Bevel Cutting Methods and Cutting Trajectory Control for Steel Laminations Used in Tooling

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Abstract

Bevel cutting of steel laminations used in profiled-edge laminated tooling allows for a more accurate representation of the intended die surface, since stair-stepping at the edges is eliminated. Based on experiments involving maximum cutting speed, bevel angle and kerf quality, the three recommended methods for bevel cutting steel are (best to worst) pulsed Nd:YAG laser with hard-optic delivery, abrasive water jet, and machining with the flute edge of an endmill. For each method, bevel angles of up to 80 degrees are possible. Further experimentation was used to determine the optimal process parameters for kerf quality, with constant cutting speed being one of the main requirements. Finally, a new technique to assure constant velocity along the entire lamination cutting trajectory is developed.

1. Introduction and Background

All lamination-based RP methods begin with intersecting a finite number of specified parallel planes (usually equally-spaced) with a 3-D solid or surface model. The resulting collection of contour or profile curves define how the laminations are to be machined and assembled. When laminations are cut perpendicular to their surface (i.e., zero-order approximation), the assembled array of laminations has a stair-stepped surface as shown in Figure 1(a). Thin laminations (e.g., 0.05 mm) that are common with some RP methods like Laminated Object manufacturing (LOM), only require surface finishing to remedy this problem. With larger models, thicker layers (e.g., 10 mm) are typically used and the stair stepping problem becomes more troublesome. Common remedies in this latter case are to perform secondary machining on the stair-stepped surface or fill in between lamination edges with some hard material (e.g., epoxy).

Many have recognized the advantages of cutting lamination edges with a varying bevel instead of simply perpendicular to the surface. Essentially, the lamination edge is defined by connecting two adjacent contour (closed curve) or profile (open curve) lines with a bevel. As shown in Figure 1(b), bevel cutting allows for a first-order or piecewise continuous approximation of the tool or model surface between lamination interfaces. Prior work with thick lamination beveling includes [Beyer et al.,1988] making laminated models with a laser; [Weaver,1991] making metal tooling with a laser; [Berman,1991] making models with a routing head; [Glozer et al.,1993] cutting aluminum with an EDM for plastic injection molds; [Walczyk,1994] cutting steel with a laser, AWJ cutter, and machining head for sheet metal forming dies; [Lee et al.,1996] cutting polystyrene foam with a electrically heated wire; [Zheng et al.,1996] making models with a laser; and [Hope et al.,1997] making models with an AWJ cutter (Styrofoam).

As seen in Figure 1(c), bevel cutting of a particular lamination requires line-of-site machining instructions defined by a series of position point and unit directional vector pairs ($P_I, \vec{V}$). There are several algorithms that have been developed for defining lamination edge bevels and suitable cutting trajectories. [Keppel,1975] developed a heuristic-based algorithm for obtaining an optimal approximation of the bevel surface defined by randomly distributed points along adjacent contour lines (i.e., contours are defined by co-planar polylines). The surface approximation is made up of a collection of triangular facets that stitch the adjacent contours together. Based on the [Keppel,1975] graph-search technique, [Fuchs et al.,1977] developed a faster algorithm that does not utilize any heuristics, thereby allowing more options for choosing optimizing criteria. A "minimum square span length" between nodes on adjacent contours was used by [Newman et al.,1995] as the cost function in an optimal surface reconstruction. Furthermore, [Newman et al.,1995] used the concept of a *cutting contour*, defined by the midpoints of the spans, to describe a constant velocity cutting trajectory. [Zheng et al.,1996] used a "smoothed" path between two adjacent contours that eliminates the rapid velocity changes corresponding to the transitions between the computed triangular facets. [Lee et al.,1996] simply defined the cutting trajectory as the exact geometry of the two adjacent contours. Instead of defining a lamination by two adjacent contours, [Hope et al.,1997] defines a lamination by its mid-contour curve and the associated surface normal, curvature, and tangency data taken directly from the NURBS surface model. This information is used to establish a linear approximation of the local surface curvature and, hence, the beveled lamination edge. Furthermore, the user can define the thickness of each lamination.

