THE ASTROPHYSICAL JOURNAL, 190:637–651, 1974 June 15

COHERENT OSCILLATIONS IN UX URSAE MAJORIS

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Received 1973 November 19

ABSTRACT

Rapid oscillations appeared in the light curve of UX UMa and were studied from 1973 March 3 through 8, during which time they varied smoothly in period from 30.03 to 28.54 s, and exhibited a rapid phase shift of $-360°$ associated with each eclipse.

The phase shift can be understood in terms of nonradial pulsations with $l = 2$ and $m = -2$ or $+2$. Numerical examples demonstrate that the model provides a qualitative agreement with the observed phase shift in UX UMa, and with the similar but positive 360° phase shift of the 71-s periodicity in DQ Her. The oscillations are present throughout the orbital cycle and are essentially undiminished in amplitude even during eclipse.

The eclipse curve is shown to be composite, with the oscillations associated with the primary; their phase change permits the centerline of this object to be identified and separated from the second object eclipsed, which is identified as the "bright spot" on the disk of gas surrounding the oscillating star.

It is proposed that the blue star in UX UMa is a pulsating white dwarf. The apparent photosphere of the blue star lies not at the surface of the white dwarf but instead lies in the optically thick disk of gas rotating around the white dwarf, and is at some distance from its surface.

Subject headings: eclipsing binaries — pulsation — stars, individual

I. INTRODUCTION

The discovery of the short-period oscillations in the light curve of DQ Her by Walker (1956) was followed by an extensive search for compact stellar objects exhibiting similar behavior (Lawrence, Ostriker, and Hesser 1967), largely without success. Two objects found to exhibit coherent oscillations, G44−32 (Lasker and Hesser 1969) and R 548 (Lasker and Hesser 1971) had unexpectedly long periods and rather complex behavior; another object studied (HL Tau 76, Landolt 1968; Warner and Nather 1972*a*) was sufficiently complex so that its coherent behavior was not recognized for some time (Warner and Robinson 1972; Fitch 1973).

In contrast, the presence of rapid, coherent oscillations in dwarf novae during outburst was uncovered in several objects in quick succession, and six are now known which show remarkably similar behavior: Z Cam, AH Her, CN Ori (Warner and Robinson 1972), SY Cnc, KT Per (Robinson 1973*b*), and VW Hyi (Warner and Brickhill 1974). In all cases the rapid oscillations are present only after outburst is well underway, suggesting that the energy released during eruption is responsible for exciting the variations.

In addition to these objects, two others not known to erupt have been shown to display rapid oscillations on some occasions but not on others: UX UMa (Warner and Nather 1972*b*) and CD $-42°14462$ (Warner 1973*b*). Considerable observational material was obtained on UX UMa in 1973 March when rapid oscillations were present; these data, combined with earlier studies of the star, show cyclic behavior on at least four widely different time scales, from 29 s to 29 years.

II. PREVIOUS OBSERVATIONS OF UX UMa

The variable star UX UMa has been observed since its discovery by Beliavski (1933), first visually by Zverev and Kukarkin (1937) who found it to have an Algol-type light curve with the unusually short period of $4^{h}43^{m}$, and later photoelectrically by Linnell (1949) who discovered the rapid "flickering" in the light curve and the "still-stand," or anomalous asymmetry during the egress portion of the eclipse. The star was classified by Kuiper (1941) as a B3 subdwarf with broad, shallow absorption lines of H and He I, and was shown by Struve (1948) to exhibit marked differences in its spectrum at different orbital aspects: the Balmer series appeared weakly in absorption prior to eclipse, while the spectrum following eclipse was largely that of a featureless, early-type continuum with Hβ in emission. *UBV* photometry by Johnson, Perkins, and Hiltner (1954) showed that the depth of eclipse varied in different colors, being deepest in the *UV* and shallowest in *V*, with large and unpredictable changes in the shape of eclipse egress. Their photometry also showed very clearly the characteristic "hump" in the light curve lasting just half the orbital period, and now known to exist in most, if not all, of the interacting binaries of this type, with eclipse located, for the most part, well after the maximum of the "hump."

Further studies by Walker and Herbig (1954) led them to suggest a model in which the hot star has a ring of thin, gaseous material around it on which a "bright spot" is present and located following the line of centers of the binary pair. Krzeminski and Walker (1963) obtained infrared observations in an attempt to find a secondary

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eclipse of the faint (presumably red) companion, without success; their infrared light curve shows the "hump" is very small if not absent entirely.

Many of the photometric and spectroscopic peculiarities of this system can now be understood in light of recent models of close, interacting binary stars undergoing mass exchange. The suggestion by Walker and Herbig of a hot disk of gas surrounding the primary was extended by Gorbatskii (1967), and later by Warner and Nather (1971) and Smak (1971) to explain the photometric behavior of U Gem. Much of the light of these interacting systems comes from a "bright spot," a region of intense luminosity formed on a ring of material surrounding the hot primary star, excited by the impact of a stream of gas from the secondary component. It is the source of the early-type continuum radiation observed by Kuiper and Struve, and accounts for the variable eclipse depth and, as we show later in this paper, for the "stillstand" during eclipse egress. Its changing aspect (and obscuration by cooler parts of the disk) as the stars revolve explains the "hump," while its location "downstream" of the line of centers explains the location of eclipse. It is the primary source of the rapid, low-amplitude flickering in the system.

III. THE PRESENT OBSERVATIONS

All of the observations reported here were obtained on the 82-inch (2-m) Struve telescope at the McDonald Observatory. They were obtained with the two-star photometer and high-speed data acquisition system (Nather 1972), through a 15″ aperture in white (unfiltered) light. The simultaneous photometric record of a constant star in the same field permits accurate extinction corrections to be made, and attests to the reality of the small variations in the light curves. The two photometric diaphragms are precisely aligned by drift scanning, with the monitor channel deliberately set slightly smaller than that for the variable, to give immediate indication of any guiding errors. The Journal of Observations (table 1) also includes, for completeness, the observations obtained in 1971 (Warner and Nather 1972*b*), which have been reanalyzed and included in the present study. Table 1 also contains the observed times of light minimum, $O - C$ values and the observed frequency of rapid oscillations which are referred to in succeeding sections.

IV. LONG-TERM VARIATIONS

The suggestion by Krzeminski and Walker (1963) that the variations in the times of eclipse minima might have a cyclic character was extended by Mandel (1965), who presented previous timings along with some of his own and showed that these variations could be well fitted by a variation with a period of 10,600 days (29.02 years). At that time barely one complete cycle had been observed and the likelihood that the variation would repeat in detail could not be assessed. Our eclipse timings from 1971 and 1973 can help decide this matter, particularly since they fall in a region of the phase diagram not previously covered. We have reproduced the observations published by Mandel in figure 1 (*filled circles*) together with our own timings (*open circles*), collected in phase based on his equation $\Psi = (\mathrm{JD}_{\odot} - 2{,}435{,}000) \times 0.0000934$ days and his orbital elements $\mathrm{JD}_{\odot}\ 2{,}427{,}341.22392 + 0^{\mathrm{d}}.196671299E$. The fit is remarkably good considering the relatively small amplitude of the total variation, and considering the difficulty of determining the exact time of minimum in a system whose light curve is somewhat variable from one cycle to the next. The continuous curve indicated in the figure is not a sinusoid but approximates one rather closely. We will examine the obvious explanation for such variation, i.e., that it is due to the presence of an unseen orbital companion, even though we have some reservations about the likelihood that such an explanation is correct.

