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REPLANNING MECHANISM FOR DELIBERATIVE AGENTS IN DYNAMIC CHANGING ENVIRONMENTS

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This paper proposes a replanning mechanism for deliberative agents as a new approach to tackling the frame problem. We propose a beliefs desires and intentions (BDI) agent architecture using a case-based planning (CBP) mechanism for reasoning. We discuss the characteristics of the problems faced with planning where constraint satisfaction problems (CSP) resources are limited and formulate, through variation techniques, a reasoning model agent to resolve them. The design of the agent proposed, named MRP-Ag (most-replanable agent), will be evaluated in different environments using a series of simulation experiments, comparing it with others such as E-Ag (Efficient Agent) and O-Ag (Optimum Agent). Last, the most important results will be summarized, and the notion of an adaptable agent will be introduced.

Key words: case-based reasoning, deliberative agents, execution time.

1. INTRODUCTION

Agents are intelligent, autonomous, adaptive systems, which live in changeable surroundings (Wooldrige and Jennings 1995). Currently, we don't have a common successful methodology for the design of agents. Strategies that are purely deliberative (Laird, Newell, and Rosenbloom 1987; Georgeff and Ingrand 1989), reactive (Agre and Chapman 1987; Brooks 1995), and hybrid (Firby 1987; Ferguson 1992), only seem to work in very specific examples. Deliberative agents use symbolic representation and abstract reasoning to generate plans with which to achieve their objectives. Reactive agents do not have internal symbolic representations of their environment, and they act by responding to the stimulus of the environment. Hybrid strategies combine both deliberative and reactive strategies. Various authors have concurred that the problems faced by designers are different manifestations of the frame problem (Fodor 1987; Shoham 1987), and it has been suggested that to avoid the situation, artificial systems should use (1) representations of the world directed toward action (Cherniak 1986) and (2) tools based on dynamic systems to formulate the relationship between the agent and its environment (Van Gelder 1998).

The first of the requisites—to have a system with representations directed toward the action—can be achieved by using a beliefs, desires, and intentions (BDI) deliberative agent model (Bratman 1987) that, together with a case based reasoning (CBR) motor (Aamodt and Plaza 1994), constitutes the base of a planning system based on previous case-based planning (CBP) (Carbonell 1983; Hammond 1989; Corchado and Laza 2003; Corchado et al. 2004). This type of model meets the conditions needed to introduce a representation and a reasoning based on the action (Pollack 1992).

As far as the second requirements—the formulation of an agent-world system—are concerned, it has been demonstrated that any dynamic model of a coupled system cannot be resolved analytically (Abraham, Abraham, and Shaw 1992). Given this situation, the alternative is to use a constructive methodology (Sloman 1994)—an agent model is constructed using iterative modeling, which will define how it behaves to changes in the environment. To make the agent model compatible with the dynamic requirements, it needs to meet two conditions: (1) necessary condition—it must be formulated using analytical techniques and meet

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the requirements of a solution—and (2) sufficient condition—the model must be complete (Hale and Koçak 1991).

With the previous hypotheses we propose the design of a MRP-Ag (most-replanable agent) architecture. First, the ***planning space*** is introduced, within which the agent represents the plans in terms of its accumulated objectives and the resources consumed. In this space, the plans, which maintain a constant efficiency, are represented by straight lines. These types of plans, known as geodesic plans, are especially interesting for our problem. On one hand, (1) they represent plans that are less risky in a changing environment, and on the other, (2) they meet the Bellman conditions of optimality (Bellman 1957), which allow us to work with dynamic programming techniques.

Second, to deal with restrictions imposed by constraint satisfaction problems (CSPs) (Sam-Haround 1995), a ***restrictions space*** is built (generally a non-Euclidean hyperplan) within the *planning space*, and a set of techniques based on variation calculation (Schutz 1993) is developed to calculate the geodesic plans within it. As a result of this, a family of plans is obtained, compatible with the restrictions imposed, which constitutes the way to pass from a current state to an objective state, maintaining the conditions of constant efficiency throughout the process.

Third, the ***notion of replanning*** and ***the concept of easily replanable plan*** is introduced, which is formulated using techniques based on Jacobi fields (Lee 1997). Given the set of geodesic plans, these types of techniques allow us to determine which of them present greatest density of alternative plans in an environment and thus, in the event of interruption, can count on the greatest number of proximate plans with which to continue. For the replanning agent design proposed to meet the hypothesis initially presented, it must fulfill the condition of being a complete solution of the dynamic agent–world system. This will be tested using the Green's theory of operators (Snieder 1994).

The last part of the paper compares the behavior of MRP-Ag with two other agents defined as, E-Ag (Efficient Agent) and O-Ag (Optimum Agent). Their behavior in environments with different levels of dynamism are evaluated, as well as the effect learning has on each one.

The paper ends with a summary of the most significant results. In the section on future work, we propose the notion of a type of agent we could call *adaptable agent* (different from adaptive agents), which may be of interest in the area of artificial intelligence systems.

The paper is structured as follows. In Section 2, we show the implications in our design to accept the hypothesis of "representations directed toward action" and the hypothesis of "dynamic agent–world interaction." The first requires an agent with a case-based planning motor, and the second implies that the behavior of the agent can be formulated as a environment-based dynamic system. In Section 3, an agent model is proposed that possesses capacity for replanning. For this, a planning space (Section 3.1), techniques for determining minimal risk plans (3.2), and a heuristic that allows us to select the most replanable plan (3.3) are introduced. The model will only be compatible with the initial hypothesis if it conforms to the requirements that make it a solution for the dynamic system presented in Section 2. This requires two characteristics: (1) the model designed should have the expressive capacity to find an invariant in the system (Section 4.1) and (2) the model should be sufficient to constitute a complete solution for the system (Section 4.2).

The paper ends with the presentation of results of experiments (Section 5) whereby the behavior of the MRP-Ag was evaluated in comparison with other designs. Last, we summarize the most significant results, which point toward the future work that needs to be done.

2. TWO HYPOTHESES TO FACE THE FRAME PROBLEM

In this paper, instead of presenting new techniques for which the model constructs successful plans, a strategy is proposed that allows us to generate flexible replanning. Given a problem E, the agents don't look for the "best plan" to resolve it, but look for the *plan that, in the event of interruption, is most easily replanned at any point of time*. To carry out this task, two hypotheses are assumed, which derive from cognitive science (Clark 1997):

Hypothesis 1: First, agents should have a type of representation system to represent knowledge that is dependent on the agent, the context, and the orientation of the action. In these cases, we say that the agents are embodied.

Hypothesis 2: Second, the agents and their environments are interpreted as coupled elements that a dynamic system defines as unique. We can say that these types of agents are embedded in the environment.

In the following section, we examine the implications of both hypotheses.

2.1. Embodied Agents: CBP-BDI Agent Model

A *representation based on an action* requires an agent architecture in which the way to acquire and process the knowledge of the world, at the reasoning stage, is closely related to the way in which plans are constructed and used, in the phase of execution. In this section, we show how such a requirement can be achieved through a BDI agent model (Bratman 1987) in which a CBP reasoning motor has been incorporated (Carbonell 1983; Hammond 1989) using plans that have already been experienced.

First, we will introduce the terminology used within the paper for a BDI agent.

(1) The *environment* M and the changes that are produced within it, are represented from the point of view of the agent. Therefore, the world can be defined as a set of variables that influence a problem faced by the agent:

$$M = \{\tau_1, \tau_2, \ldots, \tau_s\} \text{ with } s < \infty. \tag{1}$$

(2) The *beliefs* are vectors of some (or all) of the attributes of the world taking a set of concrete values:

$$B = \left\{b_i / b_i = \left\{\tau_1^i, \tau_2^i, \ldots, \tau_n^i\right\}, n \leq s \quad \forall i \in N\right\}_{i \in N} \subseteq M. \tag{2}$$

(3) A *state of the world* ($e_j \in E$) is represented for the agent by a set of beliefs that are true at a specific moment in time t.

Let $E = \{e_j\}_{j \in N}$ set of status of the world if we fix the value of t, then

$$e_j^t = \left\{b_1^{jt}, b_2^{jt}, \ldots b_r^{jt}\right\}_{r \in N} \subseteq B \quad \forall j, t. \tag{3}$$

(4) The *desires* are imposed at the beginning and are applications between a state of the current world and another that it is trying to reach

$$d : \underset{e_0}{E} \underset{\rightarrow}{\rightarrow} \underset{e^*}{E}. \tag{4}$$

(5) *Intentions* are the way that the agent's knowledge is used to reach its objectives. They are reduced to the need to determine whether there is compatibility between the knowledge

that the agent possesses and the requirements to be able to attain its desires. A desire is attainable if the application i, defined through n beliefs exists:

$$i : \overbrace{BxBx\cdots xBx}^{n)}E \to E \,. \qquad (b_1, b_2, \ldots\ldots, b_n, e_0) \to e^* \tag{5}$$

In our model, intentions guarantee that there is enough knowledge in the beliefs base for a desire to be reached via a plan of action.

(6) We define an *agent action* as the mechanism that provokes changes in the world making it change the state,

$$a_j : \underset{e_i}{E} \underset{\to}{\to} \underset{a_j(e_i)=e_j}{E} \,. \tag{6}$$

(7) *Agent plan* is the name we give to a sequence of actions that, from a current state e_0, defines the path of states through which the agent passes to reach the other world state.

$$p_n : \underset{e_0}{E} \underset{\to}{\to} \underset{p_n(e_0)=e_n}{E} \,, \tag{7}$$

$$p_n(e_0) = e_n = a_n(e_{n-1}) = \cdots = (a_n \circ \cdots \circ a_1)(e_0), \tag{8}$$

$$p_n \equiv a_n \circ \cdots \circ a_1. \tag{9}$$

Below we present the attributes that characterize the plans of a case base, which allows us to relate BDI language with the parameters of interest within a CBP. We consider CSP problems to lend the model generality (Sam-Haround 1995). These types of problems don't only search for solutions but also have to conform to a series of imposed restrictions. Based on studies on the theory of action (Davis 1979; Allen 1984), we will select the set of objectives for a plan and the resources available as a variable upon which the CSP problems impose the restrictions. A plan p is expressed as

$$p = \langle E, O, O', R, R' \rangle, \text{ where}$$

(1) E is the environment, but it also represents the type of problem faced by the agent, characterized by $E = \{e_0, e^*\}$, where e_0 represents the starting point for the agent when it begins a plan and e^* is the state or states that it is trying to attain;
(2) O indicates the objectives of the agent and O' are the results achieved by the plan; and
(3) R are the total resources and R' are the resources consumed by the agent.

