Weibull Methods for Testing Individual

Group Performance Models*

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Technical Report 273

August 1976

University of Minnesota

Saint Paul, Minnesota

*Research for this paper was supported in part by a Grant-in-Aid from the Graduate School, University of Minnesota. In addition, the research of the second author was partially supported by a Single Quarter Leave granted by the Regents of the University of Minnesota.

Abstract

Models relating individual and group solution times for problems are presented. Special consideration is given to the analysis of censored data which result from the imposition of time limits on problems. Maximum likelihood estimates of parameters and goodness-of-fit tests are given for the cases of individual solution times following gamma and Weibull distributions. The methods are illustrated on data previously analyzed by Restle and Davis [1962].

1. Introduction

In 1932 Marjorie Shaw found that her experimental groups performed better than individuals in solving certain mathematical puzzles. That is, a greater proportion of groups solved the problems before the time limits. Rather than attributing this apparent group superiority to a group synthesis, Lorge and Solomon [1955] suggested that the group's advantage might consist solely of its containing more than one individual. They proposed some probabilistic models for the group process in the case of Bernoulli observations of solution or nonsolution. The statistical analysis of group problem solving models for Bernoulli observations of success or failure was investigated further by Fienberg and Larntz [1971]. Sometimes, however, one has the additional information of latency times, the times until solution, for those individuals and groups able to solve the problem. In this case, one wants to use both information in the solution times and the numbers of solvers in making inferences about models for the individual and group performance.

In this paper we display some likelihood methods for testing the goodness of fit of some models when the data is collected with a time limit. In particular we will be looking at a generalization of the Lorge-Solomon model along with a model proposed by Restle and Davis [1962]. The testing methods will be illustrated on data previously analyzed by Restle and Davis [1962]. The general applicability of these methods will be indicated.

2. Some Group Problem Solving Models

Before describing two models which have been proposed for individual and group performance, we first need to introduce some notation for timed problem solving data. For each of the M individuals an observed time of X_i is recorded. The individual observed time, X_i, equals the solution time if the individual solves the problem or takes the value t_o, the time limit, if the individual fails to solve. We let m denote the number of individuals able to solve the problem. Similarly, the N groups yield observed times of $Y_1, \ldots, Y_N$ with n solutions. Each group consists of K individuals. The distribution of individual observed times will be denoted by F while G will be used for the group observed time distribution. These notational conventions are summarized in Table 2.1.

To illustrate the techniques we propose, we will use a reconstruction of data used by Restle and Davis [1962] and Davis and Restle [1963]. They presented individuals and groups with three puzzles (Rope, Word Tangle, and Gold Dust) which they were asked to solve. Solution times were recorded for each puzzle separately. Requests sent to both authors revealed that the raw data were no longer available. However, empirical distribution functions are plotted in each of the papers. The conversion of these empirical distributions to times was aided by a Hewlett-Packard 9107A Digitizer. The plots used were Figures 1 and 3 of Restle and Davis [1962]. A check of the closeness of the graphically generated data sets to the actual ones is provided in Table 2.2 comparing the means and standard deviations of the digitized data with those given in Table 2 of Davis and Restle [1963]. (For later computational convenience, data have been scaled down by a factor of 1000.) The comparisons in Table 2.2 are surprisingly good. The digitized data may be found in Tables 2.4, 2.5, and 2.6 of Regal [1975], and are available upon request from the authors.

Lorge-Solomon Models

Lorge and Solomon [1955] considered modeling the group as K independently working individuals. In terms of solving or not solving, the probability that a group is able to solve is then the probability that at least one of K independently working individuals is able to solve. This model can be extended to timed data by modeling the group time as the minimum time of K independently working individuals. In terms of the distribution functions F and G, the Lorge-Solomon model becomes

