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The Measurement of Small Angles Using Optical-Reading Theodolites and American-Design Transits

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

SAM HOUSTON

Cultivated mind is the guardian genius of Democracy, and while guided and controlled by virtue, the noblest attribute of man. It is the only dictator that freemen acknowledge, and the only security which freemen desire.

MIRABEAU B. LAMAR

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I. Preface

The study was conducted under a research project titled "Investigation of the Use of American-Design Repeating Transits (ordinary transits) for Measuring Distance by the Subtense Bar Method," and was sponsored by the Bureau of Engineering Research, The University of Texas.

The purpose was to develop a technique for making subtense bar measurements, using the ordinary American-design transits, to the same accuracy as may be obtained with the optical-reading theodolites such as the Wild T2 and the Kern DKM2. Also it was desired to determine if such measurements could be made as rapidly with the American-design transits as with the optical-reading theodolites.

It was proposed that the study be made in the field since it was assumed that performance of instruments under laboratory conditions was already well established and that errors would be due largely to field conditions.

The study did not develop as intended. Due to obvious systematic errors that were apparent after some of the first field comparisons of the two types of instruments, it was decided to move into the laboratory and conduct a study to determine the sources of these systematic errors. The evaluation of the optical-reading theodolites and the American-design repeating transits for measuring small angles finally became the purpose of the study.

II. Conclusions

Small angles may be measured with the American-design repeating transits with a probable error of less than one second. The instrument when operated according to the manufacturer's instructions shows no systematic errors.

Small angles when measured with the optical-reading theodolites may have a systematic error of as much as eight seconds. The systematic error results from inaccuracies in the reading micrometer and will appear in the same way when measuring angles of any size. These conclusions are based on a study of three optical-reading theodolites.

III. Procedure

All readings reported in this study were made by the author of this report, who will be referred to as the observer. The sets of readings taken as a basis for the curves were used without discarding or adjusting any one of the readings. This procedure without doubt produced erratic points for the curves but it is thought that the observer should not attempt to distinguish between blunders and errors as they appear in this study.

An examination of literature discloses no evaluation of the American-design transits for measuring small angles such as those encountered in distance measurements by the subtense bar method.

The manufacturers of optical-reading theodolites such as the Wild T2 and the Kern DKM2 state in general that when an experienced operator uses these instruments angles may be easily measured with an error of one second or less. Also articles written by practicing surveyors lead the observer to assume the surveyor is reporting measurements made with an error of one second or less when using these instruments. The observer originally assumed that angles could be measured with an error of one second or less with optical-reading theodolites such as the Wild T2 and the Kern DKM2, and he believes that most users of the optical-reading theodolites assume that the instruments are precise and that the largest contributing source of error in measuring angles is the inability of the instrument man to point the telescope and set the reading micrometer.

The data and results are presented in graphic form. This is considered desirable since a reasonable scale allows us to read the data and results to a higher precision than the precision of the measurement. Only the data and results for the final steps in the study are presented, as these are the only steps considered necessary as a basis for the essential results of this study.

Several American-design repeating transits were used. There were no significant differences between these transits. Data and results from only two of the American-design transits will be included in this report.

IV. Data and Results

Transit A1 is an American-design repeating transit owned by The University of Texas. It is about seven years old and has been in continuous use by students. The least count of the vernier is one minute. The vernier was read by using a pocket magnifier.

Transit A2 is an American-design repeating transit which was loaned by the manufacturer for the purpose of this study. The least count of the vernier is 20 seconds. The vernier was read by using an attached magnifier.

Theodolite E1 is an optical-reading theodolite owned by The University of Texas. It is about ten years old and has been used by students. The smallest division on the circle is 10 minutes. The total run of the optical micrometer is 10 minutes with the smallest division one second. Readings may be estimated to 1/10 second. The instrument is a Kern DKM2.

Theodolite E2 is an optical-reading theodolite which was loaned by the distributor for the purpose of this study. The total run of the optical micrometer is 10 minutes with the smallest division one second. Readings may be estimated to 1/10 second. The instrument is a Wild T2.

Theodolite E3 is, as theodolite E1, a Kern DKM2 loaned by the distributor for the purpose of this study.

The optical micrometer may, *for the purpose of this report,* be described as follows. (See the diagram on this page.) The line of sight of a reading telescope (A), focused on the circle to be read, is the index for reading. An optical glass plate (B) is in the line of sight of the reading telescope between the objective and the circle.