In Table 1, we show the indicators derived from the attributes that we have shown and which we will use to identify and contrast the quality of the different plans (# means cardinal of a set).

TABLE 1. Indicators of Plan Quality

Efficacy of the plan: Relationship between objectives attained and objectives proposed	$E_f = \frac{\#(O' \cap O)}{\#O}$	(10)
Cost of the plan: Relationship between the resources used and the resources available	$C = \frac{\#R'}{\#R}$	(11)
Efficiency of the plan: Relationship between the objectives attained and the resources consumed	$E_{ff} = \frac{\#(O' \cap O)}{\#R'}$	(12)

means cardinal of a set.

As a representation of a plan p in the database, we use a more compact expression,

$$p = \langle E, F(O; R) = 0\rangle, \text{ where}$$

(1) $F(O; R) = 0$ is the *restriction function of the plan* or ***quality function***. Not all plans that obtain certain objectives by consuming resources are equally acceptable. This function demands that some minimum objectives are attained and a maximum consumption of resources for the plan to be considered acceptable. F determines the level in which the solution is adequate or not, keeping in mind the relationship between objectives and resources. For simplicity, we consider a single action plan $p_1 \equiv p$, $a_1 \equiv a$, so that:

$$p \equiv a : \underset{e_0}{E} \underset{\rightarrow}{\rightarrow} \underset{a(e_0)=e_1}{E}. \tag{13}$$

From the point of view of the BDI agent, to execute a plan p, beginning from e_0 and aiming to achieve the state e_1, the agent needs to

(1) have characterized state $e_0 : \{b_1^0, \dots, b_n^0\} = \mathrm{B1} \subset \mathrm{B}$;
(2) have characterized state $e_1 : \{b_1^1, \dots, b_m^1\} = \mathrm{B2} \subset \mathrm{B}$;
(3) know how to execute action a; and
(4) know that the action allows it to pass from e_0 to e_1, in other words, to be able to guarantee that the intention exists.

If we have a defined problem $E = \{e_0, e_1\}$, a plan p to solve the problem can be characterized by the relationships between the objectives reached and the resources consumed between both states.

We can assume then that the agent doesn't just have knowledge about the world, however, given a problem, it possesses knowledge that allows it to relate its world beliefs with the objectives and resources of any state during the course of a solution plan. This condition guarantees that there is communication between the BDI structure of the agent and its CBP reasoning motor as can be seen in Figure 1. Figure 1 shows the internal structure of a CBP-BDI agent. The agent posses beliefs and has desires. The intentions are constructed to achieve desires by using the knowledge contained in the beliefs. The intentions are then processed applying CBP. If the agent receives new perceptions, the belief base changes, thus the intentions can change and a replanning action can be necessary.

The general functioning process is derived by following the typical phases of a case based system (Aamodt and Plaza 1994; eliminating the revision phase). We use the name CBP system (or methodology) to describe a procedural information process in which past experiences are used to decide how to deal with a new problem, which share similarities with previous problems (Carbonell 1983; Hammond 1989).

These experiences, stored as cases, act dynamically on each CBP cycle.

The reasoning process of a system of this type carries out the following stages (Figure 2):

(1) *Retrieval:* Given a state of the perceived world e_0 and the desire that the agent encounters in a state $e^* \neq e_0$, the system searches the case base, previously checked by the intention function through which the system gains sufficient knowledge, for plans that have resolved similar problems.
(2) *Adaptation/Reuse:* From the previous phase, we obtain a set of possible solutions for the agent $\{p_1, \dots, p_n\}$. In this phase, in accordance with the planning model G (defines how the agent is going to behave when facing changes in the environment), the system uses the possible solutions to propose a solution p^*.

$$G(e_0, p_1, \dots, p_n) = p^*. \tag{14}$$

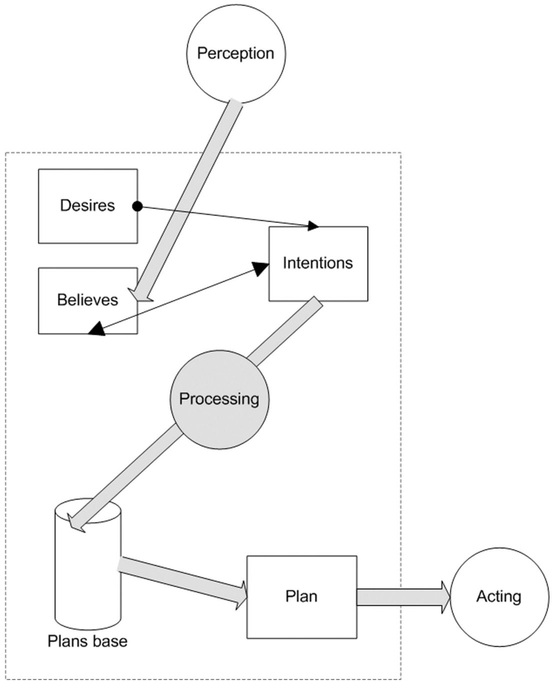

FIGURE 1. BDI-CBP based on plans.

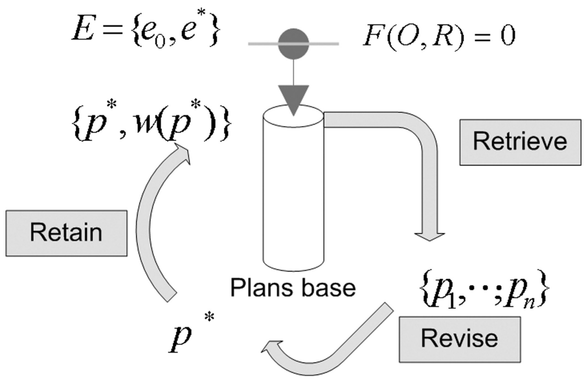

FIGURE 2. Life cycle of a CBP.

(3) *Learning/Retention:* The plan proposed may attain its objective or fail in its development. The information on the quality of the final optimum plan denoted by $w_f(p^*)$ in the cycle is stored for the future and is directly proportional to (1) the initial value of optimum plan denoted by $w_i(p^*)$ and (2) the "rate of use" $\alpha(N)$, where N is the number of times it has been used in the previous cycles of similar problems (Heckerman 1998).

$$w_f(p^*) = w_i(p^*)\alpha(N). \tag{15}$$

The model proposed conforms to the conditions required to obtain a representation and reasoning based on the action (Pollack 1992). The set of action capabilities of the system restricts what type of plans are possible that, at the same time, frame how the agent's reasoning model has ended up, ultimately fixing the type of representation of the world that the system has.

2.2. Embedded Agents: Dynamic World–Agent System

Below we model the dynamic relationship between the behavior of the agent and the changes in the environment.

We represent the behavior of ***agent A*** by its ***function action*** $a_A(t)\ \forall t$, defined as a correspondence between one moment in time t and the action selected by the agent,

$$Agent\,A = \{a_A(t)\}_{t \in T \subseteq N}. \tag{16}$$

We determine the behavior of the environment **E** by a sequence of *states* $e(t)$ through which it passes over time,

$$E = \{e(t)\}_{t \in T \subseteq N}, \tag{17}$$

which we call ***environment evolution function***. The variation of the function $e(t)$ has at least two possible sources:

(1) A term that represents the *random changes* of any environment with stochastic behavior.
(2) A term that collects the *changes in the environment provoked by the agent behavior* in search for its objectives.

From the definition of the ***action function*** $a_A(t)$, we can define a new relationship that collects the idea of an agent's ***action plan***,

$$p_A : \begin{array}{ccc} TxA & \to & A \\ (t, a_A(t)) & \to & p_A(t) \end{array}, \tag{18}$$

in the following way:

$$p_A(t_n) = \sum_{i=1}^{n} a_{iA}(t_i - t_{i-1}), \tag{19}$$

where a_{iA} indicates action i carried out by agent A.

Given the dynamic character that we want to print onto our agent, we propose as a definition of the ***agent plan***, the continuous extension of the previous expression in other words:

$$p_A(t_n) = \int_{t_0}^{t_n} a_A(t)\,dt. \tag{20}$$

The variation of the agent plan $p_A(t)$ will be provoked essentially by:

(1) The *changes that occur in the environment and that force the initial plan to be modified.*
(2) The knowledge from the success and failure of the plans that were used in the past, which are favored or discarded via *learning*.

As we have demonstrated so far, both the plan $p_A(t)$—that represents the agent—and the environment evolution function $e(t)$—that characterizes the environment—are related to each other. Together they form a system of coupled equations:

$$e(t) = F(e_0(t), a_A(t)),$$
$$p_A^* = G(e(t), p_A(t)), \tag{21}$$

where F is the abbreviation of $F(O; R)$, O are the objectives that can be expressed through the states achieved and R are the resources that can be expressed by means of agents' actions. Therefore, $F(O; R)$ and $F(e_0(t), a_A(t))$ be different forms of denoting to the same function F.

This system of equations represent a ***general world–agent model*** (the plan of actions $p_A(t)$ characterizes the agent and the evolution function $e(t)$ to the environment) and allows it to formulate its dynamic nature. The conclusions that we can obtain through the analysis of the solutions will be general and independent of the specific form F (abbreviation of $F(O; R)$), with which the world varies—in terms of the type of the environment and its level of dynamism—and the particular strategy G that the agent uses to plan $p_A(t)$.