TABLE 1

JOURNAL OF OBSERVATIONS

Run No.	Date	$\mathrm{JD}_{\odot}$ Start 244+	Integration Time (s)	$\mathrm{JD}_{\odot}$ Minimum	$O - C$	Frequency (ν) (mHz)
1144	1971 Jan. 20	971.95593	5	...	...	33.97
1159	1971 Jan. 27	978.92148	3	978.9995	−0.0023	34.58
1171	1971 Mar. 22	1032.87338	2	1032.8877	−0.0020	none
1174	1971 Mar. 23	1033.84200	2	1033.8711	−0.0020	none
1178	1971 Mar. 24	1034.82961	2	1034.8543	−0.0022	none
1184	1971 Mar. 27	1037.77531	2	...	...	none
1191	1971 Mar. 28	1036.76015	1	1038.7865	−0.0024	none
Average					−0.0022	
1322	1973 Mar. 3	1744.82950	4	1744.8380	−0.0018	33.30
1325	1973 Mar. 4	1745.73729	4	1745.8215	−0.0017	33.90
1329a	1973 Mar. 7	1748.71208	3	1748.7716	−0.0017	34.93
1329b	1973 Mar. 7	1748.71208	3	1748.9682	−0.0017	...
1332	1973 Mar. 8	1749.71759	4	1749.7549	−0.0017	34.89
1334	1973 Mar. 8	1749.92483	4	1749.9516	−0.0017	35.03
1364	1973 May 29	1831.67151	4	1831.7671	−0.0015	none
Average					−0.0017	

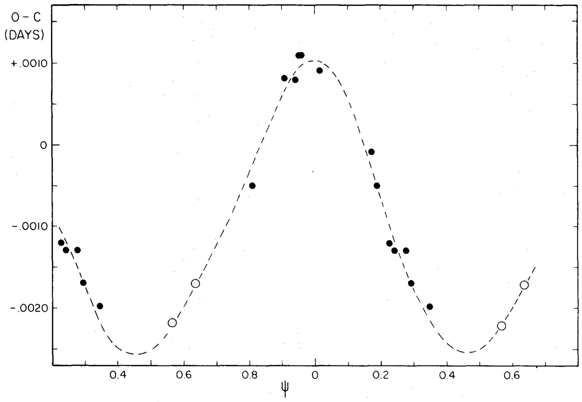

FIG. 1.—Eclipse minimum residuals, based on a period of 29.02 years. Filled circles are from Mandel (1965), open circles from our measurements in 1971 and 1973.

Using the subscript "3" to designate the unseen third member of the system, we state Kepler's third law as follows:

$$M_{ux} + M_3 = \frac{4\pi^2}{G}\frac{1}{P^2}(a_{ux} + a_3)^3$$

from which, with orbital inclination included, we derive an expression from the mass function of the system related to our observable quantities

$$\frac{{M_3}^3 \sin^3 i}{(M_3 + M_{ux})^2} = \frac{4\pi^2}{G}\frac{1}{P^2}{a_{ux}}^3 \sin^3 i\,.$$

From figure 1 we note that the light travel time across the presumed (outer) orbit is 0.0035 days, yielding a measure of the semimajor axis

$$a_{ux} \sin i = 4.54 \times 10^{12}\ \text{cm}$$

for the case of zero eccentricity, which we assume. Adopting Mandel's value of 10,600 days $= 9.16 \times 10^8$ s for P, we obtain

$$\frac{M_3 \sin^3 i}{(M_3 + M_{ux})^2} = 3.3 \times 10^{-5}\, M_\odot\,.$$

For the case where $M_{ux} \simeq 1\, M_\odot$, we find

$$M_3 \sin i = 0.032\, M_\odot\,.$$

This implies an extremely faint object incapable of sustaining hydrogen burning; this circumstance could well explain why the light from this object is not seen.

An alternative possibility, in which we assume that the mass of this object is comparable to the sum of the masses of the inner pair, i.e., $M_3 \simeq M_{ux}$, yields a value for $M_3 \sin i$ of $\sim 0.03\, M_\odot$ and demands a similar mass for the inner pair. This is in serious disagreement with, for example, Warner's (1973*a*) estimates of the masses of other similar systems, and disagrees with the observed color and luminosity of the primary of UX UMa. We conclude, therefore, that if the long-term variation is due to the presence of an unseen companion, that object is very small and is probably nonluminous in the visible region of the spectrum.

The presence of an unseen companion in orbit is not, however, the only possible explanation for long-term variations in eclipse timings, and may not even be the most likely. Smak (1972) has suggested that such variations may arise as an intrinsic part of the mass-transfer process, in which angular momentum is exchanged between the orbit of the pair and the rotation of the disk around the primary. This explanation would suggest that these variations should be present in all such systems, while the presence of a distant, unseen third component is likely to be somewhat more rare. Indeed, for those systems which have been sufficiently studied for long-term variations to be uncovered, all seem to show the effect to a greater or lesser degree. In addition to UX UMa, Mandel finds cyclic variations in RW Tri, and similar effects have been observed in U Gem, HZ 29, and DQ Her. While certainly not conclusive, the ubiquitous presence of long-term modulation of the orbital parameters suggests an origin in features common to all these systems, the most obvious of which is mass transfer. While Smak's analysis assumed

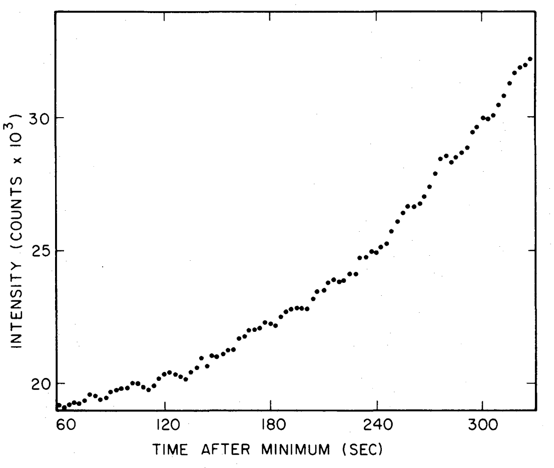

FIG. 2.—Rapid oscillations in the light curve of UX UMa just following mid-eclipse, run 1322, 4 s per reading

