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Dynamic Routing and Information Sharing for Connected and Autonomous Vehicles

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Dynamic Routing and Information Sharing for Connected and Autonomous Vehicles

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Abstract

Dynamic Routing and Information Sharing for Connected and Autonomous Vehicles

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This thesis models dynamic routing behaviors for connected and autonomous vehicles under stochastic situation of receiving incident information. Markov decision process for a single CAV and related model assumptions are introduced by the freeway instance. We formulate a generalized model based on the freeway instance and employ the value iteration algorithm to solve the problem by finding optimal policy. Numerical experiments, which are conducted on different networks, reveal the similar results about MDP model for a single CAV: if the vehicle gets the incident information, the best actions are always to travel the alternative routes to avoid the increased link cost. While for the uncertain states without receiving incident information, the best actions are always to travel on the direct links.

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Chapter 1

Introduction

1.1 Background

Traffic congestion, an important negative externality in many transportation markets, has many impacts on modern cities. Endless long queue in rush hours, for example, not only reduces our life quality by occupying leisure time and increasing anxiety, but also lowers environment quality and causes air pollution by increasing vehicles' gas consumption during congestion.

Actually, part of traffic congestion results from incidents, including car crashes, vehicle breakdown, bad weather, special events, construction and maintenance activities, on the network. However, incidents are inevitable in some extent because of the openness and complexity of transportation systems. Incidents may happen on every location of the network in any time. Complexity of system is reflected in its subtle self-regulation and self-feedback. For example, thousands of vehicles are moving in the network during different times every day, they perceive changes in the status of each link, such as increasing or decreasing in link travel time, link flow, link density, etc. by incidents, in direct or indirect ways, then they will react via their routing behaviors to the information perceiving from the changes in link status. In addition to the changes from their own self-regulation, the transportation system is open to interact with external world, for instance, bad weather like rainstorm and snowstorm will destroy pavements and make the road unavailable for vehicle to use, then the vehicles have to re-route or avoid damaged road ahead of time.

When an incident happens, either this incident is completely unpredictable, such as a car crash and vehicle breakdown, or incident-related information can not be spread efficiently among most conventional vehicles, then the incident will reduce road capacities, those uninformed vehicles that usually traverse the capacity-reduced road can not detour in time, then congestion will happen inevitably. Therefore, roadside information seems pretty significant for both individual vehicle and the whole system to reduce the congestion resulting from incidents.

On the other hand, great efforts have been made to alleviate traffic congestion. Among these efforts, connected and autonomous vehicles (CAVs) are regarded as the future form of human mobility, and are believed to bring revolution to urban transportation systems. Hence CAVs have attracted lots of attention from both academia and industry including transportation field in recent years. As the word implies, CAV is the combination of connected vehicle (CV) and autonomous vehicle (AV), which shows their close relationships; however, these two kinds of vehicles have significant differences.

Connected vehicles refer to the wireless connectivity-enabled vehicles that can communicate with their internal and external environments (Lu et al., 2014), and they provide a two-way wireless communication environment enabling vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications (Lee and Park, 2012). V2V communication can be easily understood as sharing information among themselves, while V2I communication involves that vehicles share information with infrastructure and infrastructure provides all vehicles with up-to-date traffic information. Connected vehicles can also use wireless communication to "talk" to traffic signals, work zones, toll booths, school zones, and other types of infrastructure. Therefore, connected vehicles could not only collect high-fidelity traffic data such as vehicles' origins or destinations and their trajectories, but also share important safety and mobility information with other connected vehicles and upload information to infrastructure operators.

1.2 Motivation

The above describes the importance of traffic information to overcome the uncertainty of transportation systems. In addition to CAVs sharing information among themselves, CAVs may also interface with roadside infrastructure, such as road operators. This may help to ensure vehicles to get the most up-to-date traffic information available, and can mitigate security risks caused by ”rogue” or unreliable vehicles. However, past researches (Mahmassani et al., 1993; Chiu and Huynh, 2007; Li et al., 2016) indicate that providing real-time information to all vehicles need not improve system conditions due to the re-routing and detouring which shall occur.

By combining the technology of CAV with incident-related information perception and sharing, dynamic routing and information sharing among CAV seem a promising field in future transportation systems.

1.3 Problem Statements

With the support of frontier technologies, CAVs could improve the safety and efficiency of transportation system, especially in the uncertain environment, because they could obtain and process all kinds of traffic information with less difficulty than conventional vehicles. Due to uncertainty of the transportation system, in this paper, we consider a scenario where the incident may happen in the network and hence increase the travel times on those affected roads.

To achieve the overall goal of modeling and optimizing information sharing among CAVs, this thesis addresses and models the problem into two levels: the first level is to model CAV's dynamic routing behavior when provided incident-related information for a single CAV; the second level is to extend the dynamic user equilibrium problem to a stochastic network with real-time online information. The models in these two levels are trying to answer the following questions:

1. How will a CAV make its routing decisions under different incident environments, such as regular congestion with high incident probability and car accidents with fairly low incident probability?

2. How does a CAV react facing different severity of incidents?

3. How does a CAV's capability of information gaining such as incident perception, impact its routing behavior?

4. With the routing behavior and information sharing strategy for CAVs, how will the performance of the network be when certain number of CAVs are traversing from multiple origins to multiple destinations?

1.4 Thesis Contributions

1. Extends the online shortest path problem in the environment of CAVs with 100% market penetration;

2. Incorporate CAV's cognitive learning processes for roadside information into CAVs' route choice behaviors;

3. Extends the dynamic user equilibrium to a stochastic network, models dynamic user equilibrium with recourse, and runs simulations for multiple CAVs.

1.5 Thesis Organization

The thesis is organized as follows. We first study the problems about vehicle's routing behavior with roadside information in a stochastic network, including user equilibrium with recourse and dynamic routing with real-time VMS information. We then use a motivating example to introduce the Markov decision processes and generalize the model for CAV's dynamic routing behavior when provided incident-related information. We conduct numerical experiments for a single CAV on different real world networks. Last but not least, we model multiple CAVs' dynamic routing behavior based on the dynamic user equilibrium and Markov decision processes.

In Chapter 2, literature related to vehicle's routing behavior with roadside information in a stochastic network are discussed. Chapter 3 presents the methodology of this thesis, we begin with a motivating example with a small freeway network for a single CAV to introduce the basics of Markov Decision Process, then generalize the MDP formulation for CAV's dynamic routing behavior when provided incident-related information. Chapter 4 presents the numerical experiments to explore the impacts of incidents and CAV's perception of incidents on the expected cost and optimal routing policy. Conclusions are discussed in Chapter 5.

Chapter 2

Literature Review

This chapter discusses literature related to vehicle's routing behavior with roadside information in a stochastic network. Facing hundreds of previous researches in this topic, we first separate the related literature into two categories on the basis of the way that vehicles perceive information: vehicle's optimal routing behavior and policy in a stochastic network, and vehicle's dynamic routing with real-time online information. The former category of researches originate from the traditional shortest path problem, while the latter is always accompanied by the technology developed in the intelligent transportation systems (ITS), such as advanced traveler information systems (ATIS), variable message signs (VMS) and connected vehicles (CVs). The largest difference between these two categories of researches lies in the assumption about vehicles' information perception,

First, section 2.1 reviews previous work on shortest path problem and its extended problems in a stochastic network in both static and dynamic scope. Section 2.2 then discusses studies on dynamic routing with real-time online information with an emphasis on VMS-related researches.

2.1 Shortest path problems in a stochastic network

A number of related researches based on the uncertain conditions of network were conducted since 1990s. While because of different assumptions, different research institutions differ greatly with in terminologies used.

Stochastic shortest path problems with recourse was first mentioned around 1990s (Andreatta and Romeo, 1988; Bertsekas and Tsitsiklis, 1991; Polychronopoulos and Tsitsiklis, 1993). There were some similar concepts including shortest or optimal path in probability or stochastic network. These shortest path problems defined on networks with random link costs, with following assumptions: first, information on link cost values is accumulated as the network is being traversed, the objective is to devise a policy that leads from an origin to destination node with minimal expected cost; second, the link costs are never learned or become known after a path is chosen, so we should choose a path with minimal expected value of the link lengths; third, for this class of problems, one should not look for a best path, but rather for a best policy, the rule for deciding where to go next given the currently available information. Miller-Hooks and Mahmassani (2000) extended the problem to the time-varying networks, with their concept of least expected time paths comparing to the stochastic shortest paths. Link costs are random variables with probability distribution functions that vary with time. Two procedures were presented in the paper, the first one determined a priori least expected time paths from all origins to a single destination for each departure time, and the second procedure determined lower bounds on the expected times of these a priori least expected time paths.

Online shortest path (OSP) and user equilibrium with recourse (UER) are highly pertinent area of work. OSP forms the sub-problem to UER in the same manner that the traditional shortest path problem forms the sub-problem to the static user equilibrium (UE) problem (Unnikrishnan and Waller, 2009).

Recourse can be viewed as the opportunity for a decision maker to re-evaluate his

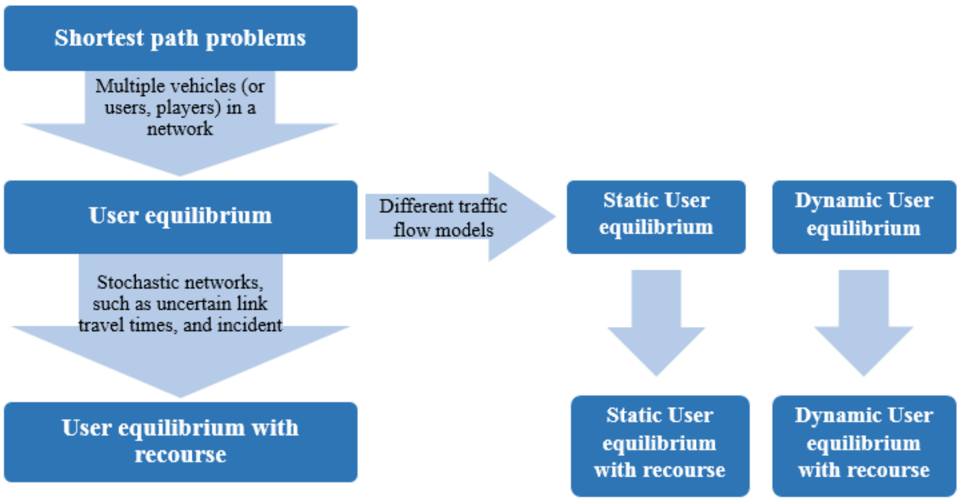

Figure 2.1: Shortest path problem and its extended problems

or her remaining path at each node based on knowledge obtained en route (Waller and Ziliaskopoulos, 2002). The UER definition implies that all used routing policies will have equal and minimum expected cost. Unnikrishnan and Waller (2009) developed a convex mathematical program for static user equilibrium under uncertain link states and update their route choice in an online manner.

The researches mentioned above are generally focused on static scope and derived from static shortest path problem.

Gao (2005) presented an algorithm when link costs are time-dependent, stochastic and have general dependency, with the concepts of adaptive routing. Same terminology is used in optimal information location for adaptive routing (Boyles and Waller, 2011) and adaptive transit routing in stochastic time-dependent networks (Rambha et al., 2016).

2.2 Real-time online information

Roadside information can generally be divided into two, a priori information and online information. A priori information is about the general description of daily fluctuations of traffic quantities. Online information is about traffic condition at a specific time, e.g., an incident just occurred on the next exit of the freeway, and it will probably last for 30 minutes. With the advancement of technology, the ways to provide vehicles with real-time traffic information and the information quality are both improving, from advanced traveler information systems (ATIS) and variable message signs (VMS) to V2V and V2I communication, or from generalized traffic condition information to personalized information, e.g., people use Google Map to get their personalized routing suggestion.

Since this thesis will model the vehicles' routing behaviors when provided with traffic information and explore the impacts of real-time information on the expected travel times, literature relevant to vehicles' response to the real-time information should be studied. We focus the previous research on VMS.

VMS are programmable traffic control devices (e.g., electronic message board) located by the roadside and convey non-personalized real-time information on network conditions (e.g., incidents, congestion, travel time or weather) to drivers. Abbas and McCoy (1999) first studied optimal VMS location problem on freeways within incident scenario. applied genetic algorithm to solve this problem. They formulated the problem with its objective maximizing the estimated delay savings due to diversion to alternative paths in response to incident information provided by VMS:

$$\max \sum_i B_i$$
$$s.t. \quad B_i = \sum_j \sum_k \frac{v_{ijk}}{\sum_i v_{ijk}} DS_j$$

where B_i is the benefit of VMS at diversion node i (with the unit of vehicle-hour); v_{ijk} is the number of vehicle trips from diversion node i through downstream freeway section j during time period k; DS_j is the total estimated delay savings due to traffic diversion during incidents on freeway section j.

Chiu et al. (2001) and Chiu and Huynh (2007) developed a bi-level stochastic integer programming framework to find optimal VMS locations with the objective of maximizing their expected benefit under stochastic traffic incident situations. The upper level was to minimize total travel time, and they used a tabu search algorithm at the upper level to generate potential solutions, which were then evaluated at the lower level by solving the dynamic user equilibrium problem. Similarly, Huynh et al. (2003) formulated optimal VMS location problem under DTA framework and tried to minimize the network travel time in response to actual incidents instead of an expected network travel time. They combined principles of greedy and drop heuristics to find the optimal solution. However, there is a limitation in their model assumptions about route choice that all users have perfect knowledge of real-time information on the incident condition, and possess the ability to anticipate other users' choice of routes and choose their optimal routes accordingly.

Actually, drivers will not be likely to change their routes according to information provided by VMS. Li et al. (2016) used an endogenous variable, utilization rate of VMS, to model this phenomenon. They also introduced a paradox that increasing the accuracy of drivers' perceived travel times leaded to the increase of total system delay. In fact, it is similar to the information saturation effect that when all users are informed of traffic condition information, the route or departure time choice behaviors make the system less efficient than if only a fraction of users get the information (Chiu and Huynh, 2007). Similar findings were first reported in Mahmassani et al. (1993).

