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A Markov Chain Model for Predicting Major League Baseball

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A Markov Chain Model for Predicting Major League Baseball

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A Markov Chain Model for Predicting Major League Baseball

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In this report, we present a Markov chain model for predicting the scores and the winning team of Major League Baseball (MLB) games. We discuss how a baseball game can be viewed as an infinite horizon discrete-time Markov chain with finite state space. We demonstrate how standard Markov chain theory can be used to obtain analytical solutions for the expected runs and win probability in a given MLB matchup. We improve upon previous models by incorporating pitching and more complex baserunning, and then demonstrate the effect of these changes by comparing our model to historical data. We also discuss computational methods for solving the model. Finally, we test our model on games from the 2015 MLB season.

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1 Introduction

Baseball, or "America's pastime", is a sport consisting of two teams made up of pitchers and batters. Each team has (at any one time) 8-9 position players (depending on whether a designated hitter is used), who are responsible for hitting and fielding, and one pitcher, responsible for preventing the opposing team's hitters from scoring runs. There are a number of characteristics which make baseball particularly amenable to mathematical modeling. First, at any given time, a baseball game can be in exactly one of a finite collection of game states. In addition, the future evolution of the system depends only on the current state and the discrete events (such as a hit) which move the game to a new state. Furthermore, statistics and data are readily available for estimating the player skills which determine in-game events.

The attributes described above suggest that a baseball game can be modeled as a Markov chain. In this article, we describe a Markov chain model for estimating runs and win probabilities for Major League Baseball (MLB) games. In what follows, we briefly survey relevant literature, describe the technical details of our model, discuss methods for solving the model, compare our model's predictions to previous models, and test our model's performance on games from the 2015 season.

2 Literature review

We briefly review some previous work which uses Markov chain models in the context of baseball. The idea for modeling a baseball game using Markov chains dates back to at least Howard [7], who, in 1960, presented a simple, one-inning, Markov decision process (MDP) which found the optimal time to bunt with the goal of maximizing expected runs scored. In his famed sabermetric tome, *The Book*, Tom Tango [16] presents a Markov chain approach for computing expected runs which does not account for differences in player skills. Bukiet et al. [1] build a Markov model for predicting the distribution of runs over a full game for a unique lineup of MLB player, and use it to analyze optimization of batting orders.

Related Markov models seek to estimate win probabilities for teams in a baseball game. In his dissertation, Null [11] uses a Markov chain framework to predict win probabilities, and discusses how the model could be extended to an MDP which could be used to make in-game decisions. Hirotsu and Wright [5] present an MDP model to evaluate the optimal time to substitute a pinch hitter. Hirotsu and Bickel [4], in an unpublished manuscript, present an MDP model to estimate the optimal time to bunt in order to maximize win probability or expected runs. They also note that a decision maker should focus on the former objective, not the latter, to correctly maximize the chances of winning.

There are also a large number of models which instead use simulation to compute expected runs, win probabilities, and other statistics. A few examples are given in [8] and [14]. This approach is used in a number of commercial software packages.

Markov models and simulation models each have their own strengths. A simulation model provides sampling distributions for various quantities of interest (such as runs scored). In addition, a simulation model can more easily track in-game changes – who specifically is on which base, substitutions, etc. Markov chain models facilitate computation of expected values, and, generally, can produce these results much faster than running simulations. It also allows for simultaneous prediction for every possible state in a given game. This feature was our primary reason for choosing the Markov chain approach – we wanted to be able to track win probabilities during a game in real time. Finally, as in [11] and [5], it is easy to incorporate optimal decision making within a Markov chain model. In the following sections, we discuss our approach to modeling a baseball game as a Markov chain.

3 The expected runs model

In this section, we introduce the objectives, the mathematical structure, and the analytical solution methods for our expected runs model. A baseball game can be fully described by a discrete set of characteristics: the current batter and pitcher, the batter due up for the defending team, the inning, the number of outs, the configuration of runners on the basepaths, and the score. Given a particular state in a baseball game, our model calculates the expected runs each team will score from that point until the end of the game.

We model a baseball game using the theory of discrete time, discrete state space Markov chains (DTMCs). Formally, a DTMC is a stochastic process consisting of a discrete state space S, and a stochastic transition matrix $\mathbf{P}$ which describes the random movement between states in S. The transition probabilities from a given state must adhere to the 'Markov property,' meaning that the probability of moving from state i to state j, $p_{ij} \in \mathbf{P}$ does not depend on the previous evolution of the system.

A baseball game fits into this framework nicely as probabilities between game states are logically assumed to be independent of past events. For example, suppose the game is in the bottom of the ninth inning, tied, with 2 outs and no one on base. The batter's chances of hitting a home run – moving the game into the "game over, home team wins" state – depend only on the current state, the current pitcher, and the batter's own talent, not on the particular path the game followed to reach that particular state. We now provide more details on calculating expected runs and win probabilities.

3.1 Computing expected runs

We first describe the model used to compute expected runs over the course of a 9 inning game for a given team. In reality, a team can potentially bat in more than 9 innings if the game goes into extra innings. On the other hand, the home team may not need to bat in the bottom of the ninth if they already lead. However, for simplicity, when computing expected runs we assume that a team will come up to bat 9 times during the game. As we are only interested in one team's performance, the elements in the state space, S, are defined completely by:

1. The player up to bat (9 total)
2. The inning (9 total)
3. The number of outs (9 total)
4. The configuration of the baserunners on the basepaths (8 total)

There are $9 \times 9 \times 8 \times 3 = 1944$ total in-game states and a single absorbing state, $\triangle$, representing the end of the game. We construct a 1945×1945 matrix P which describes the evolution of the stochastic process.

3.2 Event probabilities

Transition probabilities in any state depend on the current batter, the current pitcher, the current number of outs, and the current base configuration.

In our model, we assume the current batter/pitcher matchup will result in one of the following events: **single, double, triple, HR, walk/hit by pitch (BB+HBP), double play (with <2 outs), strikeout (K), other out (non double play)**. A sequence of calculations are performed to estimate the probabilities of these events for a given plate appearance.

We deliberately leave out a number of possible strategic baseball plays, most notably bunts, steals, and intentional walks. These events would complicate the model, as the set of possible transitions and the associated probabilities when the batter decides to bunt are completely different than those when he decides to try for a hit. In addition, treating these 'strategic' plays separately allows for the possibility of adding an optimal decision making component to our model in the future.

In each state, a pitcher from the defending team faces the hitter at the plate. The opposing starting pitcher begins the game. After the starter exits the game, we use the defending team's average bullpen performance to inform pitching for the remainder of the game. Together, the abilities of the hitter and the pitcher inform event probabilities in a given plate appearance.

The following sequence of steps are performed to calculate these probabilities. First, raw counts for batting events are obtained for both the batter and pitcher from Steamer Projections [18] or previous season data. Using projections is the default option, as these projections consider a player's previous statistics, age, and other factors to forecast future performance. For example, Steamer projects the following counting stats for Washington National's outfielder Bryce Harper for the 2016 season.

Table 1: Bryce Harper 2016 Steamer Projections.

AB	1B	2B	3B	HR	BB+HBP	K
536	94	31	2	37	110	129

Neither a player's projections nor his past season statistics are park neutral, meaning that, for hitters, the projections will be higher (lower) for players who play in home stadiums which are more (less) favorable to hitters (the opposite relations hold for pitchers). When evaluating a hitter vs. pitcher matchup, we would like to consider their talents in a neutral environment; thus we 'park normalize' the raw Steamer projections (or data) using 'park factors'.

Fangraphs.com [17] publishes park factors for every stadium in MLB. Park factors describe the frequency of batting events in a given park compared to the league average frequency. A neutral park for a given batting event, say home runs for lefties, would have a park factor of 100. If more lefty home runs were hit in that park compared to other stadiums, the park factor would be greater than 100. Park factors provided by Fangraphs are specific to the handedness of the batter. Furthermore, to facilitate calculation of park-neutral statistics, the park factors presented on Fangraphs are adjusted down to account for the fact that batters play only half of their games in their home ballpark.

We divide the projected hitting statistics for both the batter and pitcher by the park factors for each category. For example, if Bryce Harper is projected to hit 37 HR in 2016, and the National's park has a lefty-batter park factor for HR of 92, our park-neutral projection for Harper's HR total is $37/.92 = 38.54$. In Table 2 we show the park factors for National's Park, along with Bryce Harper's adjusted hitting projections.

Table 2: Bryce Harper 2016 projections, park adjusted

	1B	2B	3B	HR
Park factors	108	104	70	92
Neutralized projection	90.4	37.25	2.35	38.54

Next, we convert the park-normalized frequency projections into event probabilities for both the batter and pitcher. For a given batter or pitcher, the probability of event e, where e could be a single, double, triple, home run, walk/HBP (BB+HBP) or strikeout, is

$$P(e) = \frac{freq(e)}{AB + BB + HBP} \tag{1}$$

where $freq(e)$ is the projected frequency of event e, AB is projected at-bats, BB is projected walks, and HBP is projected hit-by-pitches. The denominator represents all possible considered outcomes of the plate appearance. We again use Bryce Harper's normalized projections for demonstration in Table 3.

