Testing proposed planetary systems – to destruction

J Horner, RA Wittenmyer, J P Marshall, T C Hinse and P Robertson examine the dynamics of exoplanet systems and find that some of them don't last long. The stability of planetary systems offers a check on the likelihood of proposed multiple-exoplanet systems.

The detection of planets around other stars is, to many, the single most exciting astronomical discovery of our times. For millennia, people have asked whether we are alone in the universe, and while we do not yet know the answer to that question, the detection of these exoplanets surely brings that answer within our grasp. Indeed, we are the first generation to be able to gaze up at the night sky and know that planets orbit most of the stars we see.

Since 1992, when the first planets or planetesimals were found orbiting a pulsar, around fifteen hundred exoplanets have been discovered, using two principal techniques: periodic radial velocity variations in the motion of their host stars determined by the mass of the unseen orbiting planet; and the slight drop in light from the host star when a planet crosses its face. As time has passed, a handful of other techniques have begun to be used in the search for exoplanets. Aside from direct imaging, all of the techniques are indirect, detecting the influence of the unseen planet on its host star or on light from background stars. At the same time, an increasing number of multiple-planet systems have been discovered – a scenario that provides for a new test to be applied to examine the veracity of the claimed planets. When more than one planet is identified in a given exoplanetary system, it is possible to examine the mutual gravitational interaction between the planets. If they are real, and the proposed orbits a fair reflection of the true architecture of the system, it is reasonable to assume that the planetary orbits must be dynamically stable on timescales comparable to the age of the planetary system. By contrast, if the proposed planets in a given system become unstable on timescales far shorter than the age of the system, then it would be reasonable to conclude that those planets are unlikely to exist – at least on the proposed orbits.

Orbital dynamics and stability

When Sir Isaac Newton published *Philosophiæ Naturalis Principia Mathematica* in 1687, he laid out the mathematical tools that allow us to understand fully the motion of objects orbiting stars. In particular, they allowed the mathematical derivation of Kepler's Laws of Planetary Motion, which Kepler had proposed almost a century earlier. The science of orbital mechanics studies the gravitational interaction between planets, stars and smaller bodies in planetary systems (comets and asteroids). Once a system contains more than two bodies, it is no longer possible to exactly solve for the motion of all those bodies as they interact under the influence of gravity. This conundrum is known as the three-body problem, and essentially means that it is impossible to predict, on long timescales, the motion of the planets around the Sun, or the dynamical evolution of exoplanetary systems.

A simple example of the problems inherent in studies of orbital mechanics is to consider a simplified version of our solar system: the Sun, the planet Jupiter, and a small object such as a comet. Consider two versions of the system, absolutely identical except for the initial location of the small object. In one system, the small object is displaced from its location in the other by a single atomic radius, with all other quantities (the velocities, masses and locations of the objects considered) remaining identical. At that instant, the two particles in the two planetary systems will experience forces acting on them that differ very slightly, as a result of their minutely different distances from the Sun and from Jupiter. As a result, they will experience slightly different accelerations, causing their orbits to gradually drift apart. The further apart the objects move, the more disparate the forces they experience, and the more dramatically their orbits will diverge.