Applying the *theory of dynamic systems* (Abraham and Shaw 1992; Norton 1995), we obtain the first result: The system is not autonomous (a system is autonomous if the properties that characterize the system are independent of time), and therefore, there is no general theory that indicates how to resolve the equations in this case. The study is made then of ***qualitative manner***, where:

(1) Function F isn't known (the behavior of the world is stochastic) and is considered simply as a generating source of changes, which are only known when they are produced.
(2) Function G, which defines how the agent is going to behave when facing changes in the environment, is obtained through a *process of constructive design:*
   (i) A sequence of stages is constructed, which determines the way in which the agent processes the information that it receives to deploy the plans in the environment.
   (ii) It demonstrates that the model G obtained is compatible with the dynamic conditions that the system demands, as shown in equation (21).

In the next section, we develop the model for behavior G of the agent and step by step we justify the reasons why the constructive process has been carried out in this way.

3. MRP-AGENT: A REPLANNING AGENT MODEL

The objective in this section is to introduce an architecture for a planning agent that behaves—and selects its actions—by considering the possibility that the changes in the environment block the plans in progress. We call this agent MRP because it continually searches for the plan that can most easily be replanned in the event of interruption.

The MRP architecture will have at its base, a CBP-BDI system (see Section 2), and will manipulate its knowledge by following a model G that allows it to determine the best plan according to the replanning criteria.

To demonstrate the process of construction for the behavior model G of the MRP agent, we need to introduce the notation and the terminology that we are going to use to represent a planning problem.

(1) Given an initial state e_0, we use the term "planning problem" to describe the search for a way of reaching a final state e^* that meets a series of requirements.

(2) We define “restrictions or requirements of a problem” as the relationship between variables of efficacy and efficiency that determines the level in which the solution is adequate or not.

$$F(O;R) = \begin{cases} f(O_1, \ldots, O_j) = 0 \\ g(R_1, \ldots, R_k) = 0 \\ h(O_1, \ldots, O_j, R_1, \ldots, R_k) = 0, \end{cases} \tag{22}$$

where f is a function that depends on the objectives that must be achieved and g is the function depending on the resources that can be used, both are lineal combinations of binary variables that take value zero or one depending on if the objective wants to be reached or not or if the resource is needed or not, and h is the hyperplan of equations of f and g.

(1) Solution is the sequence of accessible states between one that allows it to reach the final state and conforms to the restrictions,

$$S = \left\{e_0, e_1, \ldots, e_n = e^*\right\} \tag{23}$$

and “solution plan” is the sequence of actions that the agent needs to carry out to generate a solution S.

$$p_n = a_n \circ \cdots \circ a_1. \tag{24}$$

The general scheme is as follows: Given a problem $E = \{e_0, e^*\}$, for which we seek a solution S conforming to certain restrictions $F(O; R) = 0$, the agent MRP will seek a plan p that consists of a methodology CBP (Carbonell 1983) that forms the base of experienced plans and proposes a plan that considers the possibility that the changes in the environment interrupt its development. In the following subsections, the complete process is shown in detail.

3.1. Agent Planning Space

Given a discrete variable X that can take values of a numerable set that we represent as

$$X = \{x_i\}_{i \in N}, \tag{25}$$

we can define the *associated accumulated variable*, that we denote as $Ac(X)$. This is a new variable constructed by assigning each of the possible values x_i taken by variable X, the total of previous results.

If X is discrete, the value ith of the variable $Ac(X)$ is defined as

$$Ac(x_i) = \sum_{j=1}^{i} x_j \quad \forall x_i \in X. \tag{26}$$

If the variable X is continuous and its values in the interval $[a, b]$, and it is represented by function $x(t)$; we define the variable $Ac(X)$ at a point $x_i \in [a, b]$:

$$Ac(x_i) = \int_a^{x_i} x(t)\,dt \quad \forall x_i \in [a, b]. \tag{27}$$

Given a problem E and a plan $p(t)$, we can construct functions Ob and Rc accumulated from the objectives and costs of the plan. For all time points t_i we can associate two variables:

$$Ob(t_i) = \int_a^{t_i} O(t)\,dt, \tag{28}$$

$$Rc(t_i) = \int_a^{t_i} R(t)\,dt. \tag{29}$$

This allows us to construct *a planning space* (or space representing the environment for planning problems) as a vectorial hyperdimensional space, where each axis represents the *accumulative variable* associated with each objective and resource.

The planning space, defined in this way, conforms to the following properties:

Property 1: The representation of the plans within the planning space are always monotonously growing functions. Given that $Ob(t)$ and $Rc(t)$ are functions defined as positive (see definition), function $p(t)$ expressed at these coordinates is constant or growing.

Property 2: In the planning space, the straight lines represent plans of constant efficiency. If the representation of the plans is straight lines, the slope of the function is constant and coincides with the definition of the efficiency of the plan.

$$\frac{d}{dt}p(t) = cte \Leftrightarrow \lim_{\Delta\to 0}\frac{\Delta O(t)}{\Delta R(t)} = cte. \tag{30}$$

In an n-dimensional space, the extension of the straight concept line is called a geodesic curve. In this sense, we can introduce the notion of ***geodesic plans*** that are defined as those that maintain efficiency at a constant, throughout their development.

The concept of a geodesic plan can be better understood through the idea of a *plan of minimum risk*. If the environment is changeable, any other relationship with efficiency that isn't constant will imply that the agent makes plans for the future (it considers that in the future certain efficiency relationships will be met and as such, it makes sense to assume greater or lesser efficiency ratios). In an environment that changes unpredictably, any plan that is distal to the geodesic plan means that a certain risk is accepted.

3.2. Agent Reasoning Model: Construction of $G(t)$

Given a problem E, the agent must search for the plan that determines a solution with a series of restrictions $F(O; R) = 0$. To deal with the restrictions, we are going to make a change in the coordinates: Instead of seeking plans of constant efficiency that are adjusted to $F(O; R) = 0$, we construct the hyperplan that collects all such information and we calculate the straight line within it (in general non-Euclidean).

(1) In the plans base, we seek those plans that are initially compatible with the problem faced by the agent, with the requirements imposed on the solution according to the desires, and in the current state (Aamodt and Plaza 1994). If we represent all the possible plans $\{p_1, \ldots, p_n\}$ within the *planning space*, we will obtain a ***subset of states*** that the agent has already attained in the past to resolve similar problems.

(2) With the mesh of points obtained (generally irregular) within the planning space and using interpolation techniques, we can obtain a working hyperplan $h(x)$ (that encapsulates

the information on the set of restrictions from restored experiences), from which we can calculate geodesic plans.

(3) Starting from the values given for $\{f(x_i)\}_{i=1,\ldots,n}$, where $X = \{x_i\}_{i=1,\ldots,n}$ are variables in the planning space, the theory of functions of radial base as combinations of B-Splines proposes an expression of $h(x)$ in the following way (Reuter et al. 2003):

$$h(x) = m(x) + \sum \lambda_i \phi(\|x - x_i\|_2)x, x_i \in \Re^d, \lambda_i \in \Re \ \forall i, \tag{31}$$

where the coefficients vector λ_i encapsulates all the information needed to manage the restriction associated with a problem; The coefficients λ_i of the function $h(x)$ are determined by demanding that h satisfies the interpolation conditions:

$$h(x_j) = f(x_j), \quad j = 1, \ldots, n. \tag{32}$$

and functions $\phi(x)$ are a complete base of orthogonal functions. Duchon (1977) has demonstrated that the selection of cubic functions are the most suitable in interpolation problems for obtaining the smoothest function (Hegland, Roberts, and Altas 1997).

$$\phi(x) = (\|x\|_2)^3. \tag{33}$$

The system of equations (32) can be resolved by the conjugated gradient method, or directly, and the cost of the solution is at most the order $O(k^3)$ (Beatson and Light 1997). The computational pack used to make these calculations is known as JSpline+ (Spline library for Java), which uses a B-splines radial bases functions (Duchon 1997), so that the information on the restrictions space $h(x)$ can be reduced to tackling the coefficient vector λ_i.

3.2.1. Calculating Efficient Plans. The variation calculation (Schutz 1993) consists in a set of mathematical techniques that allows us to know the geodesic paths between one point in a non-Euclidean space and a set of points represented by a function that we call the *function of final states*, which we denote as $f_s f$.

In general, the simplest variation problem is given when $f_s f$ is only one point in the space, $f_s f = e^*$, and the geodesic g that links e_0 with e^* is obtained Figure 3.

In a problem where the set of end points is $n > 1$, *variation techniques with mobile frontiers* are used, which offer us a set of geodesics between the starting point and each one of the points of the final set. If $f_s f = \{e_1, \ldots, e_m\}$, we obtain a geodesia set $\{g_1, \ldots g_m\}$.

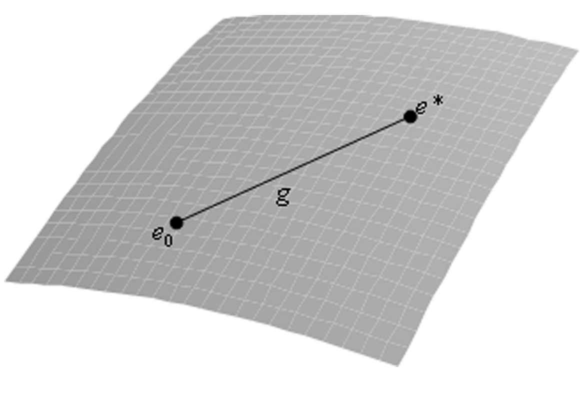

FIGURE 3. Geodesic g linking: Initial and final state.

Below we apply variation calculation techniques for the planning problem that has been set.

Given a problem E that seeks a plan that allows it to pass from one state of the world e_0 to a state $e^* \in f_s f$ conforming to restriction $F(O; R) = 0$, we can construct the hyperplan of restrictions $h(x)$, with which we apply variation calculation. Suppose, for simplicity's sake, that we have a planning space of dimension 3 with coordinates $\{O, R_1, R_2\}$.