$$(2.1) \qquad G(t) = 1 - (1 - F(t))^K .$$

In general there will be a jump in the distribution functions at the time limit, t_0, due to the nonzero probability that the individual or group fails to solve by time t_0. One possible assumption is that eventually everyone would have solved. Information on times beyond t_0 is then reduced to the fact that the solution time would have been somewhere beyond t_0. Technically, the information on solution times beyond t_0 has been censored by the time limit. The resulting distribution is a Type I censored distribution [Mann et al., 1974]. Type II censoring occurs when the experiment is continued until a predetermined number of solutions are observed. A truncated distribution occurs when the values between 0 and t_0 are known, but it is unknown how many values would have been greater than t_0. A typical Type I right-censored random variable has a distribution function

$$(2.2) \qquad F(t) = \begin{cases} F_S(t) & t < t_0 \\ 1 & t \geq t_0 \end{cases}$$

where $F_S(t)$ is the distribution function for the uncensored variable.

Instead of assuming that everyone would eventually solve, we could assume that with probability p an individual is governed by the solving distribution, $F_S(t)$, and with probability 1 - p an individual would not solve, even with unlimited time. In this case the individual distribution function is

(2.3) $$F(t) = \begin{cases} pF_S(t) & t < t_0 \\ 1 & t \geq t_0 \end{cases} .$$

The probability for t_0 is then

(2.4) $$1 - pF(t_0) = (1-p) + p[1 - F_S(t_0)] .$$

With probability 1 - p an individual would never solve, and with probability $p[1 - F_S(t_0)]$ an individual is a potential solver but fails to finish by the time limit, t_0. The group distribution function under the Lorge-Solomon model is then

(2.5) $$G(t) = \begin{cases} 1 - (1 - pF_S(t))^K & t < t_0 \\ 1 & t \geq t_0 \end{cases}$$

or

(2.6) $$G(t) = \begin{cases} \sum_{J=1}^{K} \binom{K}{J} p^J (1 - p)^{K-J} \{ 1 - [1 - F_S(t)]^J \} & t < t_0 \\ 1 & t \geq t_0 \end{cases} .$$

Equation (2.6) expresses the group distribution as a mixture of cases where J of the K people in the group are potential solvers.

<u>Restle and Davis Equalitarian Model With a Time Limit</u>

The Lorge-Solomon model does not allow for any interaction between individuals in a group. Restle and Davis [1962] introduced a model which includes a factor for the hindrance of a group by those who would never solve. In their equalitarian model a group of size K with J potential solvers has a solution time distributed as the minimum time for J independently working potential solvers multiplied by K/J. The group solution time distribution then becomes

(2.7) $$G(t) = \begin{cases} \sum_{J=1}^{K} \binom{K}{J} p^J (1 - p)^{K-J} \{ 1 - [1 - F_S(Jt/K)]^J \} & t < t_0 \\ 1 & t \geq t_0 \end{cases} .$$

The Restle-Davis model is formulated in terms of the means of exponential distributions with no censoring which implies that the mean is simply multiplied by K/J. The above formulation is a generalizaation to arbitrary distributions with time limits. Restle and Davis arrive at the factor K/J by assuming that nonsolvers waste their proportionate share of the group's time. If the group only has the fraction J/K of time to use effectively, then the time spent on the problem becomes K/J times what would have been used without the J time wasters present. Restle and Davis use the term "equalitarian" to indicate that under this model people use time equally regardless of whether or not they are on the right track.

3. Distributional Assumptions

So far the models for group problem solving have been stated in terms of unspecified distribution functions F and G. One cannot specify F and G completely, but sometimes one believes that the distributions fall into some broad family of distributions with each member of the family characterized by one or more parameter values. The various group problem solving models will then imply relationships between the individual distribution parameters and the group distribution parameters. The data give information about these parameters, and this information can be used to determine whether the relationships between the individual and group characteristics appear consistent with those of the model. The assumed parametric family will probably not contain the actual distribution but will hopefully contain sufficiently close approximations. By restricting attention to a large but manageable parametric family, one may be able to focus on the particular characteristics important to the model.

