Publications Committee

BULLETIN

OF THE

UNIVERSITY OF TEXAS

1916 : No. 58

OCTOBER 15 **1916**

The Texas Mathematics Teachers' Bulletin

(Volume 2, No. 1, October 15, 1916)

Published by the University six times a month and entered as
second-class matter at the postoffice at
AUSTIN, TEXAS

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The Texas Mathematics Teachers' Bulletin

(Vol. 2, No. 1, October 15, 1916)

Edited by

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This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

CONTENTS.

Published by the University Six times a month and entered as second-class matter at the postoffice at AUSTIN, TEXAS.

The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston.

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

President Mirabeau B. Lamar.

NOTICE

Owing to the fact that only a small number of Vol. 1, No. 1 of this Bulletin was printed, and there have been many requests for copies that could not be supplied, the articles in that number will be reprinted from time to time during the current year. One article from that number appears in this issue.

METHODS IN MATHEMATICS

By methods we shall mean the methodology of teaching, and here we begin by saying that such methods are as diverse as the capacity of the students and as varied as the personalities of the teachers. Journals concerned with the teaching of elementary mathematics are numerous in all the civilized languages, and these swarm with papers large and small on the handling of certain subjects as a whole, and in the treatment of certain topics or theorems, and while of very unequal merit they often contain suggestions that a good teacher will find worth trying. The real teacher will use such material only in so far as it harmonizes with his own individual methods, and only so far as he finds that he can get good results with it. For there is no *best* method of presenting any particular topic any more than there is any one best method of painting a picture or playing the piano. A skillful teacher is an artist whose material is the most subtile and fluid in existence, namely, the minds and characters of his pupils. It is his delicate and difficult task to induce a certain series of reactions and activities in these minds, and, since they vary between wide limits, his methods must be as varied as the material on which he operates. The learning process is complex and while the meagre data of psychology furnish useful hints, they fall far short of adequate guidance, and only endless experiments and patience can reveal the best methods of gaining the desired end.

THE END IN VIEW

The teacher of mathematics has for his main purpose to train his pupils to *think mathematically*. This does not necessarily imply the memorizing of such and such proofs but means the acquirement of certain habits of thought and the mastering of certain *processes* or *ways* of going about things. The mere ability to state and prove from memory a list of theorems and formulas may imply no mastery whatever of a subject, and the most incessant drill may accomplish nothing more than disgust and fatigue, though intelligent drill is indispensable in all good teaching. What is essential is that the pupils' interest in the essential thinking process should be aroused, directed, and developed by the careful choice of material that is within his powers of comprehension, and that he should feel steadily a pleasurable sense of effort and power in surmounting difficulties that before seemed insuperable. The feeling of effort is essential. It is poor praise of a teacher to say that he makes mathematics easy and the highest praise to say that he helps make the serious efforts to learn it pleasurable. This is the most he can do, and if he does it, his students will rise up and call him blessed and even if they do not love him will certainly respect him. There is no such thing as making the real learning of mathematics easy, and it is fortunate that it can never be made so, for then its disciplinary value, its inculcation of careful, accurate habits of thought would be gone.

There are still some people old-fashioned enough to believe that the mind can be trained, and some still more old-fashioned that believe that mathematics lends itself admirably to such training. The writer is one of these and while he believes that the so-called vocational value of mathematical knowledge is of little value outside of the careers of science and engineering he believes that it furnishes an indispensable key to the comprehension of the so-called exact sciences. There is hardly a law of Physics or Chemistry that is clearly understandable without some mathematics; not mathematics as a mysterious and incomprehensible *tool,* but as a process of thought, a way of thinking.

We do not mean to imply by this that our text-books should swarm with problems of Physics and Chemistry. The data of such problems are usually incomprehensible until these subjects have been carefully studied, but the understanding of either of them postulates mathematical training.

