Convective, Diffusive Effects on Magnetic Fields and Eddy Currents in Compulsators

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Abstract - Compulsators are being designed at ever increasing energies and energy densities and are required to deliver energy to the load in less than 10 ms. These require high speeds of operation and dense spacing of conductors. Diffusion of magnetic fields into the conductors and the formation of nonuniform, time-dependent distribution of eddy currents become dominant design considerations due to their major mechanical, thermal, and thermodynamic impact. A semi-analytical method has been developed for the two-dimensional analysis of field diffusion and eddy currents in high speed rotary machines to aid design decisions. Analytical results for fields are utilized and computations are restricted to the conductor domains alone. The semi-analytical method has been tested with two conductors (one in the stator and one in the rotor rotating at high speed). The resulting distributions of fields and eddy currents are presented.

INTRODUCTION

Rotary pulsed power machines (compulsators) are being developed for electromagnetic launching and other applications in defense, space, and other areas.[1] The design energies and energy densities are constantly increasing to suit the applications and the challenge is to design the smallest machine at the highest speed to meet the design goals and deliver the energy to the load within a few milliseconds. The requirements result in transient nonuniform magnetic fields and eddy currents with skin effects inside the conductors; these, in turn, result in nonuniform Lorentz forces, Joule dissipation, and stress concentrations. Since the conductors are pushed to the extremities of their electrical, mechanical, and thermal strengths, precise electromagnetic analyses are required for safe design and to keep the operating parameters from transgressing into regions of failure with repeated delivery of energy pulses to the load. A semi-analytical method has been developed for the analyses of convective, diffusive effects on magnetic fields and eddy currents in compulsators and is presented here with a sample analysis.

DESCRIPTION OF THE PROBLEM

A typical rotary machine to be analyzed consists of a set of conductors azimuthally distributed on the periphery of a rotor (the field coil). This system of conductors is surrounded by another set of conductors (which are static), located at a larger radius (the stator or the armature). The conductors on the stator and rotor may be distributed in more than one radial layer. There could be another set of conductors (the compensating coil) with appropriately directed currents to reduce the stored magnetic energy due to currents in the armature and thus reduce the inductance. The magnetic field produced by the currents in the rotor induces a current in the conductors of the stator which is rectified and fed back to the conductors of the rotor. The conductors in the stator are of the litz type (finely divided, individually insulated conductors). One could consider a spectrum of conductors for the rotor; densely packed solid conductors at the one end to finely divided litz variety at the other, with a continuum of rectangular Roebel conductors with different aspect ratios in between. If the litz variety were chosen, the diffusion and eddy current problem could be obviated; but, this choice introduces additional rotating mass and obviously limits the rotational speeds, energies, and energy densities that could be targeted. If densely packed solid conductors were chosen, the rotating mass is restricted to essential components and higher energies and energy densities could be targeted; but one has to contend with field diffusion, eddy currents, Ohmic heating and localized temperature rises and stress concentrations. These problems reduce in severity as one chooses the

Roebel conductors and becomes minimal with the litz type of conductors. The fabricational difficulty is high with solid conductors and reduces in different degrees as the choice traverses through the spectrum of conductors to the litz type. In order to facilitate the choice of the conductor type consistent with design goals, precise electromagnetic analyses are required to predict field diffusion, eddy currents, skin effects, and Joule dissipation at high speeds of rotation.

GOVERNING EQUATIONS

In order to illustrate the method of analysis, we will consider one rectangular conductor located at a given radius and rotating at high speeds (15,000 rpm) in a background magnetic field created by a static conductor of the litz type carrying a given current density (fig. 1). We will analyze the problem in the magneto-quasistatic approximation. The time scales of interest will be greater than the characteristic time for electromagnetic wave propagation. It will be further assumed that all the materials used will have a relative permeability equal to unity. The relevant equations with conventional notation are [2]:

$$\nabla \times H = j \tag{1}$$

$$\nabla \cdot B = 0 \tag{2}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu_0 \frac{\partial H}{\partial t} \tag{3}$$

$$B = \mu_0 H \tag{4}$$

$$j = \sigma(E + V \times B) \tag{5}$$

V denotes the velocity vector in eqn. (5).