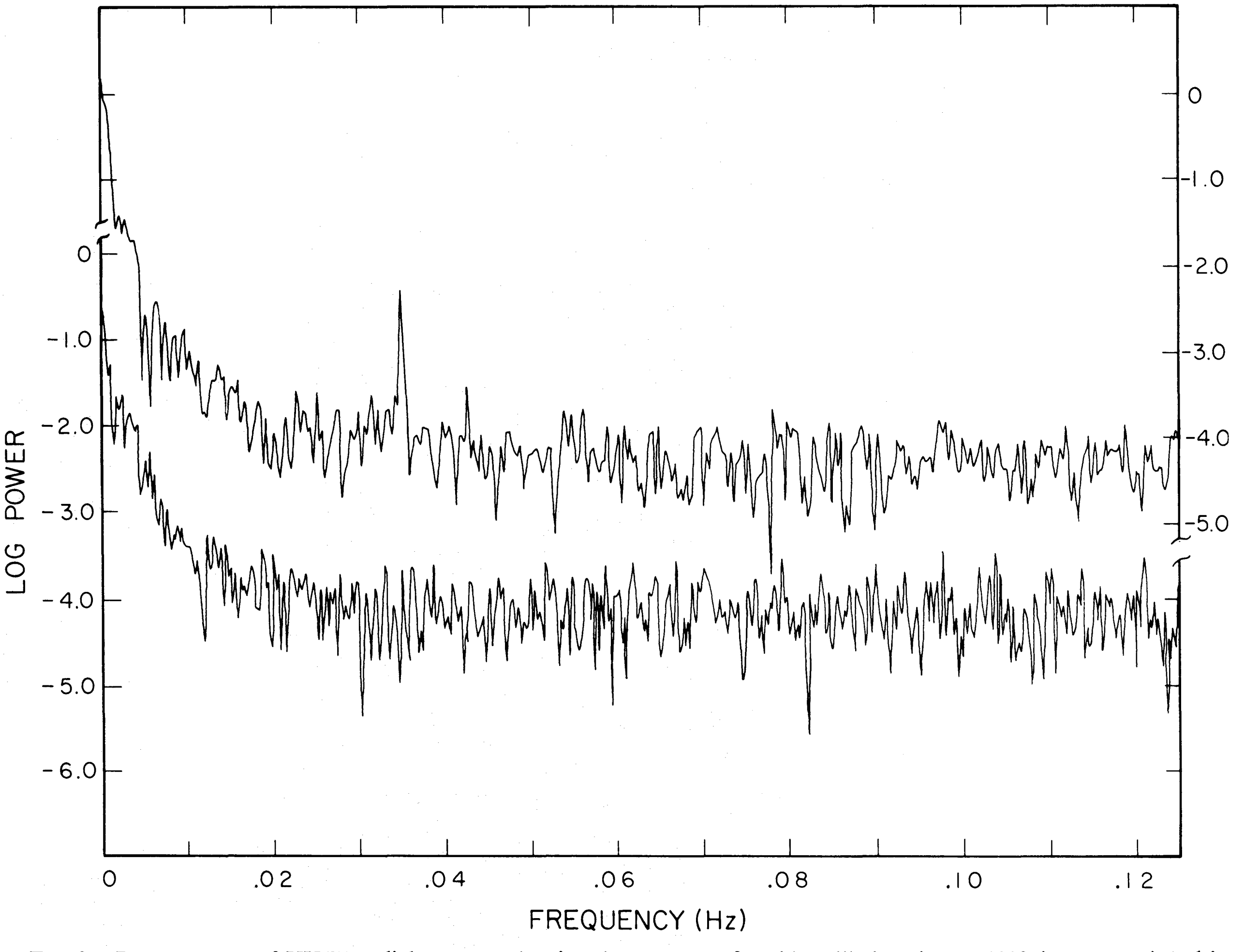

FIG. 3.—Power spectra of UX UMa light curves, showing the presence of rapid oscillations in run 1329 (*upper curve*) and its absence in run 1364 (*lower curve*) 12 weeks later. Vertical scale on the left applies to the lower curve, that on the right to the upper.

TABLE 2

TIME SCALES OF UX UMa VARIATIONS

Designation	Frequency	Period	Remarks
ν_1.............	33–35 mHz	28–30 s	Rapid oscillations
ν_2.............	588 mHz	0.196671299 days	Orbital period
ν_3.............	~100 μHz	~weeks	Long-term variation of ν_1
ν_4.............	1.09 nHz	29 years	Long-term variation of ν_2

no loss of mass (or angular momentum) from the system, recent observations of mass loss in Z Cam (Robinson 1973*c*) make such an explanation even more likely.

V. THE SHORT-PERIOD OSCILLATIONS

We have applied both power spectrum and periodogram analyses to the observed data (Robinson and Warner 1972), to search for and identify any periodic behavior present, and to determine its frequency. The discovery of fairly rapid changes in the oscillation frequency of KT Per (Robinson 1973*b*) and in CN Ori (Warner and Robinson 1972) suggested that these analytical techniques might not be entirely dependable should such changes also occur in UX UMa, and an additional computer program was written to fit, by least squares, a sinusoid of variable amplitude and phase (but preselected frequency) to the observed data. One could adjust the number of cycles fitted, as well as how far the sinusoid should be advanced between successive fittings; this latter procedure allows some overlap in the process and serves to smooth out variations between adjacent phase points. The data were first passed through a digital filtering process consisting of the successive application of simple sum and difference filters, to minimize the effects of variations outside the passband of interest; the resulting distortions in amplitude and phase have been removed from the final data as presented here.

In actual application this fitting technique proved to be considerably more sensitive than we had expected, and seemed able to detect and fit any coherent variation whose presence could be determined by the other methods. It has the further advantage that an abrupt change in frequency by the oscillating object would appear as a discontinuity in slope in the phase curve, softened by the length of fitted sinusoid, and would permit the identification of such behavior. It allowed the immediate discovery of the 360° phase shift associated with eclipse.

Observations of the UX UMa system were started on 1973 March 3, shortly after eclipse ingress had begun; the light curve, as displayed on the CRT, showed the presence of small-amplitude periodic variations as the primary emerged from eclipse. The unretouched raw data points are shown in figure 2. In all of our previous observations where rapid oscillations were present (except for those in DQ Her) the flickering made their immediate identification impossible, and extensive filtering and data massage were needed to study them.

The oscillations were most easily seen during light minimum, when the flickering noise is eliminated by the eclipse of the "bright spot," but could also be discerned in other short sections of the light curve where the flickering noise was minimal. Their raw appearance was that of a noisy sinusoid, an estimate confirmed by power-spectrum analysis which showed a single spike unaccompanied by harmonics. The oscillations persisted throughout the remainder of the run, but were not detectable 12 weeks later (fig. 3). There was no dramatic difference in the brightness of the system over this interval, such as the outburst phenomenon which accompanies the appearance of rapid oscillations in the dwarf novae and Z Cam stars.

The fact that rapid oscillations seem to come and go in the light curve (as they also did in 1971, see table 1) suggest that their appearance is a cyclic phenomenon, which may or may not be periodic. In an attempt to minimize the confusion in discussing the various cyclic or periodic changes in UX UMa, we have identified each as shown in table 2, and will hereafter refer to them in that notation. In a further attempt to avoid the awkwardness arising

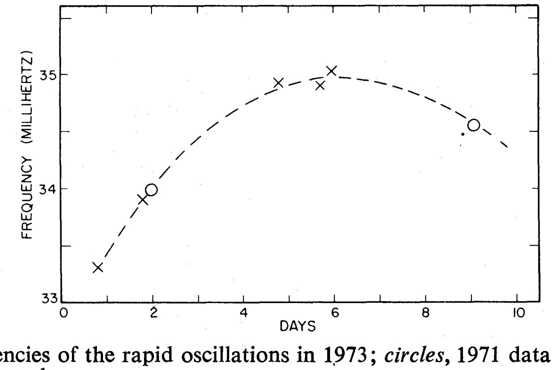

FIG. 4.—*Crosses*, observed frequencies of the rapid oscillations in 1973; *circles*, 1971 data added on the assumption the change in frequency behavior is the same for each appearance.

