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Pricing and Resale Market Strategy for Durable Goods: A Dynamic Equilibrium Model of Video Games

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Pricing and Resale Market Strategy for Durable Goods: A Dynamic Equilibrium Model of Video Games

by

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DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT AUSTIN

May 2014

Dedicated to my wife Jae-Eun.

Acknowledgments

I am deeply grateful for the guidance that I received from my advisor, Jason Duan. You always had my best interests at heart and gave me the right amount of push and support. You believed in me and gave me the confidence to persist. I would like to thank Leigh McAlister for her insights and encouragement. I admire your enthusiasm. I am grateful to Raghu Rao and Ty Henderson to whom I could turn to for friendly advices and sparkling discussions. I would like to thank Euhenio Miravete, Romana Khan, and Stephen Ryan for advising me through different stages of my program. To my friends, Seungrae Lee, Sunaina Chugani, Szu-Chi Huang, Jungeun Kim, Bomi Lee, and Donghyun Kim: thank you for the friendship that helped me through the program. You made this journey more fun and meaningful. Finally, I would like to thank my family for their love and support. Jae-Eun, you gave me strength and joy and made this period the happiest time of my life.

Pricing and Resale Market Strategy for Durable Goods: A Dynamic Equilibrium Model of Video Games

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The University of Texas at Austin, 2014

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I study the impact of the used goods market on pricing and profits in the video game industry and the implications of resale restrictions. I develop a modeling framework that incorporates (a) heterogeneous consumers who are forward looking in their buying an selling behaviors, (b) a strategic game producer who prices its products considering both inter-temporal price discrimination and price competition with used goods, (c) rational expectations on future prices by both consumers and the firm, and (d) market equilibria for both new and used-goods markets. Without observing sales data, I use equilibrium pricing solutions in my model and the varying rate of price decrease after a game's release to identify the sales volume of a game in every period as a percentage of its total demand. I develop a computationally tractable utility specification to solve the computational challenge comes with modeling the supply side equilibrium. I construct the demand function for a game from heterogeneous consumers whose valuations distribute on an interval, and partially characterize the consumers' decisions and reduce the dimensionality of the state space. Applying the model to a unique dataset of game prices collected from the Internet, I estimate the game-specific demand for multiple games released in the U.S. market. The results show significant variation across games in terms of shapes of valuation distributions, expected play time, degrees of consumers' preference for new over used games, and price sensitivities. Policy simulations show that the effects of prohibiting resale largely depend on the shape of a game's demand distribution, because most of the profits are gained from higher-valuation consumers who purchase the game when the price is high. Prohibiting resale does not dampen their willingness to pay for the game because their high utility from playing it. Moreover, higher expected future prices in the absence of the used-game market further reduces their incentives to wait. I find the predicted profit increase is significant for most games when reselling is prohibited. However, games with demand consisting mostly of low valuation consumers benefit less from this structural change, because (a) early sales increase only slightly given a much smaller proportion of high valuation consumers and (b) losing the option to resell significantly decreases the willingness to pay for low valuation consumers, forcing the firm to slash its prices dramatically over time. I find empirical evidence that a firm can be better off with the used game market. This suggests that though eliminating the resale market is generally optimal for popular games, retaining it can be more profitable for some games.

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Chapter 1

Introduction and Literature Review

1.1 Introduction

In June 2013, Microsoft announced that their new video game platform, Xbox One, would require constant online connection to play and as a result, used game reselling would be restricted.[1] What would be the effect of such a structural change, and would it be beneficial to producers unconditionally? Although the company later announced that they had decided to reverse the policy and there would be no restrictions on reselling,[2] this event shows that it is technically feasible to forbid reselling in the video game industry. This dissertation studies how the used goods market affects equilibrium outcomes and the implications of prohibiting reselling in this industry.

Reselling of software poses a serious threat to its producer since the digital software itself does not physically depreciate; there is little, if any, quality discrepancy between new and used software as long as the media disk can be read. Hence, it is hard to charge much more for new software than used one. What makes matters worse for video game producers is that video games can be consumed in relatively

[1] `http://www.ign.com/articles/2013/06/06/microsoft-details-xbox-one-used-games-always-online`
[2] `http://www.ign.com/articles/2013/06/19/microsoft-reversing-xbox-one-internet-used-game-policies`

short period of time. This differs from office related or utility software which consumers tend to use for extended period of time. Hence, video game producers are operating in a industry where previously sold copies of a game come back pretty quickly to the used goods market as competition, and the used games are almost as good as new. Naturally, some game producers desire to prohibit the reselling of the used games.[3]

I investigate these issues by developing a new model of durable goods and apply it to unique price data collected from the web. In this industry, I observe the co-existence of new and used goods markets with correlated prices. I also see initially high but dynamically decreasing game prices, suggesting that consumers' valuations for a game is heterogeneous, and firms are using the skimming pricing strategy to maximize their profits. Realizing this downward trend in prices, a game buyer is likely to be forward looking, because she can wait to buy a new or used game at significantly lower price in the future. In addition, she realizes the future opportunity to recoup part of the price she pays by reselling it after she loses interest in playing the game anymore. The resale supply will increase with time as more consumers who have purchased the game earlier lose interest in the game and resell their copies, making the producer compete heavily against its own products and further slash its prices. Incorporating these observations, I propose a model which includes the following components: (a) heterogeneous consumers who are forward looking in their buying an selling behaviors, (b) a strategic game producer who prices its products

[3]For example, *THQ* creative director said "We hope people understand that when the game's bought used, we get cheated." (`http://www.computerandvideogames.com/261330/pre-owned-cheats-developers-thq/`)

considering both inter-temporal price discrimination and price competition with used goods, (c) rational expectations on future prices by both consumers and the firm, and (d) market equilibria for both new and used-goods markets. To my knowledge, my dissertation is the first to incorporate all these components in a structural empirical model, which enables me to study a rich collection of counterfactual analyses by simulating model-based prices and sales.

Modeling the supply side equilibrium brings a significant computational burden due to the well-known *Curse of Dimensionality* (Bellman, 1957). In this context, consumers typically adopt a durable good once and exit the market, and thus as the firm sells its product, the remaining demand dynamically shrinks. Then, as the researcher tries to accommodate richer heterogeneity, the number of state variables the firm needs to keep track of increases quickly and it becomes computationally infeasible to solve the resulting dynamic problem. For example, suppose there are discrete number of consumer segments, each with a different level of valuation about the product. Then in each period, the firm needs to keep track of how many consumers are remaining in each segment. In addition, when there is the used goods market, the firm also needs to take into account how many consumers have purchased the product in previous periods for each segment as well. This is the reason why previous studies which also estimated dynamic demand in the presence of the used goods market in this industry (Ishihara and Ching, 2012; Shiller, 2012) used a couple of segments to model consumer heterogeneity, and also did not model the supply side equilibrium with the used goods market. Hence, unlike my study, they cannot generate equilibrium prices under the presence of the used game market.

In addition, what makes the modeling task especially challenging is that due to well-developed online marketplaces such as Amazon.com and eBay where consumers can individually buy and sell used goods, sales volume of used games is difficult to measure. In addition, an econometrician usually can only observe price data, whereas the sales information of new products is proprietary. Indeed, even producers are often not able to obtain the the sales information on the used products of their own. In solving these issues, my dissertation makes contributions on the following fronts.

Under market equilibrium, the observed prices provide information about the underlying demand. Intuitively, slow price decline over time implies significant demand, whereas fast price decline suggest low demand. With a fully specified supply-side model, which provides a mapping from demand to prices, we can make inference on demand without sales data. Our solution is to make use of new and used goods prices by explicitly solving the new and used-goods market equilibria and then utilize the equilibrium conditions for estimation. There are several previous studies (Feenstra and Levinsohn, 1995; Sudhir, 2001; Thomadsen, 2005) which used similar ideas in static settings. We contribute to this stream of research by extending the method to dynamic equilibrium models.

Inclusion of the market equilibria for both new and used-goods markets in the estimation means I have to solve for both consumers' and the firm's problem for each candidate parameter draw, which can make the estimation indefeasibly slow. I develop a new, computationally tractable utility specification to solve the computational challenge. First, I construct the demand function from heterogeneous consumers with their valuations distributed on a normalized, $[0, 1]$ interval. Then, I can measure demand for each period with the area under the consumer valuation distribution density curve, between the valuation of the *marginal* consumer and the maximum valuation of the entire unfulfilled demand. Here the marginal consumer is indifferent between buying a used game at current price and waiting. In this framework, I can characterize the state of the demand parsimoniously with the marginal consumer's valuation, because in turn it becomes the maximum valuation of the unfulfilled demand for the next period. In addition, using equilibrium conditions, I partially characterize consumers' dynamic video game buying, keeping and selling decisions analytically, and summarize the state of the used goods supply with another state variable. This enables me to characterize the state of the market with only two state variables, which facilitates solving the firm's dynamic problem. This approach not only enables me to incorporate the supply side equilibrium in the estimation, but also allow me to accommodate rich, continuous consumer heterogeneity distribution and non-linear price trajectories for consumer price expectations.

In this framework, the state space cannot be represented by a regular grid, making many popular approximation methods such as Chevyshev (Judd, 1988) and cubic spline interpolation (Habermann and Kindermann, 2007) inapplicable. I solve this challenge by applying the radial basis function approximation methods (Buhmann, 2000) which can approximate the surface of a function with scattered data. To my knowledge, my dissertation is the first one to apply this method to dynamic programming in the marketing literature.

The estimation of our model is through the Generalized Method of Moments

(GMM) (Hansen and Singleton, 1982). From the equilibrium conditions of the dynamic model, I solve for the unobserved aggregate demand shocks, from which we construct moment conditions together with observed prices and instrumental variables. This procedure is analogous to Berry, Levinsohn, and Pakes (1995). I extend the method to a dynamic equilibrium model with continuous control variables (prices) instead of a static discrete choice problem, using equilibrium conditions from both demand and supply sides. Numerical optimization of the GMM objective function and calculation of the standard errors are challenging due to many local minima resulting from the highly non-linear nature of the objective function. Hence I employ the Laplace-type estimator (Chernozhukov and Hong, 2003) for inference, which is robust to local minima by converting the objective function to the quasi-posterior distribution and employing MCMC methods to estimate parameters and construct intervals.

I apply this model to twenty-four video games released in the U.S. market and estimate the game-specific demand. The results show significant variation across games in the shape of the valuation distribution, expected play time, degree of consumers' preference for a new over a used copy, and demand price sensitivity. In a counterfactual analysis, I evaluate the profit change for each video game as a result of eliminating the resale market, assuming that the firms adopt optimal pricing strategies under the absence of the used game market. The results show that the effects of prohibiting resale largely depend on the shape of a game's demand distribution, because most of the profits are gained from higher-valuation consumers who purchase the game when the price is high. Since they get high utility from playing a game, prohibiting resale does not dampen their willingness to pay for the game significantly. However, the absence of the used-game market makes expected future prices higher, further reduces their incentives to wait. I find the predicted profit increase is significant for most games, which enjoy a 38% profit increase on average when reselling is prohibited. However, games with demand consisting mostly of low valuation consumers benefit less from this structural change, because (a) early sales increase only slightly given a much smaller proportion of high valuation consumers and (b) losing the option to resell significantly decreases the willingness to pay for low valuation consumers, forcing the firm to reduce its prices dramatically over time. I find empirical evidence that a firm can be better off with the used game market. This suggests that though eliminating the resale market is generally optimal for popular games, retaining it can be more profitable for some games.

This dissertation is structured in the following way. In the next section, I review the current literature on durable goods market, intertemporal price discrimination and the video game industry and elaborate on how my model differs from the prior work. In Chapter 2, I discuss the construction of a theoretical two-period model, which illustrates the fundamental ideas of dynamic demand, expectations, competition between new and used games, intertemporal price discrimination in a simple setting. I conduct comparative statics to show the effects of the used game market on equilibrium outcomes. In Chapter 3, I construct a multi-period, empirical model that I take to the data, and characterize the consumers' and the firm's dynamic decisions. Numerical details of solving dynamic problem is discussed. In Chapter 4, after I describe the data and the estimation strategy, I show the results from the estimation and the counterfactual analysis. I conclude my dissertation in Chapter 5.

1.2 Literature Review

My research builds on the empirical literature in marketing and economics which develops models on consumers' dynamic durable goods purchase decisions with aggregate level data such as Song and Chintagunta (2003), Gordon (2009), Goettler and Gordon (2011), and Gowrisankaran and Rysman (2012).

In particular, my dissertation is closely related to recent studies which use dynamic structural models to study video game adoption decisions. Nair (2007) uses a dynamic discrete choice model of adoption to estimate the demand of a game and uses equilibrium model in policy simulations to study optimal intertemporal price discrimination in the U.S. video game industry. Ishihara and Ching (2012) develops dynamic choice model with the used goods market, utilizing recently developed Bayesian estimation of dynamic structural model (Imai, Jain, and Ching, 2009) and studies the effect of the used goods market on new goods sales. They collected and used data from Japanese video game industry where they observe information about the intermediaries who buy and sell the used games. Shiller (2012) added used goods market to Nair (2007)'s approach and estimated dynamic demand with used goods and evaluated profit changes after elimination of the used goods market. He used data on used game auctions from one of the popular online marketplace and scaled up this partial information to the entire U.S. market.

I contribute to this stream of research by developing a new dynamic general equilibrium model of video game adoption and resale. Importantly, I fully specify the supply side equilibria incorporating both new and used goods markets, unlike previous studies which did not model the supply side. Hence, I can generate equilibrium prices under the presence of the used goods market, whereas previous studies cannot. In my model the equilibrium price trajectory can be directly solved from the model under different market structures, facilitating counterfactual analyses.

Also, by assuming that the observed new prices are optimally set by the firm and the used goods market is competitive, I infer sales volume in each period from the observed prices using equilibrium conditions. This enables me to make inference on demand without observing the sales data. The idea of using equilibrium conditions from the model to make inference with limited data has been employed in the literature. For example, Feenstra and Levinsohn (1995) and Thomadsen (2005) estimated their models without observing sales in static settings. Also, Sudhir (2001) used game-theoretic solutions to infer wholesale prices in the absence of information on those prices. I contribute to this stream of research by extending the method to dynamic equilibrium models.

In solving for the supply side equilibrium, I find many approximation methods which are popular in marketing and economics such as Chebyshev (Judd, 1988) and cubic spline (Habermann and Kindermann, 2007) interpolation inadequate for my application. This is due to the fact that the feasible state space of my model cannot be represented by a tensor product of state variables. Hence, I use the radial basis function approximation methods (Buhmann, 2000) which can approximate the surface of a function with scattered data. To my knowledge, my dissertation is the first one to apply this method to dynamic programming in the marketing literature.

My dissertation is also related to recent studies which estimated the video game console demand along with the software demand such as Derdenger (2014); Derdenger and Kumar (2013); Lee (2013). The scope of my dissertation is quite different from that of these studies; while my study does not include the console demand or the competition among video games, it focuses on the intertemporal price discrimination and the used goods market.

Theoretically, since the Coase conjecture (Coase, 1972), which makes an argument that a monopolist who sells a durable good to patient and heterogeneous consumers in an infinite time horizon can forfeit all his monopoly power,[4] there has been rich stream of literature on durable good monopolist's intertemporal price discrimination problem (Bulow, 1982; Huang, Yang, and Anderson, 2001; Karp, 1996; Stokey, 1979, 1981). In my model, while the monopolist does keep certain level of monopoly power because there is significant the time interval between successive price revisions, it does lose some monopoly power due to forward-looking behavior of consumers. Besanko and Winston (1990) is directly related to my study, as the authors study a stylized model of the intertemporal pricing problem for a monopolist selling a new product with forward-looking consumers, where consumers decide when to adopt the product. I extend this model to allow consumers to buy and sell used goods. Instead of discrete choice framework, my empirical model closely follows the theoretical model. This allows me to use continuous heterogeneity distribution on

[4]As the length of time interval between price revisions goes to 0 (consumers' discount rate over the time interval goes to 1), the monopolist lowers its price and eventually sets the competitive price, saturates the market in the initial period.

consumer valuation instead of the segment approach typically used in the literature. Also, the intertemporal pricing problem is related to the infinite-horizon bargaining models with one-sided incomplete information (Roth, 1985).

