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AN UNBOUND UNIVERSE?*

J. RICHARD GOTT III
California Institute of Technology
JAMES E. GUNN†
Hale Observatories, California Institute of Technology, Carnegie Institution of Washington
DAVID N. SCHRAMM
The University of Texas at Austin
AND
BEATRICE M. TINSLEY
The University of Texas at Austin and at Dallas
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ABSTRACT

A variety of arguments strongly suggest that the density of the universe is no more than a tenth of the value required for closure. Loopholes in this reasoning may exist, but if so, they are primordial and invisible, or perhaps just black.

Subject heading: cosmology

Desist from thrusting out reasoning from your mind because of its disconcerting novelty. Weigh it, rather, with a discerning judgment. Then, if it seems to you true, give in. If it is false, gird yourself to oppose it. For the mind wants to discover by reasoning what exists in the infinity of space that lies out there, beyond the ramparts of this world. . . . Here, then, is my first point. In all dimensions alike, on this side or that, upward or downward through the universe, there is no end.
[LUCRETIUS]

I. PARAMETERS

The universe appears to be on a large scale isotropic, homogeneous, matter-dominated, and with negligible pressure (Harrison 1973). Thus it can be described by one of the Friedmann models of general relativity, which, if the cosmological constant is zero, are completely specified by two parameters. Assuming that this simple description of the universe is valid, we shall see how well these parameters are determined in the dim light of present evidence.

The distance and time scales are given by the first parameter, Hubble's constant,

$$H_0 = (R_{,t})_0/R_0 \, , \tag{1}$$

where $R(t)$ is the scale factor of the universe and $R_0 = R(t_0)$ its present value.

The second parameter gives the deceleration,

$$q_0 = -(R_{,tt})_0 R_0/(R_{,t})_0{}^2 \, , \tag{2}$$

and is related to the mean density of matter

$$\rho_0 = \frac{3H_0{}^2}{4\pi G} q_0 \, . \tag{3}$$

It is useful to define a critical density ρ_c and a dimensionless density parameter Ω, by

$$\rho_c = \frac{3H_0{}^2}{8\pi G} \, , \qquad \Omega = \frac{\rho_0}{\rho_c} = \frac{8\pi G\rho_0}{3H_0{}^2} \, , \tag{4}$$

so that

$$\Omega = 2q_0 \, .$$

The significance of these quantities is that the universe is closed (and may be oscillating) if $\Omega \geq 1$ ($q_0 \geq \frac{1}{2}$, $\rho \geq \rho_c$), while it is open and monotonically expanding if $\Omega \leq 1$ ($q_0 \leq \frac{1}{2}$, $\rho \leq \rho_c$).

We shall use the two parameters H_0 and Ω to describe the set of cosmological models, and attempt to see whether present data suggest $\Omega \geq 1$ or $\Omega \leq 1$. If we could trust the simplifying assumptions made at the beginning, this would show whether the universe will eventually contract or whether it will expand forever. Homogeneity and isotropy are good approximations to use in establishing the Friedmann equations, and departures are explicitly considered in the physical arguments used to evaluate Ω. If the only significant radiation density at present is the relict radiation ("3° K background"), then this is dynamically significant only at very early times ($t \lesssim 10^4$ yr), so any deviations from the time scales used below are negligible; of course its physical effects are considered in a number of the arguments. The (unlikely) possibility of other relativistic fluids contributing significant pressure is discussed in the last section. We assume generally that the cosmological constant Λ is zero, but

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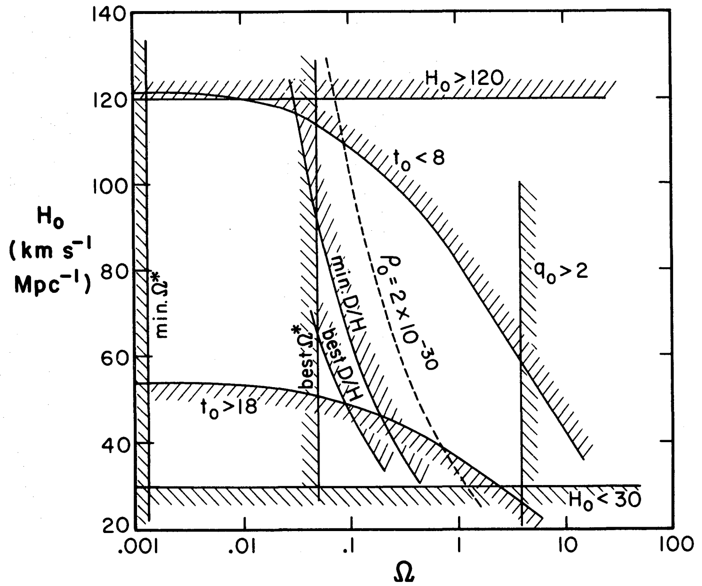

FIG. 1.—Constraints on the Hubble constant (H_0) and density parameter (Ω) that determine the Friedmann model if $\Lambda = 0$, explained in the following sections: the distance scale H_0 (§ II*a*); the age of the universe, t_0, shown in 10^9 yr (§ II*b*); the deceleration parameter q_0 (§ III*a*); minimal estimate of Ω^*, the contribution of galaxies to Ω (§ IV*a*); best estimate of Ω^* (§ IV*b*); upper limit to the present density ρ_0, from minimal estimate of the deuterium abundance and assuming standard big bang nucleosynthesis (§ V*a*); best upper limit to ρ_0 assuming standard big-bang synthesis of deuterium (§ V*a*); upper limit to ρ_0 (*dashed line*, in g cm^{-3}) from deuterium synthesis if the leptonic number may be nonzero.