Now, we need to explicitly take into consideration the handedness of the batter and pitcher. Over large samples, hitters perform worse against pitchers of the same handedness and better when facing opposite-handed hurlers. To

Table 3: Bryce Harper event probabilities, park neutral

	AB	1B	2B	3B	HR	BB+HBP	K
Projected counts	536	90.4	37.25	2.35	38.54	110	129
Event Probabilities		.15	.05	.003	.06	.17	.2

explicitly account for this in our model, we use the ZiPS splits projections, made available by creator Dan Szymborksi [15]. ZiPS provides full-season statistical projections subdivided by the hand of the opposing pitcher (for batters) and by the hand of the opposing batter (for pitchers). We use these 'splits' projections to compute handedness adjustment factors for each player in the batter-pitcher matchup.

As it may be difficult to project exact statistics by handedness for low frequency events like doubles and triples, we produce handedness splits for the following events: **Non-HR hits, HR, BB+HBP and Strikeout**. For an event e in this set, we compute the h handedness adjustment factor, $h \in \{\text{right}, \text{left}\}$ for e, $HA_h(e)$ as

$$HA_h(e) = \frac{P(e|PitcherHand = h)}{P(e)}. \tag{2}$$

If a switch-hitter (a hitter who can hit both righty and lefty) is at the plate, we assume that they will bat righty (lefty) if the pitcher is a lefty (righty).

Continuing with our example, suppose Bryce Harper is at the plate facing righty Noah Syndergaard of the division rival New York Mets. In Table 4, we show Bryce Harper's ZiPS projected event probabilities against righties, overall, and the associated handedness adjustment factors we would apply to his park-neutral event probabilities in Table 3.

Table 4: Harper 2016 split adjustment factors

	H (non HR)	HR	BB+HBP	K
vs. Righties	.1956	.063	.181	.196
Overall	.1955	.0588	.176	.201
Handedness adjustment	1.00	1.08	1.025	.975

Similarly, handedness adjustments would be calculated for the pitcher, Syndergaard, reflecting his projected performance against lefties. Now, we adjust each players projected probabilities given in Table 3 by multiplying by the relevant handedness adjustment factors in Table 4. For example, Bryce Harper's park-neutral HR probability is .06. His park-neutral HR probability against a lefty is calculated as $.06 \times 1.08 = .064$. As expected, we project that Harper will be more likely to hit home runs against a righty pitcher.

Up to this point, we have computed projected event probabilities for our batter and pitcher which reflect how we expect them to perform against the league as a whole (adjusted for handedness). However, we have not considered how the specific players in the given plate appearance will perform against each other. We would not expect Syndergaard to perform against Bryce Harper as he would against all other left-handed hitters, because Bryce Harper is one of the best hitters in MLB. To derive event probabilities for the specific head-to-head matchup, we use the so called "total Log5 rule" introduced in [3]. The total Log5 rule generalizes the well-known Log5 rule introduced by Bill James

[9] to matchups involving more than two outcomes. The logical premise for both rules is as follows: If a batter and pitcher meet, and the pitcher is worse than the league average, we would expect the batter to perform *better* against this particular pitcher than he does against the rest of the league (i.e., his performance on average). Many baseball simulators employ similar variants of the Log5 rule (see [8] and [14], for example).

We consider the following possible outcomes for the plate appearance when applying the total Log5 rule: **Ball-in-play (BIP), HR, BB+HBP, K**. These events are generally considered within the control of the pitcher; once the ball is put in play, defense and luck generally determine the outcome of the plate appearance. For an event e among these four outcomes, the Log5 probability of event e for the given plate appearance is

$$P_{Log5}(e) = \frac{\frac{p_b(e) \times p_p(e)}{p_l(e)}}{\sum_e \frac{p_b(e) \times p_p(e)}{p_l(e)}} \tag{3}$$

where $p_b(e)$ is the batter's probability of generating event e, $p_p(e)$ is the pitcher's, and $p_l(e)$ is the leaguewide probability of event e in a plate appearance with the same handedness matchup. In our example of Harper vs. Syndergaard, this latter value would be the probability of event e calculated from frequency data (as in Table 3) for all lefty-batter vs. righty-pitcher matchups from the previous season. We use season data as opposed to the projected league average because the projections often do not accurately predict playing time, which can cause distortions. In addition, it is unlikely that the average performance over hundreds of thousands of plate appearances in 2015 will be significantly different than the average in 2016.

Table 5: Harper vs. Syndergaard, Log5 probabilities

	BIP	HR	BB+HBP	K
Harper	.586	.064	.174	.1946
Syndergaard	.621	.019	.068	.292
League (L vs. R)	.685	.027	.096	.191
Total Log5 probabilities	.525	.047	.127	.303

We then distribute the log5 probability assigned to 'BIP' to the events **1B, 2B, 3B, Out (in play)** according to the hitter's own distribution of events given that he puts the ball into play. For example, Bryce Harper, against the league as a whole, is projected to put the ball in play 57% of the time, with singles making up 26.3% of these events (see Table 3). Against Syndergaard, per Table 5, Harper is expected to put the ball in play with probability .525. Thus, we would estimate that he will hit a single against Syndergaard with probability $.525 \times .263 = .138$.

We make three final adjustments to the event probabilities associated with the plate appearance. First, we readjust for the park in which the game is being played. For example, if the game is in National's Park, we would re-apply the relevant park factors for Harper. Next, we adjust for home field advantage. Using data from the previous season (obtained from Baseball-Reference), we compute a 'home-field' factor which compares the performance of players at home compared to players in all games. For example, over the 2013 and 2014 seasons, players hit HRs in 2.39% of plate appearances in all games, but 2.43% of plate appearances at home, resulting in an HR home-field adjustment of 1.014. We would then adjust the plate appearance probabilities in an analagous manner to handedness.

Finally, we would consider the number of hitters faced by the pitcher prior to the given plate appearance, as pitching performance degrades over repeated exposure to hitters. Again, we compute "times-thru-order" adjustment factors using full-season data from Baseball Reference. For example, in 2014, when the hitter faced a pitcher the third time during a game, his probability of hitting a homerun was 14% above baseline. Thus, we would increase our event probability for an HR by 14%. We do not explicitly track the batters faced by the starter, as this would need to be included in the state space to preserve the Markov property. Instead, apply the adjustment factor based on the inning in which the plate appearance takes place. Steamer provides projections for batters faced per inning for every pitcher. Thus, we can estimate the number of times a pitcher has gone through the order in a given state by:

$$\text{Batters faced} = \text{Inning} \times \text{BPI} \tag{4}$$

where BPI is the projected number of batters a pitcher will face per inning.

Finally, in some states it is relevant to consider double plays (we will discuss when double plays are applicable in 3.3). We estimate the probability of a double play for a given plate appearance using baserunning data for the hitter from the previous season. This probability is computed from frequency data available on Baseball-Reference, and the probability of a double play is simply DP/DP Opportunities.

Having described how event probabilities for a given plate appearance are derived, we now turn to the issue of baserunning, which determines transitions in the matrix **P**.

3.3 State transitions and baserunning

State transitions are determined by event probabilities, as described in Section 3.2, combined with a baserunning model which describes the movement of runners given certain events. For example, a single with a man on first could result in two possible transitions. In either case, the batter will advance to first, but the baserunner could end up on either second base or third base. We now give a full description of our baserunning model.

Previous models (see [4], [1], [5]) have used a simplified baserunning model proposed by D'Espopo and Lefkowitz [2] which is summarized in Table 6. After examining baserunning data, we felt that the model above fails to ad-

Table 6: D'Esopo and Lefkowitz baserunning model

Event	Advancement
Walk	Batter to first, baserunners advance one base if forced
Single	Batter to first, baserunners on second and third score, baserunner on first to second
Double	Batter to second, baserunners on second and third score, baserunner on first to third
Triple	Batter to third, all baserunners score
HR	Batter scores, all baserunners score
Out	No baserunners advance

equately describe the baserunning dynamics of a real MLB game. We develop a more realistic baserunning model by adding the following:

1. **Double Plays** – When there are fewer than two outs, and a runner is on first base, we consider the possibility that the batter will hit into a double play.

2. **Scoring from third base on outs** – We allow a runner on third to score on an out when the ball is put in play. We obtain the leaguewide probabilities of scoring on an out from third for a regular ball-in-play (BIP) out, and for a double play. We scale the probability by the batter's strikeout percentage, as the ball must be put in play for the runner to reach home.

3. **Runner advancement on hits** – When a single is hit, we allow a runner on first base to potentially advance to third base. Similarly, when a double is hit with a baserunner on first, we allow for the possibility that said runner will score. Finally, we assume that only some baserunners score from second on a single. Statistics for individual batters for advancements are available on Baseball-Reference and we use the team average to inform these probabilities.

4. **Runner advancement on out** – We allow for the possibility that the lead baserunner (not on third base) will advance on an out in play. We use a single leaguewide probability for this event.

