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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

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Edited by

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TABLE OF CONTENTS

ELEMENTARY AND HIGH-SCHOOL MATHEMATICS

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In teaching arithmetic it is necessary to devote much attention to the development of the *number sense,* a sort of almost instinctive feeling whether a calculation is correct or not. To this end a great deal of oral work is needed. Pupils should be drilled to answer what is approximately the product of 7.94 by 5.02. The usual method of pointing off should not be used but the pupil's number sense should tell him that the answer is near 40. Similarly, for division pointing off should be from the *right,* not the left, in all cases.

Addition and subtraction of decimals is best taught with two sets of change boxes containing counters representing dollars, dimes, cents, and mills. Then the *reasons* for the rules are seen at once and the process becomes a *rational* habit.

Common Fractions. Common fractions are best taught with a yard stick and piles of counters or pebbles, the basic property of fractions being that the only operation which does not change their value is multiplying (dividing) numerator and denominator by the same number. It does not seem desirable to teach the reason for the rules of multiplying common fractions. The reason for the rule of multiplying decimal fractions presents little trouble using a change box as indicated above.

Addition, subtraction, and division of common fractions can be rationally presented if the fact is made clear that two fractions are the same if the numerator and denominator of one fraction differ from the other only by a common multiplier or divisor, i.e., $\frac{p}{q} = \frac{2p}{2q} = \frac{\frac{p}{2}}{\frac{q}{2}}$

$$\text{Thus } \frac{\frac{p}{q}}{\frac{r}{s}} = \frac{\frac{ps}{qs}}{\frac{qr}{qs}} = \frac{ps}{qr}$$

Few pupils who have studied arithmetic and algebra can answer correctly the question what is $\frac{2}{\sqrt{2}}$ equal to? This is because they have forgotten the definition of $\sqrt{2}$. Still fewer can answer the question what is $\frac{2}{\sqrt[3]{2}}$ equal to? The reason in both cases is the same. Considerable stress should be laid on these definitions:

$$\sqrt{2}.\sqrt{2}=2,\ \sqrt[3]{2}.\sqrt[3]{2}.\sqrt[3]{2}=2, \text{ etc.}$$

After exponents have been studied the radical signs should be replaced by fractional exponents and thus a suggestive notation will replace the original logical process by a closely bonded and better understood rule.

Lastly it would be worth while to prove to the more advanced pupils of the 12th grade that every common fraction is a repetend (repeating decimal) and that every repeating decimal is a common fraction. Thus they can be shown that decimals with an infinite number of digits can be written down which are not fractions, i. e., are irrational. This is done by taking any repetend and changing the digits over wider and wider intervals.

Algebra. Some of the remarks about fractions apply here. Here first of all it is necessary to put more content into the word *formula.*

3x—6 is a formula; we can replace x by any number we please. We might better write it thus 3[ ]—6, where we place in the box any number or letter we please. Query: what number shall we place in the box so that the formula gives the number zero? i.e., solve the equation 3x—6=0.

Simple equations like this should be stated in words thus, three times what number is 6? (1) *or* one-half what number is 7? $\frac{1}{2}x=7$. (2)

Pupils will find (2) much more difficult than (1) because they solve both by the rule "divide both sides by the coefficient of x" and have forgotten how to divide by one-half.

Factoring. The central theme in factoring is the factorization of ax^2+bx+c. The usual method is a hit-or-miss process of inspection, trial, and error. This method always fails when the numbers a, b, and c are taken at random. As a matter of fact while a few problems of this sort should occur as soon as the rules of multiplication are known, such as:

$$x^2-5x+6$$
$$x^2+9x+14$$
etc.,

these problems should be taken up seriously only after much drill on *completing the square* has been given, e.g., fill the boxes in

$$x^2+6x+(\quad)^2$$
$$x^2-5x+(\quad)^2$$
etc.

As an application of this rule factor $6x^2-17x+5=$

$$6\left(x^2-\frac{17}{6}x+\frac{5}{6}\right)=6\left[x^2-\frac{17}{6}x+\frac{17^2}{12^2}-\left(\frac{17^2}{12^2}-\frac{5}{6}\right)\right]$$

$$=6\left(x-\frac{17}{12}-\frac{13}{12}\right)\left(x-\frac{17}{12}+\frac{13}{12}\right)$$

$$=6\left(x-\frac{5}{2}\right)\left(x-\frac{1}{3}\right)$$

$$=(2x-5)\,(3x-1)$$

This method looks much longer but it *always* works and in many cases when the factors are rational would be shorter than the trial and error method. It has the added advantage of giving the solution of the quadratic equation in the general case, and showing that any quadraiic can be factored into two factors of the first degree.

Observation shows that not enough stress is put on the fact that *when a set of numbers are multiplied together the product is zero when and only when one of the factors is zero.*

Thus, $3(x - 3)(2x - 5)(x + 4)$ is zero when and only when $x = 3$ or $x = \frac{5}{2}$ or $x = -4$; it would be better to write $x_1 = 3,\ x_2 = \frac{5}{2},\ x_3 = -4$. Read: first value of x is 3, second value of $x = \frac{5}{2}$, etc. The reason for this is not far to seek.

The ordinary practice of writing $x = a \mp \sqrt{b}$ is open to various objections.

Oral exercise: $x(x - a + b)(x + c + d) = 0$.

What are the values of x which make this true?

Word problems (so called) do not teach much algebra but when well graded and properly used develop the pupils' power of interpreting language and powers of analysis. They are, however, often badly worded even in good texts. As an example we might cite: the sum of two consecutive numbers is 15, what are the numbers? The objection is that there are no such things as consecutive *numbers*. The problem should read consecutive integers. Most word problems involve the distance, rate, and time relation, i.e., the proportionality relation. They are *complicated* by introducing special units of measure for each of the elements of distance, rate, and time. The hare and hounds problem is an illustration of this. Such problems test much more than the pupil's knowledge of algebra. All such problems can be skeletonized by introducing the usual units of distance, rate, and time, and thus present little difficulty. As a final example of a pursuit problem:

When will the minute and hour hand of a watch be together between 3 and 4 o'clock? Query: how much faster does the minute hand move than the hour hand? How many spaces is the hour hand ahead at 3 o'clock, how much does the minute hand gain on the hour hand in a minute? How many minutes will it take the minute hand to *gain* 15 minute spaces and thus be up with the hour hand? This is the arithmetic of the problem. Clock problems would be more interesting and less difficult if first solved in this way and then translated back into problems to be solved by algebra. The answer can be verified by setting a watch. This last step has its value as we have an example of a machine that solves equations (a calculating machine).