some empirical constraints will be mentioned in § VI; if $\Lambda \neq 0$, define q_0 by equation (2), ρ_c and Ω by equation (4), but replace equation (3) by

$$\Lambda = 3H_0^2(\Omega/2 - q_0) . \quad (6)$$

II. CONSTRAINTS GIVEN BY t_0 AND H_0

a) The Extragalactic Distance Scale

Individual methods for determining the distance scale give disparate values of H_0, ranging from as low as 40 (+15, −13) (Branch and Patchett 1973) to 100 ± 10 (de Vaucouleurs 1972) and a high of 110 ± 10 (de Vaucouleurs 1972) using a slightly different distance scale, with the most extensive recent investigation yielding 55 ± 7 (Sandage 1972*a*), in units of km s^{-1} Mpc^{-1}. (See Heidmann 1972 for a recent review.) We consider as outside limits,

$$30 < H_0 < 120 \text{ km s}^{-1} \text{ Mpc}^{-1} .$$

Figure 1 represents the set of Friedmann models (with $\Lambda = 0$) in the (Ω, H_0)-plane, and these limits are shown, with shading in the forbidden area above and below.

b) The Age of the Universe

H_0 and Ω determine the age of the universe t_0 by the relations (Weinberg 1972)

$$t_0 = f(\Omega)/H_0 \quad (H_0 \text{ in inverse time units}) ,$$

where

$$f(\Omega) = \frac{\Omega}{2}(\Omega - 1)^{-3/2} \times \left[\cos^{-1}\left(\frac{2}{\Omega} - 1\right) - \frac{2}{\Omega}(\Omega - 1)^{1/2}\right] , \quad \Omega > 1 ,$$

$$f(\Omega) = 2/3 , \quad \Omega = 1 ,$$

$$f(\Omega) = (1 - \Omega)^{-1} - \frac{\Omega}{2}(1 - \Omega)^{-3/2}\cosh^{-1}\left(\frac{2}{\Omega} - 1\right) , \quad \Omega < 1 ,$$

$$\to 1 + \frac{\Omega}{2}\ln\Omega , \quad \Omega \to 0 .$$

Useful limits to t_0 come from the age of the elements and the age of stars. The ages of *r*-process nuclear chronometers show that nucleosynthesis started in the Galaxy between 6 and 15 billion years ago (Schramm 1974). This range is derived model-independently assuming that the longest-lived nucleochronometers (^{232}Th and ^{187}Re) have lifetimes greater than the mean age of the elements prior to formation of the solar system; pathological models can be constructed with ages up to 3×10^{13} yr if this assumption is dropped (Schramm and Wasserburg 1970), but such models are most unlikely to describe the evolution of the Galaxy. The quoted uncertainties lie in meteoritic abundance ratios, nucleosynthetic production ratios, and the time-dependence of *r*-process synthesis before the solar system formed. Fowler (1972) gives a more restricted range of 9 to 15 $\times$ 10^9 yr using a more restricted model-dependent approach; however, to be on the safe side the model-independent range of (6 to 15) $\times$ 10^9 (Schramm 1974) will be used here.

Outside limits for the ages of globular clusters are currently estimated as 8 and 18 billion years (Iben 1974; Rood 1973). The main uncertainties are the primordial abundance of ^{4}He and problems of semiconvection and interior mixing.

To set a safe upper limit to the age of the universe, we suppose that *r*-process material might not have been generated until 3 billion years after the big bang (either because of a delay in galaxy formation or because *r*-process nucleosynthesis occurs in long-lived stars, neither of which seems very likely). A more realistic limit for the delay in formation of globular clusters or elements would have been 1 billion years. Consistent and generous limits to the age of the universe are therefore

$$8 < t_0 < 18 \text{ billion years .}$$

The constraints imposed on (Ω, H_0) by these limits are plotted in figure 1. It is interesting that unless Ω is very large or very small, limits on H_0 are better determined by the ages than by the direct measurements of the extragalactic distance scale mentioned above. The arguments used below to set limits on ρ_0 and Ω will also provide significant constraints on H_0.

III. ESTIMATES FROM COSMOLOGICAL EXPANSION

a) *The Redshift-Magnitude Relation for Galaxies*

The value of q_0 can in principle be determined by departures from linearity of the (log *z*, apparent magnitude)-relation at large redshifts (Sandage 1961). First-ranked cluster galaxies give a number of formal values from different sets of data and corrections, which lead to the apparent value $q_{0a} = 1.0 \pm 1$ (2 σ) (Sandage 1973). The apparent value must be corrected for the systematic effects of luminosity evolution, which are as yet inadequately understood but are likely to give a true value of q_0 that is smaller by 0.4 to 1.2 (Tinsley 1972*a*, *b*; Rose and Tinsley 1974). There is now strong spectroscopic evidence against the extreme dominance of dwarf stars in the luminosity which would be necessary to render evolution negligible (Baldwin *et al.* 1973; Tinsley 1973*a*). Until the statistical uncertainties and systematic corrections involved in this difficult determination are improved, we feel safe in concluding only that $q_0 < 2.0$, i.e., $\Omega < 4.0$. This limit is shown in figure 1.

b) *The Redshift-Magnitude Relation for QSOs*

An apparent value of $q_0 \sim 1$ is indicated by the approximately linear (log *z*, magnitude)-relation obtained for the brightest QSOs in each redshift range (Bahcall and Hills 1973), for quasars with steep radio spectra (Setti and Woltjer 1973), and for quasars with flat radio spectra (Stannard 1973). However, these results should not be interpreted in terms of a cosmological model for a number of reasons, such as statistical uncertainty and other problems of analysis (Burbidge 1973), and possibly strong dependence on unknown evolutionary effects (Petrosian 1974).

c) *The Redshift-Diameter Relation for Galaxies*

The relation between redshift and isophotal diameters of first-ranked cluster galaxies (Sandage 1972*b*) appears to be consistent with $q_{0a} \sim 1$ as a formal solution, with considerable uncertainty due to dispersion. However, this value is subject to an evolutionary correction similar to that for the redshift-magnitude relation (Tinsley 1972*c*), so it gives no better determination of Ω than argument III*a*. An attempt to measure metric diameters (Baum 1972), which would be independent of luminosity evolution, has given $q_{0a} = 0.3$ (± 0.2); but because of the controversy as to whether true metric diameters have been measured, the interpretation of this formal value is not clear.