Between the state e_0 and the objective states $f_s f$ and over the interpolation surface $h(x)$, the Euler Theorem (Glez-Bedia and Corchado 2002; Jost and Li-Jost 1998) guarantees that we will obtain the expression of the geodesic plans by resolving the following system of equations:

$$\begin{cases} \dfrac{\partial L}{\partial R_1} - \dfrac{d}{dO}\dfrac{\partial L}{\partial R_1'} = 0, \\ \dfrac{\partial L}{\partial R_2} - \dfrac{d}{dO}\dfrac{\partial L}{\partial R_2'} = 0, \end{cases} \tag{34}$$

where R_i is the function acculmulated R, O is the function of accumulated O and L is the distance function on the hyperplan $h(x)$,

$$L = \int_h dl. \tag{35}$$

To obtain all the geodesic plans that, on the surface $h(x)$ and beginning at e_0, allows us to reach any of the states $e^* \in f_s f$, we must impose as a condition of the surrounding that the initial state is $e_0 = (O_0, R_0)$.

Using variation techniques, we obtain expressions for all the geodesic plans that, beginning at e_0 allows us to attain the desired state.

3.2.2. Heuristics: Searching for Easily Replanable Solutions. Once plans have been obtained that will create efficient solutions between the current state and the set of solution states, we will be able to calculate the plan around it (along its trajectory) by a denser distribution of geodesic plans (in other words, a greater number of geodesic plans in its environment). The tool that allows us to determine this is called the *minimum Jacobi field, associated with the solution set* (Lee 1997).

With a set of geodesics $\{g_n(x)\}_{n \in N}$ over $h(x)$, we select one which we call g_0. Mathematically, the Jacobi field of g_0 over $h(x)$ is defined as the field of g_0 variations throughout the set $\{g_n(x)\}_{n \in N}$. The definition is an extension of the derived notion as shown below. We can see here, a way of introducing a derived idea that is different from the standard.

If there is a function $f : [a, b] \rightarrow \Re$, where $x \in [a, b]$ and it is $t \in (-\varepsilon, \varepsilon)$. We can build the set of functions $\{h_t(x)\}_{t \in (-\varepsilon,\varepsilon)}$ formed by functions that are small interferences of the function $f(x)$:

$$\{h_t(x)/h_t(x) = f(x + t), \forall t \in (-\varepsilon, \varepsilon)\}. \tag{36}$$

If we take the variations to a differential limit $t \in (-\varepsilon, \varepsilon)$

$$\left\{h_t(x)/\lim_{t \to 0} h_t(x) = \lim_{t \to 0} f(x + t)\right\} \tag{37}$$

we can obtain the derived definition of $f(x)$.

Impelled by this way of constructing the function derivate, we can now introduce the concept of field of variation in a geodesic g on a surface S. $g_0 : [0, 1] \rightarrow S$ be a geodesic over a surface S. Let $h : [0, 1]x[-\varepsilon, \varepsilon] \rightarrow S$ be a variation of g_0 so that for each $t \in (-\varepsilon, \varepsilon)$, the set $\{h_t(s)\}_{t \in (-\varepsilon,\varepsilon)}$:

(1) $h_t(s)\,\forall t \in (-\varepsilon, \varepsilon)$ are geodesic in S.
(2) They begin at $g_0(0)$, in other words, they conform to $h_t(0) = g_0(0)\,\forall t \in (-\varepsilon, \varepsilon)$.

In these conditions, taking the variations to a differential limit, we obtain:

$$\lim_{t\to 0}\{h_t(s) = g_0(s+t)\} = \lim_{t\to 0}\{h(s,t)\} = \left.\frac{\partial g_0}{\partial t}\right|_{(s,0)} = \frac{dg_0}{ds} \equiv J_{g_0}(s). \tag{38}$$

We give the term $J_{g_0}(s)$ to the Jacobi field of the geodesic g_0 for the set $\{g_n(x)\}_{n\in N}$, and in the same way that the definition has been constructed, we give a measurement for the distribution of the other geodesics of $\{g_n(x)\}_{n\in N}$ around g_0 throughout the trajectory.

Given a set of geodesics, some of them are always g^* that, in their environment, have a greater distribution than other geodesics in a neighboring environment. This is equivalent to saying that it presents a variation in the distribution of geodesics lower than the others and therefore the Jacobi field associated with $\{g_n(x)\}_{n\in N}$ reaches its lowest value at J_{g^*}.

Let's return to the MRP agent problem that, following the recuperation and variation calculation phase, contains a set of geodesic plans $\{p_1, \ldots, p_n\}$. If we select the p^* that has a minimum Jacobi field value, we can guarantee that in the event of interruption, it will have around it a greater number of geodesic plans to continue. To select this plan would mean to select the solution that can most easily revert to another if it is interrupted.

For our problem, the minimum Jacobi field is synonymous with the capacity for replanning. This suggests the following definition:

Given a problem E with certain restrictions $F(O; R) = 0$, and a set of final desirable states $f_s f$, and given a set of geodesic plans that define the routes of constant efficiency between the initial e_0 and each of the elements of the final states $\{e_1, \ldots, e_m\}$, we define the ***most replanable solution* (*MRS*)** as geodesic plan p^* with minimum associated Jacobi field associated with the set $\{g_n(x)\}_{n\in N}$.

In this way, the behavior model G for the MRP agent is defined. For each problem E that it represents, the agent selects the most replanable solution defined as that geodesic plan with minimum Jacobi field, that is expressed

$$G(e_0, p_1, \ldots, p_n) = p^* \Leftrightarrow \exists n \in N / J_{g_n} \equiv J_{g*} = \operatorname*{Min}_{n\in N} J_{g_n}. \tag{39}$$

With this result, we can characterize the agent's mode of behavior. If the plan p^* is not interrupted, the agent will reach a desired state $e_j \equiv e^* \in f_s f \;\; j \in \{1, \ldots, m\}$. Below, in the learning phase, a weighting $w_f(p)$ is stored where N becomes $N{+}1$ (see Section 2.2. A specific expression for Bayesian learning will be outlined in Section 5). With the updating of weighting $w_f(p^*)$, the planning cycle of the CBP motor is completed. Below we see what happens if p^* is interrupted.

3.2.3. Life Cycle of the Agent: Dynamic Replanning. Let's suppose that the agent has initiated a plan p^*, but at a moment $t > t_0$, the plan is interrupted due to a change in the environment.

The geodesic planning (the section of plans with a constant slope in the planning space) meets the conditions of the Bellman principle of optimality (Bellman 1957); in other words, each on of the plan's parts is partially geodesic between the selected points.

This guarantees that if g_0 is geodesic for interrupted e_0 in t_1 because e_0 changes to e_1, and g_1 is geodesic to e_1 that is begun in the state where g_0 has been interrupted, it follows that

$$g = g_0 + g_1 \text{ is geodesic to } e = e_0(t_1 - t_0) + e_1(t_2 - t_1).$$

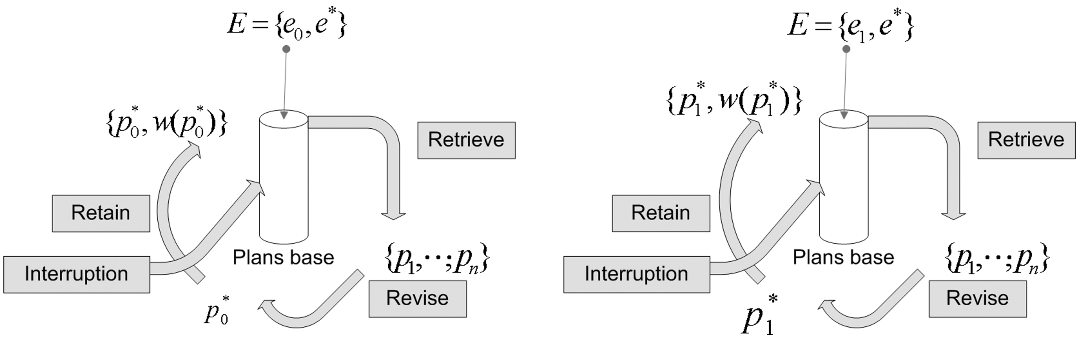

FIGURE 4. Model for behavior $G(t)$.

In other words, we can construct our global plan in "pieces." If each time the environment changes and interrupts the execution plan, a new geodesic plan is selected, the ***overall plan will be geodesic.***

The dynamic process follows the CBP cycle recurrently: Each time a plan finds itself interrupted, it generates from the state reached so far, the surroundings of the plans from the case base and adjusts them to the new problem. With this, it calculates the geodesic plans and selects the one that meets the minimum conditions of the associated Jacobi field. In this way, the dynamic planning model of the agent $G(t)$ is characterized as can be observed in (Figure 4).

In the dynamic context, the following properties of $G(t)$ are particularly relevant:

Property 1. All the Jacobi fields are variations of geodesics.

It can be demonstrated (Milnor 1973) that there exists a isomorphism among all Jacobi fields, that are constructed between the end points.

Property 2. All the geodesic variations are Jacobi fields (Milnor 1973).

These two results allow us to introduce the concept of a ***global Jacobi field***. We call ***global Jacobi field*** or ***dynamic Jacobi field*** $J(t)$ that Jacobi field, formed by a set of partial or successive Jacobi fields. The above properties allow us to ensure that the change from one partial Jacobi field and the next preserves the conditions of a Jacobi field because it produces a change between geodesics.

We can observe that a minimum global Jacobi field $J(t)$ also meets Bellman's conditions of optimality (Bellman 1957); in other words, a minimum global Jacobi field must select minimum Jacobi fields in pieces

$$J_{\min}(t) = \{J_{\min}(t_1 - t_0), J_{\min}(t_2 - t_1), \ldots, J_{\min}(t_n - t_{n-1})\}. \tag{40}$$

If on the one hand, successive Jacobi fields generate one Jacobi field, and on the other hand, minimum Jacobi fields generate a minimum Jacobi field, the MRP agent that follows a strategy of replanning $G(t)$, as indicated, to survive a dynamic environment, it generates a ***global plan*** $p^*(t)$ that, faced with all possible global plans $\{p_n(t)\}_{n\in N}$, presents a minimum value in its Jacobi field $J_{g^*}(t) \equiv J_{p^*}(t)$.

Until now, we have formally defined an agent that, in a dynamic environment, seeks plans that lend it greater capacity for replanning. In the next section, we show that the planning mechanism proposed $G(t)$ meets with the necessary and sufficient conditions for it to be a solution for an agent–world system.

