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OF THE

UNIVERSITY OF TEXAS

1915: No. 44

AUGUST 5 1915

The Texas Mathematics Teachers' Bulletin

(Vol. 1, No. 1, August 5, 1915)

Published by the University six times a month and entered as second class matter at the postoffice at Austin, Texas

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The Texas Mathematics Teachers' Bulletin

(Vol. 1, No. 1, August 5, 1915)

Edited by

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This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

CONTENTS.

Published by the University six times a month and entered as second-class matter at the postoffice at
AUSTIN, TEXAS.

The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston.

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar.

SALUTATORY

To the mathematics teachers of Texas: This bulletin is the initial number of a series to be issued three or four times a year by the Division of Mathematics at the University of Texas. It is intended to be of service to the teachers of mathematics in Texas. It is hoped that they will make use of it to a very great extent.

This publication will not be used as a medium through which the mathematicians of the University may send their preachments to the teachers of the State, rather it is to be a common forum where we may all discuss the problems that beset us and get help from those who can give it and give help to those who need it—and who of us does not need it?

Not all of us can attend the State and National meetings of our fellow teachers, not all of us can take part in the discussions there, not all of us can propose and have discussed there the questions most interesting to us. But this bulletin will be available to every teacher of mathematics in Texas, to it he may propound any question, and in it expound any good thing he may have to impart.

This publication will not be devoted especially to University Entrance Mathematics. It will be devoted wholly to elementary mathematics teaching and only incidentally to University entrance conditions to the extent that they affect the general subject.

The need of some medium of communication has been emphasized by the reading annually of some two thousand mathematics papers sent to the University by high schools seeking affiliation. These papers exhibit many errors and faults that call for a remedy, and which seem, for the most part, to be rather easy to remedy. These will be treated from time to time along with other matters.

The teaching of mathematics ought to be one of the most enjoyable pursuits for the teacher and one of most delightful and profitable for the student of the whole curriculum. Is it so in your school? If not, let's set to work to make it so.

ORDER IN GEOMETRY DEMONSTRATIONS

(With especial reference to recitations.)

What is said here in regard to the order in which a theorem should be demonstrated will be found in practice in most of our standard texts. This outline is given in order to call the attention of the teacher to the fact that at this point some definite method is necessary. He may be able to improve the suggestions made here and if he has a more successful method of procedure we hope to hear from him. At any rate, every teacher should observe some definite order in which a demonstration should take place. When pupils mix together hopelessly, hypothesis, conclusion and proof, they never get the training in logical thought that can be derived from demonstrative geometry.

The following method of procedure is suggested:

Theorem. This is a statement made up of a conditional and an assertive part. It should be made accurately and concisely.

Figure. The statement of the theorem should be followed by a certain configuration suggested by the theorem.

Concrete Analysis of the theorem:

(1) *Hypothesis.* This is a restatement of the conditional part of the theorem made in terms of the figure showing concretely what is granted in the theorem. In other words, we say concretely in terms of the figure what we say abstractly in the theorem. This is called the *hypothesis* or the *given*.

(2) *Conclusion.* This is, again, a restatement of the assertive part of the theorem which is made in terms of the figure—that is a concrete statement of what is said abstractly in the theorem. This is called the *conclusion* or *what is to be proved.*

Good examples of hypothesis and conclusion may be found on pages 58, 66. 101, 173 of the Wentworth Smith Plane Geometry.

Too much care cannot be given on the part of the teacher to the concrete expressions of the statements in the analysis of the theorem. Here neatness and accuracy of statement should be demanded. Clear statements of the hypothesis and conclusion show that the student has an accurate knowledge of what is to be done. An awkward statement of these parts usually leads to a bad course of reasoning in the proof. Half the battle is won, when the hypothesis and conclusion are neatly and accurately made.

Notice the order: Theorem, figure, hypothesis, conclusion, proof.

Proof of the theorem.