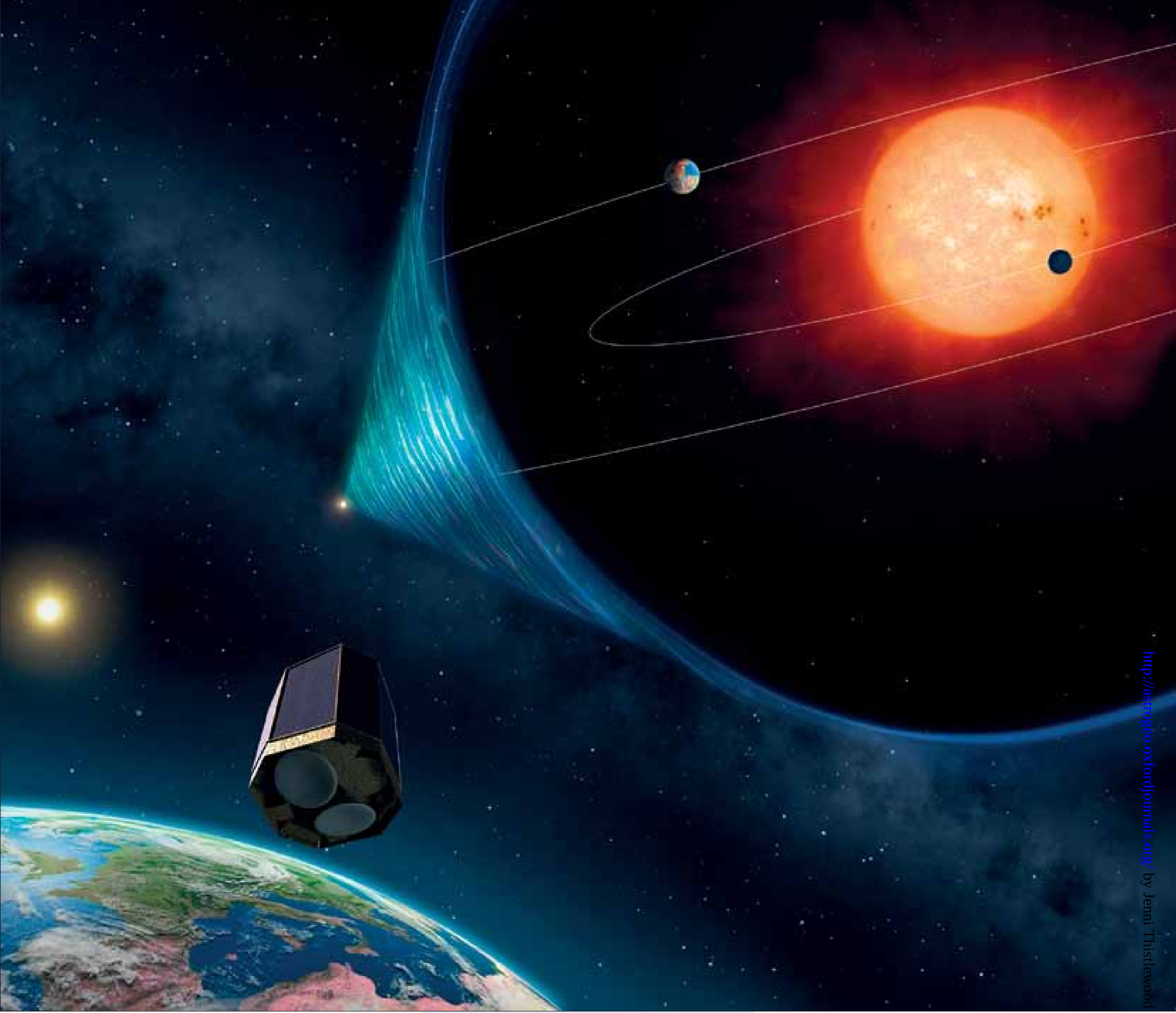
Because the initial location, mass and velocity of every single object in a given planetary system cannot be precisely known, it is impossible to determine how those objects will evolve under the influence of their mutual gravitational pulls on long timescales. But all is not lost. The gravitational evolution of planetary systems (or test particles therein) can be modelled computationally, allowing a wide range of initial conditions to be tested. The more tightly packed the initial conditions, the longer the different solutions will take to diverge and so the better we can tie down the initial motion of the objects in question, and the better we can understand their future (and past) behaviour.

In a coarse sense, two broad types of behaviour are observed for objects moving in systems of three or more bodies. Stable orbits show very little modification on long timescales, such that the objects moving on those orbits would be expected to be moving on the same (or very similar) orbits were the observer to return in a billion years, or ten billion years. Unstable orbits, by contrast, exhibit dramatic variations as time goes by.

In reality, on long enough timescales, almost all orbits become unstable; stability is actually a sliding scale, from the very unstable to the very stable. An example of a very stable orbit would be that of the Earth around the Sun. Although the orbit of the Earth is constantly nudged and tweaked by the gravitational influence of all of the other objects in the solar system, it has remained essentially unchanged since the formation of the solar system, and it is highly likely to remain so until the Sun becomes a red giant star. At the other extreme, the solar system's cometary bodies provide numerous examples of highly unstable orbits, continually being thrown on to new orbits as a result of chaotic interactions with the planets. Indeed, most comets are eventually ejected from the solar system entirely as a result of a close encounter with the giant planet Jupiter – our system's biggest, baddest bully.

Typically in these systems, objects whose orbits are widely separated exhibit significantly greater dynamical stability than those whose orbits are closely packed. Furthermore, objects moving on orbits with low eccentricity (i.e. circular, or near-circular orbits) are typically more dynamically stable than those moving on eccentric orbits. The most extreme instability typically occurs when the orbits of two objects cross one another, or approach one another particularly closely. In those cases, there is the possibility that these objects will eventually experience a close encounter, leading to significant changes in their orbits.

