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Review

Recent astronomical tests of general relativity

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This history of experimentation relevant to general relativity covers the time post-1928. Classes of investigation are the weak equivalence principle (equivalence of inertial and gravitational mass and gravitational redshift), orbital precession of a body in gravitational fields (the relativistic perihelion advance of the planets, the relativistic periastron advance of binary pulsars, geodetic precession and Lense-Thirring effect), light propagation in gravitational fields (gravitational optical light deflection, gravitational radio deflection due to the Sun, gravitational lensing, time dilation and atomic clocks) and strong gravity implications (Nordtved effect and potential gravitational waves). The results of experiments are analysed to conclude to what extent they support general relativity. A number of questions are then answered: (a) how much evidence exists to support general relativity, (b) is it a reasonable way of thinking and (c) what is the niche it may occupy?

Key words: general relativity, equivalence principle, orbital precession, gravitational fields.

INTRODUCTION

The special theory of relativity came from the mind of Albert Einstein (1879-1955) in 1905 (Einstein, 1905). In it he proposed that the laws of physics take the same form in all inertial frames and that the velocity of light is constant irrespective of the motion of the emitting body. Previously, Isaac Newton (1642-1727) had supplied the term inertial mass when treating his three laws of motion and gravitational mass in the context of his universal law of gravitation. While Newton had attempted to pursue if these conceptual terms were the same, it was Einstein in 1907 who extended his own notions and declared that acceleration and gravitation were identical, that is, objects of different composition would have identical accelerations in the same gravitational field (Einstein, 1907). This idea is now referred to as the equivalence principle. In a publication in 1916 Einstein broadened his concepts to include an accelerated frame of reference (Einstein, 1916). Within his general theory of relativity he united space and time and presented gravity as a geometrical interpretation of how bodies move in the presence of a mass.

It was claimed that there were three astronomical tests which could act as a litmus examination of general relativity: the anomalous advance of the perihelion of Mercury, the extent to which starlight could be bent as it passes the Sun and the gravitational redshift of light from the Sun. In truth, the gravitational light deflection and the gravitational redshift are derived from the equivalence principle and the Mercury situation from general relativity. This distinction will not be invoked in this paper and the term general relativity will be used to encompass the equivalence principle.

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Former work by the current author questioned the early acceptance of the results of these tests of gravitational light deflection in one paper (Treschman, 2014a) and Mercury and gravitational redshift in another (Treschman, 2014b). It was argued in those articles that insufficient evidence existed until the year 1928 for acceptance of general relativity as a reasonable explanation of the data that had been gathered.

AIM OF THIS PAPER

This paper picks up the thread post-1928. It does include the extension a number of other scientists made to general relativity from as early as 1916 and even some experiments that were conducted prior to Einstein's publications which can be interpreted within the worldview of general relativity. The history of several themes is examined to gauge at what level they support general relativity.

In order to ascertain reality, science rests on models, namely, using something known as a proxy for the unknown. Truth is not the issue but how useful is the construct in explaining phenomena and predicting outcomes. The aim in this paper is to place the theory of general relativity in the context of its suitability as a description of the cosmos.

Scientific breakthroughs are often presented as before and after. Yet, acceptance takes a long period of time. Aristarchus (c310-c230 BCE) recorded a heliocentric model which was published much later in 1543 by Nicolaus Copernicus (1473-1543). This was in contrast to the geocentric rendition of Claudius Ptolemy (90-168). Yet, even after the telescopic observations of Galileo Galilei (1564-1642) commencing in 1609, scientists correctly needed more evidence before their world picture was better presented by the earth orbiting the Sun. Interestingly, there are still vestiges of the alternative model today in terms such as "sunrise" and "sunset". The ideas of Isaac Newton (1643-1727) put to print in 1687 had initial difficulty with the notion of action at a distance which had a whiff of magic about it. It is still a practical worldview if one limits the picture to speeds much below that of light and to masses the size of the planets. So, the questions are:

(i) How much evidence exists to support general relativity,
(ii) is it a reasonable way of thinking and
(iii) what is the niche it may occupy?

Answers to these queries are attempted by tracing some selections from the historical record separated into classes based on the type of investigation. The survey of the literature is restricted mainly to journals printed in English.

WEAK EQUIVALENCE PRINCIPLE

Equivalence of Inertial and Gravitational Mass

To elucidate any difference between inertial mass and gravitational mass the Hungarian physicist, Loránd Eötvös (1848-1918), commenced measurements in 1885. He used a torsion balance which consisted of a horizontal rod suspended by a thin fibre and having two masses of different composition but the same gravitational mass at the ends of the rod. He worked firstly with copper and platinum. The rod was oriented parallel with the meridian and had an attached mirror which reflected light into a telescope so that any small twist in the fibre could be observed more easily. The rotation of the Earth created forces on the masses proportional to their inertial masses. The vector sum of the tension in the fibre, the gravitational force and the reaction to the centripetal force would result in a zero torque (beyond the rotation of the rod at the same rate as that of the Earth). For a null movement of the rod, Eötvös could claim a proportionality constant between inertial and gravitational mass.

Continuing with different materials he published his results in 1890 (Eötvös, 1890) in which he claimed an accuracy of 1 in 2 x 10^7. In 1891 he refined the model to have one of the masses suspended by its own fibre from the rod so that the system could now have measurements in two dimensions. His coworkers from 1906-1909 were Dezsö Pekár (1873-1953) and Jenö Fekete (1880-1943). The later publication by Eötvös (1909) declared an improved accuracy to 1 in 10^8. The final results (Eötvös, 1922) were printed after his death.

Later János Renner (1889-1976) (Renner 1935) who had worked with Eötvös took the results to 2-5 in10^9 and in another three decades Robert Henry Dicke (1916-1977), Peter G. Roll and R. Krotkov (Roll et al., 1964) had used improved equipment to conclude an accuracy of 1 in 10^{11}. Another avenue for testing the equivalence principle was to probe the motions of the Earth and Moon. Both bodies accelerate in the gravitational field of the Sun. To establish whether the accelerations were different, it was necessary to obtain a more accurate position of the Moon relative to the Earth. It had been proposed to bounce a laser beam off the Moon but the topography would conspire to produce spurious results. Hence, in 1969 on the first human lunar landing, the astronauts of Apollo 11 embedded a retroreflector array on the Moon. This consisted of 100 corner cube prisms in a 10 x 10 array 0.45 m square with each cube made of quartz and dimension 3.8 cm. The design of each prism had a trio of mutually perpendicular surfaces such that an incoming ray is totally internally reflected from three surfaces to generate a deviation of 180°. The array from Apollo 14 in 1971 is similar but the one also in 1971 from Apollo 15 had 300 cubes in a hexagonal array. The Soviet Union landed two rovers on the Moon: Lunokhod 1from Luna 17 in 1970 and Lunokhod 2 from Luna 21 in

1973. Each of the rovers carried 14 cubes in a triangular formation with 11 cm size apiece in an array 44 x 19 cm (Dickey et al., 1994).

A number of Earth stations have observed a reflected pulse but long term dedication belongs to the Observatoire du CERGA (Centre d'Etudes et de Recherches Géodynamiques et Astronomiques) near Cannes in France with a 1.5 m telescope and the McDonald Laser Ranging System in Texas using a 2.7 m system. The latter was replaced by a dedicated 0.76 cm instrument in 1985. The laser adopted was a neodymium-yttrium-aluminium-garnet one firing a 2×10^{-10} s pulse 10 times per second. In the early 1970s accuracies were at the 25 cm level. This was reduced to 15 cm in the mid 1970s as a result of improvements to the timing system and from 1985 to 2-3 cm. The findings were consistent with general relativity to 1 in 10^4 as well as determining the recession of the Moon from Earth by 3.8 cm yr^{-1} (Gefter, 2005). An improvement to 1 mm accuracy between the Earth and the Moon has been achieved by the 3.5 m arrangement at Apache Point Observatory in New Mexico (Murphy et al., 2008). This requires a 3.3×10^{-12} s exactitude in the one way trip or 6.7×10^{-12} s both ways. The major uncertainty in the distance is due to the libration of the Moon which, on its own, contributes to a spread of 15-36 mm in distance, equivalent to $1.0\text{-}2.4 \times 10^{-11}$ s round trip time. Accuracy has improved due to the aperture size of the telescope, altitude of 2880 m, a greater capture of photons and a timing mechanism of atomic standards to 10^{-7} s. Any violation of the equivalence principle would produce a displacement of the lunar orbit along the earth-Sun line with a variation coinciding with the 29.53 days synodic period. This has not occurred to the 0.1% level (Williams et al., 2009).

Gravitational Redshift

Measurements of the gravitational redshift of lines from the Sun followed a tortuous journey. From an apparent tangent of using the lines from Sirius B and then other white dwarfs, scientists unravelled the many factors from which the relativistic redshift emerged. Pursuing another tack, Robert Vivian Pound (1919-2010), Glen Anderson Rebka, Jr (1931-) and Joseph Lyons Snider conceived an imaginative experiment.