All three of these "classical" tests assume that the matter which is responsible for the deceleration of the expansion is smoothly distributed; gross departures from this condition yield completely spurious results.

d) *Uniformity of Expansion*

By showing that relatively local galaxies deviate negligibly from a linear Hubble flow, Sandage, Tammann, and Hardy (1972) concluded that, if the matter in the universe is distributed like visible galaxies, then $\Omega \ll 1$. A similar conclusion has been drawn by Gott and Gunn (1974) from a study of small groups of galaxies that represent density enhancements but make no appreciable perturbation of the Hubble flow. Defining Ω^* as the ratio of the smoothed-out density *in galaxies* to ρ_c, this method gives $\Omega^* = 0.04$, with about a factor 2 uncertainty. (This number is slightly dependent on the specific cosmological model.) These arguments cannot rule out a cosmologically significant density of matter distributed much *more uniformly* than visible galaxies (§ VII). Further estimates of Ω^* will be given below.

e) *Future Absorption*

Partridge (1973) has found that a microwave source radiating into free space has the same energy loss per unit time (to one part in 10^8) as it does radiating into a perfect local absorber. He suggests that according to the Wheeler-Feynman (1945) absorber theory this null result implies a complete absorption along the future light cone of his antenna. A closed universe is a complete absorber along the future light cone, but an open universe is not (Davies 1972). However, the conclusion that the universe is closed is not necessarily correct. If the universe is a perfect absorber along the *past* light cone of the antenna, then there is sufficient freedom in the initial conditions to produce the observed result. In terms of a correct analysis of the Wheeler-Feynman theory this experiment is expected to give a non-null result only when the universe is transparent along *both* the future and past light cones. Observation of the thermalized 3° K blackbody radiation shows that the universe is a complete absorber along the past light cone. Thus, the experiment provides us with absolutely no information about whether or not the universe is open or closed. The situation pertaining to this experiment is described exactly in Wheeler and Feynman (1945), p. 175.

IV. ESTIMATES OF THE PRESENT DENSITY

a) Lower Limit from Individual Galaxies

To derive an extreme lower limit to Ω^*, and thus to Ω, we adopt the luminosity density in the Local Supercluster derived by Shapiro (1971), divide by a factor 2.5 to allow for the density enhancement represented by this supercluster (Sandage *et al.* 1972), and adopt the very low estimates of mass-to-light ratios $(\mathfrak{M}/\mathfrak{L})_E = 15h$ for elliptical galaxies (Morton and Chevalier 1973) and $(\mathfrak{M}/\mathfrak{L})_S = 5h$ for spirals (Nordsieck 1973). (These ratios are in solar units, for *B* luminosities, and *h* is Hubble's constant in units of 100 km s^{-1} Mpc^{-1}.) The lower limit obtained, also shown in figure 1, is

$$\Omega \geq \Omega^* \geq 0.0013\ .$$

b) Ω^* from Group and Cluster Dynamics

Evidence has accumulated recently indicating that the $\mathfrak{M}/\mathfrak{L}$ ratios obtained from rotation-curve and velocity-dispersion data close to the centers of galaxies are too low. In particular, a massive spheroidal halo may be required for galaxies to avoid a barlike instability (Ostriker and Peebles 1973). Most galaxies may have such heavy halos, made of low-luminosity stars, extending far beyond the visible disk. Corroborating this is the fact that $\mathfrak{M}/\mathfrak{L}$ ratios obtained from double galaxies are consistently higher than those from rotation curves. Thus, we cannot treat the masses of galaxies as known, but must try to estimate virial masses. In this section, we discuss several estimates from small groups of galaxies, then some estimates from rich clusters, and conclude that $\Omega^* = 0.05 \pm 0.01$.

i) *Virial Discrepancies in the de Vaucouleurs (1974) Groups*

Using standard mass-to-light ratios [i.e., $(\mathfrak{M}/\mathfrak{L})_E = 50$, $(\mathfrak{M}/\mathfrak{L})_S = 7$], Rood, Rothman, and Turnrose (1970) calculated the expected mass in luminous galaxies, M_L, and compared it with the mass, M_{VT}, obtained by applying the virial theorem to each group. Many groups were found with $M_{VT}/M_L = 5$ to 50, and quite a few with the ratio as large as 400. Of particular interest are those with the extreme mass discrepancies, whose proportion of "missing mass" required to bind them is similar to the proportion required to close the universe. Gott, Wrixon, and Wannier (1973) have found a plausible explanation for the extreme virial discrepancies in a study of three such groups: the "groups" are nothing more than field galaxies that happen to be nearest neighbors, and their "velocity dispersions" are provided by the cosmological expansion. A histogram of the crossing times of the de Vaucouleurs groups shows two distinct peaks, one at a crossing time $\Delta t \approx 0.1H_0^{-1}$, and another broad peak centered precisely on $\Delta t \approx H_0^{-1}$. Presumably the first peak is associated with real bound groups and the second with field galaxy groups. The groups with the most extreme virial discrepancies have $\Delta t \sim H_0^{-1}$ as expected, while among the groups with $\Delta t < 0.1H_0^{-1}$ there are none with $M_{VT}/M_L > 50$. (On the field galaxy hypothesis we predict $M_{VT}/M_L \sim 200$ for those groups with $\Delta t \sim H_0^{-1}$, in good agreement with the observed values.) Thus the most extreme discrepancies can be explained in terms of the field galaxy hypothesis, leaving more moderate discrepancies reminiscent of those found in great clusters of galaxies (§ IV*b*[iv] below).

ii) *The Local Group*

Gunn (1974) has applied the virial theorem to our Galaxy and the Andromeda galaxy, obtaining $(\mathfrak{M}/\mathfrak{L}) \approx 125(10^{10}\ \text{y}/t_0)$, leading to $\Omega^* \approx 0.04$. This estimate includes all the mass associated with these galaxies out to a radius of 700 kpc, including presumably any heavy halos. Infall arguments (Gunn and Gott 1972) suggest that ≤ 20 percent of the mass in the Local Group is intergalactic gas, so it appears that most of the total mass is associated with the galaxies.