The situation is complicated by orbital resonances, such as the interaction between the giant planet Neptune and the dwarf planet Pluto. The orbit of Pluto crosses that of



1: Artist's impression of the PLATO mission to explore exoplanets. Announced in February by the European Space Agency and due to launch in 2024, PLATO will observe around a million stars to find and characterize new planets. (Mark A Garlick)

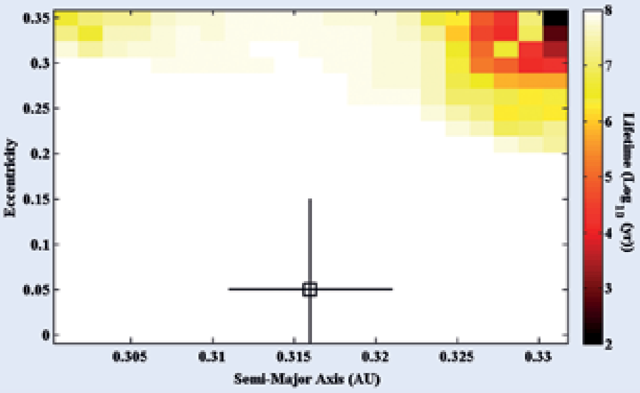
Neptune, such that the dwarf planet spends approximately 20 years of each ~248-year orbit interior to the orbit of Neptune, and the rest of the time exterior. Normally, one would expect such a situation to be untenable in the long term – eventually, the two objects would be expected to experience a close encounter and disruption of their orbits, or even a collision. However, Pluto is actually protected from experiencing close encounters with Neptune because they are trapped in mutual 3:2 mean motion resonance. In the time it takes Neptune to complete three orbits of the Sun, Pluto completes two – a commensurability between their orbital periods that ensures that, whenever Pluto is crossing the orbit of Neptune, that giant planet is far away. Despite the fact their orbits cross, the two objects never approach one another more closely than the distance between the Sun and Uranus – approximately 2 billion km.

Most of the multiple-planet systems that have been proposed in recent years orbit stars that are hundreds of millions, if not billions, of years old. As such, given that the planets orbiting them can reasonably be expected to have formed at around the same time as their host star, it is reasonable to expect that their orbits would be stable on timescales comparable to the age of their host system. Conversely, the likelihood of discovering planets as they move towards destruction on dynamically unstable orbits during the last few thousand years of their life is very low. The dynamical tools perfected for study of our solar system can provide a sanity check for newly announced exoplanetary systems, to determine whether those planets, as proposed, are dynamically feasible.

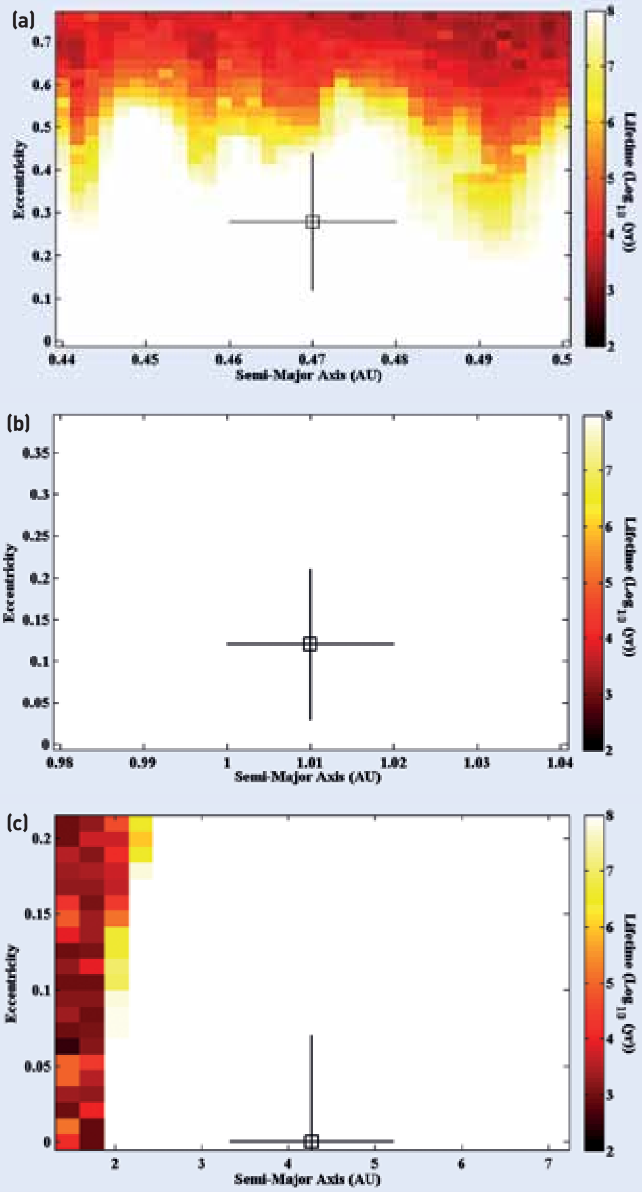
With this in mind, we have now incorporated dynamical testing as part of the exoplanet discovery process of the Anglo-Australian Planet Search programme, and have also tested a number of the multiple-planet systems proposed over the past few years to see whether they stand up to dynamical scrutiny. For some systems, dynamical simulations simply reveal that the proposed planets are dynamically stable across the whole range of orbital architectures allowed by the observational data. More interestingly, in several cases, we have found that dynamical studies can enable us to more effectively constrain the orbits of the proposed planets. Finally, in several cases, we have found that the proposed planetary systems are not dynamically feasible, indicating either that the proposed planets do not exist, or that they are moving on orbits vastly different to those proposed in the discovery work.