2. Bevel Cutting of Profiled-Edge Laminations

Profiled-Edge Lamination (PEL) tooling (see Figure 2(a)) has been shown to have distinct advantages over contoured lamination tooling (see Figure 2(b)) in terms of automated assembly, registration, securing, and re-machining of the laminations [Walczyk,1998]. Since PEL lamination edges are bevel cut, the issues of developing kinematically and dynamically desirable cutting trajectories for common tooling materials (e.g., steel) must be addressed for successful implementation of this method in industry. Therefore, the focus of this paper is to compare various methods for cutting metal laminations (specifically steel which is arguably the most common tooling material used), discuss the associated cutting trajectory requirements, and describe a cutting trajectory algorithm which fulfills these requirements.

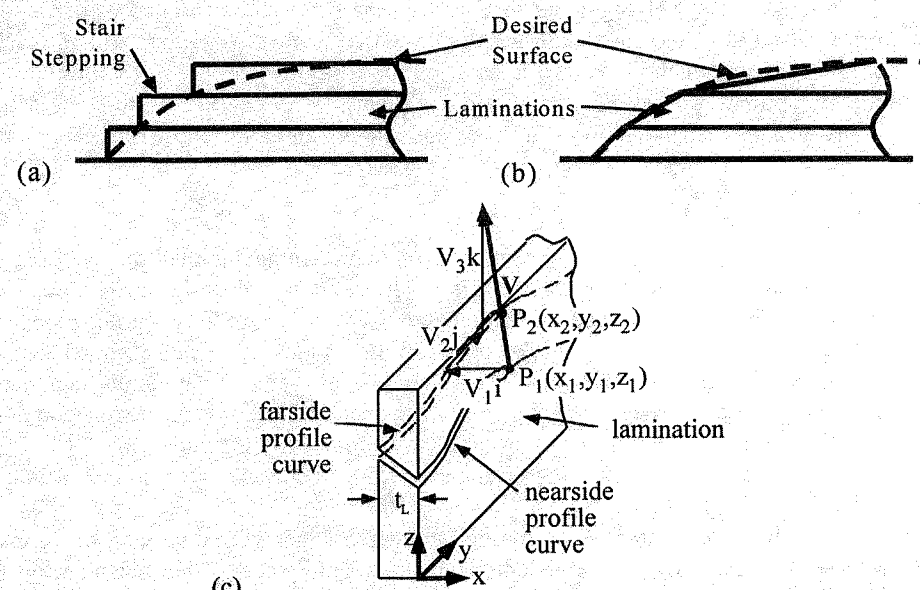

Figure 1 - (a) Stair-stepping of tool surface from perpendicular-cut lamination edges, (b) smoother surface from bevel cutting, and (c) line-of-site machining instructions needed for bevel cutting.

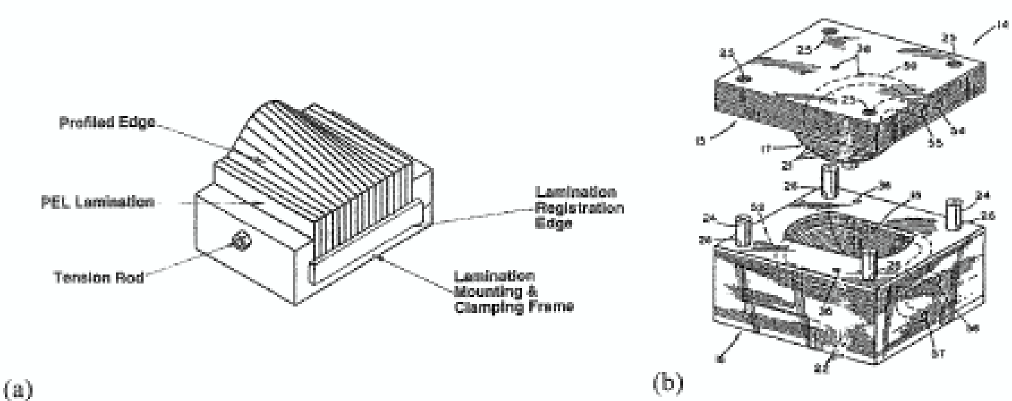

Figure 2 - (a) PEL tooling and (b) contoured lamination tooling.