Chapter 3

Methodology

We create a motivating instance for a single CAV in a freeway network to introduce the Markov decision processes (MDPs). An incident may happen on the freeway with a certain probability (when and where the incident occurs are totally random, so the probability here is a general description of randomness in these two aspects). We assume that the incident would increase travel costs on several affected road segments and thus result in congestion (the severity of congestion on affected road segment is assumed to be highly related to the average distance from this road segment to the incident location). When an incident happens, the traffic operator will quickly collect and accurately master the incident-related information and share the information with CAVs (the operator may not provide the information for all CAVs on affected road segments). Meanwhile, CAVs themselves, that are driving on affected road segments, will also perceive some incident information via recognizing vehicle speed, traffic density and so on, along with the information shared by traffic operator, CAVs can make decisions to or not to divert into alternative routes to avoid congestion. The model we introduce in this chapter is concerned with the last part of the incident story above, i.e., the CAV's routing behavior when it receives or perceives incident information in a uncertain network.

3.1 Literature review

Before we apply MDP to model CAV's dynamic routing behavior when provided incident-related information, we first should review briefly previous researches on following three topics:

1. Probabilistic description of traffic network, as well as estimation and prediction of network status and traffic conditions, including the relationship between accidents and affected link travel times;

2. Vehicles' perception of traffic information, involving travel time perception and incident perception.

3. Basics about Markov decision processes, e.g., the concepts, and the commonly used algorithms with their properties;

Several researches made efforts to estimate the link travel times in dynamic road networks with various statistic approaches.

Since the traffic volume of the object link is the result of the flows of its upstream links and its own historical series, it is reasonable to construct a Bayesian network for traffic flow of each road link at a given time. Sun et al. (2006) adopt the idea of conditional independence, that is, given the adjacent upstream traffic flows at different time delays, traffic flow at the current link is assumed to be independent of other upstream traffic flows [1]. In the presence of incidents (or accidents), the approach can still work in principle. Since it is based on pattern learning from training data, as long as given adequate data that account for these scenarios, the estimated Gaussian mixture model could represent these patterns.

[1] one-step spatial dependence in Waller and Ziliaskopoulos' online shortest path paper (2002)

To find the most reliable route, it is required to estimate the probability distribution of travel times for each link, given a link-entrance-time. Asghari et al. (2015) address the problem of computing these link travel time distributions. They show how this first step can affect the accuracy of the travel time distribution over the entire route. Since the reported travel time is not a single value but a probabilistic distribution highly depending on the departure time, it is challenging to evaluate link travel time distributions among different approaches. they thus propose a statistical test that enables us to evaluate these outcomes.

A stochastic network is often assumed in researches about vehicles' perception of traffic information.

Travelers choose their route in a network with stochastic travel times, uncertain of the travel time they will experience on any of the available alternative routes. Based on Cumulative prospect theory, Connors and Sumalee (2009) model a framework involving the perceived value and the probabilities of travel time outcomes, which are obtained via nonlinear transformations of the actual travel times and their probabilities. Based on the framework, they formulate a Perceived Value-User Equilibrium, where travelers choose the routes that maximize their perceived value in the face of uncertain travel times.

Chen and Mahmassani (2004) extend past work by further considering the travel time perception and learning process, they modeled a mechanism within Bayesian statistic inference for triggering and terminating the updating process. The findings indicated that the perceived confidence or error associated with experienced travel times is an important factor in route choice decisions.

3.2 Motivating example

As shown in figure 3.1, the freeway includes 4 nodes and 6 links with a destination, node D (black numbers are link labels). Assume that an incident may happen on link 3 with a certain probability. link 1, link 2 and link 3 are affected road segments, and their link costs are 1, 1, 4, respectively without incident, and 2, 3, 16, respectively if an incident indeed happens. Link 4, link 5 and link 6 are alternative routes which a vehicle can divert to at node A, B and C, respectively, with their link costs of 11.5, 10 and 8, respectively.

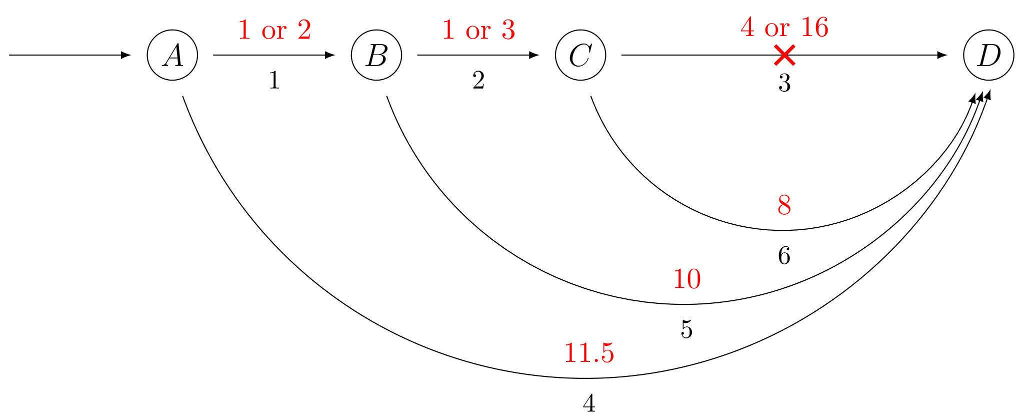

Figure 3.1: An illustrative freeway network with affected and alternative links (Red numbers are link costs and black numbers are link labels)

3.2.1 State space

When a CAV travels to the destination, it can obtain incident information at any location on the freeway, but can only make route choice at nodes. Therefore, we use node-state to define state space in our MDP. A **node-state** [1], denoted by

[1]Boyles and Waller (2011) defined node-state as a pair (i, θ) representing arrival at node i and receiving message θ. The node-state proposed in this thesis is derived from their definition.

$(j, info, inc)$, includes three components - location, information and incident.

j represents the node at which a vehicle arrives and makes route choice [1].

$info$ is a binary variable, which indicates whether the vehicle perceives information about an incident [2], i.e.,

$$\texttt{info} = \begin{cases} 1, & \text{if a vehicle perceives incident information,} \\ 0, & \text{otherwise.} \end{cases}$$

We further assume that once a vehicle gets the incident information from the operator or perceives information by itself, it has the knowledge from then on until it arrives at the destination (which can be summarized as "permanent information"). Let $P(\texttt{percv})$ and q respectively denote the probability that a vehicle could perceive an incident and its value, similarly let $P(\texttt{n\_percv})$ denote the probability that a vehicle can't perceive any incident information, then $P(\texttt{n\_percv}) = 1 - q$.

inc is a binary variable, indicates whether there indeed happens an incident, i.e.,

$$\texttt{inc} = \begin{cases} 1, & \text{if an incident indeed happens,} \\ 0, & \text{otherwise.} \end{cases}$$

Let $P(\texttt{inc})$ and p respectively be the probability of an incident and its value, then $P(\texttt{n\_inc})$ gives the probability that an incident does not occur, and $P(\texttt{n\_inc}) = 1 - p$.

Note that inc is an objective description about the traffic condition in the physical world; while $info$ describes a vehicle's subjective perceptive status, which is

[1] In conventional MDP, a decision maker makes a decision at each time step t. While in our model a vehicle makes a decision only at node, so the node can be viewed as an equivalent to the time step in conventional MDP.

[2] As mentioned in the beginning of this chapter, a CAV can perceive incident information by two ways, receiving information from the operator, or inferring the incident by its own observation and estimation.

independent of inc, so the vehicle makes routing decision only by its own perception status. For example, if a vehicle is in a state of $(A, 0, 1)$, although an incident indeed happens, for the vehicle does not perceive any incident information, it still makes its decision according to the situation without any incident.

Moreover, if no incident occurs, CAVs cannot perceive any incident information, then node-state $(j, 1, 0)$ is not legally defined, so there are only 3 possible states for each node. **State space**, denoted by $\mathcal{X}$, is a set of node-states, which contains all possible states in the model, therefore,

$$\begin{aligned}\mathcal{X} = \Big\{&(A,0,0),(A,0,1),(A,1,1),(B,0,0),(B,0,1),(B,1,1),\\&(C,0,0),(C,0,1),(C,1,1),(D,0,0),(D,0,1),(D,1,1)\Big\}\end{aligned}$$

3.2.2 Action space

In our model, an action for a state x, denoted by $u(x)$, represents a certain link that a vehicle will choose to go next. Action space for a state $x = (j, Info, Inc)$, denoted by $U(x)$, is a set of all actions for this state, so it contains all possible links that the vehicle may choose to go next from current state. For instance, a vehicle makes route choice at node A, then it can travel to link 1 or link 4 regardless of the current state, so the states at node A - $(A, 0, 0)$, $(A, 0, 1)$ and $(A, 1, 1)$ - share a common action space:

$$U\left(x = (A,0,0)\right) = U(x = (A,0,1)) = U(x = (A,1,1)) = \{1, 4\}$$

Similarly,

$$U(x = (B,0,0)) = U(x = (B,0,1)) = U(x = (B,1,1)) = \{2, 5\}$$

$$U(x = (C,0,0)) = U(x = (C,0,1)) = U(x = (C,1,1)) = \{3, 6\}$$

For the node D is the destination and the vehicle does not need to make route choices, so there is no action for states at node D.

3.2.3 Transition probability

Given a vehicle's current state and an action it will take, there is a set of probabilities for each of the next states that the vehicle may get to, and this set of probabilities are called **transition probabilities** from current state to the next possible states. For convenience, we assume that the incident will not cease until the vehicle arrives at the destination, in this case, we can avoid the complex situation where the incident state turns from $\texttt{Inc} = 1$ to $\texttt{Inc} = 0$ while the vehicle is heading for the destination. We take node A as an example to illustrate transition probabilities.

If a vehicle decides to traverse, say, link 1 at node A, it will be in one of the three states, $(A, 0, 0)$, $(A, 0, 1)$ and $(A, 1, 1)$.

1. If the vehicle is in the state $(A, 0, 0)$, when it arrives at node B, it will be in one of three possible states, $(B, 0, 0)$, $(B, 0, 1)$ and $(B, 1, 1)$.

   If the next state is $(B, 0, 0)$, which indicates no incident happens, then the transition probability from $(A, 0, 0)$ to $(B, 0, 0)$ with action of link 1 is

   $$P((B, 0, 0)|(A, 0, 0), 1) = 1 - p$$

   Likewise, if the next state is $(B, 0, 1)$ or $(B, 1, 1)$, the corresponding transition probabilities are given by

   $$P((B, 0, 1)|(A, 0, 0), 1) = p \cdot (1 - q)$$
   $$P((B, 1, 1)|(A, 0, 0), 1) = p \cdot q$$

2. If the current state is $(A, 0, 1)$, since it is assumed that the incident will not cease until the vehicle arrives at the destination, there are only two possible next states, $(B, 0, 1)$, $(B, 1, 1)$, then the transition probabilities are given by

$$P((B,0,1)|(A,0,1),1) = 1 - q$$
$$P((B,1,1)|(A,0,1),1) = q$$

3. If the vehicle is in the state $(A, 1, 1)$, according to the "permanent information" assumption, the next state can only be $(B, 1, 1)$, thus the transition probability is one, i.e.,

$$P((B,1,1)|(A,1,1),1) = 1$$

For other nodes or actions, the transition probability distribution remains the same, that is, if the vehicle decides to traverse link 4 at node A, the transition probabilities are exactly the same as those shown above, while the only differences from traversing link 1 lie in the next states.

3.2.4 Objective function

Generally, a decision maker will get a reward after one-step transition from the current state to the next state taking an action. Our MDP model replaces one-step reward with one-step cost, which represents link travel cost from one node-state to the next node-state. Let $c(x'|x, u)$ denote the one-step cost from the current state x to the successor state x' by taking an action u.

A policy (a mapping from states to actions), denoted by π, gives an exact action for each state that a decision maker must choose. Therefore, a policy can be viewed as a rule to make a vehicle decide where to go next according to the vehicle's current states.

The objective of our MDP model is to find an optimal policy among all possible policies to minimize the sum of expected travel cost for each state,

$$\min_{\pi} \ \mathbf{E}\left[\sum_{x\in\mathcal{X}} c(x) \middle| \pi \right]$$

where the expected cost, $c(x)$, represents the expected travel cost that a vehicle will get from the node in the initial state x to the destination. If a vehicle will experience a sequence of states, $x_1 \to x_2 \to ... \to x_n$, from an initial node to the destination, and choose a sequence of actions in corresponding state, $u_1 \to u_2 \to ... \to u_{n-1}$, then,

$$c(x) = c(x_1) = \sum_{i=1}^{n-1} c\left(x_{i+1}|x_i, u_i\right)$$

We can use backward induction to find the optimal policy by calculating the minimal total expected travel cost in the objective function.

3.2.5 Backward induction

Backward induction is the process of reasoning backwards in time, from the end of a problem or situation, to determine a sequence of optimal actions. It proceeds by first considering the last time a decision might be made and choosing what to do in any situation at that time. Using this information, one can then determine what to do at the second-to-last time of decision. This process continues backwards until one has determined the best action for every possible situation (i.e. for every possible information set) at every point in time.

In our model, we start from the predecessor node of the destination [1] , i.e., node C, to determine the best initial action and the minimal expected travel cost for the

[1] If there are two or more predecessor nodes, we can determine the best initial action for each branch until two of the paths meet in the same node. For more complicated situations, we will use value iteration algorithm which is shown in section 3.3.

corresponding node-states. For convenience, we use $c(x_1, u_1)$ (a combination of initial state and initial action) to represent the expected travel cost for initial state x_1 with an initial action u_1; use $u^*_{(j,Info,Inc)}$ and $c^*[(j, Info, Inc)]$ to respectively denote the best initial action and the minimal expected travel cost for the initial node-state $(j, Info, Inc)$, and $c^*[(j, Info, Inc)] = c^*\left[(j, Info, Inc), u^*_{(j,Info,Inc)}\right]$.