Simulating multiple planets

In order to test the dynamical evolution of planetary systems on timescales comparable to the lifetime of their host stars, we use the Hybrid



2: The stability of the recently proposed planet candidate HD52265c, as a function of its initial orbital semi-major axis (*a*) and eccentricity (*e*). This figure shows the results of 11025 individual simulations, each testing a different initial orbital solution for HD52265c. The lifetime shown at each location in this *a*–*e* plot is the mean lifetime of 25 unique trials that were performed for initial orbits featuring that particular combination of semi-major axis and eccentricity. The hollow box shows the location of the best-fit solution for the orbit of HD52265c, while the solid lines that radiate from that box denote the ±1σ uncertainties on the semi-major axis and eccentricity of the planet's orbit. In this case, almost the entire allowed orbital element space for HD52265c was dynamically stable – adding weight to the hypothesis that the observed variations in the radial velocity of HD52265 are the result of perturbations by at least two massive companions. The only regions of instability lie far from the nominal best-fit orbit (at the top right and top left of the plot).



3: The stability of three candidate multiple-planet systems resulting from the Anglo-Australian Planet Search programme.
(a): The dynamical stability of the planet candidate HD142d, with 30625 scenarios tested.
(b): HD159868c; 30625 scenarios tested.
(c): Planet candidate HD85390c; 11025 scenarios tested. In each case, the full ±1σ uncertainty range in semi-major axis and eccentricity solely contains orbits that are dynamically stable, with any regions of instability located far from the best-fit solution denoted by the hollow box.

integrator within the *n*-body dynamics package Mercury (Chambers 1999), and follow a procedure we first used to study the dynamics of the directly imaged four-planet system HR8799, back in 2010. We always follow the evolution of a given planetary system for a period of 100Myr within the software – this compromise allows us to run a large number of simulations on a reasonable timescale, while still spanning a sufficiently long time period to allow us to assess the stability of the proposed system.