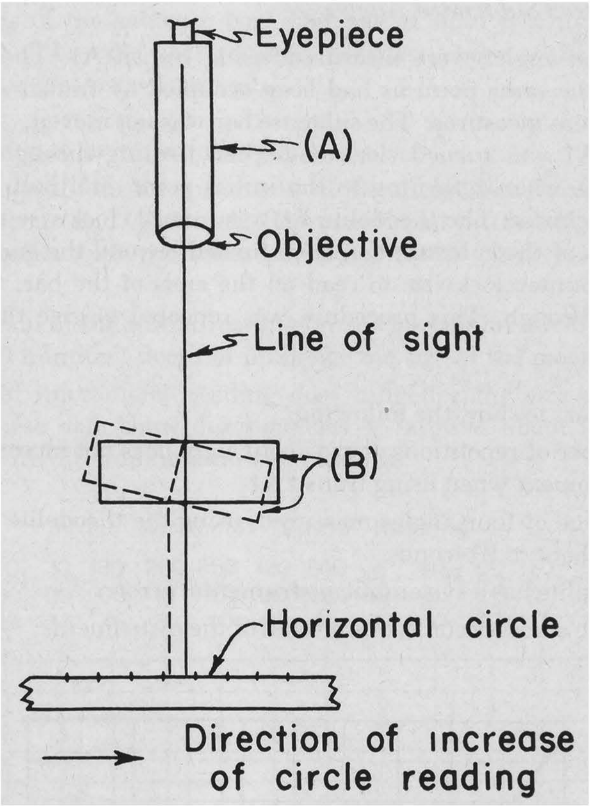


Diagram of Optical Micrometer

The index, with the glass plate in the zero position, will usually fall between two of the marks on the circle. (The zero position is shown by the solid-line outline in the sketch.) The glass plate may be rotated, using a thumb screw, to the dotted-line outline. Then the index is at coincidence with a mark on the circle. The amount of rotation of the glass plate from the zero position is the measure of the angle from the zero position of index to the mark on the circle. The amount of rotation is transcribed into angle movement of the index and read directly from the micrometer scale.

The observer practiced setting and reading the optical micrometer, and then measured his ability to set and read the micrometer. The observer's probable error in setting and reading was slightly less than one second. Thus it was obvious that, when using the optical-reading theodolite, it should be easy to measure angles with an error of less than one second if the average of several measurements is taken as the correct angle.

The data presented in Figure 1, Figure 2, and in Figure 11 were taken in the field, using a two-meter subtense bar to subtend the angle. The data for all of the other figures were taken in the laboratory, using a metal scale to subtend the angle. The line of sight was perpendicular to the scale at approximately the midpoint of the section of scale used for the set of readings. The instrument was approximately 13 feet from the scale.

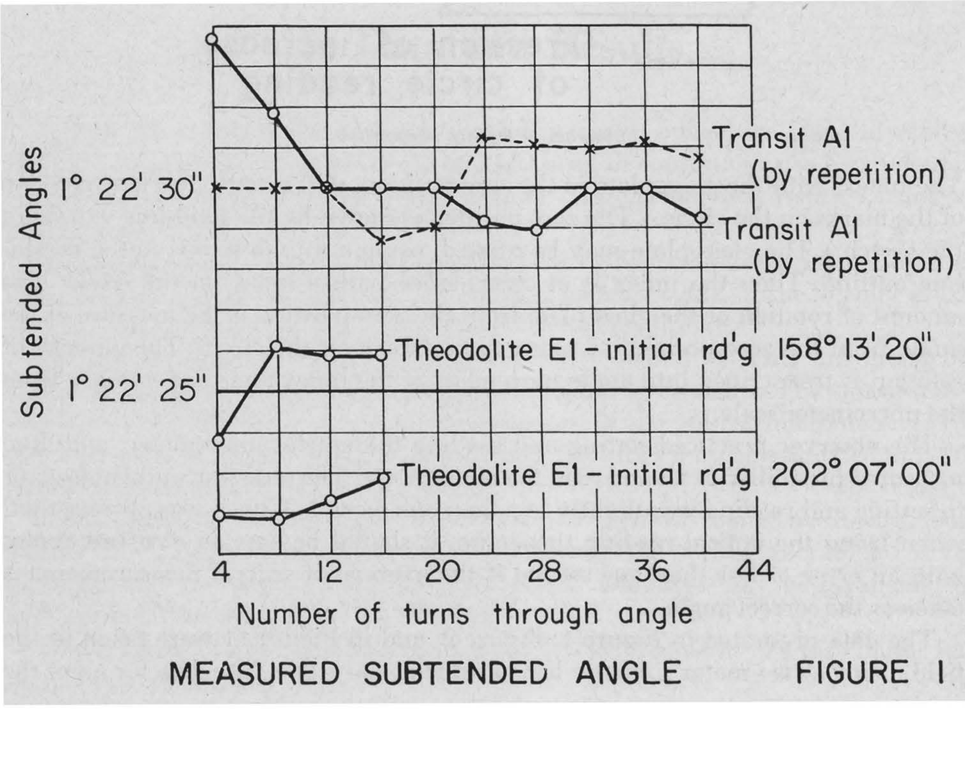
Figure 1: Measured Subtended Angles

The subtended angles were measured, using transit A1. Then theodolite E1 was placed on the same point as had been occupied by transit A1 and the subtended angles were measured. The subtense bar was not moved.

The transit A1 was turned clockwise when turning through the angle and counterclockwise when returning to the initial point, and both tangent screws were turned clockwise. The theodolite E1 was turned clockwise and angles were read to the ends of the subtense bar, then turned beyond the end of the bar and brought back counterclockwise to read on the ends of the bar, thus giving two angles turned through. This procedure was repeated to give the total turns as indicated.