The most precious results of the training we have in mind is the power of independent thought. This is often confused with the knack of invention, but the two things are really distinct, for we have some pupils with good accurate minds who are largely devoid of this power of imagination. Both classes must be cared for by the conscientious teacher. He must give to each according to his capacity and special aptitude. Both classes may be taught the meaning and essence of the method used, but the first class only will clearly apply it to new problems. The reader may ask what has become of methods; what method would you apply in teaching such and such a topic? We hope that it has been made clear from the foregoing that there is not one, but there are many methods. A good teacher *always* has a good method, no matter what it is, and a poor one has a bad method no matter what authority he can cite to prove its excellence, just as the best paint, brushes, and canvas will not make a painter out of a dauber, while some artists have done creditable work with the most primitive tools. Sense, sympathy, and observation cannot be replaced by any method, no matter how plausible, and given these attributes with the necessary knowledge of the subject, a good method with surely result. A teacher gifted with these attributes will always be a learner and while he may not greatly increase his fund of information in the subject taught, he will certainly be always learning something new concerning the complex learning process, important truths given in no text book, individual peculiarities and limitations of his pupils analogous to the constitutional peculiarites of a doctor's patients. And this is one of the principal justifications of the teacher as a useful member of society. If he cannot deal with the individual he is no better than a text-book for self-instruction, and his function degenerates at most into that of a policeman whose whole duty it is to see that his pupils learn "what is in the book." Fortunately there is much to be learned that is not in the book and the good teacher is here wonderfully helpful.

LITERAL ARITHMETIC

In the last bulletin some notion was given in regard to the manner in which students may use formulas and statements already given in letters. It was seen that all rules of Arithmetic may be given in the "short-hand" statements with letters. It is now our purpose to suggest a method of proceeding so that pupils may learn to use this new language or mode of expression and hence think in letters instead of numbers. To gain such a power on the part of the pupil some little time and training will be necessary. It must come through practice and drill by the teacher. Nothing will be so valuable here to the pupil as a skillful teacher in regular outlined lessons extending over a period of time sufficient to enable the pupile to gain this power of thought expression. One or two recitations per week during the last year before Algebra proper is begun is suggested.

After a sufficient amount of drill has been given in the substitution of numerical values for letters in formulas, the nature and purpose of literal statements are more clearly seen by the pupil and in this way he will begin to realize that he is learning to write and form general statements which may have many particular applications. The teacher will find also that the pupil gains in power in making general expression, just in the same proportion that particular applications are understood.

In regard to the signs and symbols of operation, we find the signs of addition, subtraction, etc., are now taught in the earlier grades and most pupils understand their meaning and use, especially in the earlier literal notation. The multiplication of the two numbers represented by a and b by the expression ab will be taught in the first work with formulas.

Following a sufficient practice in making numerical substitutions in well known rules expressed by letters, some work could be given by asking the pupils to write out in this "shorthand" method rules written in words or given orally by the teacher. Thus the pupil could be asked, for instance, to write out a rule:

(1) for the area of a rectangular floor,

(2) for the length of a floor when the area and the width are given,

(3) for reducing dollars and dimes to cents,

(4) for finding the volume of a room,

(5) for finding the total interior surface of a room exclusive of the floor,

(6) for finding the height of a room when the volume and area of the floor are given,

(7) for finding the principal at interest when the time, rate and per cent are given,

(8) for finding the rate when base and percentage are given,

(9) for finding the radius when the area of a circle is known.

In this manner every rule of Arithmetic may be used in different ways and studied from different viewpoints in this new way of writing them.

Formulas of a more general type may now be taken up. Examples should be taken from facts familiar to pupils. The algebraic or literal setting of any concrete fact or set of facts is possible only when a real knowledge of those facts is in the possession of the pupil. Here the skillful teacher must make the selection of the material to be used. The following examples are offered as suggestions only—the teacher with a knowledge of the ability of his class may be able to use to greater advantage other problems.

(1) A boy begins working for *a* dollars per month and has his salary increased by d dollars every month. What will be his salary in t months?

(2) Write the value of the nth term of the series.

