Essays on Sovereign Default

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Canhui Hong

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Timothy J. Kehoe and Manuel Amador

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Abstract

This paper consists three chapters that the first two chapters study the flight-to-quality sovereign debt crises while the third chapter studies the persistent market exclusion.

In the first two chapters, I characterize risk averse lenders' optimal bond holdings under which flight-to-quality debt crises can arise when there are large differences in borrowing countries' future default risks, and do this within a dynamic, stochastic general equilibrium model. In this paper, there is a substitution effect between different bonds because borrowing countries are competing with each other on international borrowing. The relative differences in countries' fundamentals, rather than the absolute fundamentals, determine the magnitude of the substitution effect, and thus the direction of lenders' cross-border capital movements. Specifically, when the differences in countries' fundamentals are large enough, international lenders would like to move toward countries with relatively low future default risks, which improves these countries' borrowing conditions and deteriorates other countries'. Furthermore, safe countries accommodate lenders' capital movements by issuing more debt, which reduces the borrowing resources available to other countries, further intensifies the difficulties faced by countries with deteriorated borrowing conditions, and may finally force them to default. Such forces were quantitatively important in explaining the empirical evidence from the recent European Debt Crisis: European peripheries had difficulty raising funds in international markets, while in countries such as Germany, and the United States, the yields declined and the debt positions rose since 2010.

In the third chapter, I characterize the lender's optimal recovery plan under which debt recovery after default is decreasing in duration of market exclusion. In this paper, the borrower's endowment realization is private information that is persistent, and she trades with the lender repeatedly. The lender has to choose a recovery plan which specifies the amount of debt recovery the borrower should repay for market reentry after default, while the borrower retains the right to decide on whether to repay the debt recovery. In equilibrium, a borrower with a high endowment realization would like to repay a higher debt recovery in order to regain market access earlier and to avoid the high output cost. In contrast, a borrower with a low endowment realization would

prefer to stay with the market exclusion in order to get a higher haircut on her debt. As a consequence, the equilibrium in this paper features decreasing debt recovery and increasing probability of market reentry over time after default, both are consistent with the empirical findings in recent studies. I show that the lender choosing such a plan to separate the borrower's states by using time. In particular, given the lender has to maintain market exclusion after default in order to support the ex ante equilibrium borrowing in the market, she would prefer to allocate the ex post inefficiency in an efficient way.

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Chapter 1

Flight-to-Quality Debt Crises

1.1 Introduction

The Greek recession around late 2009 has caused sharply different impacts on sovereign borrowers since 2010. As Figure 1.1 shows, debt yields in European peripheries increased dramatically since 2010, which is the so-called European debt crisis. However, debt yields in Germany, and the United States declined at the outset of the crisis[1] . That is, besides being immune to a crisis, these two countries[2] had significant improvements[3] in their borrowing conditions when European peripheries were struggling with international borrowing. The declined yields in Germany, and the United States are even more puzzling after considering the new bond issuance in these two countries: As Figure 1.2 shows, debt positions in these two countries increase steadily since 2010. The theoretical work on sovereign debt crises has successfully explained how and why European peripheries have a sovereign debt crisis, and how the crisis that originated in Greece was transmitted to other countries. Yet, it says nothing about why Germany, and the United States had such different experiences during the recent crisis and how

[1] Figure 1.1 plots only the average of yields for these two groups in order to demonstrate the pattern of change clearly. Yields in individual countries within the same group change in exactly the same way as the group average does. For example, yields in Germany, and the United States are hardly different since 2010.

[2] The list of countries that got benefited is longer than we have here and includes, e.g., Japan.

[3] Given that the inflation rate is around 2%, the real return is around 0 and can even be negative for more than 5 years, meaning that international lenders are paying for the privilege of holding safe assets.

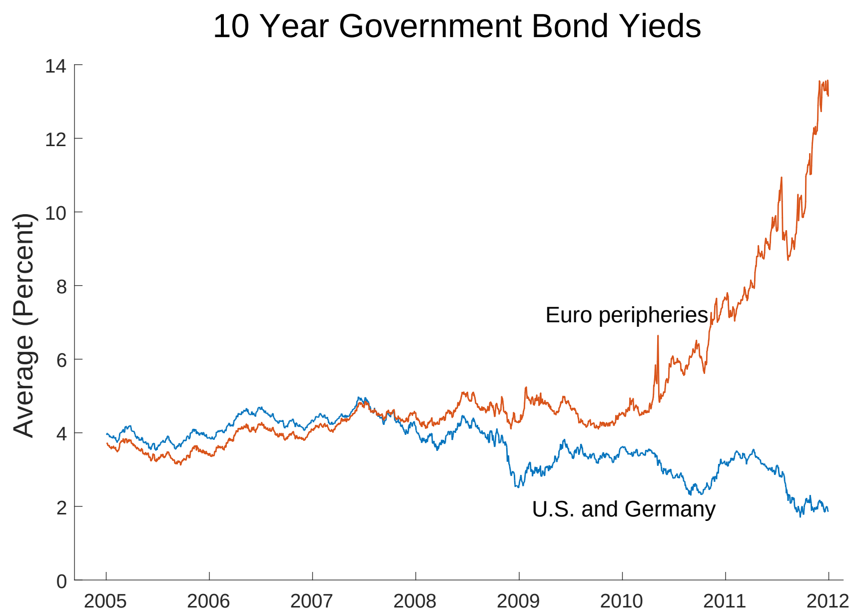

Figure 1.1: Debt Yields

such a phenomenon relates to the occurrence of a crisis in European peripheries.

This paper develops a multi-country model in which sovereign debt crises occur in some countries, and improved borrowing conditions and increased debt positions occur in other countries, and both are the consequence of international lenders' optimal portfolio choices. In particular, there exists a substitution effect between bonds issued by different countries, which motivates lenders to adjust their bond holdings across borders if borrowing countries' fundamentals such as GDP are relatively different. When international lenders are changing their bond positions across borders, countries with relatively low future default risks can have improved borrowing conditions. Furthermore, these countries also accommodate the cross-border capital movements by increasing their new bond issuance, which will intensify the difficulties faced by countries with deteriorated borrowing conditions, and may finally force these countries to default. Such forces were quantitatively important to match the observed debt yield dynamics during the recent European debt crises: from the perspective of one group of countries, both yields and default probabilities are lower when the other group of countries have a lower

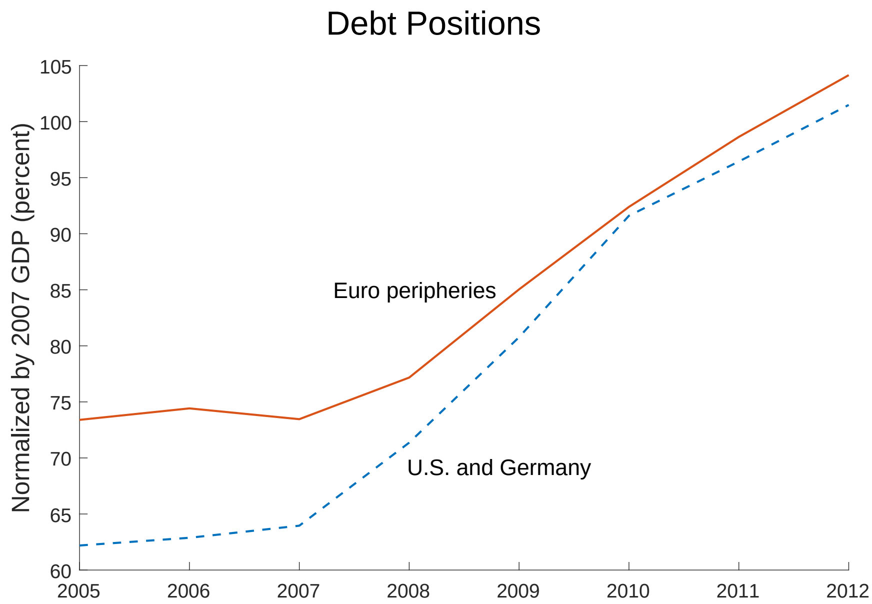

Figure 1.2: Debt Positions

endowment realization, the debt positions increase as a result of the better borrowing conditions, and the correlations of yields among these two groups of countries are lower when one group of countries is defaulting.

My framework builds on a sovereign default model with risk-averse lenders and multiple borrowing countries, in the manner of [1] and [2]. In the model, two groups of small countries are borrowing from competitive risk-averse lenders by issuing bonds with default options. Borrowing countries in both groups are connected in the sense that they are competing with each other for international borrowing. Lenders are investing in a portfolio composed of bonds issued by different countries, and the concave payoff makes them care about both the risk structure and the expected return. Hence, lenders choose their bond positions in according to the relative soundness of countries' fundamentals. If the difference in fundamentals between these two groups is large, lenders would like to change their bond positions across borders so that one group of countries have flight-to-quality crises and the other group of countries have improved borrowing conditions. Furthermore, the other group of countries accommodate the lenders' capital flight by borrowing more, and finally have a combination of lower yields and higher debt positions. By doing so, the model fully characterizes the conditions under which countries will experience different patterns of capital flows, and to what extent some of them may be forced to default.

To understand the mechanism of the model, consider a scenario in which GDP in one group of countries (say, European peripheries) unexpectedly declines. Given the fundamentals deteriorate, lenders would like to cut their bond holdings in European peripheries, and thus impose a downward pressure on these countries' borrowing conditions. If European peripheries are the only borrowers in the market, because the lenders are lack of other means to save, the magnitude of such a reduction may not be large enough to force European peripheries to default. If instead, there is another group of countries (say, Germany, and the United States) in the market, and more importantly, fundamentals in these countries signal low enough future default risks relative to the European peripheries. With better alternatives to save, lenders would have greater incentives to substitute from European peripheries to Germany, and the United States. That is, because fundamentals in Germany, and the United States are relatively strong, lenders' capital retrenchment from European peripheries has been intensified to the extent that it may bring crises to these countries. Instead of the absolute fundamentals in each group, such a behavior is driven by the relative difference between these two groups of countries' fundamentals.

In addition to the lenders' incentive to shift across borders, Germany, and the United States' endogenous responses to the capital inflows also contribute to the crises happening in European peripheries. For instance, if Germany, and the United States cannot increase the amount of bond issuance, the increased demand of bond in these countries will be fully reflected by decreases in their bond yields, which will in turn limit the lenders' incentives to shift toward these countries. In contrast, Germany, and the United States in this paper accommodate lenders' capital flight by increasing their debt issuance, meaning that in addition to relatively low future default risks, bond supplies in these countries also increase to meet the increased demand brought by international lenders. Hence, the lenders' incentive to adjust their bond positions have been greatly encouraged, and it may become large enough to force European peripheries to default. I call this type of sovereign debt crises as flight-to-quality debt crises, as lenders are retrenching out of European peripheries due to the fact that they want to move toward Germany, and the United States, which have relatively low future default risks and higher debt capacities.

The model is calibrated to Greece. Both the values of lender and borrower's risk aversion are borrowed from the standard international business cycle literature. The lender's discount factor is calibrated to match the average risk-free interest rate at 4%. The borrower's discount factor is calibrated to the average default probability at 5%. The market reentry probability is calibrated to match the average market exclusion after default to be around 4 years, as in [3]. The stochastic structure of endowment realization is borrowed from [1], with the persistence as 0.88 and the shock as 0.03. Other parameters are calibrated to match observed average yields in Greece, which is around 5% before the crisis.

The numerical exercise shows that borrowing conditions in European peripheries deteriorate due to the presence of safe countries such as Germany and the United States: When safe countries are also in the market, the average debt yields in European peripheries increases by 15%, which makes the average default probability increase by 20%, compared to the case when safe countries default. Furthermore, the presence of safe countries also crowd out sovereign borrowings in European peripheries: The debt to GDP ratio decreases by 5%. Because the substitution effect is mainly driven by the difference in countries' fundamentals, changes in these three statistics are even more dramatic when safe countries' fundamentals are strong: When GDP in safe countries has a high value, the average debt yields in European peripheries increase by 21%, the average default probability increase by 25%, and the debt to GDP ratio decreases by 10%.

To show that the substitution effect reduces debt yields in safe countries and encourages them to borrow more, I then conduct an experiment under which GDP in European peripheries unexpectedly drops by while GDP in safe countries has been held constant. The simulation result shows that because lenders are moving toward safe countries, debt yields decrease and debt positions increase in these countries even if their GDP has been held constant. To see how the safe countries' endogenous responses affect the magnitude of lenders' flight-to-quality behavior, I then compute the implied risk free interest rate by assuming that safe countries never default: The risk free interest rate also decreases when European peripheries default, but with a much lower magnitude. Since the non-defaultable nature makes the supply of bond more elastic in safe countries, safe countries respond to the increased demand mostly by increasing the bond issuance. Although the negative impact on European peripheries is the largest when bond supply is elastic, it predicts that the decrease in safe countries' debt yield is too small to match the empirical evidence. Hence, it also justifies the model setting in this paper under which safe countries are allowed to default.

The key driven factor of the model is the lenders' willingness to avoid a loss when facing with default risk, which is governed by the lenders' risk aversion and periodic income. I then conduct sensitivity analyses on how the probability of flight-to-quality crises changes when these two parameters vary. The numerical exercise shows that either with a higher risk aversion or a lower income, the changes in debt yields, default probabilities, and debt positions across countries are larger, meaning that lenders are more likely to commit flight-to-quality behavior across sovereign borders. Such an experiment can be mapped onto the recent European debt crisis. Specifically, international lenders suffer a lost on their investment after Greece defaults, which makes them poorer and more sensitive to future default risks. Because European peripheries have relatively high future default risks and smaller debt capacities compared to Germany, and the United States, they become the targets from which lenders want to retrench out. Hence, the lenders' flight-to-quality behavior leaves other European peripheries with the contagion of the Greek crisis, and at the same time, improved borrowing conditions and increased debt positions in Germany, and the United States. Such a pattern of sovereign debt crisis contagion is more consistent with empirical evidence from the recent crisis.

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1.2 Literature review

This work builds on the benchmark model of sovereign default analyzed by [4], [5] and [6]. The models presented in these papers focus on sovereign default in the case of a single borrowing country trades with risk-neutral lenders. [7, 2] and [1] study the case of risk-averse lenders' trade with multiple sovereign countries. They show that default (or renegotiation) in one country makes the lenders poorer and to request a higher premium on the other country's bond, which will force the other country to default as well. As a consequence, they are able to explain defaults and renegotiation happen in tandem, which is a feature of sovereign defaults in the data. However, the wealth effect addressed in their paper fails to explain why Germany, and the United States have seen improved borrowing conditions, when there are crises in European peripheries. In contrast, the substitution effect in this paper, which captures the lenders' incentive to distinguish bonds issued by different countries, is able to motivate lenders to commit flight-to-quality behavior, and thus leaves countries with sharply different experiences in the international borrowing market.

Another branch of sovereign debt literature examines cross-border capital flows during a crisis, such as [8], [9, 5], [10], etc. In particular, [8] document that the crisis country's capital inflow changes from positive before the crisis to negative after. Although cross-border capital outflows have long been viewed as the reason a sovereign country becomes insolvent[4] , there is a lack of theoretical studies to analyze why lenders choose to dramatically retrench out of a country, and how such retrenchments would affect other countries' borrowing conditions.

More generally, this paper also contributes to sovereign debt literature on modeling multiple borrowers and the interactions between them. The multiple big players setting in previous literature feature strategic interactions between agents, which expose the model to the risk of no pure strategy equilibrium. In this paper, risk-averse lenders trade with two groups of small borrowing countries, each with a total mass of 1, which is similar to the setting in large-game literature, including [14], [15] and [16]. As it has been proved in previous studies, each borrowing country in this paper identifies itself as a small player in the market, and takes the average response of other agents as given in making its own decisions. Therefore, strategic interactions are shut off and the existence of a pure strategy equilibrium can be guaranteed.

A recent wave of literature has examined why the flight-to-quality phenomenon emerges and how it relates to the financial crises. [17] and [18] show that investors' flight-to-quality amplifies initial negative shocks and, finally, leaves the economy in an amplified downturn. However, neither of these papers looks into the interplay between the flight-to-quality and default incentives. More recently, [19, 20] examine a problem similar to mine by considering a coordination problem between lenders on rollovering one of the two countries' debt. They also find that relative fundamentals and debt capacities determine the country from which lenders would like to retrench out. However, by focusing on a static model of default, their model fails to generate richer dynamics on debt yield, default probability and debt position. Furthermore, lenders in their model are assumed to hold only one country's bond to store the value, which contradicts common practice among international investors.

Lastly, the model in this paper is also related to the literature on how a supplier of safe asset would be affected due to investors' flight-to-quality behavior. [21] find that the US Treasury yields declined by 73 basis points, on average, from 1926 to 2008 because investors valued the liquidity and safety of the US Treasury. [22] show that

[4] See, for example, [6], [11], [12] and [13].

the low external borrowing cost in United States encourages borrowing and increases the leverage level in domestic institutions. [19, 20] show that debt yields drop in safe countries because international lenders are choosing these countries' bond as a mean to store the value. Similar to this paper, safe countries such as Germany, and the United States in their paper accommodate the increased demand by issuing more debt, and finally have a combination of improved borrowing conditions and higher debt positions.

1.3 Model

The model consists of agents who live for infinite periods, and there is a single good in each period. There are two groups of countries, denoted by $j \in \{h, f\}$. Each group contains a continuum with measure one of identical small open economies[5] . There are three types of agents in this model: households, governments, and international lenders. There is a government and a representative household in each country. For simplicity, I will mainly focus on the representative country in group h in laying out the model; everything can easily be applied to group f countries, due to the symmetry of the problem.

The household's preferences are

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u\left(c_{h,t}\right)\right]$$

where $c_{h,t}$ denotes its consumption at period t. For simplicity, the subscript t is omitted in laying out the model in the following sections. The periodic utility function $u\left(\cdot\right)$ is continuously differentiable, strictly increasing, and strictly concave. Each period, the household receives a stochastic endowment, y_h, which is public information and has a bounded support of $\left[\underline{y}, \overline{y}\right] \in \mathbb{R}$. Notice that endowment realizations are assumed to be the same across countries within one group. Thus, the only difference in endowment realizations is among these two groups. Finally, y_h follows a Markov process with transition function $f\left(y'_h \middle| y_h\right)$.