Figure 1 appears to show the following.

a. The number of repetitions above about eight does not increase the precision of measurement when using transit A1.
b. The average of four angles measured, using the theodolite E1, gives a precision of about ± 2 seconds.
c. The theodolite has a systematic instrumental error.
d. Systematic errors occur in one or both of the instruments.



MEASURED SUBTENDED ANGLE FIGURE 1

Figure 2: Measured Subtended Angles

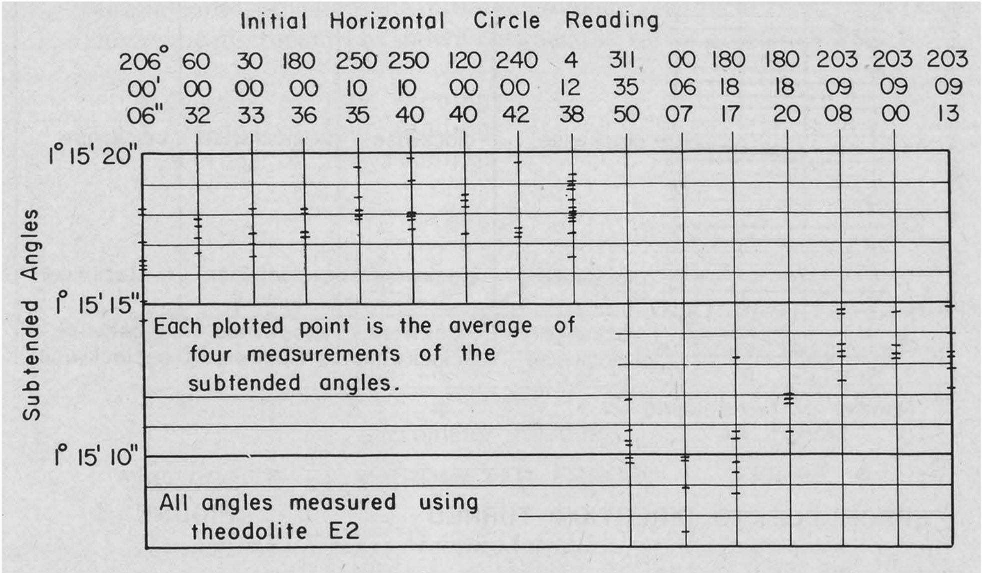
The data were collected during two different working days. The instrument and subtense bar were carefully placed the second day to the same position as on the first day. A change in subtended angle due to incorrect placing on the second day was not apparent. The theodolite E2 was turned clockwise and angles were read to the ends of the subtense bar. The line of sight was turned beyond the end of the bar and brought back counterclockwise to read at the ends of the bar, thus giving two angles turned through.

Figure 2 appears to show the following.

a. The average of four measurements of the angle has a maximum accidental error of about ± 1½ seconds. The maximum discrepancy for a group of measurements with the same initial horizontal circle reading is about 3 seconds.

b. The position of the initial reading on the horizontal circle (degree or multiple of 10 minutes) does not influence the size of the measured angle.

c. The initial micrometer reading does influence the size of the measured angle. These data show discrepancies as large as about 9 seconds due to starting with different micrometer readings.



MEASURED SUBTENDED ANGLES FIGURE 2

Figure 3: Error Due to Direction Turned

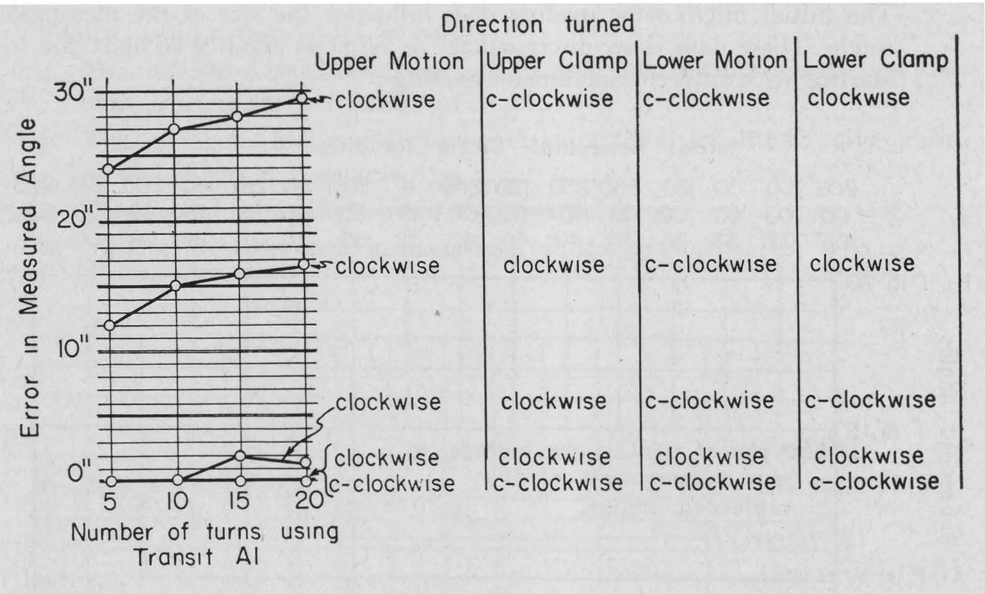
All measurements were made using transit A1. A complete circle was measured by repetition, taking readings on the fifth, tenth, fifteenth, and twentieth turn through the angle. A collimator was used to mark the point on the circle at which to start and end the 360-degree angle. The error is the measured value of the circle minus 360 degrees.

