Sample Size Determination for "Fixed Effects" ANOVA Models

by

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1. INTRODUCTION

Conscientious experimental planning demands that some thought be given to the problem of how much data to collect. For experiments in which relationships among a number of population means are of interest, and in which there is initially some degree of flexibility in the amount of data that can be collected, power analysis on the hypothesis tests can be used to choose an appropriate sample size.

Because questions about how much data to collect are not easily resolved (computational difficulties notwithstanding), there seems to be a tendency to bypass such questions by designing experiments that exhaust available resources. For example, an experiment may be designed so that all available space or experimental material is utilized, or so that the entire budget allocation for the study is used. There are reasons for supporting such a point of view. One is that experimenters by-and-large do not have a feeling for the connection between sample size and power; very powerful experiments cannot, in general, be achieved without extremely large sample sizes. Many scientists cannot do more than accept the relatively low (that is, relative to their expectations) power levels dictated by the availability of resources. If strictly adhered to, however, this procedure may often result in experiments with power levels low enough so as to render them essentially useless. It may also happen, although perhaps less often, that the experiment is very inefficient (the sample size is larger than that required). This inefficiency seems to be particularly prevalent in pilot studies. It therefore seems obvious that some consideration must be given to power even if the experiment is designed to utilize all available resources.

Another technique often used in place of power analysis is to simply choose a sample size based on that used in published studies on similar experimental material. The rationale here seems obvious. The procedure is, of course, subject to the same pitfalls as previously mentioned.

Most of the techniques that are used in place of power analysis for choosing a sample size appear to have a common failing, i.e. the sample size is chosen without much reference to the specific objectives of the experiment. These techniques are at best expedient.

Direct calculation of sample size or power is, of course, impossible without the aid of a high-speed computer. Charts for power computations have long been available (see [7]), but iterating a required sample size from these charts is an arduous task. For the t-test, tables of sample size are available [4] and for certain special cases Cochran and Cox [2] offer sample size suggestions. Recently, tables of power and sample size have been constructed (see [3], [5], [6], and [1]), but little practical use has been made of these efforts. In this report we provide a reasonably complete set of tables for determining sample size and power for experiments in which relationships among I population means are of interest. The "fixed effects" model and usual analysis of variance F-tests are assumed. Background material and the method of constructing the tables are presented in the next two sections. The use of the tables is illustrated with several examples in the last section.

2. ONE-WAY ANALYSIS OF VARIANCE

Consider a situation in which it is of interest to investigate the relationships among I treatments. Let μ_i represent the true mean for treatment i (i = 1,2,...,I) and let n represent the number of observations per treatment. Thus, there is a total of nI observations in the experiment. We restrict attention to the case in which all treatments have the same number of observations. Assuming the "fixed effects" one-way analysis of variance model, we have

$$Y_{ij} = \bar{\mu} + \alpha_i + \epsilon_{ij} , \qquad i = 1,2,\ldots,I \; , \; j = 1,2,\ldots,n \tag{2.1}$$

where

$$\bar{\mu} = \sum_{i=1}^{I} \mu_i / I ,$$

$$\alpha_i = \mu_i - \bar{\mu} = \text{effect of treatment } i$$

and

$$\epsilon_{ij} \sim N(0,\sigma^2) .$$

In this section we shall only consider the one-way ANOVA model as defined here. The tables and discussion are equally applicable to more complex ANOVA models, and sample size specifications in more complex models are illustrated in the examples.

