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OPTIMAL MANAGEMENT STRATEGIES FOR A CASCADE RESERVOIR SYSTEM

Clayton Forknall
B. Sc. with Distinction

Faculty of Health, Engineering and Sciences,
The University of Southern Queensland

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CERTIFICATION OF DISSERTATION

I certify that the ideas, experimental work, results, analyses, software and conclusions reported in this dissertation are entirely my own effort, except where otherwise acknowledged. I also certify that the work is original and has not been previously submitted for any other award, except where otherwise acknowledged.

Clayton Forknall Date

Endorsement

Signature of Supervisors Date

Abstract

Water reservoirs have long been used throughout Australia and the globe as a means of both providing communities with water security during periods of limited rainfall, as well as a form of defence against severe flooding. In recent times, the effective management of these water reservoirs has been questioned and is now, more than ever, under scrutiny.

In order to address the issue of reservoir mismanagement, this thesis demonstrates the methods and procedures undertaken in the development, formulation and application of two Mixed Integer Linear Programming (MILP) models that have the ability to determine strategies for the optimal management of a cascade reservoir system, under the two extreme environmental conditions of drought and flood. For the purposes of this thesis, the unique cascade configuration of a reservoir system was primarily considered; where cascade refers to a multiple reservoir system in which the spill from earlier reservoirs becomes a source of inflows to subsequent reservoirs. Many physical reservoir systems exhibit this type of layout including the Perseverance and Cressbrook system located near Toowoomba, which has been considered as a case study throughout this thesis.

By applying the drought and flood models to the case study of the Perseverance and Cressbrook cascade reservoir system, it was found that both models provided comprehensive approximations of the system behaviours under the differing extreme conditions considered by each model. However, in order to conduct a successful comparison of the management strategies employed by the drought and flood models, a common set of inflow records upon which both models could be considered was required. Rather than using a portion of the historic inflow records sourced for the case study considered, time series analysis was employed instead to select a time series model that suitably represented the historic records, and then from this model, an alternate set of inflows was simulated.

Using the simulated set of inflows, a comparison of the management strategies employed by the two MILP models for a drought and flood was conducted; demonstrating both similarities and differences between the optimal strategies employed for the management of the cascade reservoir system. The comparison also revealed that although "common sense" practices could be employed to operate the cascade reservoir system, these practices were not optimal and thus did not result in the

effective management of the system. Therefore, models like those developed, formulated, and utilised in this thesis are necessary to ensure that a commodity as heavily relied upon and sometimes as potentially dangerous as water is optimally governed and regulated into the future.

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Chapter 1

Introduction and Literature Review

1.1 Introduction

The management and operation of a system responsible for the storage of a valuable commodity will always be a topic open for investigation and potential improvement, especially when the commodity is valuable enough to be referred to as"liquid gold"; water. Water reservoirs have long been employed around Australia and the globe to serve two main purposes; ensure communities have access to a reliable water supply during periods of limited rainfall or drought and to help protect communities from the impact of flood. This means that it is not only the operators of the reservoir systems that are interested in their optimal management during these extreme events, but also the communities that rely upon the systems for one purpose or another.

Therefore, this thesis aims to formulate and develop two mathematical models that can be employed to determine strategies for the optimal management of a cascade reservoir system under the two opposing extreme environmental conditions of a drought and flood. In this case, cascade refers to the unique layout of the reservoirs in the system, where releases from the spillway (termed spill) of a previous reservoir in the system become a source of inflows to subsequent reservoirs; thus influencing the volume of water, or storage level, of those subsequent reservoirs. Figure 1.1, provided at the end of this chapter, gives an example of the layout exhibited by a physical cascade reservoir system and how spill from one reservoir influences the storage level of the next.

The crucial nature of water and the consequences arising from any mismanagement of reservoir systems has driven substantial research into the field of reservoir operation. One of the widely established and well defined research areas in the field is the use of mathematics, in particular Operations Research and related techniques, to develop optimal management strategies for the operation of water reservoir systems. Many different approaches exist to model the operation of a reservoir system, each with their own merits. This is demonstrated by the state-of-the-art review conducted by Yeh (1985) that explored the vast number of techniques that can be utilised to describe the operation of a reservoir system, including Linear Programming models, Dynamic Programming models, Simulation models and Nonlinear

Programming models. Although comprehensive, one shortcoming of this review by Yeh (1985) was that it only considered models for the day to day management of a reservoir system and did not investigate models capable of describing the operation of a reservoir system during extreme environmental conditions, such as a drought or flood. Sometime later, a second state-of-the-art review was released by Labadie (2004) investigating the optimal operation of multi-reservoir systems. In the work, Labadie (2004) praises Linear Programming (LP) models as being one of the most favoured optimisation techniques due to their efficiency and ability to converge to a global optimal solution. However, he goes on to discount Mixed Integer Linear Programming (MILP) models, an extension of Linear Programming and the predominant modelling technique employed in this thesis, as being less computationally efficient than pure LP models (Labadie, 2004). Although perhaps less computationally efficient, Mixed Integer Linear Programming provides an alternate method for representing constraints that would otherwise be nonlinear and, if included, severely increase the complexity of the model; a concession also made by Labadie (2004). Upon comparison to the review performed by Yeh (1985), the more recent work by Labadie (2004) investigated the use of computer based techniques such as heuristic methods alongside simulation and neural network models; an indication of the advances made in computer technology since the initial review performed in 1985.

In a research area as broad as that of environmental management, under which the operation of a reservoir system falls, there are always situations and circumstances that are yet to be explored. Some of these situations were outlined by ReVelle (2000), who provided an insight into the research challenges in environmental management and defined some problems that, at the time, remained unsolved. An example of some of the topics explored by ReVelle (2000) included techniques for the management of parallel reservoirs, water quality, and solid wastes. In the investigation of management techniques for parallel reservoirs, a basic LP model was provided as an example. This model is slightly more complex than the simple model formulated by ReVelle and McGarity (1997) that formed a foundation for research and experimentation in this thesis (explored thoroughly in Chapter 2). ReVelle (2000) also extended his original model to incorporate rationing of water from reservoirs situated in parallel, which contribute towards a common water supply. The parallel reservoir system configuration explored by ReVelle (2000) is different to that investigated in this thesis, where the reservoirs are situated in series, otherwise known as a cascade configuration. As mentioned previously, when positioned in a cascade configuration, spill from previous reservoirs in the system becomes a source of inflows to subsequent reservoirs; as demonstrated by Figure 1.1 at the end of this chapter. Chen *et al.* (2013) also considered a cascade reservoir system, though investigated the system during a flooding event. However, the methods employed by Chen *et al.* (2013) to manage the reservoir system were significantly different to those methods explored in this thesis, as they introduced the idea of a Flood Limiting Water Level as a parameter for assessing the tradeoffs between flood control and conservation (Chen *et al.*, 2013). Although the management strategies employed by Chen *et al.* (2013) are fundamentally different to those proposed in this thesis for the operation of a reservoir system in the event of a flood, the aim of both works are the same; to optimally manage a cascade reservoir system during a flood event.

1.2 Simple Model

Every thesis has a point of origin. In this case, that starting point was a simple LP model formulated by ReVelle and McGarity (1997) to describe the operation of a reservoir. The objective of the proposed model was to determine, given a known set of inflows, the minimum capacity of a reservoir required in order to ensure that a constant water demand could be confidently supplied to a community each month. This model was quite limited in its physical application; however it did provide a foundation for further research and experimentation, along with an appreciation of how such models behave and function. One suggestion that ReVelle and McGarity (1997) offered when discussing multi-reservoir systems was that, if the reservoirs are arranged in a cascade configuration, then to avoid over complicating the problem the system should be treated as a single large reservoir. The work conducted as part of this thesis suggests otherwise, and that such an assumption would be a vast over simplification, resulting in many crucial and subtle details being omitted or overlooked. For example, if the system is assumed to be one large reservoir, the model would lack the ability to monitor the amount of spill from one reservoir to the next; an important detail that influences the selection of an optimal of management strategy for the system.

1.3 Drought Model

Droughts are an environmental phenomenon that occurs worldwide, where regions experience below average rainfall for an extended period of time. These events place increased stress on the water resources of the regions affected, especially when a country like Australia is considered. Renowned as one of the driest continents and continuously battling water shortage issues, Australia faces devastating consequences from drought. To make matters worse, under predicted climate change the duration and severity of these drought events are expected to increase (Easterling *et al.*, 2000), making it of the utmost importance that reliable and accurate strategies exist to better manage water resources going into the future.

Due to the importance of ensuring that communities have access to a reliable water supply, even during the height of a drought, significant research has been conducted in order to find strategies that can be employed to better manage a reservoir system during a drought event. From the plethora of mathematical techniques presented by the research, one method suggested by the authors Shih and ReVelle (1995) formed the foundation of the work performed in this thesis. In their work, Shih and ReVelle (1995) used a discrete hedging rule to formulate a MILP model to describe the operation of a single reservoir during a drought event. This method followed on from previous work conducted by the same authors, in which they investigated the use of a continuous linear hedging rule for a single water reservoir (Shih and ReVelle, 1994). After obtaining the optimal rule, the authors converted the continuous hedging rule into multiple discrete hedging rules, a form more suitable for practical applications, which then featured in their 1995 work. The work by Shih and ReVelle (1995) is the only work found of this kind that formulates a MILP model to explore the impact of a drought event on a reservoir used solely for water supply. Many other works focused on alternate aspects of the reservoir system, such as maintaining hydroelectric output during drought (ReVelle, 2000), which are not applicable for the scenario investigated as part of this thesis.

An example of such a work is provided by the authors Tu *et al.* (2003) who developed a MILP model that considers both rule curves and hedging rules to optimize the operation of a multipurpose, multi-reservoir system. This reservoir system was not considered in isolation by Tu *et al.* (2003), but instead was assumed to be connected to a large scale water distribution network, making the system far more complex than that explored in this thesis. Although there are significant differences in the size and complexity of the system considered, along with some of the methods used by Tu *et al.* (2003), there are also similarities to the methods used in this thesis. For example, Tu *et al.* (2003) differ from this thesis by employing network diagrams and initially formulating the problem using the minimum cost method; however Tu *et al.* (2003) did develop a MILP model to optimise the operation of the reservoir system, a similar method to that employed in this thesis. The work by Tu *et al.* (2003) could also become applicable if further research was to be conducted into the reservoir system considered in this thesis, this time incorporating the entire water distribution network of the region into the model.

In their more recent work, the authors Tu *et al.* (2008) devised a multi-objective Mixed Integer Nonlinear Programming model for a multi-reservoir system that optimises water allocation and searches for new hedging rules to improve the current reservoir operation procedures. By incorporating a nonlinear objective function and nonlinear constraints into the model, Tu *et al.* (2008) have increased the complexity of their original work substantially; however at the same time increased the capabilities of the model. In the case of this thesis, the use of nonlinear constraints were avoided as the physical layout and operating procedures of the system considered were not as complex as that investigated by Tu *et al.* (2008). However, if more complex operating procedures were adopted by the managers of the reservoir system investigated as part of this thesis, the method implemented by Tu *et al.* (2008) may provide a better approximation for the behaviours exhibited by components of the system.

As mentioned previously, many different methods exist to model the operation of a water reservoir system during a drought event. One such method is described by Shiau (2003) who suggests the use of hedging rules in combination with a measure, known as the reservoir supply index, to minimise the effects of drought on water reservoir systems. This reservoir supply index was developed to determine the onset and termination of water rationing and is defined as the probability of an available reservoir water supply being sufficient to meet an established demand (Shiau, 2003). Effectively, the reservoir supply index is a probabilistic measure that evaluates the probability of a reservoir having sufficient water supplies to meet some predefined demand. In comparison to the work performed in this thesis, where the extent of water rationing is determined by the level of the reservoir itself, Shiau (2003) suggests the use of probabilities to predict the level of the reservoir into the future and thus the amount of rationing required. Although the method proposed by Shiau (2003) may provide some lead time as to when it is best to enforce rationing, the probabilities could be misleading and either see rationing imposed when it is not needed or in contrast, see no rationing imposed when it is needed. By using the level of the reservoir itself to determine when rationing is enforced, and under the assumption that the demand from the reservoir, along with the inflows to the reservoir are known, the occurrence of these types of errors have been minimised in this thesis.

Another method for modelling a water reservoir system during periods of water stress was suggested by the authors Srivastava and Awchi (2009). In their work, the authors used system analysis to develop a strategy that improved the performance of a reservoir system when the scenario of overstressed water utilisation conditions and demands was considered (Srivastava and Awchi, 2009). The technique of system analysis incorporates a series of optimisation tools including Linear Programming, Dynamic Programming, Simulation, Artificial Neural Networks and hedging rules to determine an optimal management strategy for a reservoir system. In this case, the hedging rules that appeared in the system analysis conducted by Srivastava and Awchi (2009) were based upon the series of discrete hedging rules devised by Shih and ReVelle (1995). Although the hedging rules are similar to those used in this thesis, the technique of system analysis was exceedingly complicated to warrant its application to the cascade reservoir system considered. That being said, the process of utilising the original methods and techniques proposed by Shih and ReVelle (1995) and adapting their work to function under a different set of circumstances was replicated in this thesis when developing both the drought and flood MILP models.

1.4 Flood Model

Floods are also an environmental phenomenon that occurs worldwide, where periods of above average or extreme rainfall results in rivers and streams rising to a point where the flows can no longer be contained and the excess water inundates the surrounding regions. These events can prove deadly and often cause massive amounts of damage to the regions affected; impacting natural ecosystems and communities alike. Water reservoirs can be used as a form of defence against the worst impacts of a flood, as they provide a means of stalling the progression of floodwaters. Some reservoirs are designed with flood mitigation features, such as a gated spillway, which enables the progress of a flood to be halted and the floodwaters stored until the worst of the flooding event has passed, at which time releases from the reservoir can begin. As with droughts, the severity of floods is expected to increase under projected climate change (Easterling *et al.*, 2000). Therefore, it is of vital importance that dependable strategies exist for the management of reservoir systems during a flood moving into the future.

Research into management strategies for the operation of reservoir systems during flood events has not been as widely investigated as that of strategies during drought events. This can be attributed to the fact that the behaviour of traditional reservoirs during a flood is quite simplistic; floodwaters fill the reservoir to capacity, at which time spill occurs (defined as releases from the reservoir over the spillway). This behaviour is easily modelled and there is limited scope for further research into such a system. Therefore, the research that is conducted in the area is mainly focused on developing optimal management strategies for systems containing more than one reservoir. An example of such research is presented by Karbowski (1993). In this work, Karbowski (1993) explored the problem of finding an optimal flood control technique for a system of serially connected water reservoirs (a cascade reservoir system), where the objective was to minimise the peak flow measured at some point in the system. This objective is similar to that of the models constructed in this thesis; to minimise the extent of spill from the second reservoir in a two reservoir cascade system. In order to accomplish the objective of minimising the peak flow at some point in the system, Karbowski (1993) investigated two methods; the first where the active reservoir in the system was switched in an attempt to try and minimise the number of reservoirs that are changing their storage levels over the time period considered. The second method involved the synchronised operation of all the reservoirs in the system, where continuous spills from each reservoir occurred causing long periods of gradual changes in the storage levels. Both methods proposed had limited applicability to the system investigated in this thesis; however may prove useful if a larger cascade system was to be considered in future work.

In more recent times, reservoirs have been constructed or upgraded to include flood mitigation measures such as gated spillways. These measures enable spill to be withheld until the river levels downstream of the reservoir have returned to a suitable level. Along with increasing the capabilities of reservoirs from water storage devices to flood defence mechanisms, these flood mitigation measures have also increased the scope of research that could be conducted in the area. An example of such research that has been performed into the operation of a reservoir featuring flood mitigation measures during a flood event is provided by Kearney *et al.* (2011). This work compared the existing mitigation strategy of Wivenhoe Dam in the Brisbane Valley, which features a gated spillway, to a model designed by Kearney *et al.* (2011) using the 2011 flood event in Queensland, Australia as a case study. Kearney *et al.* (2011) employed the method of model predictive control to develop a flood mitigation strategy for the reservoir and then performed a simulation study to compare the outcome of their model to the already existing mitigation strategy for Wivenhoe Dam. The method of model predictive control is significantly different to that utilised in this thesis, where the discrete hedging rules and MILP model proposed by Shih and ReVelle (1995) were adapted and altered to ensure their applicability to a flood event.

After the 2011 flood event in Queensland, Australia, the operation of the regions water reservoirs was placed under increased scrutiny. In order to make their operating procedures during a flood event more transparent to the public, the managers of one of the largest water reservoir systems in Queensland released a manual titled "Manual of Operational Procedures for Flood Mitigation at Wivenhoe and Somerset Dam" (Seqwater, 2011*b*). This manual outlined the procedures that should be followed during a flood event in the Brisbane River catchment area, in which the Somerset and Wivenhoe Dams are located. Some of the procedures described in the manual were rather complex, involving gated spillways and measurements of river levels at multiple locations downstream of the reservoir system; far beyond the scope of the operating procedures considered in this thesis. However, if further research was to be conducted, assuming that the reservoirs considered in this thesis now featured gated spillways, the operating procedures outlined by the manual could prove useful. The managers of the same reservoir system also released a report detailing how the reservoir system was actually operated during the 2011 flood event, titled "January 2011 Flood Event: Report on the operation of Somerset Dam and Wivenhoe Dam" (Seqwater, 2011*a*). The report contained information regarding how and when the operation strategies outlined in the Manual were employed, along with the measured inflows into the two reservoirs over the course of the event. These inflows were originally going to be used as a form of proxy data for the reservoir system considered in this thesis; however more suitable historic inflows were able to be sourced from the Queensland Department of Natural Resources and Mines (2012).

1.5 Time Series Analysis

Although historic inflows were able to be sourced for the cascade reservoir system considered as part of this thesis, time series analysis was utilised to generate another dataset based on the historic records. In order to formulate a time series model for the data, the methods introduced by Dunn and Addie (2008) were employed. In their work, they presented two selection tools that could be used to determine a suitable time series model to describe a given dataset. Once a model was selected, Dunn and Addie (2008) outlined a battery of diagnostic tests that could be performed in order to ensure that the model selected provided an approximate representation of the historic inflow records by capturing the important information, or signal, of the data. Dunn and Addie (2008) also outlined a method for generating a series of one-step ahead forecasts from the time series model to test its accuracy when compared to a portion of the original dataset. Each of these methods was performed in this thesis and resulted in the development of a time series model that adequately described the historic inflow records. This model was then employed to generate another series of data which could be used as a common set of inflows when performing a comparison of the management strategies employed by the drought and flood models.

1.6 Case Study: Perseverance and Cressbrook Dams

The extent of this thesis is not limited to the development of MILP models, but also includes the application of these models to an existing cascade reservoir system in order to determine optimal strategies for the management of such a system during a drought and flood event. In this case, two of the reservoirs that contribute to the water supply of the Toowoomba region exhibit a cascade configuration; being the Perseverance and Cressbrook dams. Both of these reservoirs are located approximately 35 kilometres northeast of Toowoomba and make up two thirds of the regions reservoir system, with another solitary dam located to the north of Toowoomba also contributing to the water supply network (Toowoomba Regional Council, 2013). Figure 1.1 displays the geographical layout of the Perseverance and Cressbrook cascade reservoir system, along with the location of both reservoir spillways; represented by the small black rectangles. From this figure, it can be

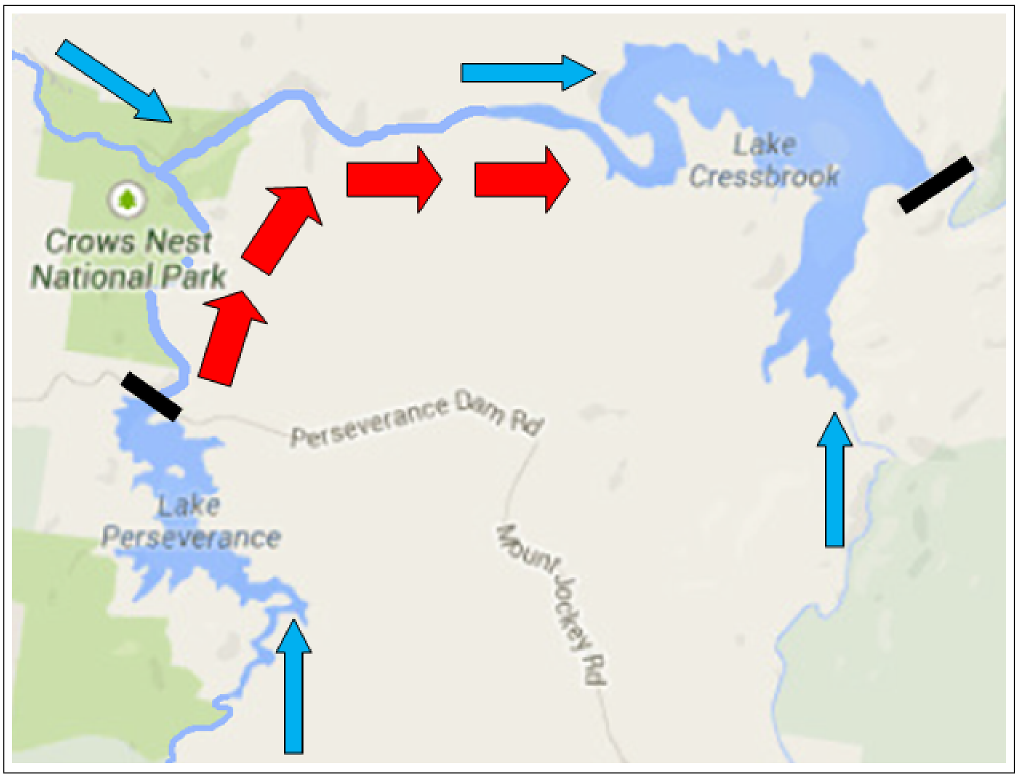

Figure 1.1: Configuration of Perseverance and Cressbrook cascade reservoir system investigated as a Case Study.
Black Rectangles = Dam Spillway, Thinner Blue Arrows = Sources of Inflows, Thicker Red Arrows = Direction of Spill from Perseverance to Cressbrook dam.

seen that Cressbrook dam is the larger of the two reservoirs, with an approximate maximum available storage of 82 000 megalitres (ML), whilst Perseverance dam is smaller with approximately 30 000 ML of maximum available storage (Toowoomba Regional Council, 2013). As stated previously, when in a cascade configuration, any spill that occurs from a previous reservoir in the system will contribute to the inflows of subsequent reservoirs. Although the orientation of the reservoirs in Figure 1.1 may suggest otherwise, any spill that occurs from Perseverance dam contributes to the inflows into Cressbrook dam, as shown by the thicker red arrows. Also demonstrated by the figure is the fact that Cressbrook dam is not reliant on the spill from Perseverance dam as a sole source of inflows; it is also fed by independent creeks and streams. The creeks and streams that feed both reservoirs are denoted in Figure 1.1 using the thinner blue arrows. The physical cascade layout of these two reservoirs, along with the features mentioned, make the Perseverance and Cressbrook cascade reservoir system an ideal case study to which the drought and flood MILP models developed in this thesis could be applied, and through the application of the models, enable optimal management strategies for the system to be determined.

The aim of this thesis is to demonstrate the process undertaken during the formulation and development of two MILP models that are applicable to a cascade reservoir system, in order to determine the optimal management strategy for the system under the two extreme environmental conditions of a drought or flood. In Chapter 2, a simple LP model is investigated and then extended to a cascade reservoir system. The exploration of this simple model formed a basis of understanding that was able to be applied throughout the rest of this thesis. Chapter 3 details the procedure of investigating and understanding a MILP model proposed to optimally manage a single reservoir system during a drought event. The original model is then adapted to be applicable to a cascade reservoir system before being applied to the case study of Perseverance and Cressbrook dams. Chapter 4 follows a similar process as that conducted in Chapter 3, however this time the models developed are applicable to the optimal management of a cascade reservoir system during a flood event. Following this, time series analysis is utilised in Chapter 5 to formulate a time series model to accurately describe the historic inflow records. This time series model was then used to simulate an alternate set of inflow data, which forms a common dataset for use in Chapter 6 where the differing management strategies employed by the drought and flood model are compared. The thesis is then concluded in Chapter 7.

Chapter 2

Simple Model

2.1 Chapter Overview

The focus of this chapter is to demonstrate where this thesis began; with the exploration and extension of a simple model used to describe the operation of a reservoir system. To begin, the simple Linear Programming (LP) model presented by ReVelle and McGarity (1997) is stated and investigated. Labelled Model 1, this model was rather limited in its applicability to a physical system and therefore required extension in order for it to adequately describe the operation of a cascade reservoir system. The extended form of the simple model, named Model 2, can also be found in this chapter along with a review of the results from an experiment conducted using the model. Although both Model 1 and Model 2 are simplistic and not as comprehensive as other models explored later in this thesis, the work conducted in this chapter formed a basis of understanding regarding how LP models can be applied to describe the operation of a reservoir system.

2.2 Models

2.2.1 Model 1 - Simple Model developed by ReVelle and McGarity (1997)

In this section, the first of two models explored in this chapter is introduced. Formulated by ReVelle and McGarity (1997) to describe the fundamental behaviours exhibited by a single reservoir system, Model 1 is given below in Equations (2.1) to (2.5). Note that a description of the constraints and variables used to compose the simple LP model are provided in Section 2.2.2:

$$\text{minimise } z = c \tag{2.1}$$

such that

$$S_t = S_{t-1} + I_t - q - W_t \qquad \forall t, \tag{2.2}$$

$$S_t \leq c \qquad \forall t, \tag{2.3}$$

$$S_T \geq S_0, \tag{2.4}$$

$$S_t, S_{t-1}, I_t, W_t, c, q \geq 0 \qquad \forall t. \tag{2.5}$$

2.2.2 Model 1 Description

Model 1 is a straightforward LP model that measures the storage level of a single reservoir system over a time period of known length. The authors of Model 1 state that the objective of this model is to determine the smallest reservoir capacity required to sustain a steady release from the reservoir for use as a water supply over the duration of the time period specified; while, at the same time ensuring that the ending reservoir condition is no worse than the condition of the system at the beginning of the time period (ReVelle and McGarity, 1997).

Variable Definitions

There are eight variables that are used in the formulation of Model 1. These variables can be defined below:

t, T = the index and the total length of the time period considered, in this case assumed to be months,

S_t = the storage level of the reservoir at the end of month t, which is unknown (megalitres, ML),

S_0 = the unknown initial storage level of the reservoir (ML),

q = the specified steady month-to-month release from the reservoir for use as a water supply (ML),

W_t = the unknown amount of spill from the reservoir in month t (ML),

c = the unknown reservoir storage capacity (ML),

I_t = the historical inflows to the reservoir in month t, which are specified (ML).

In order to investigate how these variables feature and interact in Model 1, the constraints that compose the model can be investigated.

Contraint Definitions

Model 1 consists of five constraints, including the objective function. In this case, the objective function (Equation (2.1)) contains the single variable c and aims to minimise the capacity of the reservoir subject to the other constraints in the model. Equation (2.2) is the mass balance constraint and provides the storage level of the reservoir in the current month. This value is calculated by adding the storage level of the reservoir in the previous month with the inflows to the reservoir in the current month, and then subtracting the releases made to the water supply and the spill from the reservoir in the current month. Equation (2.3) makes sure that the reservoir storage level does not exceed the capacity of the reservoir in any given month, while Equation (2.4) states that the reservoir storage level at the end of the time period considered must be greater than or equal to the storage level at the beginning of the period. This ensures that the reservoir storage level is in the same condition or better at the end of the time period than where it commenced.

Equation (2.5) is included to ensure that particular variables remain nonnegative and thus physically realistic.

2.2.3 Model 2 - Extension of Simple Model

Upon experimentation of Model 1, it was found that the model had limited applicability to a physical system and required extension in order to suitably describe the operation of a cascade reservoir system. This extended model, labelled Model 2, is given below in Equations (2.6) to (2.28). Note that variables labelled with a star (*) correspond to Reservoir 1, whilst variables without a star relate to Reservoir 2.

$$\text{minimise } z = C_D^* + C_D \tag{2.6}$$

such that

Reservoir 1 Constraints

$$S_t^* = S_{t-1}^* + I_t^* - q^* - W_t^* \qquad \forall t, \tag{2.7}$$

$$S_0^* \leq C_D^*, \tag{2.8}$$

$$S_t^* \leq C_D^* \qquad \forall t, \tag{2.9}$$

$$W_t^* \leq C_S^* \qquad \forall t, \tag{2.10}$$

$$C^* = C_S^* + C_D^*, \tag{2.11}$$

$$C_S^* \leq 0.1C_D^*, \tag{2.12}$$

$$S_{t-1}^* + I_t^* - q^* - C_D^* = p_t^{+*} - p_t^{-*} \qquad \forall t, \tag{2.13}$$

$$p_t^{-*} \leq My_t^* \qquad \forall t, \tag{2.14}$$

$$p_t^{+*} \leq M(1 - y_t^*) \qquad \forall t, \tag{2.15}$$

$$W_t^* \leq p_t^{+*} \qquad \forall t, \tag{2.16}$$

$$S_t^*, S_{t-1}^*, I_t^*, W_t^*, p_t^{+*}, p_t^{-*} \geq 0 \qquad \forall t. \tag{2.17}$$

Reservoir 2 Constraints

$$S_t = S_{t-1} + I_t - q - W_t + W_t^* \qquad \forall t, \quad (2.18)$$

$$S_0 \leq C_D, \qquad (2.19)$$

$$S_t \leq C_D \qquad \forall t, \quad (2.20)$$

$$W_t \leq C_S \qquad \forall t, \quad (2.21)$$

$$C = C_S + C_D, \qquad (2.22)$$

$$C_S \leq 0.1 C_D, \qquad (2.23)$$

$$S_{t-1} + I_t - q - C_D = p_t^+ - p_t^- \qquad \forall t, \quad (2.24)$$

$$p_t^- \leq M y_t \qquad \forall t, \quad (2.25)$$

$$p_t^+ \leq M(1 - y_t) \qquad \forall t, \quad (2.26)$$

$$W_t \leq p_t^+ \qquad \forall t, \quad (2.27)$$

$$S_t, S_{t-1}, I_t, W_t, p_t^+, p_t^- \geq 0 \qquad \forall t. \quad (2.28)$$

2.2.4 Model 2 Description

Model 2 builds upon the framework provided by Model 1 to formulate a model that is capable of describing the basic behaviour exhibited by components of a cascade reservoir system. In this sense, the model measures the storage levels of two reservoirs situated in a cascade configuration over a time period of known length. The objective function of Model 2 also builds upon that stated for Model 1; determining the smallest reservoir capacities required to sustain a steady release to a water supply, where the water supply is shared between the two reservoirs, over the duration of the time period considered.

A functionality that Model 1 did not possess was the ability to adequately describe the spill of water from the two reservoirs. Also, due to the cascade configuration of the reservoir system considered, Model 2 also needs to take into account the fact that any spill from Reservoir 1 becomes an additional source of inflows to Reservoir 2. In order to enable Model 2 to successfully describe these behaviours of a cascade reservoir system, a binary variable was added to the model; changing the nature of Model 2 from a Linear Programming model to a Mixed Integer Linear Programming (MILP) model. Another addition to Model 2 was the inclusion of a total reservoir capacity that is able to be separated into the reservoir storage capacity and the capacity of the reservoir spillway. These variables were included to better describe the spill of water from the two reservoirs.

Variable Definitions

There are 14 variables used in the formulation of Model 2. Seven of these variables were defined previously when describing Model 1 in Section 2.2.2, under the heading Variable Definitions. The seven remaining variables that have not been defined previously are listed below:

C	=	the total reservoir capacity, which is unknown (ML),
C_D	=	the unknown reservoir storage capacity (ML),
C_S	=	the unknown capacity of the reservoir spillway (ML),
p_t^+	=	an indicator variable whose value is greater than zero if spill is occurring from the reservoir in month t, which is unknown (ML),
p_t^-	=	an indicator variable whose value is greater than zero if no spill is occurring from the reservoir in month t, which is unknown (ML),
M	=	a large constant value, assumed to equal 10 000 in this case,
y_t	=	an unknown binary variable that equals 0 if spill is occurring in month t or 1 otherwise.

How these variables feature and interact can be explored through the investigation of the constraints that compose Model 2.

Constraint Definitions

Model 2 consists of an objective function and 22 constraints, eleven of which are repeated from Reservoir 1 to Reservoir 2. In this case, as Model 2 is applicable to a dual cascade reservoir system, the objective function (Equation (2.6)) contains both variables C_D^* and C_D and aims to minimise the storage capacities of the two reservoirs subject to the other constraints in the model. Equations (2.7) and (2.9) have been defined previously in Section 2.2.2 for Model 1; the first is the mass balance constraint, whilst the second ensures that the reservoir storage level does not exceed the reservoir storage capacity in any given month. Equation (2.8) ensures that the initial reservoir storage level does not exceed the reservoir storage capacity, while Equation (2.10) makes certain that the amount of spill in a month does not exceed the spillway capacity. Equation (2.11) is an equality constraint stating that the total reservoir capacity is equal to the reservoir storage capacity plus the reservoir spillway capacity. Equation (2.12) is then included to limit the size of the spillway capacity to a proportion of the reservoir storage capacity, in this case arbitrarily selected to be 10%. Equation (2.13) is used to measure the extent of spill (if any) that occurs from a reservoir in a given month.

In order for spill to occur, the reservoir storage capacity must be exceeded. This can be calculated by determining the sum of the reservoir storage level at the end of the previous month and the inflows to the reservoir in the current month, and then subtracting the steady month-to-month release of water to the community water supply. If this value is greater than the reservoir storage capacity, then water must exit the reservoir as spill. If spill occurs, the extent of the spill is measured by the variable p_t^+. On the other hand, if spill does not occur in the current month, then the extent by which the storage level is below the reservoir storage capacity is measured by the variable p_t^-. These variables are used in conjunction with Equations (2.14) and (2.15) to determine the value of the binary variable y_t. As stated earlier, the value of the variable y_t equals zero if spill is occurring in the current month or one otherwise. This can be illustrated by an example; if spill is occurring in the current month, then the variable p_t^+ will have a value greater than zero, while the variable p_t^- has a value of zero. In order to satisfy both Equation (2.14) and (2.15), the variable y_t must also have a value of zero. If the alternate situation is considered, where no spill occurs in the current month, then the variable p_t^+ will have a value of zero, while the variable p_t^- has a value greater than zero. Once again, in order to satisfy both constraints, the variable y_t must have a value of one. Equation (2.16) then states that the amount of spill that occurs in the current month must be at most equal to the variable p_t^+. Equation (2.17) is included to ensure that particular variables remain physically realistic and thus nonnegative. At this point, it should be noted that Equations (2.19) to (2.28) are the same as Equations (2.8) to (2.17); however are applicable to Reservoir 2. The only constraint that varies between the two reservoirs is the mass balance constraint; Equations (2.7) and (2.18). In the case of Reservoir 2, it contains the additional variable W_t^*; the amount of spill from Reservoir 1 in the current month. As the two reservoirs are situated in a cascade configuration, spill from Reservoir 1 becomes a source of inflows to Reservoir 2 and as such needs to be included in the mass balance constraint of the second reservoir.

2.3 Results and Discussion

In this section, an experiment to investigate how the storage capacities of two reservoirs in a cascade configuration behave given varying community water supply requirements is conducted using Model 2. This experiment not only provided a means of applying and thus testing Model 2, but also offered a broader opportunity to better understand how simple LP and MILP models can be applied to the operation of reservoir systems.

In order to perform the experiment, the optimisation modelling software LINGO version 14.0 was employed (LINDO Systems Inc, 2013). An example of the syntax required to perform this experiment, along with an excerpt of the resulting output can be found in Appendix A.

To enable a better investigation of Model 2 and assist in the interpretation of results from the experiment, a series of assumptions were made. The first of these was to assume that the experiment would be conducted over a period of six months and that both reservoirs would be subject to constant inflows of 50 megalitres (ML) per month. Additionally, it was assumed that the releases from the reservoirs to the community water supply varied in 10 ML increments from a possible demand of 0 ML to 200 ML per month.

The final assumption made regarding the parameters of the experiment was to specify that the two reservoirs shared the community water supply demand, according to a known policy. In order to explore how the reservoir capacities behaved under varying circumstances, three policies were invented; the first where the community water supply was split equally between the two reservoirs at 50% each, the second where Reservoir 1 was responsible for 70% of the community water supply and Reservoir 2 for the remaining 30%, and the final policy where Reservoir 1 was responsible for only 30% of the community water supply, while Reservoir 2 supplied 70%.

2.3.1 Policy 1

The first policy considered shared the community water supply demand equally between the two reservoirs, at 50% each. In order to investigate how the capacities of the two reservoirs behaved under varying demand from the community, a series of plots have been constructed; Figure 2.1 demonstrates the behaviour of the reservoir capacities against community water supply separately, while Figure 2.2 displays the capacities of the two reservoirs on the same axis as a means of comparison.

From Figure 2.1 it can be seen that when there was no demand for water from the community, Model 2 predicted that a storage capacity of approximately 300 ML was required for both reservoirs. This result is contrary to first thought, which would suggest that if there is no demand by the community for water, then there is no need for a reservoir. However, the model assumed that the reservoir system was already in place and as such increased the reservoir capacity due to Equations (2.12) and (2.23). These constraints limit the capacity of the reservoir spillway to 10% of the reservoir storage capacity. Therefore, Model 2 elected to increase the storage capacity of the reservoir in order to maximise the capacity of the spillway and thus the amount of water that could be released from the reservoirs as spill each month.

As the demand for water from the community increased from 0 ML per month, Figure 2.1 demonstrates that the capacity of both reservoirs decreased until the community water supply demand reached 100 ML per month. At this point, Model 2 predicted the required capacity of both reservoirs to equal 0 ML; effectively stating that a reservoir system was not necessary. Under the policy being investigated, it was assumed that the community water supply demand was divided equally between the two reservoirs at 50% each. Therefore, when the total community water supply requirement equalled 100 ML per month, the reservoirs were responsible for providing 50 ML per month each; equating to the inflows to both reservoirs of

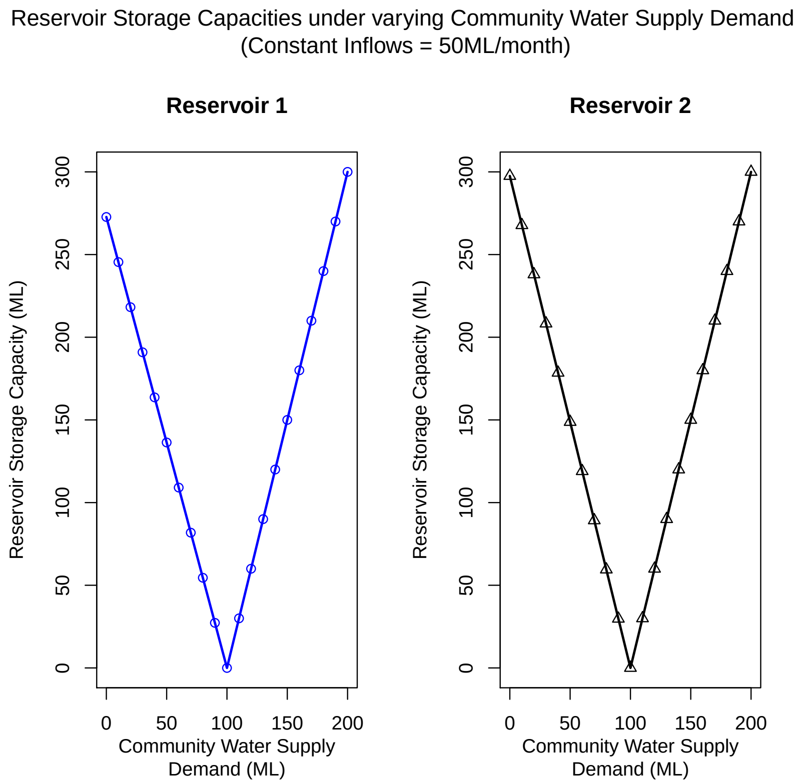

Figure 2.1: **Policy 1** - Reservoir Capacities against Community Water Demand. Community Water Demand shared - Reservoir 1 at 50% and Reservoir 2 at 50%. Left - Resevoir 1, Right - Reservoir 2.

50 ML per month and thus balancing the system. In context, this means that if it is known that 100 ML per month in total will be readily available from a river system, then the community could pump their requirements directly from the river without the need for a reservoir system. However, in practice, it is known that access to a steady water supply through rivers and streams cannot be assured and that some form of storage is necessary to ensure water security. In that sense, this component of the model is unrealistic; however this limitation is improved upon in subsequent chapters.

Figure 2.1 also shows that once the community water supply demand was greater than 100 ML per month, the capacities of both reservoirs increased at a constant rate. This was attributed to the demand from the community now exceeding the inflows to the reservoirs each month, meaning that some storage of water was required in order to meet the demand over the time period considered. From the figure, it can be seen that for every increase of 10 ML (starting from 100 ML) in community water supply demand, the reservoir capacity of Reservoir 1 and Reservoir 2 increased by 30 ML. The reason for this increase in the reservoir capacity was identified by considering the impact of increasing the community demand by 10 ML per month over a time period of six months; translating to an additional 60 ML

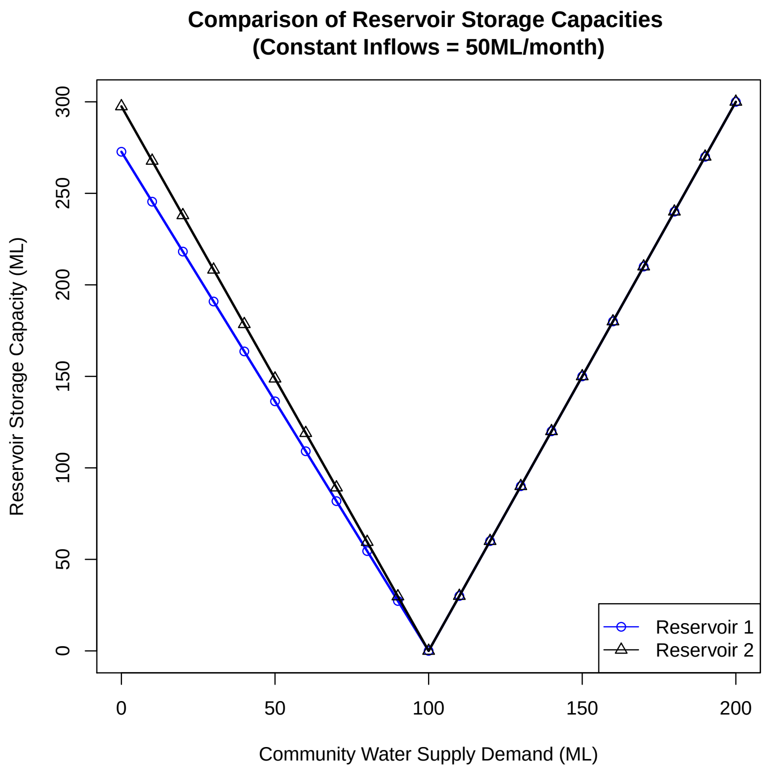

Figure 2.2: **Policy 1** - Comparison of Reservoir Capacities.
Community Water Demand shared - Reservoir 1 at 50% and Reservoir 2 at 50%.

needing to be supplied to the community in total. Under Policy 1, the demand from the community was sourced equally from the two reservoirs, resulting in the total increase of 60 ML being shared between the two reservoirs at 50% each; equating to an extra 30 ML needing to be stored in each reservoir to ensure that the community water supply demand could be met.

In addition to Figure 2.1, Figure 2.2 has been provided to compare the behaviour of the two reservoir capacities on a common set of axes. Evident from this figure is that the two reservoir capacities demonstrated much the same behaviour, except that the capacity of Reservoir 1 was slightly smaller than that of Reservoir 2 when the community water demand was less than 100 ML. This difference was attributed to the reservoirs being located in a cascade configuration, and any spill from Reservoir 1 becoming a source of inflow to Reservoir 2. Therefore, in order to combat the combined inflows to the second reservoir, the model elected to increase the storage capacity of Reservoir 2 and in doing so, the capacity of the reservoir spillway, in order to maximise the amount of spill from the reservoir when the inflows exceeded demand.

Under Policy 1, Figure 2.1 and 2.2 demonstrate that there were two key behaviours of the reservoir capacities. The first was witnessed when the inflows to the reservoir system exceeded the demand from the community. In this case, the model selected the minimum storage capacity of the reservoirs that at the same time maximised the size of the reservoir spillways, in order to release the unrequired, additional inflows. On the other hand, the second behaviour occurred when the inflows to the reservoir system were insufficient to meet the demand from the community each month. If this transpired, the reservoirs were required to store an additional volume of water in order to ensure that the community water demand could be maintained over the time period considered and, as such the capacities of the reservoirs increased proportional to this additional volume required. These two key behaviours of the reservoir storage capacities were also apparent in the other policies considered; however were influenced by the unequal sharing of the community water demand and were more difficult to identify.

2.3.2 Policy 2

The next policy to be investigated shared the community water supply demand between Reservoir 1 and Reservoir 2 at 30% and 70% respectively. This sharing of community water demand could occur as a result of convenience or for cost reasons if one of the reservoirs was to be located closer to the community, or if one of the reservoirs was supplied by more substantial tributaries. Similar to Policy 1, two plots have been constructed to explore the capacities of the reservoirs against varying community water supply demand and are given in Figure 2.3 and Figure 2.4.

From Figure 2.3, it can be seen that when there was no demand for water from the community, the model selected the same capacities for both reservoirs as those observed under Policy 1; approximately 300 ML. This behaviour is to be expected and witnessed again for the final policy, as the unequal sharing of community demand has no impact if there is no demand to share (0 ML). As the demand from the community increased, the capacity of both reservoirs decreased at constant rates until the community water demand reached approximately 80 ML per month. At this value, the community demand from Reservoir 2 began to exceed the inflows to the reservoir and as such, the behaviour of the reservoir capacity changed to reflect the need for water to be stored. On the other hand, the capacity of Reservoir 1 continued to decrease constantly until the critical demand volume of 100 ML per month was reached.

Upon the community water supply demand reaching 100 ML per month, Figure 2.3 demonstrates that the capacities of both reservoirs underwent a significant change in behaviour. In the case of Reservoir 1, the storage capacity of the reservoir spiked to 200 ML, while the storage capacity of Reservoir 2 decreased to 0 ML; indicating that under this community water supply demand, Model 2 suggested that the second reservoir was not required. This drastic change in the behaviour of the reservoir capacities was attributed to the unequal sharing of the community water supply demand. Under Policy 2, Reservoir 1 was responsible for supplying 30% of

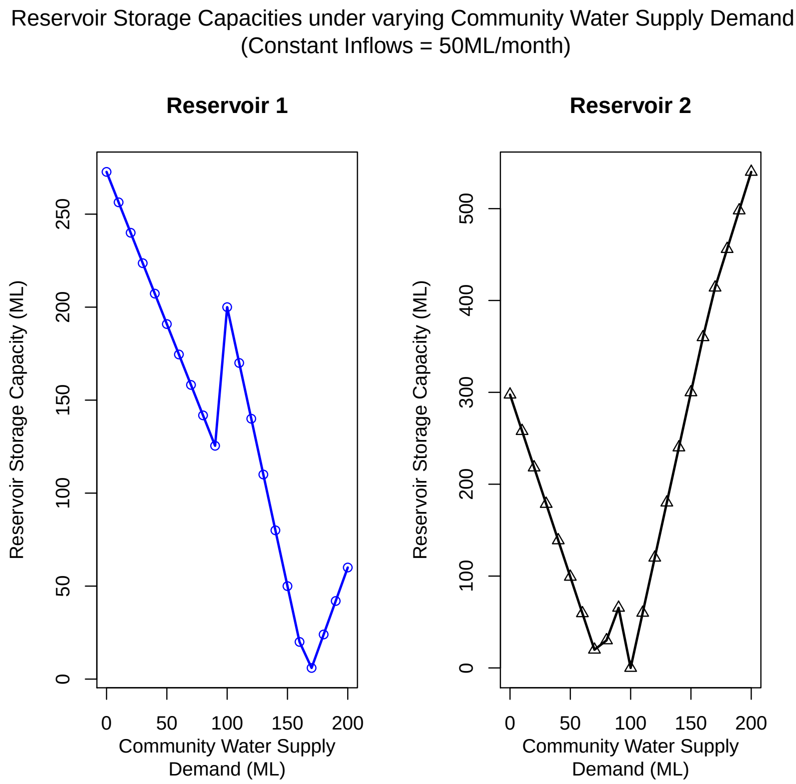

Figure 2.3: **Policy 2** - Reservoir Capacities against Community Water Demand. Community Water Demand shared - Reservoir 1 at 30% and Reservoir 2 at 70%. Left - Resevoir 1, Right - Reservoir 2.

the community water demand, while the remaining 70% was sourced from Reservoir 2; translating to 30 ML and 70 ML per month being sourced from Reservoir 1 and Reservoir 2 respectively when the total community water supply demand per month equalled 100 ML. Therefore, by increasing the capacity of Reservoir 1 to 200 ML, Model 2 also increased the capacity of the Reservoir 1 spillway to 20 ML (the spillway capacity is limited to 10% of the reservoir storage capacity by Equations (2.12) and (2.23)). This meant that the community demand of 30 ML per month from Reservoir 1 could be completely supplied out of the 50 ML of constant inflows to the reservoir each month, while the remaining 20 ML of inflows became spill from the reservoir. As the reservoirs were situated in a cascade configuration, the 20 ML of spill from Reservoir 1 combined with the constant inflow of 50 ML per month to Reservoir 2, thus providing a total inflow of 70 ML per month to the second reservoir. This combined inflow of 70 ML per month equated to the community demand of 70 ML per month from Reservoir 2; thus balancing the input and output of the reservoir and causing Model 2 to deem it unnecessary.