1. First we determine the best initial actions for three states with perfect information, i.e., $(C, 1, 1)$, $(B, 1, 1)$ and $(A, 1, 1)$.

Given state $(C, 1, 1)$, there are two actions for this state, then

$$c((C, 1, 1), 3) = c((D, 1, 1)|(C, 1, 1), 3) = 16$$

$$c((C, 1, 1), 6) = c((D, 1, 1)|(C, 1, 1), 6) = 8$$

Since $8 < 16$, we conclude that $u^*_{(C,1,1)} = 6$, and $c^*[(C, 1, 1)] = 8$.

Due to perfect information, we can consider the state $(B, 1, 1)$ similarly,

$$\begin{aligned} c((B, 1, 1), 2) &= c((C, 1, 1)|(B, 1, 1), 2) + c^*[(C, 1, 1)] \\ &= 3 + 8 = 11 \\ c((B, 1, 1), 5) &= c((D, 1, 1)|(B, 1, 1), 5) = 10 \end{aligned}$$

Since $10 < 11$, we have $u^*_{(B,1,1)} = 5$ and $c^*[(B, 1, 1)] = 10$.

For state $(A, 1, 1)$,

$$\begin{aligned} c((A, 1, 1), 1) &= c((B, 1, 1)|(A, 1, 1), 1) + c^*[(B, 1, 1)] \\ &= 2 + 10 = 12 \\ c((A, 1, 1), 4) &= c((D, 1, 1)|(A, 1, 1), 4) = 11.5 \end{aligned}$$

Therefore, $u^*_{(A,1,1)} = 4$, and $c^*[(A, 1, 1)] = 11.5$.

2. Next we turn to the state $(C, 0, 0)$. Given the current state $(C, 0, 0)$,

$$\begin{aligned}
c((C,0,0),3) &= P(\texttt{n_inc}) \cdot c((D,0,0)|(C,0,0),3) + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot c((D,1,1)|(C,0,0),3) + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{n_percv}) \cdot c((D,0,1)|(C,0,0),3) \\
&= 4(1-p) + 16p(1-q) + 16pq \\
&= 4 + 12p \\
c((C,0,0),6) &= P(\texttt{n_inc}) \cdot c((D,0,0)|(C,0,0),6) + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot c((D,1,1)|(C,0,0),6) + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{n_percv}) \cdot c((D,0,1)|(C,0,0),6) \\
&= 8(1-p) + 8p(1-q) + 8pq = 8
\end{aligned}$$

Then,

$$u^*_{(C,0,0)} = \begin{cases} 3, & \text{if } p < 1/3 \quad \text{(situation A)} \\ 6, & \text{if } p > 1/3 \quad \text{(situation B)} \\ \text{either}, & \text{if } p = 1/3 \quad \text{(situation C)} \end{cases}$$

We consider the problem in three situations.

(A) $\boldsymbol{p < \frac{1}{3}}$

Then $4 + 12p < 8$, $u^*_{(C,0,0)} = 3$, $c^*[(C, 0, 0)] = 4 + 12p$.

Consider the least expected cost for the state $(C, 0, 1)$. Note that $(C, 0, 1)$ is a special node-state, because the vehicle does not perceive any information about whether an incident indeed happens, but in the vehicle's view, it "thinks" that no incident occurs. The vehicle's route choice, in this sense, will be consistent with its behavior in the state $(C, 0, 0)$; however, one-step cost the vehicle will get corresponds to the travel cost affected by an incident [1]. Thus, $u^*_{(C,0,1)} = u^*_{(C,0,0)} = 3$, and

$$\begin{aligned}
c^*\left[(C,0,1)\right] &= c^*\left((C,0,1), u^*_{(C,0,1)}\right) \\
&= c\left((C,0,1), u^*_{(C,0,0)}\right) = c((C,0,1),3) \\
&= P(\texttt{percv}) \cdot c((D,1,1)|(C,0,1),3) + \\
&\quad P(\texttt{not_percv}) \cdot c((D,0,1)|(C,0,1),3) \\
&= 16q + 16(1-q) = 16
\end{aligned}$$

Next we determine the best initial actions and corresponding minimal expected costs for states at node B. Given the current state $(B,0,0)$,

$$\begin{aligned}
c((B,0,0),2) &= P(\texttt{n_inc}) \cdot \left\{c((C,0,0)|(B,0,0),2) + c^*\left[(C,0,0)\right]\right\} + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot \left\{c((C,1,1)|(B,0,0),2) + c^*\left[(C,1,1)\right]\right\} + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{n_percv}) \cdot \left\{c((C,0,1)|(B,0,0),2) + c^*\left[(C,0,1)\right]\right\} \\
&= (1-p) \cdot [1 + (4 + 12p)] + pq \cdot (3+8) + p(1-q) \cdot (3+16) \\
&= -12p^2 - 8pq + 26p + 5 \\
c((B,0,0),5) &= P(\texttt{n_inc}) \cdot c((D,0,0)|(B,0,0),5) + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot c((D,1,1)|(B,0,0),5) + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{n_percv}) \cdot c((D,0,1)|(B,0,0),5) \\
&= 10(1-p) + 10pq + 10p(1-q) = 10
\end{aligned}$$

$$\begin{aligned}
& -12p^2 - 8pq + 26p + 5 = 10 \\
\Leftrightarrow \qquad & -12p^2 + 26p - 5 = 8pq \\
\Leftrightarrow \qquad & 26 - 12p - 5/p = 8q
\end{aligned}$$

[1] There is an inconsistency between the vehicle's decision and the result it will get, however, it is this inconsistency that reflects the value of information.

Then, for $p < 1/3$,

$$u^*_{(B,0,0)} = \begin{cases} 2, & \text{if } 26 - 12p - 5/p < 8q \\ 5, & \text{if } 26 - 12p - 5/p > 8q \\ \text{either}, & \text{if } 26 - 12p - 5/p = 8q \end{cases}$$

We further divide the sub-problem into three situations. Let

$$A_1 = \left\{(p,q) : p < \frac{1}{3},\ 8q > 26 - 12p - \frac{5}{p}\right\}$$
$$A_2 = \left\{(p,q) : p < \frac{1}{3},\ 8q < 26 - 12p - \frac{5}{p}\right\}$$
$$A_3 = \left\{(p,q) : p < \frac{1}{3},\ 8q = 26 - 12p - \frac{5}{p}\right\}$$

(1) $\boldsymbol{(p,q) \in A_1}$

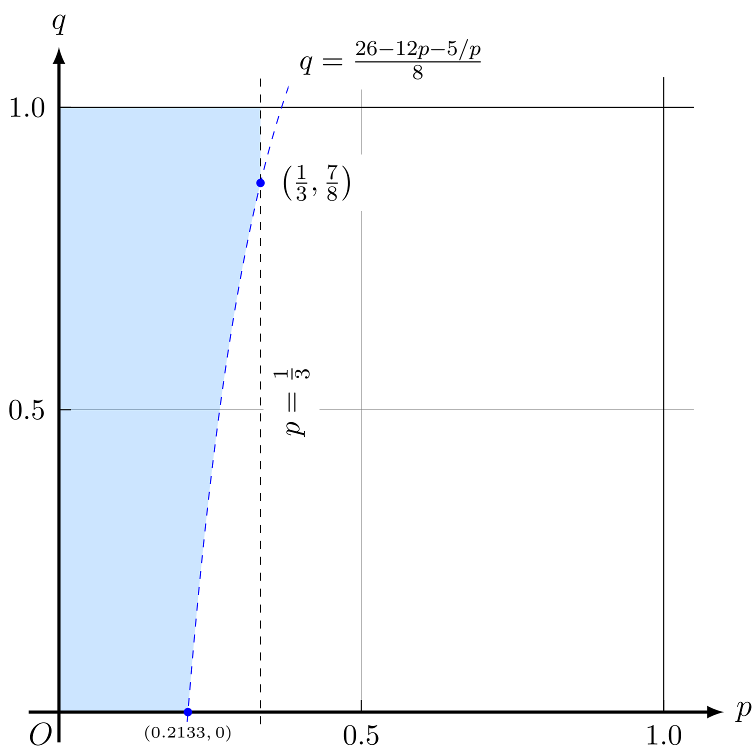

Figure 3.2: Feasible region A_1 with respect to p and q

Then, $u^*_{(B,0,0)} = 2$, $c^*[(B,0,0)] = -12p^2 - 8pq + 26p + 5$. The graph of A_1 is shown in Figure 3.2. Consider state $(B,0,1)$, similar to $(C,0,1)$, $u^*_{(B,0,1)} = u^*_{(B,0,0)} = 2$,

$$\begin{aligned}
c^*\left[(B,0,1)\right] &= c^*\left((B,0,1), u^*_{(B,0,1)}\right) \\
&= c\left((B,0,1), u^*_{(B,0,0)}\right) = c((B,0,1),2) \\
&= P(\texttt{percv}) \cdot \left\{c((C,1,1)|(B,0,1),2) + c^*\left[(C,1,1)\right]\right\} + \\
&\quad\; P(\texttt{not_percv}) \cdot \left\{c((C,0,1)|(B,0,1),2) + c^*\left[(C,0,1)\right]\right\} \\
&= q(3+8) + (1-q)(3+16) = 19 - 8q
\end{aligned}$$

Finally we determine the best initial actions and the minimal expected travel costs for states at node A. Given the current state $(A,0,0)$,

$$\begin{aligned}
c((A,0,0),1) &= P(\texttt{n_inc}) \cdot \left\{c((B,0,0)|(A,0,0),1) + c^*\left[(B,0,0)\right]\right\} + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot \left\{c((B,1,1)|(A,0,0),1) + c^*\left[(B,1,1)\right]\right\} + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{n_percv}) \cdot \left\{c((B,0,1)|(A,0,0),1) + c^*\left[(B,0,1)\right]\right\} \\
&= (1-p) \cdot \left[1 + (-12p^2 - 8pq + 26p + 5)\right] + pq \cdot (2+10) + \\
&\quad p(1-q) \cdot \left[2 + (19 - 8q)\right] \\
&= 12p^3 + 8p^2q + 8pq^2 - 38p^2 - 25pq + 41p + 6 \\
c((A,0,0),4) &= P(\texttt{n_inc}) \cdot c((D,0,0)|(A,0,0),4) + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot c((D,1,1)|(A,0,0),4) + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{n_percv}) \cdot c((D,0,1)|(A,0,0),4) \\
&= 11.5(1-p) + 11.5pq + 11.5p(1-q) = 11.5
\end{aligned}$$

$$\begin{aligned}
& 12p^3 + 8p^2q + 8pq^2 - 38p^2 - 25pq + 41p + 6 = 11.5 \\
\Leftrightarrow\quad & (8p)q^2 + (8p^2 - 25p)q + (12p^3 - 38p^2 + 41p - 5.5) = 0 \qquad (3.1)
\end{aligned}$$

We first determine whether equation (3.1) has real roots. Let

$$f(q) = (8p)q^2 + (8p^2 - 25p)q + (12p^3 - 38p^2 + 41p - 5.5), \quad (p,q) \in A_1$$

The graph of $f(q)$ with respect to q is a parabola, and its axis of symmetry is

$$q = \frac{25p - 8p^2}{16p} = \frac{25}{16} - \frac{p}{2} > \frac{25}{16} - \frac{1}{6} > 1.$$

$$f(1) = 12p^3 - 30p^2 + 24p - 5.5 \triangleq g(p), \quad 0 < p < \frac{1}{3}$$

$$\frac{dg(p)}{dp} = 36p^2 - 60p + 24 = 36(p-1)(p-\frac{2}{3}) > 0$$

So $g(p)$ is monotonically increasing with respect to p on $(0, \frac{1}{3})$, then

$$f(1) = g(p) < g(\frac{1}{3}) = 12\left(\frac{1}{3}\right)^3 - 30\left(\frac{1}{3}\right)^2 + 24 \cdot \frac{1}{3} - 5.5 = -\frac{7}{18} < 0,$$

so $f(q)$ has two real zeros [1]. Then, for $(p,q) \in A_1$,

$$u^*_{(A,0,0)} = \begin{cases} 1, & \text{if } f(q) < 0, \\ 4, & \text{if } f(q) > 0, \\ \text{either}, & \text{if } f(q) = 0 \end{cases}$$

(a) $\boldsymbol{f(q) < 0, \quad (p,q) \in A_1}$

Then

$$q > q_1 = \frac{25p - 8p^2}{16p} - \frac{\sqrt{(8p^2 - 25p)^2 - 32p(12p^3 - 38p^2 + 41p - 5.5)}}{16p}$$

where q_1 is the smaller real root of equation (3.1). Since

$$q > \frac{26 - 12p - 5/p}{8}$$

[1] We can also directly use the discriminant of (3.1) to determine the existence of real roots. $\Delta = (8p^2 - 25p)^2 - 4 \cdot (8p)(12p^3 - 38p^2 + 41p - 5.5) = p(-320p^3 + 816p^2 - 687p + 176) > 0$, for $0 < p < 1/3$

then,

$$q > max\left\{q_1, \frac{26 - 12p - 5/p}{8}\right\}$$

Let

$$A_{1a} = \left\{(p,q) : p < \frac{1}{3},\ q > max\left\{q_1, \frac{26 - 12p - 5/p}{8}\right\}\right\}$$

the graph of A_{1a} is shown in Figure 3.3.

