and eccentricity. Whilst stable zones are observed for orbits spanning the entire range of orbital eccentricities tested (between 0 and 0.3 ), the stability of the test particles is clearly most strongly influenced by the semi-major axis of the test particles orbit. We thus conclude that fine tuning of initial semi-major axis is required to obtain stable orbits within the HZ. However, while all simulations were run for $10^{8}$ years, as can be seen in Figure 5 the simulations were stable for no longer than $10^{6}$ years.

We now compare the location of the two semi-stable regions inside the HZ with the stability map analytically derived using the criterion of [9] - see Figure 6. The first remark is that planets b and d are mutually located inside their unstable regions and our work therefore supports the conclusions of [14] that the 3:2 MMR is responsible for maintaining these two planets in a stable configuration. Whilst the inner limit of the unstable zone of HD 204313 d (i.e. to the left of the innermost green curve in Figure 6) is located between 1.8 AU for highly eccentric particles and 2.8 AU for particles with $e \sim 0.3$, the inner limit of the unstable zone of HD 204313 b (i.e. to the left of the innermost red curve in Figure 6) is between 1.4 AU for highly eccentric particles and 1.8 AU for particles with $e \sim 0.25$. This map therefore explains why no stable orbits are found exterior to 1.5 AU within the HZ - beyond this semimajor axis, the proximity between additional bodies and the known planets would result in the disruption of the system.


Fig. 6: Theoretical stability map for HD 204313. The dots represent the location of the planets HD 204313 $b$ (red) and HD 204313 d (green), while the dashed lines delimit the extended orbit crossing regions around HD 204313 b (red) and HD 204313 d (green).

## Conclusions

We propose a new method to investigate the stability of potential terrestrial planets in the habitable zone of multiplanetary systems. This method combines numerical integrations and an analytical criterion to assess the suitability of the HZ to host additional long-term stable objects. Using atmospheric model criterion, we first defined the location of the HZ before testing its appropriateness to host massless test particles by numerically integrating their orbits on a timescale required for the establishment of life. We examined the resulting stable regions in terms of the presence of any resonance mechanisms, and produced high resolution maps of the test particles lifetime as a function of their initial orbital elements inside the identified stable regions. As a final step, we compared the location of the numerical stable zones with the region allowed by an analytical criteria which checks for orbit crossings. This approach provides a more complete picture of the dynamics of the HZ. We applied this method to the system HD 204313, which is mainly composed of a Sun-like star and two giant planets, HD 204313 b and d, orbiting respectively at 3.04 and 3.95 AU (we ignored HD 204313 c). We report the following results and conclusions:

- Using the 'Runaway greenhouse effect' and 'Maximum greenhouse effect', we defined the HZ of HD 204313 b and d as the region between 1.1-1.9 AU.
- By distributing massless test particles throughout the HZ of the HD 204313 system and testing their stability with numerical simulations, we found two semi-stable regions near 1.4 and 1.5 AU .
- Although no single mean motion resonance up to an order 51 was identified as controlling the evolution of the test particles in those regions, our investigations do not exclude that particles could be trapped due to overlapping weak and high-order resonances with the two outer planets of the system.
- Using the analytical criterion from [9], we confirm that the 3:2 MMR between HD 204313 b and $d$ is required to maintain this system in a stable configuration. Moreover, if not protected by any resonance mechanism, no additional planets can be located in the HZ with $a>1.5 \mathrm{AU}$ at the risk of destabilizing the orbits of the two outer giant planets.
- However, it must be noted that these two semi-stable regions remain stable for only
a short period of $\sim 9 \times 10^{6}$ years, which is less than the timescale required for the emergence of life ( $\sim 10^{9}$ years). Thus those two semi-stable regions are not suitable for a terrestrial planet to develop life.

While we did not find a zone of potential long-term stability and habitability for planets in the HZ of the HD 204313 system, this study established a framework for a larger project that will study many different systems using a similar method. One could follow this approach for all known multiple systems with well constrained orbital elements. Generally one expects low eccentricity systems would have a higher chance of hosting stable Earth-like planet in their habitable zone. If we find multiple systems which can host stable Earth-mass planets in their HZ, these systems could be targeted to search for low-mass planets in future surveys.

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[^0]:    ${ }^{1}$ Current exoplanet tally taken from the Extrasolar Planets Encyclopaedia, httest particle://www.exoplanet.eu, on the 11th November 2013
    ${ }^{2}$ Current estimation from the Habitable Exoplanets Catalog, http://phl.upr.edu on the 11th November 2013

[^1]:    ${ }^{3}$ We note that exact commensurability in the ratio of the periods is not required for the resonant angle to librate. Resonance can occur on a small distance from exact commensurability in dimensionless units of period ratio, which is called 'resonance width' [26].