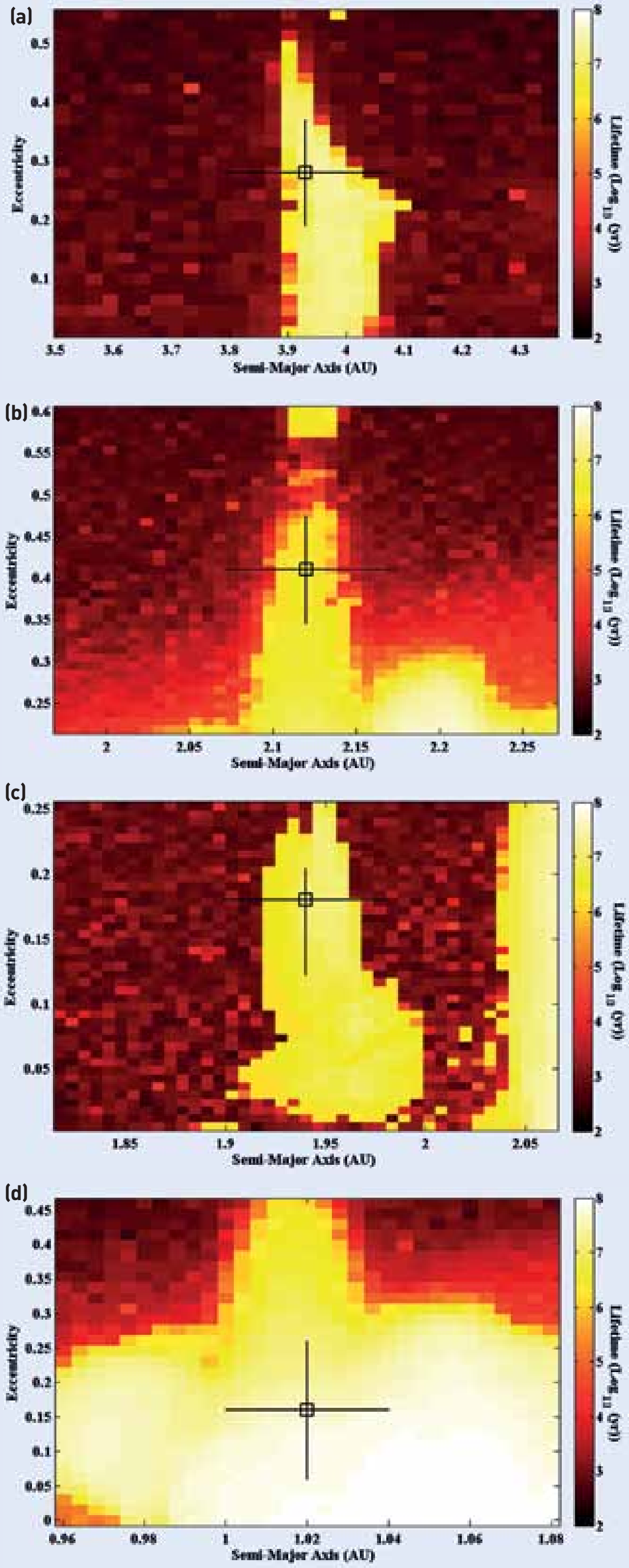
When a planet is announced around a given star, a best-fit orbital solution is given in the discovery work. For multiple-planet systems, the orbital parameters of one planet are typically better constrained than those of the others (usually, but not always, the better-constrained planet is the one with the shortest orbital period, such that the observational data span the greatest number of complete orbital periods for that planet). In our simulations, we place the better-constrained planet on its best-fit orbit, and then vary the orbit of the other planet(s) to sample the full ±3σ phase space about the best solution for that planet. This leads to the creation of a large number of unique planetary systems (in our earliest work, we studied samples of ~10000 test systems, while nowadays we typically consider ~100000 systems at a time). For each of those test systems, we follow the evolution of the planets until they either collide with one another, are thrown into the central star, or are ejected from the planetary system entirely – all common results of dynamical instability. Every time a system falls apart in this way, the time at which the ejection or collision occurred is recorded, which allows us to create dynamical maps of the system in question, showing those orbits that are most stable or unstable. An example of such a map is shown in figure 2.

Dynamically stable systems – supporting discoveries

In many cases, the second planet discovered in a given exoplanetary system moves on an orbit that is well separated from that of the first planet known in that system. As such, the gravitational interaction between the two planets will typically be weak, and so it is to be expected that most such systems will be dynamically stable on astronomically long timescales. Such has been the case for several of the systems we have studied, including the planet candidates proposed around HD 52265 (figure 2), and the three systems shown in figure 3. In each of these cases, the whole area within the ±1σ uncertainties around the best-fit solution for the planet in question is highly dynamically stable, with all solutions tested in that region being stable for the full 100million years of integration time. Any instability that might be present in these systems is found well away from the nominal best-fit orbits – either to higher orbital eccentricities (as is the case for HD 142d, figure 3a) or to smaller separations between the orbits of the two planets (as for HD 85390c, figure 3c). In such cases, while the dynamics does not tell us a huge amount of additional information on the system in question, it does reveal that planets moving on the proposed orbits would be dynamically feasible, an outcome that is clearly necessary for the presence of the planets to be taken seriously.

