Students as Toolmakers: Refining the Results in the Accuracy and Precision of a Trigonometric Activity

Igoe, D. P.[(1)*] ; Parisi, A. V.[(1)] and Wagner, S.[(2)]

[(1)] University of Southern Queensland, Toowoomba, Queensland, Australia

[(2)] University of Queensland, St. Lucia, Queensland, Australia

* Corresponding author: damienpaul@gmail.com

Address: Faculty of Health, Engineering and Sciences
University of Southern Queensland
West Street
Toowoomba, Queensland, 4350
Australia

Abstract

Smartphones used as tools provides opportunities for the teaching of the concepts of accuracy and precision and the mathematical concept of arctan. The accuracy and precision of a trigonometric experiment using entirely mechanical tools is compared to one using electronic tools, such as a smartphone clinometer application and a laser pointer. This research has demonstrated how two classroom activities based on tool making can enhance student measurement and application of accuracy and precision considerations through a trigonometric activity investigating arctan.

Introduction

A crucial skill in scientific, particularly physics-based experimental investigations, is the ability to accurately and precisely develop mathematical models of data [1]. As mathematical modelling is largely a statistical process, conceptual confusion may be eliminated by a series of tasks designed to address the random and systematic errors that affect the accuracy and precision of experimental results [2]. Accuracy is how close the measured value is to the 'true value' and precision is the degree to which measurements agree with each other [3]. The use of relatively simple and commonly used statistical tools, such as standard deviation and coefficient of determination (R^2) [3][4] can aid students in their analyses.

The use of smartphone technology as a tool in the classroom is not a new concept, with considerable research being performed in how best to use the ubiquitous mobile computing platforms these devices represent, an example being their use as an education tool to determine the relationship between relative sun angle and solar panel output [5]. Increasing levels of tool making incorporate an increase in the use of electronic, including scientific computing, tools (Table 1) [6]. The focus of this research is demonstrating how smartphone-based tool making can enhance student measurement and application of accuracy and precision considerations through a relatively simple practical trigonometric activity: investigating arctan.

<Table 1>

Methodology

A very simple example trigonometric experiment is as follows:

1. Using a wall of a known height (*h*) and measurements at a known distance from the wall (*d*), use trigonometry to calculate the angle from the horizontal for 10 different targets using:

$$\theta = tan^{-1}(h/d).$$

2. Use a standard classroom activity clinometer (Figure 1) to measure the angles to each target (level 1).

<Figure 1>

3. Then, for comparison, use a smartphone clinometer application with a laser pointer fixed on top of the phone to measure the angle to each target (level 3).
4. Repeat steps 1-3 for different distances.
5. Use Microsoft Excel or any spreadsheet program to analyse the results.

Results and Discussion

The accuracy and precision of the measurements using the hand-held clinometer (level 1) is affected by several factors that are improved using electronic-based tools (level 3). Several of these factors are compared in Table 2.

Table 2: Improvements to a level 1 trigonometry experiment by replacing the tools used.

Level 1 factors	***Level 3 improvements***
Manual aiming of the target using the straw	Laser pointer remains on target
The height the measurement is taken as required	Not required, the app displays the inclination on the LCD screen.
Practical resolution = 0.1° read by eye	Practical resolution = 0.1° digitally displayed
Movement during reading	No movement required
R^2= 0.981	R^2= 0.999

Tedeschi [3] describes the R^2 value as a measure of precision, the higher the coefficient and the smaller the error bars, the higher the precision. Accuracy can be gauged by how close the measured values have a 1-to-1 relationship with predicted values. The use of error bars clearly and visually demonstrate that the variation is greater when using level 1 (Figure 2), purely mechanical tools for this investigation, signifying less precision compared to using electronic tools in level 3 (Figure 3).

<Figure 2>

<Figure 3>

The dashed line in Figures 2 and 3 represent an exact 1-to-1 match. It can be seen that there is a consistent slight over- and under-estimation of the regression line, possibly due to errors in the physical aiming to the target. As an extensional investigation, students can determine methods of automation for positioning the measuring device (level 5).

Tedeschi [3] also describes that the mean square error (MSE) is a quick and accessible tool measuring model validity and accuracy [7] when comparing experimental data, that can be used in class based investigations. For the experiment performed, the MSE for the level 1 and 3 experiments were 0.22 and 0.0005 respectively, further emphasising the importance of the selection and use of analysis tools. Although, the use of the regression lines alone did not show any visible improvement in accuracy with the higher levels tools, the use of the MSE shows a significant reduction in the MSE with the level 3 experiments (Figure 2 compared to Figure 3).