[5] The main purpose of this setting is to address the similarities shared by the same type of countries in the sovereign debt market—for example, emerging market economies versus developed economies. The assumption can possibly be loosened by allowing idiosyncratic risk for every country. However, it will greatly complicate my analysis—without bringing any key insights—once idiosyncratic risk is not big enough to overturn the similarities shared by countries within the same group.

The government in each country trades with international lenders on defaultable bonds. It is benevolent, in the sense that its goal is to maximize its domestic households' utility. Each period, after both y_h and y_f realize, the government first chooses whether to default on the matured debt, d_h

$$d_h = \begin{cases} 0, & \text{repay} \\ 1, & \text{default} \end{cases}$$

The default decision is public knowledge once it has been made. Thus, all governments and lenders have perfect information about default decisions before taking any further action. Once a government chooses to default, it is immediately excluded from international borrowing. The defaulting country also suffers an output cost similar to [6]; that is, the endowment process changes to y_h^d after the government defaults

$$y_h^d = \begin{cases} (1-\lambda)\,\mathbb{E}\,(y_h) & \text{if } y_h \geq \mathbb{E}\,(y_h) \\ y_h & \text{if } y_h < \mathbb{E}\,(y_h) \end{cases}$$

where $\lambda \in (0,1)$. If the government repays, then given the bond price schedule q_h', it is allowed to choose the new bond issuance, b_h'. The new bond issuance is assumed to have a finite support, $\left[\underline{b}, \overline{b}\right]$[6] . The budget constraint can thus be summarized as

$$c_h + (1-d_h)\,b_h = (1-d_h)\left(y_h + q_h' b_h'\right) + d_h y_h^d$$

Furthermore, the country that defaults at period t will have a probability of $\theta \in (0,1)$ to re-access the market at period $t+1$, as in [6].

The governments are the only strategic agents in this model. Similar to the one borrower setting in [12] and [6], when the government makes its decision, it takes into account the decision's influence on the price of its own debt. Furthermore, since all countries borrow from the same lenders, there is a linkage between borrowing countries. However, the linkage in this paper differs from the one in [1], who consider only two countries, and the government in each country is a big player in the international borrowing market. In their paper, there are strategic interactions between these two big borrowing countries, which could possibly impede the existence of a pure strategy

[6] Given that the endowment process is bounded and the debt is defaultable, this assumption is innocuous if $\underline{b}$ is sufficiently low and $\overline{b}$ is sufficiently high.

equilibrium. In contrast, the model in this paper assumes a continuum of countries in each group, and each government recognizes that any single country can be ignored compared to the sum of other countries in the international borrowing market. Therefore, the government ignores its own actions' impacts on the international borrowing market and takes the average responses of other governments into account instead of any specific (other) country's. Such a large-game setting was firstly formulated by [14], then developed by [15]. It has been proved by [16] that the existence of a pure strategy equilibrium is guaranteed if there is a continuum of agents. Intuitively, because any single government ignores its own influence on other countries' borrowing conditions, the model here doesn't have the strategic interactions that may impede the existence of a pure strategy equilibrium.

To formulate the problem, define $\boldsymbol{B}$, $\boldsymbol{y}$ and $\boldsymbol{D}$ as

$$\boldsymbol{B} = (B_h, B_f), \boldsymbol{y} = (y_h, y_f), \boldsymbol{D} = (D_h, D_f)$$

where B_j is the average indebtedness of group j countries and D_j is the ratio of default in group j:

$$B_j = \int_0^1 b_j(i)\, di$$

$$D_j = \int_0^1 d_j(i)\, di$$

In equilibrium, since countries within the same group are assumed to receive the same endowment realization each period, it will be non-optimal for a single country to deviate by choosing a different default decision or a different debt level. Thus, in equilibrium, countries in group j would all default or all repay, $D_j \in \{0, 1\}$. Moreover, the individual debt level equals the average indebtedness within a group

$$b_j(i) = B_j, \forall i$$

Therefore, the net repayment from group j countries can be written as

$$\begin{aligned} NR_j &= \int_0^1 [1 - d_j(i)] \left\{ b_j(i) - q_j' \left[b_j'(i) \right] b_j'(i) \right\} di \\ &= (1 - D_j) \left[B_j - q_j' \left(s, B_j' \right) B_j' \right] \end{aligned}$$

The arguments above imply that instead of the whole distribution of initial debt level in each group, I only need to keep track of the average indebtedness and the default ratio within the group to formulate the problem. Each period, with the initial aggregate state $s = (\boldsymbol{B}, \boldsymbol{y})$, the timing of actions is similar to [6] and can be summarized as:

1. (y_h, y_f) realizes, then given (s, b_h), the government decides whether to default or not, d_h.

2. If the government defaults, it simply consumes y_h^d and will reenter the market with probability θ at the next period.

3. If the government repays, taking $q_h' = q(s, b_h')$ as given, it chooses how much to borrow, b_h'.

4. International lenders, taking the bond price schedule as given, choose how much to lend, $\tilde{b}_h'$.

5. The equilibrium condition requires $b_h' = \tilde{b}_h'$.

1.3.1 The Borrower's and Lender's Problems

To formulate the government's problem in a recursive way, I first consider the problem at the second stage faced by a government that has already made a default decision. The government takes as given the rule on the evolution on the aggregate state, s, which can be summarized as

$$s' = \mathcal{H}(s)$$

Then the government's value function, given it repays, can be written as

$$V^{nd}(s, b_h) = \max_{b_h'} \left\{ u\left(y_h + q_h' b_h' - b_h\right) + \beta \mathbb{E}\left[V\left(s', b_h'\right)\right] \right\} \tag{1.1}$$

Similarly, the value for the government that decided to default can be written as

$$V^d(s) = u\left(y_h^d\right) + \beta \mathbb{E}\left[\theta V^{nd}\left(s', 0\right) + (1-\theta) V^d\left(s'\right)\right]$$

Back to the first stage: Given the value on default or repay, the government's problem at the default decision can be written as

$$V(s, b_h) = \max_{d_h} \left[(1 - d_h) V^{nd}(s, b_h) + d_h V^d(s)\right] \tag{1.2}$$

The equilibrium outcome can be summarized by the government's default decision and new bond issuance, $d(s, b_h)$ and $b'(s, b_h)$, as well as consumption $c(s, b_h)$, and values $V(s, b_h)$, $V^{nd}(s, b_h)$, and $V^d(s)$.

There is a continuum with measure one of identical international lenders who are competitive. The representative lender is risk averse and has a preference as

$$\mathbb{E}\left[\delta^t \psi\left(c_{L,t}\right)\right]$$

where $c_{L,t}$ is the lender's consumption at period t. $\psi(\cdot)$ is continuously differentiable, strictly increasing, and strictly concave. The lender's discount factor is assumed to be greater than β, $0 < \beta < \delta < 1$, in order to guarantee borrowing in equilibrium. Each period, the lender receives a fixed amount of endowment, y_L. The lender takes as given the evolution of aggregate state $s' = \mathcal{H}(s)$. Also, taking as given the bond price schedule $Q\left(s, \tilde{b}'_j\right)$, the lender decides how much to lend to each country in group j, $\tilde{b}'_j$, and how much to consume. The lender's budget constraints can be summarized as

$$c_L = y_L + \sum_{j=h,f} \int_0^1 \left[1 - d_j(i)\right] \left[\tilde{b}_j(i) - Q'_h(i)\, \tilde{b}'_j(i)\right] di \tag{1.3}$$

where $i \in [0, 1]$ denotes an individual country within one group and $j \in \{h, f\}$ denotes the group. The equilibrium condition requires $\tilde{b}'_j(i) = b_j(i), \forall i, j$. Thus, the lender's value function is

$$V_L(s) = \max_{l'} \left\{\psi(c_L) + \delta \mathbb{E}\left[V_L(s')\right]\right\} \tag{1.4}$$

where c_L is defined in equation (1.3). From the lender's problem, we can solve for the bond price function as

$$q'_h = q\left(s, \tilde{b}'_j\right) = \mathbb{E}\left[\frac{\delta \psi'(c'_L)}{\psi'(c_L)}\left(1 - d'_h\right)\right] \tag{1.5}$$

1.3.2 Equilibrium

I focus on a recursive Markov equilibrium in which all decision rules are solely the functions of state variables (s, b_h).

Definition 1.1. A recursive Markov equilibrium for this economy consists of (i) countries' policy functions for repayment, borrowing, and consumption, $b'(s, b_h)$, $d(s, b_h)$, and $c(s, b_h)$, and values $V(s, b_h)$, $V^{nd}(s, b_n)$, and $V^d(s)$; (ii) lenders' policy functions for lending choices and consumption $\left\{\tilde{b}'(s), c_L(s)\right\}$ and value function $V_L(s)$; (iii) the functions for bond price $q\left(s, b'_j\right)$; and (iv) equilibrium prices of debt $Q\left(s, \boldsymbol{D}, \boldsymbol{B}', \tilde{b}'_j\right)$; (v) the evolution of the aggregate state $\mathcal{H}(s)$ such that, given that the initial debt levels are the same across countries within each group and $b_{j,0} = \tilde{b}_{j,0}, \forall j$:

1. Taking as given the bond price, the evolution of the aggregate state $\mathcal{H}(s)$, the evolution of default ratio $\mathcal{D}(s)$, and the evolution of average new bond issuance $\mathcal{B}(s)$, the policy and value functions for countries solve the borrower's problem in equation (1.2).

2. Taking as given the bond prices $Q\left(s, \boldsymbol{D}, \boldsymbol{B}', \tilde{b}'_j\right)$ and evolution of the aggregate states $\mathcal{H}(s)$, the policy functions and value functions for the lenders $\left\{\tilde{b}'(s), c_L(s)\right\}$ solve the lender's problem in equation (1.4).

3. Taking as given countries' policy and value functions, the bond price function satisfies equation (1.5).

4. The prices of debt $Q\left(s, \boldsymbol{D}, \boldsymbol{B}', \tilde{b}'_j\right)$ clear the bond market for every country, $b_j(i) = \tilde{b}_j(i)$, $\forall j \in \{h, f\}$.

5. The goods market clears, $c_h + c_f + c_L = y_h + y_f + y_L$.

6. The law of motions for the evolution aggregate states $s' = \mathcal{H}(s)$ is consistent with countries' decision rules and shocks.

1.4 Flight-to-Quality Sovereign Debt Crises

In this section, I construct a two-periods example to intuitively illustrate the conditions under which a sovereign debt crisis occurs in group h countries, especially when group f countries are experiencing improvements in their borrowing conditions. The numerical analysis of the full model with infinite periods and the quantitative results will be presented in section 2.1.

Consider a two-periods version of my model in which endowment realizations in both groups of countries are stochastic in period 1, (y_h, y_f), but both are fixed in sense that the endowment in period 2 equal to the endowment in period 1

$$y_h' = y_h$$
$$y_f' = y_f$$

That is, the endowment process is perfectly persistent and the only source of uncertainty is from the endowment realizations in period 1. To make the problem meaningful, in period 1, both groups are in the market with positive matured debt levels $b_h \geq 0$ and $b_f \geq 0$, and are both deciding whether to default on the debt or repay. For simplicity, I assume that $\theta = 0$, meaning that a country defaults in period 1 will be permanently excluded out of the market for both periods. Furthermore, a country under market exclusion is assumed to suffer from an output cost so that endowment process changes to be y_h^d as I've defined before. If the country chooses to repay, then it is allowed to choose the new bond issuance, b_h'.

1.4.1 Borrower's Decisions

Because the problem only has two periods, I can solve it by backward inductions. In period 2, the country either chooses to repay the debt if it borrowed in period 1 or stays with market exclusion. Specifically, the country only chooses to repay the debt b_h' in period 2 if

$$y_h - b_h' \geq y_h^d$$

$\Rightarrow$

$$b_h' \leq y_h - y_h^d$$

Let $\overline{b}_h' = y_h - y_h^d$, it is essentially a borrowing limit in period 1 that the country can leave to be repaid in period 2. Because period 2's endowment realization is fixed, there is no equilibrium default in period 2. However, the option of default in period 2 imposes a borrowing limit in period 1 such that the bond price equals to 0 if the country would like to borrow more than the limit. I further make the Assumption 1.1 below to abstract from the characterization of the borrower's new bond issuance in period 1.

Assumption 1.1. *β is sufficiently low such that a country always want to borrow up to the borrowing limit in period 1 if it already chose to repay b_h, $b_h' = \overline{b}_h'$.*

In period 1, the value of repay

$$V^{nd}(s, b_h) = u\left(y_h + q_h' b_h' - b_h\right) + \beta u\left(y_h - b_h'\right)$$

and the value of default is

$$V^d(s) = (1+\beta)\, u\left(y_h^d\right)$$

and the country chooses to repay only if

$$V^{nd}(s, b_h) \geq V^d(s) \tag{1.6}$$

From equation (1.6), it is easy to see that the interdependence between countries' default and borrowing decisions is through the channel of bond price where the endowment realization in group f countries can affect the borrowing conditions in group h countries. However, before proceeding to the discussions of the interdependence, I need to firstly state several results on the borrower's decision rules.

Lemma 1.1. *There exists a cutoff function on y_h, $\hat{y}(\boldsymbol{B}, y_f, b_h)$ such that the country repays if $y_h \geq \hat{y}(\boldsymbol{B}, y_f, b_h)$, and defaults otherwise.*

Proof. See Appendix A.1.1. □

Lemma 1.2. *At the lowest level of y_h such that the country chooses to repay in period 1, the net repayments in both periods are equalized*

$$b_h - q_h' b_h' = b_h'$$

Proof. See Appendix A.1.2. □

Lemma 1.3. *The net repayment in period 1, $b_h - q_h' b_h'$ is strictly decreasing in y_h, while the net repayment in period 2, b_h' is strictly increasing in y_h.*

Proof. See Appendix A.1.3. □

Lemma 1.4. *Under the levels of y_h such that the country strictly prefer to repay in period 1, the net repayment in period 2 is strictly higher than the net repayment in period 1*

$$b_h' > b_h - q_h' b_h'$$

and their difference, $(1 + q_h')\, b_h' - b_h$ is increasing in y_h.

Proof. See Appendix A.1.4. □

In summary, countries are more likely to choose to repay in period 1 if they have a higher endowment realization. They would choose the spread the burden equally into both periods when they are indifferent between repay and default. Furthermore, because the higher endowment increases the borrowing limit in period 1, countries would increase their new bond issuance in period 1, which is also their net repayments in period 2, and thus reduces their net repayments in period 1. As a consequence, given countries strictly prefer to repay, a higher endowment realization shifts lenders' consumption from current period to the next period.

1.4.2 Two Effects

In order to analyze the interdependence between these two groups of countries' default decisions, I first analyze how bond prices in group h countries can be affected by the endowment realization in group f countries. Then, the analysis of the default decision can be transformed into analysis of the best response functions. Because there is no further uncertainty in period 2 endowment realizations, lenders would price the bond solely base on their intertemporal discount factor. In particular, if group f countries default, then the bond price in group h countries is independent of y_f, which can be written as

$$q_h' = \frac{\delta\psi'\left(y_L + b_h'\right)}{\psi'\left(y_L + b_h - q_h' b_h'\right)} \tag{1.7}$$

If instead, group f countries repay and issue new bond, then the bond price in group h countries is

$$q_h' = \frac{\delta\psi'\left(y_L + b_h' + b_f'\right)}{\psi'\left(y_L + b_h + b_f - q_h' b_h' - q_f' b_f'\right)} \tag{1.8}$$

Equation (1.8) shows that there are two channels through which the endowment realization in group f countries can affect the borrowing condition in group h countries: the *wealth effect* and the *substitution effect*. To briefly discuss these two effects, in the next two paragraphs I will take an increase in endowment realization in group f countries to provide some intuitions on how these two effects would influence the bond price.

First, an increase in y_f improves the borrowing conditions in group h countries through the wealth effect. In particular, compared to the case when group f countries, if y_f increases to make group f countries repay, lenders become wealthier and more willing to hold group h bonds. Intuitively, the lender uses a portfolio composed of bonds from these two groups of countries as a tool to save for the next period. Compared to the case when group f countries defaults, a higher y_f encourages group f countries to repay. With a higher wealth, lenders will increase their bond holdings in both groups of countries, which increases the bond price in group h countries. As a consequence, it also encourages group h countries to repay. I call such an effect as the *wealth effect*, which has also been widely discussed in [1] and [23].

Second, given group f countries choose to repay, an increase in y_f also tightens the borrowing conditions in group h countries through the substitution effect. Specifically, because the endowment process is persistent, a higher y_f implies that y'_f is more likely to concentrate on high values, which signals lower future default risks and higher debt capacities in group f countries. Given lenders' intertemporal substitution (or saving schedule), the reduced default risk in group f countries induces the lenders to shift toward the group f bond. Such a portfolio rebalance reduces lenders' demand of the bond in group h countries, and thus tightens the bond price schedule. Intuitively, because both groups of countries are borrowing from the common international lenders, an improvement in group f's endowment implies that group h countries become less competent in the international borrowing market, which makes their bond becomes less appealing to the lenders. I call such an impact as the *substitution effect*, which is the main mechanism through which this model can generate flight-to-quality behavior by the lenders.

I use the Lemma 1.5 to demonstrate how the repayment decision by group f countries affects group h bond price positively, which is the wealth effect.

Lemma 1.5. *When group h countries strictly prefer to repay, then at the lowest level of y_f such that group f countries repay in period 1, the bond price in group h countries is strictly higher than the bond price when group f countries default.*

Proof. See Appendix A.1.5. □

Lemma 1.5 says that compared to the case when group f countries default, the fact that group f countries choose to repay improves the borrowing condition in group h countries, and I call it the wealth effect. As I've discussed before, since the lenders are investing on bonds issued by both groups of countries, if they can guarantee repayments from group f countries, their will thus be less reluctant to invest on the bond issued by group h countries, which will improve the bond price schedule provided to group h countries.

I then use the Lemma 1.6 to show that a higher endowment realization in group f countries induces these countries to borrow more in period 1, which will in turn tighten the borrowing conditions in group h countries, and I call it the substitution effect.

