Analysis of several objective functions for optimization of hexahedral meshes

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ABSTRACT

For many simulation processes, hexahedral meshes present several numerical advantages over tetrahedral ones. For example, in Finite Element Method simulations, hexahedral meshes are best suited to solve elastic, structural or fluid mechanics problems.

The quality of the mesh has high repercussion on the numerical behaviour of FEM. In some cases, mesh generators construct poor quality meshes or, even worse, with inverted elements. Indeed, a good quality mesh could degenerate into a mesh with inverted or not valid elements when the simulation requires node movement. For this reason, it is of the major importance to apply a mesh optimization algorithm to untangle the inverted elements and increase the quality of the mesh.

A widely used technique for optimizing a mesh consists on an iterative process in which each node is moved to a new position in order to improve the quality of the local mesh, that is, the set of elements connected to the free node. This new position of the node is determined by minimizing certain objective function based on the distortion of the elements of the local mesh.

In 2D, the optimization of a quadrilateral mesh is carried out by decomposing each element in four triangles. The nondegeneracy of these triangles is a necessary and sufficient condition for the validity of a quadrilateral element. Nevertheless, it is not known a set of necessary and sufficient conditions based on tetrahedra that guarantees the validity of a hexahedron. For example, the conditions proposed in [1, 2] are sufficient but not necessary conditions.

In this work we analyze the ability of several objective functions to untangle and smooth hexahedral meshes. The construction of the objective functions is addressed from different perspectives. In one hand, we form the objective function through the decomposition of the hexahedral elements into tetrahedra [1, 3, 4]. On the other hand, the objective function is built up from shape distortion measures derived from the tri-linear mapping between the reference and physical hexahedron [5].

Although, in general, the analyzed objective functions are able to optimize a not extremely distorted mesh, some of them do not work properly in complicated situations. We have designed two experiments to test the capabilities of the objective functions to achieve valid meshes.

References

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