Finally, there is a large theoretical literature on the interaction between new and used durable goods markets. These include optimal durability (Rust, 1986; Swan, 1970), leasing contracts (e.g., Desai and Purohit, 1999), channel coordination (Desai, Koenigsberg, and Purohit, 2004; Shulman and Coughlan, 2007), role of trade-ins (Rao, Narasimhan, and John, 2009), and peer-to-peer used goods (Yin, Ray, Gurnani, and Animesh, 2010) market. Also, there have been literature about optimal durability (Hendel and Lizzeri, 1999b; Swan, 1970), and adverse selection (Akerlof, 1970; Hendel and Lizzeri, 1999a).

Chapter 2

Two-Period Model

In this chapter, I introduce a two-period model, which illustrates the fundamental ideas of dynamic demand, expectations, competition between new and used games, an intertemporal price discrimination. This model can be solved analytically, which enables me to conduct comparative statics and show the effects of the used game market on equilibrium outcomes.

2.1 Two-Period Model

I consider a dynamic game played between a monopolistic firm selling a durable good and forward-looking consumers with heterogeneous valuations in the presence of the used goods market. The firm sells the durable good to a unit mass of consumers, whose flow valuations (denoted by v) follow a distribution with *c.d.f* $F(v)$, whose support is the unit interval $[0,1]$. The consumers share a discount factor δ, and I assume that they buy only one copy of the game. The initial $t=1$ represents the period when the game is released, so there is no used goods supply and the used goods market starts at period 2. The utility a consumer gets from the period when she purchases the game depends on whether she purchases new or used copy. A consumer with valuation v gets utility αv, $\alpha > 1$, from buying a new copy, whereas she gets v if she purchases a used copy. The parameter α represents consumers' taste for *newness*; for example, the additional utility they get from opening the package of a new copy of the game. In the following periods after purchase, however, both new and used copies give her the same flow utility v from playing the game. This reflects the idea that the utility consumers get from playing a video game does not depreciate physically. Since a video game essentially consists of digitally stored data, once a consumer starts playing a game, she must get the identical experience from the used and new copy as long as the game runs.

In each period after purchase period, I assume that a consumer who owns the game probabilistically *loses interest in the game*, and after that the utility from owning the game becomes zero. This can be interpreted as her completing all the scenarios in the game or just finding it not interesting anymore. Hence I am taking a discrete approach on the depreciation of consumers' utility from game holding. Note that my definition of the event of losing interest in the game is general in a sense that it includes a broad range of consumers' unobservable post-purchase behavior. The existence of the used goods market allows consumers who previously purchased the game, either new or used, to sell their used games in any period. They act as price takers, and optimally decide whether or not to sell the game at market-prevailing price.

Consumers are forward looking in both their purchase and selling decisions; consumers take into account the possibility of lower future prices for both new and used games. Moreover, they consider the potential opportunity to sell the product after losing interest in the game and expect future used game prices for the selling option. I assume that both the firm and consumers form rational expectations; their expectations about future prices are consistent with the model.

In sum, in each period, upon observing the states including prices of a new copy p_t and a used copy p_{ut}, consumers who have yet purchased the game decide whether to purchase new copy, purchase used copy, or wait based on current prices and their expectations of future prices. Consumers who already own the game decide whether or not to sell their game in the used market at market price p_{ut}. If they decide to keep the game, they get the flow utility if they have not lost interest in the game, and get zero if they have. The firm takes the consumers' behaviors as well as the interaction between its decision and the response from the used goods market into account in formulating its intertemporal pricing policy. In this model, both new and used good prices are equilibrium outcomes of a game played between the firm and its consumers.

In the two-period model, I let v be the total utility from a game instead of flow utility since I only have two periods for simplicity. If a consumer purchases the game in period 1, she loses interest in it within that period with probability λ. Then she sells her video game in the used goods market in period 2.

2.2 Solution of the equilibrium

I solve for the equilibrium by backward induction, starting in period 2. Let $\overline{v}_1$ be the valuation of the marginal consumer who is indifferent between purchasing a new game of the game in period 1 and purchasing either a new or used game in period 2. Since there are no used game transactions in period 1 and to be consistent with the notations in the empirical model, I let $\underline{v}_2 = \overline{v}_1$ and use $\underline{v}_2$ as the state in period 2. Let p^*_{u2} be the used goods market clearing price, and let $\bar{v}_2$ denote the valuation of the consumer who is indifferent between buying the new and used game in period 2. Then given period 2 new good price, p_2,

$$\alpha\bar{v}_2 - p_2 = \bar{v}_2 - p^*_{u2} \;\Rightarrow\; \bar{v}_2 = \frac{p_2 - p^*_{u2}}{\alpha - 1} \tag{2.2.1}$$

Let $\underline{v}_3$ represent the valuation of the consumer who is indifferent between buying a used game and not buying anything in period 2 and then,

$$\underline{v}_3 = p^*_{u2} \tag{2.2.2}$$

Since consumers who lost interest in the game get zero utility from owning it, they will sell their games at any $p_{u2} > 0$. On aggregate, the supply of the used game in period 2 is λs_1, where $s_1 = 1 - \underline{v}_2$, the period 1 sales. In turn, p^*_{u2} is determined at where the used goods sales $(\bar{v}_2 - \underline{v}_3)$ equals to the used goods supply:

$$\bar{v}_2 - \underline{v}_3 = \frac{p_2 - p_{u2}}{\alpha - 1} - p_{u2} = \lambda s_1 = \lambda \cdot (1 - \underline{v}_2) \tag{2.2.3}$$

$$\therefore \quad p^*_{u2} = \frac{\alpha\lambda\underline{v}_2 - \alpha\lambda - \lambda\underline{v}_2 + \lambda + p_2}{\alpha} \tag{2.2.4}$$

The firm sets p_2 to maximize the second period profit, taking into account the effect of its decision on p^*_{u2}. Note that the market size in period 2 is $\underline{v}_2$. Then, the firm's period 2 sales are,

$$\underline{v}_2 - \bar{v}_2 = \underline{v}_2 - \frac{p_2 - p^*_{u2}}{\alpha - 1} = \frac{1}{\alpha}\left((\alpha + \lambda)\,\underline{v}_2 - \lambda - p_2\right) \tag{2.2.5}$$

and the profit in period 2 is $\Pi_2 = \frac{1}{\alpha}\left((\alpha+\lambda)\,\underline{v}_2 - p_2 - \lambda\right)\cdot p_2$. From the following first-order conditions,

$$\frac{\partial \Pi_2}{\partial p_2} : (\alpha+\lambda)\,\underline{v}_2 - 2p_2 - \lambda = 0, \tag{2.2.6}$$

I derive the equilibrium prices and profit in period 2,

$$\begin{aligned} \therefore \quad p_2^* &= \frac{1}{2}\left((\alpha+\lambda)\underline{v}_2 - \lambda\right), \\ p_{u2}^* &= \frac{1}{2\alpha}\left\{\alpha\underline{v}_2 - \lambda(2\alpha-1)(1-\underline{v}_2)\right\}, \\ \Pi_2^* &= \frac{1}{4\alpha}\left((\alpha+\lambda)\underline{v}_2 - \lambda\right)^2. \end{aligned} \tag{2.2.7}$$

Now consider period 1. The indifference condition for the valuation of the marginal consumer $\underline{v}_2$ is,

$$\alpha\underline{v}_2 - p_1 + \delta\left((1-\lambda)\underline{v}_2 + \lambda E p_{u2}\right) = \delta(\max\{\alpha\underline{v}_2 - Ep_2, \underline{v}_2 - Ep_{u2}\}) \geq 0 \tag{2.2.8}$$

Assuming rational expectations, I have $Ep_2 = p_2$ and $Ep_{u2} = p_{u2}$. From the results of period 2, the value of v which satisfies $\alpha v - p_2 = v - p_{u2}$ is $\bar{v}_2$, which is clearly less than $\underline{v}_2$, because $\underline{v}_2$ is the upper bound of the valuations of consumers who have not purchased the game in period 1. Hence, I have $\max\{\alpha\underline{v}_2 - p_2, \underline{v}_2 - p_{u2}\} = \alpha\underline{v}_2 - p_2$, and from Equation (2.2.8) I have,

$$\underline{v}_2 \quad = \quad \frac{(2\alpha-1)\,\delta\lambda^2 + \alpha\delta\lambda + 2\alpha p_1}{(2\alpha-1)\,\delta\lambda^2 - \alpha^2\delta + 2\alpha\delta\lambda + 2\alpha^2} \tag{2.2.9}$$

The total sum of the discounted profit is,

$$\Pi \quad = \Pi_1 + \delta\Pi_2 = (1 - \underline{v}_2(p_1)) \cdot p_1 + \delta \cdot \frac{1}{4\alpha}\left((\alpha+\lambda)\underline{v}_2 - \lambda\right)^2 \tag{2.2.10}$$

To solve this, let

$$\Delta_1 = \alpha^3\delta^2 - 4\alpha^3\delta + 4\alpha^3 + 3\alpha^2\delta^2\lambda - 4\alpha^2\delta^2 + 4\alpha^2\delta\lambda^2 - 4\alpha^2\delta\lambda + 6\alpha^2\delta p_1$$

$$\Delta_2 = 8\alpha^2\delta - 8\alpha^2 p_1 + 7\alpha\delta^2\lambda^2 - 4\alpha\delta^2\lambda + 4\alpha\delta^2 - 8\alpha\delta\lambda^2 p_1$$

$$\Delta_3 = -4\alpha\delta\lambda^2 + 4\alpha\delta\lambda p_1 - 8\alpha\delta p_1 + \delta^2\lambda^3 - 4\delta^2\lambda^2 + 6\delta\lambda^2 p_1$$

then I can denote the first order condition as

$$\frac{\partial\Pi}{\partial p_1} = \frac{\alpha\left(\Delta_1 + \Delta_2 + \Delta_3\right)}{\left(\alpha^2\delta - 2\alpha^2 - 2\alpha\delta\lambda^2 - 2\alpha\delta + \delta\lambda^2\right)^2} = 0$$

Also, the second-order condition is satisfied,

$$\frac{\partial^2\Pi}{\partial p_1^2} = \frac{2\alpha\left(3\alpha^2\delta - 4\alpha^2 + 2\alpha\delta\lambda - 4\alpha\delta + 3\delta\lambda^2 - 4\alpha\delta\lambda^2\right)}{\left(\alpha^2\delta - 2\alpha^2 - 2\alpha\delta\lambda^2 - 2\alpha\delta + \delta\lambda^2\right)^2} < 0,$$

because $3\alpha^2\delta - 4\alpha^2 < 0$, $2\alpha\delta\lambda - 4\alpha\delta < 0$ and $3\delta\lambda^2 - 4\alpha\delta\lambda^2 < 0$ given $\alpha > 1$, $0 < \delta < 1$, and $0 < \lambda < 1$.

By solving the FOC, I have the following equilibrium values,

$$\begin{aligned}
p_1^* &= -\frac{\alpha^3\delta^2 - 4\alpha^3\delta + 4\alpha^3 + 3\alpha^2\delta^2\lambda - 4\alpha^2\delta^2 + 4\alpha^2\delta\lambda^2 - 4\alpha^2\delta\lambda}{6\alpha^2\delta - 8\alpha^2 - 8\alpha\delta\lambda^2 + 4\alpha\delta\lambda - 8\alpha\delta + 6\delta\lambda^2} \\
&\quad -\frac{8\alpha^2\delta + 7\alpha\delta^2\lambda^2 - 4\alpha\delta^2\lambda + 4\alpha\delta^2 - 4\alpha\delta\lambda^2 + \delta^2\lambda^3 - 4\delta^2\lambda^2}{6\alpha^2\delta - 8\alpha^2 - 8\alpha\delta\lambda^2 + 4\alpha\delta\lambda - 8\alpha\delta + 6\delta\lambda^2} \\
\underline{v}_2^* &= \frac{\alpha^2\delta - 2\alpha^2 - 4\alpha\delta\lambda^2 - 2\alpha\delta + 3\delta\lambda^2}{3\alpha^2\delta - 4\alpha^2 - 4\alpha\delta\lambda^2 + 2\alpha\delta\lambda - 4\alpha\delta + 3\delta\lambda^2} \qquad (2.2.11) \\
\Pi^* &= -\frac{\alpha\left(\alpha^2\delta^2 - 4\alpha^2\delta + 4\alpha^2 + 4\alpha\delta^2\lambda^2 + 6\alpha\delta^2\lambda - 4\alpha\delta^2 - 8\alpha\delta\lambda + 8\alpha\delta + \delta^2\lambda^2 - 8\delta^2\lambda + 4\delta^2\right)}{12\alpha^2\delta - 16\alpha^2 - 16\alpha\delta\lambda^2 + 8\alpha\delta\lambda - 16\alpha\delta + 12\delta\lambda^2}
\end{aligned}$$

To illustrate the effect of the used goods market on equilibrium values, I also solve for the equilibrium without the used goods market. In this case, the period 2 decision becomes ($\underline{v}_1$ and $\underline{v}_2$ no longer exist and consumers' decisions are summarized by $\overline{v}_1$ and $\overline{v}_2$ instead),

$$\alpha\bar{v}_2 - p_2 = 0. \tag{2.2.12}$$

The decision to buy in period 1 is characterized by

$$\alpha\bar{v}_1 - p_1 + \delta(1-\lambda)\underline{v}_1 = \delta(\alpha\underline{v}_1 - Ep_2) \geq 0$$

Without the used goods market, the decision to buy in period 2 becomes

$$\alpha\bar{v}_2 - p_2 = 0 \Rightarrow \overline{v}_1 = \frac{p_2}{\alpha} \tag{2.2.13}$$

and period 2 profit is:

$$\Pi_2 = p_2\left(\bar{v}_1 - \frac{p_2}{\alpha}\right) \tag{2.2.14}$$

first order conditions yields

$$\alpha\left(\bar{v}_1 - 2\frac{p_2}{\alpha}\right) = 0 \tag{2.2.15}$$

Hence, equilibrium period 2 values are

$$p_2^* = \frac{1}{2}\alpha\bar{v}_1, \quad \Pi_1^* = \frac{1}{4}\alpha\bar{v}_1^2 \tag{2.2.16}$$

period 1 decision is characterized by

$$\alpha\bar{v}_1 - p_1 + \delta(1-\lambda)\underline{v}_1 = \delta(\alpha\underline{v}_1 - Ep_2) \geq 0 \tag{2.2.17}$$

by plugging in period 2 equilibrium values, I have

$$\bar{v}_1 = -\frac{2p_1}{\alpha\delta - 2\alpha + 2\delta\lambda - 2\delta} \tag{2.2.18}$$

The total sum of discounted profit is

$$\Pi = \Pi_1 + \Pi_2\delta = \frac{1}{4}\alpha\bar{v}_1^2\delta + p_1\left(-\bar{v}_1 + 1\right) \tag{2.2.19}$$

First order conditions:

$$\frac{\partial \Pi}{\partial p_1} = \frac{2\alpha\delta p_1 + 4p_1\left(\alpha\delta - 2\alpha + 2\delta\lambda - 2\delta\right) + \left(\alpha\delta - 2\alpha + 2\delta\lambda - 2\delta\right)^2}{\left(\alpha\delta - 2\alpha + 2\delta\lambda - 2\delta\right)^2} = 0 \tag{2.2.20}$$

the second-order condition is satisfied,

$$\frac{\partial^2 \Pi}{\partial p_1^2} = \frac{6\alpha\delta - 8\alpha + 8\delta\lambda - 8\delta}{\left(\alpha\delta - 2\alpha + 2\delta\lambda - 2\delta\right)^2} < 0,$$

again because $6\alpha\delta - 8\alpha < 0$ and $8\delta\lambda - 8\delta < 0$ given $\alpha > 1$, $0 < \delta < 1$, and $0 < \lambda < 1$.