4. DEMONSTRATING THE DYNAMIC COMPATIBILITY FOR MRP-AGENT

In the section that follows, we will demonstrate that the MRP, whose planning model $G(t)$ continuously selects the geodesic $p^*(t)$ with a minimum Jacobi field value until reaching objective $J_{p^*}(t)$, is compatible with the binomic agent–world dynamic and conforms to the conditions to be the system solution.

4.1. Necessary Conditions in Non-Autonomous Dynamic Systems

In Section 2.2, we explained that in non-autonomous dynamic systems, where in general there is no quantitative solution, a constructive resolution focus is employed. We now see, in detail, what should be required of the model obtained for it to be a solution in a non-autonomous dynamic system.

In the first place, we summarize the procedural phases of analytic resolution in an *autonomous system*. An autonomous dynamic system (for simplicity's sake, one-dimensional and first-order) can be represented by

$$\frac{d}{dt}x(t) = u(x), \tag{41}$$

in which the evolution of $x(t)$ is given by $u(x)$. The source of change $u(x)$ is usually represented as if it had been obtained from a potential $V(x)$:

$$V(x) = -\int u(x)\,dx, \tag{42}$$

so that the evolution $x(t)$ over time *always follows the direction of the minimums of potential.*

$$\frac{d}{dt}x(t) = -\frac{d}{dx}V(x). \tag{43}$$

In this type of system, to calculate the solution $x(t)$ is mathematically equivalent to finding an invariant quantity of a system,

$$C(x(t), V(x), t) = cte. \tag{44}$$

The meaning of C in the problem can be understood, if we adopt a systemic perspective (Beer 1995). From this viewpoint, for a system in interaction that changes over time, to survive as a global structure, some invariant property needs to persist that will conserve the structure of the interaction (Maturana and Varela 1980; Beer 1995).

To know the value of C is equivalent to resolving the problem, and according to the Hamilton principle for autonomous systems, for all possible values of C, this will take a stationary value (maximum or minimum).

In our problem, the equation that we are trying to resolve is not autonomous,

$$\frac{d}{dt}p(t) = s(t) \tag{45}$$

and as such, we have constructed a planning model $G(t)$ that generates a plan $p(t)$ that, expressed by its coordinates $\{O_j, R_k\}_{j,k}$, conforms, at each moment of time t, to

$$\frac{d}{dt}p(t) = -\frac{\partial J(t)}{\partial O_j \partial R_K}; \tag{46}$$

in other words, the evolution of $p(t)$ follows the *minimums of the global Jacobi field*.

By analogy, with this autonomous case, in the interaction between the agent and the world, there exists an *invariant quantity in the system*,

$$C(p(t), J(t), t) = cte \tag{47}$$

that allows the *survival of the coupled set* independently of the changes that occur. Therefore, the model $G(t)$ conforms to the conditions necessary to be considered a solution for the agent–world system (Abraham et al. 1992).

Intuitively we can attempt to find a meaning for this variable C that relates to the fact that with each interruption, the system selects the plan of constant efficiency whose minimum value of $J_{\min p^*}(t)$.

In the same way that we constructed $G(t)$, the agent selects a plan that always seeks the greatest replanning capacity. The choice of $p^*(t)$ seeks the ***maximum replanning potential***, which is an end value that is reached throughout the process. Therefore, we are able to understand $C(t)$ as the measurement of the replanning capacity of the system. From this point of view, we can say that, *for an agent to survive in a dynamic environment, it must seek the solution of least risk and with greatest replanning capacity*.

We have proven that, the replanning strategy guarantees the condition required to be the solution for the system. Below we see the sufficient condition.

4.2. Sufficient Condition

Green's functions represent a way of expressing behavior that follows the response of a system $o(t)$ faced with a stimulus function $i(t)$ that comes from outside (Snieder 1994). To express the response of a system in terms of Green's functions, we need the "response–impulse" function that represents the response of the system to a momentary stimulus, written as $\delta(t)$. The stimulus function is a function that depends on t; in each moment, t is a constant that takes the value 0 if there is no interruption of the plan and the value 1 if there has been one because it only interests us to know the system response when there is a stimulus, and those stimuli take place with the interruptions of the plan.

In our case, these types of momentary stimuli are interruptions to our plans. In these cases, the system responds with a strategy that we have called $G(t)$.

Green's functions allow us to express and calculate the relationship between the stimulus and the response of a system through the product of convolution with its Green's function (Snieder 1994):

$$o(t) = \int_{-\infty}^{\infty} G(t,\alpha) i(\alpha)\, d\alpha \Leftrightarrow o(t) = G(t) \otimes i(t). \tag{48}$$

If in the planning problem, we understand plan $p(t)$ to be the response function of the system faced with stimulus in the environment $F(t)$ and that for each interruption $\delta(t)$ the agent behaves according to $G(t)$ as many times as its Green's function, then,

$$p(t) = \int_{-\infty}^{\infty} G(i,\alpha) F(\alpha)\, d\alpha. \tag{49}$$

With this result, we are going to propose a new way of resolving the equation of evolution $p(t)$ in a dynamic environment,

$$\frac{d}{dt}p(t) = s(t). \tag{50}$$

To give a specific solution $p_s(t)$ to the equation above, it isn't enough simply to have a general expression of the type of function $p(t)$, the conditions of the problem's surroundings must also impose themselves (initial state e_0, final state e^*, and restrictions $F(O; R) = 0$).

It can be demonstrated (Snieder 1994) that if the initial plan $p_0(t)$ of the system is static,

$$\frac{d}{dt}p_0(t) = 0, \tag{51}$$

which we express as

$$p_0(t) = p_0(t, e_0, f_s f, E_{ff}), \tag{52}$$

then the expression of a specific solution for the dynamic problem $p_s(t)$, conforming to the conditions of the surroundings,

$$p_s = p_s(e_0, f_s f, E_{ff}) \tag{53}$$

takes the following form in terms of the system's Green's function:

$$p_s(t) = p_0(t) + \int_{-\infty}^{\infty} G(t, \alpha)F(\alpha)\, d\alpha, \tag{54}$$

where $p_0(t)$ is the initial plan, $F(t)$ is the behavior of the environment, and $G(t)$ is the planning model.

We can conclude that model $G(t)$ contributes all the sufficient information needed to completely characterize an expression of the plan $p(t)$ throughout time, compatible with a dynamic perspective of the interaction between the agent and the environment. If we add to this the previous result, we can ensure that model $G(t)$ for the behavior of the agent ***conforms to conditions that are both necessary and sufficient*** to be considered a solution for the system.

5. EXPERIMENTAL EVALUATION

To prove the validity of our model, in this section, we design an experiment that allows us to obtain empirical information from a simulation platform. Two new agents will be defined, E-Ag and O-Ag, with the same architecture as the MRP agent, but with strategies that are *less prudent* when selecting action plan $p^*(t)$. Their results will be compared in different environments.

In the first place, the behavior models for the new agents will be introduced and then the different simulation environments will be defined. After that, the three architectures will be implemented and an analysis will be made of the results in terms of the parameters of quality (efficacy, efficiency, etc.). Finally, we define how the process of learning influences each of the agents.

5.1. Types of Agents

The architectures of E-Ag and O-Ag are equivalent to those of MRP-Ag: BDI structures with a CBP motor that generates a set of geodesic plans $\{g_n(x)\}_{n\in N}$ for a determined problem.

Nevertheless, in the adaptation phase the plan p^* that presents a minimum Jacobi field value is not selected:

(1) The efficient agent, E-Ag selects the geodesic plan p^* that allows it to pass from current state e_0 to the state $e_j \equiv e^* \in f_s f \ j \in \{1, \ldots, m\}$ that conforms to the condition of maximum efficiency. The sub-index in e indicates that goes associated to the chose state e. Each $e_j \in E$ (abbreviated by e) has a representation within the space $\{O, R\}$ in such a way that it can be associated with a measurement of its efficiency,

$$E_{ff} = \frac{O_e}{R_e}. \tag{55}$$

The agent E-Ag therefore calculates the maximum of the function E_{ff} and applies variation calculation between the two fixed states.

The system tackles the interruptions and the dynamic of the problem using a mechanism of continuous recall to the case base. The plan $p_e(t)$ (associated plan with state e) obtained through this technique, meets with Bellman's condition of optimality (to follow a constant maximum efficiency trajectory requires that maximum values are followed in pieces) and allows us to define a model $E_{ff}(t)$ characterized by selecting the most efficient plan,

$$p_e = \operatorname*{Max}_{n \in N} E_{ff}(p_n). \tag{56}$$

(2) The second of the agents that we call O-Ag, selects the geodesic plan $p_l \in \{g_n(x)\}_{n \in N}$ that allows it to pass from the current state e_0 to a state $e_j \equiv e^* \in f_s f \ j = 1, \ldots, m$ using the minimum sequence of actions. Geometrically, this minimum plan p_l is the one that unifies the initial state e_0 and a final state e^* so that the minimum value is reached within the operating distance.

To calculate this plan p_l, it is necessary for the set of final states $f_s f$ to conform to the ***equation of transversibility*** (Jost and Li-Jost 1998). This mathematical condition ensures that the plan obtained to resolve the equation is the one which passes from an initial state e_0 to a state $e^* \in f_s f$ in conditions of ortogonality. The ortogonality guarantees that the plan that results from it will have a minimum longitude.

The plan p_l obtained from this equation conforms to the condition of being

$$p_l = \operatorname*{Min}_{n \in N} L(p_n). \tag{57}$$

The O-Ag planner tackles interruption in an identical way to the other two planners described, through a mechanism of continual recall from the case base. In the same way as the ones above, the strategy meets Bellman's conditions of optimality (a minimum plan $p_l(t)$ is the plan that is also minimum for each of its pieces), which allows it to define a dynamic $L(t)$ behavior model.

In this way it is possible to characterize the three types of agents by their associated Green's function, which we describe as:

- $E_{ff}(t)$ select the plan that tends towards the most efficient state.
- $L(t)$ select the plan with least stages to reach $e^* \in f_s f$.
- $G(t)$ selects the most replanable until reaching a $e^* \in f_s f$.