As the demand for water from the community increased past 100 ML per month, the two reservoir capacities demonstrated opposing behaviours. The storage capacity of Reservoir 1 began to decrease from 200 ML at a rate of 30 ML for each

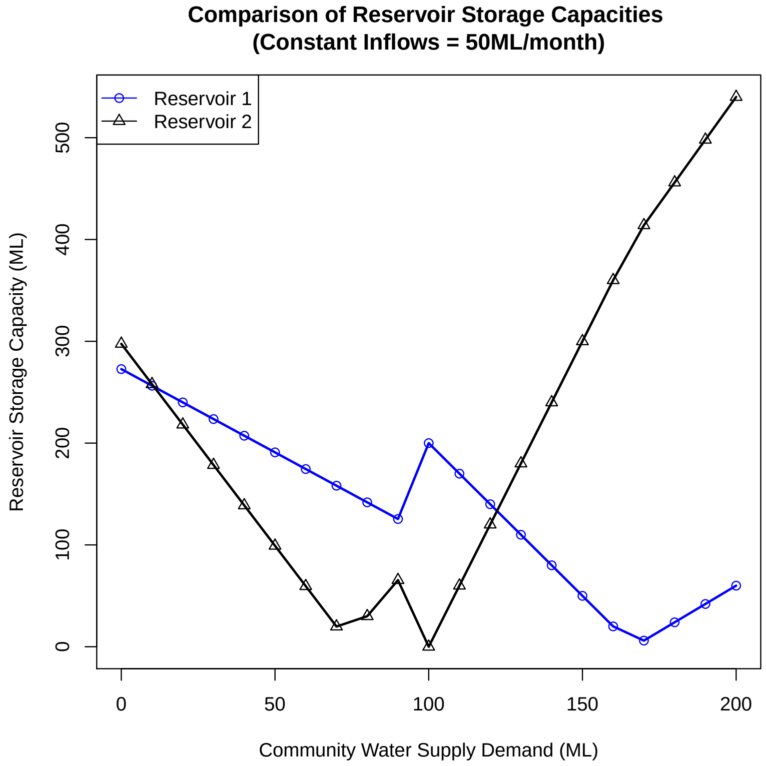

Figure 2.4: **Policy 2** - Comparison of Reservoir Capacities.
Community Water Demand shared - Reservoir 1 at 30% and Reservoir 2 at 70%.

increase of 10 ML per month in community water demand. On the other hand, the capacity of Reservoir 2 increased at a constant rate of 60 ML for each 10 ML per month increase in the demand from the community.

Although depicted in Figure 2.3, Figure 2.4 better demonstrates the point at which the inflows to Reservoir 1 no longer exceeded the community water supply demand. At this value, a demand of approximately 170 ML per month, the behaviour of both reservoir capacities changed. For Reservoir 1, this was due to the fact that water needed to be stored in order to meet the required demand from the community over the extent of the time period considered. This change in the Reservoir 1 storage level behaviour also impacted the behaviour of the Reservoir 2 storage level, as the spill from Reservoir 1 could no longer be relied upon as an additional source of inflows to the second reservoir. Therefore, past a community water supply demand of approximately 170 ML per month the storage capacity of Reservoir 1 increased by 18 ML (30% of 60 ML) for every 10 ML per month increase in demand, while the storage capacity of Reservoir 2 increased by 42 ML (70% of 60 ML) for every 10 ML per month increase in community water demand.

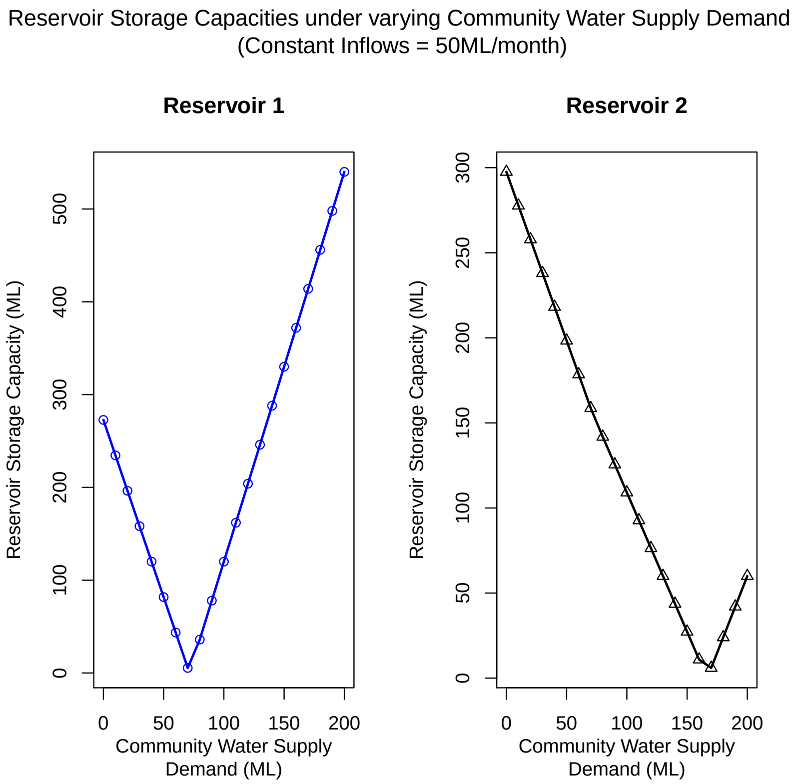

Figure 2.5: **Policy 3** - Reservoir Capacities against Community Water Demand. Community Water Demand shared - Reservoir 1 at 70% and Reservoir 2 at 30%. Left - Resevoir 1, Right - Reservoir 2.

2.3.3 Policy 3

The final policy to be investigated was when the community water supply demand was shared unequally between Reservoir 1 and Reservoir 2 at 70% and 30% respectively, or a reversal of that considered under Policy 2. As mentioned previously, this situation could occur when a more substantial river or stream system supplied one of the reservoirs, or if one reservoir was located closer to the community that the reservoir system was responsible for supplying. As before, two plots have been constructed to explore the behaviour of the reservoir capacities against varying community water demands.

From Figure 2.5 it can be seen that the behaviour of the reservoir storage capacities under this policy were easier to interpret than those witnessed under Policy 2. As already mentioned and expected for all policies, Model 2 selected the capacities of both reservoirs to equal approximately 300 ML when it was assumed that there was no demand from the community. The figure also demonstrates that as the community water supply demand increased, the capacities of both reservoirs decreased at constant rates. As Reservoir 1 was responsible for supplying 70% of the community water demand under Policy 3, the critical value where the behaviour of the reservoir storage capacity changed from attempting to release the excess inflows, to

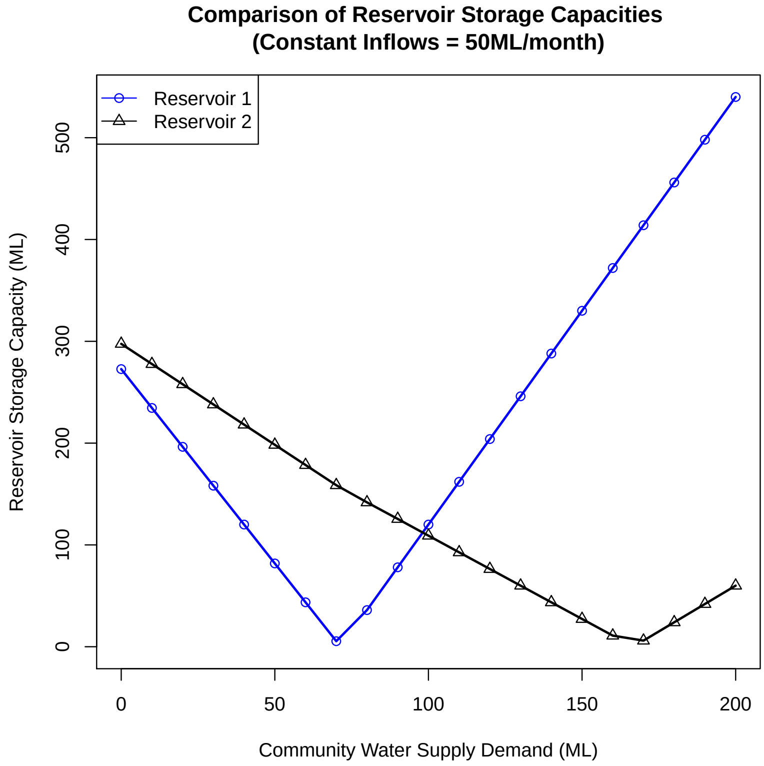

Figure 2.6: **Policy 3** - Comparison of Reservoir Capacities.
Community Water Demand shared - Reservoir 1 at 70% and Reservoir 2 at 30%.

the storage of water in order to ensure the community demand could be maintained across the time period, occurred at a smaller community water supply demand than that seen under the previous policies. In this case, once the community water demand reached approximately 70 ML per month, the inflows to Reservoir 1 were no longer sufficient to supply the community alone and the reservoir needed to begin the storage of water. On the other hand, as 30% of the community water supply demand was sourced from Reservoir 2 under Policy 3, the transition in reservoir storage capacity behaviour occurred at a much larger value of community demand than seen previously; approximately 170 ML per month in this case.

Figure 2.6 provides further evidence of the clear distinction in reservoir capacity behaviour, as the capacities of both reservoirs can be compared on a common set of axes. This behaviour was evident in each of the policies investigated and marks the point where the inflows to the reservoir were no longer sufficient to support the community water supply demand and storage needed to occur. This transition value was readily discernible in both Policy 1 and 3; however was harder to identify in Policy 2 due to the influence of spill from Reservoir 1 supplementing the inflows to Reservoir 2.

2.4 Chapter Conclusion

This chapter has presented the work, models and material that formed the origin of this thesis. To begin, the simple LP model formulated by ReVelle and McGarity (1997) was presented and described. This model was found to be limited in its ability to comprehensively describe the behaviours of a physical reservoir system and therefore required extension in order to be applicable to a cascade reservoir system. The extended model, named Model 2, was stated and a full description of the variables and constraints that compose the model provided. Following this, the results of an experiment to investigate how the capacities of two reservoirs in a cascade reservoir system behave under varying community water supply demands were provided. The experiment was replicated for three different policy types that specified how the community water demand was shared between the two reservoirs. From this experiment, it was found that the reservoir capacities behaved in two distinct ways; one when the inflows to the reservoir exceeded the demand from the community and the other when the inflows were insufficient to solely meet the community demand. Also, it was noted that Model 2 had some limitations and did not always provide informative results; in some cases suggesting that either one or both of the reservoirs in the system were not needed. In future chapters, some of these limitations are resolved.

Overall, the intention of this chapter was to demonstrate where this thesis began and how small experiments with simple models allowed for the development of knowledge and understanding that was able to be applied to more complex scenarios and situations considered later in the thesis.

Chapter 3

Drought Model

3.1 Chapter Overview

Extended periods of below average rainfall, known as drought, can place extreme stress on the water resources of regions affected. Therefore, optimal management strategies for reservoir systems, like those investigated in this chapter, are vital to ensure that communities have access to a reliable water supply throughout the extent of such an event.

To begin this chapter, a model proposed by Shih and ReVelle (1995), labelled Model 3, for the operation of a single reservoir system during drought is explored. This Mixed Integer Linear Programming (MILP) model forms the foundation not only for work performed in this chapter, but for work conducted in Chapter 4 as well. Although very informative, Model 3 is only applicable to a single reservoir system and therefore required extension in order for it to be effective in investigating the cascade reservoir system considered as a case study in this thesis. This extended form of the drought model, denoted as Model 4, is also thoroughly investigated in this chapter, with each of the constraints and variables that compose the model being identified and defined. An experiment is then conducted in order to ascertain how components of the cascade reservoir system measured by Model 4 behave under varying drought conditions. These results form the basis of a discussion and a series of conclusions are made regarding the applicability of the extended model to the case study considered of an existing cascade reservoir system.

3.2 Models

3.2.1 Model 3 - Drought Model developed by Shih and ReVelle (1995)

In this section, the Mixed Integer Linear Programming (MILP) model formulated by Shih and ReVelle (1995) for the operation of a single reservoir during a drought event is presented and explored. This model has been denoted as Model 3 and is given by Equations (3.1) to (3.19) on the following page. Note that a description of the constraints and variables used to formulate the model are provided in Section 3.2.2.

$$\text{maximise } z = \sum_{t=1}^{T} y_{1t} - \omega \sum_{p=1}^{12} (V_{1p} + V_{2p} + V_{3p}) \tag{3.1}$$

such that

$$y_{1t} \geq \frac{(S_{t-1} + \hat{I}_t) - (V_{1p} - \epsilon)}{M} \qquad \forall t, p, \tag{3.2}$$

$$y_{1t} \leq 1 - \frac{V_{1p} - (S_{t-1} + \hat{I}_t)}{M} \qquad \forall t, p, \tag{3.3}$$

$$y_{2t} \geq \frac{(S_{t-1} + \hat{I}_t) - (V_{2p} - \epsilon)}{M} \qquad \forall t, p, \tag{3.4}$$

$$y_{2t} \leq 1 - \frac{V_{2p} - (S_{t-1} + \hat{I}_t)}{M} \qquad \forall t, p, \tag{3.5}$$

$$R_t = (1 - \alpha_1).D.y_{1t} + (\alpha_1 - \alpha_2).D.y_{2t} + \alpha_2.D \qquad \forall t, \tag{3.6}$$

$$S_t = S_{t-1} + I_t - R_t - W_t \qquad \forall t, \tag{3.7}$$

$$S_t \leq C \qquad \forall t, \tag{3.8}$$

$$S_0 \leq S_T, \tag{3.9}$$

$$U_t \leq \frac{S_t}{C} \qquad \forall t, \tag{3.10}$$

$$W_t \leq MU_t \qquad \forall t, \tag{3.11}$$

$$V_{1p} \geq (1 + \beta_1)V_{2p} \qquad \forall p, \tag{3.12}$$

$$V_{2p} \geq (1 + \beta_2)V_{3p} \qquad \forall p, \tag{3.13}$$

$$V_{3p} \geq \alpha_2.D \qquad \forall p, \tag{3.14}$$

$$S_{t-1} + \hat{I}_t \geq V_{3p} + \epsilon \qquad \forall t, p, \tag{3.15}$$

$$y_{1t-1} + y_{1t+1} \leq 1 + y_{1t} \qquad \forall t, \tag{3.16}$$

$$y_{1t} \leq y_{2t+1} \qquad \forall t, \tag{3.17}$$

$$\sum_{t=1}^{T} y_{2t} = T - n \qquad \forall t, \tag{3.18}$$

$$S_t, S_{t-1}, \hat{I}_t, I_t, V_{1p}, V_{2p}, V_{3p}, R_t, W_t \geq 0 \qquad \forall t, p. \tag{3.19}$$

3.2.2 *Model 3 Description*

As can be seen from the statement of Model 3 above, Shih and ReVelle (1995) have formulated a model substantially more complex than Model 1 and 2 investigated in the previous chapter.

In order to regulate the amount of water made available from the reservoir to satisfy a community water supply during a drought, Shih and ReVelle (1995) have employed

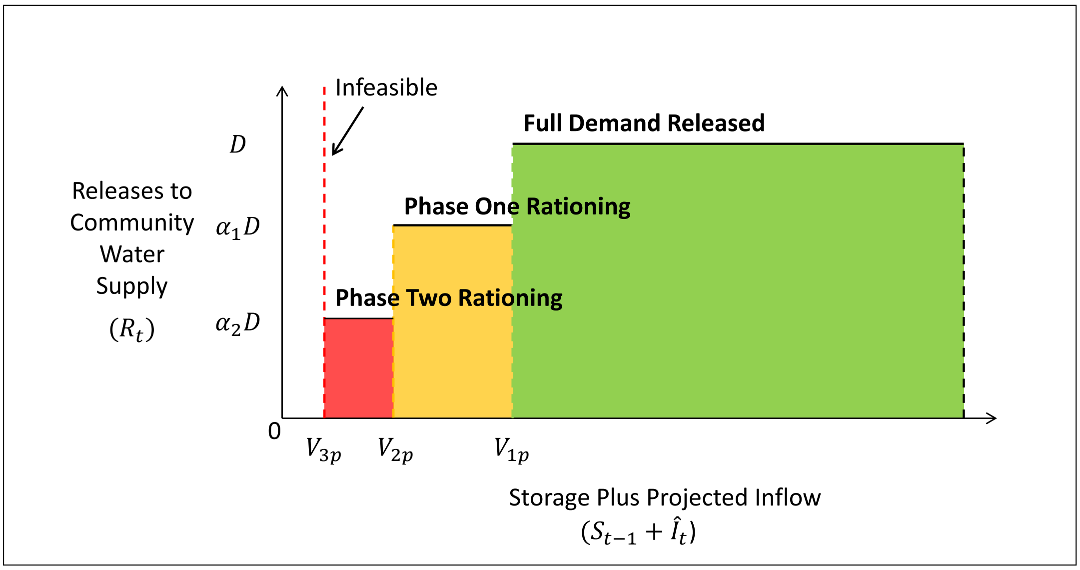

Figure 3.1: Graphical Representation of the Relationship between Reservoir Trigger Volumes and Rationing Levels.
Adapted from that presented by Shih and ReVelle (1995); where,
V_{1t}, V_{2t}, V_{3t} = *Trigger Volumes,* D = *Total Community Demand,*
α_1, α_2 = *Restrictions on the Releases to the Community.*

the technique of rationing levels. In the case of Model 3 it has been assumed that there are three rationing levels; no rationing, phase one rationing and phase two rationing. For these levels to be put into effect, the model seeks to identify three trigger volumes, V_{1p}, V_{2p}, and V_{3p}, for all months p. The values of these trigger volumes are optimally derived by the model, except for the lower boundary trigger volume, V_{3p}, which is determined through the specification of other variables by the operators of the reservoir system. The relationship between the trigger volumes and rationing levels is summarised by Shih and ReVelle (1995) as follows:

> "If the storage at the end of the previous period plus projected inflows are together greater than V_{1p}, then no water restrictions are announced. If the storage at the close of the previous period plus projected inflows fall between V_{1p} and V_{2p}, then rationing phase one is assumed to be declared, reducing demand to α_1 proportion of its expected value. If the storage plus projected inflow fall below V_{2p}, rationing phase two will be declared and only the proportion α_2 of usual demand will be expected to occur." (Shih and ReVelle, 1995)

Figure 3.1 above provides a graphical representation of the relationship between the trigger volumes and the rationing levels of the reservoir. Also depicted by the figure is the assumption that if the storage plus projected inflow decreases below the lowest trigger volume of V_{3p}, the outcome of the model is infeasible. For this model, Shih and ReVelle (1995) state that they assume that the minimum trigger volume will always be maintained in the reservoir. This assumption was felt to be unrealistic and has therefore been relaxed in the development of Model 4 later in this chapter.

In order to enforce the technique of rationing, the objective of Model 3 is twofold. The first component of the objective function (Equation (3.1)) aims to maximise the number of months in which no rationing is enforced on the community water supply, meaning that the full demand is able to be released. In conjunction with this, the second component of the objective function attempts to minimise the trigger volumes across all months p. By minimising the trigger volumes, the length of time before rationing is imposed can be maximised. By incorporating these two components in the objective function, the number of months in which rationing is enforced is minimised, and as such the releases to the community are maximised.

Variable Definitions

There are 24 variables that compose Model 3. These variables are defined below:

t, T	=	the current month and the total number of months in the event horizon which are known,
p	=	$p \in \{1, 2, ..., 12\}$, the month number of the year being considered. If the event horizon encompasses more than one year, this variable provides an index of the month number in the current year;
y_{1t}	=	an unknown binary variable that is 1 if no rationing is applied in month t or 0 otherwise;
y_{2t}	=	an unknown binary variable that is 1 if the system is at phase one rationing or better in month t, or 0 otherwise,
ω	=	a small number, which is assumed to equal 0.01 in this case,
V_{1p}	=	the unknown value of storage and inflow above which no restrictions on water use are placed (megalitres, ML),
V_{2p}	=	the unknown value of storage and inflow below which phase two rationing is implemented for month p (ML),
V_{3p}	=	the specified lower bound of storage plus inflow for month p (ML),
S_t	=	the storage in the reservoir at the end of month t, which is unknown (ML),
I_t	=	the inflow to the reservoir in month t, which is known (ML),
$\hat{I}_t$	=	the projected inflow to the reservoir in month t, which is known (ML),
ϵ	=	a small number, which is assumed to equal 0.1 in this case,
M	=	a big number, which is assumed to equal 100 000 in this case,
R_t	=	the releases to community water supply in month t, which is unknown (ML),

α_1 = the specified percentage of total demand released to the community under phase one rationing,

α_2 = the specified percentage of total demand released to the community under phase two rationing,

D = the specified total demand required by the community, which is assumed constant over the event horizon (ML),

W_t = the spill from the reservoir in month t, which is unknown (ML),

C = the capacity of the reservoir, which is known (ML),

U_t = an unknown binary variable that is 1 if the reservoir is at full capacity at the end of month t, or 0 otherwise,

β_1 = a specified separation value, assumed throughout the chapter to be 0.05,

β_2 = a specified separation value, assumed throughout the chapter to be 0.05,

n = the specified number of months in which it is deemed that phase two rationing is acceptable.

The way in which these variables interact and behave can be investigated through the exploration of the constraints featured in Model 3.

Contraint Definitions

Model 3 consists of 18 constraints, plus the objective function. As mentioned previously, the objective function (Equation (3.1)) is composed of two terms. The first and primary term of the function is to maximise the number of months in which no rationing is required. The secondary term then aims to minimise the total of the trigger volumes. By minimising the trigger volumes, the frequency at which rationing will be put into effect will be minimised. A weight is also placed on the trigger volumes to reduce the impact of this secondary term on the primary term of the function.

Equation (3.2) and (3.3) work together to calculate the value of the binary variable y_{1t}. If in the case of Equation (3.2), the reservoir storage level at the end of the previous month plus the projected inflow this month, herein referred to as available water, is greater than the trigger volume V_{1p}, then y_{1t} will equal one; indicating that no rationing is required in the current month. On the other hand, Equation (3.3) requires that y_{1t} equal zero if the available water is less than V_{1p}; meaning some amount of rationing is required. Shih and ReVelle (1995) explain that if the small value of ϵ was not included in these constraints, then when the available water equals the trigger volume ($S_{t-1} + \hat{I}_t = V_{1p}$) the value of y_{1t} could be either zero or one. To correct this, ϵ is included in the constraint to ensure that y_{1t} must equal one if this was to occur.

Similarly to Equation (3.2) and (3.3), Equation (3.4) and (3.5) work together to determine the value of the binary variable y_{2t}. Equation (3.4) sets the value of y_{2t} to

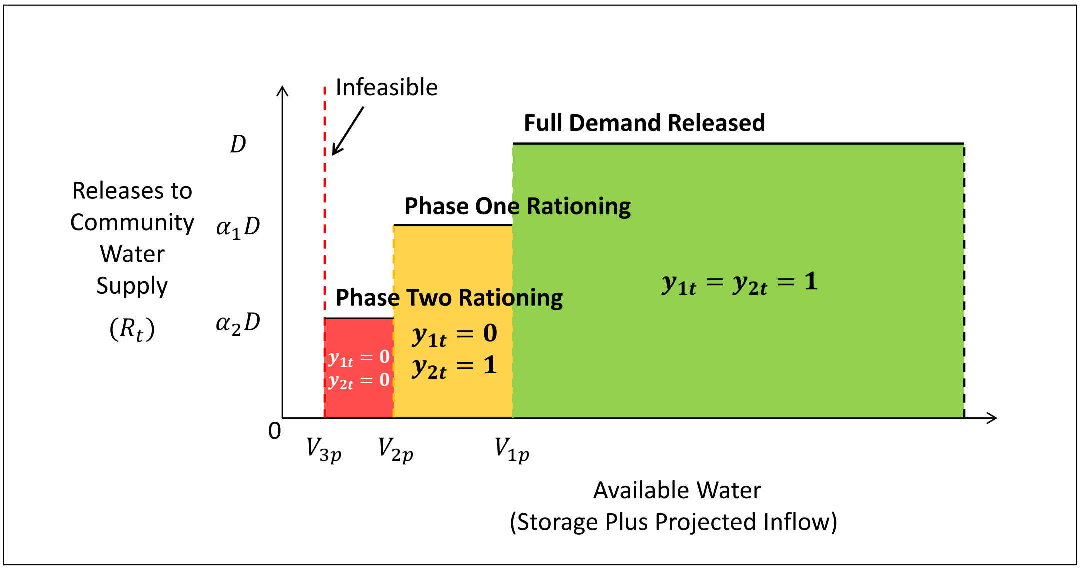

Figure 3.2: Revised Graphical Representation of the Relationship between Reservoir Trigger Volumes and Rationing Levels, showing the Values of the Binary Variables y_{1t} and y_{2t} under each Level.
Adapted from that presented by Shih and ReVelle (1995); where,
V_{1t}, V_{2t}, V_{3t} = *Trigger Volumes,* D = *Total Community Demand,*
α_1, α_2 = *Restrictions on the Releases to the Community.*

equal one if the available water is greater than the trigger volume V_{2p}. By equating the value of y_{2t} to one, this indicates that phase one rationing will be required in the current month. However, if the available water is less than V_{2p}, then Equation (3.5) ensures that y_{2t} is equal to zero, leading to phase two rationing being implemented in the current month. Shih and ReVelle (1995) point out that the four constraints, Equations (3.2) to (3.5), also assure that y_{1t} will not equal one unless y_{2t} equals one and that if y_{2t} equals zero, then y_{1t} must equal zero; special characteristics important to the formulation of the objective function. In order to better display how the binary variables y_{1t} and y_{2t} influence the transition between the three rationing levels, a revised form of Figure 3.1, labelled Figure 3.2, is presented above.

Equation (3.6) determines the amount of water to be released to the community water supply in a given month. If both the binary variables y_{1t} and y_{2t} are equal to one, then no rationing is required and the full demand of D can be released. However, if y_{2t} is equal to one while y_{1t} is equal to zero, then phase one rationing is enforced in the current month and a reduced proportion of the full demand, $\alpha_1 D$, is released. If both variables y_{1t} and y_{2t} are zero, then the constraints limit the release of water to $\alpha_2 D$, as phase two rationing is in effect.

Equation (3.7) has been encountered before in the previous chapter and is the mass balance constraint. The constraint states that the storage level of the reservoir at the end of the current month is equal to the sum of the storage level at the end of the previous month plus the inflow, minus the releases to the community water supply and any spill in the current month. Equation (3.8) limits the storage level of the reservoir each month to be less than the capacity of the reservoir, while Equation (3.9) requires that the reservoir storage at the end of the event horizon be greater than the reservoir storage to begin with. The addition of Equation (3.9) prevents the borrowing of water from the initial storage volume and ensures that the storage level of the reservoir is in the same or better condition at the end of the event horizon than where it began.

Equations (3.10) and (3.11) combine to control the spill of water from the reservoir. In a physical system, if the reservoir storage level is not at full capacity, then spill cannot occur. In this case, Equation (3.10) determines whether the reservoir is at capacity; if so, the binary variable U_t is set to one, otherwise it equals zero. Equation (3.11) then uses the value of U_t to determine the amount of spill that occurs. If U_t is zero, then the spill is also forced to be zero, as the reservoir is not at full capacity. However if U_t is one, then the right hand side of Equation (3.11) is a large number and the amount of spill is constrained to that level; though the actual extent of spill is calculated using the mass balance constraint.

In order to ensure that there is some degree of separation between the trigger volumes, and that they are not all minimised to the same value, Equations (3.12) and (3.13) are included. These constraints make certain that the trigger volume V_{1p} is at least β_1 percentage away from V_{2p} and that V_{2p} is at least β_2 percentage away from V_{3p}. Equation (3.14) specifies that V_{3p} should be equal to or greater than α_2 proportion of the full community water supply demand. This ensures that no more water than what the reservoir has in storage can be released. Equation (3.15) then states that in order to release α_2 proportion of the full demand, the available water should be greater than the lower boundary trigger volume V_{3p}. This constraint then also implies that $\alpha_2 D$, or the release under phase two rationing, can always be released from the reservoir.

Equation (3.16) is included to ensure that if in month $t-1$ and in month $t+1$ the full demand from the community is released from the reservoir, then the full demand must also be released in the current month as well. Shih and ReVelle (1995) explain that the use of this constraint is to prevent a "flip flopping" or back and forth movement between rationing and non-rationing. They go on to say that the constraint does not prevent this from happening in the real situation, but prevents it in the determination of the trigger volumes (Shih and ReVelle, 1995). Equation (3.17) then ensures that if the full demand is released in the current month, then either full demand or phase one rationing must occur in the subsequent month. This prevents the releases to the community jumping from full demand to strict phase two rationing between months; the community must at least have one month of phase one rationing before phase two rationing is enforced.

Equation (3.18) enables the managers of the reservoir system to specify the number of months in which phase two rationing is allowed through the use of the variable n. This variable n is subtracted from the total number of months in the event horizon (T) and then equated to the sum of the binary variable y_{2t}; constraining the model to select $T - n$ months where phase one rationing or better is implemented. Equation (3.19) then ensures that a range of variables remain nonnegative and thus physically realistic.

3.2.3 Model 4 - Extension of Drought Model

Model 3, the MILP model presented by Shih and ReVelle (1995), has provided a strong and comprehensive framework that can be built upon and extended in this thesis to create a model that is applicable to a cascade reservoir system. This extended model to investigate a drought event, denoted as Model 4, can be found on the following page and is given by Equations (3.20) to (3.62). As seen in the previous chapter, variables denoted with a star (*) correspond to Reservoir 1, whilst those variables without a star denote variables associated with Reservoir 2.

$$\text{maximise } z = \sum_{t=1}^{T} y_{1t}^* + \sum_{t=1}^{T} y_{1t} - \sum_{t=0}^{T} A_t^* - \sum_{t=0}^{T} A_t - \omega \sum_{p=1}^{12}(V_{1p}^* + V_{2p}^* + V_{3p}^*) - \omega \sum_{p=1}^{12}(V_{1p} + V_{2p} + V_{3p}) \quad (3.20)$$

such that

Reservoir 1 Constraints:

$$y_{1t}^* \geq \frac{(S_{t-1}^* + \hat{I}_t^*) - (V_{1p}^* - \epsilon)}{M} \quad \forall t, p, \quad (3.21)$$

$$y_{1t}^* \leq 1 - \frac{V_{1p}^* - (S_{t-1}^* + \hat{I}_t^*)}{M} \quad \forall t, p, \quad (3.22)$$

$$y_{2t}^* \geq \frac{(S_{t-1}^* + \hat{I}_t^*) - (V_{2p}^* - \epsilon)}{M} \quad \forall t, p, \quad (3.23)$$

$$y_{2t}^* \leq 1 - \frac{V_{2p}^* - (S_{t-1}^* + \hat{I}_t^*)}{M} \quad \forall t, p, \quad (3.24)$$

$$R_t^* = (1 - \alpha_1).D^*.y_{1t}^* + (\alpha_1 - \alpha_2).D^*.y_{2t}^* + \alpha_2.D^* \quad \forall t, \quad (3.25)$$

$$S_t^* = S_{t-1}^* + I_t^* - R_t^* - W_t^* \quad \forall t, \quad (3.26)$$

$$S_t^* \leq C^* \quad \forall t, \quad (3.27)$$

$$S_0^* \leq S_T^*, \quad (3.28)$$

$$S_0^* = C_0^*, \quad (3.29)$$

$$U_t^* \leq \frac{S_t^*}{C^*} \quad \forall t, \quad (3.30)$$

$$W_t^* \leq MU_t^* \quad \forall t, \quad (3.31)$$

$$V_{1p}^* \geq (1 + \beta_1)V_{2p}^* \quad \forall p, \quad (3.32)$$

$$V_{2p}^* \geq (1 + \beta_2)V_{3p}^* \quad \forall p, \quad (3.33)$$

$$V_{3p}^* \geq \alpha_2.D^* \quad \forall p, \quad (3.34)$$

$$S_{t-1}^* + \hat{I}_t^* \geq V_{3p}^* + \epsilon \quad \forall t, p, \quad (3.35)$$

$$y_{1t-1}^* + y_{1t+1}^* \leq 1 + y_{1t}^* \quad \forall t, \quad (3.36)$$

$$y_{1t}^* \leq y_{2t+1}^* \quad \forall t, \quad (3.37)$$

$$\sum_{t=1}^{T} y_{2t}^* = T - n^* \quad \forall t, \quad (3.38)$$

$$\hat{I}_t^* = I_t^* \quad \forall t, \quad (3.39)$$

$$S_t^* + A_t^* \geq \alpha_2.D^* \quad \forall t \in \{0, 1, ..., T\}, \quad (3.40)$$

$$S_t^*, S_{t-1}^*, \hat{I}_t^*, I_t^*, V_{1p}^*, V_{2p}^*, V_{3p}^*, R_t^*, W_t^*, A_t^* \geq 0 \quad \forall t, p. \quad (3.41)$$

Reservoir 2 Constraints:

$$y_{1t} \geq \frac{(S_{t-1} + \hat{I}_t) - (V_{1p} - \epsilon)}{M} \qquad \forall t, p, \quad (3.42)$$

$$y_{1t} \leq 1 - \frac{V_{1p} - (S_{t-1} + \hat{I}_t)}{M} \qquad \forall t, p, \quad (3.43)$$

$$y_{2t} \geq \frac{(S_{t-1} + \hat{I}_t) - (V_{2p} - \epsilon)}{M} \qquad \forall t, p, \quad (3.44)$$

$$y_{2t} \leq 1 - \frac{V_{2p} - (S_{t-1} + \hat{I}_t)}{M} \qquad \forall t, p, \quad (3.45)$$

$$R_t = (1 - \alpha_1).D.y_{1t} + (\alpha_1 - \alpha_2).D.y_{2t} + \alpha_2.D \qquad \forall t, \quad (3.46)$$

$$S_t = S_{t-1} + I_t - R_t - W_t + W_t^* \qquad \forall t, \quad (3.47)$$

$$S_t \leq C \qquad \forall t, \quad (3.48)$$

$$S_0 \leq S_T, \qquad (3.49)$$

$$S_0 = C_0, \qquad (3.50)$$

$$U_t \leq \frac{S_t}{C} \qquad \forall t, \quad (3.51)$$

$$W_t \leq MU_t \qquad \forall t, \quad (3.52)$$

$$V_{1p} \geq (1 + \beta_1)V_{2p} \qquad \forall p, \quad (3.53)$$

$$V_{2p} \geq (1 + \beta_2)V_{3p} \qquad \forall p, \quad (3.54)$$

$$V_{3p} \geq \alpha_2.D \qquad \forall p, \quad (3.55)$$

$$S_{t-1} + \hat{I}_t \geq V_{3p} + \epsilon \qquad \forall t, p, \quad (3.56)$$

$$y_{1t-1} + y_{1t+1} \leq 1 + y_{1t} \qquad \forall t, \quad (3.57)$$

$$y_{1t} \leq y_{2t+1} \qquad \forall t, \quad (3.58)$$

$$\sum_{t=1}^{T} y_{2t} = T - n \qquad \forall t, \quad (3.59)$$

$$\hat{I}_t = I_t \qquad \forall t, \quad (3.60)$$

$$S_t + A_t \geq \alpha_2.D \qquad \forall t \in \{0, 1, ..., T\}, \quad (3.61)$$

$$S_t, S_{t-1}, \hat{I}_t, I_t, V_{1p}, V_{2p}, V_{3p}, R_t, W_t, A_t \geq 0 \qquad \forall t, p. \quad (3.62)$$

3.2.4 Model 4 Description

The generic nature with which Model 3 was built by Shih and ReVelle (1995) has allowed for it to be easily extended to a cascade reservoir system, with very few changes required. Overall, the structure of the model has not changed from Model 3 to Model 4, as nearly all of the constraints simply needed to be replicated a second time to account for the addition of a second reservoir to the system. That being said, a few additions have been made to Model 4.

One shortcoming that was noted previously regarding Model 3 was the assumption that the reservoir storage level could not decrease below the lower boundary trigger volume, V_{3p}. This assumption was felt to be unrealistic and overly restricting the model; therefore was removed in the formulation of Model 4. In the place of this assumption, a new variable has been included in the model. The variable, denoted as A_t, measures the extent by which the reservoir storage level decreases below the lower boundary trigger volume in any given month. Though it is now acceptable for the reservoir storage level to decrease past V_{3p}, it is not favourable. Therefore, the variable A_t also features in the objective function of Model 4 (Equation (3.20)) as a penalty term to minimise the number of months in which the reservoir level falls below V_{3p}. By removing the restricting assumption and including this additional variable in the model, it is felt that Model 4 now more accurately reflects the real world behaviour of a cascade reservoir system.

Variable Definitions

There are 25 variables used in the formulation of Model 4, with all of the variables duplicated twice to account for the addition of a second reservoir to the system. All of these variables have been previously defined for Model 3 in Section 3.2.2 under the heading Variable Definitions, except for one:

A_t = the unknown extent by which the reservoir storage is below the lower boundary trigger volume (V_{3p}) in month t.

The way in which this additional variable interacts with the existing variables in the model can be investigated through the examination of the constraints used to formulate Model 4.

Constraint Definitions

Model 4 is made up of 42 constraints, plus the objective function; however 21 of these constraints are repeated from Reservoir 1 to Reservoir 2. The majority of these constraints have been defined previously in Section 3.2.2 under the heading Constraint Definitions, except for Equations (3.29), (3.39) and (3.40) of Reservoir 1 and Equations (3.50), (3.60) and (3.61) of Reservoir 2. Also, the role of the objective function of Model 4 has been altered to include the additional variable A_t. The objective function (Equation (3.20)) of Model 4 is very similar in structure to that of Model 3; however now the primary objective is to maximise the number of months in which no rationing is required from both reservoirs. A secondary component of the objective function, to minimise the total of the trigger volumes, is also the same as that seen in Model 3; though now it has been modified to minimise the trigger volumes of both reservoirs. New additions to the objective function of Model 4 are the terms containing the variable A_t. These terms are added in an attempt to minimise the extent by which the reservoir storage levels decrease below the lower boundary trigger volume, V_{3p}. Of particular note regarding these terms is the fact that the sum of A_t is from $t = 0$ to T. The reason for this is that the initial capacity of the reservoir may begin below V_{3p} in some situations and this needs to be taken into account.

Equations (3.29)and (3.50) are included in Model 4 to offer the managers of the reservoir system an opportunity to specify the initial storage levels of both reservoirs (C_0). However, if the managers do not have a defined initial capacity, this constraint can be relaxed to the form $S_0 \leq C$, where C is the capacity of the reservoir being considered.

Similar to Equations (3.29) and (3.50), Equations (3.39) and (3.60) are included to give the operators of the reservoir system the opportunity to assume that perfect prior knowledge of the monthly inflow to the reservoir system is known. On the other hand, if there is some uncertainty regarding the monthly inflow, these constraints can be relaxed.

Equations (3.40) and (3.61) are used to calculate the variable A_t, the extent by which the reservoir storage level is below the lower boundary trigger volume V_{3p}. As mentioned previously, V_{3p} is specified by the operators of the reservoir system through the selection of the variables D and α_2. In this case, the product of these variables feature in Equations (3.40) and (3.61) in favour of V_{3p} to simplify the statement of the constraint. Therefore, if the reservoir storage level is above V_{3p}, then the value of A_t is zero. However, if the reservoir storage level decreases below V_{3p}, then the value of A_t equals $\alpha_2 D - S_t$; the extent by which the reservoir level is below V_{3p}.

As seen in the previous chapter, the only constraint to vary between Reservoir 1 and Reservoir 2 is the mass balance constraint, given by Equation (3.26) and Equation (3.46) respectively. In the case of Reservoir 2, the mass balance constraint contains the additional variable W_t^*, which measures the amount of spill from Reservoir 1 in the current month. As the reservoirs are orientated in a cascade configuration, any spill from Reservoir 1 becomes a source of inflow to Reservoir 2 and therefore needs to be included in the mass balance constraint.

3.3 Results and Discussion

In this section, Model 4 is applied to the case study of the Perseverance and Cressbrook cascade reservoir system and an experiment conducted to investigate how this existing system can be optimally managed during a drought event. From the results of the experiment, the key components of the cascade reservoir system measured by Model 4 that influence the operation of the system during a drought event are explored and interpreted. At this point it is important to note that for the purposes of this experiment, Perseverance dam is referred to as Reservoir 1, while Cressbrook dam is labelled Reservoir 2.

For this experiment, as with all of the experiments conducted in this thesis, the optimisation modelling software LINGO version 14.0 was utilised (LINDO Systems Inc, 2013). An example of the syntax required to perform this experiment using LINGO, along with a portion of the output generated by the software can be found in Appendix B. In this case, the software became a limiting factor as to the number of months over which the experiment could be considered. The version of LINGO obtained to perform the experiment was under an Academic License and therefore the total number of constraints and variables that the software could consider was limited. This limitation, coupled with the size of Model 4, meant that the experiment could not be considered for a time period longer than 24 months or two years. In order to enable a more thorough exploration of the management strategies employed by Model 4, a series of assumptions were made to simplify some of the parameters of the experiment, while at the same time ensuring that the results were still applicable to the case study under investigation. The first of these assumptions regarded the inflows to the reservoir system. Rather than assuming that the reservoirs were subject to constant inflows each month, as in the previous chapter, historic inflows for the region were sourced from the Queensland Department of Natural Resources and Mines (2012). The observations from Cressbrook Creek, the major tributary flowing into Cressbrook dam, were only recorded from November 1965 to May 1981; leading to an approximate 16 year time period over which the historic inflows could be sourced. Over this time period, the inflows were screened and the years that offered the lowest flows were combined to produce a worst case drought scenario. The combined years were October 1969 to September 1970 and October 1977 to September 1978, in which the total flows were significantly lower than any of the other years considered. After further research, no inflow observations were able to be found for the tributaries that supply Perseverance dam. Therefore, due to the proximity of the two reservoirs, it was concluded that the historic inflows sourced for Cressbrook dam could be used as a proxy source of inflows for Perseverance dam; however a scale difference of 60% would be applied. This scale difference was determined by prior knowledge regarding the size of the tributaries that provide water to the two reservoirs. The monthly inflow to the two reservoirs over the 24 month event horizon is presented in Table D.1 of Appendix D, while a graphical representation of the inflows is provided by Figure 3.3 on the following page.

Another assumption made regarding the inflows was to specify that there was perfect prior knowledge of the monthly inflow to the reservoir system. This was achieved through the use of Equations (3.39) and (3.60), and as mentioned previously, was an option that could be specified by the managers of the reservoir system. Although, in practice this assumption may not be feasible, it was made in

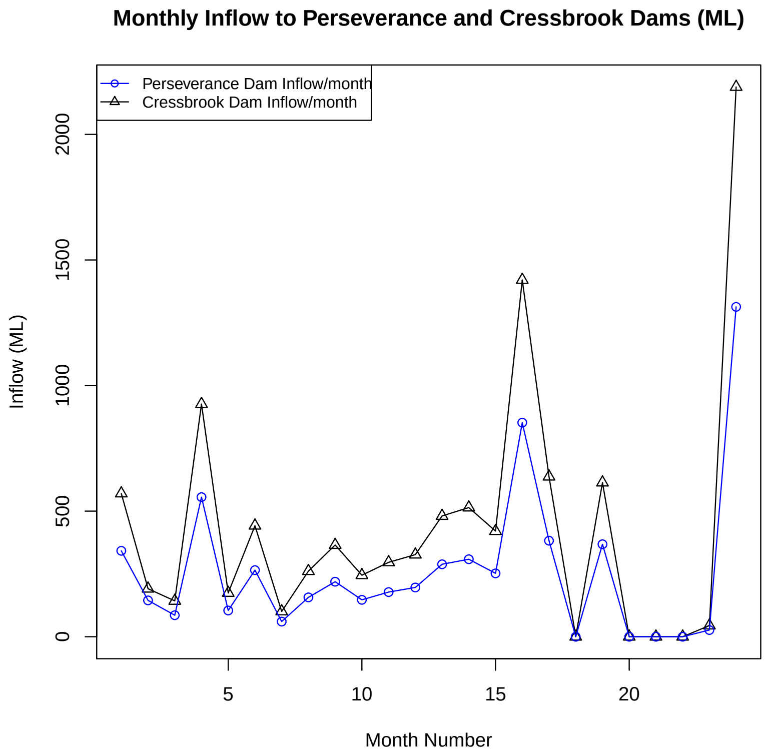

Figure 3.3: Monthly Inflow to Perseverance and Cressbrook Dams under Worst Case Drought Scenario from Historic Records.

order to reduce the complexity of the model. A comparison of the results from a model where these constraints have been relaxed or removed, to the results from this experiment provides scope for future research.

The next assumption concerned the storage capacities of both reservoirs. The Toowoomba Regional Council (2013) states that the maximum available storage of Cressbrook dam is 81 842 megalitres (ML), while for Perseverance dam it is 30 140 ML. Using this information, it was assumed for the purposes of this thesis that the capacity of Cressbrook dam and Perseverance dam were 82 000 ML and 30 000 ML respectively. Based on these values, it was concluded that the assumed capacity of Perseverance dam was approximately one third the assumed size of Cressbrook dam; a conclusion that was used to determine how the demand from the community could be shared between the two reservoirs.

From further information provided by the Toowoomba Regional Council regarding the average water usage per person, per day and the approximate population of the Toowoomba region (Toowoomba Regional Council, 2013), a calculation was performed to determine the approximate community demand for water each month. From this information it was assumed that an approximate demand of 1000 ML per month was needed in order to satisfy the community water requirements. It was assumed that this demand was shared between the two reservoirs according to the size of their respective storage capacities. Therefore, for the purposes of this experiment, Perseverance dam was responsible for supplying one third of the community demand (333 ML) while the remaining two thirds was sourced from Cressbrook dam (667 ML). These values represented a best case scenario that provided the community with a generous amount of water and enabled water use for extraneous purposes such as watering gardens and washing vehicles. However, during a drought, these generous volumes cannot always be met and water rationing needs to be enforced in order to ensure that there is sufficient water available, over the course of the event, to maintain vital water services. The three rationing levels proposed by Shih and ReVelle (1995) in Model 3 are also included in Model 4; being no rationing, phase one rationing and phase two rationing. The variables α_1 and α_2 can be used to define the percentage of the total demand released to the community each month under phase one and phase two rationing respectively. In this case, it was assumed that α_1 was equal to 60%, while α_2 was equal to 40%. This meant, for the purposes of the experiment, that each month phase one rationing was enforced, 60% of the total demand was released to the community. This translated to a release of 200 ML from Reservoir 1 or 400 ML from Reservoir 2 per month of phase one rationing. If the stricter phase two rationing was applied, then 40% of the total demand was released to the community per month. This equated to a release of 133 ML from Reservoir 1 or 267 ML from Reservoir 2 per month of phase two rationing.

Overall, it should be noted that the series of assumptions made above were selected to best approximate the case study under investigation. Depending on the type of system being explored, all or some of these values could be altered by the operators and managers of a reservoir system to suit their own requirements.

In order to investigate the different management strategies employed by Model 4, three scenarios were selected. In each of these scenarios, the initial reservoir capacities and the number of months in which phase two rationing is permitted were altered, while the remaining parameters of the model were kept constant. The results from Model 4 for each of these combinations of initial reservoir capacities and months of permissible phase two rationing can be found in the following sections.

3.3.1 Scenario 1: Unconstrained Initial Reservoir Capacities ($S_0 \leq C$), with no Phase Two Rationing Permitted ($n = 0$)

The first combination of initial reservoir capacities and months of permissible phase two rationing considered was to not constrain the initial capacity of both reservoirs, while specifying that no phase two rationing was allowed. By not constraining the initial reservoir capacities, Model 4 could select the optimal initial storage levels of the reservoirs, given that no stricter rationing level than phase one could be

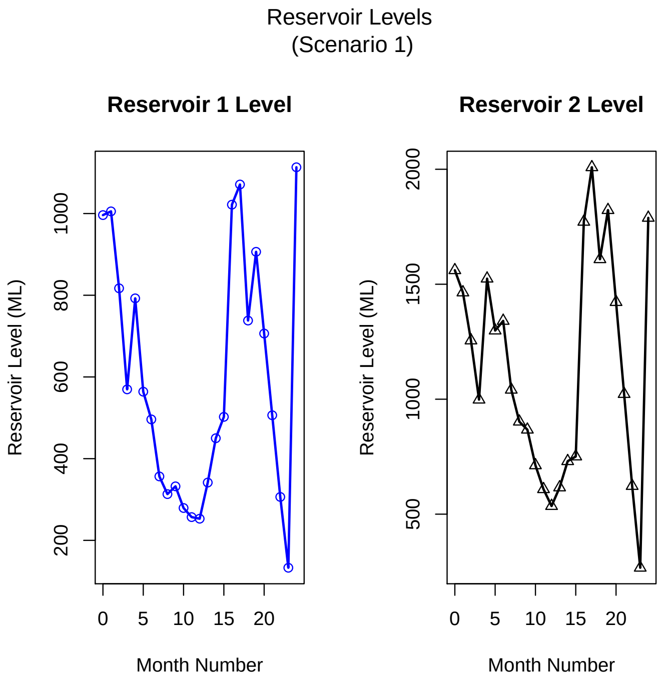

Figure 3.4: **Scenario 1** ($S_0 \leq C$ **and** $n = 0$) - Comparison of Reservoir Levels (ML).

enforced over the event horizon. In order to investigate the results of Model 4 under the conditions specified, a series of figures have been constructed. The first of these, Figure 3.4, provides a comparison of the reservoir water levels over the event horizon while Figure 3.5 displays a comparison of the monthly releases to the community from both reservoirs. Also included is Figure 3.6 which shows the trigger volumes of the two reservoirs, while Figures 3.7 and 3.8 provide a demonstration of the relationship between the available storage, trigger volumes and releases to the community for both reservoirs.

First to be considered is Figure 3.4, from which it can be seen that Model 4 selected the optimal initial reservoir capacity of Reservoir 1 to equal approximately 1000 megalitres (ML). On the other hand, the initial reservoir capacity of Reservoir 2 was optimally selected by the model to equal approximately 1500 ML. It should be noted that both of these initial storage capacities were significantly smaller than the total capacity of each reservoir, demonstrating that the assumed monthly community water demand was marginal in comparison to the overall capacities of the reservoirs. However, as a drought event is considered in this chapter, it was reasonable to assume that the water levels of the reservoirs in the system were critically low.

