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# Essays on the Effective Integration of Risk Management with Operations Management Decisions

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# Essays on the Effective Integration of Risk Management with Operations Management Decisions

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# Dedication

To my Family: Hatice, Hilmi, Mustafa and Tuba Tanrisever,

for their love and support.

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# Essays on the Effective Integration of Risk Management with Operations Management Decisions

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In today's marketplace, firms' exposure to business uncertainties and risks are continuously increasing as they strive to meet dynamically changing customer needs under intensifying competitive pressures. Consequently, modern supply chains are continuously evolving to effectively manage these uncertainties and the allied risks through both operational and financial hedging strategies. In practice, firms extensively use operational hedging strategies such as operational flexibility, capacity flexibility, postponement, multi-sourcing, supplier diversification, component commonality, substitutability, transshipments and holding excess stocks as operational means for risk management. On the other hand, financial hedging which involves buying and selling financial instruments, carrying large cash reserves or adopting conservative financial policies, changes the cash flow stream of the firms and may help to reduce the firms exposure to business risks and uncertainties. Overall, in this dissertation we explore how risk management can be integrated with operating decisions so as to improve the firm value creating more wealth for the shareholders.

In the first essay, we focus on capacity flexibility as a means of operational hedging for risk management in an MTO production environment under demand uncertainty. We demonstrate that capacity flexibility may not only be used to hedge against the demand uncertainty, but may also be employed to effectively protect against possible suboptimal operating decisions in the future. In the second essay, we focus on operational hedging in financially constrained startup firms when making short-term production and long-term investment decisions. We provide an analytical characterization of the optimal investment and operating decisions and analyze the impact of market parameters on the operations of the firm. Our findings highlight an interesting operational hedging behavior between the process investment decisions and the short-term production commitments of the firm when they are faced with financial constraints.

Our third essay focuses on the value of integrated financial risk management activities by publicly traded established firms under the risk of incurring financial distress cost. Different from the existing operations management literature, we study the risk management by a public corporation within the value framework of finance; hence our findings do not require any specific assumptions about the investors' utility functions. Moreover, we contribute to the operations management research by examining the impact of the costs of financial distress on hedging and operating plans of the firm. Overall, in this dissertation, we examine the effective integration of operational and financial risk management so as to improve the firm value creating more wealth for the shareholders.

# Table of Contents

# List of Tables

# List of Figures

# Chapter 1

## Executive Summary

Uncertainty is an integral part of most real world problems in the domain of operations management. Today, as the product markets become more and more competitive, effective management of risk associated with operational uncertainties becomes a critical factor for the economic viability of the firms. Hence, modern supply chains are continuously evolving to effectively manage their uncertainties and the allied risks through both operational and financial hedging strategies.

In this dissertation, we adopt the definition of hedging by Van Mieghem (2003): "*Mitigating risk, or hedging, involves taking counterbalancing actions so that, loosely speaking, the future value varies less over the possible states of nature*". In this respect, financial hedging refers to trading financial instruments such as options, futures or other financial derivatives to counterbalance other actions. On the other hand, operational hedging refers to mitigating risk by counterbalancing actions without using financial instruments. In practice, as discussed by Van Mieghem (2003) operational hedging may take various forms such as process flexibility, operational flexibility, dual-sourcing, component commonality, substitutability, transshipments, holding safety stocks and having warranty guarantees.

The second chapter of my dissertation specifically focuses on the value of flexible capacity, in an MTO production environment, to hedge against operational risks associated with demand uncertainty. Flexible capacity is an essential element of most MTO production environments in which it is crucial to quickly respond and satisfy diverse customer demand. Further, given the high capital investment required for flexible production equipment, deciding on an adequate level of capacity flexibility is an important strategic problem for the firms.

Motivated by a problem in computer manufacturing, we study a realistic multi-period capacity management problem where we explicitly distinguish between two types of capacity flexibility: (1) operational flexibility and (2) process flexibility. The goal of this chapter is to shed light on the connection between the value of flexibility and the operating decisions of a firm. We demonstrate that process flexibility may not only be used to hedge against demand uncertainty, but may also be employed to protect against possible suboptimal operating decisions in the future. In particular, suboptimal myopic operating policies, which are common in practice, can be hedged through process flexibility decisions prior to the beginning of the sales season. In addition, we show that the value of process flexibility depends on the operating policies as well as the length of the planning horizon. Specifically, the value of process flexibility increases with the length of the planning horizon, under optimal operating policies. However, this result is reversed if a myopic operating policy is adopted.

In the third chapter, we examine the operational hedging strategies for a financially constrained startup firm when making short-term production and long-term process investment decisions. Unlike their established counterparts, startup firms are more subject to uncertainties in firm and market characteristics, as well as in their return on investments. Further, startups are more restricted by debt and other financial considerations. Hence, they should allocate their limited amount of cash funds between operations and R&D very cautiously to avoid bankruptcy during the early phases of development. While short-term production is necessary to maintain the firm's cash flows and to keep up with the cash outflows, long-term investment is vital for the survival of the firm in the future. Therefore, whether to focus on short-term production to maximize immediate profits instead of investing in R&D is a dilemma faced by many startups early in their lifecycle.

We study this dilemma by examining the production quantity and cost-reducing R&D investment decisions in a competitive market, using a two period model. This research highlights an interesting operational hedging behavior between the process investment decisions and the short-term production commitments of the firm. That is, a change in the investment policy of the firm is always accompanied by a counter-action in the production decisions. We show that aggressive (conservative) investment plans are hedged through aggressive (conservative) production decisions.

The fourth chapter of my dissertation explores the joint financial hedging and operating decisions of a shareholder-value maximizing firms in commodity markets. Although our research is motivated by the flour milling industry, our findings can be easily generalized to other commodity processor firms which are exposed to fluctuations in commodity prices. As it is well known in the finance literature, in the absence of frictions, engaging in financial hedging is a neutral proposition. That is, it should not affect the optimal production plan, and it does not create value for firm's shareholders. However, when the firm faces capital market frictions, such as financial distress costs, bankruptcy costs, taxes and agency issues, financial hedging can contribute to shareholder-wealth creation.

We illustrate how financial hedging can be utilized to enhance firm value under the risk of incurring costly financial distress which is a common form of capital market imperfection. The risk of incurring costly financial distress changes the optimal operating decisions of the firm, and induces more conservative production decisions with respect to the first-best production levels. We first quantify this under-production problem, and then illustrate how financial futures can be used to mitigate it and generate more wealth for the shareholders. In particular, we show that a coordinated financial hedging and operating plan contributes to shareholder-wealth creation, (1) by reducing the firm's exposure to financial distress risk and mitigating the corresponding costs, and (2) by enabling the firm to operate at a higher level of output.

This research contributes to the existing operations management literature in two ways. First, we study the risk management decisions of a public corporation within the value framework of finance; hence our findings do not require any specific assumptions about the investors' utility functions. Second, we explore the impact of the costs of financial distress on hedging and operating plans

Overall, in this dissertation we examine the value of integrated risk and operations management decisions by firms under different economic and financial conditions. In Chapter 5, we summarize our results together with important managerial implications and point to directions for future research.

# Chapter 2

# Managing Capacity Flexibility in Make-to-Order Production Environments

## 2.1. INTRODUCTION

As the competition in high-tech markets becomes more and more intense, product differentiation and customization becomes a top priority for many companies. For instance, today, most of the companies in the computer manufacturing industry allow their customers to customize nearly every component of their products. While product customization is a must for strategic competition in these markets, increased levels of customization also come with their own operational-level challenges.

This chapter studies such an operational challenge recently faced by a high-tech make-to-order manufacturing firm: managing multiple flexible production lines to produce multiple product families so as to minimize the total operating cost (including the cost of managing process flexibility and the backlogged demand), over multiple production periods where the demand for the products is highly uncertain. The firm which motivated this research is a manufacturer of electronic devices that consist of a single chassis and a set of parts assembled on it. Products are grouped into families depending on the chassis that they are built onto and each family requires a different set of parts. While this work was motivated by a firm in the electronics industry, many of the same issues studied here are also faced by make-to-order manufacturing firms in other industries.

On the demand side, customers are allowed to choose almost every part of their products. In particular, a customer order includes a selection of chassis type and a set of parts that are available for that chassis. Therefore, it is possible to start the final assembly of a product only after a firm customer order is received. On the supply side, customer orders are produced on multiple production lines which may be adjusted to manufacture any set of product families prior to the start of production. The adjustments are time consuming and costly; hence it is not practical to change them once the production is started. The same set of assignments is preserved over multiple production periods, until a significant change in the demand pattern is observed (e.g., when a new product is launched or a significant price promotion is announced).

Prior to the start of production, the firm decides a product-to-line assignment which we will refer to as the *process flexibility* of the firm. Process flexibility refers to the ability of a firm to produce multiple products on multiple production facilities or lines, as described by the process-flexibility literature (see Jordan and Graves 1995). Note that each line may produce multiple products and each product may be produced on multiple lines.

As greater process flexibility is adopted by the firm, i.e., as more products are assigned to more lines, the firm's ability to match capacity with demand improves. However, process flexibility comes at a cost, in particular, assigning product $i$ to line $j$ involves a certain cost depending on $i$ and $j$ due to: (1) pre-positioning the related parts and chassis inventory next to the production line, (2) computer programming and setup, and (3) dedicating labor and material handling equipment to produce family $i$ on line $j$ during the planning horizon. Hence each assignment increases the process flexibility of the firm at a certain cost.

Once the process flexibility decision is made, operating the system by allocating capacity to demand is another practical challenge in a multi-period planning horizon. In particular, *operational flexibility* of the firm, i.e., the ability to dynamically change capacity allocations among different product families over time, plays a critical role on the selection of capacity allocations. Further, operating decisions also affect the choice of process flexibility *ex ante.*

Regarding the operational flexibility of the firm two basic modeling approaches are considered: (1) a Dynamic Allocation Model (DAM), where the allocation decisions are made after observing the demand at the beginning of each production period and (2) a Fixed Allocation Model (FAM), where the allocation decisions are made at the beginning of the planning horizon together with the assignment decisions and these decisions do not change in response to demand realizations from period to period.

The sequence of decisions for our firm is as follows (see Figure 2.1): First, based on the forecasted demand, the firm commits to a process flexibility configuration prior to the start of production and incurs a certain flexibility cost. Next, at the beginning of every production period $t$, demand is realized and the production capacity is allocated to meet that demand and the existing backlog subject to the process flexibility configuration and the operational flexibility of the firm. Unmet demand from period $t$ is backlogged and carried to the next production period. The overall objective (under both DAM and FAM) is to minimize the total operating cost over the planning horizon, which includes the cost of process flexibility and the expected cost of total backlog.

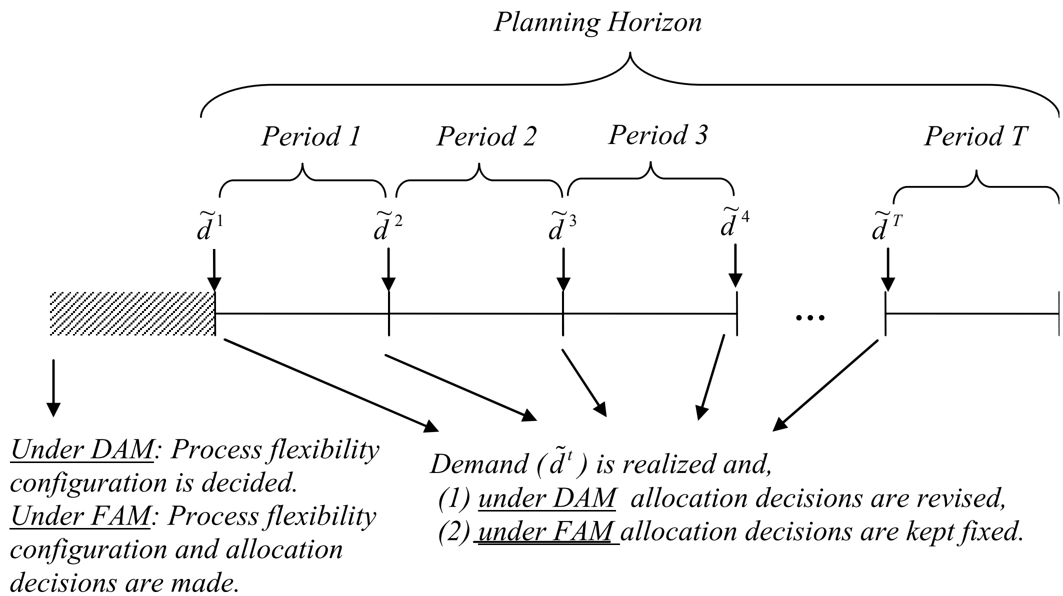



Figure 2.1: A Graphical Representation of DAM and FAM

As the sequence of decisions suggests, we model DAM as a multistage stochastic integer program with binary decisions only in the first stage and FAM is modeled as a single stage stochastic integer program. We also provide effective procedures to solve our mathematical models. Regarding DAM we assume that the product demands during the production periods are independent and identically distributed with a known distribution. The independence assumption is key in our development but the identically-distributed assumption is easily relaxed.

Note that FAM has no operational flexibility since each line is allocated a fixed time to produce a certain family, while DAM has full operational flexibility. Fixing allocation decisions may have significant operational benefits including: reduced scheduling problems, operational standardization and increased efficiency (Li and Tirupati 1997). However, in our setting, quantifying these benefits is not straightforward since it is not easy to incorporate them in a mathematical decision model. In practice, our firm employs an operating policy that is close to FAM (allocations are rarely changed in response to demand realizations). Therefore in this chapter, FAM will serve as a benchmark to evaluate the potential benefits of operational flexibility observed under DAM.

We provide two sets of computational analyses. First, we quantify the potential benefits of operational flexibility by comparing the performance of DAM and FAM, in the presence of demand uncertainty under optimal process flexibility decisions. We demonstrate that operational flexibility is most beneficial when demand is well balanced with capacity and the variability of the demand is high.

Second, we investigate the value of process flexibility in a multi-period production framework under different dynamic operating policies. For this purpose we introduce the myopic version of DAM as a third operating model (MDAM) where the firm may change the allocations at the beginning of each period, but does so without taking the impact on future periods into account. By comparing the value of process flexibility under DAM and MDAM, we show that process flexibility may not only be used to hedge against the demand uncertainty, but may also be employed to protect against possible suboptimal operating decisions in the future. In particular, firms adopting myopic operating policies need to form denser process flexibility chains prior to the beginning of the production. We further investigate the value of process flexibility as the length of the planning horizon changes.

The rest of the chapter is configured as follows: In Section 2.2, we provide a brief review of the related literature and outline our contributions. In Section 2.3, FAM is explained in detail and a solution algorithm, based on Kelley's (1960) cutting-plane method, is presented. Section 2.4 explains the DAM and presents a sampling-based decomposition method to find a near-optimal solution. Section 2.5.1 presents a computational study of the benefits of operational flexibility under various circumstances by comparing the performance of FAM and DAM. Section 2.5.2 is dedicated to the analysis of the value of process flexibility. We conclude with a brief discussion of results and future research directions in Section 2.6.

## 2.2. LITERATURE REVIEW

Our flexible assembly line management problem is most closely related to the manufacturing-flexibility and production-scheduling literatures. The literature on manufacturing flexibility is extensive. Sethi and Sethi (1990) provide a comprehensive literature review of manufacturing flexibility starting from the 1920s. More recent literature reviews can be found in Zhang et al. (2003) and De Toni and Tonchia (1998). Here, our review of the manufacturing-flexibility literature is specifically focused on capacity flexibility.

The capacity flexibility literature is also extensive, but the related literature can be categorized in two main groups. The first stream of research focuses on investment in plants or equipment that are dedicated versus totally flexible. The second stream of research allows for intermediate levels of flexibility, i.e., the capacity may be adjusted to any level of flexibility from dedicated- to fully-flexible production. The former stream includes Fine and Freund (1990), Van Mieghem (1998), Li and Tirupati (1994, 1995, 1997) and Netessine et al. (2002).

Fine and Freund (1990) consider a firm which has the option to invest in both product-dedicated capacity and flexible capacity. The latter is able to produce all kinds of products, but the firm has to make its investment decision prior to observing the demand. They model the problem as a two-stage stochastic program in which the capacity investment decisions are made in the first stage and the production decisions are made in the second stage. Their objective includes total revenues less operating and capacity investment costs. They investigate the tradeoff between the cost of investing in flexible capacity and the ability to respond to uncertain demand.

Van Mieghem (1998) shows the impact of price- and cost-mix differentials when deciding between investing in flexible and dedicated capacity. Like Fine and Freund (1990), he considers a model in which investment decisions are made at the beginning of the time horizon. Van Mieghem and Rudi (2002) extend Van Mieghem (1998) to a dynamic setting with multiple products, multiple processing and storage points, which is called a newsvendor network.

Li and Tirupati (1994) also address investment in flexible and non-flexible technology. In contrast to the previous papers, they consider a multi-period problem with deterministic demand over time. The objective is to minimize the total discounted cost, which includes the cost of technology investment and the operating cost over the planning horizon. Heuristics are provided to generate good investment strategies. Li and Tirupati (1995) consider the same problem with two products and stochastic demand. However, in this setting unlike Van Mieghem and Fine and Freund, demand uncertainty is addressed by specifying a target service level. They argue that the multi-period version of the problem is not tractable and provide a solution method for the single-period problem. Li and Tirupati (1997) extend their previous two papers by explicitly considering two kinds of operating policies, which refer to the allocation of flexible capacity among different products arising from: (1) a Static Allocation Model (SAM) and (2) a Dynamic Allocation Model (DAM). In SAM, the allocation of flexible capacity to product lines is made at the beginning of the planning horizon. In DAM they permit dynamic allocations of flexible capacity in each period after demand realizations are observed. In both models the objective is to minimize the investment cost subject to service level constraints. For SAM, an exact exponential time algorithm is provided to solve the model. Assuming a proportional allocation rule, the DAM is approximated with a single period model and a heuristic is provided to generate good solutions under special conditions.

The literature mentioned above is similar to our work in the sense that the cost and benefits of flexibility are considered explicitly. However, they only consider two types of capacity: fully flexible and non-flexible capacity. In our case, the capacity (i.e., the assembly lines) can be adjusted to any intermediate level of flexibility at a certain cost. In addition, except for Li and Tirupati (1994) who model demand as being deterministic, the above papers only consider single period models which may not be sufficiently realistic for many practical situations as mentioned by Van Mieghem (1998): "… our approach may be too highly stylized to serve as a practical decision support system, which may need to consider more complex models for which one may need to resort to numerical methods …". With its multi-period structure and stochastic demand our models are arguably more realistic for practical purposes.

The second stream of capacity-flexibility literature, which originates with Jordan and Graves (1995), allows for choosing among resources with an intermediate level of flexibility. This line of research also includes Graves and Jordan (1991), Graves and Tomlin (2003), Garavelli (2003), and Katok et al. (2003).

Jordan and Graves (1995) specifically focus on process flexibility, which is the ability of a firm to manufacture different kinds of products in the same production facility at the same time. Several principles of the benefits of process flexibility are developed, including: (1) limited flexibility (each plant builds only a few products) can achieve almost all the benefits of total flexibility (each plant builds all the products), and (2) limited flexibility should be configured to chain products and plants together as much as possible. Jordan and Graves (1995) provide analytical support for these principles. A main focus of their paper is a measure to quantify the benefits of the given product-plant configuration, and they use this measure to guide the search for a good limited-flexibility configuration. While a configuration which yields almost all the benefits of total flexibility is identified, the authors do not explicitly study associated cost trade-offs. We explicitly model the cost of process flexibility, and in this case, the flexibility measure of Jordan and Graves (1995) cannot be used to guide a search for a good configuration due to the combinatorial nature of the problem. Additionally, Jordan and Graves (1995) assume that the demand uncertainty is revealed at a single time point, i.e., immediately after the flexibility configuration decision. This limitation is recognized by the authors who say: "… in practice one must allocate production capacity to the products in real time as the demand is realized". Our model addresses this limitation by considering a multi-period model in which the unmet demand is backlogged at the end of each period.

The results of Jordan and Graves are based on the assumption that the firm optimally allocates capacity after the process flexibility decision has been made. We reconsider this issue in a multi-period framework and study the manner in which the value of process flexibility depends on the operating policies employed. Indeed, we show that a myopic operating policy (commonly practiced) may significantly reduce the value of a process flexibility configuration and increase the need for more process flexibility. We note that Bish et al. (2005) also consider the impact of allocation policies on system performance in a single period, two-product, two-firm case under fully flexible and dedicated manufacturing settings.

The work by Graves and Tomlin (2003) extends the chaining ideas of Jordan and Graves (1995) to multi-stage supply chains. They show that the effectiveness of a flexibility configuration, in multi-stage supply chains, is reduced due to stage-spanning bottlenecks and floating bottlenecks, which are not present in single-stage supply chains.

In contrast to the above work, Garavelli (2003) considers the logistics aspect of process flexibility.

Further, Gurumurthi and Benjaafar (2004) show the effectiveness of chaining in queueing systems under varying parameters and control policies. Worker cross-training (similar to process flexibility) and skill chaining are also studied in the queueing literature by Hopp et al. (2004) and Iravani et al. (2007).

Regarding the literature on production scheduling, the papers of Ahmadi et al. (1992) and Bollapragada and Rao (1999) are most closely related to our work. Similar to our setting, Ahmadi et al. (1992) consider a production facility with parallel production lines that are capable of producing all the product families. However, in their work, a setup is required to switch the production from one family to another, and this setup involves both a changeover cost and changeover time. Assuming a deterministic demand over the planning horizon, the authors develop a production schedule for each production line, which minimizes the total changeover and waiting cost over the planning horizon. Unlike our case, Ahmadi et al. (1992) assume that the changeover operation may only be performed at the beginning of a period. Hence, each line is dedicated to a single product family for that production period. Therefore, their problem is a multi-line assignment problem, where the lines are assigned (dedicated) to product families during a production period and a setup is incurred only if the assignment for a line changes between two consecutive periods. In our production system, since the number of product families exceeds the number of production lines, dedicating production lines to single products is not possible. In particular, dedicating a whole production line to a product family with relatively low demand will make our system very inefficient. So, in our problem we assign families to production lines and partial assignments are allowed, i.e., a family may be assigned to multiple lines and a line may be assigned multiple families.

Bollapragada and Rao (1999) consider allocating the production of multiple items to multiple non-identical lines under constant deterministic demand. First, the item demands are assigned to the production lines, as in our case, and partial allocations are allowed. After these assignments, in the second stage, optimal batch sizes and production schedules are generated for each line to minimize the sum of average production cost, setup cost, inventory holding cost and cost of lost sales. The assumption of deterministic constant demand over the planning period allows the authors to divide the problem into two tractable stages and solve a detailed scheduling problem in the second stage. In our problem, such a detailed two-stage analysis is not tractable due to multi-period stochastic demand.

## 2.3. FIXED ALLOCATION MODEL

In this section, we develop and analyze the fixed allocation model (FAM), which closely reflects current practice at the firm. In this model, both the product-to-line assignments (flexibility configuration) and the capacity allocations are decided before the production starts. Then, the allocation decisions as well as the assignment decisions are kept fixed throughout the planning horizon. The objective is to minimize the total assignment and expected backlogging costs. Details are presented below:

*Indices:*

$i, M$ = $i$ indexes the product families, which total $M$ in number

$j, N$ = $j$ indexes the production lines, which total $N$ in number

$t, T$ = $t$ indexes time periods, which total $T$ in number

$k$ = $k$ indexes demand realizations for period $t$

*Data:*

$K_j$ = capacity of production line $j$, per period (in time units)

$e_{ij}$ = amount of time needed to produce one unit of family $i$ on line $j$

$a_{ij}$ = assignment/flexibility cost incurred to produce family $i$ on line $j$

$s_i$ = per unit per period backlogging cost for family $i$; $s_i > 0$

$c_i(.,.)$ = backlogging cost function for family $i$ (defined below)

$\tilde{d}_i^t$ = random demand for product family $i$ in period $t$

$\tilde{d}^t$ = $(\tilde{d}_1^t,...,\tilde{d}_M^t)$ : vector of product family demands in period $t$

$\tilde{d}_i$ = $(\tilde{d}_i^1,...,\tilde{d}_i^T)$ : demand for family $i$ from period 1 to $T$

(notation $d$ without "~" refers to a general demand realization)

$d^{t,k}$ = a particular realization of demand vector in period $t$

$d_i^{t,k}$ = a particular realization of demand for family $i$ in period $t$

*Decision variables:*

$y_{ij}$ = capacity of line $j$ allocated to produce family $i$, in production units (allocation decisions)

$x_{ij}$ = 1 if product family $i$ is assigned to line $j$; 0 otherwise (assignment decisions)

**Fixed Allocation Model (FAM):**

$$z_f = \min_{x,y} \sum_{j=1}^{N}\sum_{i=1}^{M} a_{ij}x_{ij} + \sum_{i=1}^{M} s_i Ec_i(\sum_{j=1}^{N} y_{ij}, \tilde{d}_i) \qquad (2.1)$$

s.t.

$$\sum_{i=1}^{M} e_{ij}y_{ij} \le K_j \qquad \forall j \qquad (2.2)$$

$$y_{ij} \le \frac{K_j}{e_{ij}} x_{ij} \qquad \forall i,j \qquad (2.3)$$

$$x_{ij} \in \{0,1\}, y_{ij} \ge 0 \qquad \forall i,j \qquad (2.4)$$

We use $x$ and $y$ to denote the vectors whose components are $x_{ij}$ and $y_{ij}$, respectively.

The first term in the objective function is the assignment cost and the second term is the expected total backlogging cost over the planning horizon. The first set of constraints, (2.2), limits the capacity of each line $j$ to $K_j$. Constraints in (2.3) are the design constraints, which allow a line to produce only the products that are assigned to it. We develop the backlogging function as follows. Let $b_i^1(y_i, d_i^1) = [d_i^1 - y_i]^+$ be the backlog for family $i$, in period $t$ = 1, where $y_i = \sum_{j=1}^{N} y_{ij}$ and where $[.]^+$ is the larger of its argument and 0. Then, for $t = 2,\ldots,T$ the backlog is recursively defined as:

$$b_i^t(y_i, d_i^1,..., d_i^t) = [b_i^{t-1}(y_i, d_i^1,..., d_i^{t-1}) + d_i^t - y_i]^+ . \qquad (2.5)$$

Finally, $c_i(y_i, d_i) \equiv \sum_{t=1}^{T} b_i^t(y_i, d_i^1,..., d_i^t)$.

**Proposition 2.3.1:** *Assume* $\widetilde{d}_i, i = 1,..., M$, *have finite mean and that* $s_i \geq 0, i = 1,..., M$. *Then, the expected total backlogging cost, i.e.,* $f(y) = \sum_{i=1}^{M} s_i E c_i (\sum_{j=1}^{N} y_{ij}, \widetilde{d}_i)$, *is convex.*

*Proof:* It suffices to show that each $b_i^t(y_i, d_i^1,..., d_i^t)$ as defined in (2.5) is convex in its first argument. This follows immediately from inductive application of the following result: the positive part of a convex function is convex.

The expectations in the objective function, (2.1), are with respect to the joint distributions of $\widetilde{d}_i = (\widetilde{d}_i^1,..., \widetilde{d}_i^T), i = 1,..., M$. If each $\widetilde{d}_i$ has a modest number of realizations then it is straightforward to reformulate FAM as a mixed-integer linear program by introducing additional decision variables to linearize the nested positive-part terms in the definition of $c_i(y_i, d_i)$. If $\widetilde{d}_i$ has many realizations or is continuous, then this is not a viable approach. In this case, by Proposition 2.3.1 we can instead view FAM as a mixed-integer nonlinear program (MINLP) whose continuous relaxation is a convex nonlinear program. That said, it is not possible to solve such an instance of FAM by commercially-available MINLP solvers since we do not have an analytical expression for $f(y) = \sum_{i=1}^{M} s_i Ec_i(\sum_{j=1}^{N} y_{ij}, \tilde{d}_i)$. So, we instead develop a cutting-plane algorithm to solve FAM.

A cutting-plane algorithm for FAM does not require an analytical expression for $f(y)$. Rather, it requires that when $y$ is fixed to a specific value we be able to evaluate (or estimate) $f(y)$ and its gradient $\nabla f(y)$. In general, $f(y)$ is not differentiable because its definition includes nested functions involving positive-part operations. The following proposition gives conditions under which $f(y)$ is differentiable.

**Proposition 2.3.2:** *Assume* $\tilde{d}_i$ *has finite first moment and has an absolutely continuous distribution for each i* = 1,...,*M. Then,* $f(y) = \sum_{i=1}^{M} s_i Ec_i(\sum_{j=1}^{N} y_{ij}, \tilde{d}_i)$ *is differentiable.*

*Proof*: It suffices to show that each $Ec_i(y_i, \tilde{d}_i)$ is differentiable in $y_i = \sum_{j=1}^{N} y_{ij}$. We can express

$$
\begin{aligned}
c_i(y_i, d_i) = \min_{b^1,\ldots,b^T} & \sum_{t=1}^{T} b^t \\
\text{s.t. } & b^1 \geq d_i^1 - y_i \\
& b^t - b^{t-1} \geq d_i^t - y_i, \qquad t = 2,\ldots,T \\
& b^1,\ldots,b^T \geq 0.
\end{aligned}
$$

As a result, $Ec_i(y_i, \tilde{d}_i)$ may be viewed as the recourse function of a stochastic program with randomness only on its right-hand side and with complete recourse. Theorem 12 of Chapter 3 in Kall (1976) (see also Proposition 20 of Ruszczynski and Shapiro, 2003) then gives the desired result under the hypothesis that $\tilde{d}_i$ has an absolutely continuous distribution function.

Even though $f(y)$ can be differentiable, in general we cannot evaluate it (or its gradient) exactly. That said, we can estimate each expectation $Ec_i(y_i, \tilde{d}_i)$ by Monte Carlo sampling. Let $\tilde{d}_{i,r}$, $r$ = 1,…,$R$ be independent and identically distributed (i.i.d.) as

$\tilde{d}_i$ and estimate $Ec_i(y_i, \tilde{d}_i)$ via $\frac{1}{R}\sum_{r=1}^{R} c_i(y_i, \tilde{d}_{i,r})$. We let $\bar{f}_R(y) = \sum_{i=1}^{M} s_i \frac{1}{R}\sum_{r=1}^{R} c_i(y_i, \tilde{d}_{i,r})$ and we can define $FAM_R$ as FAM, except that the objective function is replaced by $\sum_{j=1}^{N}\sum_{i=1}^{M} a_{ij}x_{ij} + \bar{f}_R(y)$. The following proposition characterizes solutions of $FAM_R$ as the number of replications $R$ grows large.

**Proposition 2.3.3:** *Let* $\tilde{d}_r = (\tilde{d}_{1,r}, \ldots, \tilde{d}_{M,r}),\ r = 1, \ldots, R$ *satisfy* $\lim_{R\to\infty} \bar{f}_R(y) = f(y)$, *with probability one (w.p.1). Let* $(x_R^*, y_R^*)$ *denote an optimal solution to* $FAM_R$*. Then every limit point of* $\{(x_R^*, y_R^*)\}_{R=1}^{\infty}$ *solves FAM, w.p.1.*

*Proof*: Pointwise convergence of $\bar{f}_R(y)$ is sufficient to ensure the desired result since each $c_i(y_i, \tilde{d}_i)$ is convex and the feasible region defined by (2)-(4) is compact. See e.g., Shapiro (2003).

As indicated above, we will select $\tilde{d}_{i,r},\ r = 1, \ldots, R$, to be i.i.d. from the distribution of $\tilde{d}_i$. In this case, the pointwise convergence hypothesis of Proposition 2.3.3 holds by the strong law of large numbers for a sample mean of i.i.d. random variables. In what follows, our Monte Carlo sampling scheme will generate the demand observations according to this i.i.d. scheme. That said, the hypothesis of Proposition 2.3.3 also holds under other Monte Carlo sampling schemes designed to reduce variance. While we will not do so here, Proposition 2.3.3 allows us to generate demand observations using, e.g., latin hypercube sampling, a control variates scheme or importance sampling, provided these sampling schemes ensure $\bar{f}_R(y)$ is a strongly consistent estimator.

Proposition 2.3.3 justifies replacing FAM with $FAM_R$ when the number of replications $R$ is sufficiently large. Fortunately, given the definition of $c_i(y_i, \tilde{d}_i)$ we can choose $R$ quite large. For any finite $R$, $\bar{f}_R(y)$ is convex but nonsmooth. We can solve $FAM_R$ using Kelley's (1960) cutting-plane method, adapted to deal with integer-valued decision variables $x$ (see, e.g., Westerlund and Pettersson 1995). At iteration $\kappa$ of the algorithm the following problem (Master-$\kappa$) is solved:

$$\underline{z}_\kappa = \min_{x,y,\theta} \sum_{j=1}^{N} \sum_{i=1}^{M} a_{ij} x_{ij} + \theta \tag{2.6}$$

s.t. (2.2)-(2.4)

$$\theta \geq \bar{f}_R(y^l) + g^l (y - y^l), \qquad l = 1, \ldots, \kappa - 1, \tag{2.7}$$

where $g^l \in \partial \bar{f}_R(y^l)$, i.e., $g^l$ is a subgradient of $\bar{f}_R(y)$ at $y = y^l$.