Other models have used different baserunning frameworks. Null [11] calculates probabilities of transitioning from one base configuration to another after a certain event by looking at leaguewide historical frequency data. For example, given that an out occurs with one man on first, Null looks at all historical examples of this event type and state and computes the probability of any subsequent base state being realized. Compared to our approach, Null's estimates are more comprehensive; no limits are placed on the possible subsequent base state compositions given a certain event. However, our approach is better for incorporating readily available team and player specific data. For example, a team with fast runners is likely to advance from first to third on a single more frequently than the historical frequencies would suggest, and our baserunning model allows for those considerations.

To make the full advancement model clear, we give an example of the transitions from a representative state. Suppose that Bryce Harper is facing Noah Syndergaard with no outs and baserunners on first and third in the first inning, in a game played at National's stadium. Following Section 3.2, our event probabilities for this plate appearance are:

Table 7: Harper vs. Syndergaard, plate appearance probabilities

1B	2B	3B	HR	BB+HBP	K	DP
0.149	0.049	0.003	0.039	0.104	0.23	.095

We will use the following notation to refer to base configurations. The symbol '—' will be used to denote that all bases are empty. The symbol '–1' denotes that a man is on first, and the other bases are empty. Similarly, '32-' will be used when second and third are occupied. Notation for the remaining base configurations follows this pattern. The possible transitions from our Bryce Harper plate appearance (base configuration 3-1) are given in Table 8.

Table 8: Harper vs. Syndergaard, plate appearance transitions

Event	New state	Runs	Probability	Description
1B	0 outs, -21	1	(.149)(.78)	Single, runner on 1st to 2nd
1B	0 outs, 3-1	1	(.149)(.22)	Single, runner on 1st to 3rd
2B	0 outs, 32-	1	(.049)(.39)	Double, runner on 1st to 3rd
2B	0 outs, -2-	2	(.049)(.61)	Double, runner on 1st scores
3B	0 outs, 3–	2	.003	Triple, all runners score
HR	0 outs, —	3	.039	HR, all runners score
K	1 outs, 3-1	0	.23	Strikeout
DP	2 outs, —	1	(.095)(.74)	Double play, runner on 3rd scores
DP	2 outs, -2-	0	(.095)(.26)	Double play, runner out at home
Out (IP)	1 outs, 3-1	0	(.335)(.374)(.60)	Out in play, runners don't advance
Out (IP)	1 outs, 32-	0	(.335)(.374)(.40)	Out in play, runner on first advances
Out (IP)	1 outs, –1	1	(.335)(.626)(.60)	Out in play, runner on 3rd scores, runner on 1st stays
Out (IP)	1 outs, -2-	1	(.335)(.626)(.40)	Out in play, runner on 3rd scores, runner on first advances

A full description of all possible transitions is given in Appendix A. In Section 5.1, we compare our model results using our full baserunning model with those obtained with the D'Esopo and Lefkowitz model.

3.4 Solving for expected runs

The matrix $\mathbf{P}$ is built with possible transitions reflecting the baserunning model presented in Section 3.3 and probability values corresponding with player abilities as calculated in Section 3.2. A rewards matrix $\mathbf{R}$, which corresponds in dimension to $\mathbf{P}$ is created as follows. For an entry p_{ij} of $\mathbf{P}$, $p_{ij} = 0 \to r_{ij} = 0$. If $p_{ij} > 0$, r_{ij} is the number of runs which would score per the baserunning model (see Section 3.3), given a transition from state i to state j, $i, j \in S$. We set $r_{\triangle\triangle} = 0$ meaning that we can never score after the game has finished (See Appendix B, Section 1 for full details on the structure of the transition matrix and the rewards matrix).

We now describe how to compute the expected runs scored in a game, following the formulation in [4] and [5]. We first note that, despite assuming that a team will bat in exactly 9 innings, the time horizon for a baseball game is uncertain. The game is not over until three outs are registered in the ninth inning; the game makes an arbitrary number of transitions before this endpoint. Thus, we consider the time horizon over which rewards are aggregated to be infinite. We also note that all in-game states form a transient class, with the end-game state, $\triangle$, a single absorbing state. Let $v(i)$ be the expected runs scored from state $i \in S$ up to the end of the game. Furthermore, let $q(i) = \sum_{j \in S} p_{ij} r_{ij}$ so that $\mathbf{q} = [q(1), q(2)...]$ is the vector of one-step expected runs from any state. Following previous papers (and well known results concerning Markov chains with rewards), our expected runs vector $\mathbf{v} = [v(1), v(2), ...]$ satisfies:

$$\mathbf{v} = \mathbf{P}\mathbf{v} + \mathbf{q}. \tag{5}$$

This linear system has a unique solution once we impose the known constraint $v(\triangle) = 0$.

We now turn to the more complicated case of solving for winning percentages.

4 The win probability model

The win probability model estimates the home team's probability of winning at any state during the course of a baseball game. As in Section 3, we develop a Markov chain model to solve for these probabilities. As illustrated in previous work the state space, S, expands greatly when we are interested in estimating the probability that the home team A will beat the away team B from any point in a baseball game [4], [11]. Our new state space consists of:

1. The current home team player due to bat (9 possible players)
2. The current away team player due to bat (9 possible players)
3. The current batting team, or whether we are in the top or the bottom of the inning (2 possibilities)
4. The inning (9 regular innings)
5. The number of outs in the inning (0,1, or 2)
6. The base configuration (8 possible, ranging from empty to loaded)
7. The home team's lead, l, ($l \in [-q, q] \cap \mathbb{Z}, q \in \mathbb{Z}$ ($2q + 1$ possible values)

Techinically, the state space could be unbounded as there is no upper or lower bound for the home team's lead. However, for real games, q is finite, practically speaking. There are competing considerations when considering the cap, q, on the possible score differences in a game. From a computational standpoint, keeping q small decreases the size of the state space, making mathematical operations easier. However, setting q too low leads to unintentional bias in the model in favor of the home team. To see why, consider the ninth inning; the home team need only achieve $l > 0$ to win the game, and so a cap on the home team lead does not influence their chances of winning for any state in the final inning. However, the visiting team must prevent the home team from reaching this threshold. Thus, capping the amount by which the home team can trail artificially improves the chances that the home team will win. In our base model, we set $q = 8$ because, for two identical teams (without considering home field advantage), this cap yields a projected .500 probability of the home team winning from the beginning of the game.

We also note that the game can go into extra innings if the score is tied at the end of the 9th. At that point, the game becomes sudden death; if either team leads after an inning is completed, that team is the victor, or if not, the game continues. Thus, our state space is unbounded if we were to explicitly include all possible extra innings. However, as we show in the following section, we need only consider one extra inning to fully solve the model.

4.1 Solving for win probability

We can approach solving for win probability in a manner similar to solving for expected runs scored. We build a transition matrix $\mathbf{P}$, which, considering 9 innings and a maximum lead for either team of 8, is a $594,864 \times$

594, 864 matrix. The structure of this matrix has been previously described in [4], [5]. The baserunning model given in Section 3.3 is used to govern transitions, and event probabilities are computed as in Section 3.2. One small distinction is that scoring is now accounted for within the state space, as opposed to a scoring rewards matrix as in Section 3.4.

In the Win Probability model, the home team accrues a 'reward' only when a transition is made from a state $i \in S$ to $\triangle$, the end-of-game state. If the home team is leading when this transition is made, they have won the game, and thus accrue a reward of 1. Conversely, if they are trailing when this transition occurs, a reward of 0 is obtained. Once again let $q(i) = p_{i\triangle} r_{i\triangle}$ for $i \in S$ where $r_{i\triangle}$ is the reward of 0 or 1 accrued by the home team when the game ends. Then, as before, the vector of win probabilities from any state, $\mathbf{w} = [w(1), w(2)...]$, satisfies:

$$\mathbf{w} = \mathbf{P}\mathbf{w} + \mathbf{q}. \tag{6}$$

Again, the system has a unique solution when we impose the constraint $w(\triangle) = 0$.

Equation 6 can be thought of as a simple application of the law of total probability, with

$$w(i) = \sum_{j \in S \setminus \triangle} p_{ij} w(j) + p_{i\triangle} r_{i\triangle} \tag{7}$$

where $r_i \triangle$ is the simply the probability of winning upon transition to the end of game state.

There are a few issues with computing win probabilities by solving the linear system in Equation 6. First, when our state space is limited to nine innings, we must decide what reward to assign when the ninth is completed and the game is tied. One possibility is to simply assign a reward of .5 to the home team should the game go into extra innings. However, if one team is significantly better than the other, this can skew computed probabilities (particularly in later innings) in an unrealistic manner. Second, solving equation 6 is a computational challenge due to the size of the matrix. We now discuss our method for solving the model which addresses both issues.

Due to the sequential nature of a baseball game, Equation 6 can be solved in pieces. By sequential, we mean the following: within a given inning, we can only (potentially) transition to a later inning, and can never return to an earlier inning. Similarly, within innings, we can only advance from the top of the inning to the bottom, and not the other way around. One can see that the same pattern holds for outs within half innings as well. Mathematically, as Equation 7 makes clear, the components of vector $\mathbf{w}$ associated with inning $k \in 1, ...9$ can depend only on win probability values from innings $k+1, k+2...$

In our model, we choose to decompose Equation 6 by inning. We begin by solving for all extra-inning win probabilities simultaneously. Let $\mathbf{P}_{extra}$ be the transition probability matrix for one extra inning. In this case, as we are considering only one inning, the size of our state space is $9 \times 9 \times 2 \times 8 \times 3 \times 17 = 66,096$ (See Appendix B, Section 2 for full details on the structure of the transition matrix).