Conclusion. Today almost everyone admits that the teaching problem in algebra is a difficult one. We have to bring young and immature pupils to understand an exceedingly general and abstract language which has been won only by centuries of effort by the best minds. Certain rules and principles as the rule of signs and multiplication of fractions can only be taught as rules of the game without demonstration. Once these rules are acquired much follows by a process the pupil can *understand.* Unfortunately, too many of the rules are taught in the same way, as processes to be applied but not understood. This tendency is sufficiently illustrated by the use of such words as *cancel, clear of fractions, transpose,* which should be banished from the "bright lexicon of youth," and the reasonable processes for which they stand put in their place. Those who are interested in a detailed exposition of these matters may consult Barber's *Everyday Algebra.*

A List of Four Books for Teachers of Mathematics

Arithmetic: Introductory Book of Searchlight Arithmetics (Pub. by Ginn & Co.)

Everyday Algebra by Barber (Houghton Mifflin Co.).

Teaching of Junior High School Mathematics by Barber (Houghton .Mifflin Co.).

The Teaching of Mathematics in Secondary Schools by Schultze (Macmillan Co.).

MATHEMATICS

New and better methods of teaching mathematics are promised. It is to be hoped that the promise is fulfilled. Nothing perhaps has detracted more from the proper education of the young than the fact that no very interesting way has been discovered of teaching this fundamental subject, while very often, it seems, methods adopted have been particularly and unnecessarily tiresome to the young mind.

Contrary to what is perhaps the popular impression, mathematics has real beauties for those who study it intelligently. It is a thing which, properly entered on and pursued, will attract students to more and more study for its own sake. This view doubtless will be questioned by most people, or will be ascribed to those only who rate as "math sharks," but it is a correct view, nevertheless, and the very fact that it is not generally held illustrates the wrong impression that has been given by the wrong kind of teaching.

Mathematics is the most perfect of our sciences, and is a fundamental part of virtually all sciences. It has conquered new fields in recent years that are vast in extent. Scientists know this, of course, and the present day admiration for science is weakening the old popular animosity for mathematics.

There remains some loose talk of the uselessness of teaching algebra and geometry, of course; but when the parent realizes that to deprive his son of such courses is to cut him off forever from all scientific professions and occupations, from all fundamental knowledge of what these great sciences are about, he is not likely to conclude that the school authorities are all impractical old fogies.

The study of mathematics requires no more particular adaptation than does the study of any other subject. It is essentially consecutive and logical thinking, and the man who says he can't learn mathematics comes dangerously near saying that he can't think accurately. Its study has undoubtedly been made difficult by being made too abstract, but all mathematics is not abstract. The greater part of it can be made visual, and doubtless it is along this line that the new methods of teaching will proceed.

The study of mathematics does, and always will, make one requirement that many other studies do not make. It requires that the going be step by step, and, most important of all, that no steps be eliminated—which simply means day by day attention. We can skip chapters in history and English and most any other subject, and still come out all right in the end. But we can't do that in mathematics.

That is where the difference comes. The real difficulty in studying the subject lies not in the fact that it is dry, or abstruse, or abstract, or calls for a particular kind of mind, but solely in the fact that it requires an unbroken line of study. When better methods of teaching are developed doubtless this latter fact will be made to stand out so plainly that most everyone will hesitate to say that he "can't learn math."—Editorial published in *The Houston Chronicle.*

THE ORDER OF PROPOSITIONS IN PLANE GEOMETRY

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When Euclid wrote his "Elements of Geometry" about 300 B.C., the ideal which he set before himself was to build the geometric theorems which had been discovered by his predecessors into a scientific and logical system, in which each theorem was to be proved by means of previous theorems, the whole structure resting ultimately on a set of simple axioms and postulates. In this endeavor he was not completely successful, for he made tacitly certain assumptions which he did not state as axioms, for example the assumption that a figure can be moved about freely without change of form, on which all congruence proofs and in general all measurements depend. His system was so nearly perfect, however, that it stood unscathed the scrutiny of mathematicians for over two thousand years, and only within the last century have more strictly logical systems of geometry been built up.

This logical structure places limitations on, although it by no means completely determines, the order in which the propositions are taken up, since every theorem must come after those on which its proof is based. The first proposition must depend directly on the axioms and definitions given, and this is seen to be the case with Prop. 1 of the Wentworth-Smith text; but Props. 2, 3, 5, 7, 20, and 30 of Book I, Props. 1 and 2 of Book II, etc., also satisfy this condition, so any one of them might logically have come at the beginning. The second proposition taken up may be one which depends on the first one (together with the axioms and definitions), or it may be another of the group just mentioned; the latter is the case in the Wentworth-Smith text, and likewise with Prop. 3, Prop. 4 being the first one whose proof uses one of the preceding theorems.

It is an interesting exercise to arrange the propositions of a geometry text into groups according to the number of links in the logical chain which connects each one with the axioms. The table below shows this arrangement for the propositions in Book I of Wentworth-Smith. Those placed in Group A all depend directly on the axioms and definitions, those in Group B all depend on one or more of those in Group A, those in Group C on one or more of those in Group B, and so on. The number in parentheses following each proposition in Group B and beyond indicates the proposition of the preceding group on which its proof depends. The corollaries are not listed, but each one is regarded as belonging to the group next beyond the theorem to which it is attached[1]; thus Prop. 12, which depends on the corollary of Prop. 10, is placed in Group E, since Prop. 10 is in Group C and hence its corollary in Group D.

A: 1, 2, 3, 5, 7, 20, 30.
B: 4(2), 8(2), 9(2), 23(2).
C: 6(4), 10(9), 11(9), 13(8), 14(8), 24(23).
D: 35(11, 13).
E: 12(10 cor.), 15 (14 cor.).
F: 16(15).
G: 17(16), 18(16), 26(16), 34(12 cor.).
H: 19(18), 25(18), 27(17), 28(17), 29(26).
I: 31(26 cor.), 32(19).
J: 21(19 cor.), 33(32).
K: 22(21).