Also of note from Figure 3.4, was that the behaviour of the water levels of both reservoirs were approximately the same. That is, both reservoir storage levels exhibited a decreasing trend for approximately the first half of the event horizon until

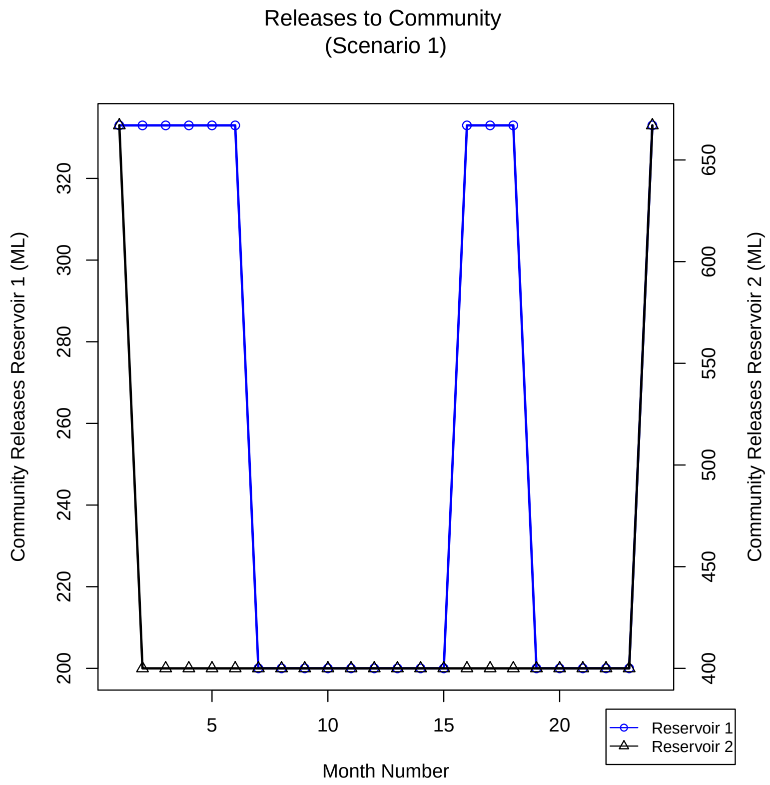

Figure 3.5: **Scenario 1** ($S_0 \leq C$ **and** $n = 0$) - Comparison of Releases to Community Water Supply (ML).

a series of inflows to the system increased the storage levels to above their initial conditions. In the subsequent months, the levels of both reservoirs decreased sharply due to minimal inflows, until the final month of the event horizon. At this time, a significant inflow occurred, replenishing the reservoir levels to above their initial capacities once more.

From Figure 3.5, it can be seen that the model observed the specification that no phase two rationing was permitted over the event horizon, as there were two distinct volumes of releases for both reservoirs. In the case of Reservoir 1, it can be seen that for the first six months of the event horizon no rationing was enforced. Following this, there was a period of nine months where the model managed the use of water from the system by applying phase one rationing. After this period, the model opted to release the full demand to the community for a further three months, before applying phase one rationing again until the final month of the event horizon where no rationing was enforced. On the other hand, when Reservoir 2 is considered, it can be seen that the only months in which no rationing was required were the first and last month of the event horizon. During the remaining months, the releases were restricted under phase one rationing, meaning a reduced volume was supplied to the community.

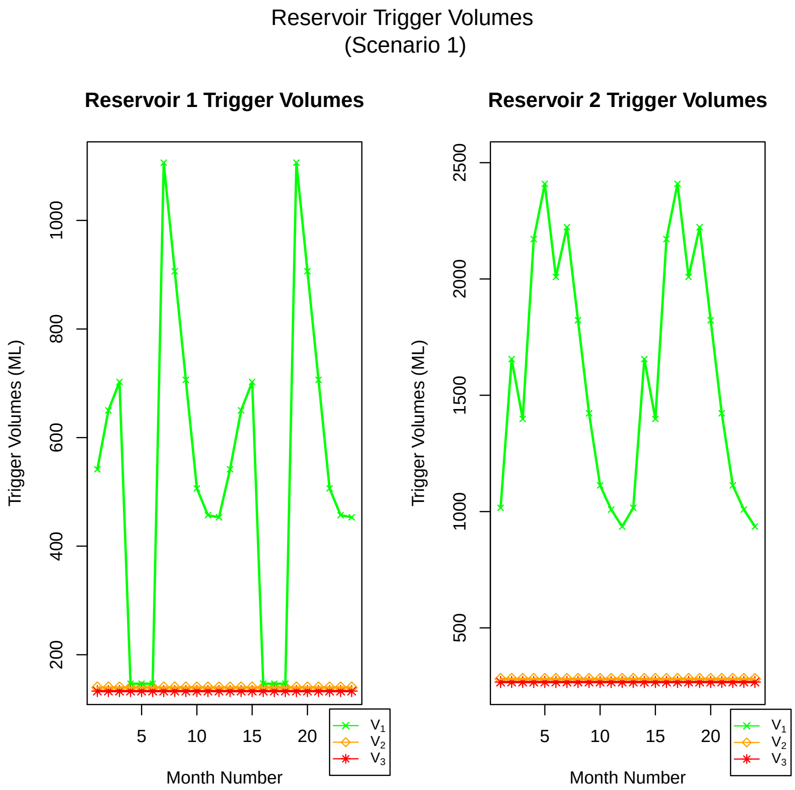

Figure 3.6: **Scenario 1 ($S_0 \leq C$ and $n = 0$)** - Comparison of Reservoir Trigger Volumes (ML).

Figure 3.6 displays the three trigger volumes, V_{1p}, V_{2p} and V_{3p}, for both reservoirs. For simplicity, from this point in the chapter the three trigger volumes will be referred to as V_1, V_2 and V_3. As mentioned previously, the trigger volumes are specified over the months p, where p ranges from 1 to 12. Therefore, the trigger volume in January of the current year will be the same as the trigger volume of January in the next and all subsequent years. In this case, the event horizon was limited to a 24 month period, therefore the trigger volumes will be duplicated twice. The repetitive behaviour of the trigger volumes is demonstrated in Figure 3.6. A figure of this type is only provided once in this chapter to demonstrate the repetitive behaviour of the trigger volumes. For the other two scenarios considered in the following sections, the trigger volumes are presented in conjunction with the available water of the reservoirs, as in Figures 3.7 and 3.8.

Figures 3.7 and 3.8 present the relationship between the available water and trigger volumes for each reservoir respectively. These figures also demonstrate how this relationship could be used to determine the level of rationing enforced, and thus the volume of water released to the community each month. As defined previously, the available water is equal to the reservoir storage level in the previous month, plus the projected inflow to the reservoir in the current month.

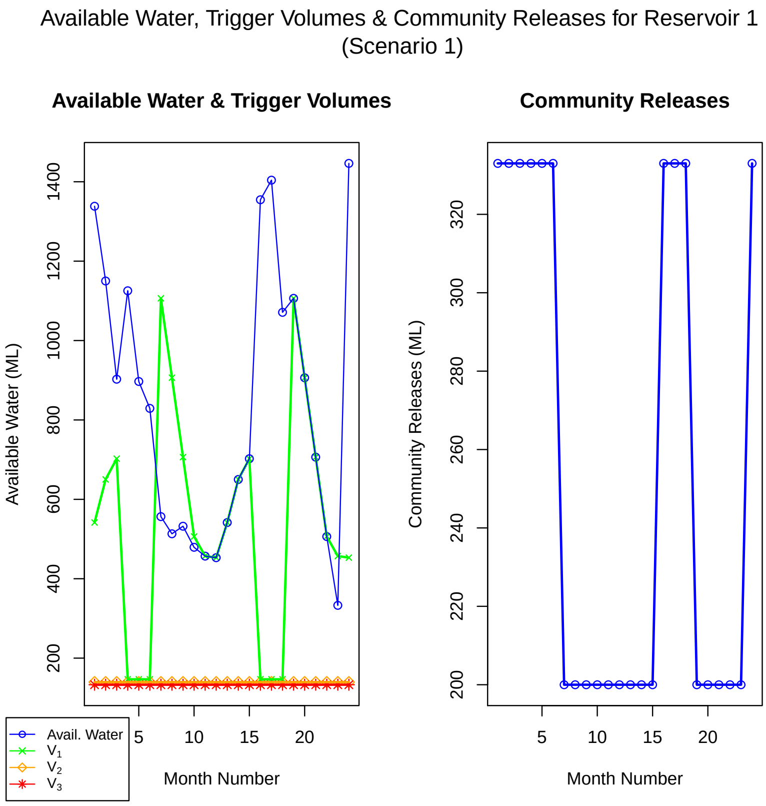

Figure 3.7: **Scenario 1** ($S_0 \leq C$ **and** $n = 0$) - Relationship between Available Water + Trigger Volumes and Community Releases for Reservoir 1.

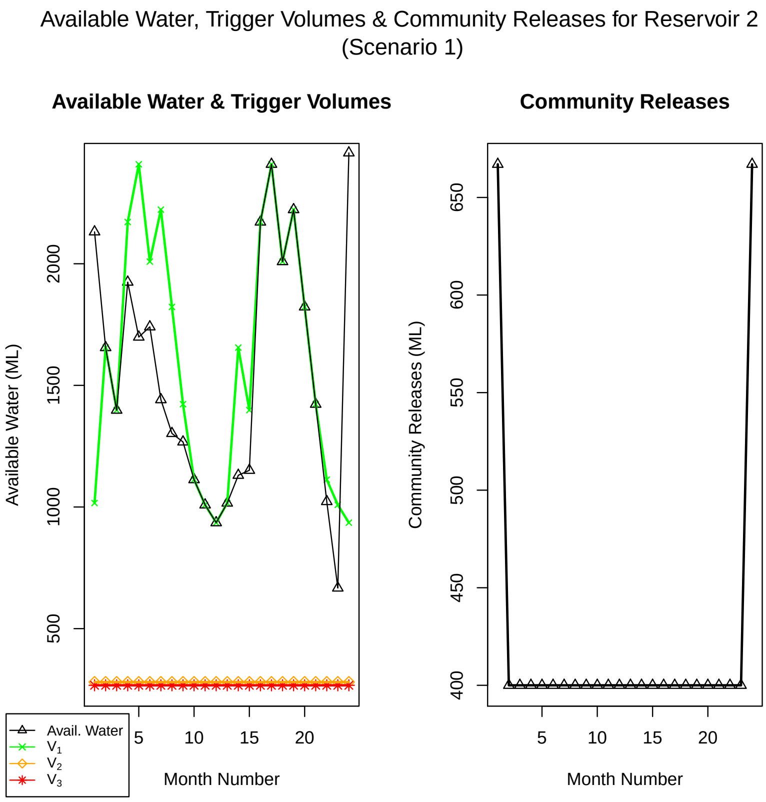

Figure 3.8: **Scenario 1** ($S_0 \leq C$ **and** $n = 0$) - Relationship between Available Water + Trigger Volumes and Community Releases for Reservoir 2.

When considering Figures 3.7 and 3.8, if the available water was above the trigger volume V_1 (described by the green line with crosses), then no rationing was enforced in that month. However, if the available water decreased below V_1, but was greater than the trigger volume V_2 (represented by the orange line with diamonds), phase one rationing was implemented for the month. In Figures 3.7 and 3.8, the available water did not decrease lower than the trigger volume V_2, indicating that the most severe rationing was not imposed over the event horizon, as specified by the conditions of the scenario under investigation. Due to this, the model has minimised the value of the trigger volumes V_2 each month in line with the objective function. Although the figures may depict that the trigger volumes V_2 and V_3 were equal, there were separation values imposed (β_1 and β_2) that required V_2 be at least 5% greater than V_3.

Figures 3.7 and 3.8 also demonstrate the relationship between the available water and trigger volumes, with the amount of water released to the community. As can be seen from Figure 3.7, no rationing was imposed on the releases to the community from Reservoir 1 for the first six months of the event horizon, as the available water was greater than the trigger volume V_1. This meant that the full demand from Reservoir 1 to the community of 333 ML could be supplied over this period. However, once the available water decreased below V_1, phase one rationing was enforced and the reduced volume of 200 ML per month, or 60% of the full demand, was provided to the community. This was reversed after a further nine months, when the available water exceeded V_1 and the full demand from Reservoir 1 to the community could be released again. This behaviour continued over the remainder of the event horizon. The same relationship is also exhibited in Figure 3.8, though for Reservoir 2 the full demand to the community of 667 ML could only be met in the first and last month of the event horizon. For the remaining months, the available water was not greater than the trigger volume V_1, meaning phase one rationing was applied and that the reduced volume of 400 ML was made available to the community.

By specifying as a condition of the scenario that phase two rationing was not permitted, the model was restricted in the number of strategies that it could explore to optimally manage the reservoir system. However, this was somewhat counterbalanced by not constraining the initial reservoir capacities, as the model could optimally select these capacities in order to ensure that the community demand was able to be met over the event horizon. In the following sections, this ability to select the initial reservoir capacities is removed from the model; though the availability to enforce phase two rationing is provided.

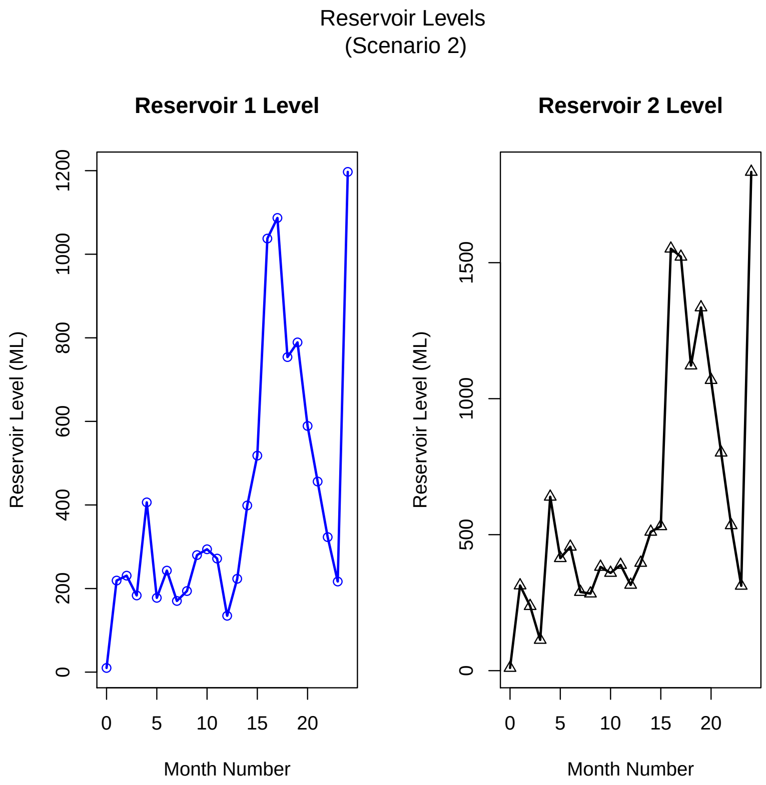

Figure 3.9: **Scenario 2 ($S_0 = 10$ and $n = 12$)** - Comparison of Reservoir Levels (ML).

3.3.2 Scenario 2: Initial Reservoir Capacities of 10 ML ($S_0 = 10$)*, with 12 Months of Phase Two Rationing Permitted* ($n = 12$)

The next combination of initial reservoir capacities and months of permissible phase two rationing was where the initial capacity of both reservoirs was specified to be near to empty or 10 ML in this case. The other condition was to deem that 12 months of phase two rationing were acceptable. Although the model was not able to optimally select the initial capacities of the reservoirs in this case, there were a wider range of management strategies that could be considered due to phase two rationing being permitted. In order to explore the results from Model 4 under Scenario 2, a series of figures have been generated.

The first figure constructed to investigate the results of Model 4 under Scenario 2 is Figure 3.9. From the figure, it can be seen that the model observed the specified initial condition that the initial reservoir capacity be set to 10 ML for both reservoirs. Also apparent, as mentioned in the previous section, was the fact that the levels of both reservoirs behave similarly, though varied according to a difference in magnitude. Under the conditions of the scenario, both reservoir levels displayed an overall increasing trend across the event horizon until approximately month 18, where a period of reduced inflows caused the reservoir levels to drop significantly. Following this low flow period, the levels of both reservoirs showed a sharp increase for the final month of the event horizon. When this behaviour was compared to Figure 3.3 presented earlier in the chapter, of the inflows under a worst case drought event, it was seen that from month 18 to 23 there was a severe shortage of water flowing into the reservoir system; thus causing the rapid decrease in reservoir level, as water was still being removed from the reservoir to supply community demand. Also clear from Figure 3.3 was that significant inflows occurred in the last month of the event horizon, larger than the inflow in any other month considered, and thus was identified to be responsible for the significant spike in the reservoir levels mentioned previously.

Figure 3.10 presents the releases from each reservoir in order to meet the community water supply demand. From the figure, it can be seen that there were three distinct levels of releases made to the community over the event horizon. These three levels corresponded to the releases when no rationing, phase one rationing or phase two rationing was imposed. In this case, by specifying as a condition of the scenario that the initial reservoir capacity equal 10 ML, the model was forced to initialise the reservoir levels in a state that was less than their respective lower boundary trigger volumes, V_3. Therefore, under this initial reservoir capacity the variable A_t would have penalised the objective function from the first month. In order to increase the level of the reservoirs above their respective lower boundary trigger volumes as quickly as possible and prevent this penalisation of the objective function, the model has selected to enforce phase two rationing in both reservoirs for the first three months. After this time, the behaviour of the releases from the two reservoirs began to vary.

Over the event horizon, the releases to community water supply from Reservoir 1 were not constrained by rationing for a total of eight months. However, the months where no rationing was enforced were not in consecutive order, with the releases fluctuating on a month to month basis. On the other hand, the releases from Reservoir 1 were constrained by phase two rationing for a further nine months, in addition to the first three months, meaning that the total permissible duration of 12 months in which phase two rationing was allowed were exploited. The releases were also subject to phase one rationing for three months throughout the event horizon, with this phase of rationing used as a "stepping stone" from no rationing to the strict phase two rationing; a behaviour specified in the construction of Model 4.

Also evident from Figure 3.10 is that the releases to community water supply from Reservoir 2 were similarly restricted by phase two rationing for the maximum permissible 12 months. Under the conditions of Scenario 2, the model selected to optimally manage Reservoir 2 by providing 10 months in which the community water supply was restricted by phase one rationing. Only in two months was no rationing applied, one of which was the last month of the event horizon where a

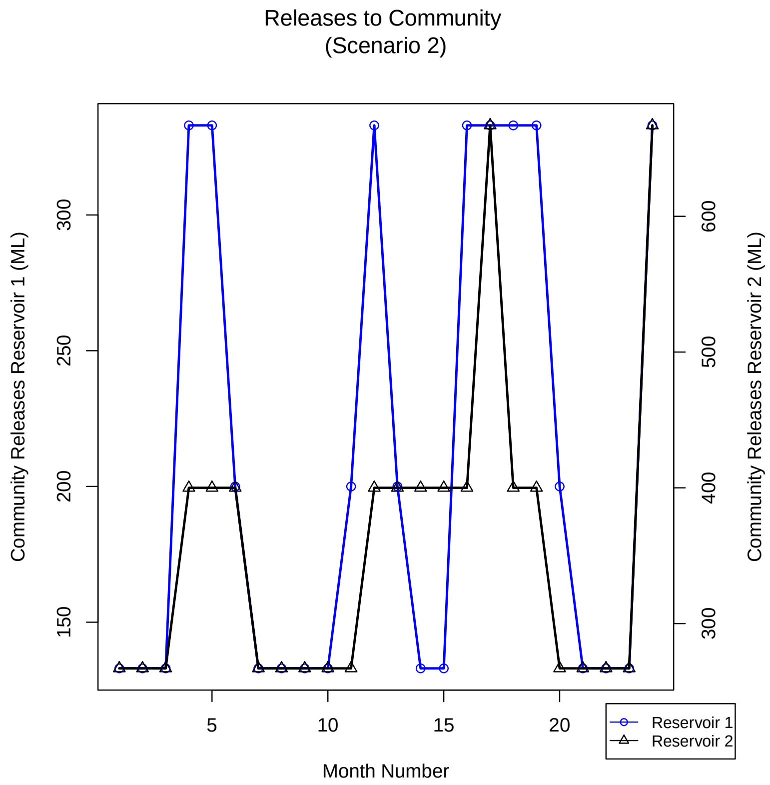

Figure 3.10: **Scenario 2 (S_0 = 10 and n = 12)** - Comparison of Releases to Community Water Supply (ML).

large inflow occurred to boost the reservoir level.

After considering the results presented in Figure 3.10, it can be seen that the model elected to use the maximum number of months in which phase two rationing was permitted for both reservoirs. By utilising the full 12 months of phase two rationing allowed under the conditions of Scenario 2, the model was able to increase the number of months in which no rationing was enforced and thus maximise the amount of water released to the community. However, there was another strategy that could have been considered. Currently, Model 4 is constructed with the primary objective of maximising the number of months in which no rationing occurs and as such, maximising the amount of water able to be released to the community. Another potential strategy that could be explored is to maximise the number of months in which a consistent supply of water could be provided to the community. An example of this would be to enforce phase one rationing over the total extent of the event horizon, rather than conserving water using a number of months of phase two rationing to later supply a month with no rationing. A comparison of the current model objective against this alternate strategy could be explored and form the basis of future research.

Figures 3.11 and 3.12 demonstrate the relationship between the available water and trigger volumes for both reservoirs. These figures also display how the relationship

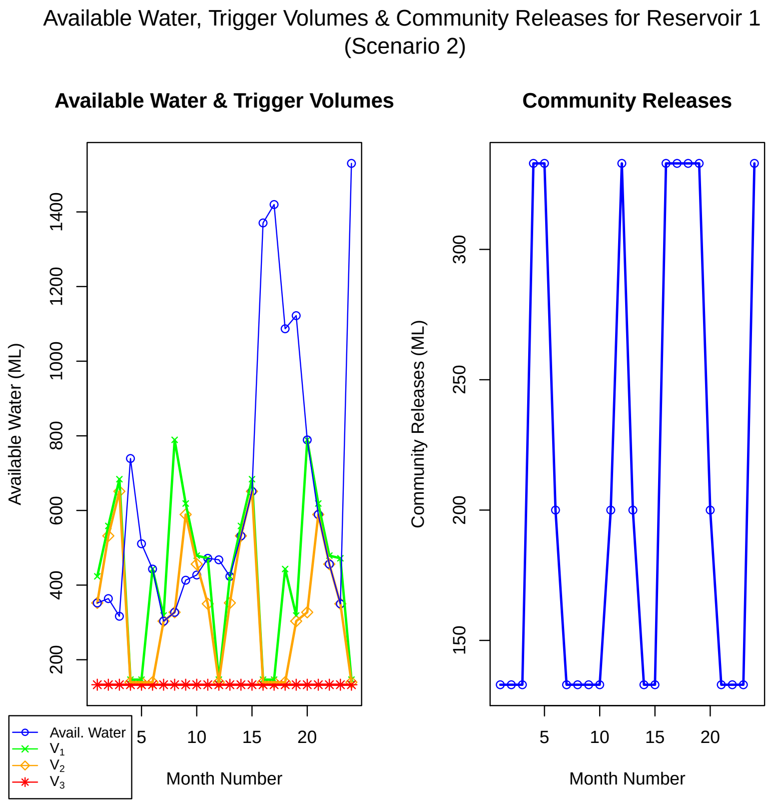

Figure 3.11: **Scenario 2 ($S_0 = 10$ and $n = 12$)** - Relationship between Available Water + Trigger Volumes and Community Releases for Reservoir 1.

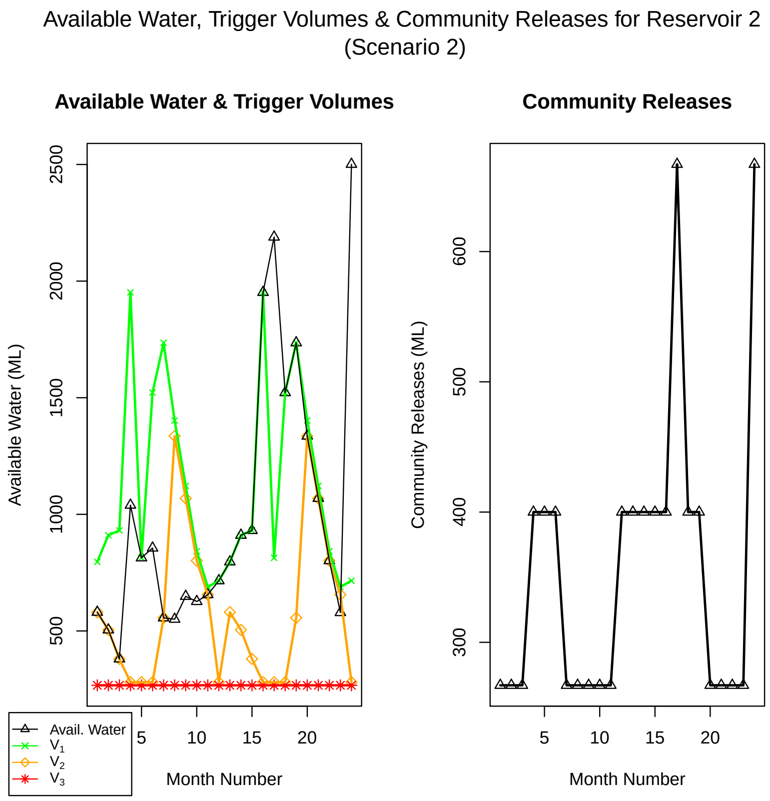

Figure 3.12: **Scenario 2 ($S_0 = 10$ and $n = 12$)** - Relationship between Available Water + Trigger Volumes and Community Releases for Reservoir 2.

between these results interacts with the rationing levels and thus, the releases made to the community each month. One feature that is apparent from Figures 3.11 and 3.12, that was not seen in the previous section due to the conditions of Scenario 1, is the fluctuation of the trigger volume V_2. In the previous section, no phase two rationing was permitted, therefore the model minimised V_2 as it did not play a part in the management strategies utilised. However, under Scenario 2, phase two rationing was permitted and the model used this fact to better manage the operation of the reservoir system. This is clearly demonstrated by both figures, where the available water can be seen to be between the trigger volume V_2 (described by the orange line with diamonds) and the lower boundary trigger volume V_3 (represented by the red line with stars or asterisks), meaning that phase two rationing was employed in those months.

Note, Figure 3.9 demonstrates that the model set the initial capacity of both reservoirs to 10 ML, however Figures 3.11 and 3.12 do not. The reason for this is that Figures 3.11 and 3.12 display the available water; defined as the sum of the reservoir level in the previous month (shown in Figure 3.9) and the inflow to the reservoir in the current month (shown in Figure 3.3). Therefore, although these figures do not show the reservoir level commencing below the lower boundary trigger volume, the objective function was still penalised for the months in which this occurred.

Under the scenario explored in this section, Model 4 was able to consider a wider range of management strategies than that seen previously, as phase two rationing was permitted. However, by specifying the initial capacities of both reservoirs to be near empty, phase two rationing was necessary for half of the event horizon in order to ensure that a feasible management strategy could be determined. In the following section, the initial capacities of the reservoirs are increased; leading to a decrease in the number of months of phase two rationing needed to determine a feasible management strategy.

3.3.3 Scenario 3: Initial Reservoir Capacities of $\alpha_2 D$ ML ($S_0 = \alpha_2 D$), with 6 Months of Phase Two Rationing Permitted ($n = 6$)

The final combination of initial reservoir capacities and months of permissible phase two rationing considered in this chapter, was where the initial capacities of the reservoirs were specified to equal their respective lower boundary trigger volumes of V_3, or $\alpha_2 D$. In this case, the initial capacities of Reservoir 1 and Reservoir 2 equalled 133 ML and 267 ML respectively. The second condition of the scenario was to deem that six months of phase two rationing were acceptable for both reservoirs over the 24 month event horizon considered. Under this scenario, by increasing the initial capacities of both reservoirs, feasible management strategies could be identified using a reduced number of months of phase two rationing. The impact of adjusting these conditions on the results from Model 4 are shown in Figures 3.13 to 3.16.

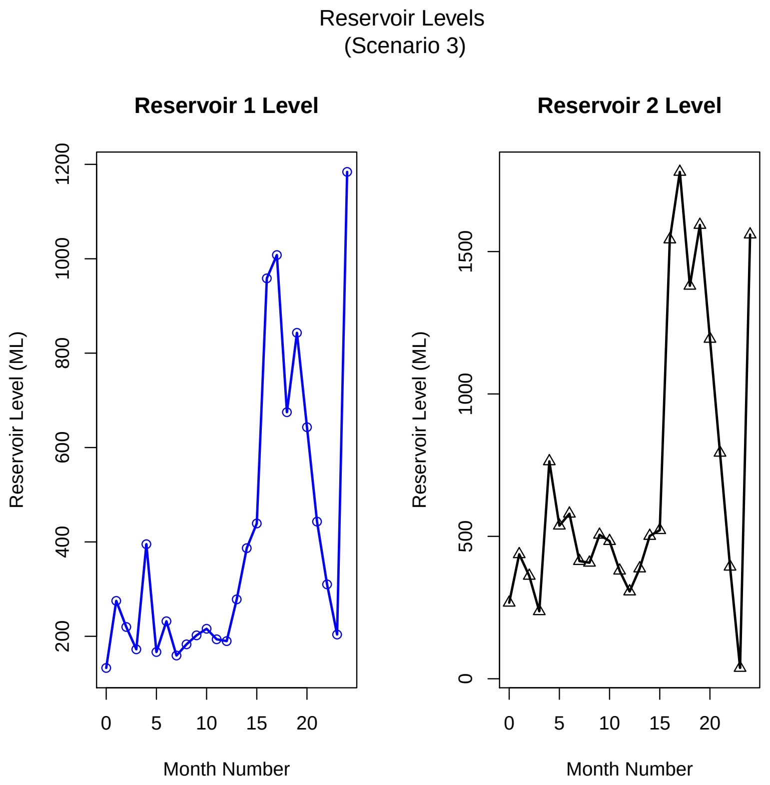

Figure 3.13: **Scenario 3** **(**$S_0 = \alpha_2 D$ **and** $n = 6$**)** - Comparison of Reservoir Levels (ML).

The first of these figures, Figure 3.13, presents the level of each reservoir over the 24 month event horizon considered. From the figure, it can be seen that the first condition of Scenario 3 was met, as the model selected the initial capacity of each reservoir to equal their lower boundary trigger volumes, $\alpha_2 D$, of 133 ML and 276 ML respectively. From these initial capacities the levels of both reservoirs displayed an overall increasing trend, until approximately month 18, where a period of limited inflows significantly reduced the levels of both reservoirs. This occurrence was also noted under the previous scenarios considered, with the cause being identified and explored in the previous section.

Of particular note from Figure 3.13 is the fact that in two months the model elected to decrease the level of Reservoir 2 below the lower boundary trigger volume of 276 ML. This occurred in month 4 and again in month 23. During these months, the objective function would have been penalised by the variable A_t. This response by the model highlights the fact that the community demand could not have been met under the second condition of Scenario 3, that six months of phase two rationing be employed, unless the reservoir level decreased below the lower boundary trigger volume in these months. Therefore, although the objective function has been penalised, this management strategy was still optimal compared to any others considered. That said, if the same situation was to be considered using Model 3, as

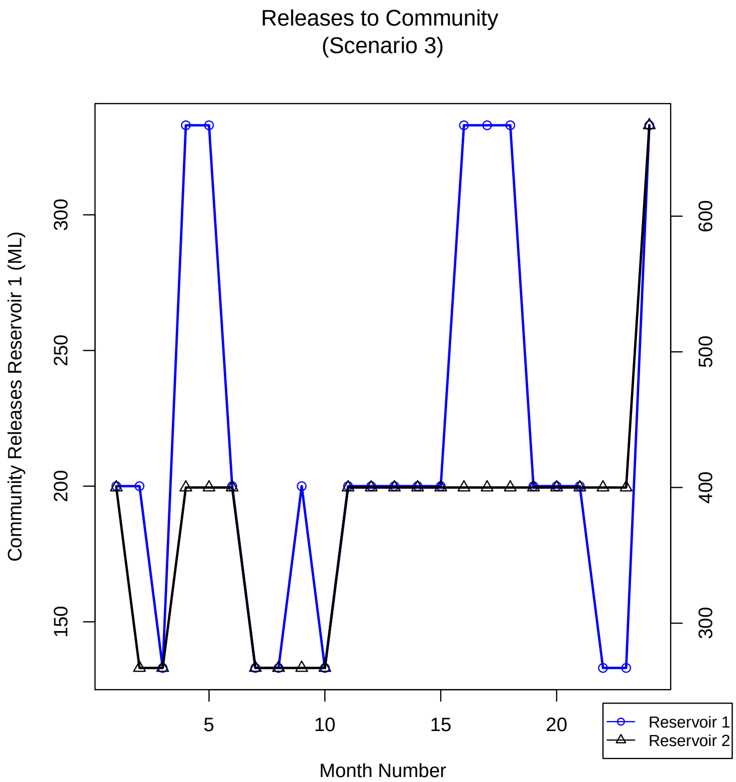

Figure 3.14: **Scenario 3** ($S_0 = \alpha_2 D$ **and** $n = 6$) - Comparison of Releases to Community Water Supply (ML).

formulated by Shih and ReVelle (1995), the current management strategy would be deemed infeasible due to the fact that the reservoir level decreased below the lower boundary trigger volume.

Figure 3.14 displays a comparison of the monthly releases from each reservoir to the community water supply. As seen in the previous section, there were three defined levels of releases corresponding to the three levels of rationing that could be enforced. In the case of Reservoir 1, it can be seen that both no rationing and phase two rationing were implemented individually for a total of six months over the event horizon. For the remaining 12 months, phase one rationing was enforced. On the other hand, there was only a solitary month in which no rationing was enforced on the releases to the community from Reservoir 2, being the last month of the event horizon. For the most part, 17 months in total, phase one rationing was implemented, while there were also six months of phase two rationing, as prescribed by the conditions of the scenario.

When considering the releases in months 4 and 23 from Reservoir 2, it can be seen from Figure 3.14 that the releases continued under phase one rationing although the reservoir level was below the lower boundary trigger volume, and as such the objective function was being penalised. Rather than restricting the releases during these two months to phase two rationing, meaning that two other months currently

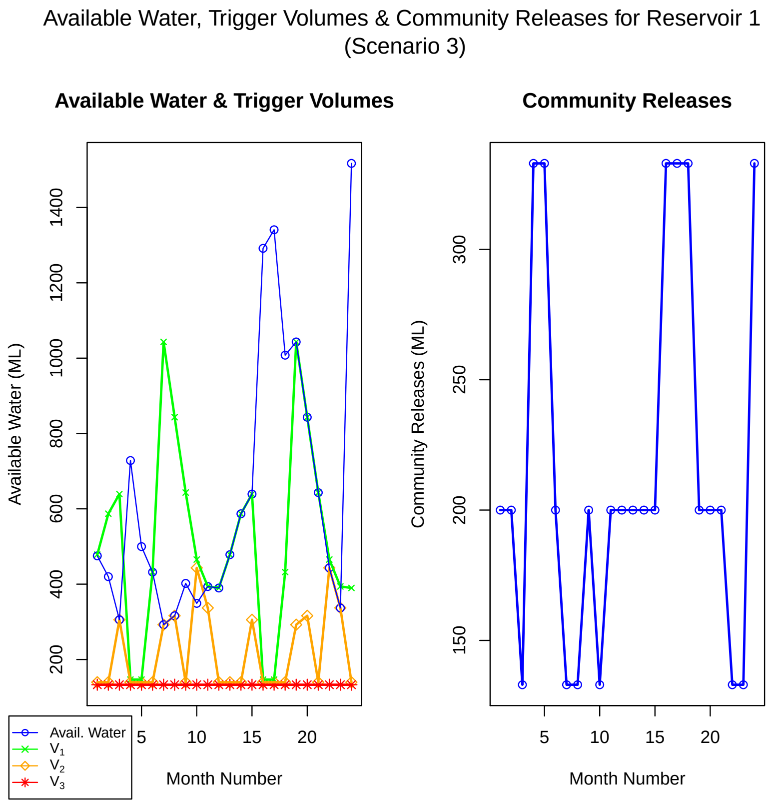

Figure 3.15: **Scenario 3** ($S_0 = \alpha_2 D$ **and** $n = 6$) - Relationship between Available Water + Trigger Volumes and Community Releases for Reservoir 1.

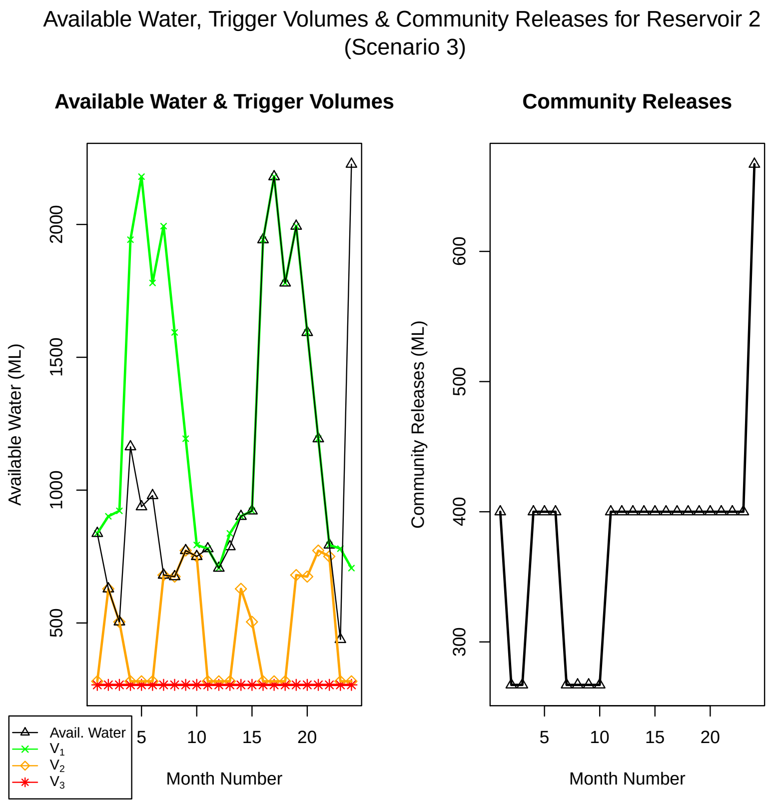

Figure 3.16: **Scenario 3** ($S_0 = \alpha_2 D$ **and** $n = 6$) - Relationship between Available Water + Trigger Volumes and Community Releases for Reservoir 2.

restricted by phase two rationing would move to the less strict phase one rationing, the model selected to decrease the reservoir level below the lower boundary trigger volume. Therefore, by selecting this management strategy as optimal, the model determined that no other combination of months and rationing levels provided a better management strategy under the conditions of Scenario 3.

Figures 3.15 and 3.16 present the relationship between the available water and the trigger volumes for each reservoir. They also demonstrate how this relationship can be used to determine the rationing levels implemented each month and as such, the volume of water available for release to the community. When Figure 3.16 is compared to Figure 3.12 in the previous section, it can be seen that there was only one month in which no rationing could be enforced under the conditions of Scenario 3, compared to two months of no rationing under the conditions of Scenario 2. This demonstrates that although the initial capacities of the reservoirs were greater under Scenario 3 than those examined under Scenario 2, there were fewer months in which the full community demand could be released. This difference can be attributed to the fact that the number of months in which phase two rationing was permitted were reduced under the current scenario. Therefore, as stricter rationing was not being enforced as often, less water was being conserved, leading to a reduction in the number of months in which the full demand could be released.

3.3.4 Comparison of Releases to Community under Scenarios 1, 2 and 3

Using the results from Model 4, under each of the scenarios considered, a comparison of the total volume of water released to the community can be made. The releases to the community, from both reservoirs, under each of the three scenarios has been presented in Table 3.1. This table also presents the number of months in which the three rationing levels were enforced for each reservoir. Using the number of months of rationing and the known volume released under each phase of rationing, the total releases under each phase could be calculated and combined to determine the total volume of water made available to the community over the 24 month event horizon.

In this case, each column of Table 3.1 corresponds to the results collected from Model 4 under each of the scenarios considered. Therefore, from the first column of Table 3.1, it can be seen that under Scenario 1 there were 10 months in which no rationing was enforced upon releases from Reservoir 1 and 14 months where phase one rationing was implemented. In this case, one of the conditions of Scenario 1 was to specify that no phase two rationing was to be enforced over the event horizon; a condition Table 3.1 confirms has been implemented. Over the 10 months in which no rationing was imposed, the full community demand from Reservoir 1 of 333 ML per month could be released, providing a total volume of 3330 ML of water to the community over the event horizon. During the remaining 14 months of the

Table 3.1: Comparison of the Releases to the Community from Reservoir 1 and Reservoir 2, across the 3 Scenarios considered.

	Scenario 1 RC[1], [# months[2]]	**Scenario 2** RC, [# months]	**Scenario 3** RC, [# months]
Reservoir 1			
No Rationing (333 ML/month)	3330, [10]	2664, [8]	1998, [6]
Phase 1 Rationing (200 ML/month)	2800, [14]	800, [4]	2400, [12]
Phase 2 Rationing (133 ML/month)	0, [0]	1596, [12]	798, [6]
Total	**6130, [24]**	**5060, [24]**	**5196, [24]**
Reservoir 2			
No Rationing (667 ML/month)	1334, [2]	1334, [2]	667, [1]
Phase 1 Rationing (400 ML/month)	8800, [22]	4000, [10]	6800, [17]
Phase 2 Rationing (267 ML/month)	0, [0]	3204, [12]	1602, [6]
Total	**10134, [24]**	**8538, [24]**	**9069, [24]**
Combined Total	**16264**	**13598**	**14265**

[1] - Release to Community (ML)

[2] - Number of Months

event, phase one rationing was applied, meaning that the releases were restricted to 60% of the total demand or 200 ML per month. Over these 14 months, 2800 ML were supplied to the community under phase one rationing. This means that under Scenario 1, the community was supplied with 6130 ML from Reservoir 1 over the event horizon, or approximately 76.7% of the potential releases that could have been made from this reservoir.

Table 3.1 also provides results pertaining to the management of Reservoir 2 and, when the first column is considered, demonstrates that of the 24 months that compose the event horizon, in two months no rationing was enforced, while for the remaining 22 months, phase one rationing was applied. Therefore, this confirms that the condition of Scenario 1, that no phase two rationing be enforced, has been implemented for this reservoir as well. Under phase one rationing, the releases from the reservoir were restricted to 60% of the total demand; equating to 400 ML per month from Reservoir 2. Therefore, over the 22 month period in which phase one rationing was enforced, the community was supplied with 8800 ML of releases. For the remaining two months, the full demand from Reservoir 2 of 667 ML per month was released, providing 1334 ML to the community. Overall, the community was provided with 10134 ML from Reservoir 2 across the event horizon, equating to approximately 63.3% of the potential releases from the reservoir. When this total figure is combined with the total figure from Reservoir 1, it can be seen that the community was supplied with 16264 ML overall, translating to 67.8% of the total potential releases that could have been made over the 24 month event horizon from the two reservoirs. This combined total community release can be compared to those calculated under the other scenarios investigated.

The second column of Table 3.1 presents a comparison of the releases made to the community from both reservoirs under the conditions of Scenario 2. Under this scenario, it was specified that a maximum of 12 months of phase two rationing was permissible. From the table, it can be seen that phase two rationing was employed to constrain the releases from both reservoirs for this maximum number of months. Under the strictest rationing, 40% of the total demand from each reservoir was able to be released; calculated to equal 133 ML per month from Reservoir 1 and 267 ML per month from Reservoir 2. This lead to a total of 1596 ML being released to the community from Reservoir 1 and 3204 ML from Reservoir 2 over the 12 months in which phase two rationing was enforced.

When the results presented for Scenario 2 are compared to those for Scenario 1, it can be seen that under Scenario 2 less water was released to the community in total over the 24 month event horizon; 13598 ML, or 56.7% of the total potential releases, compared to 16264 ML, or 67.8% of the total potential releases under Scenario 1.

This difference can be attributed to the varying number of months in which each rationing level was enforced. That being said, the model was able to provide the same number of months in which no rationing was applied to releases from Reservoir 2 under Scenario 2 compared to Scenario 1; however the number of months in which no rationing was enforced upon the releases from Reservoir 1 under Scenario 2 was reduced to eight, a decrease of two months compared to Scenario 1.

The third column of Table 3.1 presents a comparison of the releases to the community under the third and final scenario considered. As part of the conditions of this scenario, it was specified that the number of months in which phase two rationing was permitted be reduced to six, while the initial capacities of both reservoirs be increased compared to the conditions considered under Scenario 2. From the third column of Table 3.1 it can be seen that the maximum number of months in which phase two rationing was permitted under Scenario 3 (six) have been utilised for both reservoirs. As mentioned previously, under the strictest phase two rationing, 40% of the optimal community demand can be released per month. This resulted in releases of 133 ML per month from Reservoir 1 and 267 ML per month from Reservoir 2, or, total releases of 798 ML and 1602 ML to the community over the six months in which phase two rationing was implemented on the releases from the two reservoirs respectively.

Upon comparing the releases to the community under Scenario 3 to the releases under the previous two scenarios considered, it can be seen that Scenario 3 resulted in more water being provided to the community than that witnessed under Scenario 2; however less than that seen originally under Scenario 1. In the case of Scenario 3, 14265 ML was released to the community over the 24 month event horizon, or 59.4% of the total potential releases. This is an increase from 56.7% of the total potential releases seen under Scenario 2, even though the number of months in which no rationing was applied to the releases from both reservoirs has decreased. This result could suggest that although it is important to maximise the number of months in which no rationing was enforced, greater volumes can be made available to the community if more consistent releases are made. However, it is difficult to justify this conclusion based on a comparison of the results under Scenario 3 to Scenario 2, as both the initial reservoir capacities and the number of months in which phase two rationing were permitted have been altered. Therefore, it is recommended that further research and experiments be conducted to determine if this alternate strategy is indeed viable.

From reviewing the results presented in this chapter, it can be concluded that Model 4 has provided a comprehensive and detailed approximation for the operation of the Perseverance and Cressbrook cascade reservoir system considered as a case study. Under each of the scenarios considered, the model has determined the optimal strategy for the operation of the system, with each being somewhat different to the last and somewhat different to the next. This highlights the fact that for each unique set of conditions, there are adjustments that need to be made in order to optimise the way in which the system performs. However, it must be noted that a series of assumptions were made before conducting this experiment using Model 4; some of which may not be feasible when applied to a physical system. An example of such an assumption is the assumed perfect prior knowledge of the inflows to both reservoirs. Therefore, any results from the model must be treated with caution and are limited to use as a guidance tool only, in order to provide the managers and operators of a cascade reservoir system with an approximate representation of how the system will behave during a drought event.

Also demonstrated by the results from Model 4 is that there may be alternate methods of ensuring that the releases to the community are maximised. One alternate strategy is to provide releases at one consistent volume over the event horizon rather than using strict rationing to conserve water for a number of months so that at a later point in the timeline no rationing is required. As mentioned previously, before adopting this method, further testing and experimentation is required.

Overall, the results from Model 4 show that a comprehensive approximation for the operation of a cascade reservoir system during a drought event can be provided by a MILP model; although the optimal management strategy is somewhat sensitive to changes in the specified parameters.

3.4 Chapter Conclusion

In this chapter, a model was formulated to determine how the Perseverance and Cressbrook cascade reservoir system considered as a case study in this thesis could be optimally managed under a drought event. To begin, a Mixed Integer Linear Programming (MILP) model, formulated by Shih and ReVelle (1995) and named Model 3, for the operation of a single reservoir system during a drought event was explored. As Model 3 had been robustly constructed, it was easily extended to the cascade reservoir system considered in this thesis. This extended model, labelled Model 4, was stated and defined rigorously using many of the same variables and constraints that were featured in Model 3. However, there was one constraint relaxed from Model 3 to Model 4, enabling Model 4 to better approximate the real world behaviour of a cascade reservoir system. Following this, the results of an experiment to investigate how different components of the reservoir system measured by Model 4 behaved under varying scenarios were explored. The experiment was repeated three times using three different scenarios, where the initial capacities of the reservoirs and the number of months in which strict phase two rationing was permitted were varied. From these experiments it was found that Model 4 provided a comprehensive approximation to the Perseverance and Cressbrook cascade reservoir system considered as a case study; though the model was limited by some of the assumptions made preceding the conduct of the experiment and that the optimal management strategies determined by the model were sensitive to changes in the specified parameters. Also, it was noted that there may be alternate methods to that employed currently by Model 4 to maximise the releases of water to the community over the event horizon considered.

By building upon the knowledge and understanding developed in the previous chapter, and utilising an existing model, a MILP model to investigate the operation of a cascade reservoir system during drought was able to be constructed. A similar procedure is employed in the next chapter, where Model 4 forms a framework on which a model can be developed that is applicable to the same reservoir system; however this time is considered during a flood event.

Chapter 4

Flood Model

4.1 Chapter Overview

Flooding events, triggered by periods of above average or extreme rainfall, often result in vast amounts of damage to the regions affected and can sometimes prove deadly. In order to prevent the worst impacts of such an event, reservoir systems can be employed as a type of defensive measure by stalling the progression of the floodwaters. However, if used for this purpose, it is crucial that optimal strategies for the operation of these reservoir systems, like those explored in this chapter, are available to assist operators in managing the movement of water through the systems.

At the beginning of this chapter, an adapted version of Model 3, a drought model previously presented in Chapter 3 and originally proposed by Shih and ReVelle (1995), is investigated. This adapted Mixed Integer Linear Programming (MILP) model, named Model 5, uses the fundamental structure of Model 3, whilst making some significant alterations to ensure that it is now applicable to a single reservoir system during a flood event. Following this, Model 5 is further extended so that it can be applied to a cascade reservoir system. This extended flood model, labelled Model 6, is used to perform a series of experiments (similar to those seen in the previous chapter) to monitor how particular features of the cascade reservoir system measured by the model behave when the initial capacities of the reservoirs are varied. Based upon the results from these experiments, a discussion is undertaken and a series of conclusions made regarding the appropriateness and applicability of the extended flood model to the case study of the existing Perseverance and Cressbrook cascade reservoir system considered throughout this thesis.

