A snapshot of young children's mathematical competencies: Results from the Longitudinal Study of Australian Children

Amy MacDonald
Research Institute for Professional Practice, Learning and Education, Charles Sturt University
amacdonald@csu.edu.au

Colin Carmichael
Research Institute for Professional Practice, Learning and Education, Charles Sturt University
ccarmichael@csu.edu.au

This article presents a snapshot of the mathematical competencies of children aged four to five years in Australian early childhood education settings, as perceived by their educators. Data are presented from a nationally-representative sample of 6511 children participating in the Longitudinal Study of Australian Children (LSAC). The results reveal that children are seen to possess a number of mathematical competencies at 4-5 years, with the majority of children displaying interest in mathematics. Moreover, differences were noted with respect to the different program types in which the children participated. These results are discussed in relation to previous research, and implications for future research, policy and practice are presented.

Children who enter primary school with high levels of mathematical knowledge maintain these high levels of mathematical skill throughout, at least, their primary school education (Baroody, 2000; Klibanoff, 2006). Despite this, early childhood mathematics education remains a developing area of research with work yet to be done in terms of identifying young children's mathematical competencies (Peter-Koop & Scherer, 2012). Doig, McCrae and Rowe (2003) have suggested several reasons for the importance of understanding children's mathematical development in the years prior to school, including the increasing number of children participating in early childhood programs and growing recognition of the importance of mathematics in general. Furthermore, De Lange (2008) has suggested that in the years prior to commencing formal education, young children have a curiosity about scientific phenomena—including mathematics—that, for many, seems to dissipate as they enter and continue formal education.

An opportunity to explore young children's mathematical competencies has been afforded through the Longitudinal Study of Australian Children (LSAC) (Sanson, Nicholson, Ungerer, Zubrick, Wilson et al., 2002). LSAC utilises a cross-sequential design to follow two cohorts of children: a Birth cohort of approximately 5000 children aged between 6 and 12 months; and a Kindergarten cohort of approximately 5000 children aged between 4 years 6 months and 5 years. This study focuses on children from the combined Birth and Kindergarten cohorts of LSAC when they were aged four to five years and in particular the mathematical competencies of those attending a formal early childhood education program. The overarching research question guiding this study is: What are the mathematical competencies of 4-5 year old Australian children who attend formal early childhood education programs? Consideration is also given to the related question: Are there differences in mathematical competencies across prior-to-school and school sectors; and if so, what are they?

Background

In this section we provide a brief review of extant research pertaining to the mathematical skills possessed by young children, and the impact of different early childhood program types on the development of children's mathematical skills.

Young children's mathematical skills

A number of studies have demonstrated that children begin developing mathematical skills from a very young age. In a study of 1003 Norwegian children aged between 30 and 33 months, Reikerås, Løge and Knivsberg (2012) found that the toddlers showed mathematical competencies in all areas observed (encompassing number and counting, geometry and problem solving). Similarly, Björklund's (2008) study of children aged between 13 and 45 months demonstrated that toddlers interact with concepts of dimensions or proportions, location, extent, succession and numerosity, and use a range of strategies to express their understanding. The seminal Australian study, the *Early Numeracy Research Project* (see, for example, Clarke, Clarke, & Cheeseman, 2006) investigated the mathematical knowledge of over 1400 children in their first year of primary school. An important finding from the study was that much of the content which formed the mathematics curriculum for the first year of school was already understood clearly by many children on arrival at primary school (Clarke, Clarke, & Cheeseman, 2006), a finding echoed in several other studies, both in Australia (e.g. Gervasoni & Perry, 2013; MacDonald, 2010) and internationally (e.g. Aubrey, 1993; Wright, 1994).

Of course, there will be substantial variance in the mathematical competencies children develop prior to school (Peter-Koop & Kollhoff, 2015), and both standardised tests and experimental tasks reveal marked individual differences in children's mathematical knowledge by the time children enter preschool (Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010). Given the compelling research pertaining to the relationship between mathematics at the time of school entry and later school achievement (Levine et al., 2010), it is important to ascertain the mathematical competencies of children in the early years in order to understand the foundation on which subsequent mathematics education should build.