Lemma 1.6. *Given group f countries repay, the bond price in group h countries, q_h' is strictly decreasing in the endowment realization in group f countries, y_f.*

Proof. See Appendix A.1.6. □

Lemma 1.6 shows that a higher y_f deteriorates borrowing conditions in group h countries. From the borrower's problem, it is easy to see that the group h countries are more likely to default with a lower bond price schedule, q_h'. Thus, a higher y_f forces group h countries to be more likely to default, and I call it the substitution effect.

In this special case, the bond supplies in both groups are inelastic, and the bond prices are determined in the sense that lenders' demand of bond equals the total supply. A higher y_f expands the debt limit in group f countries, which induces group f countries to increase their bond supply. Given any bond price, the increased supply of bond in group f countries reduces the lenders' demand of group h bonds. Such a reduction is reflected at a declined bond price in group h countries, and thus forces group h countries to be more likely to default in period 1, as can be seen from the borrower's problem. Intuitively, the increased supply in group f countries crowds out the demand of group h bond.

Notice that the bond price in group f countries is also decreasing in y_f in this example, which contradicts to the empirical evidence during the recent crisis. The reason is because the supply of bond in group f countries is perfectly inelastic. Hence, group f countries will fully exhaust the increased demand and thus make group f bond price decrease. If instead, the endowment realization in the future is stochastic, the supply of bond in group f countries is elastic. As a consequence, group f countries would respond to an increasing demand by both issuing more bonds and having a lower yield, as in my quantitative study in later sections.

In summary, these two effects show that the endowment in group f countries affect the borrowing conditions in group h countries in two different directions. In particular, consider a case that group f countries default and the endowment increases to make these countries to choose to repay, the bond price in group h countries jumps because of the wealth effect. Furthermore, given group f countries repay, the bond price in group h countries decreases as the endowment in group f countries further increases because of the substitution effect.

In this model, the total effect through which group f countries' endowment can affect the default decisions by group h countries is a combination of the wealth effect and the substitution effect. In particular, if the wealth effect dominates, the presence of group f countries in the market benefits group h countries by improving their borrowing conditions. In contrast, if the substitution effect dominates, it implies that borrowing conditions in group h countries deteriorate because lenders are withdrawing their money from these countries and, at the same time, moving toward group f countries. With an intermediate level of y_f, an increase in y_f is more likely to encourage the group f countries to repay the debt instead of issue more debt. Hence, the wealth effect is more likely to dominate at these levels of y_f. On the other hand, if y_f is sufficiently large, a further increase in y_f will not change these countries' repayment incentives significantly. Instead, a higher y_f improve group f countries' debt capacities and makes them issue even more debt, which makes the substitution effect to dominate. Intuitively, because the substitution effect is based on the relative difference between these two groups of countries' fundamentals, it is most intensified when group f countries have a high enough endowment realization relative to group h countries. I then use Proposition 1.1 to summarize the arguments above, which will also be demonstrated by the quantitative study in later sections.

Proposition 1.1. *If $D_f = 1$, q'_h is independent of y_f. If $D_f = 0$, there exist a threshold, $\mathcal{Y} \in [\underline{y}, \overline{y}]$, such that $q'_h\,|_{D_f=1} \geq q'_h\,|_{D_f=0}$ if $y_f \geq \mathcal{Y}$ and $q'_h\,|_{D_f=1} < q'_h\,|_{D_f=0}$ if $y_f < \mathcal{Y}$.*

Proof. See Appendix A.1.7. □

1.4.3 The Interdependence

To see the interdependence between these two groups of countries' default decisions, I then map the default decisions into the space of endowment realizations. By Lemma 1.1, there exists a threshold $\hat{y}(\boldsymbol{B}, y_f, b_h)$ such that a country chooses to repay only if its endowment realization is higher than $\hat{y}(\boldsymbol{B}, y_f, b_h)$. In this sense, $\hat{y}(\boldsymbol{B}, y_f, b_h)$ is also the best response function with regard to the country's default decision. Because the bond price is the only channel through wich group h countries' default decisions can be affected by group f countries' choices, Proposition 1.1 shows that $\hat{y}(\boldsymbol{B}, y_f, b_h)$ is independent of y_f if $D_f = 1$. On the other hand, if $D_f = 1$, the shape of $\hat{y}(\boldsymbol{B}, y_f, b_h)$ depends on the value of y_f. In particular, there exist a threshold, $\mathcal{Y} \in [\underline{y}, \overline{y}]$, such that $\hat{y}(\boldsymbol{B}, y_f, b_h)\big|_{D_f=1} \geq \hat{y}(\boldsymbol{B}, y_f, b_h)\big|_{D_f=0}$ if $y_f \geq \mathcal{Y}$ and $\hat{y}(\boldsymbol{B}, y_f, b_h)\big|_{D_f=1} < \hat{y}(\boldsymbol{B}, y_f, b_h)\big|_{D_f=0}$ if $y_f < \mathcal{Y}$.

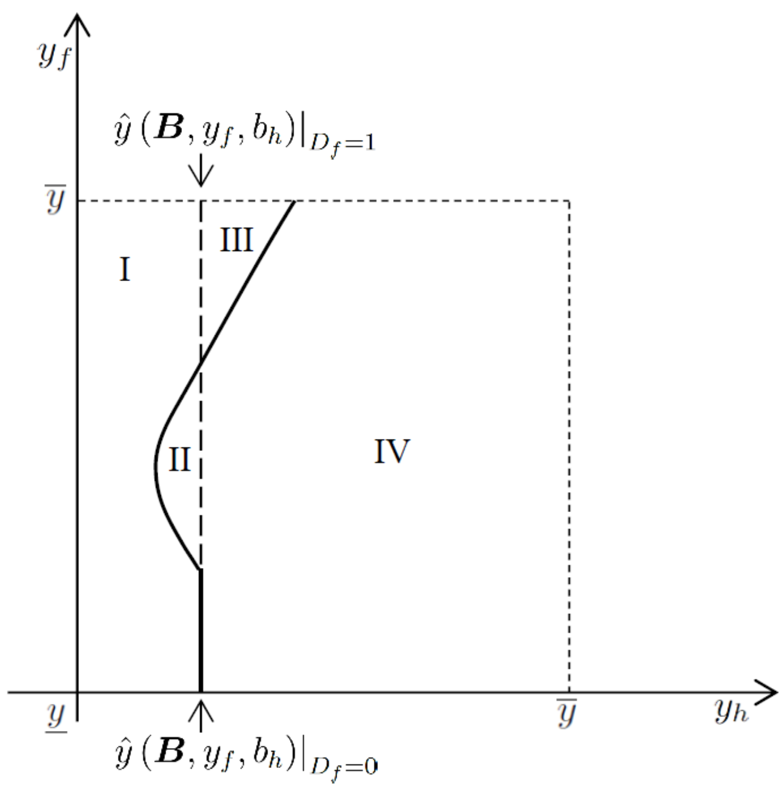

Figure 1.3: A Group h Country's Best Response

I plot both $\hat{y}(\boldsymbol{B}, y_f, b_h)|_{D_f=0}$ and $\hat{y}(\boldsymbol{B}, y_f, b_h)|_{D_f=1}$ in Figure 1.3 to summarize the analysis above, and to characterize group h countries' best response. These two lines have divided the endowment space into four regions (I, II, III, and IV), with each of them representing different default decisions in group h countries. Specifically, group h countries default in region I and repay in region IV regardless of the default decisions by group f countries. In region II, both groups of countries jointly default or repay. The most important one is region III, in which group h countries repay if group f countries default, and default if group f countries repay. By symmetry, the best response function by group f countries is similar to that for group h countries.

To characterize the equilibrium outcome, I put together the best response functions by both groups of countries in Figure 1.4. For simplicity, the graph is depicted in the case of $B_h = B_f$. The best response functions separate the endowment space into eight regions, with each of them represents different equilibrium outcomes. In equilibrium, both groups of countries repay in region II, IV, and VII, while both groups of countries default in region VI. In regions I and III, group h countries default and group f countries repay. Symmetrically, in regions V and VIII, group h countries repay, while group f countries default. These equilibrium default decisions are summarized below,

$$\begin{aligned}
\text{II} \cup \text{IV} \cup \text{VII} &= \{(y_h, y_f)|\, D_h = 0, D_f = 0\} \\
\text{VI} &= \{(y_h, y_f)|\, D_h = 1, D_f = 1\} \\
\text{I} \cup \text{III} &= \{(y_h, y_f)|\, D_h = 1, D_f = 0\} \\
\text{V} \cup \text{VIII} &= \{(y_h, y_f)|\, D_h = 0, D_f = 1\}
\end{aligned}$$

The equilibrium outcome in region III in Figure 1.3 (and, symmetrically, region V) is of particular interest in this paper because group h countries would find it optimal to repay if group f countries default[7] , $D_f = 1$. In other words, the presence of group f countries in the international borrowing market forces group h countries to default. To better illustrate the intuitions, consider two points, a and b, as both have been depicted in Figure 1.4, where $a \in$ III and $b \in$ IV. Notice that the only difference between a and b is the value of group f endowment realization: Point a has a higher y_f than point b. At both points, the low value of y_h implies that group h countries have to rollover most (or

[7] This is not true in equilibrium, but it is helpful to take as a benchmark to understand why group h countries default in this region.

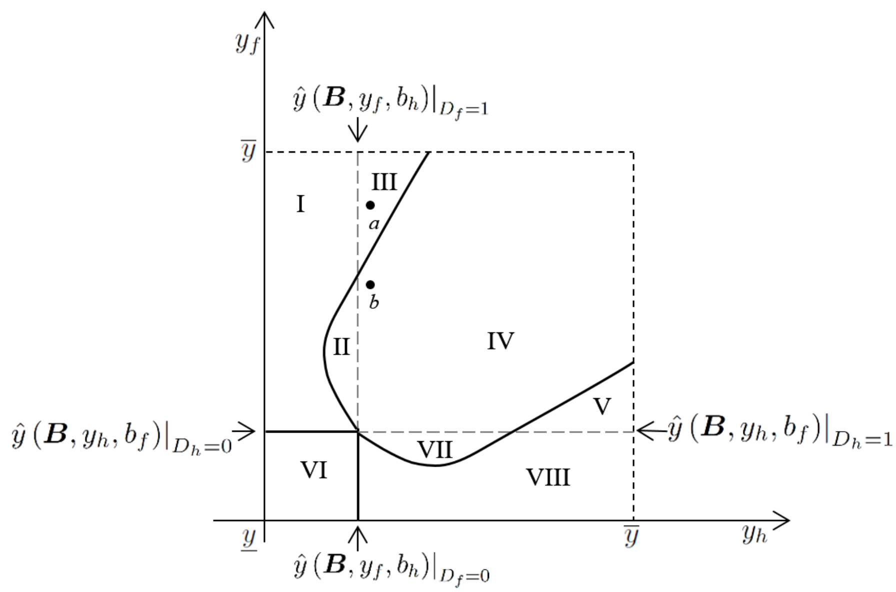

Figure 1.4: Equilibrium Outcomes ($B_h = B_f$)

accumulate more) of their debt into the next period. As a consequence, international lenders are worrying about the group h countries' abilities to service their debt at the next period. If group f countries default[8] , lenders' worries would not be turned into a large reduction in the demand of group h bond, because there is nowhere else for lenders to go. In contrast, if group f countries repay, which offers the lenders an alternative to invest. At point b, the relative difference in endowment realizations is not big enough, which implies that the group f bond is not a good substitute for the group h bond. As a consequence, the lender will not respond to high future default risk in group h countries by dramatically moving toward group f countries[9] . If instead, group f countries have a high endowment realization (at point a), the difference in endowment realizations is big enough to motivate lenders to dramatically substitute from the group h bond to the group f bond, or in other words, to commit flight-to-quality behavior. Intuitively, in this case, the group f bond serves as a safe asset for lenders to store the value, which makes lenders to dramatically retrench out of group h countries. Such a flight-to-quality

[8] Imagine that group h countries are the only borrowing countries, as in settings in the literature with a single borrowing country, e.g., [6].

[9] At point b, the presence of group f countries actually helps group h countries because of the wealth effect.

behavior by international lenders would definitely leave group h countries with sovereign debt crises and group f countries with improved borrowing conditions.

Region III in Figure 1.3 highlights the forces that lead to different changes in borrowing conditions across countries due to the substitution effect. Intuitively, risk-averse lenders are confronted by a tradeoff between future default risk and expected return in choosing their bond holdings. When y_h is relatively low, lenders' desire to earn positive returns from lending is dominated by the desire to get the promised return. The high value of the endowment realization in group f countries also intensifies such an incentive, and induces lenders to move toward group f bonds to guarantee the safety of their investment. In this sense, the substitution effect is equivalent to an additional premium requested for the group h bonds. In certain circumstances, such an additional premium is big enough to force group h countries to default, which results in sovereign debt crises. That is, the lender's incentive to take preventive measures has successfully transformed a possible default in the future into a default current period. I call lenders' behavior in this region flight-to-quality, because they are more willing to hold (relative) safe assets when there is likely to be financial turmoil. Lenders' flight-to-quality behavior is well documented by previous studies as a manifestation of international financial crises; e.g., [18] and [17]. For further reference, I will call the sovereign debt crisis that occurs in region III (and, similarly, region V) as a *flight-to-quality* sovereign debt crisis.

During the recent Euro Zone Crisis, Germany, and the United States experienced improved borrowing conditions while, some the European peripheries continued to have difficulties in borrowing abroad. In this paper, I argue that the substitution effect provides a reason for the decreased debt–yield rate observed in those countries during the European debt crisis. As I've argued above, the substitution effect in region III indicates that lenders reduce their bond holdings in group h countries and, at the same time, increase their bond holdings in group f countries. Other things being equal, capital flight toward group f countries would push up the bond prices or, similarly, push down the bond yield rate in these countries. The result is consistent with the finding by [21], in the sense that they find that US Treasury yields declined by 73 basis points, on average, from 1926 to 2008 because investors valued the liquidity and safety of US treasuries. However, [21] obtained their results by exogenously assuming that investors prefer U.S. treasuries, and thus have been able to calculate how such a preference affects the yields on U.S. treasuries. In contrast, the result in this paper is endogenous based on the lender's optimal choice on bond positions across countries.

Notice that Figure 1.4 depicts in the case of unique equilibrium. Although a pure strategy equilibrium can be guaranteed in the large-game setting in this paper, it is not necessarily the only one[10] . In contrast, the wealth effect examined in this paper could complicate my analysis by introducing multiple equilibria, as discussed by [1]. Since the wealth effect is more prominent in the case when both groups of countries have similar fundamentals, the problem of multiple equilibria is more likely to emerge in regions II, IV, and VII. However, this paper focus on region III (or region V), in which both groups of countries have sharply different fundamentals. In this region, the wealth effect is dominated by lenders' incentive to distinguish these two groups of countries when lending money. In the quantitative study in later sections, the problem of multiple equilibria has been taken care by choosing the equilibrium that maximizes the sum of both groups of borrowing countries' utilities.

[10] For discussions of the existence and properties of pure strategy equilibrium in large games, see [14], [15], and [16].

Chapter 2

Flight-to-Quality Debt Crises: Quantitative Analysis

2.1 Quantitative Analysis

I solve the infinite periods model numerically and analyze the interactions between these two groups of countries in the international borrowing market. Specifically, I analyze how the fundamentals and choices in one group of countries would affect bond prices and default decisions in the other group. I show that the substitution effect is quantitatively important, especially when one group of countries are on the brink of defaulting.

2.1.1 Calibration

The periodic utility function for the borrower is CRRA,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

Following common business cycle studies, I set the intertemporal elasticity of substitution (IES), $1/\sigma$ to $1/2$. The periodic utility function for the lender is logarithmic[1]
,

$$\psi(c_L) = \log c_L$$

[1] The lender's utility function would be loosened to be CRRA in the section of sensitivity analysis.

The length of a period is one year. The stochastic process for endowment is independent across these two groups and follows a lognormal AR(1) process,

$$\log\left(y_{t+1}\right) = \rho \log\left(y_t\right) + \varepsilon_{t+1}$$

with $\mathbb{E}\left(\varepsilon^2\right) = \eta^2$. The endowment process in one group of countries is assumed to be independent of the endowment process in the other group of countries. I use the method proposed by [24] to discretize the shock into a Markov Chain. The endowment space $\left[\underline{y}, \overline{y}\right]$ is the set of $[0.83, 1.21]$ with an unconditional mean equals to 1.04, and it has discretized into 11 grids. The debt space $\left[\underline{b}, \overline{b}\right]$ is discretized into 31 grids. The numerical algorithm is explained in detail in Appendix A.2.

Eight parameters are calibrated: I follow [1] to set up the stochastic structure for borrower's endowment process and the lender's fixed endowment. The reentry probability θ is set to be 0.282 to capture the fact that, on average, the duration of market exclusion is around 4 years; see, for example, [3]. The value of default cost λ is borrowed from [6], which has been set to be 0.016. On the other hand, the lender's discount factor, δ, is set to match with the average risk-free interest rate, which equals to 4%. Finally, the borrower's discount rate β, is set to match with two moments: the average default probability, 5%; and thus the mean of yield rate in Greece, 5.6%. Calibrated parameters and their matching moments are summarized in Table 2.1.

	Value	Target
Borrower's IES	$1/\sigma = 1/2$	Standard Value
Stochastic structure for shocks	$\rho = 0.88, \eta = 0.03$	[1]
Output cost after default	$\lambda = 0.016$	[6]
Reentry probability	$\theta = 0.282$	average Market Exclusion, 4 years
Borrower's discount factor	$\beta = 0.90$	Mean Default Probability, 5%
Lender's discount factor	$\delta = 0.96$	Mean Risk Free Interest Rate, 4%
Lender's endowment	$y_L = 1.4$	[1]

Table 2.1: Parameter Values

2.1.2 Main Results

The model is simulated for 2,500 times and statistics summarizing the debt markets in group h countries are reported. Table 2.2 reports the yield, the default probability, the debt to GDP ratio, and the implied risk–free rate for three cases: the overall mean, when group f countries repay, when group f countries default. The yield is defined as $1/q-1$, while the implied risk-free rate is calculated by

$$r_f = \frac{1}{\mathbb{E}\left[\frac{\delta\psi'(c_L')}{\psi'(c_L)}\right]} - 1$$

Table 2.2 shows that the overall mean of yield is 5.4%, the overall mean of the default probability is 4.5%, the overall mean of the debt to GDP ratio is 4.4%[2] , and the overall mean of the risk-free interest rate is 4%. These four statistics, however, are affected by the default decisions in group f countries. When group f countries default, the yield in group h countries decreases to 5.2%. This reflects the fact that default by group f countries makes the bond issued by group h countries the only asset to save for the future, and thus lenders would prefer to pay, in general, a higher price for the group h bond. The better borrowing conditions encourage group h countries to repay, as the default probability in group h countries decreases to 4.0%. Furthermore, the better borrowing conditions also encourage group h countries to borrow more: The debt to GDP ratio is 4.5% when group f countries default, higher than the value of 4.3% when group f countries repay. The pattern of changes in yield can also be demonstrated by looking into the implied risk-free rate, which decreases from 4.1% to 3.9%. That is, when group f countries default, lenders in general would request a lower interest rate from any other forms of borrowing.