4.2 Models

4.2.1 Model 5 - Flood Model adapted from that developed by Shih and ReVelle (1995)

In this section, a Mixed Integer Linear Programming (MILP) model for the operation of a single reservoir during a flood event, named Model 5, is defined and investigated. This model was adapted from Model 3 seen in the previous chapter, which itself was originally presented by Shih and ReVelle (1995) for the operation of a single reservoir during a drought event. Given by Equations (4.1) to (4.19), Model 5 is presented below, with a description of the constraints and variables used to formulate the model provided in Section 4.2.2.

$$\text{minimise} \sum_{t=1}^{T} W_t - \omega \sum_{t=1}^{T} (V_{1t} + V_{2t} + V_{3t}) \tag{4.1}$$

such that

$$y_{1t} \geq \frac{(S_{t-1} + \hat{I}_t) - (V_{1t} - \epsilon)}{M} \qquad \forall t, \tag{4.2}$$

$$y_{1t} \leq 1 - \frac{V_{1t} - (S_{t-1} + \hat{I}_t)}{M} \qquad \forall t, \tag{4.3}$$

$$y_{2t} \geq \frac{(S_{t-1} + \hat{I}_t) - (V_{2t} - \epsilon)}{M} \qquad \forall t, \tag{4.4}$$

$$y_{2t} \leq 1 - \frac{V_{2t} - (S_{t-1} + \hat{I}_t)}{M} \qquad \forall t, \tag{4.5}$$

$$R_t = D.L \qquad \forall t, \tag{4.6}$$

$$P_t = (1 - \alpha_1).K.y_{2t} + (\alpha_1 - \alpha_2).K.y_{1t} + \alpha_2.K \qquad \forall t, \tag{4.7}$$

$$S_t = S_{t-1} + I_t - R_t - W_t - P_t \qquad \forall t, \tag{4.8}$$

$$S_0 = C_0, \tag{4.9}$$

$$S_t \leq C \qquad \forall t, \tag{4.10}$$

$$U_t \leq \frac{S_t}{C} \qquad \forall t, \tag{4.11}$$

$$W_t \leq MU_t \qquad \forall t, \tag{4.12}$$

$$V_{1t} \leq (1 - \beta_1) V_{2t} \qquad \forall t, \tag{4.13}$$

$$V_{2t} \leq (1 - \beta_2) V_{3t} \qquad \forall t, \tag{4.14}$$

$$V_{3t} \leq C \qquad \forall t, \tag{4.15}$$

$$y_{1t-1} + y_{1t+1} \leq 1 + y_{1t} \qquad \forall t, \tag{4.16}$$

$$y_{1t} \geq y_{2t+1} \qquad \forall t, \tag{4.17}$$

$$\sum_{t=1}^{T} U_t = T - n \qquad \forall t, \tag{4.18}$$

$$S_t, S_{t-1}, \hat{I}_t, I_t, V_{1t}, V_{2t}, V_{3t}, R_t, P_t, W_t \geq 0 \qquad \forall t. \tag{4.19}$$

4.2.2 Model 5 Description

In the event of a flood, the role of reservoir systems, and therefore the types of strategies employed to optimally manage these systems, are different to those implemented during a drought event. For example, during a flood, reservoirs can become defensive mechanisms that are used to delay the progress of flood waters and thus better control the levels of tributaries downstream of the reservoir system that could already be at full capacity as a result of the event. Therefore, although Model 3 provided a foundation for the structure of Model 5, many of the original constraints needed to be reversed or redefined in order to ensure that Model 5 is applicable to the operation of a reservoir system during a flood event.

The first significant change from Model 3 to Model 5 is the unit of time over which the event horizon is considered. Due to the nature of flooding events and the rapidness with which they occur, it was not feasible to continue describing the system on a monthly basis, as seen in Model 3. Therefore, in order to ensure a realistic representation of the reservoir system during a flood event, the time increment t of Model 5 is now considered on an hourly basis. Another difference between the two models concerns the releases to the community water supply. In Model 3, the releases made to the community were variable and depended upon the rationing level enforced each month. In the case of Model 5, it has been assumed that the releases to the community remain constant over the event horizon. This assumption was able to be made, as during a flood event there is typically an excessive volume of water available to the community, both through rainfall and water storage in reservoir systems, meaning that no rationing of water use is required.

Although the rationing of water available to the community is not necessary during a flood, the concept of rationing levels introduced in Model 3 is still employed in Model 5 through the inclusion of a new facility. In order to provide the operators of the reservoir system with more management options, and at the same time extend the complexity of the scenario considered, it has been assumed that a pumping facility is now included as part of the reservoir system. This pumping facility can be used to move water throughout the system and in the case of Model 5, pumps water from the reservoir to an external location assumed to have infinite storage. A simple schematic of the movement of water around the reservoir system is provided by Figure 4.1, which includes the pumping of water to an external location. By transferring water from the system in this manner, the reservoir storage level can be reduced, thereby maximising the number of hours before spill occurs from the reservoir and at the same time minimising the extent of spill each hour. Although currently unrealistic and potentially unnecessarily included in Model 5, the reason for the addition of this pumping facility is made clearer in Model 6, where a cascade reservoir system is considered.

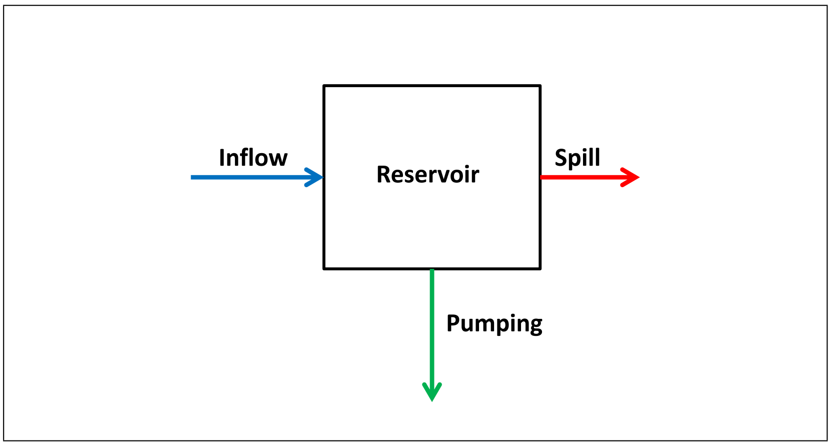

Figure 4.1: Simple Schematic of the Reservoir System considered in Model 5.

In the case of the pumping facility, it has been assumed there are three levels of restrictions that can be imposed on the volume of water pumped from the reservoir each hour; phase one pumping restrictions, phase two pumping restrictions and full capacity pumping. Analogous to Model 3, Model 5 also seeks to identify three trigger volumes, V_{1t}, V_{2t} and V_{3t} for all hours t, that denote the storage plus projected inflow (termed available water) at which changes between the pumping restrictions occur. The values of these trigger volumes are optimally selected by the model, except for the upper boundary trigger volume, V_{3t}, which is maximised to equal the storage capacity of the reservoir. Figure 4.2 provides a graphical representation of the relationship between the trigger volumes and the pumping restrictions. From this figure, it can be seen that if the available water is below the trigger volume V_{1t}, then the volume of water that can be pumped from the reservoir is restricted to α_2 proportion of the pumping facility capacity, K. Once the available water increases past V_{1t}, but is less than V_{2t}, then an additional volume of water can be pumped from the reservoir; now restricted to α_1 proportion of the pumping facility capacity. Once the available water surpasses V_{2t}, then the full capacity of the pumping facility is able to be drained from the reservoir each hour. This pumping restriction scheme is used to ensure that if the flood event is not as severe as first predicted, the reservoir is not emptied through the pumping of large volumes of water at the beginning of the event, and there is still sufficient water available in the reservoir to support the community post flood.

The objective function of Model 5 (Equation (4.1)) can be separated into two terms. The first term aims to minimise the spill from the reservoir each hour over the event horizon considered. By minimising the spill from the reservoir each hour, the floodwaters from the event are instead being stored by the reservoir, rather than being released into rivers and streams that could already be at full capacity as a result of flooding. Therefore, by storing and effectively stalling the progression of the floodwaters, the amount of damage downstream of the reservoir can be minimised.

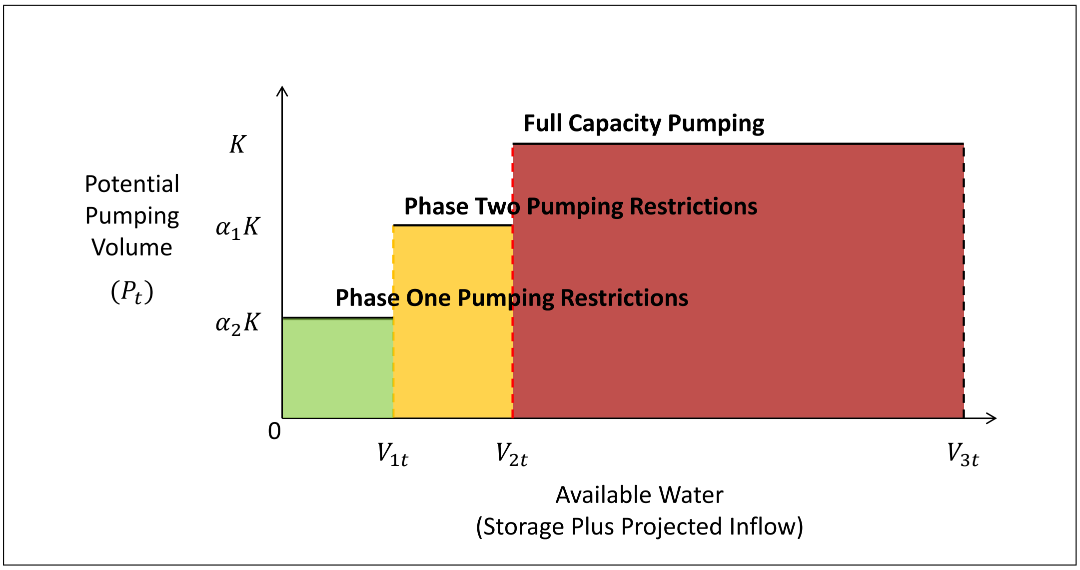

Figure 4.2: Graphical Representation of the Relationship between Reservoir Trigger Volumes and Pumping Restrictions.
Adapted from that presented by Shih and ReVelle (1995); where,
V_{1t}, V_{2t}, V_{3t} = *Trigger Volumes,* K = *Capacity of Pumping Facility,*
α_1, α_2 = *Restrictions on the Pumping Capacity.*

The second component of the objective function attempts to maximise the reservoir trigger volumes across all hours t. As mentioned previously, this ensures that the reservoir is not prematurely emptied and that appropriate storage remains in the reservoir to supply the community after the flood event has passed. By combining these two terms in the objective function, it ensures that the devastating effects of a flood event are reduced downstream of the reservoir and that, if the event was not as severe as first thought, suitable storage remains in the reservoir to supply the community.

Variable Definitions

There are 26 variables that are used in the formulation of Model 5. These variables have been defined below:

t, T = the current hour and the total number of hours in the event horizon which are known,

W_t = the spill from the reservoir in hour t, which is unknown (megalitres, ML),

ω = a small number, which is assumed to equal 0.01 in this case,

V_{1t} = the unknown value of storage and inflow, below which water pumped from the reservoir is constrained under phase one pumping restrictions for hour t (ML),

V_{2t} = the unknown value of storage and inflow, below which water pumped from the reservoir is constrained under phase two pumping restrictions for hour t (ML),

V_{3t} = the specified upper bound of storage plus inflow for hour t (ML),

y_{1t} = an unknown binary variable that is 1 if either phase two pumping restrictions, or full capacity pumping is enforced on the pumping facility in hour t, or 0 otherwise,

y_{2t} = an unknown binary variable that is 1 if no restrictions are imposed on the pumping facility and full capacity pumping can be met in hour t, or 0 otherwise,

S_t = the unknown volume of water stored in the reservoir at the end of hour t (ML),

I_t = the unknown inflow to the reservoir in hour t (ML),

$\hat{I}_t$ = the projected inflow to the reservoir in hour t, which is unknown (ML),

ϵ = a small number, which is assumed to equal 0.1 in this case,

M = a large number, which is assumed to equal 100 000 in this case,

R_t = the releases made to the community water supply in hour t, in this case calculated by known, constant values (ML),

D = the specified total demand required by the community, which is assumed constant over the event horizon (ML),

L = the percentage of the total demand required by the community that is supplied by each reservoir. For Model 5, as the system is composed of only one reservoir, the value of this variable is assumed to be 100%,

α_1 = the specified percentage of full capacity pumping that is permitted under phase two restrictions,

α_2 = the specified percentage of full capacity pumping that is permitted under phase one restrictions,

P_t = the unknown volume of water pumped from the reservoir in hour t (ML),

K = the maximum volume of water that can be pumped from the reservoir by the pumping facility each hour, assumed to be known and constant over the event horizon (ML),

C = the capacity of the reservoir, which is known (ML),

U_t = an unknown binary variable that is 1 if the reservoir is full at the end of hour t, or 0 otherwise,

β_1 = a specified separation value, assumed throughout the chapter to be 0.05,

β_2 = a specified separation value, assumed throughout the chapter to be 0.05,

n = the specified number of hours in which it is acceptable for spill to occur from the reservoir.

The way in which these variables behave and interact in Model 5 can be explored through an investigation of the constraints that compose the model.

Constraint Definitions

There are 18 constraints that compose Model 5, plus the objective function. As mentioned previously, the objective function (Equation (4.1)) consists of two terms. The first term and primary aim of the objective function is to minimise the volume of spill from the reservoir each hour. The secondary term then attempts to maximise the sum of the three trigger volumes across all hours t; however, a weight is placed on the sum of the trigger volumes in order to reduce the impact of this secondary term on the primary aim of the objective function.

Equations (4.2) and (4.3) operate together to determine the value of the binary variable y_{1t}. When considered independently, Equation (4.2) determines that the value of y_{1t} will be one if the available water is greater than the trigger volume V_{1t} in hour t; where the available water was defined in Chapter 3 to equal the reservoir storage level at the end of the previous hour, plus the projected inflow in the current hour ($S_{t-1} + \hat{I}_t$). By setting the value of the binary variable y_{1t} equal to one, the volume of water that can be pumped from the reservoir in the current hour is either limited, under phase two pumping restrictions, or unconstrained to full capacity pumping, depending upon the value of the binary variable y_{2t}. In the alternate case, in which the available water is less than the trigger volume V_{1t}, Equation (4.3) sets the value of y_{1t} to equal zero and in doing so enforces the stricter phase one restrictions on the volume of water that can be pumped from the reservoir each hour. The arrangement of pumping restrictions is reversed to that seen in the previous chapter, where phase two rationing was the most severe and phase one rationing was a "middle ground" between strict rationing and no rationing. However, throughout Models 5 and 6 proposed for a flooding event, phase one restrictions see the least amount of water being pumped from the reservoir, while phase two restrictions do not limit the pumping volume as severely.

Analogous to Equations (4.2) and (4.3), Equations (4.4) and (4.5) work together to determine the value of the binary variable y_{2t}. In this case, Equation (4.4) sets the value of y_{2t} to equal one if the available water is greater than the trigger volume V_{2t} in the current hour. By setting y_{2t} to equal one, this indicates that the full capacity of the pumping facility can be drained from the reservoir in the current hour. On the other hand, Equation (4.5) ensures that if the reverse occurs and the available water is less than V_{2t}, then the binary variable y_{2t} is set equal to zero; signifying that some form of restriction is imposed on the volume of water that can be pumped from the reservoir in the current hour. As mentioned in the previous chapter, the addition of the small value ϵ is necessary in Equations (4.2) and (4.4) to ensure that if the available water equals a trigger volume in the current hour (i.e. $S_{t-1} + \hat{I}_t = V_{1t}$), the value of either binary variable y_{1t} or y_{2t} is one. If ϵ was not included in these constraints, then the values of the binary variables could be either

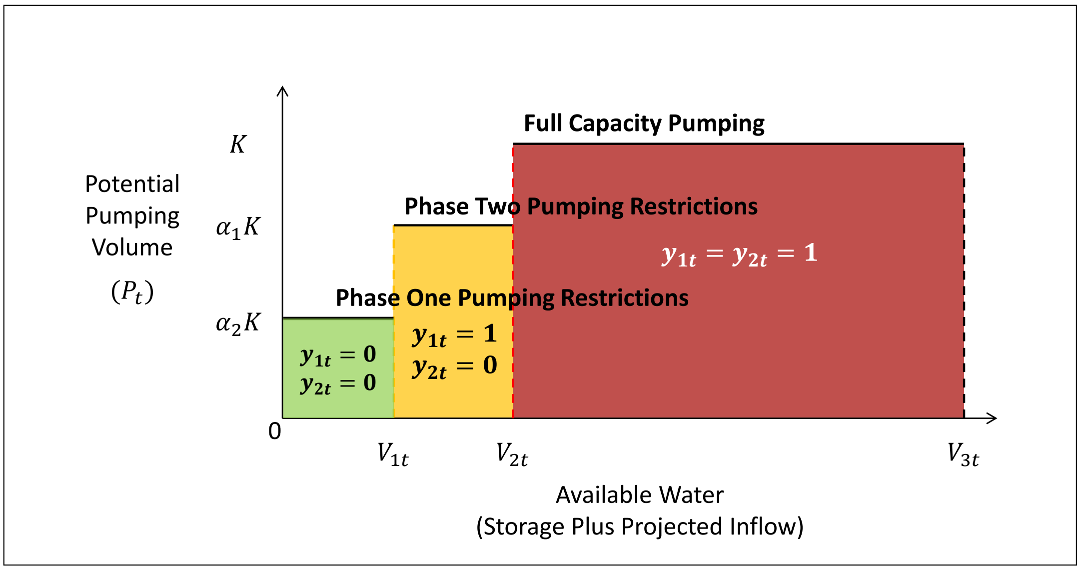

Figure 4.3: Adapted Graphical Representation of the Relationship between Reservoir Trigger Volumes and Pumping Restrictions, showing the Values of the Binary Variables y_{1t} and y_{2t} under each Phase of Restrictions.
Adapted from that presented by Shih and ReVelle (1995); where,
V_{1t}, V_{2t}, V_{3t} = *Trigger Volumes,* K = *Capacity of Pumping Facility,*
α_1, α_2 = *Restrictions on the Pumping Capacity.*

one or zero in this situation. Also, it should be noted that the relationship between these four constraints (Equations (4.2) to (4.5)) ensures that y_{2t} will not equal one unless y_{1t} equals one in the current hour and that if y_{1t} equals zero, then y_{2t} must also equal zero; an important relationship when determining the level of restriction imposed on the pumping facility. In order to demonstrate the relationship between the binary variables, y_{1t} and y_{2t}, and the level of pumping restrictions enforced, an adapted version of Figure 4.2, labelled Figure 4.3 has been provided above.

Equation (4.6) determines the volume of water released to the community each hour. In the case of a flood event, it has been assumed that the releases to the community are constant over the event horizon and equal the product of the total community demand (D) and the percentage of this total demand supplied by the reservoir (L). As the system considered during the formulation of Model 5 only consists of a single reservoir, the variable L is equal to 100%. This denotes that the total community demand is sourced from the one reservoir. The addition of the variable L in the model will be made clearer when Model 6 is considered (Section 4.2.3).

In order to determine the volume of water pumped from the reservoir in a given hour, Equation (4.7) is included. From the constraint, if the values of both binary variables y_{1t} and y_{2t} are zero, then the volume of water that can be pumped from the reservoir in the current hour is limited under phase one restrictions; translating to α_2 proportion of the pumping facility capacity (K). However, if the value of the binary variable y_{1t} is one while the value of y_{2t} is zero, then phase two restrictions are enforced on the pumping volume in hour t. This enables an increased volume of $\alpha_1 K$ to be drained from the reservoir in the current hour. In order for the full capacity of the pumping facility to be drained from the reservoir in hour t, Equation (4.7) demonstrates that both the variables y_{1t} and y_{2t} must be equal to one.

Equation (4.8) is the mass balance constraint of Model 5 and is used to determine the storage level of the reservoir each hour. From the constraint it can be seen that the reservoir storage level in the current hour is equal to the sum of the storage level in the previous hour and the inflow to the reservoir in the current hour, minus the releases to the community, spill from the reservoir, and the volume of water pumped from the reservoir in the current hour.

Equation (4.9) is included in Model 5 to offer the operators of the reservoir system the opportunity to specify the initial capacity of the reservoir. This feature was first introduced in Model 4 in the previous chapter and, in this case, enables the managers of the system to investigate how the results of Model 5 behave when the initial capacity of the reservoir is varied. However, if the operators of the reservoir system do not wish to specify the starting capacity, this constraint can be relaxed to the form $S_0 \leq C$ and the model will itself select the optimal initial condition of the reservoir level. Equation (4.10) is then included to ensure that the storage level of the reservoir does not surpass the capacity of the reservoir in any given hour.

Equations (4.11) and (4.12) combine to control the spill of water from the reservoir. In a typical physical reservoir system, where the reservoir does not possess any spillway control measures such as a gated spillway, spill cannot occur from the reservoir until the reservoir has reached full capacity. Therefore, in order to ensure that spill does not occur from the reservoir in Model 5 until the full capacity has been reached, Equation (4.11) is utilised. This constraint determines if the reservoir is at full capacity in the current hour and if so, sets the value of the binary variable U_t equal to one. On the other hand, for the hours where the reservoir has not reached full capacity, the variable is given a value of zero. The value of U_t is then utilised in Equation (4.12) to ascertain the extent of the spill that occurs from the reservoir in hour t. If the value of U_t is zero, then the right hand side of Equation (4.12) is also zero, meaning that no spill occurs in the current hour. However, if the value of U_t is one, then the right hand side of Equation (4.12) is equal to a large number, enabling spill to occur in hour t. In this case, the extent of the spill that occurs each hour is calculated by the mass balance constraint, Equation (4.8).

To ensure that the three trigger volumes are not all maximised to the same value and that there is a degree of separation between them, Equations (4.13) and (4.14) are included in Model 5. Equation (4.13) makes certain that the trigger volume V_{1t} is at least β_1 percentage less than V_{2t}, while Equation (4.14) maintains V_{2t} is at least β_2 percentage less than V_{3t}. Equation (4.15) then specifies that the upper boundary trigger volume V_{3t} is less than or equal to the capacity of the reservoir. However, as the values of the trigger volumes are maximised as part of the objective function, in practice the value of V_{3t} is consistently maximised to be equal to the reservoir capacity C.

Equation (4.16) makes sure that if in hours $t-1$ and $t+1$, the volume of water being drained from the reservoir is either limited under phase two pumping restrictions or unconstrained to full capacity pumping, then phase two pumping restrictions or full capacity pumping must also be enforced in the current hour t. This constraint was originally proposed by Shih and ReVelle (1995) in Model 3, who explained that the objective of this constraint is to prevent a back and forwards movement between the most severe and less severe phases of restrictions when determining the value of the trigger volumes. However, they also note that this constraint does not prevent the back and forwards behaviour from occurring in the real situation.

Equation (4.17) is used to ensure that if in the current hour, pumping from the reservoir is limited under phase one restrictions, then full capacity pumping cannot be enacted in the next hour. This is to ensure that the pumping facility has at least one hour at which phase two pumping restriction volumes are drained from the reservoir before moving to full capacity pumping; without which, damage could be caused to the pumping facility.

Equation (4.18) is included in Model 5 as another tool that can be utilised by the operators of the reservoir system. This constraint enables the managers of the system to select the number of hours in which they deem that it is not suitable for spill to occur from the reservoir, through the specification of the variable n. In this case, the variable n is subtracted from the total number of hours in the event horizon T. This difference is then the number of months in which it is deemed suitable for spill to occur from the reservoir.

Finally, Equation (4.19) ensures that a range of variables used in the formulation of Model 5 remain nonnegative, thus ensuring that the values of these variables remain physically realistic.

4.2.3 Model 6 - Extension of Flood Model

In this section, a MILP model for the operation of a cascade reservoir system during a flood event is formulated. This extended model, labelled Model 6, is assembled using the structure of Model 4, presented in the previous chapter, as a framework and is composed of many of the same constraints developed in Model 5. Given by Equations (4.20) to (4.63), Model 6 can be found on the following page. As seen throughout this thesis, variables denoted with a star (*) correspond to Reservoir 1, whilst the variables without a star are associated with Reservoir 2.

$$\text{minimise } J - \omega \sum_{t=1}^{T} (V_{1t}^* + V_{2t}^* + V_{3t}^*) - \omega \sum_{t=1}^{T} (V_{1t} + V_{2t} + V_{3t}) - \gamma \sum_{t=1}^{T} P_t \tag{4.20}$$

such that

Reservoir 1 Constraints:

$$y_{1t}^* \geq \frac{(S_{t-1}^* + \hat{I}_t^*) - (V_{1t}^* - \epsilon)}{M} \quad \forall t, \tag{4.21}$$

$$y_{1t}^* \leq 1 - \frac{V_{1t}^* - (S_{t-1}^* + \hat{I}_t^*)}{M} \quad \forall t, \tag{4.22}$$

$$y_{2t}^* \geq \frac{(S_{t-1}^* + \hat{I}_t^*) - (V_{2t}^* - \epsilon)}{M} \quad \forall t, \tag{4.23}$$

$$y_{2t}^* \leq 1 - \frac{V_{2t}^* - (S_{t-1}^* + \hat{I}_t^*)}{M} \quad \forall t, \tag{4.24}$$

$$R_t^* = D.L^* \quad \forall t, \tag{4.25}$$

$$S_t^* = S_{t-1}^* + I_t^* - R_t^* - W_t^* + P_t \quad \forall t, \tag{4.26}$$

$$S_0^* = C_0^*, \tag{4.27}$$

$$S_t^* \leq C^* \quad \forall t \tag{4.28}$$

$$U_t^* \leq \frac{S_t^*}{C^*} \quad \forall t, \tag{4.29}$$

$$W_t^* \leq M U_t^* \quad \forall t, \tag{4.30}$$

$$V_{1t}^* \leq (1 - \beta_1) V_{2t}^* \quad \forall t, \tag{4.31}$$

$$V_{2t}^* \leq (1 - \beta_2) V_{3t}^* \quad \forall t, \tag{4.32}$$

$$V_{3t}^* \leq C^* \quad \forall t, \tag{4.33}$$

$$B_t^* = (1 - \alpha_1).K.y_{2t}^* + (\alpha_1 - \alpha_2).K.y_{1t}^* + \alpha_2.K \quad \forall t, \tag{4.34}$$

$$y_{1t-1}^* + y_{1t+1}^* \leq 1 + y_{1t}^* \quad \forall t, \tag{4.35}$$

$$y_{1t}^* \geq y_{2t+1}^* \quad \forall t, \tag{4.36}$$

$$\sum_{t=1}^{T} U_t^* \geq 0 \quad \forall t, \tag{4.37}$$

$$\hat{I}_t^* = I_t^* \quad \forall t, \tag{4.38}$$

$$Q_t^* + S_t^* = C^* \quad \forall t, \tag{4.39}$$

$$S_t^*, S_{t-1}^*, \hat{I}_t^*, I_t^*, V_{1t}^*, V_{2t}^*, V_{3t}^*, R_t^*, P_t^*, W_t^*, B_t^*, Q_t^* \geq 0 \quad \forall t. \tag{4.40}$$

Reservoir 2 Constraints:

$$y_{1t} \geq \frac{(S_{t-1} + \hat{I}_t) - (V_{1t} - \epsilon)}{M} \qquad \forall t, \quad (4.41)$$

$$y_{1t} \leq 1 - \frac{V_{1t} - (S_{t-1} + \hat{I}_t)}{M} \qquad \forall t, \quad (4.42)$$

$$y_{2t} \geq \frac{(S_{t-1} + \hat{I}_t) - (V_{2t} - \epsilon)}{M} \qquad \forall t, \quad (4.43)$$

$$y_{2t} \leq 1 - \frac{V_{2t} - (S_{t-1} + \hat{I}_t)}{M} \qquad \forall t, \quad (4.44)$$

$$R_t = D.L \qquad \forall t, \quad (4.45)$$

$$S_t = S_{t-1} + I_t - R_t - W_t + W_t^* - P_t \qquad \forall t, \quad (4.46)$$

$$S_0 = C_0, \qquad (4.47)$$

$$S_t \leq C \qquad \forall t, \quad (4.48)$$

$$U_t \leq \frac{S_t}{C} \qquad \forall t, \quad (4.49)$$

$$W_t \leq MU_t \qquad \forall t, \quad (4.50)$$

$$V_{1t} \leq (1 - \beta_1)V_{2t} \qquad \forall t, \quad (4.51)$$

$$V_{2t} \leq (1 - \beta_2)V_{3t} \qquad \forall t, \quad (4.52)$$

$$V_{3t} \leq C \qquad \forall t, \quad (4.53)$$

$$B_t = (1 - \alpha_1).K.y_{2t} + (\alpha_1 - \alpha_2).K.y_{1t} + \alpha_2.K \qquad \forall t, \quad (4.54)$$

$$y_{1t-1} + y_{1t+1} \leq 1 + y_{1t} \qquad \forall t, \quad (4.55)$$

$$y_{1t} \geq y_{2t+1} \qquad \forall t, \quad (4.56)$$

$$\sum_{t=1}^{T} U_t \geq 0 \qquad \forall t, \quad (4.57)$$

$$\hat{I}_t = I_t \qquad \forall t, \quad (4.58)$$

$$Q_t^* \geq \alpha_2.K.z_t \qquad \forall t, \quad (4.59)$$

$$P_t \leq B_t \qquad \forall t, \quad (4.60)$$

$$P_t \leq K.z_t \qquad \forall t, \quad (4.61)$$

$$W_t + W_{t+1} \leq J \qquad \forall t, \quad (4.62)$$

$$S_t, S_{t-1}, \hat{I}_t, I_t, V_{1t}, V_{2t}, V_{3t}, R_t, P_t, W_t, W_t^*, B_t, Q_t \geq 0 \qquad \forall t. \quad (4.63)$$

4.2.4 Model 6 Description

At this point, the need to express Model 5 in a generic nature becomes clearer; to enable the model to be readily extended in order to describe a cascade reservoir system during a flood event. For the main part, the structure of Model 5 has not been substantially altered in the construction of Model 6; with many of the constraints that compose Model 5 replicated a second time to account for the addition

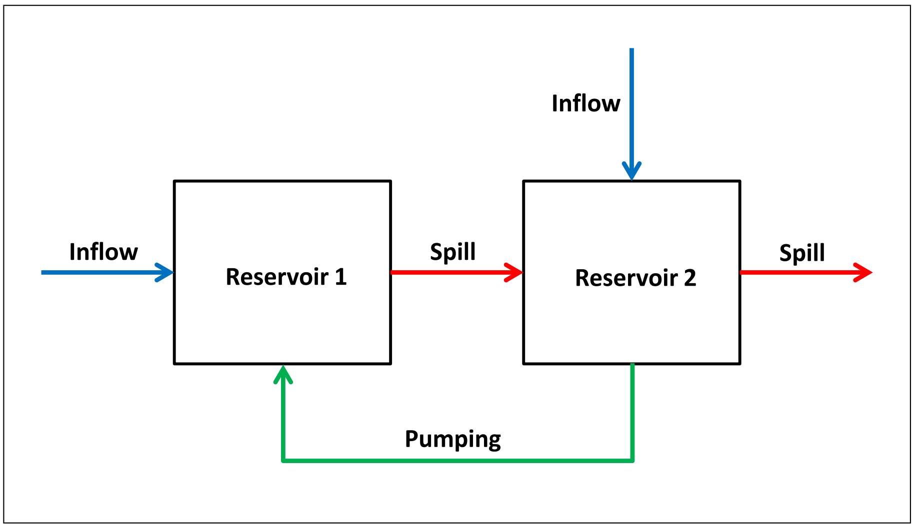

Figure 4.4: Simple Schematic of the Reservoir System considered in Model 6.

of a second reservoir to the system. That being said, some new inclusions have been made to Model 6 to ensure an accurate representation of the reservoir system during a flood event.

While describing Model 5, two components of the model were mentioned to be unrealistic or unnecessary; however it was stated that their use would become clearer upon investigation of Model 6. These components were the addition of a pumping facility to drain water from the reservoir to an unknown external location with infinite storage, and the inclusion of the variable L, the proportion of the total community water supply demand sourced from the reservoir. Although perhaps extraneous in Model 5, both of these components provide important functionality in Model 6.

The first component, whose use was somewhat unrealistic in Model 5, although offered an important extension to Model 6, was the inclusion of a pumping facility to the reservoir system. In the case of Model 5, where a single reservoir system was considered, any water pumped from the reservoir was assumed to be removed from the system; an unrealistic assumption. However, when the reservoir system is extended to include two reservoirs in a cascade configuration, the incorporation of a pumping facility provides an opportunity for additional control of the movement of water within the system. Therefore, in the formulation of Model 6, it is assumed that the pumping facility offers the one directional movement of water from Reservoir 2 backwards through the system to Reservoir 1. In order to demonstrate this movement of water through the reservoir system via the pumping facility, a simple schematic has been provided in Figure 4.4. By assuming that this pumping facility is included in the system, water can be drained from Reservoir 2, back to Reservoir 1, reducing the level of Reservoir 2 and thus the length of time before spill occurs from the system into downstream tributaries.

The second component of Model 5 that may have seemed unnecessary was the use of the variable L, the proportion of the community water supply sourced from a reservoir in the system. In the case of Model 5 where a single reservoir system was considered, the only option was for the entire community demand to be sourced from the one reservoir ($L = 100\%$). However, when a cascade reservoir system is considered, the proportion of the community demand could be equally or unequally shared between the two reservoirs, as seen in the previous chapters. Therefore, as the proportion of the community demand sourced from the reservoirs in the system could differ, the inclusion of this variable was important in the formulation of Model 6. The proportion of the community demand shared between the two reservoirs considered as a case study is defined later in Section 4.3, where the results of the experiments conducted using Model 6 are explored.

When the objective function of Model 6 (Equation (4.20)) is compared to that of Model 5, it can be seen that some additions have been made. Although some of the terms that feature in the objective function of Model 5 still appear in the objective function of Model 6, such as the maximisation of the reservoir trigger volumes for all hours t, there are two terms that were not seen previously. The first of these terms aims to minimise the volume of spill from Reservoir 2, herein referred to as the spill from the system, in the current hour plus the spill in the next hour. It is assumed that the river levels downstream of the system will be above average due to the flood event, meaning that any spill from the system in the current hour will not only increase the river levels now, but continue to influence river levels into the next hour as well. In order to account for this assumption, the spill from the system in consecutive hours needs to be minimised.

At this point, it should be noted that the spill from the system refers to the spill from Reservoir 2 alone. In the case of Model 6, it is assumed that the reservoir system is composed of Reservoir 1, Reservoir 2 and a pumping facility. Therefore, under this assumption, the spill from Reservoir 1 to Reservoir 2 and any pumping of water from Reservoir 2 to Reservoir 1 is deemed movement of water within the system. On the other hand, water that spills from Reservoir 2 exits the system and impacts upon rivers and creeks downstream, thus referred to as spill from the system.

The second additional term that features in the objective function of Model 6 aims to maximise the volume of water pumped from Reservoir 2 to Reservoir 1 each hour. As mentioned previously, by draining water from Reservoir 2 and pumping it backwards through the system to Reservoir 1, the storage level of Reservoir 2 is minimised, while the length of time before spill occurs from Reservoir 2, now termed the spill from the system, is maximised. Through the combination of the terms that compose the objective function, the overall aim of Model 6 is threefold.

Firstly, the objective of the model is to minimise the volume of spill from the system in consecutive hours, while at the same time lowering the level of Reservoir 2 through the use of the pumping facility, to maximise the number of hours before spill begins to occur. Also, by maximising the reservoir trigger volumes, the model is attempting to ensure that Reservoir 2 is not prematurely drained in the case that the flood event is not as severe as first predicted. Together, these three objectives are balanced to provide an optimal management strategy for the reservoir system.

Variable Definitions

There are 30 variables used in the formulation of Model 6, some of which are duplicated twice to account for the addition of Reservoir 2 to the system. Out of these 30 variables, 25 of these have been previously defined for Model 5 in Section 4.2.2 under the heading Variable Definitions. Of the five newly defined variables below, P_t was also used previously in Model 5, though now is utilised for a different purpose under Model 6:

J	=	an unknown variable that measures the extent of spill from the system (i.e. Reservoir 2) in the current hour t plus the spill in the next hour $t+1$. It is used to provide an approximation of the behaviour of the river levels downstream of the reservoir system (ML),
γ	=	a large number, which is known,
B_t	=	the unknown volume of water that could potentially be pumped from Reservoir 1 and Reservoir 2 in hour t (ML),
P_t	=	the unknown volume of water that is pumped from Reservoir 2 to Reservoir 1 in hour t,
Q_t	=	an unknown variable that measures the extent by which Reservoir 1 is below full capacity in hour t (ML),
z_t	=	an unknown binary variable that is 1 if there is a sufficient volume available in Reservoir 1 for pumping of water from Reservoir 2 to occur in hour t, or 0 otherwise.

The way in which these newly defined variables interact with each other and the previously defined variables can be explored through an investigation of the additional constraints included in Model 6.

Constraint Definitions

There are 43 constraints that compose Model 6, plus the objective function. Of these 43 constraints, 17 are repeated from Reservoir 1 to Reservoir 2, leaving 3 constraints unique to the operation of Reservoir 1 and 6 constraints unique to the operation of Reservoir 2. Many of the constraints that compose Model 6 have been previously described for Model 5 in Section 4.2.2 under the heading Constraint Definitions. This being said, additional constraints have been included in Model 6, being Equations (4.34), (4.38) and (4.39) of Reservoir 1 and Equations (4.54) and (4.58) to (4.62) of Reservoir 2. Also, as previously mentioned, the structure of the objective function (Equation (4.20)) has been altered. The primary objective of Model 6 is to minimise the variable J, or the volume of water spilled from the system (or Reservoir 2) in the current hour, plus the next hour. The secondary objectives of the model are to maximise the sum of the reservoir trigger volumes across all hours t, along with maximising the volume of water pumped from Reservoir 2 backwards through the system to Reservoir 1. These two secondary objectives share an inverse relationship, as to maximise the volume of water pumped between the reservoirs in the current hour, the trigger volumes need to be minimised and vice versa. In order to specify which of these secondary objectives is more favourable the weights of the terms can be adjusted. In this case, the weight (γ) of the pumping term (P_t) is larger than that of the weight (ω) of the trigger volumes. This means that the volume of water pumped from Reservoir 2 will be maximised each hour, as opposed to maximising the trigger volumes and thus minimising the drainage of water from Reservoir 2.

The first additional constraint included in Model 6 to be considered is Equation (4.34) of Reservoir 1, which is also repeated for Reservoir 2 as Equation (4.54). This constraint is very similar to Equation (4.7) in Model 5, which determines the volume of water pumped from the reservoir each hour. In the case of Model 6, these constraints measure the volume of water that could potentially be pumped from each reservoir under the pumping restrictions imposed in hour t. Therefore, although Model 6 does not have the capability of pumping water from Reservoir 1 to Reservoir 2, the volume of water that could be pumped in this direction is measured by Equation (4.34) each hour. This constraint is included to offer the operators of the reservoir system an opportunity to monitor the volume of water that could potentially be pumped from Reservoir 1 to Reservoir 2. However, the need for this type of multi-directional pumping is questionable in the reservoir system considered, as it would increase the storage level of Reservoir 2 and thus increase the spill from the system each hour, along with potentially hastening the first hour in which spill occurs.

Equations (4.38) and (4.58) for Reservoir 1 and Reservoir 2 respectively, are included in Model 6 to provide the operators of the reservoir system the opportunity to assume perfect prior knowledge of the inflows, to each reservoir, over the event horizon considered. However, if there is some uncertainty regarding the hourly inflows to the system, these constraints can be relaxed.

Although Equations (4.37) and (4.57) have not been added to Model 6, they have been altered compared to that seen in Model 5. Originally in Model 5, these constraints enabled the operators of the reservoir system to select the number of months in which they deemed it acceptable for spill to occur from the system. However, from initial experimentation with Model 6, it was found that this constraint was over restricting the model and feasible management strategies were not able to be determined. Therefore, it was decided that this constraint would be relaxed and that Model 6 would optimally determine the number of months in which spill would occur from the system.

Equation (4.39) only appears in the constraints for Reservoir 1 and is used to measure the extent by which the storage level of the reservoir is below full capacity each hour. The difference between the capacity of Reservoir 1 and the storage level in hour t is stored by the variable Q^*_t, which in turn is used in other constraints to determine the volume of water that can be pumped from Reservoir 2 to Reservoir 1.

Equations (4.59), (4.60) and (4.61) appear solely in the constraints for Reservoir 2 and work together to determine the volume of water pumped from Reservoir 2 backwards through the system to Reservoir 1 in hour t. To begin, the model checks to determine if there is sufficient storage remaining in Reservoir 1 for water to be pumped from Reservoir 2. If this check was not performed and water continued to be pumped from Reservoir 2 despite there being insufficient capacity remaining in Reservoir 1 to store the water, then any water entering Reservoir 1 would be spilled from the reservoir immediately. Due to the cascade configuration of the reservoir system, the water spilled from Reservoir 1 will return to Reservoir 2 as inflow; defeating the purpose of pumping water backwards through the system. Therefore, Equation (4.59) compares the value of the variable Q^*_t, the storage remaining in Reservoir 1, to the minimum volume that can be pumped from Reservoir 2 each hour ($\alpha_2 K$). If there is sufficient storage remaining in Reservoir 1 for pumping to occur in hour t, Equation (4.59) sets the value of the binary variable z_t equal to one. On the other hand, if the minimum volume of water that can be pumped from Reservoir 2 exceeds the storage remaining in Reservoir 1, the constraint ensures that the value of z_t equals zero. Equations (4.60) and (4.61) then employ the value of z_t to determine the volume of water pumped from Reservoir 2 in the current hour. If z_t is equal to zero, then Equation (4.61) states that the volume pumped from Reservoir 2, P_t, must also be equal to zero. On the other hand, if the value of z_t is one, then Equation (4.61) states that volume pumped from Reservoir 2 must be at most the capacity of the pumping facility, K. However, depending on the level of pumping restrictions imposed in the current hour, the full capacity of the pumping facility may not be able to be released; therefore Equation (4.60) ensures that the volumed pumped from Reservoir 2 in hour t is, at most, equal to the potential volume that could be pumped from the reservoir in that hour, B_t.

Equation (4.62) also only appears in the constraints for Reservoir 2 and is used to determine the value of the variable J. As mentioned previously, J is minimised as part of the objective function and is equal to the sum of the spill from the reservoir in the current hour, plus the spill from the reservoir in the next hour.

Of all the constraints that are replicated between the reservoirs, only the mass balance constraints vary between Reservoir 1 and Reservoir 2. Upon a comparison of the mass balance constraint of Reservoir 1 (Equation (4.26)) to Reservoir 2 (Equation (4.46)), two differences can be seen. The first is that the mass balance constraint of Reservoir 2 also contains, as an additional source of inflow, the spill from Reservoir 1 in hour t, W_t^*. Also, in the mass balance constraint of Reservoir 1, it can be seen that the volume pumped from Reservoir 2 each hour, P_t, is a source of inflows. On the other hand, the mass balance constraint of Reservoir 2 shows this volume being drained from the reservoir each hour.

4.3 Results and Discussion

In this section, Model 6 is applied to the case study of the existing Perseverance and Cressbrook cascade reservoir system in the Toowoomba region (originally presented in Figure 1.1) by means of an experiment, with the aim of determining the optimal strategy for the management of the system during a flood event. From the results of this experiment, the key components measured by Model 6 that influence the management of the reservoir system during a flood event are investigated and an interpretation of the behaviour of these components is provided. At this point, it should be noted that for the purposes of this experiment, Perseverance dam is labelled Reservoir 1, while Cressbrook dam is labelled Reservoir 2.

In order to perform this experiment, along with the experiments performed in the previous chapters, the optimisation modelling software LINGO version 14.0 (LINDO Systems Inc, 2013) was utilised. An example of the syntax employed to perform this experiment, along with a portion of the resulting output can be found in Appendix C. As mentioned in the previous chapter, the version of LINGO sourced to perform the experiments throughout this thesis was restricted under an Academic License, meaning that the number of variables and constraints that the software could consider were limited. This restriction by the software, together with the size of Model 6, meant that the event horizon over which the experiment was considered could not be any longer than 24 time units, which in the case of Model 6 equated to 24 hours or one day.

To ensure that a thorough investigation of the results from the experiment conducted using Model 6 could be performed, a series of assumptions were needed to be made. These assumptions simplify some of the conditions under which the experiment was conducted; while at the same time provide a feasible representation of the reservoir system considered as a case study. The first of these assumptions concerns the inflows to the reservoir system. As mentioned in the previous chapter, historic inflows for Cressbrook Creek, the major tributary flowing into Cressbrook dam, were able to be sourced from the Queensland Department of Natural Resources and Mines (2012), though the inflows correspond to a period from late 1965 to mid 1981 and were recorded on a monthly basis. As Model 6 is applicable to a flood event, it was assumed that the monthly inflows recorded by the Queensland Department of Natural Resources and Mines (2012) could be used as hourly inflows for the purposes of the experiment, to provide a worst case flood scenario. Therefore, after considering the 16 years of historic inflows, a two year period was selected in which the largest inflows to the region were seen. These years were October 1973 to September 1974 and October 1975 to September 1976. Also mentioned in the previous chapter was that no historic inflows were able to be sourced for the independent creeks or streams flowing into Perseverance dam. However, due to the proximity of the two reservoirs, it was assumed that the historic inflows into Cressbrook dam could be used to approximate the inflows to Perseverance dam, though a scale difference was needed to be applied. Using prior knowledge of the size of the tributaries that supply Perseverance dam, it was assumed that the inflows to the reservoir would be reduced to 60% of the monthly inflows to Cressbrook dam. This same assumption has also been made for the purposes of the experiment using Model 6. From the historic records, the assumed hourly inflow to both Perseverance and Cressbrook dams over the 24 hour event horizon is presented in Table D.2 of Appendix D, while a graphical representation of the inflows is provided by Figure 4.5 above.

Another assumption made regarding the hourly inflow to the reservoir system was to specify that perfect prior knowledge was available. This assumption was achieved through the use of Equations (4.38) and (4.58) of Model 6, and as mentioned earlier, was an option available to the managers of the reservoir system. Although perhaps not feasible in a physical system, the assumed perfect prior knowledge of the inflows to the reservoir system helped to reduce the complexity of the model and simplify the results of the experiment.

In the previous chapter, a series of assumptions were made in order to simplify some of the parameters of the experiment conducted using Model 4. In order to ensure the continuity of the methodologies employed throughout this thesis, many of the same assumptions have been made in this chapter. One such assumption, based upon information presented by the Toowoomba Regional Council (2013), was to assume that the capacities of Cressbrook dam and Perseverance dam were equal to 82 000 megalitres (ML) and 30 000 ML respectively. Also, it was assumed that the community water supply demand was shared between the two reservoirs according to their respective storage capacities. In the case of Model 6, the proportion of the demand shared between the two reservoirs has been included in the model through

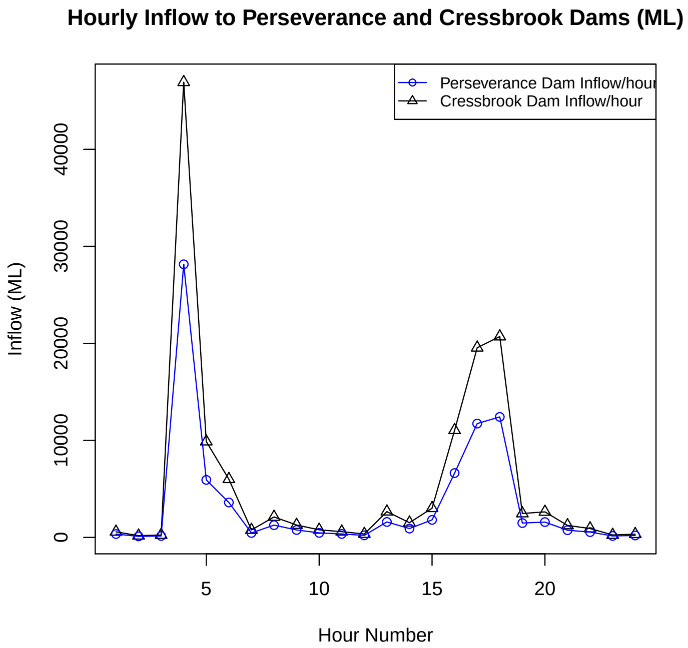

Figure 4.5: Hourly Inflow to Perserverance and Cressbrook Dams under the Worst Case Flood Scenario from Historic Records.

the use of the variable L. Therefore, under this assumption, Perseverance dam was responsible for supplying one third of the community water supply ($L^* = \frac{1}{3}$), while the remaining two thirds of the demand was sourced from Cressbrook dam ($L = \frac{2}{3}$). One significant difference between Model 6 and Model 4 was the assumption that during a flood event, the releases made to the community water supply from the reservoir system remain constant over the event horizon and that no rationing was enforced. This assumption was able to be made, as during a flood event there is typically an excessive volume of water available to the community, both in the form of rainfall and water stored in the reservoir system. Therefore, no restrictions need to be placed on the volume of water supplied to the community and as such it has been assumed that they can use up to 400 ML per hour. Although in the physical system this community usage is unrealistic on an hourly basis, for the purpose of this experiment, the usage was inflated in order for it be on a comparable scale with the size of the inflows to the system each hour. This volume was then shared between the two reservoirs according to the proportions L defined previously, meaning that 133 ML ($D.L^*$) each hour was made available to the community from Perseverance dam, while 267 ML ($D.L$) each hour was provided to the community from Cressbrook dam.