Figure 3 appears to show the following.

a. The direction of turning does affect the accuracy of the angle measurement; the instrument can be operated to produce zero error.

b. This source of error should be studied to determine the exact cause and to produce a remedy. The observer, based on additional work done during this study, believes that the cause is not in the tangent screw threads as suggested by some. The transit A1 produced the largest error of any transit studied. Most of the American-design transits produced maximum errors from this source of only about two seconds. The observer believes the error is of the same magnitude for angles of any size as for a complete circle. The observer did not make a quantitative study of the theodolites but believes, from superficial tests, they will show similar errors.

c. This source of error can explain some of the discrepancy between angles measured with transit A1 and the same angles measured with theodolite E1 as shown in Figure 1.



ERROR DUE TO DIRECTION TURNED FIGURE 3

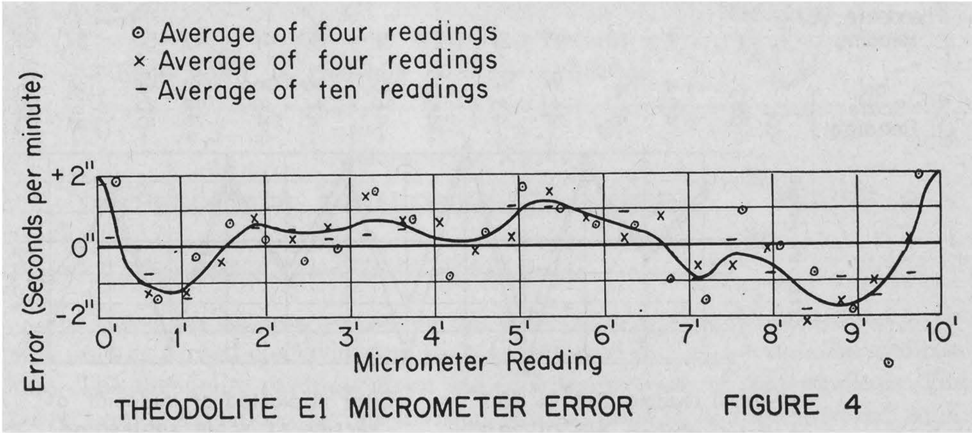
Figure 4: Theodolite E1 Micrometer Error

The errors as plotted were determined by measuring in the laboratory the angles between successive points on a scale with divisions of 1/50 inch. The angle subtended by one division was about 26 seconds. Readings were taken at each point on the scale as turned clockwise and then as returned to the beginning point in a counterclockwise direction. The average of four readings on each point was used to determine errors for two sets. The average of ten readings was used for a third set. The correct angle subtended by two divisions was assumed to be the value computed for two divisions based on the angle read when enough divisions

(23) were used to subtend an angle of about 10 minutes. The error in the micrometer per minute equals $\frac{60''}{a}$ (b—a), where *a* is the correct angle subtended by two divisions and *b* is the measured angle subtended by two divisions. The error was plotted at the micrometer reading when pointed at the midpoint of the two divisions of the scale.

Figure 4 appears to show the following.

a. The errors as measured seem fairly erratic but it is believed the curve as drawn is a fair representation of the true condition.
b. The results shown in Figure 1 indicating systematic errors in theodolite E1 are explained by this curve. An angle of 1°22′25″, measured with an initial reading of 158°13′20″, would have a terminal reading of 159°35′45″. The micrometer reading changes from 3′20″ to 5′45″. Figure 4 shows the angle is too large by the area between the zero error line and the curve from micrometer reading 3′20″ to 5′45″, which is 0.7″. Also an angle of 1°22′25″ measured with an initial reading of 202°07′00″ would have the micrometer reading change from 7′00″ to 9′25″ and the area under the curve between the readings is —1.1″. The angle measured with the initial micrometer reading of 3′20″ then should be 1.8″ (.7 + 1.1) larger than the angle measured with the initial micrometer reading of 7′00″. This result checks the discrepancy as shown between the values given in Figure 1.