The cutting-plane algorithm at each iteration forms a first-order Taylor approximation, i.e., a cut, at the current iterate $y^l : \bar{f}_R(y^l) + g^l(y - y^l)$. In what follows, the cut's gradient, $g^l$, and its intercept $\bar{f}_R(y^l) - g^l y^l$, will be called *cut coefficients*. When we solve Master-$\kappa$ we therefore have an outer piecewise linear approximation of $\bar{f}_R(y)$ given by $\max_{l=1,\ldots,K-1} [\bar{f}_R(y^l) + g^l(y - y^l)]$. The formulation in Master-$\kappa$ linearizes this piecewise linear approximation via decision variable $\theta$ and constraints (2.7). The details of the algorithm are given in Figure 2.2.

| |
|---|
| Step 0: Initialize convergence tolerance $\varepsilon > 0$, iteration count $\kappa = 1$, $\bar{z}_\kappa = +\infty$, the number<br>of replications $R$ and let $\tilde{d}_{i,r}$, $r = 1,\ldots,R$ be i.i.d. as $\tilde{d}_i$ for each $i = 1,\ldots,M$. |
| Step 1: Solve Master-$\kappa$ to obtain solution $(x^\kappa, y^\kappa)$ and value $\underline{z}_\kappa$. |
| Step 2: Let $z_\kappa = \sum_{j=1}^{N} \sum_{i=1}^{M} a_{ij} x_{ij}^\kappa + \bar{f}_R(y^\kappa)$. If $z_\kappa < \bar{z}_\kappa$ then let $\bar{z}_\kappa = z_\kappa$ and<br>$(x^*, y^*) = (x^\kappa, y^\kappa)$. If $(\bar{z}_\kappa - \underline{z}_\kappa)/\bar{z}_\kappa \leq \varepsilon$ then stop and output $(x^*, y^*)$. |
| Step 3: Let $g^\kappa \in \partial \bar{f}_R(y^\kappa)$. Add the following cut to the master problem,<br>$\theta \geq \bar{f}_R(y^\kappa) + g^\kappa(y - y^\kappa)$; Set $\kappa = \kappa + 1$ and go to step 1. |

Figure 2.2: A Solution Method for FAM

The values of $\varepsilon$ and $R$ should be selected so that they are commensurate. It is unnecessary to have the sampling error $[Var(\bar{f}_R(y))]^{1/2} << \varepsilon$, and it does not make sense to solve to a precise level of $\varepsilon$ when the sampling error exceeds the precision. Of course, the sampling error can vary with $y$, but by choosing a reasonable allocation we can obtain an estimate of the sampling error and then choose $\varepsilon$ and $R$ accordingly. This could be formalized in a two-stage procedure but we will not do so. In our computational results in Section 2.5 we use $\varepsilon = 10^{-5}$ and $R = 10^6$. Similar to the method given here, Atlason et al. (2004) present an Infinitesimal Perturbation Analysis (IPA)-based cutting-plane method to solve a staff scheduling problem.

## 2.4. DYNAMIC ALLOCATION MODEL

In this section, we develop a time-dynamic allocation model. Like the FAM of the previous section, product-to-line assignments, i.e., the process flexibility configuration, must be decided at the beginning of the planning horizon. Unlike the FAM, in our dynamic allocation model (DAM) the capacity allocation decisions can adapt to the demand in each period, i.e., the firm has operational flexibility not present in the model of the previous section. DAM is a multistage stochastic program with binary first stage decision variables representing product-to-line assignments. Each of the subsequent stages is constrained by these first stage binary decisions, and optimizes over continuous decision variables capturing period-by-period capacity allocation decisions and backlogged demand. The model is summarized below with the following additional notation.

*Additional Notation:*

$b_i^t$ = backlogged demand for family $i$ in period $t$

$y_{ij}^t$ = capacity of line $j$ allocated to produce family $i$ in period $t$

$b_i^0 \equiv 0 \ \ \forall i$

Throughout this section, we assume that $\tilde{d}^t$, $t$ = 1,…,$T$ are independent and identically distributed random vectors. The independence assumption is key to our approach but the identically-distributed assumption can be easily relaxed.

**Dynamic Allocation Model (DAM):**

$$z^* = \min_x \sum_{j=1}^{N}\sum_{i=1}^{M} a_{ij}x_{ij} + E_{\tilde{d}^1} h^1(x, b^0, \tilde{d}^1) \tag{2.8}$$

$$\text{s.t.} \quad x_{ij} \in \{0,1\}, \qquad \forall i, j,$$

where for $t$ = 1,…,$T$,

$$h^t(x, b^{t-1}, \tilde{d}^t) = \min_{y^t, b^t} \sum_{i=1}^{M} s_i b_i^t + E_{\tilde{d}^{t+1}} h^{t+1}(x, b^t, \tilde{d}^{t+1}) \tag{2.9a}$$

s.t.

$$\left.\begin{array}{ll} \sum_{j=1}^{N} y_{ij}^t + b_i^t = \tilde{d}_i^t + b_i^{t-1}, & \forall i \\ \sum_{i=1}^{M} e_{ij} y_{ij}^t \le K_j, & \forall j \\ y_{ij}^t \le \dfrac{K_j}{e_{ij}} x_{ij}, & \forall i, j \\ y_{ij}^t, b_i^t \ge 0, & \forall i, j \end{array}\right\} (y^t, b^t) \in Y(x, b^{t-1}, \tilde{d}^t) \tag{2.9b}$$

where $h^{T+1} \equiv 0$.

In the DAM, the process flexibility configuration is selected via $x$ in (2.8) to minimize the cost of that configuration plus the expected operations cost to the planning horizon. That operations cost is captured in the recursion specified by (2.9), which takes $x$ as input, and makes the allocation and resulting backlogging decisions in each time period, $t$ = 1,…,$T$. When selecting $x$ the demand process $\{\tilde{d}^t\}_{t=1}^T$ is known only through its distribution. When deciding $y^t$ and $b^t$ in period $t$, we know the current period's demand realization, $\tilde{d}^t$, demand backlog from the previous period, $b^{t-1}$ and the distribution governing the future demand process, $\{\tilde{d}^{t+1},...,\tilde{d}^T\}$. So, the timing of when we make the capacity allocation decisions, $y^t$, and when we observe the random demand differs from FAM; we have greater operational flexibility here. Beyond this important difference, the structural form of the constraints in (2.9b) is the same as that in the FAM.

Multistage stochastic programs, such as the one in (2.8)-(2.9) represent significant computational challenges, even when the demands in each period are independent. When the demands have a continuous distribution, as we will assume in our computational study in the next section, model (2.8)-(2.9) is intractable. Even if $\tilde{d}^t$ has a finite number of realizations in each time period, the size of the scenario tree grows exponentially with the number of time periods, and hence the model quickly becomes intractable. The fact that DAM has binary first stage decision variables adds further computational challenges.

When an exact solution of a multistage stochastic program is not computationally viable, we turn to approximations. If $\tilde{d}^t$ has a continuous distribution then we could replace it with a manageable number of realizations in each time period. There are multiple ways to generate such discrete approximations, and we will do so using Monte Carlo sampling. In the literature, Monte Carlo schemes for stochastic programming can be classified as either being “internal” or “external”. In the latter, one replaces expectations with, e.g., sample means based on i.i.d. observations and then uses a “standard” algorithm to solve the resulting approximating problem. Here, the sampling is external to the algorithm. In an internal sampling scheme, an algorithm for deterministic optimization is adapted to the stochastic setting replacing evaluations and gradients by sampling-based estimators. Here, the sampling is carried out with new, and often independent, observations drawn at each iteration of the algorithm.

Higle and Sen (1991) develop an internal sampling algorithm for two-stage stochastic linear programs that is an adaptation of the L-Shaped decomposition method of van Slyke and Wets (1969). In the multistage setting, the internal sampling algorithms of

Pereira and Pinto (1991), Chen and Powell (1999), Linowsky and Philpott (2005) and Donohue and Birge (2006) are adaptations of nested L-shaped decomposition (Birge 1985). They are designed for multistage stochastic linear programs with inter-stage independence and a modest number of realizations in each stage.

The requirement of having a modest number of realizations in each stage precludes direct application of the multistage algorithms discussed above to our DAM. So, we proceed in this section in four steps as follows: First we construct what we call an empirical scenario tree by replacing the true demand distribution at each stage by an empirical distribution constructed using Monte Carlo sampling. We call the dynamic allocation problem defined on this empirical tree EDAM. We construct the empirical tree in Section 2.4.1 so that EDAM is amenable to be solved using the multistage internal-sampling based algorithms we point to in the previous paragraph. Second, in Section 2.4.2 we extend the sampling-based algorithm of Pereira and Pinto (1991) to solve EDAM. We could also extend the other algorithms mentioned above but for simplicity we only consider the algorithm of Pereira and Pinto. Their algorithm requires extension because in addition to the standard staircase structure in which backlogged inventory is carried between adjacent time periods we also have binary first stage decisions, governing the process flexibility configuration, that are carried to all of the periods to the time horizon. This requires that we construct a non-standard cut, which we describe in detail. Third, a solution to a multistage stochastic program is a policy, and in Section 2.4.3 we describe how we can construct a feasible policy for DAM using the cuts generated in solving EDAM. In the forth and final step, we seek to establish whether our feasible policy is near-optimal. To do so, we first describe how to estimate the policy's expected cost in Section 2.4.4. Then, in Section 2.4.5 we show how to construct a confidence interval on the policy's optimality gap using a lower bound estimator again formed using EDAM. The solution validation ideas we use rely on Chiralaksanakul and Morton (2003), but have not been previously extended to problems with integer design decisions or decisions that directly affect all the time periods.

### 2.4.1. Empirical Scenario Tree Construction

In order to generate a sample scenario tree, we generate a set (indexed by *S*) of i.i.d. observations of the demand $\tilde{d}^{1,k}, k \in S$, in period 1. We will then use this same set of observations to represent the realizations in each time period *t*. So, the first period sampled observations are $\tilde{d}^{1,k}, k \in S$. And, in period 2, each of these realizations will have $\tilde{d}^{2,k} = \tilde{d}^{1,k}, k \in S$, as its descendent nodes, etc. In this way, our empirical scenario tree, like its "true" counterpart, exhibits inter-stage independence with identically distributed demand in each period.

If the true demand had $\tilde{d}^1, \tilde{d}^2, ..., \tilde{d}^T$ independent but not identically distributed then in period $t = 2$ we would draw $\tilde{d}^{2,k}, k \in S$, and these $|S|$ observations would form the descendent nodes of all $\tilde{d}^{1,k}, k \in S$. Repeating in this way the empirical scenario tree would, like the original, exhibit inter-stage independence.

Hence, the dynamic allocation model defined on an empirical scenario tree (EDAM) takes the following form after each expectation is replaced with the corresponding sample mean.

**(EDAM)**

$$\hat{z} = \min_{x} \sum_{j=1}^{N} \sum_{i=1}^{M} a_{ij} x_{ij} + \frac{1}{|S|} \sum_{k \in S} \hat{h}^1(x, b^0, \tilde{d}^{1,k}) \tag{2.10}$$

$$\text{s.t.} \quad x_{ij} \in \{0,1\}, \qquad \forall i, j,$$

where for $t = 1, \ldots, T$,

$$\hat{h}^t(x, b^{t-1}, \tilde{d}^{t,k}) = \min_{y^t, b^t} \sum_{i=1}^{M} s_i b_i^t + \frac{1}{|S|} \sum_{k \in S} \hat{h}^{t+1}(x, b^t, \tilde{d}^{t+1,k}) \tag{2.11}$$

$$\text{s.t.} \quad (y^t, b^t) \in Y(b^{t-1}, \tilde{d}^{t,k}), \qquad \forall k \in S.$$

where $\hat{h}^{T+1} \equiv 0$.

Solving EDAM is of central importance in generating near optimal polices for DAM. Next, we present a method using internal sampling to provide near optimal solutions to EDAM with reasonable effort.

### 2.4.2. An Algorithm to Solve EDAM

In this section, we develop a multistage nested decomposition algorithm to solve EDAM. Our algorithm is based on the idea of sequentially approximating the expected cost-to-go function in each stage with a piecewise linear function, similar to what we described in Section 2.3. Hence, at each stage one may use the approximate cost-to-go function to simply make the $x$ and $y^t$ decisions. Further, we note that due to the presence of first-stage binary assignment decisions which feed all the subsequent periods' problem, EDAM is significantly harder to solve then a standard multistage stochastic linear program. See Birge and Louveaux (1997) for a detailed review of multistage stochastic programming models and algorithms.

Here we present an algorithm extending that of Pereira and Pinto (1991) to handle binary first stage decisions which feed all the subsequent periods. First, the algorithm decomposes EDAM into a subproblem for each time period, including what we label $t$ = 0, where $x$ is selected. Then, the algorithm iteratively applies forward and backward phases. During a single forward pass, a demand realization $\tilde{d}^{t,r}$ is drawn from the sample set $S$ and the stage $t$ subproblem is solved, using the current piecewise linear approximation of the cost-to-go function. In the first iteration this subproblem is solved myopically but as the algorithm proceeds the piecewise linear function better approximates the sample-mean functions in (2.10) and (2.11). Solving the subproblems leads to a backlog demand, $b^{t,r}$, being passed to stage $t$+1, where an independent realization $\tilde{d}^{t+1,r}$ is drawn from *S*, until we reach the final period *T*. Hence, during the forward phase the multistage problem is solved along a given sample path of demand, knowing only the approximate cost-to-go function, and not the future period demands, at each period.

During the backward phase of the algorithm, given $b^{T-1,r}$ the stage *T* subproblem is first solved for all $\tilde{d}^{T,r} \in S$ and an optimality cut is passed to stage *T*-1. Next, given $b^{T-2,r}$ the stage *T*-1 subproblem is solved for all $\tilde{d}^{T-1,r} \in S$ and an optimality cut, i.e., a first-order Taylor approximation, is passed to stage *T*-2. This backward pass continues until a cut is passed to stage $t = 1$ and finally to $t = 0$. The cuts accumulated in each stage represent a piecewise outer linearization of the cost-to-go function at that stage. Hence in each iteration of the algorithm, assignment decisions, *x*, are selected to minimize the objective (2.10) where the cost-to-go function is replaced by a piecewise linear approximation. As we iterate, the approximating functions become more precise and hence the assignment and allocation policies improve.

Upon termination, the feasible policy obtained by the algorithm is evaluated by drawing independent demand samples from set *S* to provide an upper bound estimate. The details of the algorithm are given below. We will compactly denote inner products, like the first term in the objective function of (2.8), via *ax*.

**Solution Algorithm for EDAM:**

See Appendix A for the notation and the details of cut calculations, i.e., how to compute the cut coefficients $(\mu^{t,l}, \beta^{t,l})$ and $\alpha^{t,l}$. These are the cut gradient terms for *x* and $b^t$ in period *t*, along with the cut intercept term in period *t*, respectively. They form the analogy of constraint (2.7) for the multistage setting.

**Forward Pass:**

Repeat for each replication $r$ = 1,…,$R$

Repeat for each period $t$ = 0,…,$T$-$1$

If $t = 0$,

Solve:

$$\min_{x,\theta^1} \; ax + \theta^1$$

$$\text{s.t.} \quad \theta^1 + \mu^{0,l} x \geq \alpha^{0,l} \quad l = 1,\ldots,r-1,$$

$$x \in \{0,1\}^{M \times N}$$

(*Stage 0 subproblem*)

for $\hat{x}^r, \hat{\theta}^{1,r}$.

If $t > 0$,

Sample a demand vector $\tilde{d}^{t,r}$ from sample set $S$.

Solve:

$$\min_{y^t, b^t, \theta^{t+1}} sb^t + \theta^{t+1}$$

$$\text{s.t.} \quad \theta^{t+1} + \beta^{t,l} b^t \geq \alpha^{t,l} + \mu^{t,l} \hat{x}^r \quad l = 1,\ldots,r-1$$

$$(y^t, b^t) \in Y(\hat{x}^r, \hat{b}^{t-1,r}, \tilde{d}^{t,r})$$

(*Stage t subproblem*)

for $(\hat{y}^{t,r}, \hat{b}^{t,r})$

(For $t$=$T$, the cut constraints and $\theta^{T+1}$ are absent)

**Backward Recursion:**

Repeat for $t$ = $T$, $T$-1,…,1

Repeat for each scenario $k \in S$

Solve:

$$\min_{y^t, b^t, \theta^{t+1}} sb^t + \theta^{t+1}$$

$$\text{s.t.} \quad \theta^{t+1} + \beta^{t,l} b^t \geq \alpha^{t,l} + \mu^{t,l} \hat{x}^r \quad l = 1,\ldots,r-1$$

$$(y^t, b^t) \in Y(\hat{x}^r, \hat{b}^{t-1,r}, \tilde{d}^{t,r})$$

Store the optimal dual multipliers.

Calculate the cut gradient and the intercept, and add the following cut to period $t$-1:

$$\theta^t + \beta^{t-1,r} b^{t-1} \geq \alpha^{t-1,r} + \mu^{t-1,r} \hat{x}^r \qquad \text{if } t > 1,$$

$$\theta^1 + \mu^{0,r} x \geq \alpha^{0,r} \qquad \text{otherwise.}$$

Calculate lower bound for replication $r$:

$$LB_r = a\hat{x}^r + \hat{\theta}^{1,r},$$

Let $r$ = $r$+1, and repeat for a new forward run.

**Upper Bound Estimation:**

Repeat for each $r$ = 1,…, $R'$

Repeat for each period $t$ = 1,…,$T$

Sample a demand vector $\tilde{d}^{t,r}$ from sample set *S*.

$$\min_{y^t, b^t, \theta^{t+1}} sb^t + \theta^{t+1}$$

$$\text{s.t.} \quad \theta^{t+1} + \beta^{t,l} b^t \geq \alpha^{t,l} + \mu^{t,l} \hat{x}^R \quad l = 1, \ldots, R$$

$$(y^t, b^t) \in Y(\hat{x}^R, \hat{b}^{t-1,r}, \tilde{d}^{t,r})$$

for $(\hat{y}^{t,r}, \hat{b}^{t,r})$

Calculate the upper bound estimator $\tilde{UB} = a\hat{x}^R + \frac{1}{R'} \sum_{r=1}^{R'} \sum_{t=1}^{T} s\hat{b}^{t,r}$

The algorithm we have just specified approximately solves EDAM. When the algorithm terminates the gap between $LB_R$ and the upper bound estimator, $\tilde{UB}$, provides a probabilistic measure of the optimality of the identified policy for EDAM. For our purposes, it is not necessary to solve EDAM exactly, since EDAM is already an approximation of DAM. In practice, the selection of *R*, i.e., the termination condition of the algorithm, is decided by numerical experimentation. *R* may be increased if the gap between the lower bound and the upper bound estimate is not sufficiently close. The parameter $R'$ used for estimating the cost of the policy in EDAM is typically selected with $R'$>R because we can afford to run more sample forward passes in this phase then combined backward and forward passes, because of the more expensive backward recursion.

The output of the algorithm is a policy for EDAM. Specifically, at the end of the algorithm, the process flexibility configuration is given by the solution *x* to the stage 0 subproblem. Note that the subproblems replace the exact cost-to-go function with a set of linear cuts that have accumulated in the course of the algorithm. And, for the allocation policy, we first solve the stage 1 subproblem with its cuts, given *x* and $\tilde{d}^1$ to obtain $y^1$ and $b^1$ (note that when doing so $\tilde{d}^2, \ldots, \tilde{d}^T$ need not be sampled yet). Next, we solve the stage 2 subproblem with its cuts, given *x*, $b^1$ and $\tilde{d}^2$, etc., until we finally solve the stage

$T$ subproblem for $y^T$. A lower bound on $\hat{z}$, the optimal value of EDAM, is given by $LB_r$ because the cuts form an outer linearization of the cost-to-go functions determined in EDAM.

As indicated, the cuts accumulated during the backward passes of the algorithm are used to define a policy for EDAM. In Section 2.4.3, we show how these same cuts can similarly be used to construct a near optimal policy for the original problem DAM. And, the final lower bound, $LB_R$, is used in Section 2.4.5 to generate a probabilistic lower bound for the true optimal value of the DAM.

### 2.4.3. Near Optimal Policy Generation for DAM

In this section, we present a procedure for generating a near optimal feasible policy for DAM. For this purpose, we first generate an empirical scenario tree and the associated EDAM. The approximate model (EDAM) is then solved with the algorithm given in Section 2.4.2. As we have described, when the algorithm terminates, the subproblems at each stage $t$ contain a set of cuts generated during the backward passes of the algorithm. Since these cuts approximate the cost-to-go functions of EDAM, they may also be used to approximate the cost-to-go functions of DAM. Hence, they define a feasible policy for the actual model, DAM as well.

In particular, we use the following optimization models to generate a good feasible policy for DAM:

For $t = 0$:

$$\min_{x,\theta^1} \quad ax + \theta^1$$
$$\text{s.t.} \quad \theta^1 + \mu^{0,l} x \geq \alpha^{0,l} \quad l = 1,\ldots,R,$$
$$x \in \{0,1\}^{M \times N}.$$

For $t = 1,\ldots,T$:

$$\min_{y^t, b^t, \theta^{t+1}} sb^t + \theta^{t+1}$$
$$\text{s.t.} \quad \theta^{t+1} + \beta^{t,l} b^t \geq \alpha^{t,l} + \mu^{t,l} x \quad l = 1,\ldots,R,$$
$$(y^t, b^t) \in Y(x, b^{t-1}, \widetilde{d}^t),$$

(For $t=T$, the cut constraints and $\theta^{T+1}$ are absent)

Figure 2.3: Models for Generating a Feasible Policy for DAM

Note that $\beta$, $\alpha$ and $\mu$ contain the cut coefficients and $R$ is the total number of cuts obtained while solving EDAM with the method of Section 2.4.2.

Our policy determines decisions $(y^t, b^t)$ to be made at each period $t = 1,\ldots,T$, (and decision $x$ at period $t = 0$), by solving the models in Figure 2.3 for a given period $t$ and state $(\hat{b}^{t-1}, d^t)$. The overall procedure for generating a near optimal feasible policy is summarized below:

Step 1: Construct an empirical scenario tree and the associated empirical model, EDAM, as explained in Section 2.4.1.

Step 2: Solve EDAM with the algorithm of Section 2.4.2.

Step 3: When the algorithm terminates, store the generated cuts at each stage of the problem.

Step 4: Using the cuts from Step 3, construct and solve the sequence of optimization problems in Figure 2.3 to find a feasible policy.

Figure 2.4: Feasible Policy Generation Procedure for DAM

### 2.4.4. Policy Cost Estimation (Upper Bound Estimation)

Once a feasible policy is identified for DAM, the next step is to evaluate its cost to obtain an upper bound on the optimal value of DAM, $z^*$. In particular, for a given demand sample path $i, (\tilde{d}^{1,i},...,\tilde{d}^{T,i})$, our policy generates a stream of feasible solutions, $\hat{x}$, $\hat{b}^1(\tilde{d}^{1,i}),\ldots,\hat{b}^T(\tilde{d}^{T,i})$ for DAM and, the cost of the policy for that sample path is given by: $U^i = U(\tilde{d}^{1,i},...,\tilde{d}^{T,i}) = a\hat{x} + \sum_{t=1}^{T} s\hat{b}^t(\tilde{d}^{t,i})$. Since the identified policy is not necessarily optimal, the expected cost of the policy exceeds DAM's optimal value, i.e., $E\tilde{U} = EU(\tilde{d}^1,...,\tilde{d}^T) \geq z^*$.

Next, to obtain a point estimate of $E\tilde{U}$, we generate $\eta$ i.i.d. demand sample paths, $(\tilde{d}^{1,i},...,\tilde{d}^{T,i})$, $i = 1,\ldots,\eta$ and evaluate the cost of the policy, for each sample path. Then, an approximate one-sided $100(1-\alpha)\%$ confidence interval for $E\tilde{U}$ is $(-\infty, \overline{U}_\eta + z_\alpha s_u / \sqrt{\eta}]$, where $\overline{U}_\eta = \frac{1}{\eta}\sum_{i=1}^{\eta} U^i$ and $s_u^2 = \frac{1}{\eta - 1}\sum_{i=1}^{\eta}(U^i - \overline{U}_\eta)^2$. Here, $z_\alpha$ is the (1-α)-level quantile for a standard normal. In general, we may use *t*-distribution quantiles but the values of $\eta$ we use will be sufficiently large so that the difference is negligible.

### 2.4.5. Lower Bound Estimation

This section explains how to develop a probabilistic lower bound for $z^*$, the optimal value of DAM. Our goal is to combine this bound with the one in Section 2.4.4 and to develop a confidence interval for the optimality gap of the policy generated in Section 2.4.3. Our lower bound estimator is based on the following proposition.

**Proposition 2.4.1:** *Let $\tilde{L}$ denote the final lower bound generated by the algorithm of Section 2.4.2 for the optimal value of an EDAM. Then, $z^* \geq E\tilde{L}$.*

*Proof*: Let $\hat{z}$ be the optimal value of EDAM for a particular scenario tree $\Gamma$. From Theorem 2 of Chiralaksanakul and Morton (2003), $z^* \geq E\hat{z}$. Since $\hat{z} \geq \tilde{L}$, we conclude that $z^* \geq E\hat{z} \geq E\tilde{L}$.

Next, we develop a point estimate of $E\widetilde{L}$ to establish a lower bound for $z^*$. Hence, as in the previous section, we generate multiple replicates of $\widetilde{L}$. In particular, we construct $\nu$ i.i.d. sample scenario trees, $\Gamma^1,...,\Gamma^\nu$, and the associated EDAMs as explained in Section 2.4.1. Then, we solve these problems with the method of Section 2.4.2 to obtain the lower bound estimators $L^1,...,L^\nu$.

Then, by the standard central limit theorem for i.i.d. random variables, an approximate one-sided 100(1-$\alpha$)% confidence interval for $z^*$ (also for $E\widetilde{L}$) is given by $[\overline{L}_\nu - z_\alpha s_l / \sqrt{\nu}, +\infty)$, where $\overline{L}_\nu = \frac{1}{\nu}\sum_{i=1}^{\nu} L^i$ and $s_l^2 = \frac{1}{\nu - 1}\sum_{i=1}^{\nu}(L^i - \overline{L}_\nu)^2$. Finally, by combining this confidence interval with the one in Section 2.4.4, using the Boole-Bonferroni inequality, we obtain a confidence interval for the optimality gap of the feasible policy, i.e., for $E\widetilde{U} - z^*$. Specifically, an approximate 100(1-2$\alpha$)% confidence interval for $E\widetilde{U} - z^*$ is given by $[0, (\overline{U}_\eta - \overline{L}_\nu)^+ + z_\alpha s_u / \sqrt{\eta} + z_\alpha s_l / \sqrt{\nu}]$.

## 2. 5. COMPUTATIONAL RESULTS AND ANALYSIS

The purpose of the computational study in this section is two-fold. First, in Section 2.5.1, we investigate the value of operational flexibility by comparing and analyzing the expected performance of the dynamic and fixed allocation models under different real-life settings. Then, in Section 2.5.2, under dynamic operating policies, we specifically focus on the value of process flexibility in a multi-period decision environment, ignoring the cost of the assignments. Further, the computational results show that our algorithms are very effective to solve real-sized problems. Below for simplicity, we assume that production of a unit of any product requires the same amount of capacity and the production lines are identical, i.e., $e_{ij} = 1 \ \forall i, j$ and $K_j = K \ \forall j$, for the computational analyses.

### 2.5.1. Value of Operational Flexibility: Comparing DAM and FAM

In this section, we numerically compare and analyze FAM and DAM developed in Sections 2.3 and 2.4, respectively. In our experimental design we use $\tilde{d}_i \sim N(\mu_i, \sigma_i^2)$. We consider two factors that scale: (*i*) the variability of the demand ($\beta$), and (*ii*) its mean, ($\rho$). To scale the mean we use the ratio of mean-demand to capacity, i.e., $\rho = \sum_{i=1}^{M} E\tilde{d}_i \Big/ K$. Let $\tilde{d}_i = \tilde{d}_i(\beta, \rho)$. Table 2.2 contains $\mu_{Base,i} = E(\tilde{d}_i)$ and $\sigma^2_{Base,i} = \mathrm{Var}(\tilde{d}_i)$, i.e., the mean and the variance of $\tilde{d}_i(\beta, \rho)$, for $\beta = \rho = 1$. More generally, $E\tilde{d}_i(\beta, \rho) = \rho\mu_{Base,i}$ and $\mathrm{Var}\,\tilde{d}_i(\beta, \rho) = \rho^2 \beta \sigma^2{}_{Base,i}$. In this way, the coefficient of variation of $\tilde{d}_i(\beta, \rho)$ is constant as we scale $\rho$ and grows in $\beta$.

We conjecture that these two factors should have a significant impact on the performance of the policies under investigation. More specifically, we consider three mean-demand to capacity ratios ($\rho$ = 0.933, 1 and 1.067), as well as three levels of demand variability ($\beta$ = 1, 3 and 5). The demand data in this study is generated to simulate the real situation at the firm. In particular, the product families in Table 2.2 with higher mean demand have lower coefficient of variation.

Below, we summarize computational results for a problem with $M$ = 6 families, $N$ = 3 lines and $T$ = 5 production periods. In practice, the firm has 10-15 product families and 6 production lines. However, from the manufacturing point of view similar product families that share a significant number of components are aggregated. Moreover, some set of production lines are also dedicated for producing certain product families. Hence from a practical point of view it is sufficient to consider a 6-product 3-line problem.

For ease of analysis, we assumed that backlogging cost for all families is $s_i$ = \$1 while the assignment cost for any line-product pair is $a_{ij}$ = \$10 (similar results are obtained for different combinations of backlog and setup costs). A brief review of experimental settings is provided in Tables 2.1-2.2.

| $M$ | $N$ | $K$ | $T$ | $\rho$ | $\beta$ |
|---|---|---|---|---|---|
| 6 | 3 | 100 | 5 | {0.933, 1, 1.067} | {1, 3, 5} |

Table 2.1: General Experimental Settings

| | Base Mean ($\mu_{Base}$) | Base Variance ($\sigma^2_{Base}$) |
|---|---|---|
| Family 1 | 5 | 0.5 |
| Family 2 | 35 | 22 |
| Family 3 | 40 | 26 |
| Family 4 | 45 | 31 |
| Family 5 | 85 | 97 |
| Family 6 | 90 | 102 |
| Total | 300 | |

Table 2.2: Base Demand Data for the Test Problem ($\rho = 1, \beta = 1$)

The sample setting in Tables 2.1-2.2 is solved (for each experimental setting) under both FAM and DAM with the algorithms explained in Sections 2.3 and 2.4, respectively. Both of the solution methods are implemented in C++ using Concert Technology and CPLEX callable library on a Dell Precision 530 Workstation with Intel Xeon 1.8GHz processor and 1GB of RAM. From a computational point of view, under our experimental settings, it takes around 7 minutes to solve a problem instance under FAM. Identifying a near optimal policy for a problem instance under DAM takes around 20 minutes while forming a confidence interval on the optimality gap, for the identified policy, takes 6-7 hours. (See Appendix B for the computational parameters used for the solution method of DAM.)

The optimal operating costs obtained under FAM are presented in Table 2.3. These are estimates obtained with a sample size of $R$=$10^6$. The associated standard deviations are about 0.03% of the sample mean estimates. In our example, the operating cost increases with demand variability and the ratio of mean-demand to capacity. In addition, the introduction of variability (to a deterministic system) has the highest impact on the operating costs when $\rho$ =1. Intuitively, when we have plenty of capacity, demand variability can be buffered with capacity up to some extent, similarly when we have the capacity well below the demand then the variability will not affect the expected loss too much, since the capacity is already fully utilized.

| $\rho / \beta$ | Deterministic ( $\beta = 0$ ) | Low Variability ( $\beta = 1$ ) | Moderate Variability ( $\beta = 3$ ) | High Variability ( $\beta = 5$ ) |
|---|---|---|---|---|
| 0.933 | 70 | 126.45 | 214.20 | 281.33 |
| 1 | 80 | 238.95 | 353.58 | 431.98 |
| 1.067 | 370 | 443.81 | 547.68 | 626.33 |

Table 2.3: Optimal Operating Cost under Fixed Allocation Model ($z_f$)

The results for the dynamic allocation model are summarized in the next four tables. In Table 2.4, we provide the operating cost estimates of the near optimal policy identified by the solution method of Section 2.4.3. Then, in Table 2.5, in order to quantify the quality of the identified policy, we provide the lower bound estimates on the true optimal value of DAM as explained in Section 2.4.5. Finally, we provide the optimality gap estimates of the identified policies in Table 2.6. (In Table 2.6, %Gap is calculated by dividing the length of the confidence interval by the mean upper bound estimate, $\overline{U}$ .)

In the worst case, the optimality gap of the identified policy is around 5% of the estimated cost and for low and medium variability cases the gap is less than 2.5%. Hence, we conclude that, for our example, the approach presented in Section 2.4 generates near optimal policies for DAM. In addition, since our gap generation mechanism is based on sampling, it is natural to observe that the algorithm performance slightly degrades as the variability of demand grows.

| $\rho / \beta$ | Deterministic ($\beta = 0$) | | Low Variability ($\beta = 1$) | | Moderate Variability ($\beta = 3$) | | High Variability ($\beta = 5$) | |
|---|---|---|---|---|---|---|---|---|
| | $\overline{U}_\eta$ | $s_u$ | $\overline{U}_\eta$ | $s_u$ | $\overline{U}_\eta$ | $s_u$ | $\overline{U}_\eta$ | $s_u$ |
| 0.933 | 70 | 0 | 84.48 | 0.07 | 107.59 | 0.29 | 132.16 | 0.49 |
| 1 | 80 | 0 | 153.91 | 0.47 | 207.41 | 0.82 | 248.25 | 1.06 |
| 1.067 | 370 | 0 | 386.88 | 0.81 | 419.46 | 1.34 | 453.74 | 1.62 |

Table 2.4: Cost of the Identified Feasible Policy for DAM ($\overline{U}_\eta, s_u$)

| $\rho / \beta$ | Deterministic ($\beta = 0$) | | Low Variability ($\beta = 1$) | | Moderate Variability ($\beta = 3$) | | High Variability ($\beta = 5$) | |
|---|---|---|---|---|---|---|---|---|
| | $\overline{L}_\nu$ | $s_l$ | $\overline{L}_\nu$ | $s_l$ | $\overline{L}_\nu$ | $s_l$ | $\overline{L}_\nu$ | $s_l$ |
| 0.933 | 70 | 0 | 84.56 | 0.32 | 107.37 | 0.92 | 128.91 | 1.44 |
| 1 | 80 | 0 | 153.90 | 0.85 | 207.57 | 1.50 | 245.17 | 3.84 |
| 1.067 | 370 | 0 | 386.67 | 0.60 | 420.07 | 1.20 | 445.51 | 1.88 |

Table 2.5: Lower Bound for the Optimal Operating Cost of DAM ($\overline{L}_\nu, s_l$)

| $\rho / \beta$ | Deterministic ($\beta = 0$) | | Low Variability ($\beta = 1$) | | Moderate Variability ($\beta = 3$) | | High Variability ($\beta = 5$) | |
|---|---|---|---|---|---|---|---|---|
| | 95%CI | % Gap | 95%CI | % Gap | 95%CI | % Gap | 95%CI | % Gap |
| 0.933 | N/A | N/A | [0, 0.76] | 0.90 | [0, 2.60] | 2.42 | [0, 7.02] | 5.31 |
| 1 | N/A | N/A | [0, 2.61] | 1.69 | [0, 4.54] | 2.19 | [0,12.70] | 5.12 |
| 1.067 | N/A | N/A | [0, 2.99] | 0.77 | [0, 4.97] | 1.18 | [0,15.08] | 3.32 |

Table 2.6: Approximate 95% Confidence Intervals for the Optimality Gap of the Feasible Policy

Comparing the results in Table 2.3 and Table 2.4, it is clear that operational flexibility (i.e., using a dynamic allocation policy) significantly reduces the negative impact of variability on the operating cost. For example, under the low demand setting, moving from a deterministic system to a low variability system, the operating cost under DAM increases from \$70 to \$84.48 whereas, under FAM, it increases from \$70 to \$126.45.

