Expanded hesitant fuzzy sets and group decision making: slides for FUZZ-IEEE 2017

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Expanded hesitant fuzzy sets and group decision making: slides for FUZZ-IEEE 2017

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Outline

Concept of hesitant fuzzy set

Let us fix X, a non-empty set of alternatives.

Hesitant fuzzy element

A hesitant fuzzy element (also HFE) is a subset of $[0,1]$. The set of HFEs is denoted by $\mathcal{P}([0,1])$.
A typical HFE (also THFE) is a non-empty, finite subset of $[0,1]$. The set of THFEs is denoted by $\mathcal{F}^*([0,1])$.

We can represent any typical HFEs as $h = \{h^1, ..., h^{l_h}\}$, with $h^1 < \ldots < h^{l_h}$ ($l_h = |h|$ is the cardinality of h).

The definition of a hesitant fuzzy set is as follows:

Hesitant fuzzy set (Torra, 2010)

A *hesitant fuzzy set* (HFS) on X is $h_M : X \longrightarrow \mathcal{P}([0,1])$.
Particularly, a *typical HFS* on X is $h_M : X \longrightarrow \mathcal{F}^*([0,1])$.
and **HFS**(X) means the set of HFSs on X.

We also represent h_M as $M = \{(x, h_M(x)) \mid x \in X\}$.

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Criticisms to hesitant fuzzy sets

Two agents deliberate on the membership degree of x into A. One argues it should be 0.5 and the other believes it should be 0.6. Their joint position can be captured by $h = \{0.5, 0.6\}$.

1. The same THFE h is the model when their opinions are exchanged.
This modelization does not preserve the information about who expressed which opinion.

2. If another agent joins the discussion and he supports the 0.5 degree, then h is still the model that captures their joint information.
New information does not necessarily update the HFE.

Since the consultants frequently have different importances in decision making, the loss of information that derives from the issues above may lead to ineffective results.

HFEs are suitable to account for information provided by anonymous agents, whose weights are necessarily the same.

Extended hesitant fuzzy sets I

Zhu and Xu (2016) introduced Extended hesitant fuzzy sets (EHFSs), where the values provided by the agents are collected by value-groups.

Agents *must* submit their beliefs on the membership degrees, which in fact is not imposed in the definition of a HFS.

Can we truely say that EHFSs are more general than HFSs?

EHFSs are defined as the Cartesian product of HFSs. Their definition implies that HFSs can be used to produce EHFSs, and conversely, EHFSs can in some cases reduce to HFSs.

The constituents of EHFSs are Extended HFEs (EHFEs).

Furthermore, Zhu and Xu (also Lu and Liang, 2017) developed some basic operations and correlation coefficients of EHFEs.

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Extended hesitant fuzzy sets II

The following example illustrates these concepts:

Example

Two agents give their opinion on the options x and y.

(a) With respect to x, the membership provided by the first agent is $\{0.5\}$, and the second one provides $\{0.5, 0.6\}$. EHFE that captures this information: $\{(0.5, 0.5), (0.5, 0.6)\}$.

(b) With respect to y, the first agent provides the memberships $\{0.2, 0.3\}$, and the second one provides $\{0.4\}$. EHFE that captures this information: $\{(0.2, 0.4), (0.3, 0.4)\}$.

We can gather all this information into the extended HFS

$$\{\, \langle\, x, \{(0.5, 0.5), (0.5, 0.6)\}\,\rangle\,, \langle\, y, \{(0.2, 0.4), (0.3, 0.4)\}\,\rangle\,\}$$

Outline

Expanded hesitant fuzzy sets

$\mathcal{C}_m = \{1, \ldots, m\}$ are m consultants or decision makers.

Expanded hesitant fuzzy set (XHFS)

It is $h_M^m : X \longrightarrow \mathcal{P}^*([0,1] \times \mathcal{P}(\mathcal{C}_m))$ with the property that for each $x \in X$ and $(a, C), (a', C') \in h_M^m(x)$, if $a = a'$ then $C = C'$.