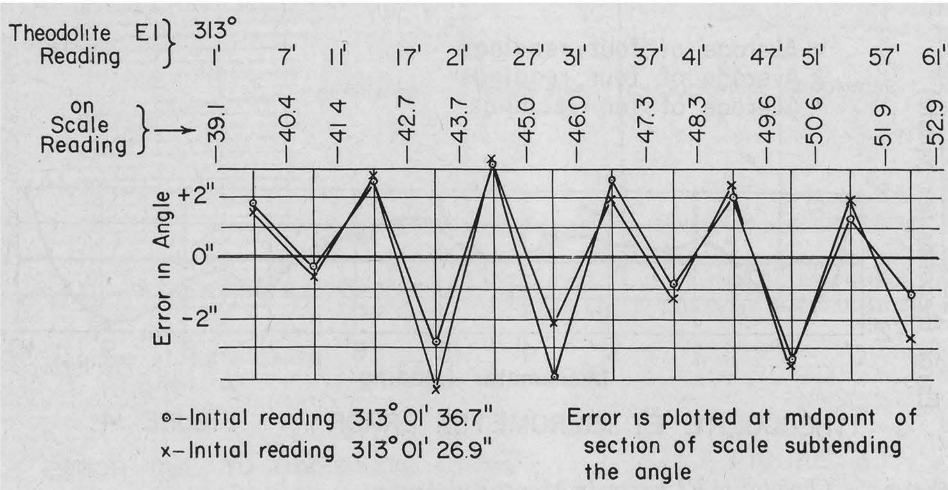
THEODOLITE E1 MICROMETER ERROR FIGURE 4

Figure 5: Theodolite E1 Error in Measured Angle

The theodolite E1 was pointed on the consecutive points 39.1, 40.4 41.4, 42.7, and so on, and the horizontal circle was read as turned clockwise. The circle was then read as returned to the same points by turning in a counterclockwise direction. The correct angle between points was assumed to be the angle computed by using the readings on the points 39.1 and 52.9 as giving a correct total angle for 13.8 divisions of the scale. For example, the correct angle subtended between 39.1 and 40.4 is 1.3 divided by 13.8 times the measured angle between 39.1 and 52.9. The points on the scale and the distance from the theodolite to scale were selected to demonstate that the curve, Figure 4, is approximately correct. The plus errors and minus errors should alternate. This arrangement should give approximately the maximum systematic error of the theodolite E1. Each theodolite reading shown is the average of four readings. The total angle subtended by 13.8 divisions of the scale is 1°00′07″.

Figure 5 appears to show the following.

a. The micrometer error curve, Figure 4, is approximately correct. The area under the error curve (Figure 4) between the consecutive micrometer readings shown in Figure 5 gives approximately the same error as shown in Figure 5.
b. The failure to get all the negative errors to be of the same magnitude and all positive errors to be of the same magnitude could be due to one or all of the following: (1) incorrect placing of the marks on the circle, (2) incorrect placing of marks on the scale, and (3) observation errors. The observer believes that items (2) and (3) are the sources for a large part of the discrepancies.
c. The maximum systematic error that theodolite E1 micrometer produces is about 3 or 4 seconds. The check between the two sets of readings indicates a maximum observation error of not more than one second. The micrometer error curve, Figure 4, indicates that the readings could be designed to give slightly larger errors than are shown by the readings of this Figure 5.

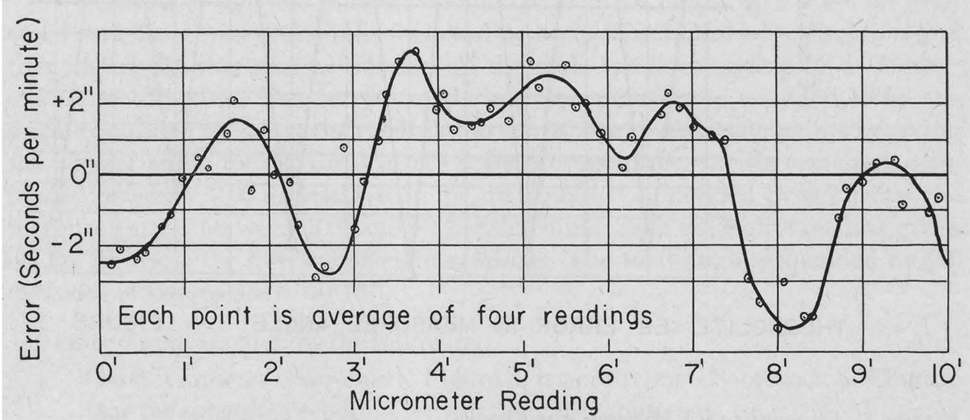


THEODOLITE E1 ERROR IN MEASURED ANGLE FIGURE 5

Figure 6: Theodolite E2 Micrometer Error

The errors as plotted were determined by measuring angles between successive points on a scale. Readings were made at each point on the scale as the theodolite turned clockwise and then as returned in a counterclockwise direction. The average of four readings on each point was used to determine errors. The correct angle subtended by one division of the scale was assumed to be the angle computed by dividing the measured angle subtended between the outside points used by the number of divisions between the points. The error is the measured value of one division minus the correct value of one division. The error is plotted at the micrometer reading midway between readings at ends of the division. The value of one division is 64 seconds and the points as plotted are actually the error in 64 seconds.