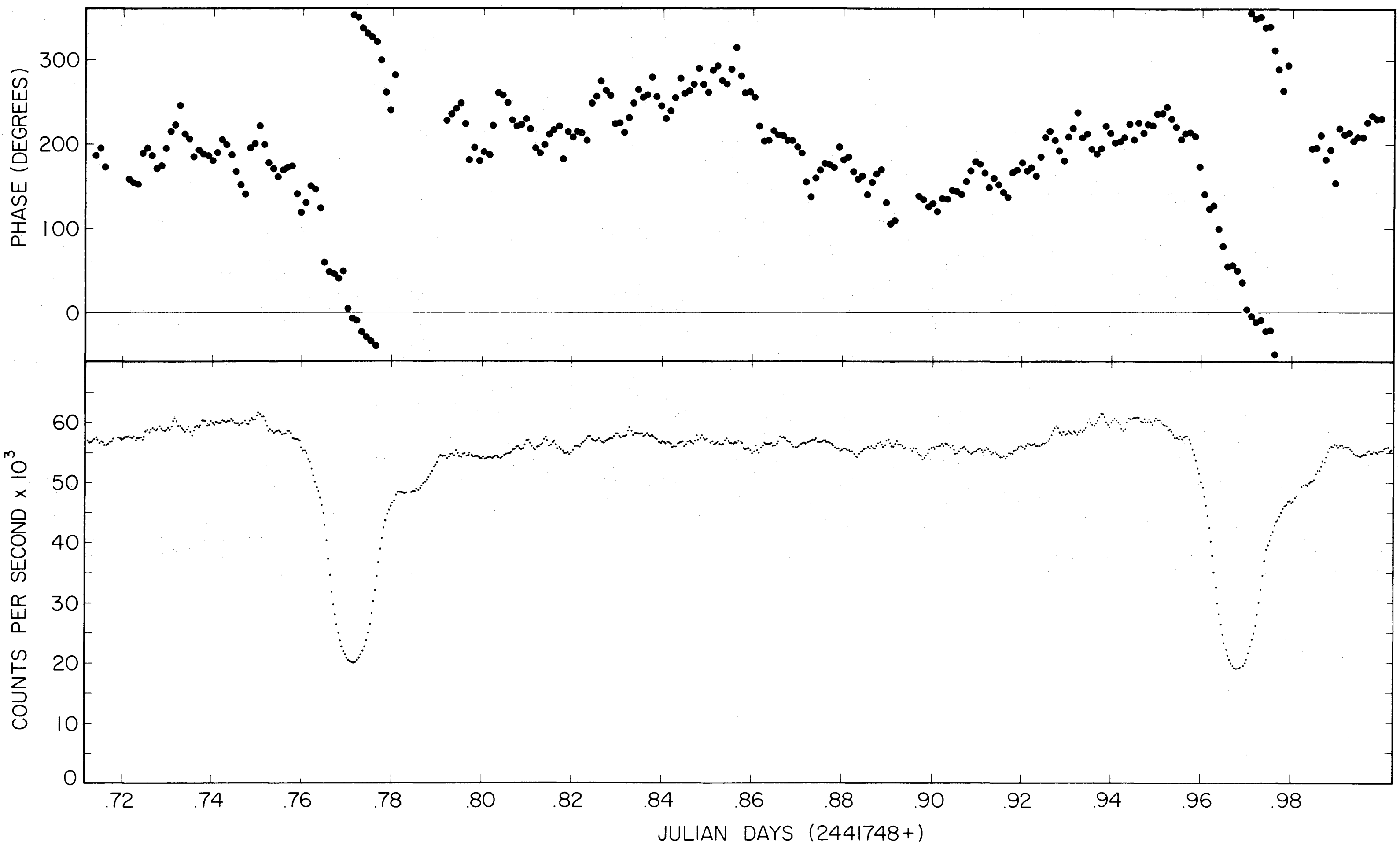

FIG. 5.—*Lower curve*, light curve of UX UMa, run 1329. Each dot is the average of 11 3-s integrations. *Upper curve*, measured phase of the rapid oscillations. Points below zero-phase line are repeated at the top (360°) for continuity. The lower scale, in fractions of a day, applies to both curves. Phase shown is actually $\Delta\Phi$, the difference between observed and computed phase.

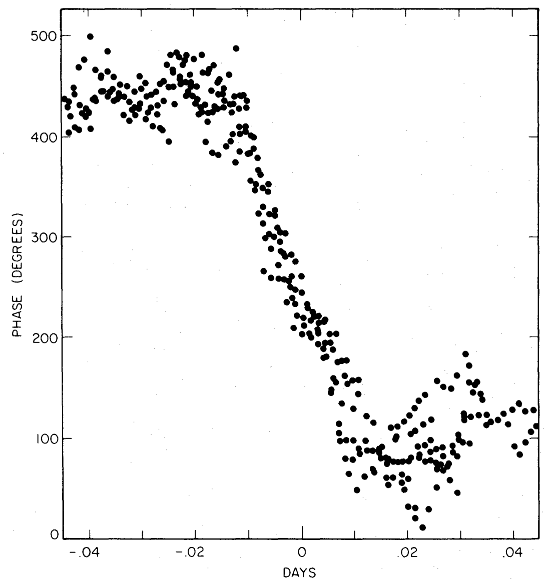

FIG. 6.—The observed phase shift, a composite of seven eclipses. Phase has been arbitrarily normalized just prior to eclipse, at −0.02 days; 0 days indicates light minimum.

from referring to the results of power spectra, normally obtained in terms of frequency, and the more conventional references to the period of stellar variations, we have included in table 2 the conversion of period to frequency in accepted, but unconventional, notation. This notation does serve to illustrate more clearly the relation between the differing frequencies found in UX UMa and provides a somewhat more uniform system of units for dealing with them.

During the five nights when rapid oscillations were present (ν_1) their frequency did not remain constant. Reference to table 1 shows that the frequency increased monotonically, at first rapidly and then at a diminishing rate, throughout this time. Such behavior explains the appearance of two different frequencies in our 1971 data (table 1) and suggests that such variation is a recurring phenomenon. It is very tempting to assume that such variations repeat from cycle to cycle, as the variations ν_2 and ν_4 seem to do, and to combine the two years' data into a single plot. There is some danger in this, in that we have no assurance that such behavior is strictly repetitive, nor that we can select the correct phase to combine the data properly. However, the 1973 data do suggest that the frequency may be approaching a maximum and, on this assumption, we combine the two sets of data and show in figure 4 the resulting variation of ν_1 with time. Such a variation can only be considered indicative, not conclusive, because of the assumptions involved in combining the data.

VI. PHASE ANALYSIS OF ν_1

After the appropriate digital filtering to minimize the contribution of unwanted frequencies and thus to enhance the apparent amplitude of the discovered rapid oscillations, the data were analyzed for both phase and amplitude. While the amplitude results showed the presence of oscillations throughout the orbital cycle they were very noisy, and little could be learned concerning any variation of amplitude with orbital phase from any individual run. Within the limits imposed by such noisy data, there did not appear to be any discernible variation.

The phase analysis immediately showed a complex pattern of behavior as a function of orbital position, as shown in figure 5. The data shown are for the only run for which the complete orbital cycle was covered, but all minima recorded that week showed the same dramatic phase shift associated with eclipse, as nearly as can be measured exactly 360° in each case, and of a sense indicating a loss of one oscillation cycle. The same phenomenon has been observed in DQ Her (Warner *et al.* 1972) but in the opposite sense: the rapid oscillation in DQ Her appears to *gain* one cycle during eclipse. This circumstance would seem to constrain any model of the process to be able to produce either sense of phase shift, and not to depend heavily on special circumstances to bring it about.

There may be additional orbit-modulated behavior in the phase diagram shown in figure 5, but since we have only the single run covering the full orbital cycle we cannot be sure of it. The slow increase in phase after the first minimum, its decline and later increase, may be such an effect since it is centered around orbital phase 0.5. There is also a suggestion of periodicity of shorter time scale in the phase diagram but this may be an artifact of the data-analysis technique and will have to be investigated further.

In an attempt to define the 360° phase shift as well as possible we have combined all of the eclipse data in a single diagram, and have included the eclipse obtained in 1971 as well, since it demonstrates the same effect, and in the

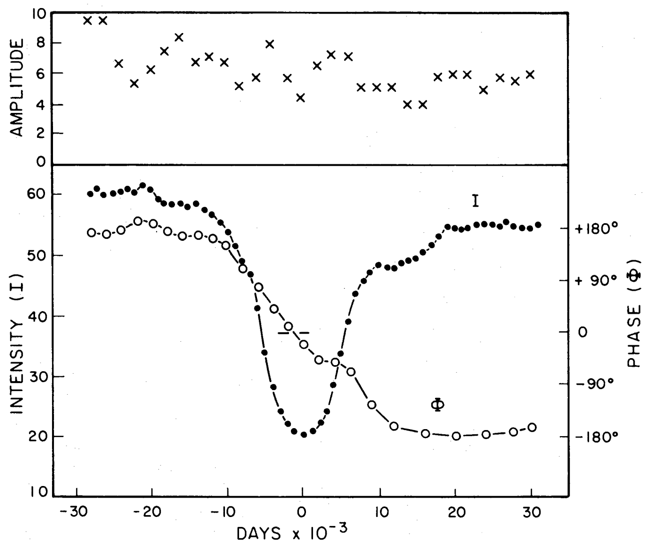

FIG. 7.—The light curve of eclipse 1329a (I) with mean phase shift (Φ) superposed. The 180° point of phase change is marked. Mean amplitude of the rapid oscillation is shown at the top.

same sense. Because there does not exist any way to determine absolute phase we have normalized all of the measures arbitrarily just prior to the onset of eclipse, and have aligned the data in time based in light minimum. The resulting combined plot is shown in figure 6. While somewhat more noisy than might be desired, the composite picture does show that the phase shift repeats quite well from cycle to cycle, displaying a nearly linear change from onset until just past light minimum, where a "lump" appears in all of the events recorded. This off-center position of the phase distortion corresponds to the position of the "bright spot" (see next section) and may be connected with obscuration by that element in the system.

We have averaged together the data shown in figure 6 to minimize the "phase noise" present, and this mean phase curve is shown in figure 7, superposed on the light curve of the first eclipse obtained on 1973 March 8. The centerline of the phase change (marked) does not correspond with light minimum, coming somewhat earlier and indicating that it is associated with the primary star which, if our model of these systems is correct, always precedes the "bright spot" in orbit.