Hence, I have the following equilibrium values:

$$\begin{aligned} p_1^* &= -\frac{\left(\alpha\delta - 2\alpha + 2\delta\lambda - 2\delta\right)^2}{6\alpha\delta - 8\alpha + 8\delta\lambda - 8\delta} \\ \bar{v}_1^* &= \frac{\alpha\delta - 2\alpha + 2\delta\lambda - 2\delta}{3\alpha\delta - 4\alpha + 4\delta\lambda - 4\delta} \\ \Pi^* &= -\frac{\left(\alpha\delta - 2\alpha + 2\delta\lambda - 2\delta\right)^2}{12\alpha\delta - 16\alpha + 16\delta\lambda - 16\delta} \end{aligned} \tag{2.2.21}$$

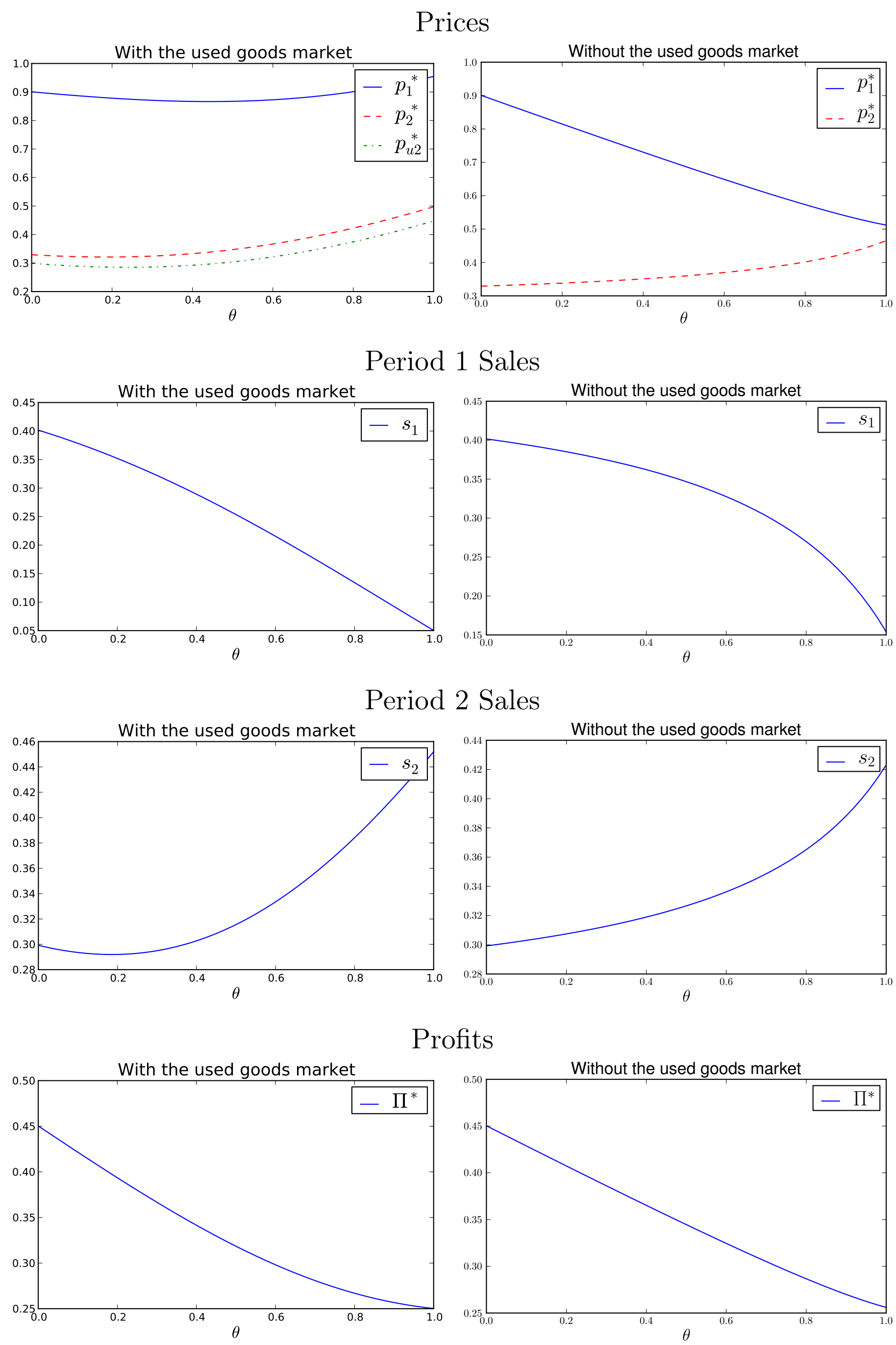
2.3 Comparative Statics

I conduct comparative statics to show the effect of the used goods market on equilibrium outcomes, and visualize them with a series of plots. I am mainly interested in how the equilibrium outcomes depend on the level competition from the used goods market, which is represented by λ. However, increase in λ decreases the durability of the game, and it has the following two effects. First, it will lower consumers' expected utility from purchasing the game earlier and hence decrease their willingness to pay in period 1. Second, given the same period 1 sales, it will increase the supply of used goods in period 2 because more consumers will lose interest in the game after one period. To highlight only the effect of increase in λ on equilibrium values through increasing used goods supply, I compare between the results from the model with and without the used goods market. I depict the results of the two models with parameter values $\alpha = 1.1, \delta = 0.9$ side-by-side in Figure 2.1.

First I discuss the changes caused by the decrease in the expected utilities with larger λ. As I described above, the increase in λ makes purchasing the game in period 1 less attractive to consumers, decreasing their willingness to pay. This explains the changes shown in the model without the used goods market. With consumers' lower willingness to pay for the game, the firm charges lower price for the game in period 1, but the firm does not reduce the price too much since it can recoup some of the lost sales in period 2. Thus the period 1 sales decline. However, since the expected utility of period 2 remains the same, period 2 sales increase due to the higher remaining demand. Overall, the total profit suffers.

With the introduction of the used goods market, I see the firm reacts differently to changes in λ. As λ increases, period 1 sales becomes less attractive for the firm, because the sold goods will come back in period 2 as the used goods supply. Thus, with the existence of the used goods market, as opposed to the absence of it, the firm responds by maintaining relatively higher price in period 1 and sells even

Figure 2.1: Comparative Statics



less. To be more precise, in the lower range of λ, the firm still decreases the price a little bit because the effects from the reduced expected utility dominate the competitive force from the used goods market. Still, the size of price decrease in this case is smaller than that in the case of the no used goods counterpart. As a part of the sales in period 1 becomes used goods supply in the next period, period 2 sales actually decrease somewhat. With higher λ, however, the competitive force from the used goods market dominates, and the firm actually *increases* period 1 price, further reducing period 1 sales.

Chapter 3

Multi-period Model for Empirical Analysis

In this chapter I introduce a multi-period model with an aggregate demand shock and additional parameters to fit the video game data and quantify the effects of used goods market on equilibrium outcomes and run policy simulations. The structure of the multi-period model is natural extension of the two-period model: I assume (i) consumers are forward-looking in their game purchasing and selling decisions, (ii) game producers apply Markov pricing strategies, and (iii) the new and used goods market outcomes follow a Markov Perfect equilibrium.

3.1 Model for Consumers

3.1.1 Demand Model

Per-period utility Let v denote a consumer's *single-period* consumption value of owning the game at period t, where I let $F(v)$ denote the *c.d.f* of this valuation distribution on the support of $[0,1]$. If the consumer has not purchased the game yet, her single-period utility for purchasing decisions in period t for is given by u_{jt}:

$$u_{jt} = \begin{cases} \xi_t \alpha v - \beta p_t & \text{if purchasing a new copy } (j=1) \\ \xi_t v - \beta p_{ut} & \text{if purchasing a used copy } (j=2) \\ 0 & \text{if no purchase } (j=0), \end{cases} \tag{3.1.1}$$

where α and β denote consumers' taste for a new copy and price sensitivity, respectively. ξ_t represents the aggregate demand shock. I assume that ξ_t's are $i.i.d$ draws from a distribution on $(0, \infty)$ with a mean equal to 1. I do not allow ξ_t to be below zero, which would yield a situation where the consumer would not buy the product at a *negative* price, which is unreasonable for a video game because possessing it does not generate negative utility and disposing it is practically costless.

Consistent with previous literature which studied video game industry (Ishihara and Ching, 2012; Lee, 2013; Nair, 2007; Shiller, 2012), I do not model the competition among different video games. As Nair (2007) argued, the large number of fairly differentiated games makes the games weak substitutes for each other, and the observed declining price trajectories cannot be explained by inter-game competition. Also, modeling competition would make the dimensionality of the state space computationally intractable.

Next, consider the consumers' selling decisions. Suppose a consumer has bought the game prior to period t, and has not lost interest in the game yet. Then, she can choose to resell the game at used market price p_{ut}, or keep the game and receive the flow utility:

$$w_{kt} = \begin{cases} \beta p_{ut} & \text{if selling the game } (k = 1) \\ \xi_t v & \text{if keeping the game } (k = 0). \end{cases} \quad (3.1.2)$$

Notice that I assume that the aggregate demand shock, ξ_t, also enters to the utility of consumers who already purchased the game. This assumption is reasonable when the source of the aggregate error is the shocks on game-playing values. For example, if potential buyers get a positive shock because it is a holiday and they have more spare time to play the game, then consumers who already have the game are likely to get more utility from the game as well. However, if the shock is from an increase in the firm's marketing expenditure (e.g., game advertisement), then it is more likely to affect potential buyers. It can still affect the utility of current game holders, however, by increasing the number of people they can interact with about the game.

Once the consumer loses interest in the game, he gets zero utility from game holding[1]:

$$z_{lt} = \begin{cases} \beta p_{ut} & \text{if selling the game } (l = 1) \\ 0 & \text{if keeping the game } (l = 0), \end{cases} \tag{3.1.3}$$

Probability of losing interest in the game For the empirical model I assume the hazard of losing interest in the game, λ, to be constant. I can extend the model by allowing the hazard that the consumer loses interest in the game after holding it for τ periods to be a function of time, $\lambda(\tau)$. Then $\lambda(\tau)$ would a discrete time hazard function, which is similar to what Farias, Saure, and Weintraub (2012) assumed in their example model.[2]

States I drop the time subscript and denote current period variables without any superscript and the next period variables with superscript $'$. Let $\mathbf{x} \in \mathbf{X}$, where $\mathbf{X}$ is the feasible set of $\mathbf{x}$, denote the current period state variables common to the

[1] I do not include the transaction cost in the model explicitly. Transaction cost is more of a nuisance parameter, as it is not a particular interest of this study. I opt to not include the parameter in my model, and instead estimating it separately and subtract the estimate from the prices before the estimation of the structural parameters.

[2] They assumed that a firm's individual state depreciates by one state with probability δ.

consumers and the firm which hold relevant aggregate level information. Specifically, I need to know the information about 1) potential used copy supply and 2) remaining demand in this period, and these potentially include the entire history of new and used copy transactions. By firstly characterizing consumers' decisions, however, I summarize this information with a couple of variables, s and $\underline{v}$. s denotes the sum of total new copy sales so far and it represents the source of the used copy supply, while $\underline{v}$ denotes the lowest valuation of the consumer who purchased the game in the previous period, and it summarizes the remaining demand. I can characterize the consumers' problem separately from that of the firm because there are many game buyers and sellers and they act as price takers in their decisions. Hence for the consumers, their state variables include the current prices, $\mathbf{p} = (p, p_u)$ in addition to $\mathbf{x}$.

Bellman Equations First consider the value function of a consumer with valuation v who has purchased the game τ periods ago. It differs by whether or not the consumer has lost interest in the game. At any period, let $W(v, \mathbf{p}, \mathbf{x})$ denote the value function where she has not lost interest in the game, and $Z(\mathbf{p}, \mathbf{x})$ denote the value function where she has. Consider the Bellman equation $Z(\mathbf{p}, \mathbf{x})$ first:

$$Z(\mathbf{p}, \mathbf{x}) = \max\{Z_0(\mathbf{p}, \mathbf{x}), Z_1(\mathbf{p}, \mathbf{x})\} \tag{3.1.4}$$

where $Z_l(\cdot),\ l \in \{0,\ 1\}$ are her alternative-specific value functions given by

$$Z_l(\mathbf{p}, \mathbf{x}) = \begin{cases} \beta p_u & \text{selling } (l = 1), \\ \delta E\left[Z(\mathbf{p}', \mathbf{x}')|\mathbf{x}\right] & \text{keeping } (l = 0) \end{cases} \tag{3.1.5}$$

Hence once she loses interest in playing the game, she does not get any utility from it anymore. Also,$\mathbf{x}$ has sufficient information about future expected prices so the expectation is only conditional on$\mathbf{x}$. Meanwhile, if she has not lost interest in the game, the Bellman equation is given by:

$$W(v,\mathbf{p},\mathbf{x}) = \max\{W_0(v,\mathbf{p},\mathbf{x}), W_1(v,\mathbf{p},\mathbf{x})\} \tag{3.1.6}$$

where $W_k(\cdot)$, $k \in \{0,\ 1\}$ are her alternative-specific value functions given by

$$W_k(v,\mathbf{p},\mathbf{x}) = \begin{cases} \beta p_u & \text{selling } (k=1), \\ \xi v + \delta\left\{(1-\lambda)E\left[W(v,\mathbf{p}',\mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}',\mathbf{x}')|\mathbf{x}\right]\right\} & \text{keeping } (k=0). \end{cases} \tag{3.1.7}$$

Note that with constant λ, $W(\cdot)$ does not depend on the number of holding periods τ.

Next, consider the purchasing decision of a potential consumer who has not purchased the game yet. Let $V(v,\mathbf{p},\mathbf{x})$ the value function for her. Then $V(v,\mathbf{p},\mathbf{x})$ satisfies the following Bellman equation:

$$V(v,\mathbf{p},\mathbf{x}) = \max\{V_0(v,\mathbf{p},\mathbf{x}),\ V_1(v,\mathbf{p},\mathbf{x}),\ V_2(v,\mathbf{p},\mathbf{x})\} \tag{3.1.8}$$

where $V_j(\cdot)$, $j \in \{0,\ 1,\ 2\}$ are her alternative-specific value functions given by

$$V_j(v,\mathbf{p},\mathbf{x}) = \begin{cases} \xi\alpha v - \beta p + \delta\left\{(1-\lambda)E\left[W(v,\mathbf{p}',\mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}',\mathbf{x}')|\mathbf{x}\right]\right\} & (j=1), \\ \xi v - \beta p_u + \delta\left\{(1-\lambda)E\left[W(v,\mathbf{p}',\mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}',\mathbf{x}')|\mathbf{x}\right]\right\} & (j=2), \\ \delta E\left[V(v,\mathbf{p}',\mathbf{x}')|\mathbf{x}\right] & (j=0). \end{cases} \tag{3.1.9}$$

where j denotes buying new copy ($j=1$), buying used copy ($j=2$), and waiting

$(j = 0)$, respectively.