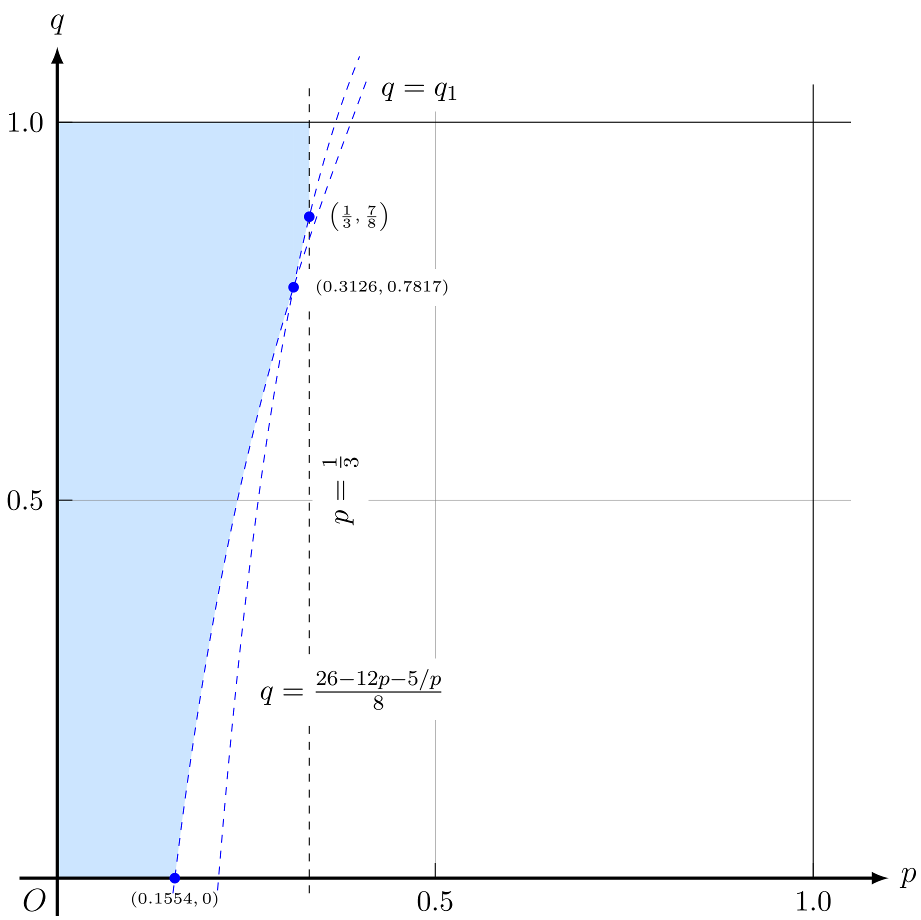

Figure 3.3: Feasible region A_{1a} with respect to p and q

Then $u^*_{(A,0,0)} = 1$, $c^*[(A, 0, 0)] = 12p^3 + 8p^2q + 8pq^2 - 38p^2 - 25pq + 41p + 6.$

Consider state $(A, 0, 1)$, similarly, $u^*_{(A,0,1)} = u^*_{(A,0,0)} = 1$,

$$\begin{aligned} c^*[(A,0,1)] &= c^*\left((A,0,1), u^*_{(A,0,1)}\right) = c((A,0,1),1) \\ &= P(\texttt{percv}) \cdot \left\{c((B,1,1)|(A,0,1),1) + c^*[(B,1,1)]\right\} + \\ &\quad P(\texttt{not\_percv}) \cdot \left\{c((B,0,1)|(A,0,1),1) + c^*[(B,0,1)]\right\} \\ &= q(2+10) + (1-q)(2+19-8q) \\ &= 8q^2 - 17q + 21 \end{aligned}$$

(b) $\boldsymbol{f(q) > 0, \quad (p,q) \in A_1}$

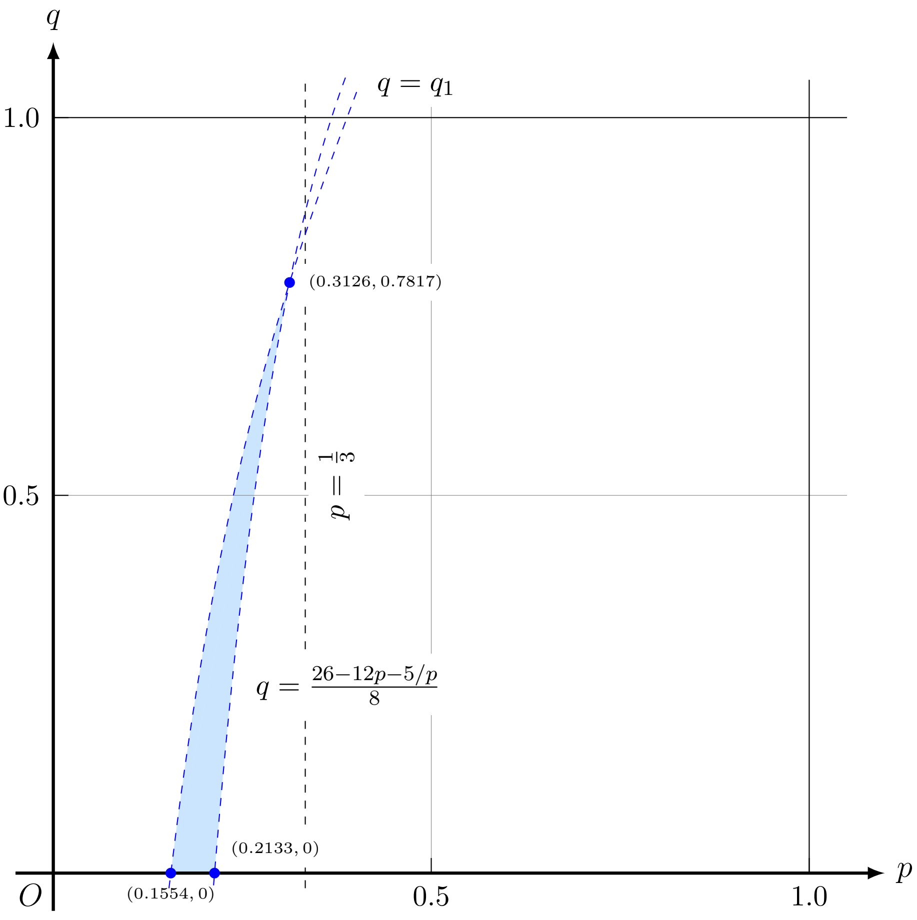

Figure 3.4: Feasible region A_{1b} with respect to p and q

Then $q < q_1$. Since

$$q > \frac{26 - 12p - 5/p}{8}$$

then

$$\frac{26 - 12p - 5/p}{8} < q < q_1$$

Let

$$A_{1b} = \left\{ (p,q) : p < \frac{1}{3},\ \frac{26 - 12p - 5/p}{8} < q < q_1 \right\}$$

the graph of A_{1b} is shown in Figure 3.4. Thus, $u^*_{(A,0,0)} = 4$, $c^*[(A,0,0)] = 11.5$. For state $(A,0,1)$, $u^*_{(A,0,1)} = u^*_{(A,0,0)} = 4$,

$$\begin{aligned} c^*\left[(A,0,1)\right] &= c\left((A,0,1), u^*_{(A,0,0)}\right) = c((A,0,1),4) \\ &= P(\texttt{percv}) \cdot c((D,1,1)|(A,0,1),4)+ \\ &\quad P(\texttt{not\_percv}) \cdot c((D,0,1)|(A,0,1),4) \\ &= 11.5q + 11.5(1-q) = 11.5 \end{aligned}$$

(c) $\boldsymbol{f(q) = 0, \quad (p,q) \in A_1}$

Then $q = q_1$, $(p,q) \in A_1$. Let

$$A_{1c} = \left\{ (p,q) : p < \frac{1}{3},\ q > \frac{26 - 12p - 5/p}{8},\ q = q_1 \right\}$$

the graph of A_{1c} is shown in Figure 3.5. However,

$$\begin{aligned} c^*((A,0,1),1) &= 8q^2 - 17q + 21 \\ &= 8(q-1)^2 + (13-q) \\ &\geq 13 - q > 11.5 = c^*((A,0,1),4) \end{aligned}$$

then $u^*_{(A,0,1)} = u^*_{(A,0,0)} = 4$, $c^*[(A,0,1)] = 11.5$. Moreover, in situation $(A1a)$, link 1 is not the "theoretical" best initial action for state $(A,0,1)$.

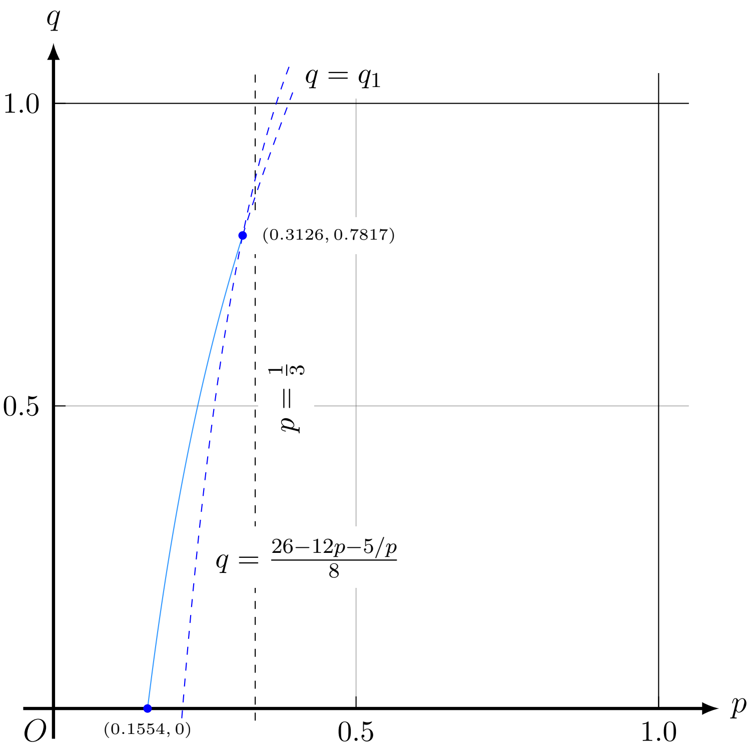

Figure 3.5: Feasible region A_{1c} with respect to p and q

Combine the two situations (A)(1)(b) and (A)(1)(c) into one, then for $(p, q) \in A_1$,

$$(A1a) \quad f(q) < 0 \;\Rightarrow \begin{cases} u^*_{(A,0,0)} = u^*_{(A,0,1)} = 1 \\ c^*_{(A,0,0)} = 12p^3 + 8p^2q + 8pq^2 - 38p^2 \\ \qquad\qquad -25pq + 41p + 6, \\ c^*_{(A,0,1)} = 8q^2 - 17q + 21 \end{cases}$$

$$(A1b) \quad f(q) \geq 0 \;\Rightarrow \begin{cases} u^*_{(A,0,0)} = u^*_{(A,0,1)} = 4 \\ c^*_{(A,0,0)} = c^*_{(A,0,1)} = 11.5 \end{cases}$$

(2) $\boldsymbol{(p, q) \in A_2}$

Then, $u^*_{(B,0,0)} = 5$, $c^*[(B, 0, 0)] = 10$. The graph of A_2 is shown in Figure 3.6.

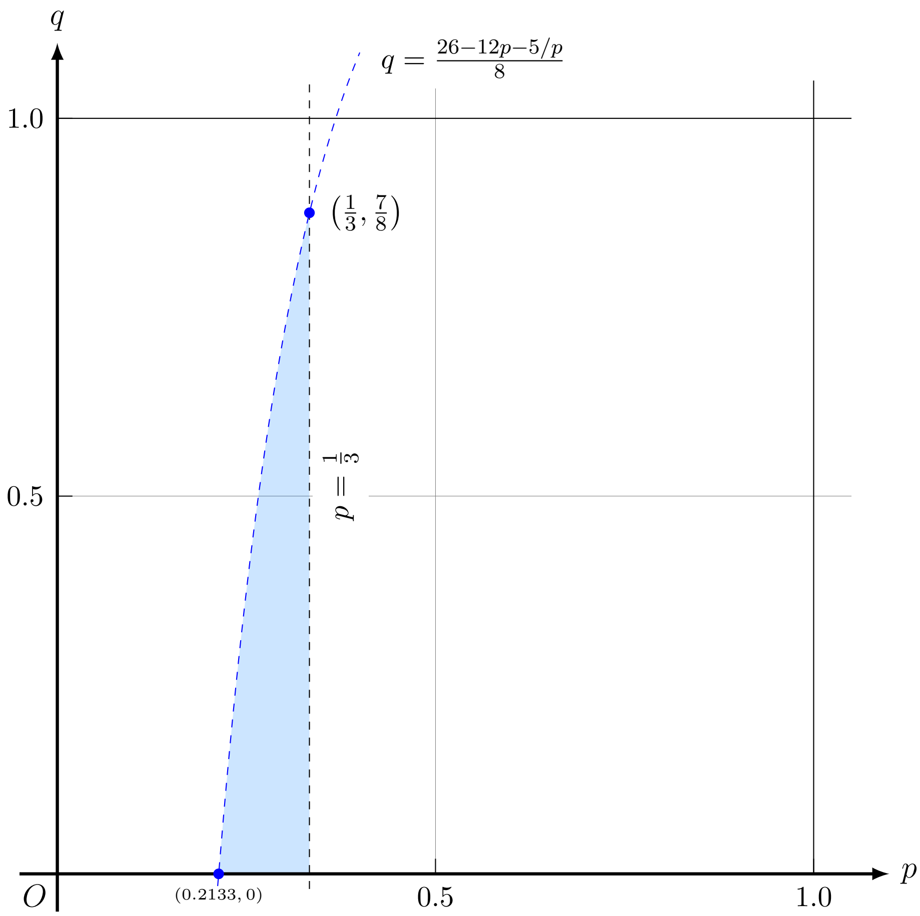

Figure 3.6: Feasible region A_2 with respect to p and q

For state $(B,0,1)$, $u^*_{(B,0,1)} = u^*_{(B,0,0)} = 5$,

$$
\begin{aligned}
c^*\left[(B,0,1)\right] = c^*\left((B,0,1), u^*_{(B,0,1)}\right) &= c((B,0,1),5) \\
&= P(\texttt{percv}) \cdot c((D,1,1)|(B,0,1),5) + \\
&\quad P(\texttt{not\_percv}) \cdot c((D,0,1)|(B,0,1),5) \\
&= 10q + 10(1-q) = 10
\end{aligned}
$$

Then we turn to node A. Given the current state $(A,0,0)$,

$$
\begin{aligned}
c((A,0,0),1) = & P(\texttt{n\_inc}) \cdot \left\{ c((B,0,0)|(A,0,0),1) + c^*\left[(B,0,0)\right] \right\} + \\
& P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot \left\{ c((B,1,1)|(A,0,0),1) + c^*\left[(B,1,1)\right] \right\} + \\
& P(\texttt{inc}) \cdot P(\texttt{n\_percv}) \cdot \left\{ c((B,0,1)|(A,0,0),1) + c^*\left[(B,0,1)\right] \right\} \\
= & (1-p)\cdot(1+10) + pq\cdot(2+10) + p(1-q)\cdot(2+10) \\
= & 11 + p < 11.5 = c((A,0,0),4)
\end{aligned}
$$

Thus, $u^*_{(A,0,0)} = 1$, $c^*[(A,0,0)] = 11 + p$. Similarly, for state $(A,0,1)$, $u^*_{(A,0,1)} = u^*_{(A,0,0)} = 1$,

$$
\begin{aligned}
c^*\left[(A,0,1)\right] &= c((A,0,1),1) \\
&= P(\texttt{percv}) \cdot \left\{ c((B,1,1)|(A,0,1),1) + c^*\left[(B,1,1)\right] \right\} + \\
&\quad P(\texttt{not\_percv}) \cdot \left\{ c((B,0,1)|(A,0,1),1) + c^*\left[(B,0,1)\right] \right\} \\
&= q(2+10) + (1-q)(2+10) = 12
\end{aligned}
$$

(3) $\boldsymbol{(p,q) \in A_3}$

The graph of A_3 is shown in Figure 3.7. Then $u^*_{(B,0,0)} = 2$ or 5 , $c^*[(B,0,0)] = 10$. However,

$$c^*((B,0,1),2) = 19 - 8q > 10 = c^*((B,0,1),5),$$

thus, $u^*_{(B,0,1)} = u^*_{(B,0,0)} = 5$, $c^*[(B,0,1)] = c^*[(B,0,0)] = 10$.