The power of the F-test for this model is a monotone function of the noncentrality parameter

$$\lambda = n \sum_{i=1}^{I} \alpha_i^2 / 2\sigma^2 \tag{2.2}$$

$$= \frac{n}{2} \sum_{i=1}^{I} (\alpha_i/\sigma)^2 .$$

We call α_i/σ the <u>standardized treatment effect</u> for treatment i. If $\Sigma\, \alpha_i^2 > 0$

and σ are known, then n (the number of observations per treatment) can be chosen to yield any desired probability p that the null hypothesis (H: $\Sigma \alpha_i^2 = 0$) is rejected. The first requirement for determining sample size is that an estimate of σ be available. Estimates of σ can often be obtained from studies on similar experimental material. When an estimate of σ is not available, the experimenter can frequently give a reasonable range for σ. Then as long as specifications about the alternative hypothesis are stated in terms of standardized treatment effects, power analysis can be carried out. Thus, we assume that σ is known or that alternatives are specified in terms of standardized treatment effects.

The quantity $\Sigma \alpha_i^2$ is simply the sum of squared deviations of the treatment means from the grand mean $\overline{\mu}$, and it measures the deviation from the null hypothesis, H: $\Sigma \alpha_i^2 = 0$. The second requirement for determining sample size is that $\Sigma \alpha_i^2$ (or $\Sigma [\alpha_i/\sigma]^2$) be specified by the experimenter. By choosing $\Sigma \alpha_i^2$ the experimenter is specifying the magnitude, in terms of differences between treatment means, of the deviation from the null hypothesis that he would like to detect. In general, as $\Sigma \alpha_i^2$ increases the sample size n required to yield a probability p that the null hypothesis is rejected decreases.

Of course, the experimenter can never be certain that an experiment will detect a deviation from the null hypothesis. The last requirement for determining sample size is that the experimenter specify his desired power, i.e. the probability p with which he would like to detect the deviation $\Sigma \alpha_i^2 > 0$. In general, as p increases the sample size also increases.

Suppose it is desired to detect with probability p a difference d, if it exists, between any two treatments. This specification is not sufficient to determine $\Sigma \alpha_i^2$. There are many ways in which at least one difference d can occur in an experiment and, generally, each will yield a different value of $\Sigma\,\alpha_i^2$. In most cases, one of the four patterns of variation in treatment means given in Table 1 should be sufficient to determine sample size. Patterns A, B, and C represent three ways in which the difference between the maximum and minimum treatment means is d. Pattern D is similar to pattern B except the difference between $\mu_{(I)}$ and $\mu_{(1)}$ is d(I-1). Note that pattern A is the most conservative pattern; namely, it gives the smallest value of $\Sigma\,\alpha_i^2$ and thus requires the largest sample size.

CORRECTED TABLE 1

Table 1

Patterns of variability and comparative factor for one-way ANOVA

Pattern	Pattern of treatment means	$\Sigma \alpha_i^2$	Comparative Factor
A	$\mu_{(I)} - \mu_{(1)} = d$ $\mu_{(i)} = \bar{\mu} = \frac{\mu_{(I)} + \mu_{(1)}}{2}$, $i = 1,2,\ldots,I-1$	$\frac{d^2}{2}$	1
B	$\mu_{(i+1)} - \mu_{(i)} = \frac{d}{I-1}$, $i = 1,2,\ldots,I-1$	$\frac{d^2 I(I+1)}{12(I-1)}$	$\sqrt{\frac{I(I+1)}{6(I-1)}}$
B'	$\mu_{(1)} = \mu_{(2)} = \ldots = \mu_{(I-1)}$ $\mu_{(I)} - \mu_{(1)} = d$	$\frac{d^2(I-1)}{I}$	$\sqrt{\frac{2(I-1)}{I}}$
C	$\mu_{(1)} = \mu_{(2)} = \ldots = \mu_{(I/2)}$; $\mu_{(I/2+1)} = \ldots = \mu_{(I)}$ $\mu_{(I)} - \mu_{(1)} = d$	$\frac{Id^2}{4}$ (I even)	$\sqrt{\frac{I}{2}}$
	$\mu_{(1)} = \ldots = \mu_{(\frac{I+1}{2})} = \mu_{(\frac{I+1}{2}+1)} = \ldots = \mu_{(I)}$ $\mu_{(I)} - \mu_{(1)} = d$	$\frac{d^2(I^2-1)}{4I}$ (I odd)	$\sqrt{\frac{I^2-1}{2I}}$
D	$\mu_{(i+1)} - \mu_{(i)} = d$, $i = 1,2,\ldots,I-1$	$\frac{d^2(I+1)I(I-1)}{12}$	$\sqrt{\frac{(I+1)I(I-1)}{6}}$