A significant assumption made when formulating the models in this chapter and upon commencing the experiment of Model 6, was that the reservoir system considered as a case study now incorporated a pumping facility. As mentioned earlier, in the case of Model 6, this pumping facility enabled water to be drained from Reservoir 2 and pumped backwards through the system to Reservoir 1, thus decreasing the storage level of Reservoir 2 and minimising the spill from the system each hour. For the purposes of this experiment, it was assumed that the maximum capacity of this pumping facility (K) was 1000 ML per hour, meaning that at most 1000 ML could be pumped from Reservoir 2 into Reservoir 1 each hour. However, as mentioned previously, the volume of water able to be pumped through the facility each hour was limited by a set of restrictions; the severity of which was determined by the available water. Analogous to the rationing levels originally proposed by Shih and ReVelle (1995) in the formulation of Model 3, there were three levels of restrictions that could have been imposed on the volume of water pumped by the facility each hour; phase one pumping restrictions, phase two pumping restrictions and full capacity pumping. In the case of Model 6, the strictest limitation on the pumping volume occurred under phase one restrictions, where the volume able to be pumped each hour was limited to α_2 percentage of the full capacity K. Under phase two restrictions, the volume of water able to be pumped each hour increased to α_1 percentage of the full capacity, while under full capacity pumping the full capacity of the pumping facility could be drained from Reservoir 2 each hour. In the case of the experiment conducted using Model 6, the values of α_1 and α_2 were assumed to equal 60% and 40% respectively. Therefore, if phase one pumping restrictions were enforced, then 400 ML per hour was able to be pumped by the facility, while under phase two restrictions this volume was increased to 600 ML per hour.

At this point, it should be noted that the series of assumptions above were made to offer the best approximation of the reservoir system considered as a case study, while not over complicating the conditions of the experiment. Depending upon the type of system being investigated, all or some of these assumed values can be changed by the operators and managers of the system to suit their own unique circumstances.

In order to enable an examination of the different management strategies employed by Model 6, the experiment of the model was repeated three times, with different initial reservoir capacities trialled under each replication. For each of these trials, all of the parameters of the model were kept the same and only the choice of the initial reservoir storage levels was altered. The results of the trials for each of the initial capacities can be found in the following sections:

4.3.1 Trial 1: Unconstrained Initial Capacities ($S_0 \leq C$)

In the first trial conducted using Model 6 it was decided to not specify the initial capacities of both reservoirs, but rather relax Equations (4.38) and (4.58) to the form $S_0 \leq C$ in order to enable the model to select the optimal starting conditions.

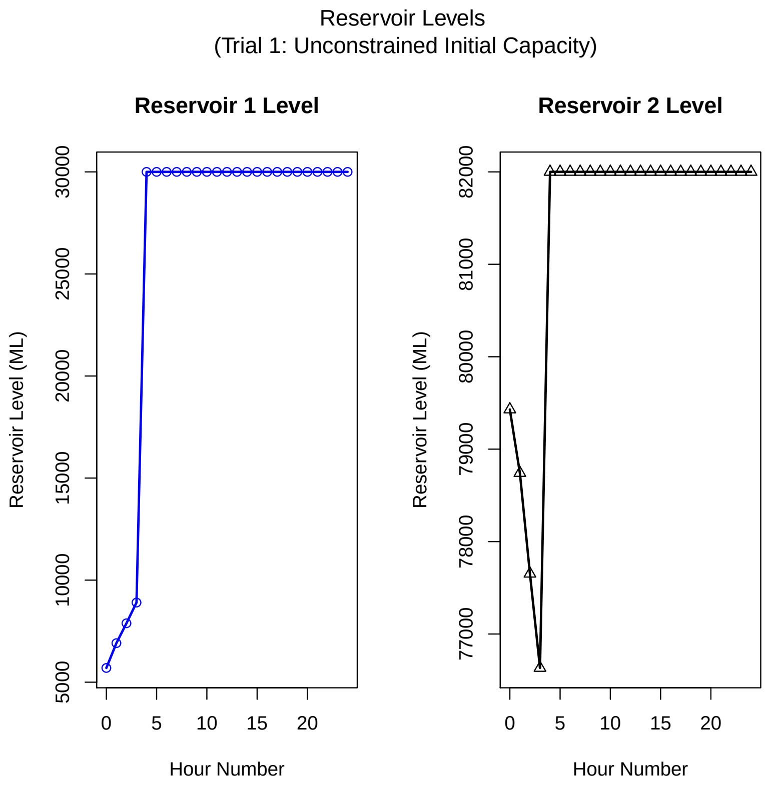

Figure 4.6: **Trial 1** ($S_0 \leq C$) - Comparison of Reservoir Levels (ML).

From this choice of initial capacity, key components of the reservoir system measured by Model 6 could then be investigated through the use of a series of figures. The first of these figures, Figure 4.6, enables a comparison to be made between the storage levels of both reservoirs over the event horizon considered. Figure 4.7 then displays the volume of water spilled from both Reservoir 1 and Reservoir 2 each hour on a common set of axes, while Figure 4.8 demonstrates the relationship between the available water, trigger volumes and pumping volume for Reservoir 2. From Figure 4.6, it can be seen that by not constraining the initial capacities of the reservoirs, Model 6 has selected to initialise the storage levels of Reservoir 1 and Reservoir 2 to approximately 6000 ML and 79 500 ML respectively. When first considering the initial capacity of Reservoir 2 it may seem counter intuitive for the storage level of the reservoir to commence at near to capacity; however upon considering Figure 4.8 later in this section, the need to initialise the storage level at this volume is made clearer.

Figure 4.6 also displays the behaviour of both reservoir levels over the event horizon. From the initial capacity of approximately 6000 ML, it can be seen that the level of Reservoir 1 increased steadily over the first three hours of the event horizon, until hour four, at which time a substantial inflow filled the reservoir to capacity. On the other hand, the level of Reservoir 2 was seen to decrease over the first three hours of the event from an initial capacity of approximately 79 500 ML. This decrease

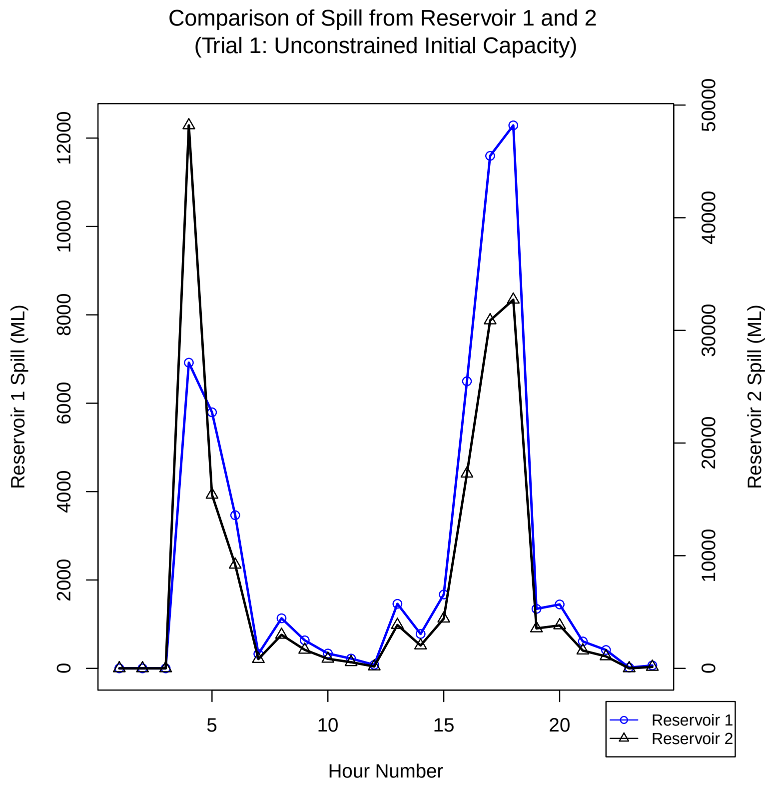

Figure 4.7: **Trial 1** ($S_0 \leq C$) - Comparison of Spill from Reservoir 1 and Reservoir 2 (ML).

in reservoir level can be attributed to water being pumped backwards through the system from Reservoir 2 to Reservoir 1. However, once again at the fourth hour, a substantial inflow filled Reservoir 2 to full capacity. Also depicted by the figure is the fact that once at full capacity, the storage levels of both reservoirs remained steady at that value. This indicates that from the fourth hour onwards, there was no hour in which the inflow to the reservoirs was less than the volume of water being released from each reservoir to the community water supply.

Figure 4.7 displays the volume of water spilled from the two reservoirs across the 24 hour event horizon considered. Over the first three hours, the figure demonstrates that no spill occurred from either reservoir. This is supported by Figure 4.6, which showed that neither reservoir reached full capacity until hour four; therefore, before this time it was physically impossible for spill to occur. From hour four, there is a period of three hours over which the behaviour of the spill from the two reservoirs varied; however at hour seven, the behaviour of the spill from both reservoirs began to approximate each other, though a difference of scale was imposed. Figure 4.7 also demonstrates that the peak spill from Reservoir 1, of approximately 12 000 ML, occurred in hour 18. On the other hand, the peak spill from Reservoir 2, or the spill from the system, occurred in the fourth hour of the event horizon and equalled approximately 50 000 ML. A comparison of how this peak spill compares

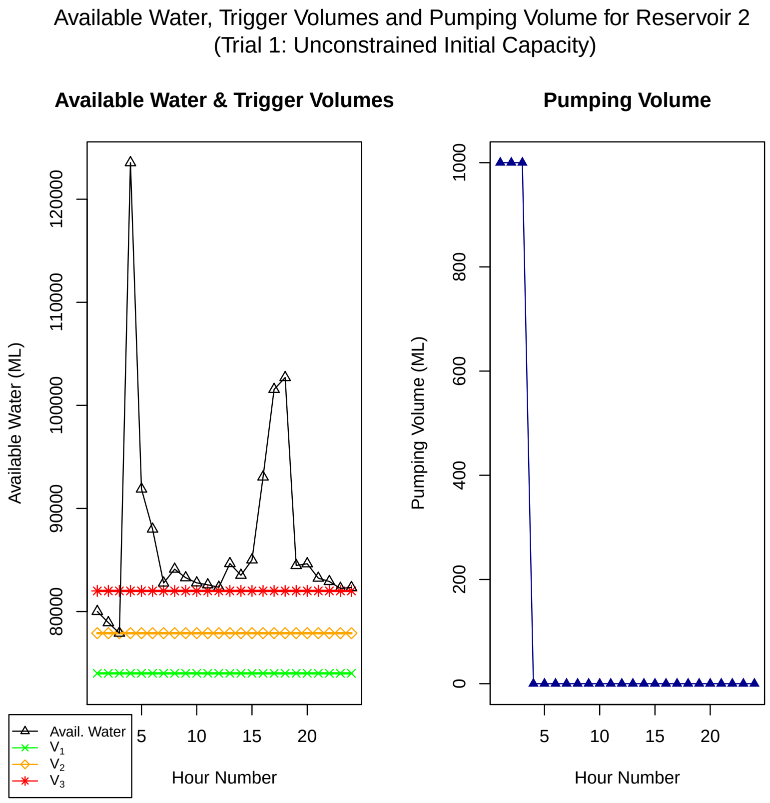

Figure 4.8: **Trial 1** ($S_0 \leq C$) - Relationship between Available Water + Trigger Volumes and Pumping Volume for Reservoir 2.

to the peak spills observed under the other trials considered is made in Section 4.3.4 later in this chapter.

Figure 4.8 provides an opportunity to examine how the relationship between the available water and trigger volumes of Reservoir 2 interact and determine the volume of water that could be pumped from the reservoir each hour. In this case, as seen in the previous chapter, the reservoir trigger volumes V_{1t}, V_{2t} and V_{3t} are represented by the green line with crosses, orange line with diamonds and red line with stars or asterisks respectively. Also, for the simplicity of notation, the three trigger volumes will, from this point, be referred to as V_1, V_2 and V_3.

The motivation behind the decision by Model 6 to initialise the storage level of Reservoir 2 at near to capacity is better illustrated by Figure 4.8. In this case, the figure shows that the available water began between the trigger volumes V_2 and V_3 for the first three hours of the event horizon, meaning that full capacity pumping could be enacted over this period. However, in the third hour the available water was seen to decrease to the value of the trigger volume V_2. Therefore, if the initial reservoir capacity was specified to be less than that selected by the model, then in the third hour, the available water would have decreased below V_2; enforcing restrictions on the pumping volume. Also, the figure demonstrates that after the third hour of the event horizon, pumping from Reservoir 2 ceased. At this point in the event horizon, Figure 4.8 shows that the available water exceeded the trigger volume V_3; however this occurrence did not limit the volume of water that could be pumped from Reservoir 2 each hour. The cause of the pumping being ceased can be seen from Figure 4.6 presented previously, which demonstrated that Reservoir 1 was filled to capacity in the fourth hour. Therefore, as Reservoir 1 had insufficient capacity to store the water being pumped from Reservoir 2, pumping stopped.

By not constraining the initial capacities of the reservoirs in this trial, Model 6 was given the opportunity to optimally select these volumes. However, from the results of this trial it has been seen that the initial capacity of Reservoir 2 selected by the model was close to full capacity and resulted in large volumes of spill from the system. Upon further consideration of these results and reviewing the structure of the objective function of Model 6, it can be concluded that the model has selected this initial capacity of Reservoir 2 as it is the minimum reservoir capacity that enables the maximisation of both the trigger volumes and the volume of water pumped from the reservoir. Although resulting in the "optimal" management strategy from a modelling point of view, this strategy may not result in the best outcome for the physical system. Once the results from the other trials have been reviewed, a comparison of the optimal management strategies employed by Model 6 will be conducted in Section 4.3.4 later in this chapter.

4.3.2 Trial 2: Initial Capacities of 0 ML ($S_0 = 0$)

In this trial of Model 6, the initial capacities of both reservoirs were no longer unconstrained but were specified to equal 0 ML; effectively, the reservoirs were both empty. Although perhaps an unrealistic starting condition for some reservoir systems, the results from this trial enable interesting comparisons to be made between the management strategies selected by the model under this "best case" scenario compared to the other starting conditions considered. Once again, a series of figures have been constructed to help explore the key components of the reservoir system measured by Model 6 during the flood event.

Figure 4.9 is the first to be considered and displays the storage levels of the two reservoirs over the event horizon of 24 hours. Starting from empty, as specified by the initial conditions of the trial, the figure shows the storage level of Reservoir 1 increased slowly over the first three hours of the horizon until hour four was reached. At this point, a large inflow event nearly filled the reservoir to capacity. Upon a review of Figure 4.5 presented earlier in the chapter, it can be seen that the fourth hour corresponded to the largest inflow of water to the system over the event horizon; an inflow of approximately 28 000 ML and 45 000 ML to Reservoir 1 and Reservoir 2 respectively. As the capacity of Reservoir 1 is 30 000 ML, this inflow effectively filled the reservoir in one hour, as depicted by Figure 4.9. In the next hour following this inflow event, the available storage volume remaining in

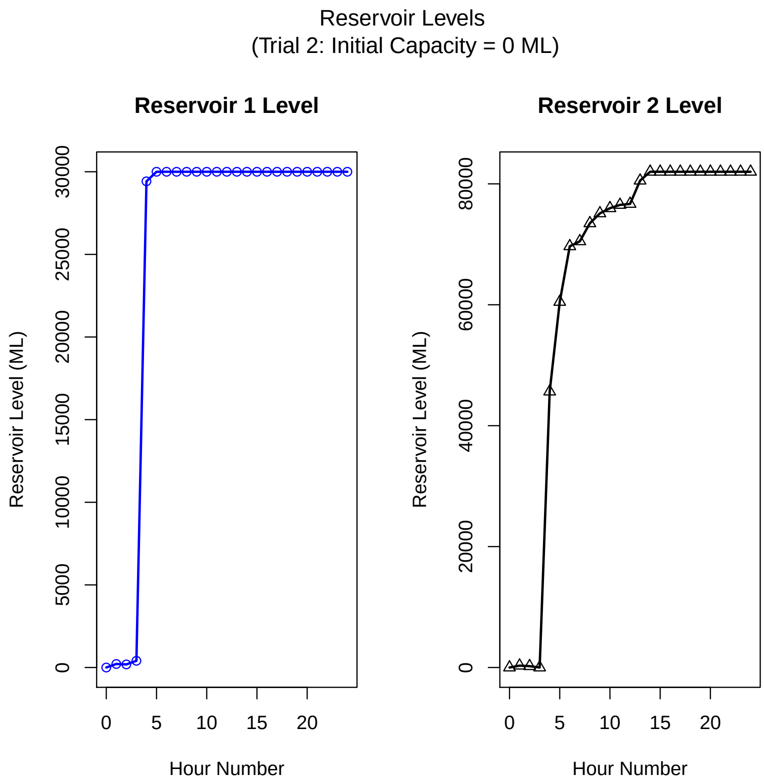

Figure 4.9: **Trial 2** ($S_0 = 0$) - Comparison of Reservoir Levels (ML).

Reservoir 1 was utilised, filling the reservoir to full capacity.
Figure 4.9 also demonstrates that the storage level of Reservoir 2 behaved in a similar manner to that of Reservoir 1. As specified by the trial parameters, at the beginning of the event horizon the reservoir was empty. Over the first two hours, the storage level of Reservoir 2 increased slowly, however the figure shows it returned to the initial capacity of 0 ML in hour three as a result of water being drained from the reservoir due to pumping. In the following hour, the reservoir was subject to the large inflow described previously, that filled the reservoir to approximately half capacity. From this point, the storage level of Reservoir 2 increased each hour until the capacity of the reservoir was reached in hour 14. As seen in the investigation of the previous trial and presented again in this figure, once the reservoirs reached full capacity, their volume was maintained at this level, indicating that the inflows to the reservoirs were greater than or equal to the releases being made to the community water supply each hour.
In order to compare the behaviour of the spill from both reservoirs over the event horizon, Figure 4.10 has been included. From the figure, it can be seen that spill began from Reservoir 1 in the fifth hour, while spill from Reservoir 2 was delayed until the fourteenth hour. These results are supported by Figure 4.9 which confirms that Reservoir 1 and Reservoir 2 reached full capacity at hour five and fourteen respectively; meaning that spill from the reservoirs before this time was a physical

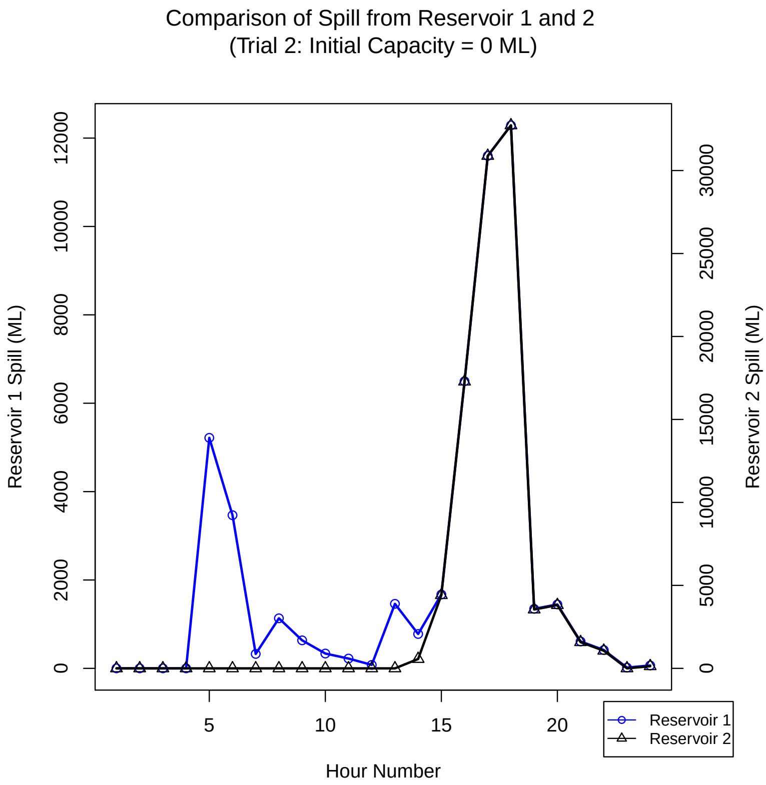

Figure 4.10: **Trial 2** ($S_0 = 0$) - Comparison of Spill from Reservoir 1 and Reservoir 2 (ML).

impossibility. Also, Figure 4.10 demonstrates that from the fifteenth hour, the spill from both reservoirs exhibited the same behaviour, though a difference in the scale of the spill was observed. Therefore, under the conditions of this trial, the peak volume of spill from both reservoirs was seen to occur in hour 18 and equalled approximately 12 000 ML from Reservoir 1 and 32 500 ML from Reservoir 2.

Figure 4.11 presents the relationship between the available water and trigger volumes of Reservoir 2 and how this relationship could be used to determine the volume of water pumped from Reservoir 2 each hour. From the figure, it can be seen that the pumping facility began under the limitations of phase one pumping restrictions, as the available water was less than the trigger volume V_1 in the first two hours of the event. This meant that the volume of water able to be pumped from Reservoir 2 each hour was constrained to 400 ML. However, also demonstrated by the figure is that during the first two hours of the event horizon, not water was pumped from Reservoir 2. This lack of pumping can be attributed to the specified initial reservoir capacity of 0 ML and the resulting lack of water in the reservoir at the beginning of the event horizon; effectively, there was no water to be pumped.

In hour three, the figure shows that the trigger volume V_1 was minimised to equal the value of the available water; changing the restrictions imposed on the pumping facility from phase one to phase two and thus enabling a maximum of 600 ML to

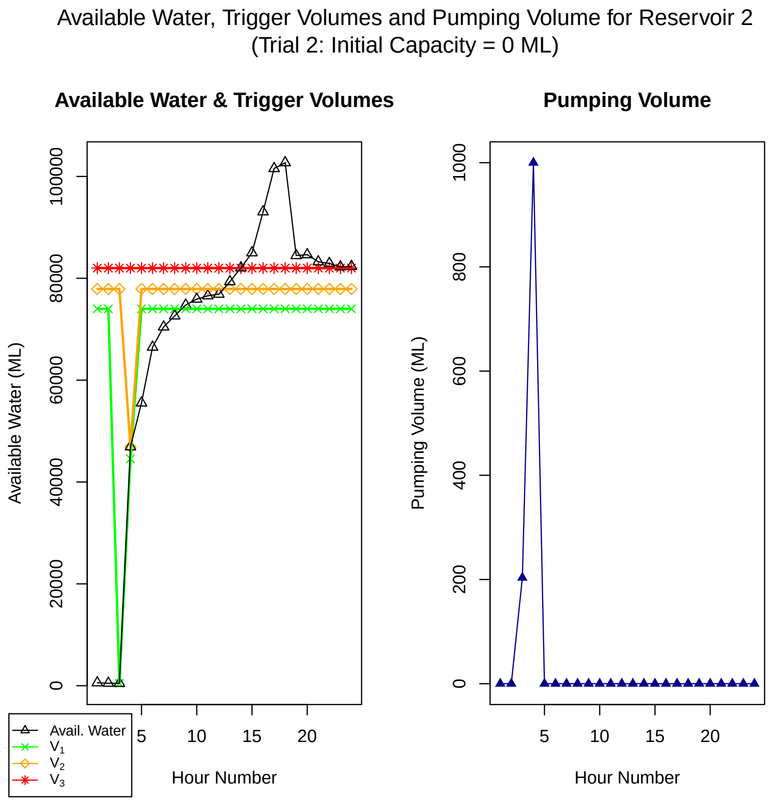

Figure 4.11: **Trial 2** ($S_0 = 0$) - Relationship between Available Water + Trigger Volumes and Pumping Volume for Reservoir 2.

be drained from Reservoir 2 each hour. Although relaxing the restrictions on the pumping facility, only approximately 200 ML was pumped from Reservoir 2 in the third hour. Upon a comparison to Figure 4.9, it can be seen that in hour three the storage level of Reservoir 2 was returned to 0 ML, meaning that the approximate 200 ML pumped from the reservoir in hour three was the total volume of water stored in the reservoir at that time. However, in order to pump this volume from the reservoir, the restrictions on the pumping facility did not have to be lifted to phase two; a maximum of 400 ML per hour could have been pumped under the more severe phase one restrictions. That being said, a requirement in the formulation of Model 6 ensures that the volume of water pumped from Reservoir 2 cannot jump from phase one pumping restrictions in the current hour to full capacity pumping in the next; a transition period of at least one hour of phase two restrictions is required. Therefore, in order to observe this requirement of the model, phase two pumping restrictions needed to be enforced in hour three to ensure that full capacity pumping could be enacted in the subsequent hour. This can be seen from Figure 4.11, as the trigger volume V_2 has been minimised to equal the value of the available water in hour four, thus enabling the full capacity of the pumping facility to be drained from Reservoir 2. Following this hour of full capacity pumping, no further water was drained from Reservoir 2 due to Reservoir 1 reaching full capacity.

In the case of this trial, by specifying the initial capacities of both reservoirs to equal 0 ML, the strategy selected by Model 6 for the optimal management of the reservoir system was different to that seen in the previous trial; resulting in the maximisation of the number of months in which no spill occurred and the minimisation of the peak spill from each reservoir compared to that seen previously. However, an initial reservoir capacity of 0 ML is unrealistic for most reservoir systems that are fundamentally designed as a secure water source for communities and only used secondarily as a defence mechanism against flood events. In the next trial, a more realistic starting condition is investigated, with the initial storage level of each reservoir selected to equal half of their respective total capacities.

4.3.3 Trial 3: Initial Capacities of Half Reservoir Capacity ($S_0 = \frac{1}{2}C$)

In the final trial of Model 6 explored in this chapter, a more realistic initial capacity to that investigated in the previous section was selected for both reservoirs. Therefore, under the conditions of the trial, it was specified that the storage level of both reservoirs be initialised to half of their respective total capacities; an initial capacity of 15 000 ML for Reservoir 1 and 41 000 ML for Reservoir 2. In order to aid in the examination of the key components of the reservoir system measured by Model 6, three figures have been provided.

The first to be investigated is Figure 4.12 which presents the storage levels of Reservoir 1 and Reservoir 2 over the event horizon of 24 hours. From the figure, it can be seen that the storage level of Reservoir 1 increased from the initial capacity of 15 000 ML over the first three hours of the horizon, until hour four, at which time the significant inflow event examined in the previous section filled the reservoir to capacity. On the other hand, the storage level of Reservoir 2 began at 41 000 ML and decreased, as a result of water being pumped from the reservoir, over the first three hours. However, as seen for Reservoir 1, Reservoir 2 was also filled to capacity due to the size of the inflows that occurred in the fourth hour. Upon comparison to Figure 4.6, it can be seen that the behaviour of the storage levels in this trial are similar to that seen under Trial 1, though the initial capacities of the reservoirs differ. This demonstrates that the strategy determined by Model 6 for the optimal management of the reservoir system under the conditions of this trial, although different to the optimal strategy selected under Trial 1, results in similar reservoir storage level behaviours.

From Figure 4.13, it is seen that no spill occurred from either reservoir over the first three hours of the event horizon, due to neither reservoir being at full capacity during this time. A difference that was noted between the behaviour of the spill from Reservoir 1 in this trial, compared to the spill seen in previous trials, was

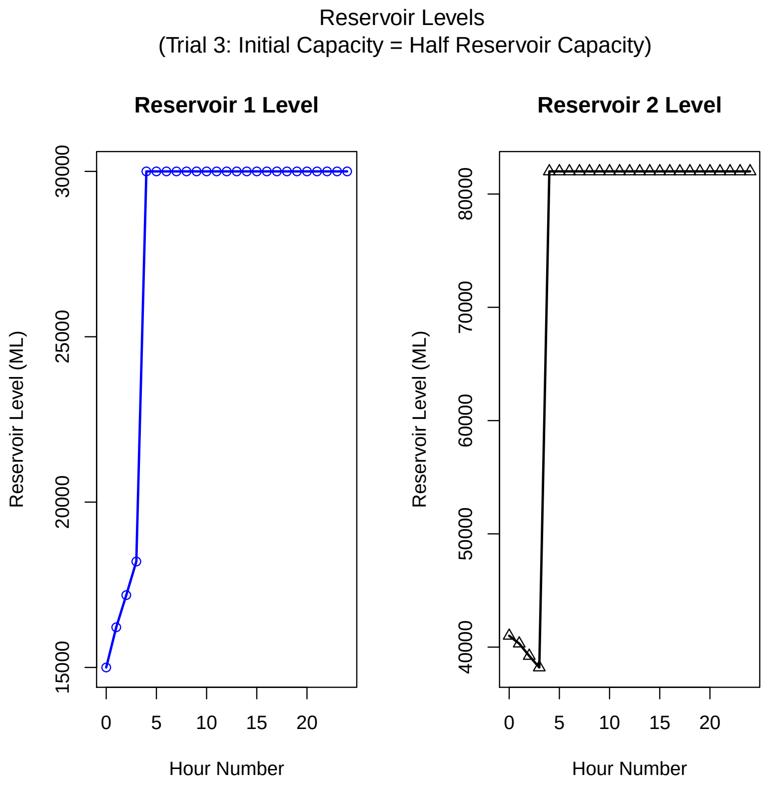

Figure 4.12: **Trial 3** ($S_0 = \frac{1}{2}C$) - Comparison of Reservoir Levels (ML).

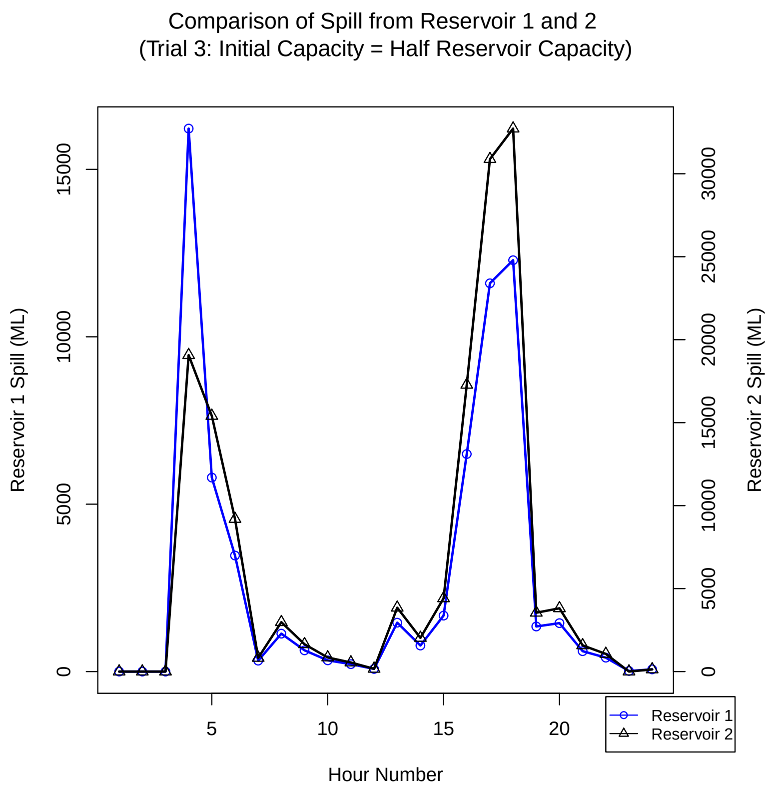

Figure 4.13: **Trial 3** ($S_0 = \frac{1}{2}C$) - Comparison of Spill from Reservoir 1 and Reservoir 2 (ML).

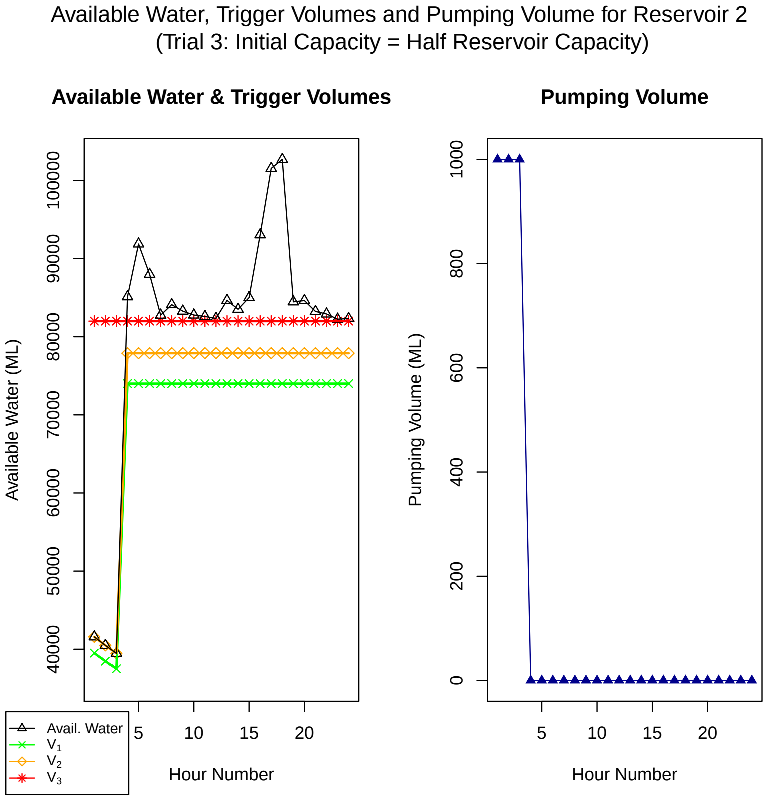

Figure 4.14: **Trial 3** ($S_0 = \frac{1}{2}C$) - Relationship between Available Water + Trigger Volumes and Pumping Volume for Reservoir 2.

that the peak spill, of approximately 16 000 ML, occurred from the reservoir in the fourth hour of the event. This difference in the timing of the peak spill can be attributed to the increase in initial capacity of the reservoir compared to that seen in previous trials. However, the initial capacity of Reservoir 2 did not affect the timing of the peak spill from the system, which occurred in hour 18 and equalled approximately 32 500 ML; the same as that seen under the conditions of the previous trial considered. Figure 4.13 also displays that from hour seven, the behaviour of the spill from the two reservoirs approximated each other, although the spill from Reservoir 2 was systematically greater than that from Reservoir 1.

The final figure considered for this trial is Figure 4.14 which presents the relationship between the available water and trigger volumes of Reservoir 2 and how this relationship interacted with the volume of water that was pumped from the reservoir each hour. In this case, the figure shows that the trigger volume V_2 was minimised to equal the value of the available water for the first three hours of the event horizon. By minimising the trigger volume to this value, it ensures that the pumping facility began in a state of full capacity pumping, with no restrictions placed on the volume of water able to be drained from Reservoir 2 each hour. By initialising the pumping facility in this state, the model has avoided the necessity to employ at least one hour of phase two pumping restrictions before enacting full capacity pumping, when transitioning from phase one restrictions; a specification of Model 6 encountered under the conditions of the previous trial. Therefore, as no restrictions were enforced, the full capacity of the pumping facility, 1000 ML, was drained from Reservoir 2 over the first three hours of the event. However, at the fourth hour, Figure 4.12 demonstrates that Reservoir 1 reached full capacity, meaning that there was insufficient storage remaining in the reservoir for pumping to continue.

The conditions considered under this final trial provided more realistic initial capacities for the two reservoirs than that seen under the previous trial, where both reservoirs were assumed to be empty at the beginning of the event. That being said, although the initial capacities of the reservoirs were increased in this trial, some of the results were similar to that seen under the conditions of Trial 2, while others were similar to those seen under the parameters of Trial 1. A more detailed comparison between the results of Model 6 from all three trials is conducted in the next section.

4.3.4 Comparison of Spill from Reservoirs 1 and 2 under Trials 1, 2 and 3

A comparison of the spill from Reservoir 1 and Reservoir 2, under the conditions of the three trials considered, can be conducted through the use of the results from Model 6 presented in the previous sections. In order to present a summary of the spill from each reservoir, under the conditions of each trial, Table 4.1 has been constructed. This table displays the minimum, median, mean and maximum spill from Reservoir 1 and Reservoir 2, for the three trials. Due to the distribution of the spill being heavily skewed, the median provides a better measure for the centre of the dataset in this case. Also presented in Table 4.1 for each trial is the total spill from each of the reservoirs over the event horizon. For ease, the total spill from each reservoir can be considered separately at first.

To begin, a comparison of the total spill from Reservoir 1, or spill within the system, can be conducted. Table 4.1 shows that the minimum total spill from Reservoir 1, of 49 611 ML, occurred under the conditions of Trial 2, while the maximum total spill, of 66 407 ML, was seen under the conditions of Trial 3. After considering these spills against the initial reservoir capacities imposed under each trial, it can be seen that there is a direct correspondence between the initial volume in the reservoir and the total spill from the reservoir. For example, under Trial 2, the minimum initial capacity of 0 ML also corresponded to the minimum volume of spill from the reservoir, while the maximum initial capacity of 15 000 ML under Trial 3 coincided with the maximum amount of spill from Reservoir 1. In the remaining trial, Trial 1, Model 6 selected the initial capacity of Reservoir 1 to be approximately 6 000 ML, falling between the other two initial capacities; a fact also reflected in the spill from the reservoir of 57 106 ML, which lies between the spill seen under the conditions

Table 4.1: Summary Statistics of Volume Spilled under each Trial from Reservoir 1 and Reservoir 2.

	Spill (ML)	
	Reservoir 1 (Within the System)	Reservoir 2 (From the System)
Trial 1		
Minimum	0	0
Median	706.46	1838.56
Mean	2379.41	7546.91
Maximum	12287.84	48198.08
Total	**57105.94**	**181125.80**
Trial 2		
Minimum	0	0
Median	621.98	0
Mean	2067.11	3999.90
Maximum	12287.84	32722.24
Total	**49610.62**	**95997.52**
Trial 3		
Minimum	0	0
Median	706.46	1838.56
Mean	2766.96	6333.23
Maximum	16219.16	32722.24
Total	**66407.12**	**151997.50**

of the other two trials.

This correspondence between the initial capacity and total volume of spill is also seen for Reservoir 2. In this case, the table demonstrates that the minimum total spill from Reservoir 2, or spill from the system, of 95 998 ML also occurred under the conditions of Trial 2, where the initial capacity of the reservoir was minimised to equal 0 ML. However, the maximum total spill from the system, of 181 126 ML, occurred under the conditions of Trial 1, where the model elected to select the initial capacity of the reservoir to equal approximately 79 500 ML; the maximum initial capacity of Reservoir 2 for the three trials. In the remaining trial, Trial 3, the initial capacity of Reservoir 2 was selected to equal 41 000 ML, while the spill from the reservoir, and thus from the system, totalled 151 998 ML; both initial capacity and total releases lying between those of the other trials.

From an exploration of the results presented in Table 4.1, it has been established that a direct correspondence exists between the initial capacity and the total spill from both Reservoir 1 and Reservoir 2; making the selection of the initial capacities of the reservoirs an important consideration when trying to minimise the spill from the system. As such, this provides an opportunity for future research, as a formula could be developed to determine the expected spill from the system given the initial capacities of both Reservoir 1 and Reservoir 2. However, in order to construct this formula, more experiments of Model 6 would need to be conducted to ensure that a better approximation of the relationship between the initial capacity and spill could be identified. Also, the usefulness of the formula could be limited, as it would only be applicable under the series of assumptions made before the conduct of this experiment.

Also of interest from the summary of the spill presented in Table 4.1 is the peak spill from each reservoir, under the conditions of the three trials. First to be considered is Reservoir 1. From the table, it can be seen that the maximum, or peak, spill from Reservoir 1 under the conditions of Trial 3 was 16 219 ML. Interestingly, the peak spill from the reservoir under the conditions of the other two trials were the same and equalled 12 288 ML. Upon considering Figures 4.7, 4.10 and 4.13 presented previously, it can be seen that the hour in which the peak spill occurred from Reservoir 1 changes under Trials 1 and 2 compared to Trial 3. In the case of Trial 3, the peak spill from the reservoir occurred in hour four and corresponded to the hour in which a significant inflow event transpired filling the reservoir to capacity. While, on the other hand, under the conditions of Trial 1 and Trial 2, the peak inflow occurred in hour 18, many hours after the reservoir had reached full capacity. Therefore, this result suggests that the smaller initial capacities of Reservoir 1 under Trial 1 and Trial 2 ensured that the reservoir had sufficient available storage remaining at the fourth hour to hold back a greater proportion of the significant inflow event, thereby minimising the spill in that hour. However, by specifying an initial capacity of 15 000 ML in Trial 3, there was less available storage remaining at the time of the inflow event, leading to an increase in the amount of spill that occurred. This result enables the conclusion that there is a tipping point relationship between the initial capacity and the peak spill from Reservoir 1, as opposed to the correspondence relationship witnessed earlier between the initial capacity and total spill. A tipping point relationship suggests that the peak spill from Reservoir 1 will be minimised to 12 288 ML and occur in hour 18 until a particular initial capacity is reached; the tipping point. At this tipping point, the peak spill from the reservoir will begin to increase as the initial capacity increases and the time at which the peak spill occurs will change to the fourth hour to coincide with the significant inflow event.

This tipping point behaviour is also exhibited upon investigation of the maximum, or peak, spill from Reservoir 2; also presented in Table 4.1. Under the conditions of Trial 1, the peak spill from Reservoir 2 occurred in the fourth hour and equalled 48 198 ML, whereas the peak spill from the reservoir under the remaining two trials was minimised to equal 32 722 ML and occurred in the eighteenth hour. From these results, it can be concluded that the initial capacities of Reservoir 2 under Trial 1 and Trial 2 were less than the tipping point, leading to the peak spill being minimised to 32 722 ML and occurring in hour 18. On the other hand, the initial capacity selected by Model 6 under the conditions of Trial 3 was greater than the tipping point, meaning that the peak spill from the reservoir occurred in the fourth hour and equalled 48 198 ML.

From a review of the results presented throughout this chapter, it can be ascertained that Model 6 has provided a thorough and comprehensive approximation for the operation of the cascade reservoir system considered as a case study, during a flood event. For each of the trials considered, the model has identified the optimal strategy for the management of the system under the constraints of the model, and given the assumptions made prior to the experiment. Although one particular solution was highlighted to be "optimal" from a modelling perspective and not completely sound in a physical sense, Model 6 still provides the operators and managers with an approximation of their reservoir system and a plausible outcome of the event. At this point, it must be reiterated that a series of assumptions were made before conducting the experiment using Model 6; some of which may not be feasible in a physical system. Examples of such assumptions are the assumed perfect prior knowledge of the inflows to both reservoirs and also the assumed size of these inflows. Therefore, the use of any results from the model must be treated with caution and should be utilised as a guidance tool; provided with the goal of supplying the managers and operators with an approximate representation of their reservoir system and an ability to investigate potential behaviours, under a variety of scenarios, that could be witnessed during a flood event.

Also, scope for future research was identified through a review of the results from Model 6. As mentioned previously, more experiments could be conducted using the model to approximate the relationship between the initial capacities and total spill from the reservoir system. This relationship could then be reduced to a formula that enables the total spill from the system to be predicted from the initial capacities of the two reservoirs. Another opportunity for future investigation is to conduct a comparison of Model 6 in its current state with a model that does not include a pumping facility. Due to the assumption made in the experiment of Model 6 regarding the size of the inflows to the system, Reservoir 1 filled to capacity very quickly, meaning that the effectiveness and impact of the pumping facility on the management strategies may have been limited. Although beyond the scope of this thesis, it would be a worthwhile investigation to determine the impact of removing the pumping facility from the system.

4.4 Chapter Conclusion

In this chapter, a model was constructed to ascertain strategies for how the existing Perseverance and Cressbrook cascade reservoir system considered as a case study in this thesis could be optimally managed during a flood event. To start, a Mixed Integer Linear Programming (MILP) model, named Model 5, was formulated using the structure of Model 3 presented in the previous chapter as a framework. Originally constructed to be applied to a single reservoir system during a drought event, the constraints of Model 3 required significant alterations to ensure that the newly formulated Model 5 would be applicable to a single reservoir system during a flood event. Also, in order to increase the complexity of the problem considered, an additional assumption was made to incorporate a pumping facility as part of the reservoir system. The usefulness of this pumping facility was made clearer upon the formulation of Model 6; an extended version of Model 5 to consider the operation of a cascade reservoir system during a flood event. In this case, it was assumed that the pumping facility enabled water to be drained from Reservoir 2 and pumped backwards through the system to Reservoir 1, thus helping to minimise the volume of spill each hour from Reservoir 2. Following this, an experiment was then performed using Model 6 to investigate how key components of the reservoir system behaved under the specification of varying initial capacities. Three different initial capacities were trialled, with the experiment repeated for each case. From the results of the trials, it was found that relationships exist between the initial capacity of the reservoirs and both the maximum spill and total spill from the reservoirs over the event horizon. Also, it was found that Model 6 provided a thorough approximation for the operation of the cascade reservoir system during a flood event; however, opportunities also exist for potential future work, both in developing a function to describe the relationship between initial capacity and total spill, along with a comparison of Model 6 in its current form to a model without the inclusion of a pumping facility.

Utilising the skills, knowledge and understanding that was developed throughout the previous chapters with regard to the application of linear programming models to reservoir systems, in this chapter, two MILP models were able to be constructed from the ground up. By employing the basic framework and some of the techniques presented in Model 3 of Chapter 3, Model 5 was able to be constructed and then extended to form Model 6. The inclusion of the pumping facility to the model increased the complexity of the situation and stretched all previous experience with MILP models. Overall, the skills and knowledge developed in the previous chapters were vital in creating the models considered in this chapter for the operation of a cascade reservoir system during a flood event.

Chapter 5

Using Time Series to Simulate Inflows

5.1 Chapter Overview

In this chapter, time series analysis is utilised to simulate an alternate set of inflow data, based upon the historic record of inflows sourced from the Queensland Department of Natural Resources and Mines (2012), for the existing Perseverance and Cressbrook cascade reservoir system considered as a case study throughout this thesis. This set of simulated inflows will then be used in the subsequent chapter to perform a comparison of the drought model (Model 4 described in Chapter 3) and flood model (Model 6 presented in Chapter 4) developed previously. In order to derive a time series model from the historic records, two model selection tools outlined by Dunn and Addie (2008) are employed. Upon the selection of an adequate model, a set of diagnostic tests are performed to ensure that the model is suitable and provides a reasonable approximation for the historic inflows. To complete the examination of the time series model, a validation test proposed by Dunn and Addie (2008) is conducted using a series of one-step ahead forecasts. The chapter is then concluded by performing a simulation using the selected time series model to generate an alternate set of inflows for the reservoir system.

5.2 Selection of Time Series Model

In this section, two model selection tools, the sample Autocorrelation Function (sample ACF) and sample Partial Autocorrelation Function (sample PACF), are utilised to deduce an appropriate time series model to describe the historic inflows to the Perseverance and Cressbrook cascade reservoir system considered as a case study. As mentioned in the previous chapters, the historic inflows were sourced from the Queensland Department of Natural Resources and Mines (2012) and corresponded to a period from November 1965 to May 1981; where monthly recordings were made of flows along Cressbrook Creek, the major tributary feeding into Cressbrook dam. However, from 1979 onwards, the recordings became inconsistent and many monthly observations were missing. Therefore, for the purposes of selecting a time series model, only the historic inflows from January 1966 to December 1978 were considered; providing a 13 year period of inflows or 156 monthly observations

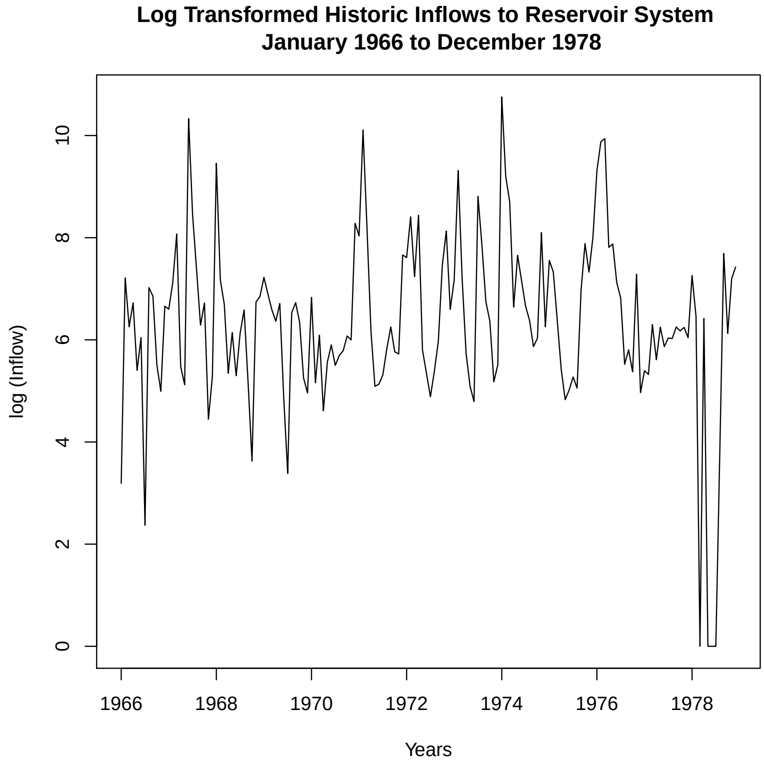

Figure 5.1: Logarithm Transformed Historic Inflows from Jan 1966 to Dec 1978. *Sourced: Queensland Department of Natural Resources and Mines (2012).*

in total.

Before considering the model selection tools, the inflow data was assessed to determine if it exhibited stationary behaviour; a behaviour that if shown, substantially reduces the complexity of selecting a time series model for any dataset. Therefore, by applying a logarithm transformation to the historic inflow records, it was found that the inflow data exhibited stationary behaviour. This meant that the statistics of the inflow records, such as the mean and variance, did not change over time; properties of the data depicted by Figure 5.1, which shows the logarithm transformed historic inflows to the Perseverance and Cressbrook cascade reservoir system over the 13 year time period considered. Also displayed by the figure is that an anomalous inflow event transpired in the final year of inflow records, where no inflows occurred to the system in a number of months. Due to this period of minimal inflow, this year was selected to form part of the worst case drought scenario considered in Chapter 3, when performing an experiment using Model 4.