Pound and Rebka (1959) reported that a fraction of gamma rays could be emitted from the nuclei of a solid without recoil momentum of the nuclei. They hypothesised that gravitational redshift could be measured from an emitter to a source at a different altitude and register the situation for maximum scattering (Pound and Rebka 1959). The emitter they chose was Co-57 electroplated onto one side of an iron disc. To ensure diffusion of the cobalt into the iron, the disc was heated up to 1000°C for one hour. The absorber was seven units of iron enriched in Fe-57 to 32% electroplated onto a beryllium disc. The absorption level was one third of the emitted gamma rays. Placed inside a space at the Jefferson Physical Laboratory of Harvard University, the source and absorber were 22.6 m apart. To reduce the absorption of gamma rays by air, helium was run through the tower continuously. The fractional change in frequency was proportional to gh/c^2 where $g = 9.8$ m s^{-2} is the acceleration due to gravity, $h = 22.6$ m is the altitude and $c = 3.0 \times 10^8$ m s^{-1} is the speed of light. The ingenious aspect was to measure the change in energy instead by having gamma rays move against gravity and then with gravity by interchanging the emitter and absorber. Thus, the change in energy down less the change in energy up $= 2gh/c^2 = 4.9 \times 10^{-15}$. The authors reported that their experimental result was 1.05 ± 0.10 times the theoretical value (Pound and Rebka, 1960a) for a frequency change of 3.27×10^{-8} s^{-1} for this altitude difference in the gravitational potential of the Earth (Pound and Rebka, 1960b) where the gradient (Hirate, 2012) is 1.1×10^{-16} c^2 m^{-1}. Improvements were effected in 1964 by Pound and Snider and their result was published as 0.999 0 ± 0.007, 6 times the predicted relativistic frequency (Pound and Snider, 1965).

From 1976, spacecraft were involved in this particular test of general relativity. Carrying a hydrogen maser, a 100 kg spin stabilised spacecraft, jointly organised by the National Aeronautics and Space Administration (NASA) and the Smithsonian Astrophysical Observatory, was launched to 10000 km almost vertically. The output frequency of $1.420\,405\,751 \times 10^9$ Hz, accurate over 100 s averaging time to 1 in 10^{14}, was compared with another maser on Earth. The agreement with general relativity was calculated to the 7×10^{-5} level (Vessot et al., 1980).

Voyager 1 was launched in 1977, flew by Jupiter in 1979 and reached Saturn in 1980. It carried an ultrastable crystal oscillator. As a result of its close approach to Saturn, a redshift of several hertz was predicted to its 2.3×10^9 Hz downlink sent by its 3.7 m antenna. Comparison was made against the three 64 m stations on Earth which are part of the Deep Space Network: Goldstone in California, near Madrid in Spain and near Canberra in Australia. Each of these stations was referenced to a hydrogen maser frequency standard. The result was in agreement with general relativity to 0.995 6 ± 0.000 4 as a formal uncertainty and ± 0.01 as a realistic uncertainty (Krisher et al., 1990).

Similar communication channels were set for Galileo which was launched in 1989 on a trajectory which included a gravity assist from Venus in 1990 and Earth in 1990 and 1992 before arriving at Jupiter in 1995. During the phase from launch to the first Earth gravity assist, regular frequency measurements of the spacecraft clock were conducted. Personnel from the Jet Propulsion Laboratory reported a 0.5% agreement with general relativity for the total frequency shift and a 1% concord with the solar gravitational redshift (Krisher et al., 1993).

However, it was the Cassini spacecraft on its way to

Saturn which has provided the closest match to general relativity at 0.0023% (Williams et al., 2004). Jointly coordinated by NASA and the Italian Space Agency, Cassini was launched in 1997, and flew by Earth, Venus and Jupiter to orbit Saturn in 2004. In 2002 it was near superior conjunction, with the Earth situated 8.43 astronomical units distant. Interference from the solar corona and the Earth's troposphere could be accounted for by two different uplink frequencies and three different downlink signals with use of Cassini's 4 m antenna. Measurements were conducted on the 18 passages of signals between Earth and Cassini (Bertotti et al., 2003). Each pulsar in a binary system is influenced by the strong gravitational field of the other. From PSR J0737 – 3039 A/B (see later), a redshift parameter of 3.856 x 10^{-4} s is compared with a relativistic calculation of 3.841 8 x 10^{-4} s to give a ratio between them of 1.003 6 (Kramer et al., 2006).

ORBITAL PRECESSION OF A BODY IN GRAVITATIONAL FIELDS

Relativistic Perihelion Advance of the Planets

Between the publication of special relativity in 1905 and general relativity in 1916, Einstein received assistance from Marcel Grossmann (1878-1936) (Einstein and Grossmann, 1913) and Michele Besso (1873-1955) (Janssen, 2002). Grossmann alerted Einstein to how tensor calculus and Riemannian geometry could be applied to general relativity and Besso worked with Einstein on solving some equations which were relevant to the perihelion advance of Mercury. Einstein incorporated into his equations Lorentz transformations named for Hendrik Antoon Lorentz (1853-1828). These involved *c* the speed of light independent of a reference frame. They showed how measurements of space and time taken by two observers were related. Thus, they gave meaning to how two observers travelling at different relative velocities may make different measures of distance and elapsed time. The Lorentz factor $\boldsymbol{\gamma}$ (gamma) was defined as

$$\boldsymbol{\gamma} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

where v is the relative velocity between inertial reference frames. In Einstein's work he used for time dilation for length contraction in the x direction.

$$\Delta t' = \boldsymbol{\gamma}\, \Delta t \tag{2}$$

and

$$\Delta x' = \frac{\Delta x}{\gamma} \tag{3}$$

for length contraction in the x direction.

In later experimentation, to ascertain how closely results may be interpreted in the worldview of general relativity, the Lorentz factor was a part of a number of equations and the closer this value is to unity, then general relativity is more supported.

It was in 1916 that Einstein wrote his gravitational field equations applying within a vacuum and chose the Sun as the origin of his coordinate system (Vankov, 1915). He made use of Huygens' principle to formulate the angular deflection of a ray of light at a certain distance from the Sun. Through a series of approximations, he derived a planetary motion equation. As long as the speed of a particle was much less than **c** the speed of light, Newton's equation could be obtained as a first approximation.

With a switch to planar orbit equations with the polar coordinates **r** and **ϕ** as the radius vector and angle respectively, the equations led to the known energy and Kepler's planetary law of areas. One result was:

$$r^2 \frac{d\phi}{ds} = \text{a constant} \tag{4}$$

where **s** is displacement. If orbital motion were described, the equation was in agreement with Kepler's third law portraying the relationship between the period of a planet and its distance from the Sun. The curvature of spacetime envisaged by Einstein was an explanation of the Mercury advance as it had further to travel than in flat space due to the distortion created by the mass of the Sun.

To obtain the secular advance of an elliptical orbit Einstein next integrated the equation containing **ϕ** over the ellipse so that **Δϕ**, the change in angle in radians per orbit, is found in terms of **a** the semi major axis and **e** the eccentricity. If this is extended to an entire passage, the result in the direction of motion for the period **T** in s is

$$\Delta\phi = 24\pi^3 \frac{a^2}{T^2 c^2 (1 - e^2)} \tag{5}$$

With conversion factors of $180/\pi$ to give °, 3 600 for ", a change of period from s to 0.240 844 45 tropical years and 100/orbital period in tropical years producing an answer in " century^{-1}, Einstein calculated a figure of 45" ± 5 century^{-1} for Mercury, the then accepted value for the anomalous advance of the perihelion of Mercury being 42".95 century^{-1}.

By 1943 Gerald Maurice Clemence (1908-1974) had examined meridian observations of Mercury totalling 10 400 in right ascension and 10 406 in declination over the period 1765-1937 and 24 transits of Mercury across the Sun spanning 1799-1940 (Clemence, 1943). From this analysis he adjusted figures for the eccentricity and perihelion of the Earth as well as for the mass of Venus. His new value for the anomalous perihelion advance of Mercury was 43".11 ± 0.45 century^{-1} against the Einstein figure at this time of 43".03 century^{-1}.

With his attention on another planet, Raynor Lockwood Duncombe (1917-2013) scrutinised meridian observations of Venus across 1750-1949 (21009 in right ascension and 19852 in declination) (Duncombe, 1956). After applying corrections to some elements of Venus and the Earth and the mass of Mercury, he deduced, for the first time, results accurate enough for the anomalous advance of the perihelion of Venus. In 1956 this was determined as 8".4 ± 4".8 century^{-1} while the relativity figure was 8".6 century^{-1} (Morton, 1956).

For Earth, HR Morgan dissected studies of the Sun over 1750-1944 from a number of observatories and applied a correction in 1945 to the eccentricity of the planet (Morgan, 1945). He combined with Clemence and Duncombe to determine by 1956 the anomalous advance of the perihelion of Earth as 5".0 ± 1".2 century^{-1} while the Einsteinian amount was 3".8 century^{-1} (Morton *op. cit.*). Kepler's third law of planetary motion for Mercury may be expressed as

$$T^2 = \frac{4\pi^2 a^3}{G(M+m)} \tag{6}$$

for **G** the universal gravitational constant, **M** is the mass of the Sun and **m** the mass of Mercury. As **m<<M**, it may be omitted. If, then, T^2 is substituted into equation (5), one may express the Einstein derivation into a similar one (Gamalath, 2012) as

$$\Delta\phi = \frac{6\pi GM}{c^2 a\left(1 - e^2\right)} \tag{7}$$

For **c** = 2.998×10^8 m s^{-1}, ***G*** = 6.673×10^{-11} m^3 kg^{-1} s^{-2}, ***M*** = 1.989×10^{30} kg, and data from a modern almanac (Seidelmann, 2006) the calculations for Mercury, Venus and Earth are juxtaposed against the observed values in Table 1. The calculated values are within the range of the observed figures.

For % difference between the calculated and observed values, the central value gives (43.11 – 42.98)/42.98 x 100 = 0.19%. However, the extreme difference is (43.11 + 0.45 – 42.98)/42.98 x 100 = 1.4%. In a similar way, the values respectively for Venus are 2.3 and 58% and Earth 32 and 62%.