Notes: $\mu_{(1)}$ = smallest mean
$\mu_{(2)}$ = second smallest mean
$\mu_{(I)}$ = largest mean

To determine necessary sample sizes for pattern A, use Table 2 directly; for patterns B, B', C, or D, enter Table using in place of $|d|/\sigma$ the value of $|d|/\sigma$ for pattern A multiplied by the comparative factor.

Table 1

Patterns of variability and comparative factors for one-way ANOVA

Pattern	Pattern of treatment means	$\Sigma \alpha_i^2$	Comparative Factor
A	$\mu_{(I)} - \mu_{(1)} = d$ $\mu_{(i)} = \bar{\mu} = \frac{\mu_{(I)} + \mu_{(1)}}{2}$, $i = 1,2,\ldots,I-1$	$\frac{d^2}{2}$	1
B	$\mu_{(i+1)} - \mu_{(i)} = \frac{d}{I-1}$, $i = 1,2,\ldots,I-1$	$\frac{d^2 I(I+1)}{12(I-1)}$	$\sqrt{\frac{I(I+1)}{6(I-1)}}$
C	$\mu_{(1)} = \mu_{(2)} = \cdots = \mu_{(I-1)}$ $\mu_{(I)} - \mu_{(1)} = d$	$\frac{Id^2}{4}$ (I even) $\frac{d^2(I^2-1)}{4I}$ (I odd)	$\sqrt{\frac{I}{2}}$ $\sqrt{\frac{I^2-1}{2I}}$
D	$\mu_{(i+1)} - \mu_{(i)} = d$, $i = 1,2,\ldots,I-1$	$\frac{d^2(I+1)I(I-1)}{12}$	$\sqrt{\frac{(I+1)I(I-1)}{6}}$

Notes: $\mu_{(1)}$ = smallest mean
$\mu_{(2)}$ = second smallest mean
⋮ ⋮
$\mu_{(I)}$ = largest mean

To determine necessary sample sizes for pattern A use Table 2 directly; for patterns B, C, or D, enter Table 2 using in place of $|d|/\sigma$ the value of $|d|/\sigma$ for pattern A multiplied by the comparative factor.

3. TABLES AND COMPUTATIONAL PROCEDURE

Tables 2a-2k give the sample sizes necessary for specified power in the one-way ANOVA with pattern A described above. The number of groups is I, d is the minimum difference between two groups, and σ is the standard deviation per observation. Sample sizes greater than fifty were omitted from the tables. For patterns other than A, multiply $|d|/\sigma$ by the comparative factor given in Table 1 and use this product as $|d|/\sigma$ in finding the sample size from Table 2.

Computations for the tables were carried out by using numerical quadrature on the University of Minnesota CDC 6400. An adaptive Simpson's rule was used with a tolerance for the final integral of $\pm 10^{-4}$. Terms of the non-central F density were computed with a tolerance of $\pm 10^{-6}$. Agreement in cases easily checked between the sample sizes given here and those of other tables is usually either exact or within one observation. The method of construction used here assures that the power specification is fully met, so that where disagreement occurs our sample size is usually larger by one observation.

4. EXAMPLES

To gain an understanding of the sample size tables, three examples will be given to illustrate their use.

Example 1

Suppose an experimenter has five treatment groups in a one-way ANOVA situation. He is interested in detecting a minimum difference of eight units between any two groups (this is pattern A). The standard deviation per observation is four units. The desired power (probability of detection) is .80. How many observation should he take in each group? Looking under I = 5 groups, $|d|/\sigma = 2.0$, we find that seven observation per group are required for $\alpha = .05$ and ten for $\alpha = .01$.