Dynamically interesting systems – constraining exoplanets

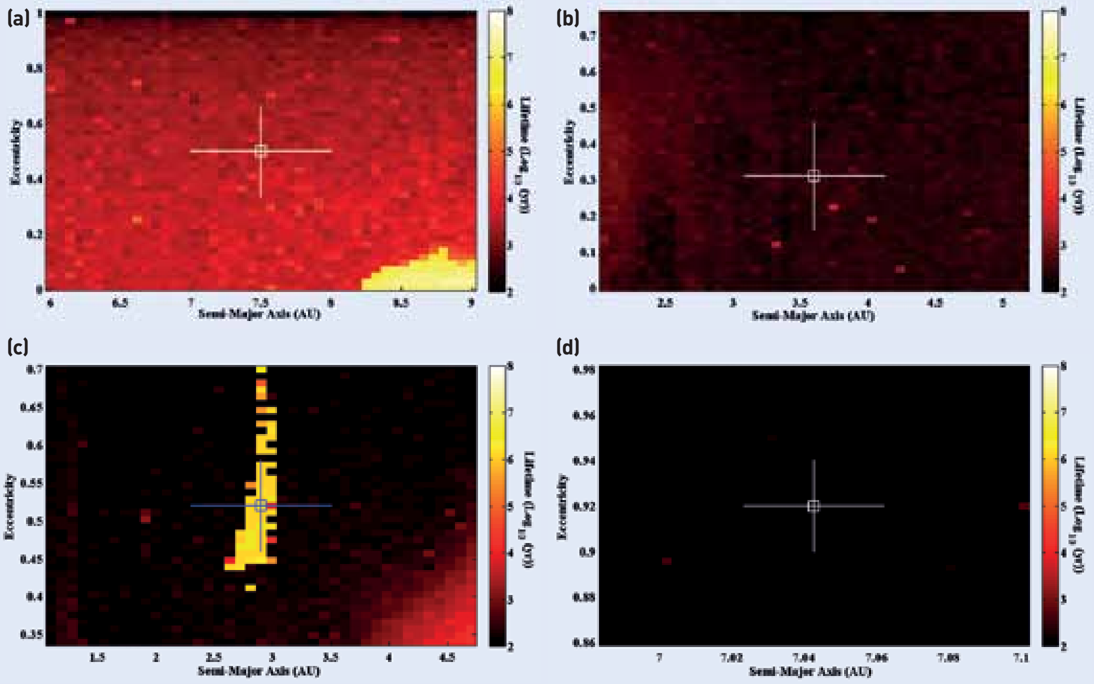
A more interesting situation occurs when the candidate planets move on orbits that are sufficiently tightly packed that the orbital period of the outermost is less than twice that of the innermost. With such tightly packed systems, significant dynamical interaction between the two planets is likely, unless their masses are very small (as is the case for the terrestrial planets in our solar system). The precise details of the orbit proposed for the second planet in such systems are clearly therefore critical in determining whether the planets are stable or unstable on long timescales. In such cases, a detailed study of the dynamics of the system can actually help to provide significant additional constraints on the orbits of the planets contained therein, over and above those that can be placed solely on the basis of the observational data. Four examples of such systems are shown in figure 4.



4: The stability of four two-planet systems for which the dynamical simulations yield significant additional constraints on the orbital parameters of the newly discovered planet.
(a): HD 204313c (Robertson *et al.* 2012a); 52 855 simulations. The newly discovered planet is only dynamically stable if it is trapped in 3:2 MMR with HD 204313b.
(b): 24 Sextantis c (discovered by Johnson *et al.* 2011); 126 075 simulations. Here, the newly discovered planet is trapped in 2:1 MMR with 24 Sex b.
(c): HD 200964c (also discovered by Johnson *et al.* 2011); 126 075 simulations. Here, the planets are only stable if they are trapped in the 4:3 MMR.
(d): HD 155358c (Robertson *et al.* 2012b); as with 24 Sex c, the best-fit orbit here lies in 2:1 MMR with HD 155358b.

In the four systems shown in figure 4, the newly discovered planets move on orbits whose uncertainties span regions of both significant dynamical stability and strong instability. In each case, however, the best-fit solution falls in the region of maximal dynamical stability, a result that once again strengthens the case for the observed radial velocity variations being the result of massive unseen companions (i.e. planets). In each case, the best stability for the system is found when the planets involved are trapped in mutual mean motion resonance. In the case of HD 204313 (figure 4a), the planetary system is only dynamically stable when the outermost planet completes two orbits in the time it takes the innermost to complete three – therefore, we say the planets are trapped in mutual 3:2 mean motion resonance (MMR). The sculpting of the stable region reveals the range of orbital solutions for HD 204313c for which the 3:2 MMR acts to stabilize the system. Beyond that region, the orbits are unstable on timescales of hundreds or thousands of years. Beyond simply showing that the planets are dynamically feasible, our simulations here add a significant extra constraint on the orbit of the newly discovered planet. Simply, it must be trapped in the 3:2 MMR.

The newly discovered planets around 24 Sextantis (figure 4b) and HD 155358 (figure 4d) are both located in the vicinity of the 2:1 MMR with the first planets discovered in those systems. In the case of 24 Sex b, the only stable solutions within ±1σ of the best fit orbit are those that are resonant – again, this allows us to more tightly constrain the orbit of that planet than is possible on the basis of observations alone. In the case of HD 155358, a large number of orbital solutions outside the 2:1 MMR offer the possibility of stability. While this means we cannot definitively state that the planets in that system must be trapped in their mutual 2:1 MMR, it seems by far the most likely solution – and we can rule out non-resonant solutions that place the newly discovered planet on more eccentric orbits. Finally, in the case of HD 200964c (figure 4c), the only stable solutions are those in which the planets are trapped in 4:3 MMR. Once again, studying the dynamics of the newly discovered system has allowed us to better constrain the orbits of the planets therein.