Our requirement on h_M^m means: if $x \in X$ and $a \in [0,1]$, there is at most one pair (a, C) in $h_M^m(x) \subseteq [0,1] \times \mathcal{P}(\mathcal{C}_m)$.

When $(a, C) \in h_M^m(x)$ we interpret that the consultants or decision makers that believe that a is a possible degree of membership for x is C.

It may be the case that $(a, \varnothing) \in h_M^m(x)$.
This case is reserved for anonymous votes.
It permits to include the HFS model into ours as a limit case.

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Typical expanded hesitant fuzzy sets

For applications we will be bound by the following constraint:

Typical expanded hesitant fuzzy set (TXHFS)

It is a mapping $h^m_M : X \longrightarrow \mathcal{F}^*([0,1] \times \mathcal{P}^*(\mathcal{C}_m))$ such that for each $x \in X$ and $(a, C), (a', C') \in h^m_M(x)$, if $a = a'$ then $C = C'$.

Observe the two differences
(1) all sets $h^m_M(x)$ are finite,
(2) the collections C with $(a, C) \in h^m_M(x)$ are non-empty.

Obviously, the constituents of XHFSs and TXHFSs are XHFEs and TXHFEs, respectively.

i) Generic XHFSs on X:
$h^m_M = \{\langle x, h^m_M(x) \rangle \mid x \in X\}$ where $h^m_M(x)$ are XHFEs.
ii) Typical XHFEs are especially useful for decision making. They may be expressed as $h_m = \{(h^1_m, C_1), \ldots, (h^l_m, C_l)\}$, with $h^1_m < \ldots < h^l_m$ and $\varnothing \neq C_i \subseteq \mathcal{C}_m$ for each i.

An example

Example. The following expression defines a TXHFS on $X = \{x, y\}$. The superindex $m = 3$ indicates that three agents submit their assessments:

$$h^3_M = \{ \langle x, \{(0.5, \{1, 3\}), (0.7, \{2, 3\})\} \rangle, \langle y, \{(0.2, \{1\}), (0.3, \{1, 3\}), (0.4, \{1, 2\})\} \rangle \}$$

For option x, the membership degree 0.5 is endorsed by agents 1 and 3, whereas the membership degree 0.7 is endorsed by agents 2 and 3.

For option y, the membership degree 0.2 is endorsed by agent 1, the membership degree 0.3 is endorsed by agents 1 and 3, and the membership degree 0.4 is endorsed by agents 1 and 2.

An example

Example. The following expression defines a TXHFS on $X = \{x, y\}$. The superindex $m = 3$ indicates that three agents submit their assessments:

$$h_M^3 = \{ \langle x, \{(0.5, \{1, 3\}), (0.7, \{2, 3\})\} \rangle, \langle y, \{(0.2, \{1\}), (0.3, \{1, 3\}), (0.4, \{1, 2\})\} \rangle \}$$

For option x, the membership degree 0.5 is endorsed by agents 1 and 3, whereas the membership degree 0.7 is endorsed by agents 2 and 3.

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For option x, the membership degree 0.5 is endorsed by agents 1 and 3, whereas the membership degree 0.7 is endorsed by agents 2 and 3.

For option y, the membership degree 0.2 is endorsed by agent 1, the membership degree 0.3 is endorsed by agents 1 and 3, and the membership degree 0.4 is endorsed by agents 1 and 2.

HFSs are XHFSs

HFSs can be embedded into XHFSs:

Proposition. Every HFE can be identified with an XHFE. Every HFS on X can be identified with an XHFS on X.

Example. Let $X = \{x_1, x_2\}$. A typical HFS is defined on X by

$$\begin{array}{rccl} h_M : & X & \longrightarrow & \mathcal{F}^*([0,1]) \\ & x & \longmapsto & h_M(x) = \{0.55, 0.7\} \\ & y & \longmapsto & h_M(y) = \{0.75, 0.8, 0.85\} \end{array}$$

The typical HFEs involved are identified with the XHFEs $h_M(x)_\varnothing = \{(0.55, \varnothing), (0.7, \varnothing)\}$ and $h_M(y)_\varnothing = \{(0.75, \varnothing), (0.8, \varnothing), (0.85, \varnothing)\}$.