If this analysis is extended to Book II, two additional groups are necessary: Group L contains Props. 7 and 8, which depend on Prop. 22 of Book I, and Prop. 21, which depends on Cor. 1, of Prop. 16, the latter in turn depending on Cor. 3 of Prop. 19 of Book I; and Group M contains Props. 31 and 32, which depend on Prop. 21.

[1]This is usually but not invariably the case, for occasionally an author states as a corollary what is really just a different wording of the theorem, or else something which has been proved incidentally in proving the theorem. Again, when a theorem has two or more corollaries, one of them may depend on one of the preceding corollaries rather than directly on the theorem.

One of the writer's students, Mr. L. F. Benson of Rosebud, Texas, has carried this analysis through all five books of Wentworth-Smith. He finds that the last group is S, which contains Props. 15, 20, and 22 of Book IV. These are construction problems, namely to construct a polygon similar to two given similar polygons and equivalent to their sum, to construct a polygon similar to a given polygon and equivalent to another given polygon, and to construct a polygon similar to a given polygon and having a given ratio to it. They may be regarded as the most recondite propositions in Wentworth-Smith, in the sense that the constructions and proofs require a longer chain of reasoning, starting from the axioms and definitions, than any of the other propositions. Mr. Benson made a similar analysis of several recent textbooks on Plane Geometry; none of them required as many groups as the Wentworth-Smith book.[2]

The group to which a theorem belongs depends of course on the particular method used in proving it. Thus in Wentworth-Smith Prop. 5 of Book I (If two angles of a triangle are equal, the opposite sides are equal), which is in Group A, is proved by a scheme which consists essentially in superposing the triangle on itself. On account of the serious logical objections which can be made to such a procedure, some other proof is usually given for this theorem. One method is to draw AD and BE bisecting the equal angles A and B respectively, prove the triangles ABD and BAE congruent, and then prove the triangles ADC and BEC congruent. If this method of proof is used, the theorem falls in Group B, since it depends now on Prop. 3. Most of the recent texts leave this theorem until later, and prove it by drawing CD perpendicular to AB and then proving the right triangles ACD and BCD congruent, using the theorem that two right triangles are congruent if a side and an acute angle of one are equal respectively to a side and an acute angle of the other. The latter theorem seems not to

[2]Mr. Benson's results are contained in his M.A. thesis: "A Study of the Order of Propositions in Plane Geometry," a copy of which is deposited in the library of the University of Texas.

be given by Wentworth and Smith; it can easily be proved by means of Props. 3 and 19 of Book I, and thus belongs in Group I of the table above, so that if Prop. 5 were proved in this way it would be in Group J.

Since logical considerations still leave a considerable degree of freedom in arranging the propositions, other factors must determine the actual choice of the sequence. The most important of these factors is the desire to group together theorems dealing with the same topics. This grouping is conspicuous in Euclid's own sequence, which has been followed more or less closely by all writers of geometry texts since until very recent times, often doubtless simply because his order had become the traditional one. The division of geometry into "Books" illustrates this grouping, as indicated by the titles "Rectilinear Figures," "The Circle," etc. Within each Book we have smaller groups of theorems, such as those dealing with the congruence of triangles, parallel lines, parallelograms, etc. The desire to bring such related groups of theorems together is a natural and commendable one. It was probably the desire to bring Prop. 5 of Book I next to Prop. 4, of which it is the converse, which led Wentworth and Smith to choose the proof which they did.

Other considerations which have influenced recent writers are of a pedagogical order. Euclid himself wrote for mature men who were particularly interested in mathematics and philosophy, and during ancient and medieval times no one except professional scholars ever studied his treatise. In this country as late as a century ago the geometry of Euclid was a university subject, and it was not until the latter part of the nineteenth century that it became a regular part of the high-school curriculum. Furthermore, the increasing prosperity of the American people and the spread of the democratic ideal of an education for all have crowded our high schools with immature boys and girls from all strata of society, many of whom have no serious intellectual interests and very mediocre ability. The attempt to teach geometry to this inchoate conglomeration by means of textbooks modelled closely on Euclid produced such unsatisfactory results that a movement set in early in the present century to reform the teaching of geometry and adapt the textbooks to the existing conditions. This movement has resulted in vast improvements, as may be seen by a comparison of a few recent texts with those in use twenty-five or thirty years ago. Among these improvements changes in the order of the propositions are of only minor importance, but some attempt has been made to give easy proofs at the beginning of the course and postpone the more difficult ones until later, or even to eliminate them altogether. When this conflicts with the requirements of logic, the device is sometimes adopted of stating a theorem in its logical position without proof, and informing the student that the proof will be taken up later on. The most important result of the reform movement is the increased emphasis on original exercises, but this lies outside the scope of the present article.

BROWN UNIVERSITY MATHEMATICAL PRIZES FOR FRESHMEN

The examination for the Brown Mathematical prize for freshmen was held this fall on October 12, at 2 P.M. There were 57 contestants entering. Only 38 papers were handed in, containing 23 correct solutions. A comparison with the corresponding statistics for last fall shows increased interest and accomplishment.

The successful contestants were: Allan D. Walker, El Paso High School, First Prize; Robert Greenwood, Navasota High School, Second Prize; Joe Muenster, Austin High School, Third Prize.

The questions were as follows:

1. Simplify:

$$\frac{\sqrt[3]{\dfrac{x}{y}}\,(xy)^{3/4}}{\sqrt{x^{1/3}\,y^{2}\,\left(\dfrac{y}{x}\right)^{-2}}}$$

2. If A pays p dollars for r neckties and B pays p+q dollars for r+q neckties of the same quality, which buys more cheaply?
3. If each of two circles is tangent to two parallel lines and also to a transversal, prove that the length of the segment of the transversal included between the parallels is equal to the distance between the centers.
4. A point P moves on the line RQ. In what position is P when it is equidistant from two points A and B? Discuss various positions of A and B.

For information in regard to the Brown University Prizes see the University of Texas Catalog and the *Texas Mathematics Teachers' Bulletin*, Vol XIII, 1.

THE EARLIEST ARITHMETICS IN AMERICA

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Latin American Librarian, University of Texas

If the average teacher of mathematics were asked when and where the first arithmetic was published in the New World, the answer would probably be Hodder's *Arithmetic,* published in Boston, in 1719. It is strange that with our typical provincialism, or is it egoism? so many still believe that everything began in the New World with the American colonies on the Atlantic seaboard. We are still only half conscious of the fact that more than a hundred years before the Pilgrims landed in Plymouth, the Spaniards and the Portuguese had not only established permanent settlements in the two Americas but had conquered, explored, and mapped the greater part of the two continents.