Gamma Distribution Assumptions

Restle and Davis [1962] and Davis and Restle [1963] discuss some formulations of problem solving models. They assume that a problem consists of r equally difficult, independent parts with r unknown. If the individual solution time for each part is modeled with a common exponential distribution with mean $1/\lambda$, the total time spent on the problem by a particular individual, being the sum of r independent exponentially distributed random variables with common mean $1/\lambda$, has a gamma distribution with parameters r and λ with probability density function

(3.1) $$g(t;\, r,\lambda) = \frac{\lambda(\lambda t)^{r-1}\, e^{-\lambda t}}{\Gamma(r)} \ .$$

The Restle-Davis forms of the Lorge-Solomon model also use these parts or stages. If the group solution time for a particular stage is the minimum time of K independently working individuals, then the group solution time for one stage has an exponential distribution with mean $(K\,\lambda)^{-1}$. If the group pools information at each stage, then the group solution time for a problem with r independent, equally difficult parts has a gamma distribution with r and $K\lambda$. Under the above assumptions, this stagewise form of the Lorge-Solomon model is necessary to maintain the relative simplicity of the parametric forms of the distributions involved.

Weibull Distribution

If the gamma family provides a good representation of solution times, this is most likely the result of the gamma family's being a broad parametric family of distributions rather than an indication of r independent, equally likely exponentially distributed stages. Another family of distributions which has been studied widely and been found useful for timed data in areas such as life-testing [Mann, et al., 1974] is the Weibull distribution. For a wide range of usual parameter values, the Weibull and gamma distributions are known to be similar [Cohen, 1973; Hager, et al., 1971]. These two families have the exponential distribution in common. The distribution function for a random variable from the Weibull family may be written

(3.2) $$W(t;\, \gamma,\, \theta) = 1 - e^{-t^{\gamma}/\theta} \qquad \text{for } t > 0$$

where γ and θ are positive parameters. The distribution of the minimum of K independent random variables each having the distribution function $W(t; \gamma, \theta)$ is given by the Weibull distribution function $W(t; \gamma, \theta/K)$. This fact yields a simple relationship between the individual and group parameters in the Lorge-Solomon without requiring the assumption of r equally difficult stages.

Method of Moments and Maximum Likelihood Estimation (Gamma)

As mentioned earlier, Restle and Davis model the individual solution times as gamma random variables. In order to estimate the parameters of the distribution, they used the method of moments. As pointed out by Fisher [1922], method of moments estimation for the gamma distribution is inefficient unless the distribution is nearly normal. Maximum likelihood estimation is more efficient but also more complicated. Another problem is caused by the time limit, t_0, since the individual solution times do not have semi-infinite support as assumed by the usual gamma distribution. The observed solution times would follow a truncated gamma distribution, the moments of which do not permit simple method of moment estimation.

To include the time limit and the possibility that some people may never solve, we will use the distribution function (2.3). In some of the models used by Restle and Davis there is a parameter $1 - p$ for the probability that an individual "makes a nonsolving decision" [Davis and Restle, 1963]. The Restle-Davis estimate of this probability is

$$(3.3) \qquad 1 - \tilde{p} = 1 - \frac{m}{M} ,$$

That is, the proportion of individuals failing to solve by the time limit, t_0. By bringing the parameter p into the model in the manner of (2.3), we allow for the fact that an individual could have been in the solving distribution but did not finish before the time limit.

For the digitized individual data from Restle and Davis [1962], Table 3.1 compares the parameter estimates obtained using maximum likelihood estimation with the model in (2.3) under the gamma distribution and using the method of moments estimates for a semi-infinite gamma distribution. The method of moments estimates are

$$\tilde{r} = (\bar{X})^2 \div s_X^2 \tag{3.4}$$

and

$$\tilde{\lambda} = \bar{X} \div s_X^2 \ . \tag{3.5}$$

The maximum likelihood estimates were found using the program MAXLIK [Kaplan and Elston, 1972] along with checks that the local maximization was actually global. The likelihood in the case of the mixed distribution like (2.3) is the product of the densities for values less than t_o and the probability of nonsolution for $t = t_0$. If we let $\underset{\sim}{\varphi}$ denote the parameters, $f_S(t)$ be the density corresponding to $F_S(t)$, and $\underset{\sim}{x} = (x_1,\ldots, x_M)$ be the vector of observed times, the first m of which are less than t_0, then the likelihood is

$$\text{lik}(\underset{\sim}{\varphi};\ \underset{\sim}{x}) = [\prod_{i=1}^{m} pf_s(X_i;\ \underset{\sim}{\varphi})]\ [1 - pF_S(t_0)]^{M-m} \ . \tag{3.6}$$