Fix m. We identify h_M with the XHFS h_M^m such that $h_M^m(x) = h_M(x)_\varnothing$, $h_M^m(y) = h_M(y)_\varnothing$.

This XHFS can be described as $\{ \langle x, \{(0.55, \varnothing), (0.7, \varnothing)\} \rangle, \langle y, \{(0.75, \varnothing), (0.8, \varnothing), (0.85, \varnothing)\} \rangle \}$.

EHFSs are XHFSs

It is also the case that our model contains Zhu and Xu's model by extended hesitant fuzzy sets:

Proposition. Every EHFE can be identified with an XHFE. Every EHFS on X can be identified with an XHFS.

Example. We return to the extended HFS

$$\{\langle x, \{(0.5, 0.5), (0.5, 0.6)\}\rangle, \langle y, \{(0.2, 0.4), (0.3, 0.4)\}\rangle\}$$

This EHFS can be identified with the following TXHFS ($m = 2$ indicates that two agents submit their assessments):

$$h^2_M = \{\langle x, \{(0.5, \{1, 2\}), (0.6, \{1\})\}\rangle, \langle y, \{(0.2, \{1\}), (0.3, \{1\}), (0.4, \{2\})\}\rangle\}$$

Outline

Additional research I

We define scores on TXHFEs that generalize well-known scores on THFEs. Let $h_m = \{(h_m^1, C_1), \ldots, (h_m^l, C_l)\}$ with $h_m^1 < \ldots < h_m^l$, and $C_i \subseteq \mathcal{C}_m$ for each i.

We measure the relative importances of the DMs by $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_m)$, $\omega_i \in [0, 1]$ for each i, $\omega_1 + \ldots + \omega_m = 1$.

The ω-*weighted score* of h_m is defined by

$$\mathcal{S}_{\boldsymbol{\omega}}(h_m) = \frac{h_m^1 \omega^1(h_m)\lambda_1(h_m) + \ldots + h_m^l \omega^l(h_m)\lambda_l(h_m)}{\omega^1(h_m)\lambda_1(h_m) + \ldots + \omega^l(h_m)\lambda_l(h_m)}$$

Let $\boldsymbol{\delta} = \{\delta(1), \ldots, \delta(n), \ldots\}$ be a monotone non-decreasing sequence of positive numbers.
The (ω, δ)-*sequence score* of h_m is defined by

$$\mathcal{S}_{\boldsymbol{\omega},\boldsymbol{\delta}}(h_m) = \frac{h_m^1 \omega^1(h_m)\delta(1) + \ldots + h_m^l \omega^l(h_m)\delta(l)}{\omega^1(h_m)\delta(1) + \ldots + \omega^l(h_m)\delta(l)}$$

Additional research II

We use scores on TXHFEs in order to propose a group decision making procedure.

It is inspired by a methodology to rank HFSs in J. C. R. Alcantud, R. de Andrés, and M. J. M. Torrecillas, “Hesitant fuzzy worth: An innovative ranking methodology for hesitant fuzzy subsets,” Applied Soft Computing (2016).

Thanks for your attention!

References I

[1] J. C. R. Alcantud.
A novel algorithm for fuzzy soft set based decision making from multiobserver input parameter data set.
Information Fusion, 29:142 – 148, 2016.

[2] J. C. R. Alcantud.
Some formal relationships among soft sets, fuzzy sets, and their extensions.
International Journal of Approximate Reasoning, 68:45 – 53, 2016.

[3] J. C. R. Alcantud, R. de Andrés, and M. J. M. Torrecillas.
Hesitant fuzzy worth: An innovative ranking methodology for hesitant fuzzy subsets.
Applied Soft Computing, 38:232–243, 2016.