(1) *Material.* A most important step is to have the pupil select the material upon which he is to base the proof. This is composed of the hypothesis of the theorem and certain definitions, axioms, postulates, and propositions previously proved. A skillful teacher will soon develop on the part of his pupils a power to select from the available material.

(2) *Abbreviations.* The use of abbreviations and symbols in written recitations should be encouraged. When possible make statements in Algebraic language using single letters for angles and lines.

(3) *Order.* A definite order or sequence of facts in the proof should be observed. Training in paragraphing will aid very much at this point. The teacher should often have the class consider the order of facts, and show how a different order, in some cases, will lead to the same proof.

(4) *References.* In giving references, the student must have in mind the sequence of previous propositions or theorems and facts that have been established or granted. He is urged to give a clear statement of the same when necessary. The teacher should not permit much abbreviation at this point, but in general require references in full. A slovenly habit in stating references will sooner or later lead to bad thinking. Nothing puts vigor and life into a demonstration as surely as accurate statement of references. At this point the careful teacher should always be on guard. Good thinking and clear statements always go together. There is an element of character training here that is very valuable.

Every assertion or statement made by the pupil should be challenged, when necessary, by the teacher. By this means memory work on the part of the pupil is easily detected, and the teacher is able to find the weak points in the students' knowledge of the theorem. Such challenges, when expected by the student, help very much in the daily preparation of the lesson giving him an insight of a true demonstration. After a time he learns to enjoy any test the teacher may make of this kind.

(5) *Training.* The training derived from this part of the work in geometry, apart from the subject matter, is of very great value. There is not a single study in the high school course that is not strengthened by having the geometry well taught.

(6) *Oral and Written Work.* The time spent in the first few weeks in developing a sense of order in demonstration will amply repay the teacher in the case with which the class will proceed after that sense of order is gained. Correct forms and order of expression are more easily impressed upon the beginner by means of written than by oral work. Oral demonstrations, however, should not be undervalued, but should alternate with written proofs of theorems. In writing out the proof of a theorem great care should be given to paragraphing and order of topics used in reaching the final conclusion. There should be no haste in the first two or three months in covering the text. After the student has developed a good notion of order in demonstration, he should be required as early as possible to develop proofs of simple originals both oral and written.

USE OF PRINTED FORMS IN GEOMETRY WORK

In reading the geometry papers sent every year to the University, one of the most common faults noted is the lack of a definite form for the arrangement of the work. In many cases the theorem to be proved or the problem to be solved is nowhere clearly stated. The information given is not set off by itself, the conclusion to be established is not clearly stated in advance. Preliminary statements, steps in the proof, description of auxilliary constructions, are all intermingled in a most confusing and illogical fashion.

Some schools use a printed form for note books and examinations, and almost without exception the papers from these schools show to great advantage over those of the schools where no such device is used. This fact seems to justify the recommendation of the use of some one of the many such forms that are to be had.

Of course, the use of such a notebook and examination paper will not enable a student to prove a theorem or solve a problem that he does not understand, but it will enable him to make the best use of the information that he has and will keep him from mixing things that ought to be kept separate.

Paper similar to the cuts below can be had from various publishers. The sort copied here is sold by Ginn & Company. The sheets are slightly different for ''theorems'' and for ''problems.'' It is not to be understood that the style used here is thereby recommended in preference to or to the exclusion of other styles that are on the market.

In his oral work the student should generally give in full any theorem cited in his proof. In written work this is laborious and introduces the appearance of diffuseness. It is, therefore, desirable to find some adequate and distinctive abbreviations for theorems that are to be cited often. A few suggestions are given, others will occur to teachers and students.

Theorem

Two triangles are congruent if two sides and the included angle of the one are equal respectively to two sides and the included angle of the other.

(In referring to this theorem as proof for a given statement, reference may be made as follows: *S*<*S*=*S* <*S*.)

Theorem

Two triangles are congruent if two angles and the included side of the one are equal respectively to two angles and the included side of the other.