Consumers' buying decisions Given the states, Consumers make decisions on new and used copy buying and used copy selling independently from the firm. I characterize consumers' decision with new copy and used copy marginal consumer; let $\bar{v}$ and $\underline{v}'$ denote the valuation of these two consumers, respectively. For any new and used prices, the new copy marginal consumer in current period is indifferent between purchasing new or used. Thus given p and p^u, $\bar{v}$ is given by the solution to the following:

$$\underbrace{\xi\alpha\bar{v} - \beta p + \delta\left\{(1-\lambda)E\left[W(\bar{v}, \mathbf{p}', \mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}', \mathbf{x}')|\mathbf{x}\right]\right\}}_{\text{buy new copy}} = \tag{3.1.10}$$

$$\underbrace{\xi\bar{v} - \beta p_u + \delta\left\{(1-\lambda)E\left[W(\bar{v}, \mathbf{p}', \mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}', \mathbf{x}')|\mathbf{x}\right]\right\}}_{\text{buy used copy}}. \tag{3.1.11}$$

The used copy marginal consumer in current period is indifferent between buying used copy now and waiting til the next period and making a decision. Hence current period used copy marginal consumer $\underline{v}'$ is given by the solution to the following:

$$\underbrace{\xi\underline{v}' - \beta p_u + \delta\left\{(1-\lambda)E\left[W(\underline{v}', \mathbf{p}', \mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}', \mathbf{x}')|\mathbf{x}\right]\right\}}_{\text{buy used copy}} = \underbrace{\delta E\left[V(\underline{v}', \mathbf{p}', \mathbf{x}')|\mathbf{x}\right]}_{\text{wait}}. \tag{3.1.12}$$

Firstly, it is straightforward to prove that for any sequence of future expected prices, if a consumer with valuation $\tilde{v}$ purchases either new or used copy in a given period, a consumer with a valuation $v > \tilde{v}$ who has not yet purchased the game will also purchase in the same period. In addition, the assumption that consumers get lower initial utility from buying a used copy than from buying a new copy implies that the new copy price p is greater than p_u in an equilibrium, since otherwise everyone will buy new copy and the demand for the used will be zero. If the new copy price is higher than used copy price, the reservation valuation of a consumer who purchased an new copy also must be greater than $\underline{v}$. Hence $\underline{v}$ denotes the lowest valuation of consumer who have purchased either new or used copy by the previous period, and it determines remaining demand in current period. The set of consumers who have purchased by the end of the last period will be on the interval $[\underline{v}, 1]$. In addition, the used copy demand in the current period is $\bar{v} - \underline{v}'$.

From equation (3.1.10), $\overline{v}$ satisfies:

$$\bar{v} = \frac{\beta\,(p - p_u)}{\xi(\alpha - 1)} \tag{3.1.13}$$

Hence, once a consumer decides to buy the game, choosing between used and new good does not have dynamic aspect, as it only depends on the relative prices between new and used game. Now consider the used copy marginal consumer. Firstly I claim that the right hand side of (3.1.12), $\delta E\left[V(\underline{v}', \mathbf{x}')|\mathbf{x}\right]$ equals to $\delta E\left[V_1(\underline{v}', \mathbf{x}')|\mathbf{x}\right]$. This is because in the next period, $\underline{v}'$ is the upper bound of the valuation of consumers who have not purchased the game yet. Hence it has to be bigger than $\bar{v}'$, which is the valuation of indifferent consumer between purchasing the new and the used copy in the next period. Hence, $V_1(\underline{v}', \mathbf{p}', \mathbf{x}') > V_2(\underline{v}', \mathbf{p}', \mathbf{x}')$ and I have

$$\xi\underline{v}' - \beta p_u + \delta\left\{(1-\lambda)E\left[W(\underline{v}', \mathbf{p}', \mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}, \mathbf{x}')|\mathbf{x}\right]\right\} = \delta E\left[V_1(\underline{v}', \mathbf{p}', \mathbf{x}')|\mathbf{x}\right] \tag{3.1.14}$$

3.1.2 Selling Decisions

By exploiting equilibrium conditions, I can study how consumers behave in an equilibrium regarding their used game selling decisions independently from the firm's problem. I found a couple of propositions which characterize their optimal decisions, and they significantly simplify their optimal used good selling behavior before I describe the firms' problem.

Proposition 1. *In equilibrium, consumers would sell the game immediately once they lose interest in the game.*

Proof. At any period, all consumers who already own the game have valuation $v \geq \underline{v}$. Hence, for any of them to sell their used copies of the game, p_u at least needs to satisfy the following:

$$\beta p_u \geq \xi \underline{v} + \delta \left\{ (1-\lambda) E \left[W(\underline{v}, \mathbf{p}', \mathbf{x}') | \mathbf{x} \right] + \lambda E \left[Z(\mathbf{p}', \mathbf{x}') | \mathbf{x} \right] \right\} \tag{3.1.15}$$

I show that at $\beta p_u = \xi \underline{v} + \delta \{(1-\lambda) E [W(\underline{v}, \mathbf{p}', \mathbf{x}')|\mathbf{x}] + \lambda E [Z(\mathbf{p}', \mathbf{x}')|\mathbf{x}]\}$, no one with $v < \underline{v}$ will buy the used copy. That is, for all consumers with $v \leq \underline{v}$, their discounted future expected utility from purchasing the used copy at p_u is smaller than that from purchasing in the next period at $E[p_u'|\mathbf{x}_t]$. This is because the difference between p_u and Ep_u' is more than enough to compensate the loss of one period flow utility. If there is no demand at this price, then there is no demand at higher prices either, and thus they cannot be supported in an equilibrium.

The expected payoff from purchasing a used copy is

$$\xi v - \beta p_u + \delta \left\{ (1-\lambda) E\left[W(v, \mathbf{p}', \mathbf{x}') | \mathbf{x} \right] + \lambda E\left[Z(\mathbf{p}', \mathbf{x}') | \mathbf{x} \right] \right\}$$

By substituting βp_u with $\xi \underline{v} + \delta \{(1-\lambda) E[W(\underline{v}, \mathbf{p}', \mathbf{x}')|\mathbf{x}] + \lambda E[Z(\mathbf{p}', \mathbf{x}')|\mathbf{x}]\}$,

$$\xi v - \xi \underline{v} + \delta \left\{ (1-\lambda) E\left[W(v, \mathbf{p}', \mathbf{x}') | \mathbf{x} \right] \right\} - \delta \left\{ (1-\lambda) E\left[W(\underline{v}, \mathbf{p}', \mathbf{x}') | \mathbf{x} \right] \right\} < 0$$

since $v \leq \underline{v}$ and $E[W(v, \mathbf{p}', \mathbf{x}')|\mathbf{x}] \leq E[W(\underline{v}, \mathbf{p}', \mathbf{x}')|\mathbf{x}]$. Hence, no consumer whose valuation is $v \leq \underline{v}$ will buy a used copy at this price, and thus it cannot be an equilibrium, and $W(v, \mathbf{p}, \mathbf{x}) = \xi v + \delta \{(1-\lambda) E[W(v, \mathbf{p}', \mathbf{x}')|\mathbf{x}] + \lambda E[Z(\mathbf{p}', \mathbf{x}')|\mathbf{x}]\}$.

□

Proposition 1 also implies the following corollary.

Corollary 1. *In equilibrium, current used copy price, p_u, is greater than future discounted expected used copy price $\delta E p'_u$. That is, I have $p_u > \delta E p'_u$ for all period.*

Proposition 2. *If $\lambda(\tau)$ is nondecreasing in τ, among consumers who own the game, those who have not lost interest in the game do not sell their copies at the market prevailing used good price in equilibrium.*

Proof. For consumers to wait instead of selling after losing interest, there must be an period where they want to wait and sell in the next period. Hence, it is sufficient to show that for any adjacent period, consumers who lost interest in the game do not have the incentive to wait and sell in the next period. In any period, for a consumer to expect to do so, the current used price needs to be

$$\beta p_u \leq \delta\beta E\left[p_u'|\mathbf{x}\right]$$

For this price to be an outcome of an equilibrium, there needs to be zero demand at this price, because otherwise there are consumers who are willing to pay more than $\beta p_u = \delta\beta E\left[p_u'|\mathbf{x}\right]$ for the used copy in this period, and at that price, suppliers are also willing to sell their copies instead of waiting. Let's compare the utility of buying now and buying in the next period for a consumer who has valuation v when the price is $\beta p_u = \delta\beta E\left[p_u'|\mathbf{x}\right]$:

$$\begin{aligned} & v - \beta p_u + \delta\left\{(1-\lambda)E\left[W(v,\mathbf{p}',\mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}',\mathbf{x}')|\mathbf{x}\right]\right\} \\ & -\delta\left[v - E\beta p_u' + \delta\left\{(1-\lambda)E\left[W(v,\mathbf{p}'',\mathbf{x}'')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}'',\mathbf{x}'')|\mathbf{x}\right]\right\}\right] \end{aligned}$$

$$\begin{aligned} \Rightarrow\quad & v - \delta\beta E p_u' + \delta\left\{(1-\lambda)E\left[W(v,\mathbf{p}',\mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}',\mathbf{x}')|\mathbf{x}\right]\right\} \\ & -\delta\left[v - E\beta p_u' + \delta\left\{(1-\lambda)E\left[W(v,\mathbf{p}'',\mathbf{x}'')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}'',\mathbf{x}'')|\mathbf{x}\right]\right\}\right] \end{aligned}$$

$$\begin{aligned} \Rightarrow (1-\delta)v + \delta\left\{(1-\lambda)E\left[W(v,\mathbf{p}',\mathbf{x}')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}',\mathbf{x}')|\mathbf{x}\right]\right\} & \\ -\delta^2\left\{(1-\lambda)E\left[W(v,\mathbf{p}'',\mathbf{x}'')|\mathbf{x}\right] + \lambda E\left[Z(\mathbf{p}'',\mathbf{x}'')|\mathbf{x}\right]\right\} & \geq 0 \end{aligned}$$

which holds because $E\left[W(v,\mathbf{p}',\mathbf{x}')|\mathbf{x}\right] \geq E\left[W(v,\mathbf{p}'',\mathbf{x}'')|\mathbf{x}\right]$ and $E\left[Z(\mathbf{p}',\mathbf{x}')|\mathbf{x}\right] \geq E\left[Z(\mathbf{p}'',\mathbf{x}'')|\mathbf{x}\right]$. Hence, at this price, consumers are better off buying now than waiting, and thus it cannot be an outcome of an equilibrium. Thus consumers do not have an incentive to wait once they lose interest in playing the game, and I have

$Z(\mathbf{p}, \mathbf{x}) = p_u$ in equilibrium. □

Propositions 1 and 2 simplify the consumers' used goods selling decisions significantly. Even though I allow consumers to optimally sell their copies, the solution of their post-purchase dynamic problem becomes trivial; in an equilibrium they will not sell their copies until they lose interest the game, and they will do so immediately on losing interest, regardless of the used copy price. Hence the supply of used copy in period t is exactly the same as the number of consumers who own the game prior to period t and just have lost interest in the game. Then, with the assumption that probability of losing interest in the game is constant λ, I have the following used copy supply:

$$q_{ut}^{*} = \lambda \cdot \left(\sum_{\tau=0}^{t-1} q_{\tau} \right) = \lambda \cdot s$$

that is, it becomes just λ times s, the cumulative new copy sales up to the previous period. This is because since the probability of losing interest is constant, all consumers who own the game has the same chance of losing interest in the game, regardless of their valuation of the game and how long they had been playing the game. Also, previous used good transactions become irrelevant because they do not affect how many people have the game, as only the ownership of the copy changes with used copy transactions. Hence, instead of keeping track of how many games are sold in each previous period, I only need to know how many games are sold so far.

In sum, as described above, the state $\mathbf{x}$ consists of $(s, \underline{v})$, where s denotes the cumulative sales up to the previous period and $\underline{v}$ is the lowest valuation of consumer who have purchased either new copy or used copy by the previous period defined in (3.1.12). The feasible set for the state variables is $\mathbf{X} = \{(\underline{v}, s)|1 \geq \underline{v}, s \geq 0, \underline{v} \leq F^{-1}(1-s)\}$. $1-s$ is the remaining demand only when all of the cumulative sales, s, occurred in the last period and there has been no used goods transaction. Thus, $F^{-1}(1-s)$ is the upper bound of $\underline{v}$.

3.1.3 Consumer Expectations

Specifying how consumers form expectations about future prices is an important component of a dynamic model. Usual treatment of the price expectations in the literature is that assuming consumers are bounded rational in a sense that they form their price expectations based on current prices and use reduced form regressions to recover the expectation parameters Gowrisankaran and Rysman (2012); Ishihara and Ching (2012); Lee (2013); Nair (2007); Shiller (2012). One of the reasons of this is that consumers' problem becomes complicated when one assumes that consumers know the states and the policy rule the firm uses and form the expectations accordingly. In my model, however, I assume consumers form rational expectations about future prices and thus they share the same aggregate state variables with the firm in the estimation. This is possible because of the following two reasons: firstly, due to propositions 1 and 2, I simplified the consumers' dynamic problem significantly, and I am able to embed the solution of their dynamic problem in the equilibrium used copy price condition. Also, my estimation strategy depends on solving the dynamic programming problem for the firm during parameter search. Hence, assuming rational expectations does not impose any additional computational burden.

3.2 Model for a Game Producer

Now I am ready to describe the firm's problem. Since the firm also has rational expectations about future prices and also it incorporates the response from consumers and the used goods market, consumers' behavior must be defined before I characterize the firm's behavior.

For implementation I use $\bar{v}$ instead of p for the firm's control, since choosing the quantity and the price is equivalent in the monopolist's problem. The new good sales are $q^*(\overline{v}, \mathbf{x}) = F(\underline{v}) - F(\overline{v})$, the equilibrium used goods supply is $q_u^{s*}(\mathbf{x}) = \lambda \cdot s$, and used goods demand is $q_u^{d*}(\overline{v}, \underline{v}', \mathbf{x}) = F(\overline{v}) - F(\underline{v}')$ and I have the following state transition rules:

$$s^*(q, \mathbf{x}) = s + q \tag{3.2.1}$$

$$\underline{v}^*(\overline{v}, \mathbf{x}) = q_u^{d*-1}(\overline{v}, q_u^{s*}(\mathbf{x}), \mathbf{x}) \tag{3.2.2}$$

where q_u^{d*-1} is the inverse function of $q_u^{d*}(\cdot)$ with respect to $\underline{v}$.

As described above, the aggregate used game supply is $\lambda \cdot s$ regardless of the used game price. Then, the used game price will be adjusted so the used copy demand is equal to the supply. From the market clearing condition, I have the following used game price equation,

$$\begin{aligned} p_u^*(\overline{v}, \underline{v}', \mathbf{x}) \quad = \quad & \beta^{-1}\left\{ \left((\xi - 1) + \frac{1-\delta}{1-\delta(1-\lambda)} - \delta(\alpha - 1) \right) \underline{v}' \right. \\ & \left. - \delta\lambda E\left[W_2(\mathbf{x}') \middle| \mathbf{x} \right] \right\} + \delta E\left[p(\mathbf{x}') \middle| \mathbf{x} \right] + \delta\lambda E\left[p_u(\mathbf{x}') \middle| \mathbf{x} \right]. \end{aligned} \tag{3.2.3}$$

for the derivation of (3.2.3), see the Appendix 1. Then, new copy price is determined from 3.1.13,

$$p^*(\overline{v}, p_u, \mathbf{x}) = \beta^{-1}\xi(\alpha - 1) \cdot \overline{v} + p_u. \tag{3.2.4}$$

Hence, when the firm maximizes its profit, it incorporates the consumers' future price expectations and their optimal behavior through the used game price. Since both the firm and consumers form rational expectations, their price expectations are consistent.

The firm solves the following problem to maximize the expected sum discounted future profit,

$$\Pi(\mathbf{x}) = \max_{\overline{v}} \left\{ p \cdot q + \delta_f \cdot E\left[\Pi(\mathbf{x}') \middle| \mathbf{x}\right] \right\}. \tag{3.2.5}$$

$$\begin{aligned} \text{subject to:} \quad & q = q^*(\overline{v}, \mathbf{x}) \\ & s' = s^*(q, \mathbf{x}) \\ & q_u^s = q_u^{s*}(\mathbf{x}) \\ & \underline{v}' = q_u^{d*-1}(\overline{v}, q_u^s, \mathbf{x}) \\ & p_u = p_u^*(\overline{v}, \underline{v}', \mathbf{x}) \\ & p = p^*(\overline{v}, p_u, \mathbf{x}) \end{aligned}$$

3.3 Market Equilibrium

To define the equilibrium, I introduce the following notations. Let $\overline{v}^*(\mathbf{x})$ denote the monopolist's equilibrium quantity strategy and $\underline{v}'^*(\mathbf{x})$ denote the valuation of the consumer who is indifferent between buying a used game in current period and buying a new game in the next period when faced with state x. Since $\overline{v}^*(\mathbf{x})$, $\underline{v}'^*(\mathbf{x})$ themselves are functions of the state variables, with slight abuse of notation, I also denote, $q_u^{d*}(\mathbf{x}) = q_u^{d*}(\overline{v}^*(\mathbf{x}), \underline{v}'^*(\mathbf{x}), \mathbf{x})$, $p_u^*(\mathbf{x}) = p_u^*(\overline{v}(\mathbf{x}), \underline{v}'^*(\mathbf{x}), \mathbf{x})$, and $p^*(\mathbf{x}) = p^*(\overline{v}^*(\mathbf{x}), p_u^*(\mathbf{x}), \mathbf{x})$.