If instead of pattern A the experimenter believes that the means are equally spaced two units apart (pattern B), we multiply $|d|/\sigma$ by the comparative factor:

$$|d|/\sigma\ (B) = |d|/\sigma\ (A) \times \text{Comparative Factor}$$

$$= 2.0 \times \sqrt{\frac{5(6)}{6(4)}}$$

$$= 2.2\ .$$

Thus, for $\alpha = .05$ and power = .80, six observations would be required, while for $\alpha = .01$ nine observations would meet the specifications.

Example 2

Suppose that a 2^3 factorial experiment is to be carried out in a randomized blocks design with blocks of size 8, $\sigma = 10$. How many blocks are needed

(a) to be 80% sure of detecting at $\alpha = .05$ an average difference of ten units between the main effects of one factor (having two levels)?

(b) to be 50% sure of discovering a difference of fifteen units between the means of any of the eight possible combinations of the three factors?

To answer (a) we will start by answering the question ignoring the structure of the experiment. Assume we have a one-way ANOVA with I = 2. Then to find with power = .80 a difference of $|d|/\sigma = 1.0$, we need 17 observations per group, or 34 observations in all. Since blocks are of size 8, an experiment with five blocks would be adequate. (It can be shown that the error degrees of freedom is relatively unimportant in deciding sample size. In this case, for the one-way ANOVA error d.f. is 32 compared to 28 for the factorial in blocks design.)

Now for (b) we are interested in the possible differences between all cell means. There are eight such means in a 2^3 factorial, so we take I = 8. To find a difference of $|d|/\sigma = 1.5$ with power = .50, we need nine observations per cell, or nine blocks in this case. Thus this specification requires a total of 72 experimental units.

Example 3

The last example will illustrate the advantage that can be gained if the experimenter can specify in advance one orthogonal comparison that is important as an alternative hypothesis. For instance, in a one-way ANOVA with seven groups, the groups may involve six different levels of a treatment plus a control.

Treatment	I	II	III	IV	V	VI	VII
Level	0	1	2	3	4	5	6

One particular comparison of interest may be the linear contrast among groups.

The comparison may be partitioned out of the total sum of squares for treatments as a single degree of freedom. Essentially this says that the treatment means are of the form

$$\mu_i = M + B\ (\text{Level})$$
$$= M + B \cdot i\ .$$

Now for any orthogonal contrast, $L = \Sigma\, \lambda_i T_i$ with $\Sigma\, \lambda_i = 0$, the associated $\Sigma\, \alpha_i^{\,2}$ in the noncentrality parameter is given by

$$\Sigma\, \alpha_i^{\,2} = \frac{(\Sigma\, \lambda_i \mu_i)^2}{\Sigma\, \lambda_i^{\,2}}\ . \tag{4.1}$$

For our example with a linear contrast, $\lambda_i = i\text{-}3$, $i = 0,1,\ldots,6$, and

$$\Sigma\, \alpha_i^{\,2} = \frac{\left[\sum_0^6 (i-3)(M + B \cdot i)\right]^2}{\sum_0^6 (i-3)^2}$$

$$= 28\ B^2\ .$$

Suppose that $\sigma = 5$ and that we want to find a slope of 1.0 or more with probability .95. To start with, a slope of 1.0 implies $d = 6.0$, and for $I = 7$ with pattern of variability B

$$|d|/\sigma \times \text{Comparative Factor} = 1.2 \times \sqrt{56/36} = 1.5$$

and 20 observations per treatment group, or 140 total observations, are required. Now taking advantage of the linear contrast that is prespecified, we can deal with only <u>one</u> degree of freedom instead of six and use the tables for $I = 2$. Also, since

$$\Sigma\, \alpha_i^{\,2} = 28\ B^2$$

compared with

$$d^2/2 = 18.0$$

for pattern A, the comparative factor is $\sqrt{28\ B^2/18}$. Using I = 2 with $|d|/\sigma = 1.2 \times \sqrt{28/18} = 1.5$, we find that 13 observations are needed per group. (Note that for a linear contrast the comparative factor is always the same as for pattern B in Table 1.) This is slightly conservative since the error degrees of freedom will be greater than the one-way ANOVA for which the I = 2 table was constructed. However, for practical purposes it is not far from the exact figure and is relatively easy to compute.