(Reference to this theorem: <S<=< <)

Theorem

If two angles of a triangle are equal, the sides opposite the equal angles are equal, and the triangle is isosceles.

(Reference to this theorem: Having 2<s=.)

Theorem

Two triangles are congruent if the three sides of the one are equal respectively to the three sides of the other.

(Reference to this theorem: 3 sides=3 sides.)

Theorem

If two parallel lines are cut by a transveral, the alternate-interior angles are equal.

(Reference to this theorem: Alt.-int.< of||lines.)

Theorem

When two lines in the same plane are cut by a transversal, if the alternate-interior angles are equal, the two lines are parallel.

(Reference to this theorem: Alt.-int.< are=.)

The following cuts are published by permission of the author, Professor H. A. Morrison.

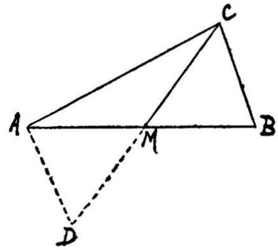
THEOREM

A median of a triangle is less than half the sum of the two adjacent sides.

Hypothesis or Given Conditions

1. $\triangle ABC$
2. CM, the median
3.
4.
5.

Construction



To Prove

1. $CM < \frac{1}{2}(AC+BC)$
2.

Auxiliary Constructions

1. Produce CM
2. Make $MD = CM$
3. Draw AD

Proof

	Argument		Reason
1.	$AM = MB$	1.	Hypothesis (2)
2.	$MD = CM$	2.	Construction (2)
3.	$\angle AMD = \angle CMB$	3.	Vertical angles
4.	$\therefore \triangle AMD = \triangle CMB$	4.	$S \angle S = S \angle S$
5.	$\therefore AD = CB$	5.	Homologous sides = $\triangle$s
6.	$CD = CM + MD = 2CM$	6.	Substitute = for =
7.	$CD < AD + AC$	7.	St. line shortest distance bet. 2 pts.
8.	$\therefore CD = 2CM < AD + AC$	8.	From 6 and 7
9.	$\therefore 2CM < BC + AC$	9.	Substitute BC for its = AD
10.	$\therefore CM < \frac{1}{2}(AC+BC)$	10.	Halves of unequals
11.		11.	
12.		12.	
13.		13.	

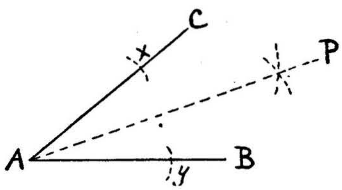
STATEMENT OF PROBLEM

To bisect a given angle.

Elements Given

1. ∠BAC
2.
3.
4.
5.

Construction



Required to Construct

1. To bisect ∠BAC
2.
3.

Method of Construction

1. With A as center and any
2. radius as Ax describe arc xy
3. With x & y as centers and any
4. radius greater than $\frac{1}{2}$ xy
5. describe arcs intersecting in P
6. Draw AP
7. Then AP bisects ∠BAC
8.

Proof

Argument	Reason
1. Ax = Ay	1. Radii of same ⊙
2. xP = yP	2. " " " "
3. AP = AP	3. Identical
4. ∴ △APC = △APB	4. 3 sides = 3 sides
5. ∴ ∠PAC = ∠PAB	5. Corres. ∠s of = △
6. ∴ AP bisects ∠BAC	6. Q.E.F.
7.	7.
8.	8.
9.	9.
10.	10.
11.	11.

SOME REMARKS ON SIMPLE ALGEBRAIC OPERATIONS

Too much care cannot be given to oral and written forms of expression in Algebraic work. Do not allow in the class room set forms that are not found in standard texts. The attention of the pupil should be repeatedly directed to methods of expression shown in the text and other supplementary books. The teacher should see that every explanation given to his pupils is carefully expressed, and never allow a bad form to be written or remain on the blackboard before his pupils. Let him ask himself whenever he places anything on the board, "how would it appear if it were in print?" Carelessness on the part of the teacher in the matter of form will soon develop the same bad habits on the part of the pupil.