Figure 6 appears to show the following. The results shown in Figure 2 can be explained in the light of this curve. For example, Figure 2 shows an angle of about 1°15′15″ measured with an initial horizontal circle reading of 250°10′40″ to be about 7 seconds larger than the same angle if the initial reading is 180°18′20″. According to Figure 6, the error in an angle with an initial reading of 250°10′40″ is the area under the curve from 00′40″ to 05′55″ which is about +4 seconds. The error in the angle with an initial reading of 180°18′20″ is the area under the curve from 8′20″ to 3′35″ which is about —3 seconds. Then the discrepancy in measured angles should be, according to Figure 6, 4 seconds plus 3 seconds, equalling 7 seconds.



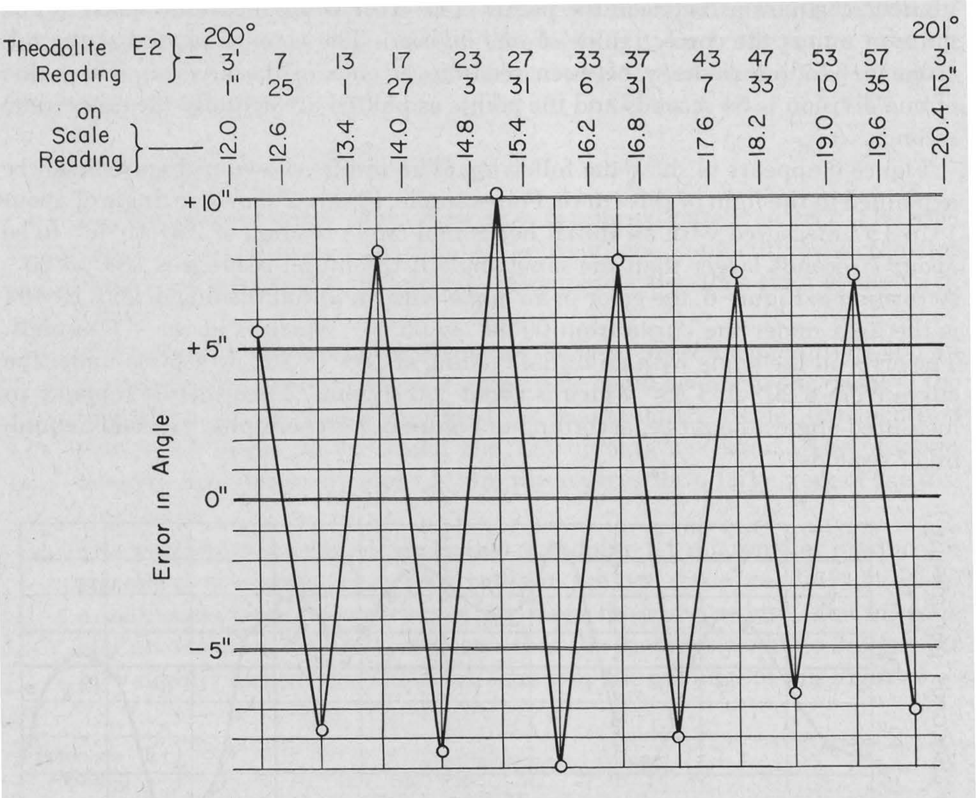
THEODOLITE E2 MICROMETER ERROR FIGURE 6

Figure 7: Theodolite E2 Error in Measured Angle

The theodolite E2 was pointed on the scale divisions and the horizontal circle was read as turned clockwise and then as returned in a counterclockwise direction. The theodolite readings given are each the average of four readings. The correct angle subtended by one division was assumed to be the measured angle between the outside points used divided by the number of divisions of scale between these points. The scale points used were selected to demonstrate that the curve, Figure 6, is approximately correct and to produce the maximum systematic error possible for this instrument. The error in measured angle should be equal to the area under the curve, Figure 6, between terminal readings of the micrometer. The error in angles should be alternately plus and minus and should be the same size as we progress along the scale.

Figure 7 appears to show the following.

a. The theodolite E2 makes systematic errors about as large as 8 seconds.
b. The micrometer error curve, Figure 6, is approximately correct.