In order to ensure the eventual validation of the time series model selected to describe the historic inflow data, the inflows needed to be separated into two portions. The first portion, which incorporated the majority of the inflow data, was referred to as the training series and in this case was composed of an 11 year period from January 1966 to December 1976. As the name of the training series suggests, the

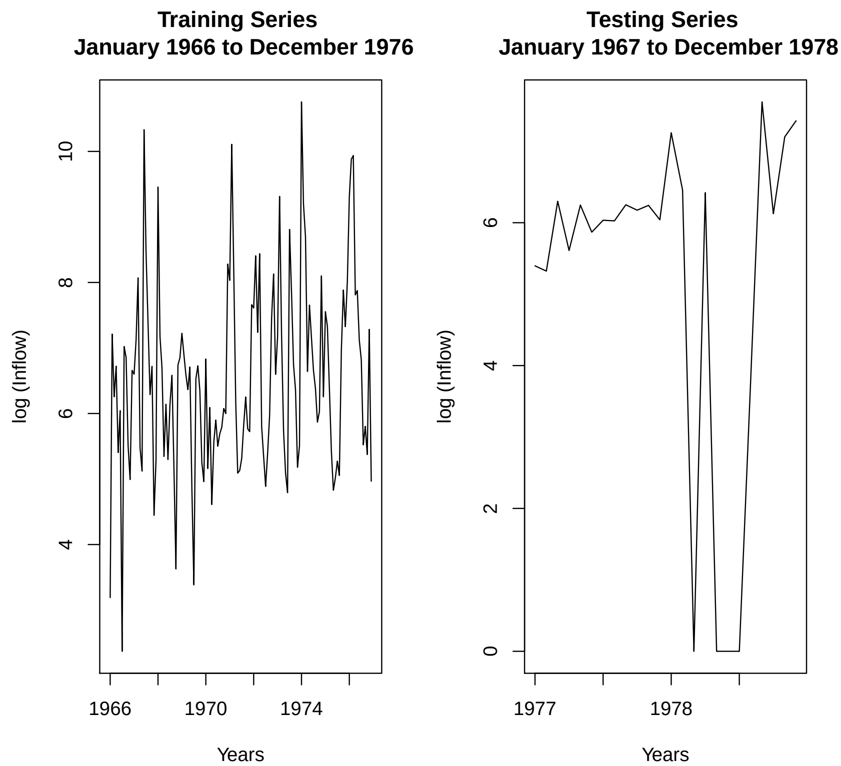

Figure 5.2: Historic Inflows Separated into Training Series and Testing Series.

time series model for the historic inflows was selected or "trained" on the basis of the information presented in this portion of the inflow records. The second portion of the inflow data, named the testing series, was much smaller and was selected to consist of only the last two years of inflow records. This portion of the historic inflow data was key in the validation of the selected time series model, as it was against the testing series that forecasts from the model were compared to assess both the forecasting ability of the model, along with how well the model "fitted" the data (performed in Section 5.4 later in this chapter). The separation of the historic inflow records into both the training and testing series is presented in Figure 5.2. From this figure, it can be seen that the testing series encompassed the months in the final year of inflow records where no inflows occurred. However, by electing to include this atypical inflow event as part of the testing series, the capability to assess the forecasting ability of the time series model may have been impaired. The limitations surrounding the use of the testing series selected are discussed more thoroughly in Section 5.4.

At this point, it should be noted that the time series analysis performed throughout this chapter was conducted using the statistical computing software environment, R version 2.15.1 (R Core Team, 2012). For each of the different analysis procedures performed throughout the chapter, a separate section is provided in Appendix E in order to document the source code utilised. In this case, the source code used to

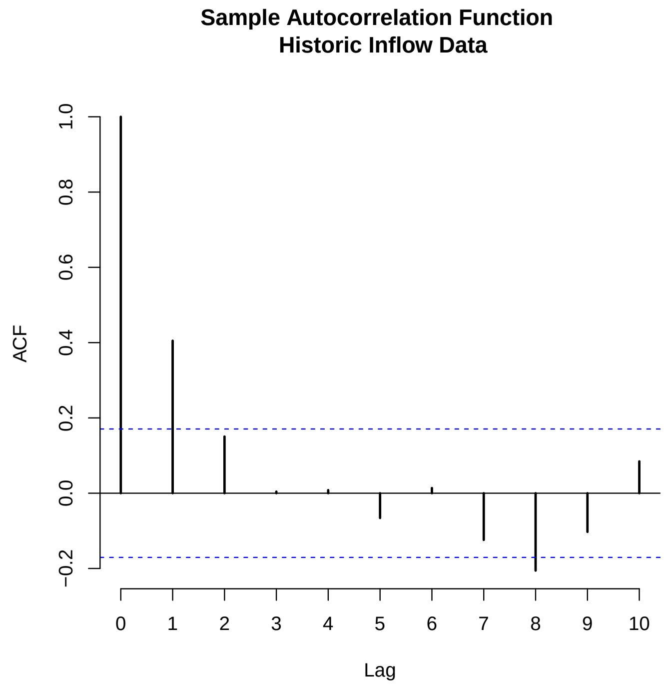

Figure 5.3: The Sample Autocorrelation Function (Sample ACF) for the Historic Inflow Training Series.

perform the formatting of the data outlined previously, such as defining the historic inflow records as a time series and separating the data into the training and testing series, can be found in Section E.1 of Appendix E.

In order to determine the type and order of the time series model that best described the historic inflow data, two model selection tools were considered; the sample autocorrelation function (sample ACF) and sample partial autocorrelation function (sample PACF). From these tools, the appropriate type of time series model under the Box-Jenkins methodology could be selected; being an autoregressive (AR) model, a moving average (MA) model or a combination of both model types (ARMA) model. It is beyond the scope of this thesis to rigorously define these model types; however, in a general sense, an autoregressive model is composed of a linear combination of the previous observations in the series, while a moving average model is expressed as a function of the previous forecasting errors (Dunn and Addie, 2008).

The first model selection tool considered was the sample ACF, from which the order, k, of a MA model that best described the historic inflow data could be determined. Dunn and Addie (2008) state that:

> "If the sample ACF has k non-zero components from 1 to k, then an MA(k) model is appropriate." (Dunn and Addie, 2008)

The sample ACF for the historic inflow data is presented in Figure 5.3. From the figure, it can be seen that the sample ACF was measured at each lag, where the term lag refers to the time difference in the ACF. The value of the sample ACF at each lag corresponded to the correlation between any term in the series, with the term x time steps prior (or after when the time series is stationary, as was the case for the historic inflow data), where x is the number of the lag being considered. Also, from the figure, it can be seen that two horizontal dashed lines appear on the plot. These lines represent the approximate 95% confidence interval of the sample ACF. Therefore, if an autocorrelation value fell within this interval, it could be considered as zero and any deviation from zero seen in Figure 5.3 was as a result of sampling error only. Figure 5.3 also demonstrates that the value of the sample ACF at lag zero was equal to one. Regardless of the data considered this value will always equal one and indicates that each term of the series is perfectly correlated with itself.

Therefore, from Figure 5.3, there were two lags at which the value of the sample ACF was greater than the 95% confidence interval. These occurred at lag one and again at lag eight; indicating that a MA(1) (first order moving average) or a MA(8) (eighth order moving average) model could have been suitable to describe the historic inflow data. However, the more parsimonious models often provide a suitable description of the data, while reducing the concerns associated with overfitting or over expressing the time series model. Taking parsimony into account, along with the small extent by which the value of the sample ACF was greater than the approximate 95% confidence interval at lag eight, it was concluded that the higher order MA(8) model would not be considered as a viable time series model for the inflow data.

The second model selection tool considered was the sample PACF. Similar to the role of the sample ACF, the sample PACF was utilised to determine the order, k, of an AR model that provided the best description of the historic inflow data. In this case, Dunn and Addie (2008) state:

> "If the sample PACF has k non-zero components from 1 to k, then an AR(k) model is appropriate." (Dunn and Addie, 2008)

Figure 5.4 presents the sample PACF for the historic inflow data. In contrast to the sample ACF, the sample PACF measured the correlation between the current term and a future term of the series, after removing the effect of the joint correlations with the intermediate terms. Similar to the sample ACF, a value of the sample PACF that fell within the 95% confidence interval (dashed lines on Figure 5.4) could again be considered to equal zero. One difference between the two autocorrelation functions was that there was no term at lag zero for the sample PACF.

From Figure 5.4, it can be seen that at lag one the value of the sample PACF exceeded the approximate 95% confidence interval. The figure also demonstrates that at lag seven, the value of the function was approximately equal to the lower bound

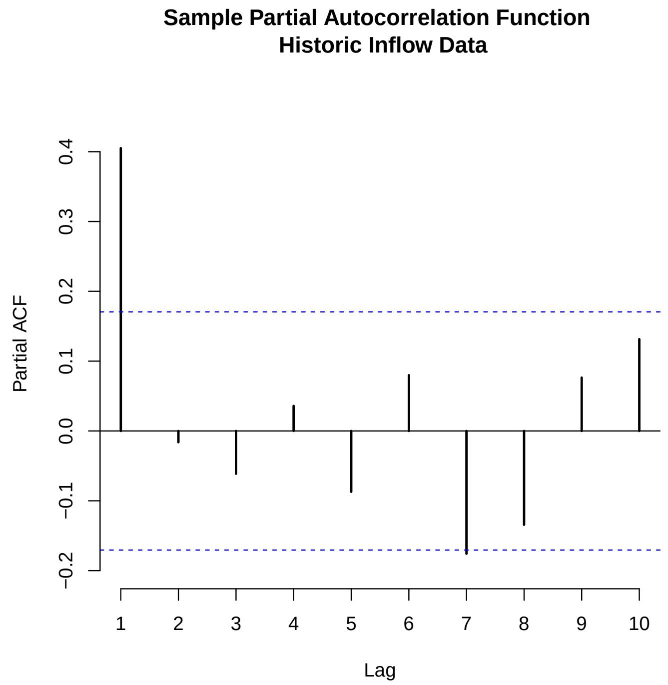

Figure 5.4: The Sample Partial Autocorrelation Function (Sample PACF) for the Historic Inflow Traning Series.

of the confidence interval. Due to the small extent by which this term exceeded the confidence interval and the approximate nature of the interval, this term was not considered to be significantly different to zero.

Therefore, after considering the sample ACF and sample PACF, it was concluded that the historic inflow data could be described through the use of a MA(1) or an AR(1) time series model. Also, these two individual components could be merged together to form an autoregressive, moving average model, or ARMA(1,1) model. In order to determine which of these three possible time series models provided the best representation of the historic inflow data, the Akaike Information Criteria (AIC) was utilised. The AIC is used in many areas of statistics and balances the size of the errors (using log-likelihood) against the overfitting of parameters in the model (using a penalty term). Therefore, while fitting additional terms to the model will always reduce the size of the errors, the AIC penalises the addition of unnecessary terms, or terms not contributing significant information.

The statistical software R was used to fit each of the three model types to the historic inflow data, with a comparison then conducted using the AIC of each model to determine which minimised the value of the AIC; thus meeting the criteria of minimising the size of the errors, whilst helping to guard against the impacts of overfitting. The source code used to fit each of these models, along with the resulting output, can be found in Section E.2 of Appendix E. An example of the output generated by the model fitting process in R can be seen in Figure 5.5, where an

```
Call:
arima(x = inflow.ts.train, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.4228     6.5073
s.e.  0.0807     0.1986

sigma^2 estimated as 1.75:  log likelihood = -224.33,  aic = 454.66
```

Figure 5.5: Example Output from the Time Series Model fitting process in R, where an AR(1) Model is fitted to the Historic Inflow Training Series.

AR(1) model was fitted to the historic inflow data.
As a result of the model fitting process, R provided the value of the coefficients included in the model, along with their respective standard errors. Also, R generated some tools that could be used in the model selection process, such as an estimate of the model variance (labelled `sigma^2` in Figure 5.5), the log likelihood, and most useful in this case, the AIC. From the output, it can be seen that the AIC of the AR(1) model was approximately 454.66. This value can be compared against the AIC of the other two model types in Table 5.1. From the table, it can be seen that the smallest AIC corresponded to the AR(1) model. This result suggests that the AR(1) model did a better job of minimising the errors, and thus approximating the historic inflow records than the MA(1) model considered, as both the AR(1) and MA(1) models contained the same number of parameters. Therefore, the difference in the AIC between the AR(1) and MA(1) models could only be as a result of the models abilities to minimise the error. However, due to the ARMA(1, 1) model containing more parameters, this model would always do a better job of minimising the errors than the AR(1) model. Thus, the difference between the AIC of the AR(1) model and ARMA(1, 1) model was due to the penalty imposed on the ARMA(1, 1) model for containing these extra parameters, one of which contributed minimal improvement to the model fit.

Table 5.1: Comparison of the AIC for each Model Type fitted to the Historic Inflow Traing Series.

Model Type	**AIC**
AR(1)	454.66
MA(1)	458.21
ARMA(1, 1)	456.55

The AR(1) model selected on the basis of the AIC for the historic inflow data can be stated below through the use of the coefficient estimates provided by R in Figure 5.5. Therefore, if the historic inflows are $\{I_t\}$, where t is the unit of time, then the fitted AR(1) model is

$$I_t - 6.5073 = 0.4228(I_{t-1} - 6.5073) + e_t$$

This can be rearranged to produce the model

$$I_t = 3.756 + 0.4228 I_{t-1} + e_t$$

for $t \geq 1$, where $\{e_t\}_{t \geq 1}$ is a series of independently, identically distributed (iid) random variables, denoting the noise in the time series.

By considering the sample ACF and sample PACF, along with the AIC, it was determined that an AR(1) model provided an adequate approximation for the historic inflow data; however, it could not be assured that the model provided a "good" representation of the inflow records. Therefore, a series of diagnostic tests needed to be performed.

5.3 Diagnostic Tests

In this section, a number of diagnostic tests are performed to ascertain whether the AR(1) model selected to describe the historic inflow records provided a "good" representation of the data set. In their work, Dunn and Addie (2008) define a "good" model as a model that is able to capture the important features, or signal, of the data. They go on to state that if the model can successfully capture the signal, then upon the signals removal from the data, all that should remain is independent and unpredictable, random noise; termed white noise. This unpredictability of the noise, or residual values, is vital, as if the noise is in some way predictable, this indicates that some element of the signal was not accounted for by the selected model and improvements, such as the addition of extra terms to the model, could be made to capture this. Dunn and Addie (2008) also indicate that a "good" model is parsimonious, and not overly complicated. In order to test for this condition, the significance of each term in a model can be considered and any insignificant terms removed. This test of significance is one of the diagnostic terms performed throughout this section; however to begin, the residual Autocorrelation Function (residual ACF) and residual Partial Autocorrelation Function (residual PACF) are considered.

Residual ACF and PACF

Figure 5.6 presents both the residual ACF and residual PACF for the AR(1) model selected to describe the historic inflow data. The residual ACF and residual PACF

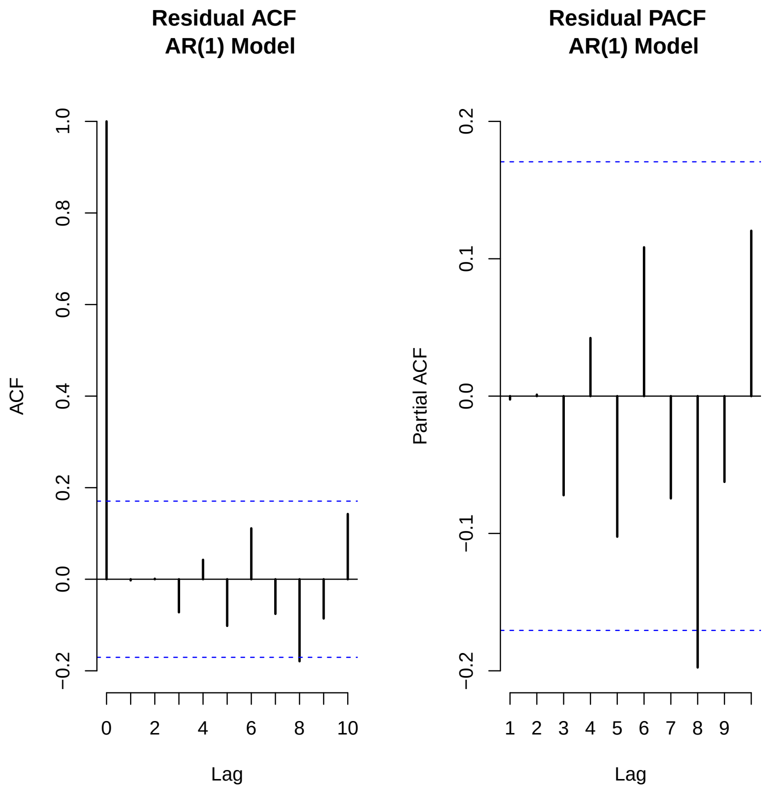

Figure 5.6: Residual ACF and PACF for the AR(1) Model fitted to the Historic Inflow Training Series.

behave in much the same way as the sample ACF and sample PACF considered previously; however in this case the autocorrelation functions were fitted to the residual values of the AR(1) time series model selected. As mentioned previously, if the selected model provided a "good" representation of the historic inflow records, the residuals of the model would appear as white noise and as such would not be predictable or able to be forecast. This would have been reflected in the residual ACF and residual PACF by all terms lying within the approximate 95% confidence interval. However, both the residual ACF and residual PACF indicated that some element of the residuals may have been able to be forecast, meaning that this forecasting ability needed to tested for its inclusion in the signal of the model; achieved through the addition of extra terms to the AR(1) model selected.

From Figure 5.6, it can be seen that the value of the residual PACF at lag eight was greater than the approximate 95% confidence interval, indicating that there was some element of the residuals that was able to be forecast from the inclusion of this term in the time series model. The residual ACF also indicated that at lag eight there was some element of the residuals able to be forecast; however due to the minimal extent by which the value of the residual ACF at that lag was greater than the 95% confidence interval, it was assumed to be as a result of sampling error only and was not considered further.

```
        Box-Pierce test

data:  resid(model.1)
X-squared = 23.7008, df = 25, p-value = 0.5367
```

Figure 5.7: Example of the Output from performing a Box-Pierce Test on the residuals of the AR(1) Model in R.

As a result of the value of the residual PACF exceeding the approximate 95% confidence interval at lag eight, an AR(8) model was fitted to the historic inflow data. The source code required to fit the AR(8) model to the data in R and the consequent output is included in Section E.3 of Appendix E. From the output of the model fitting process, the AIC of the AR(8) model was found to equal 459.29. This value was greater than the AIC of the AR(1) model (454.66, from Table 5.1), meaning that the penalty imposed on the additional terms in the AR(8) model did not outweigh the benefit gained by those same additional terms minimising the size of the errors. Therefore, the AR(1) model remained the best suited to describe the historic inflow data and was continued to be examined through the use of further diagnostic tests.

Box-Pierce Test (Q-Statistic)

The next diagnostic test, the Box-Pierce test, can be used to assess if the residuals of a time series model are independent. Dunn and Addie (2008) state that if the residuals of the model behave as a white noise process, then the Box-Pierce statistic, or Q-statistic, will approximately follow a chi-square (χ^2) distribution, with $m - N$ degrees of freedom; where m is the number of autocorrelation coefficients specified when computing the statistic and N is the number of autoregressive and moving average components estimated for the time series model.

In order to perform the Box-Pierce test on the residuals of the AR(1) model, the statistical software R was utilised. At this point, it should be noted that R does not compare the result to a chi-square distribution with $m - N$ degrees of freedom, but rather m degrees of freedom; an option, Dunn and Addie (2008) state, that is favoured by some authors. Also, the value of the parameter m is variable and is often selected by default to equal 15; however for larger time series it is recommended to increase this value. In this case, when testing the residuals of the AR(1) model fitted to the historic inflow data, the parameter m was selected to equal 25. The source code used to perform the Box-Pierce test in R is located in Section E.3 of Appendix E, while the output from the test performed on the residuals of the AR(1) model is presented in Figure 5.7.

From the output of the test, Figure 5.7 shows that R provided the test statistic, labelled `X-squared`, degrees of freedom and p-value. In the case of the Box-Pierce

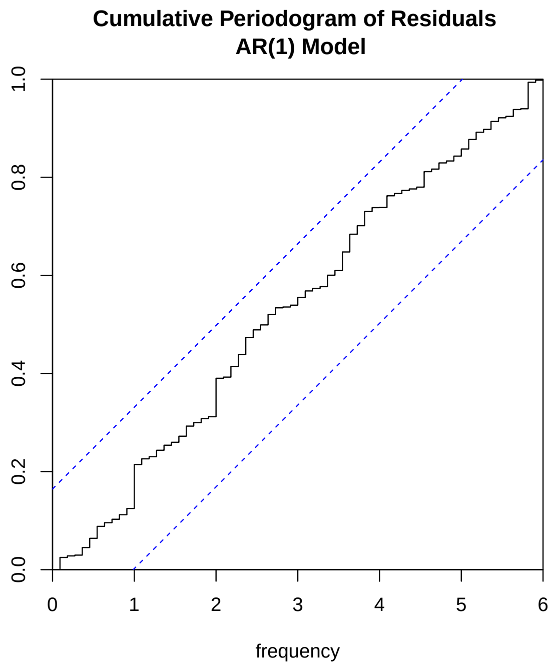

Figure 5.8: Cumulative Periodogram of the Residuals for the AR(1) Model fitted to the Historic Inflow Training Series.

test performed, the null hypothesis was that the residuals of the model were independent; while the alternative hypothesis was that they were not. Therefore, a p-value of 0.5367 indicated that there was insufficient evidence to reject the null hypothesis. This enabled the assumption that the residuals of the AR(1) model were independent. Therefore, the Box-Pierce test confirmed the independence of the residuals of the AR(1) model selected to describe the historic inflows; however more testing needed to be conducted before the model could be concluded to provide a "good" representation of the inflow records.

Cumulative Periodogram

Another diagnostic test that can be applied to the residuals of a time series model, this time to ensure that the residuals form a white noise process, is to calculate the cumulative periodogram and administer a Kolmogorov-Smirnov test. Dunn and Addie (2008) present the cumulative periodogram as a plot in their work and suggest that if the residuals form an approximate white noise process, as would be expected if the model provides an adequate representation of the data, the cumulative periodogram of the residuals will lie within the 95% confidence bands labelled on the plot.

Figure 5.8 presents the cumulative periodogram of the residuals for the AR(1) model fitted to the historic inflow data. From the figure, it can be seen that the cumulative periodogram remained within the 95% confidence bounds, presented as dashed blue lines. The function employed to generate the cumulative periodogram in R is presented in Section E.3 of Appendix E. Therefore, from this result it was concluded that the residuals of the AR(1) model reflected a white noise process; continuing to confirm that the model provided a suitable representation of the historic inflow data.

Test of Parameter Significance

The next diagnostic test considers the statistical significance of the parameters that compose a time series model. If a parameter is found to be not statistically significant, or, in other words, not significantly different to zero, then it should be removed from the model. As part of fitting a time series model in R, estimates of the parameters, along with their respective standard errors and variances are calculated. These standard errors can be utilised in determining the significance of the parameters in a model. Dunn and Addie (2008) state that as a rough guide, if the estimated value of the parameter is twice the size of its respective standard error it can be deemed to be significant. However, they also point out that in order to be more precise, a statistical test of significance (t-test) should be conducted.

In the case of the AR(1) model fitted to the historic inflow data, two parameters were estimated; the constant term, labelled as the `intercept` by R, and the AR term. The constant term was of no interest when considering the structure of the model and as such the significance of the term was irrelevant, although a t-score for the parameter was still calculated as part of the statistical test. The steps performed in order to calculate the t-scores in R are outlined in Section E.3 of Appendix E. From the output provided by R, the t-score of the AR term was found to equal approximately 5.24. As this t-score was greater than the critical value of the test (1.96), it was concluded that the AR term was statistically significant and should be retained in the model. This diagnostic test ascertained that the AR(1) model selected to describe the historic inflow data was parsimonious and did not included any redundant terms; further supporting the conclusion that the AR(1) model provided a "good" representation of the historic inflow records. However, there was one final diagnostic test that needed to be performed in order to validate a key assumption made during the conduct of the prior tests of the model.

Normality of Residuals

Throughout the diagnostic testing of the time series model conducted previously, it was assumed that the residuals of the model were normally distributed. In this final diagnostic test, this fundamental assumption of normality was examined, first through the use of a normal Q-Q plot and then by considering a histogram of the residuals.

Figure 5.9 presents the Q-Q plot of the residuals from the AR(1) model fitted to the historic inflow data. From the figure, it can be seen that the residuals were located close to the diagonal line included on the plot; however there was some

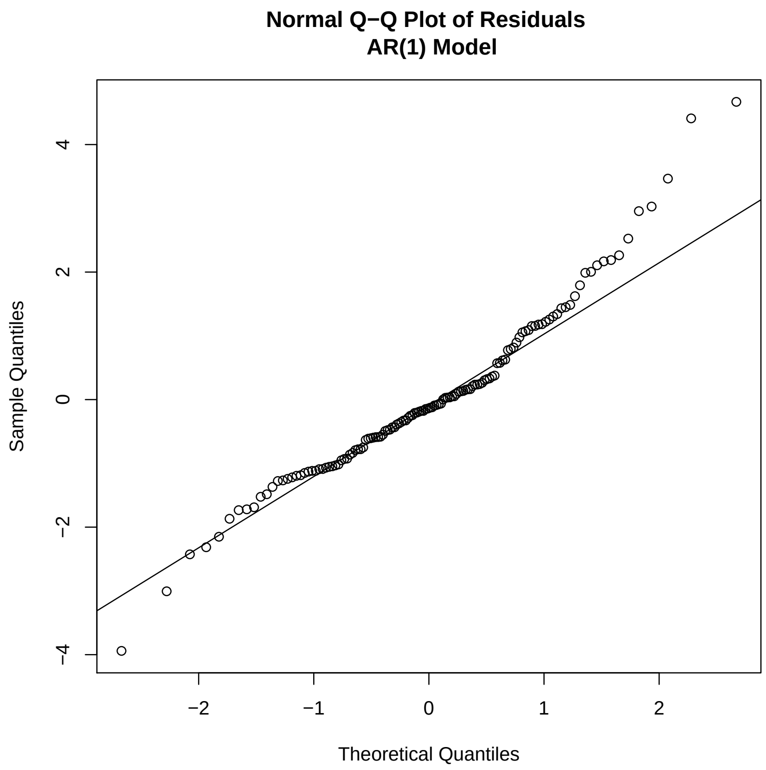

Figure 5.9: Q-Q Plot of the Residuals for the AR(1) Model fitted to the Historic Inflow Training Series.

deviation away from the line amongst the more extreme observations. Dunn and Addie (2008) state that the closer the residuals are to the diagonal line, the greater evidence there is to suggest that the residuals of the model follow a normal distribution. Therefore, from Figure 5.9 there was sufficient evidence to suggest that the residuals were approximately normally distributed.

In order to further investigate the normality of the residuals from the AR(1) model, a histogram was constructed. This histogram is presented in Figure 5.10 and demonstrates that the distribution of the residuals approximately followed the typical bell shape curve associated with a normal distribution. Therefore, from the evidence depicted in both Figure 5.9 and Figure 5.10, it was confirmed that the residuals of the AR(1) model selected to describe the historic inflow data approximately followed a normal distribution and thus, the assumption made throughout the previous diagnostic testing regarding normality was validated. Note that the source code to generate both of these plots in R is provided in Section E.3 of Appendix E.

As a result of satisfying the battery of diagnostic tests performed in this section, it was concluded that the AR(1) model was a "good" model for the historic inflows, as it captured the important information, or signal, of the data. This was confirmed by some of the diagnostic tests performed, which showed that upon removal of the signal, the residuals of the model were random, independent and not predictable.

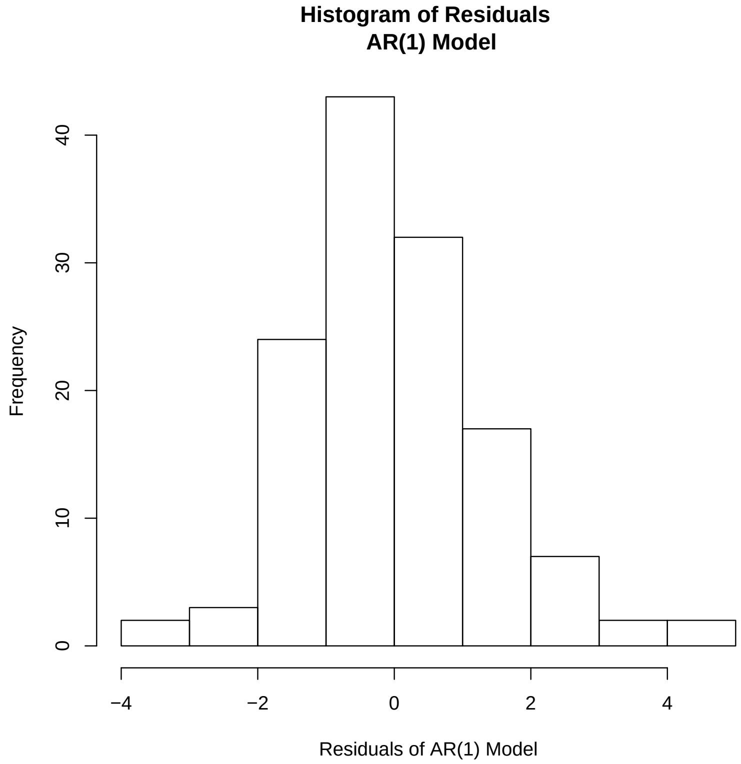

Figure 5.10: Histogram of the Residuals for the AR(1) Model fitted to the Historic Inflow Training Series.

Also, the model itself was parsimonious and proven to contain no redundant or extraneous terms. The final trial of the AR(1) model was to access its forecasting ability by performing a validation technique outlined by Dunn and Addie (2008).

5.4 Model Validation and Inflow Simulation

To begin this section, a validation technique presented by Dunn and Addie (2008) is performed to assess the forecasting ability of the AR(1) model demonstrated to provide an approximate representation of the historic inflow data. As mentioned at the beginning of this chapter, the full set of historic inflows was split into two components; the training series and the testing series. The training series was composed of the majority of the historic inflow records and, from this data, the AR(1) model was selected. On the other hand, the last two years of inflows were isolated from the data set to form the testing series and were not considered in the model selection process until this point.

The objective of this validation test is outlined by Dunn and Addie (2008) who explain that the test is to determine the forecasting ability of a model on a portion of data that the model has not "seen" before. If a model was to be trialled on a segment of data that was part of the training series, this may result in a false representation of the models forecasting ability. Therefore, it is important to validate

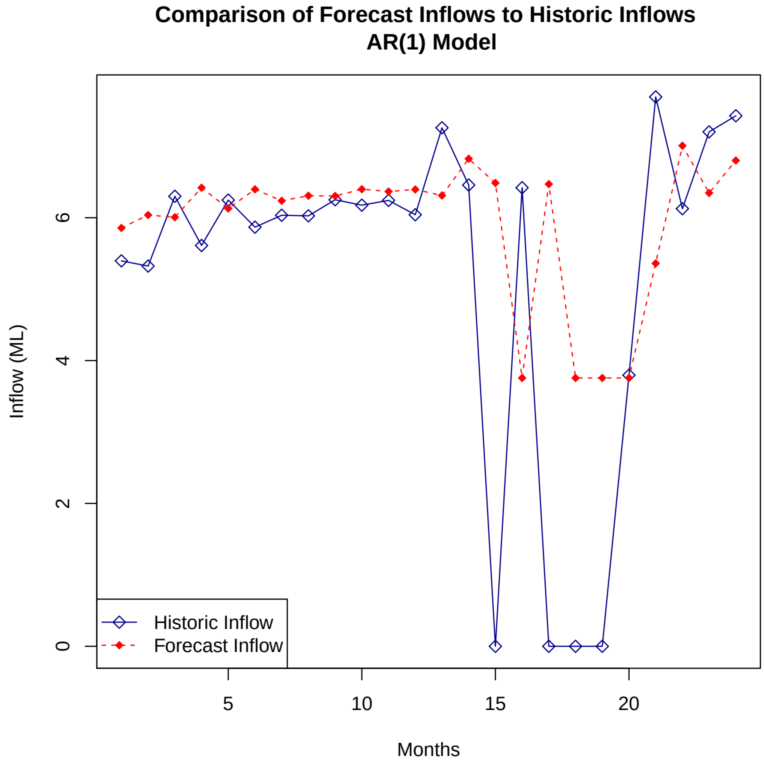

Figure 5.11: Comparison of the Forecast Inflows from the AR(1) Model to the Historic Inflows that compose the Testing Series.

a time series model on data that was not used in the formulation of the model, but remains representative of the behaviours that could be exhibited by the data set (Dunn and Addie, 2008). In this case, the testing series contained four months in which no inflows occurred. By including this period of no inflows in the testing series, the series no longer represented the inflow behaviour witnessed in the training series and therefore comparisons against that portion of data do not provide an accurate representation of the forecasting ability of the AR(1) model. However, due to time constraints and the realisation of this limitation late in the review of this thesis, the training and testing series could not be redefined to exclude this period of no inflows, and thus the testing series in its original form was utilised; though, caution was taken when interpreting the outcome of the validation test.

In order to perform this validation test, a series of one-step ahead forecasts were iteratively calculated from the AR(1) model for each of the observations that composed the testing series. The forecasts were then compared to the historic data of the testing series both through a plot and by calculating the mean and variance of the difference between the two sets of data. The source code implemented to perform this validation technique in R has been provided in Section E.4 of Appendix E.

To compare the forecasts from the AR(1) model to the historic inflows, Figure 5.11 has been included. From the figure, it can be seen that the AR(1) model provided

```
Time Series:
Start = 1
End = 24
Frequency = 1
 [1]   561.20064   875.07938   521.60832   134.62786   147.66130   519.16900
 [7]   524.22474   104.43264  1245.44917  9984.91848 13932.76839  1837.02321
[13]  1423.42558  1950.39564   616.39802   444.31853    80.25243   880.77173
[19]  1785.00120   465.27555   426.11480   149.86000   547.53137   797.62418
```

Figure 5.12: Resulting Output from the Simulation of Inflows performed using the AR(1) Model in R.

a suitable approximation of the historic inflows that composed the testing series; however the more extreme observations were not accurately forecast by the model, for example, during the months in which no inflow occurred. As mentioned previously, this inability to accurately forecast these events was to be expected, as the behaviour displayed by the testing series is atypical of the behaviour witnessed in the training series. In these months where no inflows occurred, the best one-step ahead forecast that the AR(1) model could provide was the mean, or constant value of the series, which equalled approximately 4. The mean and variance of the difference between the forecasts and the historic inflows that composed the testing series also provided some useful information; calculated to equal approximately -0.733 and 5.077 respectively. A negative mean difference indicated that the forecasts from the AR(1) model were, for the main part, over estimates of the observed inflows; a conclusion supported by Figure 5.11, which shows the forecasts being typically larger than the observed inflows. Also, the size of the variance indicated that there were sizeable differences between the forecasts from the model and the observed historic inflows of the testing series.

Therefore, due to the anomalous data included as part of the testing series, there were obvious differences between the forecasts and the historic data. Taking the limitation imposed through the selection of the testing series into account, the validation technique outlined by Dunn and Addie (2008) still demonstrated that the AR(1) model provided a reasonable approximation of the historic inflow records; an approximation that was comprehensive enough to facilitate the simulation of an alternate set of inflows based upon the time series model.

As outlined at the beginning of the chapter, the purpose for the testing and development of a time series model to approximate the historic inflow records was to enable the simulation of an alternate set of inflow data. This simulated series of inflows will provide a common set of data on which the drought model (Model 4) and flood model (Model 6), developed in previous chapters, can be tested and enable the behaviour of the two models to be compared. In order to simulate this alternate set of inflows across 24 months, R was utilised once more. The source code employed

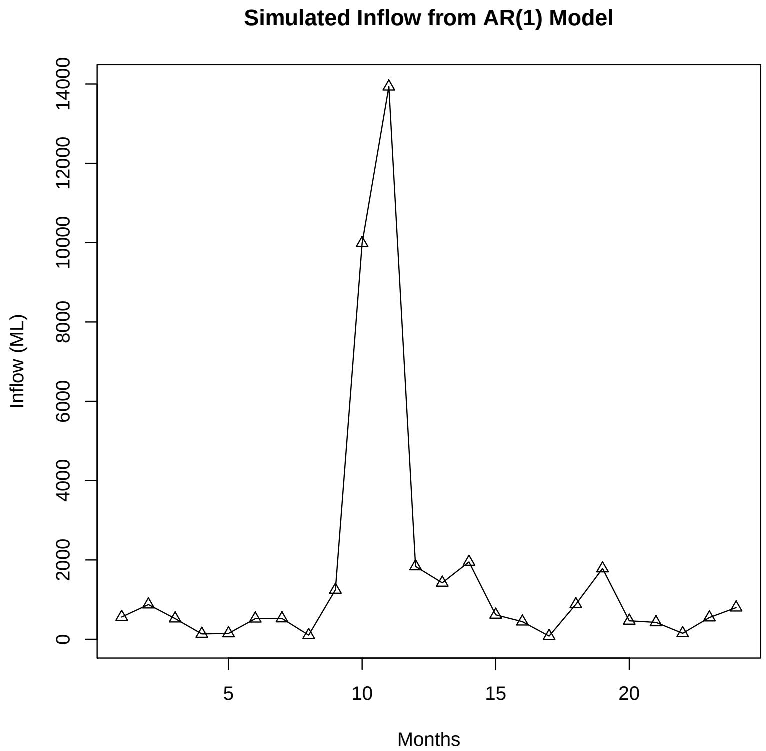

Figure 5.13: Simulated Series of Inflows from the AR(1) Model found to provide an approximate representation of the Historic Inflows.

to conduct the simulation and then to convert the results into a usable, physical unit, in this case megalitres (ML), can be seen in Section E.5 of Appendix E, while the resulting output of the simulation generated by R is presented in Figure 5.12. Therefore, using the AR(1) model selected, tested and validated, an alternate series of inflows based upon the behaviour exhibited by the historic inflow records, sourced from the Queensland Department of Natural Resources and Mines (2012), has been simulated. Along with being tabulated in Table D.3 of Appendix D, these inflows have also been presented graphically in Figure 5.13. From the figure, it can be seen that there are periods of both large and limited inflows across the 24 month time period simulated. This variation in inflow will help in the exploration of the different behaviours and management strategies employed by the drought and flood models investigated in the next chapter.

From the extent of testing performed on the AR(1) model, it can be concluded with confidence that the model provided a reasonable approximation of the historic inflow records compared to the other parsimonious model considered. However, there are a range of more complex models and techniques that could provide a better approximation of the inflow data, though it was beyond the scope of this thesis to pursue those methods.

5.5 Chapter Conclusion

The aim of this chapter was to simulate an alternate set of inflow data based upon many years of historic records, through the use of time series analysis. To begin, the historic inflow data for Cressbrook Creek, the main tributary that flows into Cressbrook dam, one of the reservoirs considered as a case study throughout this thesis, was sourced from the Queensland Department of Natural Resources and Mines (2012). Using the 13 years of complete data available, the historic records were separated into two portions. The first of these portions, which contained the majority of the inflow data, was labelled the training series and used to develop the time series model. The remaining portion of data, labelled the testing series, consisted of the final two years of records and was withheld to be utilised later in the model validation process. At this point, the sample Autocorrelation Function (sample ACF) and sample Partial Autocorrelation Function (sample PACF), two model selection tools, were employed to help determine the type and order of the time series model that best described the historic inflows. Using these tools, along with the Akaike Information Criteria (AIC), it was concluded that an autoregressive model of the first order, or an AR(1) model, provided the best approximation of the historic inflow records.

In order to ensure that the AR(1) model was a "good" model to describe the historic inflow data, a sequence of diagnostic tests were performed. These tests investigated properties of the residuals of the AR(1) model, along with tested the significance of the terms that composed the model. Upon completion of these tests, it was determined that the AR(1) model did provide a "good" description of the historic records and was able to explain the majority of the important information, or signal, of the data. The final trial of the model was to perform a validation technique outlined by Dunn and Addie (2008). This validation process was used to determine the forecasting ability of the AR(1) model by comparing a series of one-step ahead forecasts from the model against the withheld portion of the inflow data that composed the testing series. However, by including a period of atypical inflows (a period of months in which no inflow occurred) as part of the testing series, the validation process was impaired and the results from the process were not necessarily representative of the true forecasting ability of the AR(1) model. Due to time constraints and the realisation of this limitation late in the review process of this thesis, changes were unable to be made to the training and testing series to exclude this portion of the historic records. However, taking this limitation into account, the validation technique still managed to demonstrate that the AR(1) model provided a suitable approximation for the historic inflow data; an approximation that was comprehensive enough to facilitate the simulation of an alternate set of inflows using the model. These simulated inflows will be used in the subsequent chapter to provide a common set of data upon which a comparison of the drought model (Model 4) and flood model (Model 6), developed in previous chapters, can be conducted.

Chapter 6

Drought Model and Flood Model Comparison

6.1 Chapter Overview

In this chapter, a comparison is undertaken of the two Mixed Integer Linear Programming (MILP) models formulated to describe the operation of a cascade reservoir system under the extreme conditions of a drought and flood event (Model 4 from Chapter 3 and Model 6 from Chapter 4 respectively). In order to ensure a useful comparison of the Drought and Flood Models, an experiment is conducted whereby the optional parameters of the models are selected to be similar, while both consider a common set of inflows to the case study of the Perseverance and Cressbrook cascade reservoir system. This common set of inflows was simulated using time series analysis in the previous chapter and is based upon the historic inflow records sourced from the Queensland Department of Natural Resources and Mines (2012). From the results of the experiment, the behaviour of the key parameters measured by the Drought and Flood Models are explored independently. The chapter is concluded with an investigation of the management strategies employed by the two models.

6.2 Application of Models

In this section, Model 4 and Model 6 developed in Chapter 3 and Chapter 4 respectively, and herein referred to as the Drought Model and the Flood Model, are both applied to the case study of the Perseverance and Cressbrook cascade reservoir system by means of an experiment. The goal of this experiment is to ultimately compare the similarities and differences between the management strategies employed by the two models, when both are considered on a common set of inflows, with similar optional parameters. However, before conducting this comparison, the key parameters measured by the Drought Model and Flood Model can be considered separately. As seen in previous chapters, for the purposes of the experiment, Perseverance dam is referred to as Reservoir 1, while Cressbrook dam is labelled Reservoir 2. Figure 6.1, a simple copy of Figure 1.1, presents the physical layout of the Perseverance and Cressbrook cascade reservoir system and has been repeated in this chapter for ease of reference and as a reminder of the key features that compose

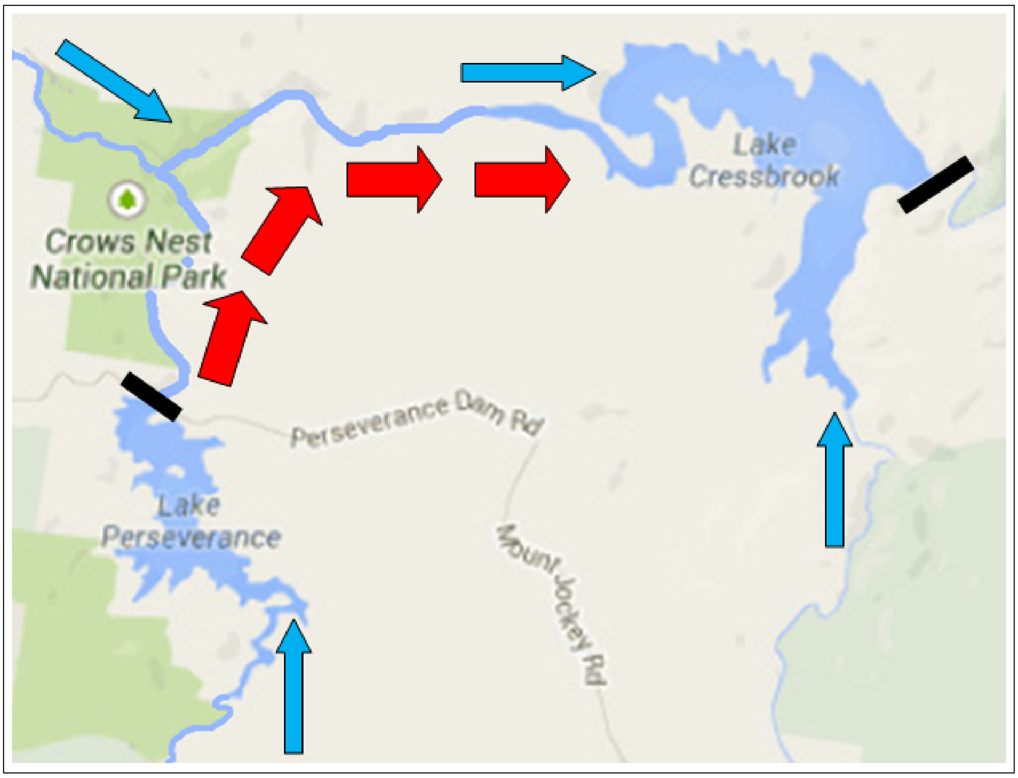

Figure 6.1: Repeated Copy of Figure 1.1 showing the Configuration of the Perseverance and Cressbrook cascade reservoir system investigated as a Case Study. *Black Rectangles = Dam Spillway, Thinner Blue Arrows = Sources of Inflows, Thicker Red Arrows = Direction of Spill from Perseverance to Cressbrook dam.*

the reservoir system considered as a case study.

In order to conduct the comparison of the Drought and Flood Models, separate experiments were performed for each using the optimisation modelling software LINGO version 14.0 (LINDO Systems Inc, 2013); where the two models were considered using the same inflows, with similar optional parameters. This software was also employed in the previous chapters when exploring each of the models individually, where it was mentioned that the Academic License acquired limited the number of variables and constraints that the software could consider. Therefore, due to the size of both the Drought Model and Flood Model, together with the restrictions imposed by the software, a total of 24 time units could only be considered for each experiment; corresponding to 24 months or two years in the case of the Drought Model and 24 hours or 1 day for the Flood Model. Examples of the syntax required to perform both the experiment of the Drought Model and Flood Model, along with examples of the resulting output, are provided in Appendix B and Appendix C respectively.

In the previous experiments of the Drought and Flood Models, the historic inflow data provided by the Queensland Department of Natural Resources and Mines (2012) was screened to determine the combination of records that corresponded to

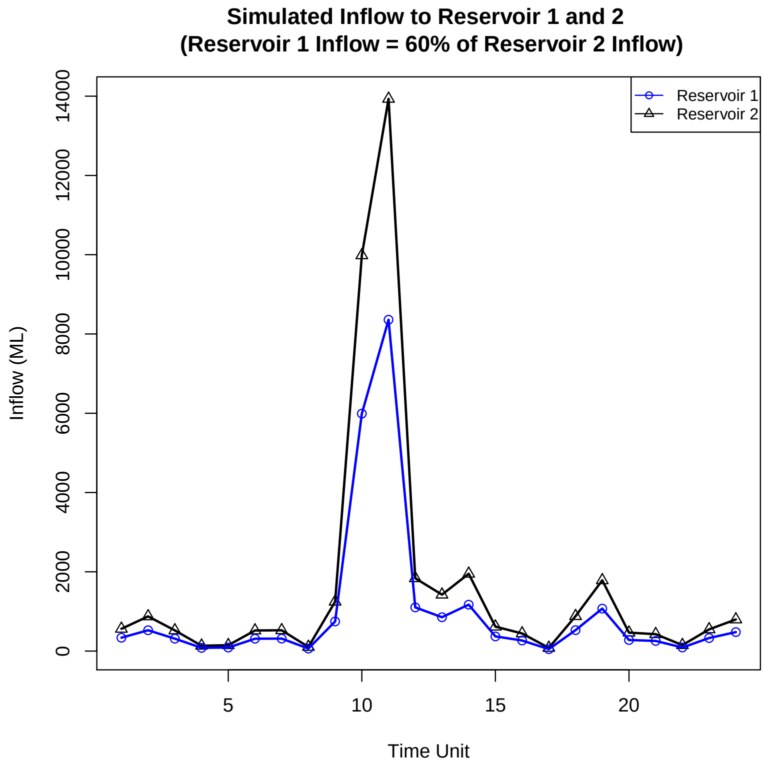

Figure 6.2: Alternate Set of Inflows to Reservoir 1 and Reservoir 2 Simulated using Time Series Analysis.

the worst case drought and flood scenario. However, in order to facilitate a useful comparison of the Drought Model and Flood Model, both need to be considered on a common set of inflow data. Therefore, utilising time series analysis, an alternate set of inflows was simulated based upon the historic records sourced from the Queensland Department of Natural Resources and Mines (2012) for the monthly inflow along Cressbrook Creek, the major tributary that flows into Cressbrook dam. The procedures followed to first determine a suitable time series model to describe the historic data, and then to simulate the alternate set of inflows can be found in Chapter 5. One assumption made in the previous chapters regarding the inflows to the reservoir system considered as a case study, was to specify that the inflows to Perseverance dam, or Reservoir 1, are equal to 60% of the inflows to Cressbrook dam, or Reservoir 2. This assumption was based upon prior knowledge of the size of the independent creeks and streams that supply Perseverance dam compared to the size of those that supply Cressbrook dam. The simulated inflows to both reservoirs are presented in Table D.4 of Appendix D and are also displayed graphically in Figure 6.2. Note that the generic "Time Unit" is used to describe the scale over which the inflows are measured in Figure 6.2. This is due to the different time scales over which a drought and flood event are measured; droughts are typically long term events and therefore have been considered on a monthly basis, whereas flood events can occur rapidly and were thus measured on an hourly scale.

Before conducting the individual experiments of the previously labelled Model 4 and Model 6 in Chapters 3 and 4 respectively, a series of assumptions were made. These assumptions were selected in order to provide the best approximation of the Perseverance and Cressbrook cascade reservoir system considered as a case study, while at the same time simplifying the conditions of the experiment. Therefore, in order to maintain the continuity of the methodologies employed throughout the research presented in this thesis, many of the same assumptions were made for the comparative experiments performed in this chapter. One such assumption was that perfect prior knowledge of the inflows to the reservoir system was available. Although this assumption may not be feasible in a physical system, perfect prior knowledge of the inflows reduced the complexity of both the Drought and Flood Model, and helped to simplify the results of the experiment.