3. Bevel Cutting Methods for Steel Laminations

The most promising methods for bevel cutting steel laminations are machining with the flute-edge of an endmill, abrasive water jet cutting, plasma-arc cutting, and laser cutting [Walczyk,1998]. To determine how rapidly and accurately they will machine steel PEL laminations, each of these methods was investigated through a series of cutting experiments at bevel angles of 0, 30, and 60 degrees. The basis of evaluation for each beveling method will be the quality characteristics of the bevel cut as defined by Figure 3. A narrow kerf width is desirable so that fine details can be cut into the lamination edge. A low surface roughness of the cut surface is desirable since this will decrease the amount of grinding and polishing time that is necessary for the assembled PEL die forming surface. Minimal or no edge burring on the farside of the kerf is desirable since it must be removed before the die laminations can be assembled into a complete die. Deburring increases the overall fabrication time. The lamination material used for every cutting experiment was 1.5 mm thick SAE 1010 cold drawn steel sheet.

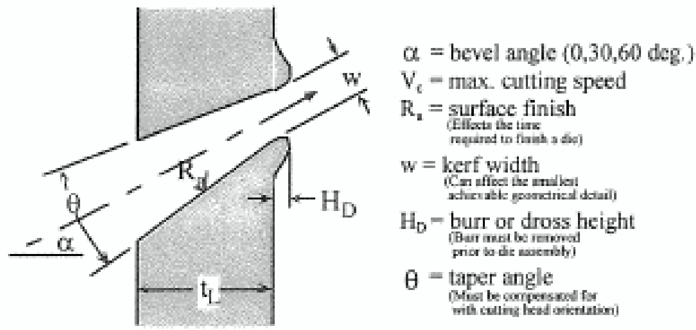

Figure 3 - Quality characteristics of a bevel cut.

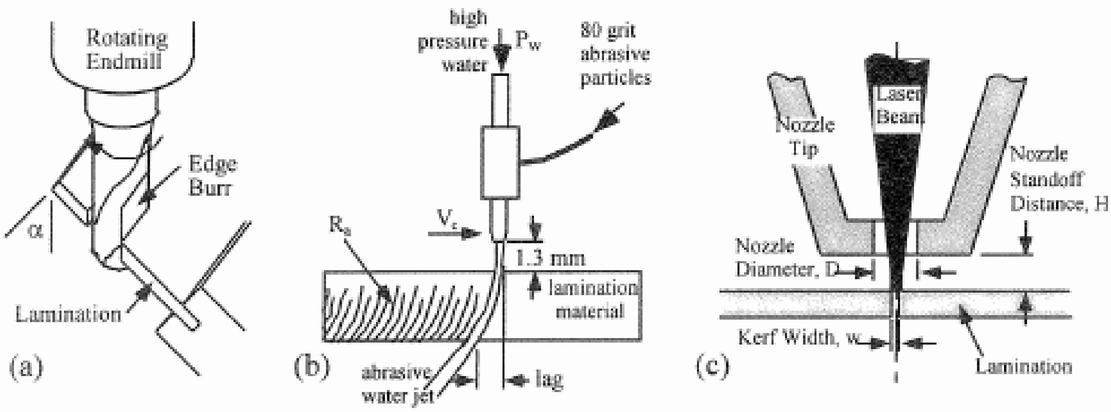

Figure 4 - Lamination cutting methods including (a) machining with an endmill, (b) abrasive waterjet cutting, and (c) laser cutting.

3.1 Machining with the Flute-Edge of an Endmill

As seen in Figure 4(a), a profiled-edge with a compound bevel can be machined using the flute-edge of an endmill mounted in a 5-axis machining center. According to a series of bevel cutting experiments using a 6.4 mm diameter endmill, beveling the 1.5 mm thick steel laminations with the endmill's flute-edge leaves very good surface finishes but cutting speeds are relatively low. Slower cutting speeds (denoted as V_c) and higher spindle speeds (denoted as N) yield the best surface finish (R_a) and the smallest burr (H_D). The maximum V_c and N cutting and spindle speeds for cutting bevels is highly dependent on the material's machinability. Although the presence of coolant did not seem to have any real effect on surface finish and edge burring, it should be used when cutting metal to extend the life of the tool and to minimize the HAZ of the cut. The extent of burring on the climb-milling side of the cut is extensive (see Figure 4a) and it gets longer as the bevel angle increases. Due to the high cutting forces from the large amount of material removed (i.e., width of the tool), there is also significant deflection of the lamination during machining which exacerbates any chattering problem and decreases machining accuracy. For comparison purposes, the average surface roughness, burr height, and maximum cutting speed (and spindle speed used) for each of the bevel angles are listed in Table 1.