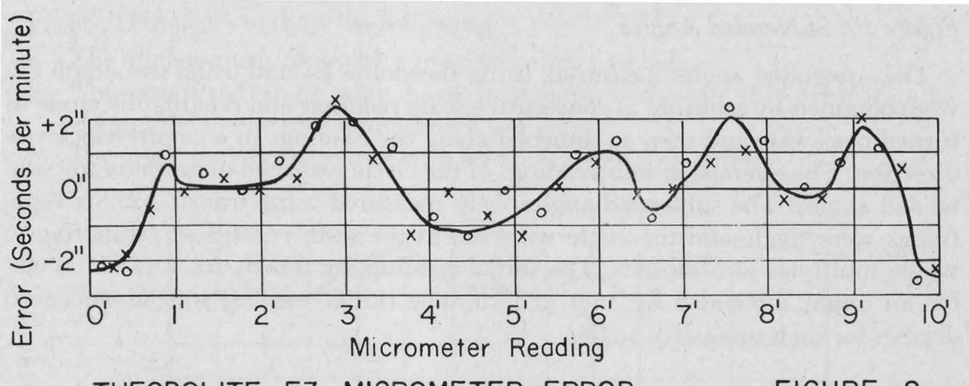


THEODOLITE E2 ERROR IN MEASURED ANGLE FIGURE 7

Figure 8: Theodolite E3 Micrometer Error

The errors were determined by measuring angles between successive points on a scale with divisions of 1/50 inch. The angle subtended by one division was about 26 seconds. Readings were taken at each point on the scale as the theodolite turned clockwise and then as returned to the points in a counterclockwise direction. The average of eight readings on each point was used to obtain errors. Two sets of readings with different initial circle readings were taken. The correct angle subtended by two divisions was assumed to be the value computed for two divisions based on the angle read when enough divisions (23) were measured to give a subtended angle of about 10 minutes. The error in the micrometer per minute equals $\frac{60''}{a}$ (b — a), where *a* is the correct angle subtended by two divisions and *b* is the measured angle subtended by two divisions. The error was plotted at the micrometer reading when pointing at the midpoint of the two divisions.

Figure 8 appears to show the following. An error of approximately 1 second, 2 seconds, or 3 seconds will result from several different combinations of initial and final readings of the micrometer: for example, 0′45″ to 3′25″, a plus error; 3′25″ to 5′25″, a minus error; 5′25″ to 9′35″, a plus error; and 9′35″ to 10′45″, a minus error. See Figure 9 for a demonstration of these errors.



THEODOLITE E3 MICROMETER ERROR FIGURE 8

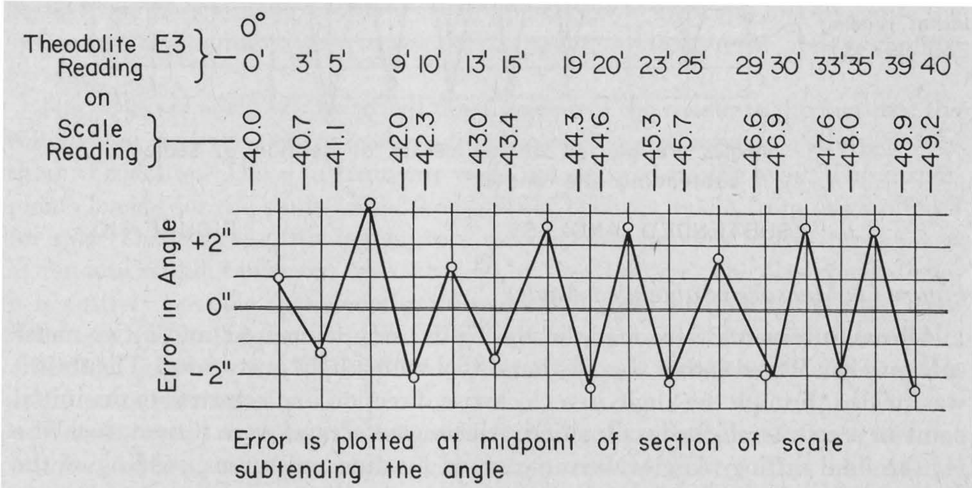
Figure 9: Theodolite E3 Error in Measured Angle

The theodolite E3 was pointed on the consecutive points of a scale as indicated and the horizontal circle was read as turned in a clockwise direction and then the circle was read as pointed on the scale while returning in a counterclockwise direction. The correct angle between points was assumed to be the angle computed by using the readings on the 40.0 and 49.2 scale points as giving the correct angle for 9.2 units of the scale. For example, the correct angle subtended between 40.0 and 40.7 scale points equals 0.70 divided by 9.2 times the measured angle between 40.0 and 49.2 scale points. Each theodolite reading given on the figure is the average of eight readings. The total angle subtended by 9.2 divisions of the scale is 0°40′13″.

Figure 9 appears to show the following.

a. The micrometer error curve, Figure 8, is approximately correct. See Figure 8 for the suggested readings for this demonstration.

b. The maximum systematic error that the theodolite E3 micrometer will make is about 3 seconds.