5.2. Definition of the Types of Dynamic Environments

A dynamic environment changes, and this is detected by the agent from the change in its beliefs. The dynamism of an environment is characterized for the agent, not only by the number of beliefs that change **K** but also by the frequency **f** of change that is measured by the period **T** between consecutive changes.

We call the unit of time in the simulation process t_{sim}. In each period of time $\Delta t = t_{\text{sim}}$ the system executes an action, it adapts beliefs B toward the state of the world and updates its plans in the case base BC compatible with the problem E and knowledge B.

$$t_{\text{sim}} = f(t_p, \dim B, \dim BP) \text{ where } t_p \text{ is the time of the timer implemented}$$

In these conditions, we define $t_{\text{sim}} = rt_p$, where r is sufficiently large for the system to update its values before carrying out the next action.

In general, an action a of an agent is executed when the system possesses a knowledge that is given by a set of beliefs:

$$\{b_1, \ldots, b_j\} \Rightarrow a_i. \tag{58}$$

For simplicity's sake we assume a unitarian relationship between a belief and an action.

$$b_i \Rightarrow a_i. \tag{59}$$

If all beliefs were true, the most complete plan would be the one that uses a number of actions equal to size m of the set of beliefs B. In these conditions we can define the variables K and T more precisely:

(1) We call the number of beliefs that change within a given period, **K** If we know that $m = \dim B$, we have limited the value of k

$$k \in \{0, m\}. \tag{60}$$

(2) We use **T** to describe the period of time between consecutive changes in the beliefs about the world. This parameter is delimited between the minimum period of time in the experiment t_{sim} and the maximum time that it could take to reflect the longest plan:

$$T \in \{t_{\text{sim}}, t_{\text{total}}\}. \tag{61}$$

To carry out specific experiments we take a set of beliefs $m = \dim B = 20$ that is considered sufficiently large to be able to see how the agents respond when faced with interruptions, but sufficiently small to be able to tackle the complete set of plans generated by the CBP. The objective of this experiment is to observe the behavior of the planner in the event of interruptions and not the efficiency in terms of response time. To be able to observe the above-mentioned we consider significant a set of 20 beliefs. If the planner is capable of replanning with a set of 20 beliefs, it will also be able to do it with a set of hundreds or thousand beliefs, and the planner also has replanning possibilities. Once the replanning ability has been demonstrate it could be of interest, for a future work, to establish a metric for the response time. This metric would depend on the similarity algorithm or the organization of the cases memory. This leads us to design experiments with four types of environments:

(1) ***Quasi-aesthetic environments:*** Very few beliefs k change, and the changes are very infrequent (in other words, the period between changes is very large)

$$E_0 = \{(k, T) \quad k \in [0, 5), T \in (15, 20]\}. \tag{62}$$

(2) ***Environments of little change:*** Few beliefs k change, and the changes produced are of a low frequency,

$$E_1 = \{(k, T) \quad k \in [5, 10), T \in (10, 15]\}. \tag{63}$$

(3) ***Changing environments:*** A significant number of beliefs k change, and changes are more frequent,

$$E_2 = \{(k, T) \quad k \in [10, 15), T \in (5, 10]\}. \tag{64}$$

(4) ***Extremely dynamic environments:*** Many changes in beliefs in each cycle and changes are very frequent,

$$E_3 = \{(k, T) \quad k \in [15, 20), T \in (1, 5]\}. \tag{65}$$

In these environments, a problem is proposed and the three types of agents are introduced. E-Ag, O-Ag, and MRP-Ag, compete to be the best planner.

5.3. Implementation

For the implementation of the three agents, an architecture is constructed of three layers: (1) *memory layer*, where the information of case base resides in the form of beliefs and plans that are periodically updated; (2) *processing layer* that controls the planning process; and (3) *presentation layer* which constitutes the interface with the user.

5.3.1. Memory Layer. Given a set of beliefs B compatible with problem E, we can generate a plan base CBP that contains all the possible plans produced by the combinations of compatible beliefs. For the experiments, only some will be used. In Figure 5, we present the algorithm used for implementing the CBR agent.

The consultations made to the memory (relational database *SQL server)* use SQL language to access the data on beliefs and plans. The database motor used is Microsoft Sql Server 2000.

Once generated, we activate some of those available to be used by the system. When the environment changes, some of these plans will remain unused and other possible plans are activated.

5.3.2. Processing Layer. The implementation of the application is developed in JAVA language, version j2sdkl.4.1., for the logic of the three types of planner $E_{ff}(t)$, $L(t)$, and $G(t)$, as well as the connection with the database.

The mathematical calculations for obtaining $h(x)$, through Duchon techniques, the set of geodesics $\{g_n(x)\}_{n \in N}$ through the resolution of the Euler and transversability equations, or for obtaining the Jacobi field, are carried out using the programme® Mathematica 5.2 and the libraries Jspline+ and Jlink for java.

In Figure 6, we present a pseudocode of the scheme for implementing the planning model of the three agents (except in the specific choice of the geodesic plan). A different geodesic plan will be determined $p_e(t)$, $p_l(t)$, and $p^*(t)$ depending on the agent being used—E-Ag, O-Ag, or MRP-Ag.

5.3.3. Presentation Layer. A last layer is developed so that the designer can specify and control the values of the simulation and its monitoring. It is an interface layer placed between the designer and the system.

Algorithm 10.7 CBR Generator

Vectors B, O, R
int i, j, k
while $i<\dim B$
    $b_i \leftarrow$ Believe_i (B)
    $B \leftarrow B - \{b_i\}$
    $O \leftarrow O(b_i)$
    R $\leftarrow R(b_i)$
    $i_k \leftarrow b_i, \{O, R\}$
    CBR ←Stores (i_k)
    k++
    IntentionConstruction (i_k, O, R, B){
    for $j = 0$ to $j < \dim B$
        $b_j \leftarrow$ Believe_j (B)
        $B \leftarrow B - \{b_j\}$
        if $(i_k, b_j) \neq 0$ **then**
            $O \leftarrow O + O(b_j)$
            $R \leftarrow R + R(b_j)$
            $i_k \leftarrow i_k \wedge b_i, \{O,R\}$
            CBR ←Stores (i_k)
            K ++
            IntentionConstruction (i_k, O, R, B)
        j ++}
    i++

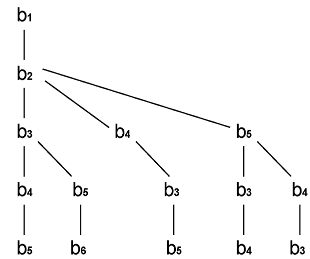

FIGURE 5. CBR generation.

Algorithm 10.6 Planning dynamics

Vectors B, O, R
Vector Plan
while planning
  if Present Believe b_i <dimPlan ***then***
    if Present Believe b_i = = True **then**
      $B \leftarrow B - \{b_i\}$
      O $\leftarrow O(b_i)$
      R $\leftarrow R(b_i)$
      Present Believe++
    if not
      /* *Present Believe b_i = = False* */
    Re-planning
  if not
    /* *serendipity* */
    if present R < available R **then**
      Re-planning
    if not
      /* Plan finished */
    Bayesian weight update (present plan)
    if ∃ plan global then
      Bayesian weight update (global plan)

FIGURE 6. Implementation of the replanning process.

Figure 7 shows how the designer can introduce the data about the problem (initial stage, restrictions function, etc.) and the type of agent being used to attain a solution plan. The system allows the possibility to make evaluations with a single agent or with various at the same time. The designer can configure the simulation, choosing the planning strategy. The designer can also interact to the agents once they have been instantiated, gaining access

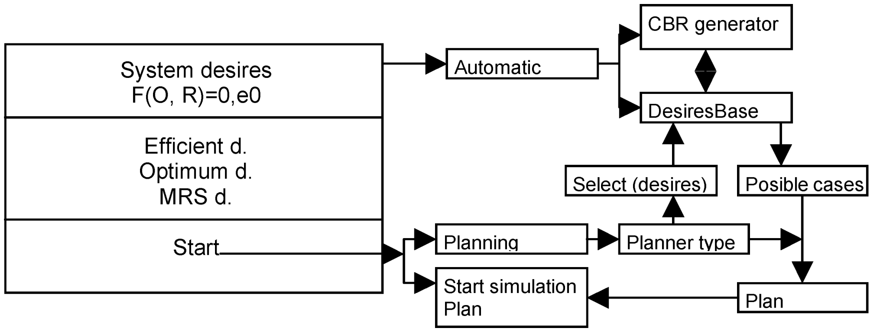

FIGURE 7. Presentation layer.

to their internal structures (beliefs, desires, intentions, plans) and to inspect the agent's state and the planning steps.

5.4. Types of Experiment

To evaluate the behavior of each agent, we measure the quality of the plans. In Table 2, we show the different attributes used to characterize the planning models. Different experiments are proposed to measure how the results of the planning change according to the strategy used and the dynamism of the environment. The conclusions are shown in the following tables where:

(1) *Files:* Characterize the different environments by $(k.T)$.
(2) *Columns:* The first column makes reference to the data related to the experiment (number of simulations *Ct*, number of series that reached the final state *Ce*, and their percentage relationship); the second shows the data of the *average case* (number of actions or beliefs used N_c, median number of interruptions N_i, efficacy E_{fo}, cost e_{fr}, and efficiency e_{ff}) and last, a third, which offer the results of the relative efficiency e_{ffr} and the percentage *Ce*02 with which the adjustments of the planner to the environment are measured.

TABLE 2. Quality Indicators Used to Characterize Plans

Ni	*Interruptions*	Number of replannings carried out up to the completion of the plan
Efo	*Efficacy*	Achieved objectives vs. possible objectives
e_{fr}	*Consumption*	Resources used vs. resources available
e_{ff}	*Efficiency*	Objectives reached vs. resources used
Nc	*Plan stages*	Number of actions or beliefs used
e_{ffr}	*Relative efficiency*	Efficiency of the plan in terms of the number of stages
Cet	*Success*	Percentage of cases that reach a solution state vs. the total number
*Ce*02	*Success* 0.20	Percentage of successful cases that reach values within the top 20%
Ct	*Simulations*	Number of simulations
Ce	*Number series*	Number of series that reached the final state

These classifications are developed from a series of simulation experiments (approximately 100) with different requirements to define its final states.