We note that all extra innings look identical in terms of computing win percentages. A reward of either 0 or 1 is given to the home team at the end of the inning depending on whether the home team is trailing or leading. If the game is tied, the game transitions to another extra inning with identical characteristics. Thus, from a modeling perspective, we can 'transition' to a new extra inning by defining non-zero transition probabilities from states at the end of the current extra inning to the beginning of the same inning. Specifically, if we are in an extra-inning state, i, where the home team is batting, the game is tied, there are 1 or 2 outs, with batter $b \in 1, 2, ...9$ due up for the away team, there (may) be a non-zero probability (if an out or double play are recorded) that the inning will end with the game tied, triggering a new extra inning to begin. In this case, we define a nonzero entry in $\mathbf{P}_{extra}$, e_{ij}, where j is a state at the top of the extra with 0 outs, no men on, and batter b at the plate.

Let $\mathbf{q}_{extra}$ be the vector of one-step rewards for the extra inning, where

$$q_{extra}(i) = \left\{ \begin{array}{ll} p_{i\triangle} & \text{Home team is leading in state } i \\ 0 & \text{o.w.} \end{array} \right\} \tag{8}$$

with $\triangle$ once again representing the end-of-game state. As before, we can solve for the home team's chances of winning from any state in extra innings by finding the unique solution to:

$$\mathbf{w}_{extra} = \mathbf{P}_{extra}\mathbf{w}_{extra} + \mathbf{q}_{extra} \tag{9}$$

To solve for the rest of the game, we work backwards inning by inning. Let $\triangle_k$ denote the end-of-inning state for inning k. Let $\mathbf{P}_k$ be the transition matrix for inning k. For the ninth inning, we can easily create $\mathbf{P}_9$ by removing the self loops in $\mathbf{P}_{extra}$ (along with potentially adjusting the values of the transition probabilities). In this case, we have:

$$q_9(i) = \left\{ \begin{array}{lr} p_{i\triangle_9} & \text{Home team is leading in state i} \\ p_{i\triangle_9}\mathbf{w}_{extra}(i, \triangle_9) & \text{Game is tied} \\ 0 & \text{o.w.} \end{array} \right\}. \tag{10}$$

where $\mathbf{w}_{extra}(i, \triangle_9)$ is the win probability associated with the state in extra innings entered upon the transition i to $\triangle_9$. For example, if $i =$\{Home batter 1, Away batter 1, Bottom of 9th, 2 outs, No baserunners, Tied\}, then $\mathbf{w}_{extra}(i, \triangle_9)$ would be the win probability associated with the extra inning state \{Home batter 2, Away batter 1, Top of 10th, 0 outs, No baserunners, Tied\}.

For inning $k, k \leq 8$, we have the following definition for the one-step rewards vector $\mathbf{q}$:

$$q_k(i) = p_{i\triangle_k}\mathbf{w}_{k+1}(i, \triangle_k). \tag{11}$$

We sequentially solve the system of equations

$$\mathbf{w}_k = \mathbf{P}_k\mathbf{w}_k + \mathbf{q}_k \tag{12}$$

for $k = 9, 8...1$. Solving Equation 12 for each inning gives estimates for the home team's win probability from every possible state in the game.

4.2 Computational experience

The model, as described in Section 4.1 was implemented in Python 2.7. There are two primary computational tasks associated with solving the model:

1. Building the matrices $\mathbf{P}_k,\ k = \text{extra}, 9, 8..., 1$

2. Solving the systems given in equation 4.1

As the matrices are large and sparse, we make use of Python's ScyPi sparse matrix functionality for both constructing the matrices and solving the linear equations.

Our decomposition approach has two computational advantages compared with solving the full model simultaneously, as in Equation 6. In terms of construction, the matrices $\mathbf{P}_k$ have an identical structure for each inning. Thus, once we have built $\mathbf{P}_{extra}$ we only have to substitute new probability values into the same matrix (to reflect a different pitcher, for example) to create $\mathbf{P}_k, k = 9, 8, ..., 1$. In addition, decomposing the problem allows for faster solution of linear equations, as computational complexity for solving Equation 4.1 grows faster than linearly in the dimension of $\mathbf{P}$.

Running the model on a MacBook Air® with a 1.4 Ghz Intel Core i5 processor, we found that, in total, building the full model and solving equation 6 takes about 33 seconds on average. In total, building the matrices and solving the linear system sequentially, as in Equation 4.1, takes just under 10 seconds.

Using an iterative solver, it takes an average of 10 seconds to find the solution for equation 6, compared to < 1 second to solve all ten systems of equations.

5 Model validation

There are three major components of our model which influence accuracy.

1. Structural – Does the hitting and baserunning model accurately reflect a real baseball game?

2. Batter vs. Pitcher matchups – Can we accurately predict the outcome of plate appearances with batters and pitchers of various skill levels?

3. Data and projections – Are the data and player projections used to inform player performance accurate?

We first consider Item 1, the accuracy of our model structure.

5.1 Structural evaluation

In this section, we explore whether our structural model of a baseball game makes sense by comparing our model to real data. By structural model, we mean the hitting events we have chosen to consider, and the baserunning model that dictates the movement of players. It's worth noting that the level of 'modeling control' in this structural sense is limited. For example, when an out is made, we *must* move to a state with one more out, or end the current half-inning. When the bottom of an inning is complete, we must move to a state in the top of the following inning.

Thus, from a modeling perspective, we are limited in decisions on what events to consider (i.e., should we include double plays?) and how runners move on the basepath (i.e., does a runner on second always score on a single?). We laid out our batting and baserunning model in detail in Sections 3.2 and 3.3, and in Appendix A. Our limited number of structural decisions means that, for validation purposes, we can limit ourselves to considering model performance in predicting runs over the course of a representative half-inning.

Baseball Prospectus [20] has data on the average runs scored in a half inning, based on the number of outs and the configuration of runners on the bases. We downloaded the data for 2015 based on a sample of $2,429$ games. Then, we used our model to predict expected runs within a half-inning using a team of average 2015 batters (assumed to be competing against average pitchers, i.e. we do not consider pitcher performance). We also compare our model's structural performance with earlier models using the Leftkowitz and D'Esopo baserunning model. A comparison of our model's predictions for expected runs and the true data is given in Table 9.

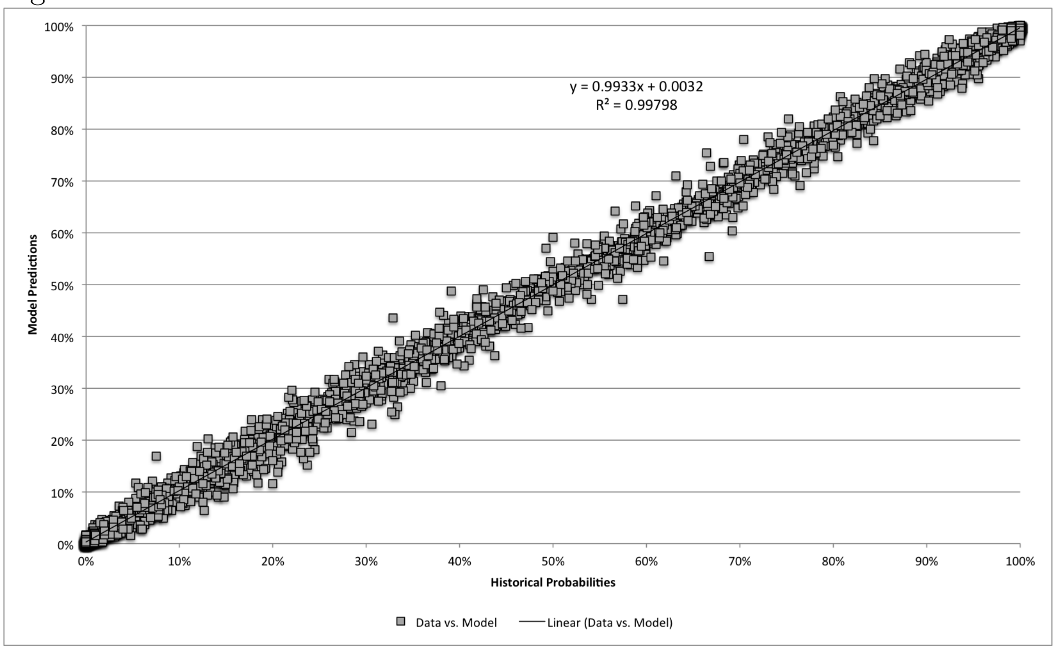
Our model does a good job at predicting runs within a half-inning, with no errors exceeding 1/10th of a run. We now demonstrate the benefit of our more complex baserunning model given in Section 3.3. We perform the same experiment as before, using the simple D'Esopo and Lefkowitz baserunning model (we will refer to it as simply LD) in place of our own. A comparison between the accuracy of our model and the simplified baserunning framework is given in Table 10.