The electric field E, the magnetic field B, the current density j could be eliminated among the above equations to obtain the following equation for the magnetic intensity H. [3]

$$\mu_0 \sigma \frac{\partial H}{\partial t} = \nabla^2 H + \mu_0 \sigma \nabla \times (V \times H) \tag{6}$$

The term on the left represents the time variation, and the terms on the right describe diffusion and convection, respectively. The diffusion of the magnetic field created by the currents in the static conductor (fig. 1) into the rotating conductor is governed by eqn. (6). When we solve eqn. (6) for H, the eddy currents distribution in conductor 2 will be determined from eqn. (1). Eqn. (6) can be nondimensionalized to assess the relative importance of the terms. Let us define the following nondimensional variables:

$$H^* = \frac{H}{H'} \; ; \; V^* = \frac{V}{V'}; \; t^* = \frac{t}{t'}; \; x^* = \frac{x}{r}; \; y^* = \frac{y}{r} \tag{7}$$

H',V' and t' are characteristic values for the problem and r is the radius of the center of the rotating conductor. Using (7) in (6), we get the following equation after grouping terms:

$$G_1 \frac{\partial H^*}{\partial t^*} = \frac{1}{Re_m} \nabla^{*2} H^* + \nabla^* \times (V^* \times H^*) \tag{8}$$

The nondimensional groups are:

$$G_1 = r/(V't')$$

$$Re_m = \mu_0 \sigma v'r \text{ (magnetic Reynold's number)}$$

The characteristic times for the problem and the nondimensional groups can be estimated using the following numbers for a typical machine: the radius of the center of the rotating conductor r = 0.36 m. The rotor rotates at 15,000 rpm; the angular velocity $\omega = 500\ \pi$ rad/s and the azimuthal velocity 565.5 m/s. The conductivity $\sigma = 3.77 \times 10^7$ mhos/m for aluminum. We will be interested in millisecond transients and t' = 0.001 s. The characteristic time for electromagnetic wave propagation τ_e, can be estimated from the least dimension (say 1 cm), and the velocity of light ; $\tau_e = 3.3 \times 10^{-11}$ s. The time for one rotation is $\tau_r = 0.004$ s. The time for magnetic diffusion $\tau_{md} = \mu_0\sigma/(1/a^2+1/b^2)$; a, b are the half sides of a rectangular conductor.[4] With a = b = 1 cm, $\tau_{md} = 0.00237$ s. The time of interest for energy delivery to the load is $\tau = 0.01$ s. The group $G_1 = 0.64$. The magnetic Reynold's number $Re_m = 9645.3$. It is seen from eqn. (8) that the term on the left and the second term on the right are of the order of unity, while the first term on the right representing diffusion is of the order of (1/10,000). Hence, the equation (dimensional) simplifies to:

$$\frac{\partial H}{\partial t} = \nabla \times (V \times H) \tag{9}$$

However, we can find very short length scales for which $Re_m < 1.0$ when the diffusion term will become larger than the convective term. In this case, eqn. (8) reduces (in a dimensional form) to:

$$\mu_0\sigma\frac{\partial H}{\partial t} = \nabla^2 H \tag{10}$$

The diffusion process occurs much slower and the rotor would have executed half a rotation before perturbations in H at a location would be felt through diffusion 1 cm away. Thus at high speeds, effects of convection and diffusion on H could be decoupled. The evolution of the field over rotational time scales and length scales is governed by eqn. (9). As the conductor on the rotor rotates, it will sense different fields at its boundary; this will initiate a diffusive transient governed by eqn. (10) at larger time scales and short length scales contributing to localized gradients in H which in turn, will lead to time-dependent eddy currents with skin effects. This is an example of a singular perturbation problem.