The estimates $\hat{r}$ and $\hat{\lambda}$ for this case are also the maximum likelihood estimates if we assume a truncated gamma model for the solution times. Then $\hat{p}$ is an extrapolation of the proportion of individuals who would have solved by infinity or

$$\hat{p} = \frac{m}{M}\ \frac{1}{F_S\ (t_0;\hat{\lambda}\ ,\ \hat{r})} \ . \tag{3.7}$$

If the estimate of $\hat{p}$ from (3.7) is greater than 1.0, estimation of all parameters reverts to the Type I censored estimation problem. For more details on this relationship, see Theorem 3.1 of Regal [1975].

In order to give a description of how far apart the two sets of estimates are, Table 3.1 includes the log likelihoods, ℓ, of the parameter values under the model (2.3). The log likelihoods from the maximum likelihood estimates are larger by definition. One way of putting these comparisons into perspective is to note that values of (p, r, λ) having log likelihoods within $\frac{1}{2} \chi^2_{1-\alpha}$ (3 df) of $\ell(\hat{r}, \hat{\lambda}, \hat{p}, x)$ form a 100 x $(1-\alpha)$% asymptotic confidence set for the true parameters. In the Word and Gold problems, $(\tilde{r}, \tilde{\lambda}, \tilde{p})$ would not be contained in a 95% confidence set determined by $(\hat{r}, \hat{\lambda}, \hat{p})$. This comparison, however, does not have an interpretation in terms of a statistical test of some hypothesis. The values given in parentheses behind $\hat{r}$, $\hat{\lambda}$, and $\hat{p}$ are asymptotic standard errors from an estimated information matrix. Part of the differences between parts (A) and (B) of Table 3.1 is in going from method of moments to maximum likelihood and part is due to including the time limits. In the Rope problem the distribution of solving times appears to have nearly died out by the time limit as evidenced by the closeness of $\tilde{p}$ and $\hat{p}$. In the Gold problem, however, the time limit appears to have kept some potential solvers from displaying their solutions.

Maximum Likelihood Estimation (Weibull)

Table 3.2 gives maximum likelihood parameter estimates and maximized log likelihoods, ℓ, for the individual data, when using a Weibull distribution in (2.3) rather than a gamma distribution. The gamma model fits slightly better in the Rope and Gold problems and slightly worse in the Word problem. Since one model is not a subset of the other, twice the difference in log likelihoods is not interpretable as an asymptotic chi-square test.

This is not to say that either model is necessarily correct, only that one is nearly as good or as bad as the other. To support their gamma distribution assumption, Restle and Davis presented plots of individual empirical distribution functions and the corresponding fitted gamma distribution functions. They claimed that one-sample Kolmogorov-Smirnov tests applied to these distribution functions were not significant. However, since the parameters for the theoretical distribution had been estimated from the data, the usual Kolmogorov-Smirnov critical values do not apply [Durbin, 1973], and, in fact, the usual critical values are much too large.

4. Goodness-of-fit Testing

Lorge-Solomon Model

Using the individual solution model of (2.3) along with the Weibull distribution, we can derive likelihood ratio tests for the Lorge-Solomon and the Equalitarian models. Assume then that the individual distribution has the form

$$(4.1) \qquad \begin{cases} p_I \, W(t;\, \gamma_I,\, \theta_I) & t < t_0 \\ 1 & t \geq t_0 \end{cases}$$

where W is the Weibull distribution function of (3.2) and that the group solution time distribution is of the form

$$(4.2) \qquad G(t) = \begin{cases} 1 - [1 - p_G W(t;\, \gamma_G,\, \theta_G)]^K & t < t_0 \\ 1 & t \geq t_0 \end{cases} .$$