Our model, equipped with the full baserunning model described in Section 3.3, outperforms the simpler baserunning model in terms of accuracy. The largest error with the simple model is .262 runs, compared to .095 for our model. On average, the absolute error between our model's predictions and the data is .031 runs, compared to .082 for the LD baserunning model. It is not hard to see how the LD model's limitations could cause inaccuracy. For example, without allowing scoring from third on outs, the LD model does not perform well in predicting runs scored when a runner is on third base with fewer than two outs.

We also validated our model against historical win probabilities. Historical win probabilities for the home team for every combination of inning, out, base configuration, and score were obtained from Greg Stoll's Win Expectancy Finder [13]. Data was obtained from all games between 1957 and 2014 (free courtesy of `retrosheet.com`). As before, we used the average hitter statistics over this time frame to inform our model, and predicted win probabilities. In Figure 1 we plot historical win percentages (for $n = 5,080$ states where at least 100 observations were available) against the win probabilities calculated from our model.

In general our model performed well at predicting win probabilities. Let p_i denote the win probability predicted for the home team in state i, and $\hat{p}_i$ be the true proportion of games a home team won when state i was reached during the game. Overall, the absolute average error between our win probability predictions and the percentages in the data (for states with

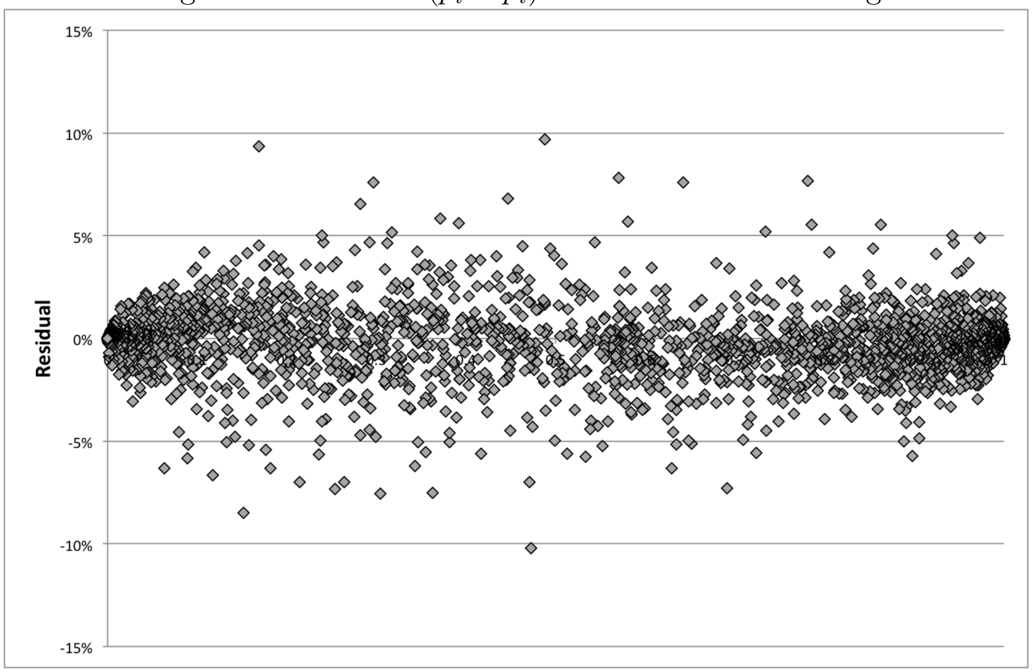
Figure 1: Historical Win Probabilities vs. Model Predicted Win Probabilities



at least 100 observations), $(\sum_i |\hat{p}_i - p_i|)/n$, was 1.12%. In 60% of the states considered $p_i \in \{\hat{p}_i \pm 1\%\}$. Delving further into the results, our model is, promisingly, more accurate on average in more commonly occurring states. In states with over 500 observations, the average absolute error between our prediction and the data, in terms of win-percentage, was .90%; in states with under 500 observations the average absolute error was 1.40%.

We also sought to verify that our model did not have notable biases in prediction errors. We looked at the residuals – the differences between historical win percentages and our model's projections, $\{\hat{p}_i - p_i, i = 1, ..., n\}$ – and searched for patterns. We first verified that we were not systematically overestimating (or underestimating) the home team win percentage from states in the bottom of the inning, and underestimating (or overestimating) win percentages in the top of the inning. If this were the case, residuals as a whole would mask the underlying flaw. We show the residuals for all states where the home team was batting in Figure 2.

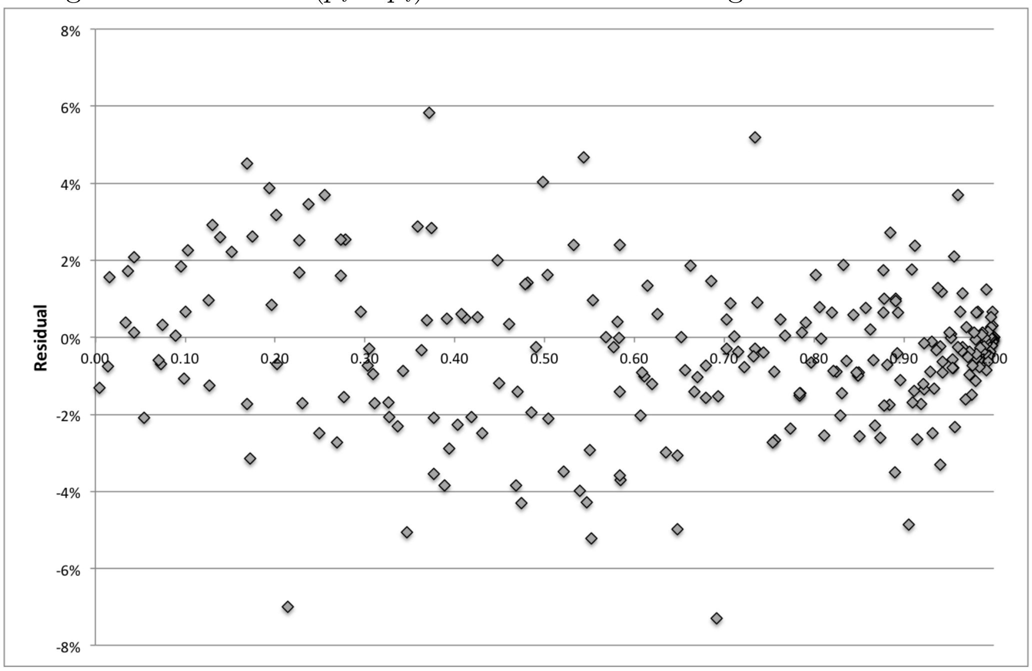
Figure 2: Residuals ($\hat{p}_i - p_i$) with home team batting



We see no pattern of overestimation or underestimation in the residuals; the average (not-absolute) residual is $-.2\%$. We can, however, continue to parse these results. Figures 3 and 4 show residuals for states with the home team batting and the bases loaded, and the home team batting, bases loaded, and 0 outs.

We see no clear patterns in the errors in either Figure 3 or Figure 4, with average residuals of $-.36\%$ and $-.62\%$ respectively. In general, we do not find

Figure 3: Residuals ($\hat{p_i} - p_i$) for home team batting with bases loaded

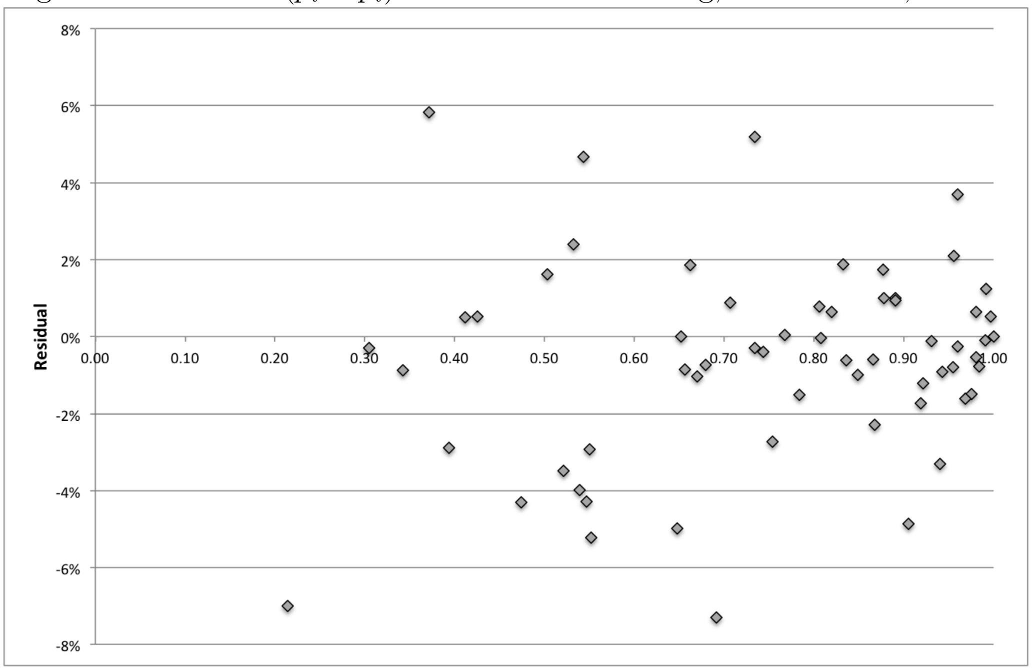


evidence that our model is systematically erroneous. Additional breakdowns yielded similar results.