Definition. A *Markov-perfect equilibrium* in this model is defined by following policy rules as functions: valuations $\overline{v}^*(\cdot)$ and $\overline{v}'^*(\cdot)$, prices $p^*(\cdot)$ and $p_u^*(\cdot)$, quantities $q^*(\cdot)$, $q_u^{s*}(\cdot)$ and $q_u^{d*}(\cdot)$ such that

1. For any period and for any $\mathbf{x} \in \mathbf{X}$, $\overline{v}^*(\mathbf{x})$ solves the firm's optimization problem defined in (3.2.5),

2. For any period and for any $\mathbf{x} \in \mathbf{X}$, and $p, p_u \geq 0$, a consumer with valuation v make a purchase of a new game in current period if and only if his current period utility from doing so exceeds his utility from purchasing a new game in any future periods, purchasing a used game in current and all future periods, or not purchasing at all,

3. for any period and for any $\mathbf{x} \in X$, and $p, p_u \geq 0$, a consumer with valuation v make a purchase of a used game in current period if and only if his current period utility from doing so exceeds his utility from purchasing a new game in

current and all future periods, purchasing a used copy in any future periods, or not purchasing at all,

4. for any period and for any $\mathbf{x} \in X$, and $p, p_u \geq 0$,the aggregate used copy supply which is result in consumers' optimal used copy selling decisions, equals to $q_u^{s*}(\mathbf{x})$,

5. both firm and consumers form rational expectations about future prices,

6. the used goods market clears every period or the used price is zero; that is, $p_u(\mathbf{x}) > 0$ only if $q_u^{s*}(\mathbf{x}) = q_u^{d*}(\mathbf{x})$, and $p_u(\mathbf{x}) = 0$ otherwise.

3.4 Numerical Details in Solving for Equilibrium

In this section, I describe the details of solving for the equilibrium numerically. I discuss the approximation of value functions, which is an essential part of solving dynamic programming with continuous state variables. Then I describe the value iteration procedure.

3.4.1 Approximation

I need to approximate for four values: $Ep_u(\mathbf{x}), EW_2(\mathbf{x}), Ep(\mathbf{x})$ are used in the calculation of used game price, and $E\Pi(\mathbf{x})$ is needed for the calculation of sum of discounted profit to solve the dynamic problem. The challenge in doing so is that in my model, the state space cannot be represented by a regular grid, which renders many popular approximation methods in marketing and economics, such as Chebyshev polynomial and spline approximation, inapplicable. This is because the state variable s, which represents the cumulative sales so far, governs the maximum value the other state variable, $\underline{v}$, can take. That is, if a firm has sold s percent of the total demand so far, by construction one cannot have remaining demand higher than $1 - s$, and thus the maximum value of $\underline{v}$ is $F^{-1}(1 - s)$. One can try to *convert* the state space into a rectangular grid. For example, one can transform $\underline{v}$ to $F(\underline{v})/(1 - s)$. I found, however, that this type of conversions introduces additional numerical error, especially when $1 - s$ is small, and yields numerically unstable approximated value functions. The bi-linear approximation, which can be used with non-rectangular grid, does not work either because the firm's control in my model is continuous. The bi-linear approximation introduces too many kink points and the optimization for continuous control breaks down. To solve this challenge, I employ recently developed the radial basis function (RBF) approximationBuhmann (2000), which can accommodate scattered data and also yields smooth approximated values. I briefly introduce the RBF approximation methods here, and I refer to Buhmann (2003) and Fasshauer (2007) for further details.

Suppose I have scattered data *centers* $\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\} \subset \mathbb{R}^k$ and associated real function values $f(\mathbf{x}_i),\ i \in \{1, \ldots, N\}$. O look for a continuous function $\hat{f} : \mathbb{R}^k \to \mathbb{R}$ which satisfies the interpolation condition,

$$\hat{f}(\mathbf{x}_i) = f(\mathbf{x}_i), \quad i \in \{1, \ldots, N\}. \tag{3.4.1}$$

In the RBF approximation, one assumes that $\hat{f}(\mathbf{x}_i)$ is of the form

$$\hat{f}(\mathbf{x}) = \sum_{i=1}^{N} w_i \phi(||\mathbf{x} - \mathbf{x}_i||)$$

where $\phi(\cdot)$ is the radial basis function and $w_i \in \{w_i, \cdots, w_N\}$ are the coefficients of the approximation. The coefficients $\mathbf{w} = (w_i, \cdots, w_N)$ are found by the interpolation condition 3.4.1,

$$\mathbf{w} = \mathbf{A^{-1}f}$$

where $\mathbf{A}_{ij} = \phi(||\mathbf{x}_i - \mathbf{x}_j||)$ and $\mathbf{f} = (f(\mathbf{x}_1), \ldots, f(\mathbf{x}_N))$. I use the multiquadratic radial basis function,

$$\phi(r) = \sqrt{1 + (\varepsilon r)^2}$$

where ε is the shape parameter, which I set to 1.5 to preserve the monotonicity of $\hat{f}$. There are a couple of additional advantages of this method. First, it helps solving the *curse of dimensionality*. Since one does not have to represent the state space by a tensor product between grids of state variables, as the dimensionality of the state increases, the number of function evaluations needed can be smaller than N^k, where N is the number of grid points and k is the dimensionality of the state. Also, one can add additional *centers* on the region in the state space where it matters for better accuracy. That is, in my model, while the neighborhood near starting point of $s = 0, \underline{v} = 1$ is important in firm's profit, the profit implication of the region with large value of s is very small. Hence, I use more centers around $s = 0, \underline{v} = 1$ so I can get better accuracy around that point.

3.4.2 Value Function Iteration

I calculate the solution of the dynamic problem through value iteration (**?**Bertsekas, 1995), and I briefly describe its steps. The detailed steps of numerical algorithm is in the Appendix 2. Firstly I choose n approximation centers in the state space. For each iteration, I loop over each center, and optimize the firm's profit with respect to $\bar{v}$. To calculate the sum of discounted profit, for any candidate $\overline{v} \in [0, \underline{v}]$, I obtain next state $s' = s + q$ and $\underline{v}' = F^{-1}\left(F(\overline{v}) - \lambda s\right)$. The latter is from the market clearing condition. Then I calculate $p_u(\overline{v}, \underline{v}', \mathbf{x})$ following (3.2.3), and in turn $p(\bar{v}, p_u, \mathbf{x})$ following (3.2.4). After each iteration, I calculate and store the coefficients for the approximation for the next iteration. I repeat these steps until the values converge.

The loop-intensive nature of the procedure makes implementation with high-level languages virtually infeasible, because it will be inadequately slow. Hence, the algorithm was programmed in Python, but the loop-intensive and the approximation part are coded in C. Specifically, I use Cython[3], which generates efficient C code from the Cython language which is close to the Python language. The calculation at one approximation center is independent from those at other centers, and I parallelized the code using openMP.[4]

3.5 Simulated Prices and Sales in Market Equilibrium

I numerically solve for the equilibrium with specific parameters and a series of draws of aggregate error to illustrate equilibrium outcomes my model generates. Comparing to the actual data I will show in the next chapter, my model can generate realistic price trajectories. I use a truncated normal distribution with parameters $\mu = 0.5$ and $\sigma = 1$, and support $[0, 1]$ for $F(v)$. I set $\alpha = 2$, $\delta = 0.95$ and use

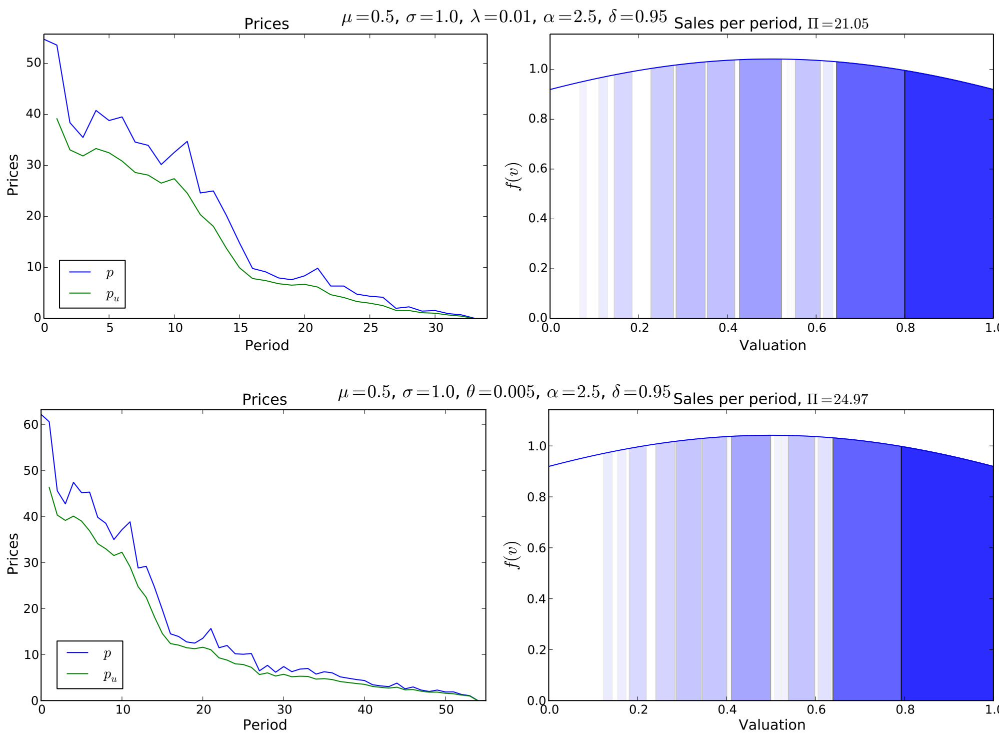
[3] `http://cython.org`
[4] `http://openmp.org`

two values of λ, 0.01 and 0.005 and compared the equilibrium paths. Figure 3.1 shows the equilibrium trajectories of the two simulations. The plots on the left side show the equilibrium trajectories of $\overline{v}$, p and p_u. The plots on the right represent the shape of the demand distribution, and each number and colored area under the density curve represent the specific period and its new sales volume, respectively. Periods with higher sales volume have darker color, and one can easily see how much penetration the firm has for each period.

In both cases the price trajectories show typical intertemporal price discrimination behavior of the monopolist. Larger λ (higher probability of losing interest in the game) has two effects on consumers' dynamic purchasing decision. Firstly, since they expect to lose interest in game earlier, consumers have lower expected utility from purchasing the game. In addition, larger λ also means there will be more supply of used copy later and lower used goods price, and thus it makes waiting option relatively more attractive. Since the early sales will become used goods supply in the later periods, the firm chooses to sell less in and $\lambda = 0.01$ case. As a result, the firm gets significantly lower profit in larger λ case ($\Pi = 1.430$ when $\lambda = 0.01$ and $\Pi = 1.651$ with $\lambda = 0.005$).

In sum, similar to the two-period model, since selling new copies of the game has additional negative effect on the profit through becoming used copy supply in the future, the firm chooses to sell relatively smaller quantity of the game initially to mitigate this. With larger λ, even though the firm optimally response to the higher competition from the used good market, the profit of the firm suffers.

Figure 3.1: Equilibrium path



Chapter 4

Data Analysis

4.1 Data

The main dataset of this study is the price data collected from Amazon.com. I have collected daily data for all games sold in Amazon between Feb 27, 2010 and April 2, 2012. For each game, I use the price of the *new* game officially sold from Amazon.com as the new price, and the minimum price[1] of the used game listings posted by individual sellers through the Amazon marketplace as the used price.

Since my estimation strategy relies on solving for the unobserved aggregate shock from the initial period, I can only use video games which I observe the data from their release period. Since many games appear in Amazon's listings before their release, I have collected release date information from the Wikipedia[2], and only use the games released after Feb 27, 2010. For estimation, I have aggregated the daily data and generated weekly price trajectories.

The plots in figure 4.1 show the trajectories of new and used prices for several example games included in the data. There are some regularities in the data. Firstly, all console games are priced at $59.99 at their release. Also, for all games, the price

[1] This is the *used from* price.

[2] 2010 in video gaming (`http://en.wikipedia.org/wiki/2010_in_video_gaming`) and 2011 in video gaming (`http://en.wikipedia.org/wiki/2011_in_video_gaming`)

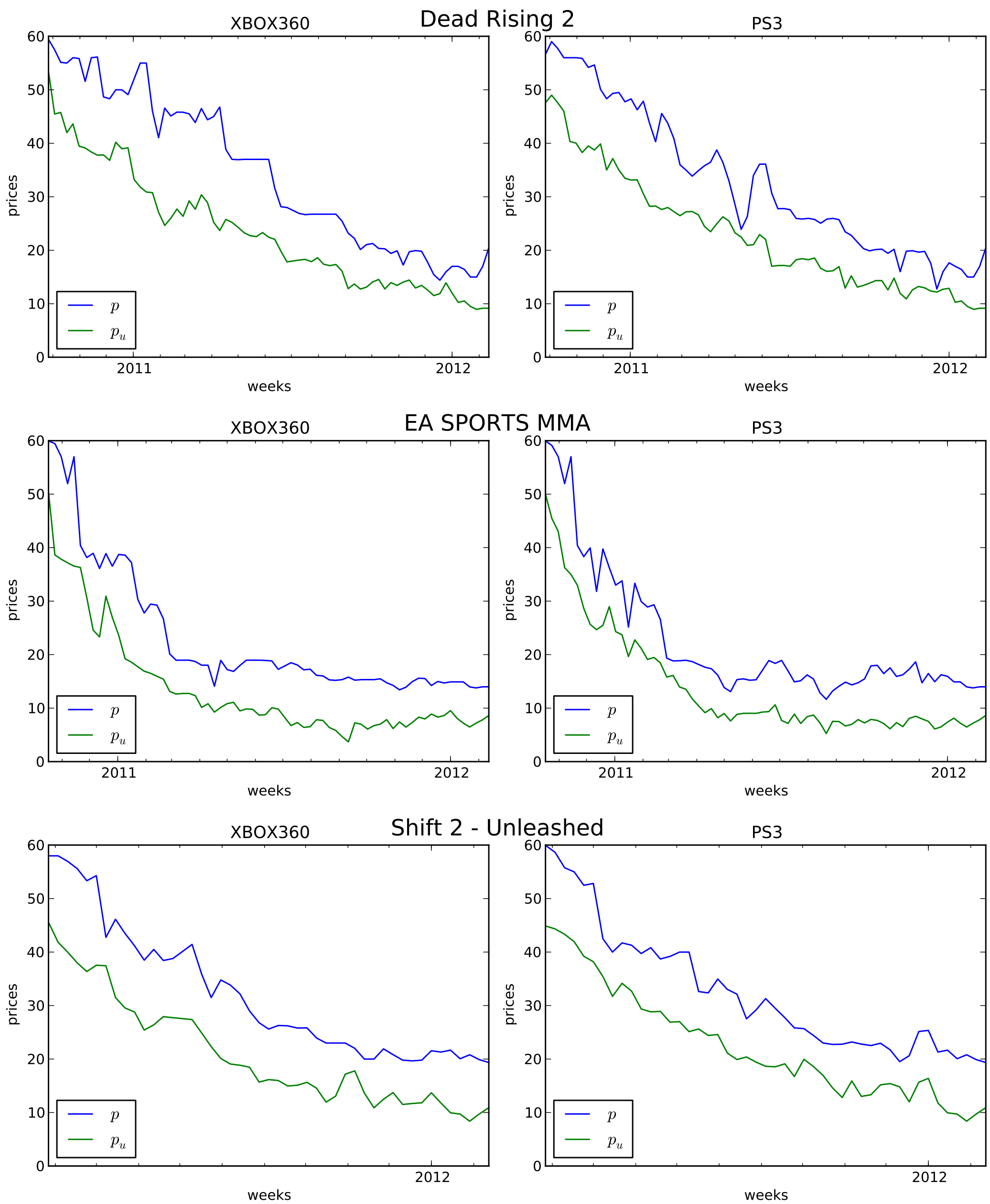
trajectories eventually converge to a stable, absorbing state, where basically the trajectories become flat. And the shape of the price trajectories and the time it takes for them to converge to the absorbing state differ across games. For example, for *Dead Rising 2*, the prices more or less decline linearly, and it takes more than a year for the new game price to hit 20 dollars. On the other hand, for *EA SPORTS MMA*, there are significant price decrease soon after its release, and it only takes about 4 months to hit 20 dollars. Since firms set their prices strategically, the shape of price decline has information about the underlying demand.