Impact of program type on mathematical opportunities and skills

In Australia, children aged 4-5 years will typically participate in either *prior-to-school programs* or *school programs*. The prior-to-school programs on offer are many and varied, and differ in the different states and territories. However, the program types can be generalised as including centre-based care (long day care or occasional care), stand-alone preschools, supported play groups, family day care, and early intervention services. School-based programs are similarly complex and diverse. In all states and territories, however, children commence school with a pre-Year 1 program, though it is termed "Kindergarten" in some jurisdictions (e.g. NSW) and a "Preparatory" year in others (e.g. Victoria).

At the time the data in this study were collected (2004-2008), each state and territory was responsible for providing curricula and policy documents for use in the various education sectors. The prior-to-school sector was the least regulated in terms of curricula frameworks. However, a common feature was a lack of explicit focus on the teaching of mathematics in the early childhood sector. On the other hand, mathematics has

been a part of the formal primary school curricula from the first year of school, with each state and territory guided by its own mathematics syllabus.

Clearly, the curricula and policy frameworks utilised in the different settings will have some impact upon the extent to which mathematics is an explicit focus of the educational program on offer, and it can reasonably be assumed that explicit teaching of mathematics is likely to occur more frequently in school settings. However, there are other factors beside curricula which will influence children's opportunities to explore mathematics in early years education settings. A study of mathematics in the childcare context by Graham, Nash and Paul (1997) has shown that children's experiences in childcare vary greatly, with differences in the physical set-up, schedule, age grouping, teacher-student ratio, teaching styles, and beliefs about child development. However, a common feature of the childcare settings investigated was the minimal amount of mathematics instruction in these settings.

Method

Sample

The sample utilized both cohorts (Kindergarten and Birth) of LSAC. Collectively this consisted of 9369 children aged from 4.2 to 5.7 years ($M = 4.8$ years, $SD = 0.2$) of whom 51.1% were male. A substantial number of the full sample ($n = 2716$), however, did not attend a formal early childhood education program. In addition, teachers of 142 children failed to provide data for their students. Consequently the sample on which this analysis is based comprises 6511 children with similar age and gender characteristics as the full sample.

Program type

These children attended a range of early childhood education programs and these are shown in Table 1, which also reports the mean age of children in each group. As is seen from the table, more than half of the children (53.7%) participated in preschool programs, which operate only during school hours and terms, and where children may attend half-days or limited sessions a week. Almost a quarter (22.8%) attended centre-based programs which operate at least eight hours a day, five days a week and most weeks of the year. Less than one fifth (17.6%) attended pre-Year 1 school programs, which are full-time, school-based programs. A small proportion attended other programs including early intervention programs, or participated in multi-age classrooms. As is also seen, children attending pre-Year 1 school programs were on average 4 months older than those attending preschool and centre-based programs.

Table 1
Participation in early childhood programs (N = 6511)

Program type	Frequency	%	Mean age (months)
Centre based childcare program	1483	22.8	57
Preschool program	3495	53.7	57
Pre-Year 1 school program	1149	17.6	61
Other	124	1.9	59
Not stated	260	4.0	

Indicators of mathematical competencies

Analysis of children's mathematical competencies was based on the mathematical skills scale (Social Development Canada, 2005), which was included in the teacher questionnaires. One significant limitation of the LSAC study design is that no opportunity was given to parents and/or other caregivers, or the children themselves, to provide a response to these six items. As such, the data reported in this article are formed on the basis of *early childhood teachers'* judgements of children's competence in relation to the following items:

1. ability to sort and classify;
2. ability to count objects;
3. ability to count to 20;
4. ability to recognise numbers;
5. ability to do simple addition; and
6. interest in numbers.

These were phrased as questions allowing a "Yes" or "No" response, with the final item asked from a negative perspective. Clearly, these items do not address all mathematical competencies a young child may possess and indeed, privilege number concepts above all other mathematical concepts. Nevertheless they do provide insight into some of the mathematical competencies 4- and 5-year-old children possess, as perceived by their educators.