One of the assumptions in Einstein's derivation was that the orbital plane of the planets coincided with the rotational equator of the Sun. This is incorrect but the technology to measure what became known as the quadrupole moment of the Sun did not exist until the 1980s and particularly into the 1990s. The splitting of spectral lines due to solar oscillations in the 1980s revealed that, with the precision of the measurements, the assumption in the derivation of Mercury's anomalous perihelion advance was acceptable (Campbell and Moffatt, 1983).

A Global Oscillations Network Group GONG was formed in 1995 to produce continuous solar velocity imaging with an aim to ascertain the spherical harmonic functions of the Sun related to its radius and latitude. Six solar observatories in the Canary Islands, Australia, California, Hawaii, India and Chile combined to analyse 33169 splits of spectral lines (Pijpers, 1998). The conclusion was that the results are currently consistent with the figure accepted for Mercury's perihelion advance determined by general relativity. This decision is also supported by the first six months of data obtained from helioseismology measurements taken by the Michelson Doppler Imager aboard SOHO, the Solar Heliospheric Observatory, launched in 1995. An interesting extension to this concept is the use of exoplanets (Zhao and Xie, 2013). Data from the Kepler space observatory launched in 2009 and future missions may give improved accuracy so the periastron advance to these other systems may be added to the information on the solar system planets.

Relativistic Periastron Advance of Binary Pulsars

There are many factors involved in determining the orbits of the planets and the positions of the perihelia. In addition, the *total* change per year in the location of the perihelion of Mercury is as small as 5".7. Fortunately, the same property applicable to the relativistic perihelion advance of the planets may be applied outside the solar system. In addition, within the solar system, the gravitational fields are comparatively weak whereas outside the solar system there are opportunities for some very strong fields. The target is a stellar binary system where at least one of the stars is a pulsar so that the periastron advance may be monitored.

The term binary pulsar is used if one or both objects are pulsars. The first such system was discovered in 1974 by Russell Alan Hulse (1950-) and Joseph Hooton Taylor, Jr (1941-) while conducting a survey at the 305 m Arecibo Observatory in Puerto Rico (Hulse and Taylor, 1975). The technology that existed at this time enabled a computer "to report on any pulsar suspects above a certain sensitivity threshold" (McNamara, 2008). The pulsar had a very short pulsation period of 5.9×10^{-2} s in a highly eccentric orbit of e = 0.615 with a period of $0^d.323\ 0$. Its companion is believed to be a neutron star. The pulsar is designated PSR 1913 + 16.

The measurement technique is a comparison between the phases of the radio pulses from the pulsar and those of atomic clocks on the Earth (Will, 1995) to register the

Table 1. Anomalous advance in the perihelia of Mercury, Venus and Earth.

Planet	**a** x 10^{10} m	**e**	**Orbit in tropical years**	$\Delta\phi$ **in " per century calculated**	$\Delta\phi$ **in " per century observed**
Mercury	5.791	0.205 6	0.240 844 45	42.98	43.11 ± 0.45
Venus	10.821	0.006 8	0.615 182 57	8.625	8.4 ± 4.8
Earth	14.960	0.016 7	0.999 978 62	3.839	5.0 ± 1.2

small changes over time with the pulse frequency. The Doppler effect alters the arrival time of the pulses. The variation was between $0^d.058\ 967$ and $0^d.069\ 045$ which amounts to 6.7 s over its cycle of $0^d.323\ 0$, that is, 7.75 h (Hulse and Taylor, *op. cit.*). The precision of measurement was such that an initial discrepancy of 2.7 x 10^{-2} s for the period of what was thought to be a single pulsar measured at different times was not considered a false value (McNamara, *op. cit.*). The speed of the orbit is highly relativistic being 10^{-3}c. The relativistic periastron advance of 4°.226 62 ± 0.000 01 yr^{-1} is 2.7 x 10^3 greater than the 5".7 y^{-1} for the perihelion advance of Mercury. This periastron advance is within 0.8% of the prediction from general relativity (Damour and Taylor, 1991). Also, this system will be revisited later in this paper as monitoring continues for how the companion's gravitational field affects the redshift of the pulses and how the relativistic time dilation is caused by the orbital motion.

A consequence of general relativity, the curvature of spacetime, is implicated in the periastron advance of binary pulsars in the same way as the perihelion advance of the planets. However, in 1918, Einstein proposed that a binary system would lose gravitational wave energy and provided a quadrupole formula for the subsequent damping on the orbital period (Einstein, 1918). However, his results are expressed here from a project which derives Einstein's conclusions (Valença, 2008). Firstly, for **E** energy, **t** time, a condition of **e** = 0, μ reduced mass where $\mu = m_1 m_2/(m_1 + m_2)$ for the individual masses, **m** representing the same mass which would be the case if e = 0 and **r** the distance between the two objects, then the change in energy over time is given by

$$\frac{d\,E\,(e=0)}{d\,t} = \frac{32\mu^2 m^3 G^4}{5c^5 a^5}\,. \tag{8}$$

Then, a correction is applied for the case when e $\neq 0$ so that

$$\frac{d\,E}{d\,t} = \frac{32\mu^2 m^3 G^4}{5c^5 a^5}\,(1+\frac{15}{2}e^2+\frac{45}{8}e^4)\,(1-e^2)^{-7/2}\,. \tag{9}$$

The change in energy per time may be extended to include a change in the period P denoted as $\dot{P}$ as

$$\frac{d\,E}{d\,t} = \frac{-\,2m_1 m_2 G}{3\,rP}\,\dot{P}\,. \tag{10}$$

From measurements on PSR 1913 + 16, the mass of the pulsar was determined as 1.441 0 ± 0.000 7 M_S (times mass of the Sun) and the companion as 1.387 4 ± 0.000 7 M_S (Will, *op. cit.*). The distance between the pair ranged from 1.1 to 4.8 solar radii. Armed with these data, Taylor, a codiscoverer, and Joel M Weisberg found, in 1989 after 14 years of measurement on the binary pulsar, that the rate of orbital decay was within 1% of that predicted by special and general relativity (Taylor and Weisberg, 1989). By 1995, improvement had reached 0.3% accuracy with a rate of (– 2.402 43 ± 0.000 05) x 10^{-12} ss^{-1}. Once a small effect caused by galactic rotation, the relative acceleration between the binary pulsar and the solar system, is subtracted, the result is (- 2.410 ± 0.009) x 10^{-12} s s^{-1} which is the prediction afforded by general relativity (Will, *op. cit.*). After 30 years of analysis in 1995, Weisberg and Taylor provided consistency between theory and observation at the (0.13 ± 0.21%) level (Weisberg and Taylor, 2005).

A further Arecibo survey operating at 4.30 x 10^8 Hz in 1990 detected another binary pulsar PSR 1534 + 12. The 3.79 x 10^{-2} s pulse of orbital period 3.64 x10^4 s has a rate of decay of 2.43 x 10^{-18} s s^{-1} and periastron advance of 1°.756 2 yr^{-1}. Due to the strong and narrow pulse, greater precision for this system was expected over time (Wolszczan, 1991). This had been achieved by 1998 with further timing observations with radio telescopes at Arecibo, 43 m Green Bank in West Virginia and 76 m Jodrell Bank and a conclusion that the results were in accord with general relativity to better than 1% (Stairs et al.,1998).

A third binary pulsar PSR 2127 + 11C (Prince et al., 1991) had its relativistic periastron advance measured at 4°.46 yr^{-1} in 1991 but more work was needed to compare this with general relativity. By 1992, 21 binary pulsars had been studied well enough for their basic parameters to be determined (Taylor, 1992).

A rare situation emerged in 2003. A pulsar discovered with the 64 m radio telescope13 beam receiver (Staveley-Smith et al., 1966) at Parkes Australia was found subsequently to have a companion which is also a pulsar. An improved position was determined with the use of the 20 cm band from interferometric observations with the Australia Telescope Compact Array (Burgay et al., 2003).

Results were published in 2006 after 2.5 years of measurements had been effected on PSR J0737 – 3039A and PSR J0737 – 3039B. Data were gathered at Parkes at 6.80 x 10^8, 1.374 x 10^9 and 3.030 x 10^{10} Hz, 76 m Jodrell Bank Observatory in the UK at 6.10 x 10^8 and 1.396 x 10^9 Hz, and 100 m Green Bank at 3.40 x 10^8, 8.20 x 10^8 and 1.400 x 10^9 Hz. A total of 131 416 arrival of pulse times for A with an uncertainty of 1.8 x 10^{-5} s were received and 507 for B with a maximum uncertainty of 4 x 10^{-3} s. The system has an orbital period of 0^d.102 251 563, respective pulse periods of 2.27 x 10^{-2} and 2.77 x 10 s and a periastron advance for A of 16°.90 yr^{-1} (Lyne et al., 2004).

Four independent tests of general relativity are obtainable with this system. The orbital decay derivative observed was - 1.252 x 10^{-12} s s^{-1}, shrinking the distance between the pulsars by 7 mm d^{-1}. The relativistic prediction was 1.247 87 x 10^{-12} s s^{-1} giving a ratio of observed to expected value of 1.003 (Kramer, op.cit.). Other results relate to gravitational redshift and time dilation.