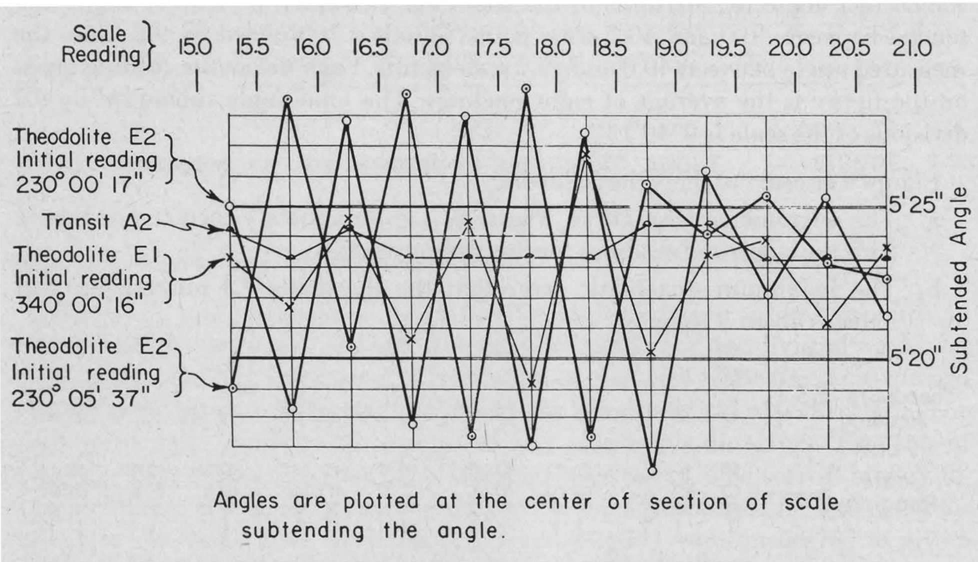
THEODOLITE E3 ERROR IN MEASURED ANGLE FIGURE 9

Figure 10: Subtended Angles

The subtended angles measured, using theodolite E1 and using theodolite E2, were obtained by pointing at consecutive scale readings and reading the circle as turned clockwise and then as returned along the readings in a counterclockwise direction. The average of four readings of the circle was used to compute the subtended angles. The subtended angles were measured using transit A2. Six repetitions were made and the angle was read at the sixth repetition, estimating to whole multiples of 5 seconds. The initial reading for transit A2 was 00°00′00″ for an angle subtended by 15.0 to 15.5. The initial reading was advanced 10 degrees for each successive angle.

Figure 10 appears to show the following.

a. Theodolite E1 and theodolite E2 give measured subtended angles that are quite erratic, obviously varying several seconds from the correct value.
b. Transit A2 gives measured subtended angles with a maximum discrepancy of about one second. This would indicate an error of about one-half second.
c. The discrepancies between measured angles, using the theodolites, are explained by the micrometer error curves of this report.



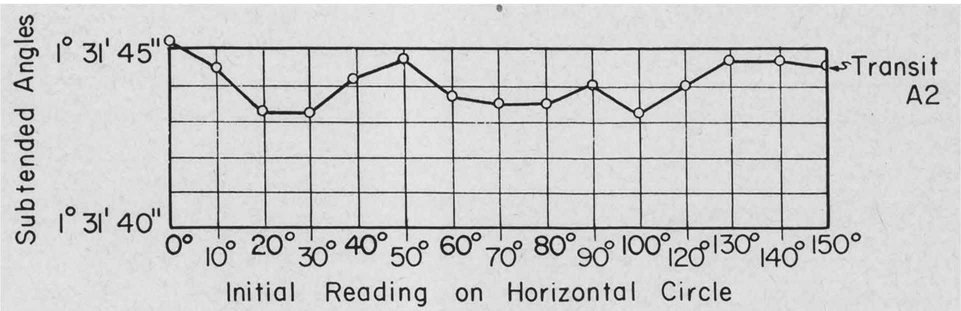
SUBTENDED ANGLES FIGURE 10

Figure 11: Measured Subtended Angles

All measurements were made in the field, using transit A2 and a two-meter subtense bar. The weather was clear and cool with a light gusty wind. The transit was turned through the angle in a clockwise direction and returned to the initial point in a counterclockwise direction. All tangent screws were turned clockwise for the final setting. Angles were measured by nine repetitions, reading on the sixth, seventh, eighth, and ninth repetition. Angle size was computed as the sum of the four readings divided by 30.

Figure 11 appears to show the following.

a. The angle is measured with a precision of ± 1 second.
b. The measured angle may have systematic error but this error can be eliminated by the procedure shown in Figure 3.
c. The angle may be measured with an accuracy of ± 1 second.



MEASURED SUBTENDED ANGLES FIGURE 11

V. Discussion of Conclusions

The observer realizes that the conclusions drawn from this study are contrary to all published data on the subject. He believes the data as presented do justify the conclusions. The observer personally collected and reported the data, and he believes this was done without prejudice.

Some have suggested that possibly the errors charged to the optical micrometer are the results of the observer's reading errors, incorrect division marks on the scale used to subtend the angles, incorrect position of marks on the horizontal circle, and so forth. The observer believes that a study of the data presented, with the idea of correlating the data and errors from other sources such as those suggested above, will easily convince the most unwilling that none of the other possible sources listed can be assigned here. The observer readily admits errors did occur from these sources, but they are of minor significance and merely cause the systematic errors due to inaccuracies of the optical micrometer to appear slightly erratic.

The observer again wishes to call the attention of the reader to the fact that the conclusions regarding the optical-reading theodolites are based on a study of only three theodolites. These instruments were not selected in any way. The instruments loaned for the study were, the observer believes, taken from stock offered for sale. These three instruments were the only instruments of this type, taken at random without previous tests, studied by the observer. The observer believes it is entirely possible that some instruments will show smaller errors than were shown by the instruments in this study. It is also possible that some intruments will show larger errors. To establish a limit of error, it would be necessary to test many more instruments.

The observer's wish is a better understanding, by all of us, of the errors encountered when measuring angles with transits and theodolites. This understanding should lead us to a more reasonable approach to field measurements and to a higher precision in manufacturing all transits and theodolites.