The arguments in the previous section show that the combination of the wealth effect and the substitution effect is not monotonic across y_f values. In particular, for medium levels of y_f, the wealth effect dominates and it improves borrowing conditions in group h countries. On the other hand, for high levels of y_f, the substitution effect dominates, which will impose a downward pressure on the group h bond price. Table 2.3 reports the same statistics as Table 2.2 conditional on endowment realization in group f countries. Specifically, the endowment realization in group f countries has been separated into three categories—low: $y_f < 0.963$; medium: $0.963 \leq y_f \leq 1.04$; and high: $y_f > 1.04$.

Table 2.3 shows that compared to the case when group f countries have a low endowment realization, the group h bond yield is lower when the endowment realization in group f countries takes a medium value and it is higher when group f countries'

[2] The model in this paper shares the same difficulties with quantitative default model in generating the right magnitude of debt to GDP ratio, see, for example, [2].

endowment realizes to be high. Furthermore, when the endowment realization in group f countries takes a high value, the group h bond yield is at its highest level among these three cases. That is, although a medium level of y_f benefits group h countries through the fact that the wealth effect dominates, the further increases in y_f would raise the importance of the substitution effect and finally motivate the lenders to request a higher premium from group h countries. In this case, group h countries respond to the worst borrowing conditions by being more likely to default: Default probability increases from 3.9% to 5.0%. Similar to the discussions before, the changing borrowing conditions also change the debt positions in group h countries: with the worst borrowing conditions when group f countries have a high endowment realization, the debt to GDP ratio is 4.1%, lower than the value of 4.6% when the wealth effect dominates. Such a pattern of changes in group h debt yield can also be justified by looking into the risk-free rate, which is also at its highest value when group f countries have a high endowment realization. In particular, with a higher endowment realization in group f countries, lenders are more heavily investing on the group f bond and are more confident about getting payments from group f countries in the next period. As a consequence, lenders would thus request a higher interest rate on any form of borrowings (even if they are risk-free), given that their payoffs at the next period are expected to be higher.

Group h countries	Overall Mean	Group f repays	Group f defaults
Yield (%)	5.4	6.0	5.2
Default Probability (%)	4.5	4.8	4.0
Debt to GDP (%)	4.4	4.3	4.5
Risk-Free Rate (%)	4.0	4.1	3.9

Table 2.2: Statistics (Default Decision)

The results in both tables show that the model in this paper is able to explain the empirical evidence from the recent European debt crisis. In particular, the results in Table 2.3 show that if the endowment realization in the European peripheries change from a high value to a low value, the yield in Germany, and the United States is reduced by 1.0% and the debt to GDP ratio in these two countries increases by 0.3%. Because the low value of endowment realization in European peripheries makes the bond issued by these countries more risky, lenders would like to shift toward Germany, and the United States. In addition, results in Table 2.2 show that the presence of Germany, and the United States in the borrowing market also intensifies the difficulties faced by European peripheries. Compared to the case in which these two countries default, the yield in European peripheries is higher by 0.8%. That is, the borrowing conditions in European peripheries are further deteriorated given the fact that international lenders are substitution from their bonds to the bond issued by Germany, and the United States. In summary, the results in both table match with the empirical evidence from the recent crisis: The bond price in Germany, and the United States increases, which worsens the difficulties faced by European peripheries.

Another interesting fact about the results in Table 2.2 and Table 2.3 is that the changes in the risk-free rate are always smaller than the changes in the yield. For example, when y_f increases from a low value to a high value, the risk-free rate increases by 0.4%, while the increase in yield is 1.0%. Such a smaller change in risk-free interest rate can be attributed to the group h countries' endogenous response to an improved borrowing condition. Notice that when calculating the risk-free interest rate, the issuer has been assumed to always repay their debt. The non-defaultable nature makes the supply of risk-free bond more elastic. That is, the risk-free bond issuer accommodates lenders' shift toward them by issuing more debt, rather than lowering the yield, and thus would have the largest impact on the demand of bond in group f countries. However, the decrease in risk free interest rate is too small to match the decrease in debt yields in Germany, and the United States. In my model, if group h countries respond to the increased demand only by issuing more bond, their default risk increase dramatically. Hence, the group h countries' endogenous response is more moderate compared to the case of risk-free bond: When facing with improved borrowing conditions, group h countries would choose to have a combination of more debt issuance and a lower debt yield.

Group h countries	Low y_f	Medium y_f	High y_f
Yield (%)	5.3	5.0	6.3
Default Probability (%)	4.3	3.9	5.0
Debt to GDP (%)	4.4	4.6	4.1
Risk-Free Rate (%)	4.0	3.9	4.4

Table 2.3: Statistics (Endowment)

2.1.3 Bond Price Schedule

Results in both Table 2.2 and Table 2.3 demonstrate the interdependence of bond price function across countries by examining the average statistics. In this section, I further study how the bond price schedule in group h countries have been affected by endowment realization in group f countries. In particular, Figure 2.1 plots the bond price schedule, q'_h, as a function of new bond issuance, b'_h. The schedules are for a group h endowment realization as $y_h = 1.04$ and initial debt levels in both groups, $B_h = B_f = 0.09$. I plot the bond price schedules at three different levels of endowment realization in group f countries, $y_f = 0.96$, $y_f = 1.04$, and $y_f = 1.12$. The bond price is a decreasing function of new bond issuance, because a higher new bond issuance implies a higher default probability in the next period, which is consistent with the theoretical discussions before.

The bond price schedule in the case of $y_f = 0.96$ is depicted as the solid line in Figure 2.1. When y_f changes to its unconditional mean, 1.04, the bond price schedule shifts up (the dashed line), which implies that borrowing conditions in group h countries improve, as a consequence of the wealth effect. Specifically, when y_f increases to 1.04, instead of defaulting, group f countries are more likely to choose to repay, but do not pay down the debt by a large amount. As a consequence, the group f bond serves as a complement to the group h bond, and thus lenders would be request a lower premium from group h countries on borrowing. On the other hand, when y_f increases further be 1.12, the bond price schedule (the dotted line) becomes tighter, and it is tightest among these three cases. In this case, further increases in y_f make the group f bond a safe asset for the lenders, and also as a good substitute for the group h bond. As a consequence, the lenders would like to flee to group f countries, and thus imposes a downward pressure on the group h bond price. Hence, it leaves group h countries with the worst borrowing conditions of the three cases.

To further investigate how the bond price in group h countries varies with the endowment realization in group f countries, Figure 2.2 plots bond price in group h countries as a function of y_f in two cases: Group f countries are in the market and group f countries are not in the market. Similar to the Figure 2.1, I plot the bond price in group h countries under the case of $y_h = 1.04$, $B_h = 0.09$, and $B_f = 0.09$ if group f countries are in the market. The new bond issuance B'_h is obtained by using the

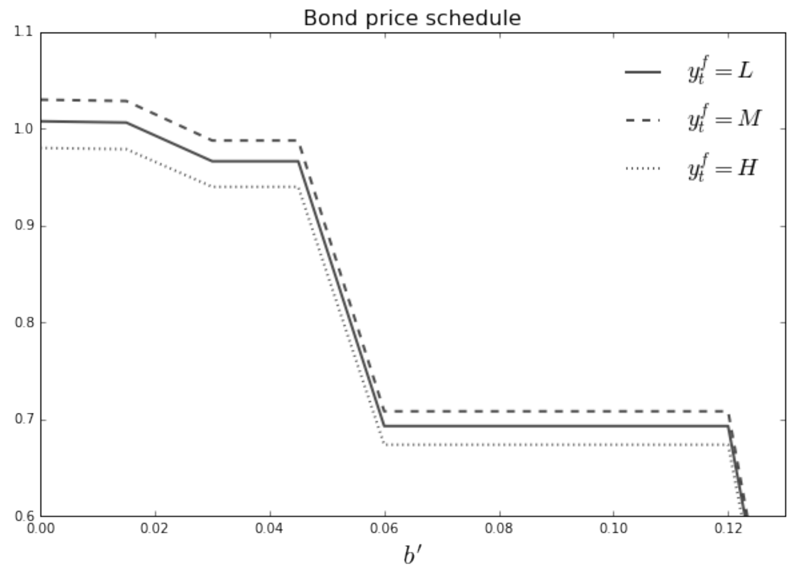

Figure 2.1: Bond price schedule

optimal rule by group h countries. The bond price when group f countries default is depicted as a straight line in Figure 2.1 because of the permanent market exclusion after default in the two-periods example. In the numerical study of infinite-periods model, defaulted countries are allowed to reenter the market with an exogenous probability. Hence, even if group f countries are under market exclusion, the value of y_f can still affect the bond price in group h countries slightly because the strictly positive future possibility of reentering the market, as it has been depicted as the solid line in Figure 2.2.

On the other hand, if group f countries are in the market with a debt level $B_f = 0.09$, the bond price in group h countries is highly related to y_f, as it has been depicted as the dashed line in Figure 2.2. Specifically, for low values of y_f, both these two curves overlap with each other because with a low value of endowment realization, group f countries would choose to default even if they are in the market. For intermediate values of y_f, the dashed line lie above the solid line because of the wealth effect I've discussed before. On the other hand, for high values of y_f, q_h' is decreasing in y_f because of the substitution effect, which makes the dashed line lie below the solid line. By comparing the Figure 2.1 to the Figure 1.3, it is easy to see that the numerical results here are consistent with my previous discussions on how the combination of the wealth effect and the substitution effect would affect the borrowing conditions in group h countries.

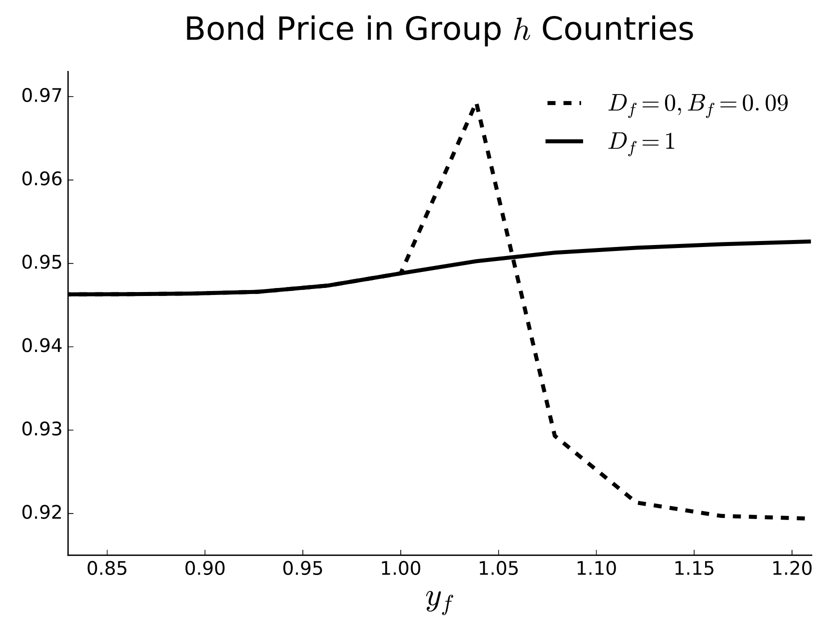

Figure 2.2: Bond price in group h countries

2.1.4 A Sudden Drop

One main goal of this paper is to see how the debt yields and debt positions in both groups countries have been affected when there is a crisis in one group. In particular, both the debt position and the bond price in group h countries increase as a response to a crisis happening in group f countries, and the lenders are shifting toward group h countries. However, such a mechanism is hard to show in the full simulation because it is difficult to disentangle the changes due to lenders' flight-to-quality behavior from the changes due to fluctuations in countries' absolute fundamentals. I then consider an experiment in which endowment realization in group f countries drops while endowment realization in group h countries has been held constant. Specifically, both y_h and y_f equal to 1.12 initially and remain unchanged for the first 50 periods. The initial debt levels at both groups of countries are set to be 0.105. At period 50, y_f is assumed to drop to the level of 1.04 unexpectedly, and y_h is held constant at 1.12.

Figure 2.3 plots the dynamics of bond prices and bond positions in both groups. In the initial period, given $y_h = y_f = 1.12$ and $B_h = B_f = 0.105$, the optimal new bond issuance in both groups are equalized, $B'_h = B'_f = 0.105$. That is, given the endowment realization in both groups, each country will simply pay the interest and maintain the same debt level. Hence, the bond prices in both groups are equalized and have a value of 0.932. Given endowment realizations in both group are set to be constant for the first 50 periods, the bond positions and the bond prices in both groups are constant and equalized for the first 50 periods. When y_f decreases unexpectedly at period 50, the spread of debt yields between these two groups increases. Specifically, with a lower endowment realization relative to group h countries, the bond price in group f countries declines from 0.932 to 0.914. On the other hand, the bond price in group h countries increases from 0.932 to 0.95 because their relatively higher endowment realization, and the lenders are shifting toward them. Both the lower endowment realization and the lower bond price make group f countries to borrow less: The bond position in group f countries decreases from 0.105 to 0.045. Although the endowment realization in group h has been held constant, the higher bond price induces them to borrow more: The bond position in group h countries increases from 0.105 to 0.12.

The results in Figure 2.3 show that the relative fundamentals, rather than absolute fundamentals, are important to motivate lenders' flight-to-quality behavior, and thus are important to determine the bond prices across countries. Specifically, the decline in y_f at period 50 makes the bond issued by group h countries more appealing to the bond issued by group f countries. Although the endowment realization in group h countries doesn't change, the bond price in these countries increases because international lenders are substituting from the group f bond to the group h bond.

Another interesting aspect of Figure 2.3 is that the short-run changes in bond price is different from the long-run changes. In particular, there is overshooting in the group h bond price, and the group f bond price increases initially despite it declines in the long run. This is because the sudden drop in y_f would firstly be reflected in changes in the bond prices before both groups of countries had optimally responded to it by changing their debt positions. When y_f decreases at period 50, the difference in relative fundamentals is large and will be firstly reflected by the group h bond price. After the group h countries increase their debt issuance, the group h bond price would drop, but it is still higher than the bond price before the change. This implies that the supply of bond in group h countries is not perfectly elastic: Group h countries accommodate the improved borrowing conditions by issuing more debt, as can be seen from Figure 2.3, but they will not increase the debt issuance enough to exhaust all the increase in the demand of their bond, and still leave rooms for their bond price to increase. Furthermore, because the group h countries are accommodating the lenders' flight-to-quality behavior, an increase in group h bond issuance implies the group f countries are now facing with decreased demand of their bond, which will finally drive down the bond price in group f countries and force these countries to reduce their debt positions, as it is shown in Figure 2.3.

The experiment here is able to explain the dynamics in debt yields and debt positions during the recent crisis. Specifically, the numerical results predict that because the GDP in European peripheries dropped at late 2009, international lenders shifted toward Germany, and the United States. The international lenders' flight-to-quality behavior left Germany, and the United States with increased demand of their bonds, while reduced the demand of bonds issued by European peripheries. On the other hand, these two safer countries accommodated such a change by increasing their bond issuance, which finally resulted at a combination of increased bond prices and debt positions in these two countries, and, at the same time, the decreased bond prices and difficulties in raising funds abroad in European peripheries. In this sense, the bonds issued by Germany, and the United States served as the global safe asset for the international lenders, which is similar to the results in [19, 20].

2.1.5 Sensitivity

The results in previous sections show that the substitution effect is quantitatively important for generating different changes in debt yields and debt positions across countries during the recent crisis. Because the flight-to-quality behavior is a consequence of lenders' willingness to avoid a loss, its magnitude is affected by how risk averse and how wealthy the lenders are. In particular, the more risk averse the lenders are, or the fewer periodic income they have, the more likely they would like to re-balance their portfolios, when there is a difference between countries' relative fundamentals.

In this section, I show that the above arguments are consistent with results from the

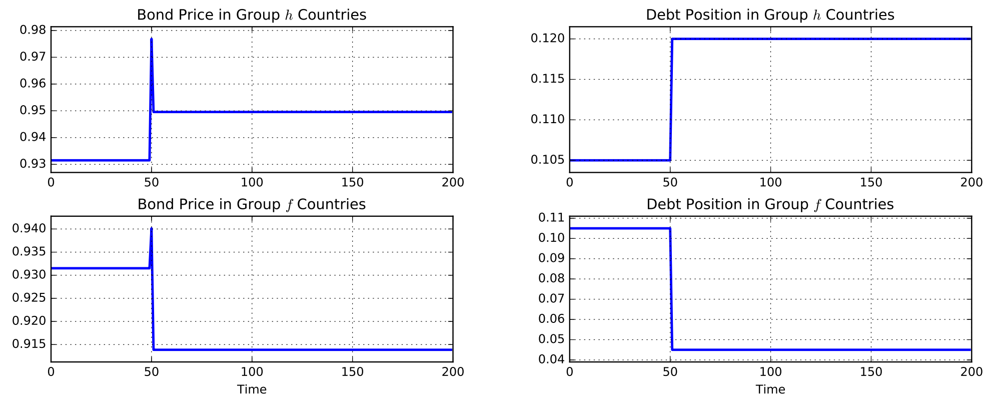

Figure 2.3: A Sudden Drop in Group f Countries' Endowment

numerical exercise by changing the lender's risk aversion or periodic fixed endowment. In this exercise, the lenders' utility function has been changed to be CRRA,

$$u\left(c_L\right) = \frac{c_L^{1-\gamma}}{1-\gamma}$$

I consider different values of lender's risk aversion or periodic fixed endowment: a linear utility function ($\gamma = 0$), a lower value of γ, a higher value of γ, or a lower value of y_L. The first four rows of Table 2.4 report results under these four variations, as well as the benchmark model. First, the overall mean of debt yield and default probability are increasing in the value of γ: The more risk averse the lenders are, the higher the premium they would request from borrowing countries, which will force borrowing countries to be more likely to default. Second, both these two statistics increase in the case of $y_L = 1.0$. Because the absolute risk aversion is decreasing in the level of wealth under a CRRA utility function, a lower periodic income in this model makes the lenders charge a higher premium in general. Furthermore, the higher yield associated with the higher risk aversion (or the lower income) also forces borrowing countries to borrow less: The overall mean of debt to GDP ratio drops, but the magnitude is lower than the change in debt yield. Finally, the overall mean of risk-free interest rate remains unchanged, because it is governed by lenders' discount rate, δ, which also remains unchanged.