Based upon information presented by the Toowoomba Regional Council (2013), another assumption made in previous chapters and thus maintained for the purposes of this experiment was to specify that the capacity of Perseverance and Cressbrook dam equalled 30 000 megalitres (ML) and 82 000 ML respectively. These capacities were also assumed to determine how the water supply demand from the community was shared between the two reservoirs. Therefore, as the capacity of Perseverance dam was assumed to be approximately one third the capacity of Cressbrook dam, it was also assumed that one third of the community water supply demand was sourced from Perseverance dam, while the remaining two thirds was acquired from Cressbrook dam.

Due to the opposing nature of drought and flood events, the Drought and Flood Models have been formulated with unique assumptions that are specific to each scenario. In the case of the Drought Model, it was assumed that the volume of water made available to the community from the reservoir system each month was subject to rationing; the extent of which was determined by the available water of the system (defined as the reservoir storage level in the previous month, plus the projected inflows in the current month). On the other hand, it has been assumed in the formulation of the Flood Model that the reservoir system featured a pumping facility that had the ability to pump water backwards through the system from Reservoir 2 to Reservoir 1. A detailed and thorough description of these assumptions and how they contribute to both models can be found in Sections 3.2.2 and 3.3 for the Drought Model and Sections 4.2.2 and 4.3 for the Flood Model.

Along with the unique assumptions specific to each of the models, both the Drought Model and Flood Model also have optional parameters that are able to be specified by the operators of the reservoir system. In order to ensure that a useful comparison could be conducted between the two models, these parameters needed to be selected to have similar values. In the case of the Drought Model, both the initial capacity of the reservoirs and the number of months in which phase two rationing is enforced are able to be specified by the reservoir operators; while, when the Flood Model is considered, the only optional parameter able to be selected is the initial capacity of the reservoirs. Therefore, to ensure that the experiment was conducted under similar conditions for both models, the initial reservoir capacities have been left unconstrained ($S_0 \leq C$), along with the number of months in which phase two rationing is permitted ($n \geq 0$) as part of the Drought Model. This meant that the models would have the opportunity to optimally select the values of these parameters, subject to the conditions of the experiment and inflows specified; in turn helping to better differentiate between the management strategies employed by the two models.

However, before comparing the differing management strategies utilised by the two models, the behaviour of key parameters measured by the Drought and Flood Model can be explored separately in the following sections.

6.2.1 Exploration of the Behaviour of the Drought Model Parameters

The first of the two models considered was the Drought Model. Of the variables measured by the Drought Model, there were four key parameters of interest from both reservoirs; being the reservoir storage level, available water, reservoir trigger volumes and the releases to the community water supply. In order to thoroughly investigate these parameters, a series of figures have been constructed. The first of these, Figure 6.3, enables a comparison of the behaviour exhibited by the storage levels of the two reservoirs, while Figures 6.4 and 6.5 display the behaviour of the available water with the reservoir trigger volumes, and how the relationship between these two parameters determines the extent of rationing enforced each month.

From Figure 6.3, it can be seen that the Drought Model selected the initial capacity of Reservoir 1 to equal approximately 29 800 ML; 200 ML below full capacity. Commencing from this initial capacity, the storage level of the reservoir fluctuates marginally in magnitude over the 24 month event horizon considered, with a minimum capacity of approximately 29 100 ML occurring in the eighth month. The figure also demonstrates that Reservoir 1 was at full capacity from month 10 to 15, and again in month 19; indicating that spill from the reservoir may have been occurring in these months.

Figure 6.3 also presents the storage level of Reservoir 2 and demonstrates that the Drought Model selected the initial capacity of this reservoir to equal approximately 2000 ML. From this starting condition, the reservoir level exhibited a decreasing trend over the first nine months of the event horizon. This trend can be attributed to the volume of water supplied to the community exceeding the inflow to the

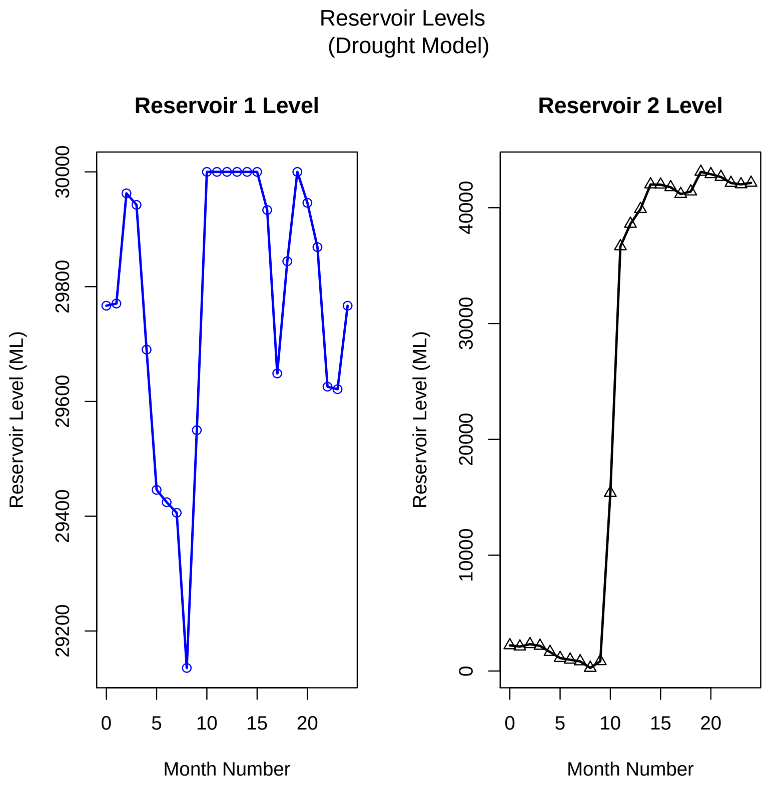

Figure 6.3: **Drought Model ($S_0 \leq C$ and $n \geq 0$)** - Behaviour of Reservoir 1 and Reservoir 2 Storage Levels (ML).

reservoir each month. However, in months 10 and 11 there was a substantial increase in the reservoir capacity. When this period is identified in Figure 6.2, it can be seen that these months corresponded to a period of significant inflows to the system; therefore resulting in the spike in reservoir level witnessed for both Reservoir 1 and 2. Following this spike, the storage level of Reservoir 2 stabilised and reached approximately 42 000 ML, or roughly half of the reservoir capacity, by the end of the 24 month event horizon.

Figures 6.4 and 6.5 have been constructed in order to explore the relationship between the available water and reservoir trigger volumes, and how this relationship influences the volume of water made available to the community each month. From Figure 6.4, it can be seen that the available water of Reservoir 1 was significantly greater than the reservoir trigger volumes across the event horizon. Due to the substantial difference in size between the available water and trigger volumes of the reservoir, the trigger volumes appear in Figure 6.4 as one solid line near 0 ML, when in fact they were separated by a difference of 5% and ranged from 133 ML (V_3) to approximately 147 ML (V_1) (for the simplicity of comparisons between the Drought Model and Flood Model, the trigger volumes have been referred to as V_1, V_2 and V_3). Therefore, as the available water was greater than the trigger volume V_1 across the event horizon, this resulted in no rationing being enforced on the volume

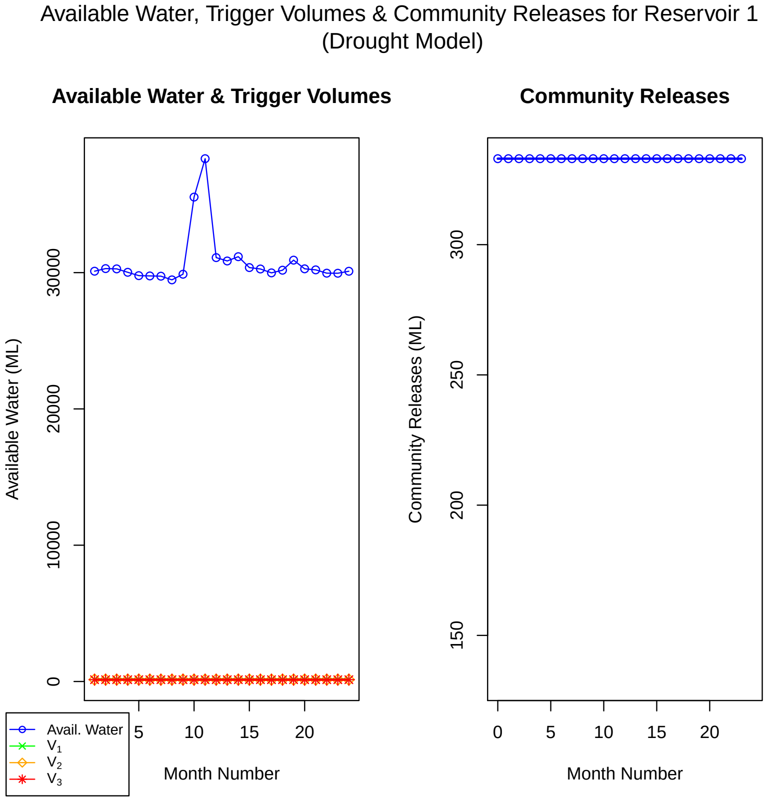

Figure 6.4: **Drought Model ($S_0 \leq C$ and $n \geq 0$)** - Behaviour of Available Water + Trigger Volumes and the Relationship with the Community Releases for Reservoir 1.

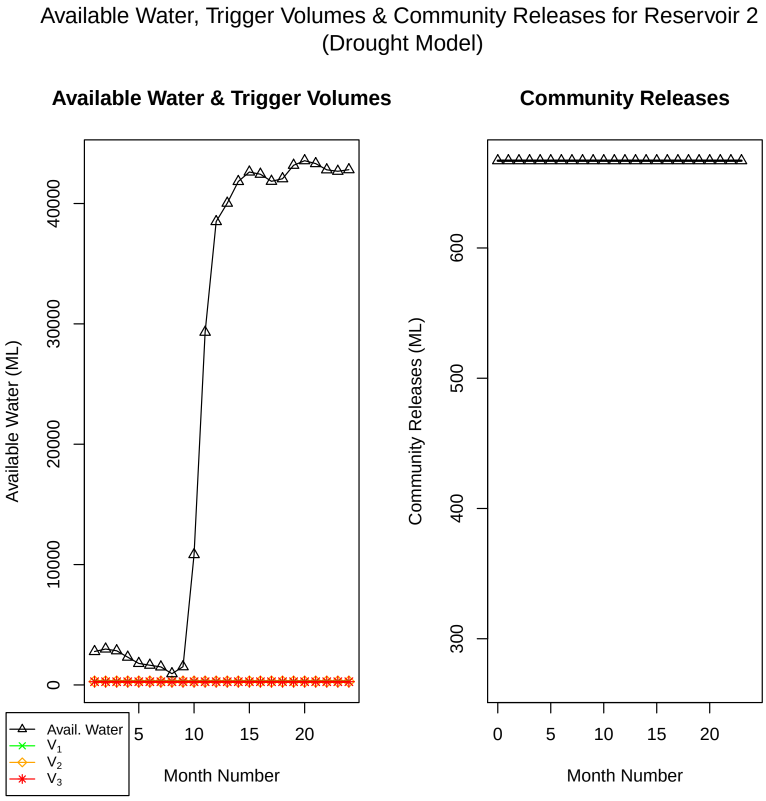

Figure 6.5: **Drought Model ($S_0 \leq C$ and $n \geq 0$)** - Behaviour of Available Water + Trigger Volumes and the Relationship with the Community Releases for Reservoir 2.

of water released from Reservoir 1 for use by the community. This is also reflected by the figure, which shows the community releases from Reservoir 1 maximised to 333 ML per month, or the most generous releases available from the reservoir, for the extent of the 24 month event horizon.

Figure 6.5 demonstrates that the available water and trigger volumes of Reservoir 2 behaved in a similar way to that seen in Figure 6.4 for Reservoir 1, with the available water being greater than the reservoir trigger volumes over the extent of the event horizon. As the available water was consistently greater than the trigger volume V_1, the full demand from the community could be made available from Reservoir 2 across the event horizon. This occurrence is also displayed by Figure 6.5, which shows the community releases maximised to a constant value 667 ML, corresponding to the most generous releases from Reservoir 2, for the duration of the event.

From a review of the results of the experiment performed using the Drought Model, it can be seen that no rationing was enforced on the volume of water made available to the community, from either reservoir, over the event horizon; although the model selected significantly different initial capacities for the two reservoirs. In the case of Reservoir 1, the Drought Model selected an initial capacity of approximately 29 800 ML, which was only 200 ML less than the full capacity of the reservoir (30 000 ML). On the other hand, the initial capacity of Reservoir 2 was selected to equal approximately 2000 ML; very close to empty for a reservoir with a total capacity of 82 000 ML. Although varying the initial capacities of both reservoirs, the management strategy employed by the model ensured that the full community demand could be made available over the extent of the 24 month event horizon, without the need for rationing. A more thorough exploration of the management strategy employed by the Drought Model will be conducted later in this chapter in Section 6.2.3.

6.2.2 Exploration of the Behaviour of the Flood Model Parameters

Similar to the exploration of the behaviour of the Drought Model parameters, there were some key parameters of interest when investigating the Flood Model. These five parameters were the storage level of both reservoirs, spill from both reservoirs, available water and reservoir trigger volumes of Reservoir 2 and the volume of water pumped from Reservoir 2 backwards through the system to Reservoir 1 each hour. At this point it is important to note the difference in the time frame under which the Drought Model and Flood Model were considered; drought events can last for long durations and therefore have been measured in months, whereas flood events can occur rapidly and thus have been measured in hours. Once again, in order to aid with the exploration of these key parameters, a series of figures have been assembled.

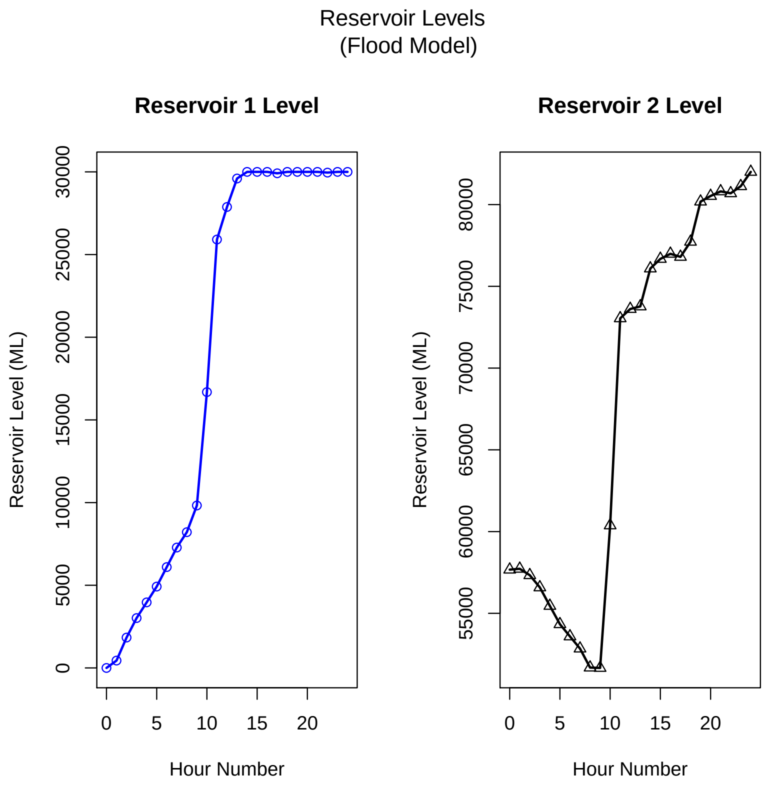

Figure 6.6: **Flood Model** ($S_0 \leq C$) - Behaviour of Reservoir 1 and Reservoir 2 Storage Levels (ML).

The first of these figures, Figure 6.6, displays the behaviour of the storage level of Reservoir 1 and Reservoir 2 over the event horizon considered of 24 hours. From the figure, it can be seen that the Flood Model selected to initialise the capacity of Reservoir 1 to be empty, or 0 ML. From this starting condition, the level of the reservoir exhibited an overall increasing trend until hour 14 was reached, at which time the reservoir was filled to capacity (30 000 ML). Reservoir 1 then remained at capacity until the seventeenth hour of the event horizon, at which time a combination of reduced inflows and the continued supply of water to satisfy the community demand (assumed to be a constant value of 133 ML per hour) resulted in the reservoir level decreasing to approximately 29 900 ML, or 100 ML below capacity (an event difficult to distinguish from Figure 6.6). Following this hour, the reservoir returned to full capacity, although a similar event reduced the reservoir level again in hour 22.

Also presented in Figure 6.6 is the storage level of Reservoir 2, which can be seen to commence at approximately 57 500 ML. From this initial capacity, the figure shows that the storage level of Reservoir 2 displayed an overall decreasing trend for the first nine hours. This decrease in reservoir level can be primarily attributed to the pumping of water from Reservoir 2 backwards through the system to Reservoir 1.

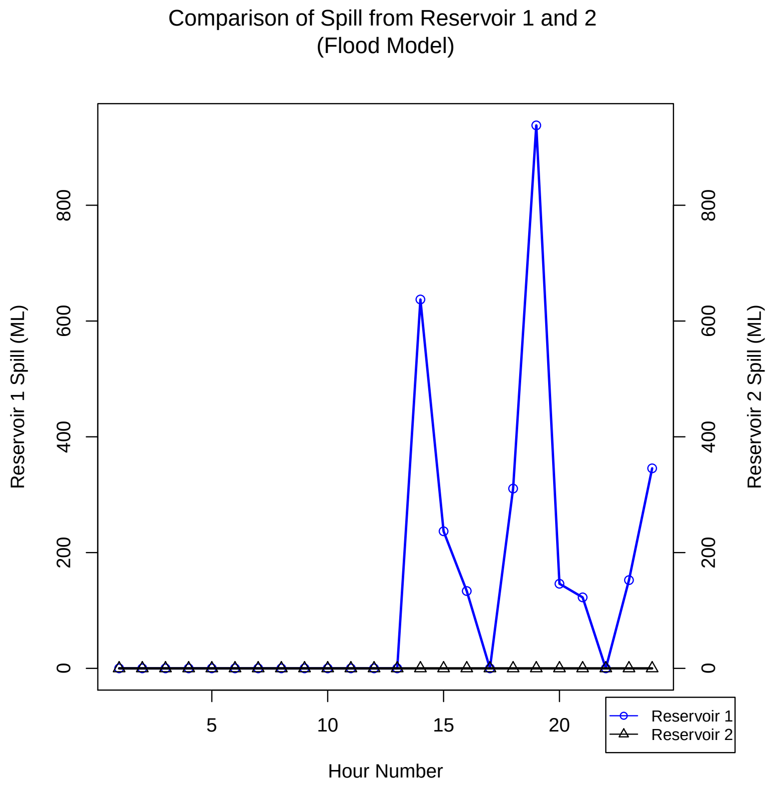

Figure 6.7: **Flood Model** ($S_0 \leq C$) - Behaviour of Spill from Reservoir 1 and Reservoir 2 (ML).

However, once hour 10 was reached, the storage level of the reservoir began to increase from a capacity of approximately 50 000 ML. Once again, this significant change in the behaviour of the reservoir storage level could be attributed to the substantial inflows that occurred in hour 10 and 11, identified previously in Figure 6.2. Following this spike, the storage level of Reservoir 2 steadily increased over the remaining 13 hours of the event horizon; reaching full capacity (82 000 ML) in the final hour.

Figure 6.7 displays the behaviour of the spill from Reservoir 1 and Reservoir 2 over the event horizon. The figure demonstrates that over the first 13 hours of the flood event, no spill occurred from Reservoir 1. As mentioned previously, Reservoir 1 did not reach full capacity until the fourteenth hour of the event, meaning that it was a physical impossibility for spill to occur prior to this time. Also mentioned previously, was that in hours 17 and 22 the storage level of Reservoir 1 decreased to below full capacity, again preventing any spill from the reservoir taking place; an occurrence reflected in Figure 6.7 which exhibits a lack of spill in those hours. The figure also demonstrates that no spill from Reservoir 2 transpired over the 24 hour event horizon. This behaviour is supported from a review of Figure 6.6, which shows that the reservoir did not reach full capacity until the final hour of the flood event, again meaning that it was physically impossible for any spill to eventuate

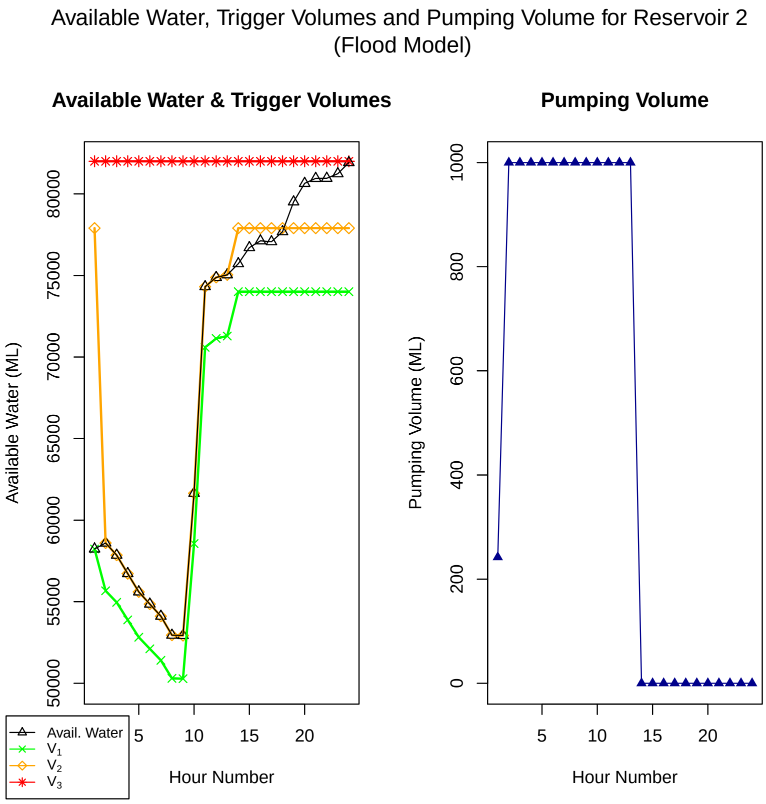

Figure 6.8: **Flood Model** ($S_0 \leq C$) - Behaviour of Available Water + Trigger Volumes and the Relationship with the Pumping Volume from Reservoir 2.

from the reservoir before this time.

The final figure to be considered in the individual exploration of the Flood Model parameters is Figure 6.8. This figure demonstrates the behaviour of the available water and trigger volumes of Reservoir 2, and how the relationship between these two parameters can be used to determine the volume of water pumped backwards through the system from Reservoir 2 to Reservoir 1 each hour. From Figure 6.8, it can be seen that in the first hour of the event horizon, the available water and the trigger volume V_1 shared the same value. This signifies that the volume of water able to be pumped from Reservoir 2 was limited under phase two restrictions to 60% of the pumping facility capacity, or 600 ML. However, when the actual volume of water drained from the reservoir in the first hour is considered (also shown in Figure 6.8), it was found that only approximately 250 ML was pumped from Reservoir 2 to Reservoir 1. The reason for not pumping the maximum volume that could have been removed from Reservoir 2 in the first hour is illustrated by considering the available water in hour two. From the figure, it can be seen that in the second hour, the available water equals the reservoir trigger volume V_2; enabling the full capacity of the pumping facility to be pumped from Reservoir 2. Therefore, if more than approximately 250 ML was to be pumped from Reservoir 2 in the first hour, the available water in the second hour would have been less than the trigger volume

V_2 (assuming it remained the same); limiting the volume of water that could have been pumped in that hour to 600 ML and potentially altering the optimal management strategy determined by the Flood Model.

From the second hour, Figure 6.8 shows that the available water was equal to the trigger volume V_2, enabling the full capacity of the pumping facility (1000 ML) to be drained from Reservoir 2 and pumped backwards through the system to Reservoir 1 each hour. However, upon reaching the fourteenth hour, it can be seen that pumping ceased. This can be attributed to Reservoir 1 reaching full capacity, meaning that any water pumped from Reservoir 2 to Reservoir 1 would immediately spill from Reservoir 1 and, due to the cascade configuration of the reservoir system, become another source of inflows to Reservoir 2; effectively defeating the purpose of the backwards pumping of water through the system.

During the exploration of Figure 6.6 previously, it was seen that in hours 17 and 22 the storage level of Reservoir 1 decreased to below full capacity, potentially enabling pumping to resume. However, upon further investigation, it was found that the storage level did not decrease by more than approximately 85 ML below the full capacity of the reservoir in either hour. As part of the formulation of the Flood Model, the model checks to ensure that there is at least 400 ML (the volume of water able to be pumped under the strictest phase one restrictions) available in Reservoir 1; if not, pumping is not initiated. Therefore, in hours 17 and 22, there was insufficient capacity available in Reservoir 1 to initiate the use of the pumping facility, as potentially, some extent of the water pumped from Reservoir 2 could be returned to the reservoir as spill from Reservoir 1.

Therefore, from a review of the behaviour of the key parameters measured by the Flood Model, it can be ascertained that the model determined an optimal strategy for the management of the Perseverance and Cressbrook cascade reservoir system during a flood event. In the next section, an assessment of how this management strategy compares to that employed by the Drought Model is conducted.

6.2.3 Comparison of Drought Model and Flood Model Management Strategies

In this section, a comparison of the strategies employed by the Drought Model and Flood Model to optimally manage the case study of the Perseverance and Cressbrook cascade reservoir system is undertaken. Using the behaviour of the key parameters of the Drought and Flood Models explored in the previous sections, this comparison outlines the commonalities and differences in the strategies utilised by the two models.

Due to the vast difference in the nature of the environmental events that the Drought Model and Flood Model describe, the objectives of, and thus strategies employed by, the two models are also vastly different. In the case of the Drought Model, the primary objective is to maximise the number of months in which no rationing is enforced on the provision of water to the community from both reservoirs; in effect maximising the amount of water available to the community each month. In order to achieve this objective, a suitable volume of water needs to be stored and maintained in each reservoir across the 24 month event horizon. On the other hand, the primary objective of the Flood Model is to minimise the amount of spill from Reservoir 2 (termed spill from the system) in consecutive hours of the 24 hour event horizon, as a proxy to minimising the extent of damage that occurs downstream of the reservoir system during the flood event. To ensure this goal is realised, the storage capacity of the reservoirs can be utilised to stall the progression of the floodwaters and to provide better control of the spill from the system.

Subject to the series of assumptions regarding the conditions under which the experiments of the Drought Model and Flood Model were conducted, including the specification of similar optional parameters and both models being considered on a common set of inflows, both the Drought Model and Flood Model were able to fully satisfy their relative objectives. When the Drought Model is considered, no rationing was enforced on the volume of water released to the community, from either reservoir, over the event horizon; thus ensuring that the entire community demand was met in each and every month of the drought event. Also, in the case of the Flood Model, the storage level of Reservoir 2 remained below full capacity for the duration of the flood event until the final hour, leading to no spill occurring from the system and the complete withholding of all floodwaters over the course of the event; successfully preventing any further damage downstream of the reservoir system. Both of these outcomes are somewhat influenced by the assumptions regarding the conditions under which the experiments were conducted; however, subject to these conditions, the models still need to select the strategy that will optimally manage the cascade reservoir system.

A key and common feature of both the Drought and Flood Model is the way in which both optimally manage the cascade reservoir system considered as a whole, rather than selecting optimal strategies for each reservoir independently. This is an important and critical element of both models, as it allows for some amount of interaction between the reservoirs and enables the sharing and balancing of water resources. For example, the initial storage level of Reservoir 1 was selected by the Drought Model to be close to full capacity. This meant that in a number of months, the excess water spilled from Reservoir 1 increased the extent of inflows to Reservoir 2 and helped to prevent the need to enforce rationing on the volume of water released to the community. Another example is from the Flood Model, where the initial capacity of Reservoir 1 was selected by the model to equal 0 ML, or empty. By minimising the initial capacity of Reservoir 1, it meant that the volume of water pumped backwards through the system from Reservoir 2 could be maximised, along with prolonging the commencement of spill from Reservoir 1; together reducing the storage level of Reservoir 2 and at the same time the number of hours before spill occurred from the system.

Although the methods employed by the models in the selection of the initial capacity of Reservoir 1 can be readily identified, the volume selected by the models for the initial capacity of Reservoir 2 can sometimes seem counterintuitive. For example, the initial capacity of Reservoir 2 was selected by the Drought Model to equal approximately 2000 ML. When first considered, the "common sense" solution would be to initialise the storage level of the reservoir at full capacity, meaning there would be plentiful water supplies to meet the community demand across the event horizon. However, instead the Drought Model selected the minimum initial capacity of Reservoir 2 that ensured the goals of the model were fully met without penalty to the objective function. A similar behaviour is exhibited when the Flood Model is considered. Again, a "common sense" approach would be to specify the initial capacity of Reservoir 2 to equal 0 ML, so that the maximum volume of floodwater could be withheld by the reservoir, ensuring that the extent of spill from the system is minimised. However, the Flood Model selected the initial capacity of Reservoir 2 to equal approximately 57 500 ML; the maximum initial storage level of the reservoir that ensured the primary aim of the model was met completely, without penalty to the objective function. Therefore, although "common sense" indicates that there are straightforward strategies that could be employed in an attempt to manage a cascade reservoir system, these strategies may not result in the optimal management of the system. Also, these strategies may not be feasible when a physical system is considered, as reservoirs are not typically empty at the time a flood event occurs, nor are they typically at full capacity when a drought commences.

At this point, it is important to reiterate that both the Drought Model and Flood Model were designed and formulated to provide an approximate representation of a cascade reservoir system under the extreme conditions of a drought and flood event, while also allowing for the provision of certain assumptions and the specification of optional parameters. In this case, the optional parameters of both models were unconstrained to enable these values to be selected by the models themselves, thus helping to differentiate between the different management strategies the models employed. However, the strength of both the Drought Model and the Flood Model lies in the ability of the reservoir system managers to be able to select the value of these optional parameters, trial the models under varying conditions, and generate an approximate representation of the system behaviour for each "what if" scenario.

6.3 Chapter Conclusion

In this chapter, an experiment was conducted to ultimately compare the management strategies of the Drought Model (Model 4 from Chapter 3) and the Flood Model (Model 6 from Chapter 4), when both Mixed Integer Linear Programming (MILP) models were subject to an equivalent set of conditions. This equivalent set of conditions involved selecting the optional parameters of both models to be similar and ensuring that the models were considered on a common set of inflows. In this case, the optional parameters of the models, being the initial reservoir capacities for both models and the number of months in which phase two rationing is permitted (the Drought Model only), were specified to be unconstrained. This enabled the models themselves to optimally select the value of these parameters and helped to better differentiate between the management strategies employed. Also, in order to ensure a useful comparison between the Drought and Flood Models, they both needed to be considered using a common set of inflows. In this case, the common set of inflows utilised were simulated using time series analysis (in Chapter 5) from the historic inflow records (sourced from the Queensland Department of Natural Resources and Mines (2012)).

In previous chapters, a series of assumptions were outlined when performing experiments using the afore labelled Model 4 and Model 6 in order to reduce the complexity of the conditions under which the experiment was conducted, while still providing an accurate approximation of the Perseverance and Cressbrook cascade reservoir system considered as a case study. To ensure the continuity of the methodologies employed throughout this thesis, many of the same assumptions were made when conducting the experiments of the models in this chapter. Using the results of the experiments, the behaviour of key parameters measured by the Drought Model and Flood Model were explored individually, before the management strategies of the two models were compared. From the investigation and comparison of the management strategies, it was found that some of the strategies employed by the two models were similar, while there were also some clear differences; primarily due to the different objectives of the two models, which in turn can be attributed to the vastly different environmental phenomena that the Drought and Flood Model consider. Also, it was noted that although there may be "common sense" strategies that could be utilised in an attempt to manage the reservoir system, these strategies may not always result in the optimal management of the system.

Chapter 7

Conclusion

The objective of the research presented in this thesis has been to demonstrate the procedure undertaken in order to develop, formulate and then utilise two Mixed Integer Linear Programming (MILP) models that have the ability to determine strategies for the optimal management of a cascade reservoir system, under the two extreme environmental conditions of a drought and flood. Throughout this research, the unique cascade configuration of a reservoir system was considered; where cascade refers to a multiple reservoir system in which the spill from earlier reservoirs becomes a source of inflows to subsequent reservoirs. Many physical reservoir systems exhibit this type of layout, such as the Somerset and Wivenhoe reservoirs in the Brisbane Valley, along with the Perseverance and Cressbrook reservoir system located near Toowoomba (presented in Figure 1.1 and again in Figure 6.1); the latter of the two systems forming the case study considered throughout this thesis. The motivation behind selecting the lesser known Perseverance and Cressbrook cascade reservoir system to form the case study of this research, over a more substantial system that exhibited the same configuration, was in part due to proximity and the greater interest that resulted from a reliance on the water supplied by the local system, and also to demonstrate that any reservoir system, no matter how small or simple, can benefit from the implementation of better management strategies.

To begin, Chapter 2 outlined where the research presented in this thesis originated; with the investigation and extension of a basic Linear Programming (LP) model that provided a simple approximation for the operation of a single reservoir system. This simple model, labelled Model 1 for the purposes of the thesis, was developed by ReVelle and McGarity (1997) and after exploration was found to be limited in its ability to comprehensively describe a physical system. That being said, the model provided a useful example of how LP models could be utilised to monitor the behaviour of key variables that contribute to the operation of a reservoir system.

As Model 1 was developed to only consider a single reservoir in isolation, it required extension in order for it to be applicable to a cascade reservoir system. However, during the formulation of the extended model, named Model 2, binary variables were added to some constraints to ensure the model could adequately describe some behaviours of a cascade reservoir system; changing the nature of Model 2 from a LP model to a Mixed Integer Linear Programming (MILP) model. An experiment was then conducted using Model 2 to investigate how the storage capacities of the two reservoirs in the system behaved under varying community water supply demand. Three different policy types were considered as part of the experiment, where each of the policies partitioned the community water supply demand differently between the two reservoirs. From the results of the experiment, it was found that two distinct behaviours were exhibited by the reservoir storage capacities; one unique behaviour when the inflows to a reservoir exceeded the community water supply demand from the reservoir, and the other when the inflows to a reservoir were insufficient to meet the demand from the community. Also, it was highlighted that Model 2 had some limitations and did not always provide a physically feasible result; under some conditions suggesting that either one or both of the reservoirs in the system were not required.

Although the models considered in Chapter 2 were simplistic and not as comprehensive as models explored in later chapters, the intention of Chapter 2 was to demonstrate how small experiments with simple models helped foster a basis of understanding that could be employed in subsequent chapters when applying MILP models to more complex and sophisticated scenarios.

In Chapter 3, the first of the more complex scenarios were explored; with the aim of developing a MILP model that could describe the operation of, along with determine optimal management strategies for, a cascade reservoir system during a drought event. To begin, a MILP model constructed by Shih and ReVelle (1995), named Model 3 for the purposes of this thesis, to describe the operation of a single reservoir system throughout the course of a drought was investigated. Due to the comprehensive nature with which Model 3 was originally developed by Shih and ReVelle (1995), the model was, with relative simplicity, extended in order to describe a cascade reservoir system during a drought event.

Following the definition and exploration of the extended drought model, labelled Model 4, the model was applied to the Perseverance and Cressbrook cascade reservoir system considered as a case study through the means of an experiment. This experiment was replicated three times, with a different plausible scenario considered under each replication of the experiment. For each of these scenarios, the optional parameters of the model were varied to explore the different behaviours exhibited by the key parameters of the reservoir system measured by Model 4. From the outcome of the experiment, it was determined that Model 4 provided a comprehensive approximation of a cascade reservoir system during a drought event, with optimal management strategies determined by the model for each scenario considered.

However, some limitations of the model were also evident; primarily attributed to the assumed conditions under which the experiment was conducted. Also, it was noted that the optimal management strategies determined by Model 4 for the Perseverance and Cressbrook cascade reservoir system were sensitive to changes in the optional parameters selected. Future scope for further research was also discovered, as it was ascertained that alternate methods to those currently employed by Model 4 could exist to determine optimal management strategies for the reservoir system.

The complexity of the scenarios explored throughout this thesis increased again in Chapter 4, with the management of a cascade reservoir system being considered under the inverse environmental phenomena to a drought; a flood. To commence Chapter 4, a model was developed, labelled Model 5, which made use of the frameworks of Model 3 and Model 4 from Chapter 3, but at the same time effectively reversed the objectives of these models to construct a MILP model with the ability to describe the operation of a single reservoir system during a flood event. Model 5 was then built upon and extended further in order to formulate Model 6; a MILP model that provides an approximation for, along with determine strategies for the optimal management of, a cascade reservoir system during a flood event. In order to extend the functionality of a typical cascade reservoir system, along with the complexity of Model 6, it was assumed that a pumping facility was included as part of the reservoir system that could be employed to drain water from Reservoir 2 (Cressbrook dam) and pump it backwards through the system to Reservoir 1 (Perseverance dam).

Model 6 was trialled using much the same technique as Model 4 in Chapter 3, with an experiment replicated three times to determine how key parameters of the Perseverance and Cressbrook cascade reservoir system measured by the model behaved under varying initial reservoir capacities (the only optional parameter in the case of Model 6). Based upon the results of the experiment, it was found that different relationships link the initial reservoir capacities selected with the spill from the reservoirs over the course of a flood event. Also, the experiment proved that Model 6 provided a comprehensive approximation for the operation of the Perseverance and Cressbrook cascade reservoir system considered as a case study during a flood event, while determining optimal strategies for the management of the system. The experiment of Model 6 also highlighted opportunities for future work. These opportunities include developing a function to describe the relationship between the initial capacities of the reservoirs selected and the total spill from the reservoirs, along with a comparison between the optimal management strategies determined by Model 6 currently, with the optimal management strategies of a model that does not consider a pumping facility as part of the reservoir system.

Therefore, the formulation of Model 4 and Model 6 resulted in two MILP models that can be employed to determine the optimal management strategies for a cascade reservoir system under the extreme environmental conditions of a drought or a flood; subject to a series of assumptions. Thus, it is important to note that the optimal strategies determined by both models under one set of assumptions will not be the same under an alternate set of assumptions; leading to the existence of an infinite set of optimal strategies for any cascade reservoir system. This existence of an infinite set of optimal strategies due to the assumptions made is unavoidable and a common occurrence in most types of modelling. Therefore, even through the adoption of one of the more complex methods mentioned in Chapter 1, such as a Mixed Integer Nonlinear Programming model or utilising the Neural Network approach, the optimal strategies determined by these methods may vary dependent upon the set of assumptions specified. Thus, the strength of MILP models, such as Model 4 and Model 6, is that with relative ease the operators of a reservoir system can generate approximate representations of their particular system under any conceivable scenario; providing an example of the behaviours that could be expected and a plausible management strategy, subject to the assumed conditions, that optimally achieves a predefined objective. Considering the relative complexity of other model types compared with MILP models, an aim of future research is to compare the optimal management strategies from other model types, such as nonlinear or inventory models, with the strategies from a MILP model and weigh the benefit from a potentially better representation of the system against the complexity of formulating and then applying the more complex model types to a cascade reservoir system.

One limitation that affected the application of both Model 4 and Model 6 was the Academic License of LINGO version 14.0 (LINDO Systems Inc, 2013) utilised. This license limited the number of constraints and variables that the software could consider and as such, the time frame over which both models could be investigated was limited to 24 time units; equating to an event horizon of 24 months duration for Model 4 and 24 hours duration for Model 6 respectively. Therefore, an aim of future research is to apply both models to the Perseverance and Cressbrook cascade reservoir system considered as a case study over a more substantial time period. For example Model 4 could be considered across a horizon of 10 years, over which time multiple drought events occur, facilitating better inference regarding when the rationing of water supplies should commence, given a drought event is imminent. On the other hand, Model 6 could be trialled under a scenario with sustained moderate inflows across multiple weeks, potentially enabling the pumping facility to have more influence on the optimal management strategies for the reservoir system.

Moving from the development and formulation of MILP models, time series analysis was utilised in Chapter 5, with the aim of simulating an alternate set of inflow data, based upon the historic records sourced from the Queensland Department of Natural Resources and Mines (2012), that could be used as a common set of inflows when performing a comparison of the management strategies employed by the drought model (Model 4) and flood model (Model 6) later in this thesis. To commence Chapter 5, the 13 year period in which complete inflow records were recorded was separated into two portions. The first and largest of the portions was labelled the training series, and it was from this data that a suitable time series model was selected. The remaining, smaller portion of the data, named the testing series, was withheld to be used later when validating the selected time series model. Two model selection tools were then employed to determine the type and order of time series model that provided the best approximate representation of the historic inflow records. Using an information criterion, in conjunction with these two model selection tools, an AR(1) model was determined to provide the best representation of the historic inflow records. Following the selection of the AR(1) model, a series of diagnostic tests were performed in order to ensure that it was a "good" model to describe the inflow records, by it capturing the important information, or signal, of the inflow data.

Before using the AR(1) time series model to simulate an alternate set of inflow data, a validation procedure outlined by Dunn and Addie (2008) was performed. However, due to the selection of an atypical period of inflows in the testing series that did not display the behaviours witnessed as part of the training series, the capability of the validation process to measure the forecasting ability of the AR(1) model was impaired. Due to time constraints, this limitation was unable to be rectified and this portion of the inflow records unable to be excluded from the testing series. Taking the limitation of the testing series into account, the results of the validation process still indicated that the AR(1) model provided an adequate approximation of the historic inflow data. Thus, from this model, an alternate set of inflows were able to be simulated for use in the Chapter 6, where a comparison of the drought and flood model was performed.

Although the AR(1) time series model selected to describe the historic inflow records did suitably capture the signal of the data for the purposes of a simulation, other types of time series models could have provided a better approximation of the inflow records. In this case, only a limited number of simple model types under the Box-Jenkins methodology were considered, while a range of more complex time series models, such as the non-stationary or non-linear time series models explored by Dunn and Addie (2008), were not. Thus, if as part of future research a more accurate representation of the historic inflow records was required, then these alternate types of time series models could be considered and compared against the parsimonious AR(1) model to determine if a better model existed to capture the signal of the inflow records.

Using Model 4 and Model 6, herein referred to as the Drought Model and Flood Model, a comparison of the management strategies employed by the two models was conducted in Chapter 6; where both considered a cascade reservoir system under a common set of inflows, and similar optional parameters. In the case of the comparison performed, the optional parameters of both MILP models were selected to be unconstrained. By not constraining these optional parameters, the models themselves were able to select their optimal values, which in turn helped to further differentiate between the management strategies employed. Also, in order to facilitate a useful comparison of the strategies employed to optimally manage the cascade reservoir system, both the Drought Model and Flood Model were considered on the common set of inflows simulated in Chapter 5 using time series analysis.

Under these similar conditions, an experiment revealed that there were some similarities between the management strategies employed by the two models, but also clear differences. These differences were primarily attributable to the different objectives of the models, which in turn were a product of the two vastly different environmental phenomena that the Drought Model and Flood Model considered. Also evident from the comparison of the Drought and Flood Models was that although "common sense" strategies exist and could be utilised to operate a cascade reservoir system; these strategies may not always result in the optimal management of the system.

Therefore, this thesis has resulted in the successful demonstration of the processes, procedures and methods undertaken in the development and formulation of two MILP models that have the ability to determine strategies for the optimal management of a cascade reservoir system, under the two extreme environmental conditions of a drought and flood. Both of these models have been tested through application to the case study of the physical Perseverance and Cressbrook cascade reservoir system; resulting in comprehensive approximations of the system behaviours. Also, time series analysis has been utilised to generate a model that provided a suitable representation of the historic inflow records sourced for the case study considered. This time series model was then employed to simulate an alternate set of inflows for use as a common data set when comparing the management strategies of the MILP models; a comparison that demonstrated both similarities and differences between the management strategies employed by the Drought Model and Flood Model. This comparison also revealed that although "common sense" strategies could be employed to operate a reservoir system, these strategies often do not result in the optimal management of the system and models like those developed, formulated and utilised in this thesis are necessary to ensure that a commodity as valuable and potentially as dangerous as water is optimally regulated into the future.

Appendix A

Example of LINGO Input and Output

Model 2: The Simple Model

An example of both the input syntax and output generated when the experiment of the simple model (Model 2) was performed using the optimisation modelling software LINGO (LINDO Systems Inc, 2013) can be found in this appendix. In order to minimise the size of this thesis, only a subset of the input and output has been provided in the following sections. A full example of the input syntax and output generated can be found in the *Example of LINGO Input and Output* directory accompanying this thesis, in the folder *Model 2 - The Simple Model*. An example of the input for Model 1 is also included in the accompanying directory, following the path *Initial Model Examples\Example of Model 1 Input*. Note that for ease of opening and reading the accompanying input and output files, text copies (extension .txt) of the original LINGO files have been included.

In order to assist with the interpretation of the LINGO input and output, each of the variables that compose Model 2 have been translated into the form in which they appear in the input syntax and listed below. In this case, the variable `q` did not appear in Chapter 2 when defining the constraints or variables that compose Model 2; however the variable was used in the input syntax to define the total community water supply demand. Also, binary variables are denoted in the input syntax by being enclosed within `@BIN()`.

t = `1, 2,..., 6`

S_t^* = `s1f, s2f,..., s6f`

S_t = `s1s, s2s,..., s6s`

S_0^* = `s0f`

S_0 = `s0s`

q^* = `qf`

q = `qs`

W_t^* = `w1f, w2f,..., w6f`

W_t = `w1s, w2s,..., w6s`

I_t^* = `i1f, i2f,..., i6f`

I_t = `i1sf, i2sf,..., i6sf`

$$C^* = \texttt{Cap1}$$
$$C = \texttt{Cap2}$$
$$C_D^* = \texttt{CapDam1}$$
$$C_D = \texttt{CapDam2}$$
$$C_S^* = \texttt{CapSp1}$$
$$C_S = \texttt{CapSp2}$$
$$p_t^{+*} = \texttt{p1pf, p2pf,..., p6pf}$$
$$p_t^{+} = \texttt{p1ps, p2ps,..., p6ps}$$
$$p_t^{-*} = \texttt{p1mf, p2mf,..., p6mf}$$
$$p_t^{-} = \texttt{p1ms, p2ms,..., p6ms}$$
$$M = \texttt{10000}$$
$$y_t^* = \texttt{y1f, y2f,..., y6f}$$
$$y_t = \texttt{y1s, y2s,..., y6s}$$

A.1 Example of LINGO Input for the Simple Model

```
MIN = CapDam1 + CapDam2;

s0f + i1f - qf - w1f - s1f = 0;
s1f + i2f - qf - w2f - s2f = 0;
...
s5f + i6f - qf - w6f - s6f = 0;

s0f - CapDam1 <= 0;
s1f - CapDam1 <= 0;
...
s6f - CapDam1 <= 0;

w1f - CapSp1 <= 0;
w2f - CapSp1 <= 0;
...
w6f - CapSp1 <= 0;

Cap1 - CapSp1 - CapDam1 = 0;
CapSp1 - 0.1*CapDam1 <= 0;
```

```
s0f + i1f - qf  - CapDam1 - p1pf + p1mf = 0;
s1f + i2f - qf  - CapDam1 - p2pf + p2mf = 0;
...
s5f + i6f - qf  - CapDam1 - p6pf + p6mf = 0;

p1mf - 10000*y1f <= 0;
p1pf + 10000*y1f <= 10000;
p2mf - 10000*y2f <= 0;
p2pf + 10000*y2f <= 10000;
...
p6mf - 10000*y6f <= 0;
p6pf + 10000*y6f <= 10000;

w1f - p1pf <= 0;
w2f - p2pf <= 0;
...
w6f - p6pf <= 0;

i1f=50;
i2f=50;
...
i6f=50;

s0s + i1sf - qs - w1s - s1s = 0;
s1s + i2sf - qs - w2s - s2s = 0;
...
s5s + i6sf - qs - w6s - s6s = 0;

s0s - CapDam2 <= 0;
s1s - CapDam2 <= 0;
...
s6s - CapDam2 <= 0;

w1s - CapSp2 <= 0;
w2s - CapSp2 <= 0;
...
w6s - CapSp2 <= 0;
```

```
Cap2 - CapSp2 - CapDam2 = 0;
CapSp2 - 0.1*CapDam2 <= 0;

s0s + i1sf - qs  - CapDam2 - p1ps + p1ms = 0;
s1s + i2sf - qs  - CapDam2 - p2ps + p2ms = 0;
...
s5s + i6sf - qs  - CapDam2 - p6ps + p6ms = 0;

p1ms - 10000*y1s <= 0;
p1ps + 10000*y1s <= 10000;
p2ms - 10000*y2s <= 0;
p2ps + 10000*y2s <= 10000;
...
p6ms - 10000*y6s <= 0;
p6ps + 10000*y6s <= 10000;

w1s - p1ps <= 0;
w2s - p2ps <= 0;
...
w6s - p6ps <= 0;

i1s=50;
i2s=50;
...
i6s=50;

i1sf - i1s - w1f = 0;
i2sf - i2s - w2f = 0;
...
i6sf - i6s - w6f = 0;

q=0;
q-qf-qs=0;
qf-0.5*q=0;

@BIN(y1f);
@BIN(y2f);
...
@BIN(y6f);
```

```
@BIN(y1s);
@BIN(y2s);
...
@BIN(y6s);
```

A.2 Example of LINGO Output for the Simple Model

```
Global optimal solution found.
 Objective value:                                   570.2479
 Objective bound:                                   570.2479
 Infeasibilities:                                   0.000000
 Extended solver steps:                                    3
 Total solver iterations:                                444
 Elapsed runtime seconds:                               0.22

 Model Class:                                           MILP

 Total variables:                   74
 Nonlinear variables:                0
 Integer variables:                 12

 Total constraints:                 97
 Nonlinear constraints:              0

 Total nonzeros:                   244
 Nonlinear nonzeros:                 0



Variable             Value        Reduced Cost
 CAPDAM1          272.7273            0.000000
 CAPDAM2          297.5207            0.000000
     S0F          0.000000           0.9917355
     I1F          50.00000            0.000000
      QF          0.000000            0.000000
     W1F          0.000000            0.000000
     S1F          50.00000            0.000000
     I2F          50.00000            0.000000
     W2F          0.000000            0.000000
     S2F          100.0000            0.000000
     I3F          50.00000            0.000000
     W3F          0.000000            0.000000
     S3F          150.0000            0.000000
     I4F          50.00000            0.000000
     W4F          0.000000            0.000000
```

Appendix B

Example of LINGO Input and Output

Model 4: The Drought Model

An example of both the input syntax and output generated when the experiment of the drought model (Model 4) under the conditions of Scenario 1 was performed using the optimisation modelling software LINGO (LINDO Systems Inc, 2013) can be found in this appendix. In order to minimise the size of this thesis, only a subset of the input and output has been provided in the following sections. A full example of the input syntax and output generated for each of the drought model scenarios considered in Chapter 3 and the model comparison experiment performed in Chapter 6, can be found in the *Example of LINGO Input and Output* directory accompanying this thesis, in the folder *Model 4 - The Drought Model*. An example of the input for Model 3 is also included in the accompanying directory, following the path *Initial Model Examples\Example of Model 3 Input*. Note that for ease of opening and reading the accompanying input and output files, text copies (extension .txt) of the original LINGO files have been included.