(a) 3, 5. 8, 11......

(b) 7, 11, 15......

(c) 1.3, 1.6, 1.8, 2.2......

(3) A tank of *V* gallons is fed by two pipes one supplying 1.5 gallons per minute and the other 2.5 gallons per minute. Give formula for finding

(a) the time *t* it will take the first pipe to fill the tank

(b) the time *t* it will take both pipes to fill the tank

(4) A tank of *V* gallons is fed by a pipe supplying 1.5 gallons per minute and is emptied by a waste pipe that carries away 2.5

gallons per minute. If the tank is full and both pipes begin to run, find the time t it will require to empty the tank.

(5) Two automobiles start from the same place and travel in opposite directions at speeds of a and b miles per hour. Find the formula for expressing

(a) the distance d they are apart after t hours

(b) the time t required in order that they may be the distance d apart

The following is taken from Prof. Nunn's book on the Teaching of Algebra: "In connection with this topic it is hardly possible to lay too much stress upon the importance of cultivating a neat and orderly way of setting down the steps in an arithmetical or algebraic argument. A piece of algebraic symbolism should be as capable of straightforward and continuous reading as a passage from a newspaper. To achieve this end the teacher will find it a sound rule never to permit a line to contain more than two expressions connected by the sign of equality and to insist upon the pupil's setting the signs of equality, in successive lines of the argument, directly underneath one another. Thus such expressions as

$$V=Bh=32.7\times 12.4=405.48 \text{ C. Cm}$$

should be excluded from the exercise book and from the blackboard in favor of the arrangement:—

$$\begin{aligned} V &= Bh \\ &= 32.7\times 12.4 \\ &= 405.48 \text{ C. Cm} \end{aligned}$$

ON DEFINITIONS

As a small boy the writer recalls reading in McGuffey's Fourth Reader that "words enclosed in a parenthesis should be read in a low tone of voice and may be left out entirely without obscuring the sense."

An examination of a good deal of work done by teachers of geometry and contact with a multitude of their students convinces him that definitions in geometry are pretty generally regarded by teachers and students in this same light. They might be left out entirely without having their absence noted.

This is exceedingly unfortunate. In the study of geometry definitions are of prime importance. There can be no real and orderly progress till this is realized and acted upon. Definitions should be

1. Composed of terms better understood than the term defined.
2. Exact in statement.

There are certain things as a point and a straight line that cannot be defined in any way that will make their meaning any clearer. In fact, any definition of them is harder to understand than the words themselves. They, therefore, should not be defined. The assumption that two points determine a line is as close to a definition as should, perhaps, be attempted. A straight line should certainly not be defined, as it seems generally to be, as the shortest distance between two points. This is not only not enlightening but is misleading. A line of any sort is not a distance at all. If two points are joined by a straight line segment the distance between them is the *length of the* straight line segment. This *length* is the ratio of the given segment to another arbitrary segment whose length is said to be 1.

Length and distances are numbers, lines are geometric figures. Hence, a *line*, straight or crooked, cannot be a *distance* either short or long.

One of the most valuable results to be got from a study of geometry should be a high degree of accuracy of expression, as a result of accuracy of thinking. This cannot be acquired where there is looseness in the matter of definitions. There should be the sharpest sort of distinction drawn between the definition of a figure and properties that it may later be shown to possess, or that may be arbitrarily assigned to it.

The writer cannot recall a single student, in his ten years of teaching students fresh from the high schools, who when asked for a definition of a right angle gave the correct one, or who could give a correct definition of "a line perpendicular to another line." The universal answer is "A right angle is an angle of ninety degrees" and "two lines are perpendicular when they make right angles with each other." Both those statements are true but neither one is a definition. One might as well define a right angle as an angle inscriptible in a semicircle. It should be clearly pointed out that, by definition, a right angle is one of the angles formed when one straight line meets another in such a way that the adjacent angles are equal. The fact that it contains ninety degrees is merely the result of using a certain system of angle-measurement.

