

# Neptuno ++: An Adaptive Finite Element Toolbox for Numerical Simulation of Environmental Problems

J. M. Cascón\*, L. Ferragut\*, M. I. Asensio\*, D. Prieto\*, D. Álvarez\*

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Neptuno++ is a finite element toolbox mainly developed by L. Ferragut at SINUMCC (Group of Numerical Simulation and Scientific Computation) and implemented in C++. It uses 4T-Rivara's refinement and error control and includes data structures to solve stationary and time dependent problems based on PDEs. Tools for assembling and solving are also provided.

In this talk, we show some of the main features of Neptuno++, through several examples. In particular, we used it to corroborates the theory of convergence and optimality of AFEM (Adaptive Finite Element Method) [2] for second order linear symmetric elliptic operators.

The Neptuno++ code is used to implement two environmental models developed by SINUMCC, namely the Physical Forest Fire Spread (PhyFire, [1, 4]) and the High Definition Wind Model (HDWind, [3, 4]). The PhyFire is a simplified 2D physical wildland fire spread model based on conservation equations, with convection and radiation as heat transfer mechanisms. It also includes some 3D effects. The HDWM arises from an asymptotic approximation of the Navier-Stokes equations, and provides a 3D wind velocity field in an air layer above the terrain surface. Finally, we show some numerical simulations that involve these former models.

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## References

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\*Fundamental Physics and Mathematics University Institute, Univeristy of Salamanca (SPAIN). Email: [casbar@usal.es](mailto:casbar@usal.es), [ferragut@utar.es](mailto:ferragut@utar.es)