REFERENCES

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Table 2a

I = 2

$\lvert d \rvert / \sigma$	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	2	3	3	3	3	4	4
	.01	3	3	4	4	4	5	6
3.00	.05	3	3	3	4	4	5	6
	.01	4	4	5	5	6	6	8
2.50	.05	3	3	4	4	5	6	8
	.01	4	5	5	6	7	8	10
2.00	.05	4	4	5	6	7	8	11
	.01	6	6	7	8	10	11	14
1.75	.05	4	5	6	7	8	10	14
	.01	7	8	9	10	12	14	18
1.50	.05	5	6	7	9	11	13	18
	.01	8	9	11	13	15	18	24
1.40	.05	6	7	8	10	12	15	20
	.01	9	10	12	14	17	20	27
1.30	.05	6	7	9	11	14	17	23
	.01	10	12	14	16	20	23	31
1.20	.05	7	8	10	12	16	20	27
	.01	11	13	16	18	23	27	36
1.10	.05	8	10	12	15	19	23	32
	.01	13	15	18	22	27	32	42
1.00	.05	9	11	14	17	23	27	38
	.01	15	18	21	26	32	38	50
0.90	.05	11	14	17	21	27	34	47
	.01	19	22	26	31	39	46	
0.80	.05	14	17	21	26	34	42	
	.01	23	27	32	39	49		
0.70	.05	17	21	27	34	44		
	.01	29	35	41	50			
0.60	.05	23	29	36	45			
	.01	39	47					
0.50	.05	32	41					
	.01							

Table 2b

I = 3

|d|/σ	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	2	3	3	3	3	4	4
	.01	3	3	4	4	4	5	6
3.00	.05	3	3	3	4	5	5	6
	.01	4	4	5	5	6	7	8
2.50	.05	3	4	4	5	6	7	8
	.01	5	5	6	7	8	9	11
2.00	.05	4	5	5	6	8	9	12
	.01	6	7	8	9	11	12	16
1.75	.05	5	6	7	8	10	12	16
	.01	7	8	10	11	13	16	20
1.50	.05	6	7	8	10	13	15	21
	.01	9	11	12	14	18	20	26
1.40	.05	7	8	9	11	14	17	23
	.01	10	12	14	16	20	23	30
1.30	.05	7	9	11	13	17	20	27
	.01	12	14	16	19	23	27	35
1.20	.05	8	10	12	15	19	23	31
	.01	13	16	18	21	26	31	40
1.10	.05	10	12	14	17	22	27	37
	.01	16	18	21	25	31	36	47
1.00	.05	11	14	17	21	27	32	44
	.01	18	22	25	30	37	43	
0.90	.05	14	17	21	25	33	40	
	.01	22	26	31	36	45		
0.80	.05	17	21	26	32	41	50	
	.01	28	33	38	45			
0.70	.05	22	27	33	41			
	.01	35	42	49				
0.60	.05	29	36	44				
	.01	48						
0.50	.05	41						
	.01							