The little care at this point on the part of the teacher will help wonderfully in developing accuracy of thought and expression on the part of the pupil. Accuracy of thought and accuracy of expression go hand in hand. Bad form indicates loose thinking, good form is an aid to clear thinking and sound, logical, reasoning.

The development of good forms of expression will repay the teacher many times over for his trouble and pains. In reading written exercises he will find relief from much drudgery.

The following examples will serve to illustrate what has just been said:

Some Expressions to Avoid:

(1) Do not use the equality sign in the sense of *is* or *are*.

(2) Do not allow such expressions as

$$\frac{3x-2=5x-10}{28} \qquad \frac{3(x-1)=4x-6}{16}$$

(3) If in the solution of two equations

$$3x+4y=18 \text{ - - - - - } (1)$$
$$7x-5y=5 \text{ - - - - - } (2)$$

we wish to multiply both members of (1) by 7 and both members of (2) by 3 in order to eliminate the x– term, do not write

$$3x+4y=18\times 7$$
$$7x-5y=5\times 3$$

(4) If both terms of a fraction

$$\frac{3x-4}{3-\sqrt{2}}$$

are to be multiplied by $3+\sqrt{2}$ do not say multiply the fraction by $3+\sqrt{3}$ or write

$$\frac{3x-4}{3-\sqrt{2}}\times(3+\sqrt{3})$$

(5) In verifying the solution of an equation, substitute in each member of the equation separately and then show the results to be equal. Thus, given the equation

$$\frac{x}{2}-\frac{3x-5}{7}=4-\frac{3x}{4} \quad \text{- - - - (1)}$$

Solution $x = 4$ - - - - - - - - - (2)

Verification $\frac{4}{2}-\frac{12-5}{7}=4-\frac{12}{4}$ - - - - (3)

or $2-1=4-3$

or $1=1$ - - - - - - - - (4)

The equality (4) would not be true if (2) is incorrect and hence the equality sign would be placed between unequal numbers. A better method is to find the value of each side. If both sides reduce separately to the same number the problem is verified.

TROUBLESOME TERMINOLOGY

In the solution of equations by beginning students there are three terms that are seldom used in an intelligent way, namely: "transposition," "clearing of fractions" and "cancellation" or "cancel." The processes that follow the use of these words are too often mechanical and very few beginners are able to give a logical answer when called upon to justify the use of the above terms.

By the use of the term transposition a number crosses the equality sign and in some mysterious way appears on the other side having changed entirely the nature of its being in making the passage. Clearing of fractions is so little understood by most beginners that the process connected with the dropping of denominators is still more mysterious than that of transposition. The term cancel or cancellation is used with so many different meanings that it means nothing to a teacher who is accurate in his forms of expression.

On account of the mechanical way in which most pupils in Algebra use these terms, it would be best if beginners were not allowed to use them at all. No false notions would be formed and the logic of the processes involved would be kept before the pupils until correct principles are fixed. A very good method is outlined below.

The solution of simple equations involves the use of one or more of the following statements called *axioms*:

I If equals be added to equals, their sums are equal.

II If equals be substracted from equals, their remainders are equal.

III If equals be multiplied by equals, their products are equal.

IV If equals be divided by equals, their quotients are equal.

These axioms are used to change the form or value of each of two equal number expressions without destroying the equality.

Thus if to both members of

$$3x-4=11$$

we add 4 we have

$$3x=15$$

In this process the two members have been changed without affecting the equality. The change in the two members was made without using the term transpose. The real principle involved is kept before the student in this way and the student is not permitted to fall into a purely mechanical way of dealing with the equation.

In like manner if both members of the equation

$$2x/3+4=12$$

be multiplied by 3 we have

$$2x+12=36$$

By the use of the third axiom given above, we obtain another equation in which no term contains a fraction. The expression "clearing of fractions" was not used. These are very simple illustrations of how we may avoid troublesome terminology and at the same time keep before the student the logic of the equation.