SOLUTION PROCEDURE

The current in the stator conductor will be prescribed and the rotor conductor will cut the resulting flux lines, as it rotates. We will assume that the field has fully penetrated the rotor conductor at time zero, when the rotor starts instantaneously rotating at 15,000 rpm. The solution of eqn. (9) is well known; it implies frozen fields, i.e., rotation, carries the field azimuthally without any change. As the conductor rotates, the background field set up by the stator conductor will be felt at the boundaries of the rotating conductor; however, the field within the rotor conductor will be the frozen field carried from the previous (elementally displaced) azimuthal location. This will induce a transient at the boundaries which will propagate inside the rotating conductor, and this process is governed by eqn. (10). We can seek a solution for eqn. (10) through separation of variables:

$$H = T(t)\, X(x)\, Y(y) \tag{11}$$

$$\begin{aligned} H &= \exp(\Omega t)(A' \sinh(\lambda_x x) + B' \cosh(\lambda_x x)) * \\ &\quad (C' \sin(\lambda_y y) + D' \cos(\lambda_y y)) \end{aligned} \tag{12}$$

$$\Omega = \frac{\lambda_x{}^2 + \lambda_y{}^2}{\mu_0 \sigma} \; ; \quad \frac{X''}{X} = \lambda_x{}^2 \; ; \quad \frac{Y''}{Y} = \lambda_y{}^2 \tag{12a}$$

It should be noted that, depending on the signs of (X''/X) and (Y''/Y), the hyperbolic and trigonometric functions in eqn. (12) will change. The eigenvalues λ_x and λ_y will be inversely proportional to the half widths of the rectangle a, b. We may obtain the solution for the entire closed domain of the rotor conductor as a series solution with a spectrum of eigenvalues, or we may obtain local solutions with the eigenvalues as functions of x, y. We will adopt the latter approach, and we will term Ω the local instantaneous growth rate.

The solution for the problem in time constitutes an initial value problem which could be obtained by the Crank-Nicholson scheme which is known to be stable. Eqn. (12) provides a better way to calculate the solution for successive time steps, knowing the local instantaneous growth rates from the initial conditions; stability can be ensured by calculating a time step at every instant such that the growth of H is $\leq$ 10% anywhere, i.e., by setting the argument of the exponential to be 0.1.

The following numerical algorithm evolves with the foregoing. The initial positions of the stator, and rotor conductors are specified. The rotor conductor can be divided into a fine grid say 0.5 mm in x and y. We have to set an upper limit on the time step; it has to be much less than the characteristic time for diffusion through 0.5 mm which is 5.9×10^{-6} s. We will set an upper limit of 5.0×10^{-7} s. The initial field at every node can be calculated (with the assumption that the field has fully penetrated). At time zero, there are no eddy currents. Let us prescribe an initial time step of 5.0×10^{-7} s. The new rotor position can be determined knowing the angular velocity. The fields within the rotor conductor will be the frozen fields; the fields at the boundary nodes will be the fields from the stator source at the new location. The X'' and Y'' and the local instantaneous growth rate Ω can be calculated (using eqn. (12a)) at every node with local quadratic splines using the updated field distribution. The fields will redistribute as a result of diffusion, and the resulting field at every node can be written down using eqn. (12):

$$H(x, y) = H_0(x, y)\ \exp(\Omega * \Delta t) \qquad (13)$$

In eqn. (13), H_0 is the field at time t and H is the field at time $(t+\Delta t)$. The eddy current at every node can be calculated using eqn. (1).

$$J_e(x, y) = \nabla \times H(x, y) \qquad (14)$$

The Joule dissipation in watts per meter at any instant is given by

$$P = \iint \frac{J_e^2}{\sigma}\ dx\ dy \qquad (15)$$

Eqn. (13) gives the fields inside the rotor conductor. The fields outside the rotor conductor will be the sum of the background field and the fields due to the eddy current sources at every node in the rotor. The next step is to obtain the maximum of the growth rates $\Omega(x,y) = \Omega_m$. The next time step Δt will be determined by

$$\Delta t = \frac{0.1}{\Omega_m} \tag{16}$$

Limiting the growth of H to 10% was found to be adequate and smaller growths could be used for finer calculations. The method sketched for obtaining the fields is analytically exact and is analogous to the problems of potential flows over spheres and cylinders described by Prandtl and Tietjens.[5]

VALIDATION

A simple test was made for the validation of the procedure with the following example with static conductors. A conductor 12 cm × 1 cm was placed symmetrically between two outer conductors 10 cm × 1 cm each. One of the outer conductors was excited by a current proportional to $j * \exp(-1000t)$; the other was excited with an equal current with the opposite sign. The dominant eigenvalue is λ_y which can be calculated from (12a) with a growth rate of 1,000. The eddy current variation with y at x = 0 will be proportional to $\sin(\lambda_y y)$ towards the end of the transient from (12). Fig. 2 shows the calculated normalized variation of the eddy current at x = 0, and the $\sin(\lambda_y y)$ prediction; the comparison is very good.