Combine the two situations (A)(2) and (A)(3) into one, then for $p < 1/3$, we

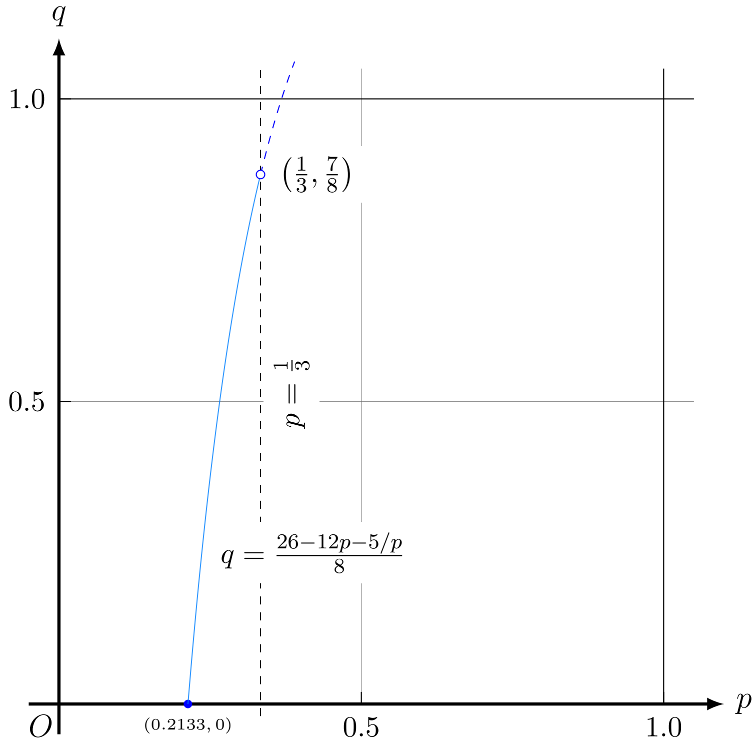

Figure 3.7: Feasible region A_3 with respect to p and q

conclude,

$$(A1) \quad 26 - 12p - \frac{5}{p} < 8q \;\Rightarrow \begin{cases} u^*_{(B,0,0)} = u^*_{(B,0,1)} = 2 \\ c^*_{(B,0,0)} = -12p^2 - 8pq + 26p + 5 \\ c^*_{(B,0,1)} = 19 - 8q \end{cases}$$

$$(A2) \quad 26 - 12p - \frac{5}{p} \geq 8q \;\Rightarrow \begin{cases} u^*_{(B,0,0)} = u^*_{(B,0,1)} = 5 \\ c^*_{(B,0,0)} = c^*_{(B,0,1)} = 10 \;; \\ \\ u^*_{(A,0,0)} = u^*_{(A,0,1)} = 1 \\ c^*_{(A,0,0)} = 11 + p \;, \quad c^*_{(A,0,1)} = 12 \end{cases}$$

(B) $\boldsymbol{p > \frac{1}{3}}$

Then $4 + 12p > 8$, $u^*_{(C,0,0)} = 6$, $c^*[(C,0,0)] = 8$. Similarly, $u^*_{(C,0,1)} = u^*_{(C,0,0)} = 6$, and

$$
\begin{aligned}
c^*[(C,0,1)] &= c^*\left((C,0,1), u^*_{(C,0,1)}\right) = c((C,0,1),6) \\
&= P(\texttt{percv}) \cdot c((D,1,1)|(C,0,1),6) + \\
&\quad P(\texttt{not\_percv}) \cdot c((D,0,1)|(C,0,1),6) \\
&= 8q + 8(1-q) = 8
\end{aligned}
$$

Next we consider node-states at node B. Given the current state $(B,0,0)$,

$$
\begin{aligned}
c((B,0,0),2) &= P(\texttt{n\_inc}) \cdot \left\{c((C,0,0)|(B,0,0),2) + c^*\left[(C,0,0)\right]\right\} + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot \left\{c((C,1,1)|(B,0,0),2) + c^*\left[(C,1,1)\right]\right\} + \\
&\quad P(\texttt{inc}) \cdot P(\texttt{n\_percv}) \cdot \left\{c((C,0,1)|(B,0,0),2) + c^*\left[(C,0,1)\right]\right\} \\
&= (1-p)\cdot(1+8) + pq\cdot(3+8) + p(1-q)\cdot(3+8) \\
&= 9 + 2p
\end{aligned}
$$

Since $c((B,0,0),5) = 10$, then for $p > 1/3$,

$$
u^*_{(B,0,0)} = \begin{cases} 2, & \text{if } p < 1/2 \\ 5, & \text{if } p > 1/2 \\ \text{either}, & \text{if } p = 1/2 \end{cases}
$$

(1) $\boldsymbol{\frac{1}{3} < p < \frac{1}{2}}$

Then $u^*_{(B,0,0)} = 2$, $c^*[(B,0,0)] = 9 + 2p$. Consider state $(B,0,1)$, then $u^*_{(B,0,1)} = u^*_{(B,0,0)} = 2$,

$$
\begin{aligned}
c^*\left[(B,0,1)\right] &= c^*\left((B,0,1), u^*_{(B,0,1)}\right) = c((B,0,1),2) \\
&= P(\texttt{percv}) \cdot \left\{c((C,1,1)|(B,0,1),2) + c^*\left[(C,1,1)\right]\right\} + \\
&\quad P(\texttt{not\_percv}) \cdot \left\{c((C,0,1)|(B,0,1),2) + c^*\left[(C,0,1)\right]\right\} \\
&= q(3+8) + (1-q)(3+8) = 11
\end{aligned}
$$

We turn to node-states at node A. Given the current state $(A, 0, 0)$,

$$\begin{aligned} c((A,0,0),1) = {} & P(\texttt{n\_inc}) \cdot \left\{ c((B,0,0)|(A,0,0),1) + c^*\left[(B,0,0)\right] \right\} + \\ & P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot \left\{ c((B,1,1)|(A,0,0),1) + c^*\left[(B,1,1)\right] \right\} + \\ & P(\texttt{inc}) \cdot P(\texttt{n\_percv}) \cdot \left\{ c((B,0,1)|(A,0,0),1) + c^*\left[(B,0,1)\right] \right\} \\ = {} & (1-p) \cdot [1 + (9+2p)] + pq \cdot (2+10) + p(1-q) \cdot (2+11) \\ = {} & -2p^2 - pq + 5p + 10 \end{aligned}$$

Since $c((A,0,0),4) = 11.5$, then

$$\begin{aligned} -2p^2 - pq + 5p + 10 &= 11.5 \\ \Leftrightarrow \qquad -2p^2 + 5p - 1.5 &= pq \\ \Leftrightarrow \qquad 5 - 2p - \frac{3}{2p} &= q \end{aligned}$$

For $1/3 < p < 1/2$,

$$u^*_{(A,0,0)} = \begin{cases} 1, & \text{if } 5 - 2p - 3/(2p) < q \\ 4, & \text{if } 5 - 2p - 3/(2p) > q \\ \text{either}, & \text{if } 5 - 2p - 3/(2p) = q \end{cases}$$

We further divide the sub-problem into three situations. Let

$$\begin{aligned} B_{1a} &= \left\{ (p,q) : \frac{1}{3} < p < \frac{1}{2},\ q > 5 - 2p - \frac{3}{2p} \right\} \\ B_{1b} &= \left\{ (p,q) : \frac{1}{3} < p < \frac{1}{2},\ q < 5 - 2p - \frac{3}{2p} \right\} \\ B_{1c} &= \left\{ (p,q) : \frac{1}{3} < p < \frac{1}{2},\ q = 5 - 2p - \frac{3}{2p} \right\} \end{aligned}$$

(a) $\boldsymbol{(p, q) \in B_{1a}}$

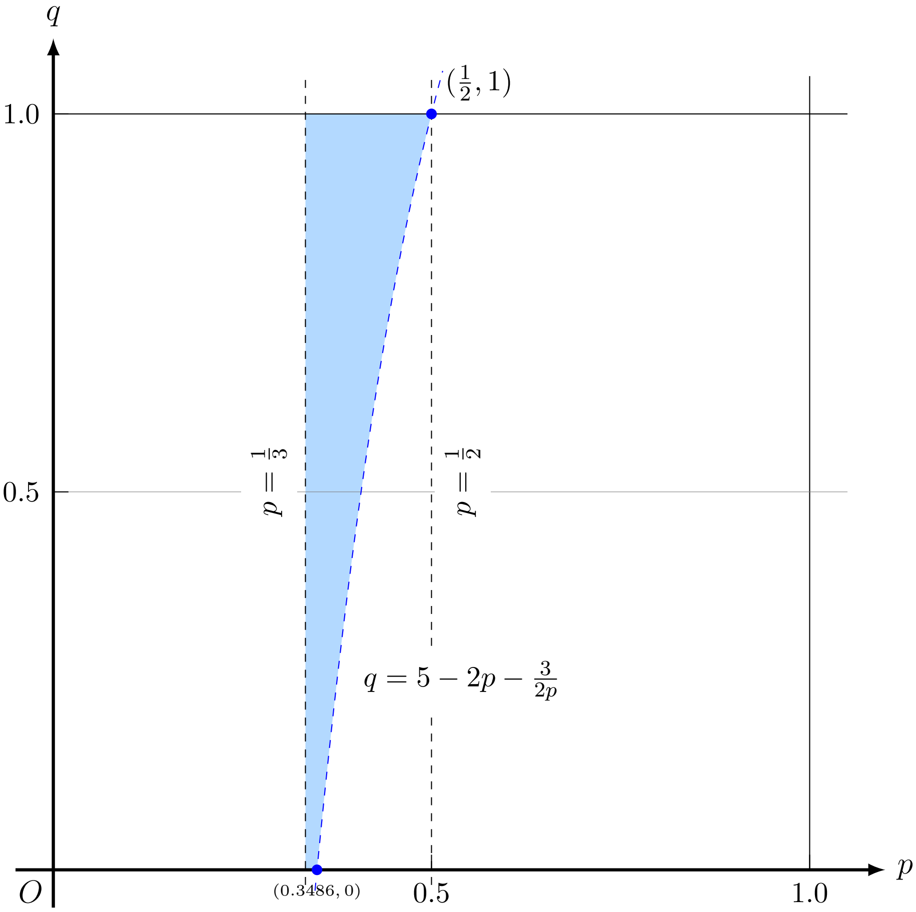

Figure 3.8: Feasible region B_{1a} with respect to p and q

Then $u^*_{(A,0,0)} = 1$, $c^*[(A,0,0)] = -2p^2 - pq + 5p + 10$. The graph of of B_{1a} is shown in Figure 3.8. Consider state $(A,0,1)$, since $u^*_{(A,0,1)} = u^*_{(A,0,0)} = 1$, then

$$
\begin{aligned}
c^*\left[(A,0,1)\right] = c^*\left((A,0,1), u^*_{(A,0,1)}\right) &= c((A,0,1),1) \\
&= P(\texttt{percv}) \cdot \left\{c((B,1,1)|(A,0,1),1) + c^*\left[(B,1,1)\right]\right\} + \\
&\quad P(\texttt{not\_percv}) \cdot \left\{c((B,0,1)|(A,0,1),1) + c^*\left[(B,0,1)\right]\right\} \\
&= q(2+10) + (1-q)(2+11) = 13 - q
\end{aligned}
$$