Another figure commonly defined so badly as to obscure rather than clarify matters is the angle. It is variously defined as the "difference of direction between two straight lines that meet" as "the amount of divergence between two straight lines that meet" as "the extent of the opening between two lines" &c, &c, &c. It would be so much better to say "An angle is the *figure* formed by two lines that meet and are terminated at their intersection." In trigonometry and calculus it is often necessary to regard an angle quantitatively and to have regard to its generation by a line rotating about a point, but in plane geometry no such view of the angle obtains. It is a figure just like a circle or a triangle is a figure.

Almost as soon as the angle is defined we begin to prove theorems involving the equality of angles. The student is then told to place angle A upon angle B, that angle C will coincide with angle D and the like. Suppose the student kept in mind the definition of an angle he had learned; (which fortunately he doesn't) what would he think of such an expression as placing one "difference of direction" or one "amount of divergence" upon another "difference of direction" or another "amount of divergence"? Luckily the student, without being conscious of it, knows better than he has been taught and knows that he is placing one *figure* upon another figure. Just here it might be well to say that the idea of direction is a far more complex notion than either straight line or angle, both of which it is often used to define.

It should be called to the attention of the student that almost all relations are matters of definition, lines are parallel, perpendicular, and oblique to each other by definition, angles are right, acute or obtuse by definition. A triangle is acute, obtuse, isosceles, &c., by definition. Is he to prove that two lines are parallel? Then he must keep in mind the definition of parallel lines. Does he wish to prove a polygon is regular? Then he must show that it agrees with the definition of a regular polygon. Does he wish to prove that two figures are equivalent? Then the definition of equivalence must be borne in mind.

One of the commonest as well as one of the most inexcusable errors, and this seems never to be corrected or criticized by the teacher, is the habit of defining things as "when" and "where". "A right angles is when......", "An isosceles triangle is where......" "A sector of a circle is when......" and similar statements greet one every time a freshman opens his mouth, and he seems shocked and grieved to be told that angles, triangles, etc., are things and not "whens" and "wheres."

This laxness in definition is especially vicious if allowed in solid geometry, for this subject is almost wholly of use as a training in logical procedure and is informational only to a very slight degree. Almost all the important metrical theorems have been anticipated in advanced arithmetic. They have been assumed, to be sure, rather than proved but they have been used and, hence, do not strike the student as new. Therefore, if he misses the logical order and accuracy of definition the subject is likely to be barren of useful results.

The school year is young, we have time to consider the proper use of definitions. Let us see to it that definitions are introduced just before we need to use them, that they are correctly stated, completely memorized, and fully understood, then let us see that the proper use is made of them.

While generally speaking the rôle of space intuition, with most people, is rudimentary and largely concerned with the most elementary properties of space such as relations of symmetry and congruence arrived at as a result of imagined experiments; in the case of the great geometers this empirical grasp of space and power of mental experiment are natural gifts refined by logical processes and often seem little short of marvelous. In the case of such men as Desargues, Pascal, Poncelet, and Steiner, it is plain that they have been a powerful auxiliary of invention. None of these geometers, with the exception of Pascal, showed much taste or aptitude for analysis, and the sure imagination that enabled them to construct the beautiful geometric theories that constitute one of the glories of the past century was little beholden to anlysis. As a method of discovery such intuition has always been frankly regarded as heuristic, rigorous proof being supplied by a subsequent *translation* of the steps involved into an appropriate algebraic symbolism.

Poncelet's *Principle of Continuity* can, in this light, be regarded as a sort of *a priori* justification of theorems when the actual spacial constructions are in default.

The greater fecundity of these methods, too, when we compare the results obtained before the elaboration of invariant theory with the relatively few new results obtained by these latter methods, is a sufficient indication that a fundamental problem is involved in the distinction between the born geometer and the analyst. The fact that all projective properties are represented by the vanishing of suitable invariants, and that any invariant can be written down by a mechanical process might even lead one to suppose that the exploration of the whole field of projective geometry was a mechanical operation. This simple view of the matter leaves out, of course, the question of *interest* and the difficulty, often a serious one, of interpreting the invariants geometrically.