iii) *Statistical Surveys of the de Vaucouleurs Groups*

Geller and Peebles (1973) used a statistical analysis of the de Vaucouleurs groups to obtain $\mathfrak{M}/\mathfrak{L} \sim 300$ for the galaxies in the sample (mostly spirals). The value of Ω^* derived, 0.12, appears to be too high for two main reasons. First, every large density enhancement was treated as if it were a bound group in virial equilibrium. But there are surely some groups that represent density enhancements and yet have not perturbed the Hubble flow significantly and are still unbound (the Local Supercluster is an example, § III*d*). Treating such groups as bound leads to an overestimate of the virial mass. Second, the method used gives not $\langle M_{VT}\rangle$ but rather $\langle M_{VT}^2\rangle/\langle M_{VT}\rangle$ so that if galaxies have a range of $\mathfrak{M}/\mathfrak{L}$ ratios the mean will be overestimated. An independent estimate can be obtained (Gott and Gunn 1974) by applying the virial theorem to just those groups representing density enhancements of over 150 times that of the Local Supercluster, which are surely bound. Calculating $\langle M_{VT}\rangle$, one finds $\mathfrak{M}/\mathfrak{L} \sim 150$, yielding $\Omega^* = 0.06$.

iv) *Rich Clusters*

Application of the virial theorem to a number of rich clusters of galaxies (Rood *et al.* 1972; Oemler 1973) gives $\mathfrak{M}/\mathfrak{L} \sim 500h$ for the galaxies (mostly ellipticals). Moreover, the concentration of brighter galaxies toward the cluster center (Oemler 1973) and the radial distributions of density and velocity dispersion (Rood *et al.* 1972) show that the hidden matter is distributed like the visible galaxies. It appears, then, that the galaxies themselves bind the clusters, not diffuse intergalactic matter. This is of course consistent with the failure of all attempts to detect a significant density of neutral or ionized hydrogen in the Coma cluster (Gunn and Gott 1972; De Young and Roberts 1973; Davidson, Bowyer, and Welch 1973). Since giant ellipticals, which are the main component of great clusters, are underrepresented in a local sample of galaxies, the estimates of Ω^* from the nearby groups should be increased; the correction is small, since spirals dominate the mass density, and it has been included in the estimate quoted in § IV*b*[ii] above.

v) *Best Estimate of* Ω^*

Three independent estimates of Ω^* (§§ III, IV*b*[ii], and IV*b*[iii]) give consistent values, 0.05 ± 0.01. The value 0.05 is shown as a "best estimate lower limit" to Ω in figure 1. Note that this value of Ω^* includes all matter *within* clusters and groups, but it remains a lower limit to Ω until we can estimate the density of intercluster matter. *Galaxies themselves cannot close the universe.*

An important question arises as to the nature of the outlying hidden matter giving $\mathfrak{M}/\mathfrak{L} \sim 150h$ for spiral galaxies and $\sim 500h$ for ellipticals, 30 times greater than in the visible regions. Faint stars in extended halos (Ostriker and Peebles 1973) are an obvious candidate. Extreme red dwarfs with masses near or even below the limit for hydrogen burning ($\sim 0.1\ M_\odot$) would have $\mathfrak{M}/\mathfrak{L} \gtrsim 300$. They would be very difficult to detect by any means other than gravitational, except locally in the Galaxy where there is now abundant evidence that the mass density (even at this radius, where $\mathfrak{M}/\mathfrak{L} \sim 5$) is dominated by very late M dwarfs (Weistrop 1972; Gliese 1972; Jones 1972; Murray and Sanduleak 1972; Pesch 1972). Possibly most of the mass of the universe resides in this silent majority of small stars.

c) *Intercluster Gas: Direct Observational Limits*

The situation is thoroughly reviewed by Field (1972), so we give a brief summary. If QSO redshifts are cosmological, the limits on Lα absorption show that neutral hydrogen contributes $\Omega_{\rm H\,I} < 1.4 \times 10^{-7}\ h^{-1}$. Lack of 21-cm absorption in radio sources with small, certainly cosmological, redshifts, shows $\Omega_{\rm H\,I} < 0.08h^{-1}$. Limits on ionized hydrogen are somewhat ambiguous. *If* the soft X-ray background is due to thermal bremsstrahlung of intergalactic gas, then its temperature is 3×10^8 ° K and $C\Omega_{\rm H\,II}^2(H_0/50)^3 \sim 1$, where C is the clumpiness $\langle \rho_{\rm H\,II}^2 \rangle / \langle \rho_{\rm H\,II} \rangle^2$ (Cowsik and Kobetich 1972). But we have no independent estimates of C or temperature, and the X-ray background can be accounted for otherwise, so no useful direct limit on $\Omega_{\rm H\,II}$ can be derived. The upper limits on $\Omega_{\rm H\,I}$ are not significant, since an intergalactic medium could well have been fully ionized since an early epoch by protogalaxies (Tinsley 1973*b*), QSOs (Arons and Wingert 1972) or pre-white dwarf stars (Hills 1972).

d) *Intercluster Gas: Confinement of Radio Sources*

The largest separation of radio components of quasars decreases with redshift faster than expected for metric diameters in any Friedmann cosmology (Kellermann 1972), but at a rate consistent with ram-pressure confinement by a fairly dense intergalactic medium (Strom 1973). This cannot be interpreted as clear evidence for a closed universe, however, because there may be intrinsic evolution affecting quasar sizes as a function of cosmic epoch (van der Kruit 1973).