In particular, it becomes more beneficial to use DAM instead of FAM as the system becomes more variable and the mean-demand to capacity ratio approaches one. The maximum difference is observed when $\beta = 5$ and $\rho = 1$. Table 2.7 summarizes the absolute benefits of using DAM over FAM under our experimental settings.

| $\rho / \beta$ | Deterministic ($\beta = 0$) | Low Variability ($\beta = 1$) | Moderate Variability ($\beta = 3$) | High Variability ($\beta = 5$) |
|---|---|---|---|---|
| 0.933 | 0 | 41.97 | 106.61 | 149.17 |
| 1 | 0 | 85.04 | 146.17 | 183.73 |
| 1.067 | 0 | 56.93 | 128.22 | 172.59 |

Table 2.7: Expected Absolute Benefits of using DAM over FAM ($z_f$-$\overline{U}$)

Intuitively, if the mean demand is well above the capacity, then operational flexibility (i.e., using DAM instead of FAM) does not provide much benefit since the system capacity is already fully utilized and dynamically changing the allocations does not help to decrease the expected backlog. Similarly, when the mean demand is well below the capacity, then again operational flexibility is not very beneficial. On the other hand, if the capacity is well-balanced with respect to the demand, then there is significant opportunity for decreasing the expected backlog by revising the allocations periodically. In addition, under all demand levels, the absolute benefits of operational flexibility are increasing with the variability of the system.

In Table 2.8, we provide the expected percentage benefits of using DAM over FAM. The effect of variability on the percentage and absolute benefits is similar: the percentage benefits also increase with the variability of the system but, from moderate to high variability the increase is not very significant. The impact of capacity availability is slightly different in this case, i.e., the percentage benefits under $\rho = 0.933$ and $\rho = 1$ are close to each other and moreover, for moderate and high variability cases percentage benefits under $\rho = 0.933$ are higher. This result may be interpreted by analogy with a queueing system with a relatively high traffic intensity of $\rho = 0.933$. In this setting, DAM has opportunities to adaptively change allocations to decrease the backlog. In addition, since for $\rho = 0.933$ we have some slack capacity, DAM is more efficient in decreasing backlog than the $\rho = 1$ case. However, as explained in the previous paragraph, as $\rho$ approaches zero, absolute and percentage difference in the performance of DAM and FAM shirks to zero.

| $\rho / \beta$ | Deterministic ($\beta = 0$) | Low Variability ($\beta = 1$) | Moderate Variability ($\beta = 3$) | High Variability ($\beta = 5$) |
|---|---|---|---|---|
| 0.933 | 0 | 33.19 | 49.77 | 53.02 |
| 1 | 0 | 35.59 | 41.34 | 42.53 |
| 1.067 | 0 | 12.83 | 23.41 | 27.56 |

Table 2.8: Expected Percentage Benefits of using DAM over FAM ($z_f$-$\overline{U}$) / $z_f$

From a computational point of view, under FAM, it is possible to solve much larger problem instances, e.g., problems up to 20 families and 10 production lines, in an effective manner. Under DAM it is also possible to quickly identify good feasible solutions for these problems. However, constructing a confidence interval for the optimality gap (under DAM) becomes computationally challenging as the problem size grows large.

On the other hand, given a certain assignment decision, i.e., with the *x*-decision pre-specified, DAM can quickly identify a near optimal allocation policy. Therefore, when solving large problems under DAM, using the assignment decision identified from the FAM may be a good heuristic approach. Another approach can involve aggregating the large problem to a manageable size and then using the optimal assignment solution of this approximate problem in the actual model.

### 2.5.2. Value of Process Flexibility

In the previous sections, for a given operating policy, we have concentrated on the joint optimization of process flexibility and capacity allocation decisions, by trading off the cost and benefits of process flexibility in a multi-period decision framework. Nevertheless, in practice, a firm may already have an incumbent process flexibility configuration (which may be difficult to fully reconfigure) and may be interested in improving over this current scheme by adding new links to it. Hence, the firm first needs to assess the value of its current flexibility configuration since the additional value of the new links will be traded off with their costs.

In this section, we specifically investigate the value of process flexibility, under different operating policies. In a single period model, it has been shown that a little process flexibility, namely a partial assignment scheme configured as a "chain", achieves almost all the benefits of full process flexibility (see Jordan and Graves 1995). Ignoring the cost of assignments in our case, allows us to extend the results of Jordan and Graves (1995) to a multi-period decision framework in which the firm must also decide how to allocate capacity to demand over time, i.e., the allocation (operating) policies.

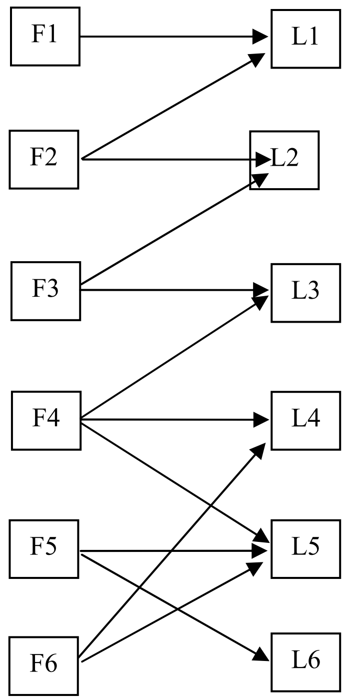
First, we consider an *optimal dynamic allocation* policy, where the firm decides allocations to minimize its expected backlogging cost over the full planning horizon (as in DAM). In this case, we solve the DAM for a given flexibility configuration to find the optimal allocations. Next, we consider a *myopic dynamic allocation* policy, where the firm minimizes its backlogging cost myopically in each period. We label this latter policy MDAM. In this case, the allocation policy is not forward-looking, but the firm still has full operational flexibility when deciding capacity allocations over time. Myopic allocation policies are quite common in practice, since they are easy to identify, understand and implement.

We start with an example with 6 families, 6 production lines (each with 100 units of capacity) and 10 time periods. Demand (again, normally distributed) and backlogging cost information for each family is provided in Table 2.9.

| | Mean | Variance | Unit backlogging cost ($) |
|---|---|---|---|
| Family 1 | 30 | 80 | 1.01 |
| Family 2 | 80 | 500 | 1.01 |
| Family 3 | 80 | 550 | 1.01 |
| Family 4 | 150 | 1000 | 1.005 |
| Family 5 | 110 | 600 | 1 |
| Family 6 | 150 | 1200 | 1 |
| Total | 600 | | |

Table 2.9: Demand and Cost Data for the Test Problem

Next, for this example, we compare the value of the partial-flexibility configuration shown in Figure 2.5.a to the full process flexibility case given in Figure 2.5.b. Here, the comparison is with respect to expected backlogging cost. In practice, Figure 2.5.a may represent the current operating configuration of the firm, or it may be implied by the cost of assignments. The corresponding results are summarized in Table 2.10.



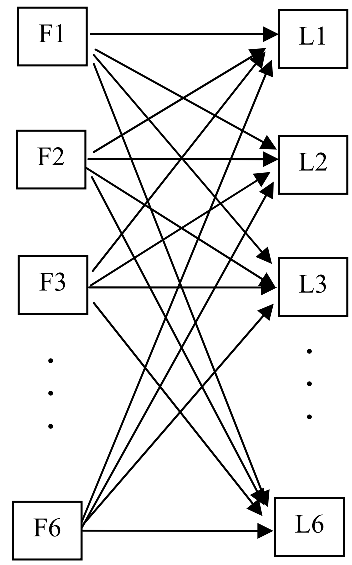




Figure 2.5.a: Partial Process Flexibility Figure 2.5.b: Full Process Flexibility

| | Backlogging Cost | | Relative Difference |
|---|---|---|---|
| Allocation Policy/Process Flexibility | Partial-Flex Case | Full-Flex Case | (Partial-Full)/Full |
| Optimal Allocation (DAM) | 1281.21 | 834.56 | 53.52% |
| Myopic Allocation (MDAM) | 1523.06 | 834.56 | 82.50% |
| (MDAM-DAM)/MDAM | 15.88% | 0.00% | |

Table 2.10: Value of Flexibility Configurations in Figure 2.5.a and 2.5.b under DAM and MDAM

Table 2.10 shows that under the optimal dynamic allocation policy, the expected backlogging cost of the partial configuration is 53.52% higher than the cost of the full flexibility configuration. However, this difference in backlogging cost grows up to 82.5% under the myopic dynamic allocation policy. Hence, the value of the partial configuration is significantly reduced under the myopic allocation case. Indeed, the expected backlogging cost for the partial configuration is 15.88% higher under the MDAM compared to DAM.

The intuition for the results in Table 2.10 is as follows. Since the myopic allocation policy minimizes the immediate cost in every period, when backlogging, families are prioritized according to their unit backlogging costs. Hence, in this particular example, the policy first tries to backlog F5 and F6, then F4 and then considers the other families. However, the assignment configuration in Figure 2.5.a implies that the total capacity accessible to produce F4, F5 and F6 (the lower group) is 400 units while their total mean demand is 410. On the other hand, for the remaining families, F1, F2 and F3 (the upper group), the total accessible capacity is 300 units while their expected demand is only 190 units. Therefore, backlogging the lower group, when it is also possible to backlog the upper group, creates an increased risk of backlogging in the future since such a policy further distorts the capacity and demand balance in the system. Hence, the myopic solution deteriorates over time.

The specific backlogging cost scheme in Table 2.10 could be driving the difference between myopic and optimal policies, since the cost structure is biased in such a way to backlog the families with restricted capacity. To investigate the impact of the cost structure, we repeat the example with a reversed backlogging cost scheme, i.e., $s_1 = s_2 = s_3 = 1$, $s_4 = 1.005$, $s_5 = 1.01$ and $s_6 = 1.01$. In this case, priority is given to families F1, F2 and F3 which have access to relatively more capacity. However, we still observe a 7.2% gap between the value of the partial configuration under DAM and MDAM. This is because, even though the upper group has more capacity at the beginning of the planning horizon, they become more restricted as backlog accumulates over time.

Moreover, even under an identical backlogging cost scheme (where $s_i = 1,\ i = 1,...,6$), the percentage gap between the value of partial configuration under DAM and MDAM is still significant at 6.1%. Indeed, the optimal policy identified by DAM is a forward-looking policy, which dynamically prioritizes families to backlog by taking the current backlog levels, backlogging cost scheme and the future demand into account. Hence, a myopic rule may fail to perform well.

In addition, we show that this performance gap under DAM and MDAM still exists even for very efficient chain flexibility configurations. Below, we investigate the value of the partial flexibility scheme, given in Figure 2.5.c, which is configured as a chain that achieves almost all the benefits of the full process flexibility under DAM. In this second example, to eliminate the influence of the non-identical backlogging cost, we further assume that the backlogging cost is identical and equal to $1 for each family.

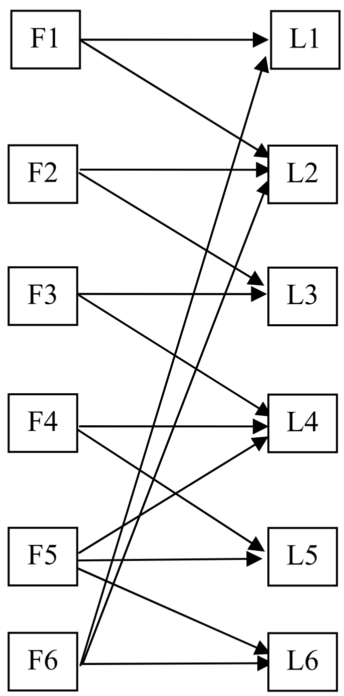



Figure 2.5.c: Chain Process Flexibility

| | Backlogging Cost | | Relative Difference |
|---|---|---|---|
| Allocation Policy/Process Flexibility | Chain-Flex Case | Full-Flex Case | (Chain-Full)/Full |
| Optimal Allocation (DAM) | 845.625 | 837.17 | **1.01%** |
| Myopic Allocation (MDAM) | 873.169 | 837.17 | **4.30%** |
| (MDAM-DAM)/MDAM | **3.15%** | 0.00% | |

Table 2.11: Value of Flexibility Configurations in Figure 2.5.c and 2.5.b under DAM and MDAM

Table 2.11 shows that, under DAM the chaining configuration in Figure 2.5.c is very effective and its cost deviates from the full flexibility case by only 1%. However, the effectiveness of the chaining configuration decreases significantly under MDAM and its cost deviates from the full flexibility case by more than 4%. Hence, even if a chain is very efficient, with respect to full flexibility configuration under DAM, its efficiency may reduce significantly under MDAM.

Next, we extend our computational results to investigate the impact of the number periods in the planning horizon. In Table 2.12 below, we analyze the effectiveness of the chaining configuration in Figure 2.5.c as the number of periods changes when the capacity is well balanced with demand.

**Results for 2-Period Problem**

| | Backlogging Cost | | Relative Difference |
|---|---|---|---|
| Allocation Policy/Process Flexibility | Chain-Flex Case | Full-Flex Case | (Chain-Full)/Full |
| Optimal Allocation (DAM) | 69.68 | 68.55 | **1.64%** |
| Myopic Allocation (MDAM) | 70.27 | 68.55 | **2.52%** |
| (MDAM-DAM)/MDAM | **0.85%** | | |

**Results for 8-Period Problem**

| | Backlogging Cost | | Relative Difference |
|---|---|---|---|
| Allocation Policy/Process Flexibility | Chain-Flex Case | Full-Flex Case | (Chain-Full)/Full |
| Optimal Allocation (DAM) | 592.85 | 586.47 | **1.09%** |
| Myopic Allocation (MDAM) | 610.68 | 586.47 | **4.13%** |
| (MDAM-DAM)/MDAM | **2.92%** | | |

**Results for 14-Period Problem**

| | Backlogging Cost | | Relative Difference |
|---|---|---|---|
| Allocation Policy/Process Flexibility | Chain-Flex Case | Full-Flex Case | (Chain-Full)/Full |
| Optimal Allocation (DAM) | 1435.47 | 1422.75 | **0.89%** |
| Myopic Allocation (MDAM) | 1488.63 | 1422.75 | **4.63%** |
| (MDAM-DAM)/MDAM | **3.57%** | | |

Table 2.12: Value of Flexibility Configuration in Figure 2.5.c for $T = 2$, 8 and 14

There are two important takeaways from Table 2.12. First, the effectiveness of the chaining configuration improves as the number of periods grows under DAM. In particular, the performance of the chaining configuration deviates from the full flexibility performance by 1.64% for the 2-period problem, while this deviation falls to 0.89% for the 14-period case. This is because the system becomes more congested over time as the demand is backlogged, and the performance of the chaining and full-flexibility configurations converges to each other.

Second, as the number of periods increases the effectiveness of the chaining configuration deteriorates under MDAM. This is due to the fact that it becomes more important to look forward when making allocation decisions in longer planning horizons. In particular, for a single period problem, a myopic allocation is optimal, but as the number of periods grows the sub-optimality of myopic decisions becomes more severe. On the other hand, the congestion argument mentioned above also applies here and eventually as the number periods keeps growing, the performance of the chaining and full flexibility configurations converge to each other under MDAM, as well. In our example, up to 14 periods, the first factor dominates and the performance gap increases with the number of periods (i.e., the gap increases from 2.52% for the 2-period problem to 4.63% for the 14-period problem). All the comparisons we make with respect to Tables 2.12 are statistically significant at the 1% level.

As a result, we numerically show that the value of process flexibility depends on the allocation policy employed by the firm to allocate capacity as the demand is revealed. That is, a suboptimal allocation policy, specifically a myopic allocation of capacity to demand, may significantly reduce the value of a particular flexibility configuration and may increase the need for more process flexibility. Hence, in the production planning phase, a firm which uses a myopic allocation policy may require the adoption of more process flexibility to hedge against demand uncertainty.

## 2.6. Discussion

In this chapter, we addressed a challenging real life production and capacity management problem which, to the best of our knowledge, has not been addressed in the literature so far. The problem is motivated by a high-tech electronic device manufacturer which produces multiple product families on multiple flexible assembly lines over multiple time periods under demand uncertainty. While our motivation stems from the electronics industry, many of the same issues considered here extend to a wide range of make-to-order manufacturing environments.

We specifically modeled and analyzed the value of process and operational flexibility, in a multi-period manufacturing environment with stochastic demand. Regarding the operational flexibility of the firm, we studied a fixed allocation model (FAM), a fully optimized dynamic allocation model (DAM) and a myopically optimized dynamic allocation model (MDAM). In the fixed allocation model, which closely represents the current practice in our firm, the capacity allocations and flexibility decisions are made at the beginning of the first period and are kept fixed throughout the planning horizon. We formulated the fixed allocation model as a single-stage stochastic program and developed a cutting plane algorithm to solve it.

In the two dynamic allocation models, the firm may change the allocation decisions from period to period after observing the demand. DAM is formulated as a multistage stochastic program with binary first stage decision variables. In Section 2.4, we outlined a method to obtain near optimal policies for DAM and to generate bounds on the optimality gap for a given feasible solution.

In contrast to FAM, DAM allows the company to utilize operational flexibility. Hence, a comparison of these two models provides the value of operational flexibility under the optimal choice of process flexibility. The computational results given in

Section 2.5.1, show that operational flexibility is most valuable when the demand variability is high and the mean-demand and capacity is well-balanced. Hence, a firm whose capacity is closely trimmed to the mean of its highly uncertain demand will have greater benefit from the dynamic allocation model.

On the other hand, managing a dynamic allocation system could be more complicated than a fixed allocation system, due to the impact of reduced standardization and increased need for additional labor, material handling equipment and advanced information systems. Hence, the benefits of DAM should be traded off with the cost implications of operating a more complex system, before deciding to use either a fixed or dynamic allocation system in practice.

Ignoring the cost of assignments, we also analyze the value of process flexibility under optimal and myopic dynamic operating policies. Our computational results show that the value of process flexibility may significantly depend on the operating policy employed by the firm to allocate capacity, in a multi-period production environment. In particular, we show that a flexibility configuration may be significantly over-valued under DAM compared to MDAM. That is, if a firm operates with a myopic allocation policy after the process flexibility decision, then more process flexibility is needed to achieve the same level of expected backlogging cost under MDAM. In other words, a firm which uses a myopic allocation policy may require the adoption of more process flexibility to hedge against demand uncertainty.

Finally, the impact of the number of periods is also revealing. If the firm employs an optimal dynamic allocation policy, then the effectiveness of a chain flexibility configuration improves as the number of periods increases, since the system becomes more utilized over time. However, if a myopic operating policy is used, then the myopic solutions become more and more distorted as the number of periods increases and hence, the effectiveness of the chain flexibility configuration decreases.

Regarding our solution methods, we require demand to be independent across time when solving DAM, but it need not be identically distributed and we can handle inter-product dependencies.

This is a reasonable assumption, for an MTO firm involved in mass customization facing an aggregate demand that comes from a large number of customers who act independently. The solution methodology developed for FAM in Section 2.3 also handles non-identical and correlated demand both across time and product families. It is also straightforward to extend our solution methods to work with non-identical production capacities.

Our computational results suggest that our methods are effective for solving real-sized problems. Decomposition methods in the two-stage setting have benefitted from the use of a trust region or a quadratic proximal term to speed convergence, and we could similarly benefit from using these in the multi-stage setting. Moreover, we have based our decomposition on the algorithm of Pereira and Pinto (1991) because of its (relative) simplicity to describe. Enhanced versions designed to reduce computational effort have more recently been developed by Chen and Powell (1999), Linowsky and Philpott (2005) and Donohue and Birge (2006), and we could also benefit from these enhancements.

In the future, we plan to extend this research with a focus on the supply side of the problem. Like demand uncertainty, supply uncertainty also creates new motivations for a firm to be flexible. Hence, we intend to design multi-stage flexible supply chains which are robust to both demand and capacity fluctuations in a make-to-order production environment.

# Chapter 3

# Production, Process Investment and Survival of Debt Financed Startup Firms

## 3.1. INTRODUCTION

According to a study by U.S. Bureau of the Census, it is estimated that over 700,000 startups are formed every year in the US (Acs and Armington 1998). However, only a small proportion of these startups are able to grow their revenues and become profitable, and even a smaller proportion of these firms can show continued growth and make initial public offerings (Acs and Armington 2003). Startup firms are endowed with unique characteristics regarding their asset structure, organization type and growth orientation (Gifford 2005), and their operational decisions are often restricted by debt and other financial considerations (Berger and Udell 2005). In practice, most startups have very limited access to capital. Most of these firms take on debt and face immediate bankruptcy in case of a payback default. Hence, for startup managers it is necessary to generate adequate short-term cash flows by exploiting immediate business opportunities in order to keep up with the cash outflows and avoid bankruptcy.

Further, startups are not merely focused on survival. They are also interested in long-term growth. Indeed, most startup firms are concerned with their ability to invest in research and development (R&D) to improve their products and services (Bhide 2000). While such investments may not generate immediate cash flows, they are likely to improve the future prospects of the firm. In general, under bankruptcy risk, long-term growth and short-term survival are two intimately linked concerns. A key area of startup decision making, involving short term survival against long term growth, is the R&D

investment that is aimed at reducing the firm's unit production cost. In this chapter, we investigate the optimal operating decisions of a startup under debt which can invest in production to exploit the current business opportunities and generate short-term cash flows, or, it can also have a strategy under which it may also invest in process improvement to secure future market share and long-term profits.

We have conducted a series of interviews in order to understand the key considerations that affect process R&D investment decisions in startup settings. For example, Faradox is an Austin based startup which provides high energy density capacitors using its niche production process. Faradox views process development to reduce unit cost as a key competitive aspect of its business. During our interviews, the VP of business operations at Faradox stated that there was tremendous amount of on-going research in the field of high energy density capacitors and, it was quite likely that new competitors might enter the market by developing new and possibly more efficient production processes with lower unit costs. He also acknowledged that while process R&D was a key element of long term survival of Faradox, it was very costly and its return was highly uncertain. Further, while making investment decisions, predicting consumer demand also imposes a serious challenge for this company since the market is evolving and the customer base is hard to analyze. Allied issues have also surfaced at other Austin startups, AccuWater, AxsTracker, Big Foot Networks etc. Managers at these firms indicated that their production and investment decisions are affected by risk created by cash flow and technology performance. These concerns are consistent with descriptions of startup decision making in the extant literature (Bhide 2000, Shane 2007).

However, in the absence of a modeling framework, these managers are not able to assess their production and process improvement risks, and underlying tradeoffs, with precision. This has motivated our effort to formalize a class of factors that have been central for startup companies while choosing their operating policies regarding production and process investment in the presence of survival considerations: *uncertainties surrounding demand, technological performance and likely entry of competition*. These factors form the core of our model, and we examine their impact on the selection of operating (production and process investment) policies and the survival chances of the startup. For ease of exposition, model specification and analysis are developed in two stages. In the first stage, we analyze a base case (BC) regarding our operating decisions under deterministic demand with a two-period model. BC provides benchmarks for more involved models. In the second stage of our analysis, we allow stochastic realization of demand. This is termed as the stochastic demand and survival constraint (SDSC) case. With stochastic demand, profits after the first period are not guaranteed and a probabilistic survival constraint comes into play. SDSC is amenable to closed form solutions under limited conditions. Hence, we explore the underlying tradeoff between expected profit and bankruptcy risk through a combination of analytical and numerical solutions.

The contributions from our work are threefold. First, we specify a deterministic-demand model for a debt financed startup firm as a base case, and characterize an optimal *invest-all-or-nothing* policy which derives the conditions for investment in process improvement in order to enhance long-term profits. Second, with demand uncertainty and the consequent probabilistic survival constraint, we find that such a startup responds to the bankruptcy risk by increasing the investment threshold, i.e., the firm looks for more favorable market conditions to invest. Indeed, while balancing the bankruptcy risk with future growth opportunities, the startup may either behave *conservatively* (*aggressively)* by investing and producing less (more) than the BC level. In effect, a probabilistic survival constraint induces the startup to produce so as to create an *operational hedge* with respect to its process investment decision. Further, we offer a probabilistic survival measure that reflects the riskiness of the startup's operating decisions under the threat of bankruptcy. Third, we explore the impact of the existence of process investment opportunities, immediate profitability of the firm and limited debt availability on the optimal operating decisions and the allied survival chances. In addition, we have circled back to some startup managers and sought their feedback on our findings. We discuss the managerial implications of these findings while we synthesize and discuss our results.

The rest of the chapter is organized as follows. Section 3.2 provides a review of the related literature. In Section 3.3 we analyze the BC and characterize a closed form solution under deterministic demand. We extend our discussion to SDSC case in Section 3.4. In Section 3.5 we discuss limited debt capacity. Section 3.6 addresses managerial implications, limitations and concludes the chapter.

### 3.2. RELEVANT LITERATURE

Here we briefly review the streams of literature that are closely related to our work: investment in process R&D, startup operations and financing, and the entrepreneurial decision-making.

Investment in process R&D and allied cost reduction and capacity management decisions have long been key issues in the manufacturing technology management literature (De Groote 1988, Fine and Porteus 1989, Chand et al. 1996, Li and Rajagopalan 1998, Carrillo and Gaimon 2000, 2004). In addition, a closely aligned literature explores the technology adoption decisions (McCardle 1985, Milgrom and Roberts 1990, Fine and Freund 1990, Gupta and Loulou 1998). R&D investment under technology uncertainty in a single firm setting (Balcer and Lippman 1984, Kornish, 1999) and in competitive settings (Mamer and McCardle 1987) usually yield an "*all-or-nothing*" type of policy: adopt the current best technology if the gap between current and state-of-the-art technology exceeds a certain threshold. In this chapter, we will show that such "all-or-nothing" policies apply under limited conditions in startup settings to avoid bankruptcy. We illustrate that the incorporation of financial limitations in a startup setting lead to joint consideration of quantity and process investment decisions.

A recently growing body of literature deals with decision models involving the financing and operations of startups. Archibald et al. (2002) argue that if the startups are more interested in surviving than maximizing their profits, they should employ conservative strategies. On the contrary, we show that profit maximizing startups under a survival constraint could follow aggressive strategies when they have investment opportunities. Babich and Sobel (2004) provide a model to maximize the likelihood of a successful IPO for debt financed startups while Buzacott and Zhang (2004) adopt an asset based financing scheme for small and start-up firms. However, they do not explicitly model for strategic investment or competition which is central to the long term growth and survival of startups. Swinney et al. (2006) build the case on how competition between startup and established firms differs from competition between two established firms and show that a startup's preference to increase its survival affects the competition. However, they consider a single period model with a survival maximizing startup. Joglekar and Levesque (2009) analyze the distribution of venture capital between product related R&D and marketing, but do not account for either survival constraint or competition explicitly. Therefore, our research extends a growing literature on the theories of startup driven R&D and operational practices (Shane and Ulrich 2004).

Finally, an established topic of research in the entrepreneurship literature explores risk bearing as the key economic role of entrepreneurs. On one hand, Kihlstom and Laffont (1979) and Cramer et al. (2002) show that entrepreneurs are more risk seeking, and on the other hand, Halek and Eisenhaur (2001) finds that entrepreneurs do not differ from wage earners and further, are more risk-averse than others in some cases. In a closely related empirical work, Wu and Knott (2006) study the entrepreneur's decision of market entry combined with two distinct sources of uncertainty: demand uncertainty and uncertainty regarding entrepreneur's own ability. They argue that entrepreneurs are risk averse with respect to demand uncertainty and risk seeking with respect to performance uncertainty. Recently, Corbett and Fransoo (2008) also empirically investigate whether entrepreneurs follow the newsvendor logic and how their risk preferences affect their inventory decisions. We contribute to this stream of literature by explicitly modeling for operating decisions and bankruptcy which derives the risk preferences of the firm together with the investment opportunities, in a framework sequentially introducing technology, competition and demand risks.

In sum, the effect of cost reducing R&D on the profitability of firms has been studied extensively for established firms that are unencumbered by bankruptcy concerns. Further, cost reduction strategies adopted after the launch of a breakthrough product to maximize the profits is a relevant problem for many startup firms that take on debt and face the cash flow related threat of survival. However, this problem has not been explored formally. In the rest of this chapter, we set up and study a startup's production and cost reducing investment decisions.

### 3.3. THE BASE CASE

When making production and investment decisions, there are three key factors a typical startup considers: customer demand, startup's technological performance and competitive pressures (Shane 2007). To understand the interrelated impact of these factors on the operating decisions, we consider a two period model of a startup firm offering a single new product. This firm is financed by debt and must generate pre-specified level of profit after the first period to ensure survival into the second period.

The objective of the firm is to maximize the total of two-period profits under the survival requirement. In this section we focus on a base case (BC) model with no demand uncertainty, and study the impact of technological performance and competitive pressures on the startup's operating decisions. In Section 3.3.1, we start with a simple model which serves as a benchmark for our analysis. Then we sequentially introduce uncertainty associated with the firm's process investment and second period competition in Section 3.3.2 and Section 3.3.3, respectively. For generality, we use the terms "return on process investment" and "technological performance" interchangeably throughout this manuscript.

### 3.3.1. A Benchmark Model

We start with some key assumptions to set up our model.

**Assumption 3.1.** *Product R&D is frozen at the beginning of the first period, i.e. at market entry.*

At least half of the startup firms in the US enter the market with a novel product (GEM Report 2007), and many of these firms continue to invest into product development effort. We do not allow for such investments, so that our analysis is not confounded by the evolution of product quality.

**Assumption 3.2.** *The startup is financed by bank loans with a constant positive interest rate.*

We consider a bank-financed startup, but our models and results trivially extend to bootstrapped startups. The interest rate is constant and positive, and upon fully paying its previous debt the startup can borrow in each period to cover its production cost and R&D investment. In general, once the loan is granted to a small firm, the loan terms including interest rate and loan limit are determined by industry practices and market conditions and do not depend on the conditions of the borrower firm (Petersen and Rajan

1994). For ease of exposure, we consider the effect of an explicit loan limit as an extension in Section 3.6. There is no time discount on the profits of the second period. The analysis is unchanged, if we consider a discount parameter between periods.

**Assumption 3.3.** *The startup goes bankrupt and gets liquidated if it cannot pay its debt at the end of each period.*

Most startups have limited access to capital markets and cannot raise additional capital other than their initial funds (Chrisman et al. 1998). In particular, informational asymmetries between the owners of the startups and the investors, and the uncertainties about the future prospects of the startup severely limit the firm's access to capital markets (Shane 2007). Hence, most new businesses are built with limited capital and face immediate bankruptcy in case of a default.

Based on these assumptions, the timing of the game is as follows. In the first period, the startup firm is a monopoly operating with a unit production cost of $c_1$ and receives funds, $y_1$, with an interest rate of *r*. It allocates these funds at the beginning of the first period between production capacity, $q_1$, and process R&D investment, *A*, which will in return linearly reduce the unit production cost in the second period to $c_2\left(A,\beta\right)=c_1-\beta A$, where $\beta$ denotes the return on investment (Gupta and Loulou 1998). At the beginning of the consecutive period, the startup realizes revenues from sales, observes reduction in unit cost due to process investment, and makes the debt payments. In case, the revenues are not sufficient to cover the debt obligations, the firm goes bankrupt and gets liquidated. If the debt is paid in full then the firm goes into the second period and could receive a second round of funding, $y_2$ to invest in production, as this is the final period. We adopt a linear inverse demand function for the startup's product as $p_t(q_t)=\theta-q_t$, *t*=1, 2, where $\theta$ denotes the constant market size. We offer the following as the benchmark model:

$$\pi_1 = \max_{q_1, A, y_1 \geq 0} (p_1(q_1) - c_1)q_1 - ry_1 - A + \pi_2(q_2; A, \beta)$$

subject to $c_1 q_1 + A \leq y_1$ (3.1a)

$$(p_1(q_1) - c_1)q_1 - ry_1 - A \geq 0 \quad (3.1b)$$

where

$$\pi_2(q_2; A, \beta) = \max_{q_2, y_2 \geq 0} (p_2(q_2) - c_2(A, \beta))q_2 - ry_2$$

subject to $c_2 q_2 \leq y_2$ (3.1c)

In this model, (3.1a) and (3.1c) represent the financial constraints in the first and second periods, respectively, such that the total expenditures of the firm in each period are limited by the amount of money borrowed. (3.1b) denotes the *survival* constraint requiring that the money borrowed in the first period should be paid back with interest at the end of the period. Based on our model assumptions, (3.1a) and (3.1c) must be binding. Therefore, we re-state (3.1) as follows:

$$\pi_1 = \max_{q_1, A \geq 0} (p_1(q_1) - (1+r)c_1)q_1 - (1+r)A + \pi_2(q_2; A, \beta)$$

subject to $(p_1(q_1) - (1+r)c_1)q_1 - (1+r)A \geq 0$ (3.2)

where $\pi_2(q_2; A, \beta) = \max_{q_2 \geq 0} (p_2(q_2) - (1+r)c_2(A, \beta))q_2$

Assuming that the return on investment $\beta$ is constant and equal to $\mu$, we characterize the operating decisions and profits in Proposition 3.3.1. We use the superscript *bc* to denote the benchmark case. Later we will use *c* for the case with competition and *t* for the case with uncertain return on investment. (See the Appendix for proofs of the lemmas, propositions and corollaries unless stated otherwise.)

**Proposition 3.3.1:** *When demand and the return on investment are deterministic, the startup's total profits are maximized at the monopoly quantity,* $q^* = q_m = \dfrac{\theta - (1+r)c}{2}$.

*Following, the process R&D investment of the startup is bounded by the discounted monopoly profits,* $A_{\max}=\frac{\pi_m}{(1+r)}=\frac{(\theta-(1+r)c)^2}{4(1+r)}$ *. Assuming that the cost cannot be driven to zero, the optimal process investment for the startup,* $A^*$*, and the optimal expected profits,* $\pi^*$*, are given by the following:*

$$A^*=\begin{cases} A_{\max} & if \quad \Delta^{bc}\geq 0 \\ 0 & o/w \end{cases}, and \quad \pi^*=\begin{cases} \left(\frac{\theta-(1+r)(c-\mu A_{\max})}{2}\right)^2 & if \quad \Delta^{bc}\geq 0 \\ \frac{(\theta-(1+r)c)^2}{2} & o/w \end{cases}$$

*where* $\Delta^{bc}=\mu^2(\theta-(1+r)c)^2+8\mu(\theta-(1+r)c)-16$.