Table 1 - Experimental results for flute-edge endmilling of bevels.

Bevel Angle (α)	0°	30°	60°
Ave. R_a (μm)	1.3	4.6	3.1
Ave. H_D (mm)	0.3	2.7	8.6
Max. V_c (m/min)	0.46 (at N=3000 rpm)	0.46 (at N=3000 rpm)	0.25 (at N=2000 rpm)

3.2 Abrasive Water Jet (AWJ) Cutting

As seen in Figure 4(b), an abrasive water jet (AWJ) is a non-contact method for cutting bevels into die laminations. The control factors that affect the cut quality are cutting velocity V_c, upstream water pressure P_w, and size of the abrasive particles. An OMAX JetMachining System was used for the AWJ cutting experiments. The cutting nozzle diameter and standoff distance from the lamination used for all experiments was 0.76 and 1.27 mm, respectively. Since the AWJ manufacturer strongly recommended that an abrasive particle of particular size be used, specifically 80 grit (0.267 mm average particle size), this was not used as a process parameter.

From the experiments, a number of general effects were noticed. It is difficult to generalize about what effect V_c, P_w and their interaction has on R_a because the trends vary with every bevel angle. However, V_c and P_w have a significant effect on the kerf width w, but in opposite directions. Specifically, w tends to increase with higher P_w and tends to decrease with higher V_c. The effect that V_c and P_w have on H_D is inconclusive. Since there appear to be no optimal process parameters for bevel cutting with AWJ in terms of R_a and H_D, the author suggests using a maximum V_c and as low a P_w as is practical. Average values of various quality characteristics from the AWJ experimental bevel cuts are listed in Table 2.

AWJ cutting is a very good non-contact method for machining narrow cuts into die laminations. Because it is non-contact and low forces are imposed on cutting area, there is negligible deflection of the lamination during cutting. Aside from a decrease in the maximum V_c, AWJ cutting seems to have no problems cutting high bevel angles. According to [Tikhomirov et al.,1992], the maximum V_c for this process is highly dependent on the yield strength of the material. However, the measured maximum V_c for 1.5 mm thick thickness lamination material made of mild steel and much harder tool steel are close in value. The surface finish of the bevel cut is slightly rougher than flute-edge endmilling, but more consistent as process parameters change. The burr left on the far edge of the kerf is much less than CNC-machining with an endmill especially at high bevel angles. However, the kerf made with AWJ has a large taper, around 10° for all the bevel angles tested. The kerf taper angle may decrease when thicker laminations are cut and/or when the upstream P_w is increased.

Even though the cut from an abrasive waterjet introduces error in the cut because of the drag line, taper angle of the kerf, and variable kerf width; [Matsui,1991] has successfully compensated for these errors to increase the precision of the AWJ process as shown in Figure 5. Compensation for these errors involves offsetting the centerline of the kerf by half of it's width which is a function of V_c, rotating the cutting nozzle slightly about the direction of travel (y-axis) by half the taper angle, and tilting the cutting head backwards (about x-axis) to minimize the effect of the lag line.

Table 2 - Experimental results for AWJ bevel cutting of 1.5 mm thick steel.

α	0°	30°	60°
Ave. R_a (μm)	4.8	5.1	4.3
Ave. w (mm)	0.62	0.63	0.80
Ave. H_D (mm)	0.061	0.100	0.053
effective thickness of material being cut (mm)	1.47	1.70	2.95
Max. V_c (m/min)	0.34	0.31	0.20
kerf taper	10°	11.5°	9°

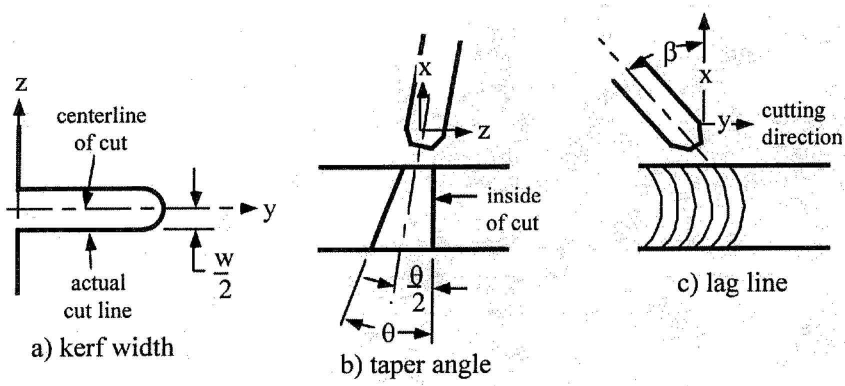

Figure 5 - Compensation methods for AWJ cutting error.