All of the available amplitude data for the eclipses have been averaged together in the same manner and are shown at the top of figure 7. While still very noisy these data do indicate little change in amplitude during eclipse, and suggest the eclipse of the oscillating object is either partial or annular. There is no discernible change in amplitude symmetrical with the center of phase change, which is puzzling; it is difficult to see how such a drastic change in phase can occur without any corresponding change in mean amplitude. Such a change may, of course, be hidden in the noise, but it is unlikely to be greater than a factor of 2. Our reanalysis of the 1971 eclipse data (Warner and Nather 1972*b*) shows that oscillations are present throughout and following eclipse, and are not, in fact, absent during the "stillstand" as stated in that paper.

VII. THE ECLIPSE CURVE

Detailed light curves of seven eclipse minima are shown in figure 8, displaced vertically so their features may be compared; the upper six were obtained while ν_1 was present and the other when it could no longer be detected. Short gaps in the light curves are present where sky readings were taken, or where observations of the monitor star indicated that imperfect guiding might have introduced error.

The eclipse is seen to reproduce best in the region of light minimum, with its widest deviations in the region of the "stillstand" on the rising portion of the curve. There appears a change in slope on ingress which repeats well from one cycle to the next, and is most clearly seen on run 1329a; it can also be seen on the eclipse curve in figure 7 (where the number of data points has been reduced by averaging several channels together) as an abrupt change in the separation of data points on the ingress portion of the curve.

The presence and phase behavior of the short-period oscillations, as shown in figure 7, offer further clues that help in understanding the complex shape of the eclipse curve. The first eclipse of run 1329 is shown in figure 9 with the time scale in minutes and eclipse minimum at time zero to aid in detailed identification of the features. We propose that the eclipse consists of the following sequence of events: (1) At -28 min the leading edge of the nebular ring surrounding the primary is first obscured, leading to the small, flat depression in the light curve at that point. (2) The primary first contact occurs at -18.4 min as this depression ends. (3) At -9.8 min the sudden change in slope corresponds to the beginning of the eclipse of the bright spot, the upper dashed curve in figure 9.

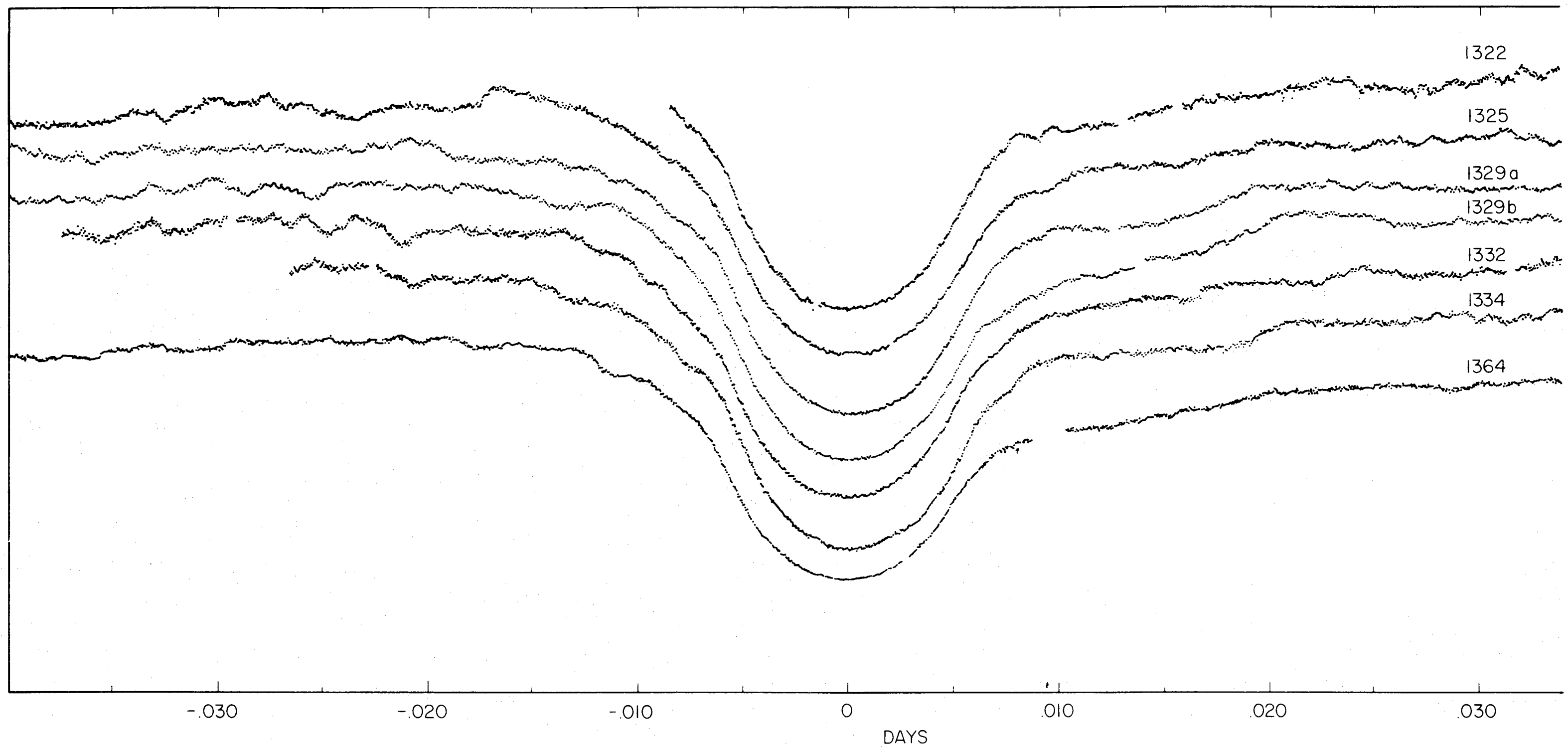

FIG. 8.—The six eclipse curves of 1973 March 3–8 (*upper curves*) and that of 1973 May 29 (*lowest curve*), displaced vertically to allow comparison of features.

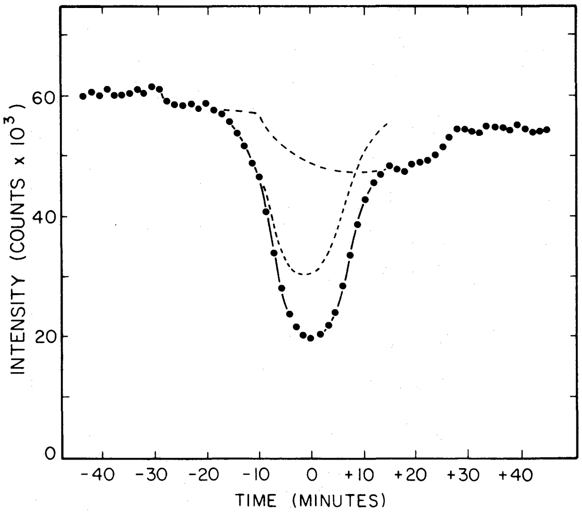

FIG. 9.—The eclipse 1329a decomposed to show the two components. The narrow (*dashed*) curve is symmetrical and has its minimum coincident with the center line of the phase shift, marked in fig. 7.

(4) Primary minimum occurs at −1.1 min, as indicated by the center of phase shift. (5) At about +8.6 min the eclipse of the bright spot reaches minimum, as the primary emerges rapidly from eclipse (*lower dashed curve* in fig. 9). (6) The end of primary eclipse occurs at +16.2 min, corresponding to the end of the 360° phase shift in the rapid oscillations. At this time the bright spot is still largely obscured and the stillstand in the light curve results. (7) At +27 min egress of the bright spot is complete. It seems likely that the nebular ring has also emerged in its entirety at this point.

This identification of eclipse curve features is consistent with our general model for these interacting binary systems, and permits a decomposition of the light curve into its basic components, as shown by the dashed curves in figure 9. This construction is based on the assumption that the bright-spot eclipse is symmetric, and that the rapid oscillations are associated with the primary object. The sum of the two dashed curves corresponds to the solid curve drawn through the observed data points. The primary eclipse curve thus derived is symmetric within the accuracy of the technique, has its minimum at the center of phase change, and strongly suggests that the eclipse is partial. Approximately two-thirds of the light of the system is removed during eclipse; of this portion ~75 percent is contributed by the primary object and ~25 percent by the bright spot and its associated ring. Clearly this ratio changes from time to time, and explains the correlation of the size of the "hump" in the overall light curve with increased eclipse depth, as seen in the data of Johnson *et al.* (1954).