Geodetic Precession

Yet another property was added to the list for testing general relativity soon after its inception. In 1916 Willem de Sitter (1872-1934) applied relativity theory to the Earth-Moon system. He realised the pair was freely falling in the gravitational field of the Sun. Since the Moon was also orbiting the Earth, he predicted that the Moon ought to undergo a non-Newtonian precession in its orbit (Sitter, 1916). His expected figure was a secular motion of the perigee and the node both of + 1".91century^{-1} (Sitter, 1917). This effect is referred to as geodetic precession.

Shapiro et al. (1988) mined the lunar laser ranging data collected over the period 1970-1986 from the retroreflectors on the Moon. A model of the Moon's motion consisted of two coupled sets of differential equations, one for its orbit and the other for its rotation. Perturbations from the gravitational fields of the Sun, Earth, and other planets as well as torques on the Moon from the Sun and Earth and the drag from tides on the Earth were factored to provide equations as a function of time. An introduced numerical factor ***h*** was related to any extra precession of the Moon's orbit about the ecliptic pole that was not included in the predicted relativistic geodetic precession. ***h*** would equal zero if it were consistent with general relativity and unity if there were 100% difference from the prediction. From the set of 4 400 echo measurements, their analysis resulted in ***h*** = 0.019 ± 0.010 (Shapiro et al., 1988).

According to general relativity the Moon should precess in its orbit by 1.9 x 10^{-2} s yr^{-1}. A data set of 8 300 lunar laser ranges over the period 1969-1993 yielded a deviation from this amount by – 0.3 ± 0.9% (Dickey et al., *op. cit.*). Gravity Probe B Relativity Mission was launched by NASA in 2004 and operated an experiment for 12 months. Its aim was to measure two effects predicted by general relativity: geodetic precession and frame dragging or Lense-Thirring effect. Geodetic precession may be described as a vector perpendicular to the orbital plane whereas frame dragging may be designated as a vector arising from rotation and acting orthogonally to the geodetic precession vector. As the two effects act at right angles to each other, the component vectors could be distinguished.

The satellite was placed in an orbit over both poles of the Earth. The mean altitude was 642 km and the orbital eccentricity was 0.001 4. A telescope was fixed on the bright star IM Pegasi, as were initially four super-conducting niobium coated, 38 mm spherical quartz gyroscopes. Each was surrounded by liquid helium at 2 K where some escaping gas caused the gyroscopes to commence spinning up to an average rate of 72 Hz. The devices were suspended electrically with two spinning clockwise and two counter clockwise. They were tested at maintaining their drift rate accuracy to 5" x 10^{-4} yr^{-1}.The gyroscope is a vector not aligned with the spin axis of the Earth. After one orbit of parallel transport of the Earth, any shift in the axis of a gyroscope would induce a current which enabled the changed to be measured (Will, 2006). The predicted Einstein drift rate was – 6".606 1 x 10^{-6} yr^{-1}. The four results were combined to give a weighted average of – (6".601 ± 0.018.3) x 10^{-6} yr^{-1}, giving an accuracy of 0.28% (Everitt et al., 2011). Across the span 1961-2003, 250 000 high precision radar observations from the USA and Russia to the inner planets and spacecraft have been examined. In addition to the perturbations of the planets and the Moon, those of 301 larger asteroids and a ring of small asteroids have been included. The result for γ was 0.999 9 ± 0.000 2 (Pitjeva, 2005). With binary pulsars, if the spin axis is not aligned with the angular momentum axis of the system, geodetic precession should occur. All the candidates that have been discovered so far need a much longer time period of measurement to arrive at definitive answers for this property.

Lense-Thirring Effect

Frame dragging refers to another effect arising from general relativity in which a massive celestial rotating body drags its local spacetime around with it. Whereas geodetic precession operates in the presence of a central mass, frame dragging is postulated to exist as a separate effect if the mass is rotating. This consequence was hypothesised by Josef Lense (1890-1985) and Hans Thirring (1888-1976) in 1918. However, Pfister (2007), in his treatment of the history of this effect, argues from evidence in the Einstein-Besso manuscript 1913, Thirring's notebook of 1917 and a letter from Einstein to

Thirring in 1917 that Einstein pointed to this phenomenon. Frame dragging is a secular precession of an orbiting object which has its orbital plane at an angle to the equator of a central entity which possesses angular momentum. The magnitude of the effect is extremely small compared with geodetic precession.

NASA launched Mars Global Surveyor in 1996 and it was inserted into its orbit in 1997. In the five year period 2000-2005, the orbital plane of the spacecraft was predicted to shift by 1.5 m due to frame dragging and the measured result was 1.6 m, giving a difference from general relativity of the order of 6% (Iorio, 2006).

Twin satellites, Laser Geodynamics Satellite (LAGEOS) launched by NASA in 1976 and LAGEOS II a joint NASA and Italian Space Agency in 1992, are passive reflectors in Earth orbit. Each contains 426 corner cube reflectors, all but four of these made of fused silica glass with the others of germanium for infrared measurements. Their respective orbital parameters are: semi-major axis 12 270 and 12 163 km; eccentricity 0.004 5 and 0.014; inclination to Earth's equator 110° and 52°.65. The expected measure of precession of their line of nodes was 3" x 10^{-4} yr^{-1} which is equivalent to a displacement of 1.9 m in that time. Monitoring was performed by 50 Earth stations as part of the International Laser Ranging Service. From 10^8 laser ranging observations over the period 1993-2003, the measure of the precession of the line of nodes was given as 4".79 x 10^{-2} yr^{-1} against the relativistic prediction of 4".82 x 10^{-2} yr^{-1}. The result of the observation was 99% ± 5 of the predicted value although the authors allow for 10% uncertainty (Ciufolini and Pavlis, 2004).

A later satellite, Laser Relativity Satellite (LARES), was launched by the Italian Space Agency in 2012. It is a spherical, laser ranged passive satellite with 92 retroreflectors made of a tungsten alloy. Its semimajor axis is 7 820 km, eccentricity 0.000 7 and orbital inclination 69°.5. Measurements are ongoing.

One of the difficulties with accurate positioning is the figure of the Earth. To ascertain deviations from spherical symmetry of the Earth's gravity field, Gravity Recovery and Climate Experiment (GRACE) consists of twin satellites of NASA and the German Aerospace Center launched in 2002 in polar orbit, 500 km above the Earth and 220 km between them. They maintain a microwave ranging link which can measure their separation to 1×10^{-5} m. Optical corner reflectors allow their position to be monitored from Earth against the GPS. Gravity Probe B results reported in 2012 gave the frame dragging effect (Everitt et al., *op. cit.*) as (– 3".72 ± 0.72) x 10^{-4} yr^{-1} compared with the Einstein value of – 3".92 x 10^{-4} yr^{-1}.

LIGHT PROPAGATION IN GRAVITATIONAL FIELDS

Gravitational Optical Light Deflection

The central equation of Einstein which led to his international fame was that the angle of deviation α of starlight in the vicinity of the Sun with mass M and distance from the centre r be given as

$$\alpha = \frac{4GM}{c^2 r} \tag{11}$$

where half that value was due to time curvature and the other half from space curvature, an intrinsic part of his general relativity (Einstein, 1916 *op. cit.*). This amounted to 1".75 at the limb of the Sun. With the technology at the time, confirmation rested on a photographic comparison of the stars near the Sun at a total solar eclipse and the same stellar field six months before or after the eclipse. The deviation for stars a little away from the limb corresponded to 1/60 mm on the plate (Eddington, 1919). Such a small measurement was difficult to ascertain with the precision instruments available in the early part of the twentieth century.

The 1919 British total solar eclipse expedition to Brazil by Andrew Claude de la Cherois Crommelin and to Principe by Arthur Stanley Eddington and Edwin Turner Cottingham demonstrated that starlight was deflected by the Sun. In 1922, with final results published in 1928, an excursion to Wallal in remote Western Australia by the Lick Observatory led by William Wallace Campbell supported the deflection at the limb of the Sun as 1".75 ± 0.09 (Campbell and Trumpler, 1928). A limitation for this technique depends on the ability of a telescope to resolve small angular separations due to refraction as light passes through the system.

$$\text{Angular resolution in arcsecond} = \frac{2.5 \times 10^5 \text{ x wavelength of light in m}}{\text{diameter of mirror in m}} \tag{12}$$

For the 33 cm telescope used and visible light, the angular resolution amounted to 0".4. Attempts at repeating the experiment have been performed at a number of total solar eclipses, now nine altogether, and the ones in 1952 and 1973 will be mentioned here.

The National Geographical Society and the Naval Research Laboratory jointly sponsored an expedition to Khartoum in Sudan in 1952 (Biesbroeck, 1953). Disappointingly, wind at the time of the eclipse induced vibrations in the 20 foot (6 m) telescope so that many of the fainter stellar images were not included in the measurement. Nevertheless, one photographic plate exposed for 60 s produced nine measurable stars in the eclipse field and eight in the auxiliary field while a second exposure of 90 s resulted in 11 and eight stars respectively. Two checkplates were secured six months later. The conclusion was 1".70 ± 0.10.

In 1973 the University of Texas mounted a mission to Chinguetti Oasis in Mauritania, Africa (Brune et al., 1976). With a 2.1 m focus, four element astrometric lens, the party prepared for a 6 min 18 s eclipse. Three plates, impregnated with a rectangular scale, were obtained with 60 s eclipse field and 30 s comparison field 10° away in declination. 150 measurable images and 60 comparison field ones were captured. After an elapse of five months, 33 calibration plates were obtained. The result extrapolated to the solar limb of 0".95 ± 0.11 serves to indicate, if general relativity is to be supported, how difficult measurements on photographic plates for the visible region of the spectrum actually is.