In Slaught and Lennes' Algebra the following helpful abbreviations are used to indicate the axioms involved in simple changes made in the two sides of an equation:

A4 indicates that 4 has been added to each member.

S4 indicates that 4 has been subtracted from each member.

M4 indicates that each member has been multiplied by 4.

D4 indicates that each member has been divided by 4.

Examples:

(1) $5m+3=18$

$5m=15$ S3

$m=3$ D5

The abbreviation at the right shows the axiom used to obtain the equation from the one above.

(2) $3x/5-8=7$

$3x-40=35$ M5

$3x=75$ A40

$x=25$ D3

The solution in each case is found without the use of the terms transposition, clearing of fractions or cancel. By this method each step is justified by an axiom and the logic of the process kept before the pupil.

SOME WEAK SPOTS IN THE TEACHING OF MATHEMATICS

(As Exhibited by Subsequent Work in the Subject.)

(Professor J. W. Calhoun, Austin.)

Reprinted from the Transactions of the T. S. T. A.

At the outset of this paper, I desire to disclaim any intent to criticise wantonly or unsympathetically the teaching of elementary mathematics. This topic was not of my suggesting and I had nothing to do with assigning myself to discuss it. All of us claim to welcome frank and honest criticism, but King David made an observation about all men that might have some point here. We all believe in whipping bad children—other people's; we all approve of the castigation of bad teaching—other people's, but it takes a good deal of nerve and grit and moral courage to see our own bubbles punctured and not hate the puncturer. Bearing in mind this very human trait, and being in no wise desirous of becoming persona non grata to this body, I approach my task with some trepidation.

But I think not one of us present will say that this subject should not be fully and freely discussed. It seems to me, therefore, that it should be done by some one whose work every day of the year, and each succeeding year brings him in contact with the results of secondary teaching and whose attention is, of necessity, constantly called to the points of excellence as well as the points of weakness in it. I shall endeavor to make all criticisms in the kindliest spirit, and will point out no fault without trying to suggest a remedy.

I shall begin my discussion with arithmetic. This branch is taught over more years of the life of every student than perhaps any other continuous subject. It ought, then, to be some index to the effectiveness of our teaching, and I have no doubt that it is. In arithmetic the student should get the foundation for all his future mathematics. Here his number sense should be developed, here his analytic powers should get their initial training, here the ability to make rapid and accurate calculations should be acquired, here he should get a training that will beget a confidence in his own mental processes.

But, alas and alack! he usually gets none of these things. In stead of having his number sense cultivated, if he ever had the rudiments of one, it is likely to be smothered, instead of developing his power of analysis he becomes a slave to a set of rules that he understands little about, instead of acquiring a confidence in his own mental powers and processes he becomes a mental cripple with no confidence at all in what he does till he has compared his results with those set down in an answer book.

The reason for this state of things is not far to seek. Arithmetic is taught as a dead formalism. The average arithmetic text and teacher are bound by as many hindering bandages as an Egyptian mummy, and are about as flexible in their movements as this same relic of the time of the Pharoahs. "Rules, rules everywhere, and not a thought to think." And while I am on this subject, I may as well say that the same charge can be made with the same degree of truthfulness against the teaching of algebra. And in this short paper I shall speak only of arithmetic and algebra, especially seeing that Dr. Bruce has hung out the necessary danger signals in geometry.

Now, I do not mean to say that all rules are bad and that we should proceed to teach arithmetic by pure reason and discard all rules entirely. This would be ideal but hardly possible, I think. For example, I can scarcely see how one could teach long division at the age and place that it seems it must be taught without a good deal of reliance on the more or less mechanical following of a rule. But working by rule should be reduced to a minimum and I hope to point out some important spots where this can be done to advantage.

In such subjects as the applications of percentage, mensuration, simple proportion, etc., the student should not be allowed to put pencil to paper to solve a problem till he can give out of his head a reasonably good approximation to the result he is to get. In other words, his number sense should be trained to a point where his written work serves merely to refine the accuracy of his mental calculation.