Thus we have a printing press established in Mexico City as early as 1536. Twenty years later, in 1556, this press brought out the first work on mathematics to be published in America. Frey Juan Diez, a religious brother, the author of the quaint and rare little book, journeyed all the way from Peru to have his work published in Mexico. The title was *Sumario compendioso de las quentas de plata y oro que en los reynos del Piru son necessarias a los mercaderes: y todo genero de trantantes. Con algunas reglas tocantes al Arithmetica.* As is evident from the title, the work is not primarily an arithmetic but a series of rules and tables for the rapid and easy conversion of the value of raw silver and gold. The book consists of one hundred and three folios, paged consecutively, the last twenty-four of which deal with arithmetic. Of these twenty-four pages, six are dedicated to Higher Arithmetic, or Algebra. This section deals primarily with quadratics. It is noteworthy, considering the general state of algebra in Europe at this time.

Apparently Diez came to Mexico purposely to publish his work, as is shown from the statement of the Viceroy in the permit to print. At this time there was no press in Lima.

Little or nothing is known concerning the good brother, Icazbalceta, the distinguished Mexican bibliographer, surmises that he must have been a merchant, but wisely adds that the knowledge revealed in the book is far above that of the average business man of his day. He apparently took back to Peru the whole edition of his book, since no copies have been found in Mexico.

According to David Eugene Smith, who reprinted the rare little book in facsimile with an English translation in 1921, only four original copies are known to exist. The Garcia Collection has only one of the facsimile reproductions.

The second arithmetic to be published in the New World was that of Pedro de Paz. This can really be called the first work printed in the New World dedicated entirely to arithmetic. Under the title of *Arte para aprender todo el menor del Arithmetica, sin Maestro,* it was printed in 1623 by Juan Ruyz, a well known printer of Mexico City. The work was dedicated by its author to Diego de Guevara y Estrada, Chancellor of the Metropolitan Church of Mexico, and consisted of one hundred eighty-one folios of text with seven preliminary leaves, a table, and five leaves of index without pagination. The only copy known to exist was owned by Medina, the well known Chilean bibliographer, who described it for the first time in his *Imprenta en Mexico.*

All that is known of the author is that he was an accountant of tithes in the Metropolitan Church of Mexico. This explains the dedication. In the foreword to the reader the author states that he has labored many years in the preparation of this book and that it was his intention to include a complete treatment of *el arte menor y mayor;* that is, the lower and higher art of arithmetic. He emphasizes the great need for a book from which the student or person interested can learn the principles of arithmetic *without a teacher,* and further commends his work by saying that fractions have been fully treated. He declares that the book should be useful both to those who cannot afford the high cost of individual instruction, and to those who employ a teacher by supplying a text that can be followed systematically.

It was not till 1649 that the first truly American arithmetic was printed. In this year Anastacio Reaton Pasamonte, a native of Mexico City, published his *Arte Menor de Arismetica y Modo de Formar Campos.* Being a native of the city, he must have received his education at the National University of Mexico. His treatise on arithmetic is, therefore, the first produced by a native, who was trained in the Mexican University, and whose work was printed in Mexico. An original copy of this extremely rare arithmetic is to be found in the Garcia Library. It consists of seventy-eight folios; that is, one hundred and fifty-six pages of text, preceded by five preliminary leaves on the second of which is the coat of arms of Pedro de Soto Lopez, who was the accountant of the Holy Office of the Inquisition and President of the Academy of Merchants of the Kingdom of New Spain. The reason for preceding the work by his coat of arms is that the author dedicated it to him, a custom much in vogue in those days when patronage was needed for the publications of scientific works more even than for the publication of literature. The colophon reads: *Con licencia, Impresso en Mexico, por la Viuda de Bernardo Calderon, en la calle de San Agustin, Año de 1649.*

It appears from the foreword to the reader that Reaton Pasamonte had made an attempt to publish an arithmetic under the title of *Contador General de Arismetica y Geometria* twenty years before, but that after he had secured the necessary license to print, he found the cost so excessive in New Spain that he had decided to send his maiden effort to the mother country, Spain, for publication. His choice was followed by misfortunes and, after many delays, it seems that the original work was lost as he says "due to the inclemency of the sea." He declares, however, that "seeing the grave necessity in New Spain and the other kingdoms of His Majesty of learning how to count with that perfection which is requisite," he has determined to bring out this brief treatise in order that "the essentials of the *Arte menor de Arismetica* be readily understood by means of the most concise method devised up to the present day." Whether the last sentence was due to the printer's influence in his desire to encourage the sale of the book, or whether it was the sincere opinion of the author concerning the true merits of his work will have to remain a question to be settled in the mind of each individual reader.

The table of contents includes twenty-four chapters. The first of these deals with the simple subject of the value of ciphers and the methods of counting. The four chapters that follow take up the four fundamentals of addition, subtraction, multiplication, and division. These are followed by a chapter on checks to verify the processes discussed. Up to this point there is nothing extraordinary. Then follow two chapters headed as follows, "How to add diverse things," "How to subtract diverse things," in which the author gives examples involving different money values, and points out methods for reducing the various coins in use to a common value. He then takes seven chapters from IX to XV to discuss fractions, discussing addition, subtraction, multiplication, and division separately, and giving lastly a chapter on checks or proofs to test the operations discussed. In chapter XVI he discusses "progressions" under the title of *De las progresiones de los numeros.* He declares there are three kinds of progressions: arithmetical, geometrical, and musical, but that he will present only the first two of these.

Of particular interest is his quaint yet thorough explanation of proportions and simple equations which he calls *Regla de Tres* (Rule of Three). In five chapters, from XVII to XXI, he explains in detail the various types of simple equations and their practical application to problems in trade and partnership. He has two chapters that deal in particular with the proportional profits in partnership. This section is very interesting.

He dedicates chapter XXII to square roots. Chapter XXIII, which he entitles *Del orden que se ha de tener para* *formar campos* (Order to be observed in forming "fields"), is on the face of it an application of arithmetic to the execution of various military formations, but in reality it is an introduction to the rudiments of plane geometry. On the whole, this rare treatise on arithmetic, appearing in Mexico only thirteen years after Harvard had opened its doors as a grammar school, and seventy years before Hodder's *Arithmetic,* is a remarkable contribution to the history of science in America.