Table 2c

I = 4

$\lvert d \rvert / \sigma$	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	2	3	3	3	3	4	5
	.01	3	3	4	4	4	5	6
3.00	.05	3	3	4	4	5	5	7
	.01	4	4	5	5	6	7	9
2.50	.05	3	4	4	5	6	7	9
	.01	5	6	6	7	8	9	12
2.00	.05	4	5	6	7	9	10	13
	.01	7	7	8	10	12	13	17
1.75	.05	5	6	7	9	11	13	17
	.01	8	9	10	12	15	17	21
1.50	.05	7	8	9	11	14	17	22
	.01	10	12	13	16	19	22	28
1.40	.05	7	9	11	13	16	19	26
	.01	11	13	15	18	22	25	32
1.30	.05	8	10	12	14	18	22	29
	.01	13	15	17	20	25	29	37
1.20	.05	10	11	14	17	21	25	34
	.01	15	17	20	23	29	33	43
1.10	.05	11	13	16	20	25	30	40
	.01	17	20	23	27	34	39	
1.00	.05	13	16	19	23	30	36	49
	.01	21	24	28	33	40	47	
0.90	.05	16	19	23	28	36	44	
	.01	25	29	34	40	49		
0.80	.05	20	24	29	36	46		
	.01	31	36	42	50			
0.70	.05	25	31	37	46			
	.01	40	47					
0.60	.05	33	41	50				
	.01							
0.50	.05	48						
	.01							

Table 2d

I = 5

$\lvert d \rvert/\sigma$	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	2	3	3	3	4	4	5
	.01	3	3	4	4	5	5	6
3.00	.05	3	3	4	4	5	6	7
	.01	4	5	5	6	7	7	9
2.50	.05	4	4	5	5	6	7	10
	.01	5	6	6	7	9	10	12
2.00	.05	5	5	6	7	9	11	14
	.01	7	8	9	10	12	14	18
1.75	.05	6	7	8	9	12	14	18
	.01	9	10	11	13	15	18	23
1.50	.05	7	9	10	12	15	18	24
	.01	11	13	14	17	20	23	30
1.40	.05	8	10	11	14	17	20	27
	.01	12	14	16	19	23	27	34
1.30	.05	9	11	13	16	20	23	31
	.01	14	16	19	22	26	31	39
1.20	.05	10	12	15	18	23	27	37
	.01	16	19	21	25	31	36	46
1.10	.05	12	15	17	21	27	32	43
	.01	19	22	25	30	36	42	
1.00	.05	14	17	21	25	32	39	
	.01	22	26	30	35	43		
0.90	.05	17	21	25	31	41	47	
	.01	27	32	37	43			
0.80	.05	22	26	32	39	50		
	.01	34	39	46				
0.70	.05	28	34	41	50			
	.01	44						
0.60	.05	37	45					
	.01							
0.50	.05							
	.01							

Table 2e

I = 6

$\lvert d \rvert / \sigma$	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	2	3	3	3	4	4	5
	.01	3	3	4	4	5	5	6
3.00	.05	3	3	4	4	5	6	7
	.01	4	5	5	6	7	8	9
2.50	.05	4	4	5	6	7	8	10
	.01	5	6	7	8	9	10	13
2.00	.05	5	6	7	8	10	11	15
	.01	7	8	9	11	13	15	19
1.75	.05	6	7	8	10	12	14	19
	.01	9	10	12	13	16	19	24
1.50	.05	8	9	11	13	16	19	25
	.01	12	13	15	18	21	25	32
1.40	.05	9	10	12	15	18	22	29
	.01	13	15	17	20	24	28	36
1.30	.05	10	12	14	17	21	25	31
	.01	15	17	20	23	28	32	41
1.20	.05	11	13	16	19	24	29	39
	.01	17	20	23	27	32	38	48
1.10	.05	13	16	19	23	29	34	46
	.01	20	23	27	31	38	44	
1.00	.05	15	19	22	27	34	41	
	.01	24	28	32	38	46		
0.90	.05	19	23	27	33	42	50	
	.01	29	34	39	46			
0.80	.05	23	28	34	42			
	.01	36	42	49				
0.70	.05	30	37	44				
	.01	47						
0.60	.05	40	49					
	.01							
0.50	.05							
	.01							