5.4.1. Experiment 1: Agents E-Ag vs. Agents O-Ag. In the first place, we propose to measure the difference in results between the strategy $E_{ff}(t)$ and $L(t)$ in different dynamic environments. The following data are obtained:

Independent of the requirement at the outset, the experiments show that (Table 3):

(1) In $E0$, E-Ag behaves better: The environment hardly changes and the strategy is able to develop without interruptions.
(2) In $E1$ and $E2$, the O-Ag agents are more successful than those of E-Ag: both quantitatively (a larger number of cases reached the desired state) and qualitatively (there are more cases which while meeting the desired requisites have values within an interval of 0–20% of their most desired values).
(3) In $E3$ and $E4$, in highly dynamic environments, the results of both structures deteriorated and were similar.

These results lead us to conclude that the optimal planning strategy $L(t)$, which seeks the plan with greater $e_{ffr} = \frac{e_{ff}}{nc}$, and the efficient $E_{ff}(t)$, which seeks the plan with greater $e_{ff} = \frac{O}{R}$, don't function in highly dynamic environments—their strategy is not designed for these types of contexts.

5.4.2. Experiment 2: Agents O-Ag vs. Agents MRP-Ag. Below, in Table 3, we present the results of the experiments that compared the planning strategy $L(t)$ with the planning strategy $G(t)$. The structure of the experiments is identical to the one previously indicated. The data found, presented in Table 4, reflects that, depending on the type of environment, the success of each strategy is distributed as follows:

From these results we can conclude that

(1) In environments $E0$, $E1$, and $E2$, with few interruptions (less that 10), the O-Ag agents are more suitable.

TABLE 3. Comparison of Results between E-Ag and O-Ag

	Strategy			Typical Case				Quality	
	Ct	*Ce*	*Cet*	*Nc*	e_{fo}	E_{fr}	e_{ff}	e_{ffr}	*Ce*02
				E-Agents					
$E0$	100	78	0.78	11	0.49	0.51	0.86	0.07	0.6
$E1$	98	55	0.55	10	0.46	0.53	0.82	0.08	0.3
$E2$	93	32	0.32	13	0.32	0.62	0.62	0.04	0.1
$E3$	97	12	0.12	12	0.28	0.69	0.40	0.03	0.02
				O-Agents					
$E0$	97	81	0.81	7	0.56	0.58	0.91	0.13	0.4
$E1$	100	61	0.61	6	0.52	0.59	0.88	0.14	0.2
$E2$	99	48	0.48	10	0.45	0.62	0.71	0.07	0.08
$E3$	98	18	0.10	8	0.26	0.71	0.36	0.04	0.06

TABLE 4. Comparison of Results between O-Ag and MRP-Ag

O-Agents									
	Strategy			Typical Case				Quality	
	Ct	*Ce*	*Cet*	*Nc*	e_{fo}	E_{fr}	e_{ff}	E_{ffr}	*Ce*02
*E*0	100	83	0.83	8	0.55	0.61	0.90	0.11	0.3
*E*1	96	69	0.69	7	0.53	0.60	0.88	0.11	0.3
*E*2	93	51	0.51	8	0.48	0.64	0.75	0.09	0.07
*E*3	98	20	0.20	9	0.28	0.76	0.38	0.04	0.05
MRP-Agents									
	Strategy			Typical Case				Quality	
	Ct	*Ce*	*Cet*	*Nc*	e_{fo}	e_{fr}	e_{ff}	e_{ffr}	*Ce*02
*E*0	98	79	0.79	11	0.51	0.64	0.79	0.07	0.4
*E*1	95	64	0.64	12	0.47	0.68	0.69	0.05	0.3
*E*2	97	56	0.56	12	0.42	0.71	0.59	0.04	0.3
*E*3	100	41	0.41	14	0.39	0.83	0.46	0.03	0.2

They don't only plan in such a way that they achieve a greater number of successful cases, but they are fundamentally different by having higher *Ce*0.2 indices than the plans proposed by MRP-Ag.

(2) In *E*3 and *E*4 environments with a higher degree of dynamism (greater variation between beliefs and greater frequency), the best results are presented by MRP-Ag architecture. The higher number of successful cases *Ce* contrast with some levels that are relatively low in terms of the *Ce*02 parameter (in comparison with the O-Ag agents in environments of little change). This result is somehow predictable, because the success of the MRP strategy is based on prudent decisions, seeking benefits in the long term at the cost of plans with more efficient levels in the final state.

The results of these environments, together with those obtained in experiment 1, allow us to confirm that the O-Ag architectures propose a better strategy for environments of *little dynamic*. However, for more dynamic environments, the ability of the MRP agents to design plans that predict situations leading to future interruption makes them more successful decision motors than the other strategies.

5.4.3. Comparison of General Results. Below we present the average results of various series of experiments with the three types of agents (in the sample 10 series have been carried out with 100 simulations). The results are compared for the following values:

(1) *Median* e_{ffr}, represented as $\overline{e_{ffr}}$, determines the average attained by the agents to reach a successful final state in relation to the number of plan stages $\overline{e_{ffr}} = \frac{\sum_i^n e_{ffr_i}}{n}$, where n indicates the size of the sample ($n = 1000$).
(2) *Standard deviation ‘,* that measures the level of concordance between the measurements and the median value.

TABLE 5. Comparison of Results E-Ag, O-Ag, and MRP-Ag

	E-Ag			O-Ag			MRP-Ag		
	$\overline{e_{ffr}}$	σ	μ	$\overline{e_{ffr}}$	σ	μ	$\overline{e_{ffr}}$	σ	μ
$E0$	0.04	0.01	0.006	0.07	0.02	0.005	0.13	0.03	0.005
$E1$	0.03	0.01	0.004	0.08	0.01	0.002	0.14	0.02	0.003
$E2$	0.01	0.008	0.002	0.04	0.006	0.002	0.07	0.008	0.002
$E3$	0.01	0.004	0.002	0.03	0.003	0.002	0.04	0.002	0.002

(3) *Interval of confidence for μ,* that determines the probability that the true median value is found at a certain distance from the average value $\overline{e_{ffr}}$. If we want reliability with a rate of error $\alpha = 00.5$, we will use *t of Student*, $t = 1.960$ as the value.

$$\mu = \overline{e_{ffr}} \pm \frac{t\sigma}{\sqrt{n}} \tag{66}$$

From results (Table 5), we can conclude the following:

(1) In static environments, the strategies $E_{ff}(t)$ and $L(t)$ achieve better results than $G(t)$ because the initial plans, in general, are the definitive plans.
(2) In dynamic environments, the strategies $E_{ff}(t)$ and $L(t)$ do not behave adequately. The interruptions ensure that the strategy $E_{ff}(t)$ cannot reach the most efficient final states while in the planning $L(t)$, the possibility to construct plans with few stages is blocked. In this case, the model $G(t)$ constructs long-term plans but ones that guarantee the objectives (although consuming more resources).
(3) Basically, we can be sure that in dynamic environments, the only strategy that can be considered successful is the replanning strategy. If the dynamism of the environment is reduced, $G(t)$ becomes a costly strategy in terms of resources and any one of the other two is preferable.

In Figure 8, we represent the behavior of the three types of agents in terms of the dynamism of the environment.

5.4.4. Experiment 3: Measuring the Effect of Learning. In the following experiments, the number of times the plan is interrupted is measured (ni = number of interruptions) until a final state is reached, to determine whether ni changes with learning (Heckerman 1998).

Each plan p has an associated indicator of its success $w(p)$ that is modified each time a CBP cycle is used, so that

$$w_f(p) = \frac{w_i(p) + \lambda(p)N}{N+1} \tag{67}$$

where $w_f(p)$ is the final weighting after being used, $w_i(p)$ is the initial weighting, N is the number of times that the plan has been used, and $\lambda(p)$ is a parameter that measures the phase of the plan where the interruption has been produced:

$$\lambda(p) = \frac{\text{number of interruptioned phase}}{\text{number of phases of the plan}}. \tag{68}$$

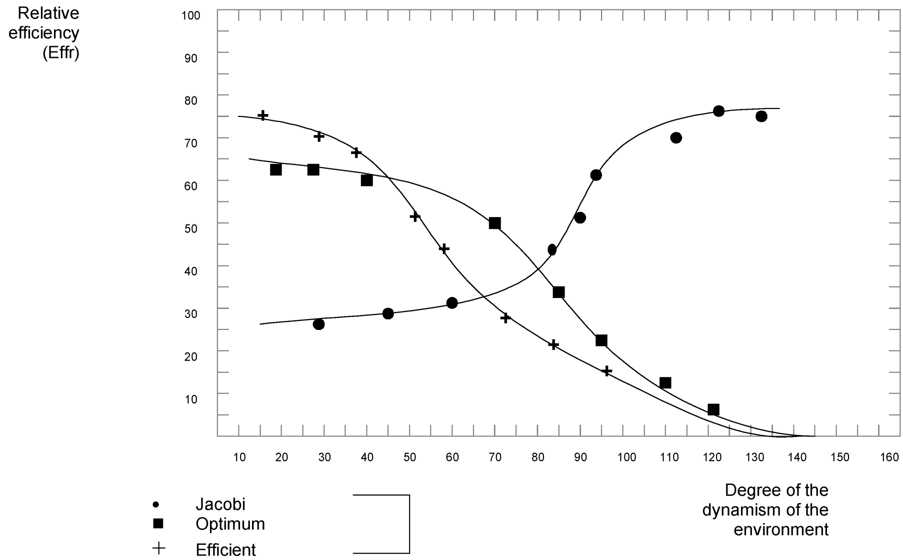

FIGURE 8. Representation of the behavior of agents according to the environment.