Since the launch of the European Space Agency spacecraft Hipparcos (high precision parallax collecting satellite) in 1989, the deflection of light at total solar eclipses has been consigned to a quaint part of history. The 29 cm aperture telescope on board has measured the position of 118 200 stars to a precision of 3" x 10^{-3} for the magnitudes 8 - 9. Any effect on the deflection of starlight by the Sun can now be measured by checking the distance between pairs of stars over time. The advantages inherent in this system were that there was no need for a total solar eclipse, bending by the solar corona could be eliminated, measurements could take place over large angular distances from the Sun and the same instrument was used well calibrated over the entire sky for 37 months. Data were collected on a set of stars chosen within 47 - 133° of the Sun. As an example, the relativistic prediction is that at 90° from the Sun the deflection would be 4".07 x 10^{-3}. As a number of theories incorporate some predictions similar to general relativity, nine so called parameterised post-Newtonian parameters have been introduced. Radiation deflected by the gravitational field of the Sun and entering a telescope on Earth is expressed as an amount equal to

$$1''.749 \, \frac{(1+\gamma)}{2} \tag{13}$$

where γ equals unity in general relativity. The result from Hipparcos was γ = 0.997 ± 0.003 (Froeschlé et al., 1997). An improved astrometric spacecraft from the ESA is Gaia which was launched in December 2013 and took up its residence at the Sun-Earth L_2 Langrangian point in January 2014. The aim of the mission is to record the position of 10^9 objects to a precision of 2".0 x 10^{-5}. A future analysis of results based on a similar method as for the Hipparcos data will improve the accuracy of this experiment.

Gravitational Radio Deflection due to the Sun

Since angular resolution is proportional to the reciprocal of the wavelength of light, the longer wavelength radio region provides an improvement over the visible spectrum. It eventually became possible to measure the position of radio sources so precisely with interferometry, even in the daytime. The blazar 3C279 is a very bright object 12' from the ecliptic and each 08 October it is eclipsed by the Sun. Deflection was measured by two groups in 1969. An Owens Valley Radio Observatory team (Seielstad et al., 1970) in California reported γ = 1.02 ± 0.23 and another Californian band from Goldstone (Muhleman et al., 1970) gave γ = 1.08 ± 0.30. This method was also employed in 1974 with three nearly collinear radio sources, 0116 + 08, 0119 + 11 and 0111 + 02, and a 35 km interferometer baseline (Fomalont and Sramek, 1975). As these radio emitters passed near the Sun, the deflection of their beams was monitored by the National Radio Astronomy Observatory at Green Bank. This comprised three steerable 26 m parabolic antennas with a maximum baseline separation of 2.7 km and a fourth element of 14 m aperture situated 35 km away. The three long baselines are 33.1, 33.8 and 35.3 km. So that the solar coronal refraction may be separated from the contribution from relativity, observations were made simultaneously at two frequencies, 2.695 x 10^9 and 9.085 x 10^9 Hz since electron refraction varies as the square of the wavelength.

The deflection at the solar limb was determined as 1".775 ± 0.019 which was 1.015 ± 0.011 times the Einstein value. This corresponds to the parameter γ = 1.030 ± 0.022. The experiment was repeated 12 months later in 1975. The combination of the 1974 and 1975 measurements (Fomalont and Sramek, 1976) produced a limb deflection of 1".761 ± 0.016 corresponding to 1.007 ± 0.009 times the general relativity prediction and γ = 1.014 ± 0.018.

The source 3C279 mentioned earlier in this area is also known as J1256 – 0547. It and three other radio emitters, J1304 – 0346, J1248 – 0632 and J1246 – 0730, were captured by the Very Long Baseline Array in 1990. This comprises 10 parabolic 25 m telescopes across the United States of America. Previous testing had shown that the system could measure relative positions to 1" x 10^5 (Fomalont et al., 2009a). The system operated at frequencies of 1.5, 2.3 and 4.3 all x 10^{10} Hz so that the effect of the solar corona was minimised. Furthermore, the relativistic bending is independent of the wavelength. The result from the four sources combined was γ = 0.999 8 ± 0.000 3 (standard uncertainty) (Fomalont et al., 2009b).

As the length of the baseline in interferometry increases, the accuracy of the determination of γ improves. A major investigation between 1980 and 1990 was conducted by personnel from the National Oceanic and Atmospheric Administration in Rockville, Maryland (Robertson et al., 1991). 74 radio sources collected by 29 very long baseline observatories produced a set of 342 810 observations. Early data used 3 000 km as the baseline, such as from Westford, Massachusetts to Fort Davis, Texas, but later ones operated between 7 000 – 10 000 km, for example, a 7 832 km stretch from Wettzell, Germany to Hartebeesthoek, South Africa. The expected deflection at the Sun's limb is 1".750, at an angle of 90° away from the Sun 4"x10^{-3} and zero deflection at 180°. The scientists concluded a value for γ of 1.000 2 ± 0.002 (standard uncertainty).

Use was made of data collected during 1979-1999 from 87 very long baseline interferometric sites and 541 radio sources (Shapiro et al., 2004). The information was intended to monitor various motions of the Earth but has been analysed to conclude γ = 0.999 8 ± 0.000 4.

Gravitational radar deflection is progressing to the planets. Measurements were taken in 2002 when Jupiter passed within 4' of the quasar J0842 + 1835, in 2008 for Jupiter 1'.4 from J1925 – 2210 and in 2009 for Saturn 1'.3 from J1127 + 0555. More arrays are devoting time to this new avenue and the results are awaiting analysis (Fomalont et al., *op. cit.* 2009b).

Gravitational Lensing

Gravitational lensing refers to the production of an image of a background object presented to an observer by another object between them. The origin of this thought has been traced to eight pages of a notebook Einstein used in 1912 (Renn et al., 1997). In it he indicated the possibility of a double image of the source due to gravitational light bending and suggested that the intensity of these images would be magnified. In 1936 Einstein returned to this idea and wrote about a background star, when bent in the gravitational field of an intermediate star, would be perceived by an observer in line with both of them not as a point-like star but as a luminous circle around the foreground object. From geometry he obtained an expression for the angular radius (later Einstein radius) of the halo (later Einstein ring) in terms of the deviation angle of light passing the lensing star, the distance of the light from the centre of the foreground object and the distance between observer and lensing star. The derivation is explained in detail by Schneider et al. (1992) as

$$\alpha = \left(\frac{4GM}{c^2} \frac{D_1}{D_2 D_3} \right) \text{ raised to 0.5 power} \qquad (14)$$

where M is the mass of the lens, D_1, D_2 and D_3 are respectively distances between source and lens, lens to observer and source to observer (Schneider et al., 1992). Einstein also noted again that the apparent brightness of the distant star would be enhanced. It is interesting to note that he saw no hope of a direct observation of this spectacle (Einstein, 1936).

An extension from a star as the lensing object was provided in 1937 by Fritz Zwicky (1937). He theorised that the gravitational fields of a number of foreground nebulae may deflect the light from background nebulae and that this might be used to determine nebular masses accurately. He also suggested that a search ought to be conducted among globular nebulae for images of globular clusters. In 1964 a proposal was published in which a supernova could be lensed by a galaxy. This would allow very faint, distant objects to produce an image much closer to the observer so measurements could be extended to much greater distances. The wait was until 1979 when the 2.2 m telescope on Mauna Kea belonging to the University of Hawaii recorded two images which, from their identical properties such as the same redshift z = 1.413, were intimated to be the twin QSO 0957 + 561 (Walsh et al., 1979). The galaxy causing the lensing was soon directly recorded along with a third image (Stockton, 1980).

With the Advanced Camera for Surveys (ACS) aboard the Hubble Space Telescope, the Sloan Lens ACS (SLACS) Survey (Bolton et al., 2008) has provided a 2008 list of 131 strong gravitational lens candidates. There are 70 systems with clear evidence for multiple imaging and another 19 probable ones. Selection was made from the spectroscopic database of an absorption dominated galaxy continuum at one redshift and nebular emission lines at a higher redshift. The lines incorporated the Balmer series and O II at 3.727 x 10^{-12} m and O III at 5.007 x 10^{-12} m.

An interesting gravitational lens system discovered in 1985 (Huchra et al., 1985) shows how it can add support to the theory of general relativity. It has been resolved by the Hubble Space Telescope to be four quasar images with z = 1.695 surrounding a 15 magnitude spiral galaxy 2237 + 0305 with z = 0.039 4. The four images are concentric but have different levels of brightness. From the application of lens models based on the lensing equation derived by Einstein along with the cosmological interpretation of redshifts, all of the data collected can be explained. The first discovery of an Einstein ring occurred in 1988 (Hewitt et al., 1988) with the radio source MG1131 + 0456 being surrounded by an elliptical ring of emission.

Time Dilation

In 1964 Irwin Ira Shapiro (1929-) proposed that with recent advances in radar astronomy, another test for general relativity would be to measure the time delay between emission and detection of radar pulses bounced off Mercury or Venus when they were near superior conjunction (Shapiro, 1964). The Doppler shift cancels on a round trip. The time delay Δt is given by

$$\Delta t = \frac{4GM_S}{c^3} \frac{1+\gamma}{2} \ln \frac{R_E + R_P + R}{R_E + R_P - R} \qquad (15)$$

where G, M_S, c and γ are as defined previously, R_E, R_P and R are respective distances between the Earth and

Sun, planet and Sun and Earth and planet (Reasenberg et al., 1979). This increase in time amounted to 1.6 x 10^{-4} s for Mercury when the beam passes by the Sun at two radii from its centre.