Although the dynamics of extended radio sources are consistent with ram-pressure confinement in regions with $\Omega \sim 1$, such a density is not necessary because the ram-pressure model is also consistent with lower densities, and other confinement mechanisms may be at work (De Young 1971; De Young and Burbidge 1973). Moreover, the same theory suggests a gas density 15 times greater within clusters of galaxies than outside (De Young 1972), which, if $\Omega = 1$, would violate the X-ray limits on intracluster gas.

e) *Intercluster Gas: Theoretical Limits*

Gunn and Gott (1972) found that unless $\Omega \lesssim 0.06$, so much gas would have fallen into the Coma cluster that its X-ray emission would be greater than observed. However, Lea (1973) has found that the pressure of the hot cluster gas may itself inhibit infall sufficiently to allow $\Omega = 1$, according to this criterion. There appear to be other strong theoretical arguments, nevertheless, against a closure density of ionized intercluster gas.

If the medium is supposed to be cold ($T_0 \lesssim 10^8$ ° K), the models must be pathological in the sense that the gas density must be essentially the same everywhere (§ III*d*), even though the galaxy density in structures like the Local Supercluster are several times that in the background, and the gas is too cold to resist the forces responsible for the clustering. The only way to satisfy the dynamical constraints (Hubble flow uniformity) is to put the clustering structures in ab initio; i.e., to arrange for more efficient galaxy formation in some large regions than in others with essentially the same density. Indeed, if $\Omega = 1$, structures like the Local Supercluster form from density perturbations $\delta\rho/\rho$ of less than 0.1 percent at decoupling, and the allowed density perturbations if the clustering is *imposed* are even smaller. On the other hand, *real* density fluctuations of order 0.5 percent are required to form the compact clusters like Coma, and these aggregates must either expel the gas or almost completely use it up in galaxy formation. The situation seems very contrived, but cannot be easily excluded.

If the medium is hot ($T_0 \gtrsim 10^8$ ° K), the material is stiff enough that it might not partake in very-large-scale clustering, but presumably would not have hindered galaxy formation, which must have occurred prior to its heating. The formation of clusters in such a scheme is very difficult, but can probably be managed with carefully chosen initial conditions. Worse objections are that there is no known energy source large enough to heat the gas to these temperatures, even if the heating is delayed as long as possible (Field 1972), and the fact that gas at the favored temperature of 3×10^8 ° K and $\Omega = 1$ exerts a pressure probably somewhat greater than the pressures in the interstellar media in galaxies, which are widely supposed to be in pressure equilibrium. It is a bit difficult to see how galaxies manage to ignore such external pressures.

f) Intercluster Dust

Reddening that could be ascribed to intergalactic grains in the Local Supercluster has possibly been detected (de Vaucouleurs, de Vaucouleurs, and Corwin 1972). It would be extremely difficult to detect dust over greater distances since its effects on the colors and luminosities of galaxies would oppose those due to evolution. Theoretically, of course, a significant intercluster density of condensable elements is extremely unlikely.

g) Other Intercluster Material

Press and Gunn (1973) have shown that the absence of predicted gravitational focusing effects suggests that black holes of galactic mass do not provide closure density for the universe. We are aware of no other *direct* constraints on the density of intercluster material. Possible forms in which it might have escaped detection are discussed in § VII.

None of the methods used for detecting intercluster matter give conclusive results, so we are left with the lower limit given by Ω^* as the only direct constraint on the present density of the universe.

V. THE ORIGIN OF DEUTERIUM

a) Standard Big-Bang Nucleosynthesis

The mass fraction of deuterium synthesized in a homogeneous, isotropic universe with present relict radiation temperature 2.7° K (the standard big bang) depends very strongly on ρ_0 (Wagoner 1973), which is related to (Ω, H_0) by equation (4) as illustrated in figure 1.

A direct measurement of the galactic interstellar deuterium abundance (Rogerson and York 1973) gives the number ratio $D/H = (1.4 \pm 0.2) \times 10^{-5}$, i.e., mass fraction $X_D = (2.0 \pm 0.3) \times 10^{-5}$.

A minimal constraint (upper limit) on the value of ρ_0 required to make this fraction in the big bang can be derived by supposing that the above value is twice too great, due to variations in sampling and experimental uncertainty, and that *none* of the primordial deuterium has been destroyed by astration (exposure to high temperatures in stellar envelopes and re-ejected) during galactic evolution. To synthesize $X_D \geq 1.0 \times 10^{-5}$ requires $\rho_0 < 8 \times 10^{-31}$ g cm^{-3} (Wagoner 1974), which is shown as an upper limit in figure 1.

For a more realistic estimate, we suppose that $X_D = 2 \times 10^{-5}$ is half the primordial value, adopting a destruction factor typical of models for galactic evolution (Audouze and Tinsley 1974) (the astration fraction is probably at least a few tenths [Tinsley 1974]). To synthesize $X_D = 4 \times 10^{-5}$ requires $\rho_0 = 4 \times 10^{-31}$ g cm^{-3}, which is also shown in figure 1. This is still an upper limit, because the D abundance could be greater if a significant fraction is tied up in interstellar molecules.

It can be seen that this low density is consistent with the value of Ω^* derived as a *lower* limit to Ω, and that, combined with the upper age limit, it defines a remarkably small range of values of Ω and H_0. The universe must be open by a wide margin ($0.05 < \Omega < 0.09$), and H_0 must lie between 49 and 65 km s^{-1} Mpc^{-1}. The best estimates of D/H and Ω^* together set a lower age limit of 14 billion years.

Using the minimal rather than the best estimates of Ω^* and D/H, one finds that H_0 may lie between 47 and 120, but that Ω still cannot exceed 0.2.