Then the Lorge-Solomon model hypothesizes

$$(4.3) \qquad H_0\colon p_I = p_G,\ \gamma_I = \gamma_G,\ \theta_I = \theta_G .$$

Following Table 2.1, let $\underset{\sim}{x} = (x_1,\ldots,x_m)$ be the individual solution times, and let $\underset{\sim}{y} = (y_1,\ldots,y_n)$ be the group solution times. Also let

$$(4.4) \qquad \mathrm{lik}_F(p,\gamma,\theta;\underset{\sim}{x}) = [\prod_{i=1}^{m} pw(x_i;\gamma,\theta)]\{1-pW(t_0,\gamma,\theta)\}^{M-m}$$

be the likelihood of (p,γ,θ) given the individual data, and let

$$(4.5) \qquad \mathrm{lik}_G(p,\gamma,\theta;\underset{\sim}{y}) = \{\prod_{i=1}^{n} Kpw(y_i;\gamma,\theta)[1-pW(y_i;\gamma,\theta)]^{K-1}\}\{1-pW(t_0;\gamma,\theta)\}^{K(N-n)}$$

be the likelihood of (p,γ,θ) given the group data. Here $w(t;\gamma,\theta)$ and $W(t;\gamma,\theta)$ are the Weibull density and distribution functions respectively. The likelihood ratio statistic for the test of H_0 in (4.3) is

$$(4.6) \qquad \lambda = \frac{\max\{ \text{lik}_F(p,\gamma,\theta;\underset{\sim}{x})\ \text{lik}_G(p,\gamma,\theta;\underset{\sim}{y})\}}{\max\{ \text{lik}_F(p,\gamma,\theta;\underset{\sim}{x})\}\ \max\{ \text{lik}_G(p,\gamma,\theta;\underset{\sim}{y})\}}$$

where the maximization in each case is over $0 \leq p \leq 1$, $\gamma \geq 0$, and $\theta \geq 0$. The Weibull distribution is sufficiently regular so that under the null hypothesis $-2\log\lambda$ is asymptotically a chi-square random variable with three degrees of freedom as long as none of the true parameter values is a boundary point, such as $p = 1$ [Regal, 1975].

Table 4.1 presents the maximum likelihood estimates and corresponding log likelihoods for the Restle-Davis data. Note that $-2\log\lambda$ for testing hypothesis (4.3) is calculated as

$$(4.7) \qquad -2\log\lambda = 2\ell(\text{Combined}) - 2\ell(\text{Individuals}) - 2\ell(\text{Groups}).$$

Only for the Gold problem is $-2\log\lambda$ less than $\chi^2_{.05}(3\text{ df}) = 7.81$, and then not by much. Since the same individuals and groups were used for each problem, we cannot combine the chi-square statistics. It should be noted that the group solution time distribution (4.2), implied by the Lorge-Solomon assumption is not a Weibull distribution unless $p_G=1$. However, the class of distributions defined by (4.2) is also wide. For example, if a Weibull distribution is fit to the group Rope data by using $K = 1$ in (4.2), the maximized log likelihood is 25.31 which is quite close to the 25.59 found using (4.2) with the "correct" $K = 4$.

Equalitarian Model

The Equalitarian model of Restle and Davis (1962) was given in section 2. For this model we still describe the individual solution times by (4.1), but now we use the group distribution function

$$(4.7) \qquad G(t) = \begin{cases} \sum_{J=1}^{K} \binom{K}{J} P_G^J (1-P_G)^{K-J} \left\{1-[1-W(\frac{Jt}{K};\gamma_G,\theta_G)]^J\right\} & t < t_0 \\ 1 & t \geq t_0 \end{cases}$$

The null hypothesis is still given by (4.3). Table 4.2 shows the results of the Equalitarian model applied to the Restle-Davis data. The chi-square statistics are very insignificant, perhaps suspiciously so. One point to check is whether the fit for the group model has changed in going from (4.2) to (4.7). One way the chi-square statistics could decrease is to have the log likelihood for groups decrease. By comparing the log likelihoods for the groups data in Tables 4.1 and 4.2, we see that this was not the case. In fact, the log likelihoods are identical (to two decimal places) for the Word and Gold problems and are very close for the Rope problem (25.59 vs. 25.90). For this particular set of data, the Equalitarian model describes the relationship between individual and group solution times much better than does the Lorge-Solomon model.