Testing began in 1967 and after three years of 1 700 measurements by the Haystack and Arecibo Observatories, Shapiro reported γ = 1.03 ± 0.04 (Shapiro et al., 1971). The first measurements made of time dilation with spacecraft were at Mars in 1969. NASA sent a dual mission of Mariner 6 and 7 and the echoes were received with the 64 m telescope at Goldstone where the accuracy of the ranging system was rated as 1 x 10^{-7} s. The respective data were total time for round trip: 44.72, 42.87 min; distance of beam from centre of Sun: 3.58, 5.90 solar radii; angle Sun-Earth-spacecraft: 0°.95, 1°.56; approximate time delay: 2.0 x 10^{-4}, 1.8 x 10^{-4} s; γ 1.003 ± 0.04, 1.000 ± 0.012. The combined figure for γ was given as 1.00 ± 0.03 (Anderson et al., 1975). This 3% uncertainty was lowered to 2% for Mariner 9 in orbit of Mars in 1971 (Reasenberg, *op. cit.*).

In 1975 NASA launched Viking 1 and Viking 2 which arrived at Mars in 1976. Each spacecraft consisted of an orbiter and lander with radio links to each other. Receiving stations on Earth were the three of the Deep Space Network. By having two set places on the Martian surface, accuracy was reduced to 0.5% (Michael et al., 1977). Two parameters from the two pulsars in a mutual orbit relate to the shape of the time delay and its range. They are given respectively followed by the Einstein comparison and ratio of observed to predicted values: 0.999 74 [0.999 87, 0.999 87] and 6.21 x 10^{-6} s [6.153 x 10^{-6} s, 1.009] (Kramer, *op. cit.*).

Atomic Clocks

In 1967 time was defined by the International Union of Pure and Applied Chemistry in terms of transitions involving the caesium-133 atom. Calibration was initially against ephemeris time where the motion of the Sun or Moon could be the standard. However, tables of motion of these bodies require many factors to be taken into account. Nevertheless, programs now exist that do give an accurate description of time.

Not long after, in 1971, four clocks containing caesium-133 were calibrated against each other and compared with the reference atomic scale at the United States Naval Observatory. As an experiment to test time changes within general relativity, they were flown on a commercial jet firstly eastward around the world. Their time losses amounted to 5.1, 5.5, 5.7 and 7.4 all x 10^{-8} s to give a mean and standard deviation of – (5.9 ± 1.0) x 10^{-8} s against the relativistic prediction with estimated uncertainty of – (4.0 ± 2.3) x 10^{-8} s. The westward round the world trip resulted in gains of 2.66, 2.66, 2.77 and 2.84 all x 10^{-7} s to result in + (2.73 ± 0.07) x 10^{-7} s against + (2.75 ±0.21) x 10^{-7} s (Hafele and Keating, 1972).

STRONG GRAVITY IMPLICATIONS

Nordtved Effect

A strong equivalence principle is known as the Nordtved effect after Kenneth Leon Nordtvedt (1939). It treats gravity as a geometric property of spacetime. Measurements described at Appache Point Observatory provide support for relativity to a few parts in 10^5 (Murphy, *op. cit.*).

Potential Gravitational Waves

As general relativity has dealt with weak fields within the solar system and stronger ones outside, it may be used to see if it will elucidate the situation with exceptionally strong fields. The conversion of rotational energy into gravitational energy would result in orbital decay in a binary pulsar. While decay has been measured, the search for gravitational waves has begun in earnest. A connection between accelerating masses and gravitational waves is hypothesised. However, compared with electromagnetic radiation from accelerating charges, the energy is extremely small. Thus, in their search for gravitational waves, scientists will firstly need to look at massive energy systems.

Towards the end of their existence, double neutron stars spiral inwards, collide and merge with a predicted enormous release of gravitational radiation. This is suggested to be strong enough to identify at the Earth. Detection is currently being attempted by VIRGO in Italy, GEO600 in Germany, TAMA in Japan and LIGO in the USA (Heuvel, 2003). As an example, (Laser Interferometer Gravitational Wave Observatory (LIGO) is on two sites. Each contains two arms four km long with weights suspended at the end of vacuum tubes. Laser beams measure the distances between the loads. The passage of a gravitational wave is expected to change the distance between the weights which would be detected with an interference pattern between the laser beams.

DISCUSSION

A summary of all the previous material is listed in Table 2. The property includes the title in this paper, the experiment performed relevant to that topic, the year of publication (not the year of the experiment) arranged chronologically for that section and percentage difference from relativity as the difference divided by the general relativity value. If there are two figures listed, the first one uses the central figure of the result against the prediction of general relativity. The second value uses the uncertainty, if it exists in the literature, and takes the larger of the difference from general relativity.

Table 2. Percentage difference from relativity for experiments conducted listed under a section, property and year of publication.

Property	Experiment	Year of Publication	% Difference from relativity
Equivalence of Inertial and Gravitational Mass	Torsion balance	1890	5×10^{-6}
	Torsion balance	1909	1×10^{-6}
	Torsion balance	1935	$2\text{-}5 \times 10^{-7}$
	Torsion balance	1964	1×10^{-9}
	Lunar laser ranging	2005	1×10^{-2}
	Lunar laser ranging	2009	1×10^{-1}
Gravitational Redshift	Gamma rays	1960	5, 15
	Gamma rays	1965	0.1, 0.9
	Hydrogen maser on rocket	1980	0.007
	Voyager 1 at Saturn	1990	0.44, 1
	Galileo spacecraft	1993	1
	Cassini spacecraft	2004	0.002 3
	Psr j0737 – 3039a/b	2006	0.36
Relativistic Perihelion Advance of the Planets	Mercury	1943	0.19, 1.4
	Venus	1956	2.3, 58
	Earth	1956	32, 62
Relativistic Periastron Advance of Binary Pulsars	PSR 1913 + 16 orbital decay	1989	1
	PSR 1913 + 16 periastron advance	1991	0.8, 1
	PSR 1913 + 16 orbital decay	1995	0.3
	PSR 1913 + 16 orbital decay + galactic rotation	1995	0
	PSR 1534 + 12 periastron advance	1998	1
	PSR J0737 – 3039A/B orbital decay	2004	0.3
	PSR 1913 + 16 orbital decay	2005	0.13, 0.4
Geodetic Precession	For Moon	1988	1.9, 2
	For Moon	1994	0.3, 2
	Planetary motions	2005	0.01, 0.03
	Gravity Probe B in Earth orbit	2011	0.28
Lense-Thirring Effect	LAGEOS and LAGEOS II in Earth orbit	2004	0.6, 0.7
	Mars Global Surveyor in orbit	2006	6
	Gravity Probe B in Earth orbit	2012	5, 24
Gravitational Optical Light Deflection	Total solar eclipse	1953	2.9, 4
	Total solar eclipse	1976	46
	Hipparchos	1997	0.3
Gravitational Radio Deflection due to the Sun	3C279 owens valley observatory	1970	2, 25
	3C279 goldstone	1970	8, 38
	3 radio sources and interferometry	1975	3, 6
	3 radio sources and interferometry	1976	1.4, 4
	74 radio sources and interferometry	1991	0.02, 0.3
	541 radio sources and interferometry	2004	0.02, 0.06
	4 radio sources and interferometry	2009	2, 5
Gravitational Lensing	Observations in accord with predictions	-	-

Table 2. Contd.

Time Dilation	Radar ranging to Mercury and Venus	1971	3, 7
	Mariner 6 in Mars flyby	1975	0.3, 0.7
	Mariner 7 in Mars flyby	1975	0, 2
	Viking – 2 orbiters and 2 landers at Mars	1977	0.5
	Mariner 9 in Martian orbit	1979	0, 2
	PSR J0737 – 3039A/B – shape of time delay	2006	0.013
	PSR J0737 – 3039A/B – range of time delay	2006	0.9
Atomic Clocks	Flying eastwards around Earth	1972	48, 73
	Flying westwards around Earth	1972	0.7, 3
Nordtved Effect	Lunar laser ranging	2003	(1) x 10^{-3}

As seen from the table, the equivalence principle has been tested to the 1 x 10^{-9} difference from relativity and the Cassini spacecraft has a measure of difference of 0.002 3% for gravitational redshift. What is significant is that from 10 properties with measurements, so many are at the 10^{-1} and 10^{-2} level.

CONCLUSION

This paper covers predominantly the period after 1928 to the present. From the three classical astronomical tests of general relativity (anomalous perihelion advance of the perihelion of Mercury, gravitational light bending and gravitational redshift), a plethora of other avenues has developed historically. Even the term relativistic astrophysics did not exist for the first 50 years following Einstein's publication of 1916. Topics covered are weak equivalence principle (equivalence of inertial and gravitational mass and gravitational redshift), orbital precession of a body in gravitational fields (the relativistic perihelion advance of the planets, the relativistic periastron advance of binary pulsars, geodetic precession and Lense-Thirring effect), light propagation in gravitational fields (gravitational optical light deflection, gravitational radio deflection due to the Sun, gravitational lensing, time dilation and atomic clocks) and strong gravity implications (Nordtved effect and potential gravitational waves). Each subject has been plumbed to determine the amount of measurement agreement with general relativity. Three questions were proposed as a guiding principle to this paper.

(i) How much evidence exists to support general relativity?