Because of the significance of this result, we next consider possible loopholes in the deuterium argument.

b) Galactic Production of Deuterium

Deuterium may possibly be synthesized in shocks in the envelopes of supernovae and/or supermassive stars (Hoyle and Fowler 1973; Colgate 1973). Unless it can be shown that there have been supermassive stars which mixed their explosion debris with ambient galactic gas, their possible contribution ("deuterium ex machina") cannot be taken seriously. The contribution of supernovae to interstellar deuterium depends largely on the poorly known energy per nucleon in the shock (Reeves 1973; Colgate 1974). However, boron and beryllium are also synthesized in the shock (Colgate 1974; Audouze and Truran 1973), and it has been shown that the resulting B/D and Be/D ratios are almost independent of shock strength and are much greater than the observed abundance ratios (Epstein, Schramm, and Arnett 1973). (The abundance of B is controversial. Values much greater than that inferred from ordinary chondrites have been proposed [Cameron, Colgate, and Grossman 1973]. Although these are apparently inconsistent with the interstellar upper limit [Audouze, Lequeux, and Reeves 1973], B could be trapped in grains; but since a great deal of B is not trapped in meteorites, that seems unlikely. The shocks [Epstein *et al.* 1973] are found to synthesize 5 times too much B even for the large proposed [Cameron *et al.* 1973] value.) This result means that even if *all* the observed B and Be are produced in supernova shocks (which is unlikely since interstellar production by cosmic rays appears adequate [Audouze and Tinsley 1974; Reeves *et al.* 1973]), the amount of D produced is still much less than that observed. In spite of uncertainties due to the detailed temperature profile in the shock, it seems in general difficult to have the conditions necessary to produce D from He without also overproducing Be and B from CNO (Epstein, Arnett, and Schramm 1974).

Deuterium is destroyed by astration more readily than Be or B, so the discrepancy is enhanced by effects of galactic evolution (Tinsley 1974). Altogether, we believe that the accompanying overproduction of B and Be makes supernovae an unlikely source of a significant fraction of the observed deuterium.

c) *Nonstandard Big Bangs*

i) *Nonzero Lepton Number*

If the lepton number is negative, the big bang can produce enough D at somewhat greater densities than in the standard case (Reeves 1972). To avoid overproduction of ^{4}He, the most negative allowed value is $\mathrm{Le} = -0.2$, where $\mathrm{Le} = [(n_\nu - n_{\bar{\nu}}) + (n_{e^-} - n_{e^+})]/n_\gamma$, and n_ν, $n_{\bar{\nu}}$, n_{e^-}, n_{e^+}, n_γ are the number densities of electron neutrinos, antineutrinos, electrons, positrons, and photons. The density limit is then $\rho_0 < 2 \times 10^{-30}$ g cm^{-3}, which is shown in figure 1. Even here, $\Omega < 1$ unless the age is as great as $t_0 = 20$ billion years. This type of model is considered further in § VII.

ii) *Hagedorn Equation of State*

If this equation of state is valid in the very early universe, it may be possible to make D but not ^{4}He (Wagoner 1974; Carlitz, Frautschi, and Nahm 1973). Some other major site of nucleosynthesis must then exist, such as supermassive stars, and we can conclude nothing about cosmological D production. However, there is no experimental evidence for the existence of the relatively long-lived massive superbaryons needed for the production of D without ^{4}He.

iii) *Inhomogeneities*

Enhanced deuterium production may occur if there are suitable inhomogeneities in the baryon number, temperature, or density. There is strong evidence against large-scale antimatter in the universe (Steigman 1972), so significant inhomogeneities in the baryon number are unlikely. It is difficult to construct temperature or density inhomogeneities in which a large abundance of deuterium would be produced, and which later would allow this to be mixed with the other matter.

Temperature variations are discussed by Harrison (1973). Epstein and Petrosian (private communication) have found that with density variations of about a factor 2, the big bang can produce the observed abundances of both D and ^{7}Li; however, they have found it impossible *both* to produce enough D in low-density regions *and* to have enough matter in high-density regions to close the universe with diffuse matter.

But a type of inhomogeneity can be contrived which would allow sufficient D production in a closed universe: if most of the mass in the universe is in isothermal density concentrations of very large amplitude, such that the mean density now exceeds the $\Omega = 1$ value but the tenuous "interclump" regions had a baryon density corresponding to $\rho_0 \leq 4 \times 10^{-31}$ g cm^{-3} nucleosynthesis would not be affected by most of the matter. The large clumps would collapse at a very early epoch, so the resulting deuterium-poor matter would not be found in galaxies.

We conclude that the deuterium argument for a very low density is valid *unless* matter is hidden in a collapsed form from a very early epoch indeed. Further comments on a universe dominated by collapsed objects are made in the last section.

VI. SUMMARY OF CONSTRAINTS

a) *A Consistent Open Universe*

The constraints on Ω given by each of the above arguments are illustrated in figure 2. Black shading shows the values allowed by the arguments we consider strongest, while values shaded in gray depend on rather more special theoretical interpretations of the data. Stars indicate allowed values of Ω^*, the contribution of galaxies to Ω. Although there are loopholes in every case, the strongest arguments taken together point to an open universe, with

$$\Omega \sim 0.06 \pm 0.02 .$$

It is remarkable that the best constraints come from very local data (ages of elements in meteorites, interstellar deuterium, dynamics of nearby aggregates of galaxies), not from observations of galaxies at large redshifts. The direct cosmological determinations of H_0 and q_0 are consistent with the local constraints, but are still subject to many statistical and systematic uncertainties.

By combining all the constraints in a straightforward manner it is possible to construct a self-consistent model for the universe: the best estimate for the deuterium made in the big bang ($X_\mathrm{D} = 4 \times 10^{-5}$) in turn yields a ^{4}He mass fraction of 0.24 (Wagoner 1973). The age of the globular clusters is extremely sensitive to the primordial ^{4}He abundance (Iben 1974), and a value of 0.24 yields globular cluster ages of $(15 \pm 1) \times 10^9$ yr (with the variation due to their different metal contents). This age is consistent with nucleochronology (Schramm 1974), and, combined with the value of ρ_0 determined by the deuterium abundance, yields a unique choice of Friedmann model, with $H_0 = 60$ km s^{-1} Mpc^{-1} and $\Omega = 0.06$.

