

**TIME-DOMAIN, STEADY-STATE TORQUE CALCULATION OF  
VOLTAGE-SOURCE, PULSE-WIDTH-MODULATED  
INVERTER FED INDUCTION MOTORS**

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**ABSTRACT**

Evaluation of torque pulsation associated with the harmonics of pulse width modulated (PWM) inverter-fed drives is important, particularly at low speed. This paper discusses an analytical method for the steady-state torque calculation of the voltage-source PWM inverter fed induction motors. Equations derived from the 1-2-0 coordinate system are used. A sample calculation is included for the illustration of practical application.

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## **DIGEST**

The pulse width modulated (PWM), inverter fed induction motors offer a number of advantages over other types of drives. The construction of induction motors is simple and robust with minimal maintenance requirements. The rapid development of advanced microcontrollers and fast-switching power electronic devices enables various optimizations of adjustable speed induction-motor drives. The currents or voltages of either voltage source or current source PWM inverter fed induction motors generally contain a certain amount of harmonics. In order to evaluate the torque pulsation [1,2] associated with harmonics of a PWM inverter-fed drive (especially at low speed) the technique for the steady-state torque calculation is important.

Apart from the initial switching on, pulse-width modulation involves a series of switching transients. The numerical method [3] can, of course, be used to investigate the steady-state torque solution. In a nonlinear situation, numerical method is the logical approach. However, for the steady-state solution that requires a full attenuation of the initial switching-on transient, additional computation time is required by the numerical method for a certain period to reach to the steady-state. Furthermore, the numerical processes introduce certain errors of various types; small computation steps are normally needed for accuracy. These small steps inevitably further prolong the computation time for the steady-state solution. Various computer-aided design programs, such as the PWLIB program developed by Bowes et. al. [5], have been reported for the analyses of the computation of PWM drives.

The frequency-domain technique, based on Fourier analysis, gives a clear picture of the harmonic content of the PWM waveform. However, for the instantaneous current and torque calculations, a large number of harmonics with their relative phase angles are required for accuracy and this can be time consuming. The time-domain technique generally takes less time for the evaluation of the current and torque [1,4,5,6,7].

There is a rich body of knowledge in the classical, induction-motor equations derived by Park [8], Stanley [9], Lyon [10], and others through various coordinate systems that were mainly used to solve problems associated with sinusoidal supplies [11-15]. This paper uses these classical equations for induction motors with a nonsinusoidal, continuously-switched supply.

The simple approach described in this paper shows the analytical logic to reach the steady state of the voltage-source PWM induction motors without a lengthy attenuation calculation. Using this method, the steady-state currents can be obtained analytically. Consequently, the fluxes associated with the currents and voltages can be calculated. Once the currents and fluxes are known, the torque can be obtained through the proper products of currents and fluxes.

The analysis is based on the following assumptions:

- a. The carrier wave is synchronized with the fundamental output wave. The negative half of voltage pulses in one cycle is the mirror image of the positive half,
- b. The rotor and load inertia is large enough to hold a constant speed. Experimental result of a six pulse, current-source drive [1] shows that even at low frequency (5 Hz), with a small oscillation of the speed, the calculated torque under this assumption is still acceptable.

- c. Saturated motor parameters are used, otherwise the motor magnetic circuit is considered to be linear.
- d. Core, friction, windage, and stray-load losses are not considered in the analysis.
- e. The commutation time is negligible to assume the heavy side unit-function nature of the voltage pulses.

This paper uses the 1-2-0 coordinate system [10, 14] and the per-unit values for the analysis. Since the value of coordinate 2 is always the conjugate complex number of the value of coordinate 1, attention can be drawn mainly to the values referring to coordinate 1. Resultant equations established in the available literature, such as the transformation from a, b, c values to 1, 2, 0 values, or vice versa [10, 14], and the definition of the operational impedances [14] are mentioned and used directly without repeating the derivations. A sample calculation is included for the illustration of practical application.

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