Einstein originally proposed that his concept could be tested by three astronomical tests. However, there was a significant hiatus between his 1916 publication and further experimentation. There was a need for technology to be developed and experimental techniques both invented and refined before more rigorous delving into the theory could ensue. Torsion balance data existed before 1916 but it continued to improve with better equipment. Lunar laser ranging and radar echoes from the inner planets improved the positioning of these solar system bodies. Allied with computer programs, scientists enhanced ephemerides and many of the perturbations were teased out to ascertain the contribution of each. By extending the reception of data from one station to several with a long base, scientists were able to use interferometry to tighten the uncertainty in their measurements. The introduction of spacecraft in Earth orbit and then venturing to the Moon and all the other planets opened up another methodology for experimentation. Precision was an essential requirement for the operation of these vehicles and so experimentation into relativity advanced. There promises to a burgeoning of data as planned spacecraft are put into service. However, with the myriad sets of results outlined in this article along with many tight constraints on the figures, general relativity has been tested well and not shown to be incorrect.

(ii) Is general relativity a reasonable way of thinking?

General relativity contains a number of simple ideas. From these, several predictions follow and these have been shown to be acceptable to usually better than a 1% level. It does not follow that general relativity is "correct" as other ideas may lead to the same forecasts. A model is judged by the fruitfulness of its operation. Against that criterion, general relativity has been shown to be superb.

A difficulty is that it does not square with notions people have, from their experience, of what reality is. However, experience tells us that the Earth neither spins nor orbits and that a body does not stay in constant motion. Yet, these ideas eventually won the day. People perceive space and time as absolute quantities and are more familiar with the geometry of Euclid than any other. Even though it is the province of scientists to understand the way the Universe operates, it is a task of all in the field to communicate these concepts to the public. Otherwise, the popularity of astrological signs in magazines and the reliance some people put on the ability of these to tell the future act as a signal of minds not thinking scientifically. General relativity is a successful concept and the public needs to have some appreciation of what it says.

(iii) What is the niche that general relativity should occupy?

Significant discussion abounds on the conflict between parts of general relativity and quantum mechanics. As a result, there is a search for a theory of everything. These models ought to be viewed as two of the greatest pieces of inspiration that have flowed from the mind of humans. It is imperative to celebrate such great thought. They are not reality but point to it. General relativity provides a worldview when masses are large and speeds approach that of the speed of light. Instead of seeing the disagreement between the two concepts, one may use whichever idea performs the role of explanation for each situation. This may involve a tension with some but the tension can be manageable. Light is light. On some occasions, its properties are better explained with a particle model and, at others, with a wave formulation. Neither holds a complete explanation; both are necessary to gain a perception of light. Perhaps, unification of general relativity and quantum mechanics may occur. In the meantime, Einstein's worldview may continually be applied to intriguing aspects of the Universe.

Formulated in 1916, general relativity was faced much later with a rapid succession of findings. In 1954 Cygnus A was a strong radio source associated with a distant galaxy that could not be detected optically. X ray sources entered the scene in 1962 followed by quasars in 1963, the 3 K background radiation in 1965, pulsars in 1967 and later further exotic objects of the cosmos. These features have been subsumed under the wing of general relativity and a scientific understanding of these phenomena would not currently exist without such a model.

Conflict of Interest

The author has not declared any conflict of interest.

REFERENCES

Anderson JD, Esposito PE, Martin W, Thornton CL, Muhleman DO (1975). "Experimental test of general relativity using time-delay data from Mariner 6 and 7". Astrophys. J. 200:221-233.

Bertotti B, Iess L, Tortora P (2003). "A test of general relativity using radio links with the Cassini spacecraft". Nature 425:374-376.

Biesbroeck G van (1953). "The relativity shift at the 1952 February 25 eclipse of the Sun". Astronol. J. 58(1207):87-88.

Bolton AS, Burles S, Koopmans LE, Treu T, Gavazzi R, Moustakas LA, Wayth R, Schlegel DJ (2008). "The Sloan lens ACS survey. V. the full ACS strong-lens sample". Astrophys. J. 682(2):964-984.

Brune RA Jr, Cobb CL, DeWitt BS, DeWitt-Morette C, Evans DS, Flloyd JE, Jones BF, Lazenby RV, Marin M, Matzner RA, Mikesell AH, Mitchell RI, Ryan MP, Smith HJ, Sy A, Thompson CD (1976). "Gravitational deflection of light: solar eclipse of 30 June 1973. I. description of procedures and final results". Astronol. J. 81(6):452-454.

Burgay M, D'Amico N, Possenti A, Manchester RN, Lyne AG, Joshi BC, McLaughlin MA, Kramer M, Sarkissian JM, Camilo F, Kalogera V, Kim C, Lorime DR (2003). "An increased estimate of the merger rate of double neutron stars from observations of a highly relativistic system". Nature 426:531-533.

Campbell L, Moffatt JW (1983). "Quadrupole moment of the Sun and the planetary orbits". Astrophys. J. 275:L77-79.

Campbell WW, Trumpler R (1928). "Observations made with a pair of five-foot cameras on the light-deflections in the Sun's gravitational field at the total solar eclipse of September 21, 1922". Lick Observatory Bulletin 397(13):130-160.

Ciufolini I, Pavlis EC (2004). "A confirmation of the general relativistic prediction of the Lense-Thirring effect". Nature 431:958-960.

Clemence GM (1943). "The motion of Mercury 1765-1937". Astronol. J. 1:126-127.

Damour T, Taylor JH (1991). "On the orbital period change of the binary pulsar PSR 1913 + 16". Astrophys. J. 366:501-511.

Dickey JO, Bender PL, Faller JE, Newhall XX, Ricklefs RL, Ries JG, Shelus PJ, Veillet C, Whipple AL, Wiant JR, Williams JG, Yoder CF (1994). "Lunar laser ranging: a continuing legacy of the Apollo program". Science 265(5171):482-490.

Duncombe RL (1956). "The motion of Venus 1750-1949". Astronol. J. 61:266.

Eddington AS (1919). "The total eclipse of 1919 May 29 and the influence of gravitation on light". Observatory 42:119-122.

Einstein A (1905). "On the electrodynamics of moving bodies". Annalen der Physik. 17:891-921.

EinsteinA (1907). "On the relativity principle and the conclusions drawn from it". Jahrbuch der Radioaktivität und Elektronik. 4:411-462.

Einstein A (1916). "The Foundations of the General Theory of Relativity". Annalen der Physik. 49:769-822.

Einstein A (1918)."Über Gravitationswellen" Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften. pp. 154–167.

Einstein A (1936). "Lens-like action of a star by the deviation of light in a gravitational field". Science 89(2188):506-507.

Einstein A, Grossmann M (1913). "Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation" (Outline of a generalised theory of relativity and of a theory of gravitation). Zeitschrift für Mathematik und Physik. 62:225–261.

Eötvös R v (1890). Mathematische und Naturwissenschaftliche Berichte aus Ungarn [Mathematical and Scientific Reports from Hungary] 8:65.

Eötvös R v (1909). l6th International Geodesic Conference in London.

Eötvös RV, Pekár D, Fekete E (1922). Beiträge zum Gesetz der Proportionalität von Trägheit und Gravität" {Contributing to the law of proportionality of inertia and gravitation}. Ann. Phys. 68:11- 66.

Everitt CW, DeBra DB, Parkinson BW, Turneaure JP, Conklin JW, Heifetz MI, Keiser GM, Silbergleit AS, Holmes T, Kolodziejczak J, Al-Meshare M, Mester JC, Muhlfelder B, Solomonik V, Stahl K, Worden P, Bencze W, Buchman S, Clarke B, Al-Jadaan A, Al-Jibreen H, Li J, Lipa JA, Lockhart JM, Al-Suwaidan B, Taber M, Wang S (2011). "Gravity Probe B: final results of a space experiment to test general relativity". Phys. Rev. Lett. 106(22):1101-1105.

Fomalont EB, Sramek RA (1975). "A confirmation of Einstein's general theory of relativity by measuring the bending of microwave radiation in the gravitational field of the Sun". Astrophys. J. 199:749-755.

Fomalont EB, Sramek RA (1976). "Measurements of the solar gravitational deflection of radio waves in agreement with general relativity". Phys. Rev. Lett. 36(25):1475-1478.

Fomalont E, Kopeikin S, Lanyi G, Benson J (2009a). "Progress in measurements of the gravitational bending of radio waves using the VLBA". Astrophys. J. 699:1395-1402.

Fomalont E, Kopeikin S, Jones D, Honma M, Titov O (2009b). "Recent VLBA/VERA/IVS tests of general relativity" Proceedings IAU Symposium No. 261 Relativity in Fundamental Astronomy. 291-295.

Froeschlé M, Mignard F, Arenou F (1997). "Determination of the PPN parameter γ with the Hipparcos data". Proceedings of the ESA Symposium at Venice. 402:49.

Gamalath KW (2012). Einstein his life and works. P. 287.

Gefter A (July 2005). "Putting Einstein to the test" Sky and Telescope. pp. 33-40.

Hafele JC, Keating RE (1972). "Around-the-world atomic clocks: observed relativistic time gains". Science 177:168-170.

Heuvel EPJ van den (2003). "Testing time for gravity". Nature. 426:504-505.

Hewitt JN, Turner EL, Scneider DP, Burke BF, Langston GI, Lawrence CR (1988). "Unusual radio source MG1131 + 0456: a possible Einstein ring". Nature 333:537-540.

Hirate CM (2012). "Lecture ix: weak field tests of GR: the gravitational redshift, deflection of light, and Shapiro delay" Caltech.

Huchra J, Gorenstein M, Kent S, Shapiro I, Smith G (1985). "2237 + 0305: a new and unusual gravitational lens". Astrono. J. 90:691-696.

Hulse RA, Taylor JH (1975). "Discovery of a pulsar in a binary system". Astrophys. J. 195:L51-53.

Iorio L (2006). "A note on the evidence of the gravitomagnetic field of Mars". Classical Quantum Gravity 23(17):5451.

Janssen M (2002). "The Einstein-Besso manuscript: A glimpse behind the curtain of the wizard" Introduction to the Arts and Science.

Kramer M, Stairs IH, Manchester RN, McLaughlin MA, Lyne AG, Ferdman RD, Burgay M, Lorimer DR, Possenti A, D'Amico N, Sarkissian JM, Hobbs GB, Reynolds JE, Freire PCC, Camilo F (2006). "Tests of general relativity from timing the double pulsar". Science 314(5796):97-132.

Krisher TP, Anderson JD, Campbell JK (1990). "Test of the gravitational redshift effect at Saturn". Phys. Rev. Lett. 64(12):1322-1325.

Krisher TP, Morabito DD, Anderson JD (1993). "The Galileo redshift experiment". Phys. Rev. Lett. 70(15):2213-2216.

Lyne AG, Burgay M, Kramer M, Possenti A, Manchester RN, Camilo F, McLaughlin MA, Lorimer DR, D'Amico N, Joshi BC, Reynolds J, Freire PCC (2004). "A double-pulsar system: a rare laboratory for relativistic gravity and plasma physics". Science 303:1153-1157.

McNamara G (2008). Clocks in the Sky: the story of pulsars.112.

Michael, WH Jr, Tolson RH, Brenkle JP, Cain DL, Fjeldbo G, Stelzried CT, Grossi MD, Shapiro II, Tyler GL(1977). "The Viking radio science investigation". J. Geophys. Res. 82(28):4293-4295.

Morgan HR (1945). "The Earth's perihelion motion". Astronol. J. 51:127-129.

Morton DC (1956). "Relativistic advances of perihelions". J. Royal Astronol. Soc. Canada 1:223.

Muhleman DO, Ekers RD, Fomalont EB(1970). "Radio interferometric test of the general relativistic light bending near the Sun". Phys. Rev. Lett. 24:1377-1380.

Murphy TW Jr, Adelberger EG, Battat JBR, Carey LN, Hoyle CD, LeBlanc P, Michelsen EL, Nordtvedt K, Orin AE, Strasburg JD, Stubbs CW, Swanson HE, Williams E (2008). "APOLLO: the Apache Point Observatory Lunar Laser-ranging Operation:Instrument Description and First Detections". Publ. Astron. Soc. Pac. 120:20-37.

Pfister H (2007). "On the history of the so-called Lense-Thirring effect". Gen. Relativity Gravit. 39(11):1735-1748.

Pijpers FP (1998). "Helioseismic determination of the solar gravitational moment". Monthly Notices of the Royal Astronomical Society. 297:L76-80.

Pitjeva EV (2005). "Relativistic effects and solar oblateness from radar observations of planets and spacecraft". Astron. Lett. 31(5):340-349.

Pound RV, Rebka GA Jr (1959). "Gravitational redshift in nuclear resonance". Phys. Rev. Lett. 3(9):439-441.

Pound RV, Rebka GA Jr (1960a). "Apparent weight of photons". Phys. Rev. Lett. 4(7):337-341.

Pound RV, Rebka GA Jr (1960b). "Gravitational red-shift in nuclear resonance". Phys. Rev. Lett. 4(7):439-441.

Pound RV, Snider JL (1965). "Effect of gravity on gamma radiation". Phys. Rev. 140(3B):788-803.

Prince TA, Anderson SB, Kulkarni SR (1991). "Timing observations of the 8 hour binary pulsar 2127 + 11C in the globular cluster M15". Astrophys. J. 374:L41-44.

Reasenberg RD, Shapiro II, MacNeil PE, Goldstein RB, Breidenthal JC, Brenkle JP, Cain DL, Kaufman TM, Komarek TA, Zygielbaum AI (1979). "Viking relativity experiment: verification of signal retardation by solar gravity". Astrophys. J. 234:L219-221.

Renn J, Sauer T, Stachel J (1997). "The origin of gravitational lensing: A postscript to Einstein's 1936 science paper". Science. 275:184-186.

Renner J (1935). *Matematikai és Természettudományi Értesítő.* 13:542.

Robertson DS, Carter WE, Dillinger WH (1991). "New measurement of solar gravitational deflection of radio signals using VLBI" Nature. 349(6312):768-770.

Roll PG, Krotkov R, Dicke RH (1964). "The equivalence of inertial and passive gravitational mass". Ann. Phys. 26:442-517.

Schneider P, Ehlers J, Falco EE (1992). Gravitational Lenses. Springer, Astronomy and Astrophysics. Library XIV

Seidelmann PK ed. (2006). Explanatory Supplement to the Astronomical Almanac. 704.

Seielstad GA, Sramek RA, Weiler KW (1970). "Measurement of the deflection of 9.602-GHz radiation from 3C279 in the solar gravitational field". Phys. Rev. Lett. 24:1373-1376.

Shapiro II (1964). "Fourth test of general relativity". Phys. Rev. Lett. 13(26):789-791.

Shapiro II, Ash ME, Ingalls RP, Smith WB, Campbell DB, Dyce RB, Jurgens RF, Pettengill GH (1971). "Fourth test of general relativity: new radar result". Phys. Rev. Lett. 26(18):1132-1135.

Shapiro II, Reasenberg RD, Chandler JF, Babcock RW (1988). "Measurement of the de Sitter precession of the Moon: a relativistic three-body effect". Phys. Rev. Lett. 41(23):2643-2646.

Shapiro SS, Davis JL, Lebach DE, Gregory JS (2004). "Measurement of the solar-gravitational deflection of radio waves using geodetic very-long-basline interferometry data, 1979-1999". Phys. Rev. Lett. 92(12):1101-1104.

Sitter W de (1916). "On Einstein's theory of gravitation, and its astronomical consequences, second paper". Monthly Notices Royal Astron. Soc. 77:155-184.

Sitter W de (1917). "Planetary motion and the motion of the Moon according to Einstein's theory". Proceedings of the Royal Netherlands Academy of Arts Sci. 19(1):367-381.

Stairs IH, Arzoumanian Z, Camilo F, Lyne AG, Nice DJ, Taylor JH, Thorsett SE, Wolszczan A (1998). "Measurement of relativistic orbital decay in the PSR B1534 + 12 binary system". Astrophys. J. 505:352-357.

Staveley-Smith L, Wilson WE, Bird TS, Disney MJ, Ekers RD, Freeman KC, Haynes RF, Sinclair MW, Vaile RA, Webster RL, Wright AE(1966). "The Parkes 21 cm multibeam receiver". Publ. Astron. Soc. Australia.13:243-248.

Stockton A (1980). "The lens galaxy of the twin QSO 0957 + 561". Astrono. J. 242:L141-142.

Taylor JH (1992). "Pulsar timing and relativistic gravity". Philosophical Trans. Royal Soc. 341:116-134.

Taylor JH, Weisberg JM (1989). "Further experimental tests of relativistic gravity using the binary pulsar 1913 + 16". Astrophys. J. 345(1):434-450.

Treschman KJ (2014a). "Early astronomical tests of general relativity: the gravitational deflection of light". Asian J. Phys. 23(1-2):145-170.

Treschman KJ (2014b). "Early astronomical tests of general relativity: the anomalous advance in the perihelion of Mercury and gravitational redshift". Asian J. Phys. 23(1-2):171-188.

Valença J de (2008). "Gravitational Waves" bachelor project online.

Vankov AA (1915). "Einstein's Paper: Explanation of the perihelion motion of Mercury from General Relativity Theory" online, translated from German to English by Rydin RA.

Vessot RFC, Levine MW, Mattison EM, Blomberg EL, Hoffman TE, Nystrom GU, Farrel BF, Decher R, Eby PB, Baugher CR, Watts JW, Teuber DL, Wills FD (1980). "Test of relativistic gravitation with a space-borne hydrogen maser". Phys. Rev. Lett. 49(26):2081-2085.

Walsh D, Carswell RF, Weymann RJ (1979). "0957 + 561 A, B – twin quasi-stellar objects or gravitational lens?". Nature 279:381-384.

Weisberg JM, Taylor JH (2005). "The relativistic binary pulsar B1913 + 16: thirty years of observations and analysis". Binary Radio Pulsars

ASP Conference Series. 328:25-31.

Will CM (2006). "The confrontation between general relativity and experiment". Living Rev. Relativity 9(3):1-100.

Will M (1995). "Stable clocks and general relativity" in Dark Matter in Cosmology, Clocks and Tests of Fundamental Laws, Proceedings of the 30th Recontres de Moriond, Moriond Workshop ed. Guiderdoni B et al, pp. 417-427.

Williams JG, Turyshev SG, Boggs DH (2004). "Progress in lunar laser ranging tests of relativistic gravity". Phys. Rev. Lett. 93(26):1101-1104.

Williams JG, Turyshev SG, Boggs DH (2009). "Lunar laser ranging tests of the equivalence principle with the Earth and Moon". Int. J. Modern Phys. 518:1129-1175.

Wolszczan A (1991). "A nearby 37.9-ms radio pulsar in a relativistic binary system". Nature 350(6320):688-690.

Zhao SS, Xie Y (2013). "Parametrized post-Newtonian secular transit timing variations for exoplanets". Res. Astronol. Astrophys. 13(10):1231-1239.

Zwicky F (1937). "On the masses of nebulae and of clusters of nebulae". Astrophys. J. 86(3):217-246.