Now, arithmetic is not taught in this way to the students that come under my observation. I have rarely found a college freshman (and I test from fifty to a hundred every year) who could tell right out of his head what would be the approximate interest on $731.59 for three years, ten months and twenty-seven days at eight per cent. per annum. I ask them if they should solve the problem and get $598.37 for the answer if they would feel any surprise. The reply is practically unanimous—they would not.

To my way o fthinking, the teacher who lets a student solve this problem and look in the book for the answer before he has taken a look at the problem and seen without the making of a single figure that the answer is in striking distance of $200.00, does the student a positive injustice. The student should be taught to see that the interest for one year is not far from eight per cent. of seven hundred dollars, and that the time is not far from four years. This will show that a rough approximation to the result will be 4 x $56.00. A student trained to look at things in this rational and independent way would not accept $50.00 or $500.00 for the answer to this problem even though he found it set down in a book as the answer.

Let us take a look at the operations on decimal fractions. Multiplication and division are the only ones where there is likely to be confusion. And here again the whole trouble comes as a result of the same dead formalism that I have mentioned before. I have never in my life seen a student in the decimal stage who could give anything approaching an explanation of the rules for ''pointing off'' in multiplication and division. Give a student taught by the rule, 2.13659 and tell him to multiply it by 13.42695 and he will get his point in the right place if he does not make some sort of slip in his calculation of where the rule says that it should be. But he will never be sure that he is right till he has looked at the answer in the book, and then if it does not agree with what he has got he will set about finding how he can get the book answer, and the fact that his error was only a misplaced point seems to him not at all a serious matter, though it means that his answer was only ten per cent. of the right result or that it was one thousand per cent. of what it should have been at the very best. Let the student be taught to look at 2.13695 as about 2, and 13.42695 as approximately 13 and then the matter of pointing off will be automatic, any rule or set of rules will be only so much rubbish, he will look at the numbers and see without any effort that the product will be about 26 and that their ratio is about 6 or 1/6.

In my trigonometry classes we come early to the place where we want to know the product of numbers like 589×.7863. I ask the freshmen to tell me offhand what the approximate value of this will be and they look at me in amazement as if they thought I looked for one to know a multiplication table of a thousand lines. The college freshman who cannot without difficulty see this this is about 450 (i. e., ¾ of 600) has been badly taught in his arithmetic.

Square and cube roots are taught in this same formal manner in both algebra and arithmetic. The average student of these subjects sees nothing beyond the algorithm employed. As soon as he has forgotten this he has no idea of the meaning of roots or any method of getting one. The idea of getting an approximate root and then testing the goodness of the approximation (a process that would suggest itself to any intelligent person who had had the good luck to escape the usual teaching of the subject) never suggests itself to him. Not over a week ago I asked two classes of college freshmen, aggregating about sixty students, to give me a rough value for the square root of .4. Not more than two of them gave an answer that came within twenty-five per cent. of being correct, and of the extreme answers given, the biggest one was eighty times the smallest. I care not how expert a student may be in getting a root by a rule of thumb, if he cannot look at .4 and in half a minute see that its square root is between .6 and .7 and that its cube root is between .7 and .8, his knowledge of the subject is negligible.

My chief criticism, then, of the teaching of arithmetic is that it is not taught as a thing that should square with common sense but is taught as a formal thing. The remedy is to discard as far as possible all formalism and teach the subject in a rational, common-sense way. Let us get away from the application of rules and use our judgment. The expressions, "about equal to, very nearly equals, is approximately equal to, is in the neighborhood of," etc., should be familiar ones in our arithmetic classes.

I shall confine the rest of my remarks to algebra. This is the branch, judging by the students that come to the University of Texas, that makes the least defiinte impression (aside from the impression that the subject is difficult) on the high school student, of any mathematical subject that he encounters. He does not even know the language of the subject. He attaches no clear-cut meaning to such terms as, "roots of an equation, equation, identity, rational, irrational, simultaneous, equivalent, solution, satisfy,'' and so on through the whole catalogue.