References II

[4] J. C. R. Alcantud and R. de Andrés Calle.
A segment-based approach to the analysis of project evaluation problems by hesitant fuzzy sets.
International Journal of Computational Intelligence Systems, 29:325–339, 2016.

[5] J. C. R. Alcantud and V. Torra.
Decomposition theorems and extension principles for hesitant fuzzy sets.
Information Fusion, 41(Supplement C):48 – 56, 2018.

[6] B. Bedregal, R. Reiser, H. Bustince, C. López-Molina, and V. Torra.
Aggregation functions for typical hesitant fuzzy elements and the action of automorphisms.
Information Sciences, 255:82 – 99, 2014.

References III

[7] N. Chen, Z. Xu, and M. Xia.
Interval-valued hesitant preference relations and their applications to group decision making.
Knowledge-Based Systems, 37:528–540, 2013.

[8] Y. Dong, C.-C. Li, and F. Herrera.
Connecting the linguistic hierarchy and the numerical scale for the 2-tuple linguistic model and its use to deal with hesitant unbalanced linguistic information.
Information Sciences, 367-368:259 – 278, 2016.

[9] B. Farhadinia.
A novel method of ranking hesitant fuzzy values for multiple attribute decision-making problems.
International Journal of Intelligent Systems, 28:752–767, 2013.

References IV

[10] B. Farhadinia.
A series of score functions for hesitant fuzzy sets.
International Journal of Intelligent Systems, 28:752–767, 2013.

[11] G. Qian, H. Wang, and X. Feng.
Generalized hesitant fuzzy sets and their application in decision support system.
Knowledge-Based Systems, 37:357–365, 2013.

[12] R. Rodríguez, B. Bedregal, H. Bustince, Y. Dong, B. Farhadinia, C. Kahraman, L. Martínez, V. Torra, Y. Xu, Z. Xu, and F. Herrera.
A position and perspective analysis of hesitant fuzzy sets on information fusion in decision making. Towards high quality progress.
Information Fusion, 29:89 – 97, 2016.

References V

[13] R. Rodríguez, L. Martínez, V. Torra, Z. Xu, and F. Herrera. Hesitant fuzzy sets: state of the art and future directions. *International Journal of Intelligent Systems*, 29:495–524, 2014.

[14] R. M. Rodríguez, A. Labella, and L. Martínez. An overview on fuzzy modelling of complex linguistic preferences in decision making. *International Journal of Computational Intelligence Systems*, 9(sup1):81–94, 2016.

[15] R. M. Rodríguez, L. Martínez, and F. Herrera. Hesitant fuzzy linguistic term sets for decision making. *IEEE Transactions on Fuzzy Systems*, 20(1):109–119, 2012.

References VI

[16] R. M. Rodríguez, L. Martínez, F. Herrera, and V. Torra.
A review of hesitant fuzzy sets: Quantitative and qualitative extensions.
In C. Kahraman, U. Kaymak, and A. Yazici, editors, *Fuzzy Logic in Its 50th Year - New Developments, Directions and Challenges*, volume 341 of *Studies in Fuzziness and Soft Computing*, pages 109–128. Springer, 2016.

[17] V. Torra.
Hesitant fuzzy sets.
International Journal of Intelligent Systems, 25(6):529–539, 2010.

References VII

[18] H. Wang.
Extended hesitant fuzzy linguistic term sets and their aggregation in group decision making.
International Journal of Computational Intelligence Systems, 8(1):14–33, 2015.

[19] M. Xia and Z. Xu.
Hesitant fuzzy information aggregation in decision making.
International Journal of Approximate Reasoning, 52:395–407, 2011.

[20] B. Zhu and Z. Xu.
Extended hesitant fuzzy sets.
Technological and Economic Development of Economy, 22(1):100–121, 2016.

References VIII

[21] B. Zhu, Z. Xu, and M. Xia.
Dual hesitant fuzzy sets.
Journal of Applied Mathematics, 2012:13, Article ID 879629, 2012.