For extended projects, the students could develop computing algorithms that increase the precision of the measured angles by increasing the amount of decimal places in the readout and constructing tools to further decrease the scatter from the mean as indicated by the error bars. The R^2 algorithm can be investigated fully in terms of variable weighting and the validity of outliers, this would require full understanding of the conditions and the mechanisms behind its calculation.

Conclusion

The research has demonstrated how two classroom activities based on tool making can enhance student measurement and application of accuracy and precision considerations through a trigonometric activity investigating arctan. Five levels of toolmaker exercises are proposed, with the first and third levels employed in this research. Level 1 is based on a purely mechanical investigation and level 3 is based on an electronic approach with no automation with the use of a smartphone clinometer app and a laser pointer. The reduction in the size of the error bars, based on two standard deviations of the mean of the set of 10 measurements at each angle were employed to illustrate the improvement in precision with the level 3 experiment compared to the level 1 experiment. The improvement in accuracy with the use of the higher toolmaker level was shown through the significant reduction in the MSE compared to the calculated values.

References

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Table caption

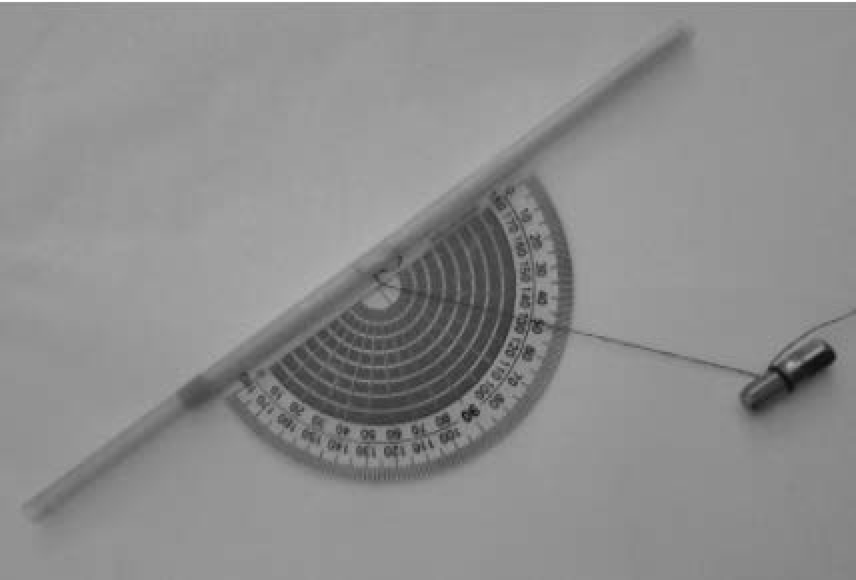
Table 1: Proposed toolmaker levels as applied to an investigation of arctan, emphasising increasing use of electronic devices (E) and decreasing use of mechanical devices (M).