One issue of using Amazon.com's data is that whether its prices are representative prices that consumers face when they make purchase decisions. If Amazon.com sets the price independently from the producers, I cannot regard its prices as the results from monopolists' intertemporal price discrimination. I use this data for two reasons; first, the observed price trajectories consistently show the patterns of price skimming. Second, since consumers have little cost of web surfing and price comparison online, it is not unlikely that Amazon.com's prices are systematically different from those of other online retailers'.

4.1.1 Initial Prices

My model starts in the initial period, where the previous sales are zero and thus there is full demand remaining, and no used goods supply. However, I observe both new and used prices in the data from the release date. The potential reasons

Figure 4.1: Weekly Price Trajectories



for this include weekly aggregation, the measurement error[3], and ineffective listings[4]. Hence I discard the first observation of the used game price. Since initial prices of all games are set at \$59.99, I cannot infer the aggregate error from this price. Hence, I assume that the draw of error is 1 at that period, and regard future errors as the relative errors to that of the initial period. Given the state of $s = 0, \underline{v} = 1$ and the $\xi_0 = 1$, I calculate the inferred control by find $\overline{v}_0$ which would yield the price level of \$59.99. Depending on the demand parameters, there are cases where even $\overline{v}_0 = 1$ cannot generate $p_0 = 59.99$. In those cases I fixed $\overline{v}_0 = 1$ and assumed that there were no sales in the period.

4.1.2 Total Sales

To convert the results from the counterfactual analysis in Chapter 4.4 to actual dollar terms, I need total number of sales figures for each game i, denoted by M_i. Since my model yields implied the sales volume for each period as a percentage of total demand, I can multiply the shares by M_i and get the implied sales volume for each period. I gather the total sales information from one of the online video game information provider. This provider only produces yearly sales figures, and I used up to two years of sales after release as the proxy for the total game sales, since after two years of release, typically new game sales are negligible.

[3]Although I have collected daily data, since I have only gathered the data once per day, I do not observe the actual data at the moment of game release.

[4]Some sellers often have used copy listings with very high prices, without any hope of selling it

4.2 Estimation

I introduce parametric assumptions for demand distribution for estimation. For $F(v)$, the *c.d.f.* of the demand distribution, I use truncated normal distribution with support $[0, 1]$. As well as the minimum and maximum value, it is characterized by its location (μ) and scale (σ) parameters, which are a part of the structural parameters I estimate.

All parameters except δ are video game specific: μ and σ, the parameters characterizing demand distribution, λ, the durability parameter, α, the newness parameter, and β, the price sensitivity. I do not attempt to estimate the discount factor, since it is not well identified in dynamic setting (Magnac and Thesmar, 2002). The inverse of interest rate is the usual value used for weekly discount factor in literature. However, recently Yao, Mela, Chiang, and Chen (2012) found consumers' estimated weekly discount factor to be much lower through a field study, and thus I chose to set $\delta = 0.95$ at the level of week. Even though consumers might have lower discount factor than the interest rate, it is unreasonable to assume that the firm would also has lower discount factor. Hence, I use 0.99 for the firm's discount factor, δ_f. Hence I am estimating 5 parameters, where I denote the game i specific structural parameters by $\theta_i = (\mu_i, \sigma_i, \lambda_i, \alpha_i, \beta_i)$, where I drop i whenever it is not ambiguous.

Unfortunately, likelihood based estimation approach is not feasible because the Jacobian for the change of variables is not available analytically due to complex and nonlinear relationship between the demand shock, ξ, and observed prices. Hence I employ the estimation strategy based on Hansen and Singleton (1982)'s generalized instrumental variables estimation of nonlinear rational expectations models. My estimation strategy is analogous to Petrin (2002) and Gowrisankaran and Rysman (2012), which supplemented approach taken by Berry, Levinsohn, and Pakes (1995) (henceforth BLP) with additional moments from the model.

Specifically, I firstly invert the observed price systems to solve for the aggregate error, ξ_t and predicted prices. Then I construct three sets of moments: 1) the first moment of ξ_t, 2) the orthogonality conditions using instruments, and 3) supply moments which match the predicted prices to the observed prices.

4.2.1 Inversion of the Equilibrium Pricing Conditions

Analogous to BLP, I invert the price system via contraction mapping to solve for the aggregate demand shock, ξ_t, and the predicted prices. The intuition for the procedure is the following: given the state, since ξ_t are serially uncorrelated[5], the expected future values only depend on the firm's control through its impact on the next state variables. Then given the state and the future expected values, I can find the ξ_t which *rationalize* the differences between observed new and used prices from the equilibrium condition.

Specifically, given the states and observed prices, I aim to find ξ^* which satisfies the equilibrium condition for the new copy marginal consumer, $\xi_t = \beta \dfrac{(p_t - p_{ut})}{(\alpha - 1) \cdot \bar{v}_t}$. Let p_t, p_{ut} be the actual prices from the data, and $\bar{v}_t^*(\xi_t, s_t, \underline{v}_t)$ be the profit maximizing quantity that firm chooses given ξ_t and the state s_t and $\underline{v}_t$. Let ξ_{ti} denote

[5]In principle I could allow ξ_t to be serially correlated, but in that case I will be introducing ξ_t as an additional state variable and it will increase computational burden significantly. This is a limitation of my model.

the ξ_t value at the i'th iteration of the contraction. I take the following steps:

Step 1 Start with $\xi_{t0} = 1$ and the tolerance ϵ.

Step 2 Given ξ_{ti-1} and the state, compute $\bar{v}^*_{ti-1}(\xi_{ti-1}, s_t, \underline{v}_t)$, the profit maximizing quantity by solving firm's problem described in (3.2.5)

Step 3 Update $\xi_{ti} = \beta \dfrac{(p_t - p_{ut})}{(\alpha - 1) \cdot \bar{v}^*_{ti-1}(\xi_{ti-1}, s_t, \underline{v}_t)}$

Step 4 If $|\xi_{ti} - \xi_{ti-1}| < \epsilon$, stop. Otherwise, go to Step 2

Since I solve the firm's optimization problem in this procedure, it also simultaneously yields the implied prices, $\hat{p}_t, \hat{p}_{ut}$, which satisfy $p_t - p_{ut} = \hat{p}_t - \hat{p}_{ut}$, and the implied next states.

Now the question is how to infer s_t and $\underline{v}_t$ when I do not observe the sales. The key observation here is that the knowledge of current state is sufficient in calculating current error, and I do know the state for a game at its release period, which is $s_0 = 0$, $\underline{v}_0 = 1$. Hence, I can calculate ξ_0, and I get the implied state for period 1 from the procedure. Then I move to the next period, and calculate ξ_1 given the implied state. I repeat this with the successive periods. Since I cannot infer the state without sales data unless I observe the prices from the beginning, this does restrict me to use only games which I observe data from their release period.

I have not been able to provide a formal proof that the procedure described above will yield unique estimate of ξ_t. At least in many experiments with simulated data with realized errors, I found that the inversion procedure reliably recovers ξ_t.

Since I assume that $E[\xi_t] = 1$, recovered ξ_t's give me the first moment condition I can use in estimation.

4.2.2 Instruments

Since Bresnahan (1981), allowing the price to be correlated with the aggregate error has been standard in the studies of industries with differentiated products. In fact, prices are explicit function of the demand shock in my model, and thus I use instruments ($\mathbf{Z}$) and the orthogonality conditions in my estimation:

$$E[\mathbf{Z}'\xi(\theta)] = 0.$$

Since I assume that the aggregate shock is serially uncorrelated, the lagged prices are valid instruments. Also, in case of games which were released in multiple platforms, I use the lagged prices of the same game from other platforms as well. While the demand for the game likely differs across platforms since consumers' choices of game console are not random, it is likely that the demand shocks are correlated across consoles for the same game, and thus the prices will be correlated. Potentially I can also include prices of other games in the same genre as well, because it is reasonable to assume that the aggregate shocks are correlated across games in the same genre.

4.2.3 Moments from Supply Side

The last set of moments I use are moments from the supply side equilibrium. Specifically, the procedure described in (4.2.1) yields predicted new and used prices associated with the error simultaneously, and I match the predicted prices to the observed prices for each period:

$$E[|p_t - \hat{p}_t|] = 0,$$

$$E[|p_{ut} - \hat{p}_{ut}|] = 0.$$

I do find that the inclusion of these additional conditions improve the fit significantly. This is analogous to the micro moments Petrin (2002) and Gowrisankaran and Rysman (2012)[6] used, in a sense that I match moment predicted from the model to the moment from the data.

4.2.4 Objective function

The three sets of moments that the GMM objective function includes are $G_1(\theta)$, the first moment of ξ_t, $G_2(\theta)$ the orthogonality conditions, and $G_3(\theta)$, the moments from the supply side. I assume that the population moment conditions uniquely equal zero at true θ_0:

$$E\left[\mathbf{G}(\theta_{\mathbf{0}})\right] = E\begin{bmatrix} \mathbf{G}_1(\theta_0) \\ \mathbf{G}_2(\theta_0) \\ \mathbf{G}_3(\theta_0) \end{bmatrix} = 0.$$

Following Hansen and Singleton (1982), my estimates of the parameters, $\hat{\theta}$

[6]They used the difference between observed and predicted increase in household penetration between two time periods to improve the identification.

are the solution of the following:

$$\hat{\theta} = \arg\min_{\Sigma} \left\{ G(\theta)' W G(\theta) \right\}$$

where W is the weighting matrix.

4.2.5 Laplace-Type Estimator (LTE)

Since my GMM objective function is a complex nonlinear function of parameters, my GMM objective function yields many local minima which complicate the optimization and can lead wrong policy implications (Knittel and Metaxoglou, 2012). Hence, I employ the Laplace-Type Estimator (LTE), developed by Chernozhukov and Hong (2003). The LTE can be especially useful in my application because it is robust to local minima through utilizing Markov chain Monte Carlo (MCMC) methods after transforming the objective function to the quasi-posterior distribution. For details of the LTE procedure, see Appendix 3.

4.2.6 Identification

Heuristically, the general difference between new and used prices over time will identify α. The general level of prices will identify β; for example, with β fixed at 1, simulations cannot generate price levels which are realistic (for example, above \$50) regardless of other parameters. The shape of price trajectories of individual games will identify μ and σ, the demand parameters. The rate of price decrease in different periods has information about underlying demand distribution; it will rapidly decrease around the region where the slope of the density of the demand is high, and it will decrease slowly around the region where the slope is more flat. The general rate of price decline, specifically the number of periods it takes for the prices to go to the absorbing state, will identify θ, the probability of losing interest in the game.

Note that the μ and σ might not be well-identified when σ is large. This is because as the scale parameter gets larger, the demand distribution increasingly resembles the uniform regardless of the value of μ. In this case, however, the estimates of the μ and σ do not matter anyway in the counterfactual analysis.

4.2.7 Estimation Using Simulated Data

Before I estimate the demand parameters with real data, I firstly estimate my model with *simulated* data for a set of parameter values and a series of realizations of the aggregate error, ξ_t. Given parameters and *i.i.d.* draws of the aggregate shocks, I simulate the equilibrium path. Taking the resulting prices as the data, I run the estimation. I use *i.i.d.* draws from truncated normal distribution with mean and scale parameters equal to (1, 0.4) as the aggregate demand shocks. Table 4.1 shows the results from the estimation. In general, the parameters are precisely estimated, except the posterior for λ has slightly larger variance.[7] Importantly, my model manages to successfully recover the true parameters; for all parameters, the true value lies within the range of standard deviation of the estimates.

[7] I suspect that this is due to suboptimal weighting matrix (identity) in the GMM objective function that I use to generate the results. Improved implementation should yield more efficient estimates.

Table 4.1: Simulation Estimates

Parameter	μ		σ		λ		α		β	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
True	0.700		0.250		0.005		3.500		0.070	
Estimated	0.720	0.136	0.309	0.105	0.006	0.004	3.646	0.564	0.072	0.025

4.3 Estimation Results

I run the estimation with real data using LTE for multiple games and report the results. Table 4.2 shows the parameter point estimates and standard deviations of the posterior draws for each game titles.

The parameters are generally precisely estimated. λ is relatively less precise, as with the simulation results. In general, the estimated demand distributions show various shapes across games; the range of μ is from 0.236 of *Duke Nukem Forever* to 0.826 of *Red Faction Armageddon*. σ also has significant variation, from 0.107 of *Duke Nukem Forever* to 0.677 of *L.A. Noire*, though in general the valuation distributions have highly concentrated mass around the mean. The additional utility consumers get from the new copy versus used copy, which is represented by α, vary across games, suggesting new copies of certain games are more *differentiated* from the used copies than others. A potential reason for this is that games with higher α might have features only applicable to new purchase, such as bonus items which can be redeemed only once. Price sensitivity (β) also varies across games, suggesting that the demand for each game consists of different types of consumers in terms of price sensitivity. For example, games with higher price sensitive demands such as *Castlevania: Lords of Shadow* (0.116), *Duke Nukem Forever (0.148), Marvel vs.*

Table 4.2: Estimation Results

Title	μ		σ		λ		α		β	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
Alpha Protocol	0.702	0.198	0.414	0.236	0.011	0.008	4.114	0.510	0.046	0.016
Backbreaker Football	0.622	0.071	0.218	0.038	0.006	0.004	3.529	0.210	0.099	0.014
Brink	0.796	0.129	0.335	0.115	0.010	0.006	3.422	0.389	0.053	0.018
Bulletstorm	0.772	0.142	0.302	0.131	0.008	0.006	3.398	0.817	0.048	0.012
Castlevania: Lords of Shadow	0.546	0.346	0.215	0.150	0.008	0.007	3.382	0.802	0.116	0.058
Dead Rising 2	0.714	0.234	0.431	0.262	0.016	0.013	3.115	0.722	0.046	0.023
Dragon Age 2	0.726	0.159	0.292	0.088	0.006	0.004	3.233	0.453	0.045	0.019
Duke Nukem Forever	0.236	0.236	0.107	0.061	0.018	0.013	2.872	0.579	0.148	0.032
EA SPORTS MMA	0.812	0.131	0.311	0.105	0.007	0.004	3.512	0.510	0.062	0.020
Fallout: New Vegas	0.749	0.153	0.262	0.096	0.007	0.004	3.073	0.481	0.053	0.015
L.A. Noire	0.481	0.262	0.677	0.058	0.003	0.001	4.238	0.403	0.033	0.007
Lost Planet 2	0.713	0.195	0.500	0.325	0.054	0.028	2.032	0.377	0.033	0.007
Marvel vs. Capcom 3: Fate of Two Worlds	0.621	0.326	0.203	0.131	0.006	0.005	3.236	0.630	0.101	0.036
Medal of Honor	0.767	0.194	0.287	0.115	0.005	0.003	3.849	0.610	0.062	0.028
Mindjack	0.752	0.167	0.263	0.099	0.005	0.003	3.716	0.530	0.067	0.028
ModNation Racers	0.440	0.276	0.281	0.258	0.013	0.010	3.497	0.860	0.084	0.049
Need for Speed: Shift 2 - Unleashed	0.743	0.195	0.507	0.235	0.020	0.012	3.080	0.639	0.046	0.018
Red Faction Armageddon	0.826	0.114	0.333	0.077	0.008	0.005	3.752	0.536	0.058	0.028
Singularity	0.693	0.240	0.235	0.201	0.021	0.015	3.107	0.816	0.091	0.060
Star Wars: The Force Unleashed II	0.611	0.365	0.250	0.144	0.007	0.009	3.839	0.891	0.107	0.032
Test Drive Unlimited 2	0.754	0.158	0.307	0.113	0.009	0.007	3.077	0.604	0.071	0.036
Transformers: War for Cybertron	0.500	0.248	0.343	0.246	0.005	0.006	4.237	0.528	0.068	0.029
TRON: Evolution	0.752	0.173	0.353	0.154	0.016	0.013	3.260	0.885	0.058	0.034
UFC Undisputed 2010	0.695	0.229	0.210	0.116	0.008	0.006	3.426	0.699	0.088	0.029

Capcom 3: Fate of Two Worlds (0.101), and *Star Wars: The Force Unleashed II* (0.107) are all action and fighting games which are more likely to attract younger consumers, who are likely to be more price sensitive.