A close examination of (3.2) reveals that the startup's optimization problem is partially separable in production quantity and process investment. Consequently, we find in Proposition 3.3.1 that it is optimal to produce the monopoly quantity, and the monopoly profits limit the investment amount due to the survival constraint.

To explain the investment decision, we define the firm's propensity to invest in process improvement as $\Delta$. In particular, if $\Delta<0$, then the firm does not invest in process improvement. Therefore, the profits in each period are identical and equal to the monopoly profits. However, if the firm's propensity to invest is sufficiently high, $\Delta\geq 0$, then it would allocate all of its funds in process improvement and make zero net profits after the debt payments, in the first period. It could later generate enough revenues with the second period sales to compensate for the missed earnings of the first period. In particular, the firm either chooses not to invest, $A^*=0$, or if it chooses to invest, it invests the maximum possible amount, $A_{max}$, which would maximize its profits without going bankrupt. Therefore, the optimal process investment decision can be characterized by an *invest-all-or-nothing* threshold policy. Further, as the mean return on investment and market size increase, the firm's propensity to invest also increases.

### 3.3.2. Technology Uncertainty

So far, we have assumed that process investment reduces the future unit cost of the firm by a deterministic amount. Nevertheless, for startups with niche processes like Faradox and BigFoot Networks return on process investment is inherently uncertain. To take this into consideration, we extend our discussion in the benchmark case to consider the impact of the return on investment uncertainty on the startup's operating decisions and profits. In particular, for every dollar invested, we assume that the second period cost is reduced by a random amount described by $\tilde{\beta}$, with a known distribution function, $\psi(\tilde{\beta})$, with a mean of $\mu$ and a variance of $\sigma^2$. The following proposition characterizes the optimal production and investment decisions in the benchmark model with technology uncertainty.

**Proposition 3.3.2:** *When demand is deterministic, but the return on investment is uncertain, then the startup's optimal production quantity is equal to the monopoly quantity. And, the optimal process investment and profits are, respectively, given by*

$$A^* = \begin{cases} A_{\max} & if \quad \Delta^{tu} \geq 0 \\ 0 & o/w \end{cases}, and \ \pi^* = \begin{cases} \dfrac{(\theta-(1+r)(c-\mu A_{\max}))^2 + (1+r)^2\sigma^2 A_{\max}^2}{4} & if \quad \Delta^{tu} \geq 0 \\ \dfrac{(\theta-(1+r)c)^2}{2} & o/w \end{cases}$$

*where* $\Delta^{tu} = (\sigma^2+\mu^2)(\theta-(1+r)c)^2 + 8(\theta-(1+r)c)\mu - 16$.

With return on investment uncertainty the firm's propensity to invest becomes larger than the case with no uncertainty in return on investment, i.e., $\Delta^{tu} = \Delta^{bc} + (\theta-(1+r)c)^2\sigma^2$. Further, the firm profits are also non-decreasing in $\sigma^2$, so when selecting among production technologies, the startup prefers technologies with more variable return compared to the ones with relatively certain returns. This may seem like a counterintuitive result, but if this technology adoption proves to be successful, then the firm could obtain significant cost reduction and have a major increase in profits. That is, the startup disproportionately benefits from upside deviation in return on process investment. In our model, this is driven by the convex monopoly profits, $(\theta-(1+r)c)^2/4$, with respect to the unit cost in the second period.

### 3.3.3. Competition

In this section, we study the case with competition. The sequence of events is precisely the same as the case without competition. The difference is that at the beginning of the second period a competitor with an identical product enters the market and firms play a Cournot game where the competitor's best response quantity is denoted by $q_c$. The updated sequence of events and the startup's decisions are summarized in Figure 3.1.

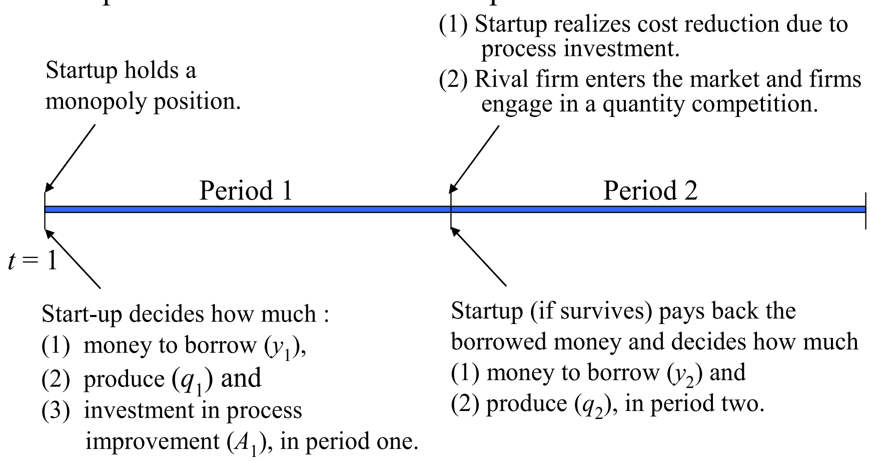



Figure 3.1: Sequence of Events and Decisions in a Two Period Model with Competition

When a competitor is to enter the market, the startup may not fully know the entrant's production system for a new product, but it may know the competitor's cost through a probability distribution function. Indeed, Faradox Inc., a producer of high energy-density capacitors, mentioned in our interview that there was tremendous amount of theoretical research in the field of capacitor technologies and it was likely that someone might enter their market by developing a new process to produce high energy-density capacitors. Hence, from the perspective of Faradox, the efficiency of the prospective competitor in the future is highly uncertain and exogenous.

To incorporate this into our benchmark model, we assume that the unit variable cost of the competitor, $\tilde{\xi}$, is distributed with a probability density function of $\phi(\tilde{\xi})$, and has a mean of $\lambda$ and a variance of $\tau^2$. For ease of exposure, we exclude technology uncertainty in this section, but our findings here also trivially extend to the case with both technology uncertainty and competition. The following proposition characterizes the startup's optimal production and investment decisions for the benchmark model with competition.

**Proposition 3.3.3:** *Under deterministic demand and return on investment, when there is competition in the future period, then the startup's optimal production quantity is equal to the monopoly quantity. The optimal process investment and the optimal profits are, respectively, given by*

$$A^* = \begin{cases} A_{\max} & if \quad \Delta^c \geq 0 \\ 0 & o/w \end{cases}, \textit{and}$$

$$\pi^* = \begin{cases} \frac{1}{9}\left\{(\theta - 2(1+r)c + \lambda)^2 + 4A_{\max}\left\{\mu(\theta - 2(1+r)c + \lambda) + \mu^2 A_{\max}\right\} + \tau^2\right\} & if \quad \Delta^c \geq 0 \\ \frac{(\theta - (1+r)c)^2}{4} + \frac{1}{9}\left\{(\theta - 2(1+r)c + \lambda)^2 + \tau\right\}^2 & o/w \end{cases},$$

*where* $\Delta^c = (\mu(\theta - (1+r)c) + 2)^2 - 4\mu((1+r)c - \lambda) - 13$.

From Proposition 3.3.3, we observe that the propensity of the startup to invest increases with the expected unit cost of the competitor, i.e., $\partial \Delta^c / \partial \lambda > 0$. In other words, when faced with a strong competitor, the startup is less willing to invest since the benefits of investment is reduced under competition. According to our investment policy, the variance of the competitor's cost would have no effect on the investment decision so long as the quantity response function is linear in the realization of competitor's cost.

However, since Cournot profits are convex in the competitor's cost, the optimal profits increase as the strength of competition gets more variable because the startup disproportionately benefits from high cost entrants.

Comparing the firm's propensity to invest with and without competition for various levels of competitor's unit cost, we can further explain the impact of the strength of future competition on the firm's propensity to invest:

**Corollary 3.3.1:** *In the presence of competition the startup's propensity to invest increases compared to the benchmark case, if the expected competitor is relatively weak. In particular:*

*i)* $\Delta^{bc} \geq \Delta^{c}$ *if* $\lambda \leq \theta - 7/4\mu$,

*ii)* $\Delta^{bc} \leq \Delta^{c}$ *if* $\lambda \geq \theta - 7/4\mu$.

Corollary 3.3.1 shows that the firm may find it optimal to invest in the presence of competition when it is better off with no investment in the benchmark case, i.e., $\Delta^{bc} < 0 < \Delta^{c}$. Therefore, the shadow of future competition may encourage investment by the startup depending on the expected strength of the competitor.

## 3.4. The Stochastic Demand and Survival Case

In the BC, we studied the startup firm's operating decisions under deterministic demand. However, in most cases the startup would have very limited information about the demand, especially for a brand new product. In this section, we examine our model with stochastic demand in each period, and replace the deterministic survival constraint of the BC with a probabilistic survival requirement.

### 3.4.1. The Model with Stochastic Demand and Survival

In this case, we assume a demand shock, $\tilde{\varepsilon}_t$, in each period $t$ ($t$ = 1, 2) with a normal probability density function, $\varphi(.)$, with mean zero and variance $v^2$, and cumulative distribution function, $\vartheta(.)$. The minimum profit level required for the survival denoted by $\underline{\pi}$ is exogenous and includes the overhead costs like rents and wages. We define the first period net expected monopoly profits, $\pi_m - \underline{\pi}$, as the immediate economical viability of the firm (Note that the expected monopoly profits is given by $\pi_m = E[(p(q_m, \tilde{\varepsilon}_1) - (1+r)c)q_m])$. Under the SDSC case with competition, technology uncertainty and stochastic demand, the two-period expected profit maximization problem of the startup is given as:

$$
\begin{aligned}
& z^* = \max_{q_1, A} E_{\tilde{\varepsilon}_1} \left\{ (p(q_1; \tilde{\varepsilon}_1) - c_1) q_1 - r(c_1 q_1 + A) - A \right\} + E_{\tilde{\varepsilon}_1, \tilde{\xi}, \tilde{\beta}} \pi_2 (A; \tilde{\varepsilon}_1, \tilde{\xi}, \tilde{\beta}) \\
& \qquad s.t. \quad q_1, A \geq 0 \\
& where \\
& \pi_2 (A; \varepsilon_1, \xi, \beta) = \max_{q_2, I} E_{\tilde{\varepsilon}_2} \left\{ (p_2(q_2 + q_c, \tilde{\varepsilon}_2) - (1+r) c_2 (A, \beta; A_0)) q_2 \right\} \\
& \qquad\qquad\qquad s.t. \quad \pi_1(\varepsilon_1) - \underline{\pi} \geq M(I-1) \\
& \qquad\qquad\qquad\qquad q_2 \leq MI \\
& \qquad\qquad\qquad\qquad q_2 \geq 0, I \in \{0,1\},
\end{aligned}
\tag{3.3}
$$

where $I$ is the first period survival indicator (= 1 if the firm survives the first period) and $M$ is a large number. In (3.3), the constraints in the second stage of the problem only hold if the startup has survived the first period. In particular, unless the first period profit for the startup meets the minimum level required for survival $\left(\pi_1 < \underline{\pi}\right)$, the startup cannot play the second period quantity game. In this case, the survival indicator variable $I$ in (3.3) has to be zero and consequently, $q_2$ is also forced to zero.

We solve the optimization problem in (3.3) by backward induction. In the second period, firms play a Cournot game to maximize their expected profits. Hence, the startup's equilibrium profit, if it could play the second period game, is given by $\pi_2\left(A; \varepsilon_1, \xi, \beta\right) = \left(\left(\theta + \overline{\varepsilon}_2\right) + \xi - 2\left(1+r\right) c_2\left(A, \beta\right)\right)^2 / 9$ where $E(\varepsilon_2) = \overline{\varepsilon}_2$. By assumption, $\overline{\varepsilon}_2 = 0$. For the startup to participate in the second period, first it has to survive in the first period only if $\pi_1(\varepsilon_1) \geq \underline{\pi}$. After substituting the optimal second period solution, the startup's problem in (3.3) becomes

$$z^* = \max_{q_1, A \geq 0} E_{\tilde{\varepsilon}_1} \left\{ (p(q_1, \tilde{\varepsilon}_1) - c_1) q_1 - r\left(c_1 q_1 + A\right) - A \right\} + E_{\tilde{\xi}, \tilde{\beta}, \tilde{\varepsilon}_1} \left\{ \frac{(\theta + \tilde{\xi} - 2(1+r) c_2 (A, \tilde{\beta}))^2}{9} \mid \pi_1(\tilde{\varepsilon}_1) \geq \underline{\pi} \right\} \qquad (3.4)$$

Unlike the BC, the maximization problem is not separable in production and investment decisions, and it is non-convex. Nevertheless, we can still prove the following important relationship for the optimal decisions.

**Proposition 3.4.1:** *When demand is stochastic, the startup firm in the first period either adopts a conservative operating policy by producing and investing less than the monopoly levels i.e.,* $q^* \leq q_m = \dfrac{\theta - (1+r)c}{2}$, $A^* \leq A_m = \dfrac{\pi_m - \underline{\pi}}{1+r}$, *or an aggressive operating policy by producing and investing more than the monopoly levels, i.e.,* $q^* \geq q_m$, $A^* \geq A_m = \dfrac{\pi_m - \underline{\pi}}{1+r}$.

Proposition 3.4.1 provides an interesting risk based justification linking production and investment decisions of a startup under stochastic demand and bankruptcy risk. The firm is aggressive in investment decision $\left(A \geq A_m\right)$, if and only if it is also aggressive in production $\left(q^* \geq q_m\right)$. Or, the firm is conservative in investment decision $\left(A < A_m\right)$ if and only if it is also conservative in production $\left(q^* < q_m\right)$. If an aggressive investment is planned, then the expected cash flows under the monopoly production plan is not sufficient to cover the debt payments. Hence, the firm increases its production quantity above the monopoly level so as to benefit from upside demand realizations and to increase its survival chances and conversely, a conservative investment reduces production below the monopoly level.

In the following proposition, we establish the intimate connection between the optimal operating policy of the firm and the survival probability.

**Proposition 3.4.2:** *An optimal operating policy is aggressive (conservative) if and only if its survival probability,* $P \doteq 1-\vartheta_{\varepsilon_1}\left(\frac{\underline{\pi}+(1+r)A}{q}+q-\theta+(1+r)c\right)$, *is less (more) than fifty percent.*

Proposition 3.4.2 provides an equivalent survival-based definition for optimal aggressive and conservative operating decisions. That is, optimal operating policies that survive less (more) than 50% chances always involve producing and investing more (less) than the monopoly levels, and vice versa. This implies that an aggressive firm is expected to go bankrupt on average while a conservative firm is expected to survive. In general, an operating policy is considered to be riskier as the survival probability decreases.

In the reminder of this section and in Section 3.5, we explore the factors that that drive the optimal operating decisions of the startup under stochastic demand. In Proposition 3.4.1 we implicitly assume that the startup would find an investment opportunity. However, that may not be the case. Corollary 3.4.1 considers the impact of the existence of investment opportunities (with positive NPV) on the operating decision of economically viable startup firms. Recall that immediate economical viability means the firm's first period net expected monopoly profits are non-negative.

**Corollary 3.4.1:** *Suppose the startup firm is immediately economically viable in the first period, i.e.,* $\pi_m - \underline{\pi} > 0$. *Then,*

*i) if there are no process investment opportunities,* $A = 0$, *the firm always adopts a conservative operating policy. That is, the firm produces less than the monopoly quantity.*

*ii) if there is an opportunity for process investment,* $A \geq 0$, *then the startup may either adopt an aggressive or conservative operating policy.*

According to Corollary 3.4.1, when there are no investment opportunities, immediately economically viable startups always choose a conservative operating policy.

To better illustrate our finding, we examine a simple situation with no minimum level of profits, $\underline{\pi}$ = 0 and we let the demand shocks in each period ($\varepsilon_t$ for $t$ =1, 2) be uniformly distributed with U[-$b$, +$b$]. Then, the optimization problem takes the following form:

$$\max_{q\geq 0, A\geq 0} f(q,A) = \left[(\theta - q)q - (r+1)(cq+A)\right] +$$

$$\left[\frac{\tau^2 + (\theta+\lambda-kc)^2 + 2(\theta+\lambda-kc)k\mu A + k^2(\mu^2+\sigma^2)A^2}{9}\right]\left[\frac{b - \left(\frac{(1+r)A}{q} + (1+r)c + q - \theta\right)}{2b}\right]$$

When there are no investment opportunities ($A$ = 0), $f(q,0)$ is concave in $q$ and the optimal quantity is given by $q^* = \frac{\theta-(1+r)c}{2} - \frac{1}{4b}\left(\frac{(\theta+\lambda-2(1+r)c)^2}{9}\right) < q_m$, which agrees with our finding that in the absence of investment opportunities, the firm always behaves conservatively. The positive second term of the optimal quantity, $q^*$, above represents the under-production amount due to stochastic bankruptcy risk in order to increase the probability of survival. In particular, if the bankruptcy risk is to be removed from the decision framework, the firm simply produces the first best production level, i.e., the monopoly quantity. That is, the bankruptcy risk drives an economically viable startup to adopt a conservative policy in the absence of investment opportunities. In addition, startups with high expected future prospects, such as a large market base, an already efficient process technology or a relatively weak competitor, focus more on survival in anticipation of future profits. That is, they deviate more from their first best operating plans and choose a more conservative policy.

We will numerically investigate the optimal operating policies for this case in Section 3.5. We now turn our attention to a firm that is not economically viable in the first period. We implicitly assume that the firm is economically viable over the planning horizon. Otherwise, it is optimal to liquidate the firm at time zero.

**Corollary 3.4.2:** *Suppose the startup firm is not immediately economically viable in the first period, i.e.,* $\pi_m - \underline{\pi} \leq 0$*, then the firm always adopts an aggressive operating policy, regardless of the existence of investment opportunities.*

When the firm is not immediately viable, e.g., due to high operating costs relative to immediate profits, its survival is contingent on the upside deviations in market demand. To benefit from these upside deviations and survive, the firm should increase its production above the monopoly quantity. Consequently, even with no investment opportunities the firm would always choose an aggressive operating policy. We summarize the effects of the immediate economical viability and investment opportunities on the operating policy of the startup in Table 3.1.

Our results show that an immediately viable startup with investment opportunities may either adopt a conservative or an aggressive operating policy depending on the market parameters. To further investigate this case and the impact of market parameters on the optimal operating decisions, we present a comprehensive computational analysis in the next section. We also note that this case is not analytically tractable. There are several reasons for this, including that the objective function in (3.4) is neither jointly convex nor concave in $q_1$ and $A$ for all feasible set of parameter settings.

| | **Immediately Viable Startup** ($\pi_m - \underline{\pi} > 0$) | **Immediately Non-viable Startup** ($\pi_m - \underline{\pi} > 0$) |
|---|---|---|
| **With Investment Opportunities** | Conservative or aggressive operating policy depending on market parameters | Aggressive operating policy |
| **With No Investment Opportunities** | Conservative operating policy | Aggressive operating policy |

Table 3.1: Operating Policy of the Startup

### 3.4.2. Computational Analysis

In this section, we focus on a set of numerical analyses to illustrate the impact of key market factors (demand uncertainty, technological performance, competition and minimum required level of profits), on the optimal operating policies (production and process investment) of the immediately viable startups with investment opportunities. We also provide insights that link the BC to SDSC.

#### *3.4.2.1. Design of Numerical Experiments*

Our design of experiment focuses on the optimal survival probability as the relevant measure of the risk taken by the firm. The survival probability is an endogenous variable determined by the firm's production and investment policy. Recall that a conservative policy survives with probability more than 50% while an aggressive strategy bankrupts with probability more than 50%. And, a conservative (aggressive) policy involves producing less (more) than the monopoly quantity and investing less (more) than the expected net monopoly profits.

We begin with examining the impact of mean return on investment in Section 3.4.2.2 in an experimental setup that has no competition, deterministic return on investment and zero interest rate as in BC. In this case the optimization problem in (3.4) reduces to

$$\max_{q,A\geq 0} f(q,A) = \left[(\theta-q)q-cq-A\right]+\frac{\left(\theta-(c-\mu A)\right)^2}{4}\left(1-F_{\varepsilon_1}\left(\frac{\underline{\pi}+cq+A}{q}+q-\theta\right)\right)$$

Following, we explore the impact of technological uncertainty and competition on the operating decisions in Section 3.4.2.3 and 3.4.2.4, respectively. For ease of exposition, throughout our numerical analysis we fix the market size and initial unit cost ($\theta$ = 10, $c$ = 7), so $q_m$ = 1.5 and $\pi_m$ = 2.25. The standard deviation of demand shock is set to $v$ = 1.2. The impact of different levels of $v$ is investigated in Section 3.4.2.5. We present a selective set of our results, but we have tested and confirmed similar results with entire sets of values that the model parameters can take.

### *3.4.2.2. Benchmark Case with Stochastic Demand*

In this section we examine the impact of mean return on investment, $\mu$, and the immediate economical viability of the firm through minimum required profits, $\underline{\pi}$. Figure 3.2 illustrates the optimal operating decisions and the associated survival probabilities for all reasonable levels of $\mu$. We observe in Figure 3.2 that it is optimal to invest if $\mu > 0.9$. Then, as in the BC, a threshold type of investment policy is optimal, but the policy does not have an invest-all-or-nothing structure. As the firm's potential efficiency in cost reduction increases, the firm raises its investment amount in process technology leading to riskier operating decisions with lower survival probability. Indeed, aggressive investment becomes the optimal policy for $\mu > 2.3$. In addition, the production quantity may either increase or decrease with the investment level to create an operational hedge in response to optimal investment decision.

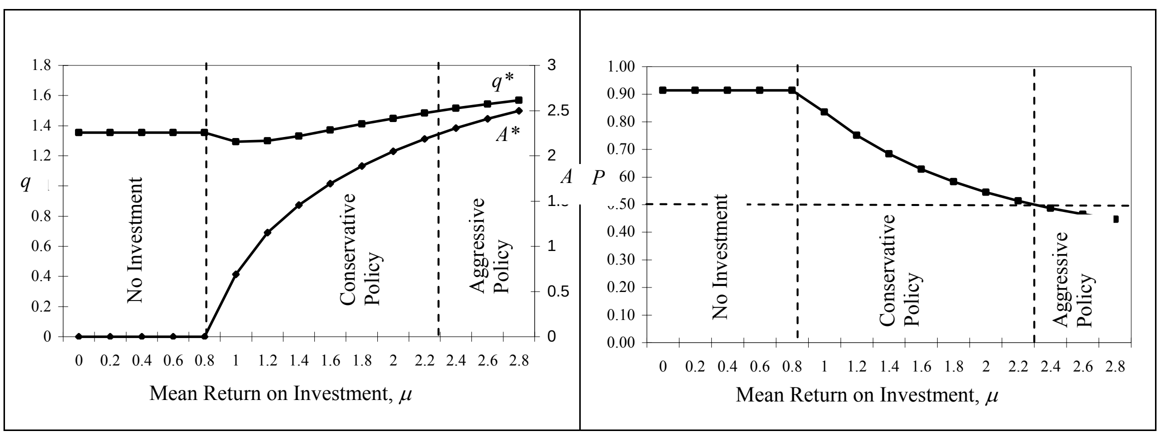



Figure 3.2: Optimal Operating Decisions and Survival Probability as a Function of $\mu$ for $\underline{\pi} = 0$

Figure 3.3 illustrates the interactive impact of $\mu$ and $\underline{\pi}$ on the optimal policy. We observe that 'no investment' region expands as $\underline{\pi}$ increases. In particular, when $\underline{\pi}$ is high, investment creates a very high bankruptcy risk consuming the limited short-term profits of the firm. Therefore, the firm avoids investment. On the hand, if $\underline{\pi}$ is low, then the firm is expected to have cash in the future and, it may invest some of this cash in process improvement without diminishing its survival probability. Further, depending on its mean return on investment, the optimal policy is either conservative or aggressive.

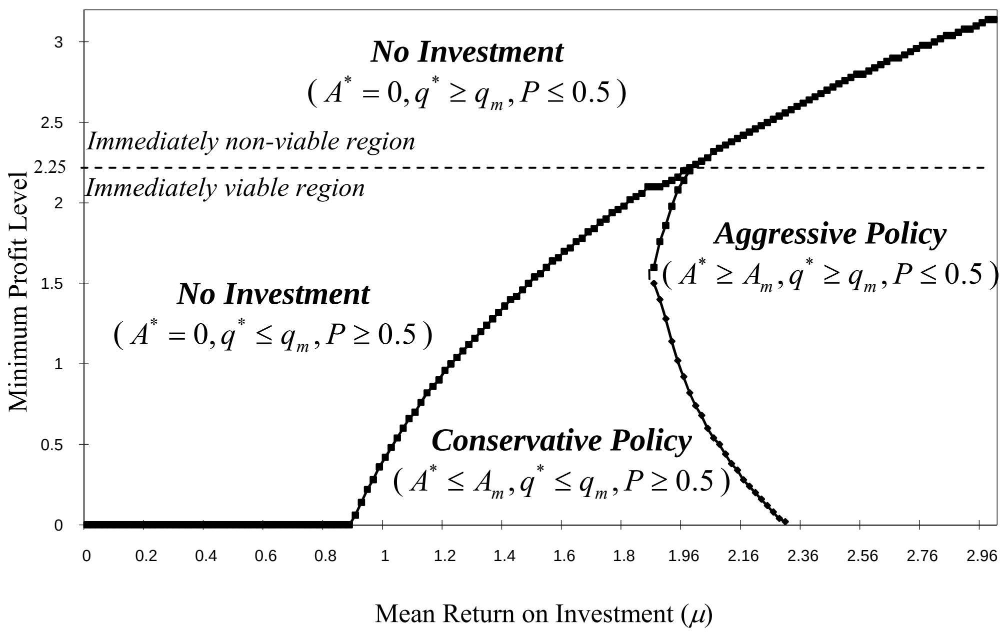



Figure 3.3: Interaction of $\mu$ and $\underline{\pi}$ under SDSC Case

#### *4.4.2.3. Technology Uncertainty*

In the previous section we discuss the impact of mean return on investment on the operating decisions of the firm, but we did not examine the associated uncertainty. We complete this discussion by illustrating the effect of technology uncertainty, $\sigma$, in Figure 3.4. For ease of discussion, in the reminder of this section we set $\underline{\pi} = 0$, but similar results can be obtained for other values. Under deterministic demand, from Proposition 3.3.2, we know that as the return on investment gets more variable and the chances of upside deviations increase, the firm is more willing to invest. Similarly, when demand is uncertain, Figure 3.4 leads to the observation that for a given level of $\mu$, an increase in technology uncertainty decreases the survival chances of the firm, by inducing more aggressive operating decisions with higher production and investment levels.

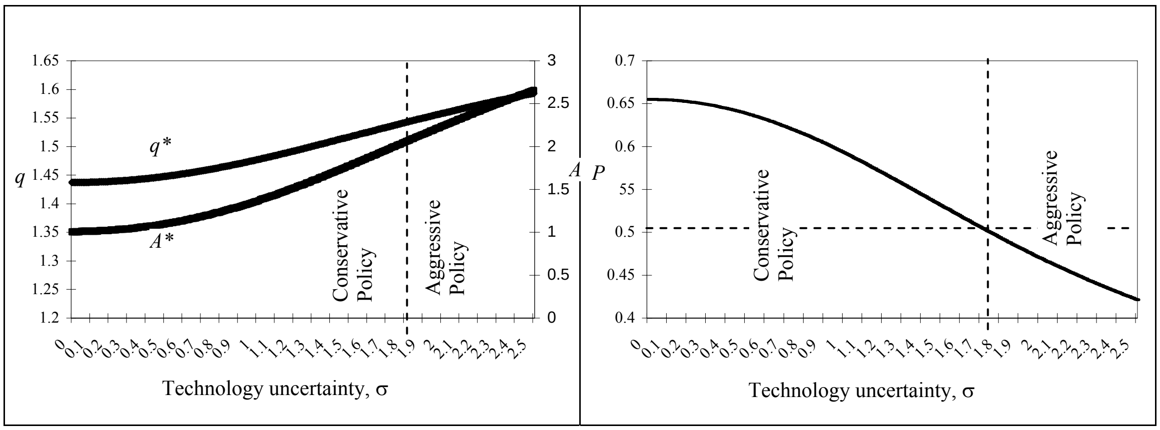



Figure 3.4: Optimal Production Quantity and Process Investment as a Function of $\sigma\,(\mu = 1.5)$

### *3.4.2.4. Competition*

In this section we explore the impact of competition on the operating decision of the firm under stochastic demand and deterministic return on investment. In Figure 3.5 we present the optimal operating decisions as well as the associated survival probabilities as the competitor's expected cost $\lambda$ changes for a given level of return on investment $\mu$.

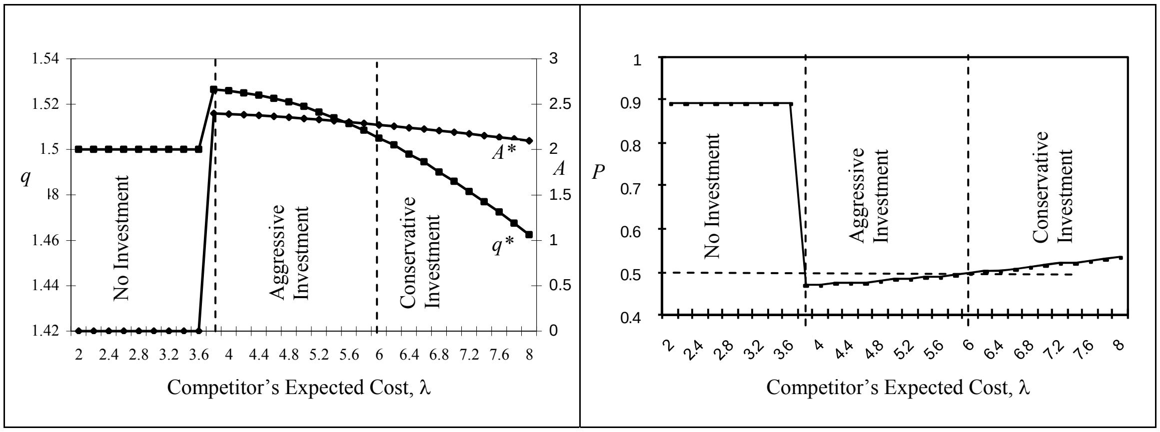



Figure 3.5: Optimal operating decisions and survival probability as a function of λ ($\mu$ = 1.5).

Similar to the BC, Figure 3.5 shows that the firm starts investing when the competitor is sufficiently weak and benefits from investment in the future period. However, the firm may invest (and produce) either aggressively or conservatively depending on the level of $\lambda$. Further, for a given level of $\mu$ it does not necessarily keep raising its investment amount as the competition gets weaker because although the expected marginal second period profit of the firm is increasing with $\lambda$, a higher investment amount also increases firm's exposure to bankruptcy. Figure 3.5 illustrates this tradeoff that firm chooses a conservative investment policy when faced with very weak competitors to control the bankruptcy risk.

Figure 3.6 further explores the interrelated impact of competition and the mean return on process improvement on the optimal operating policy of the firm. The startup makes no process investment if it is not efficient to engage in competition with a relatively strong competitor.  A conservative policy is chosen when the future entrant would not intensify competition because it has relatively high cost production process. Further, an aggressive policy is adopted when the startup is sufficiently efficient in cost reduction and the competitor is neither too strong nor too weak. In this case, the second period profits are distributed more equally between the firms. Hence, by following an aggressive strategy (if it is not too costly) the startup may significantly increase its share of expected profits in the second period and obtain a strong future market position.

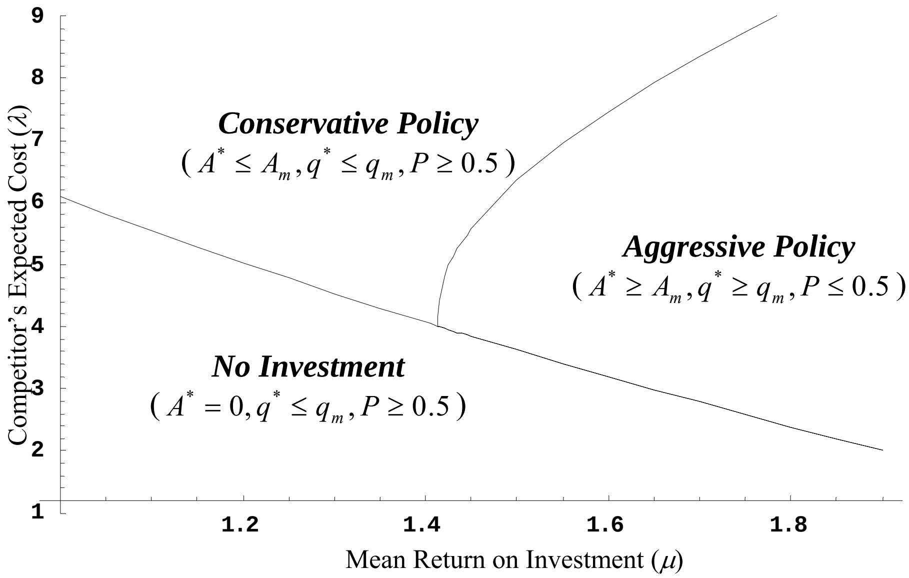



Figure 3.6: Interaction of $\mu$ - $\lambda$ under SDSC Case

### *3.4.2.5. Demand Uncertainty*

Demand uncertainty is an exogenous factor influencing the bankruptcy risk. Recall that with demand uncertainty investment amount may be either less or more than monopoly investments. Figure 3.7 illustrates that demand uncertainty shrinks investment regions when it is optimal to start investing in process R&D, and the thresholds for process investment in the SDSC are higher than the BC*,* ceteris paribus.