Analysis plan

Descriptive statistics were used to answer the overarching question in this study. These were estimated, however, through the use of a series of logistic regression models: One for each competency. These models also allowed for the later testing of program type on children's mathematical competency, whilst controlling for the influence of their ages. Given the statistical power associated with the sample size, the statistical significance of regression coefficients was assessed with a Bayesian information criterion (BIC), with values exceeding ten considered to be "very strong" effects (Pampel, 2000, p. 31). In order to account for the complex sampling design used with LSAC, all estimates and their standard errors were calculated using the R-package "Survey" (Lumley, 2012).

Results

The estimated proportions of children, who according to their teachers possessed the given mathematical competencies, are shown in Table 3. As is seen, most children were able to sort and classify, and count objects. Far fewer, however, were able to recognise numbers and undertake simple addition. Variations in these competencies, however, may have been due to differences in age and the program type that children were attending. In order to control for these factors, program-type (a four-level factor) and age (in months) centred on the mean, were regressed onto children's mathematical competencies; a series of dichotomous variables indicating whether the child had or had not met the relevant competency. The results of these models are shown in Table 4, which reports estimates of the influence of age and program type on the probability that a child will meet the given competency. More specifically, these estimates relate directly to the logit transformation (natural logarithm of the odds ratio) of this probability. In each model, the influence of program-type is relative to those children in *centre-based* programs. The specification of

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these models is shown in Equation 1, where π_i is the probability that a child will meet the relevant mathematical competency.

$$logit(\pi_i) = \beta_0 + \beta_1 Age + \beta_2 PreSchool + \beta_3 PreY1 + \beta_4 Other \text{ ------(1)}$$

Table 3
Proportion of students meeting each competency

Competency	Overall (%)
Able to sort and classify	96
Able to count objects	94
Able to count to 20	62
Able to recognize numbers	72
Able to do simple addition	32
Uninterested in numbers	2

Table 4
Results of logistic regression models

Competency	Intercept (β_0)	Age (β_1)	Preschool (β_2)	Pre-Year 1 (β_3)	Other (β_4)
Able to sort and classify	2.59	0.01	0.36	-0.03	-0.79
Able to count objects	2.69	0.07	0.25	0.16	-0.71
Able to count to 20	0.76	**0.10**	**-0.42**	-0.19	-0.49
Able to recognize numbers	0.99	**0.08**	-0.12	0.29	-0.12
Able to do simple addition	-0.83	**0.09**	-0.19	**0.67**	0.28
Uninterested in numbers	-3.65	-0.01	-0.07	-0.20	0.78

Note: emboldened coefficients report BIC>10.

As is seen from Table 4, age has a significant influence on children's ability to count to 20, recognize number, and to do simple addition. The odds ratios corresponding to each of these effects are 1.10, 1.08, and 1.09 respectively, suggesting that an increase in age of one month relative to the mean age (57.6 months) will produce small, but significant increases in the likelihood of gaining these competencies. When controlling for age, children attending preschools were less likely to be able to count to 20 than children attending centre-based programs. The corresponding odds ratio for this effect is 0.66, suggesting that preschool attendees are less likely to gain this competency than those attending centre-based programs. In addition, children attending a pre-Year 1 program were more likely to be able to do simple addition than those attending the centre-based programs. The corresponding odds-ratio for this effect is 1.95, suggesting that these children are considerably more likely to meet this competency than those attending centre-based programs.

Discussion

As reported in Table 3, the children in this study demonstrated a high level of competence on the majority of the items. This is consistent with international research showing that children develop a range of mathematical understandings in the years prior to starting school (Reikerås et al., 2012; Clarke et al., 2006).