4.4 Counterfactual Analysis: Elimination of the Used Goods Market

One of the main questions of my study is that the profit implication of the elimination of the used goods market. Since I explicitly include the supply side in my estimation, modifying my model to accommodate this counterfactual is straightforward. First, I remove the used goods related choices from consumers' dynamic decisions. Without the opportunity to sell the used copy I have $Z(\mathbf{p}, \mathbf{x}) = 0$ and the Bellman equations becomes,

$$W(v, \mathbf{p}, \mathbf{x}) = \xi v + \delta(1-\lambda)E\left[W(v, \mathbf{p}', \mathbf{x}')|\mathbf{x}\right] \tag{4.4.1}$$

$$V_j(v, \mathbf{p}, \mathbf{x}) = \begin{cases} \xi\alpha v - \beta p + \delta\left\{(1-\lambda)E\left[W(v, \mathbf{p}', \mathbf{x}')|\mathbf{x}\right]\right\} & (j = 1), \\ \delta E\left[V(v, \mathbf{p}', \mathbf{x}')|\mathbf{x}\right] & (j = 0). \end{cases} \tag{4.4.2}$$

where j denotes buying new copy ($j = 1$) and waiting ($j = 0$), respectively. Note that without the used goods market, $\mathbf{p}$ does not matter anymore after purchase and $W(\cdot)$ becomes just the sum discounted future expected utilities:

$$W(v, \mathbf{p}, \mathbf{x}) = W(v, \mathbf{x}) = \frac{1}{1-\delta(1-\lambda)} \cdot v + (\xi - 1)v$$

New copy marginal consumer condition is (note that I have $\underline{v}' = \overline{v}$)

$$\xi\alpha\underline{v}' - \beta p + \delta\left\{(1-\lambda)E\left[W(\underline{v}', \mathbf{x}')|\mathbf{x}\right]\right\} = \delta E\left[\xi'\alpha\underline{v}' - \beta p' + \delta(1-\lambda)W(\underline{v}', \mathbf{x}'')\middle|\, \mathbf{x}\right] \tag{4.4.3}$$

Hence I have

$$\alpha(\xi-\delta)\underline{v}' + \delta(1-\lambda)\frac{1}{1-\delta(1-\lambda)} \cdot \underline{v}' - \delta^2(1-\lambda)\frac{1}{1-\delta(1-\lambda)} \cdot \underline{v}' = \beta p - \delta\beta E\left[p(\mathbf{x}')\middle|\, \mathbf{x}\right]$$

$$\beta p = \left\{ \alpha(\xi - \delta) + \delta(1-\lambda)(1-\delta)\frac{1}{1-\delta(1-\lambda)} \right\} \cdot \underline{v}' + \delta\beta E\left[p(\mathbf{x}') \middle| \mathbf{x} \right]$$
$$\therefore p = \frac{1}{\beta}\left\{ \alpha(\xi - \delta) + \delta(1-\lambda)(1-\delta)\frac{1}{1-\delta(1-\lambda)} \right\} \cdot \underline{v}' + \delta E\left[p(\mathbf{x}') \middle| \mathbf{x} \right]$$

I follow procedure similar to Appendix 2 to calculate the equilibrium path under the counterfactual scheme.

Since I infer the sales volume for each period as a percentage of the total demand, to evaluate counterfactual profit changes in dollar terms, I need M_i, the *total number of sales* for a game i. That is, the estimation and the policy simulation yield implied sales share in percentage for each period, and I can calculate the implied sales figures by multiplying each share by M_i. With sales and prices for each period, producing implied profit is straightforward. I use total sales of two years since game i's release to approximate M_i[8].

Table 4.3 shows the results from the counterfactual analysis, with games ordered by the percentage change in profit. Note that I keep the fixed initial price of \$59.99 for the counterfactual scheme. The effect of eliminating the used goods market are generally positive for the firm, but *Duke Nukem Forever* shows negative profit changes after the change. On average, the profit increase is about 38%, which is about 4 million dollars. The effects widely differ across games, however. In general, the less popular a game is, in a sense that the game has higher proportion of low valuation consumers, the less the game gets benefited by the structural change. For example, games with lower value of μ such as *Duke Nukem Forever* ($\mu = 0.236$, -0.71%), *L.A.*

[8] In general after two years since release, games have negligent sales figures

Table 4.3: Profit Changes After Eliminating the Used Goods Market

Title	Resale	No Resale	Change (%)	Total Sales	Change ($)†	Genre
Fallout: New Vegas	25.160	42.355	68.34	1,627,206	$27,978	Role-Playing
UFC Undisputed 2010	15.624	25.411	62.64	628,112	$6,147	Fighting
Red Faction Armageddon	26.182	42.390	61.91	104,352	$1,691	Shooter
Bulletstorm	28.692	46.052	60.50	145,355	$2,523	Shooter
EA SPORTS MMA	27.614	43.634	58.01	143,272	$2,295	Fighting
Medal of Honor	28.631	45.198	57.86	1,263,687	$20,935	Shooter
Mindjack	27.497	43.096	56.73	43,239	$674	Shooter
Brink	28.263	44.209	56.42	489,385	$7,803	Shooter
Test Drive Unlimited 2	22.406	34.589	54.37	120,711	$1,470	Racing
Dragon Age 2	28.661	43.500	51.77	569,159	$8,445	Role-Playing
Singularity	13.481	19.891	47.55	173,231	$1,110	Shooter
Lost Planet 2	3.908	5.733	46.69	290,938	$530	Shooter
TRON: Evolution	24.497	34.832	42.19	152,878	$1,580	Action
Marvel vs. Capcom 3: Fate of Two Worlds	14.443	19.424	34.49	655,069	$3,263	Fighting
Backbreaker Football	14.754	19.408	31.54	119,115	$554	Sports
Dead Rising 2	27.896	35.615	27.67	522,010	$4,029	Action
Castlevania: Lords of Shadow	16.146	20.445	26.63	285,359	$1,226	Action
Need for Speed: Shift 2 - Unleashed	25.270	31.630	25.17	119,927	$762	Racing
Star Wars: The Force Unleashed II	15.812	19.368	22.49	530,487	$1,886	Action
Alpha Protocol	35.586	42.760	20.16	152,596	$1,094	Role-Playing
ModNation Racers	22.899	26.451	15.51	367,544	$1,305	Racing
Transformers: War for Cybertron	28.070	29.041	3.46	186,963	$181	Action
L.A. Noire	30.172	30.606	1.44	1,214,088	$527	Role-Playing
Duke Nukem Forever	22.432	22.271	-0.72	267,026	($43)	Shooter
Average	23.087	31.996	38.87	423,821	$4,082	

†: Figures in thousands

Noire ($\mu = 0.481$, 1.43%), and *Transformers: War for Cybertron* ($\mu = 0.500$, 3.46%) get small or negative profit increase. This is because the benefits from the structural change mostly come from the increase in initial sales where the prices are still high; without the opportunity of buying from the used goods market, more high valuation consumers buy in the earlier period, because their future expected prices are higher in the absence of the used goods market. However, the initial sales does not increase much for a game which does not have many high valuation consumers in its demand to start with. In addition, since $W(\cdot)$ consists of the utility from the sum discounted flow utility and the used goods sales opportunity, for given v and λ, the absence of the resale option decreases the value of buying. Moreover, for consumers with lower v, the utility from this option has relatively bigger share in $W(\cdot)$. Hence, their expected utility from buying suffers more from the absence of the option and their willingness to pay decreases more. Thus a game with higher mass of lower valuation consumers would have smaller profit increase after the structural change, and for some cases the profit change can be negative. This suggests uniformly removing the used goods market can be a suboptimal policy; the optimal strategy for games with demand consisting of large proportion of high valuation consumers is to eliminate the resale market completely (e.g., by granting ownership exclusively through downloading), whereas it can be more profitable to allow resale for games with high concentration of lower valuation consumers.

Chapter 5

Conclusion

This dissertation investigates the impact of the used game market on equilibrium market outcomes and the implications of eliminating it in the video game industry. I develop a new model which incorporates inter-temporal price discrimination by producers, a used goods market, rational expectations by both consumers and game producers, and market equilibria for both new and used games. To solve the computational challenge comes with modeling the supply side equilibrium, I develop a computationally tractable utility specification, which also allows me to accommodate continuous consumer heterogeneity and non-linear price expectations. Given the lack of sales data, I use the conditions from the supply side equilibrium to identify the underlying demand distribution without sales information.

Using this model, I estimate the game-specific demand for multiple video games released in the U.S. market. The results show significant variation in demand across games, especially for the shape of the demand distribution. I run a counterfactual analysis where I evaluate the profit change for each video game as a result of eliminating the resale market. The counterfactual results suggest that eliminating the used goods market yields significant profit increase for producers, but the size of the effects varies significantly across games, depending on the shape of the demand distribution. In fact, allowing resales can even increase profits for small number of games, suggesting differentiated optimal strategies regarding the used goods market. It has implications for platform producers such as Microsoft, as instead of employing a uniform policy to restrict the used goods market, it would be better to allow each individual producers to decide their own policies.

As a future extension, my model can be extended to incorporate additional information such as new game sales data. With the new information, I can accommodate additional aggregate shocks and make the model more flexible in fitting the data. Another potential extension could be allowing the hazard of losing interest in a game to be a function of time, $\lambda(\tau)$, instead of assuming λ to be a constant as discussed in Chapter 3. Another extension would be making the utility from purchasing a new game, α, time-varying.

Appendices

Appendix 1

Derivation of the Used Game Price Equation3.2.3

Given the proposition 1 and 2, consumer's decision after purchase is quite simple; they will hold the game and gets flow utility of v until they lose interest, and they sell at p_u:

$$W(v, \mathbf{x}) = \xi v + \delta E \left[(1-\lambda) W(v, \mathbf{x}') + \lambda \beta p_u(\mathbf{x}') \middle| \mathbf{x} \right] \tag{1.0.1}$$

Then the indifference consumer condition is:

$$\begin{aligned} \xi \underline{v}' - \beta p_u(\mathbf{x}) + \delta E \left[(1-\lambda) W(\underline{v}', \mathbf{x}') + \lambda \beta p_u(\mathbf{x}') | \mathbf{x} \right] &= \\ \delta E \left[\xi' \alpha \underline{v}' - \beta p(\mathbf{x}') + \delta \left\{ (1-\lambda) W(\underline{v}', \mathbf{x}'') + \lambda \beta p_u(\mathbf{x}'') \right\} \middle| \mathbf{x} \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad & -\beta p_u(\mathbf{x}) + \underbrace{\xi \underline{v}' + \delta E \left[(1-\lambda) W(\underline{v}', \mathbf{x}') + \lambda \beta p_u(\mathbf{x}') | \mathbf{x} \right]}_{= W(\underline{v}', \mathbf{x})} \\ & = \delta E \left[(\alpha - 1) \xi' \underline{v}' - \beta p(\mathbf{x}') + \underbrace{\xi' \underline{v} + \delta \left\{ (1-\lambda) W(\underline{v}', \mathbf{x}'') + \lambda \beta p_u(\mathbf{x}'') \right\}}_{= W(\underline{v}', \mathbf{x}')} \middle| \mathbf{x} \right] \\ & \Rightarrow \beta p_u(\mathbf{x}) - W(\underline{v}', \mathbf{x}) = -\delta E \left[(\alpha - 1) \xi' \underline{v}' + W(\underline{v}', \mathbf{x}') - \beta p(\mathbf{x}') \middle| \mathbf{x} \right] \\ & \Rightarrow p_u(\mathbf{x}) = \beta^{-1} \cdot \left\{ -\delta(\alpha - 1) \underline{v}' + W(\underline{v}', \mathbf{x}) - \delta E \left[W(\underline{v}', \mathbf{x}') \middle| \mathbf{x} \right] \right\} + \delta E \left[p(\mathbf{x}') \middle| \mathbf{x} \right] \end{aligned} \tag{1.0.2}$$

Note that in case of $s = 0$, (at the release) the new copy buying decision becomes automatically dynamic because $\underline{v}' = F^{-1}(F(\bar{v}) - \lambda s) = \bar{v}$.

For any period, the probability of losing interest in the game is λ and consumers' expected utility from any given period is $E\left[(1-\lambda)v + \lambda\beta p_u(\mathbf{x}')|\mathbf{x}\right]$. Also, when they lose interest in the game their future values are simply zero. Hence the value becomes:

$$\begin{aligned} W(v,\mathbf{x}) &= \xi v + \delta E\left[(1-\lambda)\left\{\xi' v + \delta \cdot \left\{(1-\lambda)\left\{\xi'' v + \cdots\right\} + \underbrace{\lambda\beta p_u(\mathbf{x}'')}_{\text{lose interest in 2+}}\right\}\right\}\right. \\ &\quad + \left.\underbrace{\lambda\beta p_u(\mathbf{x}')}_{\text{lose interest in 1+}}\right|\mathbf{x}\Bigg] \end{aligned} \tag{1.0.3}$$

where $\mathbf{x}'$ and $\mathbf{x}''$ denote the 1 and 2 periods future's state, respectively. I can group terms into one with v and the other with p_u:

$$\begin{aligned} W(v,\mathbf{x}) &= \xi v + \delta(1-\lambda)\left[v + \delta(1-\lambda)\left\{v + \cdots\right\}\right] \\ &\quad + \delta E\left[\lambda\beta p_u(\mathbf{x}') + \delta \cdot \left\{(1-\lambda)\lambda\beta p_u(\mathbf{x}'') + \cdots\right\}\Big|\,\mathbf{x}\right] \end{aligned} \tag{1.0.4}$$

Hence I have,

$$W(v,\mathbf{x}) = \frac{1}{1-\delta(1-\lambda)} \cdot v + (\xi - 1)v + \delta\lambda E\left[\beta p_u(\mathbf{x}') + \delta \cdot \left\{(1-\lambda)\beta p_u(\mathbf{x}'') + \cdots\right\}\Big|\,\mathbf{x}\right] \tag{1.0.5}$$

and

$$
\begin{aligned}
W(v,\mathbf{x}) &= \left((\xi-1)+\frac{1}{1-\delta(1-\lambda)}\right)\cdot v \\
&+ E\left[\delta\lambda\beta p_u(\mathbf{x}_1)+\cdot\sum_{\tau=1}^{\infty}(\delta\cdot(1-\lambda))^{\tau}\,\delta\lambda\beta p_u(\mathbf{x}_{\tau+1})\middle|\,\mathbf{x}\right] \quad (1.0.6)
\end{aligned}
$$

Hence, I can convert this to a infinite geometric series with common ratio $\delta(1-\lambda)$ and scale factor v and sum of discounted future used good prices:

$$
W(v,\mathbf{x}) = \left((\xi-1)+\frac{1}{1-\delta(1-\lambda)}\right)\cdot v + E\left[\sum_{\tau=0}^{\infty}(\delta(1-\lambda))^{\tau}\,\delta\lambda\beta p_u(\mathbf{x}_{\tau+1})\middle|\,\mathbf{x}\right] \quad (1.0.7)
$$

then I can separate $W(v,\mathbf{x})$ into two part:

$$
W(v,\mathbf{x}) = W_1(v)+W_2(\mathbf{x})
$$

where

$$
\begin{aligned}
W_1(v) &= \left((\xi-1)+\frac{1}{1-\delta(1-\lambda)}\right)\cdot v, \\
W_2(\mathbf{x}) &= E\left[\sum_{\tau=0}^{\infty}(\delta(1-\lambda))^{\tau}\,\delta\lambda\beta p_u(\mathbf{x}_{\tau+1})\middle|\,\mathbf{x}\right] \quad (1.0.8) \\
&= E\left[\delta\lambda\beta p_u(\mathbf{x}')+\delta(1-\lambda)W_2(\mathbf{x}')\middle|\,\mathbf{x}\right]
\end{aligned}
$$