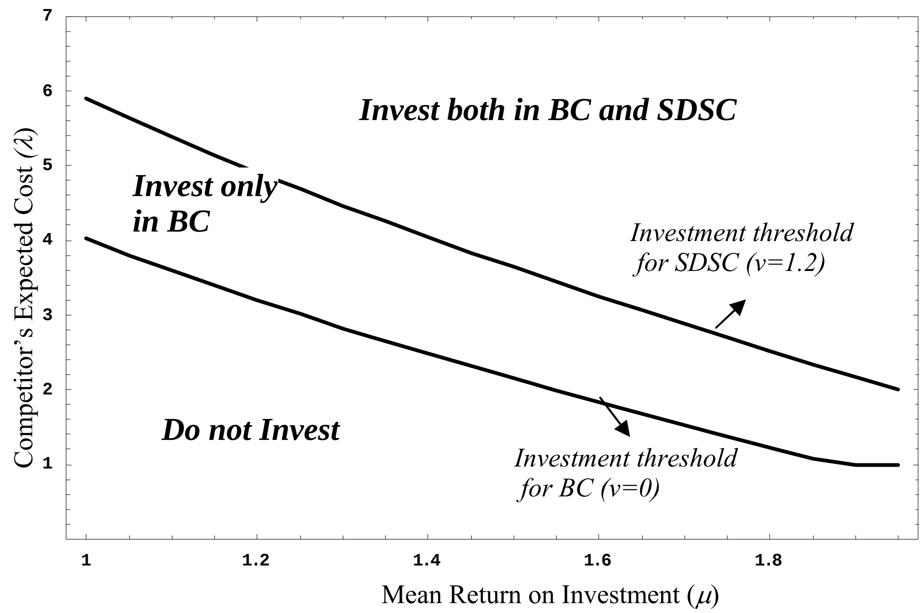



Figure 3.7: Partitioning of the Process Investment Space under the BC and SDSC

Figure 3.8 shows the impact of demand variability and mean return on investment on the optimal policy when there is no competition and technological variability. As shown in Figure 3.8, when $v$ is very low, the startup either chooses not to invest or invests conservatively. A higher variability of demand provides the firm with the opportunity of survival under aggressive investment plans. Hence, aggressive policies are only possible if demand is sufficiently variable to provide high demand and the firm is efficient in cost reduction. Indeed, when demand is deterministic as in the BC, aggressive policies are infeasible. These observations combined with our earlier findings support that demand variability is necessary to induce immediately viable firms to increase their investment amount and adopt aggressive policies if the increased second period profits due to aggressive investment compensate the excess risk taken by the firm. Also note that Figure 3.8 generalizes Figure 3.2 which is constructed for $v$ =1.2 only.

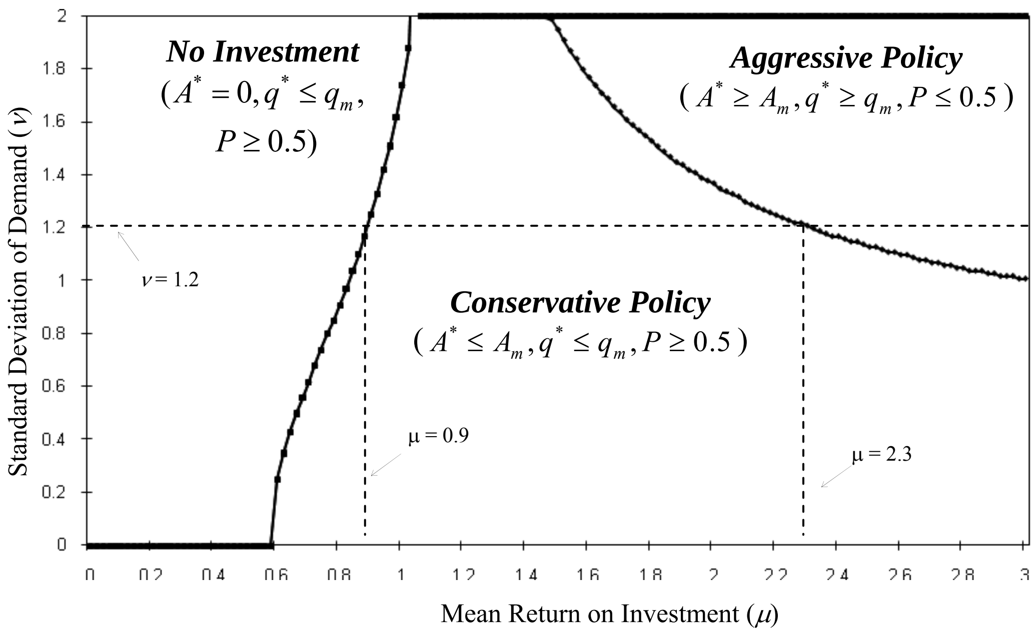



Figure 3.8: Interaction of $\mu - \nu$ under SDSC Case

### 3.5. EXTENSION: DEBT CAPACITY

To isolate the impact of bankruptcy risk, we have assumed throughout the chapter that the startup firm is able to borrow enough to finance its optimal operating policy in the first period. In this section we introduce a debt capacity, $L$, which limits the total cash available to the firm ($cq + A \le L$), and examine its effect on the risk preferences of the startup. In Proposition 3.5.1 we characterize the impact of debt capacity on the base case results under deterministic demand.

**Proposition 3.5.1:** *Under deterministic demand and return on investment, with no future competition,*

*i) if $L \ge cq_m + A_{\max}$, the debt capacity is never binding.*

*ii) if $L \le cq_m$, the debt capacity is always binding.*

*iii) if $cq_m < L < cq_m + A_{\max}$, the debt capacity may or may not be binding depending on the market parameters.*

If the debt capacity is larger than the maximum amount of cash that may be needed by the firm, i.e., $L \geq cq_m + A_{\max}$, additional cash has no value to the firm. In this case, the optimal operating decisions are characterized by Proposition 3.3.1 and the firm's propensity to invest is unaffected. However, if debt capacity is not sufficient to finance the monopoly production level, the firm may invest additional capital into production and increase profits. Also, when the debt capacity is moderately tight $\left(cq_m < L < cq_m + A_{\max}\right)$, the firm may benefit from additional cash if investment is optimal when there is no debt capacity. Further, in the following corollary we discuss the firm's propensity to invest with a binding debt capacity.

**Corollary 3.5.2:** *Under deterministic demand, when there is a binding debt capacity, startup's propensity to invest decreases.*

We show (Proposition 3.3.1) that the startup's operating policy with no debt capacity can be described as an "all-or-nothing" policy, i.e., whether to invest nothing or to invest all of the net monopoly profits, $A_{\max}$. However, under a binding debt capacity, the startup can never finance to invest as much as $A_{\max}$. Besides, since the marginal return on investment is increasing in *A*, reducing the maximum investment level decreases the benefits of scale economies in investment and hence, decreases the firm's propensity to invest. Figure 3.9 illustrates the firm's propensity to invest $\Delta^l$ as a function of the debt capacity and compare it to $\Delta^{bc}$. Note that the firm invests if $\Delta^l > 0$.

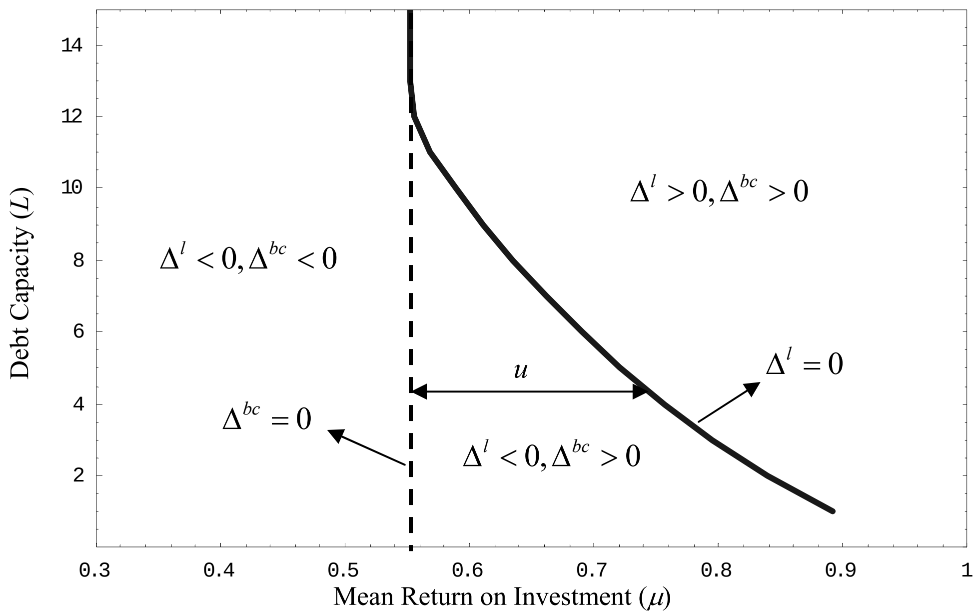



Figure 3.9: The impact of debt capacity on the propensity to invest in the BC (for $\theta$=10, $c$=7)

In Figure 3.9, the gap between solid and dashed lines denoted by $u$ corresponds to the impact of debt capacity on the propensity to invest. For example, when there is no debt capacity, the startup optimally invests if $\mu > 0.55$, but with a debt capacity of $L = 4$, the startup's mean return on investment should be at least $u + 0.55 = 0.76$ to start investing. As the debt capacity increases, the investment gap decreases and becomes zero when the debt capacity is not binding ($L > 12.75$).

Figure 3.10 presents the impact of debt capacity on the optimal operating policy of the startup under stochastic demand and survival. Note that Figure 3.10 is identical to Figure 3.6 for no debt capacity case. We observe that, our discussion in Section 3.4.2.4 still holds, but the limiting effect of a tighter debt capacity is clear. Aggressive and conservative policy regions shrink and no-investment region expands with a tighter debt capacity.

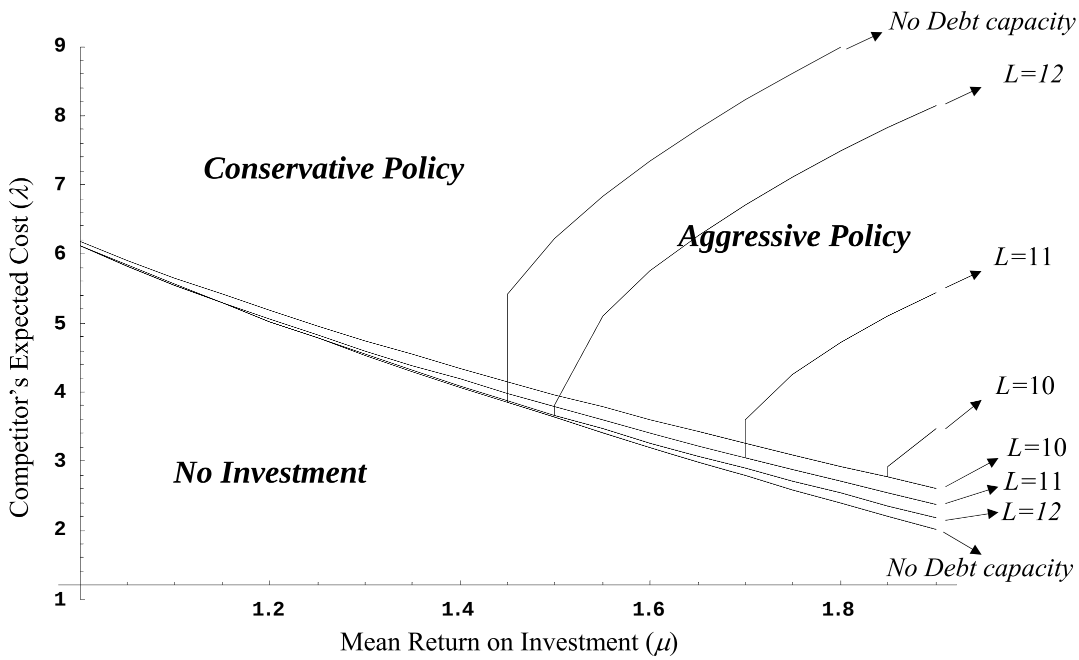



Figure 3.10: Impact of debt capacity on the operating policy in SDSC.

Overall, our observations suggest that the firm's propensity to invest is reduced with the debt capacity in the deterministic demand case, and the debt capacity makes the firm more conservative under stochastic demand. However, the basic results we have shown for the BC and SDSC remain valid under reasonably tight debt capacities.

### 3.6. Discussion and Concluding Remarks

Existing organizational theories (Bhide 2000) have marshaled evidence to argue that startup managers make myopic choices in their long term investment decisions when faced with uncertainty and financial pressure. Our analysis explores the impact of three key risk drivers (demand, technology and competition) on the short and long term production and process investment decisions of startups under the presence of explicit financial constraints. Since financial limitations alter optimal operating decisions; our results provide a risk based justification for startups linking their production with their process R&D investment.

### 3.6.1. Optimal Operating Decisions of Startups with Deterministic Demand

Under deterministic demand we find that the startup always produces the monopoly quantity and uses a process investment threshold policy involving an "invest-all-or-nothing" type of structure. The investment policy is described by the firm's propensity to invest. We investigate the impact of demand, technological performance and competition on the firm's propensity to invest in process improvement. In a large market the firm has high potential to recover the process investment. Similarly, higher expected return on investment (better expected technological performance) increases the potential benefits of investment and makes the firm more willing to invest. Further, as in new technology development, the firm disproportionately benefits more from upside deviations of return on investment. Hence, the firm's propensity to invest increases as the process technological performance gets more variable.

Impact of competition is more involved. As the expected competitor in the future gets stronger, it chips off future profits and the startup's propensity to invest decreases. However, compared to the monopoly situation, the startup may invest to mitigate the impact of competition and secure its future earnings if the competitor is not very strong. We summarize the impact of key parameters on the optimal process investment of the startup under deterministic demand (base case) in Table 3.2. Recall that in BC the firm always produces the monopoly quantity.

| Factor | Impact on Optimal Operating Policy |
|---|---|
| Return on process investment | The firm's propensity to invest $\Delta$ increases with mean ($\mu$) and standard deviation ($\sigma$) of return on process technology investment. |
| Competition | $\Delta$ decreases with expected level of competition ($\lambda$).<br>$\Delta$ increases compared to the monopoly case, if the competitor is not very strong. |
| Debt Capacity | $\Delta$ decreases with debt capacity ($L$). |

Table 3.2: Impact of Key Factors on the Optimal Operating Policy under Deterministic Demand

### 3.6.2. Optimal Operating Decisions of Startups with Uncertain Demand

In the deterministic base case the monopoly firm's profitability is guaranteed after the first period. However, in the stochastic demand and survival case, demand may be too low and profitability is not assured. Therefore, the probabilistic survival constraint becomes critical. Recall from Section 3.2 that this constraint is a unique risk driver that has not been addressed in literature. It shapes our results as follows: while the decision to invest in process development at early stages reduces the startup's profits and increases its exposure to bankruptcy, the low cost production process in subsequent periods could improve the startup's competitiveness and its market position. Hence, our core result states that under stochastic demand a central consideration in the startup's decision on the investment allocation is the tradeoff between the long-term expected profits and short-term bankruptcy risk.

When there is no demand risk, the firm always produces the monopoly level and invests nothing or all of the monopoly profits, as shown in the base case. When faced with stochastic bankruptcy risk, the startup sacrifices some short-term profits by deviating from its first best production plan. In the conservative case, the firm under-produces so as to allocate more cash to process R&D while controlling the survival chances. Further, depending upon competitor's cost, technological performance and aggregate demand, the startup may also invest aggressively by increasing the investment level above the BC level. In this case, the startup over-produces (more than the monopoly quantity) to cover the higher bankruptcy risk due to the aggressive process investment. That is, from an operational perspective, the startup hedges aggressive investment decisions by producing aggressively while conservative investment decisions involve conservative production plans.

Further, we identify two factors influencing operating decisions of startups: (1) the existence of positive NPV investment opportunities and (2) the immediate economical viability of the firm. We have shown that the startups that are not immediately economically viable adopt aggressive business plans, while immediately economically viable startups with no investment plans would always be conservative. Under stochastic demand, an intriguing case for decision making is revealed when there exists investment opportunities, *and* the firm is immediately economically viable. In this case, we numerically investigate the startup's optimal operating decisions and find that depending on demand uncertainty, success in process development, and anticipated competition, the firm could either follow an aggressive or conservative investment strategy. Our analysis indicates that the startup becomes aggressive and adopts riskier operating plans with lower survival chances when its efficiency of cost reduction increases or when faced with moderately strong competitors. Our results also demonstrate that demand uncertainty drives aggressive behavior. Since the survival of aggressive policies is dependent on the upside realizations in demand, highly variable markets create an incentive to follow aggressive policies. These results are summarized in Table 3.3.

| Factor | Impact on Optimal Operating Policy |
|---|---|
| Return on process investment | No investment for low returns<br>Conservative operating policy with moderate returns<br>Aggressive operating policy with sufficiently high $\mu$ and $\sigma$ |
| Immediate economical viability | No Investment for low levels of immediate economical viability, $\pi_m - \underline{\pi}$.<br>Conservative operating policy for high levels of $\pi_m - \underline{\pi}$<br>Aggressive operating policy for moderate levels of, $\pi_m - \underline{\pi}$ |

| | |
|---|---|
| Competition | No Investment with strong competitors<br>Conservative operating policy with weak competitors<br>Aggressive operating policy with moderately strong competitors |
| Demand | Aggressive operating policy with higher demand variability |
| Debt capacity | Conservative operating policy with tighter debt capacity |

Table 3.3: Impact of Key Factors on the Optimal Operating Policy under Stochastic Demand

We discussed our results with startup managers in search of process improvement opportunities in order to seek feedback about the model outcomes. In general, there seems to be an agreement among our respondents about the risky nature of process investment and the operational levers for hedging these risks. Some managers also pointed to additional factors that come into play into such decisions. For instance, the managers at Bigfoot Networks indicated that they currently outsourced semiconductor manufacturing and was looking for in-sourcing options, because it might provide opportunities for cost reduction. Faradox decided to use an emerging technology fund from Central Texas Regional Center of Innovation and Commercialization to develop a new fabrication process because production process has become a key part of their long-term business model. Below, we discuss the limitations of our model and potential extensions that have come up as a result of such field work.

### 3.6.3. Limitations and Extensions

We have studied process improvement which reduces the future unit cost. However, it would be interesting to study other forms of strategic investment, such as quality enhancing R&D, marketing and advertisement, which may also improve the future prospects of the firm. Further, our models and results trivially extend to bootstrapped startups. This extension is particularly important because a significant portion of the new firms are financed by bootstrapping (Bhide 2000). We also do not consider venture capital funded startups which may be an interesting future research direction. And, startups in our model do not consider exit strategies, e.g., mergers or acquisition choices, which are also central to process investment decision. It would be useful to explore how investment in process development change without financially risking the startup's survival when the startup's objective is to signal a potentially strong market presence in order to look more attractive for a takeover. This could alter the startup's decisions and yield different results. And finally, it ought to be possible to test the application frameworks in Tables 3.2 and 3.3 empirically. Investigating these issues will enhance our understanding of a startup's decision making with regards to product and process R&D management strategies.

# Chapter 4

# An Integrated Approach to Commodity Risk Management

## 4.1. INTRODUCTION AND MOTIVATION

Maximizing firm value, defined as the total properly discounted value of expected cash flows, is a central concern for the managers of publicly traded companies. In this chapter, we provide a model that links the financial risk management and the operational decisions of a value maximizing firm, under the presence of capital market imperfections. An overarching implication of our analysis is that publicly traded firms may significantly increase their market value, and generate more wealth for their shareholders by effectively integrating their financial risk management and production decisions.

Our research is motivated by the flour milling industry. Milling is the process of grinding and sifting wheat into flour which is a principal ingredient in the manufacture and production of bakery goods. Other major uses of flour include pasta, and blended and prepared flour packages (Harwood et al. 1989). The milling process also produces animal feed as a by-product.

A typical flour miller buys wheat from the market, either directly or through a third party commodity trading firm such as Cargill or ADM among others, and converts it into flour which is sold to the bakers. Historically, as milling is a weight losing activity, mills were located near wheat growing areas to save on the costs of transportation (Barber and Titus 1995). However, over the last two decades, increases in the efficiency in storage and transportation activities together with an enhanced business emphasis on responsiveness to demand fluctuations, has led mills to migrate near the consumption areas.

In our motivating example, it takes around two weeks for the miller to receive a wheat order. Once the wheat is received, it takes one to two days to convert it into flour and deliver it to the customer. To effectively meet the demand, millers usually carry inventory of wheat and convert it into flour when they receive a firm customer order. Indeed, the flour market is very competitive due to low product differentiation, and unmet demand is quickly satisfied by competitors.

On the other hand, long delivery lead times together with highly volatile wheat prices create a significant risk of holding inventory. For example, in late April 2009 wheat futures for July 09 delivery was trading around 520 cents per bushel (CPB). However, by the end of May 2009 prices quickly soared up to 670 CPB while during the following two weeks prices plummet down to 580 CPB (Figure 4.1 below shows the change in wheat prices during the first half of 2009). Hence, the millers carrying inventory during this period, experienced a sharp increase and then a decrease in the value of wheat they have in their silos.


Figure 4.1: Wheat Futures Prices for July 2009 Delivery

The sales price of flour is highly correlated with the wheat prices, and the changes in wheat prices are quickly reflected to flour prices. Hence, fluctuations in wheat prices not only impact the input costs but also affect the revenues of the miller. Table 4.1 below shows the relationship between wheat and flour prices between 2004 and 2009. As it is suggested by the table, millers operate with very thin profit margins (an average of 3-4%) and, from a financial perspective, chances in wheat prices may easily cause the millers to drain their cash reserves or default on their debt obligations. Table 4.1 also shows that despite the fluctuations in the wheat and flour prices, the profit margin of the milling business remains roughly the same.

| Mkt year and qtr 1/ | | Cost of wheat to produce flour 2/ | Wholesale price of bakery flour 3/ | Wholesale price of byproducts 4/ | Total wholesale price | Total wholesale price less cost of wheat |
|---|---|---|---|---|---|---|
| 2004/05 | Q1 Jun-Aug | 9.29 | 10.67 | 1.12 | 11.78 | 2.49 |
| | Q2 Sep-Nov | 9.59 | 11.08 | .98 | 12.07 | 2.48 |
| | Q3 Dec-Feb | 9.48 | 11.12 | .95 | 12.07 | 2.59 |
| | Q4 Mar-May | 8.97 | 10.65 | .77 | 11.42 | 2.45 |
| | MY Jun-May | 9.33 | 10.88 | .95 | 11.83 | 2.50 |
| 2005/06 | Q1 Jun-Aug | 9.07 | 10.80 | .66 | 11.46 | 2.39 |
| | Q2 Sep-Nov | 10.28 | 12.10 | 1.00 | 13.10 | 2.82 |
| | Q3 Dec-Feb | 10.59 | 11.25 | 1.40 | 12.65 | 2.07 |
| | Q4 Mar-May | 11.57 | 12.75 | 1.07 | 13.82 | 2.25 |
| | MY Jun-May | 10.38 | 11.73 | 1.03 | 12.76 | 2.38 |
| 2006/07 | Q1 Jun-Aug | 12.13 | 13.13 | 1.17 | 14.30 | 2.18 |
| | Q2 Sep-Nov | 12.44 | 13.38 | 1.47 | 14.85 | 2.41 |
| | Q3 Dec-Feb | 12.37 | 12.80 | 2.32 | 15.12 | 2.75 |
| | Q4 Mar-May | 12.69 | 13.78 | 1.33 | 15.11 | 2.42 |
| | MY Jun-May | 12.41 | 13.28 | 1.57 | 14.85 | 2.44 |
| 2007/08 | Q1 Jun-Aug | 15.14 | 16.32 | .88 | 17.20 | 2.06 |
| | Q2 Sep-Nov | 20.59 | 20.27 | 1.94 | 22.20 | 1.62 |
| | Q3 Dec-Feb | 27.70 | 27.12 | 2.74 | 29.86 | 2.16 |
| | Q4 Mar-May | 27.25 | 27.60 | 2.45 | 30.05 | 2.79 |
| | MY Jun-May | 22.67 | 22.82 | 2.00 | 24.83 | 2.16 |
| 2008/09 | Q1 Jun-Aug | 21.90 | 22.32 | 2.57 | 24.88 | 2.99 |
| | Q2 Sep-Nov | 15.96 | 16.48 | 2.35 | 18.83 | 2.87 |
| | Q3 Dec-Feb | 15.15 | -- | -- | -- | -- |

1/ June-May. Latest data may be preliminary.
2/ No. 1 hard red winter, 13-percent protein. Cost of 2.28 bushels based on a 73-percent extraction rate. cwt = hundredweight.
3/ Quoted as mid-month bakers' standard patent, bulk basis.
4/ Assumes 50-50 millfeed distribution between bran and shorts or middlings, bulk basis.
Source: USDA, Agricultural Marketing Service, Monthly Feedstuffs Prices; Milling and Baking News, Market Fax; and ERS estimates.

Table 4.1: Wheat and Flour Prices from 2004 to 2009 (USDA 2009)

When established firms run out of cash or default (either due to price or demand fluctuations), then usually they do not immediately go bankrupt or get liquidated, but they face financial distress. Indeed, flour millers typically have significant fixed (tangible) assets and equipment which enable them to raise new capital and finance their operating plans. In our motivating case, millers may borrow from a bank line of credit, or as it is often the case, from an intermediary commodity trading firm, such as Cargill, who offers fast-response short-term financing in addition to delivering wheat from market to the millers. However, there is a cost premium for using this external financing; for example, Cargill charges double digit interest rates to the millers who operate on a 3-4% average profit margin. In the finance literature, the cost of external financing is recognized as a common form of financial distress cost (FDC).

In this research, we examine how financial risk management can be integrated with the operating decisions so as to control the firm's exposure to financial distress risk and to maximize the firm value. In Figure 4.2, we demonstrate the typical sequence of decisions and the allied cash flows for a flour miller. The firm orders wheat from the wheat market at the current spot price which is delivered by the commodity trading company at a pre-negotiated delivery cost. The firm (the miller) also locks in a set of wheat futures contracts to hedge the commodity price risk. When internally generated cash is exhausted, external financing is received at a premium cost. Finally, wheat is converted into flour and sold to the bakers.

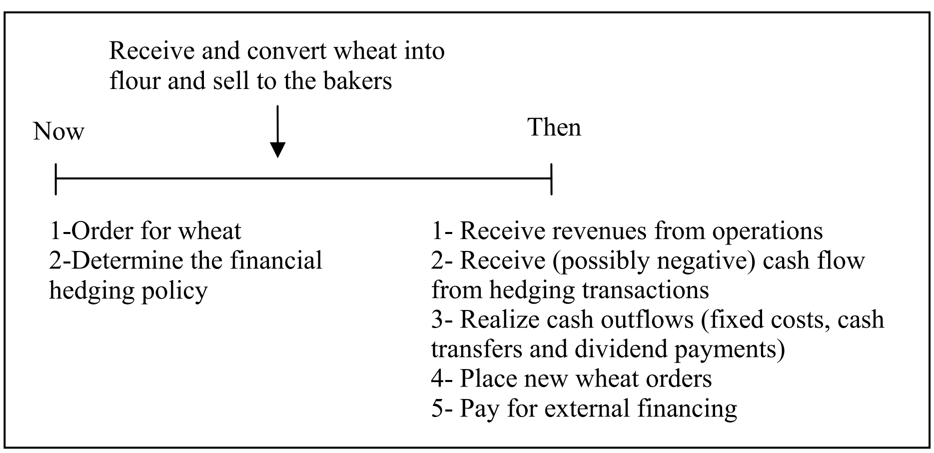



Figure 4.2: Summary of Operating and Financial Decisions of the Miller

Millers usually order wheat and produce flour in order to meet a set of deterministic customer orders as well as a set of future uncertain orders. In this research, we consider two business models for the operations of the firm. In the first case, the miller operates under fixed flour contracts in a Make-to-Order (MTO) business environment with deterministic demand. Second, we examine a case representing a Make-to-Stock (MTS) business plan with demand uncertainty. In practice, depending on their agreements with customers, and the lead time for wheat delivery, millers operate under a production system that has elements of both MTO and MTS environments.

Flour, which is mainly sold to the bakers, is a commodity-like product, i.e., product differentiation is minimal among different millers, but there is no explicit commodity market for it. Further, the flour market is very competitive and the pricing of flour strictly depends on the current wheat spot price. In particular, within a given geographical region, the competitive nature of the milling industry leads the millers to very quickly reach a single flour price by adding a standard margin (which includes a standard unit manufacturing cost and profit); in our model we assume this profit margin is an additive constant for all prices of wheat. For all practical purposes, both wheat and flour are non-perishable items and hence the leftover inventory can be easily carried over to the next production period, while the excess demand is lost.

As it is well known in the finance literature, in the absence of frictions, engaging in financial hedging is a neutral proposition. That is, it should not affect the optimal production plan, and it does not create value for firm's shareholders. However, when the firm faces capital market frictions, such as in our case costly financial distress, financial hedging can contribute to shareholder-value maximization. We show that, in the MTO case, it is optimal and possible to completely hedge for the price risk and totally eliminate the financial distress costs via taking an appropriate position in the futures market. On the other hand, in the MTS case, the uncertainties associated with the future production plans limit the effectiveness of financial hedging decisions. In general, it is not possible to completely eliminate the financial distress risk, but appropriate financial hedging plans may significantly reduce it. We show that a coordinated financial hedging and operating plan contributes to shareholder-wealth creation (1) by reducing the firm's exposure to financial distress risk and mitigating the corresponding costs, and (2) by enabling the firm to operate at a higher level of output.

The rest of the chapter is structured as follows. In Section 4.2, we provide a detailed review of the related literature in finance and operations management. Then in Sections 4.3 and 4.4, we provide a background on the evolution of stochastic commodity prices and discuss the dynamics of flour price and demand, respectively. Following, Section 4.5 details our mathematical model and Section 4.6 concludes with a summary of our results.

### 4.2. Background and Literature Review

Since the seminal paper of Modigliani and Miller (1958), it is well known in the finance literature that, with perfect capital markets, financial risk management does not create any value. In particular, when there are no capital market frictions such as informational asymmetries, taxes and costs associated with transactions, bankruptcy and financial distress, hedging for financial risk does not add value to the firm since the shareholders can replicate any risk management activities implemented by the firm at the same cost. In the finance literature, financial risk management is explained through two major theories: (1) shareholder-value maximization, and (2) managerial motives and risk aversion (agency issues) (Jin and Jorion 2006).

Financial hedging reduces the variability associated with the future cash flows of the firm. The value maximization theory argues that, fluctuations in future cash flows involve certain costs, due to capital market imperfections, and firms may reduce these costs by financial hedging. Smith and Stulz (1985) claim that due to direct and indirect costs associated with financial distress, firm value is a concave function of future cash flows and hence financial hedging increases firm value. They also argue that convex tax schemes create a similar incentive to reduce the variability of the cash flows by financial hedging. On the other hand, Stulz (1990), Bessembinder (1991), and Froot et al. (1993) show that firms may also hedge to mitigate the problem of under-investment, when there is a cost premium for external financing.

The second stream of the theory explains hedging through managerial motives derived from the managers' desire for maximizing their personal expected utility rather than maximizing wealth creation (i.e., rather than making decisions leading to Net Present Value (NPV) maximization). In particular, if the managers are risk averse, and their compensation is based on the end-of-period firm value, then the managers have a motivation to hedge (Stulz 1984, Smith and Stulz 1985).

In this chapter, financial risk management activities are motivated through the existence of financial distress costs. Direct and indirect costs of financial distress have been widely studied in the finance literature. Direct costs may include the legal costs of debt negotiations and cost of external financing while the indirect costs include lost sales, lost market share, and decreased credit ratings (see Purnanandam 2008 and Hotchkiss 2008 for a recent review of the literature on the cost of financial distress). In our motivating example, as it is predicted by the pecking-order theory, flour millers first rely on internal cash to finance their operating expenses including the fixed and variable costs associated with procurement and production, as well as other cash outflows such as dividend payments and any other cash commitments considered in the firm's financial plan. If the internally generated cash is not sufficient to cover these cash outflows, then the firm faces financial distress and incurs a cost proportional to the amount of the shortfall. In particular, in our model we consider the cost premium of external financing which is a specific form of financial distress cost.

Unlike the finance literature dealing with costs of financial distress, in this research we model the specific linkages between the operating decisions and the costs of financial distress. Hence, our results provide an explicit relationship between operating and hedging policies and their implications for value creation.

On the other hand, the connection between operational and financial decisions has recently received attention in the operations management literature. Not surprisingly, the operations management literature also follows the two main streams of finance theories to explain the value of jointly considering financial and operational decisions. Papers including Gaur and Seshadri (2005), Ding et al. (2007) and Chen et al. (2007) study the value of financial hedging from a risk-averse decision maker's perspective and consider the objective of maximizing expected utility.

A second stream of papers including Buzacott and Zhang (2004), Archibald et al. (2002), Swinney et al. (2005) and Babich and Sobel (2004) consider a firm with limited access to capital markets under the threat of bankruptcy (i.e., a small private firm). Hence, both of these research streams fall outside Miller-Modigliani framework.

Gaur and Seshadri (2005) address the problem of hedging inventory risk in a multi-period newsvendor environment when the demand is correlated with the price of a financial asset. Their objective is to maximize the expected utility of a risk-averse decision maker. Considering a wide range of utility functions and hedging strategies, they show that, a risk-averse decision maker orders more inventory when he or she hedges the inventory risk. Ding et al. (2007) consider the financial and operational hedging problems faced by a multi-national firm which has a production facility in one of the two markets it sells to. The firm commits to capacity before the selling season and faces both demand and currency exchange rate risk. They consider a real option to partially hedge against the demand uncertainty, and use financial options on the currency exchange rate to hedge against the currency risk. The objective of the decision maker is to maximize the expected utility in a mean-variance utility framework. The authors derive the optimal capacity investment and financial hedging decisions, and investigate the impact of the operational hedging (capacity allocation option) and the financial hedging on the operating decisions of the firm.

As expected, the connection between finance and operations is most easily observed when managing start-up operations. Buzacott and Zhang (2004) establish the link between the financial capacity and the operating decisions of a retailer that has no fixed assets, via incorporating an asset-based financial constraint on the firm's debt capacity. Under deterministic demand, they provide a multi-period model to maximize profits where the retailer borrows from a bank to finance its operations. In a single period newsvendor environment, they also analyze a leader-follower game between the bank and the retailer. The retailer decides the production quantity and the loan amount while the bank decides the loan limit and the interest rate.

Archibald et al. (2002) consider a multi-period inventory management problem for a startup firm with the objective of maximizing survival. The firm starts with an initial inventory and capital, and survives if its initial capital plus earnings are enough to cover the overhead cost in each period. No external funding is considered. Swinney et al. (2005) also consider a startup firm with the objective of maximizing survival probability in a single period model with competition. Financing cost is implicitly modeled as a part of the unit capacity cost, and the bankruptcy occurs if the profits at the end of the period are less than an exogenous threshold (this threshold implicitly includes the interest payments).

Babich and Sobel (2004) study the coordination of operational decisions (production and sales) and financial decisions (loan size) to maximize the expected discounted proceeds from an initial public offering (IPO). Financial and operational decisions are linked to each other through a financial constraint which requires non-negative profit in each period after the debt payments.

Consequently, in the operations management literature, the finance-operations interface is studied either in a utility maximization framework, or in a startup setting with bankruptcy risk. Hence, the findings of the literature do not necessarily extend to publicly traded firms who strive to maximize shareholder-value. Different from the above research streams, we study the finance-operations interface for a public corporation within the value framework of finance; hence our findings do not require any specific assumptions about the investors' utility functions. Moreover, we contribute to the operations management research by examining the impact of the costs of financial distress on hedging and operating plans.