3.3 Plasma-Arc Cutting

Using a plasma-arc is another non-contact cutting method that can be used to cut bevels into steel laminations. The system used for the bevel cutting experiments was a Hypertherm HD-1070 HyDefinition Plasma unit. The control variables in plasma-arc cutting are the plasma gas type (O_2 or Air), the plasma gas pressure P_G, the standoff distance of the cutting nozzle from the lamination H_{SD}, and the cutting velocity V_c. Extensive experimentation was abandoned after it became apparent that a plasma-arc was not suitable for beveled cuts. Bevel cuts at angles up to 45° yielded very large tapers (>10°) in the kerf, excessive dross (H_D=0.9 mm) that was welded to the metal, and a large heat affected zone (HAZ). The large HAZ caused the cut edge of the lamination to warp slightly. The maximum bevel angle achieved was 60° but the kerf edge closest to the cutting nozzle was consistently obliterated from an over-zealous self-burning[1] of the metal during cutting.

The attractive feature of plasma-arc cutting is the maximum allowable cutting speed which is typically 5 to 10 times faster than that of machining and water-jet cutting. Unfortunately the problems of excessive kerf taper and the tendency of kerf edge nearest to the cutting nozzle to experience self-burning make plasma-arc cutting unsuitable for cutting lamination bevels. The key to plasma-arc's successful usage in cutting bevels will be to solve the edge burning problem, minimize or even eliminate (if possible) the kerf taper, and to reduce the amount of dross. Such an effort will require more than the optimization of process parameters which is beyond the scope of this paper.

3.4 Laser Cutting

Laser cutting is another promising non-contact method for machining bevels because of the rapid cutting speeds and steep bevel angles that can be achieved. The main issue to be resolved is what combination of laser type, beam temporal mode, and beam delivery system is best for cutting lamination bevels, particularly in steel. Therefore, a series of experiments were performed with various industrial laser types (CO_2 and Nd:YAG), cutting modes (pulsed and continuous wave), and beam delivery configurations (hard optics and fiber optics) to quantify the ability of a particular laser configuration to cut bevels into lamination material. The object of these experiments was to determine what maximum bevel angles and cutting speeds were achievable.

CO_2 laser light can not be delivered by a fiber optic system. Since most glasses, including those used for fiber optic cables, have a high absorbtivity at the CO_2 laser wavelength of 10.6 μm, fiber optic delivery for a CO_2 laser is not used and is not commercially available [Powell,1993]. A Light beam from a CO_2 laser is almost always delivered with a hard optic system.

CO_2 laser cutting tests were performed using a Laser Dyne Model 780 5-axis CNC Machining System with a Rofin-Sinar 1700 SM laser. Using the laser fusion cutting method, a pulsed CO_2 laser beam with hard optic delivery and nitrogen assist gas successfully cut bevels of up to 45° with narrow kerfs (0.18 to 0.30 mm) but with an extensive recast material (a.k.a. adherent dross) that is difficult to remove. Pulsing energies of 2.7 to 6.5 joules were used. For reactive gas cutting (i.e. oxygen assist gas) in pulsed and continuous wave (CW) modes, bevel angles of only 10° were achievable and most of the cuts exhibited self-burning, presumably from an excess of oxygen in the

[1] 'Self burning' is an uncontrolled oxidation reaction caused by an excess flow of oxygen.

the cut, and taper angle θ. The control factors that affect these output responses are the focused spot diameter d which is proportional to the process lens focal length F, pulse width τ_w, pulsing frequency f_p, cutting speed V_c, type of assist gas (e.g. O_2, Air, N_2), and assist gas pressure P_g. All of these control factors (actual values used are listed in Table 3) are easily varied with the CNC laser cutter used for the experiments. Since almost all Nd:YAG lasers used for cutting have a tuned resonator, the output power of the laser beam is held constant to minimize the beam divergence. Therefore, the average laser output power was consistently held close to 0.25 kW in this case.

Table 3 - Control factor values for laser cutting experiments.