The great development in recent years of analysis and algebra, with its almost wholly arithmetical trend has resulted in a loss of interest in pure geometry except from the logical (axiomatic) aspect and while differential geometry shows considerable vitality, it may be traceable to an interest in the theory of functions of several variables rather than to space relations as such.

Space relations serve the modern geometric analyst rather as aids in his analytic transformations and are diagramatic rather than an end in themselves. The history of the integration process from Canchy and Riemann to Lebesgue and Denjoy sufficiently illustrate the diagramatic rôle of space concepts in the elaboration of a theory that in so many respects transcends geometric intuition. The fact that simplicity and ease of intuitive grasp are such important factors in geometric theorems, together with the fact that the space of the analyst so far transcends all power of mental picturing has still further tended to lessen interest in this field.

While geometry as the science of space has largely fallen from its high estate and geometer as a synoym for mathematician has long since lost its applicability, geometry still remains a discipline indispensable in the training of the modern analyst, and for pedagogic reasons, if for not other, will long hold a prominent place with the teaching faculties. Today the most elementary texts swarm with theorems useful and useless, analytic geometry hurries to the assistance of faltering space intuition before the beginner has a chance to realize his weakness in this regard, and the intuition of space (especially is this true of space of three dimensions), remains almost as rudimentary as before geometry was studied. Certain pedagogians have almost seemed to bemoan the wealth of space knowledge possessed by the average child when the real trouble has been not the abundance, but the lack of accuracy, co-ordination, and refinement of such knowledge.

It would seem that geometry might gain in interest and the student in power if a freer use of these powers of intuition were made use of. Thus it would seem that Desargues' theorem concerning perspective triangles in space might advantageously be given to students of solid geometry, or to cite still more complex illustrations, two exercises like the following.

Two unequal spheres, external to each other are touched by two sets of planes, those cutting the line of centers between the centers, and those cutting it on this line produced. The first set of planes all pass through a fixed point (internal center of similitude), here intuition may be helped by rotating the two spheres together with a common tangent plane about the line of centers, the planes of the second class, in the same way, all pass through a second point (the external center of similitude). Consider now three unequal spheres, each external to the others, and their centers not in the same straight line. They will possess six centers of similitude, all in the plane of their three centers. If a plane now be placed so as to touch all three spheres, it will pass through three centers of similitude, all external, or only one external, and as these three centers of similitude will lie in two planes they will thus be collinear. From this may be deduced at once theorems concerning the centres of similitude of three coplanar circles or the axes of similitude of four spheres. The simplicity and lack of artifice with which intersecting geometric theorems are obtained is a considerable element of interest with beginners, the fewness and intuitiveness of the steps that intervene between formulation and conclusion make it easier to grasp reasons and conclusions as a whole. A second illustration in which intuition plays a less conspicuous rôle is the following. Definition: If a point P be joined to the center of a circle O, where it cuts its chord of contact (*polar*) is the *inverse* of P, call this point P^1 with regard to the circle. Theorem 1: The relation of P and P^1 is reciprocal. Theorem 2: The curve inverse to a circle through O is a straight line. Now the locus of the centers of all the chords through P is the circle C described on O P as a diameter. From a figure we see at once that the locus of the poles of all the chords through P will be the inverse of C, hence a straight line. The same process applies to the sphere, of course. Thus we have the fundamental polar property of circles and spheres. Applying to this result the intuitive facts of shadowing (perspective) the same theorem is seen to hold for any shadow of a circle (a conic). Here, of course, the intuitive elements are in default if P is inside the circle (sphere).

Stereographic projection as a by-product of inversion in space might be used, to cite one more illustration, to establish Pascal's theorem and thus a return be made to the genetic development. The purist's objection that inversion is not a projective transformation and that the methods used are mixed will not worry the beginner. He is more interested in the theorems and the simplicity and directness of their proof than in any question of uniformity of method.