### 4.3. STOCHASTIC MODEL OF STORABLE COMMODITIES PRICES

In this section, we briefly discuss the literature on evolution of spot and futures commodity prices, and explain the concept of marginal convenience yield. We denote $s_t$ as the spot price of the commodity at time $t$, and we assume it will evolve stochastically through time as a function of only the information contained in the commodity price state space vector at time $t$, $P_t$. Thus we assume the current spot price, $s_t$, as well as the transition probabilities of the commodity price process from $s_t$ to its value at time $t+\Delta t$, denoted as $s_{t+\Delta t}$, can be obtained from the information contained in $P_t$. To preclude risk-free arbitrage opportunities, we define the time $t$ unit price of a futures contract for the commodity with delivery date $t+\Delta t$, denoted as $f_{t,t+\Delta t}$, using the risk-neutral valuation approach (Duffie 1992) as $f_{t,t+\Delta t} = E^{\mathbb{Q}}_{P_{t+\Delta t}|P_t}\left[s_{t+\Delta t}\right]$ for any $\Delta t > 0$. The superscript $\mathbb{Q}$ and subscript $P_t$ in the expectation operator indicate the expectation was calculated using the risk-neutral probability measure of the commodity price process, and this expectation is conditioned on the information in the state vector $P_t$ observable at time $t$.

The stochastic process governing the evaluation of spot price $s_t$, and futures price $f_{t,t+\Delta t}$ are obtained as (deterministic) functions of $P_t$ and $\Delta t$. We denote the risk-neutral probability density and cumulative distribution functions for $s_t$ as $\phi^{Q}_{s_t}(.)$ and $\Phi^{Q}_{s_t}(.)$, respectively. We further assume the market trades enough financial instruments to replicate the evolution of $P_t$ so that we can find a unique risk-neutral probability measure for the evolution of $P_t$.

The difference between the current spot price, $s_t$, and the futures price observed in the market for a commodity determine the economic cost of holding the commodity as inventory; in our model this difference is random, and it is also referred to as marginal convenience yield in the economics literature (Pindyck 2001). Specifically, the marginal convenience yield is the difference between the cost of buying and storing the commodity now, and the present value of the cost of buying it in the futures market for delivery next period, $s_t + \beta h_t - \beta f_{t,t+1}$, and it is the economic cost of holding a unit in inventory. Notice that to exclude arbitrage opportunities we must have, at any time, a non-negative marginal convenience yield, i.e., $s_t + \beta h_t - \beta f_{t,t+1} \geq 0$. Our models will be formulated in terms of primary, out-of-pocket costs such as $s_t$, $h_t$, and $f_{t,t+1}$; having a non-negative marginal convenience yield is an economic condition that, not surprisingly, arises as a part of the necessary conditions of optimality in our models.

### 4.4. WHEAT AND FLOUR PRICES AND THE DEMAND FOR FLOUR

Demand for flour is largely driven by the consumption of bread and other baked products. Bread consumption is affected by a number of economic, cultural and environmental factors. Anecdotal evidence indicates that weather has a large impact on short term, day to day, fluctuations in bread consumption; people are likely to increase their consumption of baked products during cold spells and are likely to decrease it during warm spells. Interestingly, even though wheat flour is the key ingredient in baked products, short term fluctuations of wheat prices do not have a significant effect on the consumption of baked products.

This phenomenon is reasonable as the cost of wheat is a very small fraction of the total price paid by the customer for the final product, and even if the price of wheat changes from one day to the next, the price for the final product does not. Consider wheat contract KWU9 (hard red wheat for delivery on Sept 09) which traded at KBOT at \$5.49/bushel on July 24, 2009. When we consider that a bushel of wheat weighs $60$ pounds, then a pound of wheat costs $\$5.49/60 = \$0.0915$. Thus in a hypothetical loaf of bread that requires 1 pound of wheat and retails at $\$2$ at the bakery, the cost of wheat represents only $4.6\%$ of its retail price. If we consider a cake with similar wheat content that retails for $\$10$, then the cost of wheat represents about $0.9\%$ of the retail price. Thus, even though wheat is the central ingredient in bread, a $10\%$ increase in the price of wheat, which represents a rather sharp short-term increase, will increase the cost of wheat by only $0.5\%$ and $0.09\%$ of the retail price for the loaf of bread and the cake, respectively.

Although bread and baked goods are usually differentiated/branded products, flour is not. Thus, even though in the short-term, the retail price of baked goods is not drastically affected by the day to day fluctuations in the price of wheat, competition for the wheat millers is cutthroat, and efficient management of their procurement and production operations is crucial.

Based on the above observations, our model assumes demand is independent of price at any time *t*. We further assume demand is uncorrelated with the returns of the stock markets, hence demand risk is fully diversifiable. This assumption is consistent with the findings in Berling and Rosling (2005). Under the above assumptions, the probability distribution of demand is independent from the probability distribution of the price process, and the risk neutral distribution of demand is equal to the historical (true) probability distribution of demand, $\Phi^{Q}_{\xi_t}(.) = \Phi_{\xi_t}(.)$.

## 4.5. MATHEMATICAL MODEL

In this section, we provide a stylized model for the operating and financial risk management decisions of a flour miller which buys wheat from the commodity market and sells flour. We consider the objective of maximizing firm value, i.e., the total discounted value of expected cash flows over $T$ time periods. At each time period $t$, the firm decides (1) how much commodity to order and, (2) the composition of financial instruments to hold[1]. We let $x_t$ denote the number of futures contracts shorted at time $t$, for delivery at $t+1$, and $q_t$ denotes the number of commodity contracts purchased at time $t$ to be shipped immediately and received in the mill at $t+1$[2]. Regarding the flour price, as it is suggested by market data (see Table 4.1), we assume that it is a linear

[1] In our model, we restrict the composition of our financial hedging portfolio to futures contracts. In practice, futures contracts (compared to financial options) are much more heavily used by the firms for hedging purposes due to their simpler nature and higher liquidity. For example, the KEU09 Sept. 09 wheat futures contract at KCBOT showed in mid-July, 2009 an open interest of over 41,000 contracts. Comparatively, the total open interest on all option contracts on this same futures contact, including calls and puts at all traded strike prices (a total of sixty different options) was less than 5,000 contracts. Typically the open interest on a specific option was between 2 and 200 contracts, a very significant drop in the liquidity of this instruments when compared to futures.

[2] One contract for wheat is defined at KCBOT as 5,000 bushels, and for shipping purposes is equivalent to a full railroad cart.

function of the spot price and given by $s_t + \lambda$, at time $t$. Our model also assumes an initial financial plan which includes a starting cash reserve, $y_0$, as well as a pre-determined sequence of cash outflows, $\vec{\eta} = [\eta_1, ..., \eta_T]$. These cash outflows account for fixed costs of operation, cash transfers to different divisions of the firm, as well as possible funding for dividend payments and other financial commitments.

Below we provide the stylized sequence of decision and events for the operations of the firm at the beginning of time $t$ :

($i$) First, the firm observes $P_t$ (which implies the current spot price, $s_t$ and the futures price, $f_t$), and the flour demand for the current period, $\xi_t$. In reality, millers continuously receive orders from customers and meet demand. For ease of exposition, we discretize the planning horizon of the firm and assume that demand is observed at the beginning of each period.

($ii$) Following this, the firm receives the wheat ordered in the previous period, $q_{t-1}$, and on hand wheat inventories increase from $I_{t-1}$ to $I_{t-1} + q_{t-1}$. Next, if available, enough wheat is converted into flour to meet the current period's demand, $\xi_t$. Excess flour demand is lost, and excess wheat inventory $I_t$ is carried to the next period, at a cost of $h$ per unit per period. In particular, the miller sells the minimum of $I_{t-1} + q_{t-1}$ and $\xi_t$ at the current flour price, $s_t + \lambda$.

($iii$) We define the initial cash reserves of the firm as the cash on hand prior to receiving the cash flows at the beginning of time $t$ and we denote it as $\hat{y}_t$; this is the cash the firm brings in its transition from period $t-1$. Also at the beginning of period $t$ the firm receives cash flows from operations, $(s_t + \lambda)\min(I_{t-1} + q_{t-1}, \xi_t) - hI_t$, receives the (possibly negative) cash flows derived from its financial hedging transactions, $(f_{t-1} - s_t)x_{t-1}$, and it pays out the cash outflows pre-committed in the financial plan, $\eta_t$.

The new pre-order cash reserves, at the beginning of period $t$ but prior to placing new orders become $y_t = \hat{y}_t + (s_t + \lambda)\min(I_{t-1} + q_{t-1}, \xi_t) - hI_t + (f_{t-1} - s_t)x_{t-1} - \eta_t$.

($iv$) The firm places new orders $q_t$, to meet the demand in period $t+1$ and decides for the composition of its financial instruments $x_t$, to hedge for the commodity price risk. After placing the new order, $q_t$, the cash reserves of the firm become $y_t - s_t q_t = [y_t - s_t q_t]^+ - [y_t - s_t q_t]^-$.

($v$) If the cash reserves of the firm at time $t$ (after ordering $q_t$) are positive, they are invested in liquid short-term assets and they grow at the risk free rate; hence in this case the initial cash reserves at $t+1$ become $\hat{y}_{t+1} = (1+r_f)\left(y_t - s_t q_t\right)$. On the other hand, if the cash reserves are negative (i.e., $y_t - s_t q_t < 0$), the firm faces financial distress, and it uses costly external funding. Our model of costly financial distress assumes a constant premium, $r > 0$, in excess of the risk free rate, $r_f$, that is paid proportionately to the amount of external financing used. Hence, the initial cash reserves at $t+1$ are $\hat{y}_{t+1} = \left(1+r\right)(1+r_f)(y_t - s_t q_t)$, which are negative. Thus in general, we can define $\hat{y}_{t+1} = \left(1+r_f\right)[y_t - s_t q_t]^+ - (1+r)(1+r_f)[y_t - s_t q_t]^-$, which simplifies to $\hat{y}_{t+1} = (1+r_f)(y_t - s_t q_t) - r(1+r_f)[y_t - s_t q_t]^-$. Hence, at the beginning of time $t+1$ the firm pays $r(1+r_f)[y_t - s_t q_t]^-$ for financial distress. At time $t$, the present value (when $q_t$ is decided) of this cost of financial distress is given by $FDC_t = r[y_{t-1} - s_t q_t]^-$.

Assuming a constant premium $r > 0$ for external financing implies that this short-term debt will be repaid in full with probability one. This assumption is reasonable, for example, when the firm's tangible assets, such as real estate and equipment, exceed its total liabilities by a large enough margin to cover the financial shortfalls implied by the short-term operational and financial decisions. In case of financial shortages, this firm will in all likelihood not go into Chapter XI bankruptcy proceedings, as it will be able to raise enough capital, and all short-term loans will be paid in full. Even if such a firm is deemed to be not economically viable, its voluntary liquidation will bring enough cash to pay all debtholders in full.

In general, financial shortage, bankruptcy and liquidation refer to three different financial states of a firm. More precisely, a publicly traded firm experiencing a financial shortage (or a default) faces financial distress which may or may not lead to filing for bankruptcy under Chapter XI. If the firm's debt can be reorganized through private negotiations with the debtholders, then the firm does not file for bankruptcy and keeps operating. If private negotiations are not viable due to agency problems and informational asymmetries then a formal bankruptcy is filed (Senbet and Seward 1995). Ideally, the bankruptcy process liquidates the firm only if it is not economically viable. Formally, a firm is not economically viable if an alternative use for its assets is more valuable; in this case the firm becomes a target for liquidation. If the firm remains economically viable, its debt is formally reorganized by the court (Hotchkiss et al. 2008). Therefore, for publicly traded firms, liquidation (or the cease of operations) does not depend on the level of debt (or financial shortage) but on the economic viability of its operations. In summary, each of the three possible outcomes, a private reorganization, Chapter XI bankruptcy reorganization, or the liquidation of the firm, may or may not lead to the full payment of debt. Therefore, in our model, the constant positive premium for short-term financial shortfalls, $r > 0$, is not driven by default probabilities, bankruptcy or liquidation, but instead the firm is willing to pay this premium due to the transaction costs and time lags associated with access to the capital markets. In Figure 4.3, we summarize the sequence of events, decisions and cash flows for the miller.

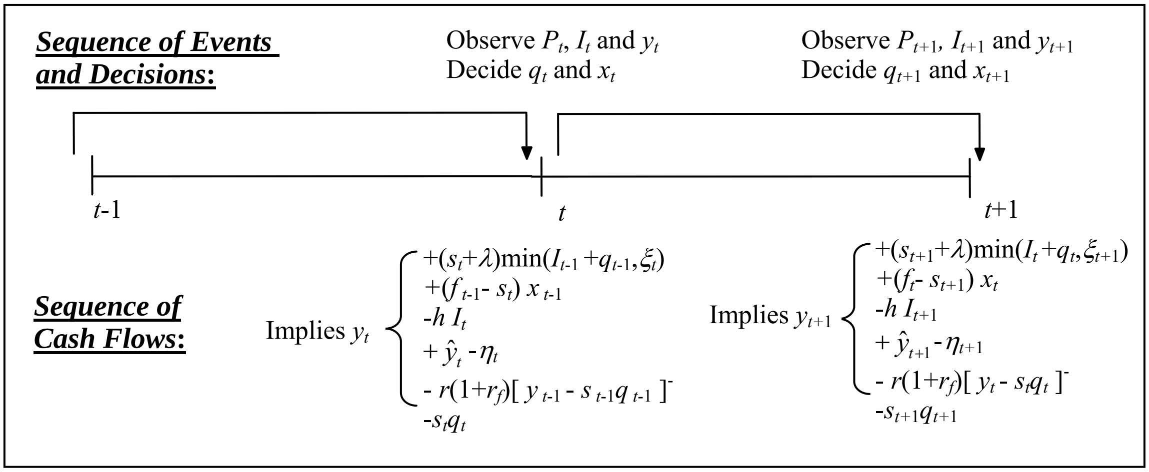



Figure 4.3: Sequence of Events, Decisions and Cash Flows for the Miller

As it is suggested by Figure 4.3, below we describe the multi-period decision problem of the firm. For notational convenience, we denote the vector of state variables at the beginning of time $t$, as $S_t=[I_t,y_t,P_t]$, and we define $\beta=1/(1+r_f)$.

For $t=1,...,T-1$,

$$V_t(S_t)=\max_{q_t\geq 0,x_t} J_t(q_t,x_t\mid S_t)$$

$$s.t.\ I_{t+1}=[I_t+q_t-\xi_{t+1}]^+ \quad (4.1)$$

$$y_{t+1}=\frac{1}{\beta}(y_t-s_tq_t-FDC_t)+(s_{t+1}+\lambda)\min(I_t+q_t,\xi_{t+1})-hI_{t+1} \\ +(f_t-s_{t+1})x_t-\eta_{t+1} \quad (4.2)$$

where,

$$J_t(q_t,x_t\mid S_t)=-s_tq_t-FDC_t+\beta E^Q_{\xi_{t+1},P_{t+1}\mid P_t}[(s_{t+1}+\lambda)\min(I_t+q_t,\xi_{t+1})-hI_{t+1} \\ +(f_t-s_{t+1})x_t+V_{t+1}(S_{t+1})],$$

$FDC_t=r[y_t-s_tq_t]^-$ and

$V_T(S_T)=s_TI_T-FDC_T,FDC_T=r[y_T+s_TI_T]^-$.

We denote the value of the firm at the beginning of period $t$ as $V_t(.)$. The first term in the objective, $-s_tq_t$, gives the cost of procurement and the following term, $FDC_t$

denotes the cost external financing. The last term describes the discounted value of the firm's future cash flows. The cash flows due to operations in period $t$ are given by $(s_t + \lambda)\min(I_t + q_t, \xi_{t+1}) - hI_{t+1}$; firm sells the minimum of demand and on-hand inventory at the current price $(s_t + \lambda)$, and pays for holding cost for the left-over inventory. Cash flows from financial transactions are given by $(f_t - s_{t+1})x_t$. Finally, the last term inside the expectation operator is the cost-to-go function. Constraints (4.1) and (4.2) in the model denote the inventory and cash flow balances, respectively. In the final period $T$, the firm does not place new orders and salvages the excess inventory at the current spot price. We start our analysis by showing that the objective function is concave in the decision variables.

**Lemma 4.5.1.** *(Convexity).* $J_t(q_t, x_t \mid .)$ *is jointly concave in decision variables,* $q_t$ *and* $x_t$, *for* $t = 1, ..., T-1$.

Next, we underline the fact that financial hedging impacts firm value and the operating decisions only if there is a cost for financial distress.

**Lemma 4.5.2.** *(Hedging without FDC). When there is no financial distress cost, i.e.,* $r = 0$, *financial hedging is immaterial to firm value and operating decisions.*

Under the risk neutral measure $f_{t,t+\tau} = E^{Q}_{P_{t+\tau}|P_t}[s_{t+\tau}]$, and the expected value of financial transactions is zero. Further, without a cost for financial distress, firm's cash flows are unaffected by financial hedging. In particular, when $r = 0$, we are in the Miller-Modigliani framework with no capital market frictions, hence buying and selling futures contracts do not affect the firm value. This argument can be generalized to other forms of correctly priced financial hedges. In the reminder of this section, we explore the interaction between financial distress cost, operating decisions and financial hedging under two common business models in practice. In Section 4.5.1, we discuss a make-to-order business plan where demand is known before committing to the purchase of wheat, and in Section 4.5.2 we analyze a make-to-stock business plan with uncertain demand.

### 4.5.1. Make-to-Order (MTO) Business Plan

In this section, we assume that the firm may observe the customer demand prior to committing for the production plan. In particular, when shipment and production lead times are relatively short compared to the delivery lead times of the customer orders, firms effectively operate in an MTO fashion. The following theorem shows the optimal production decisions of the firm when demand is known and deterministic over the planning horizon, and there is no cost for financial distress.

**Theorem 4.5.1.** *(Optimal Production without FDC). Suppose demand is known and deterministic over the planning horizon, and there is no financial distress cost, i.e.,* $r=0$. *Then, the optimal production plan is given by* $q_t^* = \xi_{t+1}$, *and* $I_t^* = 0$, *if* $-s_{t-1} + \beta(f_{t-1} - h) \le 0$ *and* $-s_t + \beta(f_{t-1} + \lambda) \ge 0$ *for* $t = 1, ..., T-1$.

Theorem 4.5.1 shows that the firm exactly orders and produces for the next period's demand and does not carry inventory between the periods, if the spread of spot and futures prices satisfies two usual economic conditions, in each period. The first condition, $\beta(f_t - h) - s_t \le 0$, states that the convenience yield is non-negative (assuming the holding cost is paid at the end of the period). This condition always holds in commodity markets since negative convenience yield creates arbitrage opportunities. The second condition, $-s_t + \beta(f_{t-1} + \lambda) \ge 0$, requires that the expected profit margin is non-negative in each period; otherwise it is not profitable to meet the demand. Note that these results are independent of the financial hedging decisions when there is no financial distress cost.

Next, we investigate the impact of financial distress cost. We first consider the following single period MTO model, where for ease of exposure we assume that the initial inventory level and cash outflows are set to zero, $I_1 = 0$, $\eta_1 = 0$, and there is no financial distress in the first period, $FDC_1 = 0$ .

$$V_1(S_1) = \max_{q_1 \geq 0, x_1} - s_1 q_1 + \beta E^Q_{P_2|P_1}[(s_2 + \lambda)\min(q_1, \xi_2) - hI_2 + (f_1 - s_2)x_1 + V_2(S_2)] \qquad \text{(MTO 1)}$$

$$s.t.\ I_2 = [q_1 - \xi_2]^+$$

$$y_2 = (y_1 - s_1 q_1)/\beta + (s_2 + \lambda)\min(q_1, \xi_2) - hI_2 + (f_1 - s_2)x_1 - \eta_2$$

where,

$$V_2(S_2) = s_2 I_2 - FDC_2, FDC_2 = r[y_2 + s_2 I_2]^-$$

**Theorem 4.5.2.** *(Optimal Production with FDC and without Hedging). Suppose the demand is known and deterministic, and the firm operates for a single period with no financial hedging. Then, when there is a cost for financial distress* $r > 0$ *, the firm*

*(i) orders strictly less than the future demand, i.e.,* $q_1^* < \xi_2$, *if*

$$a(\xi_2) = -s_1 + \beta(f_1 + \lambda) + r\beta\int_0^b (-s_1/\beta + s_2 + \lambda)\phi(s_2)ds_2 < 0 \qquad (4.3)$$

*where*,

$$b = \frac{\kappa_1}{\xi_2} + s_1/\beta - \lambda, \kappa_1 = \eta_2 - y_1/\beta,$$

*(ii) and exactly orders for the future demand, i.e.,* $q_1^* = \xi_2$, *otherwise.*

Theorem 4.5.2 describes the conditions under which it is not economical to meet all of the future demand, when there is a risk of incurring financial distress cost. In condition (4.3), $a(\xi_2)$ represents the marginal change in the firm value when $q_1 = \xi_2$. Hence, if $a(\xi_2)$ this is less than zero, the manager should order less than the future demand, and otherwise it is optimal to meet all of the demand. Note that the sum of first two terms in $a(\xi_2)$ is non-negative by assumption, and the third term may be either positive or negative depending on the net cash transfers, future demand, sales margin and current spot price. The argument of the integral in $a(\xi_2)$ represents the profit margin of the firm at $t+1$ for a particular realization of the future spot price $s_2$. When the future spot price is less than $b$, the firm faces financial distress and pays a premium $r$ for external financing.

The financial state of the firm is one of the key determinants of the financial distress threshold, $b$. The parameter $\kappa_1$ denotes the pre-committed net cash outflows of the firm, i.e., cash transfers $\eta_2$, net of pre-order cash reserves $y_1/\beta$. In particular, if the net cash outflows are very high, then the firm faces financial distress for almost all the future spot price realizations. In this case $b$ is very high and the integral term in (4.3) is positive, so the firm is unlikely to under-produce. On the other extreme, if the net cash outflows are very low, then the firm is unlikely to face financial distress in the future. So, the integral term in (4.3) is close to zero and the firm does not under-produce again. Only for moderate levels of net cash outflows, the integral term can be negative and the firm chooses an optimal quantity less than the current demand.

So far we have identified the optimal operating policy of the firm when the managers engage in no financial hedging transactions. In the following two results, we examine the impact of financial hedging on the operating decisions and firm value, when there is a cost for financial distress. First, we identify the optimal hedging policy.

**Theorem 4.5.3.** *(Optimal Hedging). Suppose the demand is known and deterministic, and the firm operates for a single period. Under a positive financial distress cost, if the firm decides to order and produce* $q_1$ *then it is optimal to short* $x_1^* = q_1$ *units of futures contracts with futures price* $f_1$. *This hedging policy completely eliminates the price risk.*

Theorem 4.5.3 shows that the optimal hedging decision is intimately linked to the operating plans. At $t = 1$, the firm commits to $q_1$ which will be sold at $t = 2$ at an uncertain price $s_2 + \lambda$. By shorting $q_1$ units of futures contracts at $t = 1$ the firm effectively fixes the future sales price to $f_1 + \lambda$. Hence, the price risk associated with future sales is eliminated.

**Theorem 4.5.4.** *(Joint Optimal Hedging and Production). Suppose demand is known and deterministic, and the firm operates for a single period. Then, under a positive financial distress cost, it is optimal to produce* $q_1^* = \xi_2$ *and hedge* $x_1^* = \xi_2$.

Theorem 4.5.4 indicates that under the optimal financial hedging policy, the under-investment problem in Theorem 4.5.2 can be totally eliminated. That is, the firm always produces to meet the demand, regardless of its financial status. Note that, financial hedging may not totally eliminate the financial distress cost. Indeed, hedged firm may still need to pay for financial distress, but it is optimal to completely hedge for the price risk. As a result, financial hedging, when optimally integrated with the operating decisions, enables the firm to increase its output level and generate more value for the shareholders.

### 4.5.2 Make-to-Stock (MTS) Business Plan

In this section, we investigate the optimal operating and financial hedging decisions of the firm when both demand and price are uncertain. As in the MTO case, we focus on a single-stage problem involving two periods only. The firm commits for wheat at the beginning of the current period, and demand is realized at the beginning of the next period. Excess demand is lost and excess inventory is salvaged at the current spot price. The single-stage MTS model is given below.

$$V_1(S_1) = \max_{q_1 \geq 0, x_1} -s_1 q_1 + \beta E^Q_{\xi_2, P_2|P_1}[(s_2+\lambda)\min(q_1,\xi_2) - hI_2 + (f_1 - s_2)x_1 + V_2(S_2)] \qquad \text{(MTS 1)}$$

$$s.t.\ I_2 = [q_1 - \xi_2]^+$$

$$y_2 = (y_1 - s_1 q_1)/\beta + (s_2+\lambda)\min(q_1,\xi_2) - hI_2 + (f_1 - s_2)x_1 - \eta_2$$

where,

$V_2(S_2) = s_2 I_2 - FDC_2, FDC_2 = r[y_2 + s_2 I_2]^-$.

In this section, we denote the probability density and cumulative distribution functions of the demand by $\phi_{\xi_2}(.)$ and $\Phi_{\xi_2}(.)$, respectively. Further, for notational convenience, we omit the subscripts for quantity and hedging decision variables. The following lemma describes a benchmark result for the MTS case when there is no financial distress cost.

**Lemma 4.5.3.** *(Optimal Production without FDC) Suppose the firm operates for a single period under both stochastic demand and price. Then, when there is no financial distress cost, i.e.,* $r=0$*, necessary and sufficient optimality condition is given by:*

$$\beta E^{Q}_{P_2|P_1}[s_2]-s_1-\beta h+\beta(\lambda+h)\overline{\Phi}_{\xi_2}(q)=0\,, \tag{4.4}$$

*and the optimal quantity* $q^{nv}=\Phi^{-1}_{\xi_2}(\dfrac{\beta E^{Q}_{P_2|P_1}[s_2]-s_1+\beta\lambda}{\beta(\lambda+h)})=\Phi^{-1}_{\xi_2}(\dfrac{\beta f_1-s_1+\beta\lambda}{\beta(\lambda+h)})$.

When there is no cost of financial distress, the firm chooses a newsvendor-type optimal quantity which depends on the spread of spot and futures prices, $\beta f_1 - s_1$. As the futures price increases, the firm is more likely to sell at a higher price and hence the optimal order quantity increases. We also refer to $q^{nv}$ as the first-best production quantity (operating level) of the firm. Note that, since the convenience yield is non-negative, i.e., $s_1+\beta h-\beta f_1\geq 0$, $q^{nv}$ is finite. Next, we quantify the impact of financial distress cost in the MTS framework.

**Theorem 4.5.5.** *(Optimal Production with FDC) Suppose the firm operates for a single period under both stochastic demand and price, with no financial hedging. And, the probability of incurring financial distress is strictly between zero and one when the firm orders for the first-best production quantity,* $q^{nv}$*. Then, when there is a positive financial distress cost, i.e.,* $r>0$*, the firm always under-produces compared to* $q^{nv}$*, i.e.,* $q^*<q^{nv}$.

When there is no risk of incurring FDC, the marginal benefit of an additional unit is zero at $q^{nv}$. However, if there is a risk of incurring FDC, when committed to the first-best production quantity, then the marginal benefit of an additional unit is negative, since it increases the firm's exposure to financial distress risk. Hence, the firm would decrease its production level below its first-best level, $q^{nv}$, so as to balance the risk of financial distress and operating profits. The following theorem characterizes the optimal hedging decision of the firm.

**Theorem 4.5.6.** *(Optimal Hedging). Suppose the firm operates for a single period under both stochastic demand and price. Then, under a positive financial distress cost, if the firm decides to order and produce q units, it is also optimal to short $x^* = q$ units of futures contracts with futures price $f_1$. This hedging policy completely eliminates the price risk.*

Theorem 4.5.6 shows that, similar to the MTO case, it is optimal to completely hedge for the price risk. Substituting for the optimal hedging policy, (MTS 1) can be stated as a function of the quantity decision only:

$$V_1(S_1) = \max_{q \geq 0} - s_1 q_1 + \beta(f_1 + \lambda) - (\lambda + h) E_{\xi_2}[q - \xi_2]^+$$
$$- rE_{\xi_2}[y_1/\beta + (f_1 + \lambda - s_1/\beta)q - (\lambda + h)[q - \xi_2]^+ - \eta_2]^-$$

Next, we examine the operating decisions of the firm under the optimal financial hedging plan.

**Theorem 4.5.7.** *(Joint Optimal Policy). Suppose the firm operates for a single period under stochastic demand and price; and there is a positive financial distress cost. Let $\xi_2^w$ denote the lowest (worst) demand scenario. Then, under the optimal hedging policy, the optimal quantity $q^f$ is*

*(i) equal to the first-best operating level $q^{nv}$, if*

$$\kappa_1 \geq (f_1 + \lambda - s_1/\beta) q^{nv} \text{ or } \kappa_1 \leq (f_1 - h - s_1/\beta) q^{nv} + (\lambda + h)\xi_2^w$$

*(ii) and less than the first-best operating level $q^{nv}$, otherwise.*

If the firm's net cash outflows are too high to be covered by the revenues from operations, even when it sells all of the first-best operating quantity $q^{nv}$, i.e., $\kappa_1 \geq (f_1 + \lambda - s_1/\beta)q^{nv}$, then the firm does not deviate from its first-best operating plan. In this case, since the firm is guaranteed to face financial distress at the first-best operating level, under-production (producing less than $q^{nv}$) does not help to reduce the firm's exposure to financial distress risk. Similarly, if the firm's net cash outflows are low enough to be covered by the revenues, even under the worst-case demand scenario, i.e., $\kappa_1 \leq (f_1 - h - s_1/\beta)(q^{nv} - \xi_2^w)$, then the firm never uses external financing and chooses the first-best operating level, $q^{nv}$.

On the other hand, if $\kappa_1$ is between these two cases, then the chances of incurring financial distress cost is strictly between 0 and 1, when the firm commits for $q^{nv}$. Consequently, in this case, a marginal reduction in the quantity commitment of the firm reduces the firm's exposure to financial distress, and hence the firm chooses a more conservative operating plan by under-producing. As in the MTO case, the financial state of the firm plays a key role in the operating decisions under the optimal hedging policy. Next, we discuss the impact of hedging on the optimal operating plans.

**Theorem 4.5.8.** *Suppose the firm operates for a single period under both stochastic demand and price. Let $q^*$ be the optimal quantity when there is a positive financial distress cost and there is no hedging. Then, $q^{nv} \geq q^f \geq q^*$ .*

Theorem 4.5.8 shows that financial hedging reduces the firm's exposure to financial distress and enables it to adopt more aggressive production plans. However, unlike the MTO case, it may not be possible to completely eliminate the under-production problem.

## 4.6. CONCLUSION

In this chapter, we examine the joint operating and financial hedging decisions of a shareholder-value maximizing firm, with costly financial distress. Although our research is motivated by the flour milling industry, our findings can be easily generalized to other commodity processor firms which are exposed to fluctuations in commodity prices. As it is well known in the finance literature, in the absence of frictions, engaging in financial hedging is a neutral proposition. That is, it should not affect the optimal production plan, and it does not create value for firm's shareholders. However, when the firm faces capital market frictions, such as in our case costly financial distress, financial hedging can contribute to shareholder-wealth creation. Our findings show that the cost of financial distress forces firms to adopt more conservative operating plans by under-producing with respect to their first-best production levels. We first quantify this under-production problem, and then illustrate how financial markets can be used to mitigate it and generate more wealth for the shareholders.

We show that in a single-period MTO production environment, the risk of incurring financial distress, due to commodity price fluctuations, may deter the firm from meeting all customer orders especially when the firm’s planned cash outflows are well balanced with the operating profits. We illustrate that the financial distress risk due to commodity price fluctuations can be totally eliminated by taking a short position in the futures market, equivalent to the production commitments. Consequently, such a financial hedging policy changes the optimal operating decisions, and further enables the firm to meet all the future demand. In other words, the under-production problem in an MTO production environment can be totally eliminated by financial hedging.

Similarly in an MTS production environment, with demand and price uncertainty, financial distress risk may lead the firm produce less than the first-best production level. In this case, due to demand uncertainty, it is not possible to completely eliminate the financial distress risk and the allied under-production problem by trading in the futures market. However, we show that hedging for the price risk still adds value to the firm by reducing the firm's exposure to financial distress, and mitigating the under-production problem.

Maximizing firm value, defined as the total properly discounted value of expected cash flows, is a central concern for the managers of publicly traded companies. In this chapter, we provide a model, within the value-framework of finance, which links the financial risk management and the operational decisions of a value maximizing firm, under the presence of capital market imperfections. An overarching implication of our analysis is that publicly traded firms may significantly increase their market value, and generate more wealth for their shareholders by effectively integrating their financial risk management and production decisions. We show that a coordinated financial hedging and operating plan contributes to shareholder-wealth creation (1) by reducing the firm's exposure to financial distress risk and mitigating the corresponding costs, and (2) by enabling the firm to operate at a higher level of output.

# Chapter 5

# Conclusions and Future Work

In this dissertation, we explore the effective integration of risk management and operational decisions so as to improve the firm value and profits, and to create more wealth for the shareholders. We present analytical models investigating the value of various operational and financial hedging strategies commonly implemented in practice. Our results demonstrate that these hedging strategies, when effectively integrated with the operating plans, may significantly reduce the firms' exposure to business uncertainties and allied risks, and hence help to create more value.

The second chapter of my dissertation focuses on the benefits of process and operational flexibility and their interaction in a multi-period MTO production environment. We observe that both process and operational flexibility create an operational hedge against the demand fluctuations and the associated risk of backlogging, through increasing the effective processing capacity of the firm by enabling dynamic capacity reallocations. We show that the value of process flexibility depends on the operating policies implemented in the production floor, the variability of product demand and the capacity availability of the firm as well as the length of the planning horizon. Indeed, our findings demonstrate that process flexibility may not only be used to hedge against the demand uncertainty, but may also be employed to protect against possible suboptimal operating decisions in the future. In particular, myopic operating policies, which are common in practice, can be hedged through adopting more process flexibility prior to the beginning of the sales season. Moreover, we demonstrate that operational flexibility is most valuable when demand and capacity is well-balanced and demand variability is high.

In this chapter, we have assumed that product demand is the only source of risk in the supply chain. We believe that an interesting and fruitful research direction is to investigate the value of capacity flexibility when there are other sources of risks and uncertainties associated with capacity, competition and technology.