The years subsequent to the collection of the LSAC data has seen the implementation of Australia's first national schooling curriculum, known as the *Australian Curriculum* (incorporating the specific *Australian Curriculum: Mathematics*) (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2014). The *Australian Curriculum: Mathematics* has content grouped into three areas: number and algebra; measurement and geometry; and statistics and probability (ACARA, 2014). A mapping exercise has been undertaken to examine the alignment of the competencies demonstrated in this study with the current expectations of children in the early years of primary school. This exercise has revealed some points of concern. For example, the *Australian Curriculum: Mathematics* mandates that simple addition is not taught until Year 1, yet one third of the LSAC children were perceived by their educators to be already doing this, either in their prior-to-school year or first year of school. Sorting and classifying, counting (including to 20) and number recognition are stated as content to be *taught* in the Foundation year; however; the majority of children in this study are already demonstrating competence in these areas. This is consistent with the findings of other Australian studies (Gervasoni & Perry, 2013; MacDonald, 2010), indicating that there is growing evidence that the early years mathematics curriculum is misaligned with children's existing competencies. Of concern is that this lack of challenge might result in children becoming disinterested in mathematics as they progress through the schooling years.

It is important to note that according to their teachers 98% of the LSAC children showed interest in numbers at 4-5 years. This is heartening because studies show a decline in levels of mathematics over the entire school period (e.g. Fredricks & Eccles, 2002). If children engage in meaningful and enjoyable mathematics education in the early childhood years, they are much more likely to appreciate and continue to engage in later mathematics education (Linder, Powers-Costello, & Stegelin, 2011).

Children attending preschools were less likely to be able to count to 20 than children attending centre-based programs. This is somewhat counter to the common perception that preschools provide "higher quality" education programs and hence are more likely to produce better outcomes (Marriner, 2013). It may be the case that preschool programs focus on developing skills beyond simple rote counting, whereas the mathematics in centre-based care is typically limited to activities such as counting and identifying shapes (Cohrssen, Church, Ishimine, & Tayler, 2013).

Children attending pre-Year 1 programs were more likely to be able to do simple addition than those attending the centre-based programs. On the one hand, it could be argued that this makes sense, given an explicit focus on mathematics education (as expressed through formal curricula) in school settings. Of note, though, is the point that simple addition typically does not feature in the formal curriculum for the first year of school; rather, it typically appears as content for teaching in Year 1 (children's second year at school). This suggests that not only is the first year of school curriculum failing to recognise the competencies children bring with them from prior-to-school settings, this lack of recognition is maintained as children progress to their second year of formal schooling.

Limitations and opportunities for further research

The analysis has been undertaken within the limits of the LSAC study design, including its measures. Although the mathematical skills scale may be viewed as limited, and there may be more appropriate measures elsewhere, the analysis could only include data from the existing study. However, this highlights the importance of future research which takes a broader view of mathematical competence and is more inclusive of other conceptual domains in mathematics.

A further limitation is that mathematical competencies were based on educators' judgements only and indeed was restricted to children enrolled in formal early childhood programs. There is much research which indicates powerful mathematical ideas are developed in home and community settings (MacDonald, 2012). Consequently further research in *all* early childhood settings, and using multiple sources, is required.

Conclusion and implications

This article has presented evidence to suggest that young children are perceived as competent by their educators in several aspects of mathematics, as assessed within the scope of the LSAC data gathering. However, given that data regarding children's mathematical competencies was *only* collected from the age of 4-5 years, this begs the question: What competencies do children possess at younger ages? Consistent with Peter-Koop and Scherer's (2012) call for further research, it seems clear that there is much work yet to be done in identifying the mathematical competencies developed by young children. Much of the extant research and existing assessment tools specifically target preschool-age children (i.e. 4-5 year old children)—as exemplified in the LSAC study—with relatively little research on the mathematical development of younger children (Mousley & Perry, 2009). As Doig et al. (2003) state, it appears that the development of an assessment instrument that gives due emphasis to the full range of young children's mathematics is long overdue. Indeed, this call for further research persists a decade later, with Peter-Koop and Scherer (2012) arguing that research leading to the development of a detailed competency model that goes beyond number and integrates the different content areas of mathematics is still needed.

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