RESULTS AND DISCUSSION

The example analyzed is shown in fig. 1. A static conductor 10 cm × 1 cm carrying a current density of 5.9×10^8 A/m^2 is located with its center at (0, 0.396 m). Another conductor 10 cm × 1 cm is located with its center at (0.36 m, 0) at time zero and starts instantaneously rotating at 15,000 rpm against the background field set up by the currents in the static conductor. The fields and eddy currents in the rotating conductor were computed for different azimuthal angles over one turn (and times ranging from 0 to 4 ms) and are shown in figs. 3 to 12 with units tesla meters and Amps/m^2, respectively. The total Joule dissipation for one turn was 1,000 Joules/m. At time zero, the eddy currents are zero, and the potential lines correspond to the background field due to the current in the stator conductor. The eddy current and potential contours at 5 μs are shown in figs. 3, 3a, respectively. The eddy current is confined to skins parallel to the 12 cm sides with the positive current flowing at the farther radius. The current and potential contours at t = 0.5 ms are seen in figs. 4, 4a. The eddy current layers have grown, and they fill the conductor; the contours are mostly parallel to the longer side with skin layers forming along the small sides. The effect of convection could be seen in the potential lines. At 0.8 ms, the rotor has turned through 72° (figs. 5, 5a); the eddy current is mostly confined to a skin layer near the leading small side, and the current near the farther long side is turning negative. We could expect local heating in the skin layer. Convection has further distorted the field lines. The figs. 6, 6a show progressive growth of these trends at t = 0.9 ms. The rotor has turned through 90° at t = 1 ms (figs. 7, 7a); the eddy current layers have now spread over the section with the current being negative all along the farther long side; this is in conformance with Lenz's law. At t = 1.2 ms (figs. 8, 8a), a skin layer has started forming near the trailing small side; we can again expect localized Joule dissipation and heating at this layer. Figs. 9, 9a show progressive growth of these patterns. Figs. 10, 10a show the contours at t = 2 ms; the current at the farther long side has completely switched sign, and the eddy current layers have spread over the section. The contours at t = 3.0 ms corresponding to a rotor angle of 270°, are shown in figs. 11, 11a. There are skin layers near the small sides with less currents filling the section. The rotor has completed one turn at t = 4 ms (figs. 12, 12a); the current at the farther long side becomes positive again. Thus, as the rotor rotates, the eddy current patterns and signs change with the formation of skin layers with different thicknesses at different times. Though the patterns of the currents (skin layers, signs etc.) are independent of the initial rotor angle, the magnitudes will depend on the initial rotor position. The computations for just one conductor executing one turn required about six interactive hours of computing on a 64-bit VAX System.

The computations for a general system with more than one stator, rotor, and compensating layer of conductors is similar but requires a parallel processor. The exciting currents need to be prescribed as functions of time and the field sensed by a conductor at a given time will be the sum of all incident fields due to applied currents and induced eddy currents in other conductors. The maximum growth rate of fields should be determined as an absolute maximum over all the conductors with eddy currents, and the variable time step should be determined with this global maximum growth rate.

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REFERENCES

[1] J.R. Kitzmiller, S.B. Pratap, M.D. Werst, C.E. Penney, T.J. Hotz, and B.T. Murphy, "Laboratory Testing of the Pulse Power System for the Cannon Caliber Electromagnetic Gun System (CCEMG)," *IEEE Transactions on Magnetics*, vol. 33, no. 1, January, 1997.

[2] W.K.H. Panofsky and M. Phillips, *Classical Electricity and Magnetism*, Addison-Wesley Publishing Company Inc., 1962.

[3] G.W. Sutton and A. Sherman, *Engineering Magnetohydrodynamics*, McGraw-Hill Book, Co., New York, 1965.

[4] H. Knoepfel, *Pulsed High Magnetic Fields*, North-Holland Publishing Co., London, 1970.

[5] L. Prandtl and O.G. Tietjens, *Fundamentals of Hydro- and Aeromechanics*, Dover Publications, Inc., New York, 1957.