Table 2.4 also reports results that showing how changes in lenders' risk attitude and endowment determine how borrowing countries have been treated differently in the international borrowing market. First, I examine the difference in these four statistics in group h countries between the case of a high y_f and the case of a low y_f. The results are consistent with the economic intuition: The differences in all these four statistics are bigger when the lenders either become more risk averse or have a lower periodic fixed endowment. That is, international lenders are more likely to commit flight-to-quality behavior when they become more risk averse. Second, I also examine the correlations of the yield rate between these two groups of countries by separating y_f into three categories in a similar way as Table 2.3. It is easy to notice that correlations become more diverse if lenders are more risk averse or have a lower endowment, which again demonstrates that lenders with a higher risk aversion are more likely to shift across borders when the relative fundamentals are different across countries.

Group h countries	Benchmark	Linear	$\gamma = 0.5$	$\gamma = 2$	$y_L = 1.0$
Mean					
Yield (%)	5.4	4.6	4.8	7.2	5.8
Default Probability (%)	4.5	4.2	4.3	8.8	4.8
Debt to GDP (%)	4.4	5.3	5.1	4.0	4.2
Risk-Free Rate (%)	4.0	4.0	4.0	4.0	4.0
Difference, High y_f - Low y_f					
Yield (%)	1.0	0.0	0.3	4.6	1.2
Default Probability (%)	0.7	0.0	0.2	2.6	1.1
Debt to GDP (%)	0.3	0.0	0.1	0.5	0.4
Risk-Free Rate (%)	0.4	0.0	0.2	0.7	0.5
Corr. of Yields					
Low y_f	0.52	-0.01	0.34	0.53	0.63
Medium y_f	0.58	0.00	0.46	0.78	0.65
High y_f	0.42	0.00	0.18	0.41	0.44

Table 2.4: Sensitivity Analyses

2.2 Discussion

The previous sections have demonstrated that the substitution effect is important in inducing lenders' flight-to-quality behavior, and finally results in different changes in borrowing conditions across countries, which is consistent with empirical evidence from the recent crisis. Although the numerical study is successful in matching the general pattern of changes in yields and debt positions in Germany, and the United States, it fails to generate the increase in yield in European peripheries with the same magnitude. In this paper, both groups of countries are symmetric, and thus the changes in yields would have same magnitude. However, countries such as Germany, and the United States are also big in their economic sizes, compared to European peripheries. As [19, 20] argue, these countries' bigger economic size implies that they have bigger debt capacities to absorb capital flights from European peripheries. By taking country size into account, I expect that the model will generate the decrease in yields in European peripheries with the correct magnitude, because the bigger debt capacities in Germany, and the United States would further motivate lenders' flight-to-quality behavior. However, such an experiment is left for future research.

Another important feature of sovereign debt crises is that they occur in tandem, e.g., the Asian financial crisis in 1997, the recent Euro Zone Crisis, etc. Prior studies address the question of how shocks in one country are transmitted to another country. [25] first noticed that the contagion of sovereign debt crisis can be explained by the basic principles of portfolio theory. [23] describe crisis contagion as a phenomenon caused by the wealth effect: when negative shocks in one particular asset brings the investor a loss, she tends to retrench her investment from the whole portfolio due to the lower level of wealth. In a numerical exercise, [1] show that home country's default probability increases dramatically when foreign country defaults due to the wealth effect.

However, if the recent crisis in European peripheries is taken as a contagion of the Greek crisis, it is easy to see that a crisis is not propagated to every country without any discrimination. [26] show that investors who suffer a lost due to a crisis are more likely to retrench their funds from countries (and thus the contagion of crisis) that share overexposed investors with the crisis country. However, their model implies that whether a country suffers from the crisis contagion is purely determined by lenders' past behavior. By inducing lenders to treat borrowing countries differently, the substitution effect (and in turn the lenders' flight-to-quality behavior) examined in this paper has the potential to explain such a pattern of the crisis propagation. I will provide a case study below to address the importance of the substitution effect in explaining why some countries suffer from contagion during the crisis, but others don't. In particular, the following analysis focuses on answering the question of what characteristics contaminated countries share with the crisis country that makes them targets for lenders to retrench their investment.

Suppose that in addition to these two groups of countries, lenders also trade with another country—say country z—which has been expected to always repay the debt. Suppose country z defaults unexpectedly, which causes lenders to lose a fixed fraction of their periodic endowment, $\mu y_L, \mu \in (0, 1)$. Lenders' lower income implies that they would charge a higher premium in general on international lending, which shifts group h countries' best response functions to the right. Moreover, as it has been demonstrated in the sensitivity analysis in section 2.1.5, the substitution effect has been intensified, because lenders are more risk averse due to the lower level of wealth. As a consequence, $\hat{y}\left(\boldsymbol{B}, y_f, b_h\right)\big|_{D_f=0}$ shifts to the right more than $\hat{y}\left(\boldsymbol{B}, y_f, b_h\right)\big|_{D_f=1}$.

Figure 2.4 plots group h's best response functions before and after the change: $\hat{y}_0\left(\boldsymbol{B}, y_f, b_h\right)\big|_{D_f=0}$ and $\hat{y}_0\left(\boldsymbol{B}, y_f, b_h\right)\big|_{D_f=1}$ are best response functions before the change, while $\hat{y}_1\left(\boldsymbol{B}, y_f, b_h\right)\big|_{D_f=0}$ and $\hat{y}_1\left(\boldsymbol{B}, y_f, b_h\right)\big|_{D_f=1}$ are best response functions after the change. Consider an output realization at point a in Figure 2.4: Group h countries don't default if country z doesn't default, but they will be forced to default after country z defaults. This is because when y_L decreases, lenders are motivated to reexamine their portfolios due to the increases in their absolute risk aversion. When the endowment realization happens to be point a, lenders notice that that group h countries' future default risks are relatively high compared to group f countries'. This property of group h bond becomes more unfavorable to the lenders under the new level of wealth, and thus lenders would like to retrench from group h countries and increase the proportion of group f bonds in their portfolios. Decreased demand will again force group h countries to default immediately. On the other hand, group f countries have better borrowing conditions, because they are experiencing capital inflows. That is, the lenders in this model are retrenching from countries which share similar risk profiles as the crisis country, instead of similar portfolio exposures as in [26].

The example in the last paragraph can be mapped onto the recent European debt crisis. Specifically, after Greece defaults, international lenders with a lower level of wealth are more sensitive to the future default risks and will reexamine the risk structure of their portfolios. Because European peripheries have a relatively higher future default risk compared to Germany, and the United States, they are the first group of countries from which lenders want to withdraw their money. At the same time, the larger debt capacities in Germany, and the United States provide the lenders safe assets to store their value. As a consequence, the lenders' flight-to-quality behavior leaves other European peripheries with the contagion of the Greek crisis, and at the same time, improved borrowing conditions and increased debt positions in Germany, and the United States, because they are relatively safe. Such a pattern of sovereign debt crisis contagion is more consistent with empirical evidence from the recent crisis.

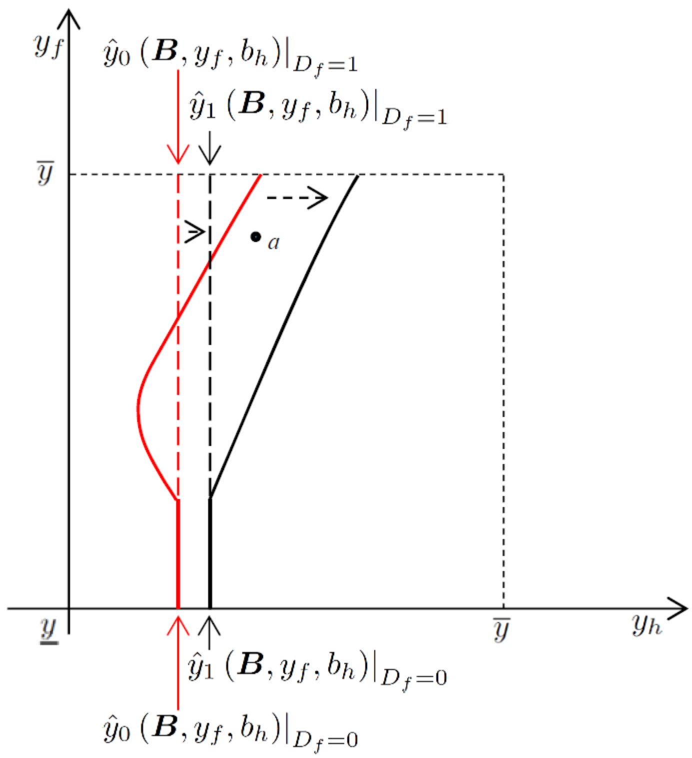

Figure 2.4: Contagion of Crisis

2.3 Conclusion

In this paper, I develop a model of sovereign default with two groups of borrowing countries and one continuum of lenders. In the model, sovereign debt crises happens in one group of countries, and improved borrowing conditions and increased debt positions in the other group of countries coexist when lenders re-balance their portfolios dramatically. A substitution effect arises between different bonds, because borrowing countries are competing with each other for international borrowing. In particular, the substitution effect indicates international lenders to reduce their bond holdings in one group of countries if countries' relative fundamentals are different, and the future default risks in these countries are relatively high. When the difference between these two groups' fundamentals is big enough, lenders' flight-to-quality behavior is dramatic enough to leave one group of countries with flight-to-quality debt crises, and at the same time, the other group of countries with improved borrowing conditions and increased debt positions, because they are experiencing capital inflows. In this sense, the model is able to explain the debt yield dynamics across countries during the recent crisis, especially the drop in debt yield in Germany, and the United States. Furthermore, because Germany, and the United States accommodate the lenders' capital flight by issuing more debt, the model also predicts that the difficulties faced by European peripheries have been intensified. Lastly, the substitution effect emphasized in this paper predicts a crisis is only proved to be contagious to countries which are likely to share similar problems as the crisis country in servicing their debt. In this way, the model in this paper is also able to explain why a crisis originating in Greece only spreads to other European peripheries, but not to Germany, and the United States.

Chapter 3

Persistent Market Exclusion

3.1 Introduction

The sovereign debt is subjected to default because it is hard to enforce the contract across borders. It is natural to argue that the after default punishment is necessary to support equilibrium borrowing in sovereign debt market. Therefore, a rational lender cares about how she should punish the defaulted borrower at the moment of lending money out. The main question in this paper is given the borrower's endowment is persistent, what is the *optimal* duration of market exclusion the international lender should choose?

In this paper, I analyze a case under which a lender trades repeatedly with a sovereign country. The sovereign country's endowment process is persistent and it is private information to the country itself. Different from traditional studies under which the duration of market exclusion has been set to be exogenous[1] , in this paper, the after default punishment is also part of the contract design. In specific, the lender decides on recovery ratio such that the borrower should repay the debt recovery specified to reenter the market (or at least to avoid the output cost). The borrower with a bad credit standing, on the other hand, decides on whether to repay the debt recovery to reenter the market and escape from the output cost due to market exclusion. By doing so, the model in this paper synthesize the choice of the duration of market exclusion with the problem of what is the recovery ratio on defaulted debt.

[1] See for example, [4], [12], [6], etc.

The lender in this paper offers to the borrower a recovery plan which is decreasing over time. Consequently, the borrower who defaulted but has a relatively high endowment realization would prefer to repay in short periods after default. On the other hand, the borrower who has a low endowment realization cannot afford a high recovery, thus she would choose to stay with the market exclusion (and so as the output cost) in order to get a higher haircut. In other words, the lender in this paper is using time to separate the borrower's states. Such a result is also consistent with [3] where they find the haircut on defaulted debt is increasing in the duration of market exclusion.

The fact that the borrower's endowment process is private information prevents the lender from making debt repayment fully contingent on the borrower's states. Instead, the lender chooses a optimal plan to elicit the highest expected value of repayments from the borrower. A harsher plan is beneficial to the lender because it brings higher repayments when the borrower repays for market reentry *ex post*, and it also discourages the borrower from defaulting *ex ante* by making the after default value lower. However, it doesn't mean that the lender should choose not to permit any haircuts on defaulted debt. A harsher plan also makes the duration of market exclusion too lengthy such that it is also non-optimal for the lender to keep it. Therefore, it is optimal for the lender to choose a plan with positive haircuts, as in [27].

The lender in this paper provides a recovery plan that features decreasing recovery ratios over time, which distinguishes this study from the invariant recovery results under an i.i.d. setting in [27]. To understand why, let's think about a case that a defaulted borrower's endowment can take two possible values, low and high. Given the lender will grant a haircut on the defaulted debt, it is more efficient for the lender to allocate the haircut to the borrower with a low endowment. However, as the borrower's endowment is unobservable, the borrower with a high endowment would like to pretend to be a low type. To make such a plan feasible, it requires the low type borrower to stay with the market exclusion longer in general. Such an arrangement is efficient also in the sense that the output cost under market exclusion is lower (and thus more bearable) for the low type borrower. In equilibrium, the high type borrower will pay more but enter the market earlier while the low type borrower should wait longer in order to pay less. In this sense, a decreasing recovery ratio enables lender to partially make debt repayment contingent on the borrower's endowment realization. By doing so, the lender's optimal plan on recovery ratio separates the borrower's states by using time.

The reason why the lender can choose such a plan is because the borrower's endowment process is persistent in this paper. A borrower refuses to repay the debt recovery signals to the lender a low current endowment realization. Because the endowment process is persistent, it also signals a low endowment in the future. Since the borrower refused to repay the debt recovery, the lender should provide a lower recovery to bring the borrower back to the market. In contrast, if the borrower's endowment process is i.i.d., as in [27], there is no longer any linkages between borrower's endowments across periods. Specifically, a low current period endowment realization doesn't necessary imply a low endowment realization in the future. As a consequence, the lender decides the recovery ratio solely based on the invariant endowment distribution, and thus the recovery ratio should be time invariant. Therefore, the model in this paper is able to explain the empirical findings in [3], where they found the haircut on defaulted debt is increasing in the duration of market exclusion.

This work builds on the model of efficient sovereign default analyzed by [27] and [28]. The models presented in these papers focus on contracting problem of sovereign default in the case of an i.i.d. endowment process. They show that despite the market exclusion is inefficiency in an ex post sense, it is necessary to support the ex ante efficiency so that sovereign debt borrowings can be sustainable. However, because the endowment process is i.i.d., the lender's way of choosing the inefficiency is invariant over time. That is, the recovery on defaulted debt is time invariant in their paper. This paper consider a similar problem with the lender's endowment process to be persistent. Different from previous studies based on i.i.d. settings, the lender in this paper no longer chooses a time invariant recovery ratio. Instead, the lender in this paper chooses a recovery plan which features a decreasing recovery ratio over time.

Another branch of sovereign debt literature examines empirical evidence regarding sovereign default and debt renegotiation, such as [29, 30], [31] and [3]. In particular, by constructing a complete database of default and debt restructurings from 1970 until 2010, [3] document that the haircut on defaulted debt is increasing in the duration of market exclusion. They also show that the probability of market reentry is increasing over time after the default. However, they tend to explain the empirical results in a way that the lender chooses a longer market exclusion to punish the borrower requests a higher haircut. In contrast, a higher haircut in this paper is not the direct cause of a higher market exclusion. In fact, the lender chooses them together to separate the borrower's states. Because in equilibrium, the recovery ratio is decreasing over time, the model in this paper can generate similar results that the probability of market reentry is increasing over time after default.

More generally, this paper also contributes to sovereign debt literature on debt restructuring since the lender takes market exclusion as an apparatus to persuade debt repayments. The existing studies model the debt restructuring as a bargaining game, ultimately concluding that the lengthy debt restructuring is a result of the lender's eagerness to get a higher recovery rate on defaulted debt. For instance, studies like [32] and [33] attribute the delay in debt restructuring to be a consequence of both parties are waiting for a better borrower's economic perspective and thus a bigger pie to split. On the other hand, some other authors look into the holdout problem in debt restructuring, see for example, [34] and [35]. Because the defaulted borrower is bargaining with a bunch of international lenders, and because any single lender can veto the whole debt restructuring plan, the lenders in these papers is seeking for last move advantage. Therefore, equilibrium holdouts by international lenders emerges. However, these papers predicts that a longer market exclusion should be associated with a lower haircut due the last move advantage, which contradicts to the empirical findings in [3]. In this paper, besides the static gains from resuming trade after default, the lender chooses the debt restructuring plan also to deliver dynamic incentives. In particular, the lender chooses a decreasing debt recovery over time so that she can bring the borrower back to the market by orders, according to the borrower's endowment realization. By doing, the lender is efficiently exploit the efficiency in choosing the debt restructuring plan.

The model in this paper is related to the classical studies on contracting the literature on contracting problems with private information, such as [36], [37, 38], etc. Similar to previous studies, the lender in this paper cannot observe the borrower's endowment state, which prevent her from making the debt recovery contingent on any observable information regarding the borrower's state. Instead, the lender relies on provide a plan which is incentive compatible for the borrower to reveal her own states to the lender. Specifically, because the plan offered by the lender has combination of market exclusions and recovery ratios, a defaulted borrower would choose a combination of both which is best for her endowment realization.