The essential thing in all these exercises is that the student should be encouraged to draw and imagine figures and when drawing is difficult, an accurate mental image of space relations should be built up in his mind so that space intuition can proceed with sureness and a sense of power to arrive at interesting theorems with the minimum amount of symbolism or other outside aids. The results arrived at would not indeed be *general,* but they would arouse an interest in geometry and the tangible properties of space as we know it by hand or eye that would possibly lead to greater confidence in the subsequent analytic work and stimulate the inventive powers in a way that the purely algebraic processes do not do.

USE OF PRINTED FORMS IN GEOMETRY WORK

Reprinted from Vol. 1, No. 1.

In reading the geometry papers sent every year to the University, one of the most common faults noted is the lack of a definite form for the arrangement of the work. In many cases the theorem to be proved or the problem to be solved is nowhere clearly stated. The information given is not set off by itself, the conclusion to be established is not clearly stated in advance. Preliminary statements, steps in the proof, description of auxilliary constructions, are all intermingled in a most confusing and illogical fashion.

Some schools use a printed form for note books and examinations, and almost without exception the papers from these schools show to great advantage over those of the schools where no such device is used. This fact seems to justify the recommendation of the use of some one of the many such forms that are to be had.

Of course, the use of such a notebook and examination paper will not enable a student to prove a theorem or solve a problem that he does not understand, but it will enable him to make the best use of the information that he has and will keep him from mixing things that ought to be kept separate.

Paper similar to the cuts below can be had from various pub lishers. The sort copied here is sold by Ginn & Company. The sheets are slightly different for "theorems" and for "problems." It is not to be understood that the style used here is thereby recommended in preference to or to the exclusion of other styles that are on the market.

In his oral work the student should generally give in full any theorem cited in his proof. In written work this is laborious and introduces the appearance of diffuseness. It is, therefore, desirable to find some adequate and distinctive abbreviations for theorems that are to be cited often. A few suggestions are given, others will occur to teachers and students.

Theorem

Two triangles are congruent if two sides and the included angle of the one are equal respectively to two sides and the included angle of the other.

(In referring to this theorem as proof for a given statement. reference may be made as follows: $S<S=S<S$.)

Theorem

Two triangles are congruent if two angles and the included side of the one are equal respectively to two angles and the in cluded side of the other.

(Reference to this theorem: $<S<=<.<$)

Theorem

If two angles of a triangle are equal, the sides opposite the equal angles are equal, and the triangle is isosceles.

(Reference to this theorem: Having $2<s=$.)

Theorem

Two triangles are congruent if the three sides of the one are equal respectively to the three sides of the other.

(Reference to this theorem: 3 sides=3 sides.)

Theorem

If two parallel lines are cut by a transversal, the alternate-interior angles are equal.

(Reference to this theorem: Alt.-int.$<$ of$\|$lines.)

Theorem

When two lines in the same plane are cut by a transversal, if the alternate-interior angles are equal, the two lines are parallel.

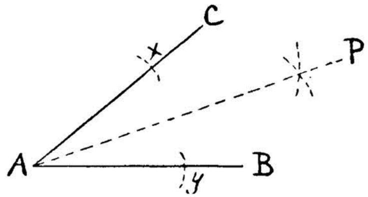
(Reference to this theorem: Alt.int.$<$ are=.)

The following cuts are published by permission of the author, Professor H.. A. Morrison.

STATEMENT OF PROBLEM

To bisect a given angle.

Elements Given	Construction
1. ∠BAC	
2.	
3.	
4.	
5.	



Required to Construct

1. To bisect ∠BAC
2.
3.

Method of Construction

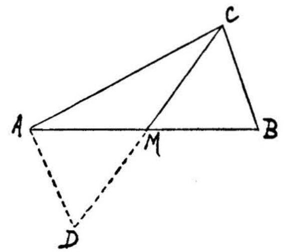
1. With A as center and any
2. radius as Ax describe arc xy
3. With x & y as centers and any
4. radius greater than $\frac{1}{2}$ xy
5. describe arcs intersecting in P
6. Draw AP
7. Then AP bisects ∠BAC
8.