(b) $\boldsymbol{(p,q) \in B_{1b}}$

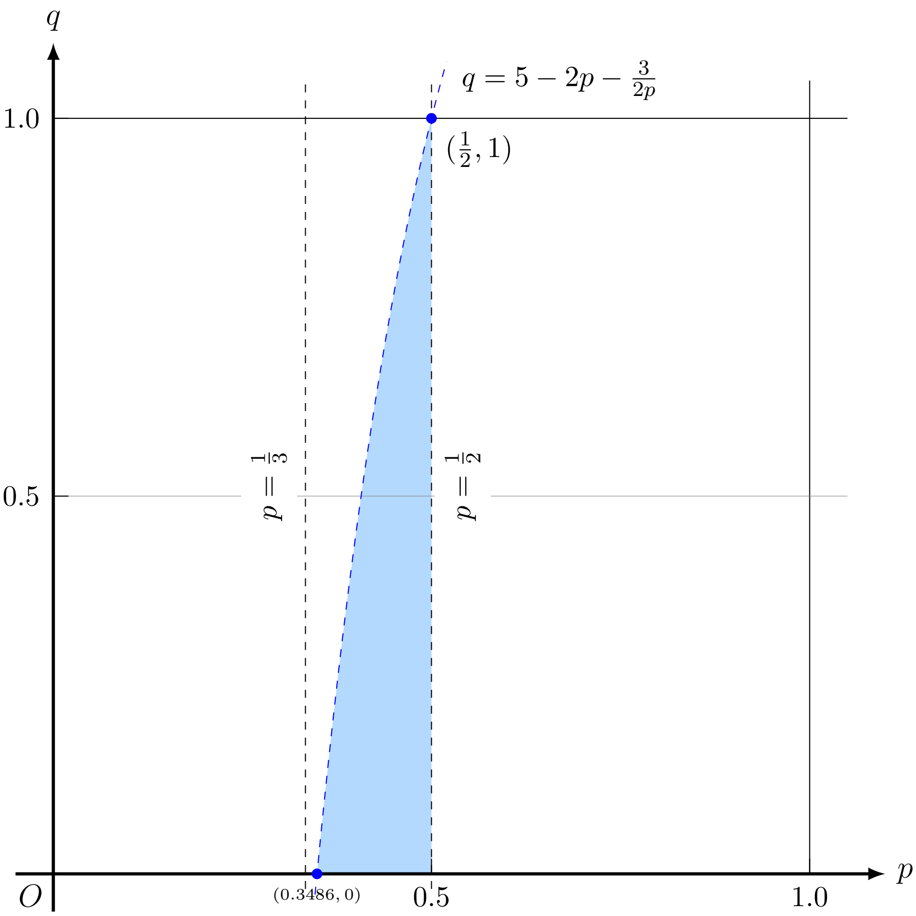

Figure 3.9: Feasible region B_{1b} with respect to p and q

Then $u^*_{(A,0,0)} = 4$, $c^*[(A,0,0)] = 11.5$. The graph of of B_{1b} is shown in Figure 3.9. Consider state $(A,0,1)$, since $u^*_{(A,0,1)} = u^*_{(A,0,0)} = 4$, then

$$
\begin{aligned}
c^*\left[(A,0,1)\right] &= c^*\left((A,0,1), u^*_{(A,0,1)}\right) = c((A,0,1),4) \\
&= P(\texttt{percv}) \cdot c((D,1,1)|(A,0,1),4) + \\
&\quad P(\texttt{not\_percv}) \cdot c((D,0,1)|(A,0,1),4) \\
&= 11.5q + 11.5(1-q) = 11.5
\end{aligned}
$$

(c) $\boldsymbol{(p,q) \in B_{1c}}$

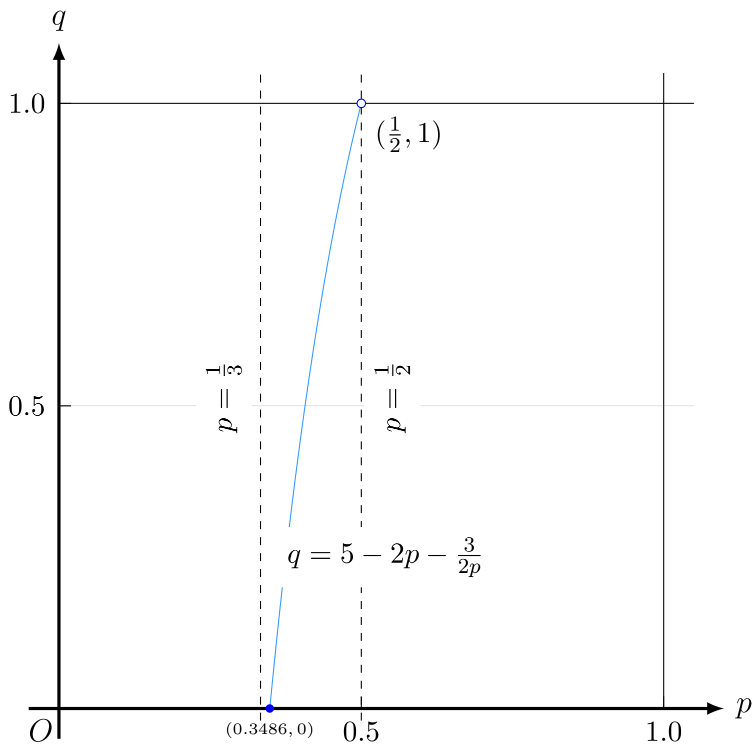

Figure 3.10: Feasible region B_{1c} with respect to p and q

The graph of of B_{1c} is shown in Figure 3.10. Then $u^*_{(A,0,0)} = 1$ or 4, $c^*[(A,0,0)] = 11.5$. However,

$$c^*((A,0,1),1) = 13 - q > 11.5 = c^*((A,0,1),4),$$

thus, $u^*_{(A,0,1)} = u^*_{(A,0,0)} = 4$, $c^*[(B,0,1)] = c^*[(B,0,0)] = 11.5$.

Combine two situations (B)(1)(b) and (B)(1)(c) to one, then for $1/3 < p < 1/2$, we conclude,

$$(B1a) \quad 5 - 2p - \frac{3}{2p} < q \;\Rightarrow \begin{cases} u^*_{(A,0,0)} = u^*_{(A,0,1)} = 1 \\ c^*_{(A,0,0)} = -2p^2 - pq + 5p + 10 \\ c^*_{(A,0,1)} = 13 - q \end{cases}$$

$$(B1b) \quad 5 - 2p - \frac{3}{2p} \geq q \;\Rightarrow \begin{cases} u^*_{(A,0,0)} = u^*_{(A,0,1)} = 4 \\ c^*_{(A,0,0)} = c^*_{(A,0,1)} = 11.5 \end{cases}$$

(2) $\boldsymbol{p > \frac{1}{2}}$

Then $u^*_{(B,0,0)} = 5$, $c^*[(B,0,0)] = 10$. Consider state $(B,0,1)$, then $u^*_{(B,0,1)} = u^*_{(B,0,0)} = 5$,

$$\begin{aligned} c^*\left[(B,0,1)\right] = c^*\left((B,0,1), u^*_{(B,0,1)}\right) &= c((B,0,1),5) \\ &= P(\texttt{percv}) \cdot c((D,1,1)|(B,0,1),5) + \\ &\quad P(\texttt{not\_percv}) \cdot c((D,0,1)|(B,0,1),5) \\ &= 10q + 10(1-q) = 10 \end{aligned}$$

We turn to node-states at node A. Given the current state $(A,0,0)$,

$$\begin{aligned} c((A,0,0),1) = &\ P(\texttt{n\_inc}) \cdot \left\{c((B,0,0)|(A,0,0),1) + c^*\left[(B,0,0)\right]\right\} + \\ &\ P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot \left\{c((B,1,1)|(A,0,0),1) + c^*\left[(B,1,1)\right]\right\} + \\ &\ P(\texttt{inc}) \cdot P(\texttt{n\_percv}) \cdot \left\{c((B,0,1)|(A,0,0),1) + c^*\left[(B,0,1)\right]\right\} \\ = &\ (1-p)\cdot(1+10) + pq\cdot(2+10) + p(1-q)\cdot(2+10) \\ = &\ 11 + p > 11.5 = c((A,0,0),4) \end{aligned}$$

Thus, $u^*_{(A,0,0)} = 4$, $c^*[(A,0,0)] = 11.5$. Similarly for $(A,0,1)$, $u^*_{(A,0,1)} = u^*_{(A,0,0)} = 4$, $c^*\left[(A,0,1)\right] = c((A,0,1),4) = 11.5$.

(3) $\boldsymbol{p = \frac{1}{2}}$

Then $u^*_{(B,0,0)} = 2$ or 5, $c^*[(B,0,0)] = 10$. However,

$$c^*((B,0,1),2) = 11^1 > 10 = c^*((B,0,1),5),$$

thus, $u^*_{(B,0,1)} = u^*_{(B,0,0)} = 5$, $c^*[(B,0,1)] = c^*[(B,0,0)] = 10$.

[1]See situation (B)(1).

Combine the two situations (B)(2) and (B)(3) into one, then for $p > 1/3$, we conclude,

$$(B1) \quad p < \frac{1}{2} \;\Rightarrow\; \begin{cases} u^*_{(B,0,0)} = u^*_{(B,0,1)} = 2 \\ c^*_{(B,0,0)} = 9 + 2p \\ c^*_{(B,0,1)} = 11 \end{cases}$$

$$(B2) \quad p \geq \frac{1}{2} \;\Rightarrow\; \begin{cases} u^*_{(B,0,0)} = u^*_{(B,0,1)} = 5 \\ c^*_{(B,0,0)} = c^*_{(B,0,1)} = 10 \;; \\ \\ u^*_{(A,0,0)} = u^*_{(A,0,1)} = 4 \\ c^*_{(A,0,0)} = c^*_{(A,0,1)} = 11.5 \end{cases}$$

(C) $\boldsymbol{p = \frac{1}{3}}$

Then $u^*_{(C,0,0)} = 3$ or 6, $c^*[(C,0,0)] = 8$. For state $((C,0,1))$, since

$$c^*((C,0,1),3) = 16 > 8 = c^*((C,0,1),6),$$

thus, $u^*_{(C,0,1)} = u^*_{(C,0,0)} = 6$, $c^*[(C,0,1)] = c^*[(C,0,0)] = 8$.

Next we consider node-states at node B. Given the current state $(B,0,0)$,

$$\begin{aligned} c((B,0,0),2) = {} & P(\texttt{n\_inc}) \cdot \left\{ c((C,0,0)|(B,0,0),2) + c^*\left[(C,0,0)\right] \right\} + \\ & P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot \left\{ c((C,1,1)|(B,0,0),2) + c^*\left[(C,1,1)\right] \right\} + \\ & P(\texttt{inc}) \cdot P(\texttt{n\_percv}) \cdot \left\{ c((C,0,1)|(B,0,0),2) + c^*\left[(C,0,1)\right] \right\} \\ = {} & (1-p)\cdot(1+8) + pq\cdot(3+8) + p(1-q)\cdot(3+8) \\ = {} & 9 + 2p = 9\frac{2}{3} < 10 = c((B,0,0),5) \end{aligned}$$

Then $u^*_{(B,0,0)} = 2$, $c^*[(B,0,0)] = 9\frac{2}{3}$. $u^*_{(B,0,1)} = u^*_{(B,0,0)} = 2$,

$$\begin{aligned} c^*\left[(B,0,1)\right] &= c((B,0,1),2) \\ &= P(\texttt{percv}) \cdot \left\{ c((C,1,1)|(B,0,1),2) + c^*\left[(C,1,1)\right] \right\} + \\ &\quad P(\texttt{not\_percv}) \cdot \left\{ c((C,0,1)|(B,0,1),2) + c^*\left[(C,0,1)\right] \right\} \\ &= q(3+8) + (1-q)(3+8) = 11 \end{aligned}$$

Finally we consider node-states at node A. Given the current state $(A,0,0)$,

$$\begin{aligned} c((A,0,0),1) = &\; P(\texttt{n\_inc}) \cdot \left\{ c((B,0,0)|(A,0,0),1) + c^*\left[(B,0,0)\right] \right\} + \\ &\; P(\texttt{inc}) \cdot P(\texttt{percv}) \cdot \left\{ c((B,1,1)|(A,0,0),1) + c^*\left[(B,1,1)\right] \right\} + \\ &\; P(\texttt{inc}) \cdot P(\texttt{n\_percv}) \cdot \left\{ c((B,0,1)|(A,0,0),1) + c^*\left[(B,0,1)\right] \right\} \\ = &\; (1-p) \cdot \left(1 + 9\frac{2}{3}\right) + pq \cdot (2+10) + p(1-q) \cdot (2+11) \\ = &\; \frac{103-3q}{9} < 11.5 = c((A,0,0),4) \end{aligned}$$

Then, $u^*_{(A,0,0)} = 1$, $c^*[(A,0,0)] = \frac{103-3q}{9}$, $u^*_{(A,0,1)} = u^*_{(A,0,0)} = 1$. However,

$$\begin{aligned} c^*\left[(A,0,1)\right] &= c((A,0,1),1) \\ &= P(\texttt{percv}) \cdot \left\{ c((B,1,1)|(A,0,1),1) + c^*\left[(B,1,1)\right] \right\} + \\ &\quad P(\texttt{not\_percv}) \cdot \left\{ c((B,0,1)|(A,0,1),1) + c^*\left[(B,0,1)\right] \right\} \\ &= q(2+10) + (1-q)(2+11) \\ &= 13 - q > 11.5 = c((A,0,1),4) \end{aligned}$$

That is, according to backward induction, when p takes the critical value of $1/3$, the best initial action for state $(A,0,0)$ is link 1, so in the vehicle's view, the best initial action for state $(A,0,1)$ is also link 1. Unfortunately, the "theoretical" best initial action for state $(A, 0, 1)$ is link 4, for the minimal expected travel cost is 11.5, which is less than $13-q$. Therefore, we cannot combine situation (B) and (C), because situation (C) reflect the inconsistency between the vehicle's decision and the result it will get as mentioned in situation (A).