Lastly, the model in this paper is also related to a recent wave of literature has examined the optimal contracting problem in dynamic environments where agents have private information that is persistent, see for example, [39], [40] and [41]. They show that the agent's consumption may grow over time under such an environment. The model in this paper generates similar results in the sense that the debt recovery offered to the defaulted borrower is decreasing over time. Similar to previous studies, such a result is due to the fact that the borrower's state is persistent, so that the lender would like to allocate the inefficiency in an efficient way.

3.2 The Model

There are two agents in the model, the borrower and the lender, each of them lives and trades for infinite periods. The borrower has a lifetime preference as

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)\right]$$

where c_t is consumption. The utility function $u\left(\cdot\right)$ is continuous, strictly increasing and strictly concave. At the beginning of each period t, the borrower receives her output endowment, $y_t \in Y = \left\{y^1, y^2, \ldots, y^N\right\}$ with $y^1 \leq y^2 \leq \ldots \leq y^N$ which follows a Markov transition function $f\left(y_t | \, y_{t-1}\right)$. I also assume that the borrower's endowment realization is private information.

The risk neutral foreign lender has a lifetime preference as

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t c_t^L\right]$$

The lender cannot observe the y_t and she receives a fixed amount of endowment y_L each period. I assume that y_L is sufficiently large in a sense that the lender is with a deep pocket. Moreover, I assume that $0 < \beta < \delta < 1$ to guarantee equilibrium borrowings.

I assume incomplete market in the international borrowing market, that is, the borrower can only trade with the lender on one period uncontingent bond with default options. At period t, b_t is the initial debt level and it is common knowledge to the lender and the borrower. With b_t and the output realization y_t, the borrower can choose to repay or default on the debt. If the borrower chooses to repay the debt, she can thus continue borrowing at the price of q_{t+1} by choosing b_{t+1}. On the other hand, if the borrower chooses to suspend the repayment (in other words, default), she will consume the endowment (suffering an output cost) and enter the debt restructuring process. I denote the default decision as d_t

$$d_t = \begin{cases} 1 & \text{default} \\ 0 & \text{repay} \end{cases}$$

By following Arellano (2008), the defaulted borrower's endowment changes to be y_t^d immediately

$$y_t^d = \begin{cases} (1-\lambda)\,\mathbb{E}\,(y_t) & y_t \geq (1-\lambda)\,\mathbb{E}\,(y_t) \\ y_t & y_t < (1-\lambda)\,\mathbb{E}\,(y_t) \end{cases}, \lambda \in (0,1)$$

Instead of assuming a prescribed length of market exclusion after default, I assume that the lender has to choose recovery ratios on the debt after default, $\Upsilon_t = \{\gamma_{t+i}\}_{i=1}^{\infty}$ with $\gamma_{t+i} \in [0,1], \forall i$ such that if the borrower default on b_t at time t, then it incurs to her the output cost immediately at period t while at period $t+i$, she can choose to repay $\gamma_{t+i}b_t$ to regain the market access right at that period or stay with the market exclusion and wait for the next period.

The timing of the sequential game between the borrower and the lender is as follows:

1. The lender sets ratio, $\Upsilon_t = \{\gamma_{t+i}\}_{i=1}^{\infty}$ such that if the borrower defaults at period t, she has to repay $\gamma_{t+i}b_t$ for market reentry at period $t+i$ for market reentry.

2. y_t is realized and privately observed by the borrower

3. The borrower picks a policy $\pi_t = (d_t, b_{t+1})$ that consists of a default rule d_t and new borrowing b_{t+1}

4. Bond price q_{t+1} is consistent with the lender's optimality.

To define the problem formally, let the public history to be $s^t = \left(s^{t-1}, \Upsilon_t, \pi_t\right), \forall t$ and $s^{-1} = (b_0)$ be the initial outstanding debt. It is convenient to define the following public histories when agents take action: $s_\gamma^t = s^{t-1}$, $s_\pi^t = \left(s^{t-1}, \Upsilon_t\right)$. The strategy for the borrower, $\{\pi_t\}_{t=0}^{\infty}$ and the price of bonds $\{q_{t+1}\}_{t=0}^{\infty}$ are all functions of the relevant histories. Notice that if the borrower default at period t, and if she decides not to repay at period $t+1$, then

$$\Upsilon_{t+1} = \Upsilon_t / \gamma_{t+1}$$

That is, the recovery ratios chose at period t will be inherited at period $t+1$ if the borrower defaulted at time t and refuses to repay for market reentry at period $t+1$. A similar result is applicable at period $t+2$ if the borrower continues to refuse to repay.

To formulate the borrower's problem, I firstly defined the problem confronted to a borrower at time $t+i$ who defaulted at time t, $W\left(s^{t+i}\left|_{b_{t+i}=\gamma_{t+i}b_i}\right., y_{t+i}\right)$ as:

$$W\left(s_\pi^{t+i}\left|_{b_{t+i}=b_t}\right., y_{t+i}\right) = \max_{d_{t+i}} \left\{ \begin{array}{c} (1-d_{t+i})\, W^r\left(s_\pi^{t+i}\left|_{b_{t+i}=b_t}\right., y_{t+i}\right) \\ +d_{t+i} W^{nr}\left(s_\pi^{t+i}\left|_{b_{t+i}=b_t}\right., y_{t+i}\right) \end{array} \right\}$$

where

$$W^{nd}\left(s_\pi^{t+i}\left|_{b_{t+i}=b_t}\right., y_{t+i}\right) = \begin{array}{c} u\left(y_{t+i} + q_{t+1}b_{t+i+1} - \gamma_{t+i}b_t\right) + \\ \beta \sum_{y_{t+i+1}\in Y} V\left(s_\pi^{t+i+1}, y_{t+i+1}\right) f\left(y_{t+i+1} \middle| y_{t+i}\right) \end{array}$$

$$W^d\left(s_\pi^{t+i}\left|_{b_{t+i}=b_t}\right., y_{t+i}\right) = \begin{array}{c} u\left(y_{t+i}^d\right) + \\ \beta \sum_{y_{t+i+1}\in Y} W\left(s_\pi^{t+i+1}\left|_{b_{t+i+1}=b_t}\right., y_{t+i+1}\right) f\left(y_{t+i+1} \middle| y_{t+i}\right) \end{array}$$

Here, I abuse the notation of d_t such that

$$d_{t+i} = \begin{cases} 1 & \text{refuse to repay} \\ 0 & \text{repay} \end{cases}$$

and thus

$$d_{t+i} = \begin{cases} 1 & W^{nd}\left(s_\pi^{t+i}\left|_{b_{t+i}=b_t}\right., y_{t+i}\right) < W^d\left(s_\pi^{t+i}\left|_{b_{t+i}=b_t}\right., y_{t+i}\right) \\ 0 & W^{nd}\left(s_\pi^{t+i}\left|_{b_{t+i}=b_t}\right., y_{t+i}\right) \geq W^d\left(s_\pi^{t+i}\left|_{b_{t+i}=b_t}\right., y_{t+i}\right) \end{cases}$$

I then define the borrower's problem given that she has a good credit standing at period t. Taking as given the price schedule, q_{t+1}, after any history $\left(s_\pi^t, y_t\right)$, the strategy for the borrower, π_t, solves the following problem:

$$V\left(s_\pi^t, y_t\right) = \max_{d_t} \left\{ (1-d_t)\, V^{nd}\left(s_\pi^t, y_t\right) + d_t V^d\left(s_\pi^t, y_t\right) \right\} \tag{3.1}$$

where

$$
\begin{aligned}
V^{nd}\left(s_{\pi}^{t}, y_{t}\right) &= \max_{b_{t+1}} \left\{ u\left(y_{t}+q_{t+1}b_{t+1}-b_{t}\right)+\beta \sum_{y_{t+1}\in Y} V\left(s_{\pi}^{t+1}, y_{t+1}\right) f\left(y_{t+1} \mid y_{t}\right) \right\} \\
V^{d}\left(s_{\pi}^{t}, y_{t}\right) &= u\left(y_{t}^{d}\right)+\beta \sum_{y_{t+1}\in Y} W\left(s_{\pi}^{t+i}\left|_{b_{t+1}=b_{t}}\right., y_{t+1}\right) f\left(y_{t+1} \mid y_{t}\right)
\end{aligned}
$$

We define d_t as

$$
d_{t}=\begin{cases} 1 & V^{nd}\left(s_{\pi}^{t}, y_{t}\right)<V^{d}\left(s_{\pi}^{t}, y_{t}\right) \\ 0 & V^{nd}\left(s_{\pi}^{t}, y_{t}\right) \geq V^{d}\left(s_{\pi}^{t}, y_{t}\right) \end{cases}
$$

The lender's problem is to choose Υ_t such that it maximizes the expected value of the debt repayment from the borrower. Thus, the lender's value can be defined as:

$$
V_{L}\left(s_{\gamma}^{t}\right) = \max_{\Upsilon_{t}} \mathbb{E}\left\{\left[1-d\left(s^{t}, y_{t}\right)\right] V_{L}^{nd}\left(s_{\pi}^{t}, y_{t}\right)+d\left(s_{\pi}^{t}, y_{t}\right) V_{L}^{d}\left(s_{\pi}^{t}, y_{t}\right)\right\} \tag{3.2}
$$

where

$$
\begin{aligned}
V_{L}^{nd}\left(s_{\pi}^{t}, y_{t}\right) &= \max_{b_{t+1}} \left\{y_{L}+b_{t}-q_{t+1}b_{t+1}+\delta V_{L}\left(s_{\gamma}^{t+1}\right)\right\} \\
V_{L}^{d}\left(s_{\pi}^{t}, y_{t}\right) &= y_{L}+\delta V_{L}\left(s_{\gamma}^{t+1} \mid b_{t+1}=b_{t}\right)
\end{aligned}
$$

3.2.1 Equilibrium

I focus on a recursive Markov equilibrium in which all decision rules are solely the functions of state variables (s, b_h).

Definition 3.1. A recursive Markov equilibrium for this economy consists of (i) the borrower's policy functions for repayment, borrowing, and consumption, $d(s_\pi, y)$, $b'(s_\pi, y)$, and $c(s_\pi, y)$, and values $V(s_\pi, y)$, $V^{nd}(s_\pi, y)$, $V^{d}(s_\pi, y)$, $W(s_\pi, y)$, $W^{nd}(s_\pi, y)$, and $W^{d}(s_\pi, y)$; (ii) lenders' policy functions for lending choices, recovery plan, and consumption $\tilde{b}'(s_\gamma)$, $\Upsilon(s_\gamma)$, and $c_L(s_\gamma)$, and value function $V_L(s_\gamma)$; (iii) the functions for bond price $q\left(s, b'_j\right)$; (v) the evolution of the aggregate state $\mathcal{H}(s)$ such that, given that the initial debt levels are the same across countries within each group and $b_{j,0} = \tilde{b}_{j,0}, \forall j$:

1. Taking as given the bond price, the evolution of the aggregate state $\mathcal{H}(s)$, the evolution of default ratio $\mathcal{D}(s)$, and the evolution of average new bond issuance $\mathcal{B}(s)$, the policy and value functions for the borrower solve her problem in equation (3.1).

2. Taking as given the bond prices $q\left(s, b'_j\right)$ and evolution of the aggregate states $\mathcal{H}(s)$, the policy functions and value functions for the lenders $\tilde{b}'(s_\gamma)$, $\Upsilon(s_\gamma)$, and $c_L(s_\gamma)$ solve the lender's problem in equation (3.2).

3. The prices of debt $q\left(s, b'_j\right)$ clear the bond market for every country, $b'(s_\pi, y) = \tilde{b}'(s_\gamma)$.

4. The goods market clears, $c + c_L = y + y_L$.

5. The law of motions for the evolution aggregate states $s' = \mathcal{H}(s)$ is consistent with the borrower and the lender's decision rules and shocks.

3.3 A Three-Periods Example

In this section, I will use a three-periods example to illustrate the main mechanism behind. There are three periods, $t = 0, 1, 2$. The borrower's lifetime preference is

$$\mathbb{E}\left[u(c_0) + \beta u(c_1) + \beta^2 u(c_2)\right]$$

The borrower's endowment is stochastic in period 0, and it is unobservable to the lender[2] . The borrower's endowment state space in period 0 is $\{y_h, y_l\}$ with $y_h > y_l > 0$. For simplicity, assume that $\Pr(y_h) = \Pr(y_l) = 0.5$. In other words, the borrower has two types, high and low, with equal probability. The borrower's endowment is further assumed to be fixed at all three periods. That is, I am considering an extreme case of persistence that the borrower's endowment is fixed once it has realized. Furthermore, assume that the initial debt level at period 0, b_0 is big enough such that at period 0 the borrower would choose to default at both states[3] . There is a fractional output cost after default

$$y_t^d = (1 - \lambda) y_t, \lambda \in (0, 1), t = 1, 2, 3$$

[2] So that the repayment plan proposed by the lender cannot be contingent on the borrower's endowment realization.

[3] This may seem weird because the borrower cannot borrow up to b_0 *ex ante* given she will default at any states *ex post*. However, it is designed for simplicity, imagine that there are actually three states, $\{y_H, y_h, y_l\}$ such that $y_H > y_h > y_l$, and the borrower would choose to repay if $y_0 = y_H$. By omitting y_H, I would be able to ignore the problem of how the optimal plan relates to the borrower's repayment incentive at period 0, as welll as the borrower's problem of rollover the debt once she chooses to repay at period 0. Or in other words, I focus on *ex post* aspect of the lender's optimal recovery plan.

The risk neutral foreign lender has a lifetime preference as

$$\mathbb{E}\left[c_0^L + \delta c_1^L + \delta^2 c_2^L\right]$$

The lender receives a fixed amount of endowment y_L each period, and I assume that y_L is sufficiently large[4] . The problem confronted to the lender is to choose $\{\gamma_1, \gamma_2\}$ in period 0 to maximize the expected value of repayment by the borrower.

To facilitate future analysis, I firstly focus on the case where the lender can (presumably) observe the borrower's endowment. Because of the time horizon is limited, the lender's plan can be derived by backward inductions. If the high type borrower didn't repay at period 1, the lender would choose $\gamma_2^h = \lambda y_h / b_0$ to make the borrower indifferent between repayment and stay with market exclusion in period 2. I further assume that, if the borrower choose to repay in period 1, then she will rollover the biggest amount of debt into period 2, which is λy_h[5] . Then in period 1, the incentive compatibility requires

$$u\left(y_h - \gamma_1^h b_0 + \delta \lambda y_h\right) + \beta u\left(y_h - \lambda y_h\right) \geq (1+\beta)\, u\left(y_h - \lambda y_h\right)$$

Because $u(\cdot)$ is strictly increasing, the equation above imposes a upper limit on the γ_1^h that the lender can choose. Since the lender would like to extract the highest debt repayment from the borrower, she would choose γ_1^h such that the equation above holds in equality. Thus, the plan for the type h borrower is

$$\gamma_1^h = \frac{(1+\delta)\,\lambda y_h}{b_0}, \gamma_2^h = \frac{\lambda y_h}{b_0}$$

Similarly, in case of $y_0 = y_l$, the lender would choose

$$\gamma_1^l = \frac{(1+\delta)\,\lambda y_h}{b_0}, \gamma_2^l = \frac{\lambda y_h}{b_0}$$

Because $y_h > y_l$, thus

$$\gamma_1^h > \gamma_1^l, \gamma_2^h > \gamma_2^l$$

Under such a plan, both types of borrower would choose to repay in period 1.

[4] Or the lender can be assumed to be able to borrower and lend at an interest rate of $1/\delta - 1$, either one can insure that the lender has a deep pocket.

[5] This can be guaranteed if β is sufficiently small, or b_0 is sufficiently big, either of these two is innocuous for my example here.

Back to the case when the borrower's endowment is unobservable and if the lender would like to bunch the borrower's states, then she would choose a plan $\{\hat{\gamma}_1, \hat{\gamma}_2\}$ such that $\hat{\gamma}_1 = (1+\delta)\lambda y_l / b_0$ and $\hat{\gamma}_2 = \lambda y_l / b_0$. A natural doubt is why the lender would choose such a plan if she want to bunch the borrower's states? The reason is because if the lender choose $\hat{\gamma}_1 = (1+\delta)\lambda y_l / b_0$ and $\hat{\gamma}_2 > \lambda y_l / b_0$, then both the borrower's repayment choice and the lender's expected value will be the same. However, if the lender chooses $\hat{\gamma}_1 > (1+\delta)\lambda y_l / b_0$ and $\hat{\gamma}_2 = \lambda y_l / b_0$, then only the type h borrower is possible to repay in period 1, which is no longer bunching. Finally, if the lender chooses $\hat{\gamma}_1 > (1+\delta)\lambda y_l / b_0$ and $\hat{\gamma}_2 > \lambda y_l / b_0$ then type l borrower would never find it optimal to repay in both periods, again, no longer bunching. Under such a plan, both types borrowers choose to repay $(1+\delta)\lambda y_l$ unambiguously in period 1 to reenter the market. Thus, the lender's expected value of repayment under this plan is plan is denoted as $\hat{V}_L$

$$\hat{V}_L = (1+\delta)\lambda y_l$$

If the lender would like to separate the borrower's states, then one extreme option for her is to abandon type l borrower's repayment by choosing $\{\tilde{\gamma}_1, \tilde{\gamma}_2\}$ such that $\tilde{\gamma}_1 = (1+\delta)\lambda y_h / b_0$ and $\tilde{\gamma}_2 = \lambda y_h / b_0$. Under such a plan, only type h borrower would repay in period 1 while type l borrower would choose to stay with market exclusion forever. The lender's expected value of repayment is denoted as $\tilde{V}_L$

$$\tilde{V}_L = \frac{(1+\delta)\lambda y_h}{2}$$

Notice that under $\{\hat{\gamma}_1, \hat{\gamma}_2\}$, I have[6]

$$u(y_h - \hat{\gamma}_1 b_0 + \delta\lambda y_h) + \beta u(y_h - \lambda y_h) \geq u(y_h - \lambda y_h) + \beta u(y_h - \lambda y_l)$$

To make the lender's problem meaningful, I further assume that the equation above is slacked, which leaves rooms for the lender to separate the borrower's states by choosing $\{\gamma_1, \gamma_2\}$. Specifically, let γ_1^* denote the value such that

$$u(y_h - \gamma_1^* b_0 + \delta\lambda y_h) + \beta u(y_h - \lambda y_h) = u(y_h - \lambda y_h) + \beta u(y_h - \lambda y_l) \tag{3.3}$$

[6] Otherwise the lender's expected value would be $\frac{(1+2\delta)\lambda y_l}{2}$, and the problem becomes trivial because the lender's problem is only to decide which one among $\{\hat{\gamma}_1, \hat{\gamma}_2\}$ and $\{\tilde{\gamma}_1, \tilde{\gamma}_2\}$ is associated with a higher expected value.