Another item of interest to the mathematician in the Garcia Library is a manuscript work of Fray Andres de San Miguel, a remarkably learned friar who came to Mexico towards the close of the sixteenth century as a barefooted Carmelite, and who was very much interested in mathematics. He was born in Medina, Spain, in 1577 and after spending many years in Mexico died in Salvatierra in 1644. He left a large folio book in manuscript with many curious and illuminating notes on arithmetic, algebra, architecture, and astronomy. The mathematician of those days included in his interests all four branches.

The manuscript work of Fray Andres de San Miguel is among the many manuscripts of the Garcia Library. It is a large folio volume, consisting of one hundred and seventy folios (that is, 340 pages) written in a close but legible hand and with numerous illustrations and full page drawings of geometric figures, architectural designs, and astronomical sketches to illustrate celestial phenomena. It is a most interesting item and one which never fails to interest the visitors that frequent the Garcia Library.

There are other items of interest to the student of the development of mathematics and its teaching in the New World which we will not describe at this time for lack of space. The student interested in the early development of science in the New World cannot afford to neglect the rich materials in the Garcia Library, where besides the works cited on mathematics, there are many other curious and interesting works on the various sciences.

EXAMINATIONS ON HIGH-SCHOOL ALGEBRA AND GEOMETRY GIVEN TO FRESHMAN MATHEMATICS CLASSES

A number of times in previous years, examinations in high-school algebra and geometry have been given to our freshman students. Sometimes the Department of Pure Mathematics as a whole has required these examinations, but more frequently it has been left with the individual instructor as to whether he would examine his sections.

These examinations have been given with the two-fold object of revealing to the instructor the needs of his students and of enabling the student himself to realize and overcome his deficiencies.

Last fall, by vote of the Department of Pure Mathematics, it was decided that each freshman section should be given an examinaiton on high-school work. Instructions were also given each student to record on his paper the name of the high school from which he received his training in mathematics. Each instructor was responsible for the grading of the papers from his sections. These grades were then turned over to a member of the department, and were filed and classified according to high schools.

While a uniform examination was not possible this year, as the various sections were examined at their regular class hours, a step was made in the direction of uniformity by giving the same examination to sections meeting at the same hour.

Of course, only tentative conclusions may be drawn from the results of these examinations; but if the same plan is continued for several years, as in the case of the Department of English, statistics of considerable value will be afforded.

The following points are of interest:

	1928–29	1929–30
Number of schools from which students come	355	356
Number of students taking the examination	880	900
Number of students failing on the examination	571	566
Number of students passing the examination	309	334
Percent of students passing the examination	33.9	37.1

A few more general remarks may be made concerning the results of the tabulation. There are only seventy-five high schools with three or more taking the examination. Several schools with four students have made a record of 75 percent passing. Alvin, Bonham, and Lockhart, with five students have 60 percent passing. Among the larger schools the best records are made by Austin, El Paso, Fort Worth, and Galveston.

Grades made by the student on these elementary examinations are not counted in determining his term grade. An investigation is in progress to discover what correlation there is between the grades on the entrance examinations and on the semester's work. A very gratifying response has been made on the part of school officials to the reports of the examinations sent out to the schools concerned. A number of individuals are at work on the problem of bridging the chasm that seems to exist between high school and college courses. We hope to have some reports to give next quarter on some of these investigations. Let us hear from you, teacher of mathematics, whether you are in high school or university.

Two of the examinations that were given the freshmen this year are appended.

1. $\frac{1}{2} + \frac{3}{7} - \frac{2}{5} = ?$

2. Reduce 5/13 to a decimal (3 places).

3. Factor completely x^4-16y^8.

4. Solve the equations:

   (a) $\dfrac{Ax}{c}+D=Bx+A$.

   (b) $3x^2-17x+10=0$.

5. The width of a room is 3/4 of its length. If the width were 4 feet more and the length 4 feet less, the room would be square. Find its dimensions.

6. In the triangle ABC, DE is parallel to AC and $AD=10$, $DB=4$, $BE=3$. Find BC.

7. Simplify:

$$\frac{\dfrac{a}{a-b}-\dfrac{a}{a+b}}{\dfrac{b}{a-b}+\dfrac{a}{a+b}}.$$

8. Rationalize the denominator of $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$.

1. Simplify $\left(\dfrac{2}{3}\right)^3$, $-\sqrt{25}$, 65×0.

2. Simplify $a-\left\{b+\overline{c-d}\right\}$, $\dfrac{1}{5+2/7}$

3. Factor $16-y^2$, $a^2-3ab+2b^2+a-2b$

4. Simplify $\left(\sqrt{7}\right)^2$. $\sqrt{3}\times\sqrt{27}$, $\dfrac{1}{\sqrt{2}}$

5. Solve for x: $2x+3=7-x$, $\dfrac{2}{x}=5+\dfrac{4}{x}$

6. Simplify $\dfrac{x^2y^7}{x^4y^3}$, $\sqrt{x^2\sqrt{x^4}}$

7. Solve: $2x^2 - x - 3 = 0$.

8. Solve simultaneously: $3x - 4y = 4$, $2x + 2y = 3$.

THE USE OF PROPORTION IN THE MATHEMATICS OF FINANCE

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It is to be regretted that skill in the use of proportion, such as may be acquired sometimes in the lower grades, soon passes away as a student pursues more advanced studies in mathematics. I do not mean that he uses proportion incorrectly—as did the Irishman who was told that if he bought a certain kind of stove he would save half his coal, and so decided to buy two stoves and save all his coal. I mean that very few students in college seem to have any knowledge of proportion at all. They cannot use proportion when the implications are obvious. They prefer to grind out formulas by algebraic processes—with no concomitant mental vibrations.

In the mathematics of finance and life insurance, *actuarial mathematics*, a great many formulas are derived by one method or another. Such formulas are extremely useful for the rapid solution of problems of a given type which arise repeatedly in routine office work. Algebra is generally regarded as the most satisfactory tool to use in carving out these formulas. But a lover of proportion "for its own sake" can get a thrill by noting how large a part of the theory of finance can be explained in terms of proportion applied to a few fundamental conceptions and definitions.

Without attempting to be exhaustive, I shall give here a few typical illustrations.