5. Discussion

Restle and Davis [1962] also arrived at the conclusion that their Equalitarian model fits the data better than the Lorge-Solomon model, so that our conclusions on this particular data set are not new. However, the methods displayed here for censored data give statistical significance to this conclusion and are methods which are easily extended to other situations. The Restle and Davis method of testing the fit of a model was to use the parameters estimated from the individual data in the hypothesized group model and compare the group empirical distribution function to the predicted group distribution. The standard one-sample Kolmogorov-Smirnov critical values do not apply to the resulting functionals, since parameters have been estimated from the data [Durbin, 1973]. Even if the added variation from the parameter estimation could be accounted for in the critical values, the resulting test would undoubtedly not be a very powerful test, since it tests H_0 against all possible alternatives. The likelihood methods displayed here have also taken into account the censoring due to the time limit.

It should be noted that the techniques and methods for the Lorge-Solomon model also have application in the area of component reliability testing, when several independent components are tested simultaneously. However, the emphasis there is on estimation of parameter values, since it can be established that the specified model (i.e. Lorge-Solomon) holds exactly. In problem solving applications, the emphasis is on the question of finding a model that describes the relationships between the individual and group solving time parameters.

Table 2.1

TIMED DATA NOTATION

	Individuals	Groups
Total number	M	N
Solvers	m	n
Observed times	$X_1, \ldots, X_M$	$Y_1, \ldots, Y_N$
Distribution	F	G
Individuals per unit	1	K

Table 2.2

MEANS AND STANDARD DEVIATIONS FROM ACTUAL RESTLE-DAVIS DATA AND DIGITIZED DATA

		Individuals		Groups	
		Mean	St. Dev.	Mean	St. Dev.
Rope	Restle-Davis	.1313	.1155	.0940	.1247
	Digitized	.1307	.1148	.0939	.1224
Word	Restle-Davis	.2648	.1541	.2690	.2162
Tangle	Digitized	.2633	.1516	.2706	.2148
Gold	Restle-Davis	.3730	.1667	.3775	.2192
Dust	Digitized	.3728	.1650	.3767	.2222

Table 3.1

A. METHOD OF MOMENTS ESTIMATES OF
GAMMA MODEL PARAMETERS FOR THE RESTLE-DAVIS INDIVIDUAL DATA

Problem	$\tilde{r}$	$\tilde{\lambda}$	$\tilde{p}$	$\ell(\tilde{r},\tilde{\lambda},\tilde{p};\ \underset{\sim}{x})$
Rope	1.3	9.92	0.75	42.87
Word	5.0	11.4	0.50	-78.52
Gold	5.1	13.7	0.44	-80.61

B. MAXIMUM LIKELIHOOD GAMMA PARAMETER ESTIMATES

Problem	$\hat{r}$		$\hat{\lambda}$		$\hat{p}$		$\ell(\hat{r},\hat{\lambda},\hat{p};\ \underset{\sim}{x})$
Rope	1.52	(0.18)	11.57	(1.61)	0.75	(0.03)	43.87
Word	1.94	(0.33)	6.34	(1.62)	0.54	(0.05)	-74.25
Gold	2.97	(0.71)	6.03	(2.39)	0.56	(0.10)	-76.30

NOTE: Estimated standard errors for the maximum likelihood estimates are given in parentheses.