3.2.6 Summary

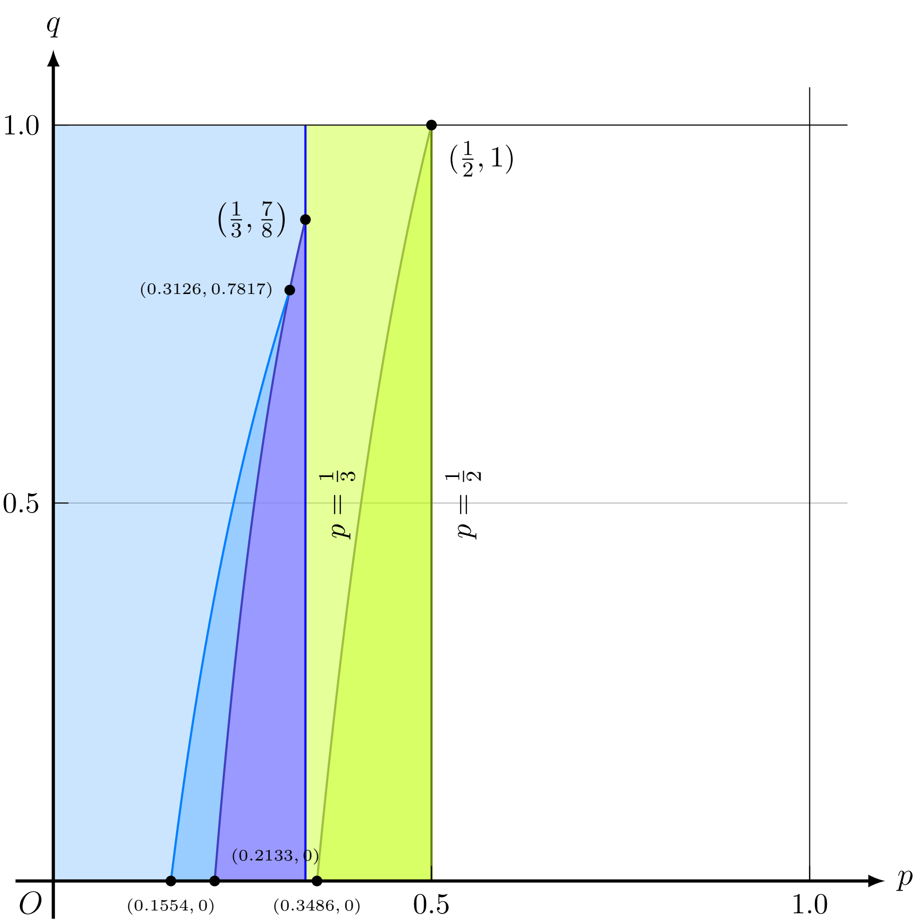

Figure 3.11: Regions for optimal policies based on different pairs of p and q

Based on backward induction, we determine the best initial action and the corresponding minimal expected travel cost for each node-state with respect to different value pairs of p and q. The best initial action and the minimal expected cost for each state, keeps unchanged in a same feasible region while differs in different regions (shaded with different colors in Figure 3.11). Particularly, the best initial action and the minimal expected cost of each state on the boundary curve could be incorporated into the region on the right side of it, except the line $p = 1/3$ which is explained in situation (C). However, as mentioned in situation $(A1a)$ and (C), the best initial action of state $(A, 0, 1)$ under the vehicle's insufficient perception, is not its "theoretical" best initial action, which reflects the inconsistency between the vehicle's decision and the real result it will get.

Additionally, notwithstanding the inconsistency, the minimal expected travel costs for each node-state in different situations are all either constants or monotonically increasing with respect to p and monotonically decreasing with respect to q, that is, the higher probability of incident, the more expected travel costs, as well as the higher probability of CAV's perception, the less expected travel costs. It is an intuitive finding, which is meanwhile consistent with the reality, and also reflects the value of information.

3.3 Model generalization

Consider a traffic network $\mathcal{G} = (N, A)$ with set of nodes N and set of links A.

Let $\mathcal{I}$ be the set of possible incidents, which represent decreases in capacity due to temporary events such as construction or vehicle collisions. Each incident $i \in \mathcal{I}$ increases travel times on one or more links. Denote by $\diamond \in \mathcal{I}$ the state of no known incident occurring. If a traveler believes the incident state is $\diamond$, then the traveler believes that no incidents are active and will choose routes accordingly.

Let $\tau_a(i)$ be the travel time on link $a \in A$ when the incident state is i. The travel time without any incidents is $\tau_a(\diamond)$.

3.3.1 Model assumptions

Before we generalize the information sharing model, some assumptions based on the freeway instance should be concluded:

Assumption 1: CAVs knows a priori probabilistic description of the network, and could get personalized information (e.g., by Google Map).

Assumption 2 (permanent information): Once CAV gets the incident information from the operator or perceives information by itself, it holds the information from then on until it arrives at the destination.

Assumption 3: If a CAV gets the incident information, then an incident definitely happens (operator would not tell a lie); otherwise, an incident may happen or not.

Assumption 4: If CAV neither receives any incident information from the system operator, nor perceives any incident information, it will follow the best initial action as if no incident occurs.

Assumption 5 (mutually exclusive incidents): Incidents are mutually exclusive, i.e. if incident i occurs then incident $i' \neq i$ does not occur. Vehicles are aware of this mutual exclusion.

The assumption that incidents are mutually exclusive is not limiting; because any incident $i \in \mathcal{I}$ may affect multiple links, multiple distinct events may be coded as one "incident" in $\mathcal{I}$. However, note that including combinations of many distinct capacity reductions in $\mathcal{I}$ will greatly increase its size. Therefore, it may be reasonable to restrict incidents in $\mathcal{I}$ to singular causes in link capacity reductions. This is likely fairly realistic: the probability of multiple distinct causes of capacity reductions occurring simultaneously is low, and furthermore travelers may not react to every distinct capacity reduction in their adaptive routing.

3.3.2 State space

A state consists of the vehicle's location in the network as well as the perception of the incident state. Let $\mathcal{X} \triangleq N \times I \times \mathcal{I}$ be the state space. A state $x(k) \triangleq (n(k), info(k), i(k))$ consists of a location $n(k) \in N$, the incident information perception $info(k) \in I$ and the incident indicator $i(k) \in \mathcal{I}$. Note that the step k is distinct from time. A step consists of traversing a link in the network.

The vehicle also has a destination, s. The states $(s, \cdot)$ are all termination states that the traveler will remain in after reaching one.

The network location is deterministic and controlled by the vehicle. The incident perception, however, is stochastic from the perspective of the vehicle. If an incident occurs, $i(k)$ may update if the system informs the vehicle. Both the occurrence of the incident, and whether a vehicle is informed, are stochastic. We assume that the system does not falsely inform vehicles of incidents. In other words, if the actual incident state is i_{net}, the vehicle will either receive information that the incident state is i_{net}, or that the incident state is $\diamond$, but not anything else.

3.3.3 Action space

At each node, the vehicle has the option to proceed on any of the downstream links. Let $\Gamma_n^+ \subseteq A$ denote the set of links outgoing from node n. Let $U(x)$ denote the action space when the state is $x = (n, info, i)$. If $n = s$, $U(x) = \{\mathscr{P}\}$, where $\mathscr{P}$ is the action to park or remain parked, because $(s, \cdot)$ is a termination state (the traveler's destination). Otherwise, $U(x) = \Gamma_n^+$. The traveler can choose any downstream links, and will be able to traverse that link deterministically.

3.3.4 Transition function

As there are two components of the state, there are two components to the transition. The vehicle's location in the network is deterministic and depends entirely on the choice of action. On the other hand, the perception of the incident state is stochastic as it depends on information propagation. Let $f(x, u)$ define the next state when the state is $x = (n, info, i)$ and the action taken is u. $f(x, u)$ is defined in three components as

$$f(x, u) = (f_{\mathrm{N}}(x, u), f_I(x, u), f_{\mathcal{I}}(x, u)) \tag{3.2a}$$

The location transition is deterministic.

$$f_{\mathrm{N}}(x, u) = \begin{cases} s, & \text{if } n = s \\ \gamma^+(u), & \text{else} \end{cases} \tag{3.2b}$$

where $\gamma^+(a) \in N$ is the downstream end of link a. Recall that if $u \neq \mathscr{P}$, then $u \in \Gamma_n^+$ is the downstream link.

The incident perception is stochastic. Let p_i be the probability that incident $i \neq \diamond$ occurs. Let q_i be the probability that the vehicle perceive the incident i, the system informs the vehicle that incident i is occurring. The probability of receiving information about i is $p_i q_i$. However, the vehicle can learn about i another way. If the vehicle enters an affected link, the higher travel times will be noticed and cause the vehicle to infer that incident i is occurring. Therefore, the transition in the incident perception is

$$f_I(x,u) = \begin{cases} \diamond, & \text{if } u = \mathscr{P} \\ \imath, & \text{if } \imath \neq \diamond \\ \jmath, & \text{w.p. } p_\jmath q_\jmath \text{ if } \tau_u(\diamond) = \tau_u(\jmath) \\ \diamond, & \text{else} \end{cases} \tag{3.2c}$$

The incident perception updates through observation because traveling through a link affected by an incident will be noticeable both to travelers and autonomous vehicles. Travelers will notice the congestion and may be able to visually identify the incident itself. Although CAVs may not visually recognize the incident, they will recognize any discrepancy in the travel time from what is expected. Furthermore, they can compare the experienced travel time with those expected for each possible incident.

$$f_{\mathcal{I}}(x,u) = \begin{cases} \diamond, & \text{w.p. } 1 - p_i \\ \imath, & \text{w.p. } p_i \\ \diamond, & \text{else} \end{cases} \tag{3.2d}$$

From a modeling standpoint, updating the incident perception with observation ensures that the vehicle's perception of travel times remains accurate.

3.3.5 One-step costs

If $u = \mathscr{P}$, then there is not any associated cost. The cost of traveling along a link is the associated travel time. Let $c(x,u)$ be the cost when the state is $x = (n, info, \imath)$ and the action is u. $c(x,u)$ is defined as

$$c(x,u) = \begin{cases} 0, & \text{if } u = \mathscr{P} \\ \tau_u(\imath), & \text{else} \end{cases} \tag{3.3}$$

Based on this definition, $u = \mathscr{P}$ is the termination state. After reaching s, the cost-to-go is 0.

3.3.6 Solution algorithm

This is a non-discounted infinite horizon MDP, which can be solved by value iteration (Bellman, 1957). Pseudocode for value iteration is shown as follow:

Algorithm 1 Value Iteration for MDP of a single CAV

Require: S, state space; $u(x)$, actions; P, transition probability matrix; C, cost functions; ϵ, the maximum error
Ensure: A utility function

```
repeat
  V' ← 0, V ← V', δ ← 0
  for each state x in S do
    for each action in u(x) do
      V'(x) ← max_u Σ_{x'} P(x'|x,u)[−c(x'|x,u) + V(x')]
    end for
    if |V(x) − V'(x)| > δ then
      δ ← |V(x) − V'(x)|
    end if
  end for
until δ < ε
```

Chapter 4

Numerical experiments

In this chapter, we conduct experiments on two networks, representing the city of Sioux Falls, and downtown Austin, Texas. Numerical results are shown based on the following two aspects: the adaptive routing behavior of a single CAV across multiple origins and destinations in both networks, including average expected link costs and the impacts of incidents and CAVs' perception of incident on the minimum expected cost and optimal routing policy by changing the probability of an incident and CAVs' perception of incident.

In addition to running base scenarios, we perform sensitivity analysis with respect to three parameters: the probability of incident occurrence, the incident severity, and CAVs' perception of incident information. This analysis focuses on the following questions:

1. How will a CAV will make its routing decisions under different incident environments, such as regular congestion with high incident probability and car accidents with fairly low incident probability?

2. How does a CAV reacts facing different severity of incidents?

3. How does a CAV's capability of perception information gaining such as incident perception, have impacts on its routing behavior?

The results should somehow demonstrate the overall value of receiving information for CAVs.

4.1 Sioux Falls network

Before the simulation, we should make some assumption about the probabilistic description of network when there indeed happens an incident. Since incidents may happen at each location of the network, it is complicated to show the entire probabilistic description of each incident throughout the network. Thus, for convenience of presentation, we just create the incident link cost data for a single incident instance, rather than use probabilistic description of the incidents throughout the whole network. For example, in Sioux Falls network (Figure 4.1), we always assume that link 46 is the only potential location for an incident, and will affect links #41, #57, #28, #32, #48, #25, #13, #21, by increasing their cost if an incident occurs. Specifically, when the incident happens on link #46, the affected links (road segments) are link #46 with the travel costs increasing 12.0 units, link #57, #41, #28 with increased 9.0 costs, link #48, #32, #25 with increased 6.0 costs, and link #13, #21 with the travel costs increasing 3.0 units.

In the base scenario, the probability of an incident and the probability of CAVs' incident perception are assumed to be the value of 0.1 and 0.6, respectively. We first explore the difference in expected travel costs across multiple origins and destinations (Figure 4.2). There are 24 nodes in the Sioux Falls network, and each time we take one node, say node 1 as the destination of this individual CAV and other nodes as the origins to explore the distribution of the average expected travel costs throughout the whole network, i.e., calculating the average expected travel costs across 23 origin-destination pairs at one time and comparing the 24 calculated average values.