A man deposits $1.00 in a bank that pays an annual interest of 4 percent. He understands that he may draw out $0.04 at the end of each year, forever—if not in person, at least through his "heirs and assigns forever." How much should he deposit if he wishes to draw out an annual $1.00 forever? The answer is simple. He wishes to increase his interest twenty-five fold, so let him increase his deposit twenty-five fold. In symbols, $\$1.00/.04=\$25$.

A donor wishes to endow a fellowship to pay $750 annually. What must he donate if money will earn 5%? $750/.05=$15,000. Interest at 5% means that money will earn one one-twentieth part of itself. Hence, multiply $750 by 20. Such periodic payment of a sum of money forever is called a *perpetuity*.

But let us write down now the main formula of *simple interest.*

Interest=Principal×Rate×Time.

Thus, *interest* is *proportional jointly* to *principal,* to *interest rate,* and to *time.* This gives rise to quite a variety of problems in proportion as found in elementary arithmetics.

For *compound interest,* the fundamental formula is quite different; but even here, the *interest* and the *amount (principal plus interest)* are *proportional* to the *principal.* And after a few preliminary definitions, *proportion* can be used effectively.

Suppose $100 is at 4% compound interest for two years.

	$100.00=Principal
Add 4%	4.00=Interest for first year
	$104.00=Amount at end of 1 year
Add 4%	4.16=Interest for second year
	$108.16=Amount at end of 2 years.

The result may be written briefly:

$\$100(1.04)(1.04)=\$100(1.04)^2=\$108.16$.

For to add 4% to a number is to take 104% of that number. In other words we multiply that number by 1.04, called here an *accumulation factor.*

In the general case, the amount S of P dollars after n periods with compound interest at rate i per period is

$$(1) \qquad S=P\,(1+i)^n$$

Indeed, we relinquish the supposition that n is a whole number and *define* the amount S to be

$$(2) \qquad S=P(1+i)^t$$

where t is the time in terms of a certain unit period. Here t may be any positive real number; in fact if by *amount* we mean merely *value* at time t, this time may be any real number, positive or negative. It is customary, however, to think of S as the value at the later date, P the value at the earlier date, thus keeping t positive. And from (1) we may now derive

$$(3) \qquad P=\frac{S}{(1+i)^n}=S(1+i)^{-n}=Sv^n,$$

where

$$(4) \qquad v=\frac{1}{1+i}.$$

This v is called the *present value* of unit of money due in unit of time. A deposit of v, improved at interest at rate i, yields by (1), (4), after unit time the amount $v(1+i)=1$. In (2), the principal P is often designated as the *present value* or *discounted value* of S due n periods later.

From equations (1)–(4) which *define* compound interest and set forth some immediate conclusions, let us revert to the use of proportion.

How much must a donor leave at 5% compound interest for 20 years so that the accumulation at *that time* will endow an annual fellowship of \$1000 with first payment 21 years from present date? A deposit of \$1 now will accumulate in twenty years to

$$(5) \qquad (1.05)^{20}=2.6532977$$

as found from a compound interest table. This would yield as interest, to be withdrawn annually

0.13266488

Hence, the initial deposit to provide for the fellowship of \$1000 is

$$\frac{\$1000.00}{0.13266488}=\$7537.79.$$

Such a periodic payment is called a *deferred perpetuity.* The deferment is here for twenty years, the first payment due twenty-one years from date, instead of being due one year from date.

A building, costing \$100,000, is already paid for. How much should be placed on deposit at 5% compound interest so that at the end of each period of 20 years, forever, there will be available \$100,000 with which to replace the building? From (5) it is seen that the *interest* on one dollar for 20 years is

$$(1.05)^{20}-1=1.6532977$$

Hence, to get \$100,000 as interest at the end of each period of twenty years, an initial deposit must be made of

$$\frac{\$100{,}000}{1.6532977}=\$60{,}485.18.$$

The *capitalized cost* of this building is

$$\$100{,}000+\$60{,}485.18=\$160{,}485.18$$

To summarize results thus far obtained, let it be assumed that the interest on \$1 for one unit of time or *period* is i, and that the amount of \$1 for n such periods—under compound interest—is $(1+i)^n$. Thus the *interest* on \$1 for n periods is $(1+i)^n-1$. It follows then by simple proportion that to get a yield of R per period, forever, requires an original investment of

$$(6) \qquad \frac{R}{i}$$

The first receipt of R in this case is at the end of the first period. If there is deferment for n periods—first receipt at end of $(n+1)^{th}$ period—then the original investment is

$$(7) \qquad \frac{R}{(1+i)^n i}=\frac{Rv^n}{i}$$

Again, if R is to be received at the end of each *nth* period, forever, the original investment required is

$$(8) \qquad \frac{R}{(1+i)^n-1}$$

But if R/i will buy a perpetuity of R, with first payment at end of first period, and if Rv^n/i will buy a perpetuity to start n periods later, then the difference

$$(9) \qquad R\left(\frac{1}{i}-\frac{v^n}{i}\right)=R\frac{1-v^n}{i}$$

will buy an *annuity* of R per period to last just the *first n periods.*

One of the most important formulas for the mathematics of finance is

$$(10) \qquad a=\frac{1-v^n}{i},$$

giving the present value of \$1 per period for n periods.

How much must a man pay at the end of each year for five years to pay off a debt of \$4000, when money is worth 8%? Now a here is the present value of \$1 per year for 5 years. Hence \$1 is the present value of $1/a$ payable for 5 years. Hence a debt of \$4000 will be discharged by annual payments of

$$\$4000.\frac{1}{a}=\$4000(0.2504564)=\$1001.83.$$

The borrower pays in all \$5009.16; of this, \$1009.15 is interest.

Let us take a simple illustration from life insurance. A man wishes to discontinue payments for life insurance at a time when his policy has a *policy value* of \$2500. How much *paid-up insurance* can he buy with this policy value, if he is 35 years old, and the American Experience Mortality Table is used with interest at 3½%? From tables it can be found that

$$A_{35} = 0.37055,$$

which means that a \$1000 paid-up policy on the life of a man of age 35 is worth in cash \$370.55 Usually insurance is secured by annual payments, rather than by a single payment—it may be noted in passing. But the above A_{35} is of significance in the problem proposed, because by simple proportion the paid-up insurance which the policy value of \$2500 will buy is

$$\frac{\$2500}{A_{35}} = \frac{\$2500}{0.37055} = \$6746.73$$

In general, A_x is the *single premium* at age x for insurance of \$1; it is thus the present value of one dollar payable at the death of a man now age *x*. Likewise, the symbol a_x is used to designate the present value to a man of age *x* of a *life annuity* of \$1 per year payable to him at the end of each year he survives. It is a function that is used to compute the value of *pensions* and also the value of the *premiums* or payments to insurance companies for insurance carried. Simple relations exist between A_x and a_x.