In the third chapter, we focus on operating and investment strategies for a financially constrained startup firm when making short-term production and long-term investment decisions under bankruptcy risk. Our results highlight an interesting operational hedging behavior between the long-term process investment decisions and the short-term production commitments of the firm. That is, a change in the process investment policy of the startup is always accompanied by a counter-action in the production decisions. We show that aggressive (conservative) investment plans are always hedged through aggressive (conservative) production decisions.

In particular, when faced with stochastic bankruptcy risk, the startup firm sacrifices some short-term profits by deviating from its first-best production plan (i.e., the monopoly quantity). In the conservative case, the firm under-produces so as to allocate more cash to process investment while controlling the survival chances. Further, depending upon the competitor's cost, startup's technological performance and aggregate demand, the firm may also invest aggressively by increasing the process investment amount above the expected monopoly profits. In this case, the startup also produces more than the monopoly level to cover the higher bankruptcy risk due to the aggressive process investment plans. We also identify and analyze two main factors driving aggressive behavior in startups: (1) the existence of positive NPV investment opportunities and (2) the immediate economical viability of the firm.

Further, we show that startup firms optimally adopting aggressive (conservative) policies go bankrupt (survive) with more (less) than a probability of 50%. Consequently, under stochastic demand, we establish the optimal operating decisions and the allied survival chances as an appropriate measure of startup's risk preferences. From this perspective, the existence of aggressive policies is interesting and consistent with empirical observations claiming that an average startup firm goes bankrupt and, only a small portion of the new firms eventually survive and grow (Gompers and Lerner 2004, Shane 2007).

In this third chapter, we have specifically studied investment in strategic process improvement which is aimed at unit cost reduction. We have excluded decisions such as new product development and brand growth. Further, startups in our model do not consider exit strategies, e.g., mergers or acquisition choices, which are also central to process investment decision. It would be useful to explore how investment in process development change without financially risking the startup's survival when the startup's objective is to signal a potentially strong market presence in order to look more attractive for a takeover. This could alter the startup's decisions, and hence, yield different results. We also do not consider venture capital (VC) funded startups which may receive multiple rounds of funding. Exploring the interaction between the startup and the venture capitalist, when making short-term and long-term business plans, is also a very promising future research direction.

In the fourth chapter of my dissertation, we extend our analyses to study the integrated operating and financial hedging decisions of a shareholder-value maximizing publicly traded firm, with costly financial distress. Similar to the effect of bankruptcy risk in Chapter 3, costly financial distress changes the optimal operating decisions for publicly traded firms, and leads them to choose more conservative operating plans via under-producing with respect to their first-best production levels. We demonstrate that financial hedging policies, when appropriately integrated with the operating plans, mitigate this under-production problem and enable firms adopt more aggressive production plans by reducing the firm's exposure to financial distress risk.

This research contributes to the existing operations management literature, by studying the risk management decisions of a public corporation within the value framework of finance; hence our findings do not require any specific assumptions about the investors' utility functions. Further we add to the literature by explicitly examining the impact of costly financial distress on hedging and operating plans.

In summary, integrating risk management and operating decisions is a new and promising research direction which offers valuable managerial insights in the field of operations management. This dissertation is one of the first works in this field, and it provides a better understanding of integrated risk management within the framework of established firms as well as small and start-up businesses. We strongly believe that our findings and analyses can be extended for many other business cases so as to develop new managerial insights.

# Appendices

# Appendix A

## Managing Capacity Flexibility in Make-to-Order Production Environments

### A.1. Explanation of Notation and Cut Calculations

$\pi_i^{t,r,k}$ = optimal dual price in period $t$ for replication $r$ under scenario $k$, for the balance constraint for family $i$

$\gamma_j^{t,r,k}$ = optimal dual price in period $t$ for replication $r$ under scenario $k$, for the capacity constraint for line $j$

$\lambda_{i,j}^{t,r,k}$ = optimal dual price in period $t$ for replication $r$ under scenario $k$, for the design constraint for assigning family $i$ to line $j$

$\rho_l^{t,r,k}$ = optimal dual price in period $t$ for replication $r$ under scenario $k$, for the cut constraint $l$

$\alpha^{t,r}$ = scalar contributing the cut intercept

$\mu^{t,r}(\mu_{1,1}^{t,r},...,\mu_{M,N}^{t,r})$ = cut gradient term

$\beta^{t,r}(\beta_1^{t,r},...,\beta_M^{t,r})$ = cut gradient term

Then, during the backward pass of the algorithm in replication $r$ and period $t$, the following cut is generated.

$$\theta^t \geq \frac{1}{|S|}\sum_{k\in S}[\sum_{i=1}^{M}\pi_i^{t,r,k}(d_i^{t,k}+b_i^{t-1})+\sum_{j=1}^{N}K_j\gamma_j^{t,r,k}+\sum_{j=1}^{N}\sum_{i=1}^{M}\frac{K_j}{e_{ij}}\lambda_{i,j}^{t,r,k}\hat{x}_{i,j}^{r}$$
$$+\sum_{l=1}^{r-1}\rho_l^{t,r,k}(\alpha^{t,l}+\sum_{j=1}^{N}\sum_{i=1}^{M}\mu_{i,j}^{t,l}\hat{x}_{i,j}^{r})]$$

Rearranging the terms we obtain:

$$\theta^t - \frac{1}{|S|}\sum_{k\in S}\sum_{i=1}^{M}\pi_i^{t,r,k}b_i^{t-1} \geq \frac{1}{|S|}\sum_{k\in S}[\sum_{i=1}^{M}\pi_i^{t,r,k}d_i^{t,k}+\sum_{j=1}^{N}K_j\gamma_j^{t,r,k}+\sum_{j=1}^{N}\sum_{i=1}^{M}\frac{K_j}{e_{ij}}\lambda_{i,j}^{t,r,k}\hat{x}_{i,j}^{r}$$
$$+\sum_{l=1}^{r-1}\rho_l^{t,r,k}(\alpha^{t,l}+\sum_{j=1}^{N}\sum_{i=1}^{M}\mu_{i,j}^{t,l}\hat{x}_{i,j}^{r})]$$

$$\Rightarrow \theta^t + \beta^{t-1,r}b^{t-1} \geq \alpha^{t-1,r} + \mu^{t-1,r}x$$

where

$$\beta_i^{t-1,r} = -\frac{1}{|S|}\sum_{k\in S}\pi_i^{t,r,k} \qquad t = 2,\ldots,T$$

$$\alpha^{t-1,r} = \frac{1}{|S|}\sum_{k\in S}[\sum_{i=1}^{M}\pi_i^{t,r,k}d_i^{t,k} + \sum_{j=1}^{N}K_j\gamma_j^{t,r,k} + \sum_{l=1}^{r-1}\rho_l^{t,r,k}\alpha^{t,l}] \qquad t = 2,\ldots,T$$

$$\mu_{i,j}^{t-1,r} = \frac{1}{|S|}\sum_{k\in S}[\frac{K_j}{e_{ij}}\lambda_{i,j}^{t,r,k} + \sum_{l=1}^{r-1}\rho_l^{t,r,w}\mu_{i,j}^{t,l}] \qquad t = 2,\ldots,T$$

$$\beta_i^{0,r} = 0 \qquad t = 1$$

$$\alpha^{0,r} = \frac{1}{|S|}\sum_{k\in S}[\sum_{i=1}^{M}\pi_i^{t,r,k}d_i^{t,k} + \sum_{j=1}^{N}K_j\gamma_j^{t,r,k} + \sum_{l=1}^{r-1}\rho_l^{t,r,k}\alpha^{t,l}] \qquad t = 1$$

$$\mu_{i,j}^{0,r} = -\frac{1}{|S|}\sum_{k\in S}[\frac{K_j}{e_{ij}}\lambda_{i,j}^{t,r,k} + \sum_{l=1}^{r-1}\rho_l^{t,r,w}\mu_{i,j}^{t,l}] \qquad t = 1$$

## A.2. Computational Settings for the Decomposition Method

While constructing an empirical scenario tree and the associated approximating problem (EDAM) we use a sample size of 100, i.e., $|S|$ = 100. Then, while solving the EDAM, with the algorithm of Section 2.4.2, the replication limit $R$ is set to 500. We observed that the improvement in the lower bound is not significant after 500 iterations. For lower bound estimation we used $\nu$ = 30, that is we have generated 30 i.i.d. sample scenario trees and the associated EDAMs and solved each model generating 30 i.i.d. lower bound estimates. Then, to estimate the cost of the feasible policy we used $\eta$ = 20,000 demand sample paths. We also employed Latin Hypercube Sampling, while generating i.i.d. sample scenario trees, for variance reduction purpose.

# Appendix B

# Production, Process Investment and Survival of Debt Financed Startup Firms

## B.1. Proofs

***Proof of Proposition 3.3.1:***

Substituting for the second period monopoly profits, and suppressing the subscripts and the superscripts yields the model:

$$\pi = \max_{q,A\geq 0} (\theta - q - (1+r)c)q - (1+r)A + (\frac{\theta - (1+r)(c-\mu A))}{2})^2$$
$$s.t.\ (\theta - q - (1+r)c)q - (1+r)A \geq 0$$

Now we observe that the single stage problem is separable in *q* and *A*, and can be written as:

$$\pi = \max_{q,A\geq 0} f(q) + g(A)$$
$$s.t.\ A \leq f(q)/(1+r)$$

where $f(q) = (\theta - q - (1+r)c)q$ and $g(A) = -(1+r)A + (\frac{\theta - (1+r)(c-\mu A))}{2})^2$.

Since $f(q)$ is concave, it is maximized at $q^* = \frac{\theta - (1+r)c}{2}$. Maximizing $f(q)$ also maximizes the right-hand side of the survival constraint, hence $q^*$ gives the optimal quantity decision. Following, by substituting $q^* = \frac{\theta - (1+r)c}{2}$ into the survival constraint, we obtain the upper bound on the first period investment as $A_{\max} = \frac{(\theta - (1+r)c)^2}{4(1+r)}$.

Here $g(A)$ is convex and the optimal solution is a boundary solution. Hence, the optimal investment amount can be found by evaluating the $g(A)$ function for the values of 0 and $A_{\max}$ and choosing the quantity that maximizes $g(A)$. More explicitly:

$$g(A_{\max}) = -\frac{(\theta-(1+r)c)^2}{4} + \left(\frac{\theta-(1+r)(c-\mu\frac{(\theta-(1+r)c)^2}{4(1+r)})}{2}\right)^2 , \; g(0) = \frac{(\theta-(1+r)c)^2}{4} \text{ and}$$

$$\Delta^{bc} = g(A_{\max}) - g(0) = \left(\frac{\theta-(1+r)(c-\mu\frac{(\theta-(1+r)c)^2}{4(1+r)})}{2}\right)^2 - \frac{(\theta-(1+r)c)^2}{2}$$
$$= \mu^2(\theta-(1+r)c)^2 + 8\mu(\theta-(1+r)c) - 16.$$

So the firm invest $A_{\max}$ if $\Delta^{bc} \geq 0$. Finally, substituting for the optimal policy we obtain:

$$\pi^* = \begin{cases} \left(\frac{\theta-(1+r)(c-\mu A_{\max})}{2}\right)^2 & if \quad \Delta^{bc} \geq 0 \\ \frac{(\theta-(1+r)c)^2}{2} & o/w \end{cases}$$

□

***Proof of Proposition 3.3.2:***

Following the developments in the proof of Proposition 3.3.1, when the return on investment is uncertain, first stage problem can be stated as:

$$\pi = \max_{q,A\geq 0} (\theta - q - (1+r)c)q - (1+r)A + \int \frac{(\theta-(1+r)c_2(A_1,\beta))^2}{4}\phi(\beta)d\beta$$
$$s.t. \quad (\theta - q - (1+r)c)q - (1+r)A \geq 0$$

Then, $g(A_{\max}) = -(1+r)A + \int \frac{(\theta-(1+r)c_2(A_1,\beta))^2}{4}\phi(\beta)d\beta$, $g(0) = \frac{(\theta-(1+r)c)^2}{4}$ and

$$\Delta^{tu} = g(A_{\max}) - g(0) = -(1+r)A_{\max} + \int \frac{(\theta-(1+r)c_2(A_{\max},\beta))^2}{4}\phi(\beta)d\beta - (1+r)A_{\max}$$
$$= \int \frac{(\theta-(1+r)(c-\beta A_{\max}))^2}{4}\phi(\beta)d\beta - 2(1+r)A_{\max}$$
$$= \int \frac{(\theta-(1+r)c+(1+r)\beta A_{\max}))^2}{4}\phi(\beta)d\beta - 2(1+r)A_{\max}$$
$$= \int \frac{(\theta-(1+r)c)^2 + (1+r)^2\beta^2A_{\max}^2 + 2(\theta-(1+r)c)(1+r)\beta A_{\max}}{4}\phi(\beta)d\beta - 2(1+r)A_{\max}$$
$$= \frac{(\theta-(1+r)c)^2}{4} + \frac{(1+r)^2(\sigma^2+\mu^2)A_{\max}^2 + 2(\theta-(1+r)c)(1+r)\mu A_{\max}}{4} - 2(1+r)A_{\max}$$

$$= \frac{(1+r)^2(\sigma^2+\mu^2)A_{\max}^2 + 2(\theta-(1+r)c)(1+r)\mu A_{\max}}{4} - (1+r)A_{\max}$$
$$= (\sigma^2+\mu^2)\frac{(\theta-(1+r)c)^2}{4} + 2(\theta-(1+r)c)\mu - 4$$
$$= (\sigma^2+\mu^2)(\theta-(1+r)c)^2 + 8(\theta-(1+r)c)\mu - 16$$
$$= \Delta^{bc} + \sigma^2(\theta-(1+r)c)^2$$

Finally, substituting for the optimal policy we obtain:

$$\pi^* = \begin{cases} \dfrac{(\theta-(1+r)(c-\mu A_{\max}))^2 + (1+r)^2\sigma^2 A_{\max}^2}{4} & \text{if} \quad \Delta^{tu} \geq 0 \\ \dfrac{(\theta-(1+r)c)^2}{2} & o/w. \end{cases}$$

□

***Proof of Proposition 3.3.3:***

*i*) Since a Cournot game is played in the second period, startup's and competitor's equilibrium quantities are given by $q_2 = \dfrac{\theta+\xi-2(1+r)c_2\left(A_1,\beta\right)}{3}$ and $q_c = \dfrac{\theta+(1+r)c_2(A_1,\beta)-2\xi}{3}$, respectively. Consequently, startup's profit in the second period is: $\pi_2(A_1,\xi,\beta) = \dfrac{(\theta+\xi-2(1+r)c_2(A_1,\beta))^2}{9}$.

Hence, substituting for the expected second period profit, the first stage problem becomes:

$$\pi = \max_{q,A\geq 0}\,(\theta-q-(1+r)c)q-(1+r)A+\int\frac{(\theta+\xi-2(1+r)c_2(A,\beta))^2}{9}\phi(\xi)d\xi$$
$$s.t.\ (\theta-q-(1+r)c)q-(1+r)A \geq 0$$

Then, following the developments in the proof of Proposition 3.3.2 above, it is still optimal to produce the monopoly level and, the investment threshold and optimal profits are obtained as follows:

$g(A_{\max}) = -(1+r)A_{\max} + \displaystyle\int\frac{(\theta+\xi-2(1+r)(c-\mu A_{\max}))^2}{9}\phi(\xi)d\xi$,

$g(0) = \displaystyle\int\frac{(\theta+\xi-2(1+r)c)^2}{9}\phi(\xi)d\xi$ and

$$
\begin{aligned}
\Delta^c &= g(A_{\max}) - g(0) \\
&= -(1+r)A_{\max} + \int \frac{(\theta+\xi-2(1+r)(c-\mu A_{\max}))^2}{9}\phi(\xi)d\xi - \int \frac{(\theta+\xi-2(1+r)c)^2}{9}\phi(\xi)d\xi \\
&= -(1+r)A_{\max} + \int \{\frac{(\theta+\xi-2(1+r)c+2(1+r)\mu A_{\max})^2-(\theta+\xi-2(1+r)c)^2}{9}\}\phi(\xi)d\xi \\
&= -(1+r)A_{\max} + \frac{2(\theta+\lambda-2(1+r)c)2(1+r)\mu A_{\max}+4(1+r)^2\mu^2(A_{\max})^2}{9} \\
&= -9+2(\theta+\lambda-2(1+r)c)2\mu+4(1+r)\mu^2 A_{\max} \\
&= -9+4\mu(\theta+\lambda-2(1+r)c)+\mu^2(\theta-(1+r)c)^2 \\
&= (\mu(\theta-(1+r)c)+2)^2-4\mu((1+r)c-\lambda)-13
\end{aligned}
$$

So the firm invest $A_{\max}$ if $\Delta^c \geq 0$. Finally, substituting for the optimal policy we obtain:

$$
\pi^* = \begin{cases} \frac{1}{9}\{(\theta-2(1+r)c+\lambda)^2+4A_{\max}\{\mu(\theta-2(1+r)c+\lambda)+\mu^2 A_{\max}\}+\tau^2\} & if \quad \Delta^c \geq 0 \\ \frac{(\theta-(1+r)c)^2}{4} + \frac{1}{9}\{(\theta-2(1+r)c+\lambda)^2+\tau^2\} & o/w \end{cases}
$$

□

***Proof of Corollary 3.3.1:***

Note that $\Delta^c$ is linear increasing in $\lambda$. Hence, solving $\Delta^{bc}-\Delta^c=0$ for $\lambda$, gives the desired result. (We assume parameters guarantee positive quantities and positive second period cost, i.e., $q_2$, $q_c$ and $c_2$ are non-negative.) □

***Proof of Proposition 3.4.1:***

Below, we evaluate the expectation step by step and simplify the problem (3.3):

$$
\max_{q_1, A_1 \geq 0} E_{\tilde{\varepsilon}_1}\{(p(q_1,\tilde{\varepsilon}_1)-c_1)q_1 - r(c_1 q_1 + A_1) - A_1\}
$$

$$
+\alpha E_{\tilde{\xi},\tilde{\beta},\tilde{\varepsilon}_1}\{\frac{(\theta+\tilde{\xi}-2(1+r)c_2(A_1,\tilde{\beta}))^2}{9} \mid \pi_1(\tilde{\varepsilon}_1) \geq \underline{\pi}\}
$$

$$
\max_{q_1, A_1 \geq 0} E_{\tilde{\varepsilon}_1}\{(p(q_1,\tilde{\varepsilon}_1)-c_1)q_1 - r(c_1 q_1 + A_1) - A_1\}
$$

$$
+\alpha \int_B \int_0^\infty \int_0^\infty \frac{((\theta+\xi-2(1+r)c_2(A_1,\beta))^2}{9}\phi(\xi)\psi(\beta)\varphi(\varepsilon_1)d\xi d\beta d\varepsilon_1
$$

where $B = \{\varepsilon_1 \mid \pi_1(\varepsilon_1) \geq \underline{\pi}\}$.

$$\max_{q_1, A_1 \geq 0} E_{\tilde{\varepsilon}_1}\{(p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1\}$$
$$+ \alpha \int_B \int_0^\infty \frac{\tau^2 + (\theta + \lambda - kc_2(A_1, \beta))^2}{9} \psi(\beta)\varphi(\varepsilon_1) d\beta d\varepsilon_1$$
$$\max_{q_1, A_1 \geq 0} E_{\tilde{\varepsilon}_1}\{(p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1\}$$
$$+ \alpha \int_B \frac{\tau^2 + (\theta + \lambda - kc_1)^2 + 2(\theta + \lambda - kc_1)k\mu A_1 + k^2(\mu^2 + \sigma^2)A_1^2}{9} \varphi(\varepsilon_1) d\varepsilon_1$$
$$\max_{q_1, A_1 \geq 0} E_{\tilde{\varepsilon}_1}\{(p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1\}$$
$$+ \alpha[\frac{\tau^2 + (\theta + \lambda - kc_1)^2 + 2(\theta + \lambda - kc_1)k\mu A_1 + k^2(\mu^2 + \sigma^2)A_1^2}{9}](1 - \vartheta_{\varepsilon_1}(\frac{\underline{\pi} + (1+r)(c_1 q_1 + A_1)}{q_1} + q_1 - \theta))$$
$$\max_{q_1, A_1 \geq 0} (\theta - q_1)q_1 - (r+1)(c_1 q_1 + A_1)$$
$$+ \alpha[\frac{\tau^2 + (\theta + \lambda - kc_1)^2 + 2(\theta + \lambda - kc_1)k\mu A_1 + k^2(\mu^2 + \sigma^2)A_1^2}{9}](1 - \vartheta_{\varepsilon_1}(\frac{\underline{\pi} + (1+r)(c_1 q_1 + A_1)}{q_1} + q_1 - \theta))$$

Finally, suppressing the subscripts and superscripts, we obtain the result:

$$\max_{q, A \geq 0} f(q, A) = \max_{q, A \geq 0} (\theta - q)q - (r+1)(cq + A)$$
$$+ \alpha[\frac{\tau^2 + (\theta + \lambda - kc)^2 + 2(\theta + \lambda - kc)k\mu A + k^2(\mu^2 + \sigma^2)A^2}{9}](1 - \vartheta_{\varepsilon_1}(\frac{\underline{\pi} + (1+r)(cq + A)}{q} + q - \theta))$$

Recall that $\pi_m = E\{(p(q_m, \tilde{\varepsilon}_1) - (1+r)c)q_m\}$, $q_m = \dfrac{(\theta - (1+r)c)}{2}$ , $A_m = \dfrac{\pi_m - \underline{\pi}}{(1+r)}$. First, we partition the feasible decision space into the following four regions:

($i$) $q \leq q_m$ and $A \leq A_m$

($ii$) $q \leq q_m$ and $A \geq A_m$

($iii$) $q \geq q_m$ and $A \leq A_m$

($iv$) $q \geq q_m$ and $A \geq A_m$

Let $D = \alpha[\dfrac{\tau^2 + (\theta + \lambda - kc)^2 + 2(\theta + \lambda - kc)k\mu A + k^2(\mu^2 + \sigma^2)A^2}{9}]$, then at the optimality it must hold that:

$$\frac{\partial f(q, A)}{\partial q} = \theta - 2q - (r+1)c - D\varphi(\frac{\underline{\pi} + (1+r)(cq + A)}{q} + q - \theta) * (1 - \frac{\underline{\pi} + (1+r)A}{q^2}) = 0\,.$$

Following, it is easy to observe $\frac{\partial f(q,A)}{\partial q} > 0$ for every decision vector in region (*ii*) and $\frac{\partial f(q,A)}{\partial q} < 0$ for every decision vector in region (*iii*). Hence the optimal policy should be either in region (*i*) or (*iv*). This concludes the desired "if and only if" argument in the proposition.

Note that the proof does not depend on the convexity of $f(q,A)$. Indeed, $f(q,A)$ is not generally convex and in this proof, we don't address the optimal investment, *A**. Recall that the joint optimization of *A* and *q* is not tractable analytically. Indeed, numeric analyses show that at the optimality, when both *A* and *q* varied simultaneously, optimal policy may be either in region (*i*) and (*iv*). □

***Proof of Proposition 3.4.2:***

Recall that the demand shock $\tilde{\varepsilon}_t$ (*t* = 1, 2) has a normal probability density function, $\varphi(.)$, with mean zero and variance $v^2$. It is sufficient to show that the argument of the survival probability, $x = \frac{\pi + (1+r)A}{q} + q - \theta + (1+r)c$, is less (greater) than or equal to zero if and only if the optimal operating policy is aggressive (conservative). Since $q_m = \frac{\theta - (1+r)c}{2}$, we can state the argument of the survival probability as $x = \frac{\pi + (1+r)A}{q} + q - 2q_m$. Finally also observe that the first order optimality condition for *q*, i.e., $\frac{\partial f(q,A)}{\partial q} = 0$, implies that $A \geq \frac{q^2 - \pi}{1+r}$ if $q \geq q_m$, and $A \leq \frac{q^2 - \pi}{1+r}$ if $q \leq q_m$.

Suppose the optimal operating policy is aggressive. Then, by definition $q \geq q_m$ and $A \geq \frac{q^2 - \pi}{1+r} \geq A_m$. Consequently, the first two terms of *x* is larger than $2q_m$ and hence,

$x \geq 0$. Now, suppose the optimal operating policy is conservative. Then, by definition $q \leq q_m$ and $A \leq \frac{q^2 - \underline{\pi}}{1+r} \leq A_m$. So, the first two terms of $x$ is less than $2q_m$ and hence, $x \leq 0$.

Next, we prove the *only if* part of the argument. Consider an optimal operating policy ($A$, $q$) such that $x = \frac{\underline{\pi} + (1+r)A}{q} + q - 2q_m \geq 0$, i.e., the survival chances are less than 50 percent. Now, suppose this optimal operating policy is conservative, i.e., $q \leq q_m$ and $A \leq \frac{q^2 - \underline{\pi}}{1+r} \leq A_m$. Then, $\frac{\underline{\pi} + (1+r)A}{q} \leq q$ and hence, $\frac{\underline{\pi} + (1+r)A}{q} + q - 2q_m \leq 0$. So, by contradiction, the policy should be aggressive. The conservative case can be proven similarly. □

***Proof of Corollary 3.4.1 and Corollary 3.4.2:***

Immediate economic viability, $\pi_m - \underline{\pi} > 0$, implies that $A_m = \frac{\pi_m - \underline{\pi}}{(1+r)} > 0$. When there are no investment opportunities firm has to set $A = 0$ and hence $A < A_m$. From Proposition 3.4.1, this implies that the optimal policy should be in region (*i*) and $q < q_m$. When there are investment opportunities, i.e., $A \geq 0$, the optimal solution may be in either region (*i*) or (*iv*). Indeed, in Section 3.4.2, we provide various numerical examples for both cases.

On the other hand, if $\pi_m - \underline{\pi} < 0$, then $A_m = \frac{\pi_m - \underline{\pi}}{(1+r)} < 0$. In this case, regardless of the existence of investment opportunities $A > A_m$ and hence, the optimal policy should lie in region (*iv*) and $q > q_m$. □

***Proof of Proposition 3.5.1:***

(*i*) Let $f(q) = (\theta - q - c)q$, and $g(A) = -A + (\frac{\theta - (c - \mu A))}{2})^2$. Then the maximization problem can be stated as:

$$\max_{q,A} f(q)+g(A)$$
$$s.t.\quad A \le L-cq \qquad (1)$$
$$A \le f(q) \qquad (2)$$
$$A, q \ge 0$$

Or equivalently,
$$\max f(q)+g(A)$$
$$s.t.\quad A \le \min\{f(q), L-cq\}$$
$$A, q \ge 0$$

Solving $f(q)=L-cq$ for $q$, we obtain $q_1 = \frac{\theta}{2}-\frac{\sqrt{\theta^2-4L}}{2}$ and $q_2 = \frac{\theta}{2}+\frac{\sqrt{\theta^2-4L}}{2} \ge q_m$.

Then, if $q_1 \ge q_m$, the debt constraint never binds as shown in Figure A.1. Hence Proposition 3.3.1 applies. More specifically, this case can be stated as:

$\frac{\theta}{2}-\frac{\sqrt{\theta^2-4L}}{2} \ge \frac{\theta-c}{2}$ ➔ $L \ge \frac{\theta^2-c^2}{4} = cq_m + A_{\max}$. This proves the first part of the proposition.

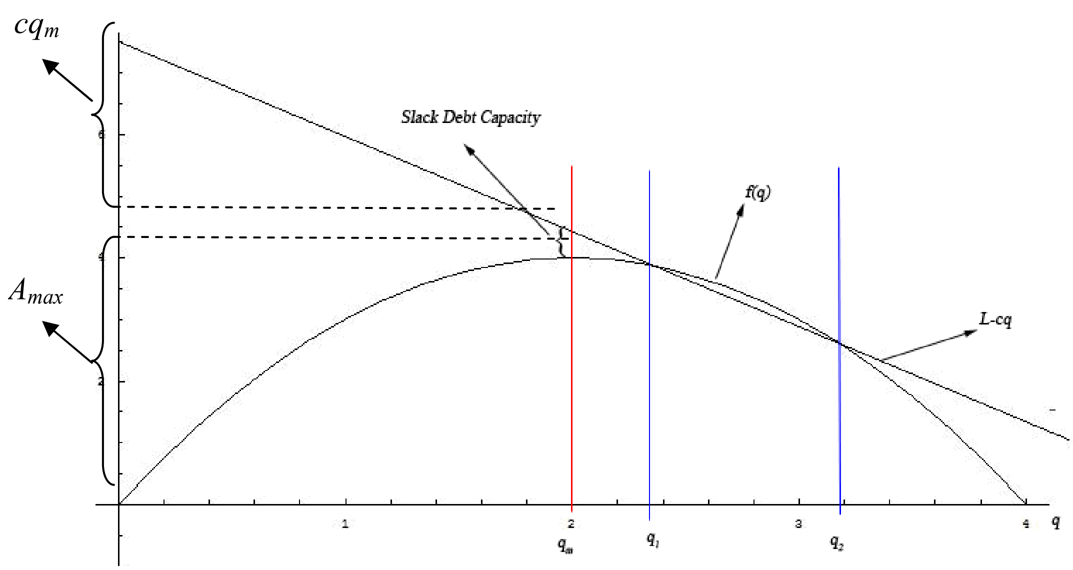



Figure A.1: An Illustration of the Debt Constraint

*ii)* Suppose $L < cq_m$, then the debt constraint should always bind since any excess cash can be used to increase production and increase profits.

*iii*) Suppose $cq_m \le L \le cq_m + A_{\max}$ and the return on investment is very close to zero. Then, the firm simply produces up to the monopoly level and does not use the rest of the debt capacity for investment. Hence, the debt capacity constraint may not be tight. □

***Proof of Corollary 3.5.2:***

From the proof of Proposition 3.3.1, when there is no debt capacity, investment threshold is given by: $\Delta = g(A_{\max}) - g(0)$. When there is an explicit debt capacity, *L*, then for any feasible of production policy *q*, maximum investment budget is given by: $A_{l\max}(q,L) = \min\{f(q), L - cq\} < A_{\max}$. Next, since *g*(.) is increasing in *A*, $\Delta^{l}(q,L) = g(A_{l\max}) - g(0) \leq \Delta$. □

## B.2. Computational Procedure

We work with problem (3.3):

$$z^{*} = \max_{q_1, A_1 \geq 0} E_{\tilde{\varepsilon}_1}\{(p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1\}$$

$$+\alpha E_{\tilde{\xi}, \tilde{\beta}, \tilde{\varepsilon}_1}\{\frac{(\theta + \tilde{\xi} - 2(1+r)c_2(A_1, \tilde{\beta}; A_0))^2}{9} \mid \pi_1(\tilde{\varepsilon}_1) \geq \underline{\pi}\}$$

Since this optimization problem has only two decision variables, we implement a search algorithm, in C++, to find the optimal decisions for each set of parameter values. This algorithm is available from the authors on request. For example, in order to construct Figure 3.6, we first fix $\mu$ and then solve (3.3) optimally for varying values of $\lambda$, and we determine the critical value(s) of $\lambda$ after which the firm switches its optimal investment strategy. Then, by connecting these critical values with a smooth line we arrive at Figure 3.6.

# Appendix C

# An Integrated Approach to Commodity Risk Management

## C.1. Proofs

***Proof of Lemma 4.5.1:***

Final period's objective, $J_{T-1}(q_{T-1}, x_{T-1} \mid .)$, is concave in decision variables $(q_{T-1}, x_{T-1})$. Therefore, by preservation of concavity under maximization, $J_t(q_t, x_t \mid .)$ is concave for $t = 1,...,T-1$. □

***Proof of Lemma 4.5.2:***

This result follows from the fact that $f_t = E^{Q}_{P_{t+1}|P_t}[s_{t+1}]$. □

***Proof of Theorem 4.5.1:***

Under deterministic demand, final period's problem with no financial distress cost is given below (note that the cash reserves and financial hedging is immaterial to the firm value):

$$V_{T-1}(S_{T-1}) = \max_{q_{T-1} \geq 0} J_{T-1}(q_{T-1} \mid S_{T-1})$$

where $J_{T-1}(q_{T-1} \mid .) = -s_{T-1}q_{T-1} + \beta E^{Q}_{P_T|P_{T-1}}\{(s_T + \lambda)\min(I_{T-1} + q_{T-1}, \xi_T) + (s_T - h)I_T\}$ and $I_T = [I_{T-1} + q_{T-1} - \xi_T]^{+}$ .

Noting that $E^{Q}_{P_T|P_{T-1}}[s_T] = f_{T-1}$, the first partial derivative can be stated as:

$$\frac{\partial J_{T-1}(q_{T-1} \mid .)}{\partial q_{T-1}} = -s_{T-1} + \beta(f_{T-1} - h)I_{\{I_{T-1}+q_{T-1} > \xi_T\}} + \beta(f_{T-1} + \lambda)I_{\{I_{T-1}+q_{T-1} \leq \xi_T\}}.$$

Hence, $q^*_{T-1} = \xi_T - I_{T-1}$ if $-s_{T-1} + \beta(f_{T-1} + \lambda) \geq 0$ and $-s_{T-1} + \beta(f_{T-1} - h) \leq 0$.

By induction this result extends for $t = 1,...,T-1$. So, $q^*_1 = \xi_2 - I_1$ and $q^*_t = \xi_{t+1}, I^*_t = 0,$ if $-s_t + \beta(f_{t-1} + \lambda) \geq 0$ and $-s_{t-1} + \beta(f_{t-1} - h) \leq 0$ for $t = 2,...,T-1$. Letting $I_1 = 0$ gives the desired result in the theorem. □

***Proof of Theorem 4.5.2:***

Substituting for $V_2(.)$, and evaluating the expectation, the single period MTO model with no financial hedging becomes:

$$V_1(S_1) = \max_{q_1 \geq 0} J_1(q_1 \mid S_1)$$

$$= \max_{q_1 \geq 0} - s_1 q_1 + \beta(f_1 + \lambda)\min(q_1, \xi_2) + \beta(f_1 - h)[q_1 - \xi_2]^+$$

$$+ r\beta E^{Q}_{P_2|P_1}[\frac{1}{\beta}(y_1 - s_1 q_1) + (s_2 + \lambda)\min(q_1, \xi_2) + (s_2 - h)[q_1 - \xi_2]^+ - \eta_2]^-$$

Now suppose $q_1 \geq \xi_2$ then,

$$J_1(q_1 \mid S_1) = -s_1 q_1 + \beta(f_1 + \lambda)\xi_2 + \beta(f_1 - h)(q_1 - \xi_2)$$

$$- r\beta E^{Q}_{P_2|P_1}[\frac{1}{\beta}(y_1 - s_1 q_1) + (s_2 + \lambda)\xi_2 + (s_2 - h)(q_1 - \xi_2) - \eta_2]^-$$

Consequently, the first partial derivative is given by:

$$\frac{\partial J_1(.)}{\partial q_1} = -s_1 + \beta(f_1 - h) + r\beta\int_0^a (-\frac{1}{\beta}s_1 + s_2 - h)\phi(s_2)ds_2 ,$$

where $a = \dfrac{(-y_1 + s_1 q_1)/\beta - \lambda\xi_2 + hq_1 + h\xi_2 + \eta_2}{q_1}$.