Table 2f

I = 7

$\|d\|/\sigma$	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	3	3	3	3	4	4	5
	.01	3	4	4	4	5	5	6
3.00	.05	3	4	4	5	5	6	8
	.01	4	5	5	6	7	8	10
2.50	.05	4	4	5	6	7	8	10
	.01	6	6	7	8	9	10	13
2.00	.05	5	6	7	8	10	12	15
	.01	8	9	10	11	13	15	19
1.75	.05	6	7	9	10	13	15	20
	.01	9	11	12	14	17	19	25
1.50	.05	8	10	11	14	17	20	26
	.01	12	14	16	18	22	26	33
1.40	.05	9	11	13	15	19	23	30
	.01	14	16	18	21	25	29	37
1.30	.05	10	12	15	18	22	26	35
	.01	16	18	21	24	29	34	43
1.20	.05	12	14	17	20	26	30	40
	.01	18	21	24	28	34	39	50
1.10	.05	14	17	20	24	30	36	48
	.01	21	24	28	33	40	46	
1.00	.05	16	20	24	29	36	43	
	.01	25	29	34	39	48		
0.90	.05	20	24	29	35	44		
	.01	31	36	41	48			
0.80	.05	25	30	36	44			
	.01	38	45					
0.70	.05	32	39	47				
	.01	50						
0.60	.05	43						
	.01							
0.50	.05							
	.01							

Table 2g

I = 8

|d|/σ	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	3	3	3	3	4	4	5
	.01	3	4	4	4	5	5	6
3.00	.05	3	4	4	5	6	6	8
	.01	4	5	5	6	7	8	10
2.50	.05	4	5	5	6	7	8	11
	.01	6	6	7	8	9	11	13
2.00	.05	5	6	7	9	11	12	16
	.01	8	9	10	12	14	16	20
1.75	.05	7	8	9	11	13	16	21
	.01	10	11	13	15	18	20	25
1.50	.05	9	10	12	14	18	21	27
	.01	13	15	17	19	23	27	34
1.40	.05	10	11	13	16	20	24	31
	.01	14	16	19	22	26	30	39
1.30	.05	11	13	15	18	23	27	36
	.01	16	19	22	25	30	35	45
1.20	.05	12	15	18	21	27	32	42
	.01	19	22	25	29	35	41	
1.10	.05	15	17	21	25	32	37	50
	.01	22	26	29	34	42	48	
1.00	.05	17	21	25	30	38	45	
	.01	26	31	35	41	50		
0.90	.05	21	25	30	37	47		
	.01	32	37	43				
0.80	.05	26	32	38	46			
	.01	40	47					
0.70	.05	34	41	49				
	.01							
0.60	.05	46						
	.01							
0.50	.05							
	.01							

Table 2h

I = 9

$\lvert d\rvert/\sigma$	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	3	3	3	3	4	4	5
	.01	3	4	4	4	5	5	6
3.00	.05	3	4	4	5	6	6	8
	.01	5	5	6	6	7	8	10
2.50	.05	4	5	5	6	8	9	11
	.01	6	7	7	8	10	11	14
2.00	.05	6	7	8	9	11	13	17
	.01	8	9	10	12	14	16	20
1.75	.05	7	8	9	11	14	16	21
	.01	10	12	13	15	18	21	26
1.50	.05	9	10	12	15	18	22	28
	.01	13	15	17	20	24	28	35
1.40	.05	10	12	14	17	21	25	32
	.01	15	17	20	23	27	32	40
1.30	.05	11	13	16	19	24	28	37
	.01	17	20	22	26	31	36	46
1.20	.05	13	16	19	22	28	33	44
	.01	20	23	26	30	37	42	
1.10	.05	15	18	22	26	33	39	
	.01	23	27	31	36	43	50	
1.00	.05	18	22	26	31	40	47	
	.01	28	32	37	43			
0.90	.05	22	27	32	38	48		
	.01	34	39	45				
0.80	.05	28	33	40	48			
	.01	42	49					
0.70	.05	36	43					
	.01							
0.60	.05	48						
	.01							
0.50	.05							
	.01							