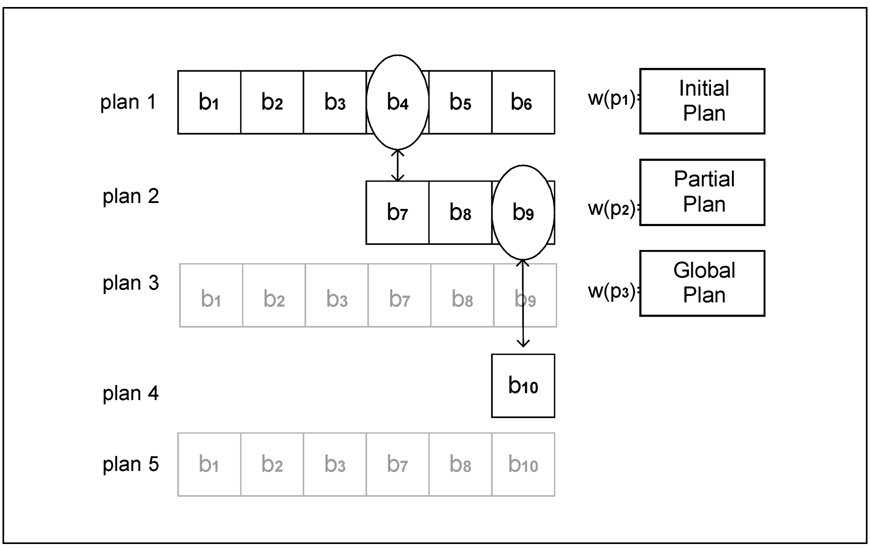

FIGURE 9. Process of updating of the Bayesian weighting.

This rule for updating the weighting of the plans is dynamically applied, both for the ***global plans*** and for each one of the ***partial plans*** that are used until reaching a final state. In Figure 9, we show how in a process where a plan p_1 is interrupted and is completed, first by a plan p_2 and finally by a plan p_3, the weightings are updates in each of the plans used and in the totals generated.

There are a number of other possible sources that may effect $\lambda(p)$:

(1) Random: $\lambda(p)$ could respond to coincidental events from the past, which have occurred, but which didn't necessarily have to be produced.

TABLE 6. Measurement of Learning in O-Ag and MRP-Ag

ni: Number of Interruptions										
	O-Agents Cycles					MRP-Agents Cycles				
	10	25	50	75	100	10	25	50	75	100
$E0$	3.3	3.1	2.9	2.8	2.3	3.6	3.2	2.8	2.5	2.4
$E1$	9.3	8.9	8.3	7.9	7.8	9.6	9.1	8.6	7.4	7.3
$E2$	14.6	14.1	13.8	13.6	12.8	14.3	14.1	13.6	13.2	12.7
$E3$	17.3	16.8	16.3	16.1	16.1	18.1	17.4	16.5	16.1	15.9

TABLE 7. Measurement of Relative Learning O-Ag and MRP-Ag

	10 cycles			50 cycles			100 cycles		
	cet	e_{ffr}	*Ce*02	*Cet*	e_{ffr}	*Ce*02	*Cet*	e_{ffr}	*Ce*02
				O-Agents					
$E0$	0.79	0.13	0.3	0.81	0.15	0.3	0.82	0.17	0.3
$E1$	0.62	0.12	0.2	0.61	0.13	0.2	0.69	0.13	0.3
$E2$	0.55	0.09	0.09	0.50	0.10	0.2	0.58	0.10	0.2
$E3$	0.22	0.04	0.05	0.27	0.06	0.09	0.26	0.05	0.09
				MRP-Agents					
$E0$	0.76	0.08	0.4	0.78	0.09	0.4	0.81	0.09	0.3
$E1$	0.63	0.06	0.3	0.65	0.07	0.3	0.62	0.09	0.3
$E2$	0.58	0.04	0.2	0.59	0.05	0.3	0.61	0.05	0.3
$E3$	0.43	0.02	0.2	0.46	0.03	0.2	0.49	0.03	0.2

(2) Critical points: $\lambda(p)$ could be responding to situations that reveal critical points in the structure of the plan (situations that are more sensitive to a change in beliefs provoked by the dynamism of the world).

Our learning model, after several cycles, allows us to detect those stages in the plans that are more sensitive to changes in the environment. These plans are penalized and gradually used less. Therefore, the learning guarantees that plans with high $w(p)$ are more robust and resistant to dynamic environments.

In principle, it seems obvious that $w(p)$ will depend on the staticity/dynamism of the environment. The measurements are carried out in different environments and the agents O-Ag and MRP-Ag are compared. From the results of 100 series, we can conclude the following:

Learning effect (*general*): A first result guarantees that both agents improve their behavior with learning. Both are benefited, on average, if they lose the plans that have behaved worse in the past and select those with the highest $w(p)$ (Table 6).

Learning effect (*relative*): We are interested in the *relative improvement* that learning produces in the two agents; in other words, which functions better when there is little information (few CBP cycles) or where the agent has learnt (many CBP cycles). In Table 7, we can see how in conditions of little information (high uncertainty) the MRP agents behave better and present greater success values in relative terms.

In terms of learning, we can conclude the in environments of high uncertainty, the MRP-Ag agents are the most successful.

5.4.5. Conclusions from the Experiments. Based upon the observations, we can summarize the results obtained in three types of environment:

(1) *Very dynamic environments*. Both the number of beliefs that change per cycle and the frequency changes that occur, are high. The original plans do not survive the dynamism of the environment and the agents need to be flexible and capable of replanning on the move. One agent with these characteristics is the MRP-Ag, presented in this paper.
(2) *Semi-dynamic environments*. There are changes in the environment that interrupt our plans, but not as much as in the very dynamic environments. The best strategy developed for this is used by the O-Ag that seeks simple plans with few stages, minimizing the possibility for interruption.
(3) *Quasi-static plans.* The plans considered at the beginning have a high probability of being completely realized. In these cases, to select plans of maximum efficiency, as done by E-Ag, while losing in flexibility, may be the most effective.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we have introduced some fundamental ideas for the theoretical analysis of the agency and theory of planning:

(1) Firstly, different agent architectures based on implemented planning tools have been considered to fail, because to a lesser or greater extent, when given a criterion, they seem the *most suitable plans* to resolve a problem. We believe that theories and agent models should be based around a concept of *flexible replanning* (in this sense, we are in accordance with Jon Elster when he comments (Elster 1999) that *"In conditions of uncertainty, the rational thing is to act as if the worst could happen"*).
(2) In the paper, we propose a replanning agent model that, using a base of plans, is able to design and redesign versatile strategies in the event that its plans are interrupted. The technique that allows us to formulate this behavior is known as the *Jacobi field* in a set of geodesics.
(3) This replanning technique should be compatible with other features demanded of dynamic agents. In this paper, we demonstrate that the proposed replanning technique based on Jacobi fields constitutes the base for an invariant property within a dynamic agent–world system. This could be described in the following way: *The condition of existence of an agent that interacts dynamically with its environment is capable of replanning*. If the contrary is true, the changes in the environment will suppress the conditions of life (and action) for the agent.
(4) Once the MRP is formulated and implemented, together with two other planning strategies—O-Ag and E-Ag–different simulation experiments have been proposed. The experimental results show how in environments with a higher level of dynamism, the only technique that offers resistant plans able to reach their objectives, are those proposed by the MRP agent model.

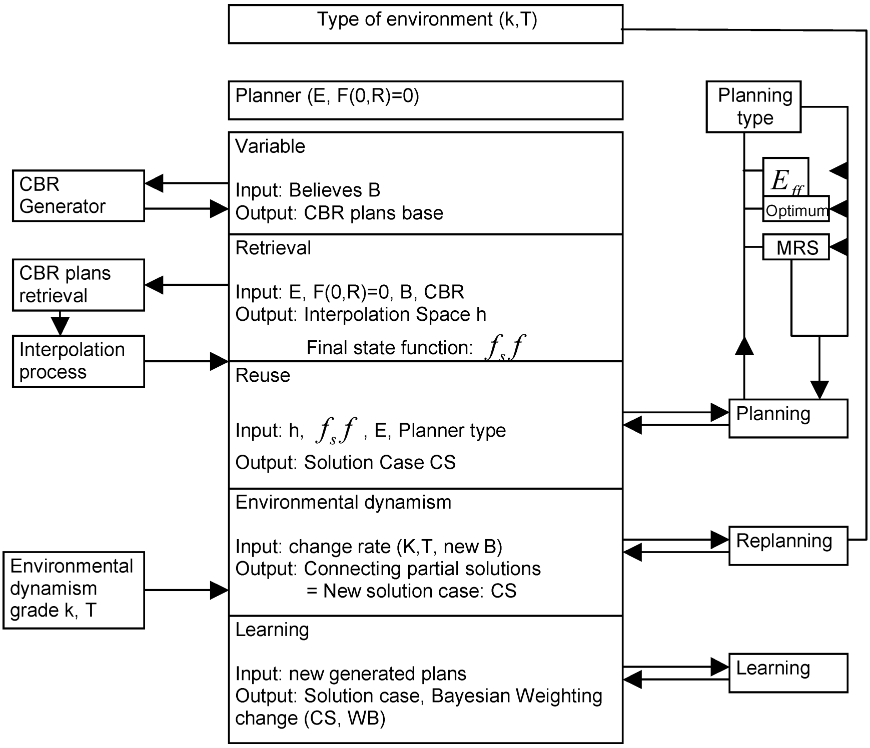

FIGURE 10. Scheme for the adaptable agent.

Using results from biological evolutionary systems (Gilbert 1987; Roy and Gilbert 2005), we can introduce the difference between *adaptive systems vs. adaptable systems*. While adaptive systems possess survival techniques that are compatible with the dynamic of a environment (perception, memory, operative capacity, or planning), *adaptable systems* possess another property: The internal ability to modify its own techniques and become systems adapted to new environments.

Given the models suggested and the results obtained, our line of future work will attempt to change the strategy according to the environment perceived. This ability in the system will have great significance in the development of artificial intelligence systems, which are flexible and adaptable to changes in the environment. In Figure 10, we can see a scheme for the adaptable system foreseen.

$$P \equiv E = \{e_0, e^*\}. \tag{69}$$

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