He does not know the most elementary principles of the subject. He does not know that $a/b=c/d$ when and only when $b\neq o$ and $c=ra$ and $d=rb$. He does not know that there are three and but three laws of exponents and that all operations with exponents not in harmony with these are absurd. He does not know that when the equation $ax^2+bx+c=o$ is solved and the roots $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$ obtained that they ought to satisfy the equation, and when he is asked to show that they do, cannot make the necessary substitution. He does not know that if $a\times b\times c=o$ that either a, b, or c must be o. He does not know that $0/5=0$, that $0/0$ is indeterminate, and that $5/0$ is meaningless.

These things are fundamental and should all be dealt with carefully and fully by the teacher before proceeding to attack the more advanced parts of the subject. This is a subject that of all things should produce and cultivate the ability and the disposition to think straight. But I am seriously inclined to doubt if algebra as at present administered aids correct thinking as much as it hinders it.

When we seek the cause of this failure on the part of algebra to produce the results that its friends feel that they have a right to look for, the specter of dead formalism again looms up. Students do not look on the symbols of algebra as things that have a real meaning apart from the subject. They look on them as things to juggle with and get out the results that the teacher knew beforehand that they would yield. A rather bright student asked me the other day when I had the identity, $x^{n+1}-1\equiv x(x^n-1)+(x-1)$ on the board if this had any meaning when applied to numerical quantities. I regarded the question as a sign that he was on the point of waking up.

A common form of error met with is in thinking that in the expression $(a+x)/(b+x)$ that we can cancel x from the numerator and the denominator and reduce the expression to a/b. The next most common one is to write $a^{\frac{1}{2}}+b^{\frac{1}{2}}=(a+b)^{\frac{1}{2}}$. These errors would evidently never occur if the student were taught to look on the algebraic symbols as representing numbers, and hence, on algebra as a sort of disguised arithmetic. Our cue is then, to make the consideration of algebra more concrete. This is to be done by means of numerical substitution. Let the student get the habit of testing all his operations by means of numerical substitution. In the cases cited above in this same paragraph let the student be constantly required to let a=4, b=9, and y=13 for example and see if the operations that he is disposed to perform agree with the results of common sense.

Algebra that is not fully understood is not understood at all and hence what is taught at all should be taught intensively. It seems to me then that if the high schools would restrict their operations to about the following topics they would do a better part by both their college-going and their non-college-going students:

(1) The fundamental operations; (2) factoring standard forms and the applications to fractions, common factors and multiples; (3) the solution of problems requiring the student to set up and solve simple equations in one and two variables; (4) quadratic equations; (5) elementary treatment of exponents where radical expressions should be written as quantities with fractional exponents; (6) graphs of simple equations.

I am firmly of the opinion that if the high school will focus attention on these topics for the usual time that algebra is taught there, and if the teacher will take as his watchword numerical substitution, *more numerical substitution, continual numerical substitution,* the average student will leave the high school with a fair degree of mastery of the elements of algebra. The above amount of algebra would be highly satisfactory to the University in the way of preparation for its courses in mathematics and for the student who is not preparing for college this amount will be sufficient for any test that he is likely to be called on to undergo.

THE STRAIGHT EDGE

Mathematics teaching has a wide reputation as a lazy man's job. What are you doing to combat the libel?

✣ ✣ ✣

Are you prepared to tell your students the "why" of studying Algebra and geometry? If not, set your wits to work.

✣ ✣ ✣

Did you ever meet a geometry class when you did not know till the class met what the lesson was about?

✣ ✣ ✣

Did you ever dismiss a class feeling that the recitation had been a failure because *you did not know your lesson?*

✣ ✣ ✣

Did you ever assign an original to be done "tomorrow" because you had not prepared to do it "today"?

✣ ✣ ✣

Do you assign "The next three theorems" for tomorrow's lesson without knowing what they are and to what they lead?