α	0°			30°			60°		
Level	low	med.	high	low	med.	high	low	med.	high
$\%O_2$ in assist gas (mass)	23	-	100	23	-	100	23	-	62
P_g (kPa)	140	280	410	140	210	280	140	210	280
F (cm)	7.6	12.7	20.3	7.6	12.7	20.3	7.6	12.7	20.3
f_p (Hz)	30	60	100	30	60	100	22	30	40
V_c (m/min)	0.13	0.25	0.38	0.13	0.25	0.38	0.13	0.19	0.25
τ_w (msec)	0.3	0.4	0.5	0.3	0.4	0.5	0.3	0.4	0.5

In the optimization of the laser cutting parameters, the object is to minimize the surface roughness R_a, kerf width w, and dross height H_D. From the experiments, R_a tends to decrease (i.e., better surface finish) with higher pulsing frequency (except at higher bevel angles), lower cutting speeds, higher percentages of oxygen in the cutting gas, lower assist gas pressure P_g, and longer laser pulses τ_w. The effect of the processing lens focal length F on surface roughness was not clear from the experimental results. The kerf width w decreases with lower concentrations of oxygen, higher pulsing frequencies, shorter pulse widths, and shorter processing lens focal lengths. This last effect may be explained by the smaller effective focused beam diameter. No significant effects on kerf width were observed for changes in assist gas pressure or cutting speed. Less dross is noticed with higher oxygen concentrations and cutting speeds. The effects of assist gas pressure, processing lens focal length, laser pulsing frequency, and pulse width on H_D are either negligible or unclear.

In the following discussion, the parameter values corresponding to the low, medium, and high levels are found in Table 3. For the smoothest surface finish, a low $\%O_2$, medium f_p, and high τ_w are recommended. For the narrowest kerf, a low $\%O_2$, high f_p, and low F are recommended. For the least amount of dross, a high $\%O_2$, medium P_g, and high V_c are recommended. Depending upon what surface qualities of the bevel cut are most important, the levels of each laser cutting parameter can be set accordingly. The test bevel cuts with the best overall kerf quality and the associated process parameters are listed in Table 4.

Table 4 - Optimal control factor levels for an Nd:YAG laser cutter.

	Control Factor Levels						Output Responses		
α	$\%O_2$	P_g	F	f_p	V_c	τ_w	R_a (μm)	w (mm)	H_D (mm)
0°	High	High	Low	High	Med.	High	2.6	0.14	0.02
30°	High	High	Low	High	Med.	High	4.4	0.10	0.04
60°	High	High	High	Med.	Low	Med.	5.9	0.07	0.39

With optimal process parameters, lamination cutting with the same general surface finish as flute-edge endmilling and AWJ are possible but surface finish deteriorates with an increase in the bevel angle. The edge-burring due to adherent dross is comparable to AWJ but much less than endmilling. The kerf width is the smallest of all three bevel cutting methods. Although bevels up to ±80° are possible, the range of parameters that yields acceptable cuts narrows with the increase in bevel angle. Since there is a very small mechanical force associated with laser cutting, the fixturing required for lamination being cut is minimal. Furthermore, the HAZ of the laser cut is very small and most of the heat-affected material is blown away during the cutting process.

cut zone. However, the kerfs are narrow; the adherent dross is porous, brittle, and easily removable; and lower pulsing energies of only 0.7 joules were needed. Higher cutting speeds were achievable in the CW cutting mode.

Initial Nd:YAG laser cutting tests were performed using a Lumonics JK701H laser with a fiber-optic delivery. Using O_2 as an assist gas, an Nd:YAG pulsed laser beam delivered with fiber-optics could cut bevels only up to 30°. Kerf widths are also narrow (e.g. 0.33 to 0.40 mm) but the cutting dross increases with the bevel angle. Cutting speeds were slightly lower than for a CO_2 laser of similar power. Pulsing energies of 2.2 joules were used. Cutting in CW mode yielded unacceptable cuts.

With the limited success of fiber-optic delivery, a Lumonics JK704 Nd:YAG laser with hard-optic delivery was used for cutting tests. When air (20% oxygen) was used as the assist gas for a pulsed Nd:YAG laser beam, bevel cuts up to 75° were achieved. Even higher angles may be achievable. Narrow kerf cuts (i.e. 0.15 to 0.30 mm) were observed with a porous dross that remains relatively constant for all bevel angles. The lower cutting speeds are attributable to the low pulsing frequency used. Faster cutting speeds will be achievable at the low bevel angles (i.e. 0 to 40°) if higher pulsing frequencies are used. For cutting experiments involving different lamination thicknesses and laser powers, the maximum cutting speed decreases non-linearly with increasing bevel angle because both the effective material thickness and laser energy absorbtivity of the material surface decreases.