b) *The Cosmological Constant*

If $\Lambda \neq 0$, the redshift-magnitude relation depends far more on q_0 than on Ω, and negative values of q_0 are possible, so the results discussed in § III*a* are consistent with $-1 < q_0 < 2$. Accepting the deuterium argument, we have $\Omega \ll 1$. From equation (6) we then find that

$$-2 < \Lambda/(3H_0^2) < +1 .$$

These limits are consistent with the existence of QSOs at a wide range of redshifts (Petrosian 1974). They are not very stringent. Until the determination of q_0 itself is more accurate, it remains possible, at least empirically, that there is a dynamically important cosmological constant. In this case, the age of the universe depends on three parameters. With the above restrictions on Λ, and with $0.05 < \Omega < 0.09$, $30 < H_0 < 120$ km s^{-1} Mpc^{-1}, the ages of all models lie between 8 and 18 $\times$ 10^9 yr. A variety of cosmological model types is allowed by this range of values of Λ and

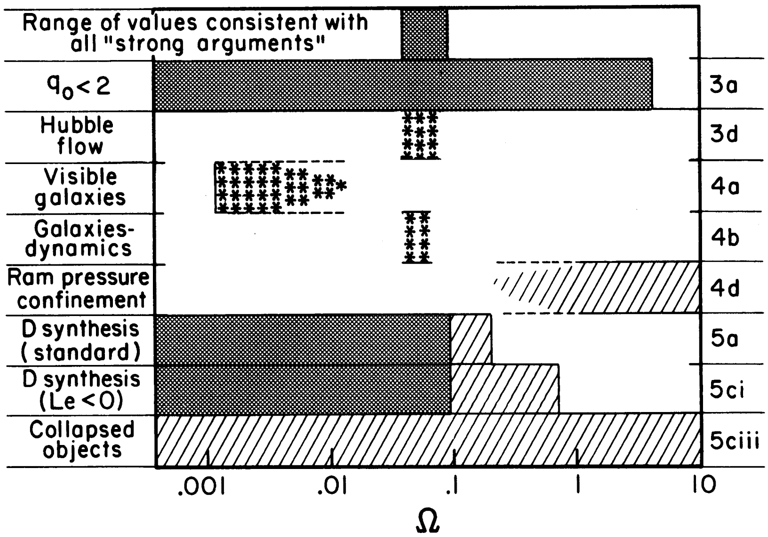

FIG. 2.—Summary of constraints on Ω, according to arguments indicated on the right and discussed in the sections given on the left. Dark shading shows values allowed by the arguments considered strongest, diagonal shading shows values allowed by weaker arguments, and stars show values of Ω* (the contribution of galaxies to Ω) allowed by the arguments indicated; the hazy boundaries indicate that the arguments provide only weakly determined limits. The top line gives the small range of values consistent with all strong arguments, including the well-determined lower bound to Ω* (line 4*a*).

Ω (Stabell and Refsdal 1966): monotonically expanding models, with positive, zero, or negative curvature, and oscillating models (i.e., collapsing in the future) with negative curvature; and, since q_0 may be slightly less than unity, within the uncertainties, models that expand from a finite radius are also possible. By contrast, if Λ is strictly zero, the smallness of Ω allows only monotonic models with negative curvature.

VII. A POSSIBLE CLOSED UNIVERSE?

Finally, let us discuss where the missing mass can be hiding if it is demanded on theological or other grounds that $\Omega \geq 1$; the possibilities are limited by the arguments presented here but are by no means absent.

a) Relativistic Fluids

There is, first, the possibility that the universe is not pressureless, but is dominated at the present epoch (and thus presumably at all previous epochs) by a relativistic fluid. Short-wavelength gravitational waves (Rees 1972) made at an early, violent, and highly nonequilibrium epoch or a degenerate sea of neutrinos or antineutrinos belonging to some (perhaps as yet unknown) lepton are obvious possibilities, but by no means exhaust the imagination.

Ruling out this case is not easily done at present. The age for all such pressure-dominated models (with $\Omega = 1$) is $\frac{1}{2}H_0^{-1}$, which is still allowed if $H \lesssim 65$. The most dramatic effect the dominating fluid would have is to speed up the expansion at a given radiation temperature in the early universe by a factor of about 100, since its present density is some 10^4 times the present photon density. A naive interpretation of the effect of this on nucleosynthesis indicates that too much helium and very much too much deuterium are produced (Peebles 1966, and § V*c*[i]). Since it is at least very difficult to destroy helium, this would seem decisive. The real situation, however, can be almost arbitrarily complicated, since the matter density in such a universe *must* have been very inhomogeneous at all epochs; the expansion is so accelerated that galaxies and clusters cannot grow from small perturbations. Estimates indicate that the regions from which galaxies form must have had baryon densities from 10 to 100 times the average; nucleosynthesis in such regions yields almost pure ^{4}He. It may be possible to circumvent this situation with anisotropy or a relativistic fluid which "participates" in the nucleosynthesis, such as electron neutrinos. The only other obvious observationally accessible effects are connected with the "freezing in" of large-scale irregularities like the Local Supercluster which accompanies these models. This might make itself felt in an increased graininess in the background light, for example (Shectman 1974), but no detailed calculations have been made to our knowledge.

b) Very-Low-Mass Particles

A variant on this idea is that the universe is closed by very-low-mass particles (neutrinos, say) (Cowsik and McClelland 1973). Since the dynamics of bound clusters can be understood on the basis of $\Omega^* \sim 0.05$ alone, the fluid must *not* partake of clustering, which demands that velocities of its particles be at least of the order of cluster virial velocities, $\sim 10^8$ cm s^{-1}. Since the fluid cannot interact with ordinary matter, it must have a temperature not higher than 2.7° K if it were ever in equilibrium. It is easy to show in this case that the present temperature is about 2.5×10^{-4}