The composite nature of the eclipse is certain to affect the measured time of minimum light, since the contribution of the "hot spot" can change and perturb the observed time of minimum. This effect is not large enough, however, to account entirely for the long-term variation in eclipse timings (ν_4), but may contribute some modulation to that process.

VIII. THE NATURE OF THE PULSATIONS

Any model of the 29-s periodicity in the light curve of UX UMa must account for the following critical characteristics of the periodicity: (1) The light variation is not measureably different from a pure sine curve either during eclipse or outside of eclipse. (2) Relative to its period outside of eclipse the periodicity loses exactly one cycle during eclipse; that is, the phase of the periodicity executes a −360° shift. The 71-s periodicity of DQ Her exhibits an eclipse-related phase shift of +360° (Warner *et al.* 1972). A conservative attitude requires that any proposed model should produce both the negative and positive 360° phase shift with equal ease. (3) The observed amplitude of the periodicity does not need to go to zero in order to produce the 360° phase shift. (4) The phase shift takes place smoothly and must begin and end near the beginning and ending of the eclipse, respectively.

The two previously published models for the 360° phase shift of DQ Her each fail to reproduce several of these characteristics. The model of Warner *et al.* (1972) uses nonradial white-dwarf pulsations with indices $l = 2$ and $m = 0$. In their model the amplitude of the light variation goes to zero twice during the eclipse, and phase shift takes place in two rapid jumps of 180° near the center of the eclipse. Thus, their model does not satisfy characteristics (3) and (4) above. The model of Bath, Evans, and Pringle (1974) uses star spots on a rapidly rotating white dwarf, which produces a light curve that is severely nonsinusoidal during eclipse. Their model can only reasonably produce a −360° phase shift, and in order to produce the phase shift the amplitude of the light variation must go to zero. Thus, their model satisfies only characteristic (4).

If we assume a star to be pulsating in a nonradial mode, then the time variation of the luminosity at each point on the surface of the star can be described by $I(\theta, \varphi, t)$, where θ and φ are the usual polar coordinates. If the star is spherical and if the pulsations are adequately described in the linear adiabatic approximation, the normal modes of the pulsation are given by

$$I(\theta, \varphi, t) = Y_{l,m}(\theta, \varphi) \exp[-i\omega t] = CP_l^m(\cos\theta) \exp[i(m\varphi - \omega t)], \tag{1}$$

where $Y_{l,m}(\theta, \varphi)$ is the spherical harmonic for indices l and m, $P_l^m(\cos\theta)$ is the associated Legendre function, C is a normalization constant, ω is the pulsation frequency for l and m, and $|m| \leq l$. The observer will see a light variation $I(t)$ given by

$$I(t) = \int_{S(t)} I(\theta, \varphi, t)(1 - \mu + \mu\cos\xi)\cos\xi \sin\theta d\theta d\varphi \tag{2}$$

$$= \exp[-i\omega t]\int_{S(t)} CP_l^m(\cos\theta)\exp[im\varphi](1 - \mu + \mu\cos\xi)\cos\xi\sin\theta d\theta d\varphi, \tag{3}$$

where ξ is the angle from the subobserver point, μ is the limb-darkening coefficient, and the integral is evaluated over the visible surface S of the star. During an eclipse S will be a function of time as more or less of the star is obscured. Equation (3) demonstrates that $I(t)$ is a sine curve given by $\exp[-i\omega t]$ modulated in amplitude and phase by a function given by the integral. If $S(t)$ varies sufficiently slowly, i.e., if the pulsation executes many cycles during the eclipse process, $I(t)$ will not be significantly different from a sine curve. In UX UMa the time scale of the eclipse is two orders of magnitude longer than the time scale of the periodicity so this condition is usually satisfied. Since their light variation is sinusoidal even when a large portion of the stellar disk is obscured, such pulsations automatically satisfy the first of our critical requirements.

Consider now the specific pulsation mode $l = 2$ and $m = +2$ or -2. We have then

$$I(\theta, \varphi, t) = C^* \sin^2\theta\cos(\pm 2\varphi - \omega t), \tag{4}$$

where we have taken the real part of equation (1), and C^* is a real-valued normalization constant. The effect of the $\sin^2\theta$ term is to restrict the light variations to regions near the equator of the star. The cosine term represents a traveling wave with two wavelengths fitting exactly around the equator of the star. Thus this pulsation mode consists of two bright regions located at opposite sides of the star which travel around the equator of the star with a frequency of exactly one-half the pulsation frequency. The ($\pm$) pair are distinguished from one another by traveling in opposite directions around the star. We emphasize that the bright regions are not moving around the equator because of rotation of the star as a whole, but instead are moving because of the traveling-wave nature of the pulsation mode.

Outside of eclipse the pulsating star will appear brightest when either of the bright regions is at the subobserver point. During partial phases of an eclipse the pulsating star will appear brightest not when a bright region is at the subobserver point, but when a bright region is in some sense centered on the unobscured part of the star's disk. Since the wave will reach the unobscured region at a different time than it will reach the subobserver point, there will be an apparent shift in the phase of the pulsation with respect to the phase outside of eclipse. In the extreme case, when only a sliver of the limb of the pulsating star is uneclipsed, the bright region must travel 90° around the equator of the star from the subobserver point in order to be most visible. Since there are two bright regions this corresponds to a phase shift of 180°.

Suppose that the pulsation wave travels around the star in the same direction as the stars are orbiting around each other. During eclipse ingress the bright region must travel progressively further in order to be most visible. They progressively lose time with respect to the uneclipsed pulsation, and they eventually fall −180° out of phase at totality. During eclipse egress the pulsations begin at −180° out of phase and must finish in phase at the end of the eclipse, but during egress the pulsation is still *losing* time with respect to the uneclipsed pulsation, so the phase shifts from −180° yet more negative to −360°. There is no *a priori* reason to expect the pulsation wave to travel in the direction of the orbital motion. If the pulsation wave travels in the opposite direction, then during eclipse the pulsation steadily gains time with respect to the uneclipsed pulsation and a +360° phase shift is produced. Thus, this pulsation mode can produce either a +360° or −360° phase shift, and our second critical requirement is satisfied.

Numerical examples demonstrate that the remaining points are also satisfied. Inserting the expression for $I(\theta, \varphi, t)$ given by equation (4) into equation (2) we have

$$I(t) = \int_{S(t)} C^* \sin^3\theta\cos(2\varphi - \omega t)\cos\xi(1 - \mu + \mu\cos\xi)d\theta d\varphi = a(t)\cos\omega t + b(t)\sin\omega t, \tag{5}$$

where

$$a(t) = \int_{S(t)} C^* \sin^3\theta\cos 2\varphi\cos\xi(1 - \mu + \mu\cos\xi)d\theta d\varphi,$$

$$b(t) = \int_{S(t)} C^* \sin^3\theta\sin 2\varphi\cos\xi(1 - \mu + \mu\cos\xi)d\theta d\varphi. \tag{6}$$