5: The stability of the four recently proposed two-planet systems for which the dynamical simulation show that the planets are simply not dynamically feasible. (a): HU Aquarii; 50 625 simulations. (b): HW Virginis; 91 125 simulations. (c): NSVS 14256825; 126 075 simulations. (d): QS Virginis; 126 075 simulations. In each case, the candidate planets lie on orbits that cross one another – a sure-fire recipe for instability without the protective influence of resonant motion. In the case of (c), there is a narrow strip of moderate stability that is the result of the 2:1 MMR between the two planets. Even then, however, at the high eccentricities proposed in the discovery work, the resonant orbits still decay on timescales of order a million years.

Not every wobble needs a planet

In recent years, a number of planetary systems have been proposed to orbit highly evolved, interacting binary-star systems – post-common-envelope binaries. The candidate planets, which are inferred on the basis of variations in the timing of eclipses between the two components of the binary-star system, are often found to move on highly eccentric orbits. For those systems in which two planets are proposed, the orbits are so extreme that they cross one another in much the same way that the orbits of comets and near-Earth asteroids cross the orbits of the planets in our solar system. In our solar system, it is well known that such orbits are highly dynamically unstable, unless the objects moving on them are protected from mutual close encounters by the protective influence of a mean motion resonance – an observation that immediately suggests that the orbital evolution of these extreme planets should be examined in some detail.

The dynamical stability of four of the proposed multiple-planet systems orbiting evolved binary stars are shown in figure 5. In each case, the systems prove incredibly unstable – often featuring collisions between the proposed planets on timescales of months or years! The systems are clearly dynamically unfeasible. Even the case of NSVS 14256825 (figure 7c), which features a narrow strip of moderate stability that coincides with the 2:1 MMR between the two proposed planets, is sufficiently unstable that it seems unlikely that the planets exist. Even those relatively stable resonant solutions fall apart on timescales far shorter than the age of the host system. Dynamical investigation of these systems has therefore revealed that the proposed planets do not exist. If the observed variations in the timings of eclipses between the stars in these binaries are the result of perturbations from massive unseen companions, then those companions must move on orbits dramatically different to those proposed in the discovery works. In light of the dynamical evidence, however, it seems much more likely that the observed variations must have some additional cause. Indeed, recent work that has built on our dynamical results has suggested that there might be a systematic under-estimation of the uncertainty with which the eclipses in these systems can be timed. Simply increasing the uncertainty on the measurements of the eclipse timings for the NSVS 14256825 system to ±5 s is enough to make all evidence for orbiting bodies disappear. Given that the eclipses in these systems are far from simple processes (a distorted secondary with significant temperature variations across its surface, orbiting a white dwarf star with a period of just a few hours; significant mass loss and accretion occurring between the two stars), such uncertainty is perfectly reasonable, and might well prove to be the simplest explanation for the observed eclipse timing variations.

Conclusions

As the search for planets around other stars advances, an ever-increasing number of multiple planet systems are being discovered. But the great majority of planets discovered are found indirectly, and the orbits proposed often feature relatively large uncertainties. We have therefore undertaken a programme of dynamical investigations in order to determine whether the candidate planetary systems are dynamically feasible. Our results fall into three broad categories.

In the simplest cases, the planets are sufficiently well separated that they do not undergo strong mutual interactions, and the systems are dynamically stable across the full range of potential orbital architectures.

More interesting are the systems in which the planets are more tightly packed. In these systems, it is often the case that the great majority of permitted orbital solutions are dynamically unstable on very short timescales. In those systems, narrow islands of stability are often found, resulting from the stabilizing influence of mean motion resonances between the proposed planets. In the case of such resonant systems, dynamical studies allow the orbits of the proposed planets to be constrained far more tightly than is possible on the basis of the observations alone – and can be used to direct future observations that will help to confirm the existence of the planets in question. They also allow us to examine planetary systems featuring gravitational interactions that are very different to those that are occurring between the planets in our own solar system. These observations can not only help us to better understand these newly discovered exoplanetary systems, but will also yield exciting insights into the early evolution of our youthful planetary system, where it has been proposed that the giant planets formed in dramatically different orbits to those they occupy at the current epoch.

Finally, a small number of systems are found to fall down under dynamical scrutiny – particularly those proposed orbiting highly evolved binary-star systems. In these cases, the results of our dynamical simulations are such that the observational data must be considered in a

Exoplanets that never were

Before the confirmed discovery of planets orbiting other stars, several other exoplanet findings had been claimed, but close scrutiny revealed that they were not real objects. Two examples are particularly illustrative – the planets proposed to orbit one of our nearest neighbours, Barnard's star, and the planet claimed orbiting the pulsar PSR B1829-10.