Because $u(\cdot)$ is strictly increasing, the equation above implies that $\gamma_1^h > \gamma_1^* > \gamma_1^l$. Thus, if the lender choose a plan $\{\bar{\gamma}_1, \bar{\gamma}_2\}$ such that $\bar{\gamma}_1 = \gamma_1^*$ and $\bar{\gamma}_2 = \lambda y_l / b_0$, then type h borrower would like to repay in period 1 while type l borrower would like to repay in period 2. The expected value of repayment under a plan is denoted as $\bar{V}_L$

$$\bar{V}_L = \frac{\gamma_1^* b_0 + \delta \lambda y_l}{2}$$

In the followings, I will show that $\{\bar{\gamma}_1, \bar{\gamma}_2\}$ is the lender's optimal plan under certain circumstance. Furthermore, under such a plan, the type h borrower repays in period 1 while type l borrower repays in period 2. In other words, the lender is using time to separate the borrower's states.

Proposition 3.1. *If $y_l < \frac{1+\delta}{1+2\delta} y_h$, then there exists a value $\bar{\beta} \in [0, \delta)$ such that the lender strictly prefers $\{\bar{\gamma}_1, \bar{\gamma}_2\}$ to $\{\hat{\gamma}_1, \hat{\gamma}_2\}$ and $\{\tilde{\gamma}_1, \tilde{\gamma}_2\}$ if $\beta < \bar{\beta}$. Under such a plan, the type h borrower would choose to repay $\bar{\gamma}_1 b_0$ in period 1 while the type l borrower would choose to repay $\bar{\gamma}_2 b_0 = \lambda y_l$ in period 2, and $\bar{\gamma}_1 b_0 > \bar{\gamma}_2 b_0$. In equilibrium, $\{\bar{\gamma}_1, \bar{\gamma}_2\}$ is also the lender's optimal plan, where $\bar{\gamma}_1 > \bar{\gamma}_2$.*

Proof. By equation (3.3)

$$u(y_h - \gamma_1^* b_0 + \delta \lambda y_h) + \beta u(y_h - \lambda y_h) = u(y_h - \lambda y_h) + \beta u(y_h - \lambda y_l)$$

$\Rightarrow$

$$u(y_h - \gamma_1^* b_0 + \delta \lambda y_h) = u(y_h - \lambda y_h) + \beta [u(y_h - \lambda y_l) - u(y_h - \lambda y_h)]$$

Since $u(\cdot)$ is strictly continuous and strictly increasing, γ^* is strictly decreasing in β. Therefore, $\bar{V}_L$ is strictly decreasing in β. Furthermore, from equation (3.3), it is easy to find that $\gamma_1^* \to \frac{(1+\delta)\lambda y_h}{b_0}$ as $\beta \to 0$, and thus $\bar{\gamma}_1 \to \frac{(1+\delta)\lambda y_h}{b_0}$ as $\beta \to 0$. As a consequence, $\bar{V}_L \to \frac{(1+\delta)\lambda y_h + \lambda y_l}{2}$ as $\beta \to 0$, which is unambiguously greater than $\tilde{V}_L$. Since $y_l < \frac{1+\delta}{1+2\delta} y_h$, it is also greater than $\hat{V}_L$. Therefore, there exists a value $\bar{\beta} \in [0, \delta)$ such that the lender strictly prefers $\{\bar{\gamma}_1, \bar{\gamma}_2\}$ to $\{\hat{\gamma}_1, \hat{\gamma}_2\}$ and $\{\tilde{\gamma}_1, \tilde{\gamma}_2\}$ if $\beta < \bar{\beta}$. The equation (3.3) also implies type h consumer would choose to repay in period 1. On the other hand, because $\bar{\gamma}_1 > \gamma_1^l$, $\bar{\gamma}_2 = \gamma_2^l$, type l borrower would prefer to repay in period 2. Finally, because type h borrower's incentive compatibility constraint holds in equality in this case so that the lender cannot increase $\bar{\gamma}_1$ to extract more repayments from type h borrower. On the other hand, because type l borrower would only like to repay in period 1 if $\gamma_1 = \gamma_1^l$, which makes the lender's expected value equal to $\tilde{V}_L$ and has been proved to be lower than $\bar{V}_L$ if $\beta < \bar{\beta}$. Therefore, $\{\bar{\gamma}_1, \bar{\gamma}_2\}$ is the lender's optimal plan on debt recovery. Because $\gamma_1^* > \gamma_1^l$ and $\gamma_1^l > \gamma_2^l$, so we also have $\bar{\gamma}_1 > \bar{\gamma}_2$ □

The Proposition 3.1 describes an equilibrium such that the high type borrower would repay $\bar{\gamma}_1 b_0$ in period 1 while the low type borrower would wait until period 2 to repay $\bar{\gamma}_2 b_0$. The fact that $\bar{\gamma}_1 > \bar{\gamma}_2$ implies that the high type borrower is paying more than the low type borrower for the early market access. In essential, the lender is separating the borrower's states by using time. Such a result is consistent with the empirical findings in [3] that the haircut on debt is increasing in the duration of market exclusion. Moreover, because the recovery plan offered to the borrower is decreasing over time. The results in this example are also consistent with the literature on optimal contracts on persistent private information, such as [39], [40] and [41], which found that the agent's consumption is increasing over time.

By assuming the borrower to default unambiguously in both states, the example here is in essential about the lender's *ex post* optimality in choosing the recovery plan. As [27] argued, as an *ex post* inefficiency, equilibrium market exclusion after default is necessary to support the *ex ante* efficiency. However, such an ignorance of the lender's *ex ante* optimality in this example is innocuous. In essential, the example here shows that even if the market exclusion after default is inefficient *ex post*, it is the lender's interest to to deliver the inefficiencies in an efficient way. In particular, the market exclusion is even more inefficient for the high type borrower because she can afford a higher debt recovery. Therefore, the example features an equilibrium under which the high type borrower repays a higher recovery to reenter the market earlier. Reciprocally, because the low type borrower can afford only a lower amount of debt recovery, it is efficient for her to wait longer, which also delivery incentives for the high type borrower to pay more.

The condition of $y_l < \frac{1+\delta}{1+2\delta} y_h$ states that if the borrower's states are too close to each other, bunching the states is more profitable for the lender. Similar assumptions can be found in previous studies, e.g., [27]. On the other hand, a low value of β implies the borrower care more about period 1 output cost than the lower debt repayment in period 2. Thus, a type h borrower would like to repay more than $(1+\delta)\,\lambda y_l$ to avoid the output cost in period 1, which again, enables the lender to separate the states. Notice that both these two conditions also interact with each other. In particular, the value of $\bar{\beta}$ is affected by the difference between of y_h and y_l.

In the following, I will do a simple numerical exercise to demonstrate previous arguments. Specifically, the borrower's utility function is natural logarithm, $u(c) = \ln c$. I assume that $y_h = 4.0$, $b_0 = 2.0$, $\lambda = 0.1$ and $\delta = 0.95$. The benchmark setting has y_l equal to 2.24, and figure 3.1 shows how $\bar{V}_L$ changes in the value of β, as well as how it compares to the values of $\hat{V}_L$ and $\tilde{V}_L$. It justifies my previous argument that given $y_l < \frac{1+\delta}{1+2\delta} y_h$, there exists a $\bar{\beta}$ such that $\{\bar{\gamma}_1, \bar{\gamma}_2\}$ strictly dominates $\{\hat{\gamma}_1, \hat{\gamma}_2\}$ and $\{\tilde{\gamma}_1, \tilde{\gamma}_2\}$ if $\beta < \bar{\beta}$, where $\overline{\beta} \simeq 0.7$ in this example.

I then analyze how the value of $\bar{\beta}$ changes in the value of y_l, or in essentially, the gap between y_h and y_l. Figure 3.2 shows a similar graph by considering the case of $y_l = 2$. The fact that $y_h = 2y_l$ implies $\hat{V}_L = \tilde{V}_L$. By comparing figure 3.2 to figure 3.1, it is easy to notice that when $y_h > 2y_l$, the value of $\bar{\beta}$ is increasing in the difference between y_h and y_l. In particular, when $y_h = 2y_l$, $\bar{\beta} = \delta$. That is, $\{\bar{\gamma}_1, \bar{\gamma}_2\}$ is the lender's optimal plan under any values of β. As I've argued before, the reason is because a larger difference between y_h and y_l leaves more rooms for the lender to separate the borrower's states. However, such an argument is not always true, figure 3.3 shows that a similar graph when $y_l = 1.78$. Instead of becoming larger, the result in this figure shows that the value of $\bar{\beta}$ is decreasing in y_l when $y_h > 2y_l$. Despite the lender chooses $\bar{\gamma}_2 = \lambda y_l / b_0$ in order to extract debt repayment from type l in period 2, it also forces her to choose $\bar{\gamma}_1$ instead of $\tilde{\gamma}_1$ because otherwise the type h borrower would find it optimal to repay in period 2. When the value of y_l becomes low enough, the benefits of extract debt repayment from the type l borrower is dominated by the benefit to extract more repayment from type h borrower in period 1, and thus makes the value of $\bar{\beta}$ lower.

One drawback of the numerical analysis above is the value of $\bar{\beta}$ is sensitive on the value of y_l. For example, $\bar{\beta}$ equals to 0.7 in the benchmark case where $y_l = 2.24$, but it equals to 0.95 when $y_l = 2$. A value of β lower than 0.7 is too small compared to the typical lender's discount rate in international business cycle studies, see for example, [6]. One caveat for such a result is the example considered here has only three periods. In particular, a borrower cannot rollover part of the debt recovery into future if a plan encourages her to repay in period 2, which in turn limits the amount of debt recovery. In contrast, if a plan makes borrower to repay in period 1, the fact that she can rollover

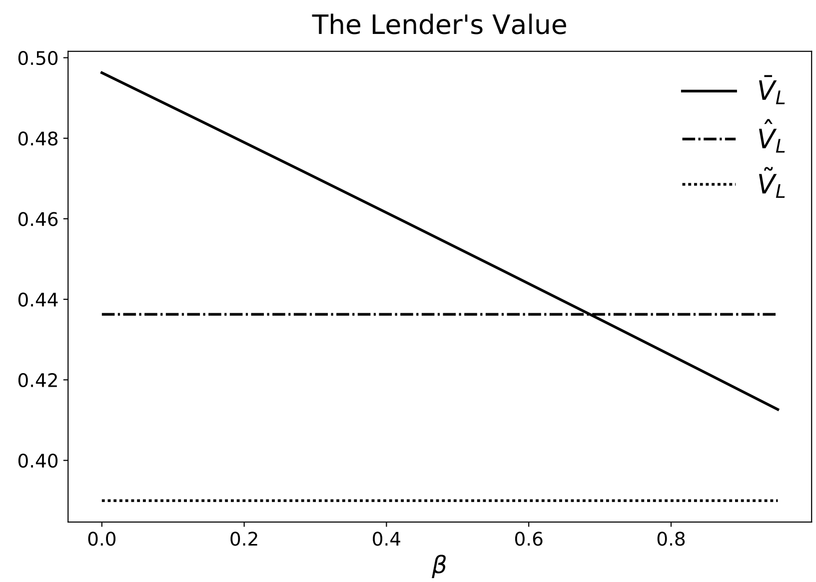

Figure 3.1: The Lender's Value, $y_l = 2.24$

the debt implies a higher debt recovery in period 1. As a consequence, in addition to the lender's discounting, the big difference between net repayments from the low type borrower in both periods also encourages the lender to adopt a plan that features debt repayments for both types of borrowers. In order to make it profitable for the lender to separate the borrower's states, such an incentive should overcome by some other benefits. In particular, to compensate the lender for letting the low type borrower to repay in period 2, debt repayment from the high type borrower should be high enough, which can only be supported if β is sufficiently small[7] .

On the other hand, if the number of periods of the example above is extended (or finally goes to infinity), the problem of $\bar{\beta}$ being too low will be alleviated (or finally eliminated). In this case, there is no longer a large difference in debt repayments across different periods due to the lack of rollover apparatus. Thus, the lender's incentive to separate states is mostly (or purely) driven by the fact that the borrower with a lower endowment cannot afford a high recovery, and thus it is profitable for her to wait for

[7] Otherwise the high type borrower would like to pretend to be a low type so that she can wait for a lower debt repayment in period 2.

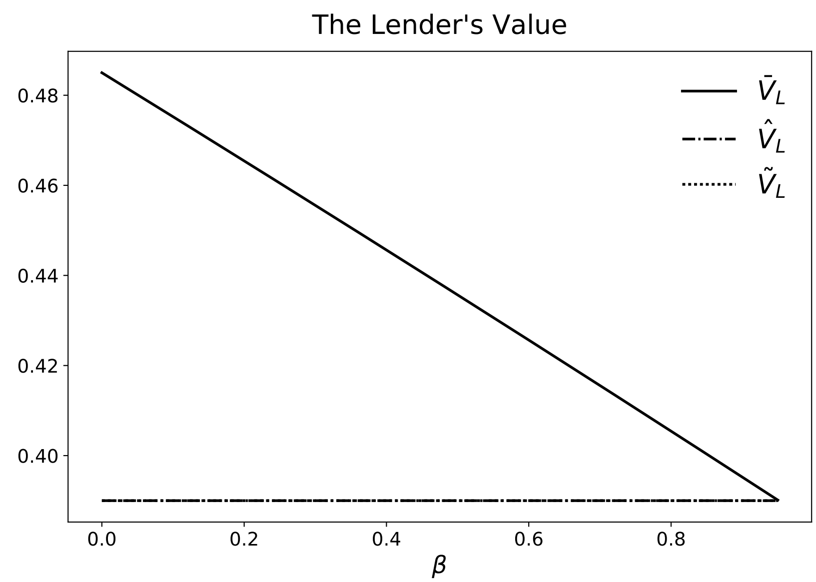

Figure 3.2: The Lender's Value, $y_l = 2$

the future. For example, in the three-periods example above, if the low type borrower is allowed to rollover part of debt in period 2 like the high type borrower will do in period 1, then she will choose to repay $(1+\delta)\,\lambda y_l$ instead of λy_l. The fact that the lender can elicit a higher debt repayment from the low type borrower reduces the high type borrower's value of deviation. In other words, the high type borrower's incentive constraint is relaxed so that γ^* becomes higher given the value of β. The higher γ^* implies that it becomes more appealing for the lender to separate the borrower's states. Reciprocally, as the number of periods has been increased, higher values of β can be supported that the lender strictly prefers $\{\bar{\gamma}_1, \bar{\gamma}_2\}$.

3.4 Conclusion

In this paper, I develop a model of optimal debt contracting between a borrower and a lender who trade in multiple periods. In the model, the borrower's endowment is private information that is persistent. By choosing an optimal recovery plan, the lender leaves the decision of reentering the market to the borrower. Unlike the efficient sovereign

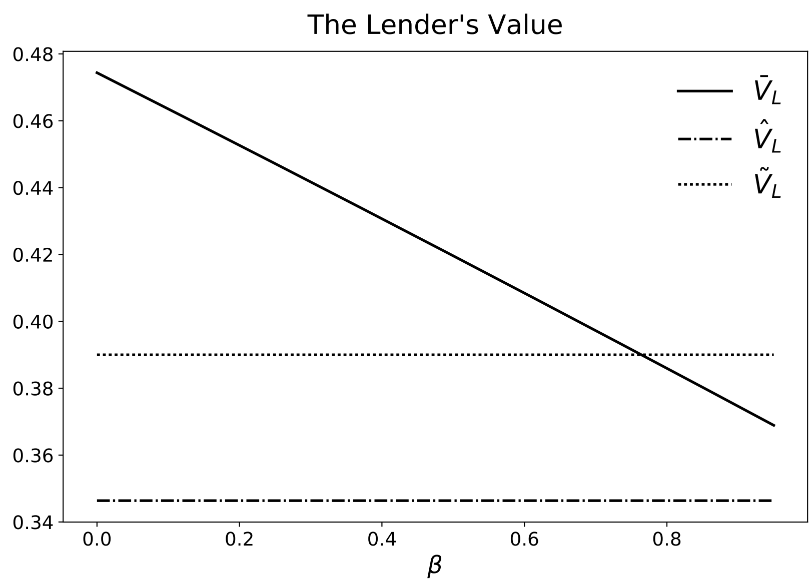

Figure 3.3: The Lender's Value, $y_l = 1.78$

default studies based on i.i.d. endowment process, the debt recovery offered by the lender in this paper is decreasing over time. As a consequence, a borrower with a low endowment realization would choose stay with the market exclusion longer in order to get a higher haircut on her debt. On the other hand, due to the higher output cost on default, a borrower with a high endowment would choose to repay a higher recovery to reenter the market earlier. The equilibrium result shows that the recovery on defaulted debt is thus decreasing in the duration of market exclusion, which is consistent with the empirical evidence regarding debt restructuring in recent studies. Such a plan is result from the fact that the lender is allocating the ex post inefficiency in an efficient way. Specifically, in order to support the equilibrium borrowing contract, the lender has to exclude the borrower for several periods after default. However, the lender would like to exercise the market exclusion on the type borrower for whom it is more bearable. In particular, because the borrower with a low endowment will suffer less output cost, it is efficient for the lender to keep her for a longer market exclusion (but with the benefit of a higher haircut). By doing so, the lender is in essentially using time to separate the borrower's states. The reason why such a plan is sustainable in this paper is because the borrower's endowment is persistent in this paper. Therefore, a default (or refuse to repay the debt recovery) by the borrower signals to the lender low endowments in both current and future periods. Once there is a linkage between endowments across different periods, it is optimal for the lender to reduce the debt recovery over time.