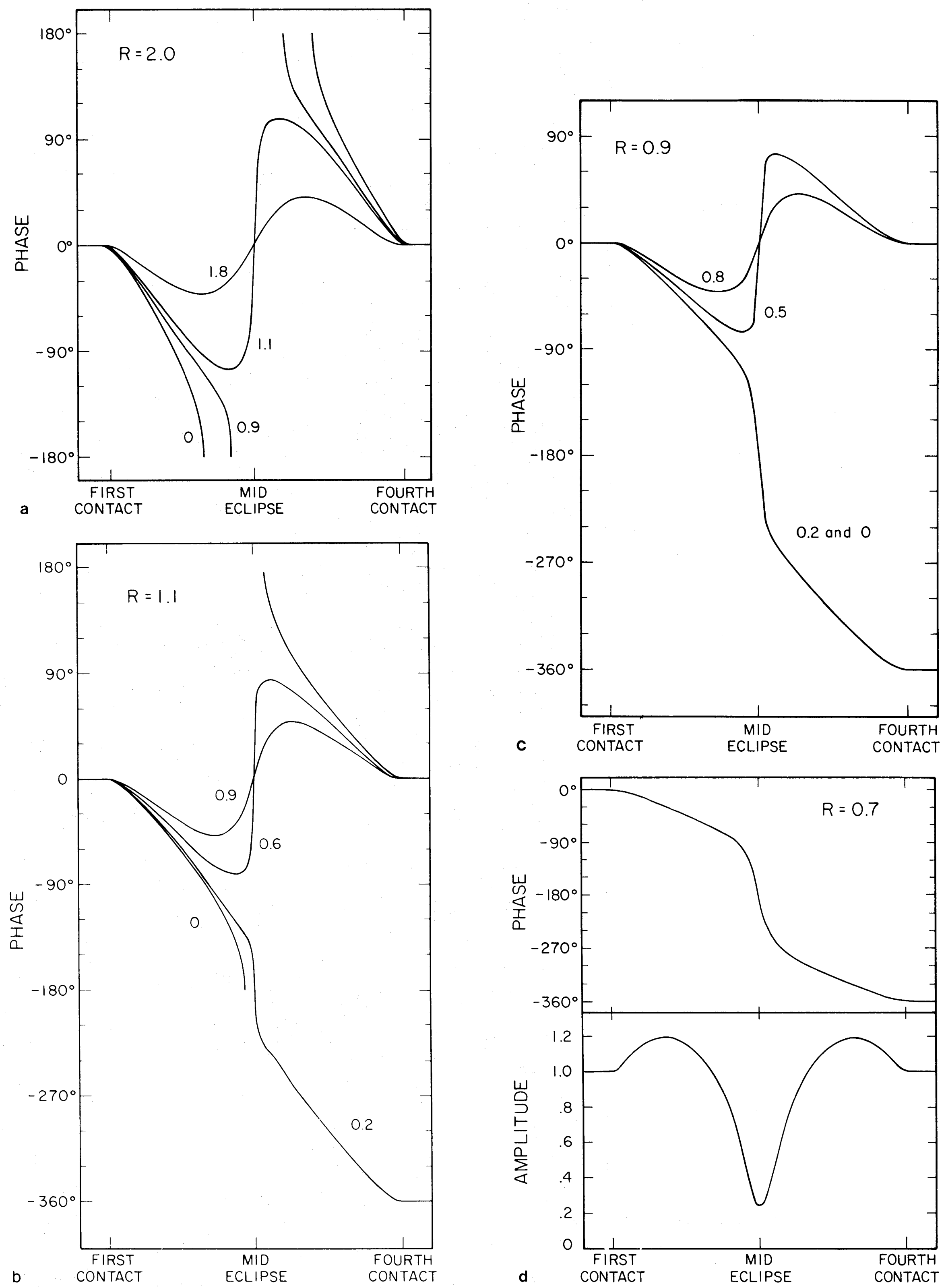

FIG. 10.—(*a*), (*b*), and (*c*) The change in the phase of the nonradial pulsation during eclipse as calculated from eqs. (6) and (7). The horizontal axis is linear in time and covers the duration of the eclipse. Each curve has been labeled with its corresponding value of Δy. (*d*) The upper curve is the change in the phase of the nonradial pulsation during eclipse as calculated from eqs. (6) and (7). The lower curve is the corresponding change in the amplitude of the light variation of the pulsation. The radius of the eclipsing star is 0.7, and $\Delta y = 0.0$.

The amplitude $A(t)$ and the phase variation $\Phi(t)$ of the pulsation are calculated from

$$A(t) = [a^2(t) + b^2(t)]^{1/2}, \qquad \Phi(t) = \arctan [a(t)/b(t)], \tag{7}$$

where $A(t)$ is defined to be positive, and care must be taken to use the correct branch of the arctangent function. The sign of m has been chosen to give the negative 360° shift. The calculations indicated in equations (6) and (7) have been carried out numerically with the aid of several simplifications. Each star is assumed to be spherical. In addition, the separation of the stars is assumed to be sufficiently large that the orbital inclinations needed to produce noncentral eclipses are negligibly different from 90°. Only results for zero limb darkening are given here; several cases of nonzero limb darkening have been calculated but the results showed only minor quantitative and no qualitative differences from the case of zero limb darkening.

The results for a variety of eclipse configurations are presented in figure 10. The vertical axes are the difference between the observed phase of the pulsation and the phase the pulsation would have if there were no eclipse. Figure 10*d* also gives the variation in the observed amplitude of the light variation during eclipse for the phase change shown above in the same figure. The unit of length is the radius of the pulsating star. The degree of centrality of the eclipse is given by Δy, the projected separation of the centers of the two stars at mid-eclipse. If $\Delta y = 0.0$, the eclipse is central; if $\Delta y > R + 1.0$, there will not be any eclipse. In figure 10*d*, $\Delta y = 0.0$, and in the other figures each curve is labeled with its corresponding value of Δy.

There are several features of note in the figures. Every total eclipse produces a 360° phase shift, and during totality the amplitude of the light variation goes to zero. If $0.6 \lesssim R \lesssim 2.0$ there are partial and annular eclipses which also produce a 360° phase shift. Numerical instabilities in the integrations occur at the ends of this range, lending a small uncertainty to the exact values of its limits. All other partial and annular eclipses produce phase shifts which may be large, but which oscillate about 0° and do not go through 360°. The phase shifts take place over the entire duration of the eclipse. The amplitude of the light variation is always greater than zero in partial and annular eclipses, so for $0.6 \lesssim R \lesssim 2.0$ there are eclipse configurations which produce a 360° phase shift and in which the amplitude of the periodicity need not go to zero. These eclipses are the best approximation to the UX UMa observations. In one of the more favorable cases, shown in figure 10*d*, the amplitude remains greater than about 0.25 of its value outside of eclipse.

While this simple model accounts to some extent for the observations presented earlier, the overall agreement with the observations is not completely satisfactory. The calculated phase-shift curves reproduce the observed curves of DQ Her reasonably well (Warner *et al.* 1972), but the observed curve in UX UMa does not have the increase in slope near mid-eclipse shown by all of the calculated curves, and the overall shape of the phase change is clearly different from that observed. The calculated amplitude curves show significant changes symmetric with respect to light minimum, which are not present in the observational data, as far as can be determined. The model has no difficulty in reproducing the low amplitude of the 71-s periodicity in DQ Her at mid-eclipse, but the mean observed amplitude of the light variation in UX UMa at mid-eclipse is considerably larger than in any of the calculated curves. It is likely, however, that a relaxation of one or more of the simplifying assumptions made in our calculations would improve the fit, and systematic obscuration by the detached matter in the system, not included in our model, may be playing a part.

IX. THE STRUCTURE OF THE BLUE STAR

There is a serious discrepancy between the estimates we may make of the radius of the blue star in UX UMa. The ~29-s period of the pulsation leads us to expect that the blue star is a hot white dwarf and that its radius is typical of white-dwarf radii. Calculations of the pulsation periods of hot white dwarfs confirm this (e.g., Harper and Rose 1970; Osaki and Hansen 1973), except for extremely hot or very low mass objects. The colors of UX UMa correspond to those of a DA white dwarf with an effective temperature near 16,000° K (Terashita and Matsushima 1966), which rules out extremely high temperatures. While the mass of the blue star is unknown, its mass would have to be much less than 0.1 $M_\odot$ to resolve the discrepancy. In both cases the pulsation would need to be an unreasonably high overtone in order to have the correct period. Thus, on the basis of the 29-s pulsation period, the radius of the blue star should be less than about 0.05 $R_\odot$.