TABLE 3.2

MAXIMUM LIKELIHOOD ESTIMATES FOR WEIBULL MODEL
OF RESTLE-DAVIS INDIVIDUAL DATA

Problem	$\hat{\gamma}$		$\hat{\theta}$		$\hat{p}$		$\ell(\hat{\gamma},\hat{\theta},\hat{p};\ \underset{\sim}{x})$
Rope	1.24	(0.08)	0.09	(0.01)	0.75	(0.03)	42.44
Word	1.62	(0.17)	0.15	(0.04)	0.52	(0.04)	-73.68
Gold	2.06	(0.29)	0.24	(0.11)	0.52	(0.08)	-77.10

Table 4.1

LIKELIHOOD RATIO TESTS OF THE LORGE-SOLOMON MODEL FOR THE RESTLE-DAVIS DATA USING THE WEIBULL FAMILY

a. Rope Problem

	$\hat{\gamma}$	$\hat{\theta}$	$\hat{p}$	$\ell(\hat{\gamma},\hat{\theta},\hat{p})$	$-2 \log \lambda$
Individuals	1.24 (0.08)	0.09 (0.01)	0.75 (0.03)	42.44	9.60 (α=.022)
Groups	0.91 (0.17)	0.22 (0.14)	0.56 (0.14)	25.59	
Combined	1.20 (0.07)	0.10 (0.02)	0.72 (0.03)	63.23	

B. Word Tangle Problem

	$\hat{\gamma}$	$\hat{\theta}$	$\hat{p}$	$\ell(\hat{\gamma},\hat{\theta},\hat{p})$	$-2 \log \lambda$
Individuals	1.62 (0.17)	0.15 (0.04)	0.52 (0.04)	-73.68	11.60 (α=.009)
Groups	1.08 (0.35)	1.46 (4.71)	0.85 (2.16)	-3.92	
Combined	1.51 (0.15)	0.20 (0.06)	0.49 (0.04)	-83.40	

C. Gold Dust Problem

	$\hat{\gamma}$	$\hat{\theta}$	$\hat{p}$	$\ell(\hat{\gamma},\hat{\theta},\hat{p})$	$-2 \log \lambda$
Individuals	2.06 (0.29)	0.24 (0.11)	0.52 (0.08)	-77.10	7.10 (α=.068)
Groups	1.57 (0.33)	1.63 (0.50)	1.00	-5.71	
Combined	1.83 (0.25)	0.43 (0.26)	0.58 (0.17)	-86.36	

Table 4.2

LIKELIHOOD RATIO TESTS OF THE EQUALITARIAN MODEL
FOR THE RESTLE-DAVIS DATA USING THE WEIBULL FAMILY

A. Rope Problem

	$\hat{\gamma}$	$\hat{\theta}$	$\hat{p}$	$\ell(\hat{\gamma},\hat{\theta},\hat{p})$	-2 log λ
Individuals	1.24 (0.08)	0.09 (0.01)	0.75 (0.03)	42.44	2.12 (α=.552)
Groups	1.11 (0.28)	0.06 (0.09)	0.59 (0.16)	25.90	
Combined	1.21 (0.07)	0.09 (0.01)	0.74 (0.03)	67.28	

B. Word Tangle Problem

	$\hat{\gamma}$	$\hat{\theta}$	$\hat{p}$	$\ell(\hat{\gamma},\hat{\theta},\hat{p})$	-2 log λ
Individuals	1.62 (0.17)	0.15 (0.04)	0.52 (0.04)	-73.68	0.44 (α=.931
Groups	1.08 (0.53)	1.62 (6.47)	0.97 (1.46)	-3.92	
Combined	1.58 (0.15)	0.16 (0.04)	0.52 (0.04)	-77.88	

C. Gold Dust Problem

	$\hat{\gamma}$	$\hat{\theta}$	$\hat{p}$	$\ell(\hat{\gamma},\hat{\theta},\hat{p})$	-2 log λ
Individuals	2.06 (0.29)	0.24 (0.11)	0.52 (0.08)	-77.10	3.00 (α=.393)
Groups	1.57 (0.33)	1.63 (0.50)	1.00	-5.71	
Combined	1.81 (0.23)	0.41 (0.26)	0.64 (0.16)	-84.31	

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