M^{-1} °K, where M is the mass of the particles in electron volts (for fermions the figure is slightly smaller). The present velocity is of order 10^8 cm s^{-1} or greater only if the mass is less than about 0.06 eV; if the particles were once in equilibrium with their antiparticles and did not annihilate, the present number density must be of the same order as the present blackbody photon density, about 400 cm^{-3}, for a total density of order 4×10^{-32}, 100 times too small. (There could, of course, be 100 independent species!) The only reasonable alternative is that the particles *are* fermions, and are degenerate; hence the original suggestion (Cowsik and McClelland 1973) that the culprits were massive neutrinos. In this case, one can get away with masses of about 0.5 eV and number densities of the order of 10^4 cm^{-3}. This choice still results in a total density 100 times the photon density at the epoch of nucleosynthesis, and hence a tenfold increase in the relevant expansion rates and concomitant overproduction of helium. (There can, again, be *direct* influence on the nucleosynthesis if the particles are *electron* neutrinos.) The effect is thus similar to but less extreme than the relativistic case above—the situations clearly become more similar as the particle masses become smaller. The age of the universe, however, remains $\frac{2}{3}H_0^{-1}$ for all masses substantially in excess of 5×10^{-3} eV (at which mass the $\Omega = 1$ density is just relativistically degenerate now).

c) *Ionized Gas*

Models in which the intergalactic medium is ionized gas are the other class in which the closure density is provided by small particles. We have shown above (§ IV*e*) that these are very difficult to reconcile with any picture of the formation of galaxies and clusters.

d) *Condensed Objects*

Finally, there is the class of models in which the missing mass is not uniform, but is in condensed bodies (Peebles 1968; Hawking 1971). The problem here is not fundamentally different from the "cold gas" models. Even if the assembly of objects (let us call them black holes; their real nature does not matter very much unless one is concerned about the deuterium question) is given random near-relativistic velocities at the end of the radiation era, their random velocities would be very small at the epoch of cluster building (now), and it would seem very difficult to avoid incorporating them into clusters and thus letting them contribute to Ω^*. This could be prevented by making the black-hole density small where the galaxy density is high, and vice versa, but this is even more repugnant than in the case of the gas: the great clusters must essentially have *none*, yet must have been dense enough to collapse, while the Local Supercluster must have essentially the same density of them as outside. This requires that the clusters form from a perturbation in which the total density is *slightly* higher (again, ~0.5 percent) than average, but the density must be made up of a completely different mix (i.e., all galaxies or potential galaxy stuff) than the ambient medium. Application of infall arguments (Gunn and Gott 1972) can probably rule this case out completely since the outside stuff mixes in rather efficiently if $\Omega \sim 1$ (there are clearly no hydrodynamical effects to worry about here). The only way out would seem to be to make the collapsed objects massive enough and hence scarce enough that they do not, on average, get incorporated into clusters. This entails masses of order $10^{15}\,\mathfrak{M}_\odot$ or bigger, and average separations of 20 Mpc or more. The dynamical and optical effects (Press and Gunn 1973) of such monster objects can almost certainly not have gone overlooked.

It is interesting to note that in any model in which the density is in the form of lumps of about the mass of galaxies or larger, the classical angular-diameter and flux-redshift tests do not work at large redshifts, since the tubes through which we receive light from a distant object do not sample the mean mass density (Zel'dovich 1965; Gunn 1967).

e) *Conclusion*

The above cases may not exhaust the list of possibilities, but other examples are not readily apparent. All the above models exhibit some peculiarity which is in principle testable without reference to the classical cosmological tests. None are compatible with the observed deuterium abundance as essentially the primordial value, except perhaps a subset of the black-hole models, which have other, probably fatal, objections. It is very difficult, but probably not impossible, to avoid helium overproduction in the light-particle models, unless the light particles are degenerate electron neutrinos, in which case nucleosynthesis can be suppressed altogether.

Perhaps the most compelling evidence is *positive*. The helium abundance, which by now is generally agreed to be primordial, comes naturally from a universe dominated by ordinary matter, and most of the "exotic" ways to have $\Omega = 1$ throw this away. The arguments for the deuterium are not so conclusive, but together with the dynamical considerations, select from the matter-dominated models those of small Ω. It is possible to construct self-consistent open models in which (*a*) the correct amounts of helium and deuterium are produced cosmologically; (*b*) the mean density in the universe exceeds that known to be in galaxies; (*c*) the age of the universe is consistent with the ages of the elements and the globular cluster stars. Satisfaction of these criteria limits the self-consistent models to the rather narrow ranges $0.05 < \Omega < 0.09$ and $49 < H_0 < 65$ km s^{-1} Mpc^{-1}. The limits on Ω imply that the universe is open by a wide margin and that most of the mass of the universe is connected with galaxies. This obviates the need for a dense intergalactic medium and is consistent with our failure to detect such a dense medium. The limits on H_0 essentially offer a prediction of H_0 which can now be tested against the observations. It is thus particularly interesting that the two most recent independent estimates of H_0 both lie within the limits mentioned above. Sandage (1972*a*, 1974), using several variants of classical techniques, finds $H_0 = 55 \pm 5$ km s^{-1} Mpc^{-1}. Kirshner and Kwan (1974), by applying the Wesselink technique to supernovae, find $H_0 = 60 \pm 15$ km s^{-1} Mpc^{-1}; this determination is completely independent of *all* steps in the derivation of the classical distance scale. Open models thus explain simply and readily a wide variety of observations, from the abundance of deuterium to the value of the Hubble constant. If a closed model is to fit the observations, a number of additional (apparently ad hoc) processes must be called into play, which mimic in a complicated fashion the same results one would obtain from a simple open model.

The objections to closed universes are formidable but not fatal; a clear verdict is unfortunately not yet in, but the mood of the jury is perhaps becoming perceptible.

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J. Richard Gott III and James E. Gunn: Robinson 105-24, California Institute of Technology, Pasadena, CA 91109

David N. Schramm: Enrico Fermi Institute, University of Chicago, 5630 Ellis, Chicago, IL 60637

Beatrice M. Tinsley: The University of Texas at Austin, Austin, TX 78712; and The University of Texas at Dallas, Box 30365, Dallas, TX 75230