Indeed, suppose that a man of age 35 deposits \$1000 in a bank that pays 4% interest, with the understanding that throughout his life-time he is to receive the annual interest of \$40, and that at the end of the year of his death the principal of \$1000 is to become due, together with the \$40 interest for the current year, and be payable to a designated beneficiary—in total \$1040=\$1000(1.04). After making this arrangement, he sells to a third party his *life interest* in the annual \$40 interest for its value

$$\$40\ a_{35} = \$1000(.04)a_{35}.$$

His net investment for the "insurance" of \$1040, which he has planned for the beneficiary, is then

$$\$1000(1-.04\ a_{35}).$$

For an insurance of an even thousand dollars, the net investment by *simple proportion,* would then be

$$\$1{,}000\frac{1-.04\ a_{35}}{1.04},$$

which, in the usual notation, has also the value

$$\$1000\ A_{35}.$$

Thus
$$A_{35}=\frac{1-.04\ a_{35}}{1.04}.$$

The general relation

$$(11)\qquad A_x=\frac{1-ia_x}{1+i},$$

is likewise established, for any age x, and interest rate i.

Such formulas are also established algebraically by use of the equations defining A_x and a_x in terms of appropriate *probabilities* with the aid of discounting factors.

Necessary are the algebraic and analytic developments and proofs in actuarial mathematics. But so often a little arithmetic throws a flood of light upon stupid-looking formulas. Arithmetic, not arithmetic that spends worlds and worlds of time in reducing pints to quarts, bushels to pecks, *et cetera, et cetera.* But arithmetic in which *proportion* plays the leading role.

A TEACHING PROBLEM

A most interesting article appears in the *Mathematics News Letter* published in Baton Rouge, La., April, 1929:

> It appears from reports of the work of Miss Adelia Palacios, professor of mathematics in the National School of Teachers and the University of Mexico City, that she has made a thorough job of the search for and application of the underlying principles governing the psychology of learning mathematics.

We agree with the *News Letter* in saying that:

> We await with interest the coming of a new method of teaching mathematics that will arouse in the pupil a keen desire to do his best work, and that will reduce the learning time of the fundamentals of arithmetic. We will welcome with open minds Miss Palacios' theories and stand ready to accept all that will be helpful in the many problems of both the teacher and the pupil in the study of mathematics.

The *News Letter* itself is a significant journal, since it is "published under auspices of the Louisiana–Mississippi Section of the Mathematical Association of America and the Louisiana–Mississippi Branch of the National Council of Teachers of Mathematics," and is devoted "to mathematics in general, to the following causes in particular: (1) the common problems of grade, high school, and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of M.A. of A. and N.C. of T. of M. projects."

THE TEACHING OF SECONDARY MATHEMATICS

J. H. SHEPPEARD
Chairman Mathematics Section T.S.T.A., 1928–1929

In recent years a great deal of literature has appeared dealing with objectives, methods, and materials in the teaching of secondary mathematics. In this literature one will find a wide divergence of opinions. If the proper objectives could be determined, perhaps the problem of adapting methods and materials would be greatly simplified. One will tell us that the major objective should be to train students in that phase of mathematics applicable to the practical affairs of life. Another will tell us that one important objective is to prepare students for a more advanced study of mathematics in the college or university. Some questions naturally arise. One is, exactly what is meant by the practical affairs of life? There are some affairs of life which, in their social and economic relations, are extremely practical, and yet the administration of these affairs requires a skill in mathematical technique far beyond that which may be acquired in the secondary school. Another question is, who are going to college, and who are going to pursue advanced studies in mathematics? Most of us, we believe, will admit that these questions cannot be answered with that degree of precision that will enable us to define specifically our objectives. Perhaps if we approach the problem from a different viewpoint we shall be able to define more clearly the objective.

If we conceive the fundamental objective in any phase of the educational process to be the furnishing of training adapted to the student at his particular level of development, this objective would apply as well to mathematics as to any other subject in the curriculum. We would not concern ourselves so much about what practical application the student will make of his mathematics, or whether or not he is going on to more advanced study of the subject; but the question would be what mathematics, if any, is adapted to his need at his particular stage of development? No doubt we shall discover some practical needs that will demand a knowledge of certain phases of mathematics. On the other hand we may discover that his intellectual need and attitude will justify our giving him training in that particular phase of mathematics that involves the more abstract processes of judging and reasoning. It is not improbable that we shall discover that some phases of mathematics will supply his emotional need in the play of his imagination and in the sense of his own achievement. We are persuaded that one of the strongest incentives one can have for pursuing the study of mathematics, or any other subject as for that matter, is the sense of his own achievement.

Some may urge that there are other subjects that will supply most of these needs equally as well as mathematics and at the same time have a greater social value. That may be true, but many of the subjects taught in the secondary schools lie in the fields of speculation and probability and do not furnish the inquiring mind that degree of mental satisfaction that is furnished by an exact science such as mathematics. There is nothing more wholesome for the mind disturbed by conflicting theories and opinions than to be able to lay hold on some truth and to say I know this is true. Mathematics lies outside the fields of opinion and speculation. For minds able to comprehend it, it is just as true for one as for another. The truths of mathematics are absolute, eternal, and universal. Man did not invent mathematics. He has discovered many of its truths and their applications, and as a result of his discoveries many other sciences have been developed without which our modern world could not exist.

In any rational system of education, materials originate in response to man's felt needs. Our problem then is to discover the needs of the student and to adapt materials and method of procedure to his needs. If we recognize the principle of individual differences, we at once understand that identically the same materials and methods are not universally adapted. The successful, growing teacher does not expect to discover some fixed formula either as to content of the course or of the method of procedure. The problem is an ever present one, to discover the needs of his students and the material adapted to their needs. The growing teacher is alive to this problem; and as he solves it from day to day, he brings to his students a spirit of enthusiasm that will inspire them to greater effort and will lead them into a fuller appreciation of the beauties of mathematics. The teacher who knows and loves mathematics, who has an abiding interest in the welfare of his students, and who understands the psychology of the adolescent mind will have little need to make inquiry concerning the content of the course or the method of procedure; but he will find his greatest delight in working out his own problems and making his entire procedure rational to himself.