Although our model does not appear to be systematically incorrect, as can be seen in Figure 1, this does not mean that it provides perfect predictions. Reiterating our earlier point, mis-prediction is most likely to occur if the structural baserunning model does not adequately reflect reality. In Table 11 we examine the average absolute prediction errors for various combinations of outs and base configurations.

As shown in Table 11, the model performs worse when there are many baserunners and a low number of outs. On the one hand, these states appear

Figure 4: Residuals ($\hat{p}_i - p_i$) for home team batting, bases loaded, one out



in real games less frequently which means that the historical data may be more noisy. On the other hand, higher errors in these states reflects known limitations of our model: we don't consider all possible baserunning outcomes, and we don't consider strategy (steals, bunts, intentional walks) which dictate state transitions when runners reach base. We also do not consider differences in hitter abilities with men on base; we use the same event probabilities when the bases are loaded as when the bases are empty. In the real world, hitters generally perform better as more runners are added to the basepath, because pitchers are forced to pitch from the stretch (to prevent steals) and pound the strikezone (to avoid walks). Adjusting hitting percentages by baserunner state could add greater fidelity.

In an absolute sense, however, our model performs reasonably well already across these scenarios with average errors lower than 3% in all cases. We now turn to our model's performance in predicting the results of individual games in the 2015 season.

5.2 2015 season prediction

In Section 5.1 we examined our model's performance against historical data obtained from many games. This did not, however, involve considering matchups between individual teams with unique lineups and pitchers. In order to test whether we could accurately predict outcomes of individual games we ran our model for $1,728$ games from the 2015 MLB season. As mentioned previously, predicting player performance in a given season (as projection systems such as Steamer attempt) is, in and of itself, an inexact science. It goes without saying that, if player projections are inaccurate, our model will be inaccurate. Thus, to separate the performance of our model with the performance of the projections, we used 2015 data for all players when predicting the games from the 2015 season. In other words, we assumed we knew exactly how well the players would perform on average.

In our sample of $2,328$ games, the home team was victorious 54.4% of the time. On average, our model predicted a win percentage for the home team of 52.6%. Per Betfirm.com [10], from 2010 to 2014 the home team won 53.7% of the time. Within our sample, the home and road teams scored an average of 4.33 and 4.12 runs per game, respectively. Our model predicted, on average, that the home team would score 4.16 runs per game, and the away team 3.88, suggesting that either our baserunning model is not adequately representing scoring, or that we are over-emphasizing pitching skill.

In order to measure the precision of our model, we separate the predictions into buckets. For example, a game where our model predicts the home team would win 53% of the time would be grouped in the $51\% - 59\%$ bucket. Then, for the set of games in each prediction bucket, we computed the actual win percentage for the home team. Ideally, we would hope that in games where the home team is predicted to win $51\% - 59\%$ of the time, they actually win a proportion of games within that range. Table 12 shows the results of this analysis.

As shown in Table 12, our model performs well, especially for buckets where predictions are made most frequently. The actual observed win percentages fell within each bucket in all but one case: when the model predicted a win percentage between 35 and 43 percent, the observed win percentage was 48.57%. However, it is worth noting that evaluating results using a bucketing approach is highly sensitive to the number of buckets, and the bucketing cutoff values. For example, when we take smaller buckets (with sufficient numbers of predictions) we can see that the model fidelity appears to degrade. See Table 13 for these results.

The results in Table 13 show that our model's predictions, which look very accurate when using large buckets, appear worse when placed under a microscope. Unlike the results in Table 12, we no longer have a clear upward trend in observed win percentages corresponding with increases in predicted probabilities. There are a few things that are going on here. First, we are looking at smaller sample sizes within buckets which could lead to more noisy real world proportions. Over small to medium samples of games, unexpected winning percentages can occur. For example, it is not uncommon for teams to outperform their hitting fundamentals over entire seasons by stringing together hits in high leverage situations.

But in addition, our model may not be accurate within this level of detail. The most likely culprit for the lack of fidelity is error in assessment of batter-pitcher matchups. As lineups in MLB are generally relatively homogeneous in talent, assessing win percentages precisely requires acurately evaluating the effect of pitching. As noted previously, the Log5 method gives only an approximation formula for quantifying matchup probabilities.

We made some attempt to more rigorously quantify the accuracy of our model. Unlike a logistic regression model, grouping by features (regressors) is not applicable for our results. When grouping by features is not possible, the Hosmer-Lemeshow (HL) test can be used to assess fit [6]. This test assesses model fidelity by comparing predicted successes (in our case, home team wins) within subgroups with observed percentages. As recommended, we split our predictions up by deciles. We then computed the HL statistic:

$$\sum_{g=1}^{10} \frac{(O_g - E_g)^2}{N_g \pi_g (1 - \pi_g)} \tag{13}$$

where O_g are the observed wins in group g, π_g is the average predicted win probabiliy for group g, N_g is the number of observations in group g, and $E_g = \pi_g N_g$. The distribution for the HL statistic approaches a chi-squared distribution with d.f. $g - 2$. The HL test states that if the probability of observing a value greater than or equal to the test statistic is less than .05 then the model does not fit the data. The test statistic for our model's predictions and the true 2015 data was .049, suggesting that our predictions don't fit the data well.

It is worth noting, however, that the HL statistic is notoriously sensitive to group size and group divisions. When testing our model's predictions, this issue came up. For example, when our predictions and the data were divided up into 12 equally sized groups, as opposed to 10, our HL statistic was greater than .05.

We also looked at classification rates based on our model's predictions. When our model gave a prediction above .5 for the home team's win probability, we categorized that game as a win, and other games we classified as losses. We then examined how many wins and losses we correctly predicted. This information is presented in Table 14.

Games we classified as home team wins were actually won by the home team 59.5% of the time. Games we classified as losses were won by the home team 46.7% of the time. In total, our model predicted the correct outcome 57.04% of the time – more accurate than predicting the home team to win every game. However, with our model most frequently predicting win probabilities within the 45% to 60% range (reflecting the relative parity among baseball teams), a model that is useful for baseball prediction model may not be useful for win/loss classification.

We also compared our predictions for expected runs with the actual score from our sample of games. Promisingly, our predictions matched up well with the observed data. The expected runs for home teams derived from our model and the true averages are shown in Table 15.

Again, all of the same caveats regarding bucketing apply. However, we see a clear trend in these results: when we predicted a higher expected run total, the average number of runs scored was indeed higher. This set of results gives us some confidence that, at the very least, our model is properly discerning lineup and pitching quality.

We also computed win probabilities for the same sample of $2,328$ games using 2015 projections to inform hitter and pitcher skill. Our model predicted on average that the home team would score 3.75 runs and the away team 3.49 runs. On average, our model predicted a win percentage of 52.8% for the home team. One immediately notices that expected runs drop considerably when using the projections, to well below the true scoring level in 2015. We recreate Table 12 for the projection-based results in Table 16.

Again, even using projections to inform the model, with the larger bucket size we appear to predict relatively accurately. At the very least, as our model's prediction for win probability increases, so does the observed win probability. Redoing the H-L test for the projection based predictions (again using deciles), we obtain a p-value for the test statistic of .214, suggesting that our model fits the data. In terms of classifying wins and losses, we compare predicted wins to actual wins in Table 17.

In this case, games we classified as a home team win were won by the home team 59% of the time. Games we classified as a home team loss were won by the home team 46.2% of the time. Overall, our model correctly predicts the outcome 57.1% of the time. Finally, we repeat our comparison of expected runs (generated by our model) and observed run scoring in Table 18.

Once again, we see that observed runs increase with our model's predictions for expected runs. However, we underestimate runs more significantly when we use projections. There are two potential reasons for this underestimation. First, projections tend to smooth out player performances towards the league average, which leads to less high-end run projections. Second, very poor spot players may not have projections available and, when projections were missing, we assumed a league average player.

The good news is that using projections appears to maintain the model's utility. Teams and hitters with better projections are predicted by the model to score more runs and win more frequently, and in the aggregate they do just that. To summarize all the results in this section: our model seems to be fairly capable at discerning at a high level which teams are better than others. However, further refinement (and perhaps related research into more accurate modeling of batter-pitcher matchups) might be necessary to increase fidelity further.