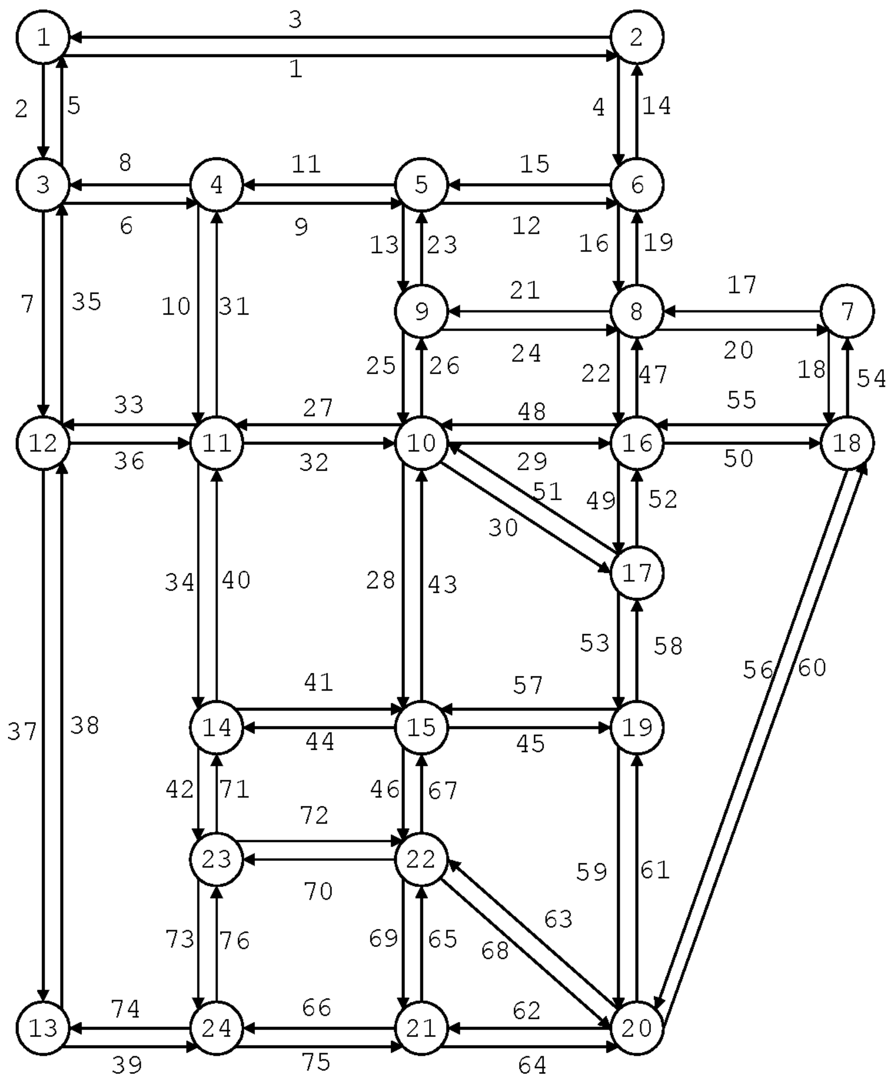

Figure 4.1: Sioux Falls network with link labels

Secondly, we examine how much the expected costs depend on the probability of an incident. Starting from the base scenario, we keep the probability of incident perception for CAVs, q, constant, and change the incident probability, p. The average expected link costs for each destination are shown in Figure 4.3. When we take node 1 to node 6 these 6 nodes as destinations, the expected shortest paths do not include those affected links, so the average expected travel costs remain the same as the

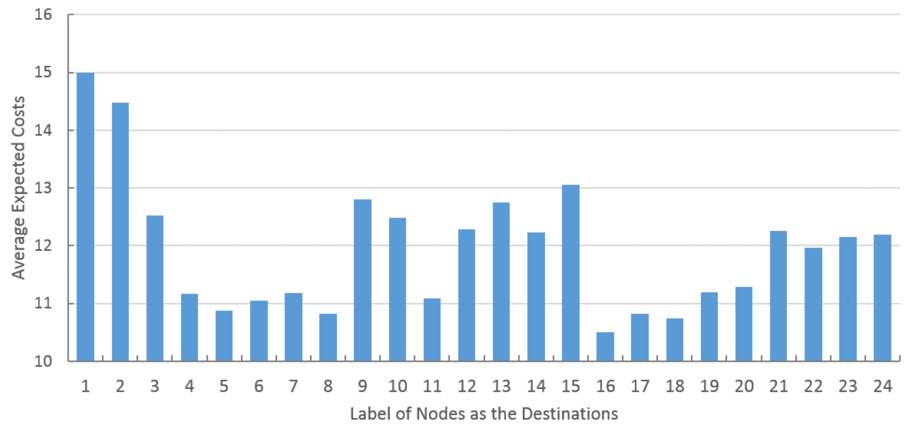

Figure 4.2: Average expected costs across multiple origins and destinations ($p = 0.1$, $q = 0.6$)

incident probability increases.

The best actions and the average expected costs remain the same for the state with the same node if node 1 to node 6 is taken as the destination, this is because the shortest paths do not include affected links for these 6 nodes. Besides those 7 nodes, the expected link costs will increase when the probability of an incident increases. In addition, the expected costs for incident-affected nodes as destinations are higher than those nodes which are not affected by the incident, and also the costs will increase more with the probability of the incident increases.

Thirdly, we explore how much the expected costs depend on the probability of CAVs' incident perception (Figure 4.4).

The main results from this analysis are as follows:

1. Out of 24 nodes as destinations, only 7 destinations witness the slight decrease in average expected costs with the CAV's perception probability increasing. However, since the average expected costs merely decreased by 0.037% for node #19

as the destination, this node is not included in Figure 4.4. For other nodes, the average expected cost remains invariant for all perception levels with the same destination.

2. The expected costs will decrease when CAV's perception probability increases. This is simply because on average, the more incident information the vehicle gets, the wiser routing decisions it will take, which directly cause decreases in travel costs. Therefore, in spite of the fact that the minimal expected cost for an individual state increases monotonically with respect to q, higher perception probability will result in lower expected costs.

3. The average expected costs are more sensitive to change when the probability of the incident changes than when the probability of CAVs' perception of incidents changes.

4.2 Downtown Austin network

We choose node #5469, which is located on the center of the downtown (Figure 4.5), as the destination to examine the extent to which both incident probability and CAV's perception probability have impact on the average expected costs.

Similar results are shown in Figure 4.6: Two average expected costs change in opposite directions for incident probability and CAV's perception probability. Along with the increase of the incident probability, the average expected costs would rise, but the rate of increase would lower. While the average expected costs will decrease when the perception probability increases with an exception of $q = 0.8$. The possible reason is that this value of $q = 0.8$ is a threshold for vehicle to choose optimal links, when q goes up, the vehicle will choose a link with lower expected cost when there

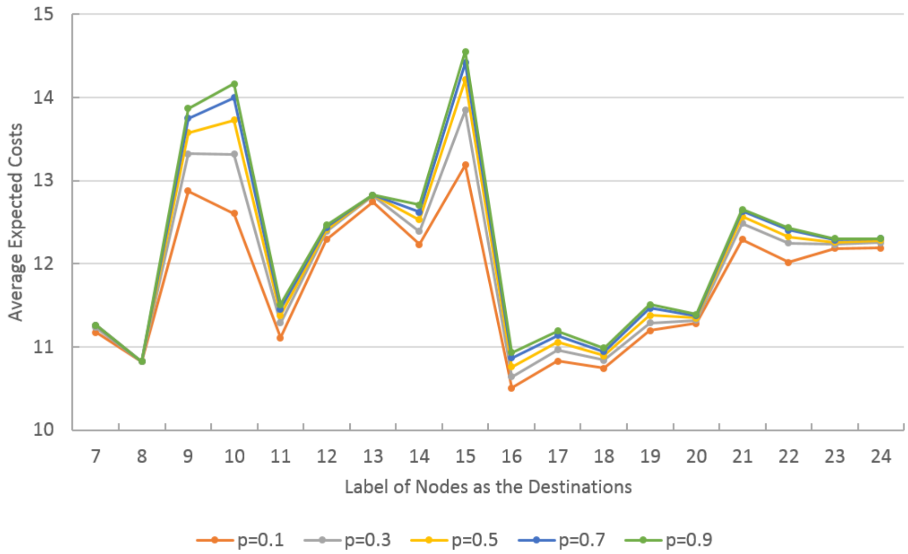

Figure 4.3: Average expected costs with different incident probabilities ($q = 0.6$)

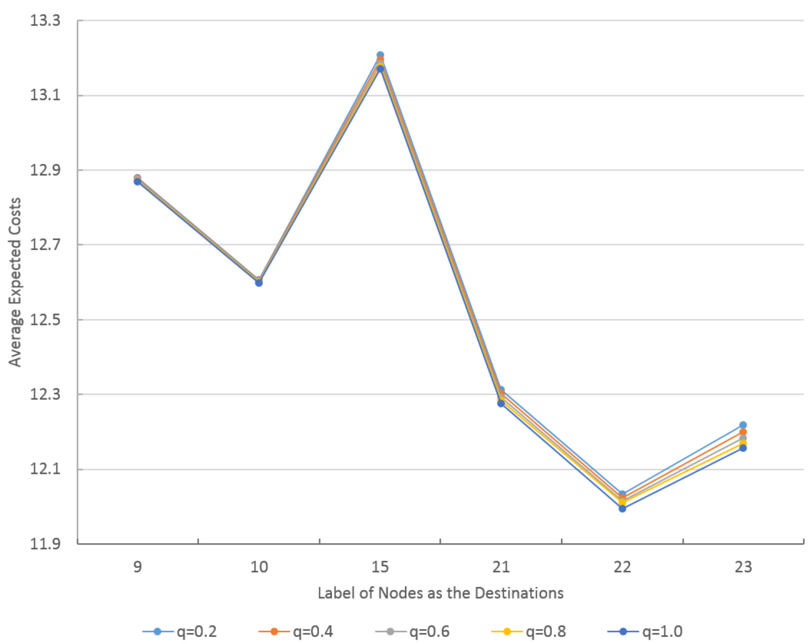

Figure 4.4: Average expected costs with different perception probabilities ($p = 0.1$)

is no incident and with high cost when there is an incident based on a certain state, then increase the expected cost of its previous states.

However, two rate of change are almost negligible, 0.03% and -0.004%, respectively. So for a single CAV, both the incident and perception of incident have slight impacts on the expected travel times in Austin network.

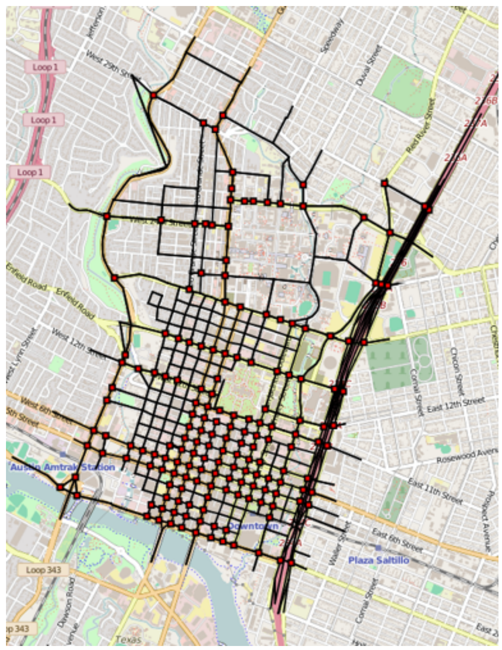

Figure 4.5: Downtown Austin network

Overall, for CAV's perception probability, the rate of change from the lowest to the highest is almost negligible, with the largest rate of 2% in Sioux Falls network and less than 0.02% in Austin network. Thus, we can conclude that the perception probabilities have minute impact on the expected costs for a single CAV. However, intuitively, the situation would be utterly different if it comes to the scenario with multiple CAVs.

However, based on the incident instance above, we get an unexpected result

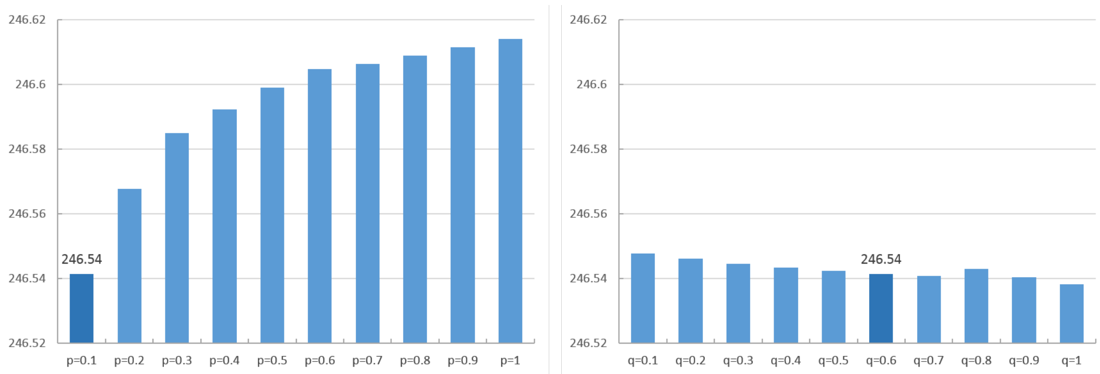

Figure 4.6: Average expected costs for downtown Austin. Left: Average expected costs with different incident probabilities ($q = 0.6$) ; Right: Average expected costs with different perception probabilities ($p = 0.1$)

that, keeping the destination node invariant, the optimal policy for a certain state will change with respect to the change in the value of the incident probability and CAV's perception probability.

Chapter 5

Conclusion

Backward induction is used to determine the best action and the minimal expected travel cost for each node-state in the motivating example. With respect to different value pairs of p and q, the best initial action and the minimal expected travel cost for each state, keeps unchanged in a same feasible region while differs in different regions. Particularly, the best initial action and the minimal expected cost of each state on the boundary curve could be incorporated into the region on the right side of it, except a certain value of $p = 1/3$.

However, the best initial action for a certain node-state under the vehicle's insufficient perception, is not its "theoretical" best initial action, which reflects the inconsistency between the vehicle's decision and the real result it will get.

Additionally, the calculated minimal expected travel cost of each node-state shows that the higher probability of incident, the more expected travel costs, as well as the higher probability of CAV's perception, the less expected travel costs, which reflects the value of information.

Value iteration is applied to solve the non-discounted infinite horizon MDP. Similar results are shown in different networks in numerical experiments that if the vehicle is more likely to get the incident information (or gets more traffic-related information), the best actions are always to travel the alternative routes to avoid congested traffic (or equivalently increased link cost).

The incident probability exerts more impacts on the average expected costs, the largest increase rate of which reaches 12% in Sioux Falls network, compared with the CAV's perception probability, with only 2% as their largest increase rate in the same network. However, intuitively, both the incident probability and the CAV's perception probability would have more significant influences on the expected costs when it comes to the scenario with multiple CAVs.

Considerable future work remains. There are two possible extensions of the MDP model described in this chapter, including the activation and deactivation of incidents, which would get more stochastic link costs based on the time the vehicle receives the incident information, as well as the simulation for multiple CAVs, which would apply the user equilibrium model to solve the problems.

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