The materials in mathematics adapted to the secondary levels are abundant, and vastly more than any one group can master within the alloted time. Instead of those responsible for the formulation of the course of study selecting materials specific in amount and kind, it would be better to compile a rather extensive list of suitable materials and then allow the teacher considerable freedom in selecting from this list such material as he may find best adapted to the needs of any particular group with which he is dealing. Formal, iron-bound courses of study that must be administered on the lock-step plan are absolutely deadening, and there is no wonder that the teaching of mathematics in the secondary schools has been subjected to severe criticism. Break the fetters that bind both teachers and students and allow them to breathe the air of freedom, and no longer will mathematics be the slaughter pen of the secondary schools.

The outstanding need is well trained teachers. No one whose chief interest lies outside the field of mathematics should undertake to teach it. One must know mathematics, love mathematics, and have faith in its educational value before his attitude toward the subject will be such as to make his teaching of it effective.

THE OLDEST BOOK IN THE LIBRARY OF THE UNIVERSITY OF TEXAS

JEANNETTE KING BAGBY

A first edition copy of the earliest printed mathematical book, Campanus' translation of Euclid's *Elements of Geometry,* which contains drawings of mathematical figures, is now in the Stark Collection of the University of Texas. It was printed in Venice in Latin by Erhard Ratdolt in 1482, ten years before the discovery of America.

The book is in its original binding of brown leather and appears to have had at one time clasps to fasten it. It contains 137 printed leaves of rough surface paper made from linen rags. In addition there are several rare block leaves inclosing the printed matter. The pages are 8½x12 inches.

There is no title page. On the first page Kästner gives a short description of the first edition of Euclid and dedicates this volume to Prince Mocenigo of Venice. Ratdolt explains in the dedication that books by ancient and modern authors were printed in Venice, yet little or nothing had appeared in mathematics due to the difficulty of reproducing figures. He adds that after much work he had discovered a method by which they can easily be printed.

Figures illustrating the propositions are produced from woodcuts and are placed in the 2½-inch margins toward the outside of the page.

At the end of the book, according to the custom of that time, is given the title of the book, name of the author and printer, and the date of printing. In this book the conclusion is as follows: "Opus elementoru euclidis megarensis in geometriā artē In id quoque Campani perspicacissimi Cōmentationes finiūt. Erhardus ratdolt Augustenis impressor solertissimus. venetiis impressit. Amo salutis. M.cccc.lxxxii. Octauis. Caleñ. Juñ Lector. Vale."

An examination of the contents will reveal little change in the subject matter of geometry as studied today. Perhaps no other subject has remained so little changed as geometry through the centuries. True there has been some rearrangement in the order of propositions with the addition of some axioms or omission of some of the theorems. Also the propositions have been analyzed to make learning easier; and the figures have been placed on the pages with the propositions. Yet the subject matter remains the same and the true mathematician still studies Euclid in preference to an abridged text prepared for pupils in elementary mathematics.

Fifteen books of propositions are included in the one volume of this early Latin edition. A rough outline will give the order and nature of the "Elements of Euclid" as they then appeared, which included:

Book I	Definitions; propositions of a straight line; angles.
Book II	Parallelograms and rectangular figures.
Book III	Circles.
Book IV	Inscribed and circumscribed polygons.
Book V	Ratio and proportion.
Book VI	Similar figures.
Book VII, VIII, and IX	Arithmetic properties of numbers.
Book X	Doctrine of commensurables and incommensurables; theory of rationals and irrationals.
Book XI	Solids.
Book XII	Use of the "method of exhaustion."
Book XIII	Construction of the five regular solids.
Book XIV	Ratio of solids.
Book XV	(a) How to inscribe certain solids within others; (b) how to calculate the number of edges and number of solid angles in the five solids; and (c) how to determine the angle of inclination between faces meeting in an edge of any one of the solids.

There has been some question as to whether Euclid wrote these last two books. Some are willing to give him the credit. Others give the credit to Hypsicles, a pupil of Euclid. It has been suggested that Hypsicles edited these books from material left by Euclid.

As is the case with so many of the great Grecian mathematicians, little is known about Euclid's life and personality. He is thought to have flourished about 300 B.C. Proclus, a mathematician who lived 410–485 A.D., gives a sketch of Euclid. Here it is stated that Euclid lived in the time of the first Ptolemy. Archimedes, who came immediately after the first Ptolemy, mentions Euclid. Also it is claimed that Ptolemy once asked Euclid if there was a shorter way to learn geometry than through the *Elements*, to which he replied there was no royal road to geometry.

Plato died in 347 B.C., so it is inferred that Euclid lived between the time of Plato and Archimedes. It is probable that Euclid received his mathematical training from pupils of Plato in Athens, for most of the geometers who could have taught him were of this school. Then most of the mathematicians on whose works Euclid's *Elements* depend lived and taught in Athens.

It is known that Euclid founded a school in Alexandria and taught there. Pappus in his writings says that Apollonius spent much time with the pupils of Euclid at Alexandria, and in this way acquired a scientific habit of thought.

Most translators and editors of the Middle Ages spoke of Euclid as Euclid of Megara. This error arose because of a confusion between Euclid the mathematician and the philosopher Euclid of Megara who lived about 400 B.C. The first trace of this confusion appears in Valerius Maximus in the time of Tiberius who mistook Euclid the mathematician as a contemporary of Plato. The misunderstanding was general down to the time of Tartaglia (Venice, 1565). However, Constantinus Lascaris, who died about 1493, made the proper distinction by stating that the mathematician was different from Euclid of Megara of whom Laertius wrote, and who wrote dialogues. Thus Constantinus is given credit as the first translator to make this point clear.

THE STRAIGHT EDGE

A "Sorter Catechism" (continued).

Do your students regard algebra as a dead formalism or a living language?

If the former, are you sure that your presentation is not to blame?

Do your students think that if a negative number is carried across an equality sign it will become a negative number?

Could your teaching possibly have given them any reason to hold such an opinion?

Do your students know that logarithms are merely exponents and, hence, subject to the laws of exponents?

Could your students when at their freshest in the subject give a good guess at what number would have for its logarithm 2.857?

Do they know that algebra is a game with a definite set of rules like baseball?

Do you teach the rules and the observance of them?