Summary: As previously mentioned, a high laser beam energy intensity at the material surface is of prime importance for cutting steep angle bevels. The limited success with the CO_2 laser beam delivered with hard optics is attributable to insufficient energy intensity at these steep angles. The energy absorbtivity of steel and other metals to CO_2 laser light is relatively low compared to Nd:YAG laser light, which contributes to this problem. The theoretical focused beam diameter is 0.102 mm in this case. The better success with Nd:YAG laser beam delivered with hard optics is attributable to the higher absorbtivity of this shorter wavelength light by the metal and a much smaller focused spot diameter, specifically 0.008 mm and 0.013 mm for the low and high powered lasers, respectively. A small focused spot diameter yields a high energy intensity.

The limited beveling success of an Nd:YAG laser beam with fiber optic delivery is due to a large focused diameter and the homogenization of the laser beam within the fiber. The spot diameter of the laser beam delivered through a 1.00 mm diameter fiber is 0.5 mm. Even though the theoretical focused spot diameter is typically 2 to 5 times larger in reality [Powell,1993], the focused spot diameter from the hard optic delivery of an Nd:YAG laser beam is at least an order of magnitude less than the 0.5 mm spot. Furthermore, the laser beam exiting a fiber optic is homogenized as compared to the incoming beam. Homogenization lessens the exiting laser light's quality to a non-gaussian spatial distribution leading to a lower energy density on the material surface.

3.4.1 Optimization of Parameters for Pulsed Nd:YAG Laser Cutting with Hard-Optic Delivery

For cutting bevels into steel sheet—the predominant material used for tooling—the optimal configuration is a pulsed Nd:YAG laser with hard-optic beam delivery. A pulsed laser beam creates a scalloped-type kerf (see Figure 6(a)) by overlapping drilled holes (see Figure 6(b)). The process parameters chosen for the previous laser cutting experiments yielded kerfs of generally poor surface finish and geometrical accuracy. In this section, the optimal parameters for better kerf quality from pulsed Nd:YAG laser cutting are determined.

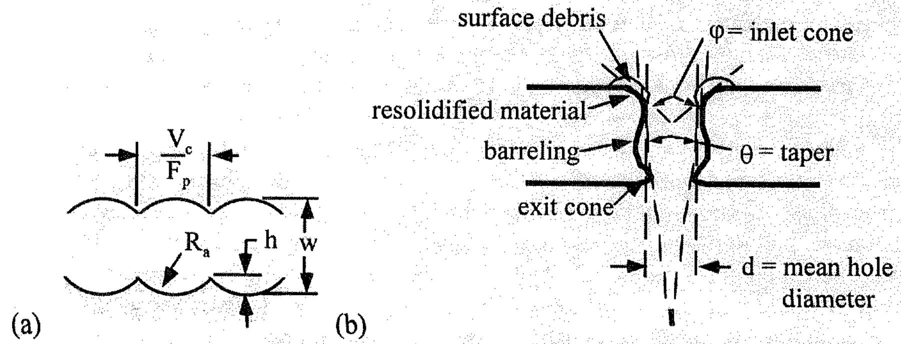

Figure 6 - (a) Top view of a pulsed laser-cut kerf and (b) features of laser-drilled holes.

As shown in Figures 3 and 6, the easily measurable output responses that help to define the kerf quality of the laser cut bevel are the kerf width w, surface roughness of the kerf R_a, the height of the burr H_D on the farside of

3.5 Comparison of Bevel Cutting Methods

Based on the previous discussion, some final comparisons can be made between flute-edge endmilling, abrasive water jet cutting and laser cutting (plasma-arc cutting not considered) for beveling steel laminations.