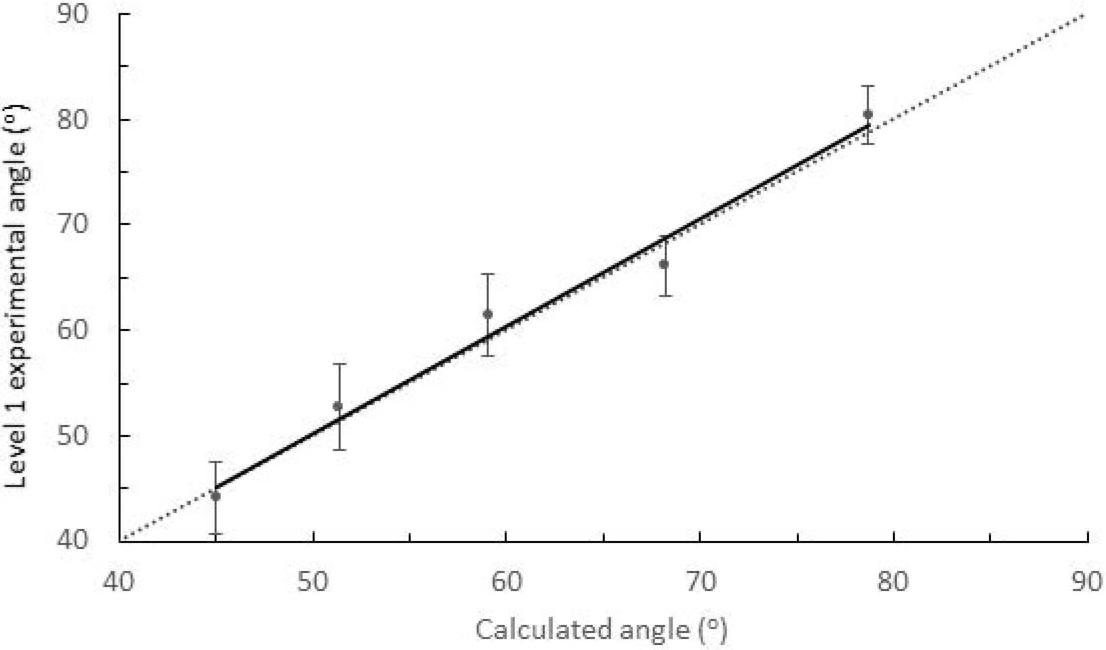
Figure caption

Figure 1: Clinometer often made in the classroom, consisting of a protractor, straw and bob (photo by 1st author).

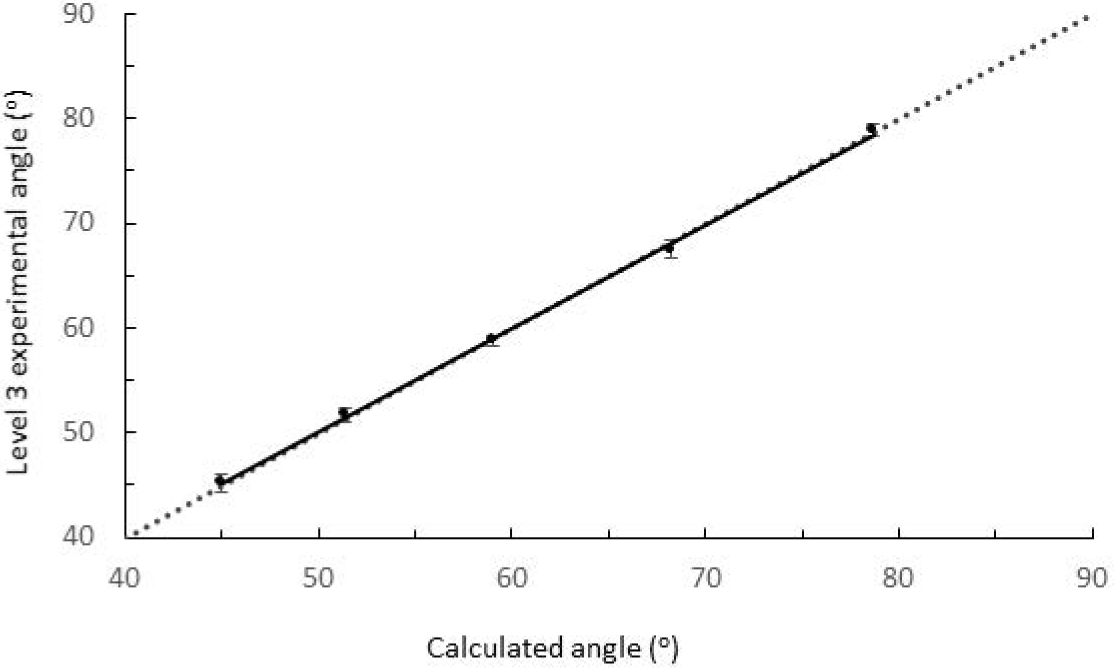
Figure 2: Comparison between the calculated and the measured angles from the handheld clinometer (level 1 experiment). The error bars indicate plus and minus two standard deviations from the mean. The dashed line represents a 1-to-1 relationship between the calculated and observed angles.

Figure 3: Comparison between the calculated and the measured angles from the clinometer app and laser pointer (level 3 experiment). The error bars indicate plus and minus two standard deviations from the mean, some are too small to be seen. The dashed line represents a 1-to-1 relationship between the calculated and observed angles.





90
80
70
60
50
40
Level 1 experimental angle (°)
40
50
60
70
80
90
Calculated angle (°)



90
80
70
60
50
40
Level 3 experimental angle (°)
40
50
60
70
80
90
Calculated angle (°)