Table 9: 2014 half-inning runs scored vs. model predictions

Outs	Bases	Data	Model WP	Difference
0	—	0.479	0.455	0.024
0	–1	0.843	0.811	0.032
0	-2-	1.076	1.04	0.037
0	3–	1.305	1.317	-0.012
0	-21	1.44	1.35	0.09
0	3-1	1.668	1.634	0.034
0	32-	1.902	1.843	0.059
0	321	2.265	2.171	0.095
1	—	0.257	0.246	0.011
1	–1	0.5	0.482	0.018
1	-2-	0.649	0.63	0.019
1	3–	0.892	0.925	-0.033
1	-21	0.892	0.856	0.036
1	3-1	1.135	1.082	0.053
1	32-	1.283	1.305	-0.022
1	321	1.526	1.505	0.022
2	—	0.1	0.094	0.006
2	–1	0.22	0.205	0.016
2	-2-	0.315	0.294	0.021
2	3–	0.361	0.347	0.014
2	-21	0.436	0.417	0.019
2	3-1	0.481	0.463	0.019
2	32-	0.576	0.552	0.024
2	321	0.697	0.734	-0.037

Table 10: Expected Runs: LD baserunning vs. full baserunning

Outs	Bases	Data	Error LD	Error Full Baserunning
0	—	0.479	0.021	0.024
0	–1	0.843	0.023	0.032
0	-2-	1.076	0.033	0.037
0	3–	1.305	0.262	-0.012
0	-21	1.44	0.021	0.09
0	3-1	1.668	0.249	0.034
0	32-	1.902	0.26	0.059
0	321	2.265	0.165	0.095
1	—	0.257	0.008	0.011
1	–1	0.5	0.018	0.018
1	-2-	0.649	-0.043	0.019
1	3–	0.892	0.2	-0.033
1	-21	0.892	-0.043	0.036
1	3-1	1.135	0.2	0.053
1	32-	1.283	0.139	-0.022
1	321	1.526	0.058	0.022
2	—	0.1	0.005	0.006
2	–1	0.22	0.02	0.016
2	-2-	0.315	-0.033	0.021
2	3–	0.361	0.013	0.014
2	-21	0.436	-0.023	0.019
2	3-1	0.481	0.023	0.019
2	32-	0.576	-0.03	0.024
2	321	0.697	-0.078	-0.037

Table 11: Average absolute error (in win %) for out/base combinations

Base Config.	**Outs**	0	1	2
—		0.58	0.57	0.55
–1		0.89	0.76	0.67
-2-		1.08	1.08	0.92
3–		1.34	1.21	0.85
-21		2.49	1.23	1.17
3-1		1.54	1.45	1.2
32-		2.52	1.39	1.54
321		1.92	1.34	1.24

Table 12: Predicted vs. actual win percentages for 2015 season

Predicted win prob. (%)	Games	Avg. prediction (%)	Actual win %
<27	24	22.17	25
27 to 35	108	32.43	36.11
35 to 43	315	40.15	48.57
43 to 51	598	47.64	47.66
51 to 59	635	55.16	55.59
59 to 67	471	62.75	63.91
67 to 75	145	70.4	71.72
75+	32	78.58	78.13

Table 13: Predicted vs. actual win percentages (2015): small buckets

Predicted win prob. (%)	Games	Avg. prediction (%)	Actual win %
41 to 45	223	43.17	0.47
45 to 49	298	46.98	0.51
49 to 53	337	50.88	0.48
53 to 57	339	54.92	0.58
57 to 61	313	59.05	0.56
61 to 65	225	63.05	0.61

Table 14: W/L Classification (2015)

	Predict Win	Predict Loss
Win	830	436
Loss	564	498

Table 15: Expected runs vs. true average scoring (2015)

Exp. Runs	n	Avg. Exp. Runs.	True Avg. in sample
1-2	1	1.77	0
2-3	177	2.74	3.11
3-4	898	3.57	3.99
4-5	902	4.44	4.4
5-6	282	5.39	5.59
6+	68	6.7	5.96

Table 16: Predicted vs. actual win percentages (2015): using projections

Predicted win prob. (%)	Games	Avg. prediction (%)	Actual win %
<32	17	0.3	0.29
32 to 39	114	0.37	0.39
39 to 45	308	0.43	0.42
45 to 52	633	0.49	0.53
52 to 58	673	0.55	0.55
58 to 64	430	0.61	0.64
64 to 71	133	0.67	0.68
71+	19	0.73	0.79

Table 17: W/L Classification (2015)

	Predict Win	Predict Loss
Win	873	393
Loss	605	457

Table 18: Expected runs vs. true average scoring (2015)

Exp. Runs	n	Avg. Exp. Runs.	True Avg. in sample
1-2	1	1.99	0
2-3	266	2.76	3.82
3-4	1262	3.52	4.09
4-5	718	4.38	4.83
5-6	75	5.26	5.51
6+	4	7.9	8.5

6 Conclusion

We begin our concluding remarks by discussing some limitations of our model. While our baserunning model is more complex than some previous examples, we certainly don't consider a completely exhaustive list of potential transitions. For example, there are rare occasions when a runner scores from first base when a single is hit – in our model, this transition is impossible. Further consideration of all possible transitions could potentially increase model accuracy further, although it is worth noting that there are diminishing returns to adding more and more possible (low probability) transitions.

In addition, our model does not have the capability of explicitly considering who is on the basepath. For example, having Dee Gordon (one of the fastest players in baseball) on first base would considerably change the possible transition probabilities for runner advancement. However, detailed inclusion of unique baserunners, and their positions on the basepath, would require a massive expansion of the state space.

There are a few limitations in our approach for incorporating pitching. For one, we don't consider changes in pitching performance contingent on the configuration of runners on the base path. In reality, pitchers may pitch worse (better) with runners on (off) base. In addition, when we make predictions from the beginning of the game for expected runs, we have to project the number of innings the opposing starter will remain in the game. However, this duration is often contingent on the runs that are scored against that pitcher. In other words, if many runs are scored, the starter will exit the game earlier;

our model, however, unlike a simulation model, has no way of taking this into account. The result would be that our model might underestimate expected runs, as relief pitchers are often worse than the starters they replace.

The Log5 method used to generate matchup probabilities for a given batter and pitcher, is, in and of itself, an approximation. The total Log5 rule used in our model has been shown to match observed event frequencies over large samples of real plate appearances (see [3]). However, testing the rule is difficult in the context of individual plate appearances, where data for a specific pitcher and batter matchup are too few to draw inferences. Similarly, while projection systems are remarkably accurate in the aggregate, for many individual players they can be far off. Large projection errors for a single player can be enough to change our model's prediction for a given matchup.

Based on the results presented in this study, we can think of a number of possible directions for future work. Adding in-game strategic decision making to our model could add insight on the specific circumstances where managers should employ bunts or steals. We could also further investigate methods beyond Log5 for modeling batter-pitcher matchups.

Overall, our model showed promise in predicting the outcome of baseball games, but failed to completely capture the interaction between batters and pitchers. We look forward to continued honing of our tool in the future.

Appendices

Appendix A

Baserunning

In this appendix, we describe completely the baserunning model presented in Section 3.3.

1 Movement on hits and walks

When a hit occurs, there are various possible ways runners on the basepaths can advance. We enumerate these possibilities here.

1.1 Home runs, triples and walks

Movement on home runs and triples is simple. On a home run, regardless of base configuration, all runners score including the batter himself. Subsequently, the bases are left empty. On a triple, the batter reaches third and all runners previously on the basepath score.

If the batter draws a walk, the batter himself advances to first. If a baserunner is blocking his path to first, that runner advances to second, pushing a runner on second to third, etc.

1.2 Doubles

If a double is hit, the following movement will occur. Runners on second and third score. The batter himself reaches second. If there is a runner on first, the runner may score, or they may reach third. Baseball-Reference provides data for every player on how often that player scores from first base when a double is hit. For the 2016 season, we would use baserunning data from 2015.

For each team in a given matchup, we compute a team-specific probability of a player on first scoring on a double. In a given lineup, for player $i \in \{1, ..., 9\}$, let s_i be the number of times a player scores from first base when a double is hit. Let $\hat{s}_i$ be the total number of times player i was on first base when a double was hit. Then our team probability of scoring from first on a double is:

$$P(\text{Score from 1st on double}) = \frac{\sum_{i=1}^{9} s_i}{\sum_{i=1}^{9} \hat{s}_i}. \tag{1}$$

Let p_{dbl} be the probability the batter at the plate hits a double (as described in Section 3.2. Now, in states with a runner on first (i.e., with base configurations –1,-21,3-1,321), the probability the runner on first scores on a double is:

$$p_{dbl} \times P(\text{Score from 1st on Double}). \tag{2}$$

1.3 Singles

In our model, when a single is hit, with certainty, the runner reaches first base and a baserunner on third scores. If a runner is on either first or second (or both), then we have a number of possibilities. A runner on second may score or they may stop at third. A runner on first may advance to second or may advance to third. In both cases, frequency data analagous to those presented in Appendix A, Section 1.2 is available from Baseball-Reference for every player. Letting ss_i be the number of times player i scores from second on a single, and $\hat{ss}_i$ be the total number of times player i is on second when a single is hit, a team probability for scoring from second on a single is calculated as:

$$P(\text{Score from 2nd on single}) = \frac{\sum_{i=1}^{9} ss_i}{\sum_{i=1}^{9} \hat{ss}_i}. \tag{3}$$

A team probability of advancing to third from first on a single is computed analagously. Clearly, when runners are on first and second, the runner on first can only reach third on a single if the runner on second has scored. Thus, for these base states, we adjust the probability of advancement from first to third given that the runner on second scores upwards until the overall probability of a runner advancing from first to third matches the value in other states.