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Appendix A

Proof and Algorithm

A.1 Proof

A.1.1 Proof of Lemma 1.1

Proof. The country chooses to repay only if

$$V^{nd}(s, b_h) \geq V^d(s)$$

which is

$$V^{nd}(s, b_h) - V^d(s) \geq 0$$

The value of repay and default, $V^{nd}(s, b_h)$ and $V^d(s)$ implies

$$\begin{aligned} V^{nd}(s, b_h) - V^d(s) &= u\left(y_h + q_h' b_h' - b_h\right) + \beta u\left(y_h - b_h'\right) - (1+\beta)\, u\left(y_h^d\right) \\ &= u\left(y_h + q_h' b_h' - b_h\right) - u\left(y_h^d\right) \end{aligned}$$

where the second equality use the property of $b_h' = y_h - y_h^d$.

Consider firstly the case of $y_h \leq (1-\lambda)\,\mathbb{E}(y_h)$, then the definition of y_h^d implies $b_h' = 0$ and $y_h = y_h^d$, thus

$$V^{nd}(s, b_h) - V^d(s) = u(y_h - b_h) - u\left(y_h^d\right) < 0$$

if $b_h > 0$. That is, the country repays the debt only if the default output cost is strictly positive, $y_h > (1-\lambda)\,\mathbb{E}(y_h)$. Therefore, I can focus on values of y_h which are greater than $(1-\lambda)\,\mathbb{E}(y_h)$.

By definition, $u\left(y_h^d\right)$ is constant for values of $y_h > (1-\lambda)\mathbb{E}(y_h)$. Notice that $q_h'b_h'$ is increasing in b_h' because otherwise the country can improve both periods' consumptions by issuing less debt. On the other hand, b_h' is strictly increasing in y_h if $y_h > (1-\lambda)\mathbb{E}(y_h)$. Because $u(\cdot)$ is strictly increasing, the arguments above implies implies that $u(y_h + q_h'b_h' - b_h)$ is strictly increasing in y_h. Hence, $V^{nd}(s, b_h) - V^d(s, b_h)$ is strictly increasing in y_h. Therefore, there exists a cutoff function on y_h, $\hat{y}(\boldsymbol{B}, y_f, b_h)$ such that the country repays if $y_h \geq \hat{y}(\boldsymbol{B}, y_f, b_h)$and defaults otherwise. □

A.1.2 Proof of Lemma 1.2

Proof. Consider a period 1 endowment realization, $\hat{y}_h \in \left[\underline{y}, \overline{y}\right]$ such that

$$u\left(\hat{y}_h + \hat{q}_h'\hat{b}_h' - b_h\right) + \beta u\left(\hat{y}_h - \hat{b}_h'\right) = (1+\beta)\,u\left(\hat{y}_h^d\right) \tag{A.1}$$

where $\hat{b}_h'$ is the borrowing limit under $\hat{y}_h$ such that $\hat{b}_h' = \hat{y}_h - \hat{y}_h^d$ and $\hat{q}_h'$ is the associated bond price. That is, $\hat{y}_h$ is the lowest endowment realization such that countries would choose to repay. The definition of $\hat{b}_h'$ implies

$$u\left(\hat{y}_h - \hat{b}_h'\right) = u\left(\hat{y}_h^d\right)$$

Hence, equation (A.1) changes to be

$$u\left(\hat{y}_h + \hat{q}_h'\hat{b}_h' - b_h\right) = u\left(\hat{y}_h^d\right)$$

Because $u(\cdot)$ is strictly increasing, the equation above implies

$$b_h - \hat{q}_h'\hat{b}_h' = \hat{y}_h - \hat{y}_h^d = \hat{b}_h'$$

That is, with an endowment equals $\hat{y}_h$, countries would choose to repay, and equally spread the burden of debt repayments into both periods. □

A.1.3 Proof of Lemma 1.3

Proof. Consider two possible endowment realizations, $\hat{y}_h$ and $\tilde{y}_h$ such that $\hat{y}_h, \tilde{y}_h \in \left[\underline{y}, \overline{y}\right]$ and $\hat{y}_h > \tilde{y}_h$. Furthermore, both $\hat{y}_h$ and $\hat{y}_h$ are large enough such that countries strictly prefer to repay their debt in period 1 under both two endowment realizations. Let $\hat{b}_h'$ and $\tilde{b}_h'$ be the associated period 1 new bond issuance such that

$$\hat{b}_h' = \hat{y}_h - \hat{y}_h^d, \tilde{b}_h' = \tilde{y}_h - \tilde{y}_h^d$$

The property of y_h^d implies that $y_h - y_h^d$ is strictly increasing in y_h, and thus

$$\hat{b}_h' > \tilde{b}_h'$$

That is, with a higher endowment realization in period 1, countries borrow more due to the fact that the borrowing limit is strictly increasing in period 1 endowment realization. Furthermore, $q_h' b_h'$ is strictly increasing in b_h' because otherwise countries can get a higher net resource transfer from the lenders by issuing less debt. Hence,

$$\hat{q}_h' \hat{b}_h' > \tilde{q}_h' \tilde{b}_h'$$

which implies

$$b_h - \hat{q}_h' \hat{b}_h' < b_h - \tilde{q}_h' \tilde{b}_h'$$

□

A.1.4 Proof of Lemma 1.4

Proof. Lemma 1.2 predicts that net repayments in both periods are equalized when countries are indifferent between repay and default. On the other hand, Lemma 1.3 predicts that period 1 net repayment is strictly decreasing, while period 2 net repayment is strictly increasing in the endowment realization. Therefore, when countries strictly prefer to repay in period 1, the net repayment in period 2 is strictly higher than the net repayment in period 1

$$b_h' > b_h - q_h' b_h'$$

Also, it is easy to see that the difference between these two periods' net repayments, $(1 + q_h')\, b_h' - b_h$ is strictly increasing in y_h. □

A.1.5 Proof of Lemma 1.5

Proof. Consider an endowment realization in group f countries, $\hat{y}_f$ such that these countries are indifferent between repay and default. Let $\hat{b}_f'$ and $\hat{q}_f'$ be the associated new bond issuance in these countries, Lemma 1.2 implies that net repayments by group f countries are the same in both periods

$$b_f - \hat{q}_f' \hat{b}_f' = \hat{y}_f - \hat{y}_f^d = \hat{b}_f' > 0$$

if $b_f > 0$. In this case, the bond price in group h countries is

$$\hat{q}'_h = \frac{\delta\psi'\left(y_L + b'_h + \hat{b}'_f\right)}{\psi'\left(y_L + b_h - \hat{q}'_h b'_h + b_f - \hat{q}'_f \hat{b}'_f\right)} \tag{A.2}$$

Consider another endowment realization in group f countries, $\tilde{y}_f$ such that $\tilde{y}_f < \hat{y}_f$, the definition of $\hat{y}_f$ implies that group f countries default under $\tilde{y}_f$, and thus the bond price in group h countries is

$$\tilde{q}'_h = \frac{\delta\psi'\left(y_L + b'_h\right)}{\psi'\left(y_L + b_h - \tilde{q}'_h b'_h\right)} \tag{A.3}$$

which is independent of $\tilde{y}_f$.

To compare $\hat{q}'_h$ and $\tilde{q}'_h$, I have to take into account the borrowing behavior in group h countries. First, if the endowment in group h countries is at a value that these countries are indifferent between default and repay, then Lemma 1.2 implies that

$$b'_h = b_h - \hat{q}'_h b'_h$$
$$b'_h = b_h - \tilde{q}'_h b'_h$$

which implies

$$\psi'\left(y_L + b'_h + \hat{b}'_f\right) = \psi'\left(y_L + b_h - \hat{q}'_h b'_h + b_f - \hat{q}'_f \hat{b}'_f\right)$$
$$\psi'\left(y_L + b'_h\right) = \psi'\left(y_L + b_h - \tilde{q}'_h b'_h\right)$$

and thus

$$\hat{q}'_h = \tilde{q}'_h = \delta$$

That is, default decisions by group f countries do not affect the bond price in group h countries in this situation. Second, if y_h is higher and group h countries strictly prefer to repay, Lemma 1.4 implies

$$b'_h > b_h - \tilde{q}'_h b'_h$$

and thus

$$\psi'\left(y_L + b'_h\right) < \psi'\left(y_L + b_h - \tilde{q}'_h b'_h\right)$$

$\Rightarrow$

$$\tilde{q}_h' = \frac{\delta\psi'\left(y_L + b_h'\right)}{\psi'\left(y_L + b_h - \tilde{q}_h' b_h'\right)} < \delta$$

Assume that $\psi'''(\cdot) > 0$, which is standard under a class of functions such as CRRA. Notice that under the case of $\hat{y}_f$, the net repayment from group f countries are equalized in both periods. That is, the fact that group f countries choose to repay under $\hat{y}_f$ would bring the lenders the same incremental in both period consumption. Assuming that the second order effect will not reduce the lenders' period 1 consumption too much to overturn the whole result[1] , then we have

$$\frac{\psi'\left(y_L + b_h' + \hat{b}_f'\right)}{\psi'\left(y_L + b_h - \hat{q}_h' b_h' + b_f - \hat{q}_f' \hat{b}_f'\right)} > \frac{\psi'\left(y_L + b_h'\right)}{\psi'\left(y_L + b_h - \tilde{q}_h' b_h'\right)}$$

and thus

$$\hat{q}_h' > \tilde{q}_h'$$

□

A.1.6 Proof of Lemma 1.6

Proof. Consider two possible endowment realizations in group f countries, $\hat{y}_f$ and $\tilde{y}_f$ such that $\hat{y}_f, \tilde{y}_f \in \left[\underline{y}, \overline{y}\right]$ and $\hat{y}_f > \tilde{y}_f$. Furthermore, both $\hat{y}_f$ and $\tilde{y}_f$ are large enough such that group f countries strictly prefer to repay their debt in period 1 under both two endowment realizations. Lemma 1.4 implies

$$\hat{b}_f' > \tilde{b}_f'$$

$$b_f - \hat{q}_f' \hat{b}_f' < b_f - \tilde{q}_f' \tilde{b}_f'$$

Assuming again that the second order effect will not increase the lenders' period 1 consumption too much to overturn the whole result[2] , the concavity of $\psi(\cdot)$ implies

$$\frac{\psi'\left(y_L + b_h' + \hat{b}_f'\right)}{\psi'\left(y_L + b_h - \hat{q}_h' b_h' + b_f - \hat{q}_f' \hat{b}_f'\right)} < \frac{\psi'\left(y_L + b_h' + \tilde{b}_f'\right)}{\psi'\left(y_L + b_h - \tilde{q}_h' b_h' + b_f - \tilde{q}_f' \tilde{b}_f'\right)}$$

That is, $\hat{q}_h' < \tilde{q}_h'$. □

[1] This is true if both b_h and y_h are sufficiently small, and thus b_h' is sufficiently small. An extreme example is $b_h = 0$ and $y_h = (1-\lambda)\mathbb{E}(y_h) + \varepsilon$ for $\varepsilon > 0$ and sufficiently small, then $b_h' = \varepsilon$. Because ε is sufficiently small, then the second order effect is sufficiently small as well.

[2] Again, this is true if both b_h and y_h are sufficiently small, and thus b_h' is sufficiently small.

A.1.7 Proof of Proposition 1.1

Proof. It is easy to see that q'_h is independent of y_f if $D_f = 1$. On the other hand, consider an extreme case of $b_f = 0$ and let $\tilde{y}_f \leq (1-\lambda)\,\mathbb{E}\,(y_f)$. Let $\tilde{b}'_f$ and $\tilde{q}'_f$ be the associated new bond issuance in these countries, Lemma 1.2 implies that net repayments by group f countries in both periods equal to 0

$$b_f - \tilde{q}'_f \tilde{b}'_f = \tilde{y}_f - \tilde{y}^d_f = \hat{b}'_f = 0$$

That is, by choosing $b_f = 0$, the wealth effect has been completely shut off in this case[3] . In fact, the only interdependence is the substitution effect. Hence, when group f countries are in the market, the bond price in group h countries is strictly lower than the bond price when group f countries are defaulting. Intuitively, the substitution effect in this case will impose a downward pressure on the borrowing condition in group h countries when group f countries have a relatively high endowment. Furthermore, by Lemma 1.3, we also know that b'_f is increasing in y_f and $b_f - q'_f b'_f$ is decreasing in y_f. Hence, the substitution effect is larger for a larger y_f, and thus the bond price in group h countries is strictly decreasing in y_f.

Notice that the discussions above are based on the extreme case of $b_f = 0$ so that group f countries never choose to default. However, in the case of $b_f > 0$, if group f net repayments in period 2 is higher than the net repayments in period 1 to a sufficient extent, the substitution effect will still dominate the wealth effect, and thus imposes a net downward pressure on the borrowing condition in group h countries. Given b'_f is strictly increasing in y_f if $y_f > (1-\lambda)\,\mathbb{E}\,(y_f)$, and we have the net repayment in period 1 equals $b_f - q'_f b'_f$, it is easy to see that such an argument is more likely to be valid if b_f is sufficiently small and y_f is sufficiently high. That is, there exist a threshold, $\mathcal{Y} \in \left[\underline{y}, \overline{y}\right]$, such that $q'_h\,|_{D_f=1} \geq q'_h\,|_{D_f=0}$ if $y_f \geq \mathcal{Y}$ and $q'_h\,|_{D_f=1} < q'_h\,|_{D_f=0}$ if $y_f < \mathcal{Y}$. □

[3] Given $b_f = 0$ in this case, the group h countries will never default under any endowment realization. However, they will be allowed to choose a positive b'_h if y_h is strictly higher than $(1-\lambda)\,\mathbb{E}\,(y_f)$. That is, the output cost after default should be strictly positive to support any strictly positive new bond issuance in period 1.

A.2 Algorithm

The endowment space $\left[\underline{y}, \overline{y}\right]$ is discretized into 11 grids using [24]'s method. Specifically, $\underline{y}$ equals to 0.83 and $\overline{y}$ equals to 1.21. The endowment process has an unconditional mean of 1.04. The debt space $\left[\underline{b}, \overline{b}\right]$ take the value of $[-0.45, 0.45]$, and it is discretized into 31 grids. The computing algorithm is as follows:

1. Make an initial guess of the borrower's value function, $V(s, b_h)$, $V^{nd}(s, b_h)$, and $V^{d}(s, b_h)$.

2. Make an initial guess on the equilibrium price function $q_h(s, b'_h)$.

3. Solve the borrower's problem to find her value function $\tilde{V}(s, b_h)$, $\tilde{V}^{nd}(s, b_h)$, and $\tilde{V}^{d}(s, b_h)$; optimal default decision, $d(s, b_h)$; optimal policy function, $b'(s, b_h)$; and the new equilibrium bond price function, $\tilde{q}_h(s, b'_h)$. This maximization involves the following sub steps:

   (a) Taking $q_h(s, b'_h)$ as given, solve the borrower's maximization problem

   $$\tilde{V}(s, b_h) = \max_{d_h} \left[(1 - d_h)\tilde{V}^{nd}(s, b_h) + d_h \tilde{V}^{d}(s, b_h)\right]$$

   where $\tilde{V}^{nd}(s, b_h)$ is the value of repay and $\tilde{V}^{d}(s, b_h)$ is the value of default

   $$\begin{aligned} \tilde{V}^{nd}(s, b_h) &= \max_{b'_h} \left\{ \begin{array}{c} u\left(y_h + q'_h b'_h - b_h\right) \\ +\beta \int_{\underline{y}}^{\overline{y}} \int_{\underline{y}}^{\hat{y}\left(\boldsymbol{B}', y'_f, b'_h\right)} V^{d}(s') f(y'|y)\, dy' \\ \beta \int_{\underline{y}}^{\overline{y}} \int_{\hat{y}\left(\boldsymbol{B}', y'_f, b'_h\right)}^{\overline{y}} V^{nd}(s', b'_h) f(y'|y)\, dy' \end{array} \right\} \\ \tilde{V}^{d}(s, b_h) &= u\left(y_h^d\right) + \beta\theta \int_{\underline{y}}^{\overline{y}} \int_{\underline{y}}^{\overline{y}} V^{nd}\left(s', 0\right) f\left(y'_h \middle| y_h\right) f\left(y'_f \middle| y_f\right) dy'_h dy'_f \\ &\quad + \beta(1 - \theta) \int_{\underline{y}}^{\overline{y}} \int_{\underline{y}}^{\overline{y}} V^{d}\left(s'\right) f\left(y'_h \middle| y_h\right) f\left(y'_f \middle| y_f\right) dy'_h dy'_f \end{aligned}$$

   The borrower's policy functions $d(s, b_h)$ and $b'(s, b_h)$ are obtained by solving the problem above.

   (b) Given $b'(s, b_h)$ and $d(s, b_h)$, compute the new equilibrium bond price function

   $$\tilde{q}_h\left(s, b'_h\right) = \int_{\underline{y}}^{\overline{y}} \int_{\hat{y}\left(B', y'_f, b'_h\right)}^{\overline{y}} \frac{\delta \psi'\left(c'_L\right)}{\psi'\left(c_L\right)} f\left(y'_h \middle| y_h\right) f\left(y'_f \middle| y_f\right) dy'_h dy'_f \qquad \text{(A.4)}$$

where

$$c_L(s) = y_L + \sum_{j=h,f} NR_j(s)$$

$$NR_j(s) = (1 - D_j)\left[B_j - q_j'\left(B_j'\right)B_j'\right]$$

(c) Compute the distance between $q_h(\cdot)$ and $\tilde{q}_h(\cdot)$. If it is bigger than the critical value, then update the guess on $q_h(\cdot)$ by using $\tilde{q}_h(\cdot)$ and repeat steps (b) and (c). Otherwise, proceed to step 4[4] .

4. Compute the distance between $V(\cdot)$ and $\tilde{V}(\cdot)$. If it is bigger than the critical value, then update the guess on $V(\cdot)$, $V^{nd}(\cdot)$ and $V^d(\cdot)$ by using $\tilde{V}(\cdot)$, $\tilde{V}^{nd}(\cdot)$, and $\tilde{V}^d(\cdot)$. Repeat step 1 to step3. Otherwise, stop.

[4] The computing speed of step (b) to step (c) can be greatly enhanced by solving a fixed point problem of equation (A.4).