University of Southern Queensland

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Dynamics of active systems with nonlinear excitation of the phase

A thesis submitted by

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Dedication

Thee my Lord to be satisfied
And then
To my Mum who passed away in
the examination period of this dissertation

Abstract

Many physical and chemical systems exhibit self-oscillatory dynamics, for example systems involving the Belousov-Zhabotinsky reaction and systems used for material synthesis by solid-phase combustion, known as self-propagating high-temperature synthesis. Phase of oscillators crucially depend on diffusion (or thermal conductivity), which is reflected in the partial differential equation governing the phase of oscillations. At first sight, the role of diffusion is to equalise the phase in space. However, more complex situations are possible; for example the phase equation may involve self-excitation such as anti-diffusion in the (Kuramoto-Sivashinsky equation). In this research we investigate a version of the phase equation based on a *nonlinear* self-excitation. Previously it was shown that *nonlinear* self-excitation can arise in chemical systems with non-local interaction.

In the present research, we analyse this kind of system in order to determine the validity range of the nonlinearly excited phase equation in the parametric space. Specifically, we numerically evaluate the values of the parameters that guarantee the assumptions of slow variations of the phase in space and time and, simultaneously, the key role of the nonlinear self-excitation.

We also numerically solve the phase equation with *nonlinear* self-excitation in two spatial dimensions by finite-difference discretization in space and subsequent numerical integration of a system of ordinary differential equation in time. Irregular dynamics intermitting with periods of slow evolution are revealed and discussed.

As a separate task, we derive a forced variant of the phase equation and present selected exact solutions – stationary and oscillatory. They are also used to verify the numerical code. In the numerical experiments, we use a range of sizes of spatial domain.

Lastly, different forms of the nonlinearly excited phase equation are investigated based on different types of dynamical balance.

Keywords

Active dissipative systems, reaction-diffusion systems, nonlinear partial differential equation, nonlinear excitation, Kuramoto-Sivashinsky (KS) equation , finite difference, irregular dynamics, Complex Ginzburg-Landau equation (CGLE), parametric space.

Publications by the candidate

The following publications were produced during the period of candidature:

- D.V. Strunin and M.G. Mohammed, Parametric space for nonlinearly excited phase equation, *ANZIAM Journal (E)* [Oxford University Press] Vol.53 (2012) pp. C236-C248.
- D.V. Strunin and M.G. Mohammed, Validity and dynamics in the nonlinearly excited 6th-order phase equation, *Discrete and Continuous Dynamical Systems - Supplement* [American Institute of Mathematical Sciences] ISBN-10: 1-60133-016-2; ISBN-13: 978-1-60133-016-1, 2013, pp. 719-728.
- M.G. Mohammed and D.V. Strunin. Finite-difference approach for a 6th-order nonlinear phase equation with self-excitation, *JPS Conference Proceedings* [The Physical Society of Japan]. Proc.1, 2014, 016003.
- D.V. Strunin and M.G. Mohammed, Range of validity and intermittent dynamics of the phase of oscillators with nonlinear self-excitation, submitted to the journal *Communications in Nonlinear Science and Numerical Simulation* [Elsevier] in January 2015.

Certification of Dissertation

I certify that the ideas, experimental work, results, analyses and conclusions set out in this dissertation are entirely my own effort, except where otherwise indicated and acknowledged.
I further certify that the work is original and has not been previously submitted for assessment in any other course or institution, except where specifically stated.

Mayada Gassab Mohammed Date

ENDORSEMENT

A/Prof. Dmitry V. Strunin, Principal Supervisor Date

Prof. Thanh Tran-Cong, Co-Supervisor Date

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Acronyms & Abbreviations

CGLE	Complex Ginzburg-Landau equation
GL equation	Ginzburg-Landau equation
NEP equation	Nonlinearly excited phase equation
KS equation	Kuramoto-Sivashinsky equation