2 Movement on outs

When an out is made, runners can sometimes advance. This occurs when the ball is put into play; the runner can advance on a groundball out or 'tag up' and advance on a fly ball out. Of course, runners can only advance when an out does not end the half-inning.

2.1 Scoring from third base on an out

A runner on third can score on an out or a double play provided the half-inning does not end. A runner cannot score on a strikeout. Leaguewide frequencies for scoring from third on an in-play out (non-double play) and on double plays are available at Baseball-Reference. Letting OS be the total number of in-play, non-double play outs where a runner scored from third, and O the total number of in-play, non-double-play outs, with a runner on third, we have

$$P(\text{Runner Score from 3rd}|\text{Out in play}) = \frac{OS}{O}. \tag{4}$$

Similarly, we can compute the probability a runner will score from third on a double play. Note, this is only an applicable possibility when there are 0 outs in the inning and the configuration of runners on the bases is such that a double play is possible – i.e. there is a runner on first. Let p_{out} be the current batter's probability of making an out (any kind), p_K the probability of a strikeout, and p_{DP} the probability of a double play. Furthermore, let p_{o3} be the league probability P(Runner Score from 3rd|Out in play), and let p_{dp3} be the league probability P(Runner Score from 3rd|Dbl. Play). For states with a runner on third, we summarize the possible movement from third base on an out in Table 1 of Appendix A.

Table 1: Scoring from third on out, 0 outs in inning

Base Configs.	Event	Probability
3–, 32-	Runner scores on in-play out	$(p_{out} - p_K)(p_{o3})$
	Runner remains at third	$(p_{out} - p_K)(1 - p_{o3}) + p_K$
3-1,321	Runner scores on in-play out	$(p_{out} - p_K - p_{DP})(p_{o3})$
	Runner scores on DP	$(p_{DP})(p_{dp3})$
	Runner fails to score on DP**	$(p_{DP})(1 - p_{dp3})$
	Non-DP out, runner stays on 3rd	$p_K + (p_{out} - p_K - p_{DP})(1 - p_{o3})$

**If runner fails to score on a double play with the bases loaded, we assume runners were out at home and first.

Besides scoring on outs, we allow the lead baserunner to advance on outs from the base configurations –1, -2-, -21, and 3-1. For simplicity, we use a single league average probability for advancement given that the ball is put in play. We use the statistics in the book Beyond Batting Average [12], along with the distribution of fly-balls vs. ground balls for the previous season (available at Baseball-Reference), to calculate the advancement probability. The base value used in our model is .3. For greater fidelity, one could segment by the base configuration, and explicitly consider a hitter's fly-ball vs. ground-ball splits.

Appendix B

Matrices

1 Matrix construction

In this appendix, we give the details of the matrix structures described in Sections 3 and 4.

1.1 Expected runs

We now give the details for building the expected runs matrix. Let $\mathbf{P}_i$ be the submatrix associated with the ith player in the batting order. Then $\mathbf{P}_i$ is a 216×216 square matrix holding transitions between all innings, all out states, and all base configurations. We organize states in our submatrix heirarchically, with innings at the top level, outs at the second level, and base configurations at the lowest level. Let $\mathbf{H}_{j,k}$ be the submatrix associated with inning j with k outs which dictates transitions on a hit, and $\mathbf{O}_{j,k}$ be the submatrix associated with transitions on outs in inning j with k outs. Then, for $i, i = 1, ..., 9$

$$\mathbf{P}_i = \begin{bmatrix} \mathbf{H}_{1,0} & \mathbf{O}_{1,0} & \mathbf{O}_{1,0} & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & \mathbf{H}_{1,1} & \mathbf{O}_{1,1} & \mathbf{O}_{1,1} & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & \mathbf{H}_{1,2} & \mathbf{O}_{1,2} & 0 & \dots & \dots & \dots & 0 \\ \vdots & \vdots & 0 & \mathbf{H}_{2,0} & \mathbf{O}_{2,0} & \mathbf{O}_{2,0} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \dots & 0 \\ 0 & & & \dots & & & \mathbf{H}_{9,0} & \mathbf{O}_{9,0} & \mathbf{O}_{9,0} \\ 0 & & & \dots & & & & \mathbf{H}_{9,1} & \mathbf{O}_{9,1} \\ 0 & & & \dots & & & & & \mathbf{0}_{9,2} \end{bmatrix}. \quad (1)$$

Now, our full expected run matrix $\mathbf{P}$ can be written as:

$$\mathbf{P} = \begin{bmatrix} 0 & \mathbf{P}_1 & 0 & \dots & 0 \\ 0 & 0 & \mathbf{P}_2 & 0 & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \mathbf{P}_8 \\ \mathbf{P}_9 & 0 & \dots & \dots & 0 \end{bmatrix}. \tag{2}$$

$\mathbf{R}$, the one-step rewards matrix also described in Section 3 is identical in structure to $\mathbf{P}$. Entries in $\mathbf{P}$ are replaced with the runs scored associated with the corresponding transition in $\mathbf{P}$.

1.2 Win probability

As described in section 4, we decompose the the full matrix for the win probability model into matrices $\mathbf{P}_k$ for inning k with states on each axis ordered heirarchically by:

Home team player up (or due up)

$\downarrow$

Away team player up (or due up)

$\downarrow$

Top/bottom of inning

$\downarrow$

Outs

$\downarrow$

Base configuration

$\downarrow$

Home team lead (-8 to 8)

For player i on the away team, $i = 1, ..., 9$, we have two associated submatrices, one for the top of the inning $\mathbf{A}_i^{Top}$, and one for the bottom of the inning, $\mathbf{A}_i^{Bottom}$. Let $\mathbf{H}_j$ be the transitions to a new base configuration and home lead given after a hit, given that there are j outs in the inning, and $\mathbf{O}_j$ be the same transitions when an out is recorded. Then

$$\mathbf{A}_i^{Top} = \begin{bmatrix} \mathbf{H}_0 & \mathbf{O}_0 & \mathbf{O}_0 & 0 & 0 & 0 \\ 0 & \mathbf{H}_1 & \mathbf{O}_1 & \mathbf{O}_1 & 0 & 0 \\ 0 & 0 & \mathbf{H}_2 & \mathbf{O}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & \dots & & & \vdots \\ 0 & & \dots & & & 0 \end{bmatrix} \tag{3}$$

and

$$\mathbf{A}_i^{Bottom} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & \dots & & & 0 \\ 0 & & \dots & & & 0 \\ 0 & 0 & 0 & \mathbf{H}_0 & \mathbf{O}_0 & \mathbf{O}_0 \\ 0 & 0 & 0 & 0 & \mathbf{H}_1 & \mathbf{O}_1 \\ 0 & \dots & & 0 & 0 & \mathbf{H}_2 \end{bmatrix}. \tag{4}$$

Note that during the bottom of the inning, all transitions to a third out are transitions to the absorbing 'end-of-inning' state, $\triangle_k$. For the extra inning, we possibly add transitions back to the beginning of the inning on outs when the game is tied and the bottom of the inning is completed. Letting E_j represent these loop transitions when there are j outs. In this case, we have

$$\mathbf{A}_i^{Bottom} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & \dots & & & 0 \\ 0 & & \dots & & & 0 \\ 0 & 0 & 0 & \mathbf{H}_0 & \mathbf{O}_0 & \mathbf{O}_0 \\ \mathbf{E}_1 & 0 & 0 & 0 & \mathbf{H}_1 & \mathbf{O}_1 \\ \mathbf{E}_2 & 0 & \dots & 0 & 0 & \mathbf{H}_2 \end{bmatrix}. \tag{5}$$

Moving one step further up in our heirarchy, for each batter $p, p = 1, ...9$ on the home team we define submatrices $\mathbf{B}_p^{Top}$ and $\mathbf{B}_p^{Bottom}$. Then

$$\mathbf{B}_p^{Top} = \begin{bmatrix} 0 & \mathbf{A}_1^{Top} & 0 & \dots & 0 \\ 0 & 0 & \mathbf{A}_2^{Top} & 0 & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & & \mathbf{A}_8^{Top} \\ \mathbf{A}_9^{Top} & 0 & & \dots & 0 \end{bmatrix} \tag{6}$$

and similarly

$$\mathbf{B}_p^{Bottom} = \begin{bmatrix} \mathbf{A}_1^{Bottom} & 0 & \dots & 0 \\ 0 & \mathbf{A}_2^{Bottom} & 0 & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \mathbf{A}_9^{Bottom} \end{bmatrix}. \tag{7}$$

Then finally, the full matrix for inning k is:

$$\mathbf{P}_k = \begin{bmatrix} \mathbf{B}_1^{Top} & \mathbf{B}_1^{Bottom} & 0 & \dots & \dots & 0 \\ 0 & \mathbf{B}_2^{Top} & \mathbf{B}_2^{Bottom} & 0 & \dots & \vdots \\ \vdots & 0 & \mathbf{B}_3^{Top} & \mathbf{B}_3^{Bottom} & 0 & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & & 0 & \mathbf{B}_8^{Top} & \mathbf{B}_8^{Bottom} \\ \mathbf{B}_9^{Bottom} & 0 & & \dots & 0 & \mathbf{B}_9^{Top} \end{bmatrix}. \tag{8}$$

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