Table 2i

I = 10

$\|d\|/\sigma$	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	3	3	3	3	4	4	4
	.01	3	4	4	4	5	5	7
3.00	.05	3	4	4	5	6	7	8
	.01	5	5	6	6	7	8	10
2.50	.05	4	5	6	6	8	9	11
	.01	6	7	7	8	10	11	14
2.00	.05	6	7	8	9	11	13	17
	.01	8	10	11	12	15	17	21
1.75	.05	7	8	10	12	14	17	22
	.01	10	12	14	16	19	21	27
1.50	.05	9	11	13	15	19	22	29
	.01	14	16	18	21	25	29	36
1.40	.05	10	12	15	17	22	25	33
	.01	15	18	20	23	28	33	41
1.30	.05	12	14	17	20	25	29	39
	.01	18	20	23	27	33	37	48
1.20	.05	14	16	19	23	29	34	45
	.01	20	24	27	31	38	44	
1.10	.05	16	19	23	27	34	40	
	.01	24	28	32	37	45		
1.00	.05	19	23	27	33	41	49	
	.01	29	33	38	44			
0.90	.05	23	28	33	40	50		
	.01	35	41	47				
0.80	.05	29	35	42	50			
	.01	44						
0.70	.05	37	45					
	.01							
0.60	.05	50						
	.01							
0.50	.05							
	.01							

Table 2j

I = 12

|d|/σ	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	3	3	3	4	4	5	6
	.01	3	4	4	4	5	6	7
3.00	.05	4	4	4	5	6	7	9
	.01	5	5	6	7	8	9	11
2.50	.05	4	5	6	7	8	9	12
	.01	6	7	8	9	10	12	15
2.00	.05	6	7	8	10	12	14	18
	.01	9	10	11	13	15	18	22
1.75	.05	8	9	10	12	15	18	23
	.01	11	13	14	16	20	23	28
1.50	.05	10	12	14	16	20	24	31
	.01	15	17	19	22	26	30	38
1.40	.05	11	13	16	18	23	27	35
	.01	16	19	22	25	30	34	43
1.30	.05	13	15	18	21	26	31	41
	.01	19	22	25	29	34	40	50
1.20	.05	15	17	21	25	31	36	48
	.01	22	25	29	33	40	46	
1.10	.05	17	20	24	29	36	43	
	.01	26	30	34	39	47		
1.00	.05	20	24	29	35	44		
	.01	31	35	41	47			
0.90	.05	25	30	36	43			
	.01	38	43	50				
0.80	.05	31	37	45				
	.01	47						
0.70	.05	40	49					
	.01							
0.60	.05							
	.01							
0.50	.05							
	.01							

Table 2k

I = 15

$\|d\|/\sigma$	α	Power .50	.60	.70	.80	.90	.95	.99
4.00	.05	3	3	3	4	4	5	6
	.01	4	4	4	5	5	6	7
3.00	.05	4	4	5	5	6	7	9
	.01	5	6	6	7	8	9	11
2.50	.05	5	5	6	7	9	10	13
	.01	7	7	8	9	11	13	16
2.00	.05	7	8	9	10	13	15	19
	.01	10	11	12	14	17	19	23
1.75	.05	8	10	11	13	16	19	25
	.01	12	14	15	18	21	24	30
1.50	.05	11	13	15	18	22	25	33
	.01	16	18	20	24	28	32	41
1.40	.05	12	14	17	20	25	29	38
	.01	18	20	23	27	32	37	46
1.30	.05	14	16	19	23	29	33	44
	.01	20	23	27	31	37	42	
1.20	.05	16	19	22	27	33	39	
	.01	24	27	31	36	43	50	
1.10	.05	19	22	26	32	39	46	
	.01	28	32	37	43			
1.00	.05	22	27	32	38	47		
	.01	34	39	44				
0.90	.05	27	33	39	47			
	.01	41	47					
0.80	.05	34	41	49				
	.01							
0.70	.05	44						
	.01							
0.60	.05							
	.01							
0.50	.05							
	.01							