In order to assist with the interpretation of the LINGO input and output, each of the variables that compose Model 4 have been translated into the form in which they appear in the input syntax and listed below. In this case, binary variables are denoted in the input syntax by being enclosed within `@BIN()`.

t = `1, 2,..., 24`

p = `1, 2,..., 12`

y^*_{1t} = `y1D1M1, y1D1M2,..., y1D1M24`

y_{1t} = `y1D2M1, y1D2M2,..., y1D2M24`

y^*_{2t} = `y2D1M1, y2D1M2,..., y2D1M24`

y_{2t} = `y2D2M1, y2D2M2,..., y2D2M24`

ω = `0.01`

V^*_{1p} = `V1D1M1, V1D1M2,..., V1D1M12`

V_{1p} = `V1D2M1, V1D2M2,..., V1D2M12`

V^*_{2p} = `V2D1M1, V2D1M2,..., V2D1M12`

V_{2p} = `V2D2M1, V2D2M2,..., V2D2M12`

$$
\begin{aligned}
V_{3p}^* &= \texttt{V3D1M1, V3D1M2,..., V3D1M12} \\
V_{3p} &= \texttt{V3D2M1, V3D2M2,..., V3D2M12} \\
S_t^* &= \texttt{D1S1, D1S2,..., D1S24} \\
S_t &= \texttt{D2S1, D2S2,..., D2S24} \\
S_0^* &= \texttt{D1S0} \\
S_0 &= \texttt{D2S0} \\
I_t^* &= \texttt{D1I1, D1I2,..., D1I24} \\
I_t &= \texttt{D2I1, D2I2,..., D2I24} \\
\hat{I}_t^* &= \texttt{D1Ihat1, D1Ihat2,..., D1Ihat24} \\
\hat{I}_t &= \texttt{D2Ihat1, D2Ihat2,..., D2Ihat24} \\
\epsilon &= \texttt{0.1} \\
M &= \texttt{100000} \\
R_t^* &= \texttt{D1R1, D1R2,..., D1R24} \\
R_t &= \texttt{D2R1, D2R2,..., D2R24} \\
\alpha_1 &= \texttt{0.6} \\
\alpha_2 &= \texttt{0.4} \\
D^* &= \texttt{333} \\
D &= \texttt{667} \\
W_t^* &= \texttt{D1W1, D1W2,..., D1W24} \\
W_t &= \texttt{D2W1, D2W2,..., D2W24} \\
C^* &= \texttt{30000} \\
C &= \texttt{82000} \\
U_t^* &= \texttt{D1U1, D1U2,..., D1U24} \\
U_t &= \texttt{D2U1, D2U2,..., D2U24} \\
\beta_1 &= \texttt{0.05} \\
\beta_2 &= \texttt{0.05} \\
n &= \texttt{0, 1,..., 24} \\
A_t^* &= \texttt{D1A0, D1A1,..., D1A24} \\
A_t &= \texttt{D2A0, D2A1,..., D2A24}
\end{aligned}
$$

B.1 Example of LINGO Input for the Drought Model under Scenario 1

```
MAX = y1D1M1+y1D1M2+y1D1M3+y1D1M4+y1D1M5+y1D1M6+y1D1M7+y1D1M8+y1D1M9
      +y1D1M10+y1D1M11+y1D1M12+y1D1M13+y1D1M14+y1D1M15+y1D1M16+y1D1M17
      +y1D1M18+y1D1M19+y1D1M20+y1D1M21+y1D1M22+y1D1M23+y1D1M24+y1D2M1
      +y1D2M2+y1D2M3+y1D2M4+y1D2M5+y1D2M6+y1D2M7+y1D2M8+y1D2M9
      +y1D2M10+y1D2M11+y1D2M12+y1D2M13+y1D2M14+y1D2M15+y1D2M16+y1D2M17
      +y1D2M18+y1D2M19+y1D2M20+y1D2M21+y1D2M22+y1D2M23+y1D2M24
```

```
      - 0.01*(V1D1M1+V1D1M2+V1D1M3+V1D1M4+V1D1M5+V1D1M6+V1D1M7+V1D1M8
      +V1D1M9+V1D1M10+V1D1M11+V1D1M12+V2D1M1+V2D1M2+V2D1M3+V2D1M4
      +V2D1M5+V2D1M6+V2D1M7+V2D1M8+V2D1M9+V2D1M10+V2D1M11+V2D1M12
      +V3D1M1+V3D1M2+V3D1M3+V3D1M4+V3D1M5+V3D1M6+V3D1M7+V3D1M8+V3D1M9
      +V3D1M10+V3D1M11+V3D1M12)
      - 0.01*(V1D2M1+V1D2M2+V1D2M3+V1D2M4+V1D2M5+V1D2M6+V1D2M7+V1D2M8
      +V1D2M9+V1D2M10+V1D2M11+V1D2M12+V2D2M1+V2D2M2+V2D2M3+V2D2M4
      +V2D2M5+V2D2M6+V2D2M7+V2D2M8+V2D2M9+V2D2M10+V2D2M11+V2D2M12
      +V3D2M1+V3D2M2+V3D2M3+V3D2M4+V3D2M5+V3D2M6+V3D2M7+V3D2M8+V3D2M9
      +V3D2M10+V3D2M11+V3D2M12)
      - (D1A0+D1A1+D1A2+D1A3+D1A4+D1A5+D1A6+D1A7+D1A8+D1A9+D1A10+D1A11
      +D1A12+D1A13+D1A14+D1A15+D1A16+D1A17+D1A18+D1A19+D1A20+D1A21
      +D1A22+D1A23+D1A24)
      - (D2A0+D2A1+D2A2+D2A3+D2A4+D2A5+D2A6+D2A7+D2A8+D2A9+D2A10+D2A11
      +D2A12+D2A13+D2A14+D2A15+D2A16+D2A17+D2A18+D2A19+D2A20+D2A21
      +D2A22+D2A23+D2A24);

D1S0 <= 30000;
D2S0 <= 82000;

!Constraint 1 - Dam 1;
100000*y1D1M1 -D1S0 -D1Ihat1 + V1D1M1 >= 0.1;
100000*y1D1M2 -D1S1 -D1Ihat2 + V1D1M2 >= 0.1;
...
100000*y1D1M24 -D1S23 -D1Ihat24 + V1D1M12 >= 0.1;

!Constraint 1 - Dam 2;
100000*y1D2M1 -D2S0 -D2Ihat1 + V1D2M1 >= 0.1;
100000*y1D2M2 -D2S1 -D2Ihat2 + V1D2M2 >= 0.1;
...
100000*y1D2M24 -D2S23 -D2Ihat24 + V1D2M12 >= 0.1;

!Constraint 2 - Dam 1;
100000*y1D1M1 +V1D1M1 - D1S0 - D1Ihat1 <= 100000;
100000*y1D1M2 +V1D1M2 - D1S1 - D1Ihat2 <= 100000;
...
100000*y1D1M24 +V1D1M12 - D1S23 - D1Ihat24 <= 100000;
```

```
!Constraint 2 - Dam 2;
100000*y1D2M1 +V1D2M1 - D2S0 - D2Ihat1 <= 100000;
100000*y1D2M2 +V1D2M2 - D2S1 - D2Ihat2 <= 100000;
...
100000*y1D2M24 +V1D2M12 - D2S23 - D2Ihat24 <= 100000;

!Constraint 3 - Dam 1;
100000*y2D1M1 -D1S0 -D1Ihat1 + V2D1M1 >= 0.1;
100000*y2D1M2 -D1S1 -D1Ihat2 + V2D1M2 >= 0.1;
...
100000*y2D1M24 -D1S23 -D1Ihat24 + V2D1M12 >= 0.1;

!Constraint 3 - Dam 2;
100000*y2D2M1 -D2S0 -D2Ihat1 + V2D2M1 >= 0.1;
100000*y2D2M2 -D2S1 -D2Ihat2 + V2D2M2 >= 0.1;
...
100000*y2D2M24 -D2S23 -D2Ihat24 + V2D2M12 >= 0.1;

!Constraint 4 - Dam 1;
100000*y2D1M1 +V2D1M1 - D1S0 - D1Ihat1 <= 100000;
100000*y2D1M2 +V2D1M2 - D1S1 - D1Ihat2 <= 100000;
...
100000*y2D1M24 +V2D1M12 - D1S23 - D1Ihat24 <= 100000;

!Constraint 4 - Dam 2;
100000*y2D2M1 +V2D2M1 - D2S0 - D2Ihat1 <= 100000;
100000*y2D2M2 +V2D2M2 - D2S1 - D2Ihat2 <= 100000;
...
100000*y2D2M24 +V2D2M12 - D2S23 - D2Ihat24 <= 100000;

!Constraint 5 - Dam 1;
D1R1 - 133*y1D1M1 -67*y2D1M1 = 133;
D1R2 - 133*y1D1M2 -67*y2D1M2 = 133;
...
D1R24 - 133*y1D1M24 -67*y2D1M24 = 133;
```

```
!Constraint 5 - Dam 2;
D2R1 - 267*y1D2M1 -133*y2D2M1 = 267;
D2R2 - 267*y1D2M2 -133*y2D2M2 = 267;
...
D2R24 - 267*y1D2M24 -133*y2D2M24 = 267;

!Constraint 6 - Dam 1;
D1S1 - D1S0 - D1I1 + D1R1 + D1W1 = 0;
D1S2 - D1S1 - D1I2 + D1R2 + D1W2 = 0;
...
D1S24 - D1S23 - D1I24 + D1R24 + D1W24 = 0;

!Constraint 6 - Dam 2;
D2S1 - D2S0 - D2I1 + D2R1 + D2W1 - D1W1 = 0;
D2S2 - D2S1 - D2I2 + D2R2 + D2W2 - D1W2 = 0;
...
D2S24 - D2S23 - D2I24 + D2R24 + D2W24 - D1W24 = 0;

!Constraint 7 - Dam 1;
D1S1 <= 30000;
D1S2 <= 30000;
...
D1S24 <= 30000;

!Constraint 7 - Dam 2;
D2S1 <= 82000;
D2S2 <= 82000;
...
D2S24 <= 82000;

!Constraint 8 - Dam 1;
D1S0 - D1S24 <= 0;

!Constraint 8 - Dam 2;
D2S0 - D2S24 <= 0;
```

```
!Constraint 9 - Dam 1;
30000*D1U1 - D1S1 <= 0;
30000*D1U2 - D1S2 <= 0;
...
30000*D1U24 - D1S24 <= 0;

!Constraint 9 - Dam 2;
82000*D2U1 - D2S1 <= 0;
82000*D2U2 - D2S2 <= 0;
...
82000*D2U24 - D2S24 <= 0;

!Constraint 10 - Dam 1;
D1W1 - 100000*D1U1 <= 0;
D1W2 - 100000*D1U2 <= 0;
...
D1W24 - 100000*D1U24 <= 0;

!Constraint 10 - Dam 2;
D2W1 - 100000*D2U1 <= 0;
D2W2 - 100000*D2U2 <= 0;
...
D2W24 - 100000*D2U24 <= 0;

!Constraint 11 - Dam 1;
V1D1M1 - 1.05*V2D1M1 >= 0;
V1D1M2 - 1.05*V2D1M2 >= 0;
...
V1D1M12 - 1.05*V2D1M12 >= 0;

!Constraint 11 - Dam 2;
V1D2M1 - 1.05*V2D2M1 >= 0;
V1D2M2 - 1.05*V2D2M2 >= 0;
...
V1D2M12 - 1.05*V2D2M12 >= 0;
```

```
!Constraint 12 - Dam 1;
V2D1M1 - 1.05*V3D1M1 >= 0;
V2D1M2 - 1.05*V3D1M2 >= 0;
...
V2D1M12 - 1.05*V3D1M12 >= 0;

!Constraint 12 - Dam 2;
V2D2M1 - 1.05*V3D2M1 >= 0;
V2D2M2 - 1.05*V3D2M2 >= 0;
...
V2D2M12 - 1.05*V3D2M12 >= 0;

!Constraint 13 - Dam 1;
V3D1M1 >= 133;
V3D1M2 >= 133;
...
V3D1M12 >= 133;

!Constraint 13 - Dam 2;
V3D2M1 >= 267;
V3D2M2 >= 267;
...
V3D2M12 >= 267;

!Constraint 14 - Dam 1;
D1S0 + D1Ihat1 - V3D1M1 >= 0.1;
D1S1 + D1Ihat2 - V3D1M2 >= 0.1;
...
D1S24 + D1Ihat25 - V3D1M1 >= 0.1;

!Constraint 14 - Dam 2
D2S0 + D2Ihat1 - V3D2M1 >= 0.1;
D2S1 + D2Ihat2 - V3D2M2 >= 0.1;
...
D2S24 + D2Ihat25 - V3D2M1 >= 0.1;
```

```
!Constraint 15 - Dam 1;
y1D1M1 + y1D1M3 - y1D1M2 <= 1;
y1D1M2 + y1D1M4 - y1D1M3 <= 1;
...
y1D1M22 + y1D1M24 - y1D1M23 <= 1;

!Constraint 15 - Dam 2;
y1D2M1 + y1D2M3 - y1D2M2 <= 1;
y1D2M2 + y1D2M4 - y1D2M3 <= 1;
...
y1D2M22 + y1D2M24 - y1D2M23 <= 1;

!Constraint 16 - Dam 1;
y1D1M1 - y2D1M2 <= 0;
y1D1M2 - y2D1M3 <= 0;
...
y1D1M23 - y2D1M24 <= 0;

!Constraint 16 - Dam 2;
y1D2M1 - y2D2M2 <= 0;
y1D2M2 - y2D2M3 <= 0;
...
y1D2M23 - y2D2M24 <= 0;

!Constraint 17 - Dam 1;
y2D1M1 + y2D1M2 + y2D1M3 + y2D1M4 + y2D1M5 + y2D1M6 + y2D1M7 + y2D1M8
+ y2D1M9 + y2D1M10 + y2D1M11 + y2D1M12 + y2D1M13 + y2D1M14 + y2D1M15
+ y2D1M16 + y2D1M17 + y2D1M18 + y2D1M19 + y2D1M20 + y2D1M21 + y2D1M22
+ y2D1M23 + y2D1M24 = 24;

!Constraint 17 - Dam 2;
y2D2M1 + y2D2M2 + y2D2M3 + y2D2M4 + y2D2M5 + y2D2M6 + y2D2M7 + y2D2M8
+ y2D2M9 + y2D2M10 + y2D2M11 + y2D2M12 + y2D2M13 + y2D2M14 + y2D2M15
+ y2D2M16 + y2D2M17 + y2D2M18 + y2D2M19 + y2D2M20 + y2D2M21 + y2D2M22
+ y2D2M23 + y2D2M24 = 24;
```

```
!Constraint 18 - Dam 1;
D1I1= 342.12;
D1I2= 144.72;
...
D1I24= 1313.28;

!Constraint 18 - Dam 2;
D2I1= 570.2;
D2I2= 191.2;
...
D2I24= 2188.8;

!Constraint 19 - Dam 1;
D1Ihat1 - D1I1 = 0;
D1Ihat2 - D1I2 = 0;
...
D1Ihat24 - D1I24 = 0;

!Constraint 19 - Dam 2;
D2Ihat1 - D2I1 = 0;
D2Ihat2 - D2I2 = 0;
...
D2Ihat24 - D2I24 = 0;

!Constraint 20 - Dam 1;
D1S0 + D1A0 >= 133;
D1S1 + D1A1 >= 133;
...
D1S24 + D1A24 >= 133;

!Constraint 20 - Dam 2;
D2S0 + D2A0 >= 267;
D2S1 + D2A1 >= 267;
...
D2S24 + D2A24 >= 267;
```

```
!Constraint 21 - Dam 1;
@BIN(y1D1M1);
@BIN(y1D1M2);
...
@BIN(y1D1M24);

!Constraint 21 - Dam 2;
@BIN(y1D2M1);
@BIN(y1D2M2);
...
@BIN(y1D2M24);

!Constraint 22 - Dam 1;
@BIN(y2D1M1);
@BIN(y2D1M2);
...
@BIN(y2D1M24);

!Constraint 22 - Dam 2;
@BIN(y2D2M1);
@BIN(y2D2M2);
...
@BIN(y2D2M24);

!Constraint 23 - Dam 1;
@BIN(D1U1);
@BIN(D1U2);
...
@BIN(D1U24);

!Constraint 23 - Dam 2;
@BIN(D2U1);
@BIN(D2U2);
...
@BIN(D2U24);
```

B.2 Example of LINGO Output for the Drought Model under Scenario 1

```
Global optimal solution found.
  Objective value:                                   -342.9380
  Objective bound:                                   -342.9380
  Infeasibilities:                                    0.000000
  Extended solver steps:                                 11236
  Total solver iterations:                              275459
  Elapsed runtime seconds:                               43.08

  Model Class:                                            MILP

  Total variables:                    414
  Nonlinear variables:                  0
  Integer variables:                  144

  Total constraints:                  701
  Nonlinear constraints:                0

  Total nonzeros:                    1946
  Nonlinear nonzeros:                   0


Variable           Value        Reduced Cost
  Y1D1M1        1.000000           -1.000000
  Y1D1M2        1.000000           -1.000000
  Y1D1M3        1.000000           -1.000000
  Y1D1M4        1.000000           -1.000000
  Y1D1M5        1.000000           -1.000000
  Y1D1M6        1.000000           -1.000000
  Y1D1M7        0.000000           -1.000000
  Y1D1M8        0.000000           -1.000000
  Y1D1M9        0.000000           -1.000000
 Y1D1M10        0.000000           -1.000000
 Y1D1M11        0.000000           -999.6700
 Y1D1M12        0.000000           -998.3400
 Y1D1M13        0.000000           -997.0100
 Y1D1M14        0.000000           -995.6800
 Y1D1M15        0.000000           -994.3500
```

Appendix C

Example of LINGO Input and Output

Model 6: The Flood Model

An example of both the input syntax and output generated when the experiment of the flood model (Model 6) under the conditions of Trial 1 was performed using the optimisation modelling software LINGO (LINDO Systems Inc, 2013) can be found in this appendix. In order to minimise the size of this thesis, only a subset of the input and output has been provided in the following sections. A full example of the input syntax and output generated for each of the flood model trials considered in Chapter 4 and the model comparison experiment performed in Chapter 6, can be found in the *Example of LINGO Input and Output* directory accompanying this thesis, in the folder *Model 6 - The Flood Model*. An example of the input for Model 5 is also included in the accompanying directory, following the path *Initial Model Examples\Example of Model 5 Input*. Note that for ease of opening and reading the accompanying input and output files, text copies (extension .txt) of the original LINGO files have been included.

In order to assist with the interpretation of the LINGO input and output, each of the variables that compose Model 6 have been translated into the form in which they appear in the input syntax and listed below. In this case, binary variables are denoted in the input syntax by being enclosed within `@BIN()`.

t = `1, 2,..., 24`

J = `J`

V^*_{1t} = `V1D1M1, V1D1M2,..., V1D1M24`

V_{1t} = `V1D2M1, V1D2M2,..., V1D2M24`

V^*_{2t} = `V2D1M1, V2D1M2,..., V2D1M24`

V_{2t} = `V2D2M1, V2D2M2,..., V2D2M24`

V^*_{3t} = `V3D1M1, V3D1M2,..., V3D1M24`

V_{3t} = `V3D2M1, V3D2M2,..., V3D2M24`

y^*_{1t} = `y1D1M1, y1D1M2,..., y1D1M24`

y_{1t} = `y1D2M1, y1D2M2,..., y1D2M24`

$$
\begin{aligned}
y_{2t}^{*} &= \texttt{y2D1M1, y2D1M2,..., y2D1M24} \\
y_{2t} &= \texttt{y2D2M1, y2D2M2,..., y2D2M24} \\
\omega &= \texttt{0.01} \\
S_t^{*} &= \texttt{D1S1, D1S2,..., D1S24} \\
S_t &= \texttt{D2S1, D2S2,..., D2S24} \\
S_0^{*} &= \texttt{D1S0} \\
S_0 &= \texttt{D2S0} \\
I_t^{*} &= \texttt{D1I1, D1I2,..., D1I24} \\
I_t &= \texttt{D2I1, D2I2,..., D2I24} \\
\hat{I}_t^{*} &= \texttt{D1Ihat1, D1Ihat2,..., D1Ihat24} \\
\hat{I}_t &= \texttt{D2Ihat1, D2Ihat2,..., D2Ihat24} \\
\epsilon &= \texttt{0.1} \\
M &= \texttt{100000} \\
R_t^{*} &= \texttt{D1R1, D1R2,..., D1R24} \\
R_t &= \texttt{D2R1, D2R2,..., D2R24} \\
\alpha_1 &= \texttt{0.6} \\
\alpha_2 &= \texttt{0.4} \\
D &= \texttt{400} \\
L^{*} &= \texttt{0.3333} \\
L &= \texttt{0.6666} \\
W_t^{*} &= \texttt{D1W1, D1W2,..., D1W24} \\
W_t &= \texttt{D2W1, D2W2,..., D2W24} \\
C^{*} &= \texttt{30000} \\
C &= \texttt{82000} \\
U_t^{*} &= \texttt{D1U1, D1U2,..., D1U24} \\
U_t &= \texttt{D2U1, D2U2,..., D2U24} \\
\beta_1 &= \texttt{0.05} \\
\beta_2 &= \texttt{0.05} \\
K &= \texttt{1000} \\
\gamma &= \texttt{100} \\
P_t &= \texttt{D2P1, D2P2,..., D2P24} \\
B_t^{*} &= \texttt{D1B1, D1B2,..., D1B24} \\
B_t &= \texttt{D2B1, D2B2,..., D2B24} \\
Q_t^{*} &= \texttt{D1Q1, D1Q2,..., D1Q24} \\
z_t &= \texttt{D2Z1, D2Z2,..., D2Z24}
\end{aligned}
$$

C.1 Example of LINGO Input for the Flood Model under Trial 1

```
MIN = J
      - 0.01*(V1D1M1+V1D1M2+V1D1M3+V1D1M4+V1D1M5+V1D1M6+V1D1M7+V1D1M8+V1D1M9
      +V1D1M10+V1D1M11+V1D1M12+V1D1M13+V1D1M14+V1D1M15+V1D1M16+V1D1M17
      +V1D1M18+V1D1M19+V1D1M20+V1D1M21+V1D1M22+V1D1M23+V1D1M24+V2D1M1
      +V2D1M2+V2D1M3+V2D1M4+V2D1M5+V2D1M6+V2D1M7+V2D1M8+V2D1M9+V2D1M10
      +V2D1M11+V2D1M12+V2D1M13+V2D1M14+V2D1M15+V2D1M16+V2D1M17+V2D1M18
      +V2D1M19+V2D1M20+V2D1M21+V2D1M22+V2D1M23+V2D1M24+V3D1M1+V3D1M2
      +V3D1M3+V3D1M4+V3D1M5+V3D1M6+V3D1M7+V3D1M8+V3D1M9+V3D1M10+V3D1M11
      +V3D1M12+V3D1M13+V3D1M14+V3D1M15+V3D1M16+V3D1M17+V3D1M18+V3D1M19
      +V3D1M20+V3D1M21+V3D1M22+V3D1M23+V3D1M24)
      - 0.01*(V1D2M1+V1D2M2+V1D2M3+V1D2M4+V1D2M5+V1D2M6+V1D2M7+V1D2M8+V1D2M9
      +V1D2M10+V1D2M11+V1D2M12+V1D2M13+V1D2M14+V1D2M15+V1D2M16+V1D2M17
      +V1D2M18+V1D2M19+V1D2M20+V1D2M21+V1D2M22+V1D2M23+V1D2M24+V2D2M1
      +V2D2M2+V2D2M3+V2D2M4+V2D2M5+V2D2M6+V2D2M7+V2D2M8+V2D2M9+V2D2M10
      +V2D2M11+V2D2M12+V2D2M13+V2D2M14+V2D2M15+V2D2M16+V2D2M17+V2D2M18
      +V2D2M19+V2D2M20+V2D2M21+V2D2M22+V2D2M23+V2D2M24+V3D2M1+V3D2M2
      +V3D2M3+V3D2M4+V3D2M5+V3D2M6+V3D2M7+V3D2M8+V3D2M9+V3D2M10+V3D2M11
      +V3D2M12+V3D2M13+V3D2M14+V3D2M15+V3D2M16+V3D2M17+V3D2M18+V3D2M19
      +V3D2M20+V3D2M21+V3D2M22+V3D2M23+V3D2M24)
      - 100*(D2P1+D2P2+D2P3+D2P4+D2P5+D2P6+D2P7+D2P8+D2P9+D2P10+D2P11+D2P12
      +D2P13+D2P14+D2P15+D2P16+D2P17+D2P18+D2P19+D2P20+D2P21+D2P22+D2P23
      +D2P24);

D1S0 <= 30000;
D2S0 <= 82000;

!Constraint to help reduce severity of releases;
D2W1 + D2W2 <= J;
D2W2 + D2W3 <= J;
...
D2W23 + D2W24 <= J;

!Constraint 1 - Dam 1;
100000*y1D1M1 -D1S0 -D1Ihat1 + V1D1M1 >= 0.1;
100000*y1D1M2 -D1S1 -D1Ihat2 + V1D1M2 >= 0.1;
...
100000*y1D1M24 -D1S23 -D1Ihat24 + V1D1M24 >= 0.1;
```

```
!Constraint 1 - Dam 2;
100000*y1D2M1 -D2S0 -D2Ihat1 + V1D2M1 >= 0.1;
100000*y1D2M2 -D2S1 -D2Ihat2 + V1D2M2 >= 0.1;
...
100000*y1D2M24 -D2S23 -D2Ihat24 + V1D2M24 >= 0.1;
!Constraint 2 - Dam 1;
100000*y1D1M1 +V1D1M1 - D1S0 - D1Ihat1 <= 100000;
100000*y1D1M2 +V1D1M2 - D1S1 - D1Ihat2 <= 100000;
...
100000*y1D1M24 +V1D1M24 - D1S23 - D1Ihat24 <= 100000;

!Constraint 2 - Dam 2;
100000*y1D2M1 +V1D2M1 - D2S0 - D2Ihat1 <= 100000;
100000*y1D2M2 +V1D2M2 - D2S1 - D2Ihat2 <= 100000;
...
100000*y1D2M24 +V1D2M24 - D2S23 - D2Ihat24 <= 100000;

!Constraint 3 - Dam 1;
100000*y2D1M1 -D1S0 -D1Ihat1 + V2D1M1 >= 0.1;
100000*y2D1M2 -D1S1 -D1Ihat2 + V2D1M2 >= 0.1;
...
100000*y2D1M24 -D1S23 -D1Ihat24 + V2D1M24 >= 0.1;

!Constraint 3 - Dam 2;
100000*y2D2M1 -D2S0 -D2Ihat1 + V2D2M1 >= 0.1;
100000*y2D2M2 -D2S1 -D2Ihat2 + V2D2M2 >= 0.1;
...
100000*y2D2M24 -D2S23 -D2Ihat24 + V2D2M24 >= 0.1;

!Constraint 4 - Dam 1;
100000*y2D1M1 +V2D1M1 - D1S0 - D1Ihat1 <= 100000;
100000*y2D1M2 +V2D1M2 - D1S1 - D1Ihat2 <= 100000;
...
100000*y2D1M24 +V2D1M24 - D1S23 - D1Ihat24 <= 100000;
```

```
!Constraint 4 - Dam 2;
100000*y2D2M1 +V2D2M1 - D2S0 - D2Ihat1 <= 100000;
100000*y2D2M2 +V2D2M2 - D2S1 - D2Ihat2 <= 100000;
...
100000*y2D2M24 +V2D2M24 - D2S23 - D2Ihat24 <= 100000;

!Constraint 5 - Dam 1;
D1R1 = 133;
D1R2 = 133;
...
D1R24 = 133;

!Constraint 5 - Dam 2;
D2R1 = 267;
D2R2 = 267;
...
D2R24 = 267;

!Constraint 6 - Dam 1;
D1S1 - D1S0 - D1I1 + D1R1 + D1W1 - D2P1 = 0;
D1S2 - D1S1 - D1I2 + D1R2 + D1W2 - D2P2 = 0;
...
D1S24 - D1S23 - D1I24 + D1R24 + D1W24 - D2P24 = 0;

!Constraint 6 - Dam 2;
D2S1 - D2S0 - D2I1 + D2R1 + D2W1 + D2P1 - D1W1 = 0;
D2S2 - D2S1 - D2I2 + D2R2 + D2W2 + D2P2 - D1W2 = 0;
...
D2S24 - D2S23 - D2I24 + D2R24 + D2W24 + D2P24 - D1W24 = 0;

!Constraint 7 - Dam 1;
D1S1 <= 30000;
D1S2 <= 30000;
...
D1S24 <= 30000;
```

```
!Constraint 7 - Dam 2;
D2S1 <= 82000;
D2S2 <= 82000;
...
D2S24 <= 82000;

!Constraint 8 - Dam 1;
!D1S0 - D1S24 <= 0;

!Constraint 8 - Dam 2;
!D2S0 - D2S24 <= 0;

!Constraint 9 - Dam 1;
30000*D1U1 - D1S1 <= 0;
30000*D1U2 - D1S2 <= 0;
...
30000*D1U24 - D1S24 <= 0;

!Constraint 9 - Dam 2;
82000*D2U1 - D2S1 <= 0;
82000*D2U2 - D2S2 <= 0;
...
82000*D2U24 - D2S24 <= 0;

!Constraint 10 - Dam 1;
D1W1 - 100000*D1U1 <= 0;
D1W2 - 100000*D1U2 <= 0;
...
D1W24 - 100000*D1U24 <= 0;

!Constraint 10 - Dam 2;
D2W1 - 100000*D2U1 <= 0;
D2W2 - 100000*D2U2 <= 0;
...
D2W24 - 100000*D2U24 <= 0;
```

```
!Constraint 11 - Dam 1;
V1D1M1 - 0.95*V2D1M1 <= 0;
V1D1M2 - 0.95*V2D1M2 <= 0;
...
V1D1M24 - 0.95*V2D1M24 <= 0;

!Constraint 11 - Dam 2;
V1D2M1 - 0.95*V2D2M1 <= 0;
V1D2M2 - 0.95*V2D2M2 <= 0;
...
V1D2M24 - 0.95*V2D2M24 <= 0;

!Constraint 12 - Dam 1;
V2D1M1 - 0.95*V3D1M1 <= 0;
V2D1M2 - 0.95*V3D1M2 <= 0;
...
V2D1M24 - 0.95*V3D1M24 <= 0;

!Constraint 12 - Dam 2;
V2D2M1 - 0.95*V3D2M1 <= 0;
V2D2M2 - 0.95*V3D2M2 <= 0;
...
V2D2M24 - 0.95*V3D2M24 <= 0;

!Constraint 13 - Dam 1;
V3D1M1 <= 30000;
V3D1M2 <= 30000;
...
V3D1M24 <= 30000;

!Constraint 13 - Dam 2;
V3D2M1 <= 82000;
V3D2M2 <= 82000;
...
V3D2M24 <= 82000;
```

```
!Constraint 14 - Dam 1;
D1B1 - 400*y2D1M1 -200*y1D1M1 = 400;
D1B2 - 400*y2D1M2 -200*y1D1M2 = 400;
...
D1B24 - 400*y1D1M24 -200*y2D1M24 = 400;

!Constraint 14 - Dam 2;
D2B1 - 400*y2D2M1 -200*y1D2M1 = 400;
D2B2 - 400*y2D2M2 -200*y1D2M2 = 400;
...
D2B24 - 400*y1D2M24 -200*y2D2M24 = 400;

!Constraint 15 - Dam 1;
y1D1M1 + y1D1M3 - y1D1M2 <= 1;
y1D1M2 + y1D1M4 - y1D1M3 <= 1;
...
y1D1M22 + y1D1M24 - y1D1M23 <= 1;

!Constraint 15 - Dam 2;
y1D2M1 + y1D2M3 - y1D2M2 <= 1;
y1D2M2 + y1D2M4 - y1D2M3 <= 1;
...
y1D2M22 + y1D2M24 - y1D2M23 <= 1;

!Constraint 16 - Dam 1;
y1D1M1 - y2D1M2 >= 0;
y1D1M2 - y2D1M3 >= 0;
...
y1D1M23 - y2D1M24 >= 0;

!Constraint 16 - Dam 2;
y1D2M1 - y2D2M2 >= 0;
y1D2M2 - y2D2M3 >= 0;
...
y1D2M23 - y2D2M24 >= 0;
```

```
!Constraint 17 - Dam 1;
D1U1 + D1U2 + D1U3 + D1U4 + D1U5 + D1U6 + D1U7 + D1U8 + D1U9 + D1U10
+ D1U11 + D1U12 + D1U13 + D1U14 + D1U15 + D1U16 + D1U17 + D1U18
+ D1U19 + D1U20 + D1U21 + D1U22 + D1U23 + D1U24 >= 0;

!Constraint 17 - Dam 2;
D2U1 + D2U2 + D2U3 + D2U4 + D2U5 + D2U6 + D2U7 + D2U8 + D2U9 + D2U10
+ D2U11 + D2U12 + D2U13 + D2U14 + D2U15 + D2U16 + D2U17 + D2U18
+ D2U19 + D2U20 + D2U21 + D2U22 + D2U23 + D2U24 >= 0;

!Constraint 18 - Dam 1;
D1I1= 347.76;
D1I2= 106.5;
...
D1I24= 198.9;
!Constraint 18 - Dam 2;
D2I1= 579.6;
D2I2= 177.5;
...
D2I24= 331.5;

!Constraint 19 - Dam 1;
D1Ihat1 - D1I1 = 0;
D1Ihat2 - D1I2 = 0;
...
D1Ihat24 - D1I24 = 0;

!Constraint 19 - Dam 2;
D2Ihat1 - D2I1 = 0;
D2Ihat2 - D2I2 = 0;
...
D2Ihat24 - D2I24 = 0;

!Constraint 20 - Dam 1;
D1Q1 + D1S1 = 30000;
D1Q2 + D1S2 = 30000;
...
D1Q24 + D1S24 = 30000;
```

```
!Constraint 21;
D1Q1 - 400*D2Z1 >= 0;
D1Q2 - 400*D2Z2 >= 0;
...
D1Q24 - 400*D2Z24 >= 0;

!Constraint 22;
D2P1 - D2B1 <= 0;
D2P2 - D2B2 <= 0;
...
D2P24 - D2B24 <= 0;

!Constraint 23;
D2P1 - 1000*D2Z1 <= 0;
D2P2 - 1000*D2Z2 <= 0;
...
D2P24 - 1000*D2Z24 <= 0;

!Constraint 24 - Dam 1;
@BIN(y1D1M1);
@BIN(y1D1M2);
...
@BIN(y1D1M24);

!Constraint 24 - Dam 2;
@BIN(y1D2M1);
@BIN(y1D2M2);
...
@BIN(y1D2M24);

!Constraint 25 - Dam 1;
@BIN(y2D1M1);
@BIN(y2D1M2);
...
@BIN(y2D1M24);
```

```
!Constraint 25 - Dam 2;
@BIN(y2D2M1);
@BIN(y2D2M2);
...
@BIN(y2D2M24);

!Constraint 26 - Dam 1;
@BIN(D1U1);
@BIN(D1U2);
...
@BIN(D1U24);

!Constraint 26 - Dam 2;
@BIN(D2U1);
@BIN(D2U2);
...
@BIN(D2U24);

!Constraint 27;
@BIN(D2Z1);
@BIN(D2Z2);
...
@BIN(D2Z24);
```

C.2 Example of LINGO Output for the Flood Model under Trial 1

```
Global optimal solution found.
  Objective value:                              -312880.0
  Objective bound:                              -312880.0
  Infeasibilities:                               0.000000
  Extended solver steps:                                2
  Total solver iterations:                           1065
  Elapsed runtime seconds:                           0.34

  Model Class:                                       MILP

  Total variables:                 507
  Nonlinear variables:               0
  Integer variables:               168

  Total constraints:               790
  Nonlinear constraints:             0

  Total nonzeros:                 2120
  Nonlinear nonzeros:                0



Variable          Value        Reduced Cost
       J       63604.80            0.000000
  V1D1M1       27075.00            0.000000
  V1D1M2       27075.00            0.000000
  V1D1M3       8035.520            0.000000
  V1D1M4       27075.00            0.000000
  V1D1M5       27075.00            0.000000
  V1D1M6       27075.00            0.000000
  V1D1M7       27075.00            0.000000
  V1D1M8       27075.00            0.000000
  V1D1M9       27075.00            0.000000
 V1D1M10       27075.00            0.000000
 V1D1M11       27075.00            0.000000
 V1D1M12       27075.00            0.000000
 V1D1M13       27075.00            0.000000
 V1D1M14       27075.00            0.000000
```

Appendix D

Tables of Inflow Records

The inflow records utilised when conducting experiments throughout this thesis, including both the inflows sourced from the Queensland Department of Natural Resources and Mines (2012) and those simulated using a time series model, have been tabulated and presented in the following sections, beginning on the next page.

D.1 Inflows under Worst Case Drought Scenario

Upon screening the historic inflow records sourced from the Queensland Department of Natural Resources and Mines (2012), the two years that resulted in the lowest inflows to the reservoir system were combined to form a worst case drought scenario. This set of inflows, presented in Table D.1 below, were then employed in Chapter 3 when performing an experiment of Model 4.

Table D.1: Monthly Inflow to Cressbrook and Perseverance Dams under Worst Case Drought Scenario from Historic Records.
Sourced: Queensland Department of Natural Resources and Mines (2012).

	Inflow (ML)	
Month Number	Cressbrook Dam	Perseverance Dam
1	570.2	342.12
2	191.2	144.72
3	142.7	85.62
4	926.3	555.78
5	174.2	104.52
6	442	265.2
7	100.6	60.36
8	261.2	156.72
9	365.1	219.06
10	244.9	146.94
11	296.2	177.72
12	326.8	196.08
13	480.8	288.48
14	514.2	308.52
15	420.5	252.3
16	1420.6	852.36
17	637.3	382.38
18	0	0
19	613.7	368.22
20	0	0
21	0	0
22	0	0
23	44.5	26.7
24	2188.8	1313.28

D.2 Inflows under Worst Case Flood Scenario

Upon screening the historic inflows records sourced from the Queensland Department of Natural Resources and Mines (2012), the two years (assumed to be 12 hour periods in this case) that resulted in the greatest flows to the reservoir system were combined to form a worst case flood scenario. This set of inflows, presented in Table D.2 below, were then employed in Chapter 4 when performing an experiment of Model 6.

Table D.2: Monthly Inflow to Perseverance and Cressbrook Dams under Worst Case Flood Scenario from Historic Records.
Sourced: Queensland Department of Natural Resources and Mines (2012).

	Inflow (ML)	
Hour Number	Cressbrook Dam	Perseverance Dam
1	579.6	347.76
2	177.5	106.5
3	247.4	148.44
4	46914.1	28148.46
5	9879.2	5927.52
6	5998	3598.88
7	766.2	459.72
8	2110.3	1266.18
9	1279.4	767.64
10	781.1	468.66
11	591.7	355.02
12	353.5	212.1
13	2656	1593.6
14	1518.8	911.28
15	3008.8	1805.28
16	11050.5	6630.3
17	19551.6	11730.96
18	20701.4	12420.84
19	2466.6	1479.96
20	2633	1579.8
21	1237.2	742.32
22	914.7	548.82
23	250.3	150.18
24	331.5	198.9

D.3 Simulated Inflows from AR(1) Time Series Model

Using the AR(1) time series model selected in Chapter 5 to provide a suitable approximation of the historic inflow record sourced from the Queensland Department of Natural Resources and Mines (2012), an alternate set of inflows were simulated. This alternate set of monthly inflows is presented in Table D.3 below:

Table D.3: Simulated Monthly Inflow from AR(1) Time Series Model.

Month Number	Inflow (ML)
1	561.20
2	875.08
3	521.61
4	134.63
5	147.66
6	519.17
7	524.22
8	104.43
9	1245.45
10	9984.92
11	13932.77
12	1837.02
13	1423.43
14	1950.40
15	616.40
16	444.32
17	80.25
18	880.77
19	1785.00
20	465.28
21	426.11
22	149.86
23	547.53
24	797.62

D.4 Simulated Inflows to Reservoir 1 and Reservoir 2

In Chapter 6 a comparison is performed between the management strategies employed by the Drought Model (Model 4 from Chapter 3) and the Flood Model (Model 6 from Chapter 4), when both models are considered on a common set of inflow records. This common set of inflow records was simulated using time series analysis and is presented below in Table D.4, where the inflows to Reservoir 1 are assumed to be 60% of the size of the inflows to Reservoir 2:

Table D.4: Simulated Inflows to Reservoir 1 and Reservoir 2 from Time Series Model, where Reservoir 1 Inflows are equal to 60% of Reservoir 2 Inflows.

	Inflow (ML)	
Time Unit	Reservoir 1	Reservoir 2
1	336.72	561.20
2	525.05	875.08
3	312.96	521.61
4	80.78	134.63
5	88.60	147.66
6	311.50	519.17
7	314.53	524.22
8	62.66	104.43
9	747.27	1245.45
10	5990.95	9984.92
11	8359.66	13932.77
12	1102.21	1837.02
13	854.06	1423.43
14	1170.24	1950.40
15	369.84	616.40
16	266.59	444.32
17	48.15	80.25
18	528.46	880.77
19	1071.00	1785.00
20	279.17	465.28
21	255.67	426.11
22	89.92	149.86
23	328.52	547.53
24	478.57	797.62

Appendix E

Time Series Analysis Source Code

The time series analysis performed throughout Chapter 5 was conducted through the use of the statistical computing software environment R (R Core Team, 2012). The source code required to perform the different formatting and analysis techniques mentioned can be seen in each of the sections below:

E.1 Formatting the Historic Data

Inputting the data to R from an Microsoft Excel Spreadsheet (.csv file):

```
inflows <- read.table("Inflow_Data.csv", header=TRUE, sep=",")
```

Calculating the logarithm of the inflows:

```
inflow.log <- log(inflows$Inflows)
inflows$log <- inflow.log
```

Separating the inflow data into the training series and testing series:

```
inflows.train <- inflows[1:132,]
inflows.test <- inflows[133:156,]
```

Defining the training series and testing series as R time series objects:

```
inflow.ts.train <- ts(inflows.train$log, start=c(1966,1), end=c(1976,12),
                      frequency=12)
inflow.ts.test <- ts(inflows.test$log, start=c(1977,1), end=c(1978,12),
                     frequency=12)
```

E.2 Fitting the Time Series Model

Calculating and plotting the sample autocorrelation function (ACF) and partial autocorrelation function (PACF):

```
temp.acf<-acf(inflow.ts.train, axes=FALSE,lag.max=10, lwd=2,
              main="Sample Autocorrelation Function \n Historic Inflow Data")
axis(side=1, at=temp.acf$lag, labels=(temp.acf$lag*12))
axis(side=2)
temp.pacf<-pacf(inflow.ts.train, ylim=c(-0.2,0.45), axes=FALSE, lag.max=10,
                lwd=2,
                main="Sample Partial Autocorrelation Function \n Historic Inflow Data")
axis(side=1, at=temp.pacf$lag, labels=(temp.pacf$lag*12))
axis(side=2)
```

Fitting the AR(1) model to the historic inflow data:

```
model.1 <- arima(inflow.ts.train, order=c(1,0,0))
model.1

Call:
arima(x = inflow.ts.train, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.4228     6.5073
s.e.  0.0807     0.1986

sigma^2 estimated as 1.75:  log likelihood = -224.33,  aic = 454.66
```

Fitting the MA(1) model to the historic inflow data:

```
model.2 <- arima(inflow.ts.train, order=c(0,0,1))
model.2

Call:
arima(x = inflow.ts.train, order = c(0, 0, 1))

Coefficients:
         ma1  intercept
      0.3664     6.5187
s.e.  0.0702     0.1592

sigma^2 estimated as 1.798:  log likelihood = -226.11,  aic = 458.21
```

Fitting the ARMA(1, 1) model to the historic inflow data:

```
model.3 <- arima(inflow.ts.train, order=c(1,0,1))
model.3

Call:
arima(x = inflow.ts.train, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.3712  0.0638     6.5083
s.e.  0.1747  0.1846     0.1939

sigma^2 estimated as 1.748:  log likelihood = -224.27,  aic = 456.55
```

E.3 Diagnostic Tests of Time Series Model

Calculating and plotting the residual autocorrelation function (ACF) and residual partial autocorrelation function (PACF):

```
par(mfrow=c(1,2))
temp.racf <- acf(resid(model.1), ylim=c(-0.2,1), lag.max=10, axes=FALSE,
                 lwd=2, main="Residual ACF \n AR(1) Model")
axis(side=1, at=temp.racf$lag, labels=(temp.racf$lag*12))
axis(side=2)
temp.rpacf <- pacf(resid(model.1), ylim=c(-0.2,0.2), lag.max=10, axes=FALSE,
                   lwd=2, main="Residual PACF \n AR(1) Model")
axis(side=1, at=temp.rpacf$lag, labels=(temp.rpacf$lag*12))
axis(side=2)
```

Fitting the AR(8) model to the historic inflow data:

```
model.4 <- arima(inflow.ts.train, order=c(8,0,0))
model.4

Call:
arima(x = inflow.ts.train, order = c(8, 0, 0))

Coefficients:
         ar1      ar2      ar3     ar4      ar5     ar6      ar7      ar8
      0.4330  -0.0207  -0.0783  0.0853  -0.1459  0.1663  -0.1117  -0.1433
s.e.  0.0882   0.0965   0.0958  0.0960   0.0960  0.0970   0.1018   0.0917
      intercept
         6.5373
s.e.     0.1380

sigma^2 estimated as 1.624:  log likelihood = -219.64,  aic = 459.29
```

Performing the Box-Pierce test on the residuals of the AR(1) model:

```
Box.test(resid(model.1), lag=25)

        Box-Pierce test

data:  resid(model.1)
X-squared = 23.7008, df = 25, p-value = 0.5367
```

Function to calculate and plot the cumulative periodogram of the residuals from the AR(1) model:

```
cpgram(resid(model.1), main="Cumulative Periodogram of Residuals \n AR(1) Model")
```

Process to test the significance of the terms in the AR(1) model:

Calculating the estimates of the parameter coefficients:

```
model.1$coef

      ar1 intercept
0.4227618 6.5072901
```

Generating the variance-covariance matrix of the parameter estimates:

```
model.1$var.coef

                    ar1     intercept
ar1        0.0065139414 -0.0007134488
intercept -0.0007134488  0.0394288712
```

Calculating the t-score of the parameters:

```
coef(model.1)/sqrt(diag(model.1$var.coef))

    ar1 intercept
5.23810  32.77125
```

Function to generate the Q-Q plot of the residuals from the AR(1) model:

```
qqnorm(resid(model.1), main="Normal Q-Q Plot of Residuals \n AR(1) Model")
qqline(resid(model.1))
```

Function to plot a histogram of the residuals from the AR(1) model:

```
hist(resid(model.1), main="Histogram of Residuals \n AR(1) Model",
     xlab="Residuals of AR(1) Model")
```

E.4 Validation of Time Series Model

Iterative procedure used to perform the one-step ahead forecasts from the AR(1) model, for the testing series:

```
predict.mat <- matrix(ncol=1, nrow=length(inflows.test$log))
model.coef <- coef(model.1)
model.pred <- predict(model.1, n.ahead=1)
predict.mat[1] <- model.pred$pred

for(i in 2:length(inflows.test$log)){
predict.mat[i] <- model.coef[2] +
                  model.coef[1]*(inflows.test$log[i-1] - model.coef[2])
}
```

Calculating the mean and variance of the differences between the forecasts from the AR(1) model and the historic data:

```
mean.pred <- mean(inflows.test$log-predict.mat)
mean.pred

 -0.7331572

var.pred <- var(inflows.test$log-predict.mat)
var.pred

5.077488
```

E.5 Simulation of Inflows

Simulating an alternate series of inflows from the AR(1) model fitted to the historic records, then converting into physical units (megalitres):

```
sim.data <- arima.sim(model=list(ar=model.coef[1]), n=24)
sim.data <- sim.data+model.coef[2]
sim.data <- exp(sim.data)

Time Series:
Start = 1
End = 24
Frequency = 1
 [1]   561.20064   875.07938   521.60832   134.62786   147.66130   519.16900
 [7]   524.22474   104.43264  1245.44917  9984.91848 13932.76839  1837.02321
[13]  1423.42558  1950.39564   616.39802   444.31853    80.25243   880.77173
[19]  1785.00120   465.27555   426.11480   149.86000   547.53137   797.62418
```

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