- The higher cutting forces associated with flute-edge endmilling significantly deflect the unsupported portion of a lamination being beveled. AWJ cutting and laser cutting, i.e. non-contact cutting methods, cause negligible deflection to the lamination.
- All three cutting methods are capable of high cutting speeds but the maximum speed is highly dependent upon the steel composition and hardness with flute-edge endmilling and less so with AWJ cutting. The maximum laser cutting speed is only dependent on laser power which means that higher cutting rates are achievable.
- The maximum bevel angle for all three methods is around ±80°.
- The kerf from AWJ cutting has a very large, consistent taper of around 10° for all bevel angles that must be compensated for during bevel cutting. Laser cutting creates only a slight kerf taper. Flute-edge endmilling leaves no appreciable kerf taper unless there is significant deflection of the lamination and cutting tool during beveling.
- The width of the kerf affects the smallest radius of curvature achievable for a PEL's profiled edge. Of the three beveling methods, laser cutting creates the narrowest kerf. The kerf from AWJ cutting is also narrow but not as much as that cut with a laser. The kerf width from endmilling is relatively large since it is the diameter of the cutting tool.
- Flute-edge endmilling leaves a very large machining burr, especially at larger bevel angles. AWJ cutting and laser cutting, on the other hand, yield much smaller machining burrs. Laser cutting with a pure oxygen assist gas yields a porous, brittle edge burr (i.e. iron oxide) which is easily removed from the cut lamination without grinding.
- Flute-edge endmilling yields the smoothest surface finish of all the methods although the finish deteriorates at higher bevel angles from more machining chatter. AWJ cutting offers the most consistent surface finish for all bevel angles.

The author ranks the suitability (best to worst) of these three bevel cutting methods for cutting steel laminations as follows:

1. Nd:YAG laser cutting because cutting speed is only dependent on laser power, tool wear is non-existent, cutting force is negligible and the kerf is the narrowest of all.
2. Abrasive water jet cutting since cutting speed is dependent on material hardness and the kerf taper is the largest among the three methods.
3. Flute-edge endmilling since the cutting force, kerf width, edge burr, and cutting speed's dependency on material hardness is the greatest overall.

4. Suitable Cutting Trajectory and Method for Cutting Steel PELs

From the preceding discussion, uniformity in cut quality and geometry of steel laminations is based on the control of cutting process parameters. The main parameter that is controlled by the chosen cutting trajectory is cutting speed. In flute-edge endmilling, the maximum V_c is constrained by the steel's machinability and the lamination stiffness. Furthermore, R_a deteriorates with increasing speed. For AWJ cutting, kerf width decreases with increasing speed but the effect of the water jet lag line becomes more pronounced. For laser cutting, surface finish gets better with increasing cutting speed but more extensive deburring is required. The ideal situation for cutting with any of these three of these cutting methods would be to cut lamination material at a constant speed for the entire cutting trajectory.

While the methods of [Newman et al.,1995], [Zheng et al.,1996], and [Hope et al.,1997] can be used to define a trajectory for cutting the top edge of a PEL at a nearly constant velocity, the author has chosen a more straight-forward procedure. As shown in Figure 7, the lamination's profiled edge is defined by two adjacent profiles resulting from the intersection of a two parallel cutting planes (separated by the lamination thickness) with the tool's CAD surface model (e.g., NURBS surface). Typically, each profile will be B-Spline curve, i.e., a piecewise polynomial made up of Bézier curves. To achieve a cutting trajectory with a nearly constant velocity, both curves are divided into the same number (n) of equal length segments using numerical procedures. Points are assigned to the end of each curve segment. This series of corresponding point pairs (P_1,P_2) are used to define corresponding line-of-site machining instructions, that is, a series of position point and unit directional vector pairs ($P_1,\vec{V}$) as shown in Figure 1(c). If a constant velocity is prescribed for the point-to-point moves performed by the CNC machine (e.g., 5-Axis laser cutter), then the cutting means will have a nearly constant velocities along both profile curves.

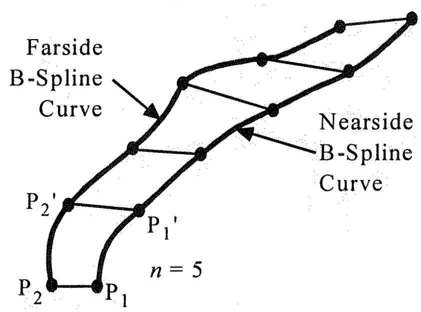

Figure 7 - Lamination's profiled-edge defined by two adjacent B-Spline curves.

Acknowledgment
This research was partially supported by the National Science Foundation CAREER award DMI-9734231.

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