On the other hand, two independent measures suggest that the radius of the blue star is within a factor of 2 of the radius of its unseen companion: the eclipse solutions by Zverev and Kukarkin (1937), by Krat (1940), and by Linnell (1950), and the pulsation model for the 360° phase shift presented in the previous section. While the eclipse solutions cannot be trusted in detail (Krzeminski and Walker 1963), they do indicate strongly that the secondary is comparable in size to the primary. Our decomposition of the eclipse light curve (§ VII) supports this conclusion. If the unseen star of UX UMa is similar to the red star in other short-period cataclysmic variables, it is a late-type main-sequence star which fills its Roche lobe and is in a circular orbit around the blue star. It has a normal radius and mass for its evolutionary state (Kraft and Luyten 1965). Faulkner (1971) has shown that with these assumptions there is a simple relation between the orbital period and the mass of the red star. An equivalent expression for the radius of the red star may be derived and is given by

$$P = 3.22 \times 10^4 \mathscr{R}^{1/2} R_r, \tag{8}$$

where P is the orbital period in seconds, R_r is the radius of the red star in solar radii, and $\mathscr{R}$ is a constant describing the evolutionary state of the red star. The quantity $\mathscr{R}$ varies from 0.87 for stars on the zero-age main sequence to about 1.7 for stars at the end of main-sequence evolution. Adopting $\mathscr{R} = 1.0$ we find $R_r = 0.53\ R_\odot$ for UX UMa. Therefore the radius of the blue star lies between $0.3\ R_\odot$ and $1.0\ R_\odot$.

We suggest an hypothesis which will partially remove this conflict:

1. The blue star is a hot white dwarf with a radius less than about $0.05\ R_\odot$. The white dwarf is the ultimate source of the luminosity and of the pulsations, but is not itself directly seen.

2. The disk of gas around the white dwarf absorbs the energy, both mechanical and optical, emitted by the white dwarf. The energy is reemitted at a lower temperature corresponding to the observed color temperature of UX UMa. The result is that the apparent photosphere of UX UMa lies not at the surface of the white dwarf, but further out and inside the disk of gas. The apparent photosphere has a mean radius in the neighborhood of $0.5\ R_\odot$.

3. The disk of gas is dynamically detached from the white dwarf and is supported by Keplerian motion around the white dwarf rather than by gas pressure. The breadth of the absorption lines of H and He I in the spectrum of UX UMa is due to rotational Doppler broadening in the disk of gas rather than to Stark broadening at the surface of the white dwarf.

This hypothesis could contain explanations for several peculiar features in the light curve of UX UMa. Since the disk is an unstable mass of gas, the eclipse light curves cannot be expected to repeat exactly from cycle to cycle. The eclipse will be intractable to ordinary methods of solution. Transfer effects may cause the apparent diameter of the blue star to differ at different wavelengths as is implied by the eclipse light curves of Krzeminski and Walker (1963). This hypothesis, however, leaves unanswered a major question: If we are not seeing light from the white dwarf, why do we see the white-dwarf pulsations? We have no convincing answer to this question, but point out that our hypothesis is open to a specific observational test. During an eclipse the behavior of the profiles of the absorption lines from the blue star will depend on the line-broadening mechanism. If the lines are Stark broadened the line profiles will not change during eclipse; but if the lines are rotationally broadened, they will exhibit an extreme rotational distortion. Since UX UMa is relatively bright, such an observation is within the range of large telescopes using modern techniques.

X. SIMILARITIES WITH Z Cam STARS

The observational characteristics of UX UMa are often considered to be unique among the cataclysmic variables. As our final point we shall compare UX UMa to the Z Cam-type stars, a small subclass of the dwarf novae which erupt every 2–3 weeks and increase in brightness by 2–5 magnitudes. The duration of the eruptions from minimum light to minimum light is usually about a week, but occasionally and unpredictably the Z Cam stars will "hang up" in their eruptions and remain just below maximum brightness for an extended period of time. Such an occurrence is known as a standstill. The standstills are variable in length and have been seen to last as long as several years as in Z Cam itself during the years 1948–1950 (Mayall 1965). We may imagine a Z Cam star in an extreme standstill lasting many decades. Such a star would be observationally indistinguishable from UX UMa.

The spectrum of Z Cam during standstill is similar to that of UX UMa. It has a blue continuum and broad, shallow hydrogen absorption lines with central emission components (Lortet-Zuckermann 1967). The colors of Z Cam during standstill are unavailable but at the peak of an eruption that are $U - B = -0.80$ and $B - V = 0.00$ (Kraft, Krzeminski, and Mumford 1969). The colors during standstill will not be much different, and will be similar to those of UX UMa. During a standstill the high-speed flickering in Z Cam has an amplitude of about 0.1 mag (Robinson 1973*a*), and the week-to-week variations may be up to a magnitude (Mayall 1965). Sinusoidal variations with periods between 16 and 31 s and amplitudes of a few thousandths of a magnitude have been detected in five Z Cam stars during their eruptions, and have been interpreted as nonradial pulsations of white dwarfs (Robinson 1973*b*). The evidence is insufficient to determine whether the pulsations ever occur during standstills. We may estimate the luminosity of the blue star in UX UMa from the expression

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T_e}{T_{e\odot}}\right)^4 .$$

Inserting $R = 0.5\ R_\odot$ and $T_e = 16{,}000°$ K we find $L \simeq 15\ L_\odot$, which compares favorably to $L \simeq 15\ L_\odot$ for Z Cam during standstill (Robinson 1973*c*).

XI. SUMMARY AND CONCLUSIONS

Our studies of UX UMa in 1971 and 1973 have revealed a number of interesting aspects to this peculiar system:

1. The long-term variation of the time of eclipse minima proposed by Mandel is strengthened by our observations, indicating the phenomenon is periodic and approximately sinusoidal in character, with a period of 29 years. If this variation is caused by the presence of a third body in the system, then it is sufficiently small so that it will not contribute any measurable light to the system in the visible. Alternatively, such variations may be due to a transfer of angular momentum between the orbiting pair and the disk of material around the primary, as suggested by Smak; the ubiquitous presence of such variations in similar systems lends weight to this alternative.

2. Short-period ($\sim$29-s) oscillations appeared in the light curve in 1971 and again in 1973, changing slowly in frequency from night to night and exhibiting a remarkable $-360°$ phase shift accompanying eclipse. Similar oscillations have been discovered in five Z Cam stars during outburst, suggesting these objects are related. The spectrum, colors, and luminosity of Z Cam in standstill and eruption are also very similar to those of UX UMa. We propose that UX UMa is, in fact, a Z Cam star at standstill, and that this condition, a form of continuous eruption, may provide the source of energy to excite the oscillations.

3. The eclipse-associated phase shift in the rapid oscillations probably arises from some type of traveling-wave pulsation associated with the primary object. We have demonstrated that a similar phase shift arises from the eclipse of a spherical body undergoing nonradial oscillations with $l = 2$ and $m = \pm 2$, the sense of the shift being determined by the sign of m. The $+360°$ phase shift discovered in the oscillations of DQ Her is as easily modeled as the $-360°$ shift in UX UMa. This form of oscillation consists of a wave of brightness traveling around the equator of the object at one-half the pulsation frequency.

4. The observed eclipse is composite, consisting of an eclipse of the luminous disk of material surrounding the primary, an eclipse of the primary object itself, and the eclipse of an incandescent bright spot in the disk of material located following the line of centers of the two stars. The eclipse curve of the primary object, when the other effects have been removed, is seen to be symmetrical and suggests that primary and secondary are of comparable size. The primary is very luminous, contributing $\sim$75 percent of the light of the system.

5. The radius of the primary deduced from eclipse data, and its radius estimated from the observed pulsation frequency are in serious disagreement. We propose that the primary object may itself be composite, consisting of a white dwarf which is oscillating, surrounded by a photosphere suspended by Keplerian motion which is being slowly accreted by the central object. This process is in equilibrium with the rate of mass transfer from the orbital companion.

While our attempts to model the various features of this stellar system are somewhat tentative they do indicate, we believe, the directions in which future understanding should be sought. Sufficient observational details now exist to suggest that this object, and others clearly related to it, are deserving of concerted theoretical attention.

We thank Alan Kipplinger for his help in obtaining some of the data presented here, and for assistance with the preliminary reductions. We have benefited from a conversation with Dr. Sumner Starrfield concerning the structure of dwarf novae. This work was supported in part by a grant from the National Science Foundation.

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