Proof

Argument	Reason
1. Ax = Ay	1. Radii of same ⊙
2. xP = yP	2. " " " "
3. AP = AP	3. Identical
4. ∴ △APC = △APB	4. 3 sides = 3 sides
5. ∴ ∠PAC = ∠PAB	5. Corres. ∠s of = △
6. ∴ AP bisects ∠BAC	6. Q.E.F.
7.	7.
8.	8.
9.	9.
10.	10.
11.	11.

THEOREM

A median of a triangle is less than half the sum of the two adjacent sides.

Hypothesis or Given Conditions	Construction
1. $\triangle ABC$	
2. CM, the median	
3.	
4.	
5.	
To Prove	
1. $CM < \frac{1}{2}(AC+BC)$	
2.	
Auxiliary Constructions	
1. Produce CM	
2. Make MD = CM	
3. Draw AD	

Proof

Argument	Reason
1. $AM = MB$	1. Hypothesis (2)
2. $MD = CM$	2. Construction (2)
3. $\angle AMD = \angle CMB$	3. Vertical angles
4. $\therefore \triangle AMD = \triangle CMB$	4. $S \angle S = S \angle S$
5. $\therefore AD = CB$	5. Homologous sides = $\triangle$
6. $CD = CM + MD = 2\,CM$	6. Substitute = for =
7. $CD < AD + AC$	7. St. line shortest distance bet. 2 pts.
8. $\therefore CD = 2\,CM < AD + AC$	8. From 6 and 7
9. $\therefore 2\,CM < BC + AC$	9. Substitute BC for its = AD
10. $\therefore CM < \frac{1}{2}(AC+BC)$	10. Halves of unequals
11.	11.
12.	12.
13.	13.

WHAT GREAT MEN SAY ABOUT MATHEMATICS

In mathematics two ends are constantly kept in view: First, stimulation of the inventive faculty, exercise of judgment, development of logical reasoning, and the habit of concise statement; second, the association of the branches of pure mathematics with each other and with applied science, that the pupil may see clearly the true relations of principles and things.
International Commission on the Teaching of Mathematics. American Report.

The idea that aptitude for mathematics is rarer than aptitude for other subjects is merely an illusion which is caused by belated or neglected beginners.

HERBART, J. F.

In the secondary schools mathematics should be a part of general culture and not contributory to technical training of any kind; it should cultivate space intuition, logical thinking, the power to rephrase in clear language thoughts recognized as correct, and ethical and esthetic effects; so treated, mathematics is a quite indispensable factor of general education in so far as the latter shows its traces in the comprehension of the development of civilization and the ability to participate in the further tasks of civilization.

Unterrictsblätter für Mathematik und Naturwissenschaft.

THE STRAIGHT EDGE

Do your students think mathematics dull? Maybe it is not the mathematics.

* * * * * * * *

Do you think mathematics in need of being "vitalized"? Maybe so, but a lot of us teachers need it worse.

* * * * * * * *

Do the students in your school do better in everything else than mathematics? That is not the fault of either the students or mathematics.

* * * * * * * *

The teacher of English knows where his subject leads, ditto the science and domestic economy teachers. How far beyond the high school course in mathematics have you explored the subject?

* * * * * * * *

Have you polished up your ideas of mathematics and the teaching of it this summer or have you added a coat of dust and rust?

* * * * * * * *

How many pages did you ever read in all your life on the teaching of mathematics?

* * * * * * * *

How much time (in hours) have you devoted in all your life to a systematic study of the problems of the mathematics teacher?

* * * * * * * *

Would you let a physician who had relatively no more training in surgery than you have in your work operate on you for an ingrowing toe nail?

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IMPORTANT NOTICE !

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