This form is intuitive in a sense that a consumer's value from keeping the game is weighted sum of the future discounted utility from holding the game and future used good price, and the weight depends on the probability of losing interest in the game. Note that $W_2(\mathbf{x})$ is the same for everybody. Then the used copy price equation becomes:

$$p_u(\mathbf{x}) = \beta^{-1}\left\{-\delta(\alpha-1)\underline{v}' + W(\underline{v}', \mathbf{x}) - \delta E\left[W(\underline{v}', \mathbf{x}')\middle|\, \mathbf{x}\right]\right\} + \delta E\left[p(\mathbf{x}')\middle|\, \mathbf{x}\right]$$

$$\begin{aligned}\Rightarrow p_u(\mathbf{x}) &= \beta^{-1}\left\{-\delta(\alpha-1)\underline{v}' + W_1(\underline{v}') + W_2(\mathbf{x}) - \delta E\left[W_1(\underline{v}') + W_2(\mathbf{x}')\middle|\, \mathbf{x}\right]\right\} \\ &+ \delta E\left[p(\mathbf{x}')\middle|\, \mathbf{x}\right]\end{aligned}$$

$$\begin{aligned}\Rightarrow p_u(\mathbf{x}) &= \beta^{-1}\left\{(\xi-1) + \frac{1-\delta}{1-\delta(1-\lambda)} - \delta(\alpha-1)\right\}\underline{v}' \\ &+ \beta^{-1}\left\{W_2(\mathbf{x}) - \delta E\left[W_2(\mathbf{x}')\middle|\, \mathbf{x}\right]\right\} + \delta E\left[p(\mathbf{x}')\middle|\, \mathbf{x}\right]\end{aligned}$$

I can simplify $W_2(\mathbf{x}) - \delta E\left[W_2(\mathbf{x}')\middle|\, \mathbf{x}\right]$ more:

$$\begin{aligned}W_2(\mathbf{x}) - \delta E\left[W_2(\mathbf{x}')\middle|\, \mathbf{x}\right] &= E\left[\delta\lambda\beta p_u(\mathbf{x}') + \delta(1-\lambda)W_2(\mathbf{x}')\middle|\, \mathbf{x}\right] - \delta E\left[W_2(\mathbf{x}')\middle|\, \mathbf{x}\right] \\ &= E\left[\delta\lambda\beta p_u(\mathbf{x}') - \delta\lambda W_2(\mathbf{x}')\middle|\, \mathbf{x}\right]\end{aligned}$$

$$\begin{aligned}\therefore p_u(\mathbf{x}) &= \beta^{-1}\left\{\left((\xi-1) + \frac{1-\delta}{1-\delta(1-\lambda)} - \delta(\alpha-1)\right)\underline{v}' - \delta\lambda E\left[W_2(\mathbf{x}')\middle|\, \mathbf{x}\right]\right\} \\ &+\delta E\left[p(\mathbf{x}')\middle|\, \mathbf{x}\right] + \delta\lambda E\left[p_u(\mathbf{x}')\middle|\, \mathbf{x}\right] \qquad (1.0.9)\end{aligned}$$

Appendix 2

Steps for the Solution of the Rational Expectations Equilibrium

In order to solve this dynamic programming problem, I need to solve for $\Pi(\mathbf{x})$, the firm's value, $p(\mathbf{x})$, the price policy function for the firm, $p_u(\mathbf{x})$, the market clearing price, and $W_2(\mathbf{x})$, the portion of future used goods selling option among the consumers' value from holding the game. The following steps describe the value iteration procedure.

Start with $Ep^0(\mathbf{x}) = 0$, $Ep_u^0(\mathbf{x}) = 0$, $EW_2^0(\mathbf{x}) = 0$, and $E\Pi^0(\mathbf{x}) = 0$. Loop over i, $i \in \mathbf{N}$, until $Ep^i(\mathbf{x}), Ep_u^i(\mathbf{x}), EW_2^i(\mathbf{x}), E\Pi^i(\mathbf{x})$ all converge (where i is the number of iteration). In each iteration,

1. Loop over each grid point $\mathbf{x} = (s, \underline{v}) \in X$. For each $\mathbf{x}$, calculate the maximized profit, $\Pi^i(\mathbf{x}) = \max_{\overline{v}} p^i(\overline{v}, \mathbf{x}) \cdot q + \delta E\left[\Pi^{i-1}(\mathbf{x}') \middle| \mathbf{x}\right]$. Specifically, for any given state $\mathbf{x}, \xi$ and control $\overline{v}$, I can calculate the implied profit as follows:

   (a) (given $\overline{v}$) update state variables: $\mathbf{x}' = (s', \underline{v}')$ with the following state transition rules:

   $$s' = s + q, \text{ where } q = F\left(\underline{v}\right) - F\left(\overline{v}\right)$$

   $$\underline{v}' = \max\left\{0, F^{-1}\left(F(\overline{v}) - \lambda \cdot s\right)\right\}$$

(b) Interpolate for $Ep_u^{i-1}(\mathbf{x}')$, $EW_2^{i-1}(\mathbf{x}')$, $Ep^{i-1}(\mathbf{x}')$, and $E\Pi^{i-1}(\mathbf{x}')$.

(c) calculate used copy prices,

$$\begin{aligned} p_u^i(\mathbf{x},\xi) &= \beta^{-1}\left((\xi-1)+\frac{1-\delta}{1-\delta(1-\lambda)}-\delta(\alpha-1)\right)\underline{v}' \\ &\quad -\beta^{-1}\delta\lambda E\left[W_2^{i-1}(\mathbf{x}')|\mathbf{x}\right] \\ &\quad +\delta E\left[p^{i-1}(\mathbf{x}')|\mathbf{x}\right]+\delta\lambda E\left[p_u^{i-1}(\mathbf{x}')|\mathbf{x}\right] \end{aligned}$$

(d) calculate new copy price and profit:

$$p^i(\mathbf{x},\xi)=\beta^{-1}\xi\cdot(\alpha-1)\cdot\overline{v}+p_u^i(\mathbf{x})$$

$$\Pi^i(\mathbf{x},\xi)=p^i(\mathbf{x},\xi)\cdot q+\delta E\left[\Pi^{i-1}(\mathbf{x}')\big|\,\mathbf{x}\right]$$

(e) when maximization is done, update consumers' value,

$$W_2^i(\mathbf{x})=E\left[\delta\lambda p_u^{i-1}(\mathbf{x}')+\delta(1-\lambda)W_2^{i-1}(\mathbf{x}')\big|\,\mathbf{x}\right]$$

2. Calculate interpolation coefficients for $E\left[p^i(\mathbf{x})\right], E\left[p_u^i(\mathbf{x})\right], E\left[W_2^i(\mathbf{x})\right]$, and $E\left[\Pi^i(\mathbf{x})\right]$

Appendix 3

Details of the Laplace-Type Estimator

Let $L_n(\theta)$ denote the GMM objective function,

$$L_n(\theta) = -n\frac{1}{2}g_n(\theta)'Wg_n(\theta)$$

where $g_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} m_i(\theta)$ and $m_i(\theta)$ is the value of moments for observation i. Following Chernozhukov and Hong (2003), I transform it to the quasi-posterior distribution $p_n(\theta)$,

$$p_n(\theta) = \frac{\exp(L_n(\theta))\pi(\theta)}{\int_\lambda \exp(L_n(\theta))\pi(\theta)d\theta}$$

which is proportional to

$$p_n(\theta) \propto \exp(L_n(\theta))\pi(\theta)$$

Then I employ the following Metropolis-Hastings algorithm with the quasi-posterior:

Step 1 Choose a starting value θ^0

Step 2 Generate the candidate θ' from $q(\theta'|\theta^j)$

Step 3 Update θ^{j+1} from θ^j for $j = 1, 2, \ldots,$ using

$$\theta^{j+1} = \begin{cases} \theta' & \text{with probability } \rho(\theta^j, \theta'), \\ \theta^j & \text{with probability } 1 - \rho(\theta^j, \theta'), \end{cases}$$

where

$$\rho(x,y) = \inf\left(\frac{\exp(L_n(y))\pi(y)q(x|y)}{\exp(L_n(x))\pi(x)q(y|x)}, 1\right)$$

I use the standard normal distribution for $q(x|y)$ and the uniform prior for $\pi(\theta)$. I dynamically choose the tune parameters so the acceptance rate of chain is on average 0.3. I make 20,000 draws with the MCMC chain, and discard first 5,000 draws for burn-in.

Bibliography

AKERLOF, G. (1970): "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84(3), 488–500.

BELLMAN, R. (1957): *Dynamic Programming.* Princeton University Press.

BERRY, S., J. LEVINSOHN, AND A. PAKES (1995): "Automobile Prices in Market Equilibrium," *Econometrica*, 63(4), 841.

BERTSEKAS, D. (1995): *Dynamic Programming and Optimal Control.* Athena Scientific.

BESANKO, D., AND W. WINSTON (1990): "Optimal Price Skimming by a Monopolist Facing Rational Consumers," *Management Science*, 36(5), 555–567.

BRESNAHAN, T. F. (1981): "Departures From Marginal-Cost Pricing in the American Automobile Industry: Estimates for 1977-1978," *Journal of Econometrics*, 17(2), 201–227.

BUHMANN, M. (2000): "Radial Basis Functions," *Acta Numerica*, 9(January 2000).

——— (2003): *Radial Basis Functions: Theory and Implementations.* Cambridge university press.

Bulow, J. (1982): "Durable-Goods Monopolists," *Journal of Political Economy*, 90(2), 314–332.

Chernozhukov, V., and H. Hong (2003): "An MCMC Approach to Classical Estimation," *Journal of Econometrics*, 115(2), 293–346.

Coase, R. (1972): "Durability and Monopoly," *Journal of Law and Economics*, 15(1), 143.

Derdenger, T. (2014): "Technological Tying and the Intensity of Price Competition: An Empirical Analysis of the Video Game Industry," *Quantitative Marketing and Economics*, pp. 1–34.

Derdenger, T., and V. Kumar (2013): "The Dynamic Effects of Bundling as a Product Strategy," *Marketing Science*, (February 2014).

Desai, P., O. Koenigsberg, and D. Purohit (2004): "Strategic Decentralization and Channel Coordination," *Quantitative Marketing and Economics*, 2(1), 5–22.

Desai, P. S., and D. Purohit (1999): "Competition in Durable Goods Markets: The Strategic Consequences of Leasing and Selling," *Marketing Science*, 18(1), 42–58.

Farias, V., D. Saure, and G. Y. Weintraub (2012): "An Approximate Dynamic Programming Approach to Solving Dynamic Oligopoly Models," *RAND Journal of Economics*, 43(2), 253–282.

Fasshauer, G. E. (2007): *Meshfree Approximation Methods With MATLAB.* World Scientific.

Feenstra, R. C., and J. A. Levinsohn (1995): "Estimating Markups and Market Conduct With Multidimensional Product Attributes," *Review of Economic Studies*, 62(1), 19–52.

Goettler, R. L., and B. R. Gordon (2011): "Does AMD Spur Intel to Innovate More?," *Journal of Political Economy*, 119(6), 1141–1200.

Gordon, B. R. (2009): "A Dynamic Model of Consumer Replacement Cycles in the PC Processor Industry," *Marketing Science*, 28(5), 846–867.

Gowrisankaran, G., and M. Rysman (2012): "Dynamics of Consumer Demand for New Durable Goods," *Working Paper.*

Habermann, C., and F. Kindermann (2007): "Multidimensional Spline Interpolation: Theory and Applications," *Computational Economics*, 30(2), 153–169.

Hansen, L. P., and K. J. Singleton (1982): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50(5), 1269–1286.

Hendel, I., and A. Lizzeri (1999a): "Adverse Selection in Durable Goods Markets," *American Economic Review.*

Hendel, I., and A. Lizzeri (1999b): "Interfering With Secondary Markets," *RAND Journal of Economics*, 30(1), 1–21.

Huang, S., Y. Yang, and K. Anderson (2001): "A Theory of Finitely Durable Goods Monopoly With Used-Goods Market and Transaction Costs," *Management Science*, 47(11), 1515–1532.

Imai, S., N. Jain, and A. Ching (2009): "Bayesian Estimation of Dynamic Discrete Choice Models," *Econometrica*, 77(6), 1865–1899.

Ishihara, M., and A. Ching (2012): "Dynamic Demand for New and Used Durable Goods Without Physical Depreciation: The Case of Japanese Video Games," *Working Paper*.

Judd, K. L. (1988): *Numerical Methods in Economics*. The MIT Press.

Karp, L. (1996): "Depreciation Erodes the Coase Conjecture," *European Economic Review*, 40(2), 473–490.

Knittel, C., and K. Metaxoglou (2012): "Estimation of Random-Coefficient Demand Models: Two Empiricists' Perspective," *Review of Economics and Statistics*.

Lee, R. (2013): "Vertical Integration and Exclusivity in Platform and Two-Sided Markets," *American Economic Review*.

Magnac, T., and D. Thesmar (2002): "Identifying Dynamic Discrete Decision Processes," *Econometrica*, 70(2), 801–816.

Nair, H. (2007): "Intertemporal Price Discrimination With Forward-Looking Consumers: Application to the US Market for Console Video-Games," *Quantitative Marketing and Economics*, 5(3), 239–292.

Petrin, A. (2002): "Quantifying the Benefits of New Products: The Case of the Minivan," *Journal of Political Economy*, 110(4), 705–729.

Rao, R. S., O. Narasimhan, and G. John (2009): "Understanding the Role of Trade-Ins in Durable Goods Markets: Theory and Evidence," *Marketing Science*, 28(5), 950–967.

Roth, A. E. (ed.) (1985): *Game-Theoretic Models of Bargaining.* Cambridge University Press, Cambridge.

Rust, J. (1986): "When Is It Optimal to Kill Off the Market for Used Durable Goods?," *Econometrica*, 54(1), 65.

Shiller, B. (2012): "Digital Distribution and the Prohibition of Resale Markets for Information Goods," *Working Paper.*

Shulman, J. D., and A. T. Coughlan (2007): "Used Goods, Not Used Bads: Profitable Secondary Market Sales for a Durable Goods Channel," *Quantitative Marketing and Economics*, 5(2), 191–210.

Song, I., and P. K. Chintagunta (2003): "A Micromodel of New Product Adoption With Heterogeneous and Forward-Looking Consumers: Application to the Digital Camera Category," *Quantitative Marketing and Economics*, 1(4), 371–407.

Stokey, N. (1979): "Intertemporal Price Discrimination," *Quarterly Journal of Economics*, 93(3), 355–371.

Stokey, N. (1981): "Rational Expectations and Durable Goods Pricing," *Bell Journal of Economics*, 12(1), 112–128.

Sudhir, K. (2001): "Structural Analysis of Manufacturer Pricing in the Presence of a Strategic Retailer," *Marketing Science*, 20(3), 244–264.

Swan, P. (1970): "Durability of Consumption Goods," *American Economic Review*, 60(5), 884–894.

Thomadsen, R. (2005): "The Effect of Ownership Structure on Prices in Geographically Differentiated Industries," *RAND Journal of Economics*, 36(4), 908–929.

Yao, S., C. F. Mela, J. Chiang, and Y. Chen (2012): "Determining Consumers' Discount Rates With Field Studies," *Journal of Marketing Research*, 49(6), 822–841.

Yin, S., S. Ray, H. Gurnani, and A. Animesh (2010): "Durable Products With Multiple Used Goods Markets: Product Upgrade and Retail Pricing Implications," *Marketing Science*, 29(3), 540–560.