Then, since the convenience yield is non-negative, $\dfrac{\partial J_1(.)}{\partial q_1} \leq 0$. This implies that the optimal quantity cannot be larger than $\xi_2$.

Now suppose $q_1 \leq \xi_2$ then,

$$J_1(q_1 \mid S_1) = -s_1 q_1 + \beta(f_1 + \lambda)q_1 - \beta E^{Q}_{P_2|P_1} r[\frac{1}{\beta}(y_1 - s_1 q_1) + (s_2 + \lambda)q_1 - \eta_2]^-$$

Consequently, the first partial derivative is given by:

$$\frac{\partial J_1(.)}{\partial q_1} = a(q_1) = -s_1 + \beta(f_1 + \lambda) + r\beta\int_0^b (-s_1/\beta + s_2 + \lambda)\phi(s_2)ds_2 ,$$

where $b = \dfrac{\eta_2 - y_1/\beta}{q_1} + s_1/\beta - \lambda.$

Hence, when there is no financial hedging, the firm under-produces if

$$a(\xi_2) = -s_1 + \beta(f_1 + \lambda) + r\beta\int_0^b (-s_1/\beta + s_2 + \lambda)\phi(s_2)ds_2 < 0 . \square$$

***Proof of Theorem 4.5.3:***

For a given quantity decision, $q_1$, the firm solves the following hedging problem:

$$V_1(S_1) = \max_{x_1} - s_1 q_1 + \beta(f_1 + \lambda)\min(q_1, \xi_2) + \beta(f_1 - h)[q_1 - \xi_2]^+$$

$$-\beta E^Q_{P_2|P_1} r[\frac{1}{\beta}(y_1 - s_1 q_1) + (s_2 + \lambda)\min(q_1, \xi_2) + (f_1 - s_2)x_1 + (s_2 - h)[q_1 - \xi_2]^+ - \eta_2]^-$$

Since the hedging decision only impacts the financial distress term:

$$x_1^*(q_1) = \arg\min_{x_1} D_1(x_1 \,|\, .)$$

$$= \arg\min_{x_1} E^Q_{P_2|P_1}[\frac{1}{\beta}(y_1 - s_1 q_1) + (s_2 + \lambda)\min(q_1, \xi_2) + (f_1 - s_2)x_1 + (s_2 - h)[q_1 - \xi_2]^+ - \eta_2]^-.$$

Now suppose $q_1 \le \xi_2$ then,

$$x_1^*(q_1) = \arg\min_{x_1} D_1(x_1 \,|\, .) = \arg\min_{x_1} E^Q_{P_2|P_1}[\frac{1}{\beta}(y_1 - s_1 q_1) + (s_2 + \lambda)q_1 + (f_1 - s_2)x_1 - \eta_2]^-.$$

Following the first partial derivate is given by:

$$\frac{\partial D_1(x_1 \,|\, .)}{\partial x_1} = \int_0^b (f_1 - s_2)\phi(s_2)ds_2 \quad \text{if } q_1 \le x_1 \text{ , and}$$

$$\frac{\partial D_1(x_1 \,|\, .)}{\partial x_1} = \int_b^\infty (f_1 - s_2)\phi(s_2)ds_2 \quad \text{if } q_1 \ge x_1.$$

Observing that $D_1(x_1 \,|\, .)$ is convex and $\frac{\partial D_1(x_1 \,|\, .)}{\partial x_1}|_{q_1} = 0,$ the optimal hedging decision $x_1^*(q_1) = q_1$. (The case for $q_1 \ge \xi_2$, can be shown similarly.) □

***Proof of Theorem 4.5.4:***

Substituting for the optimal hedging policy given in Theorem 4.5.3, we obtain the following maximization problem as a function of the quantity decision only:

$$V_1(S_1) = \max_{q_1 \ge 0} - s_1 q_1 + \beta E^Q_{P_2|P_1}\{(s_2 + \lambda)\min(q_1, \xi_2) + (f_1 - s_2)q_1 + (s_2 - h)[q_1 - \xi_2]^+\}$$

$$-\beta r E^Q_{P_2|P_1}[\frac{1}{\beta}(y_1 - s_1 q_1) + (s_2 + \lambda)\min(q_1, \xi_2) + (f_1 - s_2)q_1 + (s_2 - h)[q_1 - \xi_2]^+ - \eta_2]^-$$

$$= \max_{q_1 \ge 0} - s_1 q_1 + \beta(f_1 + \lambda)q_1 - \beta(h + \lambda)[q_1 - \xi_2]^+$$

$$- \beta r[\frac{1}{\beta}(y_1 - s_1 q_1) + (f_1 + \lambda)q_1 - (h + \lambda)[q_1 - \xi_2]^+ - \eta_2]^-$$

$$= \max_{q_1 \ge 0} (\beta(f_1 + \lambda) - s_1)q_1 - \beta(h + \lambda)[q_1 - \xi_2]^+$$

$$- \beta r[y_1/\beta + (f_1 + \lambda - s_1/\beta)q_1 - (h + \lambda)[q_1 - \xi_2]^+ - \eta_2]^-$$

Suppose $q_1 \le \xi_2$ then,
$V_1(S_1) = \max_{q_1 \ge 0} J_1(q_1 \,|\, .) = \max_{q_1 \ge 0} (\beta(f_1 + \lambda) - s_1) q_1 - \beta r [y_1/\beta + (f_1 + \lambda - s_1/\beta) q_1 - \eta_2]^-$.

Since $J_1(q_1 \,|\, .)$ is increasing in $q_1$, the optimal quantity $q_1^* = \xi_2$, for $q_1 \le \xi_2$.

Now, suppose $q_1 \ge \xi_2$ then,

$$V_1(S_1) = \max_{q_1 \ge 0} J_1(q_1 \,|\, .) = \max_{q_1 \ge 0} (\beta(f_1 - h) - s_1) q_1 + \beta(h + \lambda)\xi_2 - \beta r [y_1/\beta + (f_1 - h - s_1/\beta) q_1 + (h + \lambda)\xi_2 - \eta_2]^-.$$

In this case, $J_1(q_1 \,|\, .)$ is decreasing in $q_1$, so the optimal quantity $q_1^* = \xi_2$, for $q_1 \ge \xi_2$.

Hence $q_1^* = \xi_2$. □

***Proof of Lemma 4.5.3:***

When $r = 0$, financial hedging is immaterial to the operating decisions and the single stage problem can be stated as the following:

$$V_1(S_1) = \max_{q_1 \ge 0} - s_1 q_1 + \beta E^Q_{\xi_2, P_2 | P_1} \{(s_2 + \lambda) \min(q_1, \xi_2) + (s_2 - h)[q_1 - \xi_2]^+\}$$

Consequently, the necessary and sufficient optimality condition is given by: $\beta E^Q_{s_2|s_1}[s_2] - s_1 - \beta h + \beta(\lambda + h)\overline{\Phi}_{\xi_2}(q) = 0$ and the optimal quantity $q^{nv} = \Phi^{-1}\left(\frac{\beta E^Q_{P_2|P_1}[s_2] - s_1 + \beta\lambda}{\beta(\lambda + h)}\right) = \Phi^{-1}\left(\frac{\beta f_1 - s_1 + \beta\lambda}{\beta(\lambda + h)}\right)$. Since the convenience yield is non-negative, $q^{nv}$ is finite. □

***Proof of Theorem 4.5.5:***

Suppressing the superscripts and the subscripts (except for $s_1$ and $P_1$), the single stage problem can be stated as:

$$\begin{aligned} V_1(S_1) &= \max_{q \ge 0} J_1(q_1 \,|\, .) = \max_{q \ge 0} - s_1 q_1 + \beta E^Q_{\xi, P | P_1} \{(s + \lambda)\min(q, \xi) + (s - h)[q - \xi]^+\} \\ &\quad - \beta E^Q_{\xi, P | P_1} r [(y - s_1 q)/\beta + (s + \lambda)\min(q, \xi) + (s - h)[q - \xi]^+ - \eta]^- \\ &= \max_{q \ge 0} J(q \,|\, .) = \max_{q \ge 0} - s_1 q + \beta E^Q_{\xi, P | P_1} \{(s + \lambda) q - (\lambda + h)[q - \xi]^+\} \\ &\quad - \beta E^Q_{\xi, P | P_1} r [(y - s_1 q)/\beta + (s + \lambda) q - (\lambda + h)[q - \xi]^+ - \eta]^- \end{aligned}$$

Evaluating the expectations,

$$J(q\,|\,.) = -s_1 q + \beta(f+\lambda)q - \beta(\lambda+h)E_\xi[q-\xi]^+$$
$$+\beta r\int_0^q\int_0^{u_1}[y/\beta + (s+\lambda - s_1/\beta)q - (\lambda+h)(q-\xi) - \eta]\phi_s^Q(s)\phi_\xi(\xi)dsd\xi$$
$$+\beta r\int_q^\infty\int_0^{u_2}[y/\beta + (s+\lambda - s_1/\beta)q - \eta]\phi_s^Q(s)\phi_\xi(\xi)dsd\xi$$

where $u_1 = \dfrac{\eta - y/\beta + (s_1/\beta - \lambda)q}{q} + \dfrac{(\lambda+h)(q-\xi)}{q}, u_2 = \dfrac{\eta - y/\beta + (s_1/\beta-\lambda)q}{q}$.

Following, the necessary and sufficient optimality condition is:

$$\frac{\partial J(q\,|\,.)}{\partial q} = -s_1 + \beta(f+\lambda) - \beta(\lambda+h)\Phi_\xi(q) + \beta r\int_0^q\int_0^{u_1}[s - s_1/\beta - h]\phi_s^Q(s)\phi_\xi(\xi)dsd\xi$$
$$+\beta r\int_q^\infty\int_0^{u_2}[s - s_1/\beta + \lambda]\phi(s)\phi_\xi(\xi)dsd\xi = 0\,.$$

So, we want to show that $\dfrac{\partial J(q\,|\,.)}{\partial q}\Big|_{q=q^{nv}} \le 0$

First we define $\varepsilon$ such that:

$$\varepsilon = \arg\max_{\hat\varepsilon\in\{0,\lambda+h\}}\int_0^q\int_0^{u_2+\hat\varepsilon}[s - s_1/\beta - h]\phi_s^Q(s)\phi_\xi(\xi)dsd\xi$$

Then, from the definition of $\varepsilon$, it follows that:

$$\frac{1}{\beta r}\frac{\partial J(.\,|\,x)}{\partial q}\Big|_{q=q^{nv}} \le \int_0^q\int_0^{u_2+\varepsilon}[s - s_1/\beta - h]\phi_s^Q(s)\phi_\xi(\xi)dsd\xi + \int_q^\infty\int_0^{u_2}[s - s_1/\beta + \lambda]\phi_s^Q(s)\phi_\xi(\xi)dsd\xi$$
$$= \Phi_\xi(q)\int_0^{u_2+\varepsilon}[s - s_1/\beta - h]\phi_s^Q(s)ds + \overline{\Phi}_\xi(q)\int_0^{u_2}[s - s_1/\beta + \lambda]\phi_s^Q(s)ds$$
$$= \Phi_s^Q(u_2)(\lambda\overline{\Phi}_\xi(q) - h\Phi_\xi(q)) + \int_0^{u_2}[s - s_1/\beta]\phi_s^Q(s)ds$$
$$+\Phi_\xi(q)\int_{u_2}^{u_2+\varepsilon}[s - s_1/\beta - h]\phi_s^Q(s)ds$$
$$= \Phi_s^Q(u_2)(s_1/\beta - f) + \int_0^{u_2}[s - s_1/\beta]\phi_s^Q(s)ds + \Phi_\xi(q)\int_{u_2}^{u_2+\varepsilon}[s - s_1/\beta - h]\phi_s^Q(s)ds$$

$$= \int_0^{u_2}(s_1/\beta - f)\phi_s^Q(s)ds + \int_0^{u_2}[s - s_1/\beta]\phi_s^Q(s)ds$$
$$+\Phi_\xi(q)\int_{u_2}^{u_2+\varepsilon}[s - s_1/\beta - h]\phi_s^Q(s)ds$$
$$= \int_0^{u_2}(s - f)\phi_s^Q(s)ds + \Phi_\xi(q)\int_{u_2}^{u_2+\varepsilon}[s - s_1/\beta - h]\phi_s^Q(s)ds$$

$$\leq \Phi_{\xi}(q)\int_0^{u_2}(s-f)\phi_s^Q(s)ds + \Phi_{\xi}(q)\int_{u_2}^{u_2+\varepsilon}[s-s_1/\beta-h]\phi_s^Q(s)ds$$

(Since $\int_0^{u_2}(s-f)\phi_s^Q(s)ds < 0$ )

$$\leq \Phi_{\xi}(q)\int_0^{u_2+\varepsilon}[s-f]\phi_s^Q(s)ds \quad \text{(Since } f \leq s_1/\beta + h \text{ )}$$

$$< 0 \quad \text{(Since } f = Es \text{ )}$$

Finally note that, when ordered for $q^{nv}$, if the probability of incurring financial distress is equal to zero or one then, $q^{nv}$ is optimal. □

***Proof of Theorem 4.5.6:***

For given a given quantity decision $q_1$ , the sufficient optimality condition is given by:

$$\frac{\partial J(x_1 \mid q_1)}{\partial x_1} = \begin{cases} \beta r\int_0^{q_1}\int_0^{u_1}[f_1-s_2]\phi_{s_2}^Q(s_2)\phi_{\xi_2}(\xi_2)ds_2d\xi_2 \\ +\beta r\int_{q_1}^{\infty}\int_0^{u_2}[f_1-s_2]\phi_{s_2}^Q(s_2)\phi_{\xi_2}(\xi_2)ds_2d\xi_2 = 0 & \text{if } x_1 < q_1, \\ \beta r\int_0^{q_1}\int_{u_1}^{\infty}[f_1-s_2]\phi_{s_2}^Q(s_2)\phi_{\xi_2}(\xi_2)ds_2d\xi_2 \\ +\beta r\int_{q_1}^{\infty}\int_{u_2}^{\infty}[f_1-s_2]\phi_{s_2}^Q(s_2)\phi_{\xi_2}(\xi_2)ds_2d\xi_2 = 0 & \text{if } x_1 > q_1. \end{cases}$$

where

$$u_1 = \frac{\eta_2 - y_1/\beta - f_1x_1 + (s_1/\beta - \lambda)q_1}{q_1 - x_1} + \frac{(\lambda+h)(q_1-\xi_2)}{q_1-x_1}, u_2 = \frac{\eta_2 - y_1/\beta - f_1x_1 + (s_1/\beta-\lambda)q_1}{q_1-x_1}.$$

Observing that $\frac{\partial J(x_1 \mid q_1)}{\partial x_1}$ goes to zero as $x_1$ approach to $q_1$, proves the desired result (note that $f_1 = E_{P_2|P_1}^Q[s_2]$ ). □

***Proof of Theorem 4.5.7:***

Recall that the single period model under the optimal hedging policy is given by:

$$V_1(S_1) = \max_{q_1\geq 0} J_1(q_1 \mid .) = \max_{q_1 \geq 0} - s_1q_1 + \beta(f_1+\lambda)q_1 - \beta(\lambda+h)E_{\xi_2}[q_1-\xi_2]^+$$
$$-\beta r E_{\xi_2}[(f_1+\lambda-s_1/\beta)q_1 - (\lambda+h)[q_1-\xi_2]^+ - \kappa_1]^-$$

For the worst case (lowest) demand scenario, $\xi_2^w$, firm's operating cash flow is given by $(f_1 - h - s_1/\beta)q^{nv} + (\lambda+h)\xi_2^w$ (assuming the firm orders more than the minimum demand). If $\kappa_1 \le (f_1 - h - s_1/\beta)q^{nv} + (\lambda+h)\xi_2^w$, then the FDC term in the objective is equal to zero for $q_1 = q^{nv}$, and hence it is optimal to produce $q^{nv}$.

For the rest of the proof , we assume that the firm faces financial distress under the worst case demand scenario when $q_1 = q^{nv}$, i.e., $\kappa_1 \ge (f_1 - h - s_1/\beta)q^{nv} + (\lambda+h)\xi_2^w$. Now let $\hat{q}_1$ satisfies $(f_1 + \lambda - s_1/\beta)\hat{q}_1 - \kappa_1 = 0$. Then,

$$J_1(q_1 \mid .) = (1+r)\{-s_1 q_1 + \beta(f_1+\lambda)q_1 - \beta(\lambda+h)E_{\xi_2}[q_1 - \xi_2]^+\} - r\beta\kappa_1 \quad \text{for } q_1 \le \hat{q}_1 \text{ and,}$$

$$J_1(q_1 \mid .) = -s_1 q_1 + \beta(f_1+\lambda)q_1 - \beta(\lambda+h)E_{\xi_2}[q_1 - \xi_2]^+$$
$$+ r\beta\int_0^{q_1 - \frac{(f_1+\lambda-s_1/\beta)q_1-\kappa_1}{\lambda+h}} ((f_1+\lambda-s_1/\beta)q_1 - (\lambda+h)(q_1-\xi_2) - \kappa_1)\phi_{\xi_2}(\xi_2)d\xi_2 \quad \text{for } q_1 \ge \hat{q}_1.$$

Consequently,

$$\frac{\partial J_1(q_1 \mid .)}{\partial q_1} = (1+r)\{-s_1 + \beta(f_1+\lambda) - \beta(\lambda+h)\Phi_{\xi_2}(q_1)\} \quad \text{for } q_1 \le \hat{q}_1.$$

$$\frac{\partial J_1(q_1 \mid .)}{\partial q_1} = -s_1 + \beta(f_1+\lambda) - \beta(\lambda+h)\Phi_{\xi_2}(q_1) +$$
$$+\beta r(f_1 - h - s_1/\beta)\Phi_{\xi_2}\left(q_1 - \frac{(f_1+\lambda-s_1/\beta)q_1-\kappa_1}{\lambda+h}\right) \quad \text{for } q_1 \ge \hat{q}_1.$$

Following, if $q^{nv} \le \hat{q}_1$, then $\frac{\partial J_1(.)}{\partial q_1}|_{q^{nv}} = 0$ and it is optimal to produce $q^{nv}$. On the other hand, if $q^{nv} \ge \hat{q}_1$, then $\frac{\partial J_1(.)}{\partial q_1}|_{q^{nv}} < 0$ and hence, the firm produces less than $q^{nv}$. This proves the desired result. □

***Proof of Theorem 4.5.8:***

It suffices to prove that $q^f \ge q^*$. Recall that,

$$q^* = \arg\max_{q_1 \ge 0} J_1(s_1 \mid x_1 = 0) = \arg\max_{q_1 \ge 0} E^Q_{\xi_2, P_2|P_1} g(s_1 \mid x_1 = 0) \quad \text{and}$$

$$q^f = \arg\max_{q_1 \ge 0} J_1(s_1 \mid x_1 = q_1) = \arg\max_{q_1 \ge 0} E^Q_{\xi_2, P_2|P_1} g(s_1 \mid x_1 = q_1) \text{ , where}$$

$$g(s_1 \mid x_1 = 0) = -s_1 q_1 + \beta\{(s_2+\lambda)q_1 - (\lambda+h)[q_1-\xi_2]^+$$
$$- r[(s_2+\lambda-s_1/\beta)q_1 - (\lambda+h)[q_1-\xi_2]^+ - \kappa_1]^-\},$$

$$g(s_1 \mid x_1 = q_1) = -s_1 q_1 + \beta\{(f_1+\lambda)q_1 - (\lambda+h)[q_1-\xi_2]^+$$
$$- r[(f_1+\lambda-s_1/\beta)q_1 - (\lambda+h)[q_1-\xi_2]^+ - \kappa_1]^-\}.$$

Then,

$$\frac{\partial g(s_1 \mid x_1 = 0)}{\partial q_1} = -s_1 + \beta\{(s_2 + \lambda) - (\lambda + h)I_{\{q_1 \geq \xi_2\}} + r((s_2 + \lambda - s_1/\beta) - (\lambda + h)I_{\{q_1 \geq \xi_2\}})I_{\{A<0\}}\},$$

where $A = [(s_2 + \lambda - s_1/\beta)q_1 - (\lambda + h)[q_1 - \xi_2]^+ - \kappa_1]$.

Observing that $\frac{\partial g(s_1 \mid x_1 = 0)}{\partial q_1}$ is concave in $s_2$ and $f_{1,2} \leq_{cv} s_2$, it follows that $\frac{\partial J(s_1 \mid x_1 = 0)}{\partial q_1} \geq \frac{\partial J(s_1 \mid x_1 = q_1)}{\partial q_1}$. Hence, $q^f \geq q^*$. □

# Bibliography

Acs, Z. and C. Armington (1998). Longitudinal establishment and enterprise microdata (leem) documentation. U.S. Bureau of the Census, Center for Economic Studies.

Acs, Z. and C. Armington (2003). Endogenous growth and entrepreneurial activity in cities. U.S. Bureau of the Census, Center for Economic Studies.

Ahmadi, R. H., S. Dasu, and C. S. Tang (1992). The dynamic line allocation problem. *Management Science* 38, 1341–1353.

Archibald, T. W., L. C. Thomas, J. M. Betts, and R. B. Johnston (2002). Should startup companies be cautious? Inventory policies which maximize survival probabilities. *Management Science* 48, 1161-1174.

Atlason, J., M. A. Epelman, and S. G. Henderson (2004). Call center staffing with simulation and cutting plane methods. *Annals of Operations Research* 127, 333–358.

Babich, V. and M. J. Sobel (2004). Pre-IPO operational and financial decisions. *Management Science* 50, 935-948.

Balcer, Y. and S. Lippman (1984). Technological expectations and the adoption of improved technology. *Journal of Economic Theory* 34, 292-318.

Barber, J. and M. Titus (1996). Structure of the U.S. wheat supply chain. UGPTI Staff Paper No. 129. The Upper Great Plains Transportation Institute, North Dakota State University.

Berger, A. N. and G. F. Udell (2005). *Handbook of Entrepreneurship Research*, Chapter 13, pp. 37-54. Springer.

Berling, P. and K. Rosling (2005). The effects of financial risks on inventory policy. *Management Science* 51, 1804–1815.

Bessembinder, H. (1991). Forward contracts and firm value: investment incentive and contracting effects. J*ournal of Financial and Quantitative Analysis* 26, 519-532.

Bhidé, A. V. (2000). *The Origin and Evolution of New Businesses*. Oxford University Press.

Birge, J. R. (1985). Decomposition and partitioning methods for multi-stage stochastic linear programs. *Operations Research* 33, 989–1007.

Birge, J. R. and F. Louveaux (1997). *Introduction to Stochastic Programming*. Springer-Verlag.

Bish, E. K., A. Muriel, and S. Biller (2005). Managing flexible capacity in a make-to-order environment. *Management Science* 51, 167-180.

Bollapragada, R. and U. Rao (1999). Single-stage resource allocation and economic lot scheduling on multiple, non-identical production lines. *Management Science* 45, 889–906.

Buzacott, J. and R. Zhang (2004). Inventory management with asset-based financing. *Management Science* 50, 1274-1292.

Carillo, J. E. and C. Gaimon (2000). Improving manufacturing performance through process change and knowledge creation. *Management Science* 46, 265-288.

Carillo, J. E. and C. Gaimon (2004). Managing knowledge-based resource capabilities and under uncertainty. *Management Science* 50, 1504-1518.

Chand, S., H. Moskowitz, A. Novak, I. Rekhi, and G. Sorger (1996). Capacity allocation for dynamic process improvement with quality and demand considerations. *Operations Research* 44, 964-975.

Chen, X., M. Sim, D. Simchi-Levi, and P. Sun (2007). Risk aversion in inventory management. *Operations Research* 55, 828–842.

Chen, Z. L. and W. B. Powell (1999). Convergent cutting plane and partial-sampling algorithm for multistage stochastic linear programs with recourse. *Journal of Optimization Theory and Applications* 102, 497–524.

Chiralaksanakul, A. and D. P. Morton (2004). Assessing policy quality in multi-stage stochastic programming. *Stochastic Programming E-Print Series*.

Chrisman, J. J., A. Bauerschmidt, and C. W. Hofer (1998). The determinants of new venture performance: An extended model. *Entrepreneurship: Theory and Practice* 23, 5-29.

Corbett, C. J. and J. C. Fransoo (2007). Entrepreneurs and newsvendors: Do small businesses follow the newsvendor logic when making inventory decisions? UCLA Working Paper.

Cramer, J., J. Hartog, N. Jonker and C. van Praag (2002). Low risk aversion encourages the choice for entrepreneurship: an empirical test of altruism. *Journal of Economic Behavior and Organization* 48, 29-36.

de Groote, X. (1988). The strategic choice of production processes. Ph. D. thesis, Stanford University.

De Toni, A. and S. Tonchia (1998). Manufacturing flexibility: A literature review. *Int. J. Production Research* 36, 1587–1617.

Ding Q., L. Dong and P. Kouvelis (2007). On the Integration of production and financial hedging decisions in global markets. *Operations Research* 55, 470-489.

Donohue, C. J. and J. R. Birge (2006). The abridged nested decomposition method for multistage stochastic linear programs with relatively complete recourse. *Algorithmic Operations Research* 1, 20–30.

Duffie, D. (1992). Dynamic Asset Pricing Theory. Princeton University Press.

Fine, C. H. and R. M. Freund (1990). Optimal investment in product-flexible manufacturing capacity. *Management Science* 36, 449–466.

Fine, C. H. and E. L. Porteus (1989). Dynamic process improvement. *Operations Research* 37, 580-591.

Froot, K., D. Scharfstein, and J. Stein (1993). Risk management: Coordinating corporate investment and financing policies. *Journal of Finance* 48, 1629-58.

Garavelli, A. C. (2003). Flexibility configurations for the supply chain management. *Int. J. Production Economics* 85, 141–153.

Gaur, V. and S. Seshadri (2005). Hedging inventory risk through market instruments. *Manufacturing & Service Operations Management* 7, 103–120.

GEM (2007). Global Entrepreneurship Monitor: Executive report. Babson College, Wellesley, MA.

Gifford, S. (2005). *Handbook of Entrepreneurship Research*, Chapter 3, pp. 37-54. Springer.

Goel, A. and G. Gutierrez (2008). Procurement and distribution policies in a distributive supply chain in the presence of commodity markets. UT-Austin Working Paper.

Gompers, P. A. and J. Lerner (2004). *The Venture Capital Cycle*. MIT Press, Cambridge, MA and London.

Graves, S. C. and W. C. Jordan (1991). An analytic approach for demonstrating the benefits of limited flexibility. General Motors Research Laboratories, Research Publication GMR–7341.

Graves, S. C. and B. T. Tomlin (2003). Process flexibility in supply chains. *Management Science* 49, 907–919.

Gupta, S. and R. Loulou (1998). Process innovation, product differentiation and channel structure: Strategic incentives in a duopoly. *Marketing Science* 17, 301-316.

Gurumurthi, S. and S. Benjaafar (2004). Modeling and analysis of flexible queueing systems. *Naval Research Logistics* 51, 755-782.

Halek, M. and J. Eisenhauer (2001). Demography of risk aversion. *Journal of Risk and Insurance* 68, 1-24.

Harwood, L. J., M. N. Leath, and W. G. Heid (1989). The U. S. milling and baking industries. Agricultural Economic Report No. 611, USDA-ERS, Washington DC.

Higle, J. L. and S. Sen (1991). Stochastic decomposition: an algorithm for two-stage linear programs with recourse. *Mathematics of Operations Research* 16, 650–669.

Hopp, W. J., E. Tekin, and M. P. Van Oyen (2004). Benefits of skill chaining in production lines with cross-trained workers. *Management Science* 50, 83–98.

Hotchkiss, S., K. John, R. M. Mooradian, and K. S. Thorburn (2008). Bankruptcy and the resolution of Financial Distress. Forthcoming in *Handbook of Corporate Finance: Empirical Corporate Finance*, Vol 2, Chapter 14.

Iravani, S. M. R., B. Kolfal, and M. P. Van Oyen (2007). Call center labor cross-training: It's a small world after all. *Management Science* 53, 1102–1112.

Jin, Y. and P. Jorion (2006). Firm value and hedging: Evidence from U.S. oil and gas producers. *Journal of Finance* 61, 893-919.

Joglekar, N. R. and M. Levesque (2009). Marketing, R&D and startup valuation. *IEEE Transactions on Engineering Management* 56, 239-242.

Jordan, W. C. and S. C. Graves (1995). On the principles of the benefits of manufacturing process flexibility. *Management Science* 41, 577–594.

Kall, P. (1976). *Stochastic Linear Programming*. Springer, Berlin-Heidelberg-New York.

Katok, E., W. Tarantino, and T. Harrison (2003). Investment in production resource flexibility: an empirical investigation of methods for planning under uncertainty. *Naval Research Logistics* 50, 105–129.

Kelley, J. E. (1960). The cutting plane method for solving convex programs. *Journal of the Society for Industrial and Applied Mathematics* 8, 703–712.

Kihlstrom, R. and J. Laffont (1979). General equilibrium entrepreneurial theory of firm formation based on risk aversion. *Journal of Political Economy* 87, 719-748.

Kornish, L.J. (1999). On optimal replacement thresholds with technological expectations. *Journal of Economic Theory* 89, 261-266.

Li, G. and S. Rajagopalan (1998). Process improvement, quality, and learning effects. *Management Science* 44, 1517-1532.

Li, S. and D. Tirupati (1994). Dynamic capacity expansion problem with multiple products: technology selection and timing of capacity additions. *Operations Research* 42, 958–976.

Li, S. and D. Tirupati (1995). Technology choice with stochastic demands and dynamic capacity allocation: a two product analysis. *Journal of Operations Management* 12, 239–258.

Li, S. and D. Tirupati (1997). Impact of product mix flexibility and allocation policies on technology. *Computers and Operations Research* 24, 611–626.

Linowsky, K. and A. B. Philpott (2005). On the convergence of sampling-based decomposition algorithms for multistage stochastic programs. *Journal of Optimization Theory and Applications* 125, 349–366.

Mamer, J. W. and K. F. McCardle (1987). Uncertainty, competition and the adoption of new technology. *Management Science* 33, 161-177.

McCardle, K. F. (1985). Information acquisition and the adoption of new technology. *Management Science* 31, 1372–1389.

Milgrom, P. and J. Roberts (1990). The economics of modern manufacturing: Technology, strategy, and organization. *American Economic Review* 80, 511-528.

Modigliani, F. and M. H. Miller (1958). The cost of capital, corporation finance, and the theory of investment. *American Economic Review* 48, 261-297.

Netessine, S., G. Dobson, and R. A. Shumsky (2002). Flexible service capacity: optimal investment and the impact of demand correlation. *Operations Research* 50, 375–388.

Pereira, M. V. F. and L. M. V. G. Pinto (1991). Multistage stochastic optimization applied to energy planning. *Mathematical Programming* 52, 359–375.

Petersen, M. and R. Rajan (1994). The benefits of lending relationships: Evidence from small business data. *Journal of Finance* 49, 3-37.

Pindyck, R. S. (2001). The dynamics of commodity spot and futures markets: A primer. *The Energy Journal* 22, 1-29.

Purnanandam, A. (2008). Financial distress and corporate risk management: Theory and evidence. *Journal of Financial Economics* 87, 706-739.

Ruszczynski, A. and A. Shapiro (2003). *Handbooks in Operations Research and Management Science,* Vol. 10, Elsevier.

Schwartz, E. and J. E. Smith (2000). Short-term variations and long-term dynamics in commodity prices. *Management Science* 46, 893-911.

Senbet, L. W. and Seward J. K. (1995). Financial Distress, Bankruptcy and Reorganization. In R.A. Jarrow, V. Maksimovic and W.T. Ziemba editors, *Handbooks in Operations Research and Management Science*, Vol. 9, North Holland.

Sethi, A. K. and S. P. Sethi (1990). Flexibility in manufacturing: a survey. *Int. J. of Flexible Manufacturing Systems* 2, 289–328.

Shane, S. (2007). *A General Theory of Entrepreneurship*. Edward Elgar Publishing Limited.

Shane, S. A. and K. T. Ulrich (2004). Technological innovation, product development, and entrepreneurship in management science. *Management Science* 50, 133-144.

Shapiro, A. (2003). Monte Carlo sampling methods. In A. Ruszczynski and A. Shapiro, editors, *Handbooks in Operations Research and Management Science,* Vol. 10, Elsevier.

Smith, C. and R. Stulz (1985). The determinants of firms' hedging policies. *Journal of Financial and Quantitative Analysis* 20, 391-405.

Stulz, R. (1984). Optimal hedging policies. *Journal of Financial and Quantitative Analysis* 19, 127-140.

Stulz, R. (1990). Managerial discretion and optimal financing policies. *Journal of Financial Economics* 26, 3-27.

Swinney, R., G. Cachon, and S. Netessine (2006). Capacity investment by competitive startups. The Wharton School Working Paper.

USDA (2009). Wheat Data: Yearbook Tables, Flour Production and Prices. The U.S. Department of Agriculture, http://www.ers.usda.gov/data/wheat/YBtable32.asp.

Westerlund, T. and F. Pettersson (1995). An extended cutting plane method for solving convex MINLP problems. *Computers and Chemical Engineering* 19, 131–136.

Wu, B. and A. Knott (2006). Entrepreneurial risk and market entry. *Management Science* 52, 1315-1330.

Van Mieghem, J. A. (1998). Investment strategies for flexible resources. *Management Science* 44, 1071–1078.

Van Mieghem, J. A. (2003). Capacity management, investment, and hedging: Review and recent developments. *Manufacturing & Service Operations Management* 5, 269-302.

Van Miehem, J. A. and N. Rudi (2002). Newsvendor networks: Inventory management and capacity investment with discretionary activities. *Manufacturing & Service Operations Management* 4, 313-335.

van Slyke, R. and R. J-B. Wets (1969). L-shaped linear programs with application to optimal control and stochastic programming. *SIAM Journal on Applied Mathematics* 17, 638–663.

Zhang, Q., M. A. Vonderembse and J. Lim (2003). Manufacturing flexibility: defining and analyzing relationships among competence, capability, and customer satisfaction. *Journal of Operations Management* 21, 173–191.

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