Barnard's star is the fourth closest star to the Sun, after the three stars that make up the Alpha Centauri system (Alpha Centauri A + B, and Proxima Centauri). Less than 6 light years away, it is a tiny red dwarf star far too faint to see with the unaided eye. It is noteworthy because its proper motion isgreater than that of any other star. That is the result of both the star's proximity and its high speed relative to the Sun – of order 140 km/s. Barnard's star has been a popular target for observers over the century since E E Barnard first measured its proper motion in 1916.

The Dutch astronomer Piet van de Kamp observed the motion of many nearby stars, including Barnard's star, during his time as director of Sproul Observatory (from 1938 to 1972). As he gathered data, van de Kamp claimed to have detected Barnard's star wobbling back and forth as it tracked across the sky – evidence, he claimed, of the presence of at least two Jupiter-mass planets in orbit. As the planets and the low-mass star orbited about their common centre of mass, the star wobbled back and forth around the centre of mass of the system as it moved across the sky.

Such observations were not unprecedented – the same technique had been used a century earlier to reveal the present of Sirius B, the white dwarf companion to the brightest star in the night sky. However, van de Kamp's observations were the first time such variations had been used to propose the existence of planetary, rather than stellar, companions. Unfortunately for him, observations from other observatories failed to find any evidence for his proposed wobbles and further investigation revealed a different cause: the objective lens of the refracting telescope had been removed and cleaned, then replaced, several times during the period of his observations. More recently, observations using the Hubble Space Telescope have put the final nail in the coffin of van de Kamp's proposal, definitively ruling out such massive planets orbiting the tiny star. His technique may well see a renaissance in coming years, however: the GAIA space observatory, launched on 19 December 2013, will make incredibly precise measurements of the proper motions of approximately 1 billion stars. GAIA is expected to discover a plethora of planets orbiting nearby stars. Van de Kamp would be proud!

The announcement that a planet had been detected orbiting the pulsar PSR B1829-10 was made in the journal *Nature* in 1991. The authors proposed that observed variations in the timing of pulses arriving from the pulsar were the result of a planet-mass companion to the rotating neutron star. The proposed planet would have had an orbital period of half a year, and a mass ten times that of the Earth. In the discovery work, the authors mentioned that an alternative explanation for the observed signal would be that it was an artefact resulting from the orbit of the Earth around the Sun, but concluded that, since no such artefact could be found in the observational data they had for 300 other pulsars, the most likely conclusion was that the variations were the result of the proposed planet. Upon further analysis, however, the authors found that the variation could be explained as being the result of the Earth's elliptical orbit around the Sun. In their initial analysis, the authors had assumed the Earth's orbit to be circular when correcting for its motion around our star – and once the small but non-zero eccentricity of the Earth's orbit was taken into account, all evidence for a planet around PSR B1829-10 disappeared. The retraction of the discovery of the planet appeared in *Nature*, one week after the announcement of the discovery of planets orbiting another pulsar, PSR B1257+12, which were then acknowledged as the first confirmed planets orbiting another star.

whole new light. Most likely, the planets proposed in those systems simply do not exist, and the claimed variability in the timing of eclipses between the binary stars is simply the result of the underestimation of the uncertainty in the eclipse timings themselves.

Most importantly, however, our results highlight the importance of dynamical studies as a key component of the search for planets around other stars. As a result of our work, the Anglo-Australian Planet Search now routinely carries out detailed dynamical mapping of all new multiple-planet systems discovered as a central part of the discovery process. While such simulations are time consuming (a typical system will require more than 100 000 hours of run time on a supercomputing cluster to create a dynamical map like those presented in this work), the benefits of such work far outweigh the extra time required to perform such simulations prior to the announcement of a new planetary system.

In future years, as the search for exoplanets continues to push towards the discovery of truly Earth-like planets, such dynamical studies will only grow in importance. Not only will they allow us to confirm that any exo-Earths discovered are dynamically stable on timescales long enough that life could develop on their surfaces, they will also allow us to draw conclusions on the variability of the climates of those worlds (by examining the influence of distant perturbers driving small-scale variations in their orbital parameters), as well as enabling us to investigate the impact regimes that they might experience (by studying the influence of the planets in the system on the small object reservoirs therein). The future of exoplanet dynamics is definitely an exciting one! ●

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Acknowledgments. The work was supported by iVEC through the use of advanced computing resources located at the Murdoch University in Western Australia, and has also made use of the University of New South Wales's supercomputing cluster, Katana. This